Dodging the Leveraged ETF Bullet - An Innovative Trading Strategy

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Abstract

Two main parts, with separate purposes constitute this thesis. Firstly, ETFs and Leveraged ETFs are demystified in an indepth descriptive manor. In a short period of time ETFs have reached immense popularity, but investor knowledge and sophistication is lagging. We attempt to fill that gap of knowledge in order to help investors make better informed investment decisions. Secondly, leveraged ETFs are known for their long-term under performance and tracking error. However, unsophisticated investors, might misunderstand the nature of leveraged ETF returns hoping to gain leveraged returns over time, ending up dissapointed and perplexed. Therefore we have invented and constructed a trading strategy that provides the same leveraged exposures to markets as leveraged ETF but is suitable for buy-and-hold investing. Hence, dodging the value trap of leveraged ETF under performance and providing investors with an alternative. Our subject of study has been the ProShares UltraLong ETF SSO, providing two times the return of the S&P500 Index on a daily basis. By constructing a portfolio consisting of unleveraged ETFs, SPDR S&P500 SPY, tracking the same and gaining leverage on a portfolio level by borrowing funds, we will show both over performance realtive to the SSO and a far better market tracking leveraged position. The study is essentially a back test of our portfolio between 2006-06-22 and 2010-04-29.

Keywords S&P500, SPY, SSO, Leveraged ETF, ETF, Tracking Error, Trading Strategy, Buy-and-Hold, Portfolio Construction

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1 Introduction

In 1990 investors were introduced to the first Exchange-Traded Fund (ETF). Since then, the demand for ETFs has grown explosively and today the ETF product range includes a multitude of asset classes and exposures, serving clients searching for increasingly sophistcated investment possibilities. The U.S ETF market is well established but still growing rapidly and although the first European ETF was not made available until 2000, in late 2008 the assets under management of European ETFs amounted to around EUR 100 billion [8].

As an investment vehicle, on a qualitative level, ETFs provide easy access to many markets and asset classes at a relatively low cost. Also, unlike the units of conventional index funds, ETF units trade on stock exchanges at market determined prices combining the characteristics of mutual funds and common stocks. Having gained immense popularity in the U.S, in less than ten years, ETFs in the European markets have become a serious alternative to financial products such as futures and are at the moment venturing into more exotic markets and asset classes such as emering markets and alternative investments.

However, amidst its increasing popularity, issues have arised and will arise such as the case with any cutting-edge product. Currently, a number of ETF issuing firms in the U.S are being investigated by the Securities and Exchange Commission (SEC) [7][8] where conflicts have arisen between investor and issuer. Those cases particulariy concern leveraged- and inverse ETF returns. However, as we will demonstrate, rather than the problem stemming from the structure of the ETF, it stems from the miscommunication between investor and ETF issuer, leading to misconceptions and ultimately causing conflict.

While ETFs are clearly providing a multitude of benefits for investors, many issues still remain to be solved. Investors being well informed can be assertive of ETFs meeting expectations, but if otherwise might end up dissapointed at a high cost. Linking the double-edged nature of ETFs to their novelty in the markets and their immense popularity amongst investors, we could not think of a more interesting research topic for our thesis.

1.1 Purpose of Study

The study focuses on two main purposes. one decriptive purpose and a practical purpose presenting a trading strategy.

1.1.1 Descriptive Purpose

The first purpose is to increase investor knowledge on ETFs. ETFs and particularily leveraged ETFs are realtively modern fixtures in financial markets and while having quickly gained vast popularity, the level of investor knowledge about these products is lagging. As simple as ETFs might seem, they are highly sophisticated on a structural level incorporating risks and dynamics that are not entirely evident at first sight. [2] On a practical note, the perceived simplicity risk bestowing a potential investor with an incomplete framwork for decision making, potentially leading to an uninformed investment decision. In the U.S retail investors comprise 60% of its ETF market and many of these investors can be considered unsophisticated. Thus, the majority of U.S ETF investors run the risk of making or having already made uninformed investment decisions. The first purpose will be to enlighten these investors who might use this paper as a step in avoiding making uninformed decisions.

1.1.2 Trading Strategy

The second purpose is specifically targeted to leveraged ETFs and by extension, to a certain extent, inverse ETFs as well. The vast majority of leveraged- and inverse ETFs aim to produce the leveraged/inverted return on a daily basis. However, delivering leveraged returns on a daily basis is not equivalent to delivering the leveraged returns over any other arbitrary time period. Unfortunately in many cases, unsophisticated investors have failed to identify this distinction, becoming surprised and dissapointed when faced with underperformance of their investment. [7] This misconception is especially unfortunate for investors applying buy-and-hold strategies, which most unsophisticated investors do. The two possible causes for this misunderstanding of differences in return pattern are either the ETF issuer misinforming the investor or the investor lacking sophistication.

Previous research on the performance of leveraged ETFs has shown that over time there is a clear pattern of underperformance [5]. This underperformance is a tracking error that is a consolidation of a few factors but essentially stems back to ETF managers having to rebalance the fund at the end of the trading day to maintain a constant leverage for the morning after. Especially periods with high volatilty with no overall trending cause underperformace of leveraged ETFs. Logically, maintaining constant leverage over these periods, the ETF manager will lock in losses and cut profits, buying high and selling low. Ultimately, the fund net asset value will decrease making it increasingly hard for the fund to recover once the market turns up. Investors buying leveraged ETFs will see their investments diminish over time due to the tracking error value decay. Thus, our second purpose of the study will be to ourselves invent and evaluate alternative strategies on the same underlying asset, providing the same leverage, deliver better returns than a leveraged ETF while being more suitable for buy-and-hold investors.

To our knowledge there is no previous academic record of our strategies.

Over a period of four years, *2006-2010*, we have compared the cumulative daily return of the *ProShares Ultra S&P500 (SSO)* against holding a portfolio of *SPDR S&P500 (SPY)* where leverage is obtained by borrowing cash. The SSO is two times leveraged and the SPY is unleveraged. In order to obtain a leverage factor of two, our portfolio debt must initially and after each rebalancing event constitute 50% of the portfolio. Our rebalancing rules are based on the cumulative daily return of the underlying asset, in this case the S&P500 index. Four different rebalancing rules applied to the portfolio and evaluated. Rebalancing occurs either on a daily basis, or when the S&P500 cumulative daily return reaches $\pm 2\%$, $\pm 5\%$ or $\pm 10\%$. Below is an example that illustrates the unerlying intuition and under what circumstances the strategies theoretically would work.

Example:

When the underlying index trades up, the leverage of both the leveraged ETF and our portfolio decrease. When the index trades down the opposite occurs. This essentially creates a decreasing momentum due to leverage when the index is gaining value while an increasing momentum when it trades down. For this reason ETF managers will rebalance their leveraged ETF portfolio at end of day. The leveraged ETF will thus maintain its leverage. However, maintain constant leverage in order to conserve momentum come at a cost. If on day 1 the underlying index trades up, leverage is decreased and thus the ETF manager will need to borrow additional funds and invest them in the markets. If the underlying index on the consecutive day would trade down, leverage increases, and the fund manager would have to sell part of the portfolio and amortize debt to decrease leverage. Over the two days the manager due to rebalancing ruels has bought high and sold low which is bad investing. In addition the manager will incurr higher interest rate costs for the borrowed funds and higher transaction costs. These factors combined with the general tracking error of leveraged ETFs contribute to value lost for the investor.

Now assume one of our rebalancing rules are used instead on a leveraged portfolio using unleveraged ETFs. For the sake of the example we will assume a rebalancing rule of $\pm 2\%$. If the underlying index on day 1 trades up less than 2% no rebalancing occurs, leaving our portfolio with decreased leverage. However, when the underlying index trades down the day consequtive day, value in the portfolio will be protected since no rebalancing occured the previous trading day. Essentially, there is a trade off involved that ultimately decides which strategy will be most profitable. Generally we have understood that what one pays[gains] for less frequent rebalancing is the loss of upward[downward] momentum when the S&P500 Index trades up[down], but what one gains is the protection of value if the S&P500 Index would be volatile.

If our portfolio with alternative rebalancing rules are to provide better returns than holding the leveraged ETF the gain from having less tracking error by using unleveraged ETFs plus the gain from value protection must offset the loss of lessend upward and increased downward momentum plus the cost of

borrowing funds and rebalacing transaction costs.

◇

Having read about the cause of leveraged ETF long term under performance[2][5][7], the strategy above is a construction of our own innovative thought process. We therefore wish to summarize, in a very general hypothesized manor, the underlying intuition behind why we believe that our strategy should be successful:

> "A portfolio where leverage is transferred to portfolio level by borrowing funds, holding unleveraged ETFs and thereby eliminating leveraged ETF tracking error, will over time render a higher cumulative return than holding a leveraged ETF. We suspect this strategy will ultimately be more suitable for buy-and-hold investors who wish to gain a leveraged exposure to the underlying market. If our notion is correct investors might have been provided with an alternative way to approach leveraged exposure in the market place" /David Hu and Henrik Bark

The methodology behind the strategies are presented more thoroughly in *section 5*.

In addition to the two main purposes, it is our hope that this paper will lay ground to further research on the subject, wherefore we also will provide a section describing future research and implications.

1.2 Contribution

ETFs are relative new fixtures in financial markets. It is widely documented and used successfully with skill amongst professional practitioners and investors, but academic research is lagging. To our knowledge published academically rooted research relevant to ETFs is scarce. Most research made is published on various internet sites or at best working papers. The cause of this is most likely the novelty of ETFs. ETFs have existed for only 20 years and in Europe only since 2000. In addition, the essence of ETFs, tracking underlying assets, might not present that many interesting topics for research since historically research on the underlying assets already is extensive. Hence, from a research point of view, ETFs tracking underlying assets appear merely a vehicle for investment rather than an unique asset class. However, as true as that may be, the same does account for leveraged- and inverse ETFs. These vehicles do not track underlying assets directly, but promise multiples on the returns of those assets. In complexity the departure from unleveraged ETFs to leveraged ETFs presents a binary leap in complexity. While leveraged and inverse ETFs are even newer, their inherent complexity presents unique problems and characteristics to analyze decoupled from the nature of the underlying asset. Hence, the bulk of ETF research is concentrated to leveraged ETFs. Within the field of leveraged ETFs acadmic research is focused on performance analysis and explaining leveraged ETF tracking error. However, this paper aims to present a way investors

might be able to capitalize on that understanding, providing some alternative strategies.

This paper has one minor and two main contribution. Firstly, the minor contribution is the attempt to increase investor general knowledge on ETFs helping them making increasingly informed decisions while trading ETFs. The information and aspects we chose to present about the ETF's mechanics might not be unique revelations but still rare in academic contexts.

The first of the two main contributions is the alternative strategies presented and evaluated. Our evaluation of the strategies cover a period of time between *2006-06-22 to 2010-04-29*. For this paper we have presented the cumulative daily return of our strategies and the leveraged ETF over this period and applied the various rebalancing rules. Using the cumulative daily return our strategy applies more to a buy-and-hold investor for the holding period. Our results show that during the holding period each of our alternative strategies consistently overperform holding the leveraged ETF taking into accounting interest rate costs, but not transaction costs. Thus our portfolio with alternative rebalancing rules prove to overperform while being more suitable for a buy-and-hold strategy compared to holding leveraged ETF. We have thus provided an alternative strategy for investors looking for a leveraged exposure while applying a buy-and-hold strategy. In addition, we hope that this could provide further insight for leveraged ETF managers who might consider input in methods of rebalancing their funds.

Lastly, having presented overperformance during this period several more interesting follow-up studies can be made. The same strategies can be evaluated during other time periods with clearer underlying market trends to analyze further characteristics. It would also be interesting to empirically study what factors cause the portfolio excess returns. Understanding the conditions and dynamics of overperformance one might attempt to create market neutral strategies where one would hold one of our portfolios and short sell a leveraged ETF against and capitalize on the spread. We have chosen to evaluate one ETFs covering the S&P500 Index, but same studies can be applied to other ETFs both in Europe and across other asset classes.

1.3 Outline of Study

In section 2 we will present previous research. The bulk will concern leveraged ETFs, their performance dynamics and tracking error. However, for the sake of completeness we will also present some sources on unleveraged ETFs and ETFs in general. It might be of importance to note that some of our references used and presented here are not published papers but information extracted from different articles published on the internet. This is an effect of the scarce supply of academic research on the subject. Hence, we have, at the cost of rigour, chosen to trust some of the internet contents found. However, various comparisons have been made in order to verify the likelihood of information being correct.

In section 3 we provide an indepth description of both unleveraged ETFs and leveraged ETFs in an attempt to demystify these new innovative products.

In section 4, data used is described in detail.

In section 5, we will discuss what, how and why some data has been calculated in order to provide a good understanding the underlying dynamics of the strategies and how the strategies are constructed.

Section 6 presents the results of performance evaluation of our strategies and will also provide analysis of these results.

Section 7 presents the conclusions drawn from the result in section 6.

In section 8, having presented our results and conclusions, we open mindedly discuss the field of future research in this subject.

Section 9 contains all references that have been used to write this thesis.

2 Previous Research

Because ETFs are a rather new fixture to financial markets, the amount of previous research is scarce in comparison to the fields of asset pricing or the likes. In this section, we will present the findings from previous research on areas close to our research purposes. Formally, we have not found a previous study that specifically answers to the purposes we strive for, and but research has been in the vicinity. Because the strategies presented in the thesis concerns leveraged ETFs the presentation of previous research will be focused on leveraged ETF s and their dynamics.

In 2009, Cheng and Madhavan [7] studied the impact of leveraged- and inverse ETFs on market volatility and liquidity, unusual features of their product design, and questions of investor suitability. Their results show that leveraged funds are neither well understood amonst investors nor professionals. Some investors do not understand the point that leveraged ETFs will not necessarily replicate the leveraged index return over longer time periods. The authors shows that generally, the greater the holding period and the higher the daily volatility, the greater the deviation between the leveraged ETF's return and a statistically levered position in the same index. They conclude that the gross return of a leveraged or inverse ETF has an embedded path-dependent option that under certain conditions can lead to value destruction for buy-and-hold investors. The unsuitability of ETFs are according to the authors for long-term investors reinforced by the drag on returns from high transaction costs and tax inefficiency. In essence, their paper claim that leveraged and inverse ETFs are not suitable for buy-in-hold investors and that these products are not designed to deliver long-term performance for volatile indices.

In 2010, Dr. Giese [2] developed a general mathematical framework for assessing the long-term performance of leveraged- and short ETFs. Dr. Giese concludes that there is an optimal degree of leverage that maximizes the expected return of the daily re-balanced leveraged investment that depends on observable market parameters. Since existing leveraged- and inverse ETFs use a constant degree of leverage that does not respond to the market environment,

the paper proposes an improved design of leveraged- and inverse investment products. In particular, the theoretical and numerical results developed in this paper call for leveraged- and inverse funds where the degree of leverage is not constant in time but is adjusted according to the optimal leverage concept to ensure suffered by investors in adversely moving markets are limited.

In 2009, Lu, Wang and Zhang [5] found that inverse leveraged ETFs may deviate more from a benchmark over long time periods than a leveraged ETF. Their results caution against the use of leveraged ETFs as long-term investment substitutes for long or short position of the benchmark indices. The key findings are that since the quadric variation (further explained in *section 3.2.1*) is linearly increasing with the length of the holding period, the long-term performances of leveraged and inverse ETFs tend to underperform the multiples of the underlying benchmark over long holding periods. In their paper they present six cases of daily returns to see the different effect on the two day return of the double ETF and the inverse double ETF. Conclusively, the three authors show that:

> "Finally, the sequence of the returns in the two periods does not matter. As long as the product of the two returns is negative, the two-day return of the double (inverse double) ETF would be lower than double (negative double) of the return of the underlying benchmark. If the product of the two returns is positive, then the two-day return of the double (inverse double) ETF would be higher than double (negative double) of the return of the underlying benchmark index." [5]

Essentially, this supports the idea that ranging and volatile markets make for underperformance in leveraged and inverse ETFs, while trending provides superior returns. Lastly, Lu, Wang and Chang show in their appendix that the long term performance of a leveraged ETF is negatively correlated with the quadratic variation of the underlying index. The larger the quadratic variation the more will the leveraged ETF track below the simple multiple of the underlying index over the long term. Amenc, Goltz, Grigoriu and Schröder [8] concluded that ETFs are not yet used to their full potential, most ETFs are largely limited to passive holdings of broad market indices. They show that more than two-thirds of all ETF users frequently use ETFs to obtain broad market exposure; and for more than 50% ETFs are predominantly long-term or buy-and-hold investments. Fewer than 50% of the respondents in their survey report that they frequently rely on ETFs for short-term investment or for exposure to specific market subsegments, most practitioners do not trade options on ETFs, sell them short or lend them out.

3 Qualitative Description

3.1 The ETF

3.1.1 Background

ETFs offer investors the benefits of stocks, whereas they can go in and out of positions during trading hours. With ETFs as part of one's portfolio, investors can get exposure towards new markets, sectors and securities, in addition, with ETFs, investors are able to customize the diversification of their portfolios. ETFs are like mutual funds, open ended, hence offering new shares to investors and are obligated to buy back outstanding shares on investors' request. ETFs are traded on the stock exchanges around the world, and their prices are determined by the market. The majority of ETFs are index tracking and consist of a trust of securities with the purpose of replicating the benchmark or index as close as possible. [8] An EFT was first traded on the Toronto Stock Exchange in 1990 and ten years later in April 2000 the first ETF was launched in Europe by Merrill Lynch to track the Euro Stoxx 50 index. Even though the European ETF market was introduced later than the American, it now account for 33%, in 2008 the European ETF market amounted to approximately USD 143 billion. In 2009 about 650 different ETF products was listed in Europe and 700 in the US. The most popular ETFs among investors are still equity ETFs, tracking either sectors or national indices. They account for about 60% of the market. Fixed-income ETFs account for about 30% and commodity ETFs account for about 5%. ETFs and regular mutual funds have a lot of similarities, however, there are some important differences in how they issue and redeem shares to investors. An ETF is registered as a fund company, the assets within the fund is supported by a custodian which holds the assets. The administrator is updating the net asset value (NAV) on a regular basis and the management company handles the operations. An ETF is created when institutional investors or other authorized market participants commits to invest capital in a fund with the purpose of replicating an index or other benchmark. ETF shares are created, unlike traditional mutual funds, by authorized market participant's depositing a certain block of securities with the ETF. Based on the manager's composition of the ETF, the participant buys the block of the underlying securities on the markets. For the deposit of the shares, the participant receives a fixed number of ETF shares with a NAV that is the value of the replicating index. The ETF shares are known as creation units and usually constitutes in lots of between 10 000 to 100 000 shares. The ETF shares prices are based on the underlying securities, when the shares are traded, the prices depends on the supply and demand. If the prices differ from the NAV, market participants can step in and take positions on the spreads, making arbitrage profits. In order to make arbitrage profits, market participants seek ETFs that are undervalued compared to their NAV, buying large blocks of ETF shares, then redeem them at the custodian bank for the underlying securities. After receiving the underlying securities, the investor can sell them on the market. The same strategy may be

implemented in reverse, should the ETF be overvalued compared to its NAV. However, when differences occur between ETF prices and their underlying NAV, the market makes them disappear very fast.

3.1.2 Replication

There are ETFs attempting to replicate sectors, indices, commodities, hedge funds, currency baskets and private equity. The replication mechanism of an ETF may be structured in three different ways. An ETF manager may replicate an index by holding all of the securities in the same proportion as their part in the underlying index. This strategy offers a quite natural replication. However, when trying to track a very broad index with a lot of securities, this strategy can be very difficult to implement, due to the differences in liquidity which can make some securities difficult to acquire in the appropriate volume. High transaction costs is also a factor that makes this strategy expensive due to the large number of securities in the trust. Instead of holding all the underlying securities in the index, a manager can implement statistical sampling strategy, where the manager instead of investing in all securities, only invests in the most liquid securities in order to replicate the index. Compared to the physical replication, this strategy does not cost as much and is widely used for tracking large indices. When using a swap-based replication, the manager invests in a basket of securities and outsources the tracking error management to a third party. The basket of securities will most likely not track the index perfectly, so the differences in the ETFs performance to the underlying index are balanced by swap payments made by the swap-counterparty. These ETFs does not have the risk of large tracking errors and comes at relatively low costs. However, there is risk for the swap-counterpart. Should the ETF differ a lot from the underlying index, the swap-counterparty is obliged to make payments. [8] In order to minimize the risk, there is usually a risk limit of 10% of the fund's NAV. Should the fund reach this limit, the swap is rearranged.

3.1.3 Dividends

ETFs may handle dividends from the assets in the trust in two ways. First, they can either pay out dividends in cash to the shareholders. Dividend payments on the securities held in the fund remain in the fund in cash until they are paid out at fixed time intervals. The dividends from the underlying securities may accumulate in the fund, and create a small deviation in performance from the underlying index. ETFs can instead of paying out the dividends to the shareholders, choose to reinvest them. ETFs that use this strategy are tracking the total return of a specific underlying index (including reinvested dividends). The cash flow payments that the shareholders have to handle if this strategy is implemented, are only those occurring when the shareholder buys or sells ETF shares.

3.1.4 Diversification

The most popular kinds of ETFs are still the ones tracking specific equity indices. These ETFs tracks the return of indices such as OMX30, NASDAQ and S&P500. For an investor, these ETFs offer a much diversified alternative for exposure towards the equity markets. By using ETFs, investors now have a shortcut when they want to take positions in certain markets. There are also equity ETFs replicating the return of certain industry sectors, investment styles and recently ETF providers have created ETFs with exposure towards the emerging markets, such as the BRIC countries. Beside ETFs replicating equities, there are a wide range of other ETF alternatives for investors wanting to get exposure towards the returns of inflation protected, corporate and government bonds. The Fixed-income ETFs have expanded to alternative investments, which are more sophisticated and enables investors to invest in hedge funds, private equity, currency baskets and real estate replicating ETFs. These ETFs provides investors with more investment opportunities, and can better help them customize the diversification of their portfolios.

3.1.5 Tax

Traditional mutual funds' realized capital gains are passed on to the funds shareholders, these tax rules are disadvantaging for long-term investors. ETFs have the same tax rules as mutual funds, however, they allow investors to redeem ETF shares by swapping ETFs for the underlying stock, which does not include any cash changing hands. For example, if an investor redeems SEK 1 000 000 from an OMX30 index fund. In order for the fund to pay out cash to the investor, it must sell SEK 1 000 000 worth of stock on the market. Should the stocks be sold at an appreciated price, the fund will receive a capital gain, it will be passed on to the investor before the end of the year. Hence, the investor will pay tax for the turnover within the fund. When an ETF shareholder wishes to redeem SEK 1 000 000, the ETF manager does not have to unload any stocks in the portfolio, the manager may instead offer the shareholder in-kind redemptions in order to limit the capital gains.

3.1.6 Tracking Error

The difference in return between the ETF and the underlying index or benchmark is called the fund's tracking error. Usually the tracking error for any given ETF is small, perhaps a few basis points. However, sometimes the tracking error can increase to several hundred basis points due to a number of factors. Regulations require that ETF and mutual fund managers cannot invest more than a certain amount of its capital in one single security. When specialized ETFs tries to replicate an industry or sector, these rules can create problems, since replication of an industry index may require that the fund holds more than the allowed amount of its capital in one specific security, which is not allowed. Hence, the fund cannot replicate the index to its full extent and a tracking error will most likely occur. Other substantial contributors to tracking errors are the

fees. Even if an ETF tracks an index perfectly, the ETF will underperform the index return by the amount of fees that are deducted from the ETF's return. Returns will also be reduced by how much the fund trades securities in the market, the more it trades, the more trading fees it will accumulate which will reduce the returns. When investors take short positions in stocks, securities lending occurs. Before an investor can short sell a stock, he must in most cases first borrow it from someone, (unless he is naked short). Stocks and other securities are usually borrowed from large institutional fund managers, for example ETF index fund managers. By charging interest on borrowed stock and securities, the managers that takes part in securities lending, can generate additional returns for their investors. The ownership and dividends claims still maintains with the lending fund. The extra returns that the fees generate creates additional return for the shareholders, beyond the index performance, hence a tracking error occurs. To evaluate the tracking quality of an ETF, one can take many approaches. To start off, one may simply analyze the difference between the returns on the ETF and the underlying index or benchmark that it is trying to replicate. Another way of evaluating the tracking quality of an ETF is by analyzing the correlation between the ETF and the underlying index. Tracking errors may also be analyzed by comparing the mean returns of the ETF and the index or benchmark.

3.2 The Leveraged ETF

In order to create leverage ETFs managers use a wide range of financial products such as futures contracts, asset swaps, options and other derivatives. In many cases, especially in the U.S ETF managers simply borrow money to create a leveraged exposure for investors.

Leveraged ETFs are as multifacetted as unleveraged ETFs, but we have chosen to focus our attention in this descriptive section on two of the main distinguishing features for leveraged ETFs. The inherent tracking error will be algebraically described and the leverage dynamic will be explained with a simple example.

3.2.1 Inherent Tracking Error

The divergence of long-term return of leveraged and inverse ETFs can be explained by the quadratic variation over the holding period [5]. This can be shown by algebraically by deriving the cumulative return expressions for leveraged and inverse ETFs. To present the quadratic varition effect we begin by defining r_t^L and r_t^I as the daily returns of a double leveraged ETF and an inverse double ETF at date t. Assuming that the two ETFs perfectly deliver returns in line with what is promised the daily return should be $r_t^L = 2r_t^B$ and $r_t^I = -2r_t^B$, r_t^B being the return of the underlying index.

Define R_{tn}^B as the nth-day cumulative return of the benchmark index starting at date t. Further we define R_{tn}^L and R_{tn}^I as the nth-day cumulative return of the double leverage ETF and the double inverse ETF starting at date t. Then

$R^B_{tn} = \prod_{i=0}^{n-1}(1 + r^B_{t+i}) - 1$ (1),
$R^L_{tn} = \prod_{i=0}^{n-1}(1 + r^L_{t+i}) - 1$ (2),
$R^I_{tn} = \prod_{i=0}^{n-1}(1 + r^I_{t+i}) - 1$ (3).

To show the compounding effect we present the two-day case, since longer periods demand increasingly tedious algebra. Given that n is two, then

$R^B_{t2} = (1 + r^B_t)(1 + r^B_{t+1}) - 1 = r^B_t + r^B_{t+1} + r^B_t r^B_{t+1}$.The two day returns of the ETFs are

$R^L_{t2} = (1 + 2r^B_t)(1 + 2r^B_{t+1}) - 1 = 2r^B_t + 2r^B_{t+1} + 4r^B_t r^B_{t+1}$ for the double leveraged.

$R^L_{t2} = (1 - 2r^B_t)(1 - 2r^B_{t+1}) - 1 = -2r^B_t - 2r^B_{t+1} + 4r^B_t r^B_{t+1}$. In terms of R^B_{t2}, the return of the double leveraged ETF two-day cumulative return expression is

$R^L_{t2} = 2R^B_{t2} + 2r^B_t r^B_{t+1}$, while the double inverse ETF two day cumulative return expression becomes $R^I_{t2} = -2R^B_{t2} + 6r^B_t r^B_{t+1}$.

Neither the double leveraged ETF nor the inverse double ETF expressions are equivalent to returning twice of the underlying benchmark. Also, the inverse double ETF is farthest from the target return. Lastly, we can conclude that if the signs of the two factors in the quadratic term are unequal, there will be a negative deviation on long term leveraged returns.

3.2.2 Leverage Dynamics

Definition Portfolio Leverage

$$Portfolio\,Leverage = \frac{Portfolio\,Total\,Exposure}{Portfolio\,Debt} \quad (4)$$

This section aims to in detail describe the process of rebalancing a portfolio to keep the portfolio leverage constant. The process is refered to as constant leverage rebalancing and value lost through this process is called falling into the constant leverage trap. [2][13]

Assume a leverage portfolio with a total exposure of \$200. For a leverage factor of 2, debt must constitute 50%, \$100. The remaining \$100 is investor equity. Total exposure is the \$200 invested in the underlying market. If markets increase by 1%, the total exposure will reach \$202 dollars. Less debt of \$100 investor return on equity is 2%. This is how leverage works. However, after the 1% increase the portfolio now has a total exposure of \$202, with an equity stake of \$102. Leverage is reduced to $\frac{\$202}{\$102} = 1.98$, a decrease when underlying markets increase. To offset this 0.02 decrease in leverage the portfolio manager borrows an additional \$2 increasing debt to \$102, and invests this \$2 in the markets increasing exposure to \$204. Portfolio leverage is restored to $\frac{\$204}{\$102} = 2$. The opposite applies to underlying markets decreasing. If underlying markets decrease by 1%, the total exposure drops to \$198. Less debt of \$100 investor return on equity is now -2%, dropping to \$98. New portfolio leverage is now $\frac{\$198}{\$98} = 2.02$, an increase when underlying markets decrease. To offset this 0.02 increase in leverage, the portfolio manager sells \$2 worth of the portfolio exposure, and amortizes debt by \$2. New debt is \$98 while new exposure is \$196.

Portfolio leverage is restored to $\frac{\$196}{\$98} = 2$. Leverage is inversely proportional to market movement direction. Ultimately, if markets are moving in a strong ranging manor, this rebalancing methodology will imply locking in losses and cutting profits, in documented cases leading to value destruction. This is called the constant leverage trap. [2][13]

4 Data

In order to construct and evaluate the portfolios and strategies we have extracted daily data on the price of the S&P500 Index, the net asset value (NAV) of the *SPDR S&P500 (SPY) ETF* and NAV of the *ProShares Ultra S&P500 (SSO) leveraged ETF*. This data covers a period between *2006-06-22* and *2010-04-29*. We have a start date of *2006-06-22* because that was when the SSO first started trading. We chose to use ETFs tracking the S&P500 because it is a good general market proxy and because of good liquidity in that market. [8] The data is in time series format and all prices are in US dollars. All data on prices are collected on trading days with the underlying index and ETFs trading on the same days.

In addition to prices we have also extracted data on overnight interest rates. Implementing the strategies we have used the overnight USD LIBOR rate as a proxy for overnight lending. This rate is published on a daily basis and thus we have allocated the correct rate to the respective trading day of the ETFs and S&P500 Index.

Using this data we have computed variables and metrics needed to create our portfolios and evaluate strategies. These computations have been extensive. All variables and metrics will be defined in *section 5.1*.

Data extracted from the Bloomberg Professional service.

5 Methodology

This section will first describe how the portfolios are constructed and which rebalancing rules are used. Then we will describe the intuition behind why we have chosen the specific portfolio features and rebalancing rules. Then explain the intuition behind the characteristics of the provide information on the intution behind the strategies constructed and present the method used for construction.

5.1 Mathematical Definitions of Variables

Below we will provide the mathematical definitions of the variables computed by our model.

Defintion Daily Return of SPY

$$Daily\,Return = r^{SPY}_{t,t+1} = \frac{SPY_{t+1}}{SPY_t} - 1 \text{ , between day t+1 and t.} \quad (5)$$

Definition Return of SPY between rebalancing events

$Rebalancing\ Return = r^{SPY}_{t,t+1} = \frac{SPYt+1}{SPY_t} - 1,$ where t is a rebalancing date (6)

Definition Daily Return of the Portfolio

$Daily\ Return\ of\ the\ Portfolio = Daily\ Return$ (7)

Defintion Dollar Change of the Portfolio

$\$Delta = \frac{Daily\ Return\ of\ Portfolio}{Portfolio\ Total\ Exposure}$ (8)

Definition Percentage Equity Return

$\%ROE = \frac{\$Delta}{Portfolio\ Total\ Exposure - Debt - Interest\ Rate\ Cost}$ (9)

Definition Dollar Debt Change

$\$Debt\ Delta = \frac{(Portfolio\ Total\ Exposure)(1+Daily\ Return)}{2} - Debt$, for the debt dollar change on rebalancing dates one substitutes the Daily Return with the Rebalancing Return. (10)

Defintion Adjusted Portfolio total exposure

$Adj\ Port\ Tot\ Exp = Portfolio\ Total\ Exposure(1+Daily\ Return)+\$Debt\ Delta$, since *$Debt_ Delta* is reinvested in the SPY. (11)

Definition Adjusted Portfolio Debt

$Adj\ Debt = Debt + \$Debt\ Delta$ (12)

Definition Cumulative Equity Return on the Portfolio

$r^P_{tn} = \prod_{i=0}^{n-1}(1 + \%ROE) - 1$, on the nth day; t on a daily basis (13)

$r^P_{tn} = \prod_{i=0}^{n-1}(1 + \%ROE) - 1,$ on the nth rebalancing date; t on a rebalancing event basis (14)

5.2 The Portfolio

The portfolio constructed is initiated on *2006-06-22*, the second trading day of the SSO. On end of trading the previous day, on end of day prices *2006-06-21,* we have assumed we have bought one unit of SPY with available cash, while having borrowed funds to buy another unit. Hence, on the start of trading *2006-06-22*, we are holding two unleveraged ETFs tracking the S&P500 Index, with a portfolio leverage of 2, which in terms of leverage exposure is equivalent to holding a unit of the SSO.

5.2.1 Daily Rebalancing

From *2006-06-22* to *2010-04-29* we will monitor the portfolio value change and include rebalancing events. Four different rebalancing rules have been applied and compared for the portfolio defined above. The first will rebalance the portfolio on end of trading each trading day. This rebalancing dynamic follows that which is described in *section 3.2.2*. When markets start trading on *2006-06-22* to *2010-04-29* (975 trading days), our model re-calculates and updates the following variables on a daily basis:

1. Daily return on the SPY
2. The dollar change of the portfolio
3. The percentage equity return on the current cash position less the interest rate cost
4. The debt change necessary to maintain a leverage factor of 2
5. The leverage factor
6. Updates the portfolio value adjusting for the daily return and debt change
7. Updates debt level
8. Calculates the cumulative daily return on the portfolio equity position

In addition to the above, the daily return as well as the cumulative daily return of the SSO is computed. Important to note, is that the model is constructed in such a way that it dynamically/automatically rebalances.

5.2.2 Rebalancing Rules

The rebalancing rules are conditioned on the cumulative daily return of the underlying index of the SPY, namely the S&P500 Index. Rebalancing of our portfolio is triggered by the cumulative daily return of the S&P500 Index reaches levels outside of the following intervals:

1. $]-2\% : +2\%[$
2. $]-5\% : +5\%[$
3. $]-10\% : +10\%[$

In which case our model re-calculates and updates the following variables:

1. Daily return on the SPY
2. Daily return on the portfolio
3. Cumulative daily return of the portfolio up to the rebalancing date
4. Return on SPY between rebalancing dates

5. The dollar change of the portfolio between rebalancing dates
6. The percentage equity return on the current cash position less the interest rate cost between rebalancing dates
7. The debt change necessary to maintain a leverage factor of 2
8. The leverage factor
9. Updates the portfolio value adjusting for the daily return and debt change
10. Updates debt level
11. Calculates the cumulative daily equity return on the portfolio between rebalancing dates
12. Restarts the counter for the cumulative daily return on the S&P500 Index

The first rebalancing date is the initiation date of the portfolio, namely *2006-06-22*. As with the daily rebalancing method, the daily return as well as the cumulative daily return of the SSO is calculated. Important to note, is that the model is constructed in such a way that it dynamically/automatically rebalances.

The intuition behind the rebalancing rules is grounded in the research of Lu, Wang and Zhang, where an existence of a positive relation between the benchmark moving within a range and the leveraged ETF tracking error is suggested. By rebalancing less often, and only when the underlying index reaches above/under a certain cumulative level, the goal is for our portfolio to decreases the risk of being harmed by choppy and reversing market movements.

5.3 Interest Rate Costs

Specific to the SSO ETF, it promises to deliver a leverage factor of 2 on a daily basis, following the described leverage dynamics in *section 3.2.1*. Our portfolios will be rebalanced the same dynamic, but in addition to daily rebalancing we apply other rules. Holding leverage using debt induces interest rate costs. As mentioned in *section 4* the lending rate used is the USD overnight LIBOR. Using this rate assumes being a bank, and most probably retail investors will borrow at a higher rate. We have chosen to use the USD overnight LIBOR interest rate for the sake of generality. However, the model we have constructed to create the portfolio is flexible on interest rate input, so can be adjusted for retail investor lending rates. Having described the method of obtaining leverage we move to how our strategies eliminate the compounding effect, the main culprit of the leverage ETF tracking error.

5.4 Minimizing Leveraged ETF Quadratic Variation

In the leverage portfolio that we have constructed, we have taken tracking error effects into account and eliminated them by using unleveraged ETFs. The

unleveraged ETFs do not display the problematics of the compounding effect. As mentioned in *section 2*, research and documentation has shown that tacking error in unleveraged ETFs is small enough to be neglected [5]. In our portfolio, using unlevereged ETFs, leverage is obtained on a portfolio level rather than product level, eliminating tracking error while providing similar leveraged exposure. We believe this could be the fundamental cause of overperformance in our portfolio over the holding period.

5.5 Comparison to the S&P500 Benchmark

Even though our portfolio might show better over time performance than holding the leveraged ETF, investors who search for leveraged returns on the S&P500 Index might not be satisfied if our portfolio while providing better results that the SSO, still largely under perform twice the S&P500 Index cumulative daily return. Therefore our model will also compute a double cumulative daily return S&P500 benchmark. This modified benchmark will provide returns equivalent to twice the S&P500 Index cumulative return, thus providing a great benchmark for a buy-and-hold strategy while incorporating the multiple return of leverage.

By comparing our portfolio cumulative daily return and the SSO cumulative daily return against the double cumulative daily S&P500 benchmark we will be provided with an insight in how closely our strategy manages to track the actual underlying asset. Essentially, a perfect tracking is what we hope to provide investors with.

5.6 Model

Our model was constructed using Visual Basic programming language in Microsoft Excel 2007.

6 Results

During the holding period from *2006-06-22* to *2010-04-29*, we have observed our portfolio value on a daily basis accounting for changes induced by rebalancing. All in all the period constitutes 975 trading days. The aim with our tests have been to evaluate the cumulative daily return and daily return of the portfolio over this period and compare that to holding the *ProShares UltarLong ETF, SSO*. Also, the cumulative return on a rebalancing day basis has been calculated. The rebalancing cumulative performance metric simulates the scenario where an investor initiates the portfolio and does not observe it until it is time to rebalance. Comparing it to the daily cumulative return of the portfolio, one could get an idea of how passively the investor can manage the portfolio. We believe that these metrics can give us a good indication of how the portfolio fars over time compared to holding the ETF. Results will be sorted according to the different rebalancing rules. Tables and graphs can be found in the appendix. All spreads are not to be interpreted as absolute values. If a spread

is positive it indicates performing over the benchmark, while a negative spread indicates the opposite. The benchmarks are, depending on which case, either the SSO, S&P500 Index or a double return S&P500 Index. Lastly, all spreads are presented in units of percentage points (pps).

6.1 Daily Rebalancing

Using a daily rebalancing rule our strategy is bound by the same frameworks as the manager of the leveraged ETF. Evaluating a buy-and-hold of our portfolio with daily rebalancing compared to holding the SSO gives a good indication of value earned by using unleveraged ETFs instead. Overperformance will not be attributed to rebalancing rules but to avoiding leveraged ETF tracking errors over time.

6.1.1 Cumulative Returns

Refer to table 2 and 3 and figure 2 in the appendix for this section.

The maximum portfolio cumulative return during the holding period was *53.593%*, *2007-10-09*. The minimum cumulative portfolio return was *-76.327%*, *2009-03-09*. On the corresponding dates the SSO had returned *40.014%* and *-79.664%*. The portfolio cumulative return on the final valuation date was *2010-04-29* is *-29.198%* while the SSO cumulative return on that day was *-37.524%*. Over the holding period the average cumulative return is *-7.521%* while the SSO average cumulative return is *-16.091%*. Hence, our portfolios show better extreme returns and average returns than leveraged ETF. In addition, figure 2 in the appendix proves that the portfolio cumulative return is above the leveraged ETF every trading day. However, the SSO has a lower standard deviation, *8.683%* compared to a standard deviation of *11.076%* for our portfolio.

The largest spread between the two is *18.593*pps, on *2007-12-20* and the smallest is *0.166pps, 2006-06-25*. The average spread is *8.571pps* with a standard deviation of *4.449%*. So on average during this holding period the portfolio earns *8.571pps* higher returns. The graph of the spread shows increase up until *2008-02-22* up to *18.593pps*, with two jumps on *2006-12-22* and *2007-12-22*. Between *2007-12-22* and *2008-08-22* the spread is within a range of *14.143pps* and *17.105pps*, before showing a strong decreasing trend until *2009-02-22* to *3.472pps*. The value on the final day is *8.326pps*.

6.1.2 Daily Returns

Refer to table 1 and table 5 in the appendix for this section.

The maximum portfolio daily return was *29.040%, 2008-10-13*. The lowest portfolio daily return was *-19.690%*, *2008-10-15*. On the same days the SSO daily return was *22.414%* and *-17.109%*. The number of trading days the SSO overperforms the portfolio during the period is 477 out of 975.

The largest spread between the two is *6.626pps*, indicating that the portfolio daily return is *6.626pps* above the SSO daily return. The smallest spread is -

3.107pps, indicating that the portfolio daily return is *3.107pps* below the SSO daily return. The average spread over the entire holding period is *0.0175pps*. During *2006-06-22* to *2006-12-30* the average spread is *0.0422pps*. Between *2007-01-02* and *2007-12-30* the average spread is *0.0318pps*. Between *2009-01-02* and *2009-12-30* the average spread is *0.00965%*. Between *2010-01-02* and *2010-04-29* the average spread is *0.00075pps*.

6.2 $\pm 2\%$ Rebalancing Rule

From previous research it is clear that the tracking error of a leveraged ETF increases when the underlying index range trades. [5] Essentially, the quadratic variation term is negative. By rebalancing less frequently and only then the index return is $\geq|$ 2% $|$, our model will not rebalance if the index return is trading within a range of $range <|$ 2% $|$. Therefore, the probability of rebalancing during range trading is lessened, hence increasing portfolio performance.

6.2.1 Cumulative Return

Refer to table 2 and table 3 and fiture 3 in the appendix for this section.

The maximum cumulative return during the holding period was *53.587%, 2007-10-09*, while the minimum was *-76.328%, 2009-03-09*. For the cumulative return of the SSO on the corresponding dates please refer to *section 6.1.1*. The portfolio average cumulative return over the holding period is *-7.524%*, while the SSO average return is *-16.091%*. The portfolio cumulative return standard deviation is *11.076%* while the SSO standard deviation is less, *8.683%*. On the final valuation day, *2010-04-29*, the portfolio has returned *-29.201%*. On the same day the SSO has returned *-37.524%*. On the final day the difference between our portfolio and the SSO is *-8.323pps*. As with the case of daily rebalancing, our portfolio shows better edge returns, average returns at a higher standard deviation. Figure 3 in the appendix proves how the portfolio cumulative return is above the SSO cumulative return for every trading day.

The largest spread is *18.588pps, 2007-07-20*. The smallest spread is *0.166pps*, *2006-06-25*. The average spread for the holding period is *8.567pps* with a standard deviation of *4.448%*. On average during the holding period the portfolio earns 8.567pps higher returns. The graph over the spread essentially display a similar dynamic as in the case of the daily rebalancing portfolio, refer to *section 6.1.1*.

6.2.2 Daily Return

Refer to table 1 and table 5 in the appendix for this section.

The maximum portfolio daily return was *29.040%, 2008-10-13*. The lowest portfolio daily return was *-19.690%*, *2008-10-15*. On the same days the SSO daily return was *22.414%* and *-17.109%*. The number of trading days the SSO overperforms the portfolio during the period is 477 out of 975.

The largest spread between the two is *6.626pps*, indicating that the portfolio daily return is *6.626pps* above the SSO daily return. The smallest spread is *-3.107pps*, indicating that the portfolio daily return is *3.107pps* below the SSO daily return. The average spread over the entire holding period is *0.0175pps*. During *2006-06-22* to *2006-12-30* the average spread is *0.0422pps*. Between *2007-01-02* and *2007-12-30* the average spread is *0.0318pps*. Between *2009-01-02* and *2009-12-30* the average spread is *0.00965%*. Between *2010-01-02* and *2010-04-29* the average spread is *0.00075pps*.

Thus the descriptive metrics for the portfolio daily return are essentially equal to the daily reblancing portfolio.

6.2.3 Rebalancing Cumulative Performance

Refer to table 4 and figure 10 in the appendix for this section.

During the holding period the portfolio rebalances 201 times. The highest reblance day return is *52.416%, 2007-10-05* and the lowest is *-75.580%, 2009-03-02*. The average rebalancing date return is *-26.565%* and the standard deviation is *38.254%*.

6.3 $\pm 5\%$ Rebalancing Rule

From previous research it is clear that the tracking error of a leveraged ETF increases when the underlying index range trades. [5] Essentially, the quadratic variation term is negative. By rebalancing less frequently and only then the index return is $\geq|$ 5% $|$, our model will not rebalance if the index return is trading within a range of $range <|$ 5% $|$. Therefore, the probability of rebalancing during range trading is lessened, hence increasing portfolio performance.

6.3.1 Cumulative Return

Refer to table 2 and table 3 and figure 4 in the appendix for this section.

The maximum portfolio cumulative return during the holding period was *53.580%, 2007-10-09*. The minimum cumulative portfolio return was *-76.329%, 2009-03-09*. On the corresponding dates the SSO had returned *40.014%* and *-79.664%*. The portfolio cumulative return on the final valuation date *2010-04-29* was *-29.204%* while the SSO cumulative return on that day was *-37.524%*. Over the holding period the average cumulative return is *-7.521%* while the SSO average cumulative return is *-16.091%*. Hence, our portfolios show better extreme returns and average returns than leveraged ETF. In addition, figure 4 in the appendix proves that the portfolio cumulative return is above the leveraged ETF every trading day. However, the SSO has a lower standard deviation, 8.683% compared to a standard deviation of *11.073%* for our portfolio.

The largest spread between the two is *18.5*82pps, on *2007-12-20* and the smallest is *0.166pps, 2006-06-25*. The average spread is *8.564pps* with a standard deviation of *4.44*7%. So on average during this holding period the portfolio earns *8.564pps* higher returns. The graph over the spread essentially display a

similar dynamic as in the case of the daily rebalancing portfolio, refer to *section 6.1.1.*

6.3.2 Daily Return

Refer to table 1 and table 5 in the appendix for this section.

The maximum portfolio daily return was *29.040%, 2008-10-13.* The lowest portfolio daily return was *-19.690%, 2008-10-15.* On the same days the SSO daily return was *22.414%* and *-17.109%.* The number of trading days the SSO overperforms the portfolio during the period is *477* out of *975.*

The largest spread between the two is *6.626pps*, indicating that the portfolio daily return is *6.626pps* above the SSO daily return. The smallest spread is *-3.107pps*, indicating that the portfolio daily return is *3.107pps* below the SSO daily return. The average spread over the entire holding period is *0.0175pps.* During *2006-06-22* to *2006-12-30* the average spread is *0.0422pps.* Between *2007-01-02* and *2007-12-30* the average spread is *0.0318pps.* Between *2009-01-02* and *2009-12-30* the average spread is *0.00965*pps. Between *2010-01-02* and *2010-04-29 t*he average spread is *0.00075pps.*

Thus the descriptive metrics for the portfolio daily return are essentially equal to the daily reblancing portfolio.

6.3.3 Reblancing Cumulative Rule

Refer to table 4 and figure 11 in the appendix for this section.

During the holding period the portfolio rebalances 67 times. The highest reblance day return is *46.527%, 2007-05-18* and the lowest is *-74.592%, 2009-02-23.* The average rebalancing date return is *-32.677%* and the standard deviation is *35.473%.*

6.4 $\pm 10\%$ Rebalancing Rule

From previous research it is clear that the tracking error of a leveraged ETF increases when the underlying index range trades. [5] Essentially, the quadratic variation term is negative. By rebalancing less frequently and only then the index return is $\geq|$ 10% $|$, our model will not rebalance if the index return is trading within a range of $range <|$ 10% $|$. Therefore, the probability of rebalancing during range trading is lessened, hence increasing portfolio performance.

6.4.1 Cumulative Return

Refer to table 2 and table 3 and figure 5 in the appendix for this section.

The maximum portfolio cumulative return during the holding period was *53.580%, 2007-10-09.* The minimum cumulative portfolio return was *-76.329%, 2009-03-09.* On the corresponding dates the SSO had returned *40.014%* and *-79.664%.* The portfolio cumulative return on the final valuation date *2010-04-29* was *-29.204%* while the SSO cumulative return on that day was *-37.524%.* Over the holding period the average cumulative return is *-7.521%* while the

SSO average cumulative return is -16.091%. Hence, our portfolios show better extreme returns and average returns than leveraged ETF. In addition, figure 5 in the appendix proves that the portfolio cumulative return is above the leveraged ETF every trading day. However, the SSO has a lower standard deviation, *8.683%* compared to a standard deviation of *11.073%* for our portfolio.

The largest spread between the two is *18.5*82pps, on *2007-12-20* and the smallest is *0.166pps, 2006-06-25*. The average spread is *8.564pps* with a standard deviation of *4.447*%. So on average during this holding period the portfolio earns *8.564pps* higher returns. The graph over the spread essentially display a similar dynamic as in the case of the daily rebalancing portfolio, refer to *section 6.1.1*.

6.4.2 Daily Return

Refer to table 1 and table 5 in the appendix for this section.

The maximum portfolio daily return was *29.040%, 2008-10-13*. The lowest portfolio daily return was *-19.690%*, *2008-10-15*. On the same days the SSO daily return was *22.414%* and *-17.109%*. The number of trading days the SSO overperforms the portfolio during the period is 477 out of 975.

The largest spread between the two is *6.626pps*, indicating that the portfolio daily return is *6.626pps* above the SSO daily return. The smallest spread is *-3.107pps*, indicating that the portfolio daily return is *3.107pps* below the SSO daily return. The average spread over the entire holding period is *0.0174pps*. During *2006-06-22* to *2006-12-30* the average spread is *0.0421pps*. Between *2007-01-02* and *2007-12-30* the average spread is *0.0318pps*. Between *2009-01-02* and *2009-12-30* the average spread is *0.00965*pps. Between *2010-01-02* and *2010-04-29* the average spread is *0.00075pps*.

Thus the descriptive metrics for the portfolio daily return are essentially equal to the daily reblancing portfolio.

6.4.3 Rebalancing Cumulative Rule

Refer to table 4 and figure 12 in the appendix in this section.

During the holding period the portfolio rebalances 21 times. The highest reblance day return is *46.355%, 2007-05-18* and the lowest is *-73.728%, 2009-03-02*. The average rebalancing date return is *-36.456%* and the standard deviation is *32.150%*. Refer to table 4 in the appendix.

6.5 Comparison to the Benchmark, S&P500 Index

Refer to table 6 and figure 1 in the appendix for this section.

Figure 1 is the result of the model having computed data on cumulative daily returns of the S&P500 Index, SSO and the daily rebalancing portfolio. However, the cumulative return for the S&P500 Index we have doubled, this way we have constructed an over time cumulative daily return benchmark. The figure tells us that the our portfolio with daily rebalancing rather closely manage

to track the double cumulative daily return S&P500 Index benchmark. This is especially true in comparison to the SSO. Table 6 in the appendix tells us that the maximum spread between the portfolio and the double return benchmark is *0.00318pps*. The minimum spread for the portfolio is *-0.224pps*. For the SSO the corresponding numbers are *0.00153pps* and *-0.309pps*. These numbers are not to be interpreted as absolut values, but positive numbers indicate that the portfolio or the SSO are returning above the double return S&P500 Index benchmark. Therefore the numbers indicate that the daily rebalancing portfolio has a larger maximum difference of over performing the double return S&P500 Index benchmark than the SSO, while not under performing as much as the SSO either.

6.6 Analysis

6.6.1 Cumulative Daily Return

The portfolio cumulative daily return is essential to our study. Primarily it is the cumulative return metric that will decide whether or not our constructed portfolio and the accompanying rebalance rules will prove to be a better leveraged buy-and-hold strategy than the leveraged ETF. The results on cumulative returns presented in *section 6.4* clearly indicate that our portfolio over performs that of holding a leveraged ETF. Over the holding period, there is not a single trading day where the leveraged ETF cumulative return is above our portfolio. Also, with an average spread of of around *8.555pps* over the total holding period, we believe we have proved that investors seeking buy-and-hold investment with leveraged returns should consider holding our portfolio rather than a leveraged ETF.

Having analyzed our results and while our portfolio independent of rebalancing rule clearly is better, we expected the differences in cumulative return to be larger between the rebalancing rules. Therefore it is worth making a comment on that aspect as well. In theory, when the underlying markets trade in a range, the leveraged ETF manager must rebalance more often, and with markets reverting often, he will ultimately cut profits and lock losses. Algebraically, in *section 3.2.1,* we prove that when the quadratic term is negative, over time the leveraged ETF will not manage to deliver two times the benchmark return. . The only way for that term to be negative is if markets are trading in a range with frequent reversion. The opposite is correct for strongly trending markets, where the quadratic term takes on a positive value where the leveraged ETF will deliver more than two times the benchmark return. Using other rebalancing rules that daily, we were, ex ante, expecting the different rules to yield significantly different results, since naturally they capture returns differently. Drawing conclusions from theory, in strongly trending markets a daily rebalancing rule should provide most return, since it adjusts for the changing leverage quicker. In strongly range bound market conditions less frequent hedging should provide better returns since these do not rebalance as much, since essentially the leverage will range in a same manor, rendering it redundant to

actively rebalance.

In retrospect, we believe that the holding period is so long that it incorporates periods of both the index trading in a range and periods of trending. Therefore, overall the effects of diffferent rebalancing rules cancel out. Therefore, the results, while being different, show no significant difference in returns, where there according to conclusions drawn from theory should be one. In *section 8.1* we comment on what additional studys can be made to search for that distinction.

Derived from the above, it is therefore fair to claim that the over performance on a cumulative basis of our portfolio is more attributed to the fact of using unleveraged ETFs while gaining leverage on a portfolio basis, rather than holding a leveraged ETf gaining leverage on a product level. Essentially, the mechanism of the leverage ETF tracking error is eliminated. Rather than passively holding a leveraged ETF over time investors can simply by holding a portfolio of unleveraged ETFs and borrowing cash, implement a strategy that is historically proven much more successful.

On a humble note, it is important to mention that the results presented are only valid for the holding period, and are historically corret. While providing conceptual and quantitative indications, the results are not perfect forecast of future conditions and returns. Also, the results might not be as valid for other types of market conditions and holding periods.

6.6.2 Daily Return

The results we have obtained on the daily returns are mainly important for one reason. From theory[1], we have understood that leveraged ETFs tend to under perform their promised leverage over time. However, from an leveraged ETF managers point of view, leveraged ETFs only promise the leverage return on a daily basis. For the sake of completeness it is therefore important to measure how well they fair on a daily basis compared to our portfolio. Our results show that on a daily basis, there is no conclusive proof as with the cumulative return case, that our portfolio is better than the leveraged ETF. The peak and bottom values are better for the portfolio but all in all there were 477 our of 975 trading days where the leveraged ETF returned higher than the ETF. This is essentially 49% of the trading days. While it is not the results that we had hoped, it is inline with theory[2]. As with the case of cumulative returns, the difference between the dayly rebalancing strategies is small. This seems a bit peculiar at first glance, because one might expect the leverage factor between these to be quite different. Not having explained this result fully, we find it an interesting result to analyze further in future research.

[1] Lu, Wang and Zhang, 2009 for a more in detail description

[2] Leveraged ETFs promise to deliver the leveraged return on a daily basis. Since our portfolio is tested using some rules with less frequent rehedging it is expected that our level of leverage will lagg somewhat compared to the leveraged ETF. Therefore, on a daily basis, ex ante to our portfolio evaluation, there is no cause to expect the portfolio to over perform the leveraged ETF.

Lastly, the small average spread between the daily return of the portfolio and the leveraged ETF shows us that on a daily basis, over the holding period, the difference between the two is small anyways.

6.6.3 Rebalance Return

The rebalance returns vary widely for the dierent rebalancing rules. This is expected because the rebalancing days dier for the dierent strategies. What is observable is that the more rebalancing events that take place the higher upp will the rebalancing returns reach. This is because when the underlying index is moving the more frequent the rebalancing the more closely the portfolio cumulative return follows. In appendix figure 10 and figure 12 we see that the $\pm 2\%$ rule rebalances all the way up to *52.416%, 2007-10-05* while the $\pm 10\%$ rebalancing rule rebalances once in *2007-05-15* at *46.355%* and then in *2008-01-17* at *9.551%*. It thus misses to rebalance on the increase up to around *2007-10-05*, thus it should lose momentum from lower leverage.

6.6.4 Comparison to S&P500 Index

Figure 1 in the appendix is truly significant in this comparison. Table 6 in the appendix provides the descriptives of the spreads. The results of our comparison of the daily rebalancing portfolio cumulative daily return, the SSO cumulative daily return and 2x the S&P500 Index cumulative daily return, clearly shows, that not only does our portfolio over perform the SSO, but it actually delivers 2x the cumulative return of the benchmark apart from a period beginning *2008-10-22*. This is quite remarkable. Initiating our portfolio, has for quite a significant time period almost completely eliminated the tracking error accrued by holding leveraged ETFs, and actually deliver twice the cumulative return of the S&P500 Index. We have provided a means for investors to truly gain leveraged exposure in the markets, while applying a buy-and-hold strategy, not possible holding the SSO. With dissapointed investors in the U.S having initiated an investigation on leveraged ETFs through the SEC, our strategy using historical data, shows that if investors would have initiated our suggested portfolio strategy, the issue might not have arisen in the first place. Figure 1 in appendix, also provides a measure of the spread between the 2x S&P500 cumulative daily return and the portfolio cumulative daily return. The spread between the 2x S&P500 cumulative daily return and SSO cumulative return is also displayed. The two spreads can be read on the right y-axis, while the cumulative returns can be read on the left y-axis.

Observing the lines for the spreads we can observe an even more enticing fact. During the period *2006-06-22* to *2007-11-30*, our portfolio actually delivers a higher cumulative daily return than 2x the S&P500 Index. It does this again during *2008-10-07* to *2009-04-03*. This is while the SSO cumulative return is below the 2x S&P500 Index cumulative daily return between *2006-06-22* to *2008-11-12*, and again between *2009-03-24* and *2010-04-29*.

It is quite clear from these results that from a return perspective, all else

equal, and investor should prefer our portfolio to holding the SSO. In addition, graphs and data show that our portfolio seems to be much better at tracking twice the return of the benchmark index over time, which ultimately is the investor need.

7 Conclusions

There are four main conclusions that we have been able to draw from implementing our alternative strategy with dierent rebalancing rules.

1. Constructing a portfolio that uses unleveraged ETFs but obtains leverage on a portfolio basis by borrowing funds, proves to be a better leveraged buy-and-hold strategy than holding a leveraged ETF. Thus our initial notion was correct.

2. Constructing a portfolio that uses unleveraged ETFs, obtaining leverage on a portfolio basis by borrowing funds, proves to be a rather suitable tracker for the double cumulative return of the underlying index. Thus, not only does this portfolio outperform the leveraged ETF, it also tracks the double cumulative return of the index very well.

3. On daily return basis, there is no conclusive distinction between our portfolio and the leveraged ETF. 49% of the trading days the leveraged ETF provides better returns so the two dierent vehicles are close to equivalent.

4. Over a longer period such as the one we have examined, the underlying markets display both ranging and trending patterns. Therefore, the different rebalancing rules have not appeared to have a signicant effect on portfolio returns. However, we suspect that the effects on returns from different rebalancing rules might be signficant during other periods[3]. On a practical note, we understand that some assumptions made in our model might make it more dicult for some investors to apply our strategy or accrue more costs than we have described. These discrepancies include transaction costs and interest rate costs. Our models do not take into account transaction costs. Regarding interest rate costs, our models use USD overnight LIBOR, a rate at which banks may borrow at. For retail investors, this borrowing rate will be higher. However, we still think that our conclusions are valid on a conceptual level and are in line with what previous research might indicate.

8 Further Research

While being excited about our findings, we feel like we have only peeled the surface of ETFs, leveraged ETFs and different portfolio strategies. Writing this

[3]Periods with either strong range bound or trading market conditions

thesis has aroused even more questions and a strong interest in how we might develop our ideas further. Furthermore, we will most likely continue to create other strategies and expand the evaluation of the one presented in this thesis. In addition, we hope that investors might be able use this study to widen their basis for making investment decisions within ETFs. hopefully, our study have implictions for ETF managers as well, displaying some dynamics that might previously have been academically unexplored.

Some of the inspiration for futher research has been extracted from our own studies while other was extracted from sources we had came across that we felt brought interesting ideas but were out of the span for this thesis.

8.1 Research

Having finished our study we have identfied three main areas of interest for further research in the subject. These are:

1. Evaluation of our portfolio for other time periods, preferably with distinguished market trading characteristics. We hope these results might indicate the effect on returns from different rebalancing rules and also provide insight in the dynamics between our portfolio returns and market conditions.

2. Incorporate our model with transaction costs and a more realistic interest rate for retail investors. However, transaction costs and interest costs dffier between investors so this test might be difficult to perform with specific exactness. However, our models are flexible for different interest rate inputs thus making such as study quite viable.

3. Having observed that our portfolio over performing the leveraged ETF on a cumulative daily return basis, and benchmark tracking basis, we would like to evaluate a strategy were we take a market netutral position. A market neutral strategy would attempt to short sell [buy] the leveraged ETF while buying [short selling] the portfolio capitalizing on the difference. Results from the points above might aid in deciding which asset to buy or sell short, during what market conditions. During market conditions where it has been showed that the leveraged ETF over performs the portfolio, we would like to investigate the return from buying the leveraged ETF while short selling the portfolio. In the opposite case we would like to evaluate the opposite trading congfiguration. In addition, one might apply different rebalancing rules as well, and evaluate those results. The benefits of such a trading strategy is of course the market neutrality aspect. However, it is difficult to model such a trade, since many of the costs connected to short selling are signficant but hard to retrieve.

4. Lastly, we think that from our back test, there might be incentives in attempting to construct a forward looking model, implementing our strategy using stochastic models. However, as of now, unfortunately, that lies beyond our mathematical skills.

8.2 Implications

We believe that our paper and the results presented will have most implications for retail investors. We have clearly shown that holding our portfolio versus a leveraged ETF over a longer period provides higher cumulative returns and a superior benchmark tracker. Also, we hope that investors will take a more thorough look at leveraged ETFs and apply caution when investing in them. Clearly their dynamics are more complicated than what might be perceived at first sight. Furthermore, as previous studies have shown leveraged and inverse ETFs are not suitable for long term investing, with an increasing tracking error. Our greatest impact will have been providing alternative means to gaining leveraged exposure while applying a buy-and-hold strategy. The great accuracy in our portfolio tracking the double cumulative S&P500 benchmark indicates the latter.

On an academic level we hope that this paper might influence others to indulge in research related to leveraged ETFs. We are certain that ETFs and leveraged ETFs will gain even more popularity amongst investors when the present issues are solved. We believe that in order to drive that development forward, academic research will play a huge role. In the professional investor sphere the knowledge of ETFs is already vast, and thus academics is lagging. This gap needs to be closed, because in order for ETFs to be fully demystified, we believe that more academic rigour and legitimace needs to be applied in the field. Ultimately, this will be a gain for all stakeholders, retail investors, professionals and academics alike.

References

[1] Dave Nadig, 2010, "Capturing Trendless Volatility With ETFs", *Article available at http://seekingalpha.com/article/201354-capturing-trendless-volatility-with-etfs*

[2] Dr. Guido Giese, 2010, "On the performance of leveraged and optimally leveraged investment funds", *STOXX Ltd*

[3] Heath Hinegardner, 2009, "How Managing Risk With ETFs Can Backfire", *Article available at http://online.wsj.com/article/SB123578237239398181.html*

[4] Jonathan Bernstein, 2007, "Leverage ETF Review: How do they Stack Up One Year Later?", *Article available at http://www.etfzone.com/?template=viewarticle&article_id=837*

[5] Lei Lu, Jun Wang, Ge Zhang, 2009, "Long Term Performance of Leveraged ETFs", *Working paper available at http://ssrn.com/abstract=1344133*

[6] Marc Gerstein, 2009, "Leveraged ETFs - Intraday Pricing Trends", *Article available at http://seekingalpha.com/article/127238-leveraged-etfs-intraday-pricing-trends*

[7] Minder Cheng and Ananth Madhavan, 2009, "The Dynamics of Leveraged and Inverse Exchange-Traded Funds", *Barclays Global Investors*

[8] Noël Amenc, Felix Goltz, Adina Grigoriu, David Schröder, 2009, "The EDHEC European ETF Survey 2009", *Survey available at http://www.edhec-risk.com/indexes/etf_survey*

[9] Richard Co, Research and Product Development, 2009, "Leveraged ETFs vs. Futures : Where Is the Missing Performance?", *Article available at http://www.cmegroup.com/trading/equity-index/files/Embedded_Volatility_in_Leveraged_ETF_021709.pdf*

[10] Shiraz Lakhi, 2009, "Trading Inter Market Spreads - A Fresh Look At The Simple Pairs Trading Strategy...", *Article available at http://www.dowtrader.net/ShirazLakhi.htm*

[11] Simon Maierhofer, 2009, "Short and Leveraged ETFs - 3 Pitfalls You Shouldn't Ignore", *Article available at http://www.etfguide.com/research/210/8/Short-and-Leveraged-ETFs—3-Pitfalls-You-Shouldn't-Ignore-/*

[12] Tom Lauricella, Susan Pulliam, Diya Gullapalli, 2008, "Are ETFs Late-Day Turns? Leveraged Vehicles Seen Magnifying Other Bets; Last-Hour Volume Surge" *Article available at http://newsgroups.derkeiler.com/Archive/Misc/misc.invest.stocks/2008-12/msg01042.html*

[13] Tristan Yates, 2007, "Leveraged ETFs: A value Destruction Trap?", *Article available at http://seekingalpha.com/article/31195-leveraged-etfs-a-value-destruction-trap*

[14] Tristan Yates, 2009, "Market Neutral: The Last Path To Diversification", *Article available at http://www.tradingmarkets.com/.site/stocks/how_to/articles/Market-Neutral-The-Last-Path-To-Diversification-80548.cfm*

[15] Tristan Yates, 2007, "The Case Against Leveraged ETFs", *Article available at http://seekingalpha.com/article/35789-the-case-against-leveraged-etfs*

9 Appendix

Daily Returns

2006-2010	Dly Reblc	+/- 5%	+/- 10%	+/- 2%	SSO
Max	29,040%	29,040%	29,040%	29,040%	22,414%
Min	-19,690%	-19,690%	-19,690%	-19,690%	-17,293%
Average	-7,521%	-7,524%	-7,528%	-7,533%	-16,091%
StdDev	3,385%	3,385%	3,385%	3,385%	3,240%

Table 1: Daily Return Descriptives, period 2006-06-22 to 2010-04-29

Cumulative Returns

2006-2010	Dly Reblc	+/- 5%	+/- 10%	+/- 2%	SSO
Max	53,593%	53,587%	53,580%	53,570%	40,084%
Min	-76,327%	-76,328%	-76,329%	-76,331%	-79,664%
Average	-7,521%	-7,524%	-7,528%	-7,533%	-16,091%
StdDev	11,076%	11,076%	11,073%	11,070%	8,683%

Table 2: Cumulative Return Descriptives, period 2006-06-22 to 2010-04-29

CumRet Spread

2006-2010	Dly Reblc	+/- 2%	+/- 5%	+/- 10%
Max	18,593pps	18,588pps	18,582pps	18,573pps
Min	0,166pps	0,166pps	0,166pps	0,166pps
Average	8,571pps	8,567pps	8,564pps	8,558pps
StdDev	4,449%	4,448%	4,447%	4,445%

Table 3: Cumulative Return Spread Descriptives, portfolio vs SSO, period 2006-06-22 to 2010-04-29

ReblcDayPerf

2006-2010	Dly Reblc	+/- 2%	+/- 5%	+/- 10%
Max	53,593%	52,416%	46,527%	46,355%
Min	-76,327%	-75,580%	-74,592%	-73,728%
Average	-7,521%	-26,565%	-32,677%	-36,456%
Standard Deviation	11,076%	38,254%	35,473%	32,150%

Table 4: Rebalance Day Performance Descriptives, period 2006-06-22 to 2010-04-29

Average Dly Spread				
	Dly Rblc	**+/- 2% Rblc**	**+/- 5% Rblc**	**+/- 10% Rblc**
2006	0.0422pps	0.042pps	0.0422pps	0.0421pps
2008	0.0318pps	0.0318pps	0.0318pps	0.0318pps
2009	0.0096pps	0.0096pps	0.0096pps	0.0097pps
2010	0.00074pps	0.00075pps	0.00075pps	0.00075pps
Total Hold Period	0.0175pps	0.0175pps	0.0175pps	0.0175pps

Table 5: Daily Return Spread Averages, Yearly and Entire Holding Period

Spreads		
2xS&P500	**Dly Rblc**	**SSO**
Max	0.00318pps	0.00153pps
Min	-0.224pps	-0.309pps
Average	-0.0318pps	-0.117pps

Table 6: Cumulative Daily Return Spread, SSO and Daily Rebalancing Portfolio Cumulative Daily Return vs 2xS&P500 Index Cumulative Daily Return, period 2006-06-22 to 2010-04-29

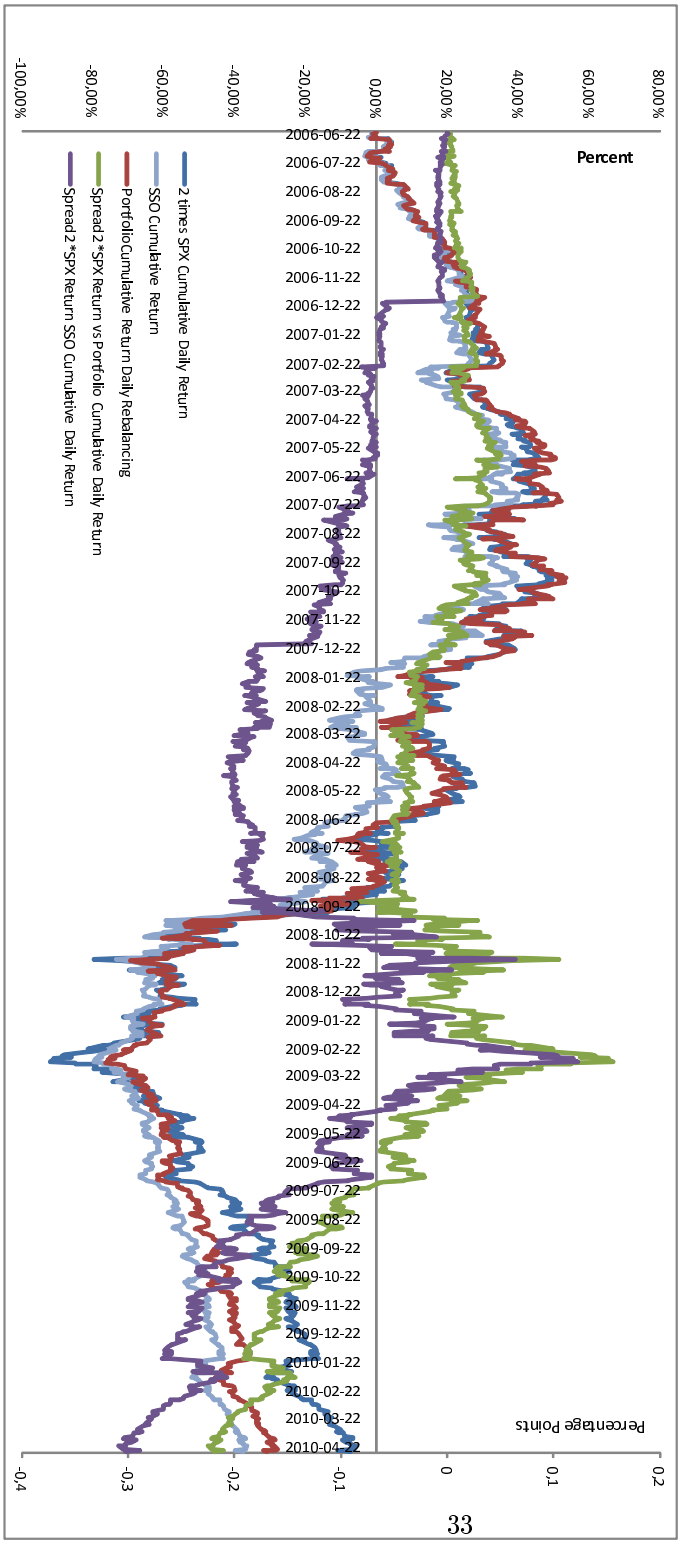

Figure 1: Cumulative Daily Return comparison, 2xS&P500, Daily Rebalance Portfolio, SSO and spreads, period 2006-06-22 to 2010-04-29

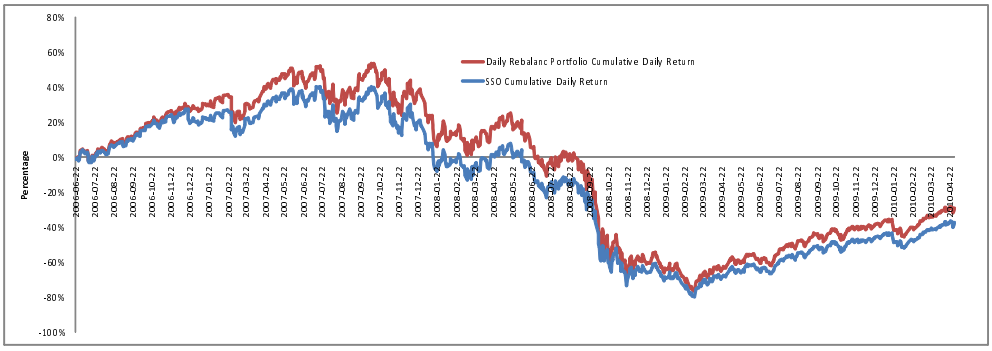

Figure 2: Comparison of Cumulative Daily Return, SSO vs Daily Rebalancing Portfolio, period 2006-06-22 to 2010-04-29

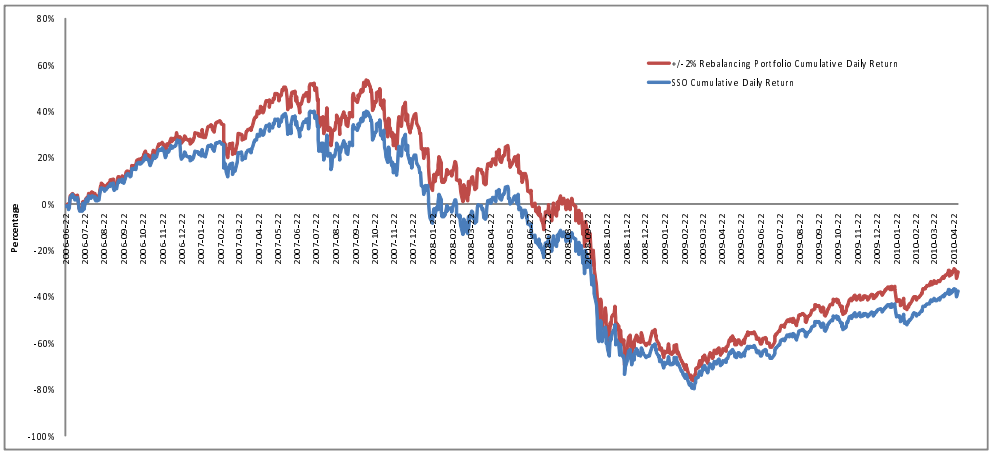

Figure 3: Comparison of Cumulative Daily Return, SSO vs $\pm 2\%$Rebalancing Portfolio Cumulative Daily Return, period 2006-06-22 to 2010-04-29

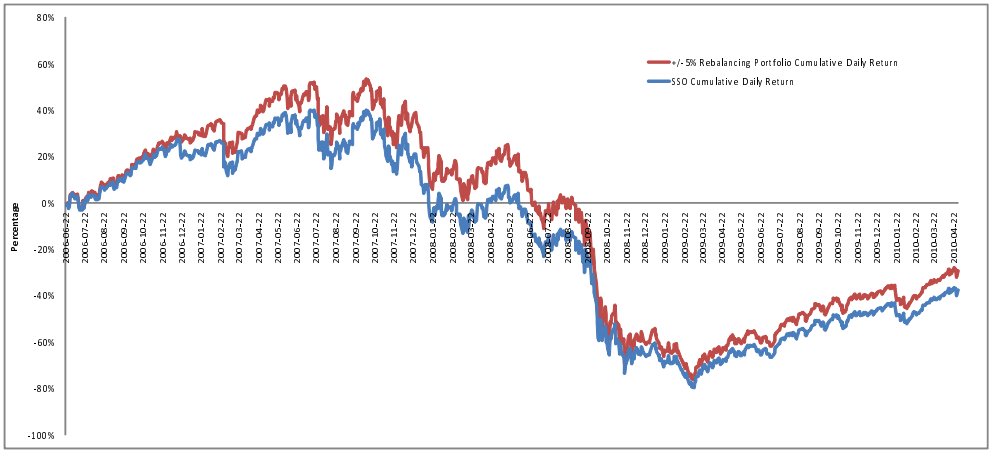

Figure 4: Comparison of Cumulative Daily Return, SSO vs ±5%Rebalancing Portfolio Cumulative Daily Return, period 2006-06-22 to 2010-04-29

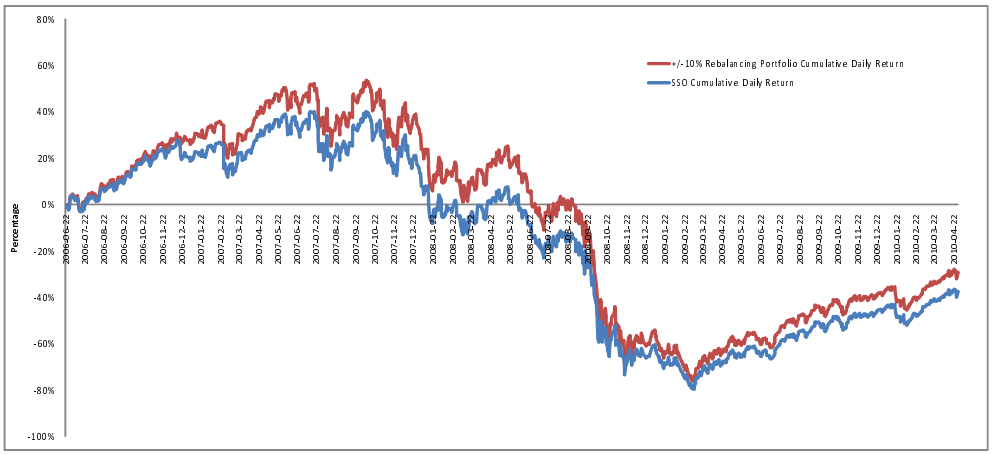

Figure 5: Comparison of Cumulative Daily Return, SSO vs ±10%Rebalancing Portfolio Cumulative Daily return, period 2006-06-22 to 2010-04-29

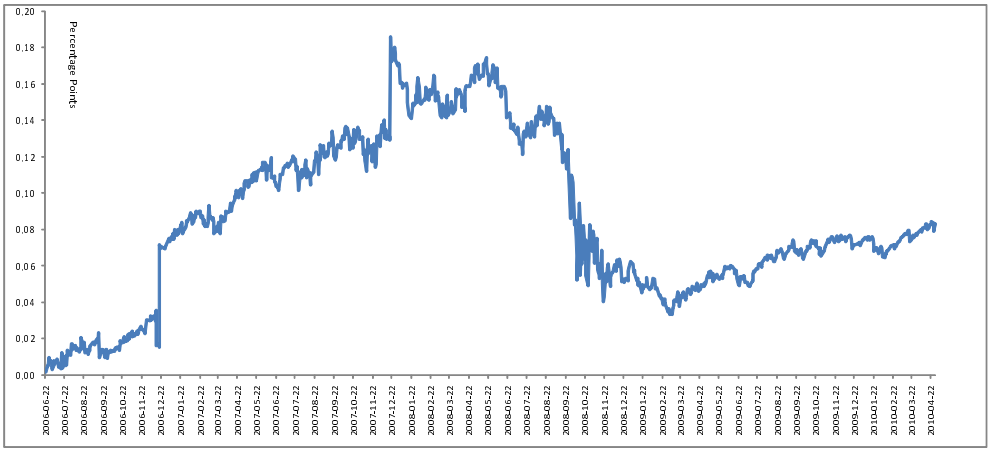

Figure 6: Spread between Cumulative Daily Return, SSO vs Daily Rebalancing Portfolio, period 2006-06-22 to 2010-04-29

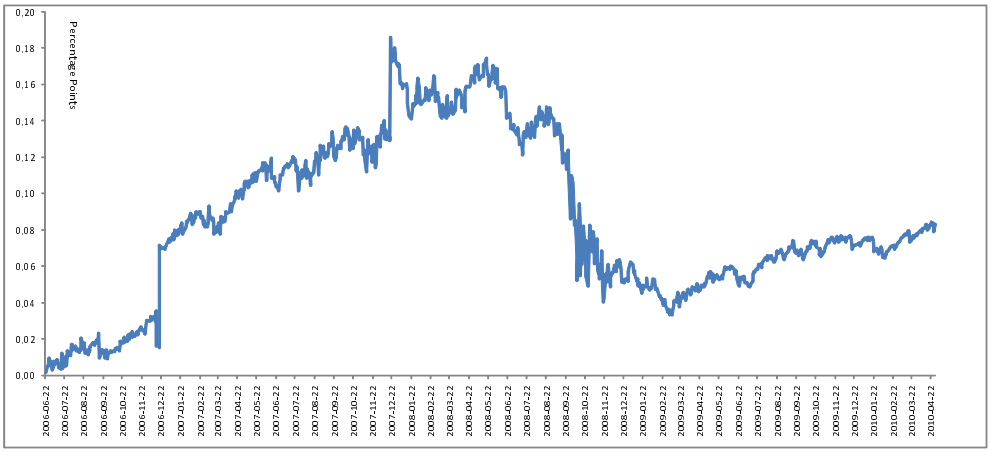

Figure 7: Spread between Cumulative Daily Return, SSO vs ±2%Rebalancing Portfolio, period 2006-06-22 to 2010-04-29

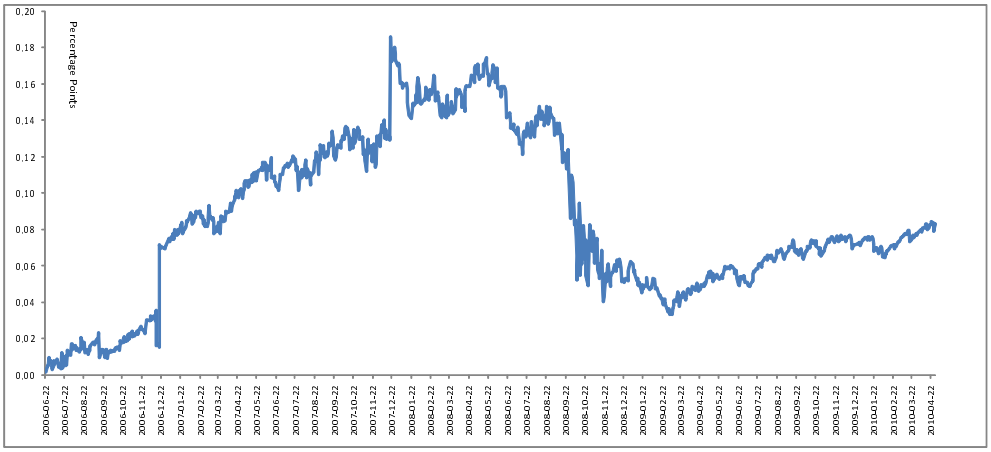

Figure 8: Spread between Cumulative Daily Return, SSO vs ±5%Rebalancing Portfolio, period 2006-06-22 to 2010-04-29

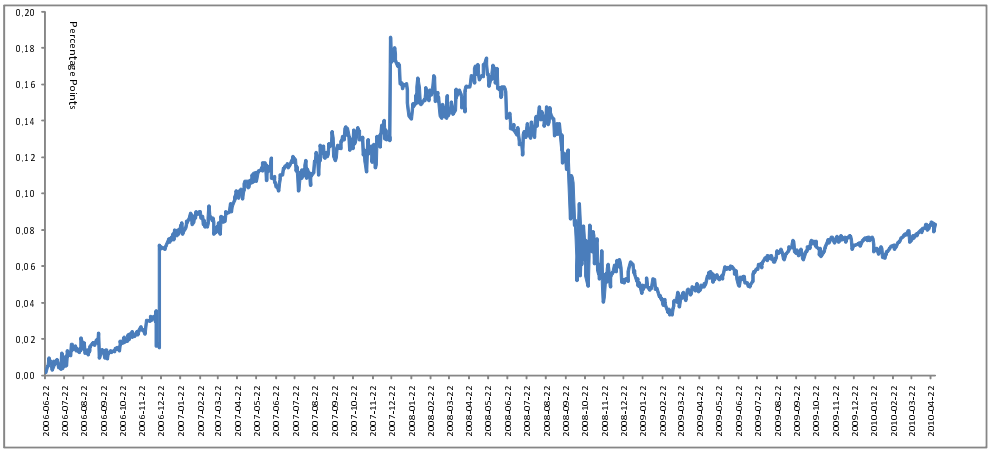

Figure 9: Spread between Cumulative Daily Return, SSO vs ±10%Rebalancing Portfolio, period 2006-06-22 to 2010-04-29

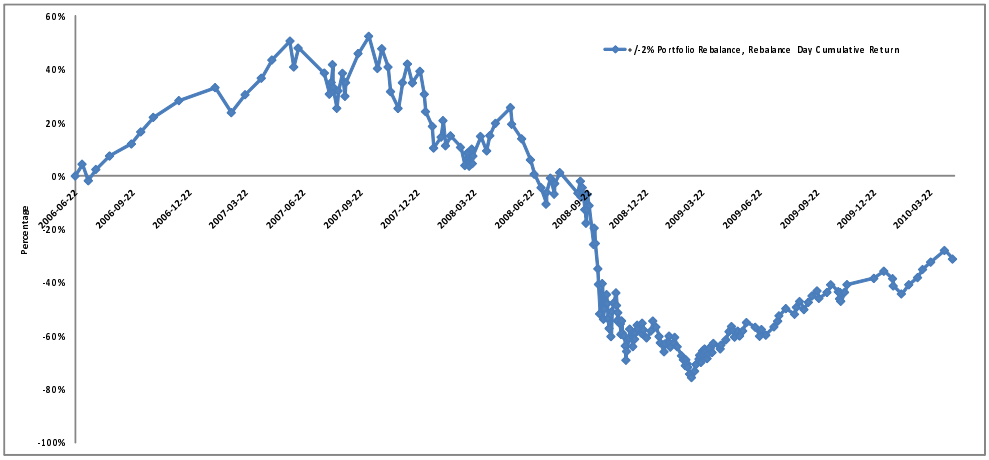

Figure 10: ±2%Rebalancing Portfolio, Rebalance Day Cumulative Return, period 2006-06-22 to 2010-04-29

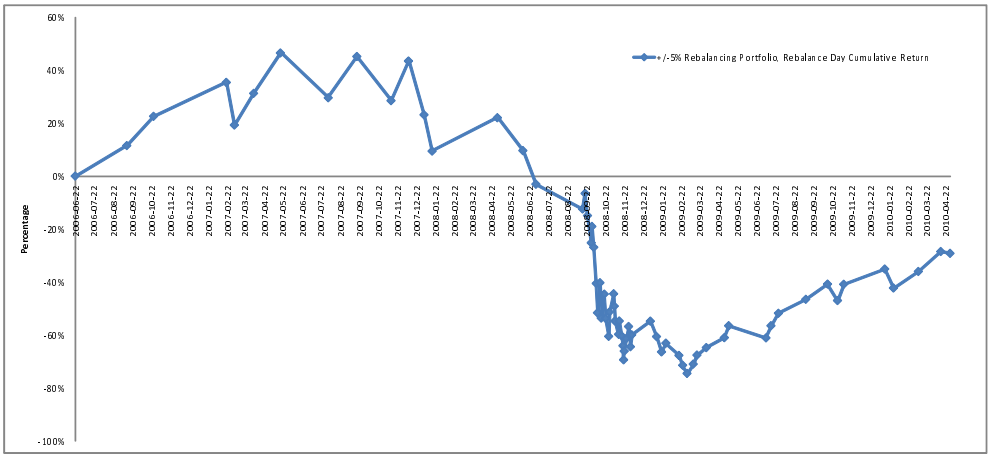

Figure 11: ±5%Rebalancing Portfolio, Rebalance Day Cumulative Return, period 2006-06-22 to 2010-04-29

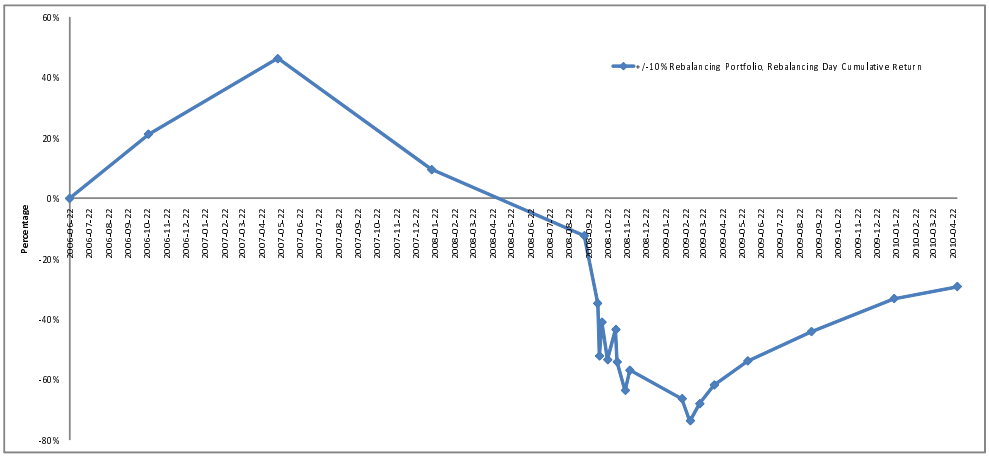

Figure 12: ±10%Rebalancing Portfolio, Rebalance Day Cumulative Return, period 2006-06-22 to 2010-04-29

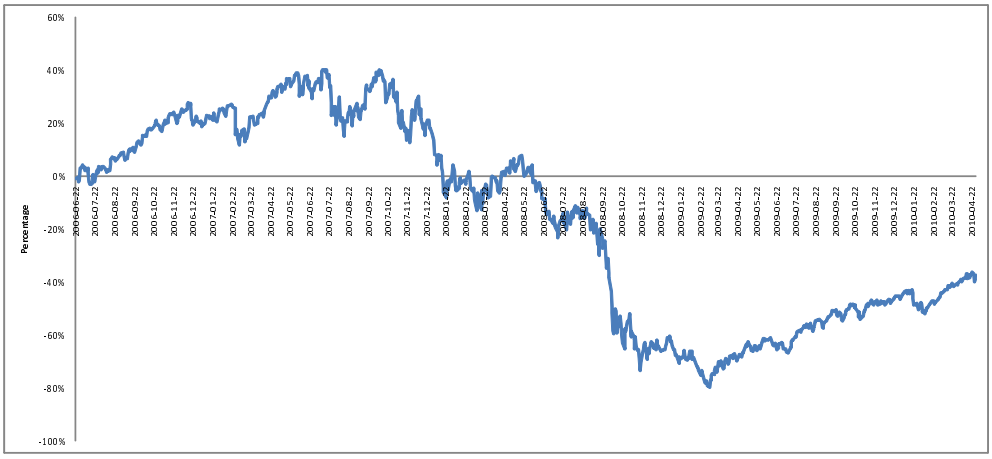

Figure 13: Cumulative Daily Return, SSO, period 2006-06-22 to 2010-04-29