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Downside Risk, Upside Uncertainty and Portfolio Selection

Master´s Thesis in Finance

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Abstract

The traditional portfolio optimization models make predictions about investors´ behavior that deviate from the empirical observations. This thesis develops a simple model for the asset allocation under risk and uncertainty. Investment decisions under risk are evaluated with regards to the following four parameters: upside profit; probability of upside profit; downside loss and probability of downside loss. The optimal asset allocation is made depending on the investor's preferences with regards to the four parameters. The model is applied to solve three paradoxes of decision making: the Allais; the Ellsberg and the St. Petersburg Paradox. The model is used to build four optimal portfolios of the stocks that constitute the OMX30 Stockholm Index. The model is also compared to the mean variance portfolio optimization model. In the sample period, the optimized portfolios dominate the OMX30 Stockholm Index by their parameter mix.

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TABLE OF CONTENTS

1 INTRODUCTION

Harry Markowitz (1952), the father of modern portfolio theory, published a paper establishing the principle of diversification in finance. He developed a theory of asset allocation under risk and uncertainty, defining risk as portfolio variance and expected return as the reward for taking that risk. Markowitz showed that by combining several securities, a portfolio could be built that would give lower risk than individual securities for the same level of expected return. The "safety first" rule, the maximum expected loss, the expected value of loss, the expected absolute deviation and the semi-variance were evaluated as a measures of risk when Markowitz (1959) developed his theory, but the variance was chosen for simplicity of calculation. A number of other models were developed for the portfolio selection. One example is the downside risk model of Tien Foo Sing and Seow Eng Ong (2000). However, the mean variance (MV) optimization is still the most common method for portfolio optimization.

The modern portfolio theory is based on the assumption that the investor is maximizing the utility function of wealth (Levy and Markowitz, 1979). The optimal portfolio for an investor with the mean variance utility function is the portfolio that gives the lowest variance for a given level of expected return.

Markowitz builds his portfolio theory on the expected utility theory. In order to solve the St. Petersburg Paradox, where people agree to pay a very moderate amount to participate in a gamble that gives infinite expected value, Bernoulli (1954/1738) proposed that people maximize expected utility instead of expected value of money assuming that money has a diminishing value. An increase in wealth is worth more for poor people than it is for rich individuals. Von Neumann and Morgenstern (1947) introduced the axiomatic ground for the expected utility theory, making it the dominating analysis of choice under risk and uncertainty.

However, empirical evidence shows that people's behavior systematically deviates from the predictions of both the expected utility and the Mean-Variance Model - some examples of the violation of the expected utility theory are demonstrated by the Allais (1953) and the Ellsberg (1961) paradoxes. A number of theories were developed to deal with each paradox, one example is the Prospect Theory of Kahneman and Tversky (1979).

In this thesis, I develop a simple model for decision making and asset allocation under risk and uncertainty that can be used regardless of the shape of the returns´ distribution. This thesis also shows that the model is consistent with people's actual behavior. In the sample period, the portfolios built using the proposed model dominate the OMX30 Stockholm Index by the parameters introduced in this thesis.

This thesis consists of four sections. The first section presents the Risk-Reward Model. The second section applies the model to solve classical paradoxes of decision making under risk and uncertainty. In the third section, the model is tested empirically by optimizing the OMX30 Stockholm Index portfolio, the model is also compared to the Mean-Variance Model. In the final section, the conclusion is presented.

2 THE RISK-REWARD MODEL

2.1 RISK-REWARD MODEL - SINGLE ASSET

The Risk-Reward Model is a one period model. It is based on one widely accepted principal; the investor prefers having more to less. A risky asset can be seen as an asset that gives a distribution of possible payoffs or returns ($x_1, x_2,...,x_n$) with probabilities ($p_1,p_2,...,p_n$), where p_1+p_2+,...,+p_n=1. Since the asset is by definition a risky asset, the payoffs x in relation to some target rate of return τ can be positive x^+, negative x^- or zero x^0. The investor can get a profit, loss or break even. For simplicity of analysis, I assume that the target rate of return is zero for the investor. The probabilities of obtaining a random positive x_i^+, negative x_i^- or zero x^0 payoff are: $p(x_i^+)$, $p(x_i^-)$ and $p(x^0)$.

If an investor makes a one-period investment decision and invests in the risky asset at time t_0, at time t_1, the investor will get one payoff from the distribution of the possible payoffs. The investor will get either one of the possible positive payoffs x_i^+, one of the possible negative payoffs x_i^- or the payoff of zero x^0. Markowitz (1952) proposed that utility should be defined through gains and losses. But the traditional way to calculate return on investment is to use expected value calculated through average return and risk calculated using variance, as it is used in the Markowitz Mean-Variance Model and the CAPM model (Sharpe (1964), Linter (1965) and Black et al., (1972)). The average return is the sum of all returns; both positive, negative and zero divided by the total number of possible returns. The average return tells us what return we will get on average, assuming that the returns distribution will stay the same, if we would make the same investment decision over and over again, infinitely (Bodie et al., 2008).

However, before making an investment decision, the investor may want to know the parameters that summarize the possible result of an investment: possible profit, possible loss, chance of profit, chance of loss and chance of getting a zero return. It is reasonable to assume that if the investor is interested in the above parameters he/she would use the most reliable method in his/her opinion to get these parameters. Feunou, Jahan-Parvar, Tédongap (2010) model risk as an average negative return multiplied by the probability of getting a negative return, and reward as an average positive return multiplied by the probability of getting a positive return, where the target rate is a risk-free rate. Following a similar approach, the parameters of interest for the investor will be defined as following: upside profit, downside loss, probability of upside profit, probability of downside loss, the neutral result and the probability of neutral result.

The probability of obtaining an upside profit can be calculated as:

$$p(x_i^+)=\frac{\mathbf{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}}\mathbf{I}\,(x_{it}\ >\tau)=\frac{\mathrm{T}_i^+}{\mathrm{T}} \quad (1)$$

Where a target rate of return is τ, T_i^+ is the number of possible upside returns and T is the total number of returns.

The probability of obtaining a downside loss can be calculated as:

$$p(x_i^-)=\frac{\mathbf{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}}\mathbf{I}\,(x_{it}\ <\tau)=\frac{\mathrm{T}_i^-}{\mathrm{T}} \quad (2)$$

Where T_i^- is the number of possible negative returns.

The probability of obtaining a neutral result can be calculated as:

$$p(x_i^0)=\frac{\mathbf{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}}\mathbf{I}\,(x_{it}\ =\tau)=\frac{\mathrm{T}_i^0}{\mathrm{T}} \quad (3)$$

Where T_i^0 is the number of possible zero returns.

The upside profit can be calculated as:

$$\frac{\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} x_{it}\,\mathbf{I}(x_{it}>\tau)}{\mathrm{T}_i^+}=\mu_{x_i^+} \quad (4)$$

Where $\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} x_{it}\,\mathbf{I}\,(x_{it}\ >\tau)$ is the sum of the positive returns.

The downside loss can be calculated as:

$$-\frac{\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} x_{it}\,\mathbf{I}(x_{it}<\tau)}{\mathrm{T}_i^-}=\mu_{x_i^-} \quad (5)$$

Where $\sum_{t=1}^{T} x_{it}\, \mathbf{I}\,(x_{it} \;< \tau)$ is the sum of negative returns.

The neutral result is:

$$\frac{\sum_{t=1}^{T} x_{it}\,\mathbf{I}(x_{it}=\tau)}{\mathrm{T}_i^0}=\mu_{x_i^0} \qquad (6)$$

Where $\sum_{t=1}^{T} x_{it}\, \mathbf{I}\,(x_{it} \;= \tau)$ is the sum of zero returns.

Since the average return is just an upside profit multiplied by the probability of obtaining an upside profit minus downside loss multiplied by the probability of obtaining a downside loss, plus a neutral result multiplied by the probability of obtaining a neutral result, it can be written as (7) below.

$$\mu_{x_i} = (\, p(x_i^+)^* \,\mu_{x_i^+})\text{-}(\, p(x_i^-)^* \,\mu_{x_i^-})\text{+}(\, p(x_i^0)^* \,\mu_{x_i^0}) \qquad (7)$$

From this equation, we see that μ_{x_i} will go up when keeping everything else equal, $p(x_i^+)$ goes up, $\mu_{x_i^+}$ goes up, $p(x_i^-)$ goes down and $\mu_{x_i^-}$ goes down.

The parameter *neutral result* is captured by the other parameters, which is why I will only concentrate on upside profit, probability of upside profit, downside loss and probability of downside loss in the analysis.

Assuming that the investor accepts the risky asset A, he/she also accepts the asset's parameter mix: $p(x_a^+)$, $\mu_{x_a^+}$, $p(x_a^-)$, $\mu_{x_a^-}$. Simultaneously, the investor is offered another risky asset B. Obviously, if $p(x_b^+)$=$p(x_a^+)$, $\mu_{x_a^+}$=$\mu_{x_b^+}$, $p(x_a^-)$=$p(x_b^-)$ and $\mu_{x_a^-}$=$\mu_{x_b^-}$, the assets A and B are identical, so the investor would be indifferent between which one to choose. But while keeping all other parameters equal between the two assets, the asset B will be preferred if it is dominating by at least one parameter: $p(x_b^+)$≥$p(x_a^+)$, $\mu_{x_a^+}$≤$\mu_{x_b^+}$, $p(x_b^-)$≤$p(x_a^-)$ and $\mu_{x_a^-}$≥$\mu_{x_b^-}$. From equation (7) we get that μ_{x_b} ≥μ_{x_a} , so the asset B is equal or dominating asset A both in the short and the long run. Following the same line of reasoning, asset B would not be preferred when keeping everything else equal, $p(x_b^+) \leq p(x_a^+)$, $\mu_{x_b^+} \leq \mu_{x_a^+}$, $p(x_b^-) \geq p(x_a^-)$ and $\mu_{x_b^-} \geq \mu_{x_a^-}$.

However, if neither asset A nor asset B is dominating by at least one parameter, while having all other parameters equal between the both assets, an investor will choose according to his or her preferences with regards to the given parameter mixes.

In other words, keeping all other parameters equal, people prefer higher profit to lower profit, lower loss to higher loss, higher probability of profit to lower probability of profit and lower probability of loss to higher probability of loss.

If parameter mix one is preferred to parameter mix two and parameter mix two is preferred to parameter mix three then parameter mix one is preferred to parameter mix three.

2.2 RISK-REWARD MODEL - PORTFOLIO SELECTION

Since the portfolio return is a linear combination of the weighted returns of individual securities, we can define the portfolio return as:

$$x_p=\sum_{i=1}^{N} R_i * w_i \qquad (8)$$

Where R_i is the return of the asset i in the portfolio and w_i is the weight allocated to asset i. In that case, the portfolios parameter mix can be calculated in the same way as for an individual security.

The probability of obtaining an upside portfolio payoff can be calculated as:

$$p(x_p^+)=\frac{\mathbf{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} \mathbf{I}\,(x_{pt} > \tau)=\frac{\mathrm{T}_p^+}{\mathrm{T}} \qquad (9)$$

Where T_p^+ is the number of possible positive portfolio returns and T is the total number of returns.

The probability of obtaining a downside portfolio payoff can be calculated as:

$$p(x_p^-)=\frac{\mathbf{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} \mathbf{I}\,(x_{pt} < \tau)=\frac{\mathrm{T}_p^-}{\mathrm{T}} \qquad (10)$$

Where T_p^- is the number of possible negative portfolio returns.

The probability of obtaining a neutral portfolio return can be calculated as:

$$p(x_p^0)=\frac{\mathbf{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} \mathbf{I}\,(x_{pt} = \tau)=\frac{\mathrm{T}_p^0}{\mathrm{T}} \qquad (11)$$

Where T_p^0 is the number of possible zero portfolio returns. Upside portfolio return can be calculated as:

$$\frac{\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}} x_{pt}\mathbf{I}(x_{pt}>\tau)}{\mathrm{T}_p^+}=\mu_{x_p^+} \qquad (12)$$

Where $\sum_{t=1}^{T} x_{pt}\mathbf{I}\,(x_{pt} > \tau)$ is the sum of the positive portfolio returns.

Downside loss can be calculated as:

$$-\frac{\sum_{t=1}^{T} x_{pt}\mathbf{I}(x_{pt}<\tau)}{\mathrm{T}_p^-} = \mu_{x_p^-} \quad (13)$$

Where $\sum_{t=1}^{T} x_{pt}\mathbf{I}\,(x_{pt} < \tau)$ is the sum of negative portfolio returns in the sample.

Since the risky portfolio is in fact a single risky asset consisting of several other risky assets, the intuition for the selection of the optimal portfolio is the same as for a single asset. The investor should build optimal portfolios depending on the investor's preferences with regards to the portfolios´ parameter mix. In line with the principals of diversification developed by Markowitz (1952), a portfolio can be built that gives a parameter mix that dominates the parameter mix of the individual securities. For example, the portfolios´ downside loss can be lower, the probability of downside loss can be lower, while the upside profit can be higher and the probability of upside profit can be higher than the corresponding parameters of individual securities.

The Risk-Reward Model defines risk as a downside loss and probability of getting a downside loss. Reward is defined as an upside profit and the probability of getting an upside profit. The model tries to minimize the risk and maximize the reward for taking such a risk by combining different assets into one optimal asset. It is another way to mathematically formulate the principal of diversification in finance. By combining different types of assets, we create an asset that carries less risk and has higher reward than individual assets. This can be made possible if the available assets move in opposite directions. A classic example is that when the stock market crashes, the bond market generally goes up. No assumptions are made about the return distribution because regardless of the return distribution, an investment will result in an outcome that can be estimated and summarized using the parameter mix. By mixing together different assets which tend to move in opposite directions, the Risk-Reward Model seeks to increase portfolios' upside profit and probability of profit, and at the same time decrease portfolios' downside loss and probability of downside loss. What is essential is how the price of each asset in the portfolio changes in relation to all other assets in the portfolio. Precisely, as in the Markowitz Modern Portfolio Theory, an asset should be included in the portfolio only if it improves a portfolio's parameter mix. Asset allocation under risk and uncertainty is a tradeoff between risk and reward for taking that risk. For an acceptable level of risk and reward, the Risk-Reward Model shows how to find a portfolio with the lowest risk and highest reward. Under the standard assumption that investors prefer more to less, the Risk-Reward Model finds the best diversification strategy for the acceptable level of risk and reward.

The parameter mix of the portfolio is a weighted sum of the parameter mix of individual securities only when all securities in the portfolio always go up or go down at the same time and by the same amount. But this is typically not the case.

Typically, all securities in the portfolio do not always go up or go down at the same time and by the same amount. That is why it is possible to diversify a portfolio with regards to the parameter mix. Typically, the upside profit of the portfolio is not a weighted sum of the upside profits of the individual securities and the downside loss is not the weighted sum of the downside losses of the individual securities. Probability of profit is not a weighted sum of the probabilities of profit of individual securities. Probability of loss is not a weighted sum of the probabilities of loss of individual securities.

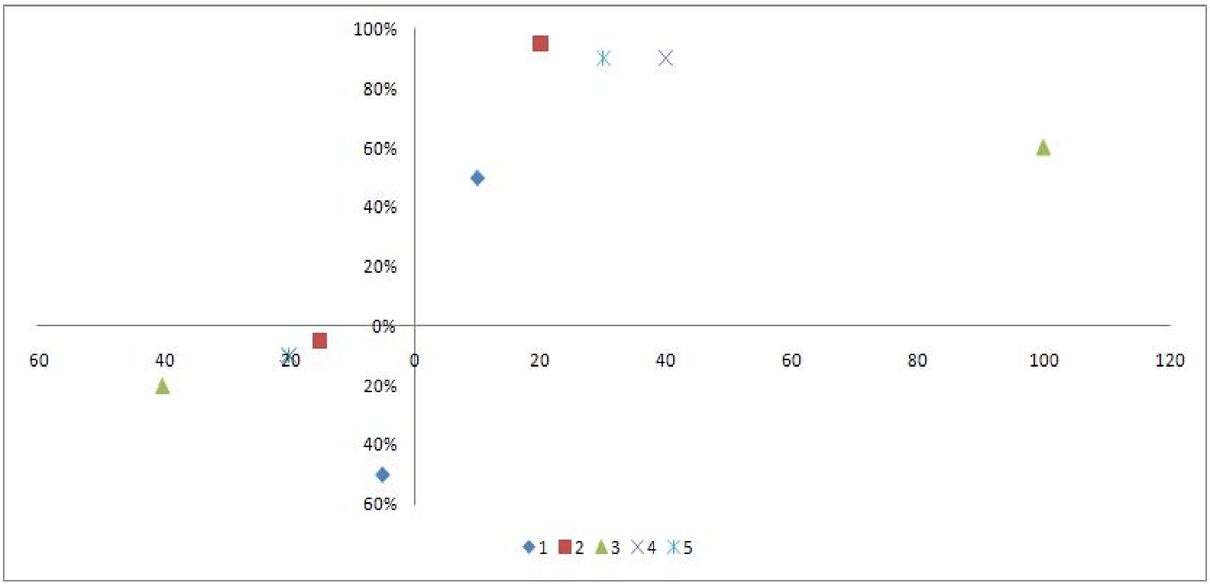
Instead of an efficient frontier with the portfolios that are mean-variance efficient as in the Mean-Variance Model, we have a universe of efficient mixes that are acceptable to an investor, see Graph 1. In the example see Table 0, the assets 1-4 are efficient because neither is totally dominating the other assets by the parameter mix. But asset 5 is totally dominated by the asset 4 since the upside profit of asset 4 is higher than asset 5 but all other parameters are equal between the two assets. The Risk-Reward model assumes that portfolios are chosen in accordance to investors' preferences for the parameter mixes. Investors can have one risky portfolio which can give a very high upside profit and very high downside loss with equal chances of getting a profit or loss, or a safer portfolio that gives a low profit with very high probability and economically insignificant loss with very low probability. The investor uses own forecast to find an efficient mix. Following this intuition, two different investors with the same acceptable mix may arrive at different stocks, since their forecasts are different.

Table 0. The five example assets' parameter mixes.

Asset	1	2	3	4	5
Upside profit (%)	10	20	100	40	30
Downside loss (%)	5	15	40	20	20
P(profit)	50%	95%	60%	90%	90%
P(loss)	50%	5%	20%	10%	10%

The numbers in the table are just example numbers

Graph 1. The five example assets' parameter mixes.



The graph illustrates the parameter mix of five different investment opportunities. The horizontal axis illustrates the upside profit or downside loss. The vertical axis illustrates the probability of getting a profit or loss.

2.3 RISK-REWARD AND MEAN-VARIANCE ALGORITHMS

An investor can make an investment decision following these two simple steps:

1) Decide what parameters are simultaneously acceptable for the investor: μ^+, μ^-, $p(x^+)$ and $p(x^-)$.

An investor should first decide how much upside profit he/she may want to have the possibility to earn. Given the desired upside profit, the investor should decide how high a probability of profit he/she can accept. Given the accepted upside profit and probability of upside profit, the investor should decide how much of a downside loss he/she can accept. Given the accepted upside profit, probability of upside profit and downside loss, the investor should make a decision on the acceptable level of the probability of the downside loss. In other words, an investor makes a decision to simultaneously accept an investment's parameter mix.

2) Find the portfolio with the optimal parameters $\mu_{x_p^+}$, $\mu_{x_p^-}$, $p(x_p^+)$ and $p(x_p^-)$, where $\mu_{x_p^+} \geq \mu^+$, $p(x_p^+) \geq p(x^+)$, $\mu_{x_p^-} \leq \mu^-$, $p(x_p^-) \leq p(x^-)$.

Given the accepted parameter mix, the next step is to find a portfolio that gives a higher or equal upside profit, a higher or equal probability of upside profit, a lower or equal downside loss and a lower or equal probability of downside loss than the accepted parameter mix.

This can be done in four different ways by using four different algorithms depending on which parameter is the most important for the investor:

Algorithm 14, minimizing downside loss, Minimize $\mu_{x_p^-}$

Algorithm 15, maximizing upside profit, Maximize $\mu_{x_p^+}$

Algorithm 16, minimizing probability of downside loss, Minimize $p(x_p^-)$

Algorithm 17, maximizing probability of upside profit, Maximize $p(x_p^+)$

Subject to the constraints

$$\sum_{i=1}^{N} w_i = 1$$

$$w_i \geq 0,\ i = 1,2 \ldots, N.$$

$\mu_{x_p^+} \geq \mu^+$, $p(x_p^+) \geq p(x^+)$, $\mu_{x_p^-} \leq \mu^-$, $p(x_p^-) \leq p(x^-)$.

I chose not to allow short sales, however short sales can be allowed. The total portfolio weight should sum up to 1. There are also constraints on each parameter. The upside profit of the portfolio should be higher or equal to the acceptable value, the downside loss should be lower or equal to the acceptable value, the probability of upside profit should be higher or equal to the acceptable value and the probability of downside loss should be lower or equal to the acceptable value.

Given a parameter mix that is acceptable to the investor, the algorithms 14-17 search for the optimal parameter mix by changing the weight of individual securities. If the program does not find the optimal portfolio given the constraints, the investor should not invest, since the desired parameter mix is unattainable. The investor can try to solve this problem by searching for new securities to add to the portfolio or look at the other time periods. Eventually, the investor can decide on a new acceptable parameter mix which is attainable.

On the other hand, a mean-variance investor first decides the desired level of the expected return and then tries to find a portfolio with the lowest variance. This can be achieved using a simple version of the Markowitz (1952) algorithm.

Algorithm 18:

Minimize σ_p^2

Subject to

$\mu_{x_p} = \mathrm{E}$

$w_i \geq 0$, i=1,2….,N.
$\sum_{i=1}^{N} w_i = 1$

Where σ_p^2 is the portfolio variance, μ_{x_p} is the portfolio mean return, w_i is the weight allocated to asset i and E is the desired level of the portfolio mean return. Given the desired level of expected return, algorithm number 18 calibrates the weight of individual securities searching for the portfolio with the lowest variance.

3 ANALYSIS OF PARADOXES

In this section, the decision problems that lead to classical paradoxes of decision making under risk and uncertainty will be analyzed. Every situation is broken down into the parameter mix: upside profit; downside loss; probability of profit; and probability of loss. The possible decision is analyzed with regards to the parameters.

3.1 THE ST. PETERSBURG PARADOX

The "St. Petersburg Paradox", which was first formulated in the 18th century by the Swiss mathematician Nikolaus Bernoulli is explained as follows:

Peter tosses a coin and continues to do so until it lands "heads" when it falls to the ground. He agrees to give Paul one dollar if he gets "heads" on the very first throw, two dollars if he gets it on the second, four if on the third, eight if on the fourth, and so on, so that with each additional throw the number of dollars he must pay is doubled (Levy et al., (1984)).

Money to win is $2^{(n-1)}$, where probability of winning is $½^{n}$. Money paid to enter the game is Y, and the monetary payoff ($2^{(n-1)}$ −Y). The gamble offers a ½ probability of winning 1 dollar, a ¼ probability of winning 2 dollars, etc, its expected value is: (½)*1+1/4*2+(1/8)*4+…=0,5+0,5….=∞

However, people are willing to pay a very small amount of money to participate in the gamble. The gamble´s payoffs is in this case x. The game can generate any monetary payoff of x_i = ($2^{(n-1)}$ −Y) from the possible distribution, and can be both positive x_i^+, negative x_i^- or zero x_i^0, with the probabilities $p(x_i^+)$, $p(x_i^-)$, and $p(x_i^0)$. We can calculate the parameter mix: upside profit μ_{x^+} ; probability of getting a profit *p(*x^+*);* downside loss μ_{x^-}; and probability of getting a loss *p(*x^-*)* The parameter mix depends on the amount paid to enter the game denoted by Y in the following way:

Y goes up **->** *p(*x^+*)* goes down. μ_{x^+}=∞

p(x^-*)* goes up. μ_{x^-} goes up

Y goes down **->** *p(*x^+*)* goes up. μ_{x^+}=∞

μ_{x^-} goes down, $p(x^-)$ goes down.

As Y goes to infinity, $p(x^-)$ goes to 1.

Higher Y thus leads to higher μ_{x^-}, higher $p(x^-)$ and lower $p(x^+)$. Lower Y leads to lower μ_{x^-}, lower $p(x^-)$ and higher $p(x^+)$.

Every rational player will be willing to pay as low a price Y for entering the game as possible. The more risk tolerant the player is, the higher the price he/she will be willing to accept to pay to enter the game. The more a player pays to enter the game, the lower is the probability of making a profit and the higher the possible monetary loss and the probability of loss.

Since different people would be willing to pay different amounts Y to enter the game, people have different preferences regarding the four parameters. This explains why some people would be willing to pay only three dollars to enter the game and why some would be willing to pay as much as 100 dollars. An investor would be more risk tolerant if he/she was willing to risk a higher loss or have a higher probability of losing for the same amount of upside profit than another investor.

3.2 THE ALLAIS PARADOX

The Allais Paradox is a paradox showing that people are in fact acting in contrast to the expected utility hypothesis. The current test is taken from Kahneman and Tversky (1979). The problem is to choose a preferred gamble from both of the following pairs of gambles. People tend to choose gamble B in the first case and gamble C in the second case, which is inconsistent with the expected utility theory (Allais, 1953).

N is the number of people who answered the problems. The numbers in brackets are the percentage of people who choose each option.

Problem 1, choose between:

A: 2500 with probability 33%	B: 2400 with probability 100%
2400 with probability 66%	
0 with probability 1%	
N=72 [18]	[82]

Problem 2, choose between:

C: 2500 with probability 33%	D: 2400 with probability 34%
0 with probability 67%	0 with probability 66%;
N=72 [83]	[17]

The 61% of participants made the modal choice in both problems.

Analysis:

Problem 1 parameter mix:

A: μ_{a^+}=2500*0,33+2400*0,66=2409 $p(x_a^+)$=0,33+0,66=99%

B: μ_{b^+}=2400 $p(x_b^+)$=100%

First, we can notice that people have to choose between parameter μ^+ and $p(x^+)$ in this example, since μ^- and $p(x^-)$ are absent in this example.

We can observe that the majority chose alternative B. However, 18% choose alternative A, because neither A nor B are dominating each other by all parameters. The only reason why 18% choose A is that it dominates alternative B by parameter $\mu_{a^+}>\mu_{b^+}$. The only reason why 82% choose the parameter B is that it dominates A by parameter $p(x_b^+)>p(x_a^+)$. People acted in accordance with their preferences to the parameter mix of the alternatives A and B.

Problem 2 parameter mix:

C: μ_{c^+}=2500, $p(x_c^+)$=33%

D: μ_{d^+}=2400; $p(x_d^+)$=34%

Again, neither C nor D is totally dominating: C is dominating by parameter $\mu_{c^+}>\mu_{d^+}$, while D is dominating by parameter $p(x_d^+)>p(x_c^+)$. Clearly the majority, 83%, choose the C because they prefer the parameter mix C to parameter mix D, while 17% choose D because they prefer parameter mix D to parameter mix C.

3.3 THE ELLSBERG PARADOX

The following is an example illustrating the Ellsberg Paradox:

We have an urn that contains 30 red balls and 60 balls that are either yellow or black. The total number of balls is 90. We do not know how many balls are black or yellow. People tend to choose gamble A in the first case and gamble D in the second case, which is inconsistent with expected utility theory (Ellsberg, 1961).

Now we have a choice between 2 gambles:

A: receive 100 dollars if a red ball is drawn

B: receive 100 dollars if a black ball is drawn

And we have also a choice between the 2 gambles:

C: receive 100 dollars if a red ball or a yellow ball is drawn

D: receive 100 dollars if a black ball or a yellow ball is drawn

Parameters:

A: μ_{a^+}=100, $p(x_a^+)$=30/90

B: μ_{b^+}=100, 0≤ $p(x_b^+)$≤60/90

C: μ_{c^+}=100, 30/90≤ $p(x_c^+)$≤90/90

D: μ_{d^+}=100, $p(x_d^+)$=60/90

Alternative A gives 100 with probability 30/90 since we know that the total number of balls is 90 and the number of red balls is 30. Alternative B gives 100 with a probability ranging from 0 to 60/90 depending on how many balls are black, since the total number of black and yellow balls is 60 and the possible number of black balls can be any number from 0 to 60.

Clearly, the only reason why people choose the alternative A over B is that there is a possibility that the number of black balls is lower than 30, and in that case, alternative A will dominate alternative B, since $p(x_a^+)$> $p(x_b^+)$, while the only reason people choose alternative B over A is that there is a possibility that the number of black balls is higher than 30, and in that case, alternative B would dominate alternative A $p(x_a^+)$< $p(x_b^+)$.

Alternative C gives 100 with a probability ranging from 30/90 to 90/90 depending on how many yellow balls are in the urn. Alternative D gives 100 with probability 60/90 since we know that the total number of black and yellow balls is 60.

Clearly, the only reason why people choose the alternative C over D is that there is a possibility that the number of black balls is higher than 30, and in that case, alternative C will dominate alternative D since $p(x_c^+)$> $p(x_d^+)$, while the only reason why people choose alternative D over C is that there is a possibility that the number of black balls are lower than 30, and in that case alternative D would dominate alternative C since $p(x_c^+)$< $p(x_d^+)$. People who choose the alternatives A and D are sure not to get a probability of profit lower than a certain level.

4 EMPIRICAL TEST

4.1 METHOD

In order to test the model empirically, the investors' acceptable parameter mix must be observed. The next step is to find an asset that has a superior or acceptable parameter mix. In order to find the required mixes, optimally all available securities should be analyzed. But it is unnecessary to analyze all available securities to show how the model functions, this can be achieved with much less effort. For that reason, the OMX 30 Stockholm Index was chosen.

The OMX 30 Stockholm Index is a market value weighted index consisting of the 30 most traded stocks on the Stockholm Stock Exchange. The index composition is public. The Risk-Reward Model is applied to analyze the OMX 30 Stockholm Index stocks to find a superior parameter mix than the OMX 30 parameter mix. It is important to know what stocks constitute the portfolio that we try to beat, because if we have at least the assets that constitute the portfolio, then in theory we can be sure to get an equal or superior parameter mix as the one that we want to beat, otherwise we may not find a parameter mix that is superior or even equal to the one we search for. There are a number of funds that track the OMX 30 Stockholm Index. People who invested in the index for a day during the sample period accepted the OMX 30 Stockholm Index parameter mix.

I use the daily returns of OMX30 Index from 2009-07-01 to 2009-12-30 to compute the OMX30 Index parameter mix: μ_{omx^+}, $p(x^+_{omx})$, μ_{omx^+}, and $p(x^-_{omx})$, during that period. OMX 30 Stockholm Index is rebalanced every six months. The period was chosen because it is the latest complete period. By construction, the model will work regardless of the chosen period or the returns' distribution.

I apply the portfolio optimization algorithms (14), (15), (16), (17) where the acceptable parameter mix is the OMX 30 Index mix, to build four optimal portfolios with 30 stocks included in the OMX30 index: MinLoss1 portfolio, minimizing downside loss; MaxProfit2 portfolio, maximizing upside profit; MaxP(Profit)3 portfolio, maximizing probability of upside profit; and MinP(Loss)4 portfolio, minimizing probability of downside loss. I also built a MV portfolio using algorithm (18) where the expected return is the OMX30 Index expected return, to compare the Risk-Reward and Mean-Variance Models. The risk-reward portfolios as well as the mean-variance portfolio are calculated using the Microsoft Excel solver function. The data is obtained from NASDAQ Nordic website.

4.2 RESULTS

The first observation that can be made from Table 1 is that the stocks in the sample that have high upside profit also have high downside loss, and stocks that have low upside profit also have low downside loss. The average probability of getting an upside profit for the stocks in the sample is around 50%, and the average probability of getting a downside loss is around 46%. Some stocks had zero returns, which is why the probabilities to get a profit and loss do not sum up to one on average. However, no portfolio had a return of zero. The highest upside profit an investor can get is 2,778% with probability 53%. At the same time he/she will get a downside loss of 2,526% with probability 43%. One could attain that parameter mix by investing in the SWED A. The lowest downside loss the investor can get in this sample is 0,816% with probability 52% and the upside profit the investor gets in that case is 0,936% with probability of 45%. One could attain that parameter mix by investing in the AZN. No individual stock had a parameter mix that totally dominated the OMX30 Stockholm Index, which demonstrates the effect of diversification.

Table 1. Sample statistics of the stocks composing OMX30 Stockholm Index.

	Mean	Median	Std. Dev.	Upside profit	Downside loss	P(profit)	P(loss)	Min	Max	Skew.	Kurt.	Range	Nr.
ABB	0,10%	0,04%	1,47%	1,259%	1,132%	0,50	0,47	-3,11%	4,01%	0,30	-0,11	7,12%	128
ASSA B	0,22%	0,17%	1,71%	1,465%	1,207%	0,52	0,45	-5,22%	4,85%	0,07	0,58	10,07%	128
ALFA	0,23%	0,27%	1,68%	1,351%	1,464%	0,59	0,38	-4,56%	4,92%	-0,12	0,19	9,49%	128
ATCO A	0,24%	0,15%	2,01%	1,777%	1,463%	0,52	0,46	-5,05%	7,10%	0,31	0,46	12,14%	128
ATCO B	0,24%	0,24%	2,19%	1,867%	1,760%	0,53	0,43	-4,56%	7,55%	0,35	0,19	12,12%	128
AZN	0,00%	-0,06%	1,10%	0,936%	0,816%	0,45	0,52	-2,56%	4,09%	0,72	1,74	6,65%	128
BOL	0,37%	0,11%	2,86%	2,450%	1,851%	0,50	0,46	-6,17%	14,45%	1,06	4,05	20,63%	128
ELUX B	0,35%	0,22%	2,38%	2,158%	1,647%	0,51	0,45	-5,84%	9,21%	0,63	1,08	15,05%	128
ERIC B	-0,13%	0,00%	1,66%	1,088%	1,434%	0,49	0,46	-7,74%	3,75%	-0,97	3,81	11,49%	128
GETI B	0,25%	0,13%	1,67%	1,489%	1,200%	0,52	0,43	-4,13%	6,84%	0,40	1,23	10,97%	128
HM B	0,02%	-0,10%	1,36%	1,133%	1,028%	0,48	0,52	-4,28%	3,41%	-0,11	0,46	7,69%	128
INVE B	0,08%	0,11%	1,34%	1,134%	1,049%	0,51	0,47	-2,69%	4,24%	0,42	0,32	6,93%	128
LUPE	-0,04%	-0,08%	2,11%	1,727%	1,668%	0,47	0,51	-5,27%	5,03%	0,07	-0,13	10,29%	128
MTG B	0,36%	0,24%	2,19%	1,981%	1,566%	0,53	0,45	-5,38%	6,63%	0,15	0,22	12,01%	128
NDA	0,15%	0,17%	2,13%	1,732%	1,740%	0,52	0,44	-5,84%	6,16%	0,07	0,19	12,00%	128
NOKI	-0,14%	0,03%	2,26%	1,282%	1,612%	0,50	0,48	-13,65%	5,31%	-2,45	13,60	18,96%	128
SAND	0,33%	0,30%	2,20%	1,986%	1,726%	0,54	0,43	-3,72%	7,05%	0,41	-0,19	10,77%	128
SCA B	0,11%	-0,03%	1,42%	1,357%	0,980%	0,45	0,50	-2,99%	3,99%	0,35	-0,01	6,98%	128
SCV B	0,16%	0,00%	2,33%	1,923%	1,776%	0,49	0,45	-5,87%	7,56%	0,58	1,03	13,43%	128
SEB A	0,23%	-0,07%	2,68%	2,342%	1,801%	0,48	0,50	-7,29%	7,87%	0,38	0,91	15,16%	128
SECU B	0,04%	-0,07%	1,30%	1,209%	0,942%	0,44	0,52	-3,03%	3,82%	0,43	0,29	6,84%	128
SHB A	0,25%	0,25%	1,95%	1,724%	1,484%	0,53	0,45	-4,30%	4,78%	0,19	-0,41	9,08%	128
SKF B	0,21%	0,00%	1,81%	1,764%	1,270%	0,46	0,48	-3,97%	6,44%	0,61	0,36	10,42%	128
SWED A	0,39%	0,34%	3,46%	2,778%	2,526%	0,53	0,43	-14,51%	12,60%	-0,09	3,22	27,10%	128
SWMA	0,18%	0,18%	1,20%	0,978%	0,922%	0,56	0,41	-2,39%	4,38%	0,51	1,06	6,77%	128
TEL2 B	0,27%	0,19%	1,89%	1,550%	1,378%	0,55	0,43	-3,77%	8,82%	0,81	2,52	12,58%	128
TLSN	0,18%	0,22%	1,60%	1,249%	1,137%	0,54	0,43	-5,06%	7,48%	0,88	4,20	12,55%	128
VOLV B	0,20%	0,00%	2,36%	2,289%	1,735%	0,46	0,49	-3,95%	7,95%	0,59	-0,05	11,90%	128
SSAB A	0,24%	-0,17%	2,55%	2,328%	1,646%	0,47	0,52	-6,38%	8,99%	0,64	1,20	15,38%	128
SKA B	0,28%	0,16%	1,47%	1,322%	1,032%	0,54	0,42	-2,71%	4,79%	0,53	0,63	7,49%	128

The mean is an arithmetic average return. P(profit) is the probability of getting an upside profit, calculated using equation 1. P(loss) is the probability of getting a downside loss, calculated using equation 2. Upside profit is calculated using equation 4. Downside loss is calculated using equation 5.

Table 2. Sample statistics of the OMX30 Index, the four optimal portfolios and the MV portfolio.

Portfolio	OMX	MinLoss1	MaxProfit2	MinP(Loss)3	MaxP(Profit)4	MVP
Mean (%)	0,131	0,171	0,254	0,177	0,177	0,131
Median (%)	0,080	0,114	0,198	0,157	0,156	0,125
Std. Dev. (%)	1,301	1,280	1,401	1,316	1,315	0,783
Mean profit (%)	1,096	1,121	1,300	1,158	1,157	0,641
Mean loss (%)	0,998	0,940	0,969	0,970	0,970	0,591
P(profit) (%)	53,906	53,906	53,906	53,906	53,906	58,594
P(loss) (%)	46,094	46,094	46,094	46,094	46,094	41,406
Min (%)	-3,267	-3,151	-3,435	-3,228	-3,228	-2,095
Max (%)	3,353	3,858	4,258	3,939	3,939	1,932
Skewness	0,037	0,143	0,083	0,155	0,155	0,053
Kurtosis	-0,381	-0,331	-0,347	-0,343	-0,343	0,100
Range (%)	6,620	7,008	7,692	7,167	7,167	4,027
Number	128	128	128	128	128	128

The mean is an arithmetic average return. P(profit) is the probability of getting an upside profit, calculated using equation 9. P(loss) is the probability of getting a downside loss, calculated using equation 10. Upside profit is calculated using equation 12. Downside loss is calculated using equation 13.

Table 3. The four optimal portfolios, the OMX30 Stockholm Index and the MV portfolio stocks weights.

Company	OMX	MinLoss1	MaxProfit2	MinP(Loss)3	MaxP(Profit)4	MVP
ABB	2,97%	7,47%	0,00%	3,45%	3,44%	0,00%
ASSA B	1,87%	3,39%	9,03%	3,44%	3,43%	0,67%
ALFA	1,56%	3,35%	4,17%	3,42%	3,41%	0,00%
ATCO A	3,26%	3,07%	2,69%	3,31%	3,30%	0,00%
ATCO B	1,37%	2,96%	1,94%	3,26%	3,26%	0,00%
AZN	4,54%	3,83%	2,84%	3,61%	3,60%	34,32%
BOL	0,80%	2,29%	12,05%	3,24%	3,24%	0,00%
ELUX B	1,62%	3,20%	5,28%	3,36%	3,36%	0,00%
ERIC B	11,45%	3,39%	0,00%	3,41%	3,57%	0,00%
GETI B	1,14%	3,47%	5,12%	3,47%	3,46%	8,95%
HM B	14,13%	3,37%	0,73%	3,41%	3,41%	0,00%
INVE B	2,72%	3,27%	1,23%	3,38%	3,37%	0,00%
LUPE	0,95%	3,04%	0,00%	3,28%	3,28%	0,00%
MTG B	0,56%	3,32%	5,99%	3,40%	3,39%	0,60%
NDA SEK	12,39%	3,08%	1,29%	3,30%	3,30%	0,00%
NOKI SEK	0,26%	3,28%	0,00%	3,37%	3,36%	1,12%
SAND	3,41%	2,98%	3,60%	3,27%	3,26%	0,00%
SCA B	2,44%	3,48%	3,02%	3,48%	3,47%	7,11%
SCV B	1,54%	3,01%	1,12%	3,28%	3,27%	0,00%
SEB A	3,70%	2,98%	2,31%	3,26%	3,25%	0,00%
SECU B	1,14%	3,53%	2,02%	3,49%	3,48%	0,00%
SHB A	4,48%	3,22%	3,88%	3,37%	3,36%	0,00%
SKF B	1,95%	3,17%	2,68%	3,34%	3,34%	0,00%
SWED A	1,16%	3,14%	5,42%	3,30%	3,30%	0,00%
SWMA	1,58%	3,84%	6,44%	3,62%	3,61%	30,40%
TEL2 B	1,57%	3,47%	5,37%	3,47%	3,46%	1,60%
TLSN	9,14%	3,44%	3,84%	3,45%	3,45%	0,24%
VOLV B	3,47%	3,00%	1,74%	3,26%	3,25%	0,00%
SSAB A	1,09%	1,51%	0,64%	1,83%	1,86%	0,00%
SKA B	1,73%	3,47%	5,53%	3,47%	3,46%	15,00%

MinLoss1 Portfolio is calculated using algorithm 14. MaxProfit2 is calculated using algorithm 15. MaxP(Profit)3 Portfolio is calculated using algorithm 16. MinP(Loss)4 Portfolio is calculated using algorithm 17. The constraints for the algorithms 14, 15, 16, 17 are the parameter mix of the OMX30 Index. The MV Portfolio is calculated using algorithm 18 where the expected return is OMX30 Index expected return.

All four optimal portfolios are totally dominating the OMX30 Stockholm Index by their parameter mix in the sample period since all portfolios give equal probabilities of upside profit and downside loss as the OMX30 Stockholm Index. However, the upside profit is higher and the downside loss is lower.

MinLoss1 Portfolio: the weights are fairly equally allocated around 3% in all stocks in the OMX30 Stockholm Index portfolio; only ABB gets a slightly higher weight of 7,47%. The downside loss is 0,94%, which is the lowest among all the portfolios. Furthermore, the upside profit is also the lowest among the optimized portfolios. The result is reasonable, since this portfolio is the one where the downside loss is minimized. The portfolio's average return is 0,17% and its standard deviation is 1,28%.

MaxProfit2 Portfolio: the four stocks are dropped: ABB, ERIC B, LUPE, NOKI SEK. Some stocks got higher allocation, such as ASSA B 9,03%, BOL 12,05%. The downside loss is 0,97% and the upside profit is 1,3%, the highest among the optimized portfolios. The portfolio's average return is 0,25% and standard deviation is 1,4%, which is the highest among all portfolios. According to the Mean-Variance portfolio theory **MaxProfit2** Portfolio carries more risk than the OMX30 Stockholm Index, but according to the Risk-Reward Model it does not carry more risk. The probability of getting an upside profit or downside loss in any given day is equal to OMX30 Stockholm Index, but the upside profit is higher and the downside loss is lower compared to the OMX30 Stockholm Index.

Portfolios **MinP(Loss)3** and **MaxP(Profit)4** are very similar, the weights allocated to all assets are all around 3%, the SSAB A has got a slightly lower weight of 1,83%. The downside loss is 0,97% and the upside profit is 1,16% in both cases. The portfolios **MinP(Loss)3** and **MaxP(Profit)4** were built to minimize the probability of downside loss and to maximize the probability of upside profit, but for both portfolios the probability of getting a profit or loss is equal to the corresponding parameters of the OMX30 Stockholm Index portfolio. This may be because given the constraints and the available securities, it is impossible to find a better portfolio with regards to the parameter mix.

MV Portfolio: around 80 % of the weight is allocated to three stocks AZN 34,32%, SWMA 30,40% and SKA B 15,00%, the majority of the remaining stocks is dropped. The portfolios' downside loss is 0,591% and the upside profit is 0,641%. The probability of downside loss is 41,406% and the probability of upside profit is 58,594%. The mean is 0,131% which is equal to the OMX30 Stockholm Index, the standard deviation is 0,783% which is lower than the standard deviation of the OMX Index.

A portfolio that has the same mean as the OMX30 Stockholm Index but a lower variance is, according to Mean-Variance Model, a superior portfolio. Mean-variance investors want $\bar{x}_{\mathrm{omx}}=\bar{x}_{\mathrm{mvp}}$ and $\sigma_{\mathrm{omx}}\geq\sigma_{\mathrm{mvp}}$ and the MV Portfolio is such a portfolio. However, if we get a portfolio that is dominating the OMX30 Stockholm Index by some parameters and is in turn dominated by the OMX30 Stockholm Index with regards to other parameters, the portfolio is not superior according to the Risk-Reward Model. It is simply a different portfolio with regards to risk and reward.

If the investor invests in the MV portfolio he/she will get a higher chance to get a profit, a lower chance to get a loss and a lower downside loss. However, the upside profit is also lower than the Index. If the investor is willing to risk losing more with the higher probability of losing in order to have the possibility to earn more, then the mean-variance portfolio is clearly not superior for the investor, because it will not give the investor what he/she may want.

The optimization process took only a few seconds using the standard Microsoft Excel spreadsheet program.

The results show that the investor, who invested in the optimized portfolios during the sample period for only one day, had an equal chance to make a profit and an equal chance to make a loss compared to the OMX30 Index investor. However, he/she could expect to get a higher upside profit if he/she got a profit and lower downside loss if he/she got a loss. If the investor would invest for one day over and over again, he/she would in the long term make a higher average profit. The optimized portfolio is thus dominating OMX30 Stockholm Index, in the short term and also in the long term. The investor should therefore choose the optimized portfolio.

I only used the same stocks that are included in the OMX30 Stockholm Index. However, if I intuitively used more stocks to try to beat the OMX30 Index, I could possibly get even better results. In an ideal scenario, all available assets should be analyzed while searching for the dominating portfolio.

An investor who makes a one period investment decision using mean-variance, mean downside variance or in fact any method for asset allocation, will get a portfolio with an acceptable distribution of payoffs. The parameter mix; $p(x^+)$, μ_{x^+}, $p(x^-)$, μ_{x^-} of all distributions can be estimated and all distributions can be compared to one another. That is why the optimal investment decision for the mean-variance or any other investor can be made using the Risk-Reward Model.

5 CONCLUSIONS AND FURTHER RESEARCH

Any situation of asset allocation under risk and uncertainty is a situation of choice between return distributions. Any distribution can be broken down into a parameter mix consisting of: upside profit, the estimated profit; probability of getting an upside profit; downside loss, the estimated loss and probability of getting a downside loss. Since any situation can be broken down into the parameter mix, all situations are comparable with regards to the parameter mix. The investor's choice of the risky asset can be observed. Since the choice is observable, the method of asset allocation is to find a better parameter mix for the investor, given the chosen parameter mix. If the better parameter mix is found, the investor should choose the better allocation. If the better parameter mix is not found, the investor should invest in the one he/she has chosen. If the chosen mix is not found, the investor should not invest in the risky asset.

In the three analyzed paradoxes, people make different decisions with regards to the parameter mix. In the St. Petersburg Paradox, some people are willing to accept a higher downside loss and a higher probability of loss in order to have the possibility to get the infinite upside profit, but most people do not accept such a parameter mix and are willing to accept a very low downside loss and a low probability of loss for the same infinite upside profit. In the Allais Paradox, some people choose the parameter mix with higher upside profit, while others choose the mix with higher probability of profit. In the Ellsberg Paradox, most people are not willing to accept a parameter mix, which can have a probability of upside profit lower than a certain level. The Risk-Reward Model successfully solves these paradoxes.

In all situations under risk and uncertainty, people always show the following preferences: keeping everything else equal, people prefer higher profit to lower profit, lower loss to higher loss, higher probability of profit to lower probability of profit, and lower probability of loss to higher probability of loss.

Since the investors´ parameter mix choice can be observed, people who invested in the OMX30 Stockholm Index accepted the index parameter mix. The Risk-Reward Model was used to find the portfolios that dominate OMX30 Index in terms of the parameter mix, and the portfolios were found in a sample which dominated both by upside profit and downside loss, while having the same probability of profit and probability of loss. The MV portfolio is dominating the OMX30

Stockholm Index by some parameters, but it is in turn dominated by the OMX30 Stockholm Index with regards to one parameter. The OMX30 investors should therefore choose the optimized portfolios, but they may not choose the MV portfolio because it will not give the investor what he/she may want.

Further research can be done to evaluate the Capital Asset Pricing Model and other financial models with regards to the parameter mix. Puzzles like the equity premium puzzle can also be analyzed using the Risk-Reward Model.

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