# An Alternative Four-Factor Model

# Abstract

In this paper, we add a liquidity factor to the Chen, Novy-Marx & Zhang (2010) three-factor model, creating an alternative four-factor model. From empirical tests we conclude that the liquidity factor is priced, and that the alternative model is overall better than the Carhart (1997) four-factor model at explaining anomalies, especially standardized unexpected earnings (SUE), financial distress and total accruals.

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# 1 Introduction

Within the field of finance, explaining cross-sectional returns has over the years been the purpose of several studies. Ever since its introduction by Sharpe (1964), the Capital Asset Pricing Model (CAPM) has served as a foundation for most subsequent papers. Numerous factors have been proposed as complements and/or alternatives to the original model, arguing that it (alone) fails to explain cross-sectional returns. The Fama & French (1992) three-factor model has since its introduction often been used a benchmark model, particularly after extended to a four-factor model by Carhart (1997).

The purpose of this paper is to present and motivate an alternative four-factor model, and to some extent show that the model outperforms previous models, in particular the Carhart (1997) four-factor model, when it comes to explaining anomalies found in the literature. We start from the Chen, Novy-Marx, & Zhang (2010) three-factor model, which includes the market factor from the CAPM model, an investment factor, and a profitability factor. While the market factor should capture returns from the consumption side of the economy, the intuition behind the other two factors is that low investments and high expected earnings, respectively, should indicate high future returns. However, none of these factors take effects of asymmetric information and market liquidity into account that arise when firms finance investments with both debt and equity instead of what is assumed in the Chen, Novy-Marx, & Zhang (2010) model – all-equity financing.

Different liquidity measures have previously been found to capture asymmetric information problems and should also capture effects from market liquidity. The fourfactor model introduced in this paper therefore includes a market, an investment, a profitability, and a liquidity factor.

In order to test the relevance of adding the liquidity factor to the model, we perform three tests. First, we sort our sample in deciles based on the individual stock's liquidity measure. In Goyenko, Holden, & Trzcinka (2009), the Amihud (2002) measure is found to be among the best price impact measures. Our results indicate that liquidity is priced since illiquid deciles show higher returns than liquid deciles. Next, we regress decile portfolio returns on the CAPM as well as the Chen, Novy-Marx, & Zhang (2010) threefactor model. Alphas are found to increase with illiquidity, both for the CAPM and the three-factor model. This gives support to the hypothesis that the liquidity factor is priced, and relevant in the four-factor model.

Second, we estimate adjusted returns following Daniel, Grinblatt, Titman, & Wermers (1997). For each stock, we control for its exposure to size, investment and profitability and find that a large portion of the liquidity spread disappears. Moreover, the test is sensitive to use of liquidity proxy as well as sample period.

Third, we run Fama-Macbeth regressions. Results indicate that liquidity is priced in the cross-section but that the investment factor could be redundant. The conclusion from the three tests is that the liquidity factor is relevant in the alternative four-factor model.

When testing the new model's ability to explain anomalies, results are mixed. The major finding, however, is that the alternative four-factor model can fully explain the momentum anomaly, considering the loadings on the profitability and liquidity factors, which both increase with momentum.

Loadings on the liquidity factor are also related to the size of firms; small firms have higher loadings than large firms. Significant contributions of the liquidity factor are also found for the standardized unexpected earnings (SUE) and failure probability anomalies. Results for the SUE anomaly are, however, not as strong as expected given earlier findings in the literature. There are also indications of that part of the abnormal returns from firms with high total accruals are explained by liquidity.

The book-to-market, net stock issues, and asset growth anomalies are likely not related to liquidity. Loadings on the investment factor increase with firms' book-to-market ratio, net stock issues as well as overall asset growth rate, which should help explain the abnormal returns. Mean returns from the factor are, however, unexpectedly low and even negative in the 1998-2010 sample period. Nonetheless, when testing the anomalies, the overall impression is that performance of the alternative four-factor model is better than the Carhart (1997) four-factor model.

The paper proceeds as follows. In Section 2 we develop our hypothesis, in Section 3 we describe the data used, in Section 4 we discuss properties of the new liquidity factor, in Section 5 we show empirical findings related to the three-factor model, in Section 6 we incorporate the new liquidity factor in the three-factor model and test its relevance, in Section 7 we test the alternative four-factor model's ability to explain a number of anomalies found in the literature, in Section 8 we compare the four-factor model to the Carhart (1997) four-factor model, before concluding in Section 9.

# 2 Hypothesis Development

In Chen, Novy-Marx, & Zhang (2010), the basic intuition behind their investment based three-factor model is that the discounted marginal benefit of investment should equal the marginal cost of investment. Thus, cross-sectional returns should be explained by firms' investment behaviour and expected profitability (ROA).

# 2.1 Liquidity and Financing Choice

#### 2.1.1 Asymmetric Information

One of the assumptions in the Chen, Novy-Marx, & Zhang (2010) model is that firms are all-equity financed. In reality, however, firms also finance investments with debt. Arguably, the choice of financing will in such case depend on the fraction of informed traders in the market. This intuition is based on Myers & Majluf (1984) where it is shown that firms' choice of financing depends on the information asymmetry between the firm and investors. Their model shows that firms tend to rely on internal financing, and to prefer debt to equity. Hence, firms subject to less asymmetric information problems should find it easier and less costly to obtain equity financing. Consequently, the required return on equity should decrease with the amount of informed investors.

When assuming all-equity financing, these asymmetric information problems would in fact be captured by the investment factor in the Chen, Novy-Marx, & Zhang (2010) model, since the discount rate would change and consequently investment. However, if the firm is allowed to finance investments with debt as well, investment would not necessarily change in order to reflect asymmetric information problems. Such problems should instead be captured by liquidity.

In accordance with Bagehot (1971), asymmetric information is arguably associated with liquidity. His theory is that since every market consists of both informed and uninformed investors, the un-informed investors will require a premium. This behavior is rational since the un-informed investors believe that when an informed investor wants to trade, he does so due to some negative/positive information about the asset.

Moreover, noise traders, who make the pricing system less informative, increase the asymmetric information problems. The reason is that informed investors will try to use noise traders as cover when slowly providing the market with their private information, making it hard for un-informed traders to interpret actions in the market place. When the fraction of informed investors is large, it is harder for them to use the cover. Thus, the more equity firms have in their capital structure, the less asymmetric information problems caused by noise traders there are.

In a model introduced by Grossman & Stiglitz (1980), there are both informed and uninformed investors where the fraction of informed investors will depend on the cost of being informed, the quality of information accessed by informed investors and the level of noise trading.

The market will be thin (low liquidity) when the portion of informed investors is almost unity or close to zero. As an example, when the amount of noise trading is low, the price system is very informative and a thin market is expected.

Thus, looking from an information-based trading perspective, cross-sectional stock returns should increase with the probability of information-based trading, reflecting costs of adverse selection due to asymmetric information (Easley, Hvidkjaer, & O'Hara (2002)). In the case of frequent trading and asymmetric information being associated with the asset, the liquidity risk is larger and a higher premium is required (Acharya & Pedersen (2005)). This premium is assumed to be captured by a liquidity factor.

#### 2.1.2 Stock Market Liquidity

The choice of financing should also be related to stock market liquidity. Lipson & Mortal (2009) find that firms tend to increase their capital with equity in times of high market liquidity. Plus, in Brunnermeier & Pedersen (2009), it is shown that investors' funding liquidity is closely related to stock market liquidity. At any point in time, an investor needs more capital than his margin requirement, or haircut (the difference between an asset's price and its collateral value when borrowed against). When accessing capital is hard, the investor is unlikely to invest in assets that require high margins. The consequence of such behavior, if collective among investors, is that market liquidity dries up. Consequently, assets in general require higher margins since financiers are imperfectly informed about the fundamental value of the assets, i.e. there is asymmetric information. From this, Brunnermeier & Pedersen (2009) thus conclude that 1) market liquidity can in fact suddenly dry up; 2) market liquidity has commonality among assets since all investors are likely to suffer from tight funding simultaneously; 3) market liquidity is related to volatility since margins increase with volatility; 4) investors, in times of crises, seek liquid assets (flight to liquidity). Hence, stocks that co-vary relatively more with

market liquidity and thus have a higher liquidity risk should carry a larger liquidity premium. This premium is assumed to be captured by a liquidity factor. Similar to the case of asymmetric information, the investment factor should also capture effects of stock market liquidity but, again, that assumes that firms are all-equity financed.

# 2.2 Concluding Comments

When debt financing is included in the Chen, Novy-Marx, & Zhang (2010) model, effects of asymmetric information (aggravated by noise traders) and stock market liquidity, resulting in an illiquidity premium, will need to be considered. These effects should be captured by a liquidity factor, but arguably not by the three factors in the Chen, Novy-Marx, & Zhang (2010) model.

# 2.3 Hypothesis

Besides the market, investment, and profitability factors from the Chen, Novy-Marx, & Zhang (2010) model, cross-sectional returns should be explained by a liquidity factor in order to capture effects of asymmetric information and stock market liquidity related to firms' financing choice.

# 3 Data Selection and Preparation

In this paper, data has been obtained from CRSP/Compustat. Data has been downloaded from January 1972 to December 2010, but due to lack of available data, all tests are not based on the entire sample period. The data consists of stocks listed at the American Stock Exchange (AMEX), New York Stock Exchange (NYSE), and the National Association of Securities Dealers Automated Quotation (NASDAQ).

Securities with negative book-to-market ratio as well as securities in financial and public utility industries (due to reasons pertaining to aberrant capital structures) have been excluded. For a list of mnemonics used from CRSP/Compustat, see Appendix Table 1. The mnemonics are also shown in square brackets when they first appear in the main text.

Data for the 25 portfolios formed on size and book-to-market as well as the size (SMB), book-to-market (HML) and momentum (WML) factors have been downloaded from Kenneth French's website<sup>1</sup>.

Furthermore, an intersectional approach has been used when constructing factors as well as portfolios. In particular, any firm for which there is insufficient data for one or more factors in the model for a given year, is entirely excluded from the model that year.

<sup>&</sup>lt;sup>1</sup> http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/, accessed 2011-04-29.

# 4 The Liquidity Factor

In the academic literature, among the first who described liquidity was Black (1971): "a liquid market is a continuous market, in the sense that almost any amount of stock can be bought or sold immediately; and an efficient market, in the sense that small amounts of stock can always be bought or sold very near the current market price, and in the sense that large amounts can be bought or sold over long periods of time at prices that, on average, are very near the current market price". A stock for which that is not the case should have a premium.

#### 4.1 Empirical Findings

Amihud & Mendelson (1986) were the first to confirm that average liquidity is priced, using the bid-ask spread as liquidity measure. Despite the results being questioned by Eleswarapu & Reinganum (1993), who argued that all explanatory power comes from the month of January, the paper has still been influential in the field of liquidity. The illiquidity premium has later been confirmed in a series of papers.

However, the magnitude of the illiquidity premium is still an open issue. Acharya & Pedersen (2005) use a liquidity-adjusted asset pricing model and find an annual premium of 4.6% while Pástor & Stambaugh (2003) use own liquidity betas and argue that the annual premium is close to 7.5%. The premium remains when controlling for the factors in Carhart's (1997) four-factor model. Significant liquidity premiums using intraday data and models developed by Foster & Viswanathan (1993) and Hasbrouck (1991) in combination and Glosten & Harris (1988) have been found by Brennan & Subrahmanyam (1996). Furthermore, Datar, Naik, & Radcliffe (1998) use a stock's turnover rate as alternative to the Amihud & Mendelson (1986) measure and find a significant illiquidity premium that also avoids the Eleswarapu & Reinganum (1993) critique.

#### 4.2 Liquidity Measures

In the literature, there are several liquidity measures. First of all, in Goyenko, Holden, & Trzcinka (2009), different measures used in the literature are evaluated and one conclusion is that commonly used liquidity proxies do measure liquidity. From this starting-point, they perform horse races between different liquidity measures. Before looking at the results from this analysis, it is important to remember that many of the

proxies are uncorrelated, which indicates that the choice of measure can be of great importance (Aitken & Comerton-Forde (2003)).

#### 4.2.1 Price Impact Measures

The Amihud (2002) measure is found to have a performance in line with the best price impact measures while still being easy to calculate. The Amihud (2002) measure is calculated as:

$$Illiq_{iy} = 1 \Big/ D_{iy} \sum_{t=1}^{D_{iy}} |R_{iyd}| / VOLD_{ivyd}$$

where  $D_{iy}$  is the number of days in year y where data is available,  $|R_{iyd}|$  is the absolute return of stock *i* on day *d* in year y and  $VOLD_{ivyd}$  is the dollar trading volume [PRC \* VOL] for stock *i* on day *d* in year y.

Another price impact measure is the Pástor & Stambaugh (2003) liquidity measure, with the following regression as starting point:

$$r_{t+1}^{e} = \theta + \phi r_t + \gamma sign(r_t^{e})(Volume_t) + \varepsilon_t$$

where  $r_t^e$  is the stock's excess return above the relevant market index on day t and *Volume*<sub>t</sub> is the dollar trading volume (in millions) on day t.

In the words of Pástor & Stambaugh (2003), the idea behind their measure is that "order flow', constructed here simply as volume signed by the contemporaneous return on the stock in excess of the market, should be accompanied by a return that one expects to be partially reversed in the future if the stock is not perfectly liquid. We assume that the greater the expected reversal for a given dollar volume, the lower the stock's liquidity."

Hence, due to the reversal theory, gamma is expected to have a negative sign. For a description of how the liquidity betas are calculated, see the Appendix, Section 11.3.

#### 4.2.2 Spread Measures

When controlling for computational difficulty, Goyenko, Holden, & Trzcinka (2009) find that their own measure, the Effective Tick, outperforms all other effective and realized spread measures, e.g. the bid-ask spread. Effective Tick is based on the idea of price clustering, which has been shown to be persistent over time. The most common explanation of this phenomenon is that it incurs lower negotiation costs between traders (Harris (1991)). The intuition behind why the measure should serve as a proxy for liquidity is first that small bid-ask spreads should indicate that assets are liquid. This is

Table 1 - Properties of the Liquidity Factor (Amihud)
Mean monthly returns in percent of the liquidity factor as well as alphas and betas from
regressions of the investment factor on the CAPM, the Fama & French (1992) three-factor
model and Carhart's (1997) four-factor model. T-statistics (in brackets) are adjusted for
heteroscedasticity and adjusted R-squared is used. The sample period is January 1972 -
December 2010. The Amihud (2002) measure has been used as proxy for liquidity.

	Mean	α	$\beta_{MKT}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{WML}$	$R^2$
$r_{LIQ}$	0.68	0.76	-0.14				0.01
	(2.43)	(2.69)	-(2.21)				
		0.48	-0.21	0.59	0.17		0.13
		(1.85)	-(2.99)	(3.74)	(1.37)		
		0.3	-0.16	0.56	0.24	0.21	0.16
		(1.08)	-(2.27)	(3.9)	(2.07)	(2.56)	

based on the arguments presented in Section 2.1. Second, increments that investors actually trade on are more informative than the bid-ask spread reported in the market. By combining these two assumptions, a stock should be regarded as liquid if relatively smaller increments are used when trading the stock. For a description on how to the Effective Tick is calculated, see the Appendix, Section 11.2.

# 4.3 Construction of a Liquidity Factor

In December in each year t - 1, the NYSE, Amex and NASDAQ stocks are sorted into three liquidity groups based on breakpoints for the low 30%, medium 40% and high 30%. Using the NYSE median as breakpoint, we split each group in two based on the firms' market capitalization [PRC \* CSHO]. Taking intersections, we form six liquidity portfolios and calculate value-weighted returns for each portfolio from January in year t to December in year t. The liquidity factor is constructed as the difference between the average returns of the low liquidity portfolios and the average returns from the high liquidity portfolios.

Following Amihud (2002), we exclude in all constructions of liquidity measures in this paper stocks that:

- 1. Have less than 200 return and volume observations in year t 1;
- 2. Stocks that have a year-end price in year t 1 of less than \$5;
- 3. Do not have data on market capitalization in year t 1;
- 4. Fulfill the requirements in 1-3 but are outliers, i.e. have liquidity measures below the 1<sup>st</sup> percentile or above the 99<sup>th</sup> percentile, in year t 1.

Table 2 - Different Sample Periods for the Liquidity Factor (Amihud)

Mean monthly returns in percent of the liquidity factor in three different sample periods (1972-1984, 1985-1997, and 1998-2010) are reported. T-statistics are reported in brackets. The Amihud (2002) measure has been used as proxy for liquidity.

	1972-2010	1972-1984	1985-1997	1998-2010
$r_{LIQ}$	0.68	0.76	-0.28	1.14
	(2.43)	(1.86)	(-0.63)	(2.24)

# 4.4 Descriptive Statistics of the Liquidity Factor

Our results indicate that the liquidity factor is priced, see Table 1. A monthly return of 0.68% (t = 2.43) is estimated for the sample period using the Amihud (2002) measure, which is higher than in most other papers. For example, it is about 0.05 percentage points higher than in Pástor & Stambaugh (2003) and 0.3 percentage points higher than in Amihud (2002). The results are, however, not directly comparable since different methods for calculating the premiums have been used. In any case, the premium is significant on the 5% significance level.

Alpha for the liquidity factor is significant also in a CAPM setting. When regressing on the Fama & French (1992) factors, alpha is 0.48 (t = 1.85) and insignificant. Not very surprisingly, the liquidity factor loads positively on the SMB factor, which confirms that part of the size effect can be explained by liquidity. In the four-factor model that also includes the momentum factor, alpha is still positive but now clearly insignificant (t = 1.08). As shown by Sadka (2006), momentum is to a large extent explained by liquidity why this result is expected. It can be noted that the HML factor is positive and significant in the four-factor model, which means that also the premium from holding value stocks (high book-to-market) could be related to liquidity.

When looking at different sample periods, the first conclusion is that the illiquidity premium varies over time. In fact, during the 1985-1997 period the premium is insignificant (albeit negative), see Table 2. Although, between 1985 and 1990, the liquidity factor reports missing values due to the intersectional approach. Returns in the other two sub-periods are on the other hand positive with p-values of 6.28 and 2.5 %, respectively. It is interesting to note that the largest illiquidity premium is in the most recent subsample, despite the market having access to a more developed technological infrastructure, which should mitigate adverse selection problems between investors. This result is to a large extent driven by the high premium around the year 2000.

January 1	972 – Decen	nber 2010. Tl	ne Effective '	Fick measure	has been used	d as proxy for	liquidity.
	Mean	α	$\beta_{MKT}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{WML}$	$R^2$
$r_{LIQ}$	0.06	-0.01	0.12				0.05
· ·	(.51)	(05)	(3.74)				
		0.18	0.02	0.06	-0.4		0.29
		(1.80)	(.87)	(1.11)	(-7.19)		
		0.18	0.02	0.06	-0.4	0.01	0.29
		(1.69)	(.95)	(1.13)	(-6.63)	(.14)	

**Table 3 - Properties of the Liquidity Factor (Effective Tick)** For a description of the coefficients reported in this table, see Table 1. The sample period is January 1972 – December 2010. The Effective Tick measure has been used as proxy for liquidity.

For the Effective Tick, the basic version of the measure only uses price as input data and is rather easy to compute. As can be seen in Table 3, the mean return of the liquidity factor when using the Effective Tick as proxy for liquidity is small and insignificant. Alpha is close to zero in a CAPM setting but increase to 0.18% in the three- and fourfactor models (t = -0.05, 1.80 and 1.69, respectively). Loadings on the SMB and HML factors are very different from when we use Amihud (2002) as liquidity proxy. The SMB factor is insignificant while the HML factor is negative and strongly significant, indicating that the value premium is compensation for liquidity rather than illiquidity.

### 4.5 Choice of Liquidity Proxy

In this paper, the Amihud (2002) measure will be used as liquidity proxy if not stated otherwise. The alternative would have been to use the Effective Tick measure, but this is not attractive for several reasons. First, the Effective Tick has not really been tested in the literature. It is true that Goyenko, Holden, & Trzcinka (2009) consider it to be the best spread measure but apart from that, there is to our knowledge no published literature that uses the measure. Second, Brunnermeier & Pedersen (2009) argue that spread measures can be noisy since large trades tend to happen outside the bid-ask spread while small trades happen inside the bid-ask spread. Third, the Amihud (2002) measure has been used in several empirical studies and is hence better for comparison reasons. The Effective Tick measure and the Pástor & Stambaugh (2003) measure will thus serve as robustness checks.

# 5 The Three-Factor Model

The economic intuitions and portfolio implications of the investment, profitability and market factors in the Chen, Novy-Marx, & Zhang (2010) model are explained in the original paper, limiting the need for a thorough review in this paper. However, important results from the literature as well as results that are important in our analysis are discussed below.

# 5.1 The Investment Factor

#### 5.1.1 Economic Intuition

From capital budgeting, it can be shown that firms that face high costs of capital are more likely to reduce investments since, ceteris paribus, they have less positive net present value projects to choose from. Low investments (relative to the asset base) should thus indicate high future returns (Liu, Whited, & Zhang (2009)).

A second argument is promoted by Carlson, Fisher, & Giammarino (2004). The authors argue that assets in place are less risky than various expansion options. In other words, high investments make the firm's assets less risky. Given that investors are risk-compensated, high investments should indicate lower future returns.

#### 5.1.2 Empirical Findings

In the literature, there is consensus regarding the high investments-low returns relationship. On one hand, it has been shown that corporate actions related to asset expansion are followed by low future returns. Examples of such corporate actions are acquisitions (Agrawal, Jaffe, & Mandelker (1992)), debt offerings (Spiess & Affleck-Graves (1999)), and share issuances (Pontiff & Woodgate (2008)).

On the other hand, corporate actions related to asset contraction are followed by high returns. Examples of such actions are divestments/spin-offs (Cusatis, Miles, & Woolridge (1993)), share repurchases (Ikenberry, Lakonishok, & Vermaelen (1995)), and calls of debt (Affleck-Graves & Miller (2003)).

In a more general setting, it has been shown that firms with low asset growth deliver substantially higher returns than other firms. The effect is not limited to the base year but persists for up to five years. The effect is also consistent over time and the relationship is found to be particularly strong for small firms (Cooper, Gulen, & Schill (2008)).

Low returns from high investments can also be explained by managers' empire building (Jensen (1986)). There is evidence that the relationship is mitigated in times of heavy company oversight, which supports the empire building theory. Also, firms that face high investment discretion (low debt or high cash flows) have a more pronounced relationship (Titman, Wei, & Xie (2004)).

In contrast, there could also be arguments to why high investments should indicate high future returns. For example, high investments could signal good investment opportunities as well as high confidence in current management from the capital markets. The risk is, however, that firms publicly announce only investments they believe will be looked at favorably, at times when the stock price is high and monitoring is low (Titman, Wei, & Xie (2004)). This could be the answer to why *announcements* of large investments are indeed looked at favorably by the stock market (Blose & Shieh (1997)).

#### 5.1.3 Construction of the Investment Factor

The investment (I/A) factor should reflect effects from both short- and long-term investments and be comparable between firms. In order to do this, investments, defined as the annual change in property, plant and equipment [PPEGT] and inventories [INVT], are divided by one-year-lagged total assets [AT].

In June in each year t, the NYSE, Amex and NASDAQ stocks are sorted into three I/A groups based on breakpoints for the low 30%, medium 40% and high 30% of I/A in year t - 1. Using the NYSE median as breakpoint, we split each group in two based on the firms' market capitalization. Taking intersections, we form six I/A portfolios and calculate value-weighted returns for each portfolio from July in year t to June in year t + 1. The investment factor is constructed as the difference between the average returns of the low investment ratio portfolios and the average returns from the high investment portfolios.

#### 5.1.4 Descriptive Statistics of the Investment Factor

The mean monthly return of the investment is positive, 0.09% (t = 0.90), although insignificant, see Table 4. This is different from the Chen, Novy-Marx, & Zhang (2010) paper where there is a significantly positive mean return, 0.28% (t = 3.21). The difference could be explained by the intersectional approach used in this paper where some stocks are excluded due to insufficient liquidity data (see Section 4.3). If we exclude the liquidity

#### Table 4 - Properties of the Investment Factor

For a description of the coefficients reported in this table, see Table 1. The sample period is January 1972 – December 2010.

	Mean	α	$\beta_{MKT}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{WML}$	$R^2$
$r_{INV}$	0.09	0.14	-0.08				0.03
	(.90)	(1.36)	-(3.54)				
		0.00	-0.03	0.03	0.26		0.16
		(.01)	-(1.33)	(.99)	(7.69)		
		0.02	-0.03	0.03	0.26	-0.02	0.16
		(.16)	-(1.5)	(1.03)	(7.4)	-(.73)	

factor from the intersection check, the mean return of the investment factor is 0.22% (t = 2.08). The remaining difference is likely due to somewhat different sample periods.

The question that arises is then why the investment factor goes from significant before the intersection check to insignificant after. Stocks that are excluded after the intersection check do not fulfill the criteria listed in Section 4.3, where missing volume data is the most probable cause. Thus, stocks that have missing volume data seem to drive returns of the investment factor. One possibility is that the data for some reason is not reported, but a more plausible reason is that the excluded stocks are very illiquid or at least trade infrequently. Whatever the reason, it is interesting to note that the factor is sensitive to sample selection.

In a CAPM setting, alpha is again positive and insignificant. In a Fama & French (1992) setting, alpha is very close to zero. The reason for this could be that the investment factor is more or less entirely explained by the HML factor. This is no surprise given that the investment and HML factors show the highest correlation of all factor pairs in this paper (see Table 10).

#### 5.2 The Profitability Factor

#### 5.2.1 Economic Motivation

That future profitability should indicate future returns is intuitive; firms that are expected to be relatively more profitable, ceteris paribus, should deliver higher returns. From discounting theory, it can be shown that high expected cash flows and low market equity has to be explained by high discount rates (Fama & French (2006)). From the Chen, Novy-Marx, & Zhang (2010) model, the profitability factor is theoretically not independent of the investment factor. However, empirically, correlation between the two is low (0.03), see Table 10.

#### 5.2.2 Empirical Findings

Profitability has been shown to have a positive relation to future returns, even though investors tend to underreact to cash flow news (Cohen, Gompers, & Vuolteenaho, (2002)). The relative importance of cash flow and expected return news has also been investigated. In fact, "information about future cash flows is the dominant factor driving firm-level stock returns" (Voulteenaho, (2002)). Hence, expected future cash flows should be a good indicator of cross-sectional returns.

#### 5.2.3 Construction of the Profitability Factor

In Fama & French (2000), current profitability is the best indicator of future profitability. The profitability factor is thus based on current profitability rather than forecasts of future profitability. Profitability is measured as return on assets (*ROA*), defined as current net income [IBQ] divided by one-quarter lagged total assets [ATQ].

In the beginning of each month t, the NYSE, Amex and NASDAQ stocks are sorted into three ROA groups based on breakpoints for the low 30%, medium 40% and high 30% of ROA in the current month t, using the most recently announced quarterly earnings. Quarterly earnings are used in portfolio sorts in the months immediately after the most recent public earnings announcement month<sup>2</sup> [RDQ]. Using the NYSE median as breakpoint, we split each group in two based on the firms' market capitalization. Taking intersections, we form six ROA portfolios and calculate value-weighted returns for each portfolio in the current month t. The profitability factor is constructed as the difference between the average returns of the high investment ratio portfolios and the average returns from the low investment portfolios.

#### 5.2.4 Descriptive Statistics of the Profitability Factor

The summary statistics from the profitability factor indicate that there is, as expected, a positive relationship between expected profitability and future returns. The mean monthly return of the factor is 0.43% (t = 2.90), see Table 5. This is lower when comparing to the Chen, Novy-Marx, & Zhang (2010) paper where a mean return of 0.76% (t = 3.84) is reported. Again, this difference could be explained by the intersectional approach, but also by different methods since all necessary assumptions regarding the factor construction are not reported in the original paper.

<sup>&</sup>lt;sup>2</sup> If there is no new announcement within three months for a particular firm, the ROA for that firm is considered missing from month t + 3 until there is a new announcement.

#### Table 5 - Properties of the Profitability Factor

For a description of the coefficients	reported in this table,	, see Table 1. The	sample period is
January 1972 – December 2010.			

	Mean	α	$\beta_{MKT}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{WML}$	<i>R</i> <sup>2</sup>
r <sub>ROA</sub>	0.43	0.51	-0.18				0.07
	(2.90)	(3.55)	-(4.36)				
		0.62	-0.15	-0.29	-0.11		0.16
		(4.46)	-(3.72)	-(4.95)	-(1.69)		
		0.43	-0.11	-0.29	-0.04	0.20	0.25
		(3.22)	-(2.89)	-(4.63)	-(.59)	(5.29)	

Alpha exceeds the mean return of the factor in both traditional CAPM and Fama & French (1992) settings. Loadings are significant for the market (MKT) and size (SMB) factors. When including the momentum factor, alpha is back at 0.43% (t = 3.22) due to the significantly positive loading on the factor. It can also be noted that  $R^2$  increases substantially when adding the momentum factor to the model.

#### 5.3 The Market Factor

More or less all asset-pricing models have its origin in the Sharpe (1964) and Lintner (1969) capital asset pricing model (CAPM). Over the years, the model has, however, been criticized and shown not to explain cross-sectional returns very well (e.g. Lewellen & Nagel (2006)). The market beta from the CAPM model is thus not enough to explain cross-sectional returns, but can play an important role in multi-factor models.

The investment and profitability factor both have their origin in the production side of the economy. In order to also explain effects that origin from the consumption side, the market factor should serve as a good proxy. Besides, the importance of the market factor should persist even after including a liquidity factor in the model (Acharya & Pedersen (2005)).

Returns from the S&P 500 [VWRETD] are used as a proxy for the market factor. The mean monthly return of the S&P 500 between January 1972 and December 2010 is 0.46% (t = 2.14).

#### 5.4 Alternative Sample Periods

From Table 6 it is clear that returns from the profitability and investment factors vary over time. The profitability factor is positive in all sample periods but only significant in the 1985-1997 period. The mean returns of the investment factor give support to the hypothesis that the somewhat lower mean return in this paper compared to the Chen,

Table 6 - Mean Returns of the Investment Factor and the Profitability Factor Mean monthly returns in percent of the profitability and investment factors in three different sample periods (1972-1984, 1985-1997, and 1998-2010) are reported. T-statistics are reported in brackets.

	1972-2010	1972-1984	1985-1997	1998-2010	
r <sub>ROA</sub>	0.43	0.33	0.63	0.31	
	(2.90)	(1.63)	(3.48)	(0.87)	
r <sub>INV</sub>	0.09	0.37	0.2	-0.25	
	(.90)	(2.13)	(1.48)	-(1.24)	

Novy-Marx, & Zhang (2010) paper is partly explained by different sample periods. The reason is that the investment factor, unexpectedly, has a negative mean return in the most recent subsample (1998-2010). For the other two periods, the mean return is positive but only significant in the first sub-period.

# 6 Adding Liquidity to the Three-Factor Model

#### 6.1 Empirical Tests

In order to confirm the liquidity factor's relevance in the four-factor model, we perform three tests. The tests have in common that they try to exclude effects from other factors than liquidity. Then, if the hypothesis is true, portfolios that include more illiquid stocks should have higher returns (an illiquidity premium) than portfolios containing more liquid stocks.

#### 6.1.1 Pástor & Stambaugh Alphas

In December in each year t - 1 we sort NYSE, Amex and NASDAQ stocks into deciles based on liquidity in year t - 1 and calculate value-weighted returns for each portfolio from January in year t to December in year t and regress the portfolio returns using the CAPM and the three-factor model with as well as without momentum. For robustness reasons, we not only use the Amihud (2002) measure, but also the Pástor & Stambaugh (2003) liquidity betas as well as the Effective Tick as proxies for liquidity.

When using the Amihud (2002) measure (Panel A in Table 7), alphas from the CAPM regressions increase more or less linearly with illiquidity. The difference in returns between liquid and illiquid deciles is substantial; the spread between decile 1 and 10 is more than 8% (t = 3.25) per year. The spreads persist when regressing on the (Chen, Novy-Marx, & Zhang (2010) factors with as well as without momentum. The interpretation of this test is that the liquidity factor should play an important role in explaining cross-sectional returns when added to the three-factor model.

When instead using Pástor & Stambaugh (2003) liquidity betas we obtain similar results. The trend is again that alphas increase more or less linearly with illiquidity. The major difference is that deciles 8 and 9 show unexpectedly low alphas in all regressions. This deviation could to some extent be driven by outliers. The difference between decile 1 and 10 is on the other hand large and significant, 8% (t = 2.93) per year with CAPM for example, no matter which factors that are included in the regression.

Results from deciles sorted on the Effective Tick are not as clear as for the Amihud (2002) and Pástor & Stambaugh (2003) liquidity betas sorted decile portfolios. The spread between the extreme portfolios persists but is now significantly negative. Results

are, however, only significant for three deciles and there is no clear trend between deciles.

The overall impression from the analysis of Pástor & Stambaugh (2003) alphas is that illiquid stocks have higher returns than liquid stocks, no matter if we include factors from the existing three-factor model or when we also add momentum. However, it should be noted that the returns of the decile portfolios do not increase linearly between deciles. Rather, the liquidity measures tend to increase/decrease exponentially for the most illiquid/liquid deciles and be quite stable for the middle deciles. The interpretation is thus that a (small) group of stocks are much more liquid than other stocks, presumably the most traded stocks (often a part of knowledgeable indices such as S&P 500). Meanwhile, another (small) group of stocks are seldom traded and because of this have relatively large returns. Most stocks are, however, in neither of these two groups.

# 6.1.2 Adjusted Returns

Another test to confirm the relevance of liquidity in the four-factor model is the Daniel, Grinblatt, Titman, & Wermers (1997) test. In their original setting, returns from individual stocks are adjusted for portfolio returns that are captured by the size (market capitalization), book-to-market and prior-year return (momentum) factors. However, instead of book-to-market and momentum, we use the investment and profitability factors to adjust returns. If our hypothesis is true, we should see an illiquidity premium even after returns have been adjusted and sorted into deciles based on liquidity.

More specifically, in the beginning of each month t, NYSE, Amex and NASDAQ stocks are sorted into quintiles based on size (market capitalization), investment (I/A), and profitability (ROA), respectively, using the most recently announced quarterly earnings.<sup>3</sup> Quarterly earnings are used in portfolio sorts in the months immediately after the most recent public earnings announcement month. Using the NYSE median as breakpoint, we split each group in two based on the firms' market capitalization. Taking intersections, we form 125 (5 x 5 x 5) portfolios and calculate value-weighted returns for each portfolio for the current month t.<sup>4</sup> The adjusted return for a particular stock in month t is the (raw) stock return in month t minus the return in month t of the benchmark portfolio for which the stock is associated. Thereafter, in December in each year t - 1 we sort

<sup>&</sup>lt;sup>3</sup> For definitions of investment and profitability, see Section 5.

<sup>&</sup>lt;sup>4</sup> For investment, we base the portfolio returns from July in year t to June in year t + 1 on the investment in year t - 1.

NYSE, Amex and NASDAQ stocks into deciles based on liquidity in year t - 1 and calculate value-weighted returns for each portfolio from January in year t to December in year t, using the adjusted returns for each stock. For robustness reasons, we not only use the Amihud (2002) measure, but also the Pástor & Stambaugh (2003) liquidity betas as well as the Effective Tick as proxies for liquidity.

As can be seen in Table 8, returns from the liquidity sorted deciles increase with illiquidity. The spread between decile portfolios 1 and 10 is substantial, 8.64% (t = 3.30) yearly (raw returns). When instead looking at adjusted returns, the spread is surprisingly much smaller and insignificant, 1.32% (t = 0.7) yearly and there is no clear difference between illiquid and liquid deciles. One explanation could be that each stock's exposure to the profitability and investment factors is directly controlled for in Section 6.1.1 while portfolio returns are deducted from the return of each individual stock in this section. The alternative liquidity proxies show a larger spread between decile portfolios 1 and 10 but these results are completely driven by the adjusted return for decile 10 and 1 for Pastor & Stambaugh (2003) and the Effective Tick, respectively. The spread for the Effective Tick is in fact still negative. All other deciles show similar results and no clear tendency can be seen. When separating the results into subsamples, results are mixed. For the first two subsamples (1972-1984 and 1985-1997), the spread between decile portfolio 1 and 10 is positive, 3.96% (t = 1.46) and 1.68% (t = 0.61) yearly, respectively. For the second period, all decile portfolio returns are significant except for decile portfolio 10 that supports an illiquidity premium. In the most recent subsample (1998-2010), the spread is in fact negative, -1.56% (t = -0.38) yearly. All in all, the spreads between decile 1 and 10 is insignificant for all sample periods.

#### Table 7 - Alphas from Regressions using the CAPM and the Chen, Novy-Marx& Zhang Model (With and Without Momentum)

The table reports alphas with t-statistics (in brackets, adjusted for heteroscedasticity) for liquidity sorted deciles regressed on the CAPM, the Chen, Novy-Marx, & Zhang (2010) three-factor model (with and without momentum). All results are in percentages per month and decile portfolio 1 is the most liquid. The sample period is January 1972 – December 2010.

					De	ecile Portfo	olio				
	1	2	3	4	5	6	7	8	9	10	10 - 1
					A. /	Amihud (20	002)				
CAPM Alpha	-0.04	0.16	0.13	0.19	0.17	0.26	0.37	0.42	0.60	0.66	0.70
	-(.6)	(1.94)	(1.4)	(1.69)	(1.47)	(2.04)	(2.45)	(2.54)	(3.48)	(3.64)	(3.25)
Chen et al. Alpha	-0.09	0.21	0.21	0.28	0.22	0.33	0.41	0.58	0.70	0.71	0.80
	-(1.48)	(2.52)	(2.05)	(2.32)	(1.92)	(2.47)	(2.55)	(3.24)	(3.79)	(3.8)	(3.66)
Chen et al. + Momentum Alpha	-0.08	0.20	0.20	0.25	0.24	0.30	0.37	0.51	0.65	0.68	0.75
	-(1.24)	(2.44)	(2.01)	(2.12)	(2.02)	(2.23)	(2.3)	(2.89)	(3.49)	(3.5)	(3.35)
		B. Pastor & Stambaugh Liquidity Betas									
CAPM Alpha	-0.31	-0.07	0.02	0.10	0.05	0.16	0.38	0.01	-0.02	0.39	0.70
	-(1.7)	-(.58)	(.19)	(.91)	(.52)	(1.34)	(3.69)	(.08)	-(.18)	(2.04)	(2.93)
Chen et al. Alpha	-0.22	-0.16	-0.09	-0.01	0.06	0.14	0.36	0.09	0.04	0.65	0.88
	-(1.15)	-(1.19)	-(.78)	-(.05)	(.56)	(1.18)	(3.33)	(.65)	(.29)	(3.43)	(3.7)
Chen et al. + Momentum Alpha	-0.28	-0.15	-0.10	0.01	0.07	0.18	0.38	0.10	0.09	0.62	0.90
	-(1.53)	-(1.15)	-(.84)	(.11)	(.64)	(1.49)	(3.53)	(.72)	(.68)	(3.3)	(3.79)
					C. Th	e Effective	e Tick				
CAPM Alpha	0.48	0.02	0.04	0.16	0.19	0.05	0.11	-0.03	-0.44	-0.01	-0.49
-	(2.92)	(.19)	(.38)	(1.52)	(2.04)	(.47)	(1.)	-(.22)	-(2.72)	-(.09)	-(2.3)
Chen et al. Alpha	0.47	-0.03	-0.01	0.04	0.13	-0.03	0.14	-0.02	-0.39	0.03	-0.44
	(2.83)	-(.27)	-(.06)	(.36)	(1.35)	-(.26)	(1.32)	-(.14)	-(2.36)	(.2)	-(2.05)
Chen et al. + Momentum Alpha	0.48	-0.02	0.01	0.05	0.13	-0.03	0.14	-0.04	-0.41	0.01	-0.47
	(2.91)	-(.19)	(.05)	(.45)	(1.44)	-(.24)	(1.32)	-(.3)	-(3.02)	(.11)	-(2.53)

#### Table 8 - Raw and Adjusted Returns Following Hou & Robinson (2006)

The Amihud (2002) measure is used as liquidity measure. T-statistics (in brackets) are adjusted for heteroscedasticity. Adjusted returns are reported for three different subsamples (1972-1984, 1985-1997, and 1998-2010). The Effective Tick and Pastor & Stambaugh measures are used as robustness checks. All results are in percentages per month and decile portfolio 1 is the most liquid.

					Dec	ile Portfol	ios				
	1	2	3	4	5	6	7	8	9	10	10 - 1
Raw returns 1972-2010	0.45	0.73	0.72	0.79	0.75	0.84	0.93	1.00	1.13	1.17	0.72
	-(2.1)	-(2.81)	-(2.66)	-(2.8)	-(2.7)	-(3.01)	-(3.23)	-(3.31)	-(3.93)	-(4.1)	(3.3)
Adjusted returns 1972-2010	-0.48	-0.42	-0.46	-0.37	-0.59	-0.39	-0.43	-0.46	-0.40	-0.37	0.11
	-(28.9)	-(8.04)	-(9.72)	-(7.22)	-(9.)	-(5.26)	-(5.95)	-(5.51)	-(3.81)	-(2.4)	(.7)
Adjusted returns, alternative sample periods											
1972-1984	-0.77	-0.60	-0.66	-0.63	-0.68	-0.56	-0.68	-0.74	-0.53	-0.43	0.33
	-(21.18)	-(5.89)	-(6.54)	-(7.37)	-(6.23)	-(4.85)	-(6.13)	-(5.90)	-(3.97)	-(1.92)	(1.46)
1985-1997	-0.45	-0.49	-0.48	-0.37	-0.68	-0.45	-0.34	-0.43	-0.65	-0.31	0.14
	-(31.22)	-(8.38)	-(7.49)	-(5.44)	-(6.85)	-(4.72)	-(3.19)	-(3.53)	-(4.25)	-(1.34)	(.61)
1998-2010	-0.27	-0.19	-0.28	-0.15	-0.40	-0.20	-0.33	-0.27	-0.02	-0.39	-0.13
Adjusted returns, alternative liquidity measures 19	-(13.45) 072-2010	-(1.76)	-(3.42)	-(1.41)	-(3.13)	-(1.22)	-(2.19)	-(1.55)	-(.09)	-(1.16)	-(.38)
The Effective Tick	-0.05	-0.47	-0.60	-0.35	-0.35	-0.40	-0.44	-0.34	-0.69	-0.35	-0.31
	-(.27)	-(6.12)	-(8.03)	-(5.24)	-(5.12)	-(5.25)	-(5.59)	-(2.81)	-(6.57)	-(2.93)	-(1.32)
Pastor & Stambaugh	-0.57	-0.54	-0.42	-0.42	-0.52	-0.45	-0.36	-0.46	-0.48	-0.11	0.46
	-(3.65)	-(6.)	-(5.04)	-(5.71)	-(6.74)	-(5.95)	-(4.6)	-(5.14)	-(5.18)	-(.66)	(2.28)

Table 9 – Fama & Macbeth (1973) Cross-Sectional Regressions

The table reports betas from Fama & Macbeth (1973) cross-sectional regressions estimated yearly between 1972 and 2010. Time-series average values of the yearly regression coefficients are reported with time-series t-statistics in brackets. The Amihud (2002) measure has been used as liquidity measure.

MKT	ROA	INV	LIQ	<i>R</i> <sup>2</sup>
1.08	-0.2	-0.02		0.78
(24.06)	-(3.30)	-(.11)		
1.1	-0.17	0	0.14	0.78
(22.84)	-(2.98)	(0.00)	(3.49)	

#### 6.1.3 Fama-Macbeth Regressions

In order to examine the relationship between liquidity and average stock returns even further, we run Fama & Macbeth (1973) regressions. Characteristics included in the regressions are the market, investment, profitability, and liquidity factors. The regressions should be seen as a robustness check of the relationship between liquidity and average stock returns where no breakpoints between quintiles are needed and alternative explanations of liquidity can be tested (Hou & Robinson (2006)).

From Table 9, it is clear that liquidity is priced in the cross-section when included in the existing three-factor model. The beta coefficient is 0.14 (t = 3.49) and significant. An important observation is that the investment factor is small already in the three-factor model but zero in the four-factor model, which indicates that the investment factor could be redundant. The fact that the adjusted  $R^2$  does not increase when adding the (significant) liquidity factor gives support to this interpretation. The profitability factor is on the other hand highly significant, -0.17 (t = -2.98).

#### 6.1.4 Concluding Comments

The impression from the tests performed in Sections 6.1.1-6.1.3 is that liquidity is priced and that the liquidity factor is relevant when added to the three-factor model. In fact, the liquidity factor seems to be more important than the investment factor. Additional tests would, however, be needed in order to exclude the investment factor from the new model. Another observation is that the illiquidity premium is to a large extent driven by returns from highly illiquid stocks. In any case, the empirical tests form a basis to include the liquidity factor in the Chen, Novy-Marx, & Zhang (2010) three-factor model and to test the new model's ability to explain a number of anomalies previously found in the literature.

	$r_{MKT}$	$r_{INV}$	$r_{ROA}$	$r_{LIQ}$	$r_{SMB}$	$r_{HML}$
r <sub>INV</sub>	-0.18					
	-(3.3)					
r <sub>ROA</sub>	-0.35	-0.03				
	-(6.89)	-(.58)				
r <sub>LIQ</sub>	-0.14	0.10	-0.07			
C C	-(2.57)	(1.78)	-(1.21)			
r <sub>SMB</sub>	0.26	-0.09	-0.39	0.29		
0112	(5.03)	-(1.6)	-(7.7)	(5.62)		
r <sub>HML</sub>	-0.32	0.41	0.11	0.06	-0.26	
	-(6.17)	(8.19)	(1.99)	(1.13)	-(4.92)	
<b>W</b> ML	-0.18	-0.08	0.37	0.23	0.08	-0.19
,, <i>D</i>	-(3.26)	-(1.49)	(7.31)	(4.39)	(1.53)	-(3.58

#### Table 10 - Correlation Matrix

A correlation matrix of the market, investment, profitability, liquidity, size, book-to-market and m factors. The Amihud (2002) measur a has he . . . .

#### 6.2 **Alternative Factors**

Instead of the suggested four-factor model, an alternative would have been to instead add a liquidity factor to the Fama & French (1992) three-factor model. However, seeing that the factors in the Fama & French (1992) three-factor model are relatively more correlated with liquidity than the factors in the Chen, Novy-Marx, & Zhang (2010) model, that alternative is less interesting. As can be seen in Table 10, correlations between the liquidity factor and the three factors from the Chen, Novy-Marx, & Zhang (2010) model are low and significant only for the market factor. As a side note, the liquidity factor is correlated with the momentum factor, which is in accordance with Sadka (2006) and should be important when testing the momentum anomaly.

In particular, it has been argued that small firms' relatively high returns partly are explained by liquidity. The intuition is that small firms suffer more from asymmetric information problems, as they are not as closely monitored as larger firms (Amihud (2002)). The relationship between liquidity and the HML factor is not that clear; e.g. Acharya & Pedersen (2005) find only small empirical support, and no theoretical motivation has been found in the literature. In the last regression of Table 1 there are, however, indications of that the HML factor is related to liquidity.

Furthermore, instead of using liquidity, the momentum factor from Carhart (1997) could have been added to the Chen, Novy-Marx, & Zhang (2010)) three-factor model. However, that three-factor model is in fact able to explain momentum profits rather good. In particular, loadings on the profitability factor are increasing linearly with momentum. Although, given that liquidity and momentum are somewhat related, *both* momentum and liquidity effects could be captured by adding a liquidity factor. In other words, by adding momentum, we would take away explanatory power from the existing factors while not gaining new explanatory power. This theory has also gained empirical support in tests performed (not tabulated), where performance of the liquidity factor is better than the momentum factor in near to all cases.

# 7 Tests of Anomalies

In order to test a number of anomalies found in the literature, we use the following factor regression (the alternative four-factor model):

$$r_i - r_f = \alpha_i + \beta_{i,MKT} r_{MKT} + \beta_{i,INV} r_{INV} + \beta_{i,ROA} r_{ROA} + \beta_{i,LIQ} r_{LIQ} + \epsilon_i$$

where  $r_i - r_f$  is the portfolio excess return,  $\beta_{i,MKT}$ ,  $\beta_{i,INV}$ ,  $\beta_{i,ROA}$ , and  $\beta_{i,LIQ}$  are betas for the market, investment, profitability and liquidity factors, respectively, and  $r_{MKT}$ ,  $r_{INV}$ ,  $r_{ROA}$ ,  $r_{LIQ}$  are return premiums from zero-cost portfolios.

This model will primarily be evaluated for its ability to return alphas that are not significantly different from zero. In all regressions, the Amihud (2002) measure has been used as proxy for liquidity. Unless stated otherwise, the five percent significance level has been used.

#### 7.1 Size and Momentum

Momentum profits arise from a strategy where an investor buys stocks that have had high returns in the past and sells stocks that have had low returns during the same period. Jegadeesh & Titman (1993) were the first to document momentum profits and argued that it is an anomaly since common risk factors, including the CAPM model, were not able to explain the returns. Later, momentum profits have been found in several markets and within different asset classes (Asness, Moskowitz, & Pedersen (2009)). The size effect was discovered by Banz (1981), where especially very small firms were found to have high risk-adjusted returns.

We construct 25 size and momentum portfolios using the "11/1/1" convention (Fama & French (1996)), meaning that at the beginning of each month t, we sort NYSE, Amex, and NASDAQ stocks into quintiles based on their prior returns from month t - 2 to t - 12, excluding month t - 1, and calculate the value-weighted portfolio returns for the current month t. We use NYSE market equity breakpoints to sort the stocks independently each month into quintiles. Taking intersections of the size and prior 12-month returns quintiles, 25 portfolios are formed monthly.

To save space, only results from quintiles 1, 3 and 5 are reported in Table 11. From Table 11, it is clear that there are large momentum profits. The spread between winner and loser portfolios are in the range of 0.46% (t = 1.34) to 1.02% (t = 1.05) monthly

where the smallest firms have the largest spread. It is also clear that small firms earn higher returns than large firms.

The CAPM is not able to explain the spreads. The reason is that beta loadings are similar across all portfolios even though small firms tend to have somewhat higher loadings than large firms, which decreases the size spread. Seven out of the 25 portfolios have significant alphas. When adding the investment and profitability factors, the number of significant alphas decreases to six, but they are on average larger than in the CAPM. Winners have less negative loadings on the ROA factor than losers, but the negative sign is still surprising (see, however, Section 6.1.3). Given the intuition behind the ROA factor, we should see positive beta loadings. In fact, only large winners show the expected positive loading on the profitability factor. Loadings on the investment factor show no clear trend except that the extreme portfolios tend to have negative loadings while portfolios in between have positive loadings. Altogether, the profitability and investment factors explain returns from losers but further aggravate alphas for winner portfolios.

In the alternative four-factor model, only three alphas are significant and smaller than in the CAPM and the three-factor model. All significant alphas in the four-factor model are in the quintile with the smallest firms, which means that the four-factor model can be said to explain the momentum anomaly, while the size effect to some extent remains unexplained. Loadings on the liquidity factor increase with momentum, which indicates that part of the momentum profits are related to liquidity. Additionally, loadings decrease with firm size. Hence, the liquidity factor also helps explaining the size effect even though loadings are not large enough to fully explain returns from the smallest firms. The GRS test indicates that alphas are jointly different from zero under all factor specifications, but F-statistics for the Chen, Novy-Marx, & Zhang (2010) model and the four-factor model are lower than for the CAPM.

### 7.2 Book-to-Market

Fama & French (1992) argue that the CAPM is not able to explain returns from portfolios sorted on the book-to-market equity ratio. Firms with higher ratios seem to deliver abnormally high returns and vice versa.

The 25 book-to-market portfolios are downloaded from Kenneth French's website<sup>5</sup>. When looking at mean returns, it is clear that firms with high book-to-market equity and low market capitalization deliver higher returns than firms with low book-to-market equity and high market capitalization.

Performances of all factor models including the CAPM, the Chen, Novy-Marx, & Zhang (2010) three-factor model, and the four-factor model are poor. Alphas are large and significant, high minus low book-to-market alpha spreads are wide, and most factor loadings do not show clear patterns. The only factor where there are linear trends between high and low book-to-market portfolios is the investment factor. Unfortunately, the positive effects from this factor are not large in magnitude and often shadowed by noise from other factors. The liquidity factor has little to offer when it comes to explaining returns from the book-to-market sorted portfolios. The poor performance is confirmed by the GRS test, which reports highly significant F-statistics.

<sup>&</sup>lt;sup>5</sup> http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/, accessed 2011-04-29.

In panel A of the table, mean monthly returns in percent and corresponding t-statistics of the 25 (nine tabulated) size and momentum sorted portfolios are reported. Momentum and size are sorted horizontally and vertically, respectively. Below the mean returns in panel A, alphas from regressions of portfolio returns on the CAPM and the Chen, Novy-Marx, & Zhang (2010) three-factor model are reported. For the three-factor model, beta loadings of the investment and profitability factors are also reported. In panel B, alpha and beta coefficients from regressions of the 25 (nine tabulated) portfolio returns on the four-factor model are reported. F-statistics and p-values (in brackets) from the (Gibbons, Ross, & Shanken, 1989) test are reported in the headline above each model's alpha t-statistics. The sample period is January 1972-December 2010.

		Panel A: Means - CAPM and Chen, Novy-Marx, & Zhang									Panel B: The four-factor model									
	Loser	3	Winner	W-L	Loser	3	Winner	W-L	Loser	3	Winner	W-L	Loser	3	Winner	W-L				
		N	1ean			$t_1$	nean				α		t	$t_{\alpha}(F_{GRS} = 3)$	3.79, p = 0.00	)				
Small	0.20	0.66	1.22	1.02	3.76	3.77	6.74	4.12	0.21	0.29	0.60	0.40	0.68	1.55	2.46	1.33				
3	0.10	0.65	0.92	0.82	3.77	5.17	7.28	2.81	0.14	0.26	0.17	0.04	0.52	1.75	0.76	0.11				
Big	-0.04	0.30	0.80	0.84	2.78	2.63	5.91	2.26	-0.40	-0.05	0.24	0.64	-1.24	-0.40	1.12	1.45				
		α	CAPM		$t_{\alpha C}$	$_{APM}(F_{GRS}$	= 4.58, <i>p</i> = 0	.00)		β	МКТ			$t_l$	ИКТ					
Small	-0.38	0.22	0.67	1.05	-1.43	1.28	3.08	4.20	1.22	1.04	1.28	0.06	13.94	13.87	20.71	0.66				
3	-0.49	0.20	0.38	0.88	-2.10	1.55	1.89	2.97	1.21	1.07	1.29	0.08	16.40	21.08	22.40	0.94				
Big	-0.54	-0.06	0.36	0.90	-1.99	-0.61	1.90	2.38	1.06	0.90	1.08	0.02	15.21	23.34	19.52	0.21				
		α	Chen		$t_{\alpha c}$	$hen(F_{GRS})$	= 3.64, p = 0.	00)		/	3 <sub>INV</sub>			t	INV					
Small	0.07	0.32	0.80	0.73	0.26	1.83	3.44	2.57	-0.21	0.15	-0.12	0.09	-1.42	1.70	-1.01	0.62				
3	-0.06	0.23	0.49	0.55	-0.24	1.82	2.13	1.63	-0.15	0.10	-0.46	-0.31	-1.24	1.32	-3.78	-1.87				
Big	-0.34	-0.18	0.39	0.73	-1.16	-1.55	1.85	1.75	0.11	0.09	-0.53	0.09	0.73	1.48	-4.10	-2.88				
			B <sub>INV</sub>			1	- INV			f	ROA			$t_{j}$	ROA					
Small	-0.18	0.17	-0.10	0.09	-1.35	1.94	-0.77	0.59	-0.84	-0.16	-0.33	0.51	-5.52	-1.96	-3.55	3.22				
3	-0.17	0.11	-0.41	-0.25	-1.36	1.54	-3.11	-1.35	-0.85	-0.09	-0.11	0.74	-6.06	-1.29	-1.28	4.14				
Big	0.08	0.07	-0.46	-0.53	0.57	1.22	-3.46	-2.39	-0.43	0.18	0.12	0.55	-2.96	3.27	1.19	2.51				
		ļ	3 <sub>ROA</sub>			t	ROA			ļ	3 <sub>LIQ</sub>			t	LIQ					
Small	-0.83	-0.20	-0.35	0.48	-5.93	-2.69	-3.44	2.97	0.05	0.21	0.31	0.26	0.66	4.88	4.88	2.46				
3	-0.80	-0.11	-0.16	0.64	-5.81	-1.81	-1.53	3.18	-0.12	0.05	0.33	0.45	-1.97	1.63	3.97	3.93				
Big	-0.43	0.18	0.08	0.51	-3.03	3.26	0.82	2.35	-0.13	-0.09	0.21	0.33	-1.91	-1.82	2.44	2.71				

#### An Alternative Four-Factor Model

		Panel A: Means, CAPM and Chen, Novy-Marx, & Zhang									Panel B: The four-factor model							
	Low	3	High	H-L	Low	3	High	H-L	Low	3	High	H-L	Low	3	High	H-L		
	Mean					$t_m$	ean		α				$t_{\alpha}(F_{GRS} = 7.81, p = 0.00)$					
Small	0.09	0.82	1.11	1.02	0.24	2.95	3.86	5.10	-0.31	0.49	0.80	1.11	-1.23	2.77	4.05	5.14		
3	0.42	0.77	1.07	0.65	1.32	3.29	4.13	3.02	-0.06	0.47	0.79	0.85	-0.36	3.55	4.29	3.58		
Big	0.39	0.49	0.58	0.19	1.70	2.30	2.46	0.76	-0.19	0.08	0.27	0.46	-1.91	0.66	1.46	3.37		
		$\alpha_{C_{A}}$	APM		$t_{\alpha}$	$CAPM(F_{GRS})$	7.16, p = 0.0	)))	$\beta_{MKT}$					$t_{MKT}$				
Small	-0.56	0.34	0.63	1.19	-2.52	2.11	3.53	6.42	1.32	1.01	1.03	-0.29	20.21	16.58	15.03	-4.42		
3	-0.18	0.32	0.62	0.80	-1.26	2.96	4.08	3.90	1.23	0.95	0.97	-0.26	27.12	21.07	16.00	-3.50		
Big	-0.07	0.09	0.19	0.26	-0.88	0.89	1.29	2.17	1.01	0.89	0.81	-0.20	32.50	26.17	15.23	-17.27		
		$\alpha_{C}$	hen		$t_{\alpha c}$	$t_{\alpha chen}(F_{GRS} = 7.82, p = 0.00)$			$\beta_{INV}$				t_{INV}					
Small	-0.12	0.56	0.80	0.91	-0.45	3.21	4.54	4.54	-0.35	0.09	0.32	0.67	-2.36	0.84	3.34	5.10		
3	0.05	0.35	0.66	0.62	0.29	3.12	4.44	2.94	-0.40	0.18	0.40	0.80	-3.97	2.86	3.87	5.83		
Big	-0.20	0.01	0.21	0.41	-2.39	0.11	1.30	3.69	-0.06	0.18	0.42	0.48	-1.50	2.65	4.12	5.62		
-		$\beta_I$	NV			$t_I$	NV		$\beta_{ROA}$			tROA						
Small	-0.29	0.10	0.33	0.62	-1.87	0.93	3.42	5.10	-0.60	-0.24	-0.28	0.32	-5.25	-2.78	-3.26	3.20		
3	-0.62	0.18	0.54	1.16	-4.04	3.00	4.18	6.08	-0.27	0.02	-0.09	0.18	-3.06	0.26	-0.93	1.31		
Big	-0.06	0.25	0.44	0.50	-1.50	2.74	3.88	5.38	0.17	0.10	-0.08	-0.25	4.49	1.94	-0.89	-5.38		
	$\beta_{ROA}$			t <sub>ROA</sub>			$\beta_{LIQ}$				$t_{LIQ}$							
Small	-0.64	-0.29	-0.35	0.29	-5.32	-3.42	-4.58	2.90	0.34	0.23	0.21	-0.13	3.97	3.60	4.84	-1.66		
3	-0.26	0.00	-0.12	0.13	-3.09	-0.01	-1.45	1.00	0.12	0.02	0.01	-0.10	1.97	0.62	0.27	-1.33		
Big	0.20	0.11	-0.08	-0.28	6.13	2.33	-0.92	-7.05	-0.06	-0.05	-0.04	0.02	-3.22	-1.21	-0.77	2.45		

Table 12 - Descriptive statistics and Factor Regressions for Monthly Percent Excess Returns of 25 Size and Book to Market Portfolios

For a description of the coefficients in this table, see Table 11. Returns of the 25 book-to-market and size sorted portfolios (nine tabulated) are obtained from Kenneth French's website. Book-to-market and size are sorted horizontally and vertically, respectively. The sample period is January 1972-December 2010.

### 7.3 Financial Distress

Financially distressed firms should, ceteris paribus, on average deliver higher returns than firms that are not subject to financial distress, this in order to compensate for the higher risk. However, Campbell, Hilscher, & Szilagyi (2008) find that firms in financial distress have, since the beginning of the 1980s, delivered abnormally *low* returns despite higher return standard deviations and market betas. Additionally, the findings are not explained by the size and book-to-market factors from the Fama & French (1992) three-factor model. Ohlson's (1980) *O*-score is used as an alternative to the Campbell, Hilscher, & Szilagyi (2008) failure probability measure.

Using what is regarded to be the best model in the Campbell, Hilscher, & Szilagyi (2008) paper, the distress measure (12 month lagged) is defined as:

$$\begin{split} Distress(t) &\equiv -9.164 - 20.264 NIMTAAVG_t + 1.416TLMTA_t - 7.129EXRETAVG_t + \\ 1.411SIGMA_t - 0.045RSIZE_t - 2.132CASHMTA_t + 0.075MB_t - 0.058PRICE_t \\ NIMTAAVG_{t-1,t-12} &\equiv \frac{1-\phi^3}{1-\phi^{12}} \left( NIMTA_{t-1,t-3} + \dots + \phi^9 NIMTA_{t-10,t-12} \right) \\ EXRETAVG_{t-1,t-12} &\equiv \frac{1-\phi}{1-\phi^{12}} \left( EXRET_{t-1} + \dots + \phi^{11}EXRET_{t-12} \right) \end{split}$$

where  $\phi = 2^{-1/3}$  so that the weight is halved each quarter. *NIMTAAVG* is a moving average of *NIMTA* defined as net income [IBQ] divided by the sum of market equity [PRC\*CSHO] and total liabilities [LTQ], used to reflect that a series of losses is a better indicator of distress than a single quarterly loss. *TLMTA* is a solvency ratio defined as total liabilities divided by the sum of market equity and total liabilities. *EXRETAVG* is a moving average of the stock's performance relative to the relevant market index, *EXRET* =  $log(1 + R_{i,t}) - log(1 + R_{s\&P500,t})$ , and is used to capture that long-term underperformance arguably a better indicator of future bankruptcy than sudden abnormal returns. S&P 500 [TOTVAL] is used as market index. *SIGMA* is the past 3 month daily return volatility calculated as the annualized three-month rolling sample standard deviation:  $\sqrt{\frac{252}{N-1}\sum_i r_i^2}$ , where  $i_{\in(t-1,t-2,t-3)}$  is the index of trading days in month t - 1, t - 2 and t - 3, is the firmlevel simple daily return, and *N* the number of trading days in the three months. Should *N* be less than 6, then *SIGMA* is treated as missing. *RSIZE* is the logarithmic

Table 13 - Descriptive Statistics and Factor Regressions for Monthly Percent Excess Returns of Deciles Formed on Campbell, Hilscher, & Szilagyi's (2008) Failure Probability and on Ohlson's (1980) O-score

In Panel A, mean monthly returns in percent from failure probability sorted deciles are reported. To save space, only the low, median, and high failure probability deciles are shown. Coefficients from the CAPM, Chen, Novy-Marx, & Zhang (2010), and the four-factor model are reported. F-statistics (p-value) from the GRS (Gibbons, Ross, & Shanken, 1989) test are reported for each model. In Panel B, corresponding results from O-score sorted deciles are reported.

$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		Panel		ilure prob iles	ability	Par	el B: The	O-score d	eciles
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	-	Low	5	High	H-L	Low	5	High	H-L
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	М	1.06	0.58	-0.16	-1.22	0.55	0.53	0.40	-0.14
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$t_M$	6.55	7.04	9.62	-3.64	2.49	2.07	1.11	-0.52
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$F_{GRS} =$	10.34 (0.00	))	CAPM		$F_{GRS} = 1.9$	8 (0.03)	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	α	0.49	-0.04	-1.00	-1.49	0.03	-0.06	-0.34	-0.37
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$t_{lpha}$	2.34	-0.19	-3.29	-4.78	0.32	-0.53	-1.65	-1.49
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	β	1.10	1.22	1.62	0.52	0.92	1.05	1.32	0.40
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$t_{eta}$	22.22	22.93	20.32	6.18	42.57	38.20	21.93	5.49
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$F_{GRS} =$	= 7.00 (0.00	)) Chen,	& Zhang	$F_{GRS} = 2.4$	3 (0.01)		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	α	0.62	0.14	-0.28	-0.90	-0.07	-0.04	0.02	0.09
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$t_{lpha}$	2.83	0.64	-1.08	-3.16	-0.98	-0.32	0.09	0.38
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\beta_{MKT}$	1.06	1.15	1.35	0.29	0.95	1.03	1.18	0.23
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$t_{\beta MKT}$	18.56	21.90	20.79	3.68	49.95	38.02	20.40	3.51
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\beta_{ROA}$	-0.22	-0.33	-1.23	-1.01	0.20	-0.08	-0.62	-0.82
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$t_{\beta ROA}$	-2.14	-3.14	-10.94	-6.86	6.69	-1.46	-6.10	-7.32
F_{GRS} = 7.64 (0.00)The four-factor model $F_{GRS} = 1.91 (0.04)$ $\alpha$ 0.620.20-0.19-0.81-0.070.010.040.11 $t_{\alpha}$ 2.690.78-0.61-2.50-0.810.110.180.42 $\beta_{MKT}$ 1.051.141.370.330.951.021.180.23 $t_{\beta MKT}$ 15.0617.1717.563.3441.6230.3917.932.95 $\beta_{ROA}$ -0.27-0.34-1.22-0.950.21-0.07-0.60-0.81 $t_{\beta ROA}$ -2.56-3.03-9.86-5.966.21-1.22-5.96-7.03 $\beta_{INV}$ 0.040.00-0.06-0.11-0.220.08-0.25-0.03 $t_{\beta INV}$ 0.330.01-0.45-0.69-5.041.27-1.79-0.22 $\beta_{LIQ}$ 0.130.050.04-0.10-0.02-0.040.180.20	$\beta_{INV}$	0.03	0.02	-0.04	-0.07	-0.20	0.08	-0.17	0.03
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$t_{\beta INV}$	0.21	0.14	-0.29	-0.45	-4.96	1.30	-1.24	0.19
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$F_{GRS} =$	= 7.64 (0.00	)) Th	e four-factor	model	$F_{GRS} = 1.9$	01 (0.04)	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	α	0.62	0.20	-0.19	-0.81	-0.07	0.01	0.04	0.11
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$t_{lpha}$	2.69	0.78	-0.61	-2.50	-0.81	0.11	0.18	0.42
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\beta_{MKT}$	1.05	1.14	1.37	0.33	0.95	1.02	1.18	0.23
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$t_{\beta MKT}$	15.06	17.17	17.56	3.34	41.62	30.39	17.93	2.95
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\beta_{ROA}$	-0.27	-0.34	-1.22	-0.95	0.21	-0.07	-0.60	-0.81
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$t_{\beta ROA}$	-2.56	-3.03	-9.86	-5.96	6.21	-1.22	-5.96	-7.03
$\hat{\boldsymbol{\beta}}_{LIQ}$ 0.13 0.05 0.04 -0.10 -0.02 -0.04 0.18 0.20	$\beta_{INV}$	0.04	0.00	-0.06	-0.11	-0.22	0.08	-0.25	-0.03
-	$t_{\beta INV}$	0.33	0.01	-0.45	-0.69	-5.04	1.27	-1.79	-0.22
$t_{\beta LIQ}$ 1.50 0.74 0.50 -0.88 -1.12 -1.11 2.00 2.26	$\beta_{LIQ}$	0.13	0.05	0.04	-0.10	-0.02	-0.04	0.18	0.20
	$t_{\beta LIQ}$	1.50	0.74	0.50	-0.88	-1.12	-1.11	2.00	2.26

ratio of the stock's size to the size of the market index used, in this case the S&P 500. *CASHMTA* is used to capture the liquidity status of the firm and is defined as cash and other short-term investments [CHEQ] divided by the sum of market equity and total liabilities. *MB* is the market-to-book equity [BKVLPS] where book equity is defined as in

Cohen, Polk and Voulteenaho (2003). We adjust *MB* by adding 10% of the difference between the firm's market equity and book equity to adjust for firms with negative book equity. Firms that still have negative book equity will have book equity of \$1. *PRICE* is the logarithmic price per share. Following Chen, Novy-Marx, & Zhang (2010), we only include stocks between \$1 and \$15.

The definition of Ohlson's (1980) O-score is taken from Model 1 in Table 4 of his paper:

$$O - score = -1.32 - 0.407 \log(\frac{MKTASSET}{CPI}) + 6.03TLTA - 1.43WCTA + 0.076CLCA - 1.720ENEG - 2.37NITA - 1.83FUTL + 0.29INTWO - 0.52CHIN$$

where *MKTASSET* is defined as total liabilities + market equity + 0.1 x (market equity - book equity) to make sure that assets are not too close to zero (following Campbell, Hilscher, & Szilagyi's (2008)). The construction of book equity follows Fama & French (1993). *CPI* is the consumer price index [CPI1]. *TLTA* is the ratio of total liabilities to *MKTASSET*, *WCTA* is the ratio working capital (ACTQ - LCTQ) to *MKTASSET*, *CLCA* is the ratio of current liabilities [LCTQ] to current assets [ACTQ], *OENEG* is one if total liabilities exceeds total assets and zero otherwise, *NITA* is the ratio of net income to *MKTASSET*, *FUTL* is the ratio of funds provided by operations [PIQ] to total liabilities. *INTWO* is one if net income was negative for the last two years and zero otherwise, and *CHIN* is ( $NI_t - NI_{t-1}$ )/( $|NI_t| + |NI_{t-1}|$ ), where *NI* is the net income.

In the beginning of each month, NYSE, Amex and NASDAQ stocks are sorted into deciles based on their Campbell, Hilscher, & Szilagyi (2008) failure probability as well as the Ohlson (1980) *O*-score. We then calculate value-weighted returns for each portfolio in the current month. Earnings and other accounting data for a fiscal quarter are used in portfolio sorts in the months immediately after the quarter's public earnings announcement month. Due to lack of data, the sample period for Campbell, Hilscher, & Szilagyi (2008) failure probability begins in February 1976, and the sample period for the (Ohlson, 1980) O-score begins February 1975.

Results show that market beta loadings increase with failure profitability, indicating that firms with high failure probability are associated with relatively high systematic risk – the spread is 0.52 (t = 6.18), see Table 13. The mean returns from the failure probability sorted portfolios confirm previous findings; returns decrease linearly with failure probability. This trend is even more evident when looking at CAPM alphas, where the spread between the lowest and highest decile portfolios is as large as -1.49 (t = -4.78) monthly. Thus, the additional market risk is not compensated with higher returns.

When including the investment and profitability factors, alpha for the high failure probability deciles in particular move closer to zero. This is more or less entirely explained by loadings on the profitability factor, which are very negative for the high failure probability deciles. The interpretation is that relatively low expected earnings can explain the low returns from these deciles. The liquidity factor has little to offer when added to the three-factor model.

When instead sorting on Ohlson's (1980) *O*-score, the spread between the high and low *O*-score deciles is much smaller -0.14% (t = -0.52). It should, however, be noted that deciles 1 through 5 are significantly different from zero while deciles 6 through 10 are not. The tendency is thus the same as for the failure probability sorted deciles. The CAPM increases the spread (due to higher betas for firms with high *O*-score) but report mostly low and insignificant alphas. The relatively good performance is indicated by the GRS test, which reports a F-statistic of 1.98. The three-factor model improves performance regarding the firms with the highest *O*-score but alphas in general increase somewhat. The spread for the profitability factor is again negative, which is the major explanation for the low alpha in the highest *O*-score decile. The best performing model is the four-factor model, which reports even lower alphas and GRS F-statistic. The reason for this additional explanatory seems to come from the high *O*-score deciles. The liquidity factor loads positively on these deciles, which indicates that they include an illiquidity premium.

#### 7.4 Total Accruals

Firms with high total accruals have been found to deliver abnormally low returns. Since this is public information, it is inconsistent with an efficient market where prices should fully reflect all available information (Sloan (1996)). Traditional asset pricing models have not been able to explain this difference, making it an anomaly.

Total accruals is defined as:

$$Accruals = (\Delta CA - \Delta Cash) - (\Delta CL - \Delta STD - \Delta TP) - Dep$$

where  $\Delta CA$  is the change in current assets,  $\Delta Cash$  is the change in cash and cash equivalents,  $\Delta CL$  is the change in current liabilities,  $\Delta STD$  is the change in debt included

# Table 14 - Descriptive statistics and Factor Regressions for Monthly Percent Excess Returns of Deciles formed on Total Accruals

In the table, returns from total accruals sorted deciles are reported. For a description of the coefficients reported, see Table 13.

			Т	'he total ac	cruals dec	iles			
	Low	5	High	H-L		Low	5	High	H-L
Μ	0.62	0.70	0.30	-0.32	M	0.62	0.70	0.30	-0.32
$t_M$	2.89	2.43	1.18	-2.47	$t_M$	2.89	2.43	1.18	-2.47
		CAPM					our-facto		
		$_{RS} = 6.08$ (					$_{RS} = 5.24$ (		
α	0.12	0.07	-0.28	-0.39	α	0.06	0.18	-0.35	-0.41
$t_{lpha}$	1.57	0.49	-2.58	-3.09	$t_{lpha}$	0.75	1.15	-2.70	-2.59
β	0.89	1.13	1.02	0.13	$\beta_{MKT}$	0.90	1.07	1.03	0.13
$t_{eta}$	46.51	34.43	34.94	3.84	$t_{\beta MKT}$	41.13	26.41	30.32	3.11
	Chen, N	lovy-Marx	, & Zhang	5					
	$F_{GI}$	$_{RS} = 5.31$ (	0.00)		$\beta_{ROA}$	0.16	-0.26	0.15	-0.00
α	0.05	0.25	-0.31	-0.36	$t_{\beta ROA}$	4.98	-3.93	2.65	-0.07
$t_{lpha}$	0.61	1.76	-2.90	-2.73	$\beta_{INV}$	-0.09	-0.21	-0.24	-0.16
$\beta_{MKT}$	0.91	1.06	1.03	0.11	$t_{\beta INV}$	-2.21	-2.02	-4.22	-2.34
$t_{\beta MKT}$	48.26	33.27	36.63	3.29	$\beta_{LIQ}$	-0.03	0.11	-0.03	0.00
$\beta_{ROA}$	0.14	-0.26	0.14	0.00	$t_{\beta LIQ}$	-1.48	1.93	-1.13	-0.06
$t_{\beta ROA}$	4.74	-3.90	2.59	-0.10					
$\beta_{INV}$	-0.07	-0.20	-0.22	-0.15					
$t_{\beta INV}$	-1.79	-2.11	-3.98	-2.45					

in current liabilities [DLC],  $\Delta TP$  is the change in income taxes payable [TXP], and *Dep* is the depreciation and amortization expense [DP].

In June in each year t, NYSE, Amex and NASDAQ stocks are sorted into deciles based on total accruals in year t - 1. We calculate value-weighted returns for each portfolio from July in year t to June in year t + 1.

As can be seen in Table 14, mean returns differ by -0.32% (t = -2.47) between the high and low deciles. The effect is, however, limited to the decile that includes firms with extremely high total accruals; all other deciles show similar mean returns. Actually, decile six has the highest mean return, 0.97% (t = -3.19).

Three alphas are significant in both the CAPM and three-factor model but only two when including the liquidity factor. From not tabulated results, the liquidity factor has positive and significant loadings from decile seven through decile nine, indicating that the somewhat higher returns from these deciles are associated with liquidity. The high total accruals decile, however, is not explained.

#### 7.5 Net Stock Issues

An abnormally strong positive relationship between share repurchases and subsequent high returns has been found in the literature (Ikenberry, Lakonishok, & Vermaelen (1995)). In contrast, stock issues are negatively related to subsequent returns (Pontiff & Woodgate (2008)).

Net stock issues is defined as "the natural log of the ratio of the split-adjusted shares outstanding at the fiscal year end in t - 1 divided by the split-adjusted shares outstanding at the fiscal year end in t - 2" (Fama & French (2008)). The split-adjusted shares outstanding is shares outstanding [CSHO] times the adjustment factor [ADJEX C].

In June in each year t, NYSE, Amex and NASDAQ stocks are sorted into deciles based on net stock issues in the end of year t - 1. We calculate value-weighted returns for each portfolio from July in year t to June in year t + 1. We group all firms with negative net stock issues in decile 1, and firms with zero net stock issues in decile 2, because a disproportional number of firms have zero net stock issues. Firms with positive net stock issues are sorted in the remaining eight (equal-numbered) deciles.

From panel A in Table 15, it is clear that firms with negative stock issues and zero stock issues earn higher returns than firms with high stock issues, the spread is -0.41% (t = -2.48). The CAPM is not able to explain the negative and high net stock issue deciles while the investment factor does (in the Chen, Novy-Marx, & Zhang (2010) three-factor model). Firms with high net stock issues tend to have more negative loadings relative to other firms, indicating that some of the spread can be explained by investment behavior. The profitability as well as the liquidity factor explains little, but nonetheless merely one alpha is significant in both the three- and four-factor models.

#### 7.6 Asset Growth

Cooper, Gulen, & Schill (2008) find that asset growth rates are "strong predictors of future abnormal returns", especially among small firms. The relationship between asset growth rate and future returns should be negative, given the intuition for the investment factor and in Cooper, Gulen, & Schill (2008). The results are robust to firm size and the book-to-market ratio, which indicates that our model has more to offer than the Fama & French (1992) three-factor model.

	Panel A: Net stock issue deciles					]	Panel B: Asset growth deciles			
-	Neg.	Zero	Low	High	H-N	Lo	w 5	High	H-L	
М	0.76	0.76	0.57	0.35	-0.41	0.9	2 0.73	0.17	-0.75	
$t_M$	3.55	2.50	2.32	1.25	-2.48	2.9	0 3.19	0.50	-3.04	
	$F_{GRS} = 5.12 \ (0.00)$			CAPM			$F_{GRS} = 5.75 \ (0.00)$			
α	0.26	0.14	0.03	-0.27	-0.53	0.2	6 0.21	-0.58	-0.85	
$t_{lpha}$	3.39	0.83	0.22	-2.09	-3.34	1.4	8 2.07	-3.60	-3.39	
β	0.89	1.10	0.96	1.10	0.22	1.1	7 0.91	1.34	0.17	
$t_{eta}$	38.25	24.93	33.78	29.84	4.49	25.0	9 33.52	31.82	2.30	
		$F_{GRS} = 4.41$	(0.00)	Chen, No	ovy-Marx,	& Zhan	g $F_{GRS} = 4$	.80 (0.00)		
α	0.14	0.45	-0.05	-0.24	-0.38	0.3	7 0.13	-0.40	-0.77	
$t_{\alpha}$	1.92	2.44	-0.39	-1.85	-2.40	2.0	1 1.32	-2.76	-3.31	
$\beta_{MKT}$	0.93	0.95	0.98	1.09	0.15	1.1	4 0.95	1.23	0.09	
t <sub>βMKT</sub>	39.62	25.01	32.89	29.62	3.02	25.9	0 35.58	36.23	1.52	
$\beta_{ROA}$	0.20	-0.45	0.14	0.02	-0.17	-0.3	0.13	-0.23	0.07	
$t_{\beta ROA}$	5.65	-5.20	2.73	0.45	-2.67	-2.9	0 2.71	-3.43	0.54	
$\beta_{INV}$	0.07	-0.28	-0.08	-0.36	-0.44	0.3	0 0.18	-0.77	-1.07	
$t_{\beta INV}$	1.86	-2.61	-1.23	-4.70	-4.80	2.8	1 3.28	-9.77	-8.43	
	$F_{GRS} = 4.43 \ (0.00)$		The four-factor model			$F_{GRS} = 4$	$F_{GRS} = 4.93 \ (0.00)$			
α	0.19	0.37	-0.02	-0.26	-0.45	0.4	3 0.15	-0.45	-0.89	
$t_{lpha}$	2.24	1.89	-0.15	-1.71	-2.43	2.0	1 1.28	-2.64	-3.15	
$\beta_{MKT}$	0.91	1.01	0.96	1.11	0.20	1.1	4 0.91	1.25	0.11	
t <sub>βMKT</sub>	29.55	21.60	27.26	24.84	3.10	20.5	55 31.72	30.10	1.45	
$\beta_{ROA}$	0.20	-0.42	0.15	0.07	-0.12	-0.2	.0.11	-0.23	0.05	
$t_{\beta ROA}$	5.41	-4.52	2.79	1.24	-1.72	-2.5	2.10	-3.18	0.36	
$\beta_{INV}$	0.07	-0.33	-0.10	-0.41	-0.49	0.2	9 0.20	-0.79	-1.08	
$t_{\beta INV}$	1.85	-2.83	-1.49	-5.01	-5.02	2.4	8 3.41	-9.15	-7.72	
$\dot{\beta}_{LIQ}$	-0.04	0.09	-0.02	0.02	0.06	-0.0	0.00	0.00	0.01	
$t_{\beta LIQ}$	-2.12	1.21	-0.74	0.63	1.73	-0.1	1 -0.13	-0.09	0.04	

 Table 15 - Descriptive statistics and Factor Regressions for Monthly Percent Excess

 Returns of Deciles formed on Net Stock Issues and Asset Growth

Panel A and B show results from net stock issues and asset growth sorted deciles, respectively. For a description of the coefficients reported, see Table 13.

Using the definition from Cooper, Gulen, & Schill (2008), asset growth is defined as the difference between total assets in year t - 1 and total assets in year t - 2 divided by total assets in year t - 2.

In June in each year t, NYSE, Amex and NASDAQ stocks are sorted into deciles based on asset growth in the end of year t - 1. We calculate value-weighted returns for each portfolio from July in year t to June in year t + 1.

In panel B of Table 15, results from the asset growth deciles are reported. The mean returns of the deciles are shown to decrease more or less linearly with asset growth rate,

the spread is -0.75% (t = -3.04). The CAPM fails to explain this pattern since betas are similar for most deciles and highest for the high asset growth decile.

It is intuitive to expect similar patterns for the net stock issue deciles as for the asset growth deciles since both are related to changes in the size of the balance sheet. Hence, it is no surprise to see that the profitability nor the liquidity factor can help explain the asset growth spread. The investment factor is, however, even more related to asset growth than stock issues, why the large significant loadings are expected. Even though beta loadings on the factor show a nice trend, returns from the factor are too low to explain the spread.

#### 7.7 Standardized Unexpected Earnings (SUE)

In Ball & Brown (1968), a drift after earnings announcements was noticed for the first time. In Chordia & Shivakumar (2006), this drift, often referred to as earnings momentum or the post-earnings-announcement drift (PEAD), is defined as follows: "[e]arnings momentum refers to the fact that firms reporting unexpectedly high earnings subsequently outperform firms reporting unexpectedly low earnings". The outperformance lasts for about nine months and remains unexplained, making it an anomaly.

We use the standardized unexpected earnings (SUE) measure from Chan, Jegadeesh, & Lakonishok (1996) and model two in Foster, Olsen, & Shevlin (1984) to sort all stocks in deciles each month. The measure is defined as:

$$SUE_{i,t} = \frac{e_{i,q} - E_{q-4}[e_{i,q}]}{\sigma_{i,t}}$$

where  $e_{i,q}$  is earnings per share  $[EPS_1]$  in quarter q,  $E_{q-4}[e_{i,q}]$  is forecast of EPS in quarter q announced four quarters ago  $[EPS_2]$ , and  $\sigma_{i,t}$  is the standard deviation of unexpected earnings  $(e_{i,q} - E_{q-4}[e_{i,q}])$  over the last eight quarters, in the current month t.

In the beginning of each month, we sort NYSE, Amex and NASDAQ stocks in the into deciles based the most recent past SUE and calculate monthly value-weighted returns for the current month.

				SUE o	deciles				
	Low	5	High	H-L		Low	5	High	H-L
М	-0.30	0.51	1.18	1.48	М	-0.30	0.51	1.18	1.48
$t_M$	-0.72	1.28	2.70	3.16	$t_M$	-0.72	1.28	2.70	3.16
		CAPM			The four-factor model				
	$F_{GF}$	$s_{s} = 5.68(0.1)$	.00)		$F_{GRS} = 5.00 \ (0.00)$				
α	-0.83	-0.02	0.59	1.42	α	-0.77	0.30	0.52	1.29
$t_{lpha}$	-2.82	-0.09	1.89	2.98	$t_{\alpha}$	-2.55	0.98	1.51	2.49
β	1.06	1.05	1.16	0.10	$\beta_{MKT}$	0.96	0.90	1.13	0.17
$t_{eta}$	15.73	16.41	11.41	0.77	$t_{\beta MKT}$	12.12	9.99	11.08	1.15
	Chen, N	ovy-Marx,	& Zhang						
	$F_{GR}$	s = 4.56 (0	.00)		$\beta_{ROA}$	-0.16	-0.19	0.03	0.19
α	-0.74	0.04	0.56	1.30	$t_{\beta ROA}$	-1.25	-1.24	0.19	.96
$t_{lpha}$	-2.36	0.12	1.85	2.64	$\beta_{INV}$	0.26	-0.33	-0.62	-0.88
$\beta_{MKT}$	1.04	1.00	1.13	0.09	$t_{\beta INV}$	1.93	-2.07	-4.21	-3.78
$t_{\beta MKT}$	15.22	14.12	13.09	0.81	$\beta_{LIQ}$	-0.11	-0.06	0.05	0.16
$\beta_{ROA}$	-0.16	-0.09	0.07	0.23	$t_{\beta LIQ}$	-2.17	-0.64	0.66	1.57
$t_{\beta ROA}$	-1.27	-0.63	0.52	1.16					
$\beta_{INV}$	0.34	-0.35	-0.60	-0.94					
$t_{\beta INV}$	2.45	-2.25	-4.39	-4.21					

 Table 16 - Descriptive statistics and Factor Regressions for Monthly Percent Excess

 Returns of Deciles formed on SUE.

For a description of the coefficients reported, see Table 13.

In Table 16, the mean returns give support to the previous empirical findings. The spread in returns between firms that report the most unexpectedly high earnings and firms that report the most unexpectedly low earnings is as large as 1.48% (t = 3.16). The CAPM is not able to explain this spread since beta loadings are similar across the deciles. The Chen, Novy-Marx, & Zhang (2010) three-factor model lowers the spread to 1.30% (t = 2.64). Loadings on the profitability factor increase with reported unexpected earnings, which is the explanation to the three-factor model's superiority to the CAPM. The investment factor shows the opposite spread, which might decrease the gain from the profitability factor. The liquidity factor helps explaining returns from the deciles that contain firms with low unexpected earnings. The spread between the extreme deciles is thus positive and there is only one significant alpha when using the four-factor model (the other models have two or more significant alphas).

## 8 Comparisons to The Carhart (1997) Four-Factor Model

When adding the momentum factor to the Fama & French (1992) three-factor model, performance improves significantly. This is in line with Carhart (1997) and gives support to the hypothesis that the three-factor model does not explain momentum. When instead including the liquidity factor as a fourth factor in the Fama & French (1992) model, loadings on the new factor are generally insignificant and close to zero. As argued in Section 6.2, this is expected since the SMB factor is more correlated with liquidity than any other factor. Thus, Carhart's (1997) four-factor model is superior to the Fama & French (1992) three-factor model plus a liquidity factor (based on the Amihud (2002) measure). The question that remains is whether results from our four-factor model are comparable or even better than Carhart's (1997) four-factor model.

Returns from the size and momentum sorted portfolios are well explained by the Carhart (1997) model. This is no surprise given that the sorts are based on two factors that are included in his model. Compared to our four-factor model, results are slightly better when looking at alphas and the GRS F-statistic. The differences are, however, relatively small. One way to interpret this is that our four-factor model explains momentum, while merely its existence is noticed in Carhart (1997) (and explained by the momentum factor). The size effect is of course explained by the SMB factor, but the liquidity factor does a relatively good job in comparison.

When instead sorting on book-to-market and size, our four-factor model has a hard time explaining the results. By comparison, only five (out of the 25) portfolios show significant alphas in the Carhart (1997) model, whereas our four-factor model show 13 significant alphas. The better performance of the Carhart (1997) model is as expected explained by the HML factor, which increases with book-to-market ratio.

However, for the remaining anomalies, our four-factor model shows better or about equal results regarding the financial distress, total accruals, asset growth, net stock issues, and SUE anomalies, compared to the Carhart (1997) model.

In particular, the financial distress and total accruals anomalies are by far better explained in our four-factor model. The total accruals anomaly is a bit harder to interpret already when looking at mean returns, but alphas are less significant in the four-factor model. Alphas for the asset growth and net stock issues deciles are similar in the two models. Although the HML factor shows a nice spread between firms with high and low investments, the investment factor does an even better job. The HML factor is, however, larger in magnitude, which means that the end results are similar. Moreover, the four-factor model explains earnings momentum (SUE) better than the Carhart (1997) model. This is no surprise given the results in Sadka (2006) where liquidity explains at least a part of the anomaly.

## 9 Conclusions

Based on the empirical tests, the liquidity factor is priced, and relevant in the Chen, Novy-Marx, & Zhang (2010) three-factor model, especially when liquidity is measured according to price impact measures such as the Amihud (2002) measure.

When it comes to explaining anomalies, results from the size and momentum sorted portfolios are of most interest. In previous models, a momentum factor has been needed to control for momentum profits. In this paper, however, momentum profits are explained by factors already included in the model, not directly related to momentum. In particular, the profitability and liquidity factors increase linearly with momentum.

The explanatory power gained from the liquidity factor is for all other anomalies relatively modest. The financial distress and total accruals anomalies are somewhat better explained when including the liquidity factor but the net stock issues, asset growth, and book-to-market anomalies show only small improvements. That the SUE anomaly is not fully or at least close to fully explained when adding the liquidity factor is surprising given that it previously has been associated with liquidity.

When comparing to Carhart's (1997) four-factor model, performance of the four-factor model is superior or at least comparable for the anomalies tested, with the exception of the book-to-market anomaly. Nonetheless, results could indicate superior performance also for other anomalies. Hence, our model can be used as an alternative four-factor model when trying to explain cross-sectional returns.

# 10 Suggestions for Future Research

It would be interesting to further investigate the underlying risk factor that creates the illiquidity premium. In this paper, good candidates have been proposed but no tests of their relative importance have been performed. For example, the probability of informed trading (PIN) measure from Easley, Kiefer, O'Hara, & Paperman (1996) could be used to see whether adverse selection problems are a driver of returns. Such an analysis could also help when choosing liquidity proxy.

Returns from the investment factor are, as has been shown above, sensitive to sample periods and overall relatively small. Thus, it would be interesting to see what happens when the factor is excluded from the model, replaced by some other factor, measured differently or tested using other samples.

Furthermore, the anomalies tested are just a sample of all anomalies found in the literature. More anomalies need to be investigated to show the relevance of liquidity in other settings, as well as compare the model to previous four-factor models.

# 11 Appendix

## 11.1 CRSP/Compustat Mnemonics

Appendix Table 1 - Data types used and corresponding CKSF Minemonics.						
Data Type	Mnemonic	Data Type	Mnemonic			
Book Equity	BKVLPS	Quarterly Earnings	IBQ			
Cash & Equivalents	CHEQ	Number of Shares	CSHO			
Cash From Operations	PIQ	Pastor & Stambaugh innovation in liquidity	PS_INNOV			
Consumer Price Index	CPI1	Price	PRC			
Current Assets	ACTQ	S&P 500 Return	VWRETD			
Current Liabilities	LCTQ	S&P 500 Market Value	TOTVAL			
Depreciation & Amortization	DP	Short-term Debt	DLC			
Earnings Announcement Date	RDQ	Total Assets	AT, ATQ			
$EPS_1$ Actual	EPS	Total Inventories	INVT			
EPS <sub>2</sub> Detail (1 year)	EPS	Total Liabilities	LTQ			
Gross PPE	PPEGT	Volume	VOL			
Income Taxes Payable	TXP	•				

Appendix Table 1 - Data types used and corresponding CRSP Mnemonics

## 11.2 The Effective Tick Measure

Following Holden (2009), first "let  $A_j$  and  $A_{J+j}$  be the total number of (trade) prices and (no-trade) midpoints, respectively, corresponding to the *jth* spread (j = 1, 2 ... J). Second, let  $B_j$  and  $B_{J+j}$  be the number of special prices and special midpoints, respectively, corresponding to the *jth* spread (j = 1, 2 ... J). At last, define  $O_{jk}$  as the number of price increments for the *jth* spread (j = 1, 2 ... J) which overlap the price increments of the *kth*spread and do *not overlap* the price increments of any spreads between the *jth* spread and the *kth* spread. Similarly, let  $O_{J+j,k}$  be the number of overlapping midpoints for the *jth* spread(j = 1, 2 ... J) which *overlap* the midpoints of the *kth* spread and do *not overlap* the midpoints of any spreads between the *jth* spread and the *kth* spread" (Holden (2009)). Panel A and B in Appendix Table 2 report coefficients for decimal and fractional price grids, respectively.

From Appendix Table 2, a general formula of the unconstrained probability of the *jth* spread can be calculated as:

# Appendix Table 2 - $A_j$ , $B_j$ and $O_{jk}$ for the Decimal Price Grid and the Fractional Price Grid

Panel A and B reports the number of possible exclusive observations on every dollar for the  $A_j$ ,  $B_j$  and  $O_{jk}$  variables using decimal and fractional price grids, respectively. P/M indicates whether it regards prices or midpoints.

Panel A: Decimal Price Grid								
j	Spread	P/M	$A_j$	$B_j$	$O_{jk}$			
1	0.01	Р	100	80				
2	0.05	Р	20	8	$O_{21} = 20$			
3	0.1	Р	10	8	$O_{31} = 0, O_{32} = 10$			
4	0.25	Р	4	3	$O_{41} = 0, O_{42} = 2, O_{43} = 2$			
5	1.00	Р	1	1	$O_{51} = 0, O_{52} = 0, O_{53} = 0, O_{54} = 1$			
6	0.01	М	100	80				
7	0.05	М	20	16	$O_{71} = 20$			
8	0.1	Μ	10	10	$O_{81} = 0, O_{82} = 10$			
9	0.25	Μ	4	4	$O_{91} = 0, O_{92} = 4, O_{43} = 0$			
10	1.00	М	1	1	$O_{10,1} = 0, O_{10,2} = 0, O_{10,3} = 0, O_{10,4} = 0$			
	Panel B: Fractional Price Grid							
j	Spread	P/M	$A_j$	$B_j$	$O_{jk}$			
1	1/16	Р	16	8				
2	1/8	Р	8	4	$O_{21} = 20$			
3	1/4	Р	4	2	$O_{31} = 0, O_{32} = 10$			
4	1/2	Р	2	1	$O_{41} = 0, O_{42} = 2, O_{43} = 2$			
5	1	Р	1	1	$O_{51} = 0, O_{52} = 0, O_{53} = 0, O_{54} = 1$			
6	1/16	Μ	16	8				
7	1/8	М	8	4	$O_{71} = 20$			
8	1/4	Μ	4	2	$O_{81} = 0$ , $O_{82} = 10$			
9	1/2	М	2	2	$O_{91} = 0, O_{92} = 4, O_{43} = 0$			
10	1	Μ	1	1	$O_{10,1} = 0, O_{10,2} = 0, O_{10,3} = 0, O_{10,4} = 0$			

$$U_{j} = \begin{cases} \left(\frac{A_{1}}{B_{1}}\right)F_{1} + \left(\frac{A_{J+1}}{B_{J+1}}\right)F_{J+1} & j = 1\\ \left(\frac{A_{j}}{B_{j}}\right)F_{j} - \sum_{k=1}^{j-1}\left(\frac{O_{jk}}{B_{k}}\right)F_{k} + \left(\frac{A_{J+j}}{B_{J+j}}\right)F_{J+j} - \sum_{k=1}^{j-1}\left(\frac{O_{J+jk}}{B_{J+k}}\right)F_{J+k} & j = 2,3,\dots,J \end{cases}$$

The effective tick measure is then simply:

$$Effective Tick = \sum_{j=1}^{J+j} \frac{\widehat{U}_j s_j}{\overline{P}_l}$$

Where  $s_j$  is the corresponding spread and  $P_i$  is the average price during time interval *i*.

### 11.3 Pástor & Stambaugh (2003) Liquidity Betas

Pástor & Stambaugh's (2003) gamma ( $\gamma$ ) is obtained from the regression

$$r_{t+1}^{e} = \theta + \phi r_t + \gamma sign(r_t^{e})(Volume_t) + \varepsilon_t$$

where  $r_t^e$  is the excess return of stock *i* above the relevant market index on day *t*, and *Volume*<sub>t</sub> is the dollar volume (in millions) on day *t*. 15 consecutive observations are needed each month for the stock to be included while stocks with a share price below \$5 and above \$1000 are excluded. In order to obtain liquidity betas, innovations in liquidity is constructed. To avoid biases arising from the growth of the stock market, a scaling factor defined as  $\frac{m_t}{m_1}$  is used.  $m_t$  is total market value at t - 1 of all stocks included at *t* and  $m_1$  is total market value in August 1962. The difference between the current and the previous month is averaged for all stocks in each month:

$$\Delta \hat{\gamma}_t = \left(\frac{m_t}{m_1}\right) \frac{1}{N} \sum_{i=1}^{N_t} (\hat{\gamma}_t - \hat{\gamma}_{i,t-1})$$

 $\Delta \hat{\gamma}_t$  is then used as the dependent variable in a regression with lagged  $\Delta \hat{\gamma}_t$  and scaled lagged  $\Delta \hat{\gamma}_t$  as independent variables.:

$$\Delta \hat{\gamma}_t = a + b \Delta \hat{\gamma}_{t-1} + c \left(\frac{m_{t-1}}{m_1}\right) \Delta \hat{\gamma}_{t-1} + u_t$$

The residuals from the last regression are serially uncorrelated and are used construct innovation in liquidity<sup>6</sup> (scaled by 100):

$$L_t = \frac{1}{100}\hat{u}_t$$

To obtain liquidity betas, we regress NYSE, Amex and NASDAQ stocks on innovation in liquidity and the Fama-French Factors. The liquidity beta,  $\beta_i^L$ , comes from the regression:

$$r_{i,t} = \beta_i^0 + \beta_i^L L_t + \beta_i^M M K T_i + \beta_i^S S M B_i + \beta_i^H H M L_i + \varepsilon_{i,t}$$

where  $r_{i,t}$  is the excess return of stock *i* at time *t*, MKT is the excess return of a market index above the risk-free rate and SMB and HML are size and book-to-market zero-cost portfolios, respectively (Fama & French (1992)). In December each year, we regress the stocks using the most recent five years of data continuing through the current year-end<sup>7</sup>.

<sup>&</sup>lt;sup>6</sup> The innovation of liquidity is downloaded from CRSP [PS\_INNOV].

<sup>&</sup>lt;sup>7</sup> The liquidity betas obtained using this procedure are in Pástor & Stambaugh (2003) referred to as "Historical Liquidity Betas".

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