STOCKHOLM SCHOOL OF ECONOMICS

BACHELOR THESIS IN FINANCE

# Predictive Power of the Volatility Smile

Henry Li[■] Johan Möller[□]

**Abstract**

The Black-Scholes option pricing formula yield lower volatility than volatility observed in the market when looking at option prices. Several theories have been presented to explain this phenomenon and how the world of finance can use this information. Parameterized volatility smile coefficients have been examined in several recent studies but the findings are inconclusive and the alleged predictive power of the smile has not yet been fully understood or proved. We contribute to the field of forecasting finance by proposing a model where coefficients from a parameterized volatility smile contain information that predict the probability and amplitude of geometric Brownian motion jump diffusion. Using S&P 500 options- and index data from 2006 to 2011 we found a negative relationship between curvature on the volatility smile and jump risk indicating that the smaller curvature on the volatility smile, the higher the probability for positive asset price jumps and vice versa.

**Keywords:** Volatility smile, Predicting power, jump risk, Brownian Motion with Jump diffusion, Maximum Likelihood Estimation.

**Tutor:** Jungsuk Han

**Date:** June 1st 2011

**Discussants:** Fredrik Hallgren and Rasmus Rehn

**Acknowledgements:** We would like to thank Jungsuk Han for his enthusiastic support and mentorship. We thank Kathryn Kaminski and John Sjödin for providing with inspirational ideas. Finally we want to thank Fang Li, Jian Kang, Daniel Åhlfeldt and Julia Appelgren for helpful comments.

[■] 21294@student.hhs.se
[□] 21768@student.hhs.se

## 1. Introduction

In the mid 1980s very few traders knew about the phenomenon called the volatility smile in the option prices. Trading with options was considered difficult as the price is set by the market and the underlying mechanics of the asset price development are not accounted for. Options were priced according to a Nobel prize winning formula, the Black-Scholes option pricing model[1] (BS model). The model uses a perfectly hedged option to calculate the price of the option, which doesn't include any volatility smile but rather a flat volatility curve. The volatility smile anomaly was discovered after the stock market crash of October 1987 and left many practitioners and academics wondering why this shape of the implied volatility over strike prices existed. Ever since the October 1987 market crash the volatility smile, that still haunts the minds of traders, has served as a reminding grin of the historical devastating crash.

The BS model assumes constant volatility and would therefore apply historical volatility as the future volatility, but using this value one soon finds that far out- and far in-the-money options on the market are overpriced relative to the at-the-money option. This implies that if the BS model is the true model then arbitrageurs should want to sell these overpriced options and wait for the market to restore its flat volatility curve. The problem is that these "mispricings" persists and would therefore imply that options do not use the historical volatility but rather another measure (or other option pricing formula than BS) for volatility. When first calculated in the late 1980s, the implied volatility revealed that the constant volatility assumption in the BS model does not seem to hold. The prices indicated that the implied volatilities calculated by inverting the BS model increased as the option was out-of-the-money. The further out-of-the-money the option is, the higher the implied volatility. The phenomenon of the volatility smile that prevails as a result of higher implied volatilities in the out- and in-the-money options, or so called fat-tails in the return distribution, has since its discovery been a topic of many finance papers.

The smile as a whole can be seen as a deviation from the Black-Scholes option pricing formula and there has been discussions about whether or not this somewhat unexplainable and mystical smile contains any valuable information or not. Theories have been developed to explain the existence of the volatility smile. Some argue that the smile exist due to market inefficiencies such as transaction costs, illiquidity etc. while other choose to explain the smile with behavior biases in traders' risk appetite. Irrespective of which explanation is true they all seem to converge to the conclusion that the smile occurs from a violation of the simplistic assumption of log-normal

---

[1] Henceforth referred to as the BS model

return distribution in the BS model. The purpose of this paper is to retrieve some of the information that is contained in the market by using the smile anomaly in option prices that should not exist if prices were set efficiently and follows the BS model. It might be argued that the option traders' future looking view have some predictive power. When the market expects higher future volatility and larger jumps in prices the tails of the volatility smile would increase and thus create a steeper smile. The steeper the smile the larger is the probability of a high volatility regime in the future. Our hypothesis is that there are market inefficiencies and information in the volatility smile, for whatever reason, that could be used to understand future jumps in stock prices and volatility regime shifts.

This paper examines the possibility of using the second derivative of a squared line fit to the volatility smile as a market mood proxy to foresee future price jump regimes in the market. Our hope is that this study might help practitioners and academics to foresee when markets switch between high- and low volatility regimes in order to understand the underlying factors of large market movements.

The efficient market hypothesis proposed by Fama (1965) suggests that markets on average correctly price assets and that historical data cannot foresee future stock movements but this hypothesis has been questioned by the behavioral finance literature as stated by, among others, Kahneman & Tversky (1979). Previous research has been made in the field of predicting market behavior by using implied volatility derived from option prices. There have however not, to our knowledge, been studies using the whole range of options prices, i.e. the volatility smile, to predict future high/low market volatility periods. This study contributes to previous studies by offering a thorough examination of whether, or not, options prices in the form of a volatility smile can predict when the market will enter in to a certain volatility regime where traditional Geometric Brownian Motion[2] (GBM) model cannot describe the fat-tails that occur in the return distribution. We argue that there are two types of volatility regimes. The first regime contains no jumps and moves according to the underlying mechanics of the GBM model while the second regime is defined as a GBM universe with abnormal jumps that cannot be explained by the GBM model. The abnormal price jumps are measured through Geometric Brownian Motion with jump diffusion[3] (GBMJ) as a parameterization of the stock return dynamics.

[2] Henceforth referred to as GBM
[3] Henceforth referred to as GBMJ

Our study uses the volatility smiles' parameterized coefficients to try to predict future S&P 500 irrational behavior through abnormal price jumps. Instead of launching new theories about asset price dynamics in our paper we attempt to find a proxy of the underlying explanatory factors in the GBMJ model (i.e. information) and test if an anomaly said to contain information could say anything about a model that moves according to information arrival. It is important to point out that this paper has not analyzed trading strategies based on the findings and thus the method of creating arbitrage on this information has not been tested.

We test the volatility smile parameter correlation on the next period's GBMJ model's jump probabilities and amplitudes. In order to find whether or not the volatility smile can predict which of two models (GBM or GBMJ) better predict asset price development, several steps were required. Firstly a continuous dataset of second derivative coefficients from a squared fitted line to the volatility smile were calculated as described in section 4.1. The resulting coefficients were then regressed against several sets of jump coefficients derived from applying Maximum Likelihood Estimation[4] (MLE) for the GBMJ model on S&P 500 index returns.

Statistically significant results were found for longer maturities implying that curvature for longer time-to-maturities could predict high/low regimes for our sample period. Although this is a positive result it remains unclear why similar results could not be found for other maturities which leave the question of the legitimacy of the test. This study concludes that there is weak evidence for the curvature of the volatility smile in helping to predict low and high volatility regimes in the sense of asset price jumps. Nevertheless more research has to be done within this topic to make more certain conclusions. This study contributes to the forecasting finance literature by making an extensive study in forecasting future jumps in stock prices with the volatility smile coefficient for the period May 2006 to April 2011.

## 1.1 Outline

This paper begins with a brief study of relevant earlier research in the models used and papers explaining the intuition behind their respective uses in section 2. The paper then states the data sources used and the methods by which the data has been handled in section 3. We then explain how we extracted the option informational content by a parameterization of the volatility smile and deriving dynamics of the underlying asset movement by numerical methods in section 4. The volatility smile parameter was tested as an explaining factor of the dynamics parameters through a series of regressions in section 5. In section 6 we discuss the implications of the findings in

[4] Henceforth referred to as MLE

section 5 and then the validity and potential problems of these results are discussed in section 7. The findings of the paper are finally concluded in section 8.

## 2. Previous literature

Earlier studies have tried to explain the change in volatility for options with same expiry date over different strike prices. The resulting volatility smile from such data has been parameterized and analyzed but findings have been difficult to interpret. The increase in implied volatility in out- and in-the-money options that causes the volatility smile has been a strenuous subject of study for many academics in finance. Several papers have been trying to explain the smiles by extending the traditional BS model with models that incorporate stochastic volatility (Hull & White, 1987), implied binomial tree (Derman & Kani, 1994) or discontinuous jumps (Merton, 1976) in the stock price. This field of study is in itself a very challenging and intriguing subject which requires further research to find a model that is coherent with the observed smile; however this paper will not make any attempt in explaining the smile but rather study if there is information in the smile that can forecast future volatility regime shifts in stock prices.

Since option markets are forward looking it forces the traders in these markets to estimate the future volatilities to correctly price the options. If information about future events is priced in the option market before it is priced in the equity market it naturally follows that the implied volatility extracted by inverting the BS model can be a feasible predictor of future volatility. Many studies have been done within this subject to test this proposition and the results have been very varying, both positive, indicating that the implied volatility is a feasible predictor (Malz, 2001) and negative (Chakravarty et al., 2004). Overall the empirical studies on the forecasting power of the implied volatility have come to contradicting results but compared with other models it is the most accurate forecasts on future volatility for all forecast horizons and performance measures considered (Blair et al., 2000). The implied volatility has been used by practitioners as a predictor for future volatility since the early eighties (Beckers, 1981) and it is regarded so important that it has even been given its own index, the Market Volatility Index[5] (VIX). The VIX has been found to be a good proxy for market mood as found by Sun & Yu (2010). Sun & Yu found that implied volatilities contain information about the future market behavior and the general mood of the market. They conclude that if the well informed traders' mood is contained by the implied volatility, the implied volatilities should be able to predict future abnormal returns given that the fears of traders are rational. Rational traders are loss averse and will be cautious of e.g. short sales

[5] Henceforth referred to as VIX

when the volatility is high. A pessimistic or cautious mood is shown through higher prices due to volatility and higher implied volatility reflects this cautiousness.

The problem of pricing options such as calls and puts was presented with a solution in 1973 when Fischer Black and Myron Scholes presented a paper with the idea to create a perfectly hedged risk-neutral portfolio that eliminated systematic risk (Black & Scholes, 1973). As with most financial models, the BS model requires some assumptions to make its derivation possible. According to the BS model, the stock prices move according to a GBM model as shown in equation 1.

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{1}$$

The GBM model states that stock prices develop according to a constant drift or return $\mu$ and a constant volatility $\sigma$. The dt term is time and dW is a Wiener random walk parameter.

The issues with using GBM as the underlying model for pricing options was early recognized for its flaws. Option prices in the market did not follow the rationale of the theory and showed that the returns were leptokurtic or fat-tailed which was an intrusion to the log normal assumption in the model. Merton extended the GBM model by adding a theory to consider the possibility of asset price jumps (Merton, 1976) to explain the fat-tails. The resulting model was the GBMJ model which adds a new jump variable J to the classical GBM model:

$$\frac{dS}{S} = \mu dt + \sigma dW + J dq \tag{2}$$

The dq term is a Poisson counter with intensity $\lambda$, i.e. the probability that dq=1 is $\lambda$dt.

This alternation of the standard GBM model implies that there are normal and non-normal market returns. The return when dq=1 is $\mu$+J where J is the addition by the jump. J is assumed to be normally distributed with a mean of $\mu_J$ and variance $\sigma_J$.

The GBMJ is not a closed form function and in order to obtain the GBMJ parameters it needs to be numerically solved for. A method historically used to obtain population parameters from samples is Maximum Likelihood Estimation (MLE) in which candidate parameters are tested for their likelihood to be the true population parameters by maximizing a log likelihood function.

The log likelihood equation for estimating the parameters in the GBMJ model as defined by Craine et al. (2000) is:

$$log\ likelihood(\theta, y) = \sum_{t=0}^{T} \ln \left[ \sum_{q=0}^{Q} \frac{e^{-\lambda}\lambda^q}{q!} \frac{1}{\sqrt{2\pi\left(\sigma_B^2 + q\sigma_J^2\right)}} \exp\left( \frac{-\left(y_{t+1} - \mu_B - q\mu_J\right)^2}{2\left(\sigma_B^2 + q\sigma_J^2\right)} \right) \right] \quad (3)$$

The log likelihood equation is a function of the unknown parameters θ. The parameters are:

$$\mu_B = Drift\ in\ the\ GBMJ\ model$$
$$\mu_J = Average\ jump\ amplitude$$
$$\sigma_B = Volatility\ of\ the\ underlying\ asset$$
$$\sigma_J = Volatility\ of\ jump\ amplitude$$
$$\lambda = Intensity\ of\ Poisson\ counter\ \left(Prob(dq = 1)\right)$$

The variances of the parameters are inversely proportional to the observations as described in equation 8 in section 4.2. MLE is therefore a very efficient estimation method even for small samples. Derivation of the log likelihood function can be found in an article by Jorion (1988).

In order to build a continuous database with GBMJ coefficients one needs to divide the data into periods. Moving windows have been used in financial research earlier as found in the research by Pesaran and Timmermann (1995). Pesaran and Timmermann try to predict future returns by using moving window periods. In this paper we have used moving windows as explained in section 4.2.

Models that price options with GBMJ result in a visible volatility smile (Kou, 2002). A drawback of such models is that options are more difficult to price than with the traditional BS model and need to be numerically solved for. Another method for dealing with fat-tails in return distributions and the deviation from the log normal assumption is to use stochastic volatility. The stochastic volatility method requires extreme parameters that are implausible for options (Bates, 1998). Bates found that the jump diffusion model is a better model to predict option prices. The jump diffusion model does have its drawbacks. The jump itself is assumed to be normally distributed and implicit distributions from the model is inconsistent with large stock index returns between 1988-93 (Bates, 1998). The GBMJ model parameters have to be numerically estimated and the best current method for estimation is the MLE method (Eraker, 2004; Pratt, 1976).

As discussed earlier the implied volatility has a forward looking power and might forecast future volatility. However implied volatility contains merely the information from a single option for each expiry without taking into account a wider range of strike prices. Using all option prices over the strike prices as summarized by the volatility smile, the volatility smile might be viewed as an extension of the implied volatility with more exhaustive information incorporated. The following paragraph will extend the discussion on the volatility smile.

In accordance with its component parts, i.e. implied volatility, the volatility smile should be forward looking and contain information about the market expectations. There have been papers written on the predictive power of the smile with regards to market disturbances such as crashes. Gemmil (1996) discusses the predictive power in the skew of the volatility smile for anticipating the 1987 crash. By a parameterization process of the volatility smile, weighing the relative bias of the skew, Gemmil found that his model could not predict the crash by option pricing. However, later studies have found a predictive power of the slope of the smile by modeling with in and out-of-the money options to deduct a smile slope that could predict future stock returns (Deuskar et al., 2007; Shu yan, 2011). Using data between January 1996 to June 2005 Shu yan (2011) used the curvature of the volatility smile as a proxy of the jump risk from the GBMJ model and found that there is a negative relationship between the return of a portfolio and the curvature of the slope, i.e. low slope portfolios earn a high return and vice versa; high slope portfolios earn a low return.

This paper is an extension to previous studies and contributes to the research on the predictive power of the volatility smile. The approach in this study is to parameterize the smile by extracting the second derivative coefficient of the curvature for options using a wide range of strike prices. It might be argued that the intensity of the curvature of the smile contains information about future GBMJ jumps in stock prices much like the assumption made by Shu yan (2011). Previous research by Merton (1976) suggests that there are two types of jumps. The first category includes jumps that might be expected from information arrival to the market. These jumps can result from new information to the market or due to periods without trade. The jumps can also occur when markets are experiencing abnormal shocks and returns are non-normal. The second category consists of jumps in markets which result from unexpected events such as terrorist attacks, earthquakes etc. Jumps from the first category are sometimes described as predictable and might be found by modeling in the historical data. These jumps will be the subject of study and also serve as the definition of jumps henceforward.

## 3. Data

In order to construct the coefficients of the volatility smiles for different dates and time-to-maturities, data on plain vanilla options on the S&P500 Composite Index was used. With Thomson DataStream Advance 4.0, option-, VIX-, and S&P 500 index-data was extracted for the period between May 2006 and present day, April 2011. May 2006 represented the earliest day for which option data was available in DataStream and consequently May 2006 was chosen as the first date in the data set to maximize the amount of data used in the analysis and capture as much information as possible in the historical data.

### 3.1 Option data

Data is given for the exchange traded stock index option[6] comprising of daily closing prices of each option maturing on the third Friday in every month for the period May 2006 to April 2011. This means that the data contains 60 expiry dates and a series of strike prices for every expiry date. From initial data there were 10 505 options with daily prices ranging from at least 80 days to maturity to expiry date. Prices with longer time-to-maturity than 87 days were excluded from the analysis since data did not exist for some options earlier than this date and the further away the option got from its expiry date the more prices were missing in the data as can be seen in figures 1 and 2 in the appendix wherein the data is visualized. After excluding longer time-to-maturities we obtained a time series data of 913 935 observations.

In order to standardize the options data to address the fact that the underlying index itself moves, we defined moneyness as percentage in and out-of-the-money relative to its spot price,

$$M = \frac{K}{S_0} \tag{4}$$

where M is the moneyness of an option, K the strike price and $S_0$ index spot price. Trading volumes for the options were not used to check for illiquid options that could be mispriced and be far away its market equilibrium value owing to low liquidity. Instead moneyness of ± 20%, i.e.

$$0.8 < M < 1.2 \tag{5}$$

were used to ensure that illiquid far out- and in-the-money options were not included in the dataset since near the money options are more traded than far out- and in-the-money options.

[6] Put options were chosen since there were more data points for puts than for calls. Because of the put-call-parity it does not matter which is used when calculating implied volatility so puts and calls can thus be used interchangeably.

### 3.2 S&P 500 & VIX

The Standard & Poor 500 Composite Index was used to calculate the GBMJ parameters. The index measures American companies only and weight the returns with regards to market capitalization. The reason for using the S&P 500 was that historical option data is difficult to find for other indices and the S&P 500 has been the subject of many analyses, indicating that traders on the S&P 500 options might be better informed and convey this information through option prices. In order to use MLE as described in section 4.3, the log-returns of the S&P 500 were calculated.

We used the Chicago Board Options Exchange[7] (CBOE) VIX with daily index values for the whole period to estimate the market expectation of the implied volatility of the S&P 500 Index option and the future stock market volatility.

## 4. Methodology

### 4.1 Implied Volatility Smile

All option prices obtained from DataStream were converted into BSM implied volatility calculated using a Newton-Raphson iterative algorithm. Furthermore, a certain set of time-to-maturities were chosen to plot the implied volatilities around the index, thus creating a volatility smile for each set. As described in section 3.1 strike prices with moneyness ± 20% were used to control for illiquid far out- and in-the-money options. Therefore the window and data points for each plotted smile were moved according to its prevailing daily spot price. Since there is a change in the volatility smile owing to the time parameter, different time-to-maturities options were not compared with each other to avoid the mismatch problem in many prior studies (Lopes, 2000). There are some techniques used by financial engineers to neutralize the time effect by discounting the inverse of the square-root of the time parameter (Natenberg, 1994). These methods have shown to make the smiles less dependent of the time-to-maturity but will however not be employed in this study. The time-to-maturities chosen to be analyzed were 10 days, 21 days, 30 days, 45 days and 87 days to maturity. It is arguable that shorter time-to-maturity options are more liquid than longer dated time-to-maturities and that is why more weight was put on the shorter dates. Longer dated options such as 87 days to maturity were included to extend the prediction window and in addition capture more information incorporated in the options.

[7] Henceforth referred to as CBOE

Implied volatility data for each time-to-maturity was extracted within the moneyness boundary and modeled in STATA 11. The nature of the plotted implied volatility data for every time-to-maturity seems to follow a positive second degree curvature which creates the characteristic smile-effect. A fitted line regression was run in STATA to find the second degree coefficient of the smile; the $\tau$ in equation 6.

$$\sigma_{imp,i,t} = \alpha_{it} + \beta_{1,i,t} strike + \tau_{i,t} strike^2 + \varepsilon_{i,t} \qquad (6)$$

Using the $\tau$ coefficient for each month $i$ and each time-to-maturity $t$, a time-series of coefficients over time was created. The information in this coefficient should help in predicting the future market behavior as the volatility smile is forward looking. In order to estimate market jumps in stock prices we chose to model the market behavior according to the GBMJ model.

$\tau$ coefficients that were t-tested and found to be insignificant on the 5% level were removed from the data as a rule to eliminate potentially biased data points. The choice is debatable being the case that the insignificance in the smiles itself could contain important information. Further tests need to be done to exclude the insignificance owing to erroneous data rather than valuable information. This analysis is however out of the scope for this paper.

### 4.2 Geometric Brownian Motion with Jump diffusion

Using only GBM to describe stock price movements has been contradicted by several researchers (Merton, 1976; Eraker, 2004; Craine, 2000) and furthermore options prices deviate from the fair price according to a model that prices options according to an underlying asset that moves according to GBM, i.e. The BS-model. In order to capture the fat-tails that result from the informational jumps as mentioned in section 2 we used GBMJ as suggested by Merton (1976). The model estimation obtains the drift $\mu$, volatility $\sigma$ for the stock price and the jump components respectively. For the jump coefficient we also obtain the intensity of the Poisson counter, the $\lambda$ value. The $\lambda$ is the probability that the jump coefficient is active and there is a jump in the market as described in equation 2.

It is unclear exactly what information the curvature of the volatility smile contains but a model to match information sending from the volatility smile and information received by the market is proposed in equation 7.

$$\frac{dS}{S} = \mu dt + \sigma dW + J(X)dq \qquad (7)$$

The jump coefficient J depends on X that represent some unobserved factor that temporarily affect the market by asset price jumps due to new information arrival as stated by Merton (1976). A part of this factor might be mood as suggested by Sun & Wu (2010), it could also be due to some other factor that is not observed but partly observable through parameterization of the volatility smile shape.

In order to divide the S&P 500 returns in jump regimes and no-jump regimes we estimated the jump probability $\lambda$, the jump magnitude $\mu_J$, and the jump volatility $\sigma_J$ for moving windows. Each window starts the day after the date we use to extract the volatility smile and extends to the exercise date the following month.

Example: If the smile was extracted for the 2008-02 option (strike date was February 15th 2008) with time-to-maturity 30 (i.e. January 16th), the smile was assumed to contain information from the day after the date we calculated it until the exercise of the next option (i.e. the 2008-03 option expiration date: March 21st). The relevant moving window was from Jan 16th 2008 to March 21st 2008 (65 days window with 47 trading days). The procedure is visualized in figure 3.

Moving windows are necessary as the MLE of parameters become more dependable the more observations one use. The windows are overlapping as shown in figure 3 but using values several times did not seem to create biased coefficients (coefficients with no overlap were almost identical to the ones with overlap). Using moving windows has the positive effect of lowering the variance of the coefficients. There is little literature on how few values that could be used in MLE as the method often uses entire populations and not subsamples of it. A reassuring fact is that the MLE method is asymptotically effective as it yields a parameter with variance as depicted by Pratt (1976) in equation 8.

$$Variance = \frac{sample\ Variance}{n} \tag{8}$$

Using the time span described above is intuitive as an option with expiry in 10 days should not say much about the period before but rather the expectations for the next period. This is due to the options being reflections of forward market conditions conditional on the present conditions.

This method of using moving windows for jump diffusion modeling by MLE has not, to our knowledge, been used before but results in section 5.2 suggest that it can correctly model the jumps of the relevant periods. The moving windows let us create continuous data to match with the curvature of the smile.

## 4.3 Maximum Likelihood Estimation

MLE was used to find the GBMJ model parameters for each window. The method uses a numerical maximizing function[8] and delivers the parameters that have the highest likelihood of being correct according to the predefined log-likelihood function. The method is sensitive to the length of the estimation windows and too short windows can lead to biased results as the variance of the parameters is inversely dependent on the number of observations as seen in equation 8.

### *4.3.1 MLE initial values*

Deciding on initial values in numerical methods can have a large impact in the efficiency of the algorithms. A problem with a random choice of initial parameters when using numerical methods is that one might find a local max or min and takes that value for the global maximum or minimum. Testing for different initial values can show if the true maximum for the log-likelihood function is found and not a local one. All initial values were changed to test for this and the result showed that there were only global maxima. The only significant difference when changing the initial values were the number of iterations needed and the convergence of the algorithm.

*Parameters*

The initial values have to be carefully chosen in order for the log-likelihood function to converge. Earlier researchers have had difficulties with non-convergence probably due to this. The standard initials of all zeros or all ones (or a permutation of these) generate non converging estimates and it is advisable to have non equal and positive initial values in order for the method to converge to one set of parameter values for each period.

*Maximum daily jumps*

The maximum number of daily jumps in equation 3 (see section 2), Q, is set to 5. The number of daily jumps can be altered and in principle set to an infinite amount but this alteration has a diminishing effect on the parameters as can be seen in the log-likelihood function in section 2. Brigo et al. (2009) used 5 while Crain et al. (2000) used 10.

## 4.4 Regression

To analyze the predictive power of the volatility smile ordinary least squared (OLS) regressions were run between variables extracted in the volatility curvature in section 4.1 and the jump

[8] We used Matlab's MAX function for maximization.

diffusion parameters from the MLE algorithm mentioned in section 4.3. The regressions followed the set-up of our parsimonious model in equation 9.

$$\delta_t = \alpha + \beta \cdot \tau_{t-1} + \varepsilon \tag{9}$$

Where $\delta_t$ is the jump variable from MLE in the moving estimation window and $\tau_{t-1}$ is the smile coefficient variable taken from the first date of the estimation window. The sign and magnitude of $\beta$ will be the parameter in focus when making inference on the forecasting power of the smile.

In the first regressions the curvature variables $\tau_{t-1}$ for TTM 10, TTM 21, TTM 30, TTM 45 and TTM 87 were regressed on some more primitive variables such as S&P500 return and VIX - representing $\delta_t$in equation 9 - to check for correlation between curvature and index. This is mainly to explore if the indexes can be explained by the forward looking curvatures in the volatility smile.

In the second regressions parameters from the MLE were incorporated as the dependent variable $\delta_t$ in equation 9 and ran on the same curvature variables with different time-to-maturities. A new variable was created at this stage to capture the probability, size and direction of the jumps; $\mu_J \cdot \lambda$ is the product of lambda $\lambda$ and jump amplitude $\mu_J$ and gives a notion of an average expected jump. To make the inference of the predictive power of the curvature some lagging windows needed to be modeled to capture the correct explanatory rule. The curvatures were given for a certain date in every given month, e.g. for the parameter TTM 10, ten days before the expiry date (third Friday in each month, see section 3.1) a curvature was generated for the option. Furthermore, from section 4.3 jump diffusion parameters extracted from the S&P500 Composite Index using moving windows were used as the independent variables in the regressions on the dependent curvature variables. The way the windows were constructed is such that for each different time-to-maturity the time horizon for which the jump variables were estimated follows a logically constructed pattern. The pattern for the windows started from the day after the date for which the curvature was extracted and extended to the expiry date for the option with expiration one month later. Taking TTM 10 as example figure 3 shows how the moving windows were created for TTM10 and thus the time period for which the jump parameters were estimated from. To make the regressions consistent, the same pattern was followed for the other time-to-maturities hence prolonging the estimation windows for longer maturities relative its shorter counterparts.

Our third analysis was conducted on the volatility smiles on a more aggregate level. New variables were created from the earlier extracted coefficient parameters as the independent variable $\tau_{t-1}$ in equation 9. The parameters average smile coefficient and standard deviation of the smile coefficient were generated by simply calculating the mean and standard deviation of the smiles for each option. This follows the intuition that the average smile coefficient and standard deviation for each option during its life time has information contained in a more aggregate level. To make the mean and standard deviation calculations more consistent and robust nine more time-series of smile coefficients were added between the TTM10 and TTM87 with more weight on the shorter dates.[9] These results were subsequently regressed on the indexes and jump parameters with the same moving windows as the TTM 10 since the new parameters include information up until the date of TTM 10.

## 5. Results

This section presents our results in a logical order following the set-up described in section 4.4. Section 5.1 reports the results from the volatility smile dynamics, the coefficients and how the smile changes over time with different maturities. Section 5.2 focuses on the jump diffusion parameter estimations from MLE with graphs to show when jumps occurred on the S&P500 Composite index for the sample period. Finally section 5.3 puts the two results from 5.1 and 5.2 together to examine if there were any relationships and consequently predictive power between the volatility smile coefficients and jump parameters.

### 5.1 Volatility smile dynamics

Table 1 reports the descriptive curvature statistics for different maturities. The second degree coefficients become smaller and less volatile the further away the option gets from the expiry date. Figures 4 and 5 show the decreasing smile coefficient graphically. This follows the intuition that the longer (shorter) the maturity the more (less) flat the curvature of the smile. This implies that traders become more concerned about a jump in prices the nearer they come to the expiry date as more information becomes available about the future. Insignificant values with p-values over 5 percentages were removed from the sample when regressing the second degree curvatures which explains the difference in number of observations in table 1.

[9] The time-to-maturities used was TTM10, TTM 15, TTM 18, TTM 21, TTM 23, TTM 28, TTM 30, TTM 35, TTM 42, TTM 45, TTM 50, TTM 60, TTM 74 and TTM 87.

### 5.2 Jump diffusion dynamics

Tables 2-6 summarize the descriptive statistics for the jump components obtained from the MLE method. The method used to obtain the jump diffusion parameters yielded noisy data as shown in figures 6-8. This paper focuses on jump diffusion components and of these we especially looked at the jump amplitude $\mu_J$, and the Poisson intensity $\lambda$. $\mu_J$ varies over time seemingly at random as depicted in figure 6. $\mu_J$ does, however, contain information on upwards or downwards jumps and how big the jumps are and should thus not be completely omitted from the study. In section 5.3 we therefore created a variable that captures the effect and the probability of the jumps. Figures 9, 10, and 11 show the results from the MLE method for TTM 45 against the S&P 500 market returns, and $\mu_J$ indeed seem rather independent from the daily returns as depicted in figure 9. Figure 10 indicate that the $\lambda$ coefficient captures the market abnormal returns defined as returns more than two standard deviations from the mean return. Market abnormal returns from figure 10 are shown with its direction and size in figure 12. The high $\lambda$ values seem to be well timed to the abnormal returns in figures 10 and 11 and the correlation between the two examined factors in figure 11 is -0.44. All these findings suggest that the method of using moving windows for MLE to estimate market jumps has some validity and that that the regression of the curvature as independent variable and $\lambda$ as dependent have information about the predictive power of the volatility smile. This partly confirms the assumption made by Shu yan (2011), i.e. that volatility curvature proxy GBMJ jumps in the market.

### 5.3 Combined dynamics

Table 7 shows the correlation matrix between the parameters as described in section 4.4. It demonstrates a varying correlation between the smiles' coefficients and MLE jump parameters both in size and direction.

Since correlation in itself says little about the statistical significance when trying to make inference table 8 with p-values instead of correlations was created with OLS regression. The tables were created from regressing each of the dependent variables (horizontal) on the dependant variables (vertical). P-values with significance under the five percentage label are highlighted in bold. Although the significant values are numerous yet still ambiguous with no clear pattern over the series of different variables there seems to be a consistent significant relationship between the coefficients in TTM 30, TTM 45, and TTM 87 on the λ and $\mu_J \cdot \lambda$.

Given these results, we focused on the forecasting power for the smiles coefficients of TTM 30, TTM 45 and TTM 87 on parameters $\lambda$ and $\mu_J \cdot \lambda$. The following paragraphs present and discuss the results of the regressions with the main interest on the direction of the relationship i.e. significant positive or negative relationships. Table 9 reports extended results from the regression on the chosen variables including coefficients and t-values. In equation-form $\delta_t = \alpha + \beta \cdot \tau_{t-1} + \varepsilon$ the regression results are reported as following:

$$\lambda_t^{TTM30} = 35.263 - 39.363 \cdot \tau_{t-1}^{TTM30} + \varepsilon \quad (10)$$

$$\lambda_t^{TTM45} = 20.716 - 22.311 \cdot \tau_{t-1}^{TTM45} + \varepsilon \quad (11)$$

$$\lambda_t^{TTM87} = -4.903 + 9.176 \cdot \tau_{t-1}^{TTM87} + \varepsilon \quad (12)$$

$$\lambda_t^{TTM30} \cdot {\mu_J}_t^{TTM30} = 1.437 - 1.588 \cdot \tau_{t-1}^{TTM30} + \varepsilon \quad (13)$$

$$\lambda_t^{TTM45} \cdot {\mu_J}_t^{TTM45} = 0.675 - 0.804 \cdot \tau_{t-1}^{TTM45} + \varepsilon \quad (14)$$

$$\lambda_t^{TTM87} \cdot {\mu_J}_t^{TTM87} = 0.034 - 0.065 \cdot \tau_{t-1}^{TTM87} + \varepsilon \quad (15)$$

where $\lambda_t$ is the Poisson probability of a jump in the estimation period, $\tau_{t-1}$ represents the second degree coefficient of the smile in the beginning of the estimation period and ${\mu_j}_t$ corresponds to the magnitude and direction of a jump. Equations 10-12 describe the relationship between the smiles coefficient and the Poisson jump probability. For shorter maturities there seem to be a negative relationship between $\lambda_t$and $\tau_{t-1}$ i.e. the flatter the coefficient is in the beginning of a period, the larger is the probability of a jump during the whole period. TTM 87 on the contrary shows a positive relationship between parameters $\lambda_t$and $\tau_{t-1}$ which implicates that the steeper the smile 87 days to maturity the larger is the possibility of a jump. The magnitude of the coefficient seems to follow a pattern of increasing absolute value the nearer to maturity. Since equations 10-12 only describe the relationship of the smile coefficient and the probability of a jump in any direction, equations 13-15 incorporate the direction, magnitude and probability in the jump by multiplying the parameters $\lambda_t$ and ${\mu_j}_t$as the dependent variable to describe a notion of expected average jump. As $\lambda_t \geq 0$, the term $\mu_J \cdot \lambda_t$ will keep the sign of $\mu_j$.

Equations 13-15 describe the forecasting power of the smile coefficient on the average expected jump and this time showing a consistent negative relationship between the parameters meaning that the flatter the smile is the higher probability there will be for a positive jump in the estimation window following and vice versa, the more curvature on the volatility smiles the larger the probability for a negative jump in prices. The magnitude of the coefficients seems to follow the same as equation 9-12 i.e. the nearer maturity the larger the magnitude and importance the coefficient has on its predictive power.

Although the coefficient of equation 15 is not significant on a 5 percentage level of significance (see table 9) the sign and magnitude of it is consistent with the pattern of earlier maturities.

## 6. Implications

The results from section 5.3 show that there is a predictive power of the volatility smile on the jump variables in the following period. Even though it's puzzling why the results are varying in significance over different TTM and variables there is a somewhat consistent pattern over the statistical significant values to proceed in making inference. Equations 10-15 all show the same pattern of a lower magnitude of the smile coefficient the further away from expiry date. These results might be explained by differences in the values of both the dependent and independent variable and as described in earlier sections (see section 4.1) different time-to-maturities will not be compared and elaborated but rather the sign of the coefficient is important for inference.

Equations 10-12 portrait a negative relationship between the smile coefficient and the probability of a jump in the following period. This rather peculiar and intuition contradictory result implies that when the option market discounts a high variability in future prices (i.e. high smile coefficient hence a steeper smile) the smaller the probability of a jump will be in the underlying during the following period.

When looking at equations 13-15 the results became more interesting. It followed that the beta coefficient to $\tau_{t-1}$ is negative as before, however in these equations the dependent variable incorporates the sign and magnitude of the jump as a new variable $\mu_J \cdot \lambda$. $\mu_J \cdot \lambda$ is interpreted as an expected average jump and takes negative values for expected negative jumps and positive values for expected positive jumps. The same logic follows the implications made from equations 10-12 however since positive and negative jumps were incorporated inference about the direction of the jump could be deduced. When the smile coefficient is high (i.e. a steep volatility smile) the larger is the probability of a negative jump and vice versa, the smaller the smile coefficient the larger the probability of a positive jump. It is remarkable that there was a threshold for the size of the smile coefficient for when a negative jump forecast becomes a positive jump forecast. This implies that trading strategies with long/short rules can be induced for a certain holding period by extracting the smile coefficient in the beginning of the same period. Why there would be a higher risk for a positive jump for a flat smile is questionable. Indirectly the results from equations 13-15 tell that the market is more predictive of a negative jump than a positive given that a steep smile means that the markets discounts larger moves on the market while flat smile is usually seen during less distressed periods during a lower volatility regime. Even though this might seem to be contradictory the results are in line with previous research made by Shu yan (2011). Shu Yan argued that the implied volatility smile curvature can be regarded as a proxy to the jump risk and found similar results to our study, i.e. low slope portfolios earn a high return and vice versa; high slope portfolios earn a low return (we obtain the same relationship but with jumps instead of returns in the same positive or negative direction). By showing the significant relationship between the volatility smile and future jump risk our results consequently confirm Shu Yan's assumption that the curvature of the volatility smile is a feasible proxy for market mood and jump risks.

## 7. Problematization

As pointed out earlier in this study there are several of problems with running the MLE algorithm that might create bias in the extracted jump parameters. Firstly it is a complex method that makes advanced estimations from a single input (i.e. log return values). It is questionable if there is enough information in such an input to create jump estimations for a certain window. The algorithm in the numerical method to maximize the log-likelihood requires input of a start value which itself can create bias depending on the unknown distribution of the likelihood function. Furthermore, large jumps are rare events so long time series and estimation windows are required to make estimations of when jumps occur. Misspecifications in the model can create computation errors that are difficult to control for. For example in the estimations made in this thesis the $\lambda$ probability values took extreme values for some time windows which is questionable and will be heavily weighted when using OLS. Some of these concerns have been expressed in earlier research (Shu Yan, 2011) which is why the existence of academic papers about jump diffusion models using MLE is very limited.

When modeling the curvatures of the second degree derivative of the volatility smile statistically insignificant curvatures over the 5 percentage level were removed from the time-series data. It is debatable whether or not insignificant curvature originates from data error, occurring from calculation- or input errors in the dataset, or if it in fact contains valuable information that should be taken into account. A systematic study of the cause of these curvatures is needed to draw further conclusions.

The jump parameters in the final regressions used different length of the moving windows for different time-to-maturities as described in section 4.2. The difference in window range gave different jump values both in magnitude and probability and could be a source of problems in the OLS regressions and the following inferences.

The smile coefficient in itself incorporates both a positive view and negative view of the underlying asset and no information of the direction (i.e. left or right skew) of the smile can be interpreted. This might be a problem for the inference. Another approach that is out of the scope of this paper would be to make a parameterization of the smile (Gemmil, 1996) by measuring the relative weight of the direction of out-of-the-money puts and calls skew to make such a conclusion.

Even though more than 900 000 observations of option prices for the period May 2006 to April 2011 were used in this study there are still limitations to which conclusions can be drawn from the data set. It would be preferable to have a longer time-series to get more robust results and more reliable jump estimations from the MLE.

## 8. Conclusions

Mispricing in the option prices and deviation from the Nobel prize winning Black-Scholes option pricing model has long been taunting and intellectual challenging for academic scholars and practitioners. Previous studies have shown that option traders get information earlier than equity traders and other literature has concluded that the implied volatility might be a good predictor of realized volatility. In order to extend previous research this study conducted a research on the predictive power of the implied volatility smile extracted from S&P option prices to forecast certain movements and jumps on the underlying S&P 500 Composite Index.

As a first step using daily option prices with a large range of strike prices the volatility smile was plotted and regressed to extract the second degree coefficient of the smile. To avoid the time decay factor same time-to-maturities has been used to construct continuous time series of coefficient smiles over the period May 2006 to April 2011.

Next, jump parameters from the GBMJ model were estimated using MLE for different moving windows. These estimations showed a certain set of jumps in the market for the period May 2006 to April 2011 however with extreme Poisson probabilities for certain periods which might have created bias in later regressions.

Using the steepness of the smile (i.e. the second degree coefficient) as a proxy for the mood of the future looking option market regressions were made on the jump parameters from MLE. Although many significant relationships were found the results proved to be varying for different time-to-maturities and jump parameters which makes it hard to make clear inferences. This study concludes that there is some weak proof of a predictive power of the volatility smile on the future volatility regime for longer dated options with between 30 to 87 days to maturity. Nevertheless more research has to be done within this topic to make more certain conclusions. This study contributes to the forecasting finance literature by making an extensive study in forecasting future jumps in stock prices with the volatility smile coefficient for the period May 2006 to April 2011.

## 9. Suggestions for further research

The findings of this study show that there are some, although unclear, relationships between the volatility smile and future market movements. It would be interesting to conduct further studies in this subject using longer time-series and modeling with other dependent variables than GBMJ parameters.

Improvements on the techniques in the parameterization of the volatility smile could be another field of study. Weighting market views on the tails of the smile would improve the precision by taking into account how much more positive or negative the smile is (i.e. left or right skewed). This approach would be more intuitive and clearer about future up- or downside risks. Another way to proxy for the smile curvature could be to use the newly launched CBOE Skew Index which uses CBOE's own definition and parameterization of the smile slope.

Further research could be made using options on futures. Derivatives of a higher order have an ever more future looking horizon and studies (Szakmary et al., 2003) have shown that the implied volatility of options on futures outperform other predictors of the subsequently realized volatility on the underlying.

## Bibliography

Afef, A., Olfa, B.O. (2009, August). How Option Markets Affect Price Discovery on the Spot Markets: A Survey of the Empirical Literature and Synthesis, *International Journal of Business and Management*, Vol. 4(8)

Bates, D.S., (2000, January), Post-'87 crash fears in the S&P 500 futures option market. *Journal of Econometrics*, Vol. 94(1-2), 181-238.

Beckers, S. (1981, September). Standard deviations implied in option prices as predictors of future stock price variability. *Journal of Banking & Finance*, vol. 5(3), 363-381.

Black, F., & Scholes, M. (1973, June). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* , vol. 81(3), 637-659.

Blair B.J., Poon S.H., Taylor S.J. (2001, Spring). Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns. *Journal of Econometrics*, Vol. 105(1), 5-26.

Brigo, D., Dalessandro, A., Neugebager, M., & Triki, F. (2009, January). A stochastic processes toolkit for risk management: Geometric Brownian motion, jumps, GARCH and variance gamma models. *Journal of Risk Management in Financial Institutions,* Vol. 2(4), 365-393.

Chakravarty, S., Gulen, H., & Mayhew, S. (2004, June). Informed Trading in Stock and Option Markets. *Journal of Finance*, Vol. 59 (3), 1235-1257.

Craine, R., Lochstoer, L., & Syrtveit, K. (2000, June). Estimation of a Stochastic-Volatility Jump-Diffusion Model. *Economic Analysis Review*, Vol. 15(1), 61-87.

Derman, E. & Kani, I. (1994, January). The volatility smile and its implied tree. *Goldman Sachs Quantitative Strategies Research Notes*

Deuskar, P., Gupta, A., Subrahmanyam, M.G. (2008, May). The Economic Determinants of Interest Rate Option Smiles. *Journal of Banking & Finance*, Vol. 32(5), 714-728.

Duque, J. L. C. & Lopes, P. T. (2000, February). Maturity and volatility effects on smile or dying smiling?

Ederington, L. & Guan, W. (2004, August). The information frown in option prices. *Journal of Banking & Finance*, Vol. 29 (2005), 1429–1457.

Eraker, B. (2004, June), Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. *The Journal of Finance,* vol. 59(3), 1367-1404.

Fama, E. (1965). The Behavior of Stock Market Prices. *Journal of Business,*Vol.38, 34–105.

Gemmil, G. (1996, December). Did Option Traders Anticipate the Crash? Evidence from Volatility Smiles in the U.K. with U.S. Comparisons, *The Journal of Futures Markets*, Vol. 16(8), 881-897.

Jorion, P. (1988, Winter), On Jump Processes in the Foreign Exchange and Stock Markets, *The Review of Financial Studies*, Vol. 1(4), 427-445.

Kahneman, D. & Tversky, A. (1979, March). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*. Vol. 47 (2), 263–291.

Kou, S. G., (2002, August), A Jump Diffusion Model for Option Pricing. *Management Science,* vol. 48(8), 1086-1101.

Malz, A.L. (2001, October), Financial crises, implied volatility and stress testing. *Risk Metrics Group,* Working Paper Number 01-01.

Merton, R. (1976, January). Option Pricing when Underlying Stock Returns Are Discontinuous. *Journal of Financial Economics*, Vol. 3(1-2), 125-144.

Natenberg, S. (1994). Option Pricing and Volatility: Advanced Trading Strategies and Techniques, 2nd edn. McGraw-Hill, 1994.

Pesaran, M. H., & Timmermann, A. (1995, September) Predictability of Stock Returns: Robustness and Economic Significance, *The Journal of Finance*, Vol. 50(4), 1201-1228.

Pratt, J. W. (1976, May), F. Y. Edgeworth and R. A. Fisher on the Efficiency of Maximum Likelihood Estimation. *The Annals of Statistics*, Vol. 4(3), 501-514.

Shu, Y. (2011, January), Jump risk, stock returns, and slope of implied volatility smile. *Journal of Financial Economics 99 (2011) 216–233.*

Sun, Y. & Wu, X. M. (2010, April), A Nonparametric Study of Dependence Between S&P 500 Index and Market Volatility Index (VIX). *European Financial Management Symposium April 2010.*

# Appendix

## Tables

Table 1: Descripive Statistics of the second degree coefficient of the Volatility Smile for different maturities*

| | TTM 10 | TTM 15 | TTM 18 | TTM 21 | TTM 23 | TTM 28 | TTM 30 | TTM 35 | TTM 42 | TTM 45 | TTM 50 | TTM 60 | TTM 74 | TTM 87 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean | 0.53964 | 0.39727 | 0.41568 | 0.35453 | 0.31914 | 0.29306 | 0.25571 | 0.19581 | 0.19233 | 0.18105 | 0.16844 | 0.15336 | 0.13413 | 0.11237 |
| Standard Deviation | 0.28586 | 0.23166 | 0.15437 | 0.16104 | 0.14305 | 0.12619 | 0.14756 | 0.17079 | 0.11855 | 0.09801 | 0.10326 | 0.10603 | 0.08191 | 0.05412 |
| Max | 1.34900 | 1.04060 | 0.79090 | 0.63830 | 0.61450 | 0.59110 | 0.49980 | 0.42470 | 0.39010 | 0.35120 | 0.31900 | 0.46580 | 0.50730 | 0.21960 |
| Min | -0.21250 | -0.25880 | 0.05030 | -0.10590 | -0.06360 | -0.03670 | -0.13540 | -0.65120 | -0.18970 | -0.14470 | -0.10200 | -0.14100 | -0.02180 | -0.04910 |
| Observations | 57 | 55 | 49 | 52 | 55 | 54 | 54 | 52 | 53 | 56 | 53 | 51 | 48 | 41 |

*All coefficients given in 1/1000

Table 2: Descripive Statistics of the GBMJ coefficients from the MLE method for time to maturity 10 days

| | $\mu$ | $\sigma$ | $\mu_j$ | $\sigma_j$ | $\lambda$ | $\log \lambda$ |
|---|---|---|---|---|---|---|
| Mean | -0.00091 | 0.23829 | 0.27666 | 0.11988 | 0.35939 | -5.84068 |
| Standard Deviation | 0.91710 | 0.17163 | 0.24880 | 0.10987 | 2.67914 | 1.68027 |
| Max | 2.46804 | 0.94100 | 0.93341 | 0.49945 | 20.75653 | 1.31715 |
| Min | -3.43840 | 0.06496 | -0.35857 | 0.00000 | 0.00000 | -8.49242 |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 |
| Average number of datapoints used to calculate coefficients | 40 | 40 | 40 | 40 | 40 | 40 |

Table 3: Descripive Statistics of the GBMJ coefficients from the MLE method for time to maturity 21 days

| | $\mu$ | $\sigma$ | $\mu_j$ | $\sigma_j$ | $\lambda$ | $\log \lambda$ |
|---|---|---|---|---|---|---|
| Mean | 0.00534 | 0.24148 | 0.19675 | 0.16175 | 0.18445 | -5.72139 |
| Standard Deviation | 0.89777 | 0.16705 | 0.41561 | 0.27571 | 1.35242 | 1.70334 |
| Max | 1.75152 | 0.93290 | 0.77810 | 1.86627 | 10.47723 | 1.02025 |
| Min | -3.79940 | 0.08209 | -1.98438 | 0.00000 | 0.00000 | -8.92530 |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 |
| Average number of datapoints used to calculate coefficients | 52 | 52 | 52 | 52 | 52 | 52 |

Table 4: Descripive Statistics of the GBMJ coefficients from the MLE method for time to maturity 30 days

| | $\mu$ | $\sigma$ | $\mu_j$ | $\sigma_j$ | $\lambda$ | $\log \lambda$ |
|---|---|---|---|---|---|---|
| Mean | 0.00853 | 0.23480 | 0.20747 | 0.12806 | 6.07180 | -5.66664 |
| Standard Deviation | 0.71187 | 0.14266 | 0.24912 | 0.11506 | 42.59050 | 2.05957 |
| Max | 0.98275 | 0.77818 | 0.65379 | 0.54229 | 329.85688 | 2.51833 |
| Min | -2.86105 | 0.07820 | -0.57382 | 0.00000 | 0.00000 | -7.89477 |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 |
| Average number of datapoints used to calculate coefficients | 60 | 60 | 60 | 60 | 60 | 60 |

Table 5: Descripive Statistics of the GBMJ coefficients from the MLE method for time to maturity 45 days

| | $\mu$ | $\sigma$ | $\mu_j$ | $\sigma_j$ | $\lambda$ | log $\lambda$ |
|---|---|---|---|---|---|---|
| Mean | -0.00153 | 0.23941 | 0.25601 | 0.11539 | 4.71549 | -5.43325 |
| Standard Deviation | 0.73605 | 0.15528 | 0.22824 | 0.08945 | 20.77926 | 2.31577 |
| Max | 1.63473 | 0.75922 | 1.07379 | 0.37833 | 140.95267 | 2.14907 |
| Min | -2.94564 | 0.07447 | -0.16614 | 0.00000 | 0.00000 | -9.42414 |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 |
| Average number of datapoints used to calculate coefficients | 75 | 75 | 75 | 75 | 75 | 75 |

Table 6: Descripive Statistics of the GBMJ coefficients from the MLE method for time to maturity 87 days

| | $\mu$ | $\sigma$ | $\mu_j$ | $\sigma_j$ | $\lambda$ | log $\lambda$ |
|---|---|---|---|---|---|---|
| Mean | -0.00389 | 0.24355 | 0.19974 | 0.12895 | 3.09280 | -5.01901 |
| Standard Deviation | 0.58301 | 0.15739 | 0.30325 | 0.11927 | 11.70816 | 2.68180 |
| Max | 0.85692 | 0.77481 | 1.53927 | 0.54422 | 56.52993 | 1.75228 |
| Min | -1.93889 | 0.07591 | -0.53349 | 0.00000 | 0.00000 | -8.33238 |
| Observations | 60 | 60 | 60 | 60 | 60 | 60 |
| Average number of datapoints used to calculate coefficients | 116 | 116 | 116 | 116 | 116 | 116 |

**Table 7: Correlations matrix for Smile Coeffients and MLE Jump parameters**

| | TTM 10 | TTM 21 | TTM 30 | TTM 45 | TTM 87 | Mean smile | Standard dev |
|---|---|---|---|---|---|---|---|
| SP500 Index* | -0.260 | -0.470 | -0.288 | -0.191 | 0.116 | -0.317 | -0.648 |
| SP500 Return* | 0.179 | 0.086 | 0.289 | 0.324 | 0.150 | 0.245 | -0.103 |
| VIX* | -0.019 | 0.041 | -0.205 | -0.267 | -0.305 | -0.233 | 0.472 |
| VIX Change* | -0.142 | 0.005 | -0.169 | -0.121 | 0.061 | -0.140 | -0.084 |
| Jump Coefficient, $\mu$** | 0.000 | 0.084 | -0.062 | -0.042 | -0.041 | -0.042 | -0.063 |
| Sd Jump, $\sigma$** | -0.082 | -0.131 | 0.180 | 0.200 | -0.200 | 0.078 | -0.044 |
| Lambda, $\lambda$** | -0.213 | 0.061 | -0.371 | -0.399 | 0.319 | -0.314 | -0.023 |
| Log Lambda, log($\lambda$)** | -0.129 | 0.056 | -0.237 | -0.169 | 0.173 | -0.221 | -0.087 |
| $\mu \cdot \lambda$** | 0.007 | -0.077 | -0.373 | -0.450 | -0.181 | 0.005 | 0.040 |

* Using Monthly Data

** Using Moving Windows (Smile Date plus one day - Expiry Date following month)

**Table 8: P-value for regressions on Smiles Variables**

| | TTM 10 | TTM 21 | TTM 30 | TTM 45 | TTM 87 | Mean smile | Standard dev |
|---|---|---|---|---|---|---|---|
| SP500 Index* | 0.051 | **0.000** | **0.035** | 0.159 | 0.470 | **0.014** | **0.000** |
| SP500 Return* | 0.186 | 0.551 | **0.036** | **0.016** | 0.350 | 0.064 | 0.442 |
| VIX* | 0.887 | 0.771 | 0.136 | **0.047** | 0.053 | 0.076 | **0.000** |
| VIX Change* | 0.296 | 0.972 | 0.226 | 0.377 | 0.703 | 0.295 | 0.531 |
| Jump Coefficient, $\mu$** | 0.998 | 0.556 | 0.653 | 0.761 | 0.798 | 0.753 | 0.636 |
| Sd Jump, $\sigma$** | 0.548 | 0.359 | 0.192 | 0.143 | 0.209 | 0.563 | 0.742 |
| Lambda, $\lambda$** | 0.115 | 0.672 | **0.006** | **0.003** | **0.042** | **0.016** | 0.865 |
| Log Lambda, $\log(\lambda)$** | 0.343 | 0.697 | 0.084 | 0.218 | 0.281 | 0.096 | 0.518 |
| $\mu \cdot \lambda$** | 0.108 | 0.593 | **0.005** | **0.001** | 0.257 | **0.015** | 0.837 |

* Using Monthly Data

** Using Moving Windows (Smile Date plus one day - Expiry Date following month)

P values < 5% in bold font

**Table 9: Smiles coefficient predicitve power on Lambda, $\lambda$ and average expected jump, $\mu \cdot \lambda$**

| | TTM 30 | TTM 45 | TTM 87 |
|---|---|---|---|
| Lambda, $\lambda$† | -39.363** | -22.311** | 9.176* |
| | (-2.88) | (-3.17) | (-2.10) |
| $\mu \cdot \lambda$† | -1.588** | -0.804** | -0.065 |
| | (-2.90) | (-3.67) | (-1.15) |
| Observations | 54 | 55 | 41 |

† Using Moving Windows (Smile Date plus one day - Expiry Date following month)

t-statistics in parentheses. * $p<0.05$, ** $p<0.01$, *** $p<0.001$

# Figures

*Figure 1. Option dataset illustration 1.*

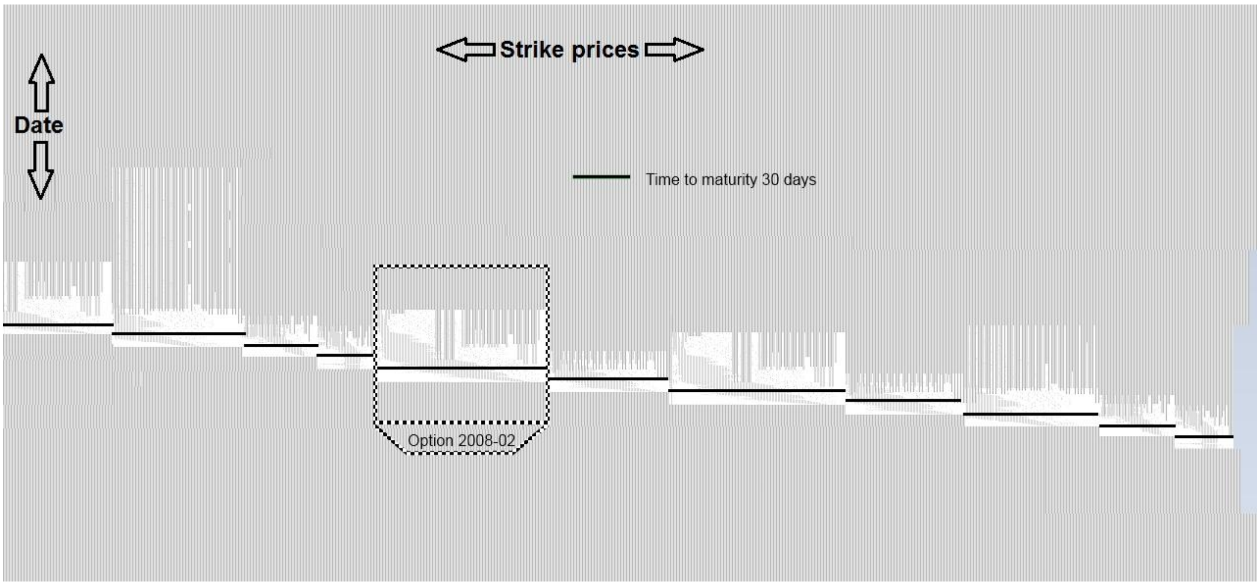


*Figure 2. Figure 1 magnified: Option dataset illustration 2.*

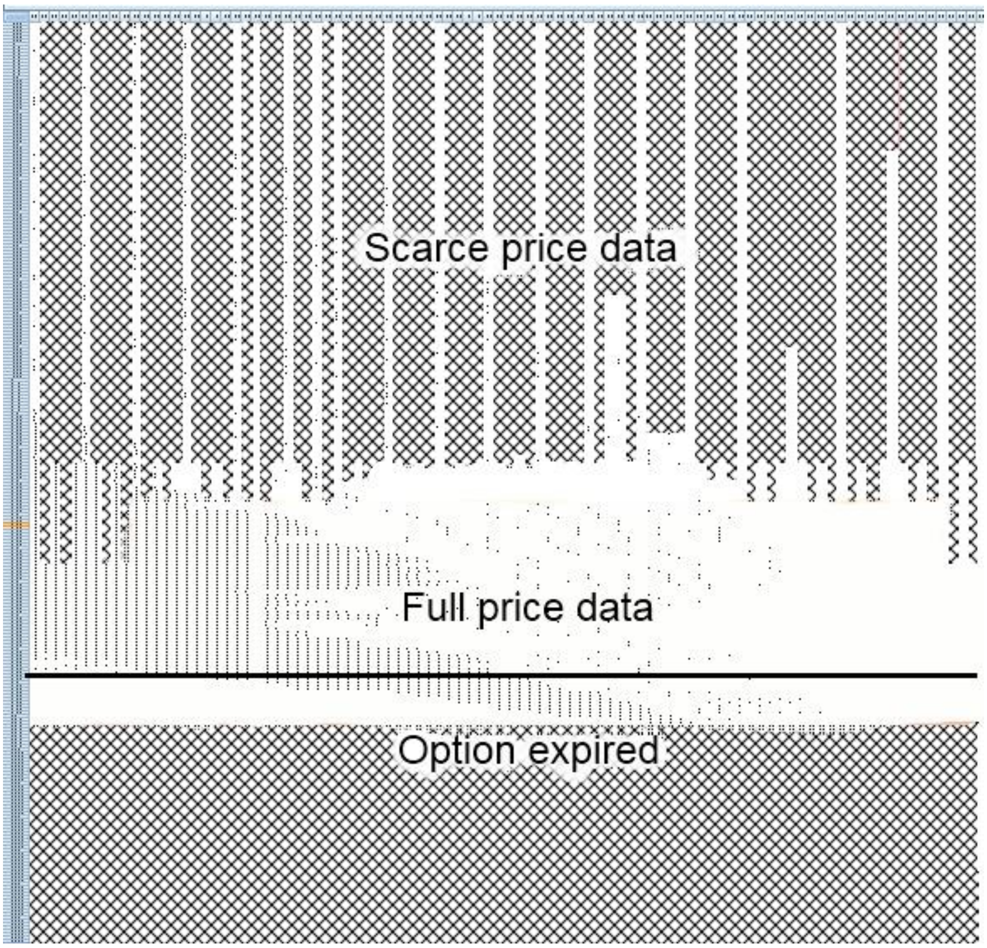

*Figure 3. Moving windows.*

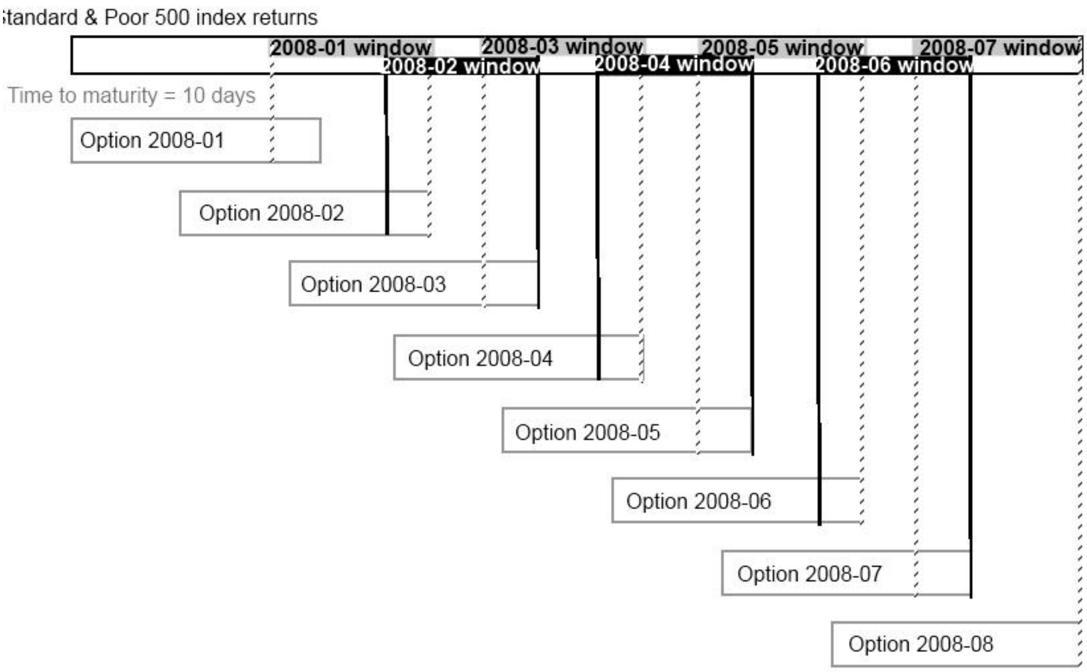


*Figure 4. Mean and Standard Deviation of the Smile Coefficients.*

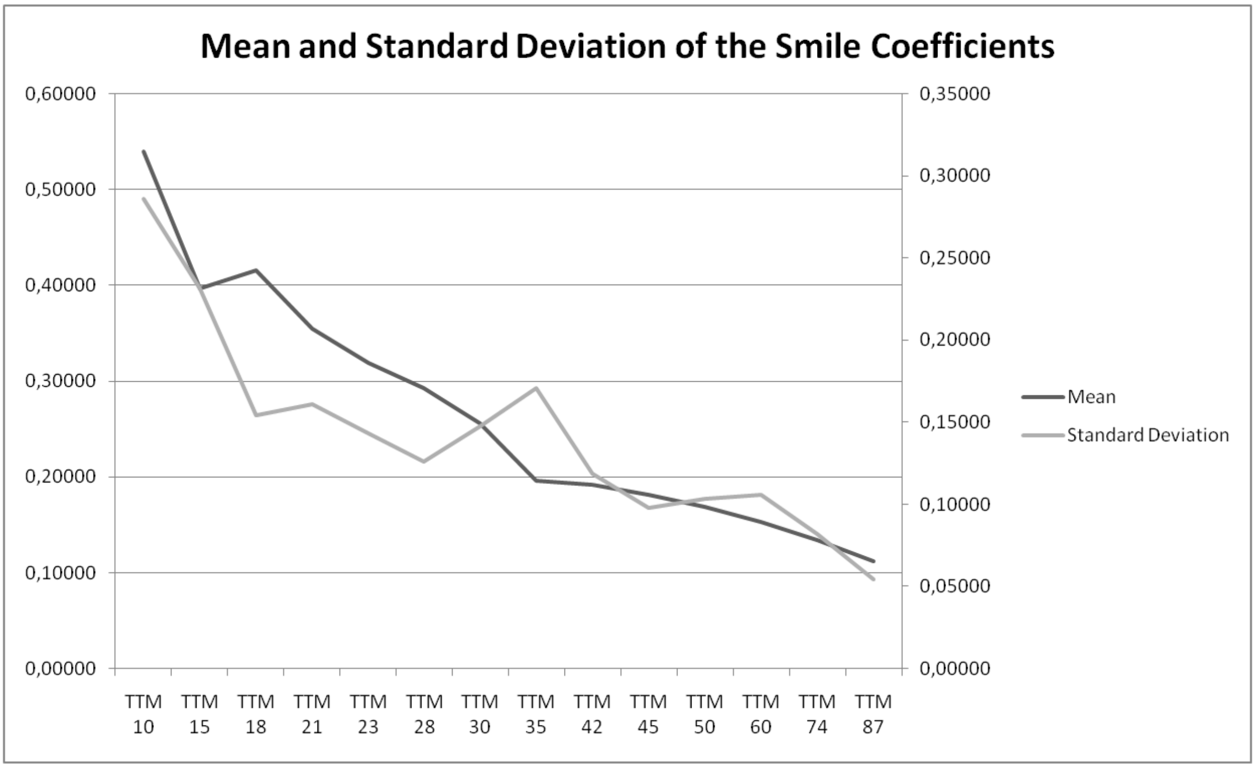

*Figure 5. Smile development when time-to-maturity decreases.*

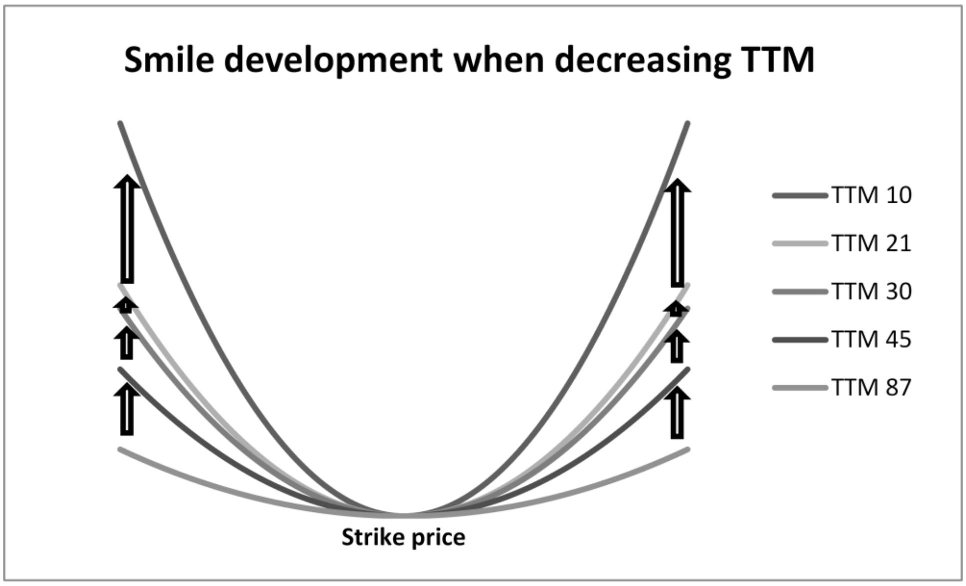

*Figure 6. Log λ development over time for different time-to-maturity.*

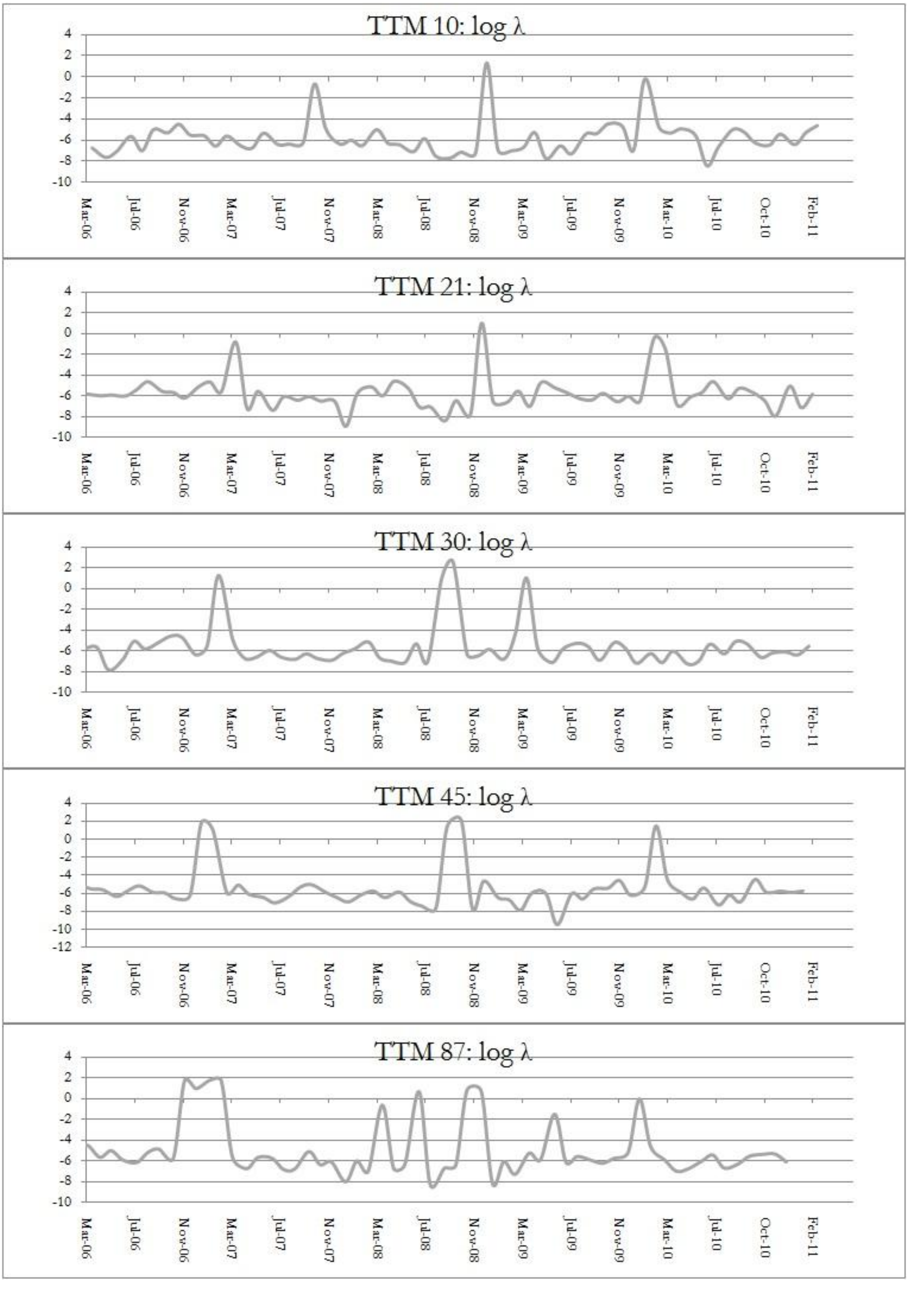

*Figure 7. $\mu_j$ development over time for different time-to-maturity.*

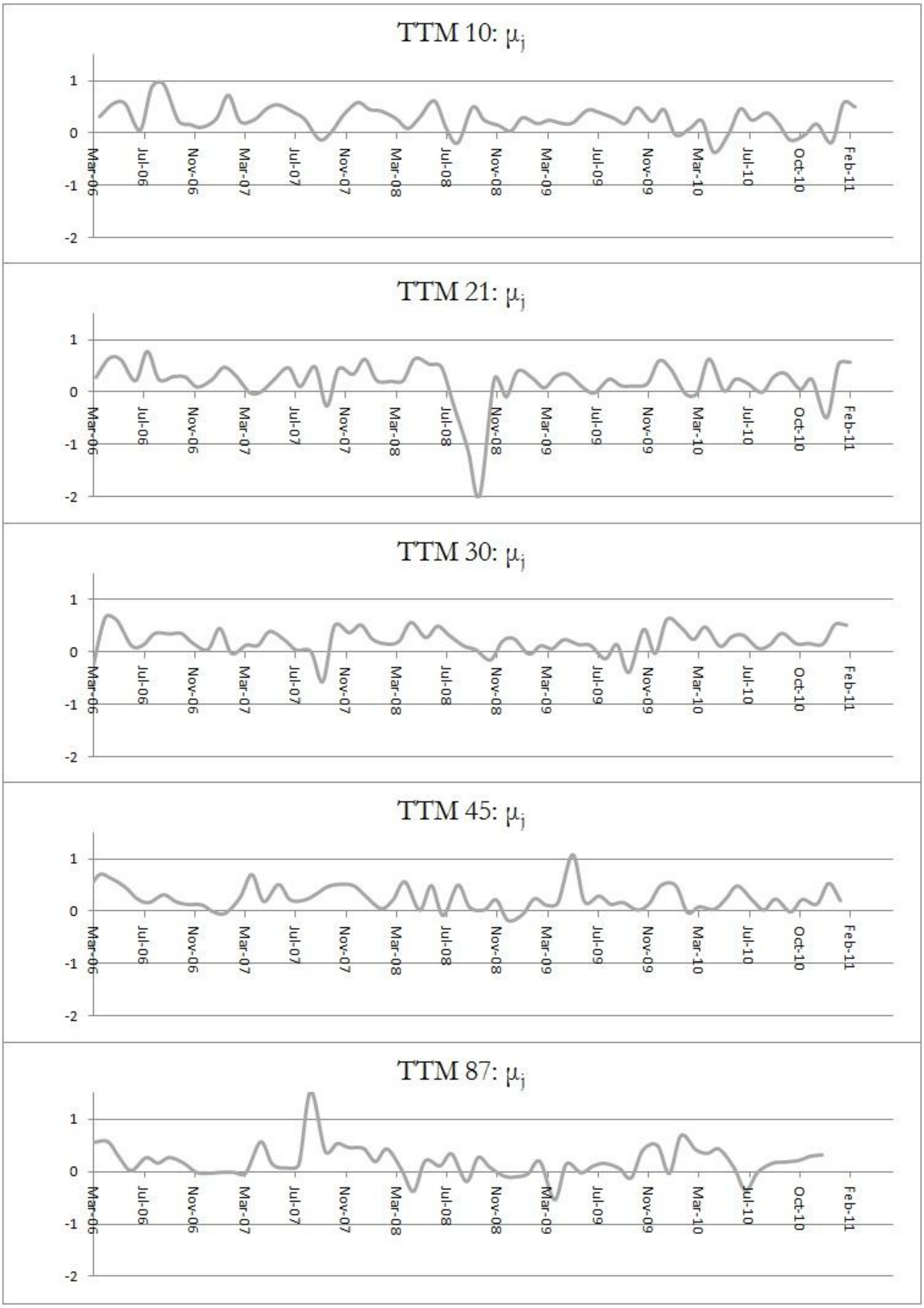

*Figure 8. $\sigma_J$ development over time for different time-to-maturity.*

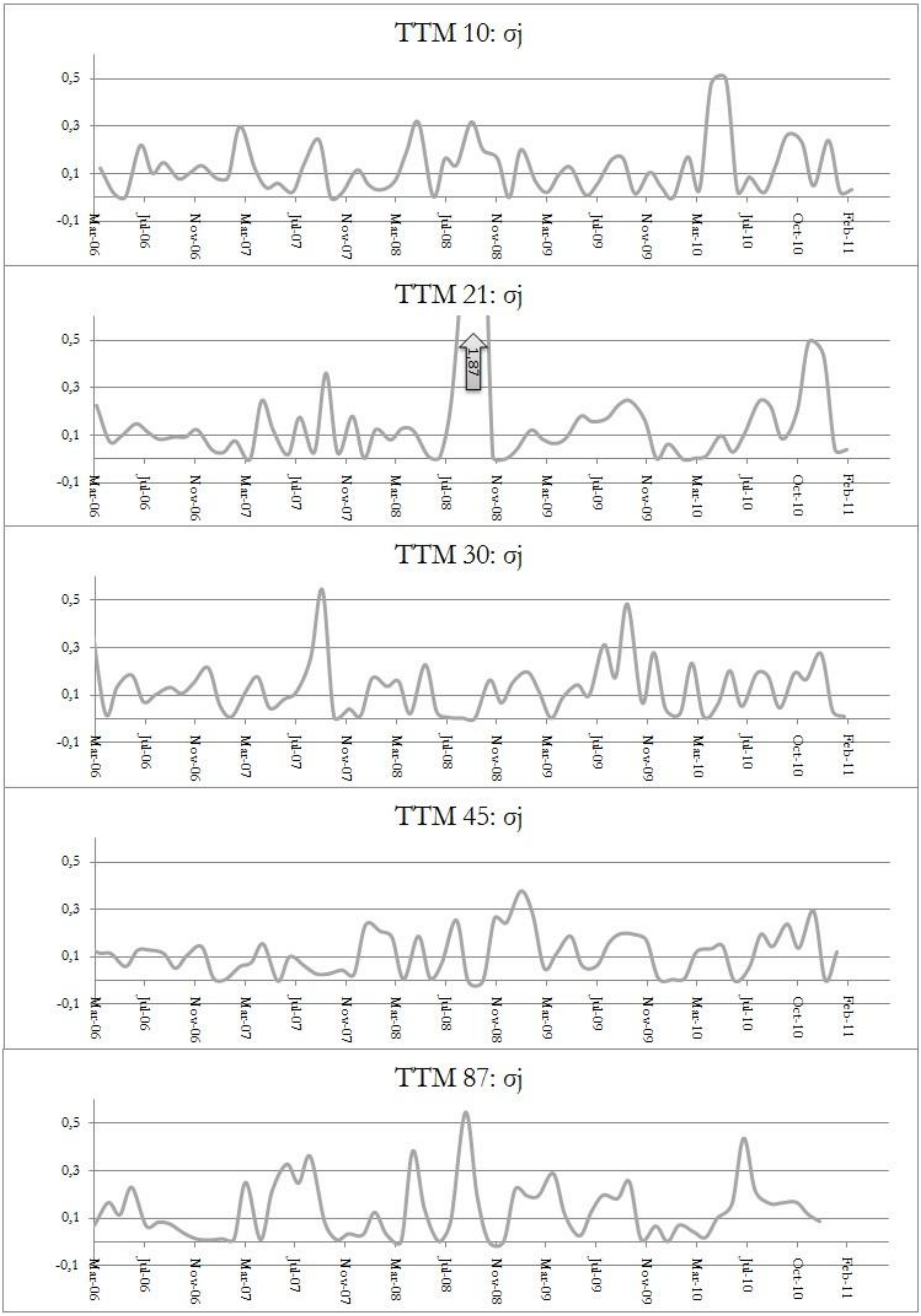

*Figure 9. Average jump amplitude per month from MLE and the S&P daily returns for TTM 45.*

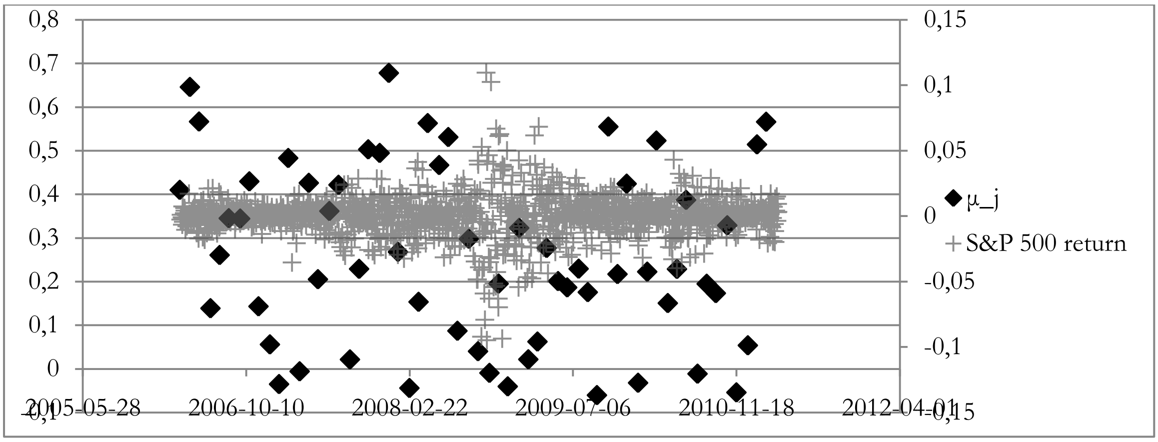


*Figure 10. Probability of jumps (λ) for TTM 45 according to MLE versus the real market "jumps" calculated as returns deviating more than 2 standard deviation s from the mean.*

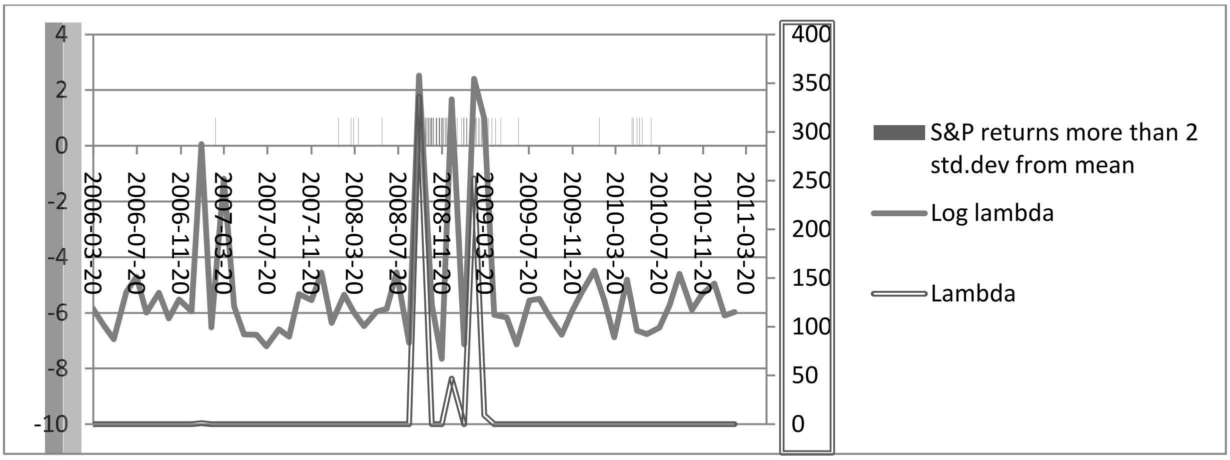


*Figure 11. Probability of jumps (λ) for TTM 45 according to MLE and the average S&P returns for the corresponding periods.*

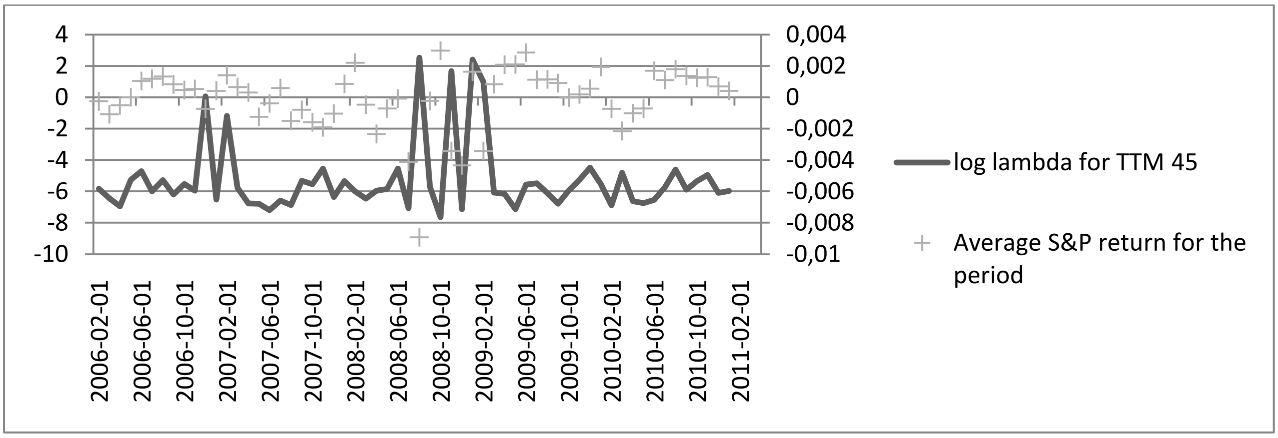

*Figure 12. Abnormal daily returns in the periods defined by TTM 45*