Stockholm School of Economics
Master's Thesis in Finance

Hedging Effectiveness of Index Options in Sweden

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Abstract

We test hedging performance of five different hedging techniques on the OMX Stockholm 30 (OMXS30) call options. Four of the hedging techniques applied are based on the Black-Scholes-Merton (BSM) model and the fifth is a regression-based model that adjusts the original BSM Greeks. Using market data from June 2007 to March 2011 we show that returns on the OMXS30 index are non-normally distributed and are inversely related to implied volatility. Analysis shows that hedge ratio adjustment for the inverse relation between implied volatility and stock price yields significantly better performance than the classical BSM delta and delta-gamma hedge. Delta adjustments for non-normal skewness and kurtosis provide better hedging performance than the BSM hedge ratio for all but ITM and deep ITM options. Empirical tests prove that the regression-based hedge ratio returns the lowest levels of hedging errors across all moneyness levels, maturities, and market regimes. We also find that the optimal hedge ratio is consistently lower than that suggested by the BSM model.

JEL Classification: G11, G12, G13

Keywords: Black-Scholes-Merton model, Regression-based hedge, Hedging errors, Hedging performance and effectiveness

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Table of Contents

1 Introduction 1

2 Characteristics of the OMXS30 Index Returns 2

2.1 OMXS30 Implied Volatility 5

3 Hypotheses.. 7

4 Literature Review 7

4.1 Black-Scholes-Merton Model 7

4.2 The Optimal Delta.. 9

4.3 Skewness and Krutosis Adjustments to the BSM Delta and Local Volatility Models.. 10

4.4 Stochastic Volatility Models 12

5 Methodology.. 13

5.1 The Dividend Adjusted BSM Model 13

5.2 Local Delta 14

5.3 Kurtosis and Skewness-Adjusted Model 16

5.4 Regression-Based Hedge Ratio.. 19

5.5 Measurement of Hedging Performance 19

6 Data Description.. 21

7 Empirical Findings.. 24

7.1 Summary of Results 31

8 Conclusions 31

9 Limitations and Further Research 32

References 34

Appendix.. 38

1 Introduction

The history of financial instruments dates back to as far as 1750 B.C. when forward contracts were created using Mesopotamian clay tablets. Gelderblom and Jonker (2004) report evidence that options and forwards were used in grain deals in Amsterdam already in 1550. By the beginning of 1900 active option markets had developed in New York, Paris, London and several other European cities. There was a decent level of sophistication in these markets and Kairys and Valerio (1997) show that tail events were priced already in 1870s in the US financial markets.

Since then, derivative markets have grown exponentially and the value of outstanding contracts reached USD 583 trillion in June 2010 (Bank for International Settlements, 2010). Even though derivatives allow transferring risks to market participants who are able and willing to bear them, inaccurate assessment and limitation of risks can lead to financial disasters (e.g. cases of Metallgeselschaft, Barings, and Long Term Capital Management). Some of these failures can arguably be attributed to lack of understanding of the complex and non-linear risk structure of derivative instruments. In this outset, elimination of risk or hedging in derivative market has become of utmost importance. Ideally, theory of derivative pricing provides a framework for elimination of risks associated with the derivative position that can be achieved using both dynamic and discrete hedging techniques. However, practical implementation of various hedging strategies is extremely complex due to different market restrictions and market microstructure characteristics (e.g. transaction costs, liquidity constraints). The various characteristics of markets dictate that there is no uniform hedging framework that would be equally efficient across different market classes and regions (see Sim and Zurbruegg (1999), and Rao and Thakur (2008)). Thus, pricing and hedging derivative instruments is subject to risk associated with the use of misspecified financial models or the so called model risk. According to Stix (1998), it can account for 20-40% of losses on derivative positions. Apparently, the use of more sophisticated derivative pricing and risk management tools has given rise of yet another type of risk.

This paper aims at evaluating performance of different hedging strategies in option market in Sweden. In particular, by deploying five different hedging techniques we will try to answer the question of which type of non-stochastic models and in what market regimes provides the best hedging performance for OMX Stockholm 30 (OMXS30) index plain vanilla call options traded on the NASDAQ OMX Stockholm. To capture different market characteristics we apply not only the traditional Black-Scholes-Merton (BSM) model, but also hedging techniques that account for non-normal skewness and kurtosis of index returns, as well as negative correlation between implied volatility and the index. Due to the fact that derivative markets across regions differ in their characteristics a separate analysis and application of various hedging techniques in the Swedish market is relevant and could provide useful insights for individuals or institutions willing to trade on the NASDAQ OMX Stockholm.

The paper begins with characterization of the OMXS30 returns and volatility. Next, we develop three hypotheses on model hedging performance. Section four provides a comprehensive literature review and is followed by methodology. Section six provides data description, which is followed by empirical findings. Section seven includes a review of empirical findings on hedging performance. At the end of the paper we provide concluding remarks, research limitations, and suggestions for further research.

2 Characteristics of the OMXS30 Index Returns

Visual inspection of the plot of OMXS30 index returns (natural logarithm; see Figure 1) indicates that large movements are followed by large movements, and low movements are observed after low movements. This is the so called volatility clustering, which implies that the OMXS30 index return volatility varies over time and tends to be mean-reverting.

Figure 1: Natural Logarithm of OMXS30 Historical Returns

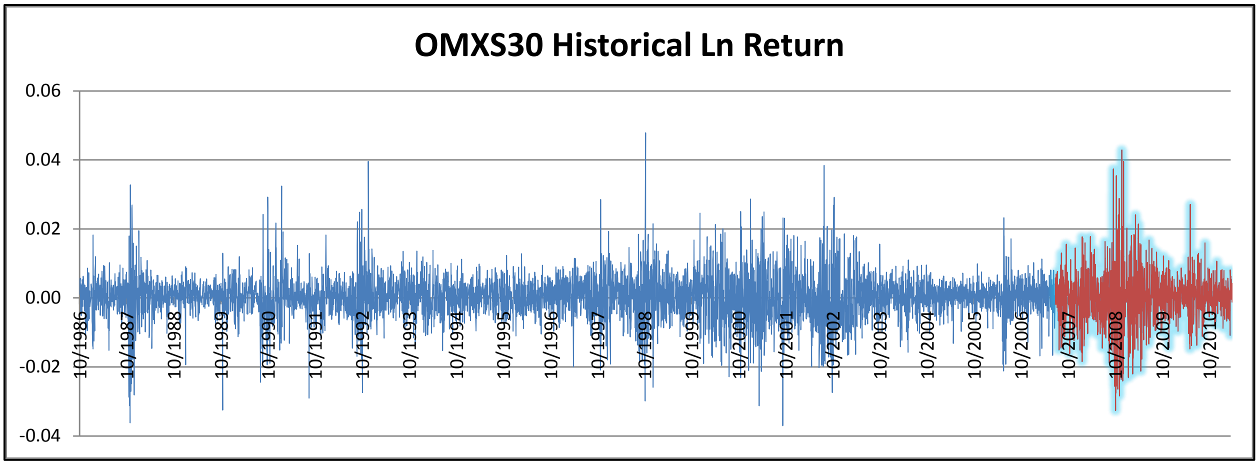
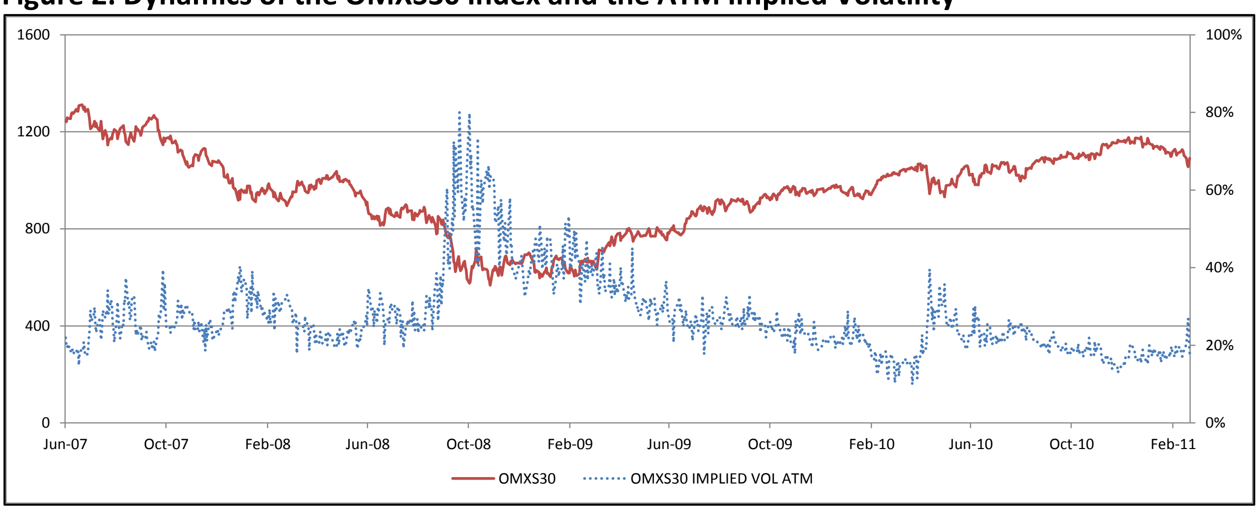


Table 1 and Figure 2 show the correlation and dynamics of the OMXS30 index and the ATM implied volatility of option contracts over the sample period. As predicted by Derman (1999), the two are negatively correlated. This implies that adjustments to the original BSM delta that account for this relationship (i.e. local delta) are likely to provide significant improvements in hedging performance.

Table 1: Correlation Between the OMXS30 Index and the ATM Implied Volatility

Full Sample	-0.76
June 2007 - June 20008	-0.43
July 2008 - June 2009	-0.77
July 2009 - June 2010	-0.64
July 2010 - March 2011	-0.88

Figure 2: Dynamics of the OMXS30 Index and the ATM Implied Volatility



Descriptive statistics for absolute and natural logarithm of both the OMXS30 and the ATM implied volatility are reported in Table 2.

Table 2: Descriptive Statistics for Absolute and Logarithmic Values of the OMXS30 Index and the ATM Implied Volatility

	Level		Logarithmic First Differences	
	OMXS30	ATM Implied Volatility	OMXS30	ATM Implied Volatility
Mean	955	0.28	-0.0001	-0.0002
Median	966	0.25	0.0000	-0.0043
Minimum	568	0.10	-0.0751	-0.5923
Maximum	1312	0.80	0.0987	0.5364
Standard Deviation	171	0.11	0.0183	0.1296
Skewness	-0.33	1.79	0.2762	0.0302
Excess Kurtosis	-3.57	1.02	0.3399	-1.3197

Both descriptive statistics and the histograms below imply that returns of the OMXS30 index are not normally distributed and have excess kurtosis and fat tails. Non-normality in the third and fourth moments indicate that hedging approach adjusting for skewness and excess kurtosis (i.e. skewness and kurtosis adjusted delta) can deliver a better result than the BSM delta hedge. Figure 3 represents the distribution of the OMXS30 index returns since its inception on September 30, 1986. These returns have excess kurtosis of 1.35 and non-normal skewness of 0.06.

Figure 3: Log Returns of the OMXS30 Index Since 1986 vs. Normal Distribution

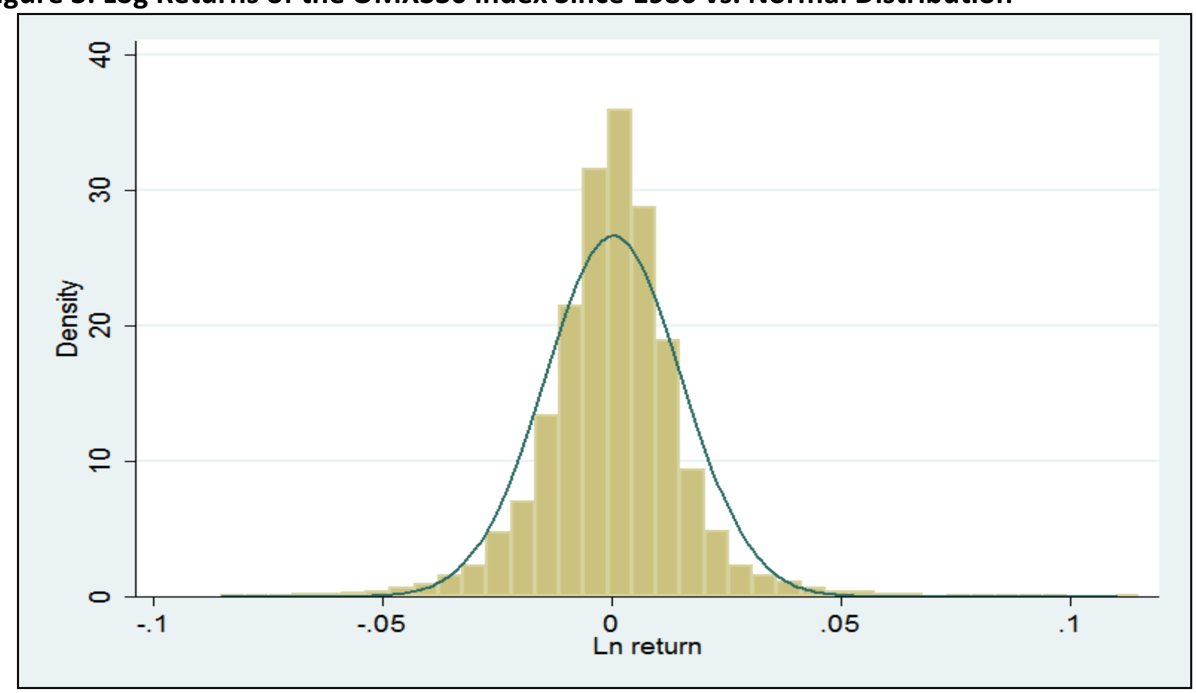
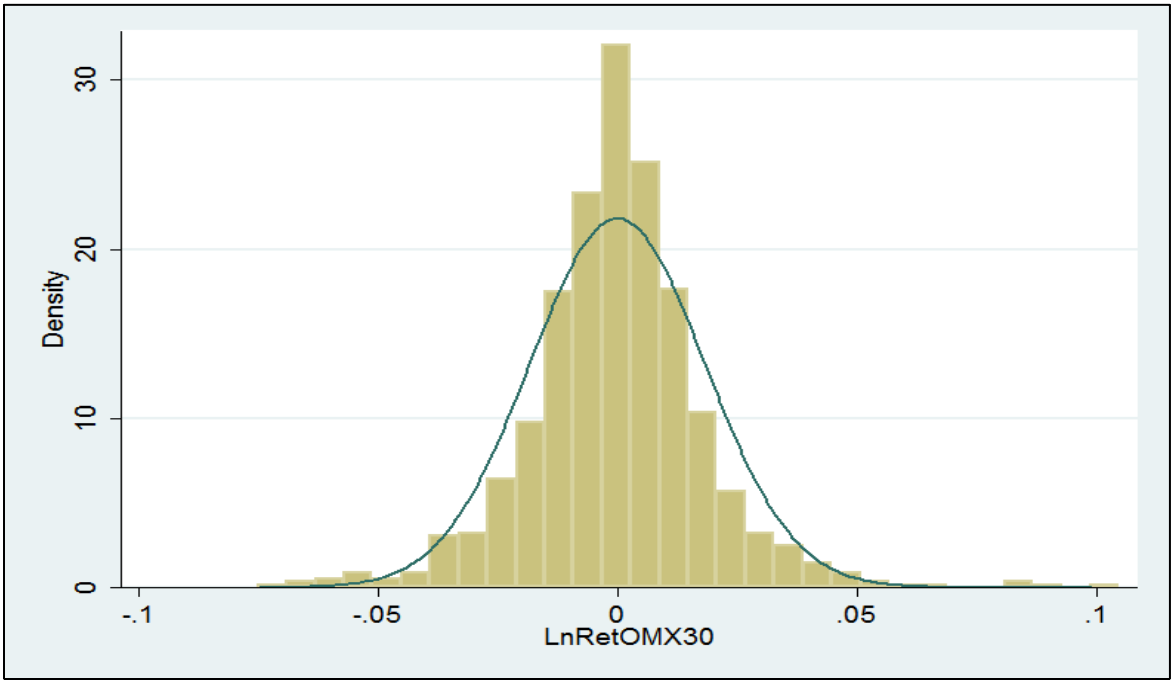


Figure 4 represents the distribution of log returns of the OMXS30 index for the sample period. The plot clearly shows that these returns are not normally distributed.

Figure 4: Sample Period Log Returns of the OMXS30 Index vs. Normal Distribution



2.1 OMXS30 Implied Volatility

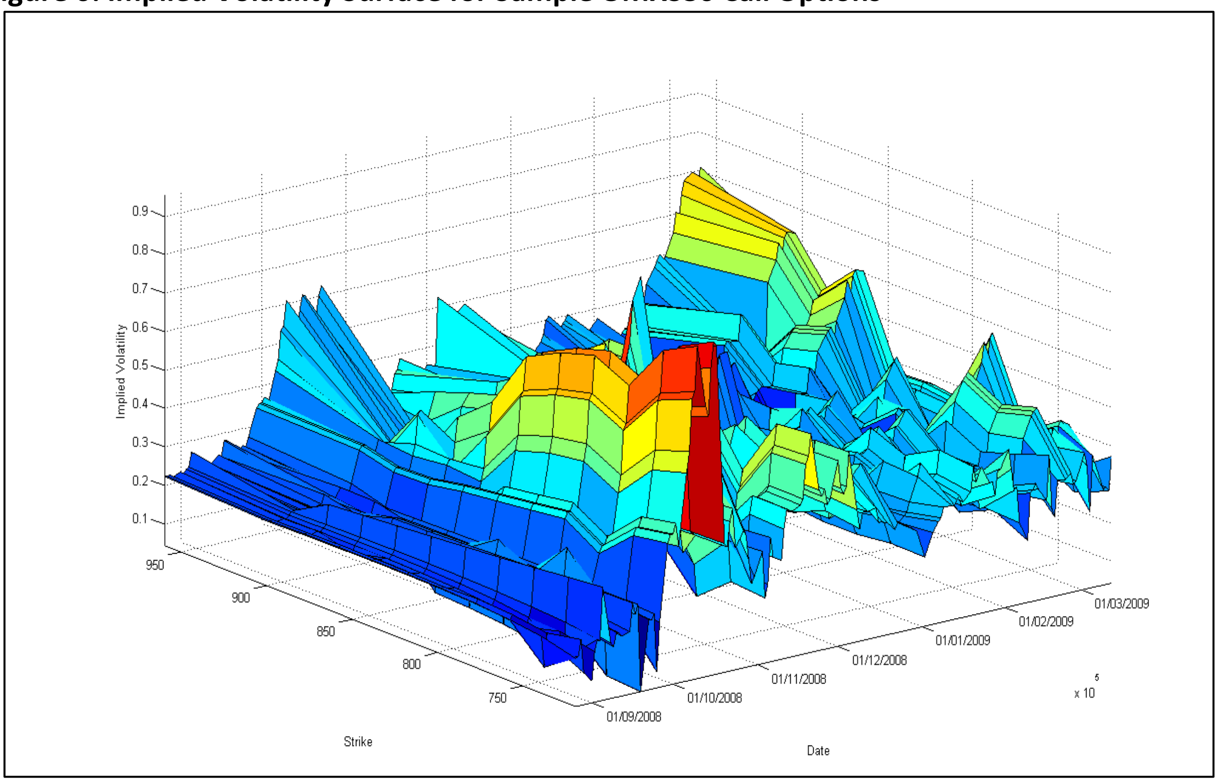
Table 3 in the Appendix illustrates the average BSM implied-volatility values across five moneyness and three maturity categories for the entire sample period and selected four sub periods, which are selected one year in length.

BSM implied volatilities demonstrate a U-shaped pattern which corresponds to volatility smirk in options market both looking at the whole sample and sub sample periods. Deep ITM and OTM calls return the highest implied volatility values. Furthermore, volatility smirks are the most pronounced for short-term options indicating higher mispricing compared to the BSM model. For given sub periods long-term option implied volatilities demonstrate more of a linear shape (see Figure 5 in the Appendix for illustration). Bakshi, Cao, and Chen (1997) report that the volatility smirk is indicative of negatively-skewed implicit return distributions and excess kurtosis. Thus, an appropriate model for hedging purposes should be based on a distributional assumption that controls for negative skewness and excess kurtosis.

One can observe that the implied volatility spread between short-term and long-term options widens during the crisis period (July 2008 – June 2010), with implied volatility for short-term options being higher than that for long-term options. An explanation for this pattern might be that during turbulent market periods investors are less exposed to gamma risk on long-term options, thus, their position is less affected by market movements.

Figure 6 demonstrates the implied volatility surface for OMXS30 options in the sample period. The exact shape changes from day to day and across contracts with different maturities. We observe particular swings in the surface that correspond to periods of market turmoil. Options with shorter maturity are more volatile across different strike levels compared to longer dated options. We see elevated volatility levels as options approach their maturity.

Figure 6: Implied Volatility Surface for Sample OMXS30 Call Options



Figures 7, 8, and 9 in Appendix plot implied volatility surface of long-, medium-, and short-term maturity options during the second half of 2008 and are representative of general pattern observed. Based on the plots, short-term options are more volatile than medium- and long-term options over comparable sub periods of the sample. The pattern is observed across all sample periods. During market turbulences short-term options are also more sensitive to changes in price of the underlying.

3 Hypotheses

Characteristics of the OMXS30 index returns and call options suggest that traditional hedging models that assume normal distribution of index returns and/or constant volatility might not be optimal for hedging OMXS30 call options. There is a need for hedging techniques that account for non-normal return distribution and negative correlation between implied volatility and the OMXS30 index.

Hence, we form three hypotheses regarding hedging model performance:

Hypothesis 1: Skewness and kurtosis adjusted hedge delivers lower hedging errors than the BSM hedge;

Hypothesis 2: Hedging model accounting for negative correlation between implied volatility and the underlying asset delivers lower hedging errors than the BSM hedge;

Hypothesis 3: On average, regression-based hedge delivers lower hedging errors than the BSM hedge.

The third hypothesis stems from Jarrow (2011) who shows that theoretical estimation of hedge parameters using historical estimation procedure is always correct. Thus, a regression based hedge that uses market data as inputs would not account for any market characteristics on a separate basis, but rather it would include various effects that are priced by the market. We expect this approach to provide better results on average compared to the BSM model (with performance being less superior in times of high volatility).

4 Literature Review

4.1 Black-Scholes-Merton Model

Despite the fact that Bachelier (1900) had documented several fundamental option pricing and hedging concepts already in the beginning of the twentieth century, they were not united into one comprehensive model until the beginning of the 1970’s. A revolutionary step in formalization of option pricing and hedging was done by Black, Scholes (1973), and Merton (1973) with the introduction of the BSM model. Assumptions behind the BSM model are:

- possibility to borrow and lend cash at a known and constant risk-free interest rate;
- price of the underlying asset follows geometric Brownian motion with constant drift and volatility, returns of the underlying are lognormally distributed;
- no transaction costs, taxes or bid-ask spread, no dividend payments;
- it is possible to buy any fraction of a share;
- no restrictions on trading and no arbitrage opportunities.

Even though the practical applicability of the model has been questioned due to its restrictive assumptions, the BSM model has served as one of the benchmark tools for pricing and hedging options since its inception in 1973 thanks to its straightforward and simple application. In order to fit the BSM model to option characteristics observed in the markets several extensions of the model have been designed.

The need for dynamic hedging is one of the most controversial assumptions of the BSM model. Black and Scholes (1973) conclude that "it is possible to create a hedged position, consisting of a long position in the stock and a short position in calls on the same stock, whose value will not depend on the price of the stock." They argued that in order to make the portfolio riskless one has to perform a stream of dynamic hedges. Derman and Taleb (2005) claim that in reality dynamic hedging is virtually impossible due to limits on continuous trading. They also find that from an economic point of view it is difficult to maintain a zero cost position due to high transaction costs. Additionally, if hedging is performed in discrete time, the portfolio bares risk between rebalancing periods. However, Wilmott (2006) considers that high frequency hedging is realistic in highly liquid markets where cost of buying and selling is close to nil. However, in less liquid markets the trader would incur loss due to the bid-ask spread and hence would be forced to hedge less frequently.

Similarly to Black and Scholes (1972), Boyle and Emanuel (1980) use the BSM framework to analyze the distribution of returns on a hedged portfolio that is rebalanced at discrete time intervals and consists of a European call option and its underlying stock. They find that increasing hedging frequency reduces the variability of excess hedge returns. Galai (1983) finds that hedging at discrete time intervals does not significantly affect mean returns from a hedging position but it increases its variance. The issue of hedging derivative risks in a similar way has also been addressed by Leland (1985) and Bhattacharaya (1980). Boyle and Vorst (1992) developed a perfect hedging strategy with transaction costs using binomial tree that was based on Cox, Ross, and Rubinstein (1979). Whereas Zhao and Ziemba (2007) use simulations to prove that an exact hedge at the limit cannot be achieved even if rebalancing intervals approach zero.

As to the BSM assumption regarding volatility, Melino and Turnbull (1995) find that the BSM hedging framework with constant volatility assumption produces relatively small errors for short- and medium-term options, but it performs worse for hedging long-term options. Bron (2005) tests hedging with constant and non-constant volatilities on the Dutch AEX and S&P 500 index and finds that hedging performance using non-constant volatility is not superior to that of constant volatility. Meanwhile, Lam, Chang, and Lee (2002) report that hedging performance of a more advanced asymmetric variance gamma option pricing model is poor compared to the traditional BSM model.

4.2 The Optimal Delta

Realizing that volatility is not constant over the life span of an option, a trader using the BSM model has to continuously change the volatility assumption in order to match the market price. Crépey (2004) shows that this leads to hedge ratios that are effectively out of the trader's control. Thus, it is often observed that trader's positions are affected even if they are perfectly hedged according to the BSM $N(d_1)$ hedge ratio. Hull and Suo (2002) show that delta-neutral hedge does not immunize position because of misspecification in the BSM model. Derman (1999) proposes that the size of delta should depend on market conditions. He claims that under highly volatile market conditions when index and volatility are likely to move in opposite directions, the optimal hedge ratio should be smaller than the BSM delta. The opposite holds in trending markets when volatility and index are likely to move in the same direction – the optimal hedge ratio should be larger than the BSM delta. Based on empirical evidence, Mixon (2002) suggests that the optimal delta should be lower than the BSM delta. Evidence that optimal deltas are different from the BSM delta is provided also by Bakshi, Cao, and Chen (2000a), Coleman et al. (2001), and Lam, Chang, Lee (2002). Vähämaa (2003) uses data on the FTSE 100 index to show that adjustments to the BSM delta to account for the inverse relationship between volatility and price of the underlying can significantly improve performance of delta hedging. Moreover, he finds that the optimal delta is smaller than the one based on the BSM model.

Based on the BSM theory, volatility of an option should be independent of its strike and expiration. Thus, plotted as a surface, it should be flat. Rubinstein (1994) shows that this assumption performed reasonably well up to the stock market crash in 1987. However, since the crash, the volatility surface of index options has become skewed, reflecting higher probability of extreme events and pricing in these events. The volatility surface varies over time and across different strike levels. Derman (2003) claims that the volatility smile phenomenon has become even more important over time, as it has spread to stock options, interest-rate options, currency options, and almost every other volatility market. Since the BSM model can match neither the volatility structure, nor stock return distributions observed in the market, trading desks have come up with more complex models to hedge their positions.

4.3 Skewness and Krutosis Adjustments to the BSM Delta and Local Volatility Models

To match the non-normal distribution of returns on the underlying asset observed in the market, academia and practitioners have come up with alternative models that adjust the BSM delta to match non-normal skewness and kurtosis. One of the approaches in doing that is to expand the lognormal and normal density function in terms of its moments; hence, Jarrow and Rudd (1982) defined option price as a function of the third and fourth moments of the terminal price distribution. Based on the same idea, Corrado and Su (1996) and Brown and Robinson (2002) propose a model using the Gram-Charlier expansion of the normal density function. These adjustments have proven to be of significant value in pricing options that are deeply out of the money.

Breeden and Litzenberger (1978) figured that the risk-neutral partial differential function can be derived from European option market prices. Afterwards, Dupire (1994) and Derman and Kani (1994) found that conditional on risk neutrality there was a unique diffusion process consistent with these distributions. The coefficient that is consistent with current European option prices is known as the local volatility coefficient with the functional form of $\sigma_L(S,t)$. This implies that in local volatility models realized volatility of the stock varies deterministically as a function of future time *t* and the future stock price *S*.

Vähämaa (2004) shows that the local delta must control not only for the direct impact of the underlying's price change on the option price, but also for the indirect impact of the volatility change which is correlated with the underlying's price change. A relatively simple adjustment of implied delta can correct for this:

$$\Delta^{LOC} = \frac{\partial c}{\partial S} + \frac{\partial c}{\partial \sigma}\frac{\partial \sigma}{\partial S}$$

Most of the time one can observe negative correlation between stock returns and volatility changes. Thus, the local delta should be smaller than the BSM delta. Local delta utilizes the volatility skew of an equity index together with the vega of the option to account for the volatility changes with respect to changes in the underlying instrument.

Crépey (2004) suggests that local delta provides a better hedge on average, as well as on average conditionally on the fact that the market is in a fast regime, or on average conditionally on the fact that the market is in a slow regime. Overall, he recommends using the local delta rather than the implied delta hedge. The same recommendation in a persistently positively skewed market is made. Derman (1999) also observes that in negatively skewed markets local delta should be better on average conditionally on the fact that the market is in a fast regime. Further research by Coleman et al (2001) and Vähämaa (2003) also conclude that, on average, local delta provides better hedge than the implied delta, especially in jumpy markets. In addition, Alexander and Kaeck (2010) find that the sticky-strike deltas outperform locally- calibrated deltas only for options with low strike prices.

4.4 Stochastic Volatility Models

Volatility smile and the peaked, fat-tail distribution of returns gave rise to a group of models that, in contrast to the BSM model, treat volatility as a stochastic variable. This characteristic allows stochastic volatility models to provide a self-consistent way of explaining why options with different strikes and maturities have different implied volatilities compared to the BSM model. Cox and Ross (1976) developed the constant elasticity of variance (CEV) model in the form of

$$dS = (r - q)Sdt + \sigma S^{\alpha} dz.$$

The model allows the stock price volatility to fluctuate if $\alpha \neq 1$. If $\alpha = 1$, the model takes the form of the geometric Brownian motion model that was used in derivation of the BSM model. Using the stochastic volatility model by Cox, Ingersoll, Ross (1985), Bakshi, Cao, and Chen (2000b) show that allowing for stochastically varying volatility is important for pricing options but further accounting for stochastic interest rates does not improve pricing performance.

Heston (1993) created a model

$$d\sigma = -\gamma\sigma dt + \delta dX_2$$

incorporating arbitrary correlation between the underlying asset and its volatility. Kim and Kim (2004) show that Heston's model outperforms other stochastic volatility models in terms of hedging effectiveness. Evidence of solid hedging performance of the Heston model in comparison to other hedging alternatives is also found by Alexander, Kaeck and Nogueira (2010).

A considerable part of research related to hedging effectiveness has focused on determining regression-based hedge ratios. Thus, generalized autoregressive conditional heteroskedasticity (GARCH) - an econometric model for an asset and its volatility – was first used by Duan (1995) to model stochastic volatility, and he also shows that the BSM model can be treated as a specific form of the GARCH model. It is found that the pricing impact of stochastic volatility is rather small for options with maturity of less than a year. Hull (2002) claims that the effect of stochastic volatility on delta hedging is quite significant. However, Yung and Zhang (2003) show that an Exponential GARCH model performs well in terms of pricing options but its hedging performance is not superior commpared to more traditional models (including the BSM).

5 Methodology

The methodology of the paper is based on hedging error calculations as a benchmark for hedging effectiveness. Five hedging approaches are tested in the report out of which four are based on the BSM delta and the fifth is based on regression adjusted BSM Greeks. All calculations are done using average implied volatility which is backed out from each option price in the sample and then equally weighted for call options on a given maturity-moneyness level.

5.1 The Dividend Adjusted BSM Model

A call price in the BSM model incorporating continuous dividend yield is given by

$$c = Se^{-q(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2)$$

where *S* is the stock price, *K* is the strike price, *q* is the dividend yield, *(T-t)* is time to maturity, and *r* is the risk free interest rate and

$$d_1 = (\ln\left(\frac{S}{K}\right) + (r - q + \frac{\sigma^2}{2}))/\sigma\sqrt{(T-t)} \ and \ d_2 = d_1 - \sigma\sqrt{(T-t)}$$

Hedging according to the BSM model implies holding Δ^{BS} of shares where

$$\Delta^{BS} = \frac{\partial c}{\partial S} = N(d_1).$$

The hedged portfolio includes a short position in a European call option and a long position in shares of the underlying stock

$$\frac{\partial \Pi}{\partial S} = -\frac{\partial c}{\partial S} + \Delta\frac{\partial S}{\partial S} = -\frac{\partial c}{\partial S} + \frac{\partial c}{\partial S}\frac{\partial S}{\partial S} = 0 \qquad (1)$$

Equation 1 shows that the price of the portfolio is not sensitive to variations in price of the underlying, so the portfolio is riskless as long as the number of shares held is adjusted to Δ. If continuous hedging was realistic, it would be possible to eliminate all the risk from the portfolio making it riskless.

Additionally, we also test gamma hedging. The BSM gamma is given by

$$\Gamma = \frac{\partial^2 c}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T}}.$$

Delta-gamma hedge ratio equals

$$\Delta^{BS}\mathrm{S} + \Gamma\mathrm{S}^2.$$

5.2 Local Delta

Crépey (2004) shows that gains and losses on a portfolio in the BSM world can be approximated using Taylor expansion to

$$\delta\Pi = -\delta\Pi + \Delta\delta S = \frac{1}{2}S^2\Gamma(\Sigma^2\tau - \left(\frac{\delta S}{S})^2\right) + o(\tau)$$

where Π represents the option's price, $\Gamma \equiv \partial^2_{S^2}\Pi$ and $\Delta \equiv \partial_S\Pi$ is the option's gamma and delta, respectively. Thus, $P\&L = \sum \delta P\&L$ represents the distribution of profit and loss on the portfolio, which is asymptotically symmetric and centred as τ tends to 0. European vanilla call or put option *P&L* volatility of is dominated by $\sqrt{\tau}$.

With fixed local volatility

$$\delta P\&L^{loc} = -\delta\Pi + \Delta^{\mathrm{loc}}\delta\mathrm{S} = \frac{1}{2}\mathrm{S}^2\Gamma^{loc}(\sigma(t,S)^2\tau - \left(\frac{\delta S}{S})^2\right) + o(\tau)$$

where Π is the option's price, Δ^{loc} and Γ^{loc} are the option's local delta and gamma, respectively. Crépey (2004) shows that Γ^{loc}>0 because both vanilla call and put options are convex in the spot price of the underlying asset. Hence, up to the order $o(\tau)$ the profit or loss on the portfolio depends on the following:

– If $\left({}^{\delta S}/_{S}\right)^2$ is larger or smaller than $\sigma(t,S)^2\tau$;

– On the relative position of the realized volatility $|\delta S|/S\sqrt{\tau}$ with respect to the local volatility $\sigma(t,S)$.

Up to the order $o(\tau)$, the expectation of the realized volatility squared is equal to the square of the local variance $\sigma(t,S)^2$. Hence, in a local volatility model the distribution of $\delta P\&L^{loc}$ is asymptotically centred as $\tau \to 0$.

Crépey (2004) shows that $\delta P\&L^{loc}$, in contrast to $\delta P\&L^{BS}$, is not directional and driven by factors in δS since

$$\delta P\&L^{BS} - \delta P\&L^{loc} = (\Delta^{BS} - \Delta^{LOC})\delta S.$$

Crépey (2004) concludes that "the fluctuations (such as measured by the standard deviation) of $\delta P\&L^{BS}$ are one order of magnitude greater than those of $\delta P\&L^{LOC}$".

As $\Pi_{T,K}(t,S;\sigma) = \Pi_{T,K}^{BS}(t,S;\Sigma_{T,K})$, then

$$\Delta^{LOC} = \Delta^{BS} + \vartheta^{BS}\partial_S\Sigma$$

$$\vartheta^{BS} = \partial\sigma/\partial S.$$

If changes in stock price at some point are negatively correlated with changes in volatility, then the local delta is smaller than the BSM delta because the option's vega is always positive.

Due to the complexity of expressing $\partial\sigma/\partial S$ numerically, we use the proof by Derman, Kani, and Zou (1995) who show that implied volatility can be expressed as an average of the local volatilities on the most probabilistic paths between *(t, S)* and *(T, K)*. Moreover, if the value of local volatility is independent of time and changes linearly with the underlying asset, the implied skew gives a proxy for $\partial_S\Sigma$, i.e.,

$$\partial_S\Sigma \approx \partial_K\Sigma.$$

Effectively, this implies that $\frac{\partial\sigma}{\partial S}$ can be approximated by the slope of the volatility smile, $\frac{\partial\sigma}{\partial K}$. Hence,

$$\Delta^{LOC} = \frac{\partial c}{\partial S} + \frac{\partial c}{\partial\sigma}\frac{\partial\sigma}{\partial K} = \Delta^{BS} + \vartheta^{BS}\frac{\partial\sigma}{\partial K}.$$

This approximation assumes that a unit change in *S* is associated with a parallel shift of $\frac{\partial\sigma}{\partial K}$ units in volatility smile.

5.3 Kurtosis and Skewness-Adjusted Model

Jarrow and Rudd (1982) developed an option pricing model which assumes that at expiration the underlying asset follows distribution *F* that is known only by its moments. The pricing formula was derived from an Edgeworth series expansion[1] of the distribution *F* about an approximating distribution, *A*. A call price *c(F)* based on the unknown distribution *F* is given by

$$c(F) = c(A) - e^{-rt}\frac{k_3(F)-k_3(A)}{3!}\frac{da(K)}{dS_t} + e^{-rt}\frac{k_4(F)-k_4(A)}{4!}\frac{d^2a(K)}{dS_t^2} + \varepsilon(K) \qquad (2)$$

where *c(A)* is a call price based on a known distribution, *A*; it is followed by adjustment terms for distributions *F* and *A* that are based on the cumulants[2] $k_j(F)$ and $k_j(A)$, respectively. $\frac{da(K)}{dS_t}$ and $\frac{d^2a(K)}{dS_t^2}$ are derivatives of the density function of *A* that are calculated at the strike price *K*. The density function of *A* is given by $a(S_t)$, where S_t is a random stock price at expiration. $\varepsilon(K)$ continues the series with terms based on higher order cumulants, which are negligible if the known distribution, *A*, is a good choice.

In this setting, the cumulants resemble distribution moments, and they are independent of translation in description of the shape of the function. Hence, the first and second cumulants

[1] The Edgeworth series are series that approximate a probability distribution in terms of its cumulants. The Edgeworth series are used to write the characteristic function of the distribution (with probability density function F) in terms of the characteristic function of a known distribution with suitable properties (*A)*. Afterwards, *F* is recovered through the inverse Fourier transform. Source: http://en.wikipedia.org/wiki/Edgeworth_series

[2] Cumulants of a probability distribution are a set of quantities that provide an alternative to the moments of the distribution. The moments determine the cumulants in the sense that any two probability distributions whose moments are identical will have identical cumulants as well, and similarly the cumulants determine the moments. Source: http://en.wikipedia.org/wiki/Cumulant#Cumulants_and_moments

are equal to the mean and variance of a distribution, respectively, and $k_2(F) = \mu_2(F)$. The third moment measures the lopsidedness of the function (skewness), which, if defined, is set to be zero if the function is symmetric, and $k_3(F) = \mu_3(F)$. The fourth moment measures the kurtosis of the function; thus, $k_4(F) = \mu_4(F) - 3\mu_2^2(F)$. μ_2^2, μ_3, μ_4 are the squared variance, third, and fourth moments, respectively. If the known distribution (*A)* is set to be lognormal, then the formula becomes the BSM pricing formula. In application of the risk neutral valuation, Jarrow and Rudd (1982) set the means (that are the same as first cumulants) of *F* and *A* equal, i.e.,

$$k_1(F) = k_1(A) = S_0 e^{rt}.$$

They express call price as in Equation (2) by setting the variances of the distributions *F* and *A* equal, which in effect implies that the second cumulants are equal, i.e., $k_2(F) = k_2(A)$. With *A* set as lognormal distribution, the parameter for volatility is expressed as

$$k_2(F) = k_1^2(A)\left(e^{\sigma^2 t} - 1\right).$$

Corrado and Su (1996) show that when the $\varepsilon(K)$ term in Equation (2) is dropped, the option price can be expressed in a more compact form as

$$c(F) = c(A) + \lambda_1 Q_3 + \lambda_2 Q_4$$

where the terms λ_j and Q_j are defined as

$$\lambda_1 = \varphi_1(F) - \varphi_1(A), \qquad Q_3 = -(S_0 e^{rt})^3 (e^{\sigma^2 t} - 1)^{3/2} \frac{e^{-rt}}{3!} \frac{da(K)}{dS_t}$$

$$\lambda_2 = \varphi_2(F) - \varphi_2(A), \qquad Q_4 = (S_0 e^{rt})^4 (e^{\sigma^2 t} - 1)^2 \frac{e^{-rt}}{4!} \frac{d^2 a(K)}{dS_t^2}$$

Q_3 and Q_4 measure the deviations of skewness and kurtosis from the lognormal distribution. The terms $\varphi_1(F)$ and $\varphi_1(A)$ are coefficients of skewness for the *F* and *A* distributions, respectively, while the terms $\varphi_1(F)\ and\ \varphi_1(A)$ are coefficients of excess kurtosis. Skewness and excess kurtosis coefficients are defined in the following way:

$$\varphi_1(F) = \frac{k_3(F)}{k_2^{\frac{3}{2}}(F)} \; and \; \varphi_2(F) = \frac{k_4(F)}{k_2^2(F)}$$

Brown and Robinson (2002) published a correction of the Corrado and Su (1996) model, so that the correct skewness and kurtosis adjusted price of a call option is

$$c = SN(d_1) - Ke^{-rT}N(d_2) + \mu_3 Q_3 + (\mu_4 - 3)Q_4$$

where

$$Q_3 = \frac{1}{3!} S\sigma\sqrt{T}\left[\left(2\sigma\sqrt{T} - d_1\right)n(d_1) + \sigma^2 T N(d_1)\right]$$

$$Q_4 = \frac{1}{4!} S\sigma\sqrt{T}\left[\left(d_1^2 - 1 - 3\sigma\sqrt{T}\left(d_1 - \sigma\sqrt{T}\right)\right)n(d_1) + \sigma^3 T^{\frac{3}{2}} N(d_1)\right]$$

The skewness and kurtosis adjusted delta is expressed as

$$\Delta^{ADJ} = N(d_1) + \mu_3 q_3 + (\mu_4 - 3)q_4$$

where

$$q_3 = \frac{1}{3!}\left[\sigma^3 T^{\frac{3}{2}} N(d_1) + \left(\frac{\phi_1 d_1}{\sigma\sqrt{T}} + \sigma^2 T - 1 - \phi_1\right)n(d_1)\right]$$

$$q_4 = \frac{1}{4!}\left[\sigma^4 T^2 N(d_1) + \sigma^3 T^{\frac{3}{2}} n(d_1) + \frac{n(d_1)}{\sigma\sqrt{T}}\left(\phi_2 - 2\sigma^2 T + 2rT + 2\ln\left(\frac{\mathrm{S}}{\mathrm{K}}\right)\right) - \frac{\phi_2 d_1 n(d_1)}{\sigma^2 T}\right]$$

with

$$\phi_1 = rT - \left(\frac{3}{2}\right)\sigma^2 T + \ln(\frac{S}{K})$$

$$\phi_2 = r^2T^2 - 2r\sigma^2T^2 + \left(\frac{7}{4}\right)\sigma^4T^2 - \sigma^2T + \ln(\frac{S}{K})(2rT - 2\sigma^2T + \ln(\frac{S}{K}))$$

Hence, the skewness and kurtosis adjusted delta above consists of the BSM delta and two additional terms which measure the effects of non-normal skewness and kurtosis.

5.4 Regression-Based Hedge Ratio

Jarrow (2011) distinguishes between theoretical and statistical models. One can create a theoretical model by developing economic reasoning that is applied in estimation of the model by using historical data. Statistical models try to identify patterns in historical data hoping that they will persist in the future. In order to fit parameters of a theoretical model to market prices one has to use calibration, which means that the respective theoretical model is transformed into a statistical model. As calibration is applied to fit the theoretical model to market prices, it means that the theoretical model has been rejected in the first place. Jarrow (2011) argues that the new (statistical) model serves as a tool for pricing only, since it has been validated for this purpose using market prices. Hence, this calibrated model should not be used for anything else besides pricing.

However, one can still use a statistical model for hedging purposes. For instance, in regard to a calibrated BSM model, Jarrow (2011) suggests running a regression in the form

$$dc_t = \beta_0 + \beta_1(\Delta dS_t) + \beta_2(\Gamma dS_t^2) + \beta_3(\vartheta d\sigma_t) + \varepsilon_t,$$

where $\beta_0, \beta_1, \beta_2$, and β_3 are constants, c is the market price of a call option, S is the stock price, σ is the volatility, and ε is the estimation error. The constants returned by regression analysis are then used as adjustment factors for the original BSM Greeks. This hedging method is solely a statistical one, which uses partial derivatives from the BSM model as inputs. As it is based on patterns observed in market data, there is no need for separate adjustments that would fit the data patterns (e.g. adjustments for excess kurtosis or non-normal skewness).

We test the Jarrow's (2011) suggested BSM Greeks' adjustment running the regression specified above and use the first 120 observations in our sample as historic data. To our best knowledge, regression ratios of such a specification have not been tested before.

5.5 Measurement of Hedging Performance

To measure hedging performance, we assume that hedging is done without any long or short positions in other option contracts. Thus, a self-financed delta-hedged portfolio in the form of

$$\Pi_t = \Delta_t S_t + B_t - c_t$$

is created, where Δ represents the number of units of the underlying asset, *B* represents the number of units of a risk-free bond, and – *c* stands for one short unit in an option.

Initially,

$$\Pi_0 = 0 \text{ as } B_0 = c_0 - \delta_0 S_0.$$

Whenever the position is rebalanced, the self-financing portfolio is recalculated, and the hedging error from time *t-1* to *t* is reckoned as

$$\varepsilon_t = \delta_{t-1} S_t - c_t + e^{rt} B_{t-1}.$$

Hence, the cumulative hedging error is given by Π_T, the value of the portfolio at the end of the hedging horizon

$$\varepsilon_\tau = \textstyle\sum_{t=1}^{T} \varepsilon_t = \Pi_T.$$

To assess the hedging performance of different models, we use the mean absolute hedging error (MAHE) and the root mean squared hedging error (RMSHE).

$$\mathrm{MAHE} = \frac{1}{n}\sum_{i=1}^{n} |\varepsilon_{ti}| \qquad \mathrm{RMSHE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \varepsilon_{ti}^2}$$

MAHE measures the average magnitude of the errors without considering their direction. MAHE returns the average over the sample of the absolute values of the differences between observations and all the individual differences are weighted equally in the average. RMSHE measures the average magnitude of the error. The hedging error first is squared, then averaged over the sample and afterwards the square root of the average is taken. RMSHE gives a relatively high weight to large errors. This means RMSHE is most suitable if large errors are particularly undesirable. The greater difference between MAHE and RMSHE, the greater the variance in the individual errors in the sample. As risk minimization is the primary goal of hedging process, we consider RMSHE to be the primary evaluation criterion.

6 Data Description

In the paper, we analyze call options on the OMXS30 index, which is the leading and most comprehensive OMX Nordic Exchange Stockholm's share index. The index includes 30 companies with the highest trading volume on the NASDAQ OMX Stockholm. This ensures that all the stocks OMXS30 consists of are highly liquid.

The composition of the OMXS30 index is revised twice a year. The OMXS30 index is a market weighted price index. The index was introduced in September 30, 1986, with a base value of 125 (NASDAQ OMXa). The primary objective of OMXS30 is to create an index based on a limited number of stocks which develops in close correlation with the stocks listed on the Exchange thus reflecting the development of the portfolio of stocks included (NASDAQ OMXb). The index is calculated as follows:

$$I_t = \frac{\sum_{i=1}^{n} q_{i,t} * p_{i,t} * r_{i,t}}{\sum_{i=1}^{n} q_{i,t} * (p_{i,t-1} - d_{i,t}) * r_{i,t-1} * j_{i,t}} * I_{t-1}$$

where

I_t is the index level at time t, $q_{i,t}$ is the number of shares of company i applied in the index at time t, $p_{i,t}$ is the price in quote currency of a share in company i at time t, $d_{i,t}$ is the dividend for company i at time t (only used for total return indexes and special dividend in price indexes), $r_{i,t}$ is the foreign exchange rate of index quote currency to quote currency of company i at time t, $j_{i,t}$ is the adjustment factor for adjusting the share price of a constituent security due to corporate actions by the issuing company at time t.

As the OMXS30 is a price-weighted index, no cash dividends are reinvested. Thus, the OMXS30 index only reflects the performance of stock prices. To account for the dividend yield, we extracted the difference in growth between the total return and the price-weighted version of the index. NASDAQ OMX (2010) reports that the difference between the two types of indexes is attributable to the dividend yield of the OMXS30 index. Brenner, Courtadon, and Subrahmanyam (1987) report that dividend adjustment can have a significant impact on option value calculations. We obtained time series of the total return index SIX30RX with reinvested dividends from SIX Telekurs[3]. Hence, the difference in implied growth between the two indexes was used to account for the dividend yield.

Investors can trade European call and put option contracts with cash settlement on the OMXS30 index. Our analysis specifically focuses only on plain vanilla call options. We do not find this a fundamental limitation of our study, as Jiang and Oomen (2002) report that using only European call options can provide insights into general risks of derivatives because of the following reasons. First, risks of put options are similar to those of call options based on put–call parity and put-call symmetry (see Carr, Ellis and Gupta (1998)). Second, adjusting for early exercise premium, American option prices can be reckoned from European option prices. Third, numerous exotic derivatives can be created from portfolios of plain vanilla call and put options.

Options contracts on the OMXS30 index with terms of three, twelve and thirty six months are listed with the third Friday of the expiration month as the day of option maturity. The trading ends on the expiration day at the close of the electronic trading system. The sample was organized in three maturity categories: short-term options with maturity less than 60 days; medium-term options with maturity between 60 and 180 days; long-term options with maturity more than 180 days.

The sample period extends from June 2007 through March 2011. Index data was obtained from the NASDAQ OMX Stockholm, and data on option contracts was obtained from the Thomson Reuters Datastream database. Trading on the NASDAQ OMX Stockholm ceases at 5:25 pm CET for derivative instruments and at 5:30 pm CET for equities. Closing prices of both the index and option contracts were used in the analysis to minimize nonsynchronous price issue in the data.

The three-month Stockholm Interbank Offered Rate (STIBOR) is used as a proxy for the risk-free interest rate. The STIBOR data is obtained from the Central Bank of Sweden – the Riksbank.

[3] SIX Telekurs is the third largest provider of financial information in Europe.

To avoid liquidity-related and close-to-expiration issues, contracts with less than 10 observations as well as the last 5 observations for each option were deleted from the sample.

Further, each observation was classified according to its moneyness (defined as spot price at time *t* divided by option's strike - S_t/K) in five categories: deep out-of-the-money options with moneyness less than 0.9; out-of-the-money options with moneyness between 0.9 and 0.97; at-the-money options with moneyness from 0.97 to 1.03; in-the-money options moneyness range from 1.03 up to 1.1; and deep in-the-money options are defined with moneyness greater than 1.1.

Similarly to Bakshi and Kapadia (2003), to avoid errors in calculation, we deleted days with missing observations and with implied volatility less than 1% or greater than 100%. After adjustments the total number of observations in our data set is 75,401, which is 56% of the initial number (135,122 initial observations).

Table 4 describes sample properties of the OMXS30 call prices used in the study. Summary statistics are reported for the closing price and the total number of observations for each moneyness and maturity category. Out of 75,401 call option observations, OTM options constitute 58%. The average call price ranges from 2.58 SEK for short-term deep OTM options to 169.66 SEK for long-term deep ITM calls.

Table 4: Average Price and Number of Observations for Sample Options

	Moneyness	**Days-to-Expiration**			
	S/K	**< 60**	**60 - 180**	**> 180**	**Subtotal**
xOTM	< 0.9	2.58	7.41	20.73	
		5285	9102	10347	24734
OTM	0.9 - 0.97	9.86	18.6	39.42	
		3625	10496	4465	18586
ATM	0.97 - 1.03	32.04	41.78	66.98	
		2183	9308	3350	14841
ITM	1.03 - 1.1	69.11	80	101.69	
		1928	6858	2500	11286
xITM	> 1.1	156.85	130.68	169.66	
		1789	2322	1843	5954
Subtotal		14810	38086	22505	75401

We define market regimes in terms of volatility of the OMXS30 (see Table 5). Period from June 2007 to June 2009 is defined as high volatility market and period from July 2009 to March 2011 as low volatility market because the daily volatility is above and below historic realized volatility, respectively.

Table 5: OMXS30 Average Daily Volatility

Oct 1986 - June 2007	1.4%
June 2007 - June 2009	2.2%
July 2009 - March 2011	1.2%

7 Empirical Findings

Based on theory and empirical research related to vanilla call option hedging, we form expectations on possible patterns in hedging performance in the Swedish market.

Expectations:

1) Hedging errors are inversely correlated with rebalancing frequency across all maturities and moneyness levels;
2) Hedging errors vary across different market regimes and are higher in periods of high volatility;
3) Gamma hedge improves delta hedge performance:
   a. in periods of high volatility;
   b. as rebalancing frequency decreases;
4) Long-term options have higher hedging errors;
5) The optimal hedge ratio is smaller than the one implied by the BSM model.

Table 6 reports the average BSM, skewness and kurtosis adjusted, and local delta values across different moneyness levels. As expected, delta value is increasing with moneyness of the options. The S-shape of delta is most evident for short-term contracts. The results show that, compared to the BSM delta, the adjusted delta returns smaller hedge ratio for deep OTM, OTM, and ATM options across all maturity levels. On the other hand, ITM and deep ITM options return a marginally higher hedge ratio. While the BSM delta and adjusted delta values closely follow each other, local delta provides significantly different and smaller hedge ratio. The last column of Table 6 shows the difference between the BSM delta and local delta. Long-term options are adjusted downwards the most, with the average differences in delta values of 0.18 for long-, 0.17 for medium-, and 0.07 for short-term options. Based on the magnitude of the change, deep OTM option delta is the most affected across all maturities. Option vega peaks for ATM options, because value of ATM options can be most adversely affected by fluctuations in price of the underlying (i.e. higher risk of ending OTM). The difference between the two deltas is the highest for ATM and ITM options in most cases.

Table 6: Average BSM Delta, Kurtosis and Skewness Adjusted Delta, Local Delta

Maturity	Moneyness	DELTA	ADJ	LOC	DELTA-LOC
LT	**xOTM**	0.16	0.15	0.04	0.12
	OTM	0.34	0.32	0.13	0.21
	ATM	0.55	0.54	0.35	0.20
	ITM	0.74	0.75	0.54	0.19
	xITM	0.87	0.88	0.71	0.16
MT	**xOTM**	0.12	0.10	0.05	0.06
	OTM	0.27	0.24	0.15	0.11
	ATM	0.53	0.52	0.33	0.20
	ITM	0.75	0.77	0.53	0.22
	xITM	0.90	0.91	0.73	0.16
ST	**xOTM**	0.05	0.05	0.03	0.02
	OTM	0.17	0.16	0.17	0.01
	ATM	0.52	0.51	0.40	0.12
	ITM	0.79	0.81	0.66	0.14
	xITM	0.93	0.94	0.86	0.07

Table 7 reports BSM delta, gamma, and vega adjustment factors based on regression analysis. A similar pattern to local delta is observed with largest adjustment needed for long-term deep OTM options. Irrespective of moneyness or maturity level, the BSM delta is adjusted downwards indicating that it exceeds the optimal hedge ratio. Except in 3 cases, both gamma and vega hedge ratios are also reduced according to regression results. Overall, regression analyses indicate that the BSM hedge values are overstated and should be adjusted downwards. Similarly to results with local delta, regression results indicate that deep OTM and OTM options' delta needs the highest level of adjustment compared to the BSM delta.

Table 7: Average BSM Delta, Gamma, Vega, and Respective Regression-Based Adjustment Factors

Maturity	Moneyness	DELTA	BETA1	GAMMA	BETA2	VEGA	BETA3
LT	**xOTM**	0.16	-0.01	0.0013	0.25	160	0.09
	OTM	0.34	-0.03	0.0028	0.15	230	0.91
	ATM	0.55	0.20	0.0035	-0.28	240	1.07
	ITM	0.74	0.06	0.0027	1.22	177	0.69
	xITM	0.87	0.12	0.0013	1.32	123	0.41
MT	**xOTM**	0.12	-0.15	0.0016	0.05	80	0.41
	OTM	0.27	-0.07	0.0034	-0.06	123	0.79
	ATM	0.53	0.13	0.0050	0.03	146	0.95
	ITM	0.75	0.16	0.0033	-0.27	115	0.99
	xITM	0.90	-0.07	0.0013	-0.76	54	0.90
ST	**xOTM**	0.05	-0.39	0.0011	-0.01	27	-0.14
	OTM	0.18	-0.06	0.0033	0.39	64	0.60
	ATM	0.52	0.15	0.0063	0.27	100	0.98
	ITM	0.79	0.13	0.0036	0.82	72	0.93
	xITM	0.93	0.18	0.0010	0.30	34	0.48

Figures 10, 11, and 12 in the Appendix graphically depict Table 6 delta values for ATM options. All three modeled delta values move in parallel, with the BSM delta having the highest values. Local delta is trailing below and, because of an additional volatility term, it demonstrates higher fluctuation amplitude. Local delta is dependent on correlation between volatility and index level and, in line with Derman's (1999) observation, in most cases it is smaller than the BSM delta.

Hedging performance using 1 day rebalancing period is reported in Table 8. First, gamma hedge does not improve hedging efficiency and is redundant in combination with the BSM delta, as the latter provides lower hedging errors on its own. Adjusting the BSM delta for skewness and kurtosis reduces hedging errors for deep OTM, OTM, and ATM options; however, adjusted delta hedge ratio delivers worse results than the BSM delta hedging ITM and deep ITM options. Thus, we find Hypothesis 1 only partly satisfied in the sample analysis. In line with Hypothesis 2, local delta provides the lowest hedging errors across all maturities and moneyness levels compared to the previous three hedge ratios. Still, the local delta hedge is outperformed by the regression-based hedge strategy. The results confirm Hypothesis 3. Additionally, the regression-based hedging errors are comparatively smaller for ITM and deep

ITM options, outperforming the local delta by a higher margin. Considering regression-based hedge ratio, hedging errors are larger for long-term options across all moneyness levels, except deep ITM options - as predicted by Expectation #4.

Table 8: Hedging Errors for the 1-Day Hedging Horizon

1 DAY		MAHE					RMSHE				
Moneyness	**Maturity**	**DELTA**	**D+G**	**ADJ**	**LOC**	**REG**	**DELTA**	**D+G**	**ADJ**	**LOC**	**REG**
xOTM	**ST**	1.10	1.11	1.09	0.84	0.59	2.22	2.22	2.12	1.86	1.24
	MT	2.02	2.02	1.92	1.55	1.15	3.36	3.36	3.19	2.80	2.09
	LT	2.92	2.93	2.75	1.76	1.62	5.00	5.00	4.80	3.76	3.54
OTM	**ST**	3.34	3.35	3.13	3.02	1.57	6.18	6.20	5.82	5.78	2.97
	MT	4.59	4.60	4.32	3.42	2.07	7.28	7.30	6.93	5.96	3.16
	LT	6.77	6.78	6.56	4.43	3.93	12.48	12.49	12.25	7.56	6.25
ATM	**ST**	8.50	8.53	8.45	6.90	4.13	13.89	13.96	13.82	12.17	7.40
	MT	7.82	7.84	7.75	5.39	3.58	11.18	11.22	11.10	8.08	5.28
	LT	8.23	8.24	8.19	5.98	5.05	15.08	15.09	15.04	10.08	8.27
ITM	**ST**	10.97	10.98	11.17	8.63	5.13	16.74	16.76	16.98	13.50	8.64
	MT	10.47	10.48	10.63	8.11	5.98	16.70	16.72	16.88	14.06	11.37
	LT	9.98	9.99	10.11	7.98	5.65	16.32	16.33	16.50	12.17	8.81
xITM	**ST**	11.96	11.95	12.07	11.22	5.11	18.24	18.25	18.40	17.74	11.34
	MT	14.07	14.07	14.20	12.53	7.33	23.16	23.16	23.30	20.41	14.17
	LT	10.03	10.03	10.16	8.43	4.39	16.01	16.02	16.18	12.52	7.11

Darker shading indicates lower hedging error.

Results in Table 8 indicate that the variance in individual errors (in money terms) increases with moneyness of the options for all hedging approaches, as observed by the increasing difference between MAHE and RMSHE. The distribution of error variance also grows with option maturity in most cases, with long-term options having the highest variance in individual errors.

Based on the results, we confirm Expectation #5 that the BSM delta is overstated and lower hedge ratios deliver considerably lower hedging errors. F-test analyses (see Appendix Table 9) show that the regression-based hedge ratio is significantly different from the BSM delta at 1% level across all maturity and moneyness levels. Local delta is also different at 1% level across all maturity and moneyness levels, exception for being significant at 10% level for short-term OTM options and not significantly different for long-term deep OTM and short-term ATM options. The adjusted delta is significantly different from the BSM delta at 10% level for medium-term OTM and long-term ITM options; at 5% level for long-term and medium-term deep ITM options; not different for long-term OTM and ITM options, medium- and short-term ITM options and different at 1% level for all the rest observations. The results show that ITM options hedge ratios are not statistically different using these two hedge ratios. Further analysis showed that adding vega to the BSM delta-gamma hedge does not alter hedging performance and the difference is insignificant; thus, we do not report the BSM delta-gamma-vega hedge.

The distribution of hedging errors is leptokurtic and asymmetric (see Table 10). Errors on long-term options have high excess kurtosis with the BSM delta, delta-gamma, and adjusted delta hedging strategies. Medium- and short-term option hedging errors have the highest excess kurtosis using the regression-based hedge ratio indicating that errors are concentrated around the mean and variation in hedging errors is relatively low. Negative skewness in errors is primarily observed under the delta, delta-gamma, adjusted delta, and regression-based hedging strategies across different maturities indicating higher possibility of extreme hedging errors using these methods.

Table 10: Distribution of Hedging Errors

EXCESS KURTOSIS		DELTA	GAMMA	ADJ	LOC	REG
LT	xOTM	5.21	5.17	6.31	19.40	21.25
	OTM	26.04	25.98	27.45	9.69	6.20
	ATM	17.56	17.53	17.74	12.32	19.86
	ITM	12.98	13.00	13.10	2.93	4.36
	xITM	23.06	23.12	22.71	4.67	0.94
MT	xOTM	4.03	4.01	4.13	6.93	15.81
	OTM	4.14	4.13	4.58	7.21	3.06
	ATM	1.15	1.13	1.20	4.21	5.39
	ITM	6.86	6.84	6.56	11.77	22.53
	xITM	8.40	8.40	8.18	4.59	10.23
ST	xOTM	16.21	16.05	15.45	21.11	26.90
	OTM	8.06	8.07	7.94	9.85	16.94
	ATM	10.13	10.26	10.36	17.31	18.26
	ITM	3.18	3.19	3.03	6.85	7.46
	xITM	6.95	6.96	6.84	9.21	25.55

SKEWNESS		DELTA	GAMMA	ADJ	LOC	REG
LT	xOTM	0.04	0.01	0.04	0.17	0.02
	OTM	-0.34	-0.36	-0.32	-1.06	-0.46
	ATM	-0.07	-0.07	-0.06	0.96	1.60
	ITM	-1.02	-1.03	-1.05	0.50	0.74
	xITM	-1.74	-1.75	-1.73	1.14	0.35
MT	xOTM	0.45	0.45	0.43	0.38	-0.68
	OTM	0.43	0.42	0.45	0.58	-0.14
	ATM	-0.27	-0.27	-0.28	-0.33	-0.73
	ITM	-0.07	-0.06	-0.08	0.03	0.24
	xITM	-0.59	-0.59	-0.59	0.41	0.82
ST	xOTM	0.55	0.48	0.39	0.42	-1.46
	OTM	-1.12	-1.13	-1.08	-0.99	-1.23
	ATM	-0.48	-0.49	-0.48	-0.93	-2.23
	ITM	-0.64	-0.65	-0.63	-0.67	-0.07
	xITM	-1.18	-1.18	-1.19	-1.14	-0.29

Darker shading indicates highest excess kurtosis and lowest skewness.

Tables 11, 12, and 13 report hedging errors as rebalancing frequency is decreased to a 5, 10, and 20 trading day interval, respectively. We observe that the general pattern does not change compared to the previous results. The adjusted delta hedge performs marginally better than the delta and delta-gamma hedge for deep OTM, OTM, and ATM options. As the rebalancing frequency is further reduced, we see that the skewness and kurtosis adjustment gains its significance also for ITM and deep ITM options. Interestingly, the gamma hedge does not prove to deliver better results, which contradicts Expectation #3. Local delta proves to be a better hedge compared to the previous three ratios but still considerably lags behind performance of the regression-based hedge ratio. With decreasing rebalancing frequency, the regression-based hedge delivers better results, and the difference increases significantly also for ATM, OTM, and deep OTM options.

We also find proof that supports Expectation #1, namely, hedging errors are inversely related to the frequency of portfolio rebalancing.

Comparing hedging errors and rebalancing frequency of short- and long-term options we find that error growth is slower for long-term options. Thus, the rebalancing is less frequently required for long-term options because the delta of the options is more static due to lower gamma risk.

Table 11: Hedging Errors for the 5-Day Hedging Horizon

5 DAY		MAHE					RMSHE				
Moneyness	**Maturity**	**DELTA**	**D+G**	**ADJ**	**LOC**	**REG**	**DELTA**	**D+G**	**ADJ**	**LOC**	**REG**
xOTM	**ST**	2.33	2.33	2.30	1.71	1.22	4.19	4.23	4.05	3.42	2.32
	MT	3.45	3.48	3.26	2.71	2.17	5.20	5.26	4.95	4.43	3.65
	LT	5.64	5.68	5.29	3.20	2.92	8.31	8.40	7.86	5.62	5.28
OTM	**ST**	6.84	6.83	6.40	6.17	3.23	11.71	12.00	11.01	11.07	5.70
	MT	8.39	8.53	7.86	6.15	3.46	12.32	12.55	11.63	9.92	5.02
	LT	13.39	13.36	12.98	7.54	5.97	22.95	23.10	22.45	11.26	8.81
ATM	**ST**	16.76	17.08	16.62	13.85	8.12	26.54	27.31	26.43	21.90	12.84
	MT	14.32	14.47	14.20	9.75	5.91	18.93	19.32	18.80	13.44	7.81
	LT	13.35	13.38	13.28	9.35	7.29	20.51	20.70	20.48	14.10	11.65
ITM	**ST**	23.10	23.29	23.55	18.38	9.24	31.39	31.73	31.94	25.93	15.10
	MT	18.99	19.07	19.30	14.10	9.21	27.37	27.58	27.79	20.98	13.99
	LT	18.25	18.30	18.49	13.16	9.58	28.80	28.97	29.17	19.53	13.94
xITM	**ST**	26.63	26.64	26.86	25.88	13.90	39.22	39.30	39.54	39.33	22.56
	MT	26.07	26.05	26.33	22.97	14.30	38.50	38.56	38.78	34.13	22.06
	LT	20.98	21.00	21.21	16.16	8.98	31.87	31.94	32.17	21.24	11.60

Darker shading indicates lower hedging error.

Table 12: Hedging Errors for the 10-Day Hedging Horizon

10 DAY		MAHE					RMSHE				
Moneyness	Maturity	DELTA	D+G	ADJ	LOC	REG	DELTA	D+G	ADJ	LOC	REG
xOTM	ST	3.36	3.43	3.32	2.32	1.62	5.78	5.89	5.52	4.58	3.08
	MT	4.43	4.60	4.19	3.23	2.68	6.06	6.27	5.71	4.77	4.00
	LT	7.39	7.58	6.88	4.63	4.53	10.88	11.14	10.30	7.87	7.71
OTM	ST	10.67	10.97	9.97	9.63	4.06	17.13	18.05	16.12	17.14	7.28
	MT	10.69	11.30	10.00	7.46	5.17	15.62	16.30	14.72	12.26	7.54
	LT	18.35	18.60	17.86	9.02	7.53	30.91	31.35	30.29	13.91	10.67
ATM	ST	25.44	26.76	25.25	22.08	12.39	37.12	39.16	36.99	30.93	18.63
	MT	19.84	20.67	19.68	13.29	7.70	26.58	27.66	26.40	19.35	10.69
	LT	18.00	18.24	17.93	11.21	8.98	25.33	25.87	25.31	14.52	13.31
ITM	ST	34.65	35.17	35.40	25.62	10.32	43.37	44.31	44.31	32.27	14.30
	MT	28.93	29.48	29.47	21.51	12.04	37.45	38.02	38.06	29.81	18.72
	LT	21.94	31.92	22.29	15.93	10.62	31.20	89.98	31.65	21.28	14.36
xITM	ST	39.38	39.44	39.72	37.23	19.72	53.08	53.26	53.44	52.86	28.35
	MT	37.94	37.99	38.31	33.27	21.05	53.53	53.67	53.92	47.11	31.06
	LT	28.85	30.16	29.20	19.40	43.79	42.77	46.20	43.19	25.60	66.14

Darker shading indicates lower hedging error.

Table 13: Hedging Errors for the 20-Day Hedging Horizon

20 DAY		MAHE					RMSHE				
Moneyness	Maturity	DELTA	D+G	ADJ	LOC	REG	DELTA	D+G	ADJ	LOC	REG
xOTM	ST	4.04	4.52	4.03	2.96	2.13	6.94	7.76	6.61	5.81	3.68
	MT	5.21	5.62	4.99	3.65	2.80	7.17	7.96	6.72	5.03	4.34
	LT	9.95	10.20	9.41	6.43	5.91	14.51	15.26	13.78	10.76	10.09
OTM	ST	12.90	14.30	11.95	12.01	5.47	18.30	21.06	16.96	18.18	9.24
	MT	13.56	14.36	12.36	9.34	6.04	18.99	21.08	17.57	14.55	7.99
	LT	22.12	22.38	21.50	11.63	10.36	33.28	34.59	32.54	18.16	14.57
ATM	ST	35.17	37.88	34.80	29.79	15.84	47.71	51.83	47.28	39.66	21.67
	MT	26.78	27.47	26.70	18.64	9.59	36.07	39.01	35.71	26.07	13.26
	LT	24.94	25.81	24.81	15.56	10.87	32.42	34.00	32.43	21.50	16.13
ITM	ST	45.52	45.87	46.53	37.29	16.72	59.98	62.57	61.42	45.76	20.87
	MT	38.11	38.63	38.83	29.94	18.22	56.49	57.92	57.31	48.79	27.64
	LT	28.20	28.30	28.71	19.63	11.72	36.50	37.62	37.24	24.10	15.53
xITM	ST	63.32	63.77	63.32	67.10	32.86	91.55	91.83	91.55	97.58	46.63
	MT	38.69	38.57	39.19	28.26	23.99	63.01	63.52	63.45	41.44	31.88
	LT	42.71	39.21	43.62	30.40	17.03	127.93	60.09	132.10	114.50	23.19

Darker shading indicates lower hedging error.

Further data examination of four sub-periods (see Table 14 in Appendix) reveals that hedging errors are significantly larger in the first and second sub periods which are explained by market turbulence in the respective periods. During the first two periods of the sample (June 2007 - June 2009) the market experienced large fluctuations and an increase in volatility. We observe that gamma hedge gains strength as the rebalancing frequency is decreased and it outperforms pure delta hedge on some occasions. Gamma hedge improves hedging performance for ITM options in highly volatile market conditions, as it delivers lower hedging errors than the BSM hedge ratio. Still, its performance is considerably poorer in comparison to local delta or the regression-based hedge ratio. On separate occasions, the BSM delta proves to be a better hedge for long-term options; however, in most cases it is outperformed by the skewness and kurtosis adjusted delta, local delta, and the regression-based hedge ratio. The regression-based RMSHE errors are considerably lower than those of other hedge ratios, particularly in high volatility conditions.

In most cases Expectation #2 is fulfilled, as hedging errors change along with market conditions and are higher in periods of high volatility.

7.1 Summary of Results

To finalize, the analysis proves that hedging errors are inversely related to rebalancing frequency across all maturities and moneyness levels but is less pronounced for long-term options. We find evidence that hedging errors are positively correlated with volatility and are higher in turbulent markets. Delta-gamma hedge did not prove to reduce hedging errors and only on separate occasions improved the delta hedge performance in turbulent markets with lower rebalancing frequency. The results of the study demonstrate that hedging errors increase with maturity and moneyness of the options. The skewness and kurtosis adjusted delta hedge marginally outperforms the BSM delta hedge only for deep OTM, OTM, and ATM options, while the local delta hedge proved to deliver significantly lower hedging errors compared to the BSM delta hedge across all options. Hedge constructed using the regression-adjusted BSM Greeks returned the lowest hedging errors across all maturity and moneyness levels.

8 Conclusions

Our study is based on empirical evidence that fundamental differences in markets dictate that there is no uniform hedging technique that would be equally efficient across all regions and option markets. The choice of hedging technique is especially important in markets that possess characteristics that are not in line with assumptions of the classical option pricing models. Five different models were tested to find the most appropriate hedge ratio for the Swedish stock market using OMXS30 index, options based on the index, and money market account. The hedge ratios tested were the BSM delta, the BSM delta-gamma, skewness and kurtosis adjusted BSM delta, local volatility adjusted BSM delta, and regression-based adjusted BSM Greeks.

Empirical analysis shows that accounting for negative relationship between implied volatility and index returns can improve hedging performance significantly. While adjustment for non-normal skewness and kurtosis does not provide considerable enhancement of hedging performance compared to the BSM model.

Benchmarking root mean squared hedging error values, the regression-based hedge ratio delivered significantly lower hedging errors across all maturities and moneyness levels. This approach also suggests that the optimal delta and gamma levels are considerably lower than those suggested by the BSM model. While the regression-based model delivered lowest errors overall, hedging error decrease was higher for ITM options compared to other hedging strategies. Consistent with previous research, hedging frequency is inversely related to hedging errors for all models tested.

9 Limitations and Further Research

The limitations of the study are mainly related to data availability. Using index bid-ask price midpoints could yield more precise results and eliminate the non-synchronous price risk. Our results are subject to bias arising from empirical evidence that trading volume increases at market close, creating significant fluctuations in volatility and price of the underlying.

We encourage further research on the topic by using the regression-based hedge ratios on stock indexes other than the OMXS30 in order to test if the results are persistent across different markets. Additionally, one could test several regression methods including the use of Weighted Least Squares regressions in periods of high volatility. Further tests could also be done using a hedge portfolio that includes not only the underlying asset and money market account, but also other option contracts. This would allow for enhanced gamma and possibly also vega hedge. Additionally, if the midpoint of bid-ask prices is obtained, one could create and test a stochastic volatility model and compare its hedging performance to the constant volatility models. Previous research indicates that stochastic volatility models provide considerable improvements in option pricing, while evidence on hedging performance is contradictory. Additionally, regression-based hedge ratios using e.g. GARCH models could be calculated and compared to the more traditional approaches. This would also allow testing hedging predictability power of these models that could be of significant usefulness for practitioners.

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Appendix

Table 3: Average Implied Volatilities for the Sample

Sample Period	Moneyness S/K	Days-to-Expiration < 60	60 - 180	> 180
June 2007 - March 2011	< 0.9	33.02	31.45	30.24
	0.9 - 0.97	25.6	24.11	21.96
	0.97 - 1.03	27.99	25.02	22.5
	1.03 - 1.1	30.99	27.68	23.73
	> 1.1	39.72	29.29	22.3
June 2007 - June 2008	< 0.9	30.96	28.07	27.25
	0.9 - 0.97	22.51	22.06	20.43
	0.97 - 1.03	23.23	23.01	21.29
	1.03 - 1.1	26.77	25.51	22.8
	> 1.1	29.68	25.27	19.61
July 2008 - June 2009	< 0.9	39.8	40.58	36.68
	0.9 - 0.97	36.15	32.29	27.66
	0.97 - 1.03	39.75	34.99	26.76
	1.03 - 1.1	41.79	35.77	26.04
	> 1.1	37.93	29.24	20.19
July 2009 - June 2010	< 0.9	29.36	22.47	20.36
	0.9 - 0.97	20.91	21.39	20.51
	0.97 - 1.03	19.07	22.7	21.54
	1.03 - 1.1	24.9	26.48	22.13
	> 1.1	44.98	36.19	28.01
July 2010 - March 2011	< 0.9	21.95	21.36	20.19
	0.9 - 0.97	16.63	17.92	19.48
	0.97 - 1.03	-	19.84	21.6
	1.03 - 1.1	44.66	24.11	25.51
	> 1.1	41.6	25.4	26.6

Figure 5: Implied Volatility Smirk July 2009 – June 2010

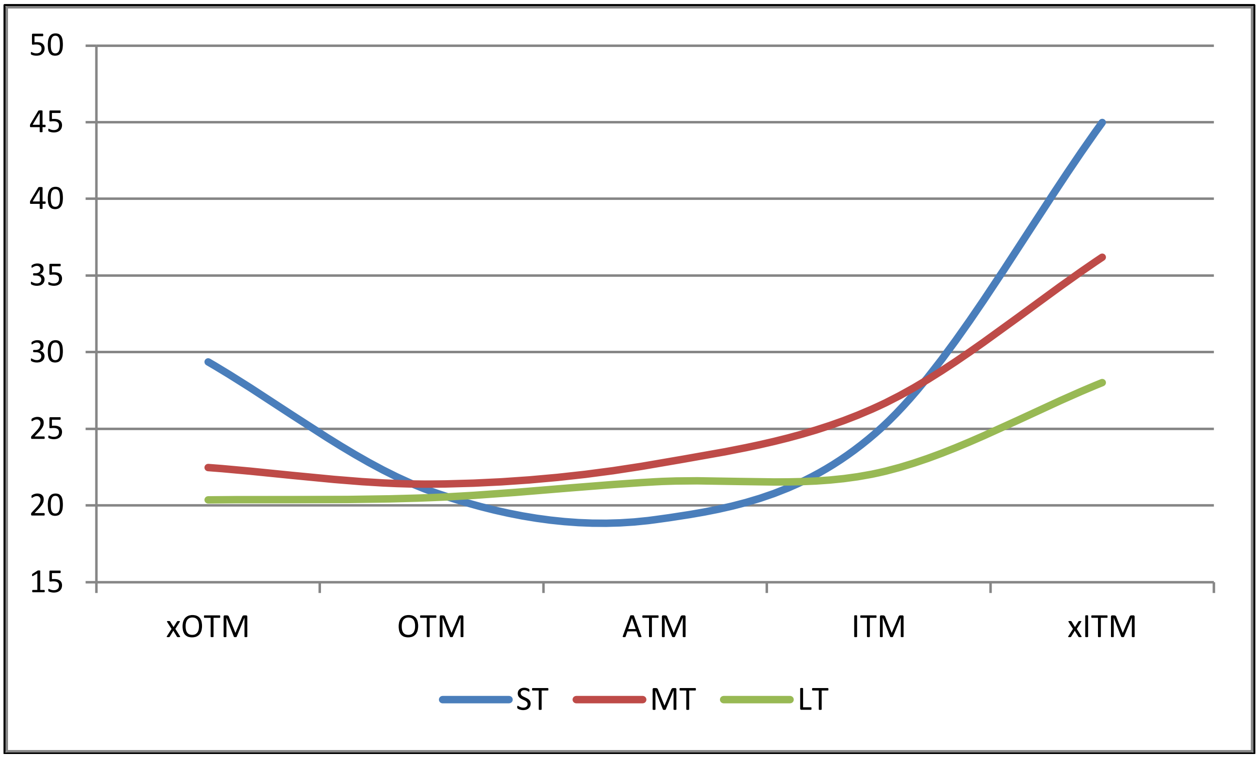

Figure 7: Long-term Options Implied Volatility Surface

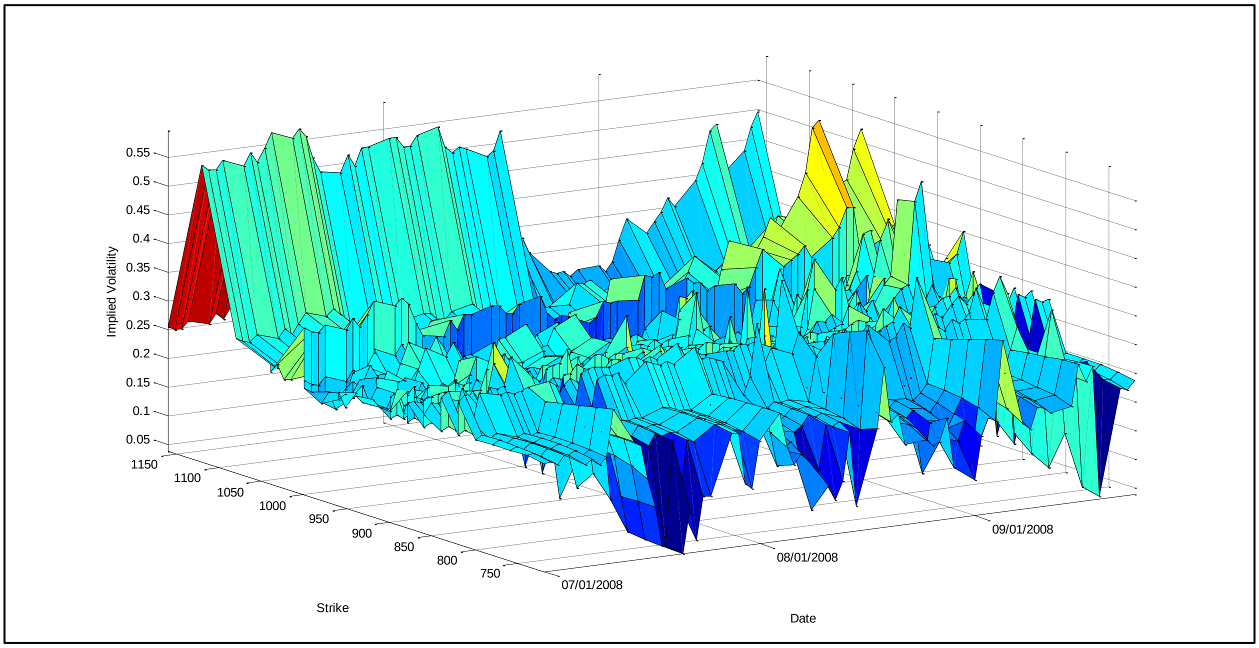

Figure 8: Medium-term Options Implied Volatility Surface

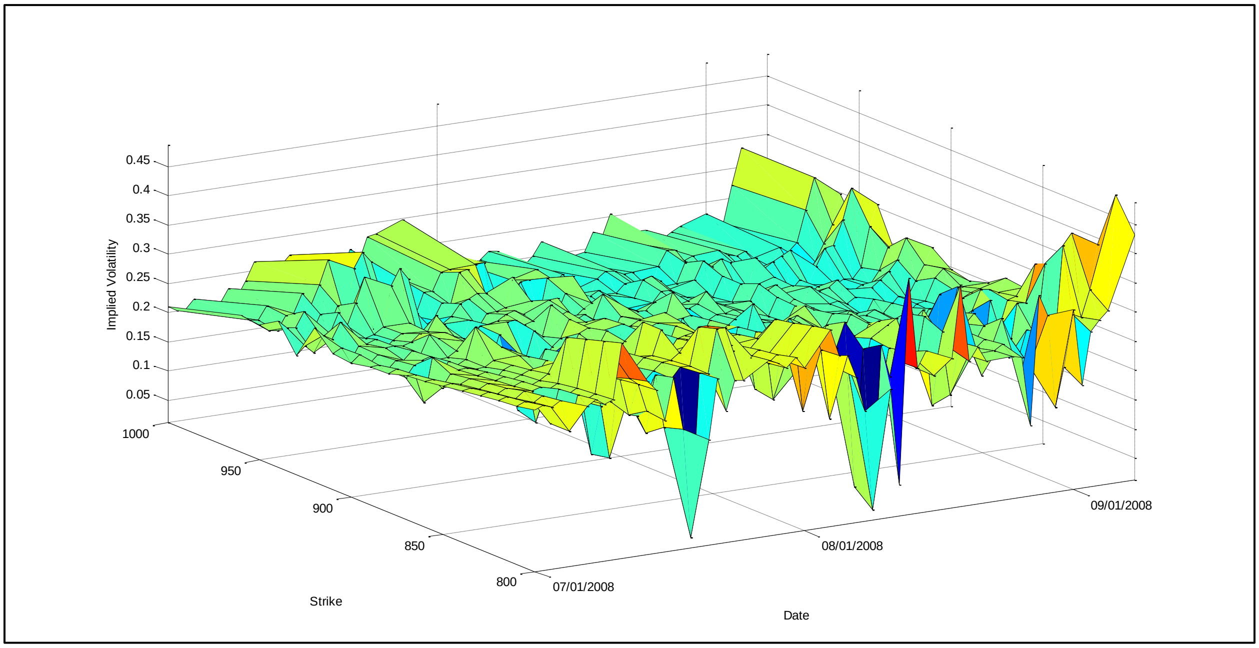

Figure 9: Short-term Options Implied Volatility Surface

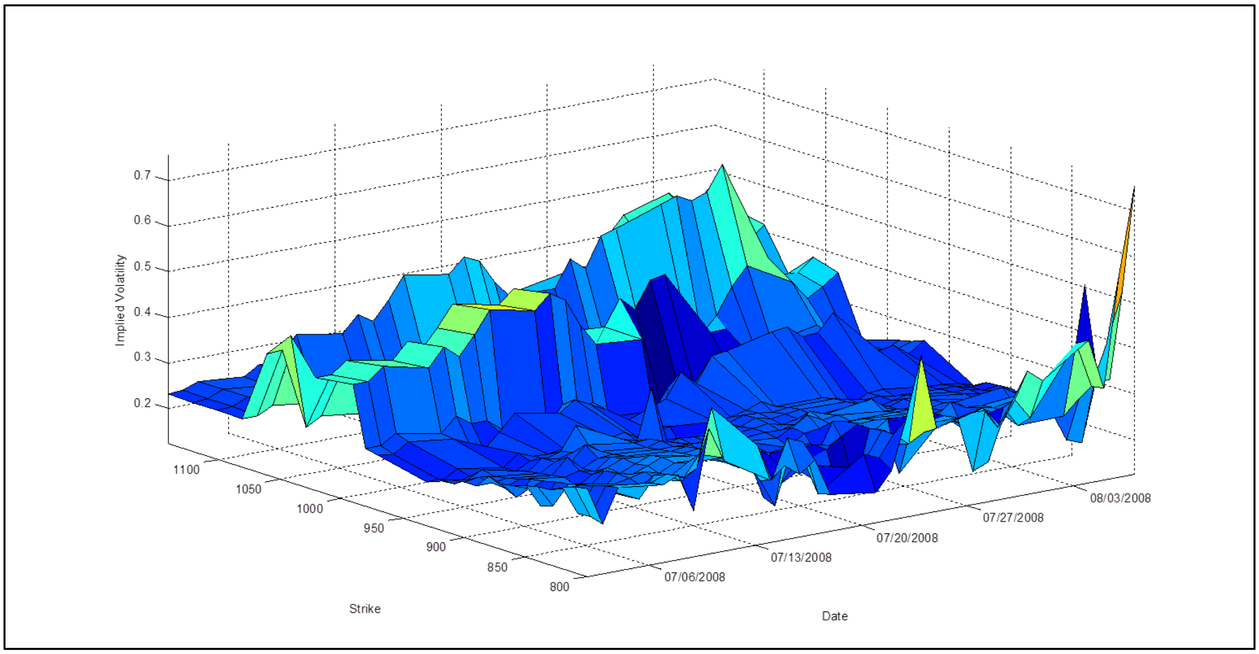

Figure 10: Long-term ATM BSM, Kurtosis and Skewness Adjusted, and Local Delta

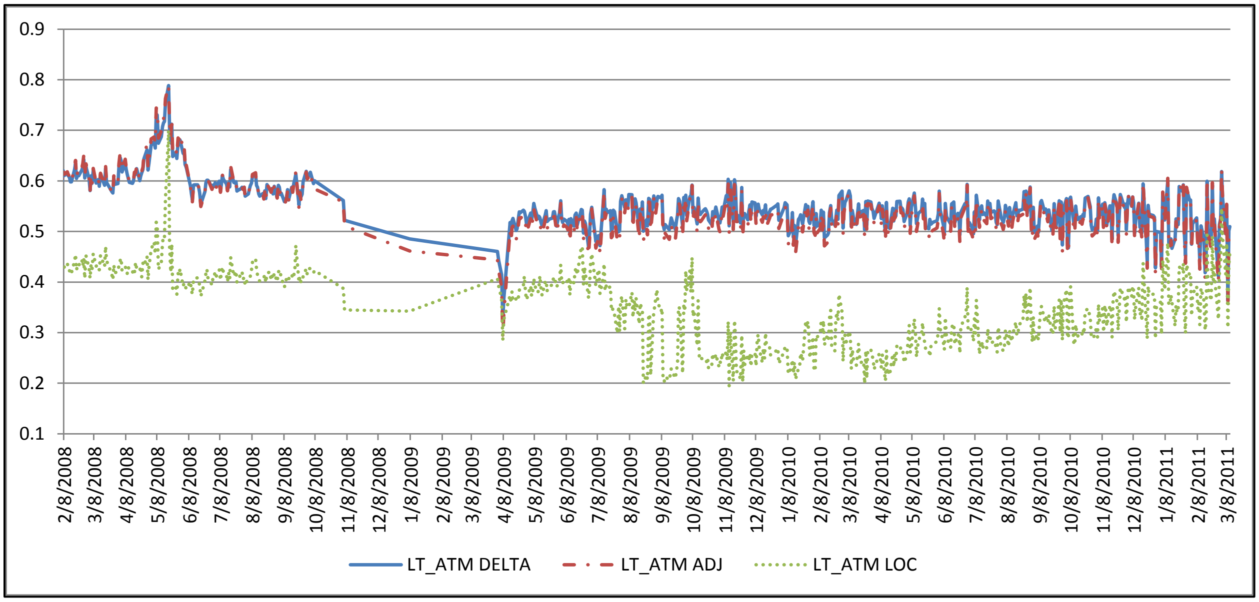

Figure 11: Medium-term ATM BSM, Kurtosis and Skewness Adjusted, and Local Delta

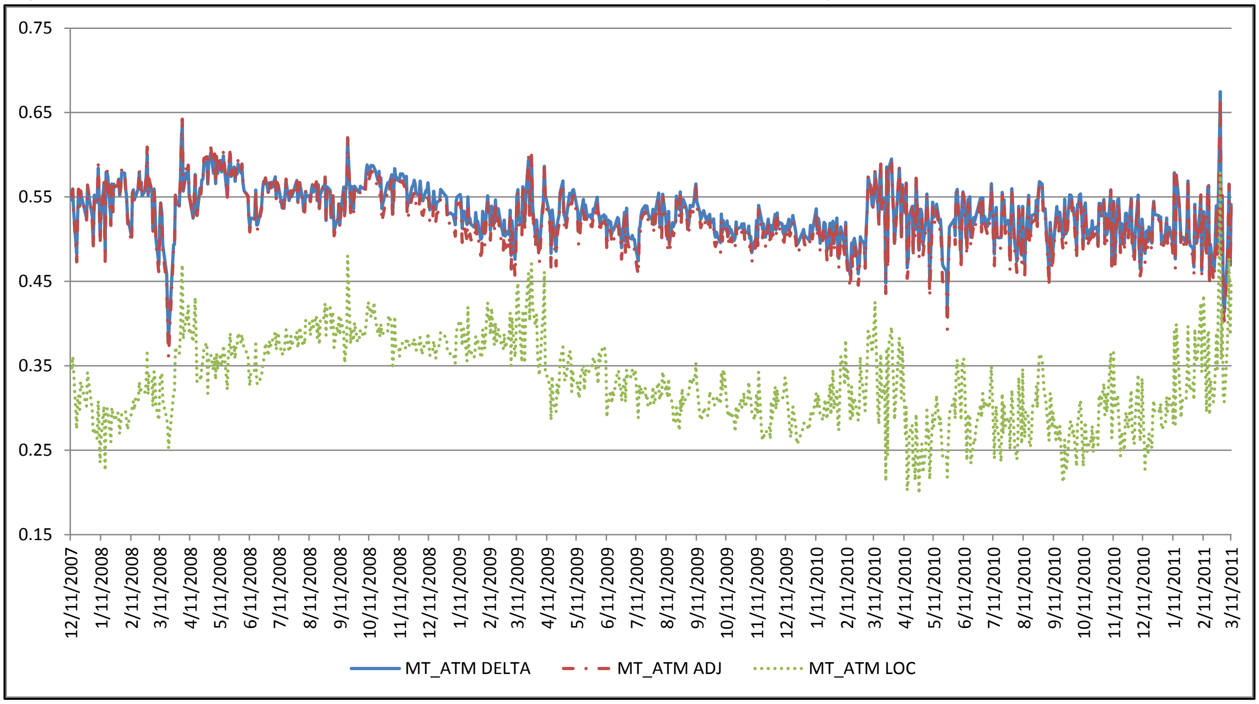

Figure 12: Short-term ATM BSM, Kurtosis and Skewness Adjusted, and Local Delta

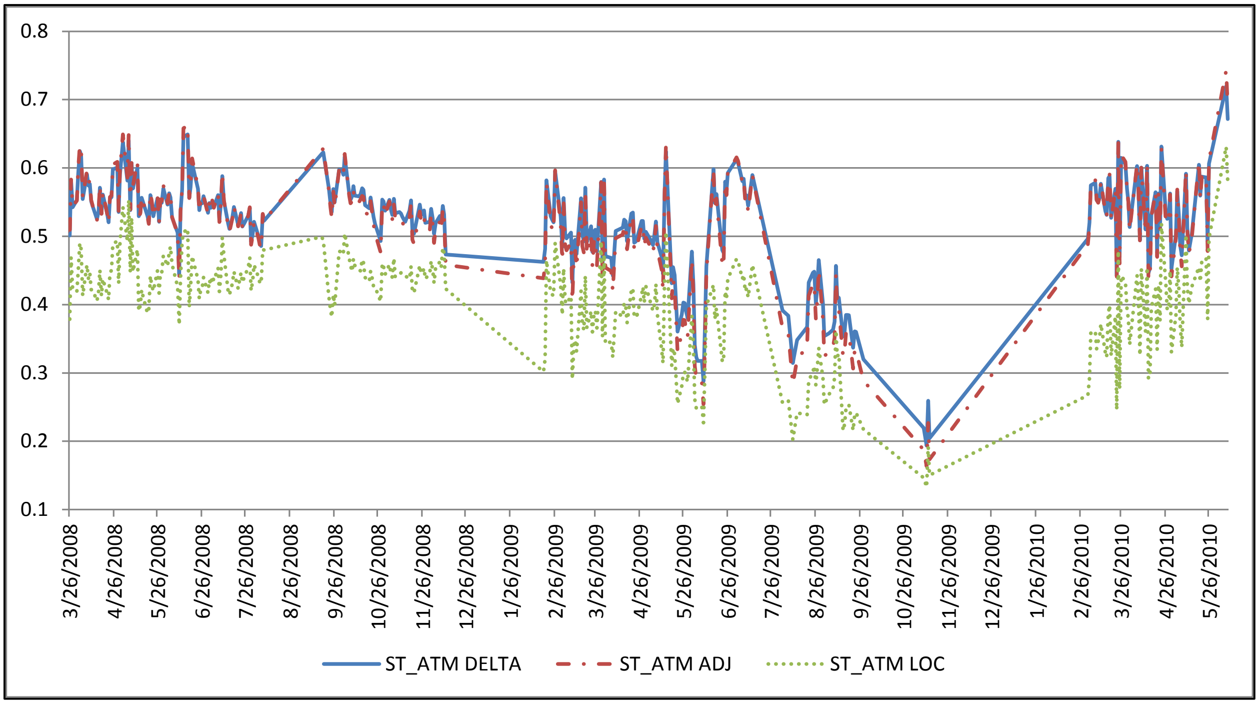

Table 9: Adjusted, Local, Regression-Based Delta F test (the two-tailed probability that the variances are not significantly different)

		REG	LOC	ADJ
LT	**xOTM**	0.00 *	0.26	0.00 *
	OTM	0.00 *	0.00 *	0.77
	ATM	0.00 *	0.00 *	0.00 *
	ITM	0.00 *	0.00 *	0.10 ***
	xITM	0.00 *	0.00 *	0.04 **
MT	**xOTM**	0.00 *	0.00 *	0.00 *
	OTM	0.00 *	0.00 *	0.06 ***
	ATM	0.00 *	0.00 *	0.00 *
	ITM	0.00 *	0.00 *	0.19
	xITM	0.00 *	0.00 *	0.03 **
ST	**xOTM**	0.00 *	0.00 *	0.00 *
	OTM	0.00 *	0.08 ***	0.01 *
	ATM	0.00 *	0.90	0.01 *
	ITM	0.00 *	0.00 *	0.98
	xITM	0.00 *	0.00 *	0.00 *

* significant at 1%   ** significant at 5%   *** significant at 10%

Table 14: Hedging Errors for Sample Sub Periods (1: June 2007 – June 2008; 2: July 2008 – June 2009; 3: July 2009 – June 2010; 4: July 2010 – March 2011)

1 DAY			MAHE					RMSHE				
Moneyness	Maturity		DELTA	D+G	ADJ	LOC	REG	DELTA	D+G	ADJ	LOC	REG
xOTM	ST	1	1.14	1.16	1.14	0.69	0.45	1.91	1.92	1.88	1.03	0.69
		2	2.21	2.21	2.10	1.81	1.11	3.65	3.66	3.45	3.10	1.99
		3	0.59	0.59	0.62	0.43	0.45	0.99	0.98	0.98	0.81	0.77
		4	0.18	0.19	0.24	0.12	0.16	0.33	0.33	0.38	0.27	0.31
	MT	1	2.20	2.22	2.09	1.57	1.13	3.12	3.13	2.98	2.37	1.66
		2	3.09	3.09	2.92	2.56	1.55	4.86	4.87	4.61	4.18	2.82
		3	1.70	1.70	1.62	1.26	1.22	2.72	2.72	2.59	2.09	2.01
		4	0.64	0.64	0.63	0.49	0.51	1.04	1.03	1.01	0.88	0.93
	LT	1	4.94	4.96	4.71	4.74	3.99	7.93	7.94	7.72	8.61	7.88
		2	2.80	2.81	2.60	1.82	1.72	4.27	4.28	4.00	3.07	3.03
		3	2.43	2.43	2.26	1.25	1.24	3.48	3.48	3.27	2.00	2.02
		4	1.08	1.08	1.02	0.63	0.61	1.94	1.93	1.82	1.16	1.11
OTM	ST	1	4.11	4.13	3.86	3.78	1.74	6.60	6.64	6.19	5.04	2.38
		2	5.78	5.78	5.40	5.69	2.73	9.00	9.02	8.51	8.98	4.57
		3	1.30	1.29	1.20	1.13	0.86	2.20	2.20	2.04	1.98	1.42
		4	0.35	0.35	0.37	0.28	0.30	0.55	0.54	0.54	0.45	0.57
	MT	1	5.09	5.14	4.81	3.70	2.28	7.11	7.15	6.75	5.58	3.25
		2	7.38	7.39	7.00	6.12	3.08	10.90	10.91	10.43	9.27	4.47
		3	3.16	3.16	2.92	2.05	1.59	4.66	4.66	4.34	3.17	2.19
		4	1.97	1.97	1.85	1.36	1.20	2.96	2.96	2.78	2.07	1.71
	LT	1	10.47	10.52	10.30	6.35	6.19	18.91	18.93	18.70	9.72	8.69
		2	7.81	7.81	7.57	6.47	4.95	12.69	12.71	12.40	10.81	8.12
		3	5.36	5.36	5.11	3.53	3.33	7.71	7.71	7.40	5.03	4.62
		4	2.91	2.90	2.77	2.37	2.32	5.21	5.20	5.01	3.97	3.65
ATM	ST	1	8.76	8.84	8.76	6.03	3.73	12.59	12.73	12.56	8.11	5.91
		2	9.31	9.31	9.21	8.03	4.45	15.21	15.27	15.11	13.59	8.13
		3	6.59	6.59	6.51	5.49	3.86	13.77	13.77	13.69	11.79	6.95
		4	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	MT	1	8.98	9.02	8.97	4.67	3.42	12.62	12.69	12.60	7.26	5.21
		2	10.21	10.23	10.09	7.95	4.73	14.32	14.34	14.18	11.50	6.91
		3	5.52	5.51	5.44	3.69	2.48	7.42	7.42	7.32	5.00	3.40
		4	6.12	6.14	6.07	4.79	3.67	8.06	8.08	8.00	6.24	4.90
	LT	1	11.14	11.16	11.17	6.94	5.34	21.62	21.64	21.63	11.54	9.02
		2	10.53	10.55	10.48	8.63	6.20	16.98	17.00	16.95	14.78	11.58
		3	5.89	5.89	5.81	4.52	4.36	9.58	9.58	9.46	7.56	6.85
		4	6.32	6.32	6.27	5.58	5.04	9.22	9.22	9.15	7.83	6.59
ITM	ST	1	12.41	12.43	12.65	8.20	3.55	18.13	18.18	18.43	10.38	4.54
		2	11.46	11.46	11.66	10.00	5.83	17.63	17.65	17.86	16.00	8.94
		3	8.38	8.38	8.52	7.42	5.43	13.46	13.46	13.64	12.35	10.11
		4	9.58	9.58	9.73	8.27	4.62	12.06	12.08	12.18	11.19	10.47
	MT	1	10.80	10.82	11.02	4.91	3.82	15.72	15.77	16.00	8.03	5.88
		2	11.95	11.95	12.10	9.92	6.41	17.57	17.59	17.75	14.87	10.47
		3	9.95	9.95	10.09	8.46	6.51	17.49	17.49	17.62	15.54	13.50
		4	8.87	8.87	9.00	7.13	5.96	15.56	15.56	15.69	13.55	11.68
	LT	1	11.91	11.95	12.10	8.31	5.64	21.60	21.63	21.85	11.87	7.69
		2	12.09	12.10	12.25	10.56	6.41	18.03	18.05	18.22	16.20	10.72
		3	8.08	8.08	8.19	6.94	5.12	12.20	12.20	12.32	10.73	8.47
		4	9.06	9.06	9.17	7.62	5.89	12.92	12.93	13.06	11.13	8.37
xITM	ST	1	12.14	12.12	12.29	11.58	4.92	16.93	16.94	17.19	14.64	9.24
		2	15.34	15.34	15.53	14.89	4.97	23.45	23.48	23.68	22.73	7.31
		3	12.67	12.67	12.76	12.01	6.32	20.41	20.41	20.54	19.57	13.80
		4	9.59	9.59	9.65	8.77	3.87	13.84	13.84	13.91	13.00	9.94
	MT	1	14.94	14.94	15.16	NA	NA	27.22	27.25	27.43	NA	NA
		2	14.96	14.96	15.09	13.82	7.50	21.79	21.79	21.94	19.51	12.20
		3	15.85	15.85	15.94	14.53	9.78	26.37	26.37	26.47	25.00	19.00
		4	10.51	10.50	10.59	8.85	4.48	16.39	16.38	16.49	14.56	8.73
	LT	1	10.91	10.92	11.06	8.99	5.45	18.54	18.56	18.74	12.08	7.47
		2	11.50	11.49	11.69	9.81	5.99	16.81	16.80	17.03	14.73	9.10
		3	9.18	9.18	9.29	8.29	3.92	15.04	15.03	15.15	14.04	6.79
		4	8.84	8.84	8.93	7.65	3.43	12.13	12.13	12.25	10.56	6.14

5 DAY Moneyness	Maturity		MAHE					RMSHE				
			DELTA	D+G	ADJ	LOC	REG	DELTA	D+G	ADJ	LOC	REG
xOTM	ST	1	2.62	2.59	2.63	1.40	0.69	3.77	3.76	3.77	1.82	0.96
		2	4.31	4.36	4.10	3.52	2.35	6.66	6.77	6.34	5.57	3.75
		3	1.35	1.35	1.39	1.00	0.97	2.32	2.31	2.27	1.92	1.50
		4	0.46	0.44	0.56	0.28	0.29	0.77	0.75	0.86	0.50	0.54
	MT	1	3.34	3.44	3.15	2.82	2.20	4.37	4.57	4.15	3.85	3.05
		2	5.48	5.53	5.19	4.48	3.08	7.71	7.76	7.38	6.61	5.02
		3	3.06	3.05	2.88	2.15	2.11	4.33	4.31	4.09	3.12	3.29
		4	1.19	1.12	1.14	0.81	0.90	1.71	1.64	1.66	1.40	1.61
	LT	1	9.02	9.16	8.56	7.54	6.72	11.80	12.00	11.28	11.07	9.94
		2	5.51	5.58	5.14	3.85	3.41	8.21	8.31	7.71	5.95	5.79
		3	4.74	4.73	4.42	2.37	2.34	6.71	6.70	6.28	3.59	3.55
		4	2.65	2.58	2.47	0.91	0.79	3.78	3.73	3.48	1.56	1.38
OTM	ST	1	8.42	8.36	7.88	7.60	3.11	12.35	12.91	11.49	9.03	3.83
		2	12.23	12.33	11.55	12.24	5.94	17.41	17.63	16.50	17.56	9.02
		3	2.19	2.17	1.93	1.76	1.74	3.14	3.12	2.73	2.57	2.45
		4	0.74	0.67	0.72	0.50	0.55	1.00	0.98	0.91	0.65	0.91
	MT	1	8.84	9.22	8.28	5.74	3.63	10.68	11.28	9.99	7.02	4.48
		2	14.00	14.15	13.26	11.62	5.41	19.23	19.46	18.31	16.22	7.49
		3	5.66	5.65	5.16	3.44	2.27	7.78	7.76	7.14	4.74	3.10
		4	3.82	3.83	3.60	2.69	2.33	5.52	5.46	5.11	3.52	2.90
	LT	1	23.27	23.27	22.82	11.29	7.91	37.13	37.41	36.56	14.45	11.16
		2	13.12	13.13	12.79	11.04	8.36	18.87	19.05	18.37	15.62	11.47
		3	9.81	9.80	9.33	5.59	4.87	13.30	13.30	12.63	7.72	6.96
		4	6.10	5.96	5.78	4.42	3.87	10.44	10.36	9.96	7.20	5.92
ATM	ST	1	16.28	16.95	16.29	11.34	7.52	26.38	27.83	26.43	17.97	13.12
		2	18.73	18.95	18.52	16.20	8.65	26.34	26.83	26.17	22.90	13.15
		3	14.18	14.04	13.93	11.36	7.62	27.18	27.17	26.86	22.44	12.08
		4	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	MT	1	16.33	16.66	16.32	9.84	6.73	20.92	21.80	20.92	13.57	8.42
		2	18.94	19.04	18.72	14.40	7.88	23.67	23.99	23.41	18.37	9.79
		3	10.03	9.96	9.88	6.19	3.72	13.54	13.50	13.36	8.39	4.88
		4	11.17	11.44	11.07	8.20	5.62	14.69	14.81	14.56	10.72	7.56
	LT	1	16.67	16.91	16.78	8.10	5.49	26.63	27.04	26.79	11.02	7.80
		2	20.81	21.05	20.71	17.10	11.13	28.74	28.93	28.66	25.30	21.00
		3	9.04	9.03	8.86	6.70	6.44	12.22	12.21	12.01	8.58	7.86
		4	10.11	9.79	10.00	8.16	6.67	12.90	12.96	12.77	9.92	7.71
ITM	ST	1	25.53	26.01	26.12	16.23	6.38	33.50	34.07	34.22	21.05	7.75
		2	22.80	22.86	23.25	19.52	8.28	31.72	32.02	32.28	27.93	12.16
		3	20.00	19.94	20.28	18.33	11.68	27.91	27.88	28.20	26.32	20.12
		4	21.77	21.74	21.89	21.03	15.99	28.07	28.24	28.16	27.66	22.33
	MT	1	22.10	22.36	22.55	14.44	10.38	31.11	31.65	31.78	23.09	14.56
		2	22.10	22.17	22.39	18.12	9.99	29.72	29.93	30.15	24.35	13.09
		3	18.19	18.15	18.45	14.32	10.36	28.23	28.22	28.51	22.25	17.66
		4	12.32	12.35	12.58	8.45	5.95	15.57	15.58	15.88	10.82	7.51
	LT	1	22.99	23.18	23.30	10.60	7.44	39.62	39.96	40.21	14.71	8.97
		2	25.39	25.57	25.61	22.85	16.25	34.53	34.70	34.80	31.47	22.10
		3	11.10	11.04	11.30	8.61	8.19	16.77	16.74	17.05	13.19	11.75
		4	18.28	18.25	18.50	14.34	8.27	23.34	23.41	23.60	18.85	11.75
xITM	ST	1	23.09	23.16	23.32	18.55	13.87	30.77	31.01	31.27	20.47	17.09
		2	38.19	38.27	38.62	37.28	10.21	61.11	61.34	61.53	60.03	13.19
		3	27.75	27.74	27.98	26.13	14.87	39.94	39.95	40.21	38.00	23.93
		4	23.84	23.78	24.00	21.88	14.39	33.19	33.16	33.39	31.01	24.71
	MT	1	27.43	27.40	27.88	NA	NA	43.32	43.56	43.66	NA	NA
		2	26.83	26.84	27.12	23.16	14.76	35.54	35.62	35.88	30.92	19.87
		3	30.68	30.67	30.88	27.82	19.25	46.49	46.47	46.75	43.36	29.54
		4	19.21	19.17	19.36	16.11	8.03	25.97	25.91	26.13	22.76	11.93
	LT	1	24.98	25.09	25.27	15.13	8.50	41.12	41.28	41.51	19.21	10.20
		2	22.92	22.78	23.34	18.79	7.33	26.38	26.34	26.80	22.13	8.90
		3	16.82	16.77	16.98	15.78	9.13	26.75	26.67	26.91	25.31	12.68
		4	17.71	17.72	17.83	16.09	9.79	20.80	20.79	20.95	18.99	12.50

10 DAY Moneyness	Maturity		MAHE					RMSHE				
			DELTA	D+G	ADJ	LOC	REG	DELTA	D+G	ADJ	LOC	REG
xOTM	ST	1	4.17	4.33	4.15	1.84	1.41	5.63	5.54	5.54	2.51	1.69
		2	6.15	6.23	5.82	5.19	3.24	9.27	9.56	8.68	7.75	5.13
		3	1.60	1.61	1.70	1.07	0.98	2.24	2.21	2.29	1.60	1.39
		4	0.89	0.91	1.03	0.37	0.42	1.42	1.45	1.54	0.55	0.83
	MT	1	4.44	5.08	4.16	2.96	2.76	5.11	5.85	4.77	3.21	3.72
		2	5.92	6.08	5.57	4.67	3.29	8.24	8.39	7.80	6.79	4.71
		3	4.86	4.79	4.62	3.25	2.87	6.21	6.15	5.83	4.35	4.40
		4	1.78	1.64	1.71	1.40	1.53	2.41	2.21	2.33	2.14	2.17
	LT	1	12.18	12.80	11.25	12.17	9.89	14.80	15.36	14.04	14.83	13.17
		2	6.86	7.08	6.50	5.22	6.10	11.16	11.44	10.68	9.15	9.76
		3	7.15	7.10	6.58	3.44	3.18	9.71	9.69	9.01	4.53	4.32
		4	2.31	2.26	2.21	1.35	1.18	3.79	3.64	3.53	1.99	1.78
OTM	ST	1	12.35	13.57	11.49	9.02	4.93	15.65	17.40	14.46	10.69	6.24
		2	20.44	20.37	19.20	20.62	7.33	27.74	28.69	26.32	28.43	11.50
		3	3.62	3.43	3.32	3.23	2.08	4.38	4.22	3.92	3.83	2.77
		4	1.29	1.16	1.28	1.07	0.50	1.59	1.32	1.55	1.31	0.77
	MT	1	12.59	14.81	11.74	7.74	5.84	15.30	16.88	14.26	9.72	6.81
		2	14.74	14.82	13.90	12.13	7.73	22.68	23.42	21.66	19.42	11.23
		3	9.38	9.31	8.58	5.56	3.49	11.81	11.73	10.77	7.07	4.73
		4	4.58	4.69	4.43	3.64	3.51	6.24	6.05	5.94	4.71	4.30
	LT	1	37.85	38.94	37.11	15.44	10.93	52.41	53.22	51.62	18.84	12.42
		2	15.34	15.67	15.03	12.97	11.38	24.85	25.40	24.16	20.73	16.04
		3	10.61	10.57	10.14	6.40	6.10	13.01	12.98	12.34	7.71	7.59
		4	7.61	7.14	7.23	4.92	3.68	11.81	11.52	11.14	7.22	5.16
ATM	ST	1	21.83	24.63	21.82	16.58	10.53	34.93	38.93	35.00	24.59	15.50
		2	29.26	30.20	29.01	25.07	13.84	37.73	39.09	37.45	31.24	19.15
		3	25.04	24.38	24.67	19.99	11.09	39.75	39.70	39.55	33.65	19.67
		4	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	MT	1	23.48	26.24	23.48	12.03	8.23	28.49	30.93	28.50	17.21	9.65
		2	27.43	27.61	27.14	21.45	11.05	36.64	37.60	36.32	29.24	15.51
		3	13.14	13.00	12.94	8.60	5.28	17.28	17.16	17.06	10.89	6.59
		4	13.90	14.31	13.72	9.57	6.07	16.33	16.72	16.15	11.52	7.55
	LT	1	24.68	25.50	24.89	8.04	7.14	36.83	37.84	37.01	10.31	9.20
		2	21.91	22.11	21.86	18.81	15.76	26.16	27.06	26.13	23.07	23.39
		3	13.31	13.16	13.07	8.92	7.40	16.49	16.44	16.23	10.82	9.24
		4	15.53	14.84	15.35	11.31	7.57	19.12	18.40	18.90	13.80	9.49
ITM	ST	1	41.17	42.14	42.26	25.43	8.32	49.14	50.64	50.41	28.89	10.36
		2	35.98	36.62	36.69	30.68	12.11	44.36	45.24	45.14	37.44	17.34
		3	24.19	23.97	24.58	20.11	8.47	33.09	33.00	33.63	28.22	11.07
		4	34.43	34.99	34.48	34.20	30.98	34.43	34.99	34.48	34.20	30.98
	MT	1	31.23	32.95	32.05	19.47	13.95	37.65	39.20	38.62	27.15	16.41
		2	35.75	35.99	36.27	28.79	10.25	42.96	43.53	43.58	35.21	14.41
		3	30.01	29.87	30.50	23.74	16.12	41.02	41.00	41.47	33.70	26.65
		4	15.91	16.40	16.21	10.26	7.51	20.72	20.79	21.09	13.42	9.13
	LT	1	27.12	27.87	27.76	18.17	11.22	38.77	39.68	39.49	19.55	13.49
		2	22.73	22.83	22.96	19.97	15.49	31.67	32.31	32.08	26.96	19.76
		3	17.04	16.93	17.34	13.08	9.40	24.97	24.89	25.31	19.10	13.48
		4	22.48	60.16	22.69	16.37	9.12	29.48	167.77	29.75	20.76	11.82
xITM	ST	1	37.79	38.04	38.24	31.73	20.38	43.94	44.52	44.54	33.21	24.63
		2	56.19	56.37	56.65	55.26	11.79	84.90	85.19	85.15	84.00	14.69
		3	42.52	42.54	42.82	39.77	23.37	53.93	53.95	54.32	50.64	32.04
		4	30.00	29.91	30.25	27.14	20.11	37.41	37.35	37.71	34.50	30.31
	MT	1	45.06	45.96	45.85	NA	NA	65.59	66.21	66.13	NA	NA
		2	37.75	37.58	38.15	34.13	22.20	53.56	53.69	54.04	49.50	31.42
		3	40.02	40.02	40.20	37.86	27.35	56.81	56.78	57.10	53.79	39.56
		4	30.55	30.26	30.80	25.12	12.76	37.41	37.25	37.67	32.81	16.20
	LT	1	43.29	45.44	43.70	27.27	103.09	59.12	65.25	59.56	29.14	114.10
		2	11.58	18.47	11.99	7.84	82.15	14.16	20.40	14.60	10.41	87.75
		3	26.39	25.57	26.93	24.06	13.78	38.47	37.07	39.25	34.96	17.70
		4	17.83	17.47	17.97	15.85	11.31	21.78	22.03	21.94	19.17	15.33

20 DAY Moneyness	Maturity		MAHE					RMSHE				
			DELTA	D+G	ADJ	LOC	REG	DELTA	D+G	ADJ	LOC	REG
xOTM	ST	1	5.15	6.68	4.94	2.97	2.33	6.69	8.48	6.33	3.69	2.87
		2	7.53	7.84	7.32	6.65	4.75	11.37	12.20	10.72	9.95	6.23
		3	1.67	1.64	1.88	1.06	0.72	2.03	1.99	2.30	1.32	1.00
		4	1.23	1.32	1.47	0.62	0.41	1.75	1.69	2.10	0.78	0.50
	MT	1	5.14	6.61	4.60	2.58	2.41	6.61	9.06	5.86	3.24	2.84
		2	6.32	6.85	5.85	4.58	2.57	8.43	9.26	7.84	6.21	3.57
		3	5.90	5.73	5.83	4.28	4.22	7.97	7.85	7.53	5.52	6.51
		4	2.21	2.41	3.14	2.16	1.55	3.41	2.90	4.52	3.04	2.17
	LT	1	18.48	19.40	17.65	21.15	18.09	23.92	25.42	22.84	26.79	23.80
		2	10.44	10.71	9.87	7.34	7.06	13.40	14.22	12.69	9.70	10.09
		3	6.63	6.66	6.10	3.89	3.86	8.71	8.64	8.10	4.54	4.44
		4	4.19	3.98	4.00	1.97	1.58	5.79	5.34	5.37	2.58	2.19
OTM	ST	1	13.19	17.59	11.88	9.08	5.11	16.49	22.15	14.67	10.42	6.14
		2	24.81	25.65	23.26	24.94	10.44	29.33	32.03	27.60	29.87	15.15
		3	6.42	6.00	5.88	5.75	2.72	7.69	7.15	6.93	6.74	3.23
		4	2.56	2.26	2.66	2.36	1.81	3.32	2.71	3.40	3.14	2.94
	MT	1	14.86	17.71	13.85	11.04	8.94	18.99	23.77	17.55	13.99	9.44
		2	20.76	21.96	19.47	16.71	9.11	27.89	30.54	26.35	22.81	11.08
		3	10.48	10.21	9.48	5.85	4.29	11.78	11.47	10.72	7.10	5.72
		4	5.27	5.43	4.78	2.93	2.09	7.08	6.37	6.42	3.74	2.66
	LT	1	37.79	38.85	37.05	9.27	8.61	49.40	51.69	48.63	10.93	9.53
		2	31.40	32.18	30.37	22.59	19.87	41.91	43.92	40.78	32.38	24.38
		3	13.35	13.22	12.97	10.02	9.36	17.03	16.84	16.32	11.10	11.39
		4	7.91	7.35	7.51	5.04	3.98	10.99	10.06	10.46	7.51	6.04
ATM	ST	1	31.23	35.70	30.99	24.91	14.28	42.32	49.88	41.83	32.38	16.70
		2	33.78	36.76	33.49	28.44	16.00	48.66	52.27	48.22	38.31	21.72
		3	44.74	43.80	44.01	35.15	16.81	54.64	54.41	54.31	45.58	24.84
		4	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	MT	1	28.23	29.54	28.26	17.63	10.98	39.46	45.77	39.57	28.16	14.33
		2	39.38	41.49	38.90	30.21	13.05	48.11	51.51	47.61	37.10	17.49
		3	22.34	21.73	21.99	13.81	7.81	26.72	26.44	26.33	17.53	10.97
		4	14.75	13.67	14.60	10.01	6.27	18.51	18.75	18.32	12.32	7.28
	LT	1	32.05	34.35	32.30	13.20	5.11	39.37	42.67	39.77	15.80	5.84
		2	32.85	33.94	32.82	25.47	18.27	42.75	45.42	42.78	32.91	25.39
		3	15.98	15.89	15.68	10.65	8.36	18.34	18.10	18.07	13.08	11.67
		4	23.83	24.33	23.43	16.02	11.30	30.42	30.15	30.09	22.22	15.50
ITM	ST	1	43.35	42.84	44.56	24.08	14.73	62.87	67.05	64.57	31.23	18.40
		2	50.30	51.87	51.50	40.69	15.58	62.37	65.30	64.01	48.76	22.81
		3	48.39	48.41	49.10	44.37	17.56	57.08	56.83	58.00	51.80	18.51
		4	9.02	10.31	8.39	13.46	29.84	9.02	10.31	8.39	13.46	29.84
	MT	1	42.17	44.03	43.41	38.24	21.52	56.37	60.66	58.09	48.95	24.17
		2	35.61	35.80	35.96	31.15	15.49	67.91	69.07	68.21	62.26	22.06
		3	43.74	43.60	44.38	35.83	26.01	59.58	59.50	60.31	50.07	39.35
		4	28.10	28.33	28.74	15.34	8.10	30.87	30.97	31.51	19.01	9.46
	LT	1	36.80	38.38	37.71	22.46	18.27	46.74	49.26	47.86	23.30	19.08
		2	31.12	30.48	31.51	26.47	13.68	42.65	44.56	43.54	32.95	22.01
		3	27.65	27.35	28.14	18.89	11.31	33.83	33.62	34.38	23.87	14.45
		4	19.37	19.07	19.62	15.56	9.07	22.34	22.50	22.69	17.40	9.84
xITM	ST	1	40.95	43.14	40.95	63.60	31.84	51.24	53.33	51.24	63.60	42.28
		2	129.82	129.19	129.82	128.90	14.51	184.59	184.65	184.59	184.11	18.89
		3	58.01	57.88	58.01	54.83	31.55	68.81	68.67	68.81	65.87	39.03
		4	63.26	63.24	63.26	57.79	40.36	86.80	86.77	86.80	81.32	58.69
	MT	1	32.50	33.42	33.53	NA	NA	45.71	48.31	47.13	NA	NA
		2	74.59	74.38	75.29	49.45	38.07	107.28	107.85	107.72	63.87	44.30
		3	25.82	25.71	26.03	23.99	22.62	31.52	31.38	31.71	29.45	28.83
		4	19.53	18.85	19.79	15.12	11.28	24.17	23.45	24.42	20.36	16.01
	LT	1	63.79	64.46	64.41	24.53	27.52	86.80	88.06	87.69	40.46	32.88
		2	6.51	7.10	6.74	5.97	8.63	9.76	8.63	9.86	9.07	12.02
		3	40.82	39.95	41.32	37.18	20.56	48.35	47.52	48.83	45.06	28.57
		4	38.15	18.63	39.35	33.15	10.86	158.14	21.79	163.83	138.51	12.33

Darker shading indicates lower hedging error.