Minimum Variance Portfolios in the Swedish Equity Market

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ABSTRACT

Recent research in finance has suggested an investment strategy aimed at reducing volatility without sacrificing returns. This is the concept of minimum variance trading strategies, which has gained popularity in both academia and the industry in recent years. According to the supporters of the theory, a portfolio of stocks constructed for the sole purpose of minimizing risk will on average generate the same returns as a capitalization stock market index with substantially lower risk. In this paper we focus on the Swedish equity market and construct minimum variance portfolios of OMXS30 members from 1991-2010, and compare the properties of this strategy to the OMXS30 value weighted index and an equally weighted index constructed from the same OMXS30 members. The study finds that a covariance estimation using twelve months of data is the optimal strategy and that the performance is improved with shorter holding periods. Adjustments to the covariance matrix estimation through shrinkage or exponentially weighted moving average (with any decay factor) ads no further value to the portfolio construction in the analysis. Over the last 20 years, a minimum variance portfolio with a 3 month holding period dominates portfolios with 1 or 6 month within the 0.106 percent to 0.237 percent rebalancing cost range. In addition, the minimum variance portfolios with leverage to equal the volatility of the benchmark (44 % for portfolios with 1 month holding period, 41 % for 3 month and 36 % for 6 month) are only resilient at a 0.28 percent cost for a one month holding period compared to 0.56 percent for a 3 month window in order to beat the index on a risk adjusted basis. We find that the minimum variance strategy with a 3 month holding period and a 12 month estimation window generates persistent positive alphas over the last decade, but that the excess returns are strongly correlated with a value factor. The study finds that in absolute terms the equally weighted index produces greater returns (15.5 %) than the OMXS30 (14.9 %) and the minimum variance strategy (13.5 %) for the full period. However, the minimum variance portfolio exhibits the highest risk adjusted returns with consistently and significantly lower volatility.

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1 Introduction

Efficient market theory has been challenged by relatively simple investment strategies that generate higher returns than the market portfolio. Strategies such as momentum, value and size have all been studied in various markets and seem to outperform broad market indexes. Recent interest, both from academia and the finance industry, has suggested that a relatively simple investment strategy based on minimizing risk will match market returns and potentially beat the market. Low volatility equity strategies, sometimes referred to as managed volatility or minimum variance, seek to deliver equity market returns with significantly less return variability than capitalization-weighted market indices. Compared to traditional equity strategies, they have similar return potential, but vary on the risk dimension. Low volatility strategies seem to provide a smooth pattern of returns over time, but can deviate from the capitalization-weighted index. The attractiveness of low volatility strategies to potentially generate the same level of returns while reducing risk should interest all investment academics and professionals.

The pioneer in minimum variance research is Professor Robert Haugen who first analyzed the phenomena in 1991. His and later research have been based on large developed stock markets and these researches have concluded that minimum variance strategies significantly reduce portfolio volatility without reducing returns. In this thesis we look to the Swedish stock market, which in addition to being relatively small compared to leading global stock markets, has displayed significantly more volatility than more established markets have over the past decades .

The paper strives to compare the performance of a minimum variance portfolio in the Swedish equity market to a value-weighted index reflecting the overall market. It quantifies whether constructing a minimum variance portfolios can add a measurable benefit for investors in listed Swedish equities by applying a minimum variance optimization of the OMXS30 members from 1991 until 2010. Furthermore, the study compares a naive equally weighted strategy to the minimum variance portfolio and the OMXS30 value weighted index.

The evaluation of the investment strategy is based on the minimum variance portfolios' ability to create a substantial and consistent lower standard deviation than the benchmark portfolio, although still generating equally sized returns. Thus, producing a risk adjusted return that is excess of what an investor would get through a passive investment strategy in the index. Furthermore, to make the investment strategy plausible it also needs to be implementable and sustain its outperformance even after rebalancing costs are applied. Therefore the study seeks to find the parameter values of the portfolio optimization that determines the feasibility of the strategy.

The paper is structured in the following manner. We start with a brief overview of the theoretical background of Modern Portfolio Theory and how it relates to the minimum variance portfolio, and the desirable properties of the minimum variance portfolio. We explain the main findings of the most relevant previous research on both minimum variance and equally weighted strategies. A short overview of the use of minimum variance strategies in the asset management and index industry follows. We finish the essential background information with a short-term focus on minimum variance strategies and how to use the basic minimum variance framework in addition to a more fundamental approach. We then provide a theoretical framework in the methodology section and a description of the data. We conclude this paper with our main empirical evaluations and results and our final interpretations on the performance of the minimum variance strategy on the Swedish equity market.

2 Theoretical Background

Harry Markowitz (1952) first presented the theories that formed the basis for what is today known as Modern Portfolio Theory. His findings gathered little interest to start with but Modern Portfolio Theory is today considered to be one of the most important theories in finance and has become the norm in the area of investment management. For his contributions to financial economics, Markowitz was awarded the 1990 Nobel Prize in Economics along with William Sharpe and Merton Miller.

2.1 Modern Portfolio Theory

The main idea behind Modern Portfolio Theory is to minimize the risk of a portfolio of assets while not sacrificing the portfolio´s expected return. Portfolio Theory introduced the statistical notation of covariance and correlation, the relationship that exists between two or more assets. If an investor holds two securities, X and Y, which have equal standard deviation of returns a portfolio consisting of these two assets, will have a lower standard deviation than the individual securities unless their returns are perfectly correlated. If security X goes broke and is perfectly correlated with security Y, security Y will also go broke, leading to the entire portfolio going broke, a clearly undesirable characteristic of a portfolio of assets. Putting all your investments in assets whose returns are highly correlated is not a practical investment strategy. Therefore the motivation behind Portfolio Theory is the benefits of diversification. The main insight of the theory is that through diversification, investors can decrease the risks they face; i.e. the riskiness of a portfolio of assets is lower than the risk of the individual assets in the portfolio. Portfolio Theory attempts to maximize the expected return of a portfolio for a given

2.2 The Markowitz Portfolio Selection Model

According to Markowitz´s Portfolio Theory investors face a risk-return tradeoff which can be summarized by a mean-variance frontier of risky assets. This frontier is the lowest possible variance for a given portfolio expected return. Risky assets lie to the right of the frontier, and therefore portfolios consisting of a single risky asset are inefficient since a lower standard deviation can be obtained for a given asset to the right of the frontier by diversifying risk. Through portfolios of many assets investors are able to diversify the risks they face. The key result of Portfolio Theory is that the risk of a portfolio is less than the weighted average of the risk of the individual securities in the portfolio. Investors should not only be concerned about the expected returns and variance of their assets but also the way these assets co-vary with each other. Using time series data one can calculate expected returns, variances and covariance's and construct a mean variance portfolio for any specific expected return. The part of the frontier that lies above the minimum variance portfolio is called the efficient frontier of risky assets, which is a set of all mean variance efficient portfolios. For any portfolio on the lower portion of the minimum variance frontier there exists a portfolio with the same standard deviation but a higher expected return directly above it. Therefore, the bottom part of the mean variance frontier is always inefficient.

The part of the mean variance frontier that is of interest in this paper is positioned on the far left of the mean variance efficient frontier. This minimum variance portfolio assigns weights to securities so that overall portfolio risk is minimized and therefore describes a portfolio with the lowest return-variance for a covariance matrix of stock returns. Due to the diversification effect the minimum variance portfolio does not consist of only the lowest risk stock in the investment universe but may even contain all the stocks given that their correlation is such that it leads to the lowest possible variance.

The Capital Asset Pricing Model first presented by Sharpe (1964) further developed the ideas of Markowitz into a mean variance efficient market portfolio, which according to the model is the only portfolio of risky assets an investor should hold. In practice broad capitalization weighted stock indexes such as the S&P 500 or global indexes such as the MSCI World Index are used as a proxy for this mean variance efficient market portfolio. To maximize return per unit of risk, investors should hold the market portfolio or a combination of the market portfolio and a risk free asset. This portfolio will always dominate the minimum variance portfolio according to the CAPM. If investors hold a well-diversified market portfolio the risk of individual securities will be diversified away.

2.3 Imp Implementation of Modern Portfolio Theory

Modern portfolio theory is used in practice for asset allocation purposes. Investors select the assets they wish to invest in and the constraints that investors wish to apply, such as turnover constraints and maximum weights of individual securities. Investors obtain input estimates from historical data of returns, correlations and variances of the specified securities, which are then optimized to construct the mean variance efficient frontier. The CAPM is the model of choice to estimate expected return. The attractiveness of the model is its simplicity and it is widely used in many financial decisions such as those relating to portfolio management, capital budgeting, and performance evaluation.

2.4 Weakness of Modern Portfolio Theory

As the mean variance optimization is extremely sensitive to the expected return input, any errors in the estimation might significantly undermine the performance of the out of sample mean variance portfolio. A weak link in Portfolio Theory is that portfolio optimization is dependent on the expected return forecast. The CAPM is the model of choice to predict expected returns but it makes many simplifying and unrealistic assumptions and has been shown to be a less than perfect forecaster of future returns by Fama and French (2004).

Another problematic input in mean variance optimization is the covariance matrix. The simplest way to estimate it is to assume that covariance matrix in the future will be the same as the sample covariance matrix. However, Ledoit & Wolf (2004) pointed out that such estimation method was subject to errors caused by outliers and non-stationary parameters that tended to be different from period to period. This will cause the estimation of the covariance matrix to be unrealistic in the out of sample performance. A widely used approach to reduce these estimation errors is Bayesian

Shrinkage. It is designed to pull the most extreme parameters toward universally constant values and in that way systematically improves the out of sample performance.

2.5 Desirable properties of the minimum variance portfolio

All portfolio´s on the efficient frontier are designed to minimize risk for a given return level, all but one, namely the minimum variance portfolio which is derived through minimizing risk without the need for an expected return input.

As previously explained, to use the basic Markowitz model one needs to estimate the expected returns and covariances for individual securities and then minimize the portfolio risk for any given expected return by adjusting the weights of each security in the portfolio. The minimum variance portfolio however has the desirable property that security weights are independent of the forecasted expected returns. Because of these desirable properties, the minimum variance approach is less affected by the shortcomings of Portfolio Theory. Additionally, Jagannathan & Ma, (2003) found that using empirical data the minimum variance portfolio performed better than any other mean variance efficient portfolio.

Because of these attractive properties empirical studies of Modern Portfolio Theory have been increasingly focused on the minimum variance portfolio. Asset managers and index providers have in recent years launched minimum variance products but companies focusing on another mean variance efficient strategy derived from Modern Portfolio Theory are hard to find. Furthermore, due to the high volatility in equities markets in recent years a strategy developed for the sole purpose of volatility reduction should be appealing to equity investors.

3 Previous Research and Industry Practice

A number of research papers have been published on the properties of minimum variance portfolios, but most of them have been focused on the US stock market or other highly developed markets. Professor Robert Haugen was the first to suggest that portfolios located on the left hand side of Markowitz's efficient frontier would offer investors the best returns and lowest variance in the long run i.e. portfolios with lower volatility performed better, a great contradiction to Markowitz's efficient frontier. Haugen and his research partner Nardin Baker were the first to publish studies of minimum variance portfolios and their research has served as a basis for later research.

Haugen & Baker (1991) used a minimum variance optimization to test the efficiency of the Wilshire 5000 stock index, which they believed to be the most comprehensive capitalization weighted index in the US. To construct the minimum variance portfolio they used a population of the largest 1000 US stocks. In order to ensure diversification of the minimum variance portfolio, they placed constraints on maximum weight of a stock in the minimum variance portfolio (1,5%) and a maximum weight of a single industry sector (15%). At the beginning of each quarter from 1972-1989, they computed weights of the minimum variance portfolio using covariance matrices calculated from two-year historical stock returns, which would have minimized the volatility of the 1000 stock portfolio in the previous 24 months. As a basis for comparison, they constructed a portfolio from the 1000 stock sample with weighting structure similar to that of the Wilshire 5000.

Haugen and Baker found that capitalization weighted stock portfolios are inefficient investments. For the period 1972-1989 they showed that repeatedly investing in a portfolio of stocks constructed to minimize risk would outperform a proxy for the Wilshire 5000 index They found that the minimum variance portfolio consistently outperformed the Wilshire 500 stock index proxy in terms of both higher returns (average 23% higher) and lower volatility (11% lower on average).

Clarke, de Silva & Thorley (2006) extended Haugen and Baker's research to include data from 1968 to 2005 as well as use Bayesian shrinkage and principal component analysis for constructing covariance matrices. They used 60-month historical data to estimate covariance matrices, and constructed a new minimum variance portfolio monthly. They compared the resulting minimum variance portfolio to a market portfolio of 1000 stocks that was a close approximation to the Russell 1000 stock index. They found that the minimum variance portfolio had 75% of the realized risk of the general market and this lower risk did not come at the expense of lower returns. They found the Sharpe ratio of the minimum variance portfolio for the period to be 0,55 compared to a 0,36 Sharpe ratio for the market proxy.

On a global scope, Nielsen & Aylursubramanian (2008) developed a minimum volatility strategy for the MSCI World Index. They simulated a minimum variance portfolio from 1995-2007. Their results were consistent with earlier research of U.S minimum variance portfolios. The MSCI minimum volatility index experienced approximately 30% lower volatility than the MSCI World Index for the period June 1995 to December 2007. The outperformance of the minimum volatility index as measured by the Sharpe ratio´s for the period was 0.67 compared to 0.45 for the MSCI World Index for the same period.

3.1 Properties of the Minimum Variance Phenomena

While there is empirical evidence for the outperformance of minimum variance portfolios, few have researched the theoretical justifications of the strategy. Academics and practitioners alike have questioned whether minimum variance investing is a new phenomenon in the financial universe or can established theories explain the seemingly attractive characteristics of minimum variance strategies.

According to Robert Haugen (2010) there is little small cap bias in minimum variance portfolios. He believes that small cap stocks tend to exhibit higher volatility than large cap stocks. However he believes that the strategy has a bias towards value stocks since value stocks have lower volatility. Haugen also believes that an explanation for the minimum variance phenomena is that too much money is chasing previous volatility in the race for higher returns, which appreciates the price of these assets. Therefore the price of low volatile assets does not rise, leading them to be relatively more attractive and can generate superior returns.

According to Nielsen & Aylursubramanian (2008) minimum variance portfolios show across a number of empirical studies common characteristics that may explain their superior risk adjusted returns. They often have low portfolio beta relative to capitalization weighted index, have a bias towards small and value stocks and have a bias towards stocks with low total and idiosyncratic risk.

Similarly, Scherer (2010) claims that the portfolio construction behind minimum variance investing tends to hold low beta and low residual risk stocks. Scherer concludes that 84% of the excess returns from minimum variance strategies can be attributed to the Fama/French factors and two characteristic portfolios, low beta and low residual risk. This indicates that the focus on volatility in minimum variance investing captures the characteristics of value, low beta and low residual risk stocks and the minimum variance effect might be a proxy for these factors.

3.2 Research on Equally Weighted Strategies

Modern Portfolio theory is only one of many asset allocation theories within the finance spectrum. Optimal asset allocation has long been a topic of debate and there is numerous asset allocation strategies used within asset management. Despite sophisticated theoretical models developed in recent decades many investors continue to use simple asset allocation models. DeMiguel, Garlappi, & Uppal (2007) evaluate 14 different asset allocation strategies and find that the simplest strategy performs best in terms of Sharpe ratios. They evaluate the out of sample performance of the sample based mean variance portfolio and its various extensions, and compare this performance relative to a naïve equally weighted portfolio. The naïve portfolio is such that of N available assets, each asset is allocated a weight of 1/N at each rebalancing date. The researchers compare the out of sample performance of 14 different asset allocation models including, minimum variance, sample based mean variance and a value weighted market portfolio to name a few. Different datasets are considered, the S&P 500, eight country indices and the MSCI World Index and numerous factor models. De Miguel et al. find that of all the models evaluated none performs consistently better than the naïve 1/N strategy. This outperformance holds for Sharpe ratios and lowest portfolio turnover. According to De Miguel et al. naive diversification works better than minimum variance strategies and more sophisticated diversification models in terms of Sharpe ratio´s.

3.3 Industry Practice

A number of banks and asset managers offer minimum variance portfolios. Due to estimation errors of implementing minimum variance strategies some market practitioners prefer the term managed volatility strategies, since given the estimation error of forecasting future volatility from past data the strategy will never be truly minimum variance. To ensure the investability of the strategy, practitioners implement a number of portfolio constraints on the minimum variance portfolio while still aiming for the lowest volatility. The maximum and minimum weight of a constituent is restricted to a fixed percentage of the total portfolio. The turnover of the portfolio must also be restricted to a reasonable percentage; otherwise transaction costs could possibly wipe out any relative gain from the strategy.

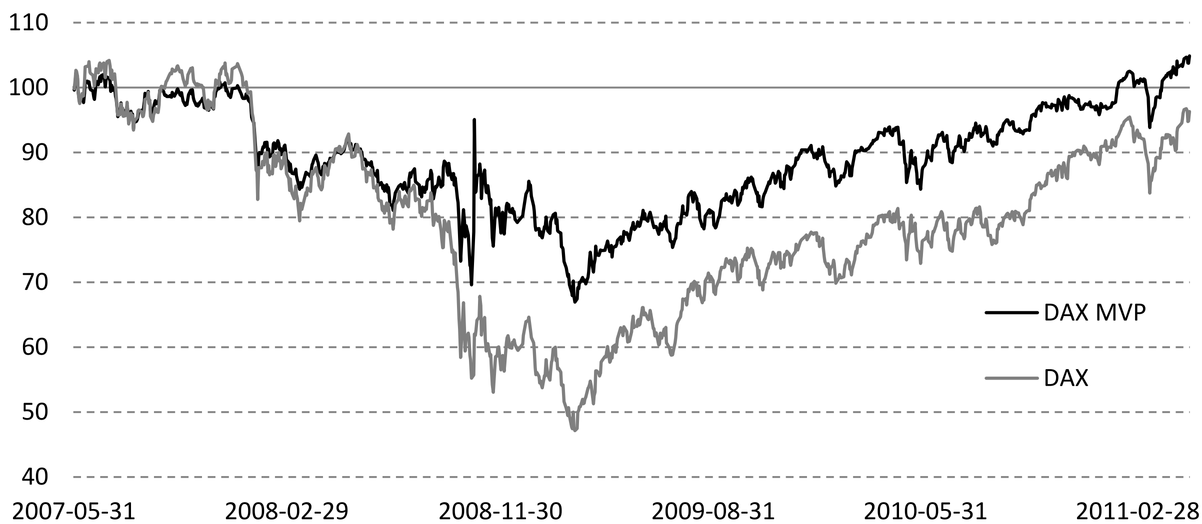
The largest provider and the one with the longest track record is Unigestion of Switzerland. Unigestion introduced minimum variance portfolio’s for the Swiss market in 1997 followed by portfolios for the European, US, Japanese, global and emerging markets. Unigestion’s European strategy has outperformed the wider market by more than 4% a year since 1999. Other noteworthy firms in minimum variance strategies are Acadian Asset Management, Martingale Asset Management, State Street Global Advisors, Robeco, Crédit Agricole, Lazard and Scandinavian asset manager Alfred Berg to name a few.

3.4 Indices using Minimum Variance

The increasing popularity of minimum variance strategies can well be seen from the introduction of minimum variance indices by market leaders MSCI Barra and Deutsche Börse. MSCI Barra launched minimum volatility portfolio indices in 2008 designed to serve as a transparent and relevant benchmark for managed volatility equity strategies. The indices aim to reflect the performance characteristics of a minimum-variance strategy, focused on absolute return and volatility with the lowest absolute risk. MSCI World Minimum Volatility indices are constructed with a number of constraints such as sector and individual security weights. As of 2011, MSCI calculates minimum variance indices for the world index, Europe, US, emerging markets and the EAFE world index (Europe, Australia, Asia and Far East). MSCI Barra is in talks with providers of exchange-traded funds to replicate its indices to make them easily investable for market participants.

Deutsche Börse (DAX) offers minimum variance indices for the German, French, Japanese, Swizz and US markets. The DAX minimum variance indices are constructed to include no more than 30 liquid stocks (50 for the US portfolio) and are quoted in EUR, USD and GBP. Although Deutsche Börse did not calculate these indices until 2007, historical simulation show that for each country, a minimum variance portfolio would have outperformed the relevant country benchmark on a risk adjusted bases for the period 2001-2006. From its inception in May 2007 through May 2011, the DAX Germany Minimum Variance Portfolio (DAX MVP) has outperformed the benchmark DAX index as can be seen in Figure 3-1.

Figure 3-1: The DAX MVP Index vs. the DAX from inception to May 2011



However, implementing minimum variance strategies on a short-term basis can be nerve wracking, which is the topic of the next section.

3.5 Performance of minimum variance strategies short term

Minimum variance portfolios tend to over perform benchmark indices during weak markets and underperform in strong markets, therefore generating much lower volatility and higher Sharpe ratios than the benchmark as can be seen in Figure 3 1 above. Implementing minimum variance strategies in the short run can be quite frustrating for investors. During the financial crisis, the DAX index hit bottom on March 6th 2009. From that time the DAX has far outperformed the DAX MVP, up 105% while the minimum variance index has gained 55%. A minimum variance investor holding the DAX MVP for the year ended March 2010, with the DAX up 60%, would have fallen behind the benchmark by 25%. However a minimum variance investor holding the DAX MVP for the year ended March 2009 would have beaten the benchmark by 22%.

In general, minimum variance strategies can be expected to deliver weak returns compared to market indices in a rapidly rising market led by high volatility such as the rebound of 2009 after the global financial system narrowly escaped meltdown. Another example of the structure of the minimum variance approach is because of it has a tendency to stay away from highly correlated stocks it will therefore underperform in a market driven by one thing such as oil prices or a price bubble in technology. Unigestion, the world's largest minimum variance provider, had its worst year in 1999 when the technology bubble was at an all-time high. Conversely in sharply falling markets where the most volatile highly correlated stocks fall the most, minimum variance strategies are structured to shy away from these correlated stocks and are expected to outperform the benchmarks.

It is evident that minimum variance strategies do not necessarily function well as short-term strategies, but can provide downside protection and higher Sharpe ratios compared to market benchmarks over a longer investment horizon.

3.6 Minimum variance strategies with a twist

To diversify the strategies' returns, and compensate for the anticipated under performance in rising markets, most active minimum variance managers add quantitative stock picks to the basic methodology for volatility control and performance enhancement.

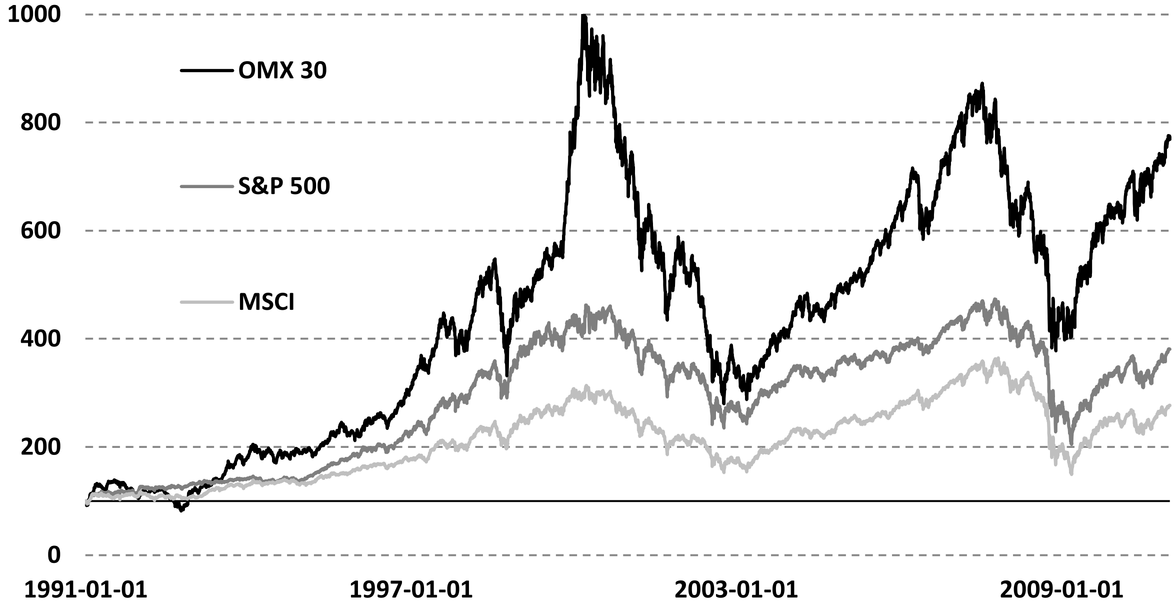
Minimum variance providers such as Credit Agricole and Lazard offer products that are based on the minimum variance strategy but factor in stock market and company fundamentals. Instead of minimizing the risk of a certain market, they minimize the risk of a portfolio of stocks, which they believe to be good investments through fundamental analysis. Many managers also shy away from stocks with low liquidity and credit difficulties. Through combining quantitative and fundamental methods these providers hope to offer equity exposure with reduced risk.

4 Data in the Study

4.1 The Swedish Equity Market

As discussed earlier in the paper previous research has focused on minimum variance portfolios in the world's most developed equities markets, but as can be seen in Figure 4-1 the OMX 30 has behaved very differently in terms of returns and volatility than the major indices over the past two decades. It is therefore an interesting study to see if the minimum variance approach delivers lower volatility in this market as research has shown that it does in more developed markets.

Figure 4-1: Performance of OMXS 30 & Leading Equity Indices (ex. dividends)

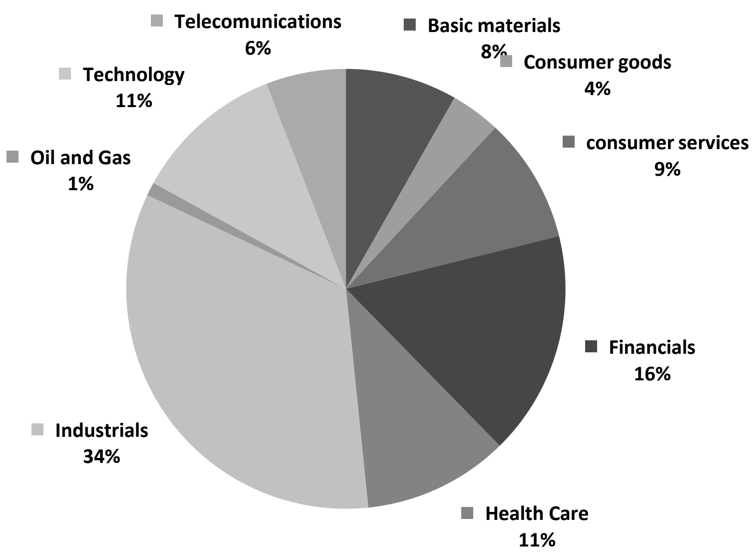


4.2 Index selection

The OMX 30 index is one of the Nordic regions most well-known equity indices and consists of the 30 most actively traded stocks on the Nasdaq OMX Stockholm Stock Exchange and serves as a benchmark for the Swedish equities market. It is a capitalization-weighted index and is developed to display the general movements in the stock market. The index is rebalanced semiannually and the composition of the index is dependent on two factors. First, if a stock that is not currently in the OMX 30 index is among the 15 most traded stocks during the previous 6 months, the stock will replace the index stock with the lowest turnover. Second, if an index stock is not amongst the 45 most traded stocks the stock will be replaced by the non-index stock with the highest turnover during the previous 6 months.

The sector weights of the OMX 30 as of May 2011 can be seen in Figure 4 3, and the number of stocks in each sector in parenthesis. A third of the index is comprised of industrial stocks, and roughly half the index is in the industrial or financial sector.

Figure 4-2: OMXS 30 Sector Weights by Industry as of May 20th 2011



We have chosen to restrict our analysis to stocks included in the OMX 30 market index. This is obviously a simpler and less time consuming approach than to use all listed shares for our analysis , but by restricting our analysis to the OMX 30 index we focus on the most actively traded stocks that display the general movements in the Swedish stock market. Furthermore, this population of OMX 30 stocks has good liquidity and does not suffer from the same survival bias as many smaller stocks, which may only be listed on the exchange for a short time. All stocks are quoted in Swedish kronor and therefore are not subject to exchange rate fluctuations. Additionally, all datasets are corrected for dividends and stock splits so that they can be viewed as a total return time series. Most of the data we gathered for our research was obtained from DataStream, however some data was missing and we obtained additional data from the Nasdaq OMX and Bloomberg. For the risk free rate, we used 3 month STIBOR. Furthermore, MSCI indices for large cap value and growth are used in the creation of the HML factor implemented in the performance evaluation.

4.3 Historical Sample

Our analysis covers 40 semiannual periods from 1991 throughout 2010. At the start of every 3-month period, we compute portfolio weights of the OMX 30 with the constituents in that period that would attain the lowest volatility portfolio in the estimation period subject to constraints to ensure diversification of the minimum variance portfolio. These weights then form the minimum variance portfolio in the following 3-month period. This process is then repeated and at the start of each period a new efficient portfolio is computed from the historical data of the OMX30 constituents as they are in the 6-month period under observation.

We perform the following four steps to compute the minimum variance portfolio. First, we gather the historical return series of the member stocks. Second we use historical data to calculate the individual covariance and variance matrix. Third we feed this data into the optimizer and determine the weights of the minimum variance portfolio. Finally, we monitor the out of sample performance of the minimum variance portfolio and compare the results to other strategies.

5 Methodology

5.1 Covariance Estimation

Creating an optimal portfolio in a mean variance framework requires a measure of covariance between all assets that are available in the investment space. This variance-covariance-matrix is at the center of optimizing the risk adjusted return, but cannot be observed in the market. Therefore, it is necessary to estimate it using statistical techniques on historical data, which creates two distinct problems. First, the variances of assets are time dependent making old observation less reliable estimators than current. Mandelbrot (1963) and Fama (1965) presented evidence for serial correlation in the returns of securities, which must be taken into account to arrive at the best possible prediction for future volatility. Second, the estimation might contain estimation errors that will subsequently distort the optimization.

When selecting historical data, the assumption that market volatility and correlations are time dependent will make it feasible to focus on shorter horizons with higher frequencies in estimating the risks of the assets. Incorporating too old volatilities will contaminate the estimates with irrelevant data according to Litterman (2003). To strengthen the predictability of our estimation we therefore use a daily return series for the stocks that are included in our benchmark.

Taking the two problems previously mentioned into consideration we focus on shorter time horizons and also utilize two different approaches for estimating the covariance matrix. Firstly an exponentially weighted moving average (EWMA) approach is used to capture the time varying aspect of the covariance and secondly a shrinkage method suggested by Ledoit and Wolf (2003) is implemented to minimize the errors in the estimation.

5.1.1 Ledoit and Wolf

The standard statistical method in estimating the covariance matrix is to gather a sample of historical stock returns and use this sample to compute the correlations. As the covariance matrix is at the center of the optimization of minimum variance portfolios, this creates problems in the portfolio weights. Jobson and Korkie (1980) found that when the number of stocks in the sample is large and the numbers of historical observable returns are limited, the estimated covariance matrix contains significant errors. These errors are often more substantial in the stocks that take on extreme values in the estimation and the portfolio optimization will subsequently lead to a phenomenon called “error-maximization” according to Michaud (1989). The problem is that the stocks with the biggest errors will attain the highest weights in the model and subsequently lead to investments in the stocks that have estimated covariance’s that are not representative of their actual return characteristics. To counter this problem a shrinkage method with a shrinkage target that is the average of all the sample correlations, combined with the vector of the sample variances for each security is employed. Ledoit and Wolf (2003) determine the shrinkage intensity by the scalar δ*, which optimal value is derived through a quadratic measurement of the distance between the true and the estimated covariance matrix.

$$\sum^{Shrink} = \sum^{est.} \times \delta^{*} + \prod^{ident.} \times (1-\delta^{*})$$

5.1.2 Exponentially Weighted Moving Average

Considering several methods of predicting future volatility, Akgiray (1989) discovered that the EMWA was superior to ARCH models and Tse (1991) showed that GARCH forecast are slower to react to changes in volatility. Consequently, the EWMA approach is used to predict the volatility, where the time varying aspect of volatility is taken into account by assuming that more recent volatility in the sample is a better predictor for the future. This is instigated in the approach by assigning exponentially declining weights to older observations in the sample through a constant decay parameter λ. The optimal value of this constant has been estimated to be 0.94 by RiskMetrics (1996) for daily observations and we utilize this value in the covariance matrix estimation. This has the distinct advantage of using relatively little data to estimate the covariance matrix, as any volatility beyond 90 days will have no impact on the results. However, we do not use this method for holding periods greater than 3 months as this measure is used for estimating current changes in the volatility.

$$\sigma_t^2 = (1-\lambda)\sum_{j=0}^{\infty} \lambda^{j-1}\left(r_{t-j} - \bar{r}\right)^2$$

5.1.3 Restrictions

The time series of the minimum variance portfolios are estimated only with long positions and without an unrestricted ability to short securities in the sample universe. Furthermore, the portfolios are also evaluated with a 5, 10 and 100 percent cap on the holding in any specific asset. At least a 10 percent cap is necessary for a trading strategy to be implemented in a common mutual fund. Implementing these restrictions has been shown to induce an error reducing effect on the minimum variance estimation in itself according to Jagannathan and Ma (2002). Finally, the weights of the portfolio have to add up to unity to have a fully invested portfolio, which is necessary for the comparative performance analysis.

5.2 Equally weighted strategy

An additional comparison to evaluate the strategy is an equally weighted, so called 1/N portfolio, which should be viewed as a strategy implemented by many individual investor who wish to diversify their wealth across assets. This strategy is easy to implement since it is not dependent on estimation of historical returns or optimization. We construct the index by allocating a 1/30 portion of the portfolio to each of the 30 member stocks of the OMX 30 in each semiannual period when the OMX 30 value weighted index is rebalanced

5.3 Performance evaluation

To evaluate the attractiveness of implementing a minimum variance trading strategy on the Swedish equities market we measure the returns compared to the benchmark portfolio and on a risk-adjusted basis. The performance is evaluated in specific periods with emphasis on performance during special circumstances; including the dot-com crash and the credit crisis. This measures centers on Sharpe ratios, volatilities and the total return of the portfolio versus the benchmark during these specific time horizons. In addition to this analysis a statistical analysis is made of the portfolios ability to generate positive alpha in the CAPM and in a multiple regression including its correlation with a value factor.

5.3.1 Jensen's Alpha

The most commonly used measure in describing the value of an investment strategy is Jensen's Alpha. It is the one factor market models beta (1) parameter, which describes the relationship between the excess return of the portfolio by a beta (2) parameter of the market return.

5.3.2 Modigliani and Modigliani

Modigliani and Modigliani is a technique that is closely related to the Sharpe ratio. The idea is to lever or de-lever a portfolio, i.e. move along the capital market line, to make its standard deviation identical to the market portfolio. The Modigliani and Modigliani of a portfolio is the leveraged return that the portfolio earned in a period, which can be directly compared to that of the market return. Its usefulness is that it characterizes how well a portfolio's return rewards an investor for the amount of risk taken units of percent as opposed to the sharp measure. An extension of this measure that takes the covariance between the cash position and the market position into account, and thus the curvature of the capital market line, is the Graham-Harvey measure.

$$M2 = \left({}^{\sigma_m}/_{\sigma_p}\right) \times \left(R_p - R_f\right) + R_f$$

5.4 Information Ratio

The information ratio is also similar to the Sharpe ratio, but divides the portfolios excess return over its benchmark with its tracking error. This returns an estimate of the portfolios risk adjusted return within tracking error space, and produces an alternative measure for the strategies ability to generate returns that compensate for its risk. This measures the relative return of the portfolio strategy divided by its relative risk compared to the benchmark. It is important to note that it ignores the leverage in the portfolio and is therefore most useful when the benchmark mirrors the strategy.

$$IR = \frac{E\left(R_p - R_b\right)}{\sqrt{var\left(R_p - R_b\right)}}$$

5.5 Value Factor

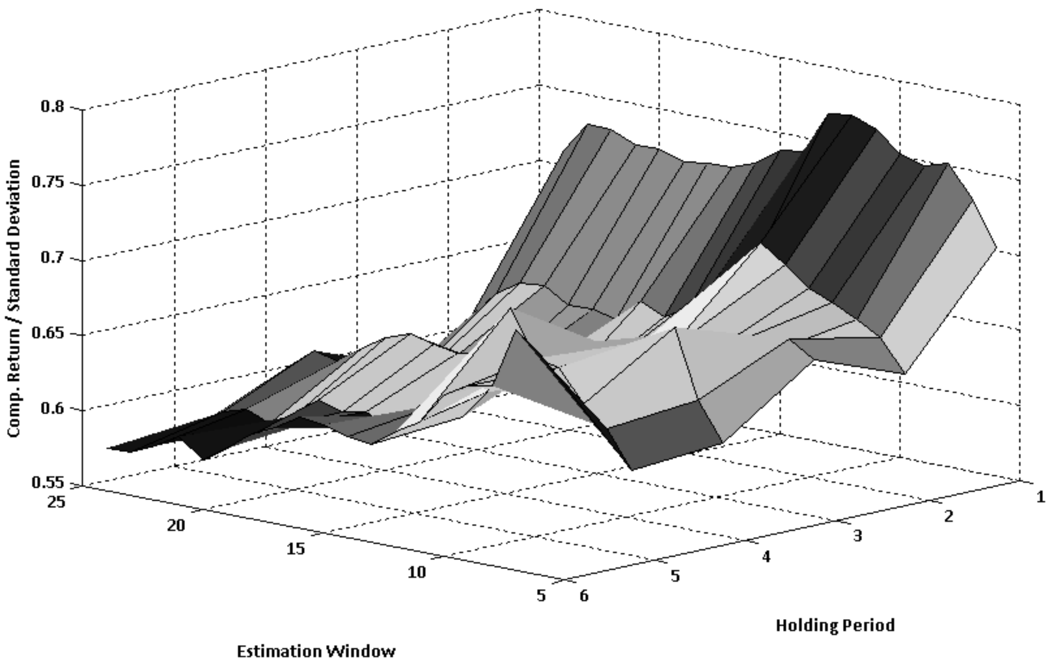
In previous research by Haugen (2010), it is argue that the minimum variance strategy has a bias towards value stock (no exposure to small caps), which can help to explain a portion of its performance. We therefore test the return series of the optimal portfolio versus a two factor model with a parameter measuring its loading on a value variable. This model is then evaluated with F-statistics and the parameter with a t-statistics to determine its explanatory power on the returns of the portfolio.

6 Empirical Findings

6.1 The Optimal Minimum Variance Portfolio Strategy

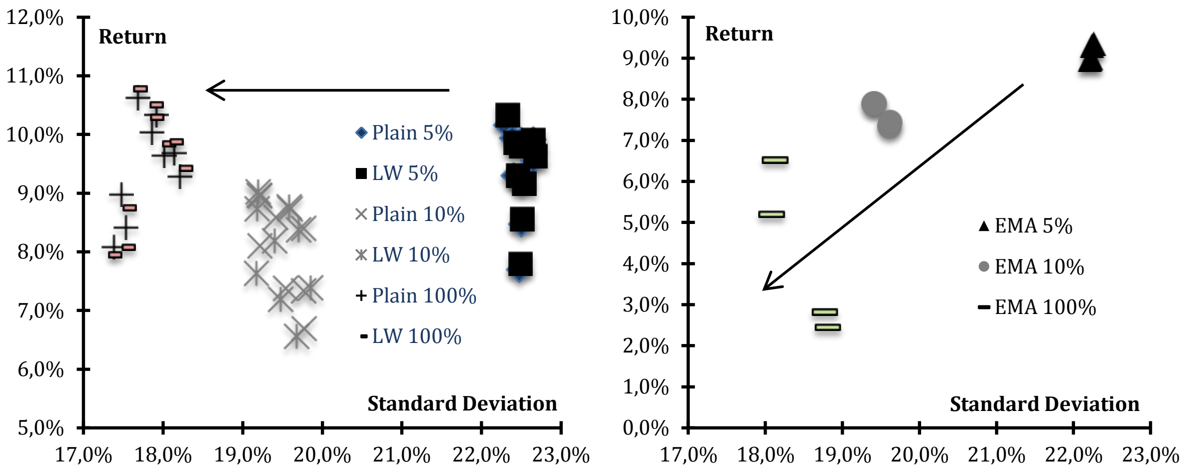
To arrive at the optimal construction methodology for the minimum variance portfolio investment strategy, different time series indexes are constructed that are compiled of the resulting daily returns. This includes portfolios with varying number of months for the estimation period (the window of historical data used in the calculation) in the covariance matrixes and all are evaluated with varying holding periods. Furthermore, the portfolios are constructed with 5, 10 and 100 percent caps on a single security holding. Ultimately these parameters are all implemented on portfolios constructed with plain, L&W and EWMA covariance matrixes. Figure 6-1 depicts the resulting surface of the plain covariance matrix with a 10 percent cap on any single security holding in the portfolios.

Figure 6-1: Return per unit of S.D. for Est. Windows & Holding Periods over 20 Years



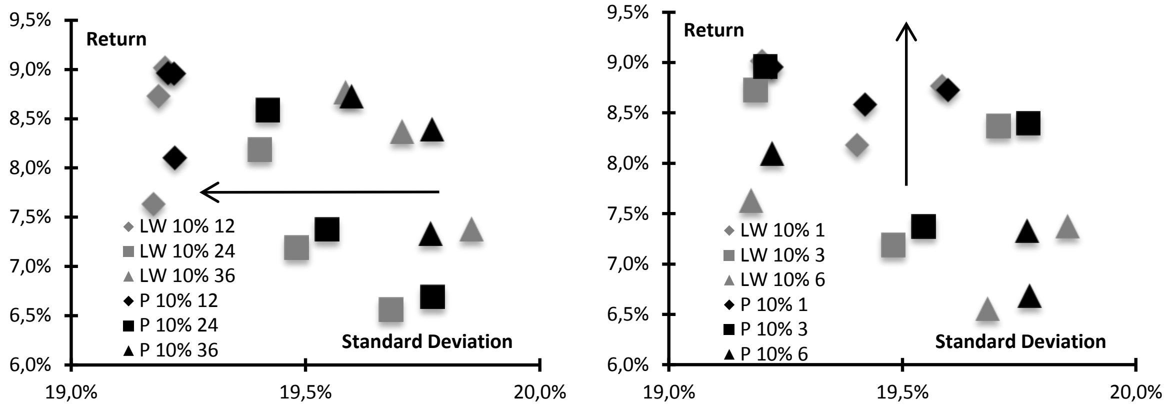
What can be observed from this figure is the optimality of shorter holding periods and a tendency for the estimation period to be superior in a 12 month estimation window. To further analyze the impact that different parameters in the estimation have on the performance of the investment strategy we turn to a 2 dimensional description of the properties within a mean variance space. Figure 6-2 presents the distinct clusters that are created with step-vice varying of different portfolio construction techniques. These clusters are compiled of all portfolios sharing the same restrictions of the amount that can be invested in any single security. What can clearly be observed is a strong tendency of improved portfolio performance with lesser restrictions. However, this is not the case for the exponentially weighted moving average which exhibits considerably diminished returns with lesser restrictions. Varying the decay factor around the 0.94 initially used, only draws the pattern closer or further away from the regular covariance matrix portfolio, but never succeeds in surpassing it, which leads us to abandon the usage of this estimation technique. Furthermore, since the objective is to evaluate a potential trading strategy, we employ the 10 percent cap on any single security holding. This is to some extent a reflective restriction of what an investment manager implementing the strategy (in a normal fund) would face and is therefore used in the optimization.

Figure 6-2: Portfolio Optimization Restrictions



Error! Reference source not found. describes the impact of changing the estimation window and the holding period of the portfolios with clear directional influence for both parameters. Separating the different horizons for the estimation period, we obtain a very clear pattern of improved performance with shorter horizons. These portfolios are better representations of the coming volatility in the stocks, a pattern that is analogues with the observed time dependence of stock returns. The impact of the holding period is also very distinct with higher returns for shorter periods of time that amounts to several percentage points of compounded annual returns.

Figure 6-3: Effect of Holding and Estimation Period



Finally, analyzing the impact of the Ledoit & Wolf shrinkage approach, we can see no improvement or distinct difference from the regular covariance estimation and therefore we conclude that this methodology adds no additional value to the portfolio construction.

6.2 Trading Cost

Although it is impossible to determine a given trading cost, as it fully depends on the type of investor, we calculate the historical breaking points for the optimal values with respect to different parameters. When looking at portfolios with 1, 3 and 6 month holding periods over the last 20 years we find that 3 month portfolio is optimal between 0.106 percent and 0.237 percent rebalancing cost. In addition, when leveraging the investment strategy in order to reach the equivalent volatility of the OMXS30 (se section 6.4), we find that the cost necessary for the 1 month portfolio to beat the market is 0.279 percent whilst the 3 month portfolio outperforms the index up to 0.56 percent (the different portfolios are leveraged with 44 %, 41 % and 36 %). Taking a conservative approach to the trading cost we therefore choose to implement a 3 month portfolio together with a 12 month estimation window for the minimum variance strategy.

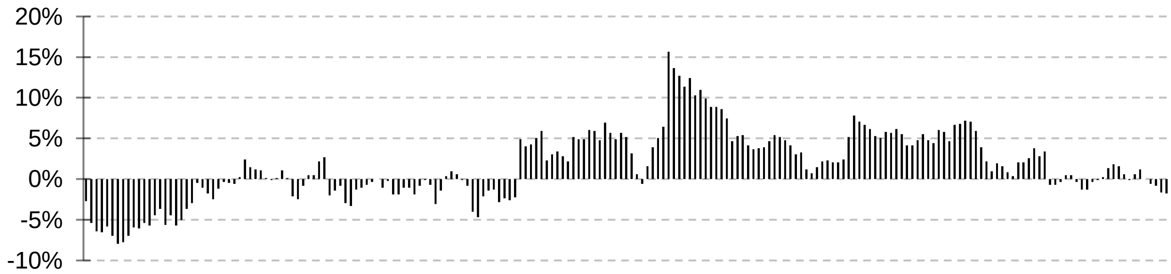
6.3 Portfolio Performance

To analyze the performance of our investment strategy we look at the excess returns generated over the full period and the sensitivity of these parameters within 36 months rolling periods for the minimum variance portfolios.

6.3.1 Jensen's Alpha

The investment strategy generates an alpha of 2.43% annually over the full period. However, this alpha exhibits strong seasonality when estimated over rolling 36 month horizon. Interpretation of this pattern could center on exogenous variables that if effectively integrated into the investment strategy would improve the performance and stabilize the return series alpha generation. Alternatively, the model does not capture all explanatory variables and subsequently misrepresent the performance of the portfolios.

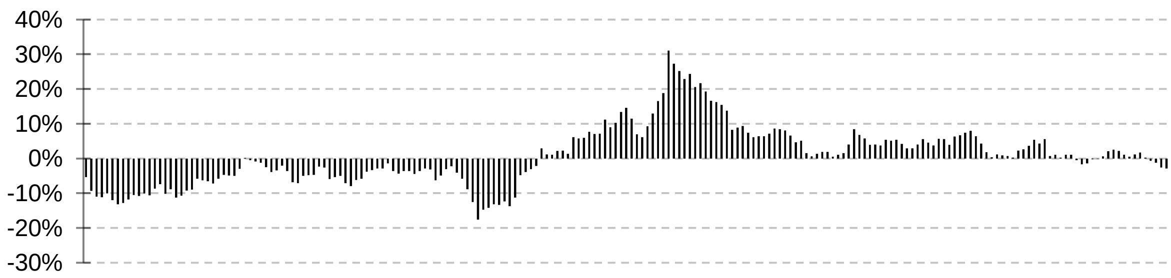
Figure 6-4 36 Months Rolling Alpha from 1991 to 2011



6.3.2 Modigliani & Modigliani

The M2 measure, which mirrors the SML analysis with full market risk shows that the investment strategy has generated substantial risk adjusted gains throughout the 21th century. An investment into the portfolios within a 36-month time frame has consistently returned positive risk adjusted excess returns in this period (illustrated in excess of the market). These results are stable with GH-tests that take the covariance between cash and the market portfolio into account.

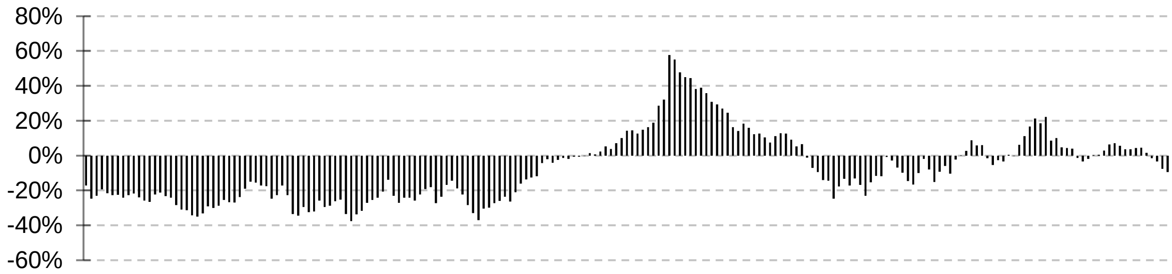
Figure 6-5 36 Months Rolling Risk Adjusted Performance from 1991 to 2011



6.3.3 Information Ratio

The information ratio of the investment strategy is slightly less positive reflecting that the portfolios are less leveraged risk when generating its active return. For the full period of monthly portfolios the tracking error is -0.053% and during 36 months periods with large price corrections downwards, the unleveraged portfolio has positive information ratio.

Figure 6-6 36 Month Rolling IR from 1991 to 2011



6.3.4 Value Factor

The value factor is composed of MSCI Sweden Value subtracted by MSCI Sweden Growth on a monthly basis from 1994-05-31 to 2010-12-31 (both indices are for large cap). This represents the monthly return discrepancy of companies with high versus low book to market values. This becomes the third beta and the second parameter in the 2x199 matrix representing the monthly explanatory returns.

Figure 6-7: Rolling 36 Months of Annual CAPM Alpha

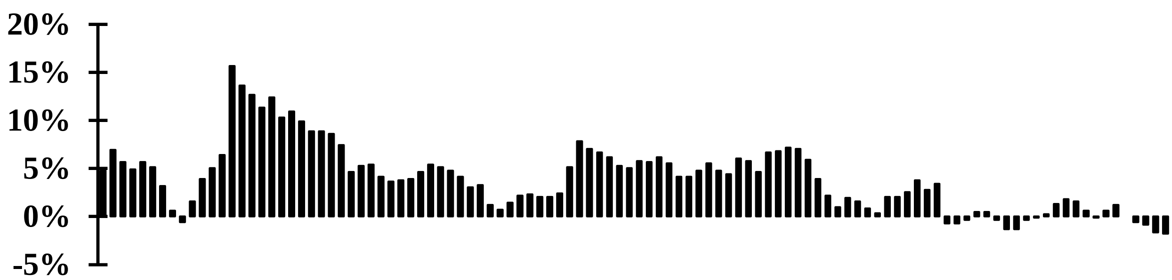
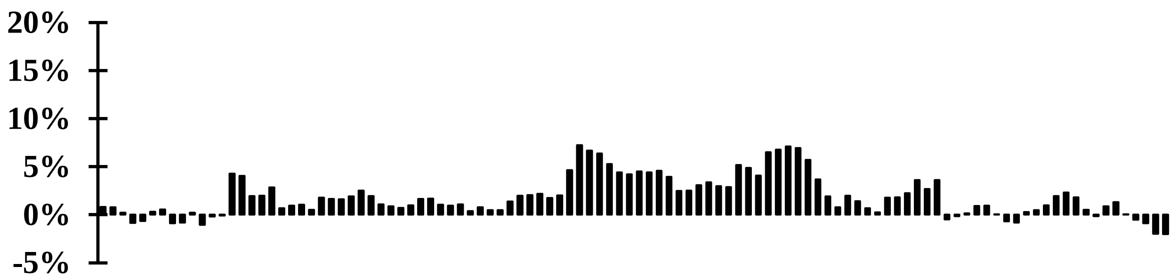


Figure 6-8: 36 Months Rolling Annual Alpha with Value



The investment strategy returns a full period alpha of 1,41% annually under the two-factor model including the explanatory HML parameter. The second beta loading for the market portfolio is at a factor of 0,70 and the loading for the HML parameter is 0,33. This explains a portion of the over-performance and results in a reduced alpha parameter for the investment strategy. However, the remaining value inherent in the portfolio performance is still at a significant level.

Table 6-1 Multiple Regression with Value Parameter (monthly)

Beta 1	Beta 2	Beta 3
0.00117	0.70196	0.3293

6.3.5 Confidence of Beta1 and Beta3

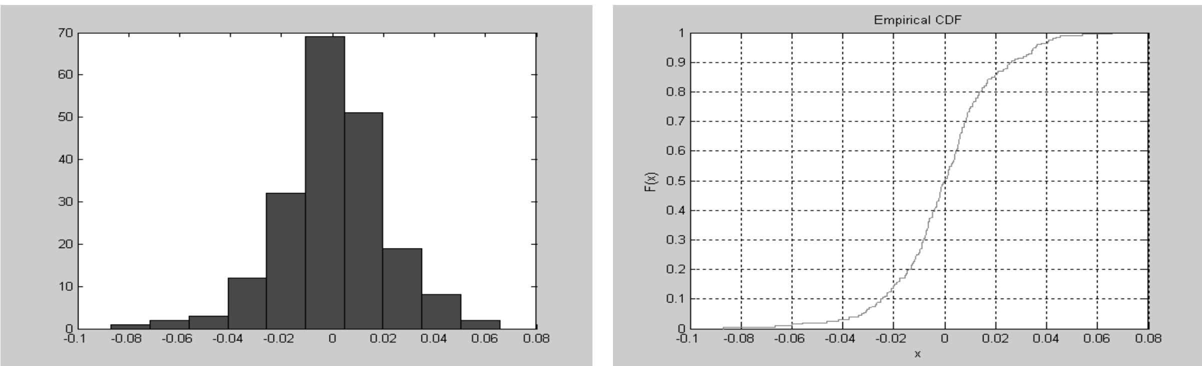
The parameters are estimated with 196 degrees of freedom and both the B2 and B3 parameters are significantly different from zero. The F-test for the model also gives high significance for the explanation power of the model as it greatly explains the returns of the investment strategy.

Table 6-2 Multiple Regression t-statistics

Parameter	t-stat	SE	p-value
B1	0.77	0.0015	0.44
B2	26.91	0.0261	0.00
B3	11.20	0.0294	0.00

The residual is not normally distributed according to a Bera-Jarque test, which implies that there are variations in the return series that are not explained by the independent variables.

Figure 6-9 Residual Distribution

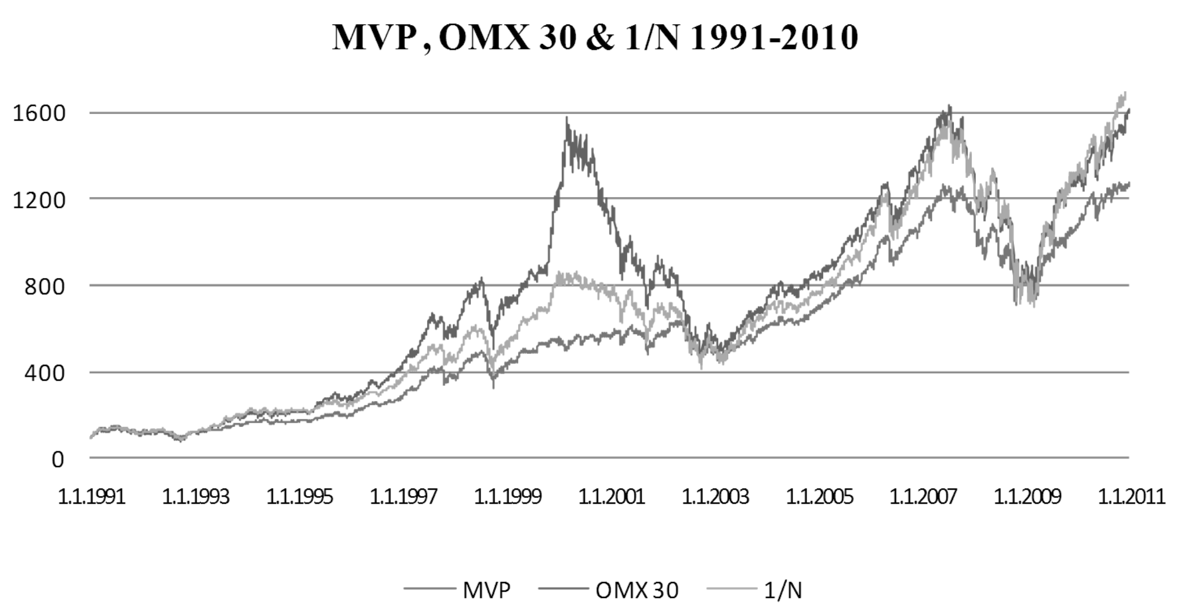


6.4 Empirical Results

6.4.1 Characteristics

In the timeframe under observation the OMX 30 stock index has fluctuated enormously as can be seen in Figure 4-1. The Swedish stock market was hit hard by the internet bubble of the late 90´s and the OMX 30 index dropped significantly during the 2007-2008 global financial crisis. However, as Figure 6-10 demonstrates the Minimum Variance Portfolio (MVP) displayed much more stable returns during this two decade period. It avoided the highly volatile internet stocks during the turn of the new millennium and exhibited far less volatility than the two other indexes.

Figure 6-10: Total Return of MVP, OMXS30 & 1/N 1991-2011



On an annualized basis, the OMX 30 index returned 14.86% total return (9.35% excess return) per year while the MVP returned 13.52% (8.07%) on a total return basis, but this higher return came with substantially higher risk. The annualized standard deviation of returns for the OMX 30 was 24.3% while the MVP had an annualized standard deviation of 18.0% per year, or only 75% of the standard deviation that the index had. We compare the risk reward of each strategy by looking at their Sharpe ratio´s. Over the two decade period the MVP outperformed the OMX 30 benchmark on a risk adjusted basis, with a Sharpe ratio of 0.45, somewhat higher than the OMX 30 Sharpe ratio of 0.38.

During the full period, the equally weighted index (1/N) displayed some similarities with the MVP as can clearly be seen from the trend lines in Figure 6-10. Over the period the 1/N index returned 15,47% (9,93% excess return) annually, which is considerably higher than the MVP did, and interestingly higher than the benchmark OMX 30 value weighted index. However, these higher returns of the 1/N index came at a cost of higher volatility and the 1/N index had a standard deviation of 22,7% (26% higher than the MVP). The MVP and 1/N index have similar Sharpe ratio´s for the full period, while the OMX 30 has the lowest value as can be seen in Table 6-3.

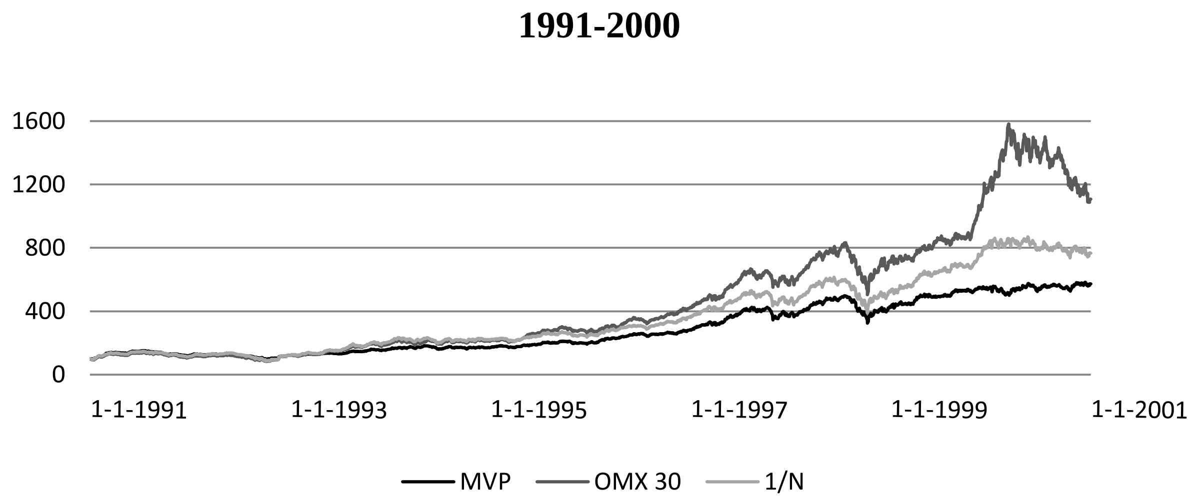
Table 6-3: Total Return of MVP, OMXS30 & 1/N 1991-2000

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	2010	13,5%	14,9%	15,5%	8,07%	9,35%	9,93%	18,0%	24,3%	22,7%	0,45	0,38	0,44

6.4.2 Analysis of Characteristics

Over the past two decades the Swedish stock market and economy experienced many ups and downs. During the first decade under observation the Swedish banking system went through a credit crisis, with widespread insolvency. The economy and stock market quickly bounced back and the OMX 30 appreciated tremendously from 1991 through 2000. During this prosperous time, the MVP index lagged the 1/N portfolio and lagged the OMX 30 benchmark substantially. The OMX 30 and 1/N portfolio returned annually 27,19% and 22,58% respectively while the MVP returned 19,06% annually during the 1990´s. On a risk adjusted basis, the OMX 30 displayed the highest Sharpe ratio of 0,84 with the MVP and 1/N generating a Sharpe ratio of 0,67 and 0,73 respectively.

Figure 6-11 Total Return of MVP, OMXS30 & 1/N 1991-2000

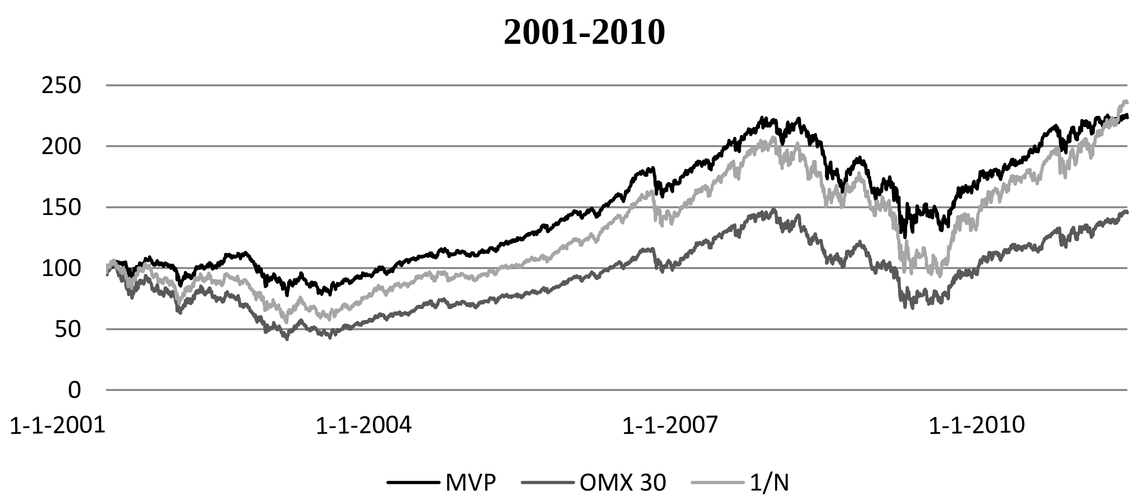


Since the burst of the dot-com bubble at the start if the 2000´s the MVP has significantly outperformed the OMX 30 as can be seen in Figure 6-11 and Table 6-4, returning annually 8,37% from 2001 through 2010 while the benchmark has averaged 3,82% during the same timeframe. The 1/N underperformed the MVP for the decade until mid-year 2010 and has earned an average annualized return of 8,95% during the decade. In risk adjusted terms, the MVP has outperformed both strategies with a 0,28 Sharpe ratio over the 10 year period.

Table 6-4: Total Return of MVP, OMXS30 & 1/N 2001-2010

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	2000	19,1%	27,2%	22,6%	11,02%	18,61%	14,30%	16,5%	22,2%	19,5%	0,67	0,84	0,73
2001	2010	8,4%	3,8%	9,0%	5,33%	0,90%	5,89%	19,3%	26,1%	25,5%	0,28	0,03	0,23

Figure 6-12.MVP, Total Return of MVP, OMXS30 & 1/N 2001-2010



When we divide the sample into four 5 year periods, the MVP has substantially lower volatility than the other strategies as can be seen in Table 6-5, but no clear pattern is visible in terms of the consistently best Sharpe ratio. In terms of risk adjusted returns, the MVP only over performs both the OMX 30 and 1/N in the 2001-2005 period, and over performs the OMX 30 for both 2001-2005 and 2006-2010. The 1/N has the highest Sharpe ratio in the 2006-2010 period and the OMX 30 locks in superior risk adjusted returns for 1991-1995 and 1996-2000.

Table 6-5: Return Characteristics over Five-Year Periods

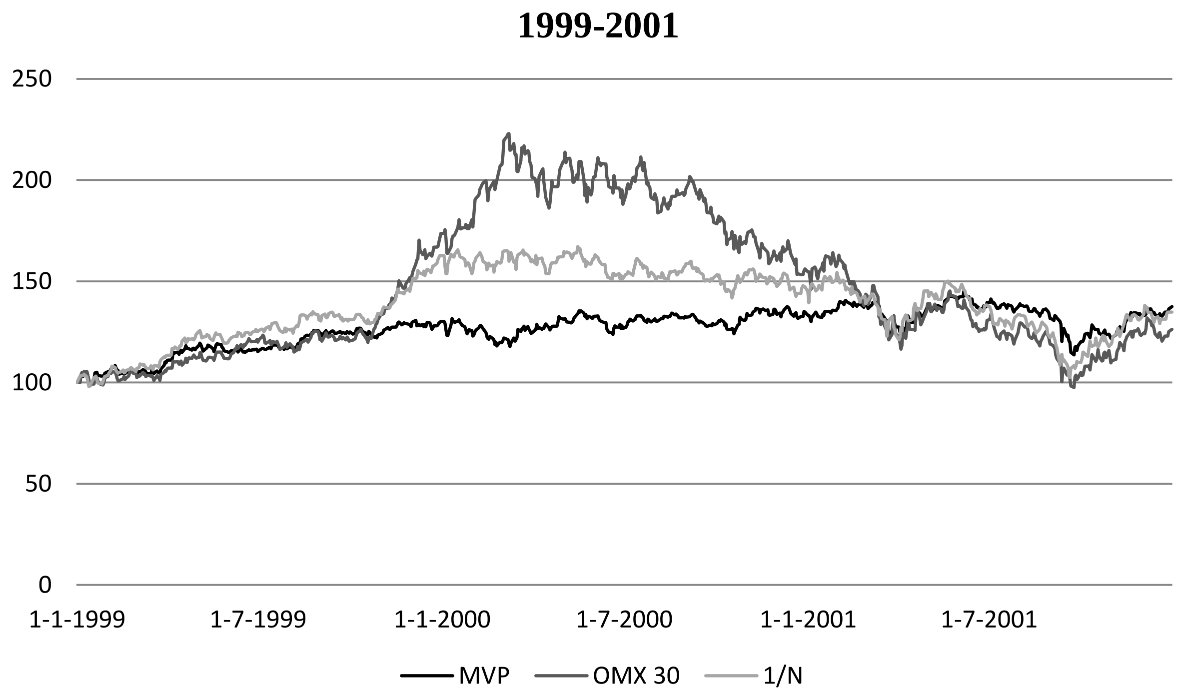
Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	1995	15,6%	22,8%	20,4%	4,9%	11,4%	9,2%	15,0%	19,6%	19,2%	0,327	0,57878	0,4814
1996	2000	22,2%	31,5%	24,4%	17,2%	26,1%	19,3%	17,9%	24,6%	19,8%	0,961	1,05879	0,9755
2001	2005	10,2%	0,7%	7,3%	6,7%	-2,4%	4,0%	17,0%	25,3%	22,3%	0,396	-0,0955	0,179
2006	2010	7,1%	7,2%	10,8%	4,3%	4,5%	8,0%	21,4%	27,0%	28,3%	0,203	0,16534	0,283

An interesting comparison of the performance of each strategy is to look to the two main causes of equity market turbulence during the past 20 years, namely the burst of the dot-com bubble and the global financial crisis of 2007-2009. The OMX 30 was severely hit by the dot-com craze of the late 1990´s. The OMX 30 more than doubled from 1999 until mid-year 2000 with the MVP and 1/N rising 23% and 50% respectively. From the summer of 2000 the OMX 30 dropped close to 70% during the next 28 months while the MVP and 1/N fell 21% and 46% respectively. Figure 6-13 visualizes how the MVP is structured to shy away from highly correlated and volatile stocks such as those that lead the rise and subsequent fall of the OMX 30 during the dot-com period. From 1999 through 2001 the MVP returned annually 11,2% while the OMX 30 returned 8,0% per year and its annual standard deviation was 72% higher than the MVP´s from 1999 through 2001. The 1/N underperformed the MVP during the period returning annually 10,46%, as can be seen Table 6-6 the MVP strategy displayed significantly lower volatility than the other strategies during these years. For the 5-year period 1998 through 2002, the MVP had a significantly superior Sharpe ratio.

Table 6-6: Properties of the three Strategies during Dot-com Years

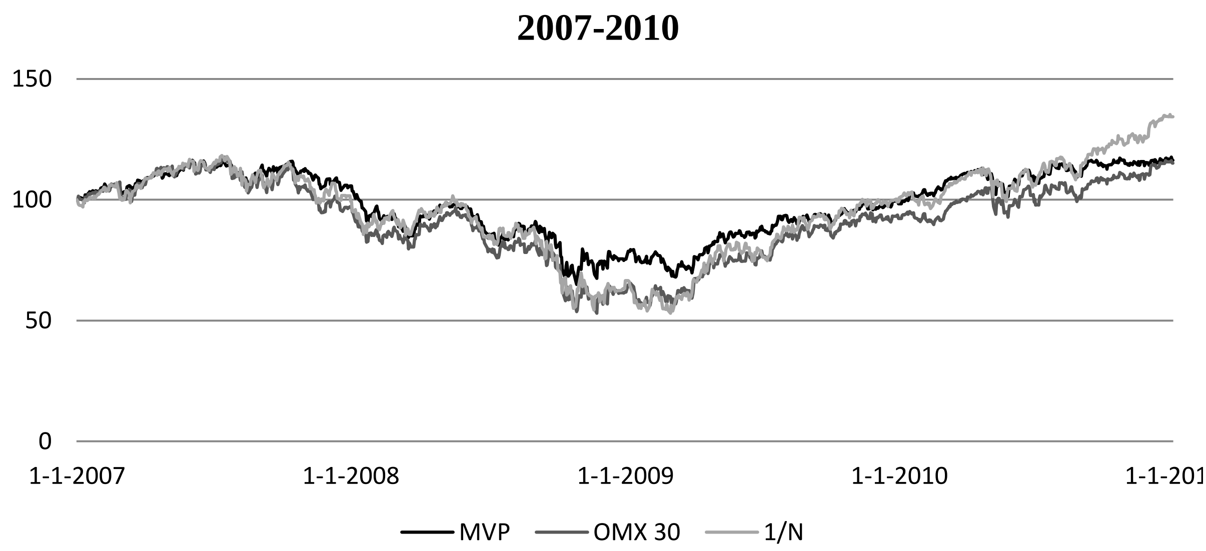
Period		Nr of	Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	Years	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1998	2000	3	14,3%	23,0%	17,8%	10,0%	18,3%	13,3%	18,6%	27,6%	21,2%	0,54	0,66	0,63
1999	2001	3	11,2%	8,1%	10,5%	1,3%	0,4%	1,5%	17,0%	29,1%	21,2%	0,08	0,01	0,07
2000	2002	3	-2,1%	-24,4%	-16,7%	-8,6%	-27,41%	-20,04%	20,4%	33,0%	26,4%	-0,42	-0,83	-0,76
1998	2002	5	4,5%	-2,2%	0,8%	0,42%	-6,01%	-3,07%	20,1%	30,4%	24,8%	0,02	-0,20	-0,12

Figure 6-13: Total Return of MVP, OMXS30 & 1/N 1999-2001



The global financial crisis affected all developed stock markets from the summer of 2007 up until equity prices finally stabilized 18 months later. From its peak the OMX 30 fell 53% while the MVP and 1/N fell 40% and 48% respectively from July 2007 through November 2008. The difference in performance of the MVP between the dot-com crash and the credit crisis can in our mind be explained by the nature of the events that drove the stock market. In the late 90´s non-profitable fast growing companies drove the market to new heights. These stocks tended to be highly volatile and not correlated with more established blue chip stocks and therefore the MVP methodology automatically does not include them because they would add to the overall risk of the portfolio without any positive diversification effect. However, while the financial crisis affected banks more than other companies, all companies are affected by the scarcity of liquidity and closing of debt markets.

Figure 6-14: Total return of MVP, OMXS30 & 1/N 2007-2010



During the rebound after the financial crisis, the MVP has underperformed both the 1/N and OMX 30 considerably. From its lowest level in November 2008 through 2010 the OMX 30 has gained 110% and the 1/N has beaten the benchmark OMX by returning 124% in the same timeframe. The MVP lags its counterparts, returning 68%, but with far lower annualized standard deviation of 20% while 1/N and OMX 30 have had 28,9% and 27,2 respectively. This return should not come as a surprise since the MVP is structured to minimize the overall risk of the portfolio. Historical data is used to construct the MVP, and because stocks, which dropped a great deal in price naturally, showed high volatility in the estimation period they are not a feasible investment for the MVP methodology. Therefore the stocks that rebounded greatly after the financial crisis were not allocated a substantial weight in the MVP because of their drop and volatility in the estimation period.

Table 6-7: Properties of Strategies during the Credit Crisis & Rebound

Period			Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	Nr of Years	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
2007	2008	2	-13,1%	-21,4%	-20,5%	-16,8%	-24,6%	-23,8%	26,4%	31,8%	32,7%	-0,6	-0,774	-0,7
2007	2009	3	-0,6%	-2,7%	-0,3%	-3,6%	-5,7%	-3,4%	24,7%	31,1%	32,8%	-0,15	-0,18	-0,10
2007	2010	4	3,8%	3,6%	7,7%	1,2%	1,0%	4,9%	22,6%	28,6%	30,0%	0,05	0,03	0,16
2009	2010	2	24,1%	36,5%	42,2%	23,0%	35,2%	40,9%	17,9%	24,9%	27,0%	1,29	1,42	1,52

When looking at the return properties from the 2007-2010 period of the three strategies, the MVP consistently dominates the other strategies in terms of low volatility. However it only has the highest Sharpe ratio in the worst performing period, 2007-2008.

6.4.3 Analysis of volatilities

An unfailing feature of the MVP strategy is its low volatility in every period we observed. The strategy displayed as low as 45% of the volatility that the OMX 30 exhibited during each semiannual period as demonstrated in Figure 6-30. Additionally, the MVP demonstrated lower volatility than the 1/N index in all but one semiannual period.

Figure 6-15 Volatility of the MVP as a Percentage of the other Strategies

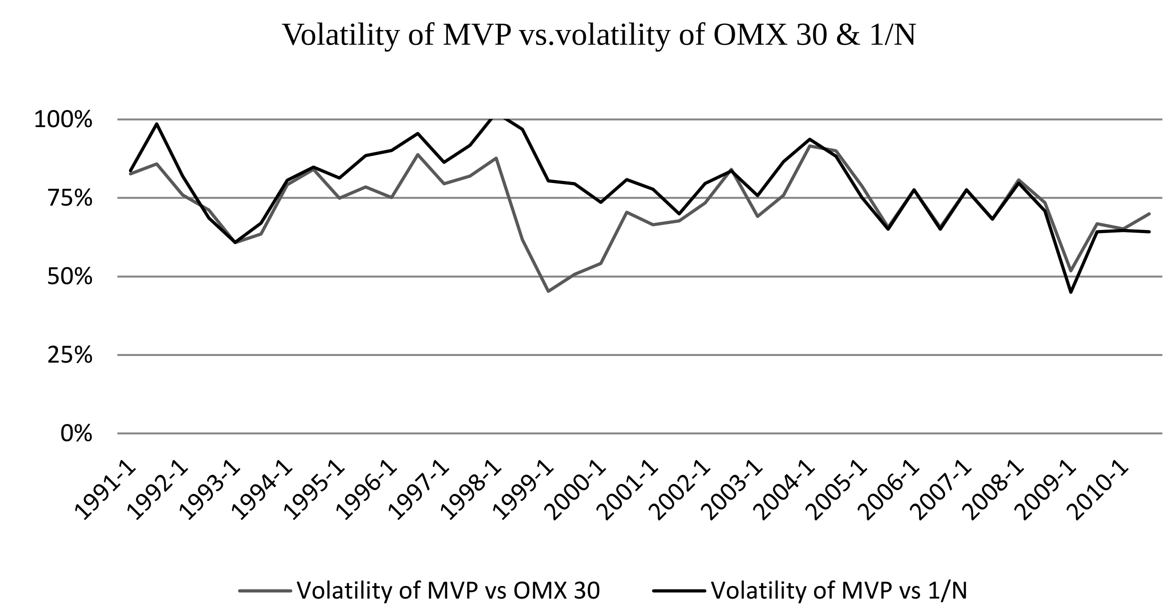
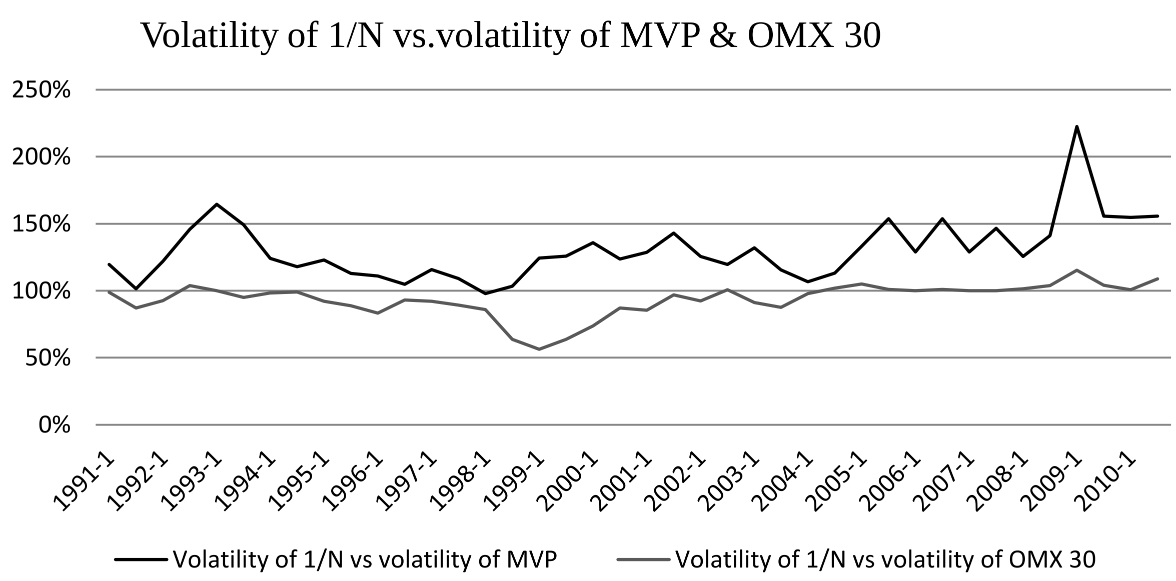


Figure 6-16: Volatility of 1/N as a percentage of the other Strategies



6.5 Similarities with previous research

Figure **Error! Reference source not found.** is interesting because it has the same time frame as the Nielsen & Aylursubramanian (2008) research. In this period for the OMX 30, as in their research of the MSCI, the MVP far outperformed its benchmark in risk adjusted terms. Nielsen & Aylursubramanian (2008) found the MSCI MVP had a Sharpe ratio of 0,67 compared to a 0,45 for the MSCI for the period 1995-2007. We find that during the same 13 year period the OMX 30 MVP significantly outperformed the benchmark OMX 30 on a risk adjusted basis as can be seen in table below.

Figure 6-17: The different Strategies 1995-2007

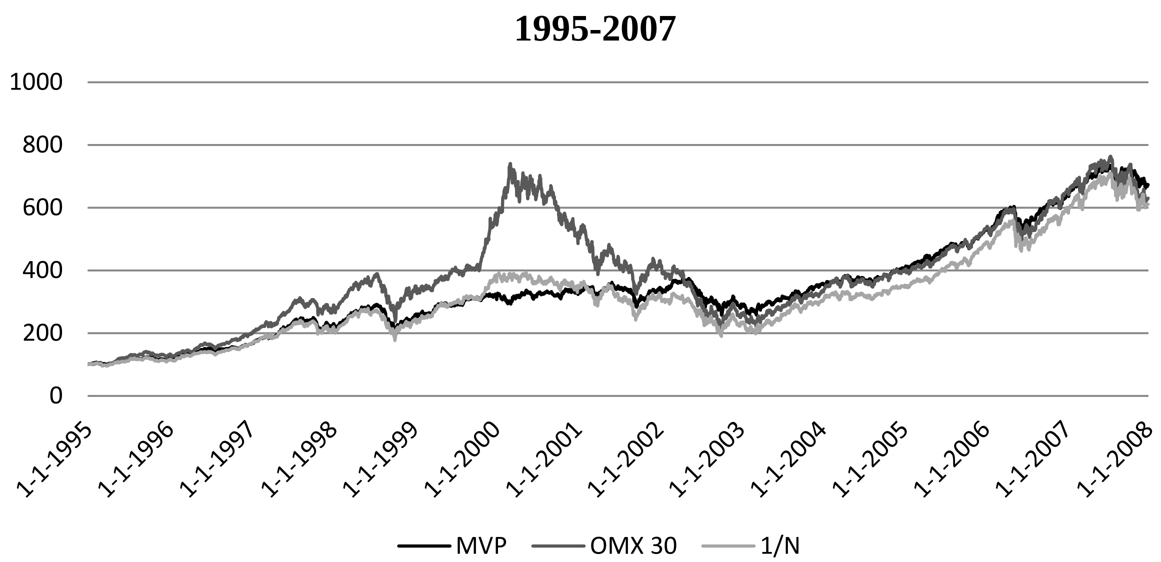


Table 6-8: Properties & Sharp for the different Strategies

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1995	2007	0,157922	0,152069	0,149308	0,112378	0,106753	0,104099	0,16824	0,235661	0,204452	0,667961	0,52214	0,509159

As to the similarities with Haugen´s (1991) analysis of minimum variance portfolios in the US , we find that the MVP consistently outperforms the benchmark in terms of lower volatility, but this lower volatility is at the expense of a lower return in our Swedish equity sample. Our results are similar to those of Clarke et al (2006) research of the US equity market in terms of lower volatility. Clarke et al. and our results are that the minimum variance portfolio had 75% of the realized risk of the general market) Additionally Clarke et al. found the Sharpe ratio of the minimum variance strategy to be 0,55 compared to a 0,36 Sharpe ratio for the market proxy, while our finding is that the MVP´s Sharpe ratio is 0,45 compared to 0,38 for the market. When comparing our findings to those of DeMiguel et al (2007), they find that the equally weighted strategy performs better than any other strategy in terms of Sharpe ratios. We find that in terms of Sharpe ratios the two strategies are almost equivalent, the MVP having a slightly higher Sharpe ratio. However, since we have not factored in turnover costs, the 1/N might well be the top performer on a risk adjusted basis since our MVP is rebalanced on a quarterly basis but the 1/N on a semiannual basis. Turnover costs would of course also affect the OMX 30.

6.6 Summary of Empirical Results

6.6.1 Performance Evaluation

The investment strategy consistently generates positive alphas throughout the later part of the 1991 to 2011 period and has only reoccurring negative B1 parameters in the beginning of the 90's. The strategy is correlated with value stocks, but is still generating a positive alpha of 1,4% annually for the last 20 years. The seasonality in the performance and the statistics of the residual parameter however suggest that there are more factors affecting the returns of the portfolios. Furthermore, the M2 measure suggests a clear out of sample underperformance for the strategy, since it's optimized in the 2002 to 2011 time horizon.

6.6.2 Investment Results

For the full period, The OMX 30 is the worst performing of the three strategies on a risk-adjusted basis. In terms of absolute returns over the full period the MVP underperforms the OMX 30with an annual return of 13,5% while the benchmark returns 14,9%, both have to be considered as very impressive returns over a two decade period (8,07% and 9,35% respectively in excess of the risk free rate). In terms of the riskiness of the two strategies, the MVP generates far less volatility (18% compared to 24,3%) as measured by the annual standard deviations. On a risk-adjusted basis, the MVP dominates with a Sharpe ratio of 0,45 for the two decade period while the OMX 30 generates a Sharpe ratio of 0,38. When the sample is divided into two decade long periods, the results are not as clear cut, but the OMX 30 displays the highest volatility and the MVP the lowest in both period, while the MVP outperforms on a risk adjusted basis in the second 10 year period (0,28 compared to 0,03) and the OMX 30 in the first (0,84 compared to 0,67). When looking at smaller time periods, the MVP always displays lower volatility, be it in 5 year, 4 year, 2 year, 1 year or 6 month periods.

The equally weighted index dominates the value weighted OMX 30 index in all categories for the full period. It returns an annual return of 15,5% whereas the OMX 30 delivers 14,9% annually in the full period (9,93% and 9,35% annual excess returns). The annualized standard deviation of the equally weighted is 22,7% noticeably lower than the 24,3% standard deviation of the benchmark OMX 30. In terms of Sharpe ratios, the 1/N outperforms with a 0,44 ratio compared to a 0,38 of the OMX 30. Looking at the two strategies decade by decade, the 1/N outperforms on a risk adjusted basis during the 2000´s (0,23 compared to 0,03) while the OMX 30 displays superior risk adjusted returns during the 1990´s (0,84 compared to 0,73) . When looking at the volatility of the two strategies over smaller time periods, the 1/N exhibits lower volatility in all 5, 4 and 2 year periods up until 2005. Since then the 1/N strategies has been somewhat more volatile than the OMX 30.

When comparing the MVP and 1/N strategies it is difficult to nominate a clear winner in terms of risk adjusted returns for the full period, since the MVP has a Sharpe ratio of 0,45 and the 1/N Sharpe ratio of 0,44. The MVP´s lower volatility is offset by its lower annualized returns and vice versa for the 1/N strategy. In every 20,10,5, 4, 2, 1 year period the MVP has a lower annualized standard deviation and has lower volatility in all but one 6 month period compared to the 1/N.

7 Conclusions

Previous research in the area of minimum variance investing has so far focused on the most developed, largest stock markets, and the strategy has exhibited impressive risk adjusted returns. The Swedish stock market is different from many larger established markets in terms of having outperformed most developed markets over the span of the past two decades, but it has displayed far greater volatility during the timeframe. The Swedish market accelerated substantially more than the world's leading stock indices during the late 1990´s and then subsequently declined more during most of the 2000´s. After the burst of the dot-com bubble in mid-2000 it wasn't until the beginning of 2009 that the OMXS30´s returns for the decade caught up with those of the S&P 500 and MSCI World.

The empirical study of divergent methodologies for portfolio construction on the OMXS30 constuents has reached several conclusions regarding the estimation method and the parameters:

I. The two alternative approaches to estimate the covariance matrix (Ledoit & Wolf shrinkage and exponentially weighted moving average) have been shown to add no improved performance in finding portfolios with lower standard deviations. This is persistent even when removing the security holding restriction on the portfolios and thus infers that there is no shrinkage influence on the regular covariance estimation.

II. The performance is improved by lower standard deviation, with little change to returns, when removing restrictions on the portfolios. This leads us to believe that the optimization process is a very good tool in finding lower future volatility.

III. The optimal estimation window for historical data is at 12 months of data and its strongest prediction power is for the following 1 month volatility. This is also ture from an investment strategy perspective as the return is also increased, thus creating the best risk-adjusted returns that can be achieved. However, this is only true if the rebalancing cost is below 0.106 percent, above this level up to 0.237 percent the 3 month holding period is superior. Consequently the 3 month portfolio is most likely the optimal choice for an investor.

The study finds that the minimum variance portfolio has generated greater risk adjusted returns than the value weighted benchmark. The strategy has persistently generated returns that exhibit lower volatility and throughout the last decade demonstrated significant positive alphas. The alphas of the strategy have been shown to have a high correlation with value stocks, which intuitively means that the optimization process latches on to these types of companies. This is evident from the performance of the portfolios throughout the full sample period when looking at the growth period of the Dot-com era and the following Value period.

This analysis has been focused on an executable investment strategy and has taken into account the possible cost associated with higher frequencies of rebalancing. In the end of the appendix, there is a presentation of the 20 year realized returns of a leveraged 1 month strategy, which at the volatility of the OMXS30 (not taking any cost into account) can be seen to substantialy outperform.

Interestingly, implementing a simple equally weighted strategy over the past two decades has produced less volatile returns than investing in the market index. In addition to the attractive volatility properties, the equally weighted index has outperformed the market benchmark in terms of total returns. The same is not true for shorter time periods, where the equally weighted, capitalization weighted and minimum variance strategies have taken turns in outperforming one another.

The general conclusion is that the OMX 30 has been an efficient investment strategy over the past two decades and primarily in the 21th century. However, Investors would have been able to achieve similar risk adjusted returns over the full sample period with better absolute returns by investing an equal amount of their assets into each stock in the portfolio.

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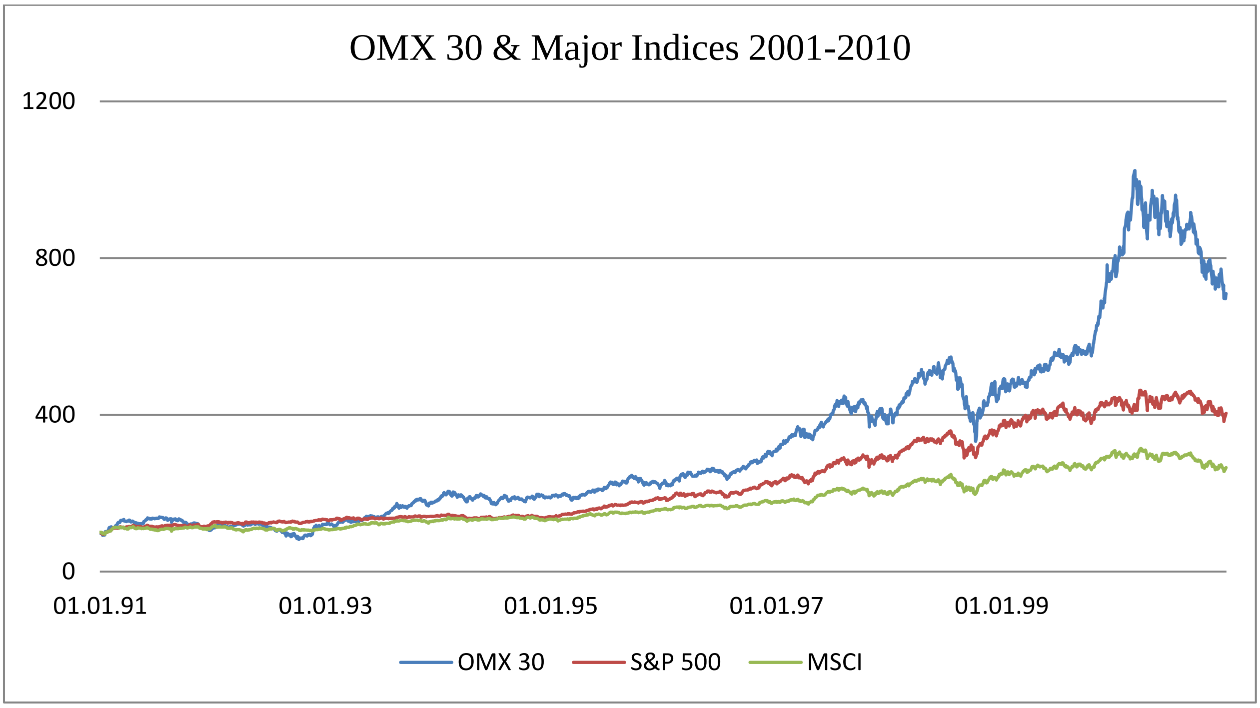
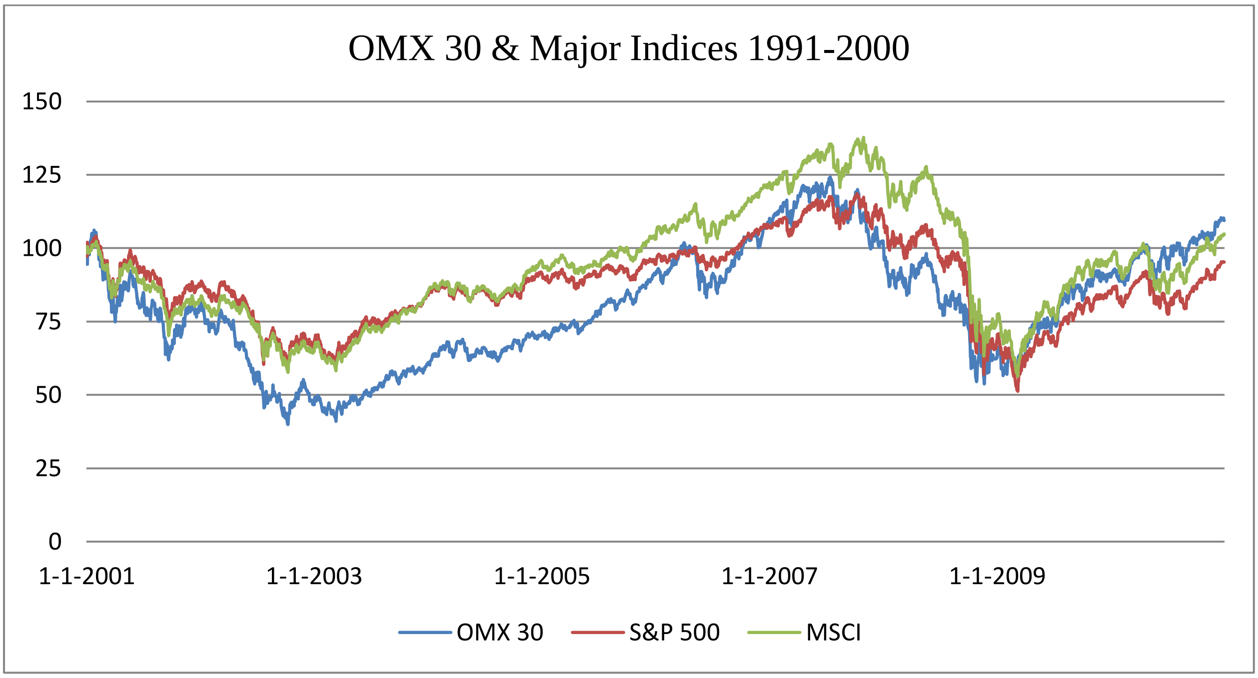
9 Appendix

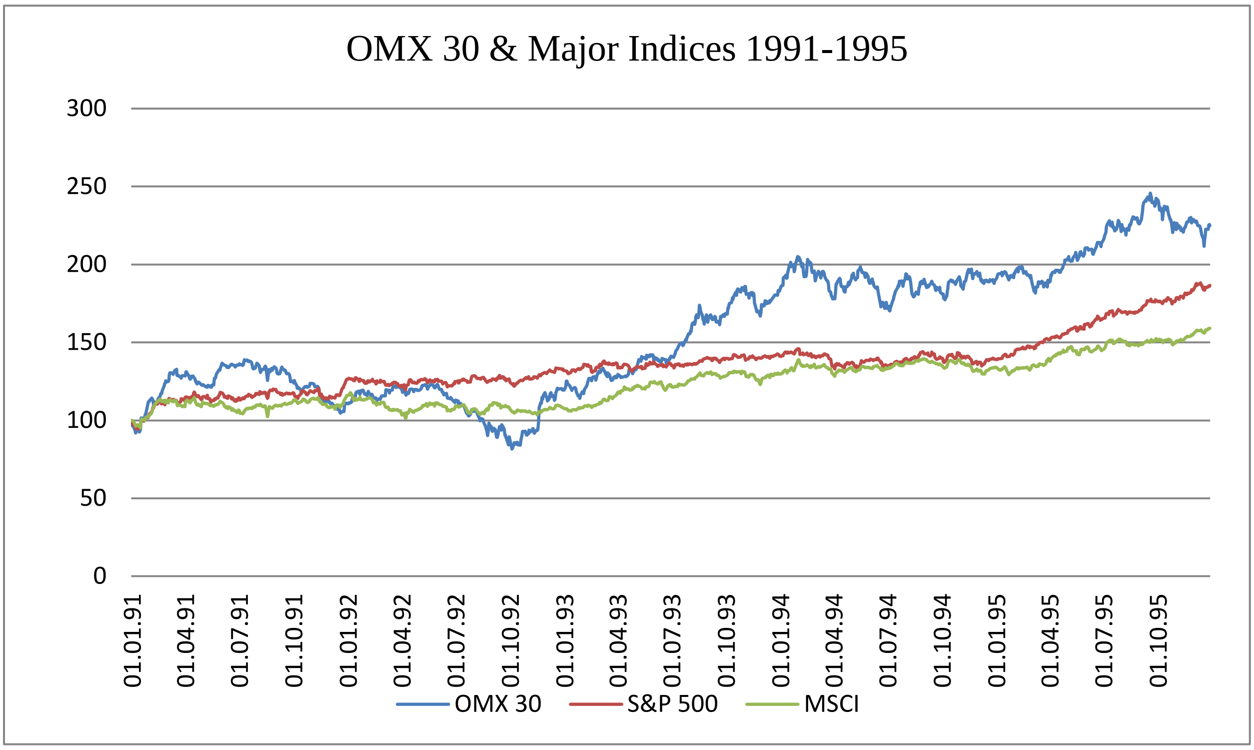
As can be seen in the following graphs the OMX 30 displayed greater volatility than the world's leading stock market indices during the two decades under observation in this thesis. This was especially true during the 1990´s and early 2000´s. All three indices displayed highly volatile returns during the latter part of the 2000´s due to the global financial crisis.



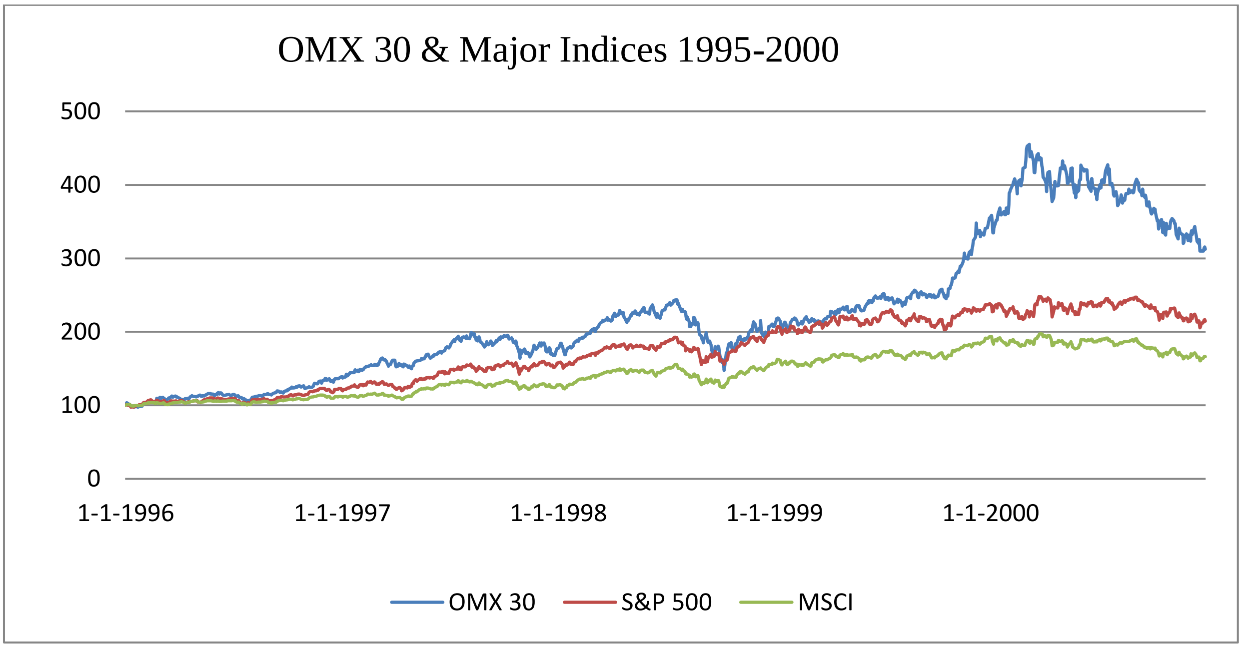
Appendix Chart 1: OMX 30, S&P 500 and MSCI World Index 2001-2010.



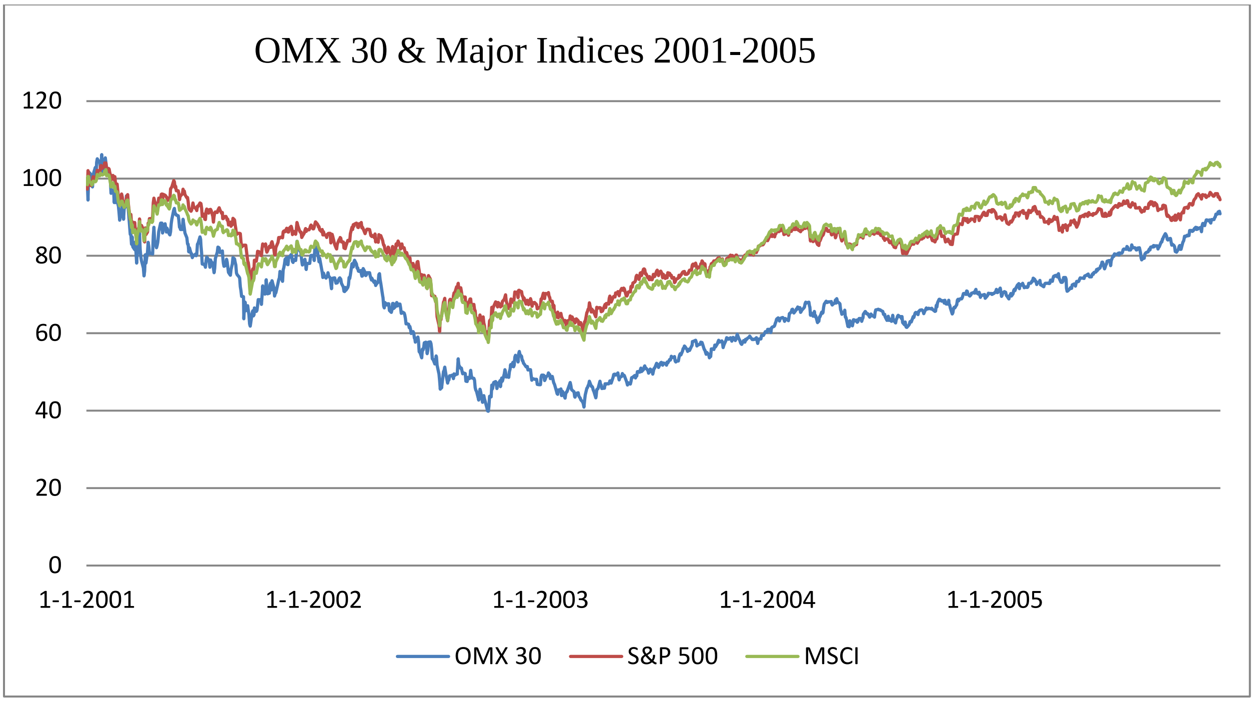
Appendix Chart 2: OMX 30, S&P 500 and MSCI World Index 1991-2000.



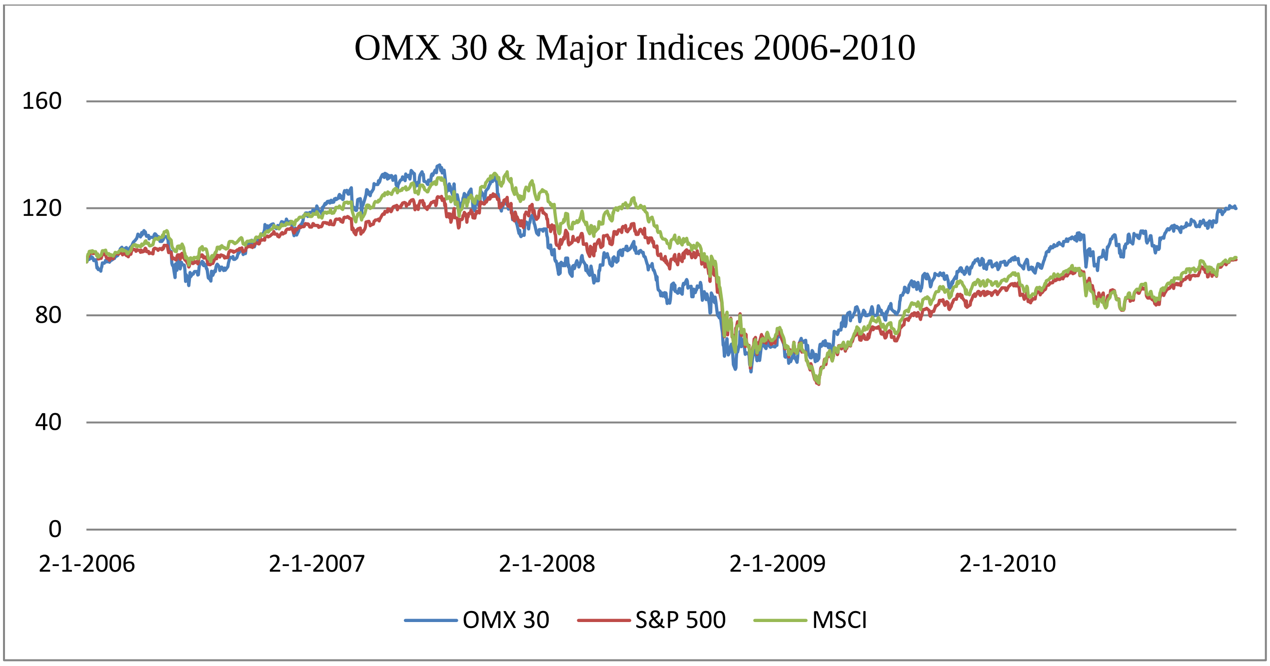
Appendix Chart 3: OMX 30, S&P 500 and MSCI World Index 1991-1995.



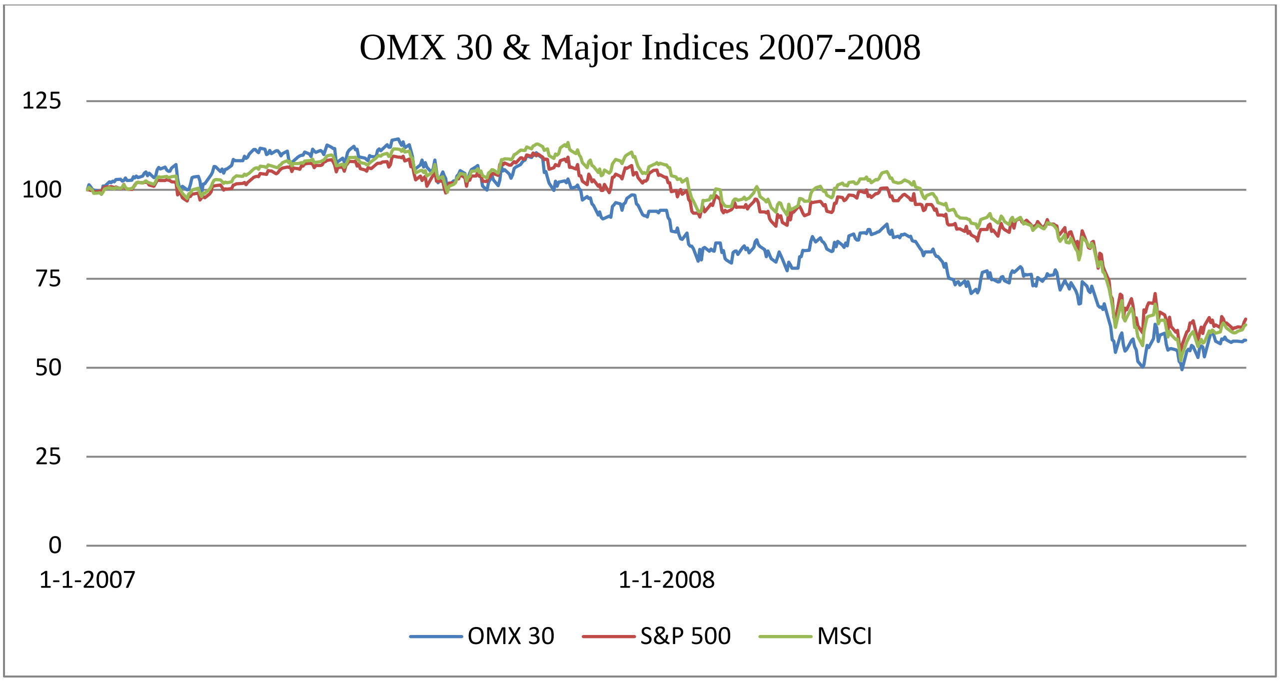
Appendix Chart 4: OMX 30, S&P 500 and MSCI World Index 1995-2000.



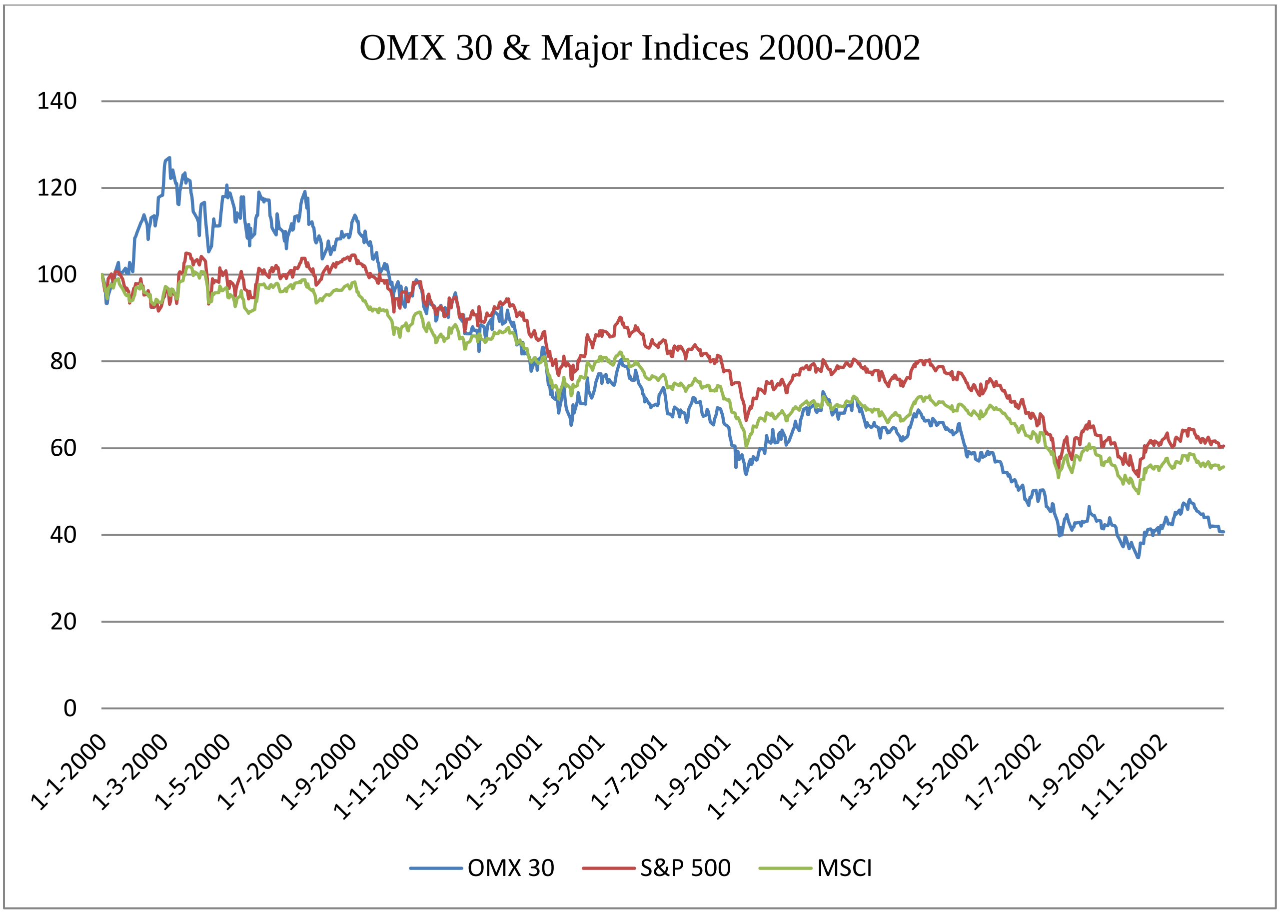
Appendix Chart 5: OMX 30, S&P 500 and MSCI World Index 2001-2005.



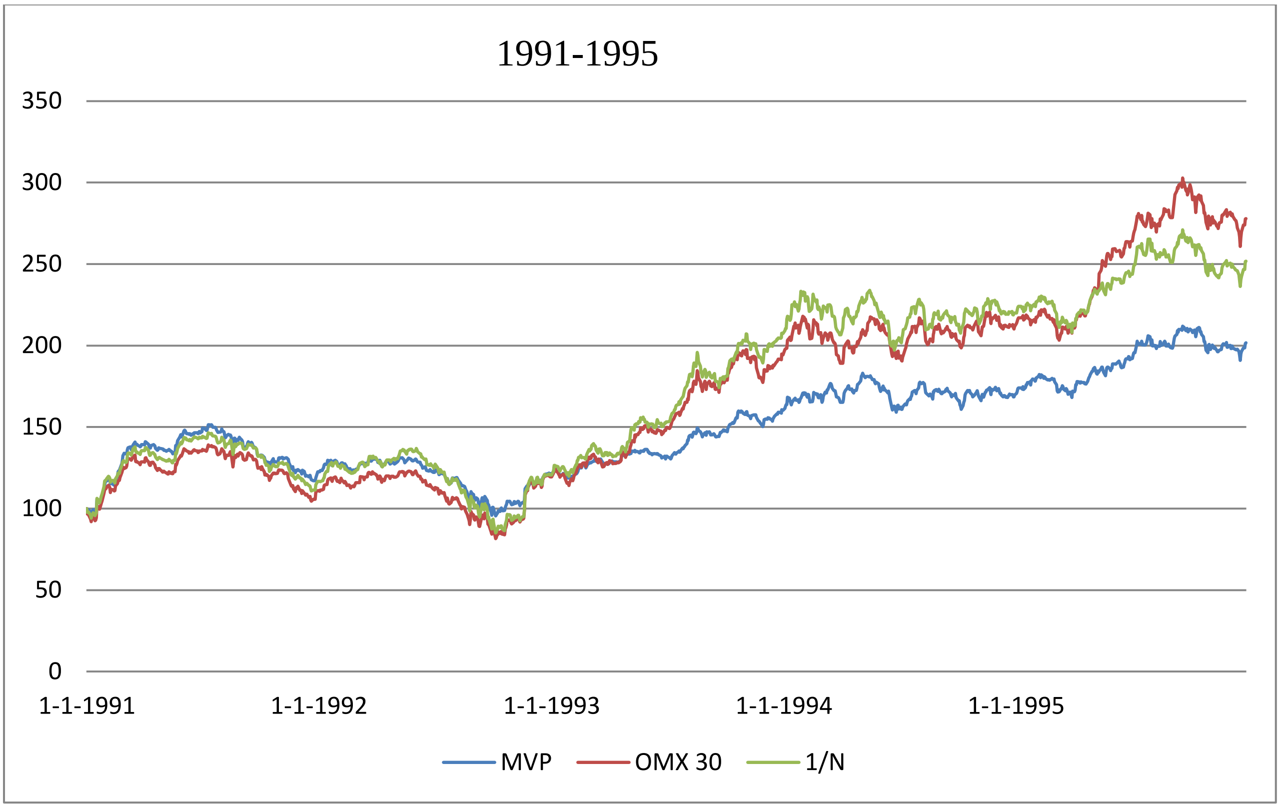
Appendix Chart 6: OMX 30, S&P 500 and MSCI World Index 2006-2010.



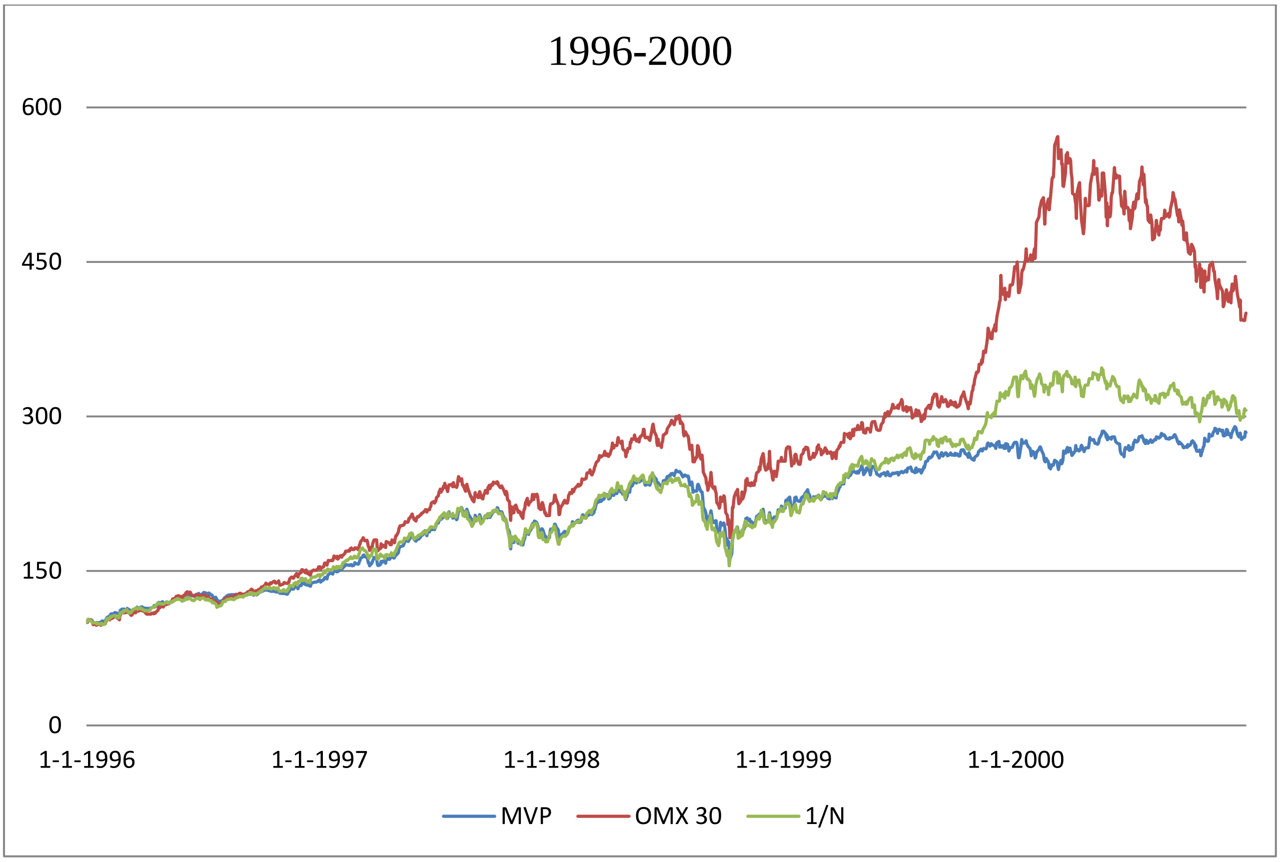
Appendix Chart 7: OMX 30, S&P 500 and MSCI World Index during the global financial crisis of 2007-2008.



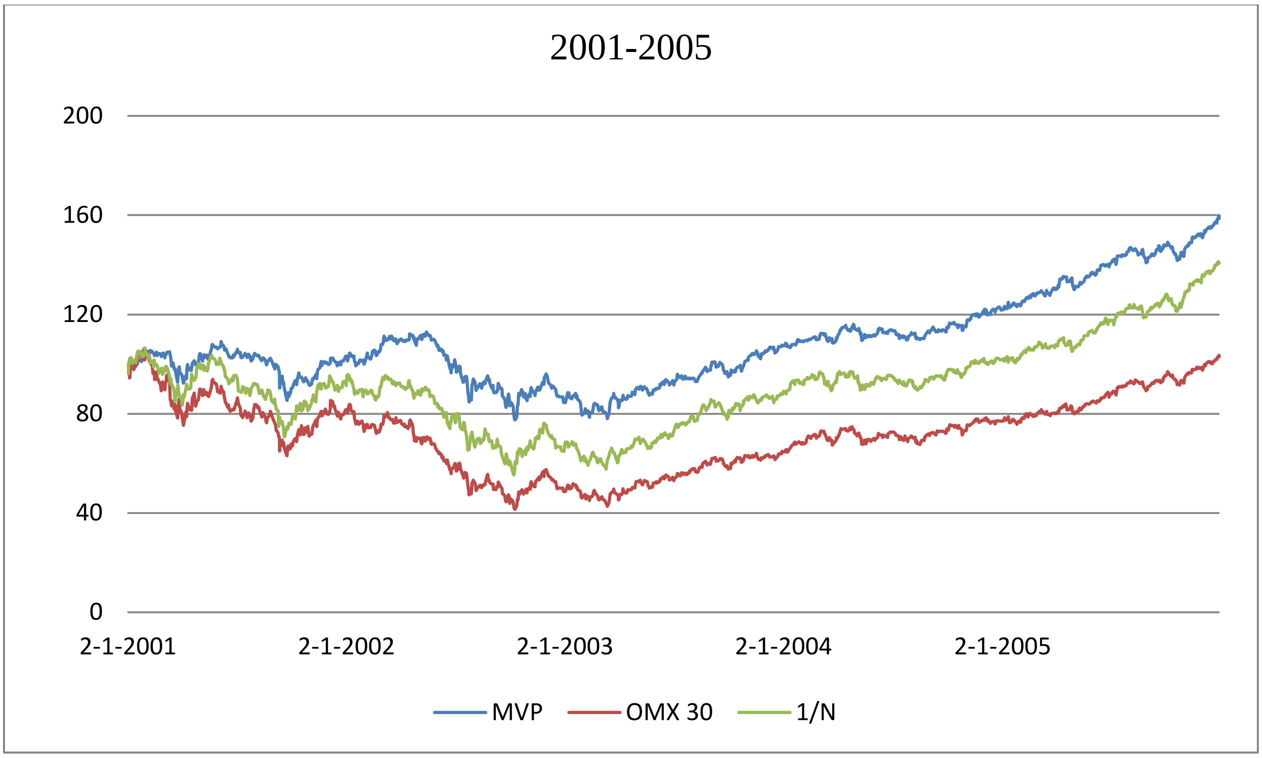
Appendix Chart 8: OMX 30, S&P 500 and MSCI World Index during the burst of the dot com bubble.



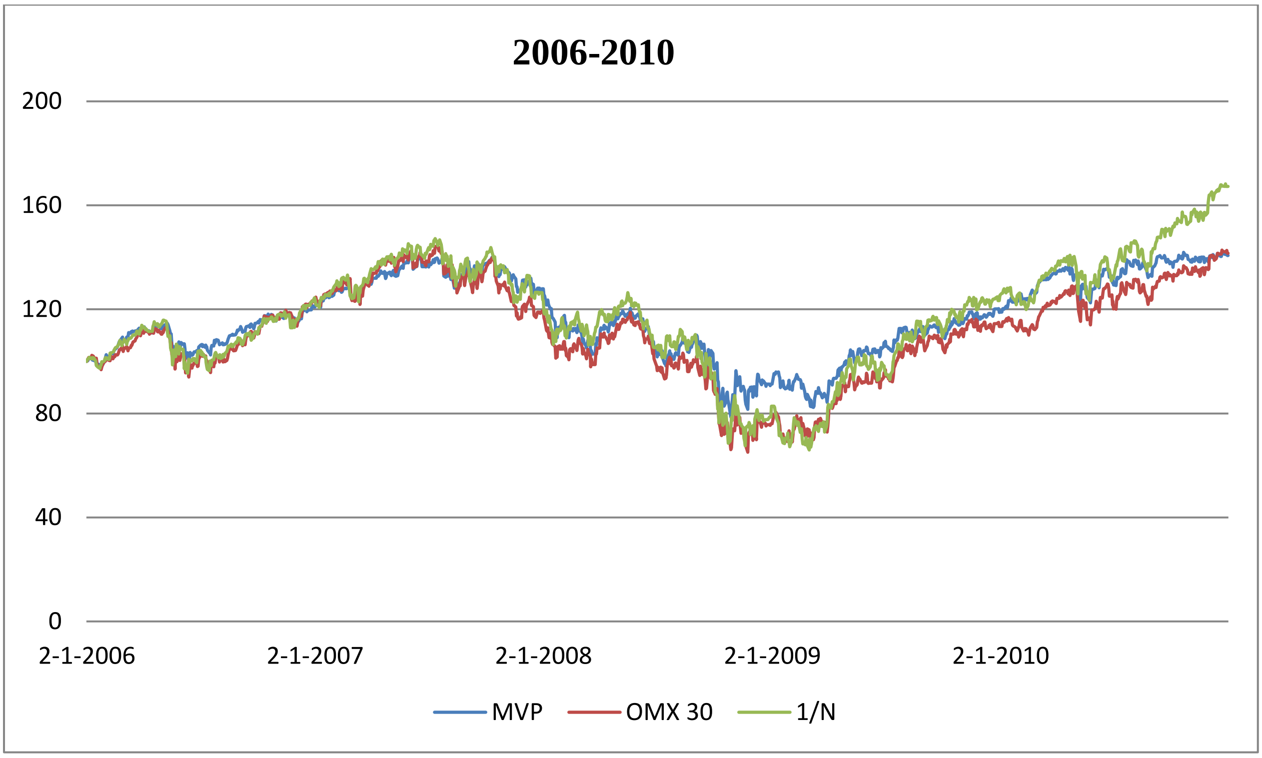
Appendix Chart 9: MVP, OMX 30 and 1/N from 1991-1995.



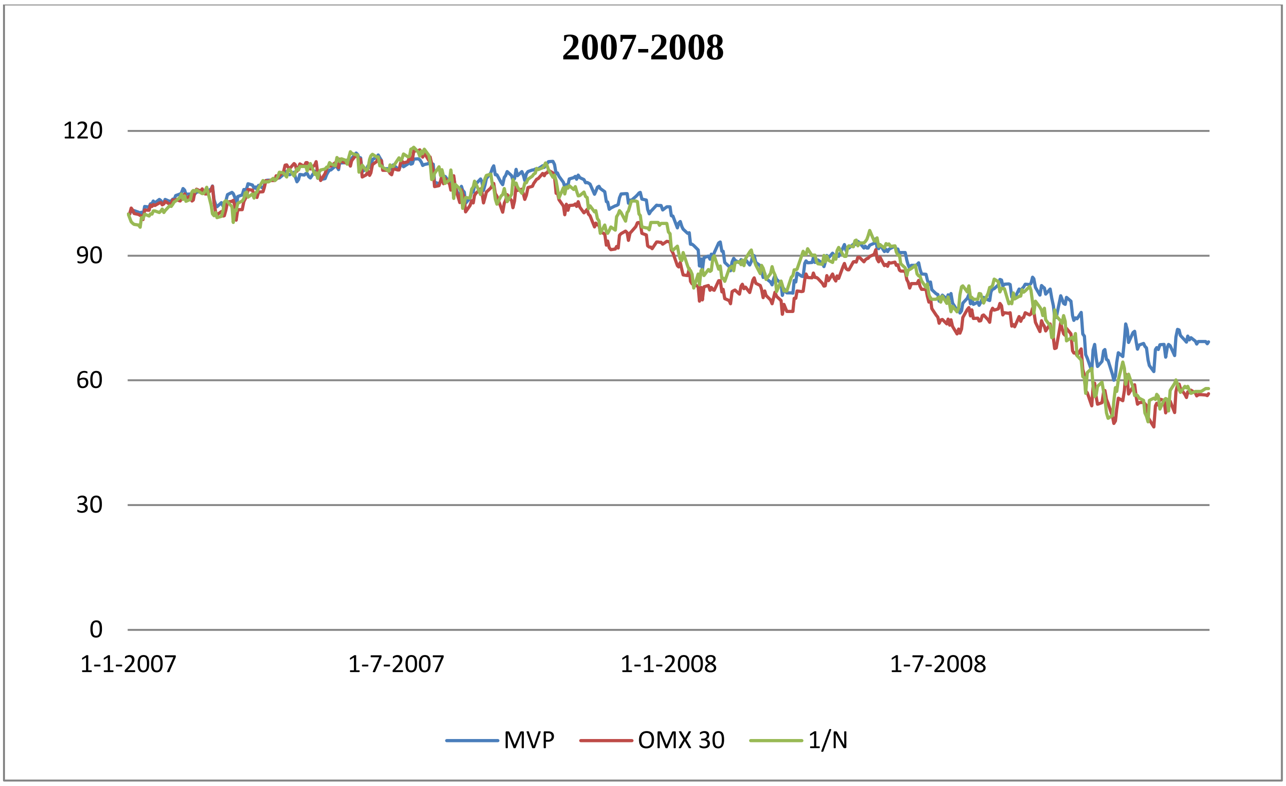
Appendix Chart 10 : MVP, OMX 30 and 1/N from 1996-2000.



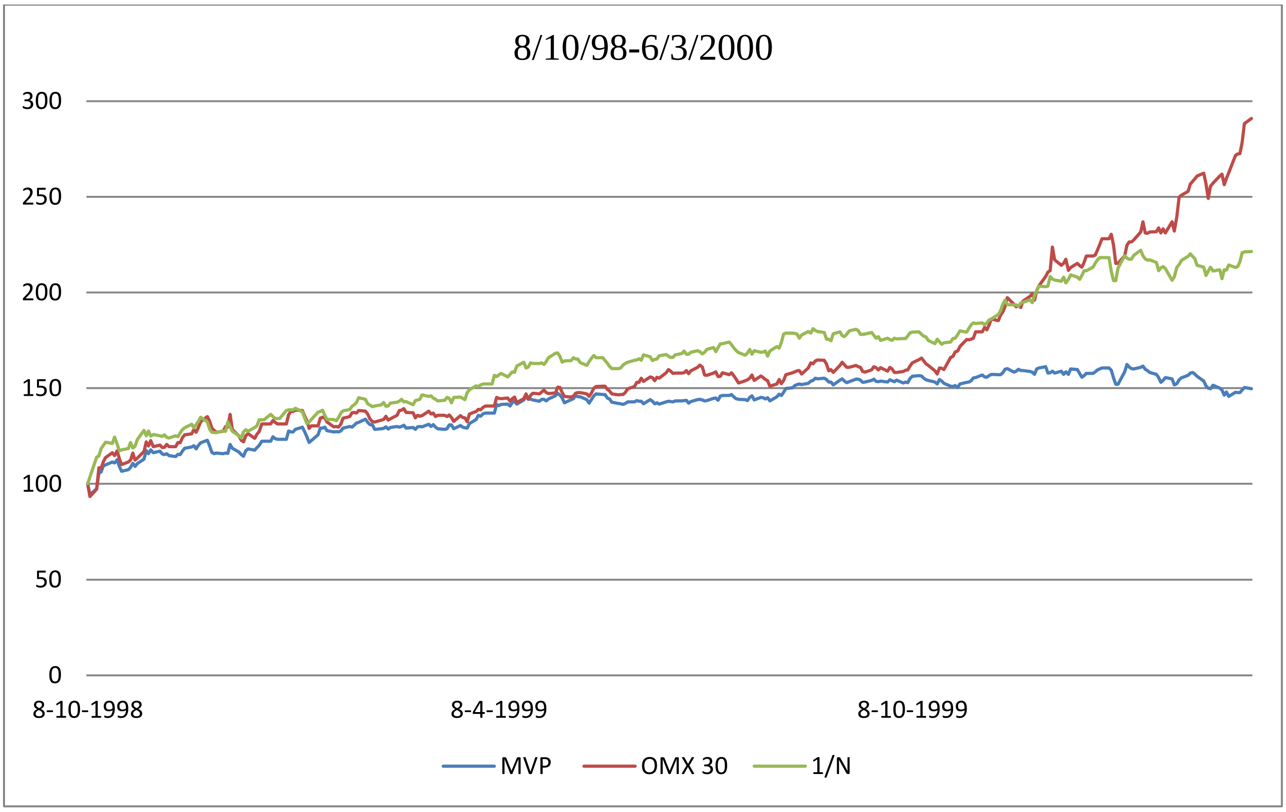
Appendix Chart 11: MVP, OMX 30 and 1/N from 2001-2005.



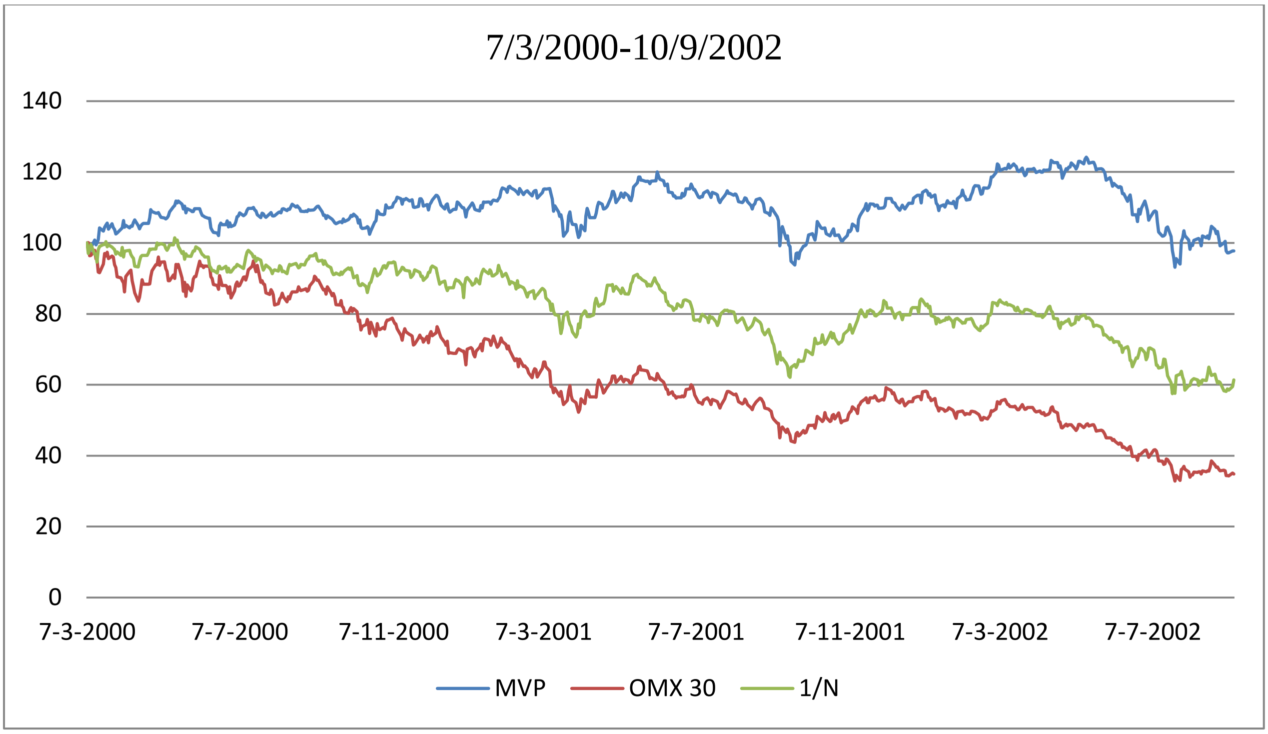
Appendix Chart 12: MVP, OMX 30 and 1/N from 2006-2010.



Appendix Chart 13: MVP, OMX 30 and 1/N during the 2007-2008 financial crisis.



Appendix Chart 14: MVP, OMX 30 and 1/N during the biggest rise in the OMX 30 in late 90´s .



Appendix Chart 15: MVP, OMX 30 and 1/N during the subsequent fall of the OMX 30.

	Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
Period	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	22,6%	11,0%	16,7%	9,6%	-0,8%	4,3%	17,6%	20,8%	19,6%	0,54	-0,04	0,22
1992	-0,9%	7,6%	3,3%	-12,6%	-5,1%	-9,0%	18,6%	25,9%	26,5%	-0,68	-0,20	-0,34
1993	32,0%	62,6%	72,0%	21,4%	49,6%	58,3%	10,6%	17,0%	16,6%	2,01	2,91	3,52
1994	5,1%	7,1%	5,2%	-2,3%	-0,5%	-2,3%	14,1%	17,5%	17,1%	-0,17	-0,03	-0,13
1995	21,1%	30,5%	15,2%	10,6%	19,2%	5,1%	12,3%	15,2%	14,0%	0,86	1,26	0,37
1996	35,4%	51,3%	42,6%	28,6%	43,7%	35,4%	12,0%	14,7%	12,8%	2,38	2,98	2,76
1997	35,2%	39,9%	28,0%	29,4%	34,0%	22,6%	20,2%	22,4%	20,6%	1,46	1,51	1,10
1998	10,7%	19,1%	11,0%	6,0%	14,1%	6,3%	24,0%	30,5%	27,1%	0,25	0,46	0,23
1999	19,7%	107,5%	59,2%	15,1%	99,6%	53,1%	15,3%	21,4%	16,7%	0,99	4,66	3,18
2000	12,5%	-24,8%	-8,6%	8,8%	-27,3%	-11,6%	14,8%	31,0%	18,4%	0,59	-0,88	-0,63
2001	3,2%	-18,4%	-7,0%	-0,9%	-21,6%	-10,7%	20,4%	34,3%	27,0%	-0,04	-0,63	-0,40
2002	-18,2%	-40,5%	-29,0%	-21,5%	-43,0%	-31,9%	24,0%	33,6%	31,5%	-0,90	-1,28	-1,01
2003	27,3%	33,1%	31,1%	23,3%	28,9%	27,0%	16,1%	21,9%	20,8%	1,44	1,32	1,29
2004	13,5%	19,4%	14,6%	10,9%	16,8%	12,0%	10,4%	15,9%	14,0%	1,05	1,05	0,86
2005	31,0%	34,2%	37,5%	28,6%	31,7%	35,0%	9,1%	11,5%	11,0%	3,14	2,77	3,18
2006	21,4%	23,2%	23,0%	18,3%	20,1%	19,9%	15,6%	19,2%	19,8%	1,18	1,04	1,01
2007	5,7%	-2,9%	1,6%	1,8%	-6,6%	-2,2%	15,7%	20,3%	20,5%	0,11	-0,32	-0,11
2008	-28,6%	-36,3%	-36,4%	-31,9%	-39,2%	-39,3%	33,9%	40,2%	41,5%	-0,94	-0,98	-0,95
2009	30,3%	49,2%	49,1%	29,1%	47,9%	47,8%	20,7%	29,4%	32,7%	1,40	1,63	1,46
2010	18,3%	24,9%	33,3%	17,2%	23,7%	32,0%	14,6%	19,3%	19,6%	1,18	1,23	1,63

Appendix Table 1. Return properties and Sharpe ratio´s for the three strategies when the full sample is split into annual periods.

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	1992	5,0%	4,6%	4,8%	-1,1%	-1,5%	-1,3%	18,1%	23,5%	23,3%	-0,06	-0,06	-0,05
1993	1994	8,7%	15,4%	16,2%	4,5%	10,9%	11,8%	12,5%	17,3%	16,9%	0,36	0,63	0,70
1995	1996	13,5%	18,8%	13,6%	9,6%	14,7%	9,6%	12,2%	15,0%	13,5%	0,78	0,98	0,71
1997	1998	10,6%	13,6%	9,2%	8,2%	11,2%	6,8%	22,2%	26,8%	24,1%	0,37	0,42	0,28
1999	2000	7,8%	11,8%	10,2%	5,8%	9,8%	8,2%	15,1%	26,0%	17,5%	0,39	0,38	0,47
2001	2002	-8,1%	-30,3%	-19,5%	-11,8%	-33,1%	-22,7%	22,3%	34,0%	29,3%	-0,53	-0,97	-0,77
2003	2004	20,2%	26,1%	22,5%	16,9%	22,7%	19,2%	13,6%	19,1%	17,8%	1,24	1,18	1,08
2005	2006	25,7%	28,1%	30,2%	23,0%	25,3%	27,4%	12,8%	15,8%	16,0%	1,80	1,60	1,71
2007	2008	-13,1%	-21,4%	-20,5%	-16,8%	-24,6%	-23,8%	26,4%	31,8%	32,7%	-0,63	-0,77	-0,73
2009	2010	9,0%	13,3%	15,1%	8,6%	12,8%	14,7%	17,9%	24,9%	27,0%	0,48	0,52	0,54

Appendix Table 2. Return properties and Sharpe ratio´s for the three strategies when the full sample is split into ten 2 year periods.

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	1994	14,1%	20,6%	21,8%	3,4%	9,2%	10,3%	15,6%	20,6%	20,4%	0,22	0,45	0,51
1995	1998	25,6%	35,0%	24,0%	18,6%	27,5%	17,1%	17,9%	21,7%	19,5%	1,04	1,27	0,88
1999	2002	3,0%	-6,9%	-1,5%	-0,9%	-10,5%	-5,3%	19,0%	30,3%	24,2%	-0,05	-0,34	-0,22
2003	2006	22,9%	27,1%	26,4%	19,9%	24,0%	23,3%	13,2%	17,6%	16,9%	1,51	1,37	1,38
2007	2010	3,8%	3,6%	7,7%	1,2%	1,0%	4,9%	22,6%	28,6%	30,1%	0,05	0,03	0,16

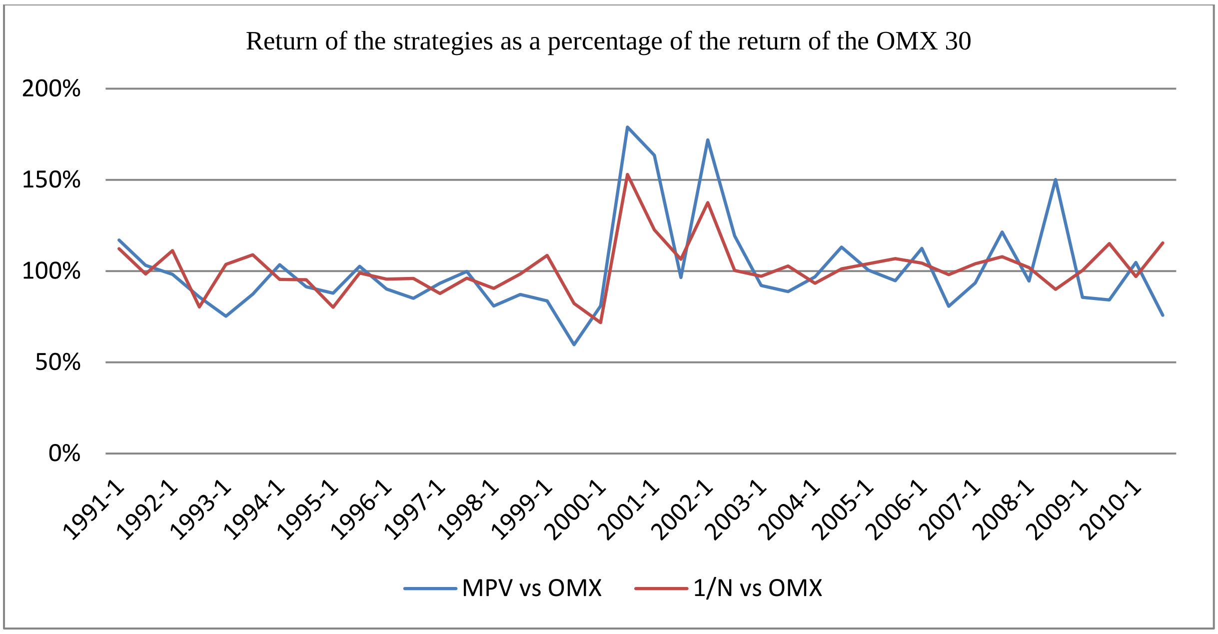
Appendix Table 3. Return properties and Sharpe ratio´s for the three strategies when the full sample is split into five 4 year periods.

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	1995	15,6%	22,8%	20,4%	4,9%	11,4%	9,2%	15,0%	19,6%	19,2%	0,327	0,57878	0,4814
1996	2000	22,2%	31,5%	24,4%	17,2%	26,1%	19,3%	17,9%	24,6%	19,8%	0,961	1,05879	0,9755
2001	2005	10,2%	0,7%	7,3%	6,7%	-2,4%	4,0%	17,0%	25,3%	22,3%	0,396	-0,0955	0,179
2006	2010	7,1%	7,2%	10,8%	4,3%	4,5%	8,0%	21,4%	27,0%	28,3%	0,203	0,16534	0,283

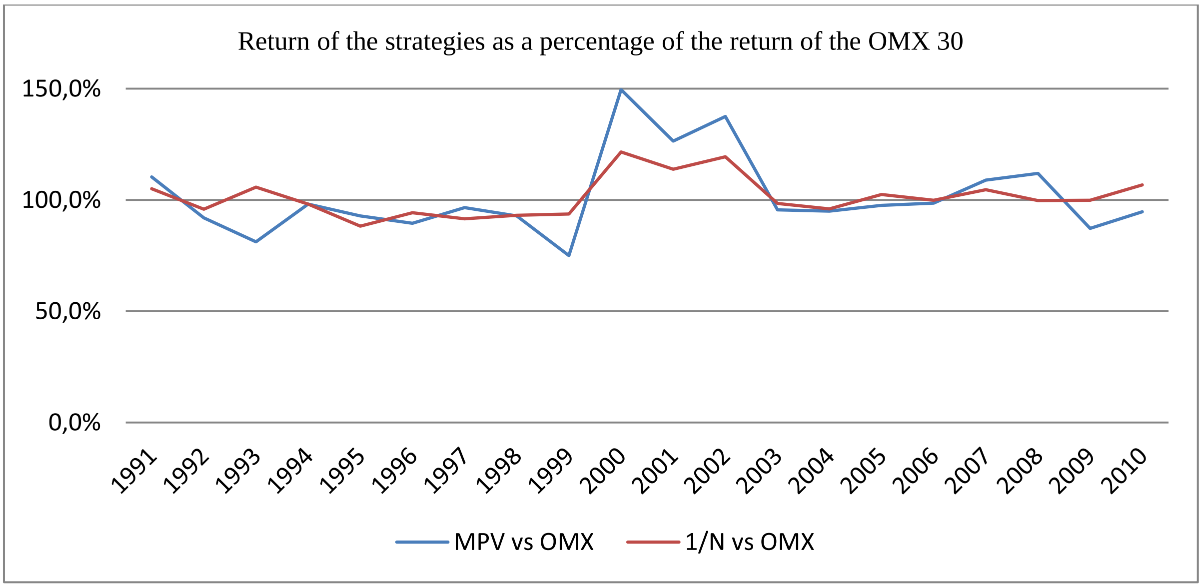
Appendix Table 4. Return properties and Sharpe ratio´s for the three strategies when the full sample is split into four 5 year periods

Period		Annualized Return			Annualized Return over Rf			Standard Deviation			Sharpe Ratio		
From	To	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N	MVP	OMX 30	1/N
1991	2000	19,1%	27,2%	22,6%	11,02%	18,61%	14,30%	16,5%	22,2%	19,5%	0,67	0,84	0,73
2001	2010	8,4%	3,8%	9,0%	5,33%	0,90%	5,89%	19,3%	26,1%	25,5%	0,28	0,03	0,23

Appendix Table 5. Return properties and Sharpe ratio´s for the three strategies during 1991-2000 and 2001-2010.



Appendix Chart 16: Shows the annualized return of the MVP and 1/N in each semiannual period as a percentage of the annualized return of the OMX 30 in the same period



Appendix Chart 17: Shows the return of the MVP and 1/N in each year as a percentage of the return of the OMX 30 in the same year

#	40-42 Periods	#	20-39 Periods	#	10-19 Periods	#	4-9 Periods	#	1-3 Periods
42	ABB	39	INVESTOR B	19	ABB A	9	BOLIDEN	3	ALFA LAVAL B
42	AZTRAZENECA	38	SANDVIK	19	AUTOLIV	8	SYDKRAFT	3	GETINGE
42	ATLAS COPCO A	37	H&M B	19	PHARMACIA	7	AVESTA SHEFFIELD	3	LB ICON
42	ELECTROLUX	34	STORA ENSO	19	TELIASONERA	7	SSAB A	3	MTG
42	ERICSSON B	33	SKANDIA	18	ASTRA B	6	FABEGE	3	SAAB SCANIA A
42	SCA B	29	SWEDBANK	18	SANDVIK B	6	KINNEVIK B	2	BILSPEDITION B
42	SKANSKA	27	NOKIA	17	AGAB	6	LUNDIN PETROLEUM	2	ESSELTE B
41	SEB A	25	NORDEA	16	SWEDISH MATCH	6	WM DATA	2	LBI INERNATIONAL
41	SKF B	23	HOLMEN B	15	ALFA LAVAL	6	VOSTOK GAS	2	LUNDBERGSFORETAGEN
41	VOLVO B	23	TELE2	15	STORA B	5	GAMRO B	2	N&T ARGONAUT
40	ATLAS COPCO B	22	SECURITAS B	14	ENIRO	5	TRYGG HANSA	2	OSTGOTA ENSKILDA
40	HANDELSBANKEN	22	TRELLEBORG	13	INVESTOR A	4	CELCIUS B	2	PROVENTUS B
		20	ASSA ABLOY B	13	SCANIA B	4	EUROPOLITAN	1	AGA A
						4	PROCORDIA	1	KINNEVIK A
								1	PROVIDENTIA A
								1	VOLVO A

Appendix Table 6 shows in each period how many of the OMX 30 stocks had sufficient historical data available to estimate the minimum variance portfolio

#	1991-1	1991-2	1992-1	1992-2	1993-1	1993-2	1994-1	1994-2	1995-1	1995-2	Average
12 months	25	26	25	24	28	28	28	28	28	28	26,8
24 Months	22	24	24	24	27	27	28	27	27	28	25,8
36 Months	21	22	22	22	26	27	27	26	27	27	24,7
48 Months	20	22	21	21	24	25	27	26	26	27	23,9
60 Months	20	21	20	21	24	24	24	24	26	27	23,1
#	1996-1	1996-2	1997-1	1997-2	1998-1	1998-2	1999-1	1999-2	2000-1	2000-2	Average
12 months	29	28	28	29	28	29	30	28	30	30	28,9
24 Months	29	27	27	28	26	28	29	28	29	29	28
36 Months	29	27	27	27	25	25	27	27	28	28	27
48 Months	28	27	27	27	25	24	25	23	26	27	25,9
60 Months	28	27	27	27	25	24	25	22	24	24	25,3
#	2001-1	2001-2	2002-1	2002-2	2003-1	2003-2	2004-1	2004-2	2005-1	2005-2	Average
12 months	29	29	29	30	29	30	30	30	30	30	29,6
24 Months	28	28	28	29	29	29	29	30	30	30	29
36 Months	27	28	28	28	27	28	29	29	29	30	28,3
48 Months	26	28	28	28	27	27	27	28	29	29	27,7
60 Months	25	27	27	28	27	27	27	27	27	28	27
#	2006-1	2006-2	2007-1	2007-2	2008-1	2008-2	2009-1	2009-2	2010-1	2010-2	Average
12 months	30	30	30	30	30	30	30	30	30	30	30
24 Months	30	30	30	30	30	30	30	30	30	30	30
36 Months	30	30	30	30	30	30	30	30	30	30	30
48 Months	29	30	30	30	30	30	30	30	30	30	29,9
60 Months	29	29	30	30	30	30	30	30	30	30	29,8

Appendix Table 7 Shows which securities are included in the minimum variance portfolio, and in how many periods they are included (12 month estimation period). Total of 68 securities

1 month holding period without cost leveraged to equal to OMXS30