Stockholm School of Economics
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Master's Thesis

# Marriage and Unemployment within a Lifecycle Portfolio Allocation Model

We address the problem of optimal portfolio allocation for individuals who are exposed to uninsurable flows of risky non-financial income. We adopt a normative approach and calibrate our model using German data. Our contribution lies primarily with explicitly modeling unemployment risk and quantifying the effect of changes in marital status on the riskiness of human capital. We show that the size and riskiness of human capital have significant influence on optimal portfolio composition. Marriage-induced reduction in riskiness of human capital causes adjustments in allocation of financial assets at the scale of 4-12%. We also show that the utility costs of deviating from the optimal allocation policy are economically large.

Key words: portfolio optimization, human capital, unemployment risk, marital status, SOEP

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## 1. Introduction

This paper contributes to the growing Household Finance literature and examines optimal portfolio choice of agents who earn non-financial income. Our analysis starts with recognizing that exposure to risky flows of non-financial income, such as wages or pensions, should affect portfolio allocation of individuals and households. Realistically, such exposure is not tradable and represents a source of background risk (as in Kimball, 1993), in a sense that individuals are endowed with it, and that it cannot be hedged. Another way to look at the problem is to visualize human capital, a notional asset that yields non-financial income and is an unavoidable part of everyone's total wealth 'portfolio'. Then the amount and riskiness of human capital should leave an effect on the optimal composition of the financial portfolio.

Inspired by recent literature, we formalize the problem within a life cycle consumption and portfolio composition model that owes a lot of credit to the work of Cocco, Gomes, and Maenhout (2005). To the best of our knowledge, we are the first to explicitly incorporate the age structure of unemployment risk into the non-financial income process. Compared to the model of Cocco, Gomes, and Maenhout (2005), this enables us to better capture how the riskiness of human capital evolves over age. We find that young individuals in our data set are more exposed to unemployment risk, which, cet. par., reduces their willingness to hold risky assets in their financial portfolio.

We are making another contribution by allowing agents to change their marital status. In our model, income sharing between the spouses results in a reduction in riskiness of each other's human capital in a similar way as diversification reduces portfolio risk. We find that the reduction in background risk that follows the transition from single to married state results in a greater tolerance of financial risks. We estimate that the impact of marriage on the optimal risky asset allocation is moderate and is typically in the range of 4-12% of financial assets.

A common prediction that normative portfolio allocation models, such as ours, make is that young investors heavily tilt their financial portfolios to equities. This prediction does not stand an empiric check: in reality young

investors tend to have low equity market participation rates, and when they do invest in equities allocations are modest (Poterba and Samwick, 2003; Heaton and Lucas, 2000). Several modifications to the life cycle model have been proposed, including equity market entrance costs (Gomes and Michaelides, 2005), a wedge between borrowing and lending rates (Davis, Kubler, and Willen, 2006), and habit utility formation (Polkovnichenko, 2007), to better account for behavior of young investors. We add to this discussion by showing that young individuals, who are exposed to a higher risk of unemployment and are likelier to be single, have a higher background risk and, therefore, have also a rational reason to hold more conservative portfolios.

With a goal of drawing representative normative inferences about the influence of human capital on portfolio choice, we calibrate the income process to the data from the German Socio-Economic Panel Study (SOEP), the longest-running household panel in Germany. Similar to models that are calibrated to U.S. data, we obtain the 'humped' shape of expected non-financial income profiles over agents' lifetimes. Since we explicitly build in unemployment risk into the income process, our estimates of human capital riskiness are not directly comparable to previous research; however we are able to notice qualitative similarities, for example, that dispersion of transitory shocks is approximately two-three times larger than that of persistent shocks.

We identify three forces which jointly determine the optimal risky share along the dimension of age. First, the background exposure to human capital creates diversification demand for risky financial assets above the level predicted by Merton's (1969) optimal policy. Second, the amount of financial wealth, which we find to peak around the retirement age, affects the relative importance of human capital in the determination of the composition of the financial portfolio. Third, the reduction in riskiness of human capital which occurs as agents marry and approach retirement results in increased risk appetite with regard to their financial portfolios.

Given our calibration results and our choice of benchmark parameters, the model's predictions are qualitatively similar to the ones of Cocco, Gomes, and Maenhout (2005). On average the optimal risky asset share is 100% early in life.

As age increases, the optimal risky share declines until the level of 30-40% by the retirement age. During retirement, optimal allocation to the risky asset increases again as the increasing force of mortality motivates rapid depletion of financial assets. Quantitatively, in our model, with unemployment risk and the distinction of family composition, we generate considerably less aggressive portfolio allocation profiles for younger investors when compared to Cocco, Gomes, and Maenhout (2005).

We find that the value of human capital is large. On average, it accounts for more than 90% of total wealth, i.e. the sum of financial wealth and human capital, for agents younger than 35. This ratio on average does not fall below 40% thoughout one's life. Given that human capital is large, ignoring it when making portfolio allocation decisions is costly. We compare the constant consumption-equivalent of the utility achieved under alternative asset allocation policies to the one achieved from following the optimal one. We find that ignoring human capital altogether or following a popular rule of thumb for life cycle investing (Malkiel, 1996) leads to utility losses equivalent to a reduction in constant certain consumption between 1% and 5%. We consider these losses economically significant.

Actual behavior of German private investors differs from the predictions of our model. In Germany, stock market participation rates as well as risky asset shares are very low (Barasinska et al., 2008; Börsch-Supan and Essig, 2002). If the model employed here accurately represents investors' preferences and income processes, actual behavior leads to significant welfare losses.

## 2. Theory and Previous Research

In this section we give a brief review of the life cycle portfolio allocation literature, which a reader might find useful for better understanding the context of our work. A sophisticated reader might prefer to skip this section.

### 2.1. Classical portfolio optimization

Markowitz (1952) pioneers the formalization of the portfolio allocation problem. His framework is set up in a mean-variance space and is limited to a single investment period. Mossin (1968) and Samuelson (1969) show that under the assumptions of constant relative risk aversion (CRRA) utility, stationary asset return distributions, and complete and frictionless markets, optimal portfolio allocation is invariant to investment horizon, wealth, and optimal level of saving. Within the same setup, but in continuous time and with normally distributed returns, Merton (1969) finds the closed form solution to the problem for a two-asset investment universe as

$$\varphi^* = \frac{\mu}{\sigma^2 \gamma}, \tag{1}$$

where $\varphi^*$ is the optimal share of the financial portfolio invested in the risky asset, $\gamma$ is the coefficient of relative risk aversion, and $\mu$ and $\sigma^2$ are the first and second moments of the distribution of risky asset excess returns, respectively. The above solution is straightforwardly extendable to a multi-asset case. Assuming homogenous investor expectations, such an extension is, however, redundant: according to the mutual fund separation theorem, a case with many risky assets is equivalent to the two-asset case in Merton (1971), with the risk-free asset and the market portfolio.

The fact that investors earn labor and other non-financial income calls for the recognition of human capital, i.e. the present value of future non-financial income, as an embedded portfolio component. The efficient allocation to the risky asset, along the lines of Merton (1971), is then defined as

$$\varphi^* = \frac{\mu}{\sigma^2 \gamma} \left(1 + \frac{W^{HC}}{W^F}\right), \tag{2}$$

where $W^F$ and $W^{HC}$ stand for financial wealth and human capital, respectively. In the complete markets setup of Merton (1971), exposure to non-financial income could be replicated by some portfolio of marketable securities and turned into a synthetic $W^{HC}$ position in the risk-free asset. Thus, everything else being equal, a larger exposure to non-financial income increases the optimal share of financial capital invested in the risky asset. This leads to the idea that as the value of human capital evolves over the life cycle, so does the optimal

portfolio allocation. For example, Jagannathan and Kocherlakota (1996) argue that older people should invest less in stocks than younger ones since the present value of future labor income generally declines over the lifetime.

When exposure to non-financial income cannot be completely hedged, either because markets are incomplete or because agents face liquidity constraints, equation (2) does not generally hold. In such a case, exposure to human capital is also a source of unavoidable background risk, which at best can only be diversified (Kimball, 1993). For example, Gollier and Pratt (1996) show that exposure to independent uninsurable background risk induces CRRA investors to become more risk-averse with respect to risky assets. The optimal exposure to the risky asset portfolio is thus decreasing in the riskiness of human capital. This finding is confirmed by Elmendorf and Kimball (2000), Koo (1999), and Heaton and Lucas (2000), who estimate optimal saving and risky asset allocation for two-period, finitely, and infinitely lived investors, respectively.

### 2.2. Life cycle portfolio optimization

Contemporary life cycle portfolio optimization literature operates within a setup with finitely lived investors who are subject to uninsurable income risk. The allocation problem is defined in terms of maximizing the expected utility of consumption over the remaining lifetime given the specification of the income process, shape of the utility function, life span expectations, and investment opportunities. Such models do not normally have a closed form solution and are solved numerically by dynamic programming techniques. Because of computational intensity, two-asset models are common, even though the weights attributed to individual risky financial assets should not necessarily be proportional to their market capitalizations when non-financial income is not tradable.

Cocco, Gomes and Maenhout (2005) are the first to use an empirically calibrated non-financial income process in the context of life cycle portfolio optimization. By fitting the income process to U.S. data using the Panel Study of Income Dynamics (PSID), the authors are able to discuss the evolution of realistic saving and portfolio allocation rules over the life cycle. The authors find

that empirical correlation of income shocks and stock returns is low and insignificant, implying that, in terms of its impact on the financial portfolio, human capital resembles an implicit holding of the risk-free asset rather than of the stock market portfolio. Cocco, Gomes and Maenhout (2005) estimate that it is optimal for a typical investor to invest solely in the risky asset until approximately the age of 40. Later in life, approximately until the assumed retirement age of 65, the optimal allocation to the risky asset gradually falls to around 50%. It rises again slightly during retirement, but overall, the optimal risky asset share is downward-sloping over the life cycle. The authors find that ignoring the effect of non-financial income on portfolio allocation leads to a loss equivalent to around 2% of annual consumption on a certainty-equivalent basis. The findings are robust to the introduction of a small probability of disastrous labor income shocks, uncertainty in retirement income, a bequest motive, Epstein-Zin utility, and endogenous labor supply.

The prediction of Cocco, Gomes and Maenhout's (2005) that young investors should have large exposures to equities does not match the empirical evidence. First, most households were shown to hold at least part of their wealth in bonds (Poterba and Samwick, 2003). Second, nonparticipation in the stock market is common, particularly among young households (Heaton and Lucas, 2000). Only when Cocco, Gomes and Maenhout's (2005) allow for disastrous labor income draws, the predicted optimal risky asset share for young investors decreases substantially.

Various other approaches to improving life cycle models with respect to this particular issue have been discussed. Gomes and Michaelides (2005) assume heterogeneity in investor preferences and a fixed cost that has to be borne by households when they first invest in the risky asset. In their setup, more risk-averse households start investing in the risky asset sooner, but hold more conservative portfolios. This matches the observed low stock market participation rates among young households. Davis, Kubler, and Willen (2006) use a wedge between borrowing and lending rates to reduce demand for equity. Polkovnichenko (2007) uses habit formation preferences to explain why young investors, who have not yet accumulated enough wealth to confidently sustain

consumption above habit, invest more conservatively than middle-aged households. Benzoni, Collin-Dufresne and Goldstein (2007) explain late entry to the stock market by assuming cointegrated labor income and stock return processes, which results in human capital losing its 'stock-like' character as investors age.

### 2.3. Non-financial income process

It has become increasingly common in the literature on life cycle portfolio allocation and savings to calibrate a non-financial income process to longitudinal data. Common specifications of non-financial income dynamics have a two-component structure: income consists of a deterministic and a stochastic component (MaCurdy, 1982).

#### 2.3.1. Deterministic component

The deterministic component is often represented by a function of age and individual characteristics such as education and gender. Non-financial income on average peaks during the middle age after a period of high growth in the early years of individuals' working lives. As retirement approaches, real income declines on average. During retirement, the deterministic part of non-financial income is relatively constant and generally lower than income before retirement. A smooth representation of the age-dependence of non-financial income is often obtained by fitting a polynomial to the loadings on age dummies from earnings regressions (for example, Hubbard, Skinner, and Zeldes, 1994, 1995, Storesletten et al., 2004, Cocco, Gomes, and Maenhout, 2005, Gomes and Michaelides, 2005).

Life cycle patterns of labor income are shown to differ depending on demographic parameters. Hence, many authors estimate processes separately for education, occupation, and/or gender groups (Hubbard, Skinner, and Zeldes, 1994, 1995; Gourinchas and Parker, 2002; Cocco, Gomes, and Maenhout, 2005; Gomes and Michaelides, 2005; Polkovnichenko, 2007; Carroll, 1997; Davis and Willen, 2000). Other characteristics such as household size or marital status are controlled for (Carroll and Samwick, 1997, Cocco, Gomes, and Maenhout, 2005).

### 2.3.2. Stochastic component

With respect to the stochastic component, MaCurdy (1982) shows that common specifications found in the literature (until the publication of his work) are all special cases of an ARMA process. The stochastic component is often decomposed into a persistent, AR(1), and a transitory, MA(0), component. In some contributions, the distinction between transitory and persistent shocks is not made (e.g. Davis and Willen, 2000) and sometimes only permanent shocks are considered (Koo, 1998, Viceira, 2001).

While some authors allow for a higher order MA process for the transitory shocks (Carroll and Samwick, 1997), they are most often assumed to be i.i.d. normal.

Authors report varying estimates of the standard deviation of transitory shocks and find them to differ across education and occupation groups. The magnitude of estimated transitory shocks is economically high, and seems to generally decrease in the level of education. Cocco, Gomes and Maenhout (2005) estimate annualized standard deviations of 0.24 (i), 0.27 (ii) and 0.32 (iii) measures as a fraction of earnings for household heads with a college degree (i), with a high school degree but without a college degree (ii), and without a high school degree (iii), respectively. Gourinchas and Parker (2002) report standard deviations in the range of 0.18 to 0.26, and Heaton and Lucas (1997) estimate 0.24. Hubbard, Skinner, and Zeldes (1994) report estimates ranging from 0.12 to 0.20. Carroll (1992) estimates a standard deviation of 0.15 without distinguishing between education groups but uses only 0.10 in his model for reasons related to potential measurement error.

The persistent stochastic component is commonly assumed to follow an AR(1) and to have normally distributed innovations. Many authors (e.g. Carroll (1992, 1997), Carroll and Samwick (1997), Gourinchas and Parker (2002), Cocco, Gomes, and Maenhout, 2005, Gomes and Michaelides, 2005, Polkovnichenko, 2007) restrict the coefficient of autocorrelation to 1, i.e. impose a random walk process. Authors usually justify the choice of the random walk over an AR(1) specification by referring to Hubbard, Skinner, and Zeldes' (1994) result that the first-order autocorrelation coefficient is close to one (0.95 – 0.96).

Apart from that, the random walk assumption simplifies the solution of the optimal consumption and portfolio choice problem for which the income process is used. Campbell et al. (2001) argue that the random walk assumption should have no material effect on the estimation of the optimal consumption and portfolio allocation policies.

Estimates of the standard deviation of permanent income shocks are generally smaller than those of transitory shocks. Cocco, Gomes, and Maenhout (2005) report standard deviations between 0.10 and 0.13 as a fraction of earnings, while Carroll's (1992) and Gourinchas and Parker's (2002) estimates over all education and occupation groups are around 0.15. Hubbard, Skinner, and Zeldes' (1994) estimates range from 0.13 to 0.18. In order to account for potential measurement error, Carroll (1992) again reduces the estimate from 0.15 to 0.10 when using it as parameter input to his buffer stock saving model. Unlike for transitory shocks, estimates of the variability of permanent shocks do not seem to be related to education.

These estimates are not always directly comparable because they are found within different models and pertain to different sample periods; however, all of them are at least partly based on PSID data. In two cases, other U.S. datasets are used alongside the PSID.

### 2.3.3. Correlation with financial asset returns

In addition to the variability of shocks, the correlation between labor income shocks and innovations to risky asset returns is a factor in portfolio allocation decisions. Results reported by authors who realistically calibrate portfolio choice models generally suggest that the contemporaneous correlation between income shocks and aggregate equity returns is close to zero (Cocco, Gomes, and Maenhout, 2005, Davis and Willen, 2000). Campbell et al. (2001) confirm this finding, but yet find a correlation of 0.15 between permanent labor income shocks and one-year lagged stock returns. Davis and Willen (2000) show that the Fama-French SMB factor is significantly correlated with occupation-level income shocks for a number of occupations in a setting where no distinction is made between transitory and permanent shocks.

## 3. Model

We use the partial equilibrium model of Cocco, Gomes, and Maenhout (2005) as a base for our theoretical framework. While many characteristics of the model are borrowed from Cocco, Gomes, and Maenhout (2005), we adjust their model most notably by allowing unemployment and marital status to influence the properties of non-financial income process.

### 3.1. Budget constraint

We specify the life cycle consumption and saving model in discrete time, where agents' lives span for up to $A$ periods. We set period length equal to one year, and throughout the rest of this paper refer to age $a$ as the $a + 1$-th year of life, i.e. the number of full years that an individual has lived.

Agents start independently financing their consumption at age $a_1$ when they do not have any financial wealth. Up to age $K - 1$, agents earn risky non-financial income, and, starting from age $K$, they receive risk-free retirement income ($0 < a_1 \leq K \leq A$). At every point in time there is an infinite number of agents at all admissible ages.

At each age $a$, an agent receives real non-financial income $y_a$ (for clarity we suppress agent-specific subscripts throughout this section). Non-financial income is assumed to be exogenous and non-tradable. We denote the sum of financial wealth and non-financial income realized in the beginning of period $a$ by $x_a$ and, following Deaton (1991), we call it cash on hand. In the beginning of period $a$, the agent chooses the amount $c_a$ to spend on consumption in that period. After the amount allocated to consumption is 'set aside', the agent invests the remaining cash on hand, $x_a - c_a$, into the financial portfolio. The investment yields a simple gross after-tax rate of return $R_{a+1}$ in the beginning of period $a + 1$. The intertemporal budget constraint then is

$$x_{a+1} = (x_a - c_a)R_{a+1} + y_{a+1}. \quad (3)$$

### 3.2. Investment constraints and financial asset universe

The agents allocate a proportion $\varphi_a$ of financial wealth to the risky asset and the rest to the risk-free asset. The risk-free asset yields a time-invariant periodic after-tax net log rate of return $r^f$ and the risky asset yields an after-tax net log rate of return $r^f + \mu + \eta_{a+1}$ in the beginning of period $a+1$, where $\eta_{a+1}$ is marginally distributed as $N(0, \sigma_\eta^2)$. Hence the return on the financial portfolio is

$$R_{a+1} = e^{r^f}\left(1 - \varphi_a(1 - e^{\mu+\eta_{a+1}})\right). \quad (4)$$

Agents face liquidity, borrowing, and short sale constraints. Thus financial wealth and positions in the investment assets are non-negative at all ages:

$$x_a - c_a \geq 0 \text{ and } 0 \leq \varphi_a \leq 1, \quad (5)$$

### 3.3. Non-financial income

The income process that we assign to our agents is in many ways similar to the ones used by, for example, Cocco, Gomes, and Maenhout (2005), Polkovnichenko (2007), Hubbard, Skinner, and Zeldes (1994, 1995), and Gomes, Kotlikoff, and Viceira (2008). However, our income process has two noticeable distinctions from the processes that have been widely used in the literature so far.

First, we notice that occurrence of unemployment must cause sizeable but rare negative adjustments to income realizations. Such shocks cannot be well modeled by AR(0) or AR(1) Gaussian noise structures that have been widely used in the life cycle consumption and allocation literature. Hence, we explicitly model rare but large discrete negative adjustments to the level of income with Bernoulli noise. Incidence of unemployment has been shown to be higher at young (Guiso, Japelli and Pistaferri, 2002) and also old age (Berthold and Fehn, 2003). This is why we recognize the age structure of unemployment incidence and, through it, are able to better model the age structure of non-financial income's risk.

Second, in a way that is similar to Cubbedu and Ríos-Rull (2003) and Love (2010), we allow our agents to be either single or married and allow them to change their marital status in either direction as they age. We implicitly assume that married spouses share their non-financial income in accordance to

a constant pre-specified proportion. The sharing proportion is defined in such a way that marital status choice does not affect the expected level of individual log income. Meanwhile, we allow married agents to have a different level of volatility in their income innovations compared to that of married agents.

Formally, we define the pre-retirement non-financial income received at age $a$, as

$$y_a = \exp(f_a + g_a + v_a + \varepsilon_a), \qquad a < K. \tag{6}$$

In equation (6), $f_a$ is a deterministic function of an agent's age and other demographic characteristics; it is equal to the unconditional mean of the log of non-financial income at age $a$ if the agent is employed. We limit the space of demographic characteristics that are arguments of $f_a$ to gender and education. The impact of both gender and education on life cycle income is empirically well established in the literature (Dustmann and van Soest, 1998; Rosenfeld, Trappe, and Gornick, 2004; Beblo and Robledo, 2008).

In equation (6), $g_a$ is a random unemployment shock. Its magnitude depends on the marital status while the likelihood of its realization being distinct from zero depends on an agent's age:

$$g_a = g(m_a)j_a$$

Here, $j_a$ and $m_a$ are Bernoulli variables that are equal to 1 if the agent is respectively jobless and married at age $a$. Since $m_a$ is a Bernoulli variable, the magnitude of unemployment adjustment, $g(m_a)$, can take only two values, corresponding to an agent's marital status.

We allow the probability distribution of $j_a$ to depend on the current age,$a$, and the probability distribution of $m_a$ to depend on both $a$ and $m_{a-1}$, marital status of the agent in the previous period. Formally,

$$\mathrm{P}(j_a = 1) = p^j(a) \text{ and}$$
$$\mathrm{P}(m_a = 1) = p^m(a,\ m_{a-1}).$$

The stochastic persistent component, $v_a$, follows the process

$$v_a = v_{a-1} + w_a,$$

where $w_a$ is marginally distributed as $N\left(0, \sigma_w^2(m_a)\right)$. That is, we allow the variance of persistent shocks to depend on the marital status of the agent. At the initial adult age, the value of the persistent component is random: $v_{a_1} \sim N(0, \sigma_v^2)$.

The innovation $w_a$ to the stochastic persistent component is correlated with the abnormal risky asset return, $\eta_a$; the coefficient of correlation is $\rho$. The transitory shock to income, $\varepsilon_a$, is i.i.d. $N\left(0, \sigma_\varepsilon^2(m_a)\right)$, i.e. its variance is also dependent on the marital status of the agent.

By specifying the model in this way, we allow marriage to reduce riskiness of income flows through three channels: by reducing the magnitude of the unemployment shocks and by reducing the variance of both transitory and persistent shocks. In line with that we expect that

$$g(m_a)|_{m_a=0} < g(m_a)|_{m_a=1},$$
$$\sigma_w(m_a)|_{m_a=0} > \sigma_w(m_a)|_{m_a=1}, \text{ and}$$
$$\sigma_\varepsilon(m_a)|_{m_a=0} > \sigma_\varepsilon(m_a)|_{m_a=1.}$$

After retirement, non-financial income is constant and equal to a fraction $\exp(\lambda)$ of income before unemployment and transitory shock in the last pre-retirement period:

$$y_a = \exp(f_K + v_K + \lambda), \qquad a \geq K. \tag{7}$$

While ignoring a certain degree of uncertainty about real retirement income, the approach does account for the main properties of retirement insurance in Germany's social security system at least for the sample period used.

In the model we do not let households to have other income earners apart from heads or spouses. Given that instances of additional income earners typically refer to kids in their twenties who live with their parents, we hypothesize that income earned by non-head-or-spouse members is not shared with other household members and is used to finance one's own consumption.

Also, we treat households with kids or other relative or non-relative members in the same way as households without additional members. In this regard we notice a significant population of not working spouses. Assuming that income of these people is closely linked to income of the working partner, our choice of CCRA utility function secures that portfolio allocation results for such people are numeraire independent and identical to the results obtained for their working partners.

### 3.4. Optimization problem

In period $a$, $a_1 \le a \le A$, an agent's inter-temporal utility function is given by

$$U_a = \sum_{j=a}^{A} \delta^{j-a} \left( \prod_{k=a+1}^{j} p_k \right) E_a u(c_j), \tag{8}$$

where $\delta$ is a subjective risk-neutral discount factor, $E_a$ is the expectation operator conditional on the information available in period $a$, and $p_k$ is the probability of being alive at age $k$ conditional on being alive at $k-1$; we use a convention that $\prod_{k=i}^{j} p_k = 1$, $i > j$. Controlling for uncertainty in life expectancy in such a way, we implicitly assume that the force of mortality, $p_k$, is independent of financial or non-financial income realizations. We choose power utility for the indirect utility function, where $\gamma > 0$ is the coefficient of relative risk aversion:

$$u(c_a) = \begin{cases} \dfrac{c_a^{1-\gamma}}{1-\gamma}, & c_a > 0 \\ -\infty, & c_a \le 0 \end{cases}. \tag{9}$$

The problem of optimal saving and allocation faced by an agent at age $a$ is formalized as maximizing $U_a$ in (8) subject to constraints (3)-(7) and (9). The set of policy variables is $c_a, \varphi_a$, and the state variables are $a, x_a, v_a, m_a$ $a$ for $a = a_1, \dots, A$. The problem can be stated in the form of a stochastic dynamic programming equation:

$$\mathrm{V}(a, x_a, v_a, m_a) = \max_{c_a, \varphi_a} \left\{ u(c_a) + \delta p_{a+1} E_a \left( \mathrm{V}(a+1, x_{a+1}, v_{a+1}, m_{a+1}) \right) \right\}, \tag{10}$$

where $V(a, x_a, v_a, m_a) = \max_{\{c_j, \varphi_j\}_{j=a}^{A}} U_a$. Given $V(a+1, x_{a+1}, v_{a+1}, m_{a+1})$ and the contemporaneous state variables $\{a, x_a, v_a, m_a\}$, solving (10) amounts to optimization with respect to only the contemporaneous policy variables $\{c_a, \varphi_a\}$ for each $a$, $a_1 \le a$, at a time.

Here we give a brief description of the numeric solution method that we use to solve (10). First, using the fact that $V(a, x_a, v_a, m_a)$ is homogeneous with respect to $x_a$, we normalize $x_a$ and $c_a$ by $e^{v_a}$:

$$\dot{x}_a = x_a e^{-v_a} \text{ and}$$

$$\dot{c}_a = c_a e^{-v_a}$$

By doing so, we withdraw the $v_a$ dimension from the state space and reduce the complexity of the problem. The Bellman equation can then be expressed in

terms of the modified value function $\dot{V}(a, \dot{x}_a, m_a)$ that is equal to $V(a, x_a, v_a, m_a)e^{-v_a(1-\gamma)}$ (please refer to Appendix 1 for the proof):

$$\dot{V}(a, \dot{x}_a, m_a) = \max_{\dot{c}_t, \varphi_a}\{u(\dot{c}_a) + \delta p_{a+1} E_a\big(\dot{V}(a+1, \dot{x}_{a+1}, m_{a+1})e^{w_{a+1}(1-\gamma)}\big)\} \quad (11)$$

Since consumption only up to period $A$ matters, $\dot{V}(a, x_\alpha, m_a)\big|_{\alpha>A} = 0$ and $\dot{V}(A, x_A, m_A) = u(\dot{x}_A)$, i.e. agents consume all their remaining wealth in the last period. Knowing this, we find $\dot{V}\big(A\text{-}1, x_{A-1}, m_{A-1}\big)$ by numerically solving (10), and continue the induction backwards until we find $\dot{V}\big(a_1, x_{a_1}, m_{a_1}\big)$. At each step of induction, we record the estimated optimal consumption and allocation policies as functions of the corresponding age, cash on hand, and marital status: $\dot{c}^*(a, \dot{x}_a, m_a)$ and $\varphi^*(a, \dot{x}_a, m_a)$, respectively.

Numerical solution of (10) amounts to the execution of the following steps. Starting with $a = A - 1$ we approximate the next period's value function, $\dot{V}(a+1, \dot{x}_{a+1}, m_{a+1})$, by discretizing it over the grid of $\dot{x}_{a+1}$ with grid gaps following exponential series. Using an exponentially spaced grid allows us to cover a larger range with fewer grid points without compromising the relative precision of the solution. We use third-order spline interpolation in ${\dot{x}_{a+1}}^{1-\gamma}$ in between the grid points for a closer approximation. We also approximate the multivariate normal distribution of $\eta_{a+1}$, $w_{a+1}$, and $\varepsilon_{a+1}$ by a discrete distribution with equally probable realizations, as proposed by Adda and Cooper (2003), to estimate the stochastic integrals necessary for finding the expected value in (11). Finally, we optimize numerically using grid search over the policy variables $\dot{c}_a$ and $\varphi_a$. We continue by incrementing $a$ by $-1$ and repeating the steps above until $a$ reaches $a_1$.

## 4. Data

In this section we give a review of the data that we use for calibrating the non-financial income process discussed in the previous section; we identify the estimation sample and define variables that we use further in the empirical analysis.

### 4.1. Data source

For the calibration of the income process, we use data on individual and household income and other variables from the German Socio-Economic Panel Study (SOEP). The SOEP is a research-driven, representative household panel study that contains, among other things, detailed information on individual and household income over extended periods of time. To date, the SOEP encompasses 27 years of annual survey responses (1984 – 2010), making it the "longest-running longitudinal survey of private households and persons in the Federal Republic of Germany" (Frick, 2010). Interested readers find a discussion of the purpose, structure and development of the panel study in Wagner et al. (2007). To obtain our sample, we use a SOEP data release for calendar years 1983 to 2008 (Socio-Economic Panel (SOEP), Data for years 1984-2009, Version 26, SOEP, 2010).

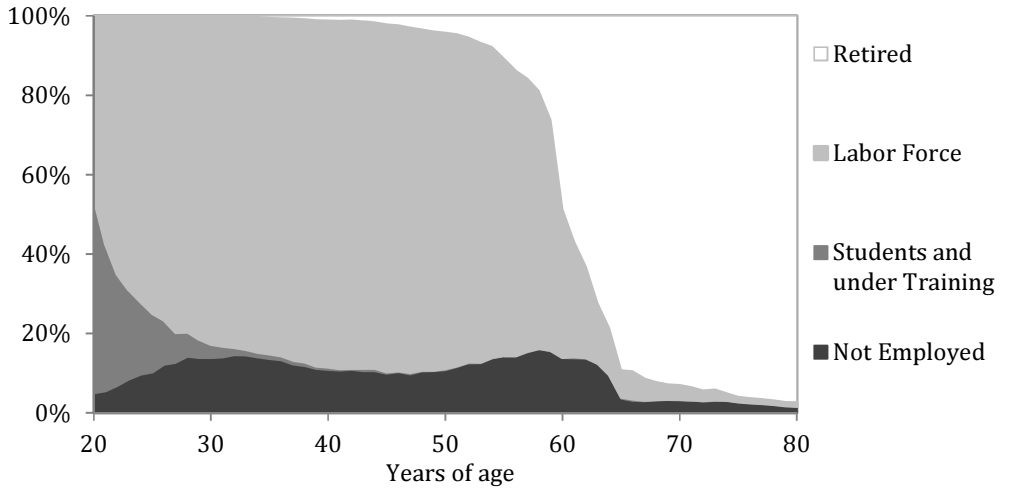
### 4.2. Sample identification and restriction

The raw SOEP sample is non-representative due to selection by design and heterogeneous panel attrition; therefore we weight observations by the inverse sampling probability. When applicable – for example, when using differenced observations – we use 'longitudinal' weights that take into account unequal attrition probabilities.

In Figure 1 we show the distribution of individuals in the raw SOEP sample by their labor force status and age. The agents in our theoretical model can be employed, unemployed or retired. Therefore we consider for inclusion in our gross sample individuals who are either in the labor force (which we define as being either employed or registered unemployed) or are pensioners, and whose age is between $a_1 = 20$ and $A = 100$. We disregard observations of students (also individuals in military or community service, apprenticeships or internships) because of the structurally different and more complicated optimization problem that these people are faced with: they are forgoing employment income during studies and are compensated by higher expected income in the future. With a purpose of calibrating the income process for individuals in the labor force, we also exclude observations of voluntarily

unemployed (defined as neither employed nor registered unemployed) individuals. This exclusion does not compromise our results with regard to life cycle risky asset allocation.

**Figure 1 Labor force status by age**




The figure shows stacked proportions of people within each of the labor force status groups by age. Individuals are assigned to groups in the following order of hierarchy: Retired, defined as individuals receiving any kind of retirement pension; Students and under Training, defined as individuals in education, military or community service, apprenticeship or internship; Labor Force, defined as individuals who are either employed or registered unemployed; Not Employed, defined as individuals without employment and not belonging to any other group.

Our theoretical model is limited to single and married individuals; for this reason we only consider observations pertaining to individuals who are a head or spouse in their household. We further require that age and gender information is reported for each observation to be included in the gross sample. Having defined the gross sample in this way, we apply a number of additional restrictions before we are left with the net sample that we use for the calibration of the income process. Below, we describe the reasons for applying these restrictions and the corresponding effects on the composition of the net sample.

First, up to 1999, we exclude observations pertaining to individuals living in those federal states that formerly belonged to East Germany. During the 1990s, the level and growth rate of real income in eastern federal states were significantly different from those in the rest of the country (Grabka, 2000). We argue that the structural factors underlying both the large initial income gap at the time of unification in 1991 and the subsequent income growth differential do

not reflect the factors driving income processes today and in the future. While real income continues to be lower in the eastern part of Germany (Table 1), the last year for which we exclude observations from former East Germany is 1999. This is also the last year for which the German Federal Statistics Office reported separate CPI indices for former East and West Germany.

**Table 1 Descriptive statistics**

| | Gross sample | Excluded subsample | | | Net sample |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | |
| Number of observations | 307,413 | 36,206 | 39,812 | 21,678 | 191,646 |
| Median age | 50 | 49 | 47 | 45 | 50 |
| Median household nonfinancial income per household member | 11,926 | 10,028 | 12,559 | 14,880 | 12,036 |
| Mean household size | 2.37 | 2.47 | 2.54 | 2.80 | 2.22 |
| | (0.00) | (0.01) | (0.01) | (0.01) | (0.00) |
| Proportion females | 0.49 | 0.52 | 0.53 | 0.33 | 0.48 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Proportion with low education | 0.16 | 0.08 | 0.13 | 0.06 | 0.16 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Proportion with middle education | 0.67 | 0.67 | 0.67 | 0.64 | 0.68 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Proportion with high education | 0.17 | 0.25 | 0.19 | 0.29 | 0.15 |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |

The table shows descriptive statistics for the gross sample, excluded subsamples and the net sample. Subsample 1 contains observations of individuals living in those federal states that formerly belonged to East Germany up to 1999; subsample 2 contains observations with more than 25% imputed post-government household income; subsample 3 contains observations pertaining to self-employed individuals. Where applicable, standard errors of the estimates are shown in parentheses.

Second, we find that our measures of income risk are sensitive to the inclusion of observations with imputed income information. We exclude observations for which more than 25% of household-level post-government income is imputed. Summary statistics for this group are reported in Table 1.

Third, as it is sometimes done in the literature (Carroll, 1992; Davis and Willen, 2000), we exclude observations pertaining to self-employed individuals. Table 1 shows that individuals in this group are on average younger, likelier to

be male, have better education and higher income per household member compared to not self-employed individuals.

Fourth, we exclude observations with missing education information. The relatively small subgroup that isdropped as a result of this restriction exhibits relatively low income and young age.

Fifth, we exclude observations of individuals living in households with one or more income earners who are neither a head nor a spouse in that household. This is done to match the specification of our theoretic model, in which we do not allow for more than two income earners in a household. While this group is characterized by a relatively low level of education and increased household size by around one person, the average level of income per household member in this group is commensurate to that of the rest of the sample.

**Table 2 Descriptive Statistics (2)**

| | Gross sample | Excluded subsample | | Net sample |
|---|---|---|---|---|
| | | 4 | 5 | |
| Number of observations | 307,413 | 1,354 | 33,098 | 191,646 |
| Median age | 50 | 39 | 52 | 50 |
| Median household nonfinancial income per household member | 11,926 | 10,556 | 11,635 | 12,036 |
| Mean household size | 2.37 | 2.25 | 3.54 | 2.22 |
| | (0.00) | (0.03) | (0.01) | (0.00) |
| Proportion females | 0.49 | 0.47 | 0.47 | 0.48 |
| | (0.00) | (0.01) | (0.00) | (0.00) |
| Proportion with low education | 0.16 | | 0.24 | 0.16 |
| | (0.00) | | (0.00) | (0.00) |
| Proportion with middle education | 0.67 | | 0.65 | 0.68 |
| | (0.00) | | (0.00) | (0.00) |
| Proportion with high education | 0.17 | | 0.11 | 0.15 |
| | (0.00) | | (0.00) | (0.00) |

The table shows descriptive statistics for the gross sample, excluded subsamples and the net sample. Subsample 4 contains observations with missing education information; subsample 5 contains observations of individuals who live in households with non-head-or-spouse income earners. Where applicable, standard errors of the estimates are shown in parentheses.

We show summary statistics for the gross and net samples as well as the five excluded subsamples that we have discussed so far in Table 1 and Table 2. While

dropping some of the subsamples has a seeming potential to cause a sampling bias in our estimates, we find that differences in demographic characteristics of the gross and net samples, although significant statistically, are small economically.

**Table 3 Descriptive statistics (3)**

| | Excluded subsample | Receivers of pension income |
|---|---|---|
| Number of observations | 11,391 | 98,301 |
| Median age | 68 | 68 |
| Median household nonfinancial income per household member | 11,023 | 11,463 |
| Mean household size | 1.87 | 1.83 |
| | (0.01) | (0.00) |
| Proportion females | 0.57 | 0.58 |
| | (0.01) | (0.00) |
| Proportion with low education | 0.27 | 0.25 |
| | (0.00) | (0.00) |
| Proportion with middle education | 0.64 | 0.64 |
| | (0.01) | (0.00) |
| Proportion with high education | 0.09 | 0.11 |
| | (0.00) | (0.00) |

This table shows descriptive statistics for the excluded subsample of individuals who received certain types of pension income in years 1983, 1984, 2001, and 2002. For comparison we show descriptive statistics for all the receivers of the same types of pension income. Where applicable, standard errors of the estimates are shown in parentheses.

Finally, we drop a sizeable group of observations pertaining to recipients of certain types of pension income in calendar years 1983, 1984, 2001 and 2002. This is done because for these observations the level of aggregation in retirement income items prevents a reliable separation of financial and non-financial income and/or a reliable allocation of taxes to these items. In Table 3, we present descriptive statistics for the dropped observations and, for comparison, for all the receivers of the same types of pension income as in the dropped observations. We find that the excluded subsample of pensioners is homogenous with other receivers of pension income.

After excluding certain subsamples, as it discussed in this section, we are left with the net sample of around 191 thousand data points corresponding to

approximately 32 thousand individuals. Around three fifths of the net sample pertain to individuals who live in households with two income earners; one third of the sample are pensioners.

## 4.3. Variable definitions

### 4.3.1. Panel dimensions

We identify observations in our sample by a combination of the unique personal index $i$ and age $a$, defined as the maximum age achieved by individual $i$ during the calendar year to which an observation pertains.

### 4.3.2. Income

We are interested in obtaining an estimate of real post-tax individual non-financial income. Within our data set we are however only able to observe post-tax income at the household level. Therefore, we define $y_{i,a}^{h}$ – real disposable non-financial income of the household to which individual $i$ belongs at age $a$ – as the after-tax sum of all forms of labor income, including infrequent payments such as bonuses and severance packages, public transfers[1], private transfers and all social security pension income discounted using the CPI for 2005. Additionally, we include private pension income related to former employment, because – unlike other private pension income – it is not the outcome of an explicit savings and portfolio allocation decision but rather a compensation for prior service provision and hence a form of non-financial income. In Appendix 2, we give more detail on the income components available in the SOEP data set and show how we classify them. The level of detail at which income information is provided generally allows us to make the necessary distinctions and estimate household non-financial income before taxes.

Income and social security taxes, however, are estimated using a procedure outlined in Schwarze (1995) and are available only at the aggregate household level for both financial and non-financial income. We deal with this issue in the

[1] Unlike other public transfers, support for owner-occupied housing is not included in $y_{i,a}^{h}$. Given the subsidy's design, it constitutes a discount on an asset purchase, and hence we classify it as financial income.

following way: for each household, we 1) estimate its taxable income, 2) using SOEP-estimated income taxes, find each household's effective tax rate and 3) apply the estimated tax rate to the pre-tax non-financial income. Thus, we obtain an estimate of after-tax non-financial income at the household level.

Detailed information on the tax estimation procedure applied by SOEP, i.e. information that goes beyond that given in Schwarze (1995), is not available to us. Therefore, we use the relevant features of the German income tax code for the calendar years 1983 to 2008 in order to reverse-estimate taxable income of the households in our sample. In Appendix 3, we give more detail on this procedure.

The approach to cleaning the SOEP-provided income taxes adopted in this paper potentially introduces a bias. The simplifying assumptions applied by the SOEP for the tax estimation possibly result in biased tax estimates, too. The main concern is the overestimation of taxes for households that itemize expenses, particularly with respect to labor income.

### 4.3.3. Demographic variables

We differentiate individuals in the sample by their gender and level of education. We distinguish three education groups – 'low', 'middle', and 'high' – according to the highest level of education that individuals have achieved during their lifetimes. Low level of education refers to inadequate completion of basic schooling and to degrees from tier-two and tier-three high schools[2]. Middle-level education encompasses all forms of vocational degrees as well as tier-one high school degrees, and high-level of education is assigned to individuals who have completed any degree of higher education. These education groups correspond to the broad education groups in the CASMIN classification ("Comparative Analysis of Social Mobility in Industrial Nations"), with two exceptions. First, we assign individuals with basic vocational degrees and a degree from a tier-three high school to the middle group. Second, we attribute individuals with tier-two

[2] The German high school system distinguishes three types of high schools according to the educational achievement they are geared towards. For the purpose of our education classification, we refer to these schools as tier-one, tier-two and tier-three high schools (in order of decreasing educational achievement).

high school degrees to the low-education group. In the CASMIN scale, they are instead assigned to the low and middle level, respectively.

We assign the value of married dummy $m_{i,a} = 1$ to individuals if they belong to a household in which both spouses are either in the labor force or are retired, and $m_{i,a} = 0$ otherwise. We are interested in the economic effects of cohabiting on diversification of non-financial income and, hence, only assign $m_{i,a} = 1$ to individuals who are exposed to two income sources, rather than just to one. Unemployed dummy $j_{i,a}$ is assigned the value of one for individuals who were jobless at any time during the year when they turned $a$ years old.

## 5. Calibration

The goal of the empirical calibration is to estimate realistic parameter values for the theoretical model that is described in Section 3. In this section we discuss statistical approaches that we use.

### 5.1. Earnings regressions

The income process in equations (6) and (7) is defined for individual income while in our sample we observe household income $y_{i,a}^{h}$. We address this issue by adding a marriage dummy to the first stage income regression in a similar way as it is done by Cocco, Gomes and Maenhout (2005). Doing so is reasonable if we assume that family composition transitions leave no effect on the expected level of individual income and that income within a household is shared according to a constant ratio. In the result we obtain estimates of income level that are conditional on individuals being single.

With these thoughts in mind, we run the following first-stage individual fixed effects OLS regression for observations of non-retired individuals before the age of $K = 65$, separately for each gender-education group:

$$\ln\left(y_{i,a}^{h}\right) = \phi_a + g^{S} j_{i,a} + g^{M-S} j_{i,a} m_{i,a} + \beta^{M} m_{i,a} + \alpha_i + \epsilon_{i,a}, \qquad (12)$$

where the set of $\hat{\alpha}_i$ is restricted so that its sample mean is zero. Specifying the regression in this way gives us the estimates of age dummy loadings, $\hat{\phi}_a$, that we can directly use further to fit the functional form of the deterministic component

of non-financial income, $f_a$ . We also find $g(m_a|m_a = 0) = \hat{g}^S$ and $g(m_a|m_a = 1) = \hat{g}^S + \hat{g}^{M-S}$.

Table 4 summarizes the results of the first-stage regressions. As we would have expected, we find a large negative effect of unemployment on income of singles. The same effect is on average half as large for married individuals. All our estimates are significant using heteroskedasticity-robust and cluster-robust standard errors.

**Table 4 First-stage earnings regression**

| | males | | | females | | |
|---|---|---|---|---|---|---|
| | low | medium | high | low | medium | high |
| $j_{i,a}$ | -0.21 | -0.24 | -0.22 | -0.25 | -0.22 | -0.35 |
| | (0.03) | (0.02) | (0.06) | (0.05) | (0.03) | (0.06) |
| $j_{i,a}m_{i,a}$ | 0.00 | 0.10 | 0.13 | 0.12 | 0.15 | 0.25 |
| | (0.04) | (0.02) | (0.07) | (0.06) | (0.03) | (0.08) |
| $m_{i,a}$ | 0.24 | 0.21 | 0.17 | 0.55 | 0.60 | 0.58 |
| | (0.02) | (0.01) | (0.01) | (0.04) | (0.02) | (0.04) |
| F-statistic for $\phi_a$ | 26,711 | 132,627 | 60,250 | 3,270 | 21,571 | 7,094 |
| Adjusted R-squared | 0.75 | 0.77 | 0.81 | 0.77 | 0.83 | 0.83 |
| Number of observations | 8,748 | 50,532 | 16,852 | 9,367 | 40,392 | 11,257 |
| Number of persons | 1,507 | 8,155 | 2,963 | 1,872 | 7,469 | 2,279 |

The table shows the results of regressing log non-financial household income on age dummies ($\phi_a$), jobless dummy ($j_{i,a}$), interaction of jobless and married dummies ($j_{i,a}m_{i,a}$), married dummy ($m_{i,a}$), controlling for individual fixed effects. Age dummy loadings are omitted. Where applicable heteroskedasticity-robust and clustered (across individuals) standard errors are reported in parentheses. F-statistics pertain to a test of joint significance of the age dummy loadings. Columns correspond to gender-education groups.

In order to have a smooth $f_a$, we specify it to be polynomial in $a, a < K$. We estimate polynomial coefficients by regressing the estimated age dummy loadings, $\phi_a$, obtained in the first-stage regression on the powers of age separately for each gender-education group. We then estimate $\lambda$ as the difference between $\ln(f_K)$ and mean log non-financial income of pensioners in the corresponding gender-education groups. We report the results of these calculations in Table 5.

**Table 5 Polynomial fit for the deterministic component of income**

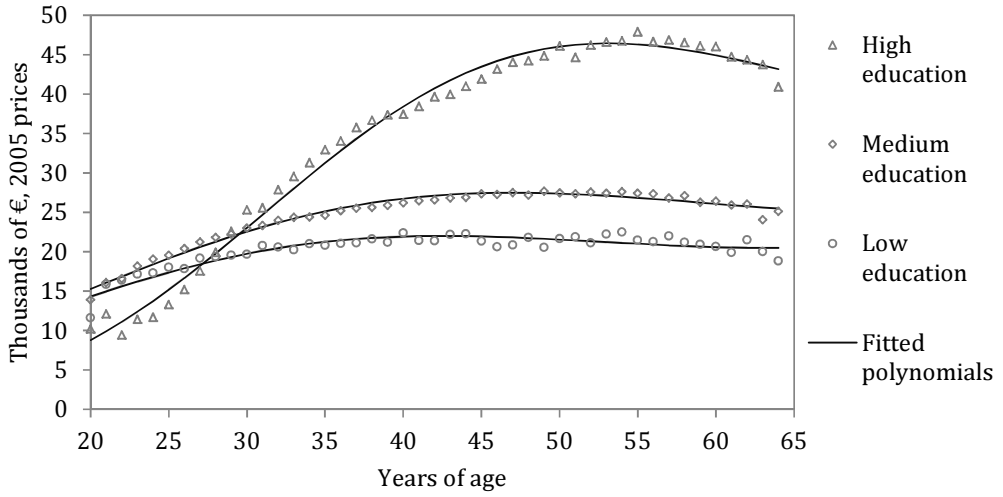
| | males | | | females | | |
|---|---|---|---|---|---|---|
| | low | medium | high | low | medium | high |
| intercept | 7.90 | 7.89 | 5.30 | 6.89 | 8.28 | 5.22 |
| $a$ | 0.13 | 0.13 | 0.26 | 0.17 | 0.08 | 0.28 |
| $a^2/10^2$ | -0.25 | -0.22 | -0.41 | -0.34 | -0.10 | -0.52 |
| $a^3/10^4$ | 0.16 | 0.12 | 0.21 | 0.21 | 0.03 | 0.32 |
| R-squared | 0.85 | 0.98 | 0.98 | 0.98 | 0.98 | 0.99 |
| $\lambda$ | -0.49 | -0.48 | -0.65 | -0.48 | -0.33 | -0.44 |

The table shows the results of the regressions of age dummy loadings obtained in the first-stage income regressions on the powers of age. The log of the replacement ratio, $\lambda$, is found as the difference between $\ln(f_K)$ and mean log non-financial income of pensioners in the corresponding gender-education group.

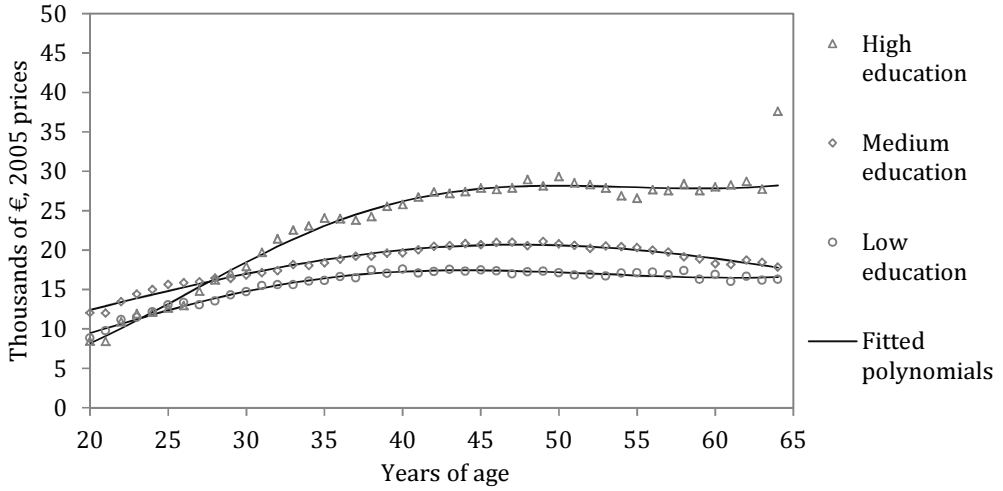
We find that age-income profiles follow a typical hump-shaped pattern for all subsamples (Figure 2). As expected, the profiles differ across gender-education groups, where a higher level of education and being a male is associated with a higher level of income. The shape of the profiles is qualitatively similar for the low-education and middle-education subsamples, while persons in the high-education group, especially males, demonstrate considerably stronger income growth during the early years of their working lives.

**Figure 2 Fitted polynomials through age dummy loadings from the first-stage regressions**

**Males**




**Females**




The graphs plot exponents of the estimated age dummy loadings from the first-stage regressions and fitted third order polynomial functions for each of the education groups separately for males and females.

## 5.2. Variance decomposition

We closely follow the variance decomposition method proposed by Carroll and Samwick (1997) to estimate the variances of the transitory and persistent shocks of the income process. Let $d_{i,a,l}$ denote the $l$-period differenced residual in (12), i.e. $d_{i,a,l} = \epsilon_{i,a} - \epsilon_{i,a-l}$. Notice that

$$d_{i,a,l} = \sum_{\alpha=a-(l-1)}^{a} w_{i,\alpha} + \varepsilon_{i,a} - \varepsilon_{i,a-l}\ ,$$

so that

$$Var\left(d_{i,a,l}\right) = 2\sigma_{\varepsilon}^2 + l\sigma_w^2. \tag{13}$$

When estimates of $Var\left(d_{i,a,l}\right)$ for more than two distinct values of $l$ are available, the system of equations as in (13) is over-specified. In order to avoid uncertainty associated with picking some two values of $l$, we estimate the sample variance estimates $\widehat{Var}\left(d_{i,a,l}\right)$ across the dimensions $i$ and $a$ for $l = 1, 2, \dots, L$. Then we regress the vector of estimated $\widehat{Var}\left(d_{i,a,l}\right)$, $l = 1,2, \dots L$, on the vector $\{1, 2, \dots, L\}$ and a vector of constants $\{2, 2, \dots, 2\}$:

$$\widehat{Var}\left(d_{i,a,l}\right) = \sigma_{\varepsilon}^2 2 + \sigma_w^2 l + \ \psi_l, \tag{14}$$

where $\psi_l$ is the regression residual. Assuming that errors in the estimates of $Var\left(d_{i,a,l}\right)$ are i.i.d., the OLS regression coefficients are efficient estimates of $\sigma_{\varepsilon}^2$ and $\sigma_w^2$. We run variance decomposition regressions separately for single and married individuals and report the main results of these regressions in Table 6.

**Table 6 Variance decomposition regression**

| Single | | | Married | | |
|---|---|---|---|---|---|
| $\sigma_\varepsilon$ | $\sigma_w$ | R-squared | $\sigma_\varepsilon$ | $\sigma_w$ | R-squared |
| 0.20 | 0.10 | 0.94 | 0.13 | 0.08 | 0.97 |
| (14.3) | (10.7) | | (15.0) | (15.3) | |

The table reports the results of the variance decomposition regressions for single and married individuals. We regress sample variance estimates of differenced residuals with a maximum lag $L = 10$ on the vectors $\{1, 2, \dots, L\}$ and $\{2, 2, \dots, 2\}$. We use square root half intercept and square root slope coefficient as the estimators of $\sigma_\varepsilon$ and $\sigma_w$, respectively. We report t-statistics in parenthesis where applicable.

Although we notice some variation in non-financial income shock variance estimates across gender-education groups, we do not find it systematic and substantially large and allow $\sigma_\varepsilon$ and $\sigma_w$ to vary only along the single-married dimension. As we expected, we find that married individuals have less risky non-financial income, whereas most of decrease in income uncertainty comes through a reduction in transitory shock variance. The persistent shock variance of married individuals is only slightly lower than that of the single ones.

Unlike Cocco, Gomes, and Maenhout (2005), Carroll (1992), Gourinchas and Parker (2002), and Hubbard, Skinner, and Zeldes (1994), we control for unemployment risk in the first-stage regressions. That is why income volatility due to unemployment is not reflected by our estimates of shock variances, which are generally lower than the corresponding estimates for the U.S. data reported by these authors. We still find our estimates consistent with previous research to the extent that the volatility of transitory shocks is two to three times larger than that of the persistent ones.

In our variance decomposition regressions we set $L = 10$, that is, we use series of differenced first-stage regression residuals with up to a ten-period lag. We notice however that increasing the maximum lag length $L$ results in larger estimates of transitory shock volatility and smaller estimates of persistent shock volatility. Here we choose $L = 10$ so as not to use variance estimates from either tail of the estimate distribution that can be obtained from our sample. Our findings are generally robust to the choice of $L$. In Section 9.2 we give more details regarding this issue.

Since by construction we have

$$\alpha_i + \epsilon_{i,a_1} = v_{i,a_1} + \varepsilon_{i,a_1},$$

a straightforward way to estimate the standard deviation of the persistent random component of income at the initial age is

$$\hat{\sigma}_v = \sqrt{\widehat{Var}\left(\hat{\alpha}_i + \hat{\epsilon}_{i,a_1}\right) - \hat{\sigma}_\varepsilon^2}.$$

In the above equation we use our estimate of $\sigma_\varepsilon$ for singles, and correspondingly estimate sample variance of $\hat{\alpha}_i + \hat{\epsilon}_{i,a_1}$ that pertain to single individuals, assuming that agents start their independent lives single.

### 5.3. Asset returns and correlation with income shocks

Assuming taxes on capital income are 25%, which is in line with current interest, dividend and capital gains withholding taxes in Germany (ignoring additional surcharges and exemptions for simplicity), we use a real after-tax log risk-free rate of return, $r_f$, of 1.0%. Using Dimson et al.'s (2003) historical estimate of the equity risk premium and adjusting it for taxes, we set $\mu$ to 3.0%. We follow their

argument that the historical volatility of excess returns is unlikely to be an unbiased indicator of future variability given the nature of the historical events included in their sample period, and we adopt their approach of limiting $\sigma_\eta$ to 15% (20% pre-tax).

We allow for a non-zero correlation $\rho$ between abnormal stock returns and persistent shocks to non-financial income. We represent the risky asset from our model by the CDAX, a broad, value-weighted index for the German equity market (Thomson Reuters, 2011), and compute $\eta_t$ as a time series of CDAX excess returns for all years $t$ in which we have observations in the sample. Since we do not observe $w_{i,a}$ directly, we estimate $\rho$ as follows. First, for every year $t$ we calculate a sample mean of $d_{i,a,1} = w_{i,a} + \varepsilon_{i,a} + \varepsilon_{i,a-1}$, which we denote $\overline{w}_t$. By construction, $\rho$ is equal to the correlation coefficient of $\overline{w}_t$ and $\eta_t$. Instead of using the contemporaneous correlation coefficient, we follow Campbell et al. (2001) and use one year lagged return series to avoid simultaneous causality issues. We estimate $\rho$ at 0.17, which is consistent with the view that demand for labor is positively related with past returns on capital. Our estimate of the coefficient of correlation is similar in magnitude to that of Campbell et al. (2001) but is larger than estimates of contemporaneous correlation reported by Cocco, Gomes, and Maenhout (2005).

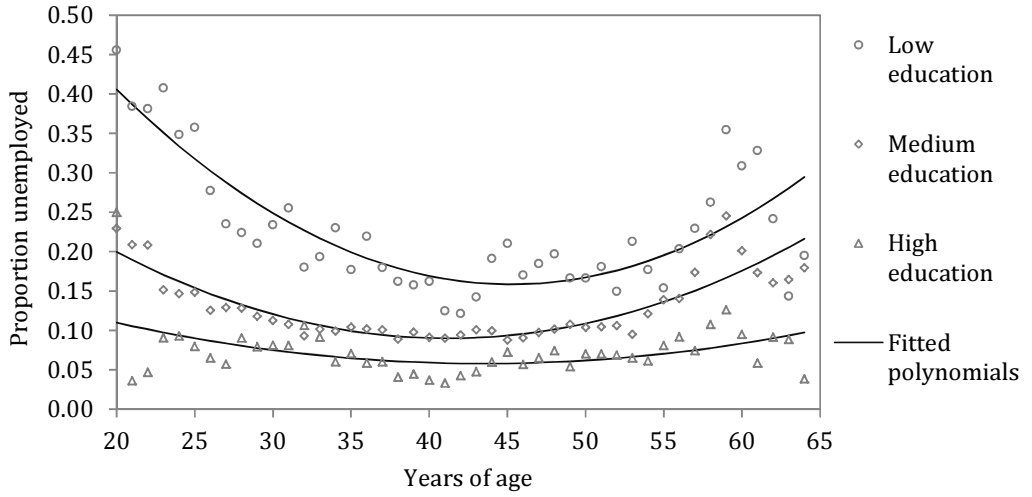
### 5.4. Unemployment and marital status transitions

We model the age structure of unemployment probability by regressing unemployed dummy, $j_{i,a}$ on the set of age dummies interacted with education group dummies. In the regression we control for calendar year fixed effects. Then, by education group, we fit a second order polynomial function through the loadings on age dummies. The results of both regressions are illustrated in Figure 3, where we plot unemployment probabilities conditional on age and education group together with the fitted polynomials.

We can notice that unemployment probability decreases as the level of education improves. We see that the average unemployment rate is decreasing in the first half of the working life and that it is increasing afterwards. Berthold

and Fehn (2003) notice the same feature in the age structure of unemployment in Germany.

**Figure 3 Probability of unemployment**

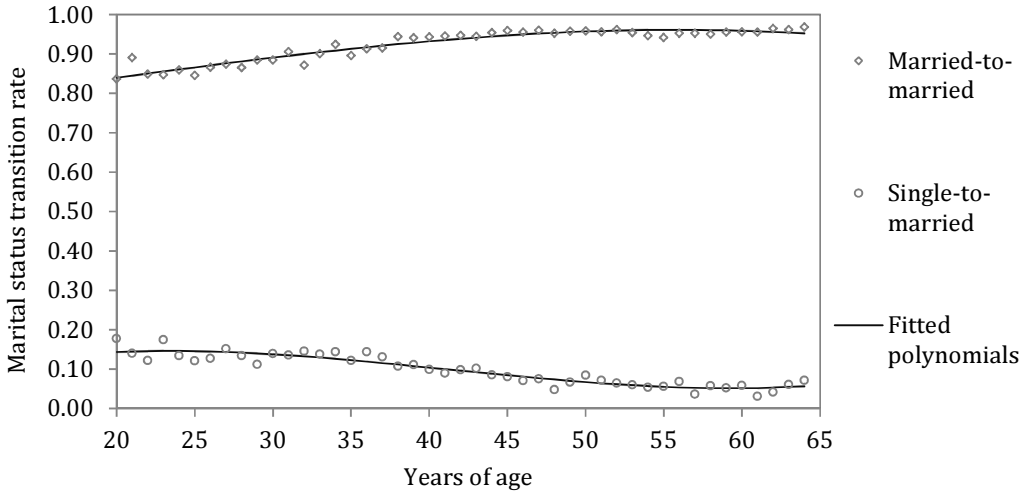



The graph plots estimated unemployment probability conditional on age and education group together with the fitted polynomial functions.

The U-shaped pattern is present in every education group and is most evident for individuals with low education. This could be explained by larger difficulties that these people face when finding their first job and by earlier obsolescence of their job skills.

We proceed with modeling marital status probabilities in a similar way: first, we regress single-to-married and married-to-married transition dummies on age dummies and then fit a third order polynomial to the estimated age dummy loadings to obtain a transition probability as a smooth function of age. The results of these operations are illustrated in Figure 4. We observe that family status is highly persistent. Individuals who are married at age $a-1$ tend to remain such at age $a$, and the same is true for singles. Another observation is that changes in marital status in either direction are likelier to occur at a younger age.

**Figure 4 Marital status transition probabilities**




The graph plots estimated age-conditional marital status transition probability together with the fitted polynomial functions. We show both married-to-married and single-to-married transition probabilities.

Similar to Lauer (2003), who reports very similar age structures of unemployment for males and females in Germany, we do not find evidence of significantly different unemployment-age profiles for males as opposed to females. Neither do we find significant differences in age profiles of family status transition probabilities between males and females. That is why in our theoretical model we restrict these probabilities to be the same across genders. Additionally, we cannot confirm that marital status transition probabilities exhibit apparent differences among the education groups, so we use the same probability sets for individuals with different levels of education.

## 5.5. Other parameters and the benchmark parameter set

We obtain transitional survival probabilities $p_a$, for $a = a_1, \dots A$, from the mortality tables published by the German Statistics Office (Statistisches Bundesamt, 2010). Following Cocco, Gomes, and Maenhout (2005), we set the base-case risk aversion to $\gamma = 10$ and the inter-temporal discount factor to $\delta = 0.96$. In the benchmark case, we solve the model for individuals of both genders and both marital statuses with middle-level education. This education group is by far the largest in our sample and is most representative of the

underlying population. Table 7 summarizes the parameters used in the benchmark case.

**Table 7 Benchmark case parameters**

| Parameter | Notation | Value |
|---|---|---|
| Real risk-free rate | $r^f$ | 0.01 |
| Equity risk premium | $\mu$ | 0.03 |
| Standard deviation of equity returns | $\sigma_\eta$ | 0.15 |
| Standard deviation of transitory shocks to log non-financial income | $\sigma_\varepsilon$ | 0.20 (0.13) |
| Standard deviation of persistent shocks to log non-financial income | $\sigma_w$ | 0.10 (0.08) |
| Standard deviation of persistent component of non-financial income at the initial age | $\sigma_v$ | 0.51 |
| Correlation of persistent shocks to log non-financial income with abnormal equity returns | $\rho$ | 0.17 |
| Initial age | $a_1$ | 20 |
| Retirement age | $K$ | 65 |
| Terminal age | $A$ | 100 |
| Subjective discount factor | $\delta$ | 0.96 |
| Risk aversion coefficient | $\gamma$ | 10 |

The table lists the set of benchmark model parameters. Where applicable, parameters for married individuals are shown in parentheses.

## 6. Policy Functions

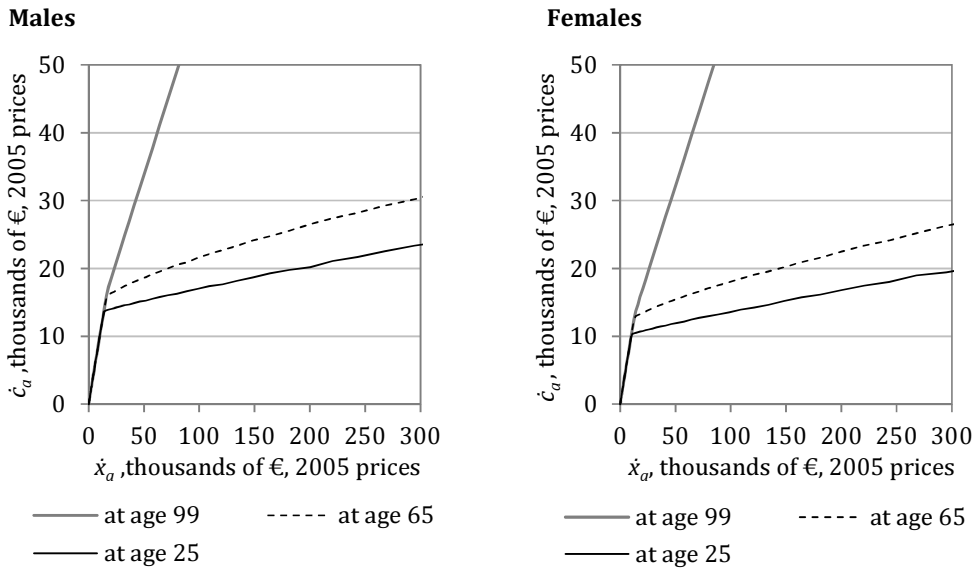
The result of solving the consumption and portfolio allocation optimization problem, which we parameterize with estimates discussed in the previous section, are consumption and allocation policy functions. In this section we discuss the shape of these functions.

### 6.1. Consumption policy

Figure 5 plots projections of optimal consumption policy function $\dot{c^*}(a, \dot{x}_a, m_a)$ for males and females at selected ages. The function is concave in $\dot{x}_a$ and has a general shape similar to that in Deaton's (1991) precautionary savings model. For low values of cash on hand, the liquidity constraint $x_a \geq 0$ is binding. In such cases, the impatience motive dominates the behavior, and it is optimal to

spend all of the cash on hand on consumption, hence the slope of the projected policy function is 1 for low values of normalized cash on hand. When cash on hand is relatively high, the precautionary savings motive is dominant, and it is optimal to save some wealth to smooth consumption in later periods.

**Figure 5 Optimal consumption policy**



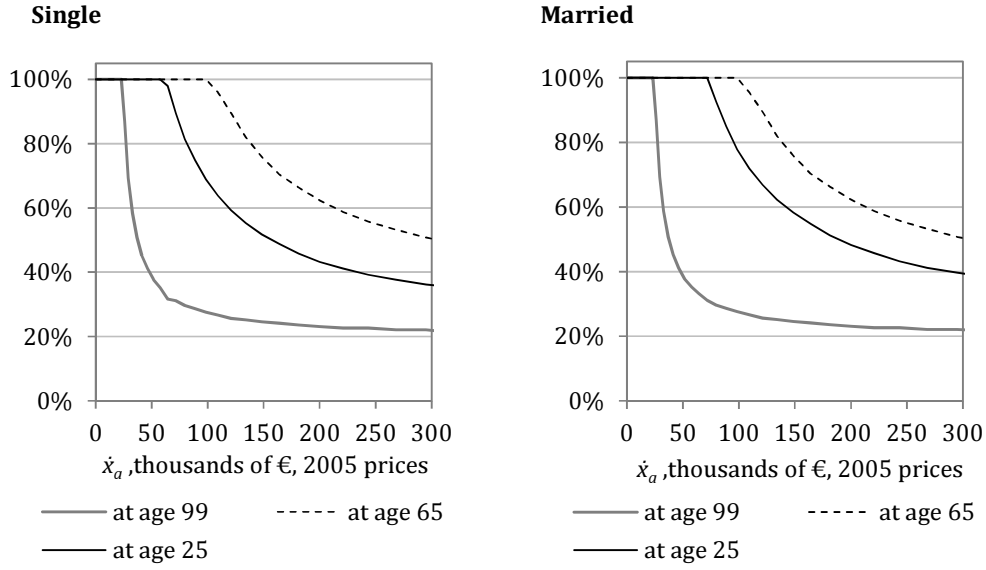

The figure plots projections of $\dot{c^*}(a, \dot{x}_a, m_a)$, the optimal normalized consumption, along the dimension of normalized cash on hand, $\dot{x}_a$, for selected ages for single males and females in the middle-education group.

As agents age and their life horizon shortens, saving becomes less attractive. This is why the optimal consumption function becomes less concave and approaches a straight line with a slope of 1 as $a$ approaches $A$. As we have shown previously, males on average have a higher level of expected future income, i.e. larger human capital, which is why their consumption is higher than that of females for any given level of cash on hand. As age increases and human capital is being depleted, the difference in average consumption levels between the representatives of the two genders converges to zero. Consumption policies of married individuals are very close to those of single individuals, and we do not report them.

## 6.2. Allocation policy

Figure 6 plots projections of the optimal allocation policy functions $\varphi^*(a, \dot{x}_a, m_a)$ for single and married males at selected ages. As we establish empirically, persistent innovations to non-financial income are only moderately correlated with risky asset returns – non-financial income risk is largely idiosyncratic – and hence human capital covaries with the risky asset only to a limited extent. Therefore, the diversification motive induces agents to allocate a larger fraction of their financial wealth to the risky asset when the ratio of human capital to total wealth is large. When the level of cash on hand is low, human capital accounts for a large fraction of total wealth. This is why the optimal allocation to the risky asset is decreasing in cash on hand at every age. These findings are supported by the widespread notion in the literature (for example, Jagannathan and Kocherlakota 1996, Cocco, Gomes, and Maenhout, 2005, Calvet and Sodini, 2010) that human capital affects allocation as if it was comprised of the risk-free rather than the risky asset.

**Figure 6 Optimal allocation policy**




The figure plots projections of $\varphi^*(a, \dot{x}_a, m_a)$, the optimal allocation to the risky asset, along the dimension of normalized cash on hand , $\dot{x}_a$, for single and married males in the middle-education group.

In the absence of borrowing constraints, agents with sufficiently low cash on hand and sufficiently large human capital would find it optimal to borrow and

hold leveraged positions in the risky asset to diversify away the exposure to human capital. In our model, with the borrowing constraint, $\varphi_a \leq 1$, agents with relatively low cash on hand – young and/or poor agents – invest 100% of their financial wealth in stocks. As agents become older, the L-shaped allocation policy projections in Figure 6 'move' up and to the right until the agents reach the age of retirement. After retirement, the projection of the optimal allocation function 'moves' back down and to the left. Thus, the willingness to take financial risks, given the size of the financial portfolio, increases approximately until the retirement age and decreases afterwards.

To understand the intuition behind this result, consider first an agent aged 99 (grey line in Figure 6). The last remaining retirement income stream at $a = 100$ is deterministic; exposure to it is equivalent to holding the risk-free asset. Hence, the optimal allocation function equals Merton's (1971) result as in (2) where $W^F = x_{A-1} - c_{A-1}$ and $W^{HC} = y_A p_A e^{-r^f}$. Since non-financial income is risk-free throughout the whole retirement period, Merton's result holds for every age from $K$, the last pre-retirement year, to $A - 1$. The optimal allocation functions for these ages are therefore hyperbolas with a horizontal asymptote of $\mu/\sigma^2\gamma$: as cash on hand increases, human capital plays a less important role in the allocation of the financial portfolio and the optimal allocation converges towards the result with zero non-financial income exposure. Younger retirees have larger human capital (i.e. more future expected pension income), causing the allocation functions in Figure 6 to compress as retirees age.

Prior to retirement, a significant part of human capital consists of the present value of retirement income, uncertainty in which is solely dependent on $v_K$. The conditional variance of the persistent component $v_a$ is larger for more distant income flows. That is why as age approaches $K$, uncertainty about the amount of retirement income decreases and so does the riskiness of human capital. Consistent with findings of Gollier and Pratt (1996), the reduction in background risk reduces the risk-aversion towards the risky asset. Consequently, agents closer to retirement find it optimal to invest more heavily in the risky asset for any level of cash on hand.

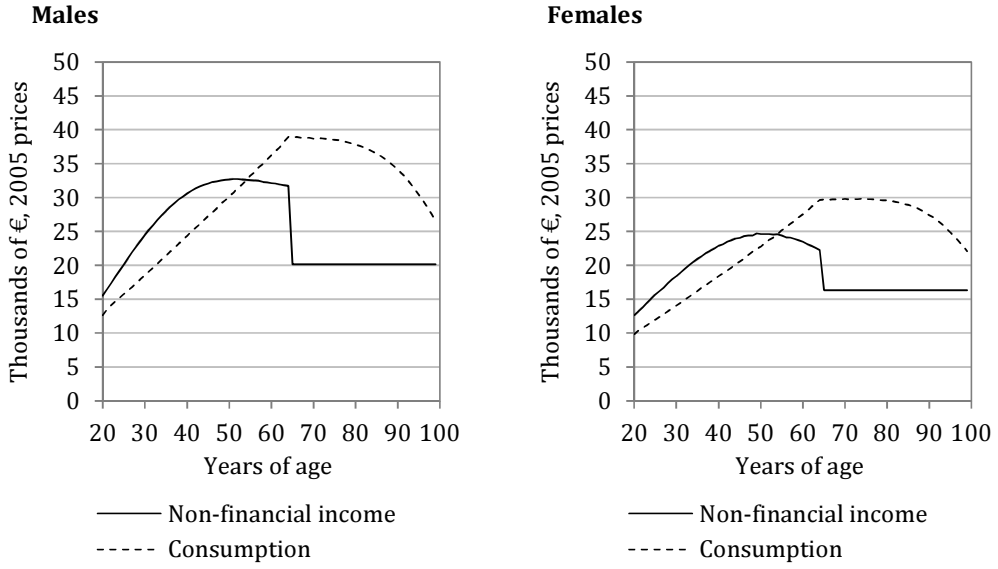
For any given level of cash of hand at every age before the retirement period, human capital is riskier for single individuals than it is for married individuals. Therefore, the optimal risky share is lower for single individuals than it is for the married ones. The difference in allocation policies gradually converges to zero as age approaches $K-1$, when human capital becomes riskless and the optimal allocation rule is universal for both single and married individuals. Hence, in Figure 6, only the solid line (age 25) in the right panel (married) is located higher and to the right from the solid line in the left panel (single). Allocation policies for females are qualitatively similar to those for males and we do not report them.

## 7. Simulation Results

We run Monte-Carlo simulations of our model for $N = 10{,}000$ agents that live from period $a_1$ to $A$. For each agent $i$ and at each age $a$ we draw a vector of random variables $\{\eta_a, w_{i,a}, \varepsilon_{i,a}, j_{i,a}, m_{i,a}\}$. For the initial age, we draw $v_{a_1}$ from the Gaussian distribution with mean zero and variance $\sigma_v^2$. We calculate the simulated trajectories of non-financial income, $\left\{\{y_{i,a}\}_{a=a_1}^{A}\right\}_{i=1}^{N}$, and the returns on the risky asset $\{r_a\}_{a=a_1}^{A}$.

The agents start independent lives with no financial wealth, and in the first period their cash on hand is equal to the first realization of non-financial income. Agents then make their saving and allocation decisions according to the estimated optimal policy functions. Given the allocation choice, we find the rate of return on the financial portfolio (from (4)) and the corresponding realizations of cash on hand (from (3)) in the subsequent period. We iterate the calculation over age and in the end obtain a set of realizations of cash on hand, consumption, and optimal allocation: $\left\{\{x_{i,a}, c_{i,a}, \varphi_{i,a}\}_{a=a_1}^{A}\right\}_{i=1}^{N}$. The average life cycle trajectories of non-financial income and consumption for agents in the middle-education group are plotted in Figure 7.

**Figure 7 Expected trajectories of non-financial income and consumption**




The graphs show average simulated realizations of non-financial income and consumption of males and females in the middle-education group over the life cycle.

We find the pattern of average consumption to be generally similar to the one reported by Cocco, Gomes, and Maenhout (2005). Allthough expected non-financial income in the second half of the working life is high relative to its level early in life, it is on average optimal to sustain a moderate level of consumption in the early years. Average consumption follows the growth in income, and, as uncertainty about future income shrinks, it gradually increases towards the retirement age. Consistent with the classical microeconomic life cycle consumption model, agents in our model accumulate financial wealth in the first half of their lives and use it to finance consumption in the second half.

Our findings regarding the savings behavior over the life cycle differ from those of Cocco, Gomes, and Maenhout (2005) on one account. Cocco, Gomes, and Maenhout (2005) find that early in the life cycle, when income is low and agents have not yet accumulated substantial savings, consumption is very close to 100% of income. On the contrary, we find that it is optimal to make sizeable savings during the early years. The average ratio of savings to non-financial income lies in the range 20%-25% for individuals aged 20 to 40.

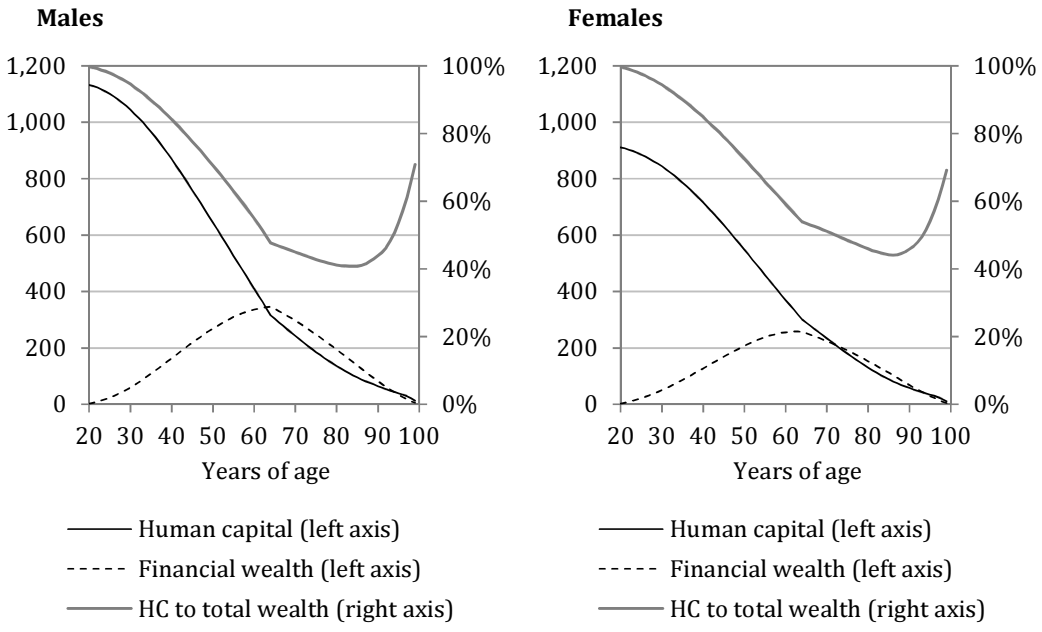
### 7.1. Pattern of human capital and financial wealth

We find the approximate expected value of human capital by discounting the expected future non-financial income streams at the risk-free rate:

$$E_{a_1}(W_a^{HC}) \approx \sum_{j=a+1}^{A} E_{a_1}(y_j)\left(\prod_{k=a+1}^{j} p_k\right) e^{-r^f(j-a)}, \qquad (15)$$

where $E_{a_1}(y_j)$ is estimated as the sample mean of simulated income realizations for age $j$ across agents. Since retirement income is risk-free, (15) gives the precise estimate for $K \le a \le A$ and overestimates it for $a_1 \le a \le K-1$.

**Figure 8 Expected trajectories of human capital and financial wealth**



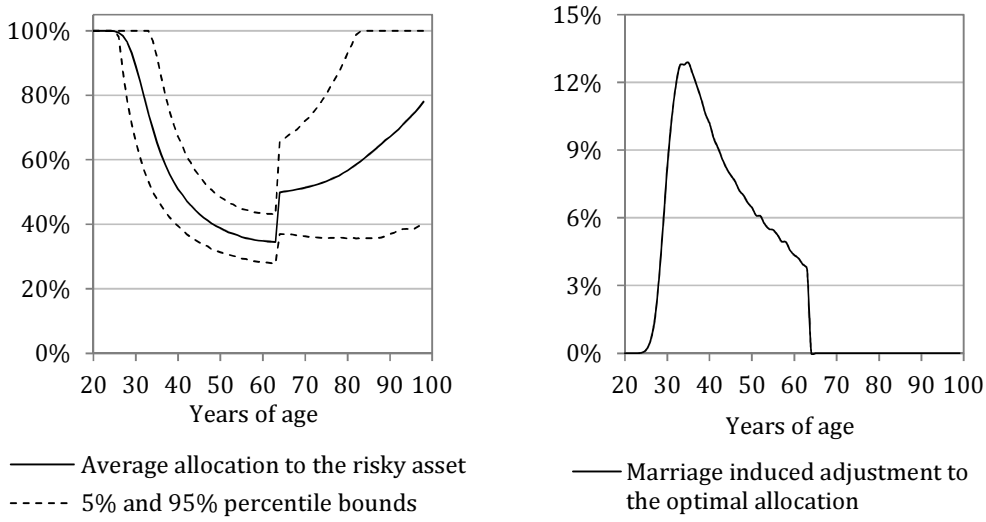

The graph shows average simulated realizations of financial wealth and human capital (left axis, measured in thousands of € at 2005 prices), and the ratio of average human capital to total wealth (right axis) over the life cycle for males and females in the middle-education group.

Figure 8 plots the expected trajectories of human capital and financial wealth over the life cycle. We find that human capital follows a downward-sloping trajectory as opposed to a hump-shaped trajectory in Cocco, Gomes, and Maenhout (2005). We also find that the ratio of human capital to total wealth, $W^{HC}/(W^{HC}+W^F)$, is decreasing all the way from its maximum early in life until approximately the age of 85 when the increasing force of mortality encourages rapid consumption of financial wealth.

## 7.2. Portfolio allocation pattern

In the left panel of Figure 9 we plot the mean and 90% confidence bounds of the simulated allocation to the risky asset over the life cycle. Young agents with relatively large values of human capital and low financial wealth aggressively invest in the risky asset, which follows directly from the shape of the optimal allocation policy shown in Figure 6. As agents age and accumulate financial wealth, the allocation to the risky asset decreases and reaches its lowest point of approximately 35% right before the retirement age. During retirement, the predicted risky share increases gradually again.

**Figure 9 Expected trajectory of the allocation to the risky asset**




The graph on the left shows the mean and the 90% confidence bounds of the simulated allocation to the risky asset over the life cycle for single males in the middle-education group. The graph on the right plots the average adjustment to the optimal allocation to the risky asset caused by marriage.

We distinguish three forces that are at work shaping the optimal allocation to the risky asset prior to retirement. First, as the level of human capital decreases over the lifetime, so does the diversification demand for the risky asset, which drives down its share in the portfolio. Second, as agents accumulate a stock of financial wealth towards the middle of the life cycle, the impact of human capital on the allocation of the financial portfolio becomes weaker, and allocation tilts towards Merton's (1969) policy as in (1), that is towards a lower allocation to the risky asset. Both these forces determine the overall downward-sloping

pattern of average optimal allocation to the risky asset during the first half of life.

The third force stems from the changes in riskiness of human capital. As agents turn retired, human capital becomes risk-free and agents become less risk averse towards the risky asset and allocate more financial wealth to it. Hence the age profile of average optimal allocation has a 'discontinuity jump' at the retirement age. Notice also the convex shape of the optimal allocation trajectory for several years before retirement. We know that a large fraction of human capital risk is associated with the uncertainty about the realization of $v_K$ (the persistent stochastic component at age $K$), which determines the level of retirement income. Close to retirement, when each passing year reduces the conditional variance of $v_K$ by an increasingly large fraction, agents experience a yet larger reduction in human capital risk. Thus, over several years right before retirement the trajectory of average optimal allocation to the risky asset is convex.

During retirement, income is risk-free and the 'riskiness' of human capital does not change anymore. As both human capital and financial wealth decrease during retirement, the two former forces affect allocation in opposite directions. The net effect is an increase in average allocation to the risky asset during retirement.

On the right side of Figure 9 we show the average adjustment to the simulated risky asset share that is caused by marriage. As it is illustrated by the optimal allocation rules in Figure 6, married agents act in a less risk-averse way towards their financial portfolio since their human capital is less risky. Hence the marriage-induced adjustment to the average allocation is never negative. For both single and married agents the borrowing constraint is binding during the first years of adulthood, and human capital is riskless during retirement. Therefore, the adjustment is zero both early in life and in retirement. We find that for agents of both genders the marriage-induced adjustment to the average allocation trajectory peaks around the age of 35 and then gradually decreases towards the retirement age. This decrease is caused by the diminishing riskiness

of human capital as agents are reaching the retirement age. The size of the adjustment is similar for both males and females and lies in the range 4%-12%.

The predicted average risky asset share of single individuals in our model is less than 100% starting from the mid-twenties, which is about a decade earlier than in Cocco, Gomes, and Maenhout (2005). We also predict substantially less aggressive portfolio allocations throughout most of working life. These findings are partly attributable to unemployment risk, which we model to be high early in life, and partly to higher savings rates early in life predicted by our model.

## 8. Welfare Metrics

We are interested in measuring the welfare effect of divergence from the optimal allocation policy. The methodology we employ for this purpose is taken from Cocco, Gomes, and Maenhout (2005). We examine and compare three suboptimal policy rules apart from the estimated optimal consumption and allocation policies. First, we consider a nonparticipation policy, i.e. $\varphi_a = 0$ for every $a$. Second, we examine Merton's (1969) policy as in (1). This policy ignores any effect of human capital on portfolio choice. Finally we consider a rule of thumb, proposed in Malkiel (1996), defined as $\varphi_a = (100 - a)/100$, so that the allocation to equities is 80% at age 20 and linearly decreases to 0% by the age of 100.

Each of the suboptimal policies assigns $\varphi_a$ exogenously, which leaves consumption as the only policy variable in the utility maximization problem. For each of the suboptimal allocation rules we solve the dynamic programming equation in (10), where for each $a$ we maximize only with respect to $c_a$ and treat $\varphi_a$ as given. We find the optimal consumption policies subject to the given allocation policy and estimate the value function for single individuals who have received an average amount of log non-financial income in the beginning of their independent life. That is, we find the maximum achievable level of utility at age $a_1$ for the optimal and each of the suboptimal policies:

$$U^{max} = V\left(a_1, x_{a_1}, v_{a_1}, m_{a_1}\right) \text{ for } x_{a_1} = \exp\left(f_{a_1}\right), v_{a_1} = 0, \text{ and } m_{a_1} = 0.$$

We then find a certain constant level of consumption which results in the same level of utility as the maximum achievable utility for each of the allocation

policies considered. From (8), the constant level of consumption that results in the given level of utility is

$$\bar{c} = \left( \frac{(1-\gamma)U^{max}}{\sum_{j=a_1}^{A} \delta^{j-t_1} \left( \prod_{k=a_1+1}^{j} p_k \right)} \right)^{\frac{1}{1-\gamma}}.$$

We compare $\bar{c}$ that is associated with each of the suboptimal allocation policies to $\bar{c}$ for the optimal policy in Table 8.

**Table 8 Allocation policy comparison on a constant-consumption equivalent basis**

| | nonparticipation | Merton (1969) | Malkiel (1996) |
|---|---|---|---|
| Males | 95.4% | 97.5% | 99.1% |
| Females | 95.1% | 97.4% | 99.0% |

The table presents the ratios of utility-equivalent certain consumption levels $\bar{c}$ of three suboptimal allocation policies to $\bar{c}$ of the optimal policy for males and females in the middle-education group.

We find that suboptimal allocation decisions are costly. Ignoring the effect of human capital and following Merton's (1969) portfolio rule results in around 2.5% loss on a constant-consumption equivalent basis, which we consider economically large.

Malkiel's (1996) portfolio rule reflects the idea that as agents become older, their human capital decreases and their capacity to endure large financial losses diminishes as the financial portfolio plays an increasingly important role in funding consumption. The rule, however does not take into account that the size of financial portfolio evolves with age, and neither does it recognize the changing riskiness of the human capital. We find that Malkiel's (1996) rule is around 1% inferior to the optimal policy on a constant-consumption equivalent basis.

## 9. Parameter Sensitivity and Robustness Checks

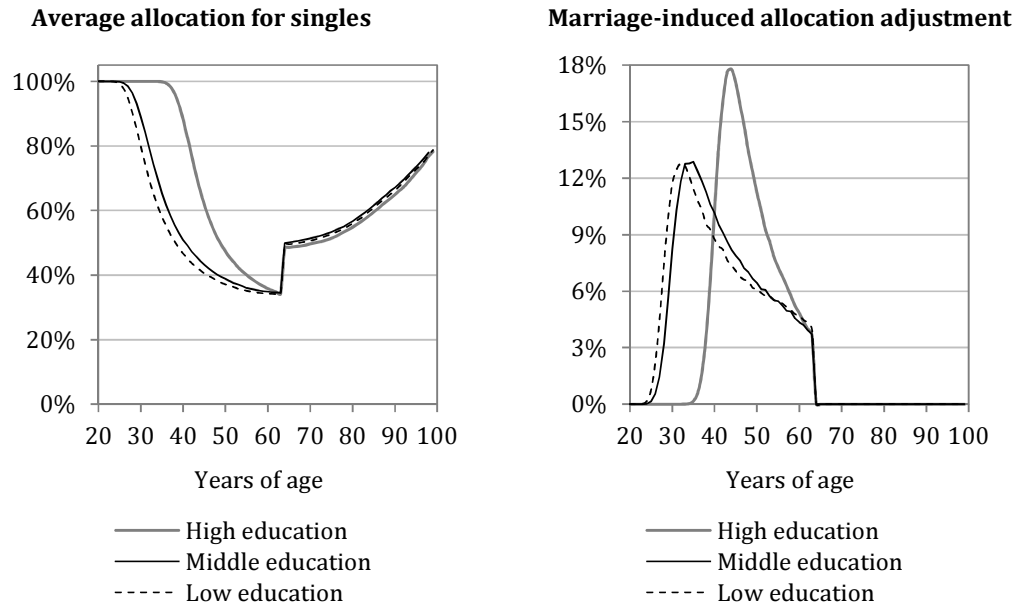
The age-income profiles estimated in Section 5 differ across education groups. In this section we examine how dissimilar our results are across these groups. Additionally we assess the robustness of the main results to changes in variance

estimation approach, withdrawing the unemployment shocks, and the choice of the coefficient of relative risk aversion.

### 9.1. Education groups

It is a relevant question whether the differences in age-earnings profiles across education groups lead to variation in optimal allocation across these groups. The results across genders are similar and for brevity, we report findings for males only.

**Figure 10 Expected risky asset allocation for different education groups**




The graph on the left shows the mean simulated allocation to the risky asset for single males in different education groups. The graph on the right shows average marriage-induced adjustment to the risky asset allocation for males in different education groups.

Figure 10 shows that the overall life cycle pattern of the optimal risky share is similar among education groups; the benchmark case is almost indistinguishable from the low-education group. From the age-earnings profiles in Figure 6, we know that the amount of human capital is significantly higher for individuals in the high-education group. This results in higher diversification demand for risky financial assets at all ages before retirement and more aggressive portfolio allocations for agents in this group. Yet, given the width of confidence intervals around the predicted allocations and the similarity of their shapes, particularly

for the two lower education groups, it is not obvious that education-specific portfolio advice would be justified.

We notice that the results for high-education group in our sample resemble the base-case results of Cocco, Gomes, and Michaelides (2005) more than the results for our base-case group do. In particular, we find that agents in high-education group start building their financial wealth later in life, around the age of 30 for males and 25 for females, and on average consume all of the non-financial income in years preceding the mentioned ages. We also notice that our high-education group on average fully allocates its portfolio to the risky asset until older in life. Such a behavior is explained by two facts.

First, agents in the high-education group have a relatively high growth rate of income in the first half of working life, which is more comparable to the one exhibited by the base-case sample of Cocco, Gomes, and Maenhout (2005). High expected future income causes these people to be willing to borrow when young. That is why, when young, these individuals maintain small financial portfolios. Then, given the small size of their financial portfolio, the effect of human capital on allocation is large and it is optimal for high-education individuals to invest solely in the risky asset.

Second, the age structure of unemployment is almost flat for agents in the high-education group. When high-education individuals are young, their human capital is not as risky as the one of individuals in the low-education and middle-education group. That is why agents with high education are willing to maintain a higher level of risk in their financial portfolio until relatively late in life.

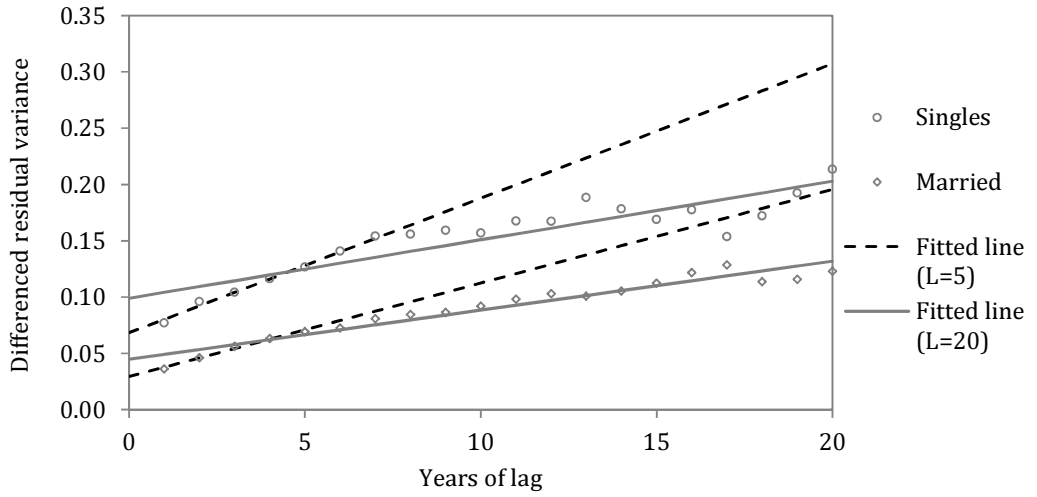
The effect of marriage on optimal allocation for individuals in the high-education group is qualitatively similar to that for the two lower education groups. The combination of larger human capital, and delayed accumulation of financial wealth that is typical for agents in this group, results in a more pronounced effect that marriage exerts on allocation.

### 9.2. Maximum lag length in the variance decomposition

The choice of $L$ determines how many estimates of sample variances of differenced first-stage regression residuals we use in variance decomposition

regressions, described in Section 5.2. Contrary to the theoretic result in (14), which is based on our specification of the non-financial income process, we find that the series of $\widehat{Var}(d_{i,a,l})$ are not linear in $l$. The relationship appears to be concave for both single and married individuals.

**Figure 11 The influence of the choice of $L$ on estimates of shock variance**




The graph shows OLS fits of equation (14) for single and married individuals; the relationship between $\widehat{Var}(d_{i,a,l})$ and $l$ is concave, and the choice of larger values of $L$ results in larger intercepts and smaller slopes of the fitted lines, which corresponds to larger estimated variances of transitory shocks and lower estimated variances of persistent shocks.

The slope of a straight line fitted to the $\widehat{Var}(d_{i,a,l})$ series estimates $\sigma_w^2$; and half of its vertical intercept measures $\sigma_\varepsilon^2$. In the context of a concave relationship, a larger $L$ reduces the slope of the fitted line and increases its intercept, thus 'shifts' estimated variance from the persistent shocks to the transitory ones. Figure 11 illustrates this point by plotting sample variances of differenced first-stage regression residuals and fitted straight lines for $L = \{5, 20\}$. Table 9 reports the estimates of $\sigma_w$ and $\sigma_\varepsilon$ for different choices of $L$.

**Table 9 Estimates of shock standard deviation for different values of $L$**

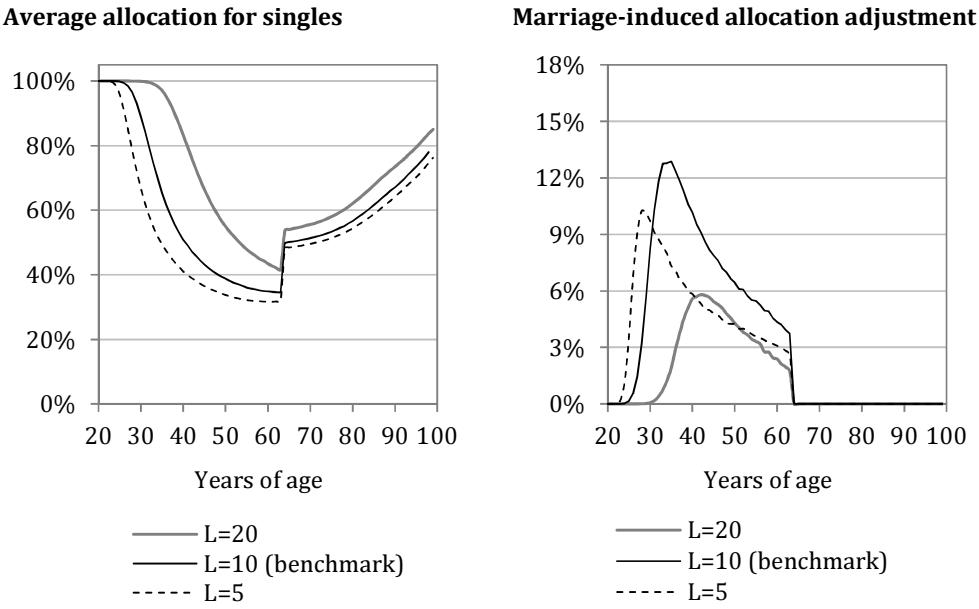
| $L$ | single | | | married | | |
|---|---|---|---|---|---|---|
| | $\sigma_\varepsilon$ | $\sigma_w$ | R-squared | $\sigma_\varepsilon$ | $\sigma_w$ | R-squared |
| 5 | 0.18 | 0.11 | 0.98 | 0.12 | 0.09 | 0.99 |
| 10 | 0.20 | 0.10 | 0.94 | 0.13 | 0.08 | 0.97 |
| 15 | 0.21 | 0.08 | 0.88 | 0.14 | 0.07 | 0.97 |
| 20 | 0.22 | 0.07 | 0.80 | 0.15 | 0.07 | 0.94 |

The table reports the estimates of standard deviation of the innovations to the non-financial income process for different values of $L$ for single and married individuals. Choice of larger values of $L$ results in 'shifting' estimated variance from persistent to transitory shocks.

The larger variance estimates of persistent shocks obtained for smaller values of $L$ result in higher assumed riskiness of human capital. Increases in the riskiness of human capital in turn, make agents less willing to invest in the risky asset. That is why, as we illustrate in the left panel of Figure 12, the average allocation to the risky asset is lower for lower values of $L$. The magnitude of the marriage-induced adjustment to the optimal allocation depends primarily on the differential between $\sigma_w$ estimates for single and married individuals. This difference is the largest for $L = 10$ and so is the size of the adjustment (right panel of Figure 12).

The overall life cycle pattern of the risky share and the marriage-induced adjustment to allocation is unchanged from the benchmark case. In that sense, the benchmark result is robust to the choice of $L$. However, the choice of $L$ does affect the level of the average allocation to the risky asset. The differences in predicted risky asset shares for $L = 5$ and $L = 20$ reache almost 50 percentage points for certain ages.

**Figure 12 Sensitivity of the expected allocation to the risky asset to the choice of *L***




The graph shows the average simulated realizations of the allocation to the risky asset over the life cycle for different choices of $L$ for males in the middle-education group. Higher values of $L$ result in lower estimates of $\sigma_w^2$ and consequently in higher average allocation to the risky asset.

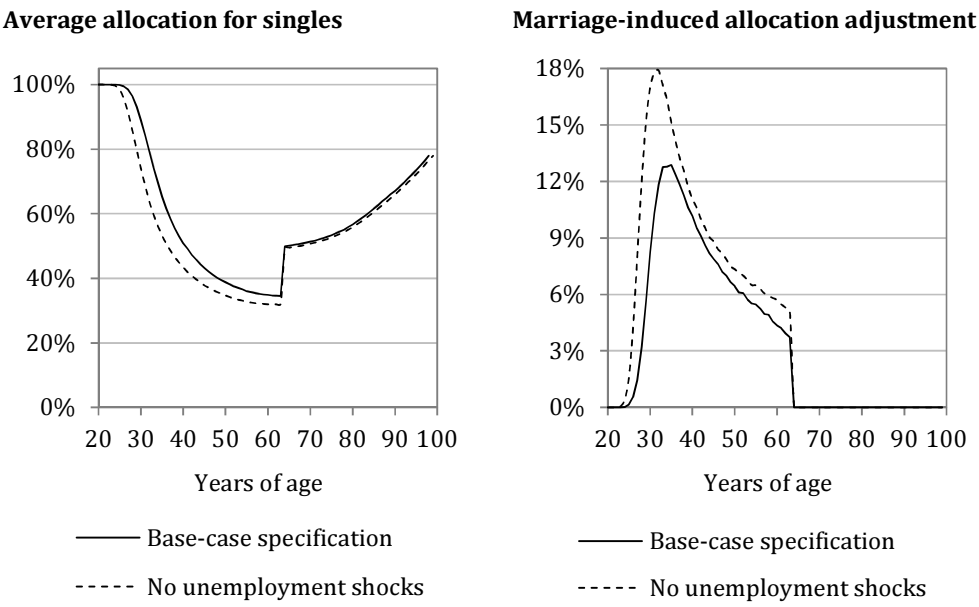
### 9.3. Withdrawing unemployment shocks

In this paper we adjust the specification of the non-financial income process that is used by Cocco, Gomes, Maenhout, 2005 by including unemployment shocks. We are, thus, interested in seeing if our results regarding the average allocation pattern and marriage-induced adjustment to it are driven by this adjustment. For this purpose we calibrate our model in which we restrict $g_a = 0$, that is, we disallow unemployment shocks. The volatility of the income process then is only captured by persistent and transitory shocks. In the restricted version of the model estimates of $\sigma_w$ ($\sigma_\varepsilon$ for $L = 20$) for the single individuals are around 1% larger than those in the unrestricted version of the model and that are reported in Table 6.

Figure 13 plots the average simulated realizations of the allocation to the risky asset in the base-case and for the specification of the model without unemployment shocks. While the results are qualitatively similar, the restricted version of the model predicts lower allocation to the risky asset for singles and a larger impact of marriage on allocation behavior. This could be explained by the

fact that the estimated volatility of persistent shocks for singles is larger in the restricted version of the model, and the corresponding reduction in volatility of the persistent shocks that follows marriage is more pronounced.

**Figure 13 Sensitivity of the expected allocation to modeling of the unemployment shocks**




The graph shows the average simulated realizations of the allocation to the risky asset over the life cycle for the base-case and for the specification of the model without unemployment shocks.

### 9.4. Risk-aversion

One of the widely-discussed phenomena in finance is the so-called "equity premium puzzle" (Mehra and Prescott, 1985). With CRRA utility, estimated risk premia in equity markets can only be rationalized under the assumption of risk aversion coefficients that are considered empirically implausible. The value of the risk aversion coefficient $\gamma = 10$ that we use in the benchmark specification of our model is the same as the one used by Cocco, Gomes, and Maenhout (2005). At $\gamma = 5$, the average optimal risky asset share is very close to or equal to 100% over the entire life cycle, and does not deviate by more than 10% from a full-equity portfolio at any age. At $\gamma = 2$, the average allocation to the risky asset invariably equals 100% throughout the whole lifetime. We conclude that our benchmark results are not robust to a change in the assumed coefficient of relative risk aversion.

## 10.Discussion

The findings regarding saving and allocation behavior that we present in the preceding sections are purely normative. That is, given a set of assumptions regarding the income process and the shape of the utility function, we show the utility maximizing behavior of rational individuals. As a discussion of our results, we examine how our predictions compare to actual investor behavior.

### 10.1.Savings behavior

The results of the life cycle optimization exercise are largely in line with the classical life cycle savings and consumption model in the sense that agents save when young and then use savings to finance their consumption when old. Börsch-Supan and Essig's (2005) discussion of household savings behavior in Germany suggests that the prediction of our model is broadly consistent with observed savings behavior for the first half of agents' lives, even though actual savings rates seem to be lower than predicted by the model.

In the second half of agents' lives, however, predicted consumption behavior does not match the observed one. High-income and median-income households in Germany appear to keep saving even during retirement, albeit at a considerably lower rate. Börsch-Supan et al. (2001) call this "the German savings puzzle". This behavior might partly be due to the fact that in the past households were faced with high replacement ratios in statutory pension insurance. The availability of sufficient levels of non-financial income should have reduced the expected need to dissave during retirement, but this only raises the question why substantial savings are accumulated in the first place and why consumption is not higher during retirement.

One potential explanation for the difference between predicted and actual behavior during the second half of the life cycle is that the precautionary savings motive remains very strong because agents perceive income and/or expenditures during retirement as risky while our model assumes risk-less retirement income and ignores increased probability of large healthcare-related expenditures. Incorporating these risks in the income process during retirement could allow the model predictions to match actual savings behavior more

closely. At the same time, it is conceivable that actual behavior is driven by biased perceptions of risks during retirement and that altering savings policies would increase utility.

### 10.2.Asset allocation behavior

Information on risky asset shares in German household portfolios is limited, particularly with respect to current life cycle allocation behavior. Barasinska et al. (2008) report very low stock market participation rates of about 30% (for combined direct and indirect equity holdings) of households using SOEP data for 2001 to 2005. Guiso et al. (2003) show a relatively stable age structure of stock market participation in Germany in 1998, yet at the level of around 20%. Börsch-Supan and Essig (2002) also report lower rates for the 1990s. They show that participation increases with education, but they also show that most of the effect is driven by higher income and higher wealth as opposed to education itself. In our model, agents always participate in the stock market. For those households that do participate, Börsch-Supan and Essig (2002) show data on risky shares for 1993 and 1998 that do in fact suggest a U-shaped pattern of the risky asset allocation over age.

We are not aware of empiric research into how marriage affects asset allocation that would be based on German data. A study by Agnew, Balduzzi, and Sunden (2003), based on U.S. data, finds that, on average, individual investors maintain a 6.4 percentage points higher allocation to equities when married than when single. Agnew, Balduzzi, and Sunden's (2003) result is estimated across all ages and compares well with the 4-12% marriage-induced adjustment to the risky asset allocation predicted by our model.

More recent findings of Christiansen, Joensen, and Rangvid (2010), which are based on a Danish dataset, suggest however that the effect of marriage on portfolio allocation is gender specific. They find that "marriage causes men to reduce the fraction of wealth they hold in risky assets", and the opposite is true for females. While Christiansen, Joensen, and Rangvid (2010) do not suggest an explanation for such a behavior of males, an indication of its possible irrational nature might be conjectured from their finding that the effect is stronger for

younger and less educated men. On the other hand, if we modified our model in such a way that marriage on average caused a drop in the disposable income of males – that is their human capital decreased post marriage – reducing their exposure to the risky asset would be a rational response.

Overall, while a complete assessment of the model's ability to describe actual allocation behavior is not feasible, we conclude based on the available evidence that large discrepancies exist between predictions and observed behavior. It is conceivable that actual behavior cannot easily be rationalized because it might be suboptimal. Equity market nonparticipation, which seems to be the most common policy actually followed by German households, underperforms remarkably on a consumption-equivalent basis in our model even when compared to policies like the classical Merton rule that ignores non-financial income altogether.

## 11. Conclusion

We numerically solve a life cycle problem of portfolio allocation and consumption in discrete-time and within a two-asset investment universe with CRRA-utility maximizing agents receiving flows of exogenous stochastic non-financial income. The non-financial income process is calibrated to German data. While to a large extent we follow Cocco, Gomes, and Maenhout (2005) in terms of the model specification and parameter choice, we contribute by explicitly modeling the age structure of unemployment risk and marriage-caused effects on the riskiness of human capital.

Despite perceived differences in labor market structure and welfare policies between Germany and the U.S., we obtain comparable shapes of age-earnings profiles and qualitatively similar estimates of income risk. We find that this risk is largely idiosyncratic, even though we do estimate a non-zero correlation between labor income innovations and excess stock returns.

Within the life cycle model we identify three ways how human capital affects allocation. First, cet. par., larger human capital leads to a larger optimal risky share in the financial portfolio through increased diversification demand for the risky asset. Second, the magnitude of this effect depends on the ratio of human

capital to financial wealth. Larger financial wealth reduces the impact of human capital on portfolio allocation. Third, cet. par., higher riskiness of human capital (higher background risk) exerts downward pressure on the optimal risky share.

Young agents, whose total wealth consists almost entirely of human capital, are predicted to allocate 100% of their financial portfolio to the risky asset. On average the risky asset share then decreases until retirement, 'jumps' up at retirement age and then gradually rises during the retirement period. The expected downward drift in the risky asset share in the first half of life is driven by depletion of human capital and build-up of financial wealth. The dominating factor behind the 'jump' in average risky asset allocation when agents become retired is the reduction in the riskiness of human capital. The gradual rise in the predicted risky asset share during retirement is driven by the reduction in financial wealth during this period. We show that income sharing between spouses results in reduced riskiness of human capital and, cet. par., higher optimal allocation for married individuals.

Similar to earlier normative life cycle allocation models, our calibration results and choice of benchmark parameters on average lead to very high risky asset shares for young investors. Nevertheless, modeling the empirically verified high young-age unemployment risk results in less radical age-allocation profiles for single agents, compared to profiles generated by Cocco, Gomes, and Maenhout (2005).

Qualitatively, the life cycle pattern of allocation is robust to investor heterogeneity related to education and gender as well as to differences in shock variance estimation. As it is common for the earlier literature, our result is sensitive to the choice of the coefficient of relative risk aversion. In junction with "equity premium puzzle", we are able to generate moderately aggressive allocation profiles only for relatively high values of the risk aversion coefficient.

Failing to pursue the optimal allocation policy leads to economically significant utility losses. Our results suggest that rational investors whose preferences and choices are accurately reflected in the model stand to lose substantially from nonparticipation in the equity market or from following classical portfolio policies and rules of thumb.

We notice that model-implied optimal behavior is inconsistent with observed savings behavior during the second half of agents' lives and with allocation behavior over the entire life cycle. Actual equity market participation rates are exceptionally low and the risky asset share of those households that do participate is considerably lower than predicted by the model. There are two possible explanations, and we deem it likely that both are relevant to some extent. First, the model might need to be extended or altered to reflect investor preferences and choices more accurately. Second, investors' preferences and income processes are adequately captured by the model, and the observed behavior is irrational. In the latter case, we predict that investors could gain substantially from following the life cycle consumption and allocation policies discussed in this paper.

## Appendices

### Appendix 1

We define the adjusted value function in the dynamic programming equation, which we are in fact solving instead of (10), as

$$\dot{V}(a, \dot{x}_a, m_a) = \max_{\dot{c}_a, \varphi_a} \{u(\dot{c}_a) + \delta p_{a+1} E_a(\dot{V}(a+1, \dot{x}_{a+1}, m_{a+1}) e^{w_{a+1}(1-\gamma)})\}$$

Let us show that $\dot{V}(a, \dot{x}_a, m_a) = V(a, x_a, v_a, m_a) e^{-v_a(1-\gamma)}$, $a_1 \leq a \leq A$.

The identity is true in the last period, i.e. for $a = A$:

$$\dot{V}(A, \dot{x}_A, m_A) = u(\dot{x}_A) = u(x_A e^{-v_A}) = u(x_A) e^{-v_A(1-\gamma)}$$
$$= V(A, x_A, v_A) e^{-v_A(1-\gamma)}.$$

Then, by induction, it is also true in all preceding periods, i.e. for $a_1 \leq a < A$:

$$\dot{V}(a, \dot{x}_a, m_a) = \dot{V}(a, x_a e^{-v_a}, m_a) =$$
$$= \max_{\dot{c}_a, \varphi_a} \{u(\dot{c}_a) + \delta p_{a+1} E_a(\dot{V}(a+1, \dot{x}_{a+1}, m_{a+1}) e^{w_{a+1}(1-\gamma)})\} =$$
$$= \max_{c_a, \varphi_a} \{u(c_a e^{-v_a}) + \delta p_{a+1} E_a(\dot{V}(a+1, \dot{x}_{a+1}, m_{a+1}) e^{(v_{a+1}-v_a)(1-\gamma)})\} =$$
$$= \max_{c_a, \varphi_a} \{u(c_a) e^{-v_a(1-\gamma)} + \delta p_{a+1} E_a(V(a+1, x_{a+1}, v_{a+1}, m_{a+1}) e^{-v_a(1-\gamma)})\} =$$
$$= V(a, x_a, v_a, m_a) e^{-v_a(1-\gamma)}.$$

### Appendix 2

The first-stage regressions that we use to characterize $f_a$ use a measure of non-financial disposable income as a dependent variable. We construct this income measure based on an SOEP-provided measure of household post-government income. Table 10 presents income and tax items that household post-government income is based on and documents whether we classify these items as financial or non-financial. Income items classified as financial are excluded from our measure of non-financial disposable income. Tax items classified as financial are added back.

Several items have been aggregated for the purpose of this appendix. For exact references to each individual SOEP variable used, please contact the authors.

**Table 10 Classification of income items**

| SOEP Category | Income item | Classification | Comment |
|---|---|---|---|
| Labor earnings | Wages/salary from primary job | Non-financial | |
| | Income from secondary employment | Non-financial | |
| | Income from self-employment | Non-financial | If the household head reports being primarily self-employed, we exclude the head (and hence the household) from our sample |
| | Military/community service pay | Non-financial | |
| | 13th/14th monthly salary | Non-financial | |
| | Christmas bonus, vacation bonus | Non-financial | |
| | Profit-sharing, other bonuses | Non-financial | |
| | Severance payments | Non-financial | |
| | Commuting expense allowances | Non-financial | |
| Private Transfers | Alimony payments | Non-financial | |
| | Other private transfers | Non-financial | |
| Public Transfers | All forms of public unemployment benefits including related payments | Non-financial | |
| | Subsistence allowance | Non-financial | |
| | Old-age transition benefit | Non-financial | |
| | All forms of social welfare ("social assistance") benefits and related payments | Non-financial | |
| | Maternity benefit | Non-financial | |
| | Student grants | Non-financial | |
| | Child allowance | Non-financial | |
| | Nursing allowance | Non-financial | |
| | Housing support for owner-occupiers | Financial | A discount on an asset purchase; assumption: home ownership as a portfolio allocation decision |
| Social Security Pensions | Statutory old-age and disability pension, including widows/orphans payments | Non-financial | |
| | Civil servants pensions,including widows/orphans payments | Non-financial | |
| | Miners and farmers pensions, including widows/orphans payments | Non-financial | |
| | War victims pensions, including widows/orphans payments | Non-financial | |
| | Statutory accident insurance, including widows/orphans payments | Non-financial | |
| Private Retirement Income | Company pensions, including widows/orphans payments | Non-financial | Related to prior service provision; a form of deferred labor income |
| | Supplementary benefits for civil servants, including widows/orphans payments | Non-financial | Related to prior service provision; a form of deferred labor income |
| | Private and other pensions, including widows/orphans payments | Financial | Outcome of a portfolio allocation decision |
| Asset Income | Income from rental and leasing net of operating expenses | Financial | |
| | Interest/dividend income, capital gains | Financial | |
| Taxes | Federal Taxes ( = straight income taxes) | Partly financial, partly non-financial | Separated into a part pertaining to financial income and a part pertaining to non-financial income; see separate appendix |
| | Social Security Taxes | Non-financial | Social security taxes are related exclusively to those income items that we classify as non-financial |

The table shows the classification of (categories of) income items into financial and non-financial income. The item "federal taxes" is separated into taxes pertaining to financial income and taxes pertaining to non-financial income. The required disaggregation of income taxes is described in Appendix 3.

## Appendix 3

The income taxes in the SOEP data are estimated by the SOEP and reported for all forms of income and all household members combined, i.e. they are reported as one number per household per year. The same applies to social security taxes, but we do not discuss them since they are related exclusively to income items that we classify as non-financial and hence do not require adjustment.

When calculating household disposable non-financial income, we seek to remove financial income from total household disposable income. This requires that we allocate part of the SOEP-provided income taxes to the financial income items we remove.

We adopt the simplifying assumptions described in Schwarze (1995). Most importantly, we assume that filers never itemize expenses (except for expenses related to rental income) but rather take standard deductions wherever possible. We ignore losses from capital and rental income. However, unlike Schwarze (1995), we treat each household as a single filing unit.

Under German tax law, taxable income is determined in a multi-step procedure outlined in Schwarze (1995). The major steps are (1) the determination of the taxable part of each form (category) of income a filer receives and (2) the subtraction of deductions or itemized expenses that apply to overall income (as opposed to applying to any particular form of income dealt with under (1)). Depending on the form of income, (1) may involve applying some fraction determining the taxable portion of income reported under a particular category, applying a tax-exemption and/or subtracting either an expense deduction or a sum of itemized expenses from income in that category. For our purposes, only step (1) is relevant, because considering deductions applied after the sum of the taxable parts of each form of income is calculated only complicates the computations while having no impact on our allocation of SOEP-provided taxes to individual income categories.

To implement step (1), we apply standard deductions and/or exemptions to different forms of labor income, to interest and dividend income and capital gains and to civil servants pensions. For rental income, we deduct reported operating expenses from gross rental income. In case of civil servants pensions, we extrapolate one tax exemption for years for which the correct parameter value is not known. For all other forms of taxable pension income, we calculate taxable income using specific ratios that determine the taxable part of each type of pension. We choose the according ratios adopting all simplifying assumptions explicitly stated in Schwarze (1995) so that there is no variation in those ratios across individuals and households within a specific pension category. For later

survey years, major changes in pension taxation require similar simplifying assumptions. We ignore one minor standard expense deduction related to non-civil servants social security pensions and private pensions.

This procedure yields taxable income by income category. The sum of these estimates is overall taxable income before any deductions that would usually be applied in step (2). We then allocate the SOEP-provided, estimated taxes proportionally to different forms of income, which allows us to quantify after-tax financial income and hence to move from household disposable income to household non-financial disposable income.

Our calibration results are robust to alternative approaches to allocating taxes to financial income and to higher variation in tax rates.

For calendar years 1983, 1984, 2002 and 2003, certain pension items subject to different taxation rules are reported in aggregated form, preventing the estimation of taxable income. We drop the affected observations. The effects of this are discussed in 4.2.

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