Per Bäckman Frank Nieuwendijk

Master's Thesis Economics Stockholm School of Economics Tutor: Erik Lindqvist

MORAL HAZARD IN FINANCIAL NETWORKS

Abstract: We develop a model of moral hazard within financial networks. By combining network theory, financial contagion theory and game theory we show that players in financial markets have the potential to exploit their positioning within the network. To illustrate the theory, we construct a simulation model where one bank has a defaulting external asset. As this asset has been repackaged and resold prior to the shock, the default spreads through the system affecting every single bank. As a result, the bank initiating the default shock only carries part of the consequences and can therefore have an incentive to engage in overly risky behaviour. Our findings suggest that the way banks are connected in financial networks have an effect on moral hazard for the individual player. After obtaining this result, we elaborate on suggestions for further research and applications of the model.

Keywords: financial contagion, network theory, moral hazard, simulation, interbank markets

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Introduction

A commonly held view on the recent financial crisis is that investors have been taking on too much risk. Not only the public, but also legislators tend to be of this opinion: "The collapse of financial markets in autumn 2008 and the credit crunch that followed can be attributed to multiple, often interrelated, factors at both macro- and micro-economic levels, as identified in the De Larosière Report, and in particular to the accumulation of excessive risk in the financial system." (European Commission, 2012)

The current literature on financial network stability focuses on systemic defaults and sensitivity to external factors. Research that focuses on the aggregate market does not fully take into account individual behaviour of different firms within the network. By taking the behaviour of firms in a financial market as given, the possibility that one bank can set off a financial crisis can be overlooked.

In standard economic models, there is a balance between the risk an investor takes on and the return he receives. However, when advanced financial products are introduced, the risk-return relationship becomes more complex. This happens because of a loss of information through the repackaging and reselling of assets in a repeated number of steps, with a large number of banks (Chamley, 2004).

In our research we try to find if individual banks in a financial network can exploit this loss of information by taking on high amounts of risk, and if so, what the characteristics of such a bank would be. To find these particular firms, we simulate a financial shock for every single permutation of the way banks can be interconnected in a market. We define a shock as the consequence of the external product defaulting, i.e. the cost equal to the value of the product. This then travels on in the system.

To illustrate how a financial network might look like in our setting. We show a simple banking structure in Figure 1. Three banks of comparable size and organization are displayed. All are fully connected (to the other two banks) and have determined their respective risk exposure according to the relative size of the receiving bank. The way the banks interact and deal with market dynamics is as described in the theory section. The red line represents the initial shock being sent through the system, initiating a chain reaction. For clarity purposes, this diagram only shows the first 3 steps of this particular shock being sent initially from 1 to 2.



Figure 1: Graphical representation of a financial network

Application of the model

Most papers on Financial Contagion use arbitrary interbank systems, and make few, or no, direct connections to real world applications. A typical case is illustrated by the following excerpt from Gai & Kapadia (2010): "In particular, our assumption that the network structure is entirely arbitrary carries the advantage that our model encompasses *any* structure which may emerge in the real world or as the optimal outcome of a network formation game."

Our research, although a purely theoretical exercise, has direct potential for application to real life markets when applied to data. Due to limitations in data availability we have found ourselves unable to apply this model to actual interbank data. Below we will give two types of financial products for which the model can be applied.

First off, a CDO, or Collateralized Debt Obligation, is a financial product in which a fixed-income security, for example a mortgage, is repackaged and sold. The dividends from the mortgage are distributed to the buyers of the CDO according to different tranches¹. The underlying asset defaults and Bank 1 is forced to write off the value and take a loss. Banks 2 and 3, having bought, and in turn repackaged and resold the product, are also forced to do the same, thus spreading the shock throughout the system. However, since Bank 1 sold bonds in the CDO, the cost of the default of the asset is not as high as it would otherwise be, as some of the loss is offset by the profit from the sale. The same holds, to a lesser extent, for other banks in the network.

Another way for shocks to spread is through interbank lending. Banks lend to and borrow from each other frequently as liquidity is needed. While this is a vital function for banks, and a requirement for a healthy banking system, it can also serve as a way for shocks to spread. "The stronger the connections between the institutes, the larger the secondary effects of from an initial disturbance. Interbank loans are a classic example. The failure of a bank to fulfil its contractual obligations imposes losses on correspondent banks." (Sheldon and Maurer, 1998). Regardless of the source of the initial default, interbank linkages help propagate the shock throughout the system, if the shock affects the initiating bank's ability to maintain its obligations to other banks in its network.

In the model, we will use the term 'capital buffer', which is a requirement of capital that banks need to set aside to protect against losses from credit exposures. It is regulated by the Basel Accords, commonly referred to as Basel I and Basel II, and is mandated by the Basel Committee on Banking Supervision, a part of the Bank for International Settlements. Spurred by the latest financial crisis, Basel III is also under development, which aims "to strengthen the regulation, supervision and risk management of the banking sector" (Basel III, 2012). We will use Basel II, which states that the capital ratio, or the ratio between a bank's capital and its risk-weighted assets, be no less than 8% (Basel II, 2012).

¹ A Tranche essentially dictates the order in which dividends are made. It typically includes a senior tranche, which gets paid first, a junior tranche and mezzanine, or equity, tranches. Different tranches can have different risks and maturities.

Banks and other institutional investors rely on credit rating agencies such as Moody's, Standard & Poor and Fitch to help them measure the risk of investments. These ratings range from, with some variations between them, AAA ('Investment grade' or 'Prime') to C (substantial risk or 'Extremely Speculative'), with everything under BBB considered non-investment grade or speculative. Many of the larger institutions, such as pension funds, have strict risk guidelines and cannot invest in anything below AAA. One would think that this would lead to banks being conservative in taking on risk, but as seen in the 2008 crisis this is not necessarily the case. Instead, banks came up with increasingly complicated products, often with questionable collateral (such as Sub-prime Mortgage Backed Securities). But these products still tended to receive AAA-ratings by the rating agencies. Whether this was due to the complexity of the products or intent is unclear, but the rating agencies are one source of failure that Barnett-Hart (2009) identifies as leading to the CDO meltdown in her thesis, "The Story of the CDO Market Meltdown – An Empirical Analysis" (Barnett-Hart, 2009). It seems clear that risk management failed, in the sense that banks engaged in overly risky behaviour with no regard for consequences, and rating agencies failed to properly assess these risk.

The rest of this paper is organised in the following manner:

In the literature review, different elements from network theory and financial contagion theory are explained to form a body of theory on how a shock travels through a financial market. To support the game-theoretical arguments for the morally hazardous behaviour, we investigate a game-theoretical study of repeated credit market games.

After reviewing the current state of the literature on the topic, the central problem statement splits up the main issue into a general research question and two more specific sub-questions. This will guide the research process and allow for a more detailed view.

Building upon the literature review, the theoretical framework combines financial contagion theory, network theory and game theory into a model where one of the players in a randomly generated network faces an external default shock and only has to bear part of the negative consequences because of the network dynamics. Subsequently, the initiating firm can optimise risk-taking according to the magnitude of the negative effects they experience from the external default shock.

The method section gives an exact representation of the model constructed in the theoretical framework and how the simulation generates the results. A short example is provided to aid the understanding of the process.

Having simulated the model, the results section will provide data and interpretation to answer the research questions. To aid the understanding of the results, graphical representations will be included.

Finally, in the conclusion we answer our two sub-questions which will enable us to conclude the research and answer the main research question. After having concluded, the discussion section elaborates on the application of the model, the limitations in obtaining data and suggestions for further research.

Literature review

There is quite an extensive literature dealing with financial contagion and financial networks. However, most of it looks only at the effects of, or the reasons for, defaults of banks on the system as a whole, and how the shocks spread, rather than how individual banks handle risk.

The inspiration for our research comes from the paper "The Social Structure of a National Securities Market" Baker (1984), which investigates how systemic risk is affected by the structure of the financial system, and develops a model for simulating inter-firm connections and network reactions to shocks. Baker concludes that once a market, in this case a Securities Exchange, grows too large, price volatility increases due to inefficient communication between players.

In "Network models and financial stability", Nier et al (2007), present a theoretical model of how actors within a securities market interact from a sociological point of view, and how one might deviate from the ideal-typical prescription. They find that four parameters are of interest when describing the banking system. "These are net worth, the size of the interbank market, the degree of connectivity and the concentration of the system." (Nier et al, 2007).

Nier et al. draw four conclusions: First, for high net-worth systems the risk of default is virtually nonexistent. But that decreases in net-worth leads to a higher number of contagious defaults. Second, "increases in the size of the interbank liabilities tend to increase the risk of knock-on default" (Nier et al., 2007). Furthermore, a capital buffer for interbank assets might not offer protection against systemic failure. Third, contagion is a function of the number of interbank linkages, in a non-monotonic fashion. They conclude that this relationship is also dependent on the level of capital in the system. For systems with low levels of capital, an increase in interbank connections increases the chance for contagious defaults. For systems with a high level of capital, increases in interbank linkages improve the system's ability to handle shocks. And fourth, more concentrated networks tend to be more susceptible to systemic shocks. In small, but highly concentrated networks shocks spread easily and are able to affect a larger number of banks. Whereas in less concentrated networks, some banks will be insulated.

"Financial Contagion" (Allen and Gale, 2000), and "Contagion in Financial Markets" (Gai and Kapadia, 2010) look at how shocks spread throughout the financial system after the default of one bank. Allen and Gale (2000) provide microeconomic foundations on financial contagion and the spread thereof. Allen and Gale also introduce a concept of short, or liquid, assets and long, or illiquid, assets. When banks receive a shock, they have to liquidate assets to meet the demands of creditors. If their stock of short assets isn't enough to cover these demands, they will have to prematurely sell off long assets. Here the short assets act as a capital buffer of sorts. Banks will go bankrupt if the shock in the region is greater than its capital buffer, and then spread on to other banks in the network.

Gai and Kapadia (2010) look at an arbitrary system of banks linked by financial claims on each other. They find that the system exhibits a "robust-yet-fragile-tendency", (Gai and Kapadia, 2010), in that a high level of connectivity can protect the system from systemic shocks but also spread the shocks leading to defaults. If the system is small, a high level of connectivity will spread the shocks throughout the system, and potentially leading to a system-wide collapse. In larger systems, the increased connectivity will help the system to absorb shocks.

Sheldon and Maurer's (1998),"Interbank Lending and Systemic Risk: An Empirical Analysis for Switzerland" analysis is based upon two assumptions. First, a bank defaults due to a single event forcing the bank to lapse on its interbank liabilities. Second, the default leads to the loss of the entire value of the loan on behalf of the lending bank. This paper uses data of actual interbank loans in Switzerland. However, difficulties in data collection are inevitable in this particular niche, in that aggregate data is readily available, but that data on individual interbank relationships are generally not disclosed. Sheldon and Maurer solve this by entropy (a measure of how much information a given message contains) maximisation. This approach essentially sets the probability of an event occurring to 50:50, which leads to a uniform distribution of the interbank loan matrix. I.e. banks spread their loans evenly across all other banks. This method is also discussed in Upper (2007), "Using counterfactual simulations to assess the danger of contagion in interbank markets". Banks spread their exposures, given the positions of other banks, as evenly as possible.

(Sheldon and Maurer, 1998) conclude that a system-wide default due to a single bank failure is unlikely. However, their results are limited in that it only deals with domestic interbank lending, and that only a single idiosyncratic shock occurs at a time.

In "Using Counterfactual Simulations to Assess the Danger of Contagion in Interbank Markets", Upper (2007) surveys existing literature and summarizes results of default stemming from idiosyncratic shocks as well as aggregate, system wide, shocks. Upper concludes that while danger from contagion exists, system-wide defaults are very unlikely. He also mentions that lack of data is a limitation in using counterfactual simulations.

In "Moral Hazard and Secured Lending in an Infinitely Repeated Credit Market Game" Boot and Thakor (1994) provide a game-theoretical construct where profit is maximised with respect to the potential cost of default in an external contract. Their conclusion is that, even when assuming risk neutrality and an absence of learning, "a durable bank relationship benefits the borrower" (Boot and Thakor, 1994)

Central problem statement

In our research, we aim to determine whether the composition of a financial market can influence morally hazardous behaviour. Moral hazard in economic theory refers to when players take on high risk because they do not themselves bear the costs.

As observed in the late financial crisis, players in financial markets engaged in financial transactions with third parties that had a very high risk of defaulting and a high sensitivity to market fluctuations. This behaviour suggests that players had incentives to engage in these risky activities because the potential negative consequences were not borne by the player alone.

In a hypothetical market where risk is shared perfectly, overly risky behaviour does not occur. As a result, network theory and financial contagion theory lead us to suggest that the spatial organisation of the market is fundamental to market stability. Extending these theories to a more individualistic game theoretical optimum leads to the following research question:

Can players in financial markets exploit their position in the network? If so, what are the defining characteristics of these players?

The characteristics can be divided into two categories, variables relating to the player and variables relating to the market. These are: player integration, relative player integration, relative size and in/out ratio for the individual players. The market specific characteristics are market integration and size of the market. These measures will be explained further in the theoretical framework section.

Theoretical framework

Outline of the theory

Firms in a financial market can be of multiple types (banks, insurance, pension funds, etc.). To keep a certain level of abstraction, we view financial markets as networks where each player is a node (n), and the links (I) between the different nodes represent financial assets between firms. The terms 'financial markets' and 'financial networks' will therefore be used interchangeably. We will refer to firms in the network as 'players' or 'banks' interchangeably.

Every player in the network maximises their profit coming from external assets (e.g. mortgages, loans, etc.). In this profit maximisation, the players in the network weigh higher returns from risky external assets against the potential negative cost of these assets defaulting. The starting position of the model is the case where all the players have set up interbank contracts with a number of other players and have determined the value of the contract to each respective player. These contracts function as a tool to mitigate risk and diversify portfolios.

At this point, one of the players (the 'initiating player') has an external asset that defaults, creating the external default shock (X). Initially, the player receiving this shock tries to catch the shock by using their capital buffer. If the capital buffer does not suffice in capturing the shock, the player is forced to sell off some of the assets in the financial network (Allen and Gale, 2000) to raise capital. Selling off (or 'liquidating') these assets is a costly procedure.

After the first assets are liquidated, the banks connected to the initiating player will then receive the shock, by, for example, credit losses or defaults on payments from the shock initiating bank. Another way of the shock spreading is through the premature liquidation of assets. The market receives a signal that the asset is overpriced. Therefore the price of that asset goes down, which in turn devalues those assets in the portfolio of other banks. The same order of catching the shock with the capital buffer and passing on the shock through the liquidation of assets then continues through the system until the shock is reduced to such an amount that it no longer exceeds the capital buffer of the receiving players.

After the shock has been fully absorbed in the network, the players add up all the costs of the subsequent shocks they have received after the one external default hit the network. The expected cost of a default can then be found by multiplying the total cost of an external default by the probability of an external default occurring. The initiating player then maximises its profit by increasing risk to such a point where the additional revenue of a higher return on assets is outweighed by the expected cost of an external default.

The obtained values for external risk default can now be mapped against market characteristics to find when moral hazard occurs. To pinpoint this accurately, we map it against some key market and player characteristics later referred to as 'identifying coefficients'.

The following sections will provide an exact definition of the simulated financial network and its internal dynamics.

Institutional setting

In our research, we simulate a financial market where players determine their interactions with one another randomly. Within one interaction, there are no changes made to the network, implying no possibility for entrants, mergers or exits. We also assume the financial input parameters to be constant. That is, the risk-free rate² and the risk-return relationship are predetermined and fixed. Not only to achieve internal consistency within between simulations, but also because the aggregate economy reacts too sluggishly to still apply to the same shock.

There are three intuitive arguments for why we assume market organisation to be fixed and predetermined. First of all, establishing and ending contracts takes time, a lengthy technical and legal procedure is required. The span of this procedure is very likely to exceed that of the shock, implying that even if the market changes, it will be mapped and simulated as a different case (with *l*+1 connections). As a second argument, despite advances in information technology, the organisation of financial markets is still dependent on geography. Therefore the number of financial partners and possible connections is limited. Finally, social connections form an important basis for establishing financial markets and part of each other's social network are perhaps more likely to establish financial connections. The social factor, in its place is dependent on geography.

Within the predetermined and fixed network, every player has the same information and is identical in preferences and rationality, but only one player at the time has the defaulting external assets. This assumption makes it possible to pinpoint the exact effects of the default on the initiating player and what the resulting effects are. Furthermore, the firm and its employees are homogeneous in their preferences, rationality and information. Therefore, principal-agent problems cannot exist within the firm. This assumption is motivated not only by its necessity for the model but also by the fact that these types of niche product departments are relatively small and integrated.

Where in reality banks can altogether default or be bailed out by the state, the model being used in this research does not take this possibility into account. Not only is the exclusion of this possibility a useful simplifying assumption, it also ensures the market stability and aim of the banks to exploit the market structure without causing a systemic breakdown.

² The risk-free rate is the return an investor could receive by investing in risk free assets, generally government bonds such as the U.S. T-bill.

Setup of the model

The financial system being modelled has the following basic characteristics:

- n players
- I connections between the n players in the market
- S_n size of the *n* banks in the network
- X size of external shock to the initiating player

Using Sheldon and Maurer's (1998) assumption on interbank exposure where there is no information available, we assume a weighted uniform distribution of exposures across connected players. The weighting is done according to the size of the connected bank relative to the other connected banks.

Formally:
$$E_{i,j} = \frac{b_{i,j} \cdot s_j}{B_i \cdot S}$$

i.e. Exposure from player i to $j = \frac{\text{connection bank i to } j \cdot \text{size of the connected bank}}{\text{total size of all the connected banks}}$

In words, the exposure of player *i* to player *j* equals the value of the connection between the two (1 if they are connected, 0 otherwise) times the size of the receiving bank divided by the connections of all the connected banks in the network.

When a player receives a shock and is forced to use the capital buffer and liquidate assets, there are negative consequences for the bank's balance sheet as several assets need to be sold off in order to acquire more liquid assets (Allen and Gale, 2010).

First of all, the capital buffer (legally required minimum) should be 'refilled'. The money to do this has to come out of the bank's assets meaning that the value will be deducted from the total value of the bank. Second, not only does the bank lose value via the capital buffer but also liquidating assets costs money. This value again will be deducted from the bank's balance sheet. Combining the two costs above, for every shock the bank will receive the bank size decreases with an amount:

- If buffer > cost of the shock: X
- If buffer < cost of the shock: buffer + cost of liquidating assets \cdot (X buffer)

When adding all of the costs (through external shock and lagged shocks through network effects), the total impact of the shock on the initiating bank can be defined as $c_{default}$. This number is depicted as a fraction of the external shock, denoted as:

Formally: $\chi_{p_1} = \frac{c_{default, p_1}}{X}$

i.e.: cost of default to player $1 \text{ ratio} = \frac{\text{cumulative cost of default to player } 1}{\text{size of the initial shock}}$

When the cost for the individual player is known, a profit maximisation formula can be constructed using the cost of defaulting ratio χ_{p1} and the return on risky contracts $r_m(\alpha)$:

- Cost of defaulting: The cost of having the external contract with the external party defaulting is given by *X*_{default,p1}. Together with the probability of this external contract defaulting, the expected value of defaulting cost is given by: α · *X*_{default,p1}
- Return on risky contracts: We define the return on risky contracts (r_m) as a function of default risk (α) and the risk free rate (r_f).

$$r_{m} = f(r_{f}, \alpha)$$
$$r_{m}(\alpha) = r_{f} + \frac{\ln(100\alpha)}{\ln(100)}$$

Combining the return on the external contract and the expected losses from the default shock, we come to the following profit maximisation problem where the player tries to obtain the highest possible profit π by manipulating external contract default risk α :

$$\max_{\alpha} \left[\text{profit} = \text{return} - \text{expected cost} \right]$$
$$\max_{\alpha} \left[\pi_i = r_m(\alpha) - \alpha \chi_{def, p1} \right]$$
$$\max_{\alpha} \left[\left(r_f + \frac{\ln(100\alpha)}{\ln(100)} \right) - \alpha \chi_{def, p1} \right]$$

Maximising the profit function for the initiating firm with respect to α :

$$\frac{\partial \pi}{\partial \alpha} = \frac{1}{\alpha \ln (100)} - \chi_{def, p1} = 0$$
$$\alpha = \frac{1}{\ln (100) \chi_{def, p1}}$$

The value found for external default risk (α) is the essential variable in pinpointing which player in the financial network is most likely to engage in overly risky behaviour.

Mapping the external default risk to the identifying coefficients

By conducting a correlation study between the identifying coefficients, one can find the relationships between the different player- or market characteristics and incentives to engage in risky behaviour. The mapping of the coefficients will be controlled for network size and number of connections within the particular market.

In relating the obtained external default risk values to each of the following identifying coefficients:

– Market integration coefficient:

Market integration is a relatively common factor to investigate, as mentioned in Nier et al (2007). The argument behind mapping the market integration coefficient to the risk optimum is that well integrated markets have better risk sharing and therefore the shock gets absorbed faster and in a more consistent way (the propagation and absorption of the shock is not as dependent on the initiating player as it would be in a less integrated market).

$$\omega_{market} = \frac{total \ number \ of \ connections \ in \ the \ market}{maximum \ number \ of \ connections \ in \ the \ n \ market}$$

– Player integration coefficient:

The theory behind player integration coefficient is simply that a more integrated player has more channels to which it can dissipate risk. However, if the player has a lot of risk exposure from other players coming in as well, there will also be a large share of cost turning back to this player.

 $\omega_{p1} = \frac{number \ of \ connections \ to \ the \ player \ initiating \ the \ shock}{maximum \ number \ of \ connections \ to \ the \ rest \ of \ the \ n \ player \ market}$

Relative integration coefficient

This coefficient is a combination of the market- and player integration coefficient. It depicts the integration of the player relative to the market integration. If a player is relatively more integrated than the market, it will have a hub function in financial contagion whereas a player with relatively low integration will have to opportunity to send a shock into a system through the hub and let most of the shock be absorbed through this hub.

$$\omega_{rel} = \frac{number of connections to the player initiating the shock}{average number of connections per player}$$

Incoming/outgoing exposure coefficient

Although every player has the same total amount of direct risk exposure coming in and out, there are differences in networks between the number of incoming and outgoing risk exposure connections. This coefficient has a strong explanatory power because firms have control over this parameter and can adapt it to maximise their risky opportunities.

$$inout = \frac{incoming \ connections \ to \ p_1}{outgoing \ connections \ from \ p_1}$$

- Relative size coefficient:

This coefficient maps alpha among the relative size of the initiating bank. The intuition behind this is not that a bigger bank initiates a bigger shock (because the cost coefficient is relative) but it is about the absorptive capacity of the network and buffer requirements for different sizes of banks.

$$relsize = \frac{s_1}{(\sum_{2}^{n} s)/(n-1)}$$

Method

Overview

Following the logical steps made in the theoretical framework, the computer model replicates the market with random connections between players, then simulates the shock and subsequent profit maximisation. It then maps these results for α by the different identifying coefficients. The simulation repeats this process one thousand times for every number of players and the different numbers of connections in the network possible. This is a computationally very demanding process and can require multiple days to complete.

First off, the market needs to be set up. Having predetermined how many players there are and how many connections they have between them, the simulation randomly generates players in different sizes and connections between them while making sure that each player is connected to at least one other. When the connections are set up, the players determine relative exposure to each other using the relative size of the connected players as weights.

Now that the market is set up, the simulation introduces a shock to a random player. This player absorbs an amount of the shock that is equal to the capital buffer times the size of that particular bank. The rest of the shock is sent on to the connected players according to the exposures determined earlier. This process of absorbing and sending on shocks continues until the shock is fully absorbed. For the firm that had the external default in the beginning of the shock, a profit maximisation can now be calculated using the total cost of the number of times the shock hit that firm. This delivers α , our main coefficient of interest. Having obtained α , the mapping among different identifying coefficients can be done for every single time the market is simulated.

After constructing the pattern for the one particular shock in that market setup, the process is repeated multiple times and then averaged out over these iterations. When repeated often enough, the sample average will form a good representation of the population. Since finding an analytical solution is not only very complex, it also has to be recalculated whenever a parameter changes or the network structure is changed. Therefore, mapping the network on a computer and letting it generate a large number of random paths is a faster way to approach the right answer that is less prone to errors. This is called the Monte Carlo method and commonly used in financial simulation.

Simulation

This section gives an exact technical description of all the steps made in writing a code for simulation in Matlab[®] for those interested. The reader that is more interested in the results and interpretation can skip ahead to the next section 'Results'.

To show the process in a structured and transparent manner, the simulation description has been split up in 6 steps that correspond to the logical steps in the Matlab[®] code (appendix B) and the theoretical constructs made in the previous paragraph.

- 1) Set up a list of definitions and parameters (Matlab[®]: line 12-26)
 - a) N: maximum number of firms for which the market is being simulated Since the simulation becomes exponentially more demanding to compute as the number firms increases, N is currently set at 30 but any value equal to or higher than 2 firms will give results.
 - b) *I*: number of iterations to which the simulation should be run
 - For results that produce smooth results a number of 500 or higher is required. As the number of firms increases the number of iterations should increase with it as the number of different possible forms of the market increases. The number of different possible market forms for N=30 equals N^2 -N=870. In this simulation, a value of I=1000 is used.
 - c) S: size of players in the network (market capitalisation of the bank's division in the niche market)

Because the simulation is qualitative, the value of the bank size does not need to represent any real value. The deviation of the parameter is the important feature. In the current simulation, *S* is randomly drawn from a uniform distribution $S \in (100, 150)$. The deviation is set relatively low to ensure most results stem from market organisation and not from outliers in player size.

d) X: size of the shock being started by the initiating bank equal to the size of the initiating bank division (S)

The shock size equals the initiating bank's market capitalisation (S) in this particular setup because the size of the division is assumed to equal the size of the external assets.

- e) μ: buffer requirement for banks:
 Defined as a fraction of the bank's market capitalisation: 8%. This value is the minimum reserve requirement taken from the Basel II accord.
- f) γ : the cost of refilling the bank's buffer:

As explained in the theory section, the cost of refilling the buffer exceeds the buffer size itself because the firm needs to increase liquid assets by selling them off. In the current simulations, the cost quotient to refilling is γ times the size of the buffer. In this simulation, the γ coefficient equals 5.

In the following steps, the explanation is extended with an example for the simple case of a perfectly integrated market of n=3 players.

- A bank connections matrix (matrix B) and a connected exposure matrix (matrix E) are created. This matrix depicts all the banks and their respective links (Matlab[®]: line 48-76)

Visualised in a Matlab[®] biograph:



Figure 2: biograph of (*n*=3, *l*=6) market Create a vector of random bank sizes

S: [200 150 220];

Create the interbank exposure matrix using the following formula. The interbank exposure

between bank i and j is: $E_{i,j} = \frac{b_{i,j} \cdot s_j}{B_i \cdot S}$

in this case: the first column of matrix E is

$$E(1,1) = \frac{200 \cdot 0}{200 \cdot 0 + 150 \cdot 1 + 220 \cdot 1} = 0$$

$$E(2,1) = \frac{150 \cdot 1}{200 \cdot 0 + 150 \cdot 1 + 220 \cdot 1} \approx .405$$

$$E(3,1) = \frac{220 \cdot 1}{200 \cdot 0 + 150 \cdot 1 + 220 \cdot 1} \approx .595$$

$$E = \begin{bmatrix} 0 & .4762 & .5714 \\ .4054 & 0 & .4286 \\ .5946 & .5238 & 0 \end{bmatrix}$$

- 3) Run simulations for different levels of market integration for $n \in (3,N)$ players (Matlab[®]: line 78-102)
 - a) First, simulate player 1 receiving a shock from an external contract of size X,
 - b) After absorbing part of the shock (μ^*s_1) itself, the bank sends the shock through the network (if $x_1 > \mu s_1$)
 - i) According to the randomly determined interbank exposures, p₁ sends a financial shock to one or more players in the network
 - ii) These players absorb μs_1 of the shock, then send it on through other banks following the randomly determined interbank exposure matrix **E** (*if* $x_{2,i} > \mu s_i$).
 - iii) This logic continues until (*if* $x_{i,i} \le \mu s_i$) and the bank fully absorbs the shock.
 - c) Putting the logic above into practice: After having created the exposure matrix, the actual simulation of the shock can be done by creating the X_{in} and X_{out} matrices depicting respectively the shocks coming into the different players in the market and what their sending out onto the market.
 - i) generating the initial shock:

 $\mathbf{X}_{in}(1,1) = initial shock (X)$

 $\mathbf{X}_{out}(1,1) = x_{in} - \mu(S_1)$

ii) and subsequent transmission

 $\begin{aligned} \mathbf{X}_{in}(r,1:n) &= x_{out}(r-1,:)^*E' \\ \mathbf{X}_{out}(1,1) &= x_{in}(r,b) - \mu^*S_b \\ if \quad x_{in}(r,b) < mu^*S_b \\ \quad x_{out}(r,b) = 0 \end{aligned}$

iii) in the case of the example in 2), leading to the following matrices (with on the horizontal axis the bank and on the vertical axis the r^{th} round :

$$X_{in} = \begin{bmatrix} P_1 & P_2 & P_3 \\ 200 & 0 & 0 \\ 0 & 74.5946 & 109.4054 \\ 82.2672 & 39.3452 & 32.7876 \\ 21.7002 & 33.3741 & 53.7258 \\ 30.8214 & 17.7934 & 14.5852 \\ 2.7587 & 6.0087 & 11.8474 \\ 0 & 0 & 0 \end{bmatrix}$$
$$X_{out} = \begin{bmatrix} 184 & 0 & 0 \\ 0 & 62.5946 & 91.8054 \\ 66.2672 & 27.3452 & 15.1876 \\ 5.7002 & 21.3741 & 36.1258 \\ 14.8214 & 5.7934 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

d) After obtaining the shock-in matrix one can sum the total shock received (over the multiple rounds) by the particular bank. Since every time a bank has to refill the buffer, assets have to be sold prematurely. This means that the total cost to a bank for initiating a shock can be found by maximising the shocks in the first column of the X_{in} matrix to the size of the buffer $\mu(s_1)$ and then summing up the resulting vector. In the case of our example:

$$\mathbf{X}_{buf} = \mathbf{X}_{in}(:,1) = X_{buf} = \begin{bmatrix} 16 \\ 0 \\ 16 \\ 16 \\ 16 \\ 2.7587 \end{bmatrix}$$

The X_{buf} vector can be translated into C_{def} , the value depicting the cost of initiating a shock to the bank in question. In this case, $C_{def} = \gamma \cdot \sum x_{buf}$. In the optimisation stage of the model, C_{def} will be used as a fraction of the initial shock. That is:

$$\chi_{def} = \frac{C_{def}}{S_1}$$

Applying the cost calculation to the specific case:

$$\chi_{def} = \frac{C_{def}}{s_1} = \frac{333.7937}{200} = 1.6690$$

4) Initiating player optimisation: (Matlab-: line 115-121)

$$\pi_{i} = r_{m}(\alpha) V_{ext_asset} - \alpha \chi_{def}$$
$$B_{i} : \max \left[\pi_{i} = r_{m}(\alpha) V_{ext_asset} - \alpha \chi_{def} \right]$$

Depending on the form of the return on risk $(r_m(\alpha))$ function, optimisation of the profit (π) function can be done algebraically or by linearly generating a string of different values for π dependent on α in Matlab[®].

Since the analytical answer provides a more accurate estimate of π and is less computationally demanding in the Matlab[®] estimation process, solving the optimisation algebraically is preferred at all times.

$$\max\left[\pi_{i} = r_{m}(\alpha) X - \alpha \chi_{def}\right]$$
$$r_{m}(\alpha) = r_{f} + \frac{\ln(100\alpha)}{\ln(100)}$$
$$\frac{\partial \pi}{\partial \alpha} = \frac{1}{\alpha \ln(100)} - \chi_{def, p1} = 0$$
$$\alpha = \frac{1}{\ln(100) \chi_{def, p1}}$$

With the optimal number for α obtained, this value can be documented in a matrix that captures all the alphas among the dimensions *n* (*number of players*), *I* (*total number of connections between the players*) and *i* (*number of iterations*).

Applying the α optimisation to the example:

$$\alpha = \frac{1}{\ln(100) \cdot 1.6690} = 0.1301$$

An α of 0.1301 implying a level of 13% risk for this particular setup.

- Structure the results in a form so that conclusions can be drawn. (Matlab[®]: line 123-155)
 In order to obtain intelligible results from the data, the following structural form modifications are applied to the raw output:
 - a) Defining different coefficients to which the external default risk should be measured, and then mapping alpha among these axes. The coefficients to be mapped are given in the theoretical framework section and defined respectively as:
 - i) Coefficient of market integration: $\omega_{market} = \frac{total number of connections in the market}{maximum number of connections in the n market} = \frac{l}{n^2 n}$ As the coefficient increases, the market will get more integrated with $\omega_{market} = 1$ being perfect intragration.

ii) Coefficient of player integration:

<i>(</i>) –	number of connections to the player initiating the shock	$\underline{sum}(B(:,1)+B(1,:))$
ω_{p1} –	maximum number of connections to the rest of the n player market	2(n-1)

iii) $\omega_{p1} \in (0,1)$

As for market integration, the coefficient varies between zero and one with zero being no single connection and one depicting perfect integration.

iv) $\omega_{rel} = \frac{number \ of \ connections \ to \ the \ player \ initiating \ the \ shock}{average \ number \ of \ connections \ per \ player \ (not \ including \ player \ initiating \ shock)} = \frac{sum(B(:,1)+B(1,:))}{l}$ $\omega_{rel} > 1$: the player is more integrated than the average player on the market

 ω_{rel} < 1: the player is less integrated than the average player on the market

v) In/out ratio:

The in/out ratio is can be defined as the number of incoming connections over the number of outgoing connections:

 $inout = \frac{incoming \ connections \ to \ p_1}{outgoing \ connections \ from \ p_1} = \frac{sum(B(:,1))}{sum(B(1,:))} inout \in \left(\frac{1}{n-1}, n-1\right)$

vi) Relative size:

The relative size coefficient is defined as the size of the initiating player (p_1) relative to the average size of the rest of the market.

$$relsize = \frac{s_1}{(\sum_{2}^{n} s)/(n-1)}$$
$$relsize \in \left(\frac{2}{3}, 1\frac{1}{2}\right)$$

b) By showing the relation between the player integration, market integration and the external default risk coefficient in a graph, one can detect the relation between the spatial organisation of the market and the resulting risk-taking behaviour. Apart from depicting this graphically, the relation can also be estimated by using correlation graphs.

Results

In appendix A, a larger version of the figures in this section can be found. This is to aid the understanding of sometimes complicated figures with three axes.

Figure 3 shows the relationship between α , or external default risk, and number of players and connections. Here we see a clear trend of α increasing as the number of connections and the number of players increases. The interpretation of this is that as the network increases in size and connectedness, the initiating player's ability to take on more risk increases as the network can bear more of the cost. As seen in Figure 4, the average cost of the initiating player goes up in small and less integrated networks. As the network size and connectedness increases, the cost to the initiating player decreases because of the network absorbing the shock for the initiating player.

Relating back to the literature, the consensus is that the larger and more connected the network, the more robust it becomes, and thus more capable of dissipating shocks. This due to that in a large and well connected network, there are more paths for the shock to be spread to other players, and hence the cost to the initiating bank decreases.

These findings are in line with Nier et al. (2007)'s observations that for systems with a high level of capital, increases in interbank linkages improve the system's ability to handle shocks. In response to the system's absorptive capacity, we can see the firm maximizing the risk taken in this market setup. Which firm in this particular market depends on the firm-specific characteristics, the following paragraphs will focus on these.



Market integration

Market integration measures the degree that the market is interconnected. The higher the market integration coefficient, the more the individual players are connected to one another. In our setting, all

players must at least have one connection to another. In a perfectly integrated market, all players are connected to every single other player.

In the scatter plot showing the calculated values of α versus the different values of market integration, Figure 5, the data is smoothed with a moving average procedure and the fit is generated by a rational estimation with a quadratic polynomial numerator and a linear polynomial denominator.







figure 6: a vs. market integration correlation

Figure 5 depicts a clear upward trend for α with respect to market integration. This implies that in a more integrated market, the initiating player takes on more external default risk than it would do a in a less integrated market. The surface graph depicting the correlations between market integration and α under different numbers of players and connections between them, Figure 6, confirms this data trend and adds that the positive correlation especially holds for networks with relatively few connections.

A potential reason for market integration to be positively correlated with external default risk is that a more integrated market has a higher absorptive capacity for the shock being spread. The slightly negative correlation for large and integrated markets suggests there is a maximum for nearly-perfectly integrated markets.

Player integration

Player integration is the degree to which the initiating player is connected to the rest of the network.

In Figure 7, the scatter plot depicting the levels of external default risk (α) taken relative to the player integration coefficient, smoothed data has been generated using a moving average procedure and the fit has been estimated using a rational procedure with a cubic polynomial numerator and a linear polynomial denominator .





figure 8: α vs. player integration correlation

In less connected networks, regardless of size, player integration is strongly negatively correlated with external default risk (α), as shown in Figure 8. But as the network grows larger and more connected, the correlation between player integration and α slowly turns positive. The strong negative correlation between player integration and external default risk taken can be found in the scatter plot with estimated values.

Relative player integration

The relative player integration measures how the player is connected relative to other banks in the network. Where 1 signifies that the initiating player is equally as connected as the average other bank in the network. Values above 1 indicate that the initiating bank is more connected than the average bank, and vice versa.





figure 10: α vs. relative player integration correlation

Figure 9 shows the data of the calculated external default risk (α) in relation to relative player integration. In this scatter plot, data is smoothed using a moving average procedure and the fit is estimated using a rational estimation with a linear polynomial numerator and a cubic denominator polynomial.

In figure 10, when comparing the correlation graphs of relative player integration and player integration, one can see that the correlations of relative player integration and player integration with α roughly display the same path. The strongest correlations are found in the least integrated markets, this pattern of strong negative correlation shows over all market sizes. A possible explanation for this strong correlation in the less integrated markets would be that relatively less integrated markets generally have a higher sensitivity to placement of the initiating player. In other words, if one is in a market that is not very well integrated, having a high level of player integration will leave the initiating player more exposed to shocks then when the general market around the initiating player is relatively large and more integrated.

There is a slight positive correlation found in the almost perfectly integrated markets with a large number of players, this is a relatively weak relation compared to the negative correlation found among the lesser integrated markets. This lower correlation means that in less integrated markets player positioning is generally not as important.

In/out ratio

The in/out-ratio measures the relationship between ingoing and outgoing connections for the initiating bank.

In the in/out ratio scatter plot, Figure 11, the smoothed data is generated with following a moving average procedure and the fit is generated using a rational estimate with a linear polynomial numerator and denominator. The in/out ratio is a measure of the initiating player's relationship between ingoing and outgoing connections.





figure 12: a vs. in/out ratio correlation

In networks with a large number of players, but not fully interconnected, the in/out-ratio is highly positively correlated with external default risk (α). As the network becomes more interconnected, the correlation drops. This is because the in/out-ratio converges to 1 as the network becomes more connected, thus less differentiating. In the fitted graph over the scatter plot, a rather counter-intuitive relation becomes visible: when the in/out ratio increases, the external default risk increases with it. The

positive relation between the in/out ratio and α seen in Figure 12 implies that the initiating player starts to behave more risky when it has relatively more connections coming in than going out (this relation especially holds in relatively well connected markets). Because it holds especially in relatively large and connected markets, the estimated relation in Figure 11 is likely to be fundamentally caused by market integration. There might be a high number of incoming connections, but this does not imply that all of these carry a shock.

Relative size of the initiating player

The relative size measures how large the initiating bank is compared to the rest of the banks in the network.





figure 14: α vs. relative size correlation

The relative size of the initiating bank is negatively correlated with external default risk (α) in small networks and networks of intermediate integration, as seen in Figure 13. When studying the scatter plot of the external default risk (α) and relative size of the initiating player, there seems to be no general pattern over all different number of players and connections. A possible explanation for the lack of general pattern among all firm sizes can be the relatively small difference in player sizes: $S \in (100, 150)$

Conclusion

To answer the main research question as defined in the central problem statement, we divide the variables from the results section into two categories: player specific characteristics and market specific characteristics.

The player specific characteristics include: player integration, relative player integration, relative player size and the in/out ratio.

Looking at player integration, we see that less integrated networks lead to lower external default risk taken on. But as the player becomes more integrated, and hence has more paths by which to spread the shock, the risk taken increases. However, in very large and well-integrated markets, risk is negatively correlated with market integration, as there are more paths for the shock to travel back to the initiating player.

The relationship between relative player integration and external default risk indicates that the position of the player in the market has a strong relation to the amount of risk taken on. In large, but less integrated networks there is a strong negative correlation between relative player integration and external default risk. As the network integration increases the relationship becomes positive.

The market specific characteristics consist of market integration and market size.

Market integration is a strong driver of external default risk (α). This also holds for the size of the market. In small markets less risk is initially taken on, as the potential to offload the cost of the default to the network is limited. As the market grows larger, its ability to dissipate more of the shock also grows.

Combining the player and market specific results, we can conclude the general case and answer the main research question:

Can players in financial markets exploit their position in the network?

From our results we conclude that players can use their network of connected banks to offset some of the cost of defaulting external products, and thus increase the potential for profit by taking on higher external default risk.

If so, what are the defining characteristics of these players?

The most important variables are the market characteristics ones, i.e. market integration and size of the market. This leads us to conclude that the composition of the market itself is the leading driver in risk taking behaviour. The characteristics of the individual bank are less important, but still play a role, as the bank has to be well connected to be able to offload the cost of the defaulting external product.

Discussion

Data acquisition

The reason there is no comparative study between the simulated model and data observed from the market is because of a lack of availability. Cerutti et al (2012) investigate what data is available, and what more data is needed, to measure and analyse systemic risk. They find that while aggregate data is publicly available, bank level data is not. In most markets, the financial authority will have extensive records of interbank exposures and all transactions made, but they deliberately choose to not reveal this information for competition policy reasons.

As discovered during the recent crisis, aggregate data is not enough during market turmoil. Supervisors and researchers need access to much more detailed information to gauge effects of policy decisions or to predict failures.

"Detecting these types of stresses early on requires detailed breakdowns of banks' assets and liabilities (i.e. by currency, instrument, residual maturity, and, if possible, counterparty type and country), and their joint analysis across many banks." Cerutti et al (2011) .There are initiatives in place to reduce the information deficit described in Cerutti.

Suggestions for further research

Applying the model to actual data to predict which bank is most likely to default on its assets More specifically, a data-driven research question leads us to the following hypotheses based on the three sub questions stated in the central problem statement:

- H1: Integration

The key in explaining risk taking behaviour is integration of the market and the initiating player, this implies that there are two key variables in the regression:

 ω_{market} : integration coefficient of the market

 ω_1 : integration coefficient of the initiating player

$$\alpha = \beta_0 + \beta_1 \omega_{market} + \beta_2 \omega_{p1}$$
$$H_1: \quad \beta_1 = \beta_2 = 0$$
$$A_1: \quad \beta_1 \neq 0$$
$$\beta_2 \neq 0$$

H2: player specific characteristics

The second hypothesis ignores market characteristics and focuses on the initiating player. The explanatory factors in this case are relative player integration, relative player size and in/out ratio of the player:

$$\alpha = \beta_0 + \beta_1 \omega_{rel} + \beta_2 relsize + \beta_3 inout$$
$$H_2: \quad \beta_1 = \beta_2 = \beta_3 = 0$$
$$A_2: \quad \beta_1 \neq 0$$
$$\beta_2 \neq 0$$
$$\beta_3 \neq 0$$

- H3: market specific characteristics:

After focusing on specific players, H3 states that only market specific characteristics matter in explaining the external default risk taken:

$$\alpha = \beta_0 + \beta_1 \omega_{market} + \beta_2 n$$
$$H_3: \quad \beta_1 = \beta_2 = 0$$
$$A_3: \quad \beta_1 \neq 0$$
$$\beta_2 \neq 0$$

Adapting the model to other financial interbank products / interbank markets

We set up the model to represent the market for CDOs, and interbank lending but one could adapt it to fit other specific product characteristics. An interesting example to where the financial contagion model can apply is in the case for sovereign debt, as researched by Acharya et al. (2011) in "A Pyrrhic Victory? – Bank Bailouts and Sovereign Credit Risk". In private markets, the limiting factor when applying the model to actual products is data availability. The market for sovereign debt should have better data availability allowing for a more thorough and applied research including testing of the model.

Expanding the model with more specific network dynamics

This paper focuses on the CDO and loan market between banks. However, with slight adaptations the market with other type of players can be simulated. These different types of players all have their unique behaviours in the market we are simulating:

In the case of a hedge fund, there is no capital reserve requirement and the firm can be highly speculative and relatively less connected. This allows for more volatile markets. On the other side of the spectrum there is the insurance market. This market is very connected and taking in a lot of contracts. Combining banks, hedge funds, insurance companies and other players such as sovereigns allows for more diverse networks and can generate very interesting results.

Applying the model to within-bank dynamics (principal agent problem)

One of the problems in the past has been that individual traders have taken positions that have lead to substantial losses and write-downs for the banks where they have worked. One example is the \$2 billion loss taken by UBS, stemming from the actions of one trader at their 'Delta-one' desk. Here, the problem is that individual traders do not necessarily work for the good of the bank, but rather to maximize their own salary and bonuses. For this exact reason the assumption of uniform preferences between firm and employee might not apply. There are two ways in which the principal agent problem can apply:

- While keeping the general market framework the same as in this research, but by adding an intra-firm principal agent problem, one can find an alternate explanation for why the firms take on excessive risk and exploit their market position.
- Intra-firm dynamics: by scaling down this research, one can replace the market with the firm and replace the firms in the market with players in the firm. Because of a lack of transparency and different places in the firm's internal network, traders have the potential to take on excessive risk and take the profits while the rest of the firm takes the large part of the potential risk.

Setting up a maximisation strategy for banks to exploit their market position

Since banks have a great deal of information on their counterparties in the market, there is a potential to exploit this potential and set up a maximisation strategy. In other words, banks can use financial simulation of our type to generate profit while others in their network take the excess risk.

Setting up an optimisation model for central banks to determine certain goals by adjusting financial market parameters.

Central Banks could examine how the external default risk reacts to different capital buffer sizes, connections and number of players.

By using our research as a tool to tweak financial economy parameters, central banks and other financial authorities can influence the market by setting up restrictions for certain types of players in a financial market, or test how their planned policies influence market dynamics. A typical aim of a financial authority in this case would be minimising default rates, or the variance in external default risk taken between different players in the financial network.

Make a model for market entry and where this new player is bound to position itself

In our model, new players do not enter the market, but it could be extended to account for this. For example, given the positions and actions of other players, where would it be optimal to position given in- and outgoing connections, size of banks, etc. This extension has applications not only for those scientifically interested, but also for firms seeking to enter the market, or financial authorities.

Extend the market to account for full firm defaults (across all products)

In the current state, the model does not handle defaults of firms. Generally the shocks aren't large enough to deplete the buffer and then fully deplete the banks entire capitalisation. However, we only let one bank at a time default, if the model were extended to allow for multiple banks to simultaneously default, the combination of shocks could lead banks to fully default.

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Appendices

Appendix A – Graphs

Figure 3: external default risk surface plot



Figure 4: average cost of default surface plot



Figure 6: α vs market integration correlation



number of connections between the players



















Appendix B – Matlab[®] code

General simulation code

```
1
     clc
 2
3
     clear
     colormap('jet')
 4
     pause on
 5
 6
7
     currentfolder=pwd
     cd C:\Users\fnieuwendijk\Dropbox\thesis\matlab\results;
8
     date=(sprintf(datestr(now, 30)))
9
     mkdir (sprintf(date))
10
     cd (date)
11
12
     %number of banks: N
13
     N=30;
14
     %number of iterations: I
15
     I=1000;
16
     %max number of links: L
17
     L=N^2-N;
18
     %initial cost of default (part of initial shock):
19
     gamma_init=0;
20
     % cost ratio of buffer money: gamma
21
     gamma=5;
22
     %avg and sd bank size: S
23
     avg S=100;
24
     sd_S=50;
25
     %reserve requirement banks: mu
26
     mu=.08;
27
28
     time=zeros(N,2);
29
     xcounterror='fine';
30
     links=zeros(L,N,I);
31
     p inte=zeros(L,N,I);
32
     m inte=zeros(L,N,I);
33
     rel inte=zeros(L,N,I);
34
     relsize=zeros(L,N,I);
35
     inout=zeros(L,N,I);
36
     size=zeros(L,N,I);
37
     cost=zeros(L,N,I);
38
     nmap=zeros(L,N,I);
39
     lmap=zeros(L,N,I);
40
     alph=zeros(L,N,I);
41
     j=1;
42
43
     for n=3:N
44
         tic
45
         for l=n:n^2-n
46
              for i=1:I
47
48
                  B1=zeros(1, n^{*}(n-2));
49
                  Ri=randperm(n*(n-2));
50
                  Ri=Ri(1:l-n);
51
                  B1(Ri)=1;
52
                  B1=reshape(B1, n-2, n);
53
                  B2=zeros(n-1,n);
54
                  for n1=1:n
55
                      Re=randperm(n-1);
56
                      Re=Re(1);
57
                      B2(:,n1)=[B1(1:Re-1,n1);1;B1(Re:end,n1)];
58
                  end
59
                  B3=zeros(n);
60
                  for n1=1:n
                      B3(1:n,n1)=[B2(1:n1-1,n1);0;B2(n1:end,n1)];
61
62
                  end
```

```
63
                   B=B3;
 64
 65
                   %set bank sizes
 66
                   S=avg S+sd S*rand(1,n);
 67
 68
                   %calculate exposure matrix
 69
                   for b=1:n
 70
                       for a=1:n
 71
                           E(b,a) = (S(b) * B(b,a)) / (S*B(:,a));
 72
                           if (S*B(:,a))==0
 73
                                E(b, a) = 0;
 74
                           end
 75
                       end
 76
                   end
 77
 78
                   %defining the size of the shock
 79
                   X=S(1,1);
 80
                   %calculate the shock matrix X
 81
                   xin=zeros(20,n);
 82
                   xout=zeros(20,n);
 83
                   xin(1,1)=X;
 84
                   xout(1,1)=X-mu*S(1,1);
 85
                   for r=2:20
 86
                       if sum(xout(r-1,:))>0
 87
                           %define X in matrix
 88
                           xin(r,1:n) = xout(r-1,:)*E';
 89
                           %define X out matrix
 90
                           xout(r,:)=xin(r,:)-mu*S;
 91
                           xout(xout<0)=0;
 92
                       end
 93
                   end
 94
 95
                   %adding up the buffer values in the incoming shocks matrix (excluding
 96
      the initial shock)
 97
                   xcount=xin(:,1);
 98
                   xcount(xcount>mu*S(1,1))=mu*S(1,1);
 99
                   C=gamma*sum(xcount);
100
                   if xcount(20,1)>0
101
                       xcounterror='ERROR in xcount!'
102
                   end
103
104
                   %documenting all important variables into result matrices
105
                   %number of links of bank 1
106
                   links(l,n,i)=sum(B(:,1))+sum(B(1,:));
107
                   %integration coefficient
108
                   p inte(l,n,i) = (sum(B(:,1))+sum(B(1,:)))/(2*(n-1));
109
                   m inte(l,n,i)=(sum(sum(B(2:end,2:end)))/((n-1)^2-(n-1)));
110
                   rel inte(l,n,i)=p inte(l,n,i)/m inte(l,n,i);
111
                   %size of the bank
112
                   rel size(l,n,i)=S(1,1)/mean(S);
113
                   %incoming/outgoing connections for p1
114
                   inout(l,n,i)=sum(B(:,1))/sum(B(1,:));
115
116
                   %cost to player 1 of starting a shock
117
                   cost(l,n,i) = C/S(1,1);
118
                   beta=C/X;
119
120
                   %maximising pi wrt other variables - the algebraic solution
121
                   alph(l,n,i)=(1/cost(l,n,i))/log(100);
122
123
                   %mapping n to other match to alph for scatter plot
124
                   nmap(l,n,i)=n;
125
                   lmap(l,n,i)=l;
126
                   %alph mapping:
127
                   %omega=[alph;omega market;omega player;omega relative]
```

```
128
129
      omega(:,j)=[alph(l,n,i);m inte(l,n,i);p inte(l,n,i);rel inte(l,n,i);rel size(l,n,i)
130
      ;inout(l,n,i);nmap(l,n,i)];
131
                  alph_j(:,j)=alph(l,n,i);
132
                  minte_j(:,j)=m_inte(l,n,i);
133
                  pinte j(:,j)=p inte(l,n,i);
134
                  relinte_j(:,j)=rel_inte(l,n,i);
135
                  relsize_j(:,j)=rel_size(l,n,i);
136
                  inout_j(:,j)=inout(l,n,i);
                  nmap_j(:,j)=nmap(l,n,i);
137
138
                  lmap_j(:,j)=lmap(l,n,i);
                  j=j+1;
139
140
141
                     %end of i loop
              end
142
      143
              %correlation mapping along 1 and n
144
              %market
145
              prelim_corr_m=corrcoef(alph(l,n,:),m_inte(l,n,:));
146
              corr_m(l,n)=prelim_corr_m(2,1);
147
              %player
148
              prelim corr p=corrcoef(alph(l,n,:),p inte(l,n,:));
149
              corr p(l,n)=prelim corr p(2,1);
150
              prelim corr relinte=corrcoef(alph(l,n,:),rel inte(l,n,:));
151
              corr relinte(1,n)=prelim corr relinte(2,1);
152
              prelim_corr_relsize=corrcoef(alph(l,n,:),rel_size(l,n,:));
153
              corr_relsize(l,n)=prelim_corr_relsize(2,1);
154
              prelim_corr_inout=corrcoef(alph(l,n,:),inout(l,n,:));
155
              corr_inout(l,n)=prelim_corr_inout(2,1);
156
157
              %show progress within loop
158
              clc
159
              cd
160
              progress n=[roundn(((l-n+(i/I))/(n^2-2*n+1))*100,-4) n]
161
162
          end
163
          time(n,1)=n;
164
          time(n,2)=toc;
165
          time(n,:)
166
          total time=sum(time(:,2))
167
          save results.mat
168
      end
169
170
      xcounterror
171
      save results.mat
172
173
      %create averages
174
          avg alph=mean(alph,3)
175
          avg cost=mean(cost,3)
176
177
      %set zero to NaN and add N+1=Nfor plotting purposes
178
          avg alph(avg alph==0)=NaN;
179
          avg alph(:,N+1) = avg alph(:,N);
180
          avg cost(avg cost==0)=NaN;
181
          avg cost(:,N+1)=avg cost(:,N);
182
          corr_m(corr_m==0)=NaN;
183
          corr_m(:,N+1)=corr_m(:,N);
184
          corr_p(corr_p==0)=NaN;
185
          corr p(:,N+1)=corr_p(:,N);
186
          corr_relinte(corr_relinte==0)=NaN;
187
          corr_relinte(:,N+1)=corr_relinte(:,N);
188
          corr relsize(corr relsize==0)=NaN;
189
          corr relsize(:,N+1)=corr relsize(:,N);
190
          corr inout(corr inout==0)=NaN;
191
          corr inout(:,N+1)=corr inout(:,N);
192
193
      %plotting the alpha graph
```

```
194
      surf alph=surf(avg alph, gradient(avg alph));
195
           title('alpha')
196
          xlabel('number of players (n)')
197
          ylabel('number of connections (1)')
198
          zlabel('external default risk (alpha)')
199
      pause
200
      saveas(surf_alph,'surf_alpha.bmp')
201
202
      %plotting the avg cost graph
203
      surf_cost=surf(avg_cost, gradient(avg_cost));
204
          title('average cost')
205
          axis([0 30 0 870])
206
          xlabel('number of players (n)')
207
          ylabel('number of connections (1)')
208
          zlabel('average cost of default to player 1')
209
      pause
210
      saveas(surf cost,'surf cost.bmp')
211
212
      %making surface plots for the correlation coefficients
213
      surfcorr m=surf(corr m);
214
          title ('market integration correlation with alpha')
215
          xlabel('number of players')
216
          ylabel ('number of connections between the players')
217
          zlabel('correlation coefficient')
218
      pause
219
      saveas(surfcorr m, 'surfcorr m.bmp')
220
221
      surfcorr_p=surf(corr_p);
222
          title ('player integration correlation with alpha')
223
          xlabel('number of players')
224
          ylabel('number of connections between the players')
225
          zlabel('correlation coefficient')
226
      pause
227
      saveas(surfcorr_p,'surfcorr_p.bmp')
228
229
      surfcorr_relinte=surf(corr_relinte);
230
          title ('relative player integration correlation with alpha')
231
          xlabel('number of players')
232
          ylabel('number of connections between the players')
233
          zlabel('correlation coefficient')
234
      pause
235
      saveas(surfcorr_relinte,'surfcorr_relinte.bmp')
236
237
      surfcorr_relsize=surf(corr_relsize);
238
          title ('relative size of the initiating player correlation with alpha')
239
          xlabel('number of players')
240
          ylabel('number of connections between the players')
241
          zlabel('correlation coefficient')
242
      pause
243
      saveas(surfcorr relsize, 'surfcorr relsize.bmp')
244
245
      surfcorr inout=surf(corr inout);
246
          title ('in/out ratio of the initiating player correlation with alpha')
247
          xlabel('number of players')
248
          ylabel('number of connections between the players')
249
          zlabel('correlation coefficient')
250
      pause
251
      saveas(surfcorr inout,'surfcorr inout.bmp')
252
253
      %generating curve fitting graphs
254
      fit=figure;
255
      cd (currentfolder);
256
      fit_inout(inout_j,alph_j)
257
      axis([0 1 0 0.55])
258
      xlabel('in/out ratio')
259
      ylabel('alpha')
```

```
260
      cd C:\Users\fnieuwendijk\Dropbox\thesis\matlab\results;
261
      cd (date);
262
      saveas(fit,'fit_inout.bmp')
263
264
      cd (currentfolder);
265
      fit_minte(minte_j,alph_j)
266
      axis([0 1 0 0.55])
267
      xlabel('market integration')
268
      ylabel('alpha')
269
      cd C:\Users\fnieuwendijk\Dropbox\thesis\matlab\results;
270
      cd (date);
271
      saveas(fit,'fit_minte.bmp')
272
273
      cd (currentfolder);
274
      fit_pinte(pinte_j,alph_j)
275
      axis([0 1 0 0.55])
276
      xlabel('player integration')
277
278
      ylabel('alpha')
      cd C:\Users\fnieuwendijk\Dropbox\thesis\matlab\results;
279
      cd (date);
280
      saveas(fit,'fit_pinte.bmp')
281
282
      cd (currentfolder);
283
      fit relinte(relinte j,alph j)
284
      axis([0.2 10 0.07 0.55])
285
      xlabel('relative integration')
286
      ylabel('alpha')
287
      cd C:\Users\fnieuwendijk\Dropbox\thesis\matlab\results;
288
      cd (date);
289
      saveas(fit,'fit relinte.bmp')
290
291
      close all
292
      cd (currentfolder)
```

Curve fitting code: Market integration data

```
1
     function createFit(minte j,alph j)
 2
     %CREATEFIT Create plot of data sets and fits
 3
     8
         CREATEFIT (MINTE_J, ALPH_J)
 4
     8
         Creates a plot, similar to the plot in the main Curve Fitting Tool
 5
     % Number of data sets: 2
 6
         Number of fits: 1
     8
 7
 8
     % Data from data set "alph j vs. minte j":
 9
     8
           X = minte j:
10
     8
           Y = alph_j:
11
     00
           Unweighted
12
13
     % Data from data set "alph_j vs. minte_j (smooth)":
14
     8
           X = minte j:
15
            Y = alph_j:
     8
16
     8
           Unweighted
17
18
     % Set up figure to receive data sets and fits
19
     f = clf;
20
     figure(f );
21
     set(f_, 'Units', 'Pixels', 'Position', [451 49 912 592]);
22
      % Line handles and text for the legend.
23
     legh_ = [];
24
     legt = {};
25
     % Limits of the x-axis.
26
     xlim_ = [Inf -Inf];
27
     % Axes for the plot.
28
     ax = axes;
     set(ax_, 'Units', 'normalized', 'OuterPosition', [0 0 1 1]);
29
     set(ax_, 'Box', 'on');
30
31
     axes(ax );
32
     hold on;
33
34
     % --- Plot data that was originally in data set "alph j vs. minte j"
35
     minte_j = minte_j(:);
36
     alph_{j} = alph_{j}(:);
     h_ = line(minte_j,alph_j,'Parent',ax_,'Color',[0.333333 0 0.666667],...
37
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
38
39
40
     xlim (1) = min(xlim (1),min(minte j));
41
     xlim (2) = max(xlim_(2), max(minte_j));
42
     legh_(end+1) = h_;
43
     legt_{end+1} = 'alph_j vs. minte_j';
44
45
     % --- Plot data that was originally in data set "alph_j vs. minte_j (smooth)"
46
     sm_.y2 = smooth(minte_j,alph_j,5,'moving',0);
47
     h_ = line(minte_j,sm_.y2,'Parent',ax_,'Color',[0.333333 0.666667 0],...
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
48
49
50
     xlim (1) = min(xlim (1), min(minte j));
51
     xlim(2) = max(xlim(2), max(mintej));
     legh(end+1) = h;
52
53
     legt_{end+1} = 'alph_j vs. minte_j (smooth)';
54
55
      % Nudge axis limits beyond data limits
56
     if all(isfinite(xlim ))
57
          xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
58
          set(ax ,'XLim',xlim )
59
     else
          set(ax_, 'XLim',[-0.01, 1.01]);
60
61
     end
62
63
      % --- Create fit "fit 1"
64
     ok = isfinite(minte j) & isfinite(alph j);
65
     if ~all( ok_ )
66
          warning( 'GenerateMFile:IgnoringNansAndInfs',...
```

```
67
                'Ignoring NaNs and Infs in data.' );
 68
       end
       st_ = [0.0056677695070871392 0.16149302168494162 0.17933927937658833
 69
 70
       0.86447251063913622 ];
 71
       ft_ = fittype('rat21');
 72
 73
       % Fit this model using new data
 74
       cf_ = fit(minte_j(ok_),alph_j(ok_),ft_,'Startpoint',st_);
 75
       % Alternatively uncomment the following lines to use coefficients from the
% original fit. You can use this choice to plot the original fit against new
 76
 77
       % data.
 78
            cv = { 0.21936868351106498, 0.31184834319708604, 0.042403634469485321,
       8
 79
       0.095953558350834162};
 80
       % cf_ = cfit(ft_,cv_{:});
 81
 82
       % Plot this fit
 83
       h_ = plot(cf_,'fit',0.95);
 84
       set(h (1), 'Color', [1 0 0], ...
            'LineStyle','-', 'LineWidth',2,...
'Marker','none', 'MarkerSize',6);
 85
 86
 87
       \% Turn off legend created by plot method.
 88
       legend off;
 89
       % Store line handle and fit name for legend.
 90
       legh_(end+1) = h_(1);
       legt_{end+1} = 'fit 1';
 91
 92
 93
       % --- Finished fitting and plotting data. Clean up.
 94
       hold off;
 95
       % Display legend
 96
       leginfo_ = {'Orientation', 'vertical'};
 97
       h_ = legend(ax_,legh_,legt_,leginfo_{:});
98
       set(h_,'Units','normalized');
 99
       t_ = get(h_, 'Position');
100
       t(1:2) = [0.653874, 0.142976];
101
       set(h_,'Interpreter','none','Position',t);
102
       % labels x- and y-axes.
      xlabel(ax_, 'market integration');
ylabel(ax_, 'alpha');
103
```

104

Curve fitting code: Player integration data

```
1
     function createFit(pinte j,alph j)
 2
     %CREATEFIT Create plot of data sets and fits
 3
     8
         CREATEFIT (PINTE_J, ALPH_J)
 4
     8
         Creates a plot, similar to the plot in the main Curve Fitting Tool
 5
     % Number of data sets: 2
 6
         Number of fits: 1
     8
 7
 8
     % Data from data set "alph j vs. pinte j":
 9
     8
           X = pinte j:
10
     8
           Y = alph_j:
11
     00
           Unweighted
12
13
     % Data from data set "alph_j vs. pinte_j (smooth)":
14
     8
           X = pinte j:
15
           Y = alph j:
     8
16
     8
           Unweighted
17
18
     % Set up figure to receive data sets and fits
19
     f = clf;
20
     figure(f );
21
     set(f_,'Units','Pixels','Position',[469 119 680 474]);
22
     % Line handles and text for the legend.
23
     legh_ = [];
24
     legt = {};
25
     % Limits of the x-axis.
26
     xlim_ = [Inf -Inf];
27
     % Axes for the plot.
28
     ax = axes;
     set(ax_, 'Units', 'normalized', 'OuterPosition', [0 0 1 1]);
29
     set(ax_, 'Box', 'on');
30
31
     axes(ax );
32
     hold on;
33
34
     % --- Plot data that was originally in data set "alph j vs. pinte j"
35
     pinte_j = pinte_j(:);
     alph_j = alph j(:);
36
     h_ = line(pinte_j,alph_j,'Parent',ax_,'Color',[0.333333 0 0.666667],...
37
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
38
39
40
     xlim (1) = min(xlim (1),min(pinte j));
41
     xlim (2) = max(xlim_(2), max(pinte_j));
42
     legh_(end+1) = h_;
43
     legt_{end+1} = 'alph_j vs. pinte_j';
44
45
     % --- Plot data that was originally in data set "alph_j vs. pinte_j (smooth)"
46
     sm_.y2 = smooth(pinte_j,alph_j,5,'moving',0);
47
     h_ = line(pinte_j,sm_.y2,'Parent',ax_,'Color',[0.333333 0.666667 0],...
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
48
49
50
     xlim (1) = min(xlim (1), min(pinte j));
51
     xlim (2) = max(xlim (2), max(pinte j));
     legh(end+1) = h;
52
53
     legt_{end+1} = 'alph_j vs. pinte_j (smooth)';
54
55
     % Nudge axis limits beyond data limits
56
     if all(isfinite(xlim ))
57
          xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
58
          set(ax ,'XLim',xlim )
59
     else
          set(ax_, 'XLim',[0.016578947368421051, 1.0097368421052633]);
60
61
     end
62
63
     % --- Create fit "fit 1"
64
     ok = isfinite(pinte j) & isfinite(alph j);
65
     if ~all( ok_ )
66
          warning( 'GenerateMFile:IgnoringNansAndInfs',...
```

```
67
                'Ignoring NaNs and Infs in data.' );
 68
       end
       st_ = [0.54688151920498385 0.9575068354342976 0.96488853519927653
 69
 70
       0.15761308167754828 ];
 71
       ft_ = fittype('rat21');
 72
 73
       % Fit this model using new data
 74
       cf_ = fit(pinte_j(ok_),alph_j(ok_),ft_,'Startpoint',st_);
 75
       % Alternatively uncomment the following lines to use coefficients from the
% original fit. You can use this choice to plot the original fit against new
 76
 77
       % data.
 78
            cv = { 0.24564816828160418, 0.27073577576659003, 0.019770547721634286,
       8
 79
       0.020383397364387951};
 80
       % cf_ = cfit(ft_,cv_{:});
 81
 82
       % Plot this fit
 83
       h_ = plot(cf_,'fit',0.95);
 84
       set(h (1), 'Color', [1 0 0], ...
           'LineStyle','-', 'LineWidth',2,...
'Marker','none', 'MarkerSize',6);
 85
 86
       \% Turn off legend created by plot method.
 87
 88
       legend off;
 89
       % Store line handle and fit name for legend.
 90
       legh_(end+1) = h_(1);
       legt_{end+1} = 'fit 1';
 91
 92
 93
       \% --- Finished fitting and plotting data. Clean up.
 94
       hold off;
 95
       % Display legend
 96
       leginfo_ = {'Orientation', 'vertical', 'Location', 'NorthEast'};
 97
       h_ = legend(ax_,legh_,legt_,leginfo_{:});
 98
       set(h_,'Interpreter','none');
 99
       % labels x- and y-axes.
100
       xlabel(ax_,'player integration');
101
       ylabel(ax_, 'alpha');
```

Curve fitting code: relative player integration data

```
1
     function createFit(relinte j,alph j)
 2
     \ensuremath{\texttt{\sc CREATEFIT}} Create plot of data sets and fits
 3
         CREATEFIT (RELINTE J, ALPH J)
 4
     8
         Creates a plot, similar to the plot in the main Curve Fitting Tool
 5
     % Number of data sets: 2
 6
         Number of fits: 1
     8
 7
 8
     % Data from data set "alph j vs. relinte j":
 9
     8
           X = relinte j:
10
     8
           Y = alph_j:
11
     00
           Unweighted
12
13
     % Data from data set "alph_j vs. relinte_j (smooth)":
           X = relinte_j:
14
     8
15
            Y = alph_j:
     8
16
     8
           Unweighted
17
18
     % Set up figure to receive data sets and fits
19
     f = clf;
20
     figure(f );
     set(f_,'Units','Pixels','Position',[469 119 680 474]);
21
22
     % Line handles and text for the legend.
23
     legh_ = [];
24
     legt = {};
25
     % Limits of the x-axis.
26
     xlim_ = [Inf -Inf];
27
     % Axes for the plot.
28
     ax = axes;
     set(ax_, 'Units', 'normalized', 'OuterPosition', [0 0 1 1]);
29
     set(ax_, 'Box', 'on');
30
31
     axes(ax );
32
     hold on;
33
34
     % --- Plot data that was originally in data set "alph j vs. relinte j"
35
     relinte_j = relinte_j(:);
36
     alph_j = alph j(:);
37
     h_ = line(relinte_j,alph_j,'Parent',ax_,'Color',[0.333333 0 0.666667],...
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
38
39
40
     xlim (1) = min(xlim (1),min(relinte j));
41
     xlim (2) = max(xlim_(2), max(relinte_j));
42
     legh_(end+1) = h_;
43
     legt_{end+1} = 'alph_j vs. relinte_j';
44
45
     % --- Plot data that was originally in data set "alph_j vs. relinte_j (smooth)"
46
     sm_.y2 = smooth(relinte_j,alph_j,5,'moving',0);
47
     h_ = line(relinte_j,sm_.y2,'Parent',ax_,'Color',[0.333333 0.6666667 0],...
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
48
49
50
     xlim (1) = min(xlim (1),min(relinte j));
51
     xlim (2) = max(xlim (2), max(relinte j));
     legh(end+1) = h;
52
53
     legt_{end+1} = 'alph_j vs. relinte_j (smooth)';
54
55
     % Nudge axis limits beyond data limits
56
     if all(isfinite(xlim ))
57
         xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
58
          set(ax ,'XLim',xlim )
59
     else
          set(ax_, 'XLim',[-0.0038095238095238043, 10.099047619047619]);
60
61
     end
62
63
     % --- Create fit "fit 1"
64
     ok = isfinite(relinte j) & isfinite(alph j);
65
     if ~all( ok_ )
66
         warning( 'GenerateMFile:IgnoringNansAndInfs',...
```

```
67
               'Ignoring NaNs and Infs in data.' );
 68
      end
      st_ = [0.43874435965639824 0.3815584570930084 0.76551678814900237
 69
 70
      0.79519990113706318 0.1868726045543786 ];
 71
      ft_ = fittype('rat13');
 72
 73
      % Fit this model using new data
 74
      cf_ = fit(relinte_j(ok_),alph_j(ok_),ft_,'Startpoint',st_);
 75
      % Alternatively uncomment the following lines to use coefficients from the
 76
      % original fit. You can use this choice to plot the original fit against new
 77
      % data.
 78
           cv = { 0.82120375908517562, -0.075547865684368221, -1.7902037076763417,
      8
 79
      2.707322024940185, -0.2628383862203188};
 80
         cf_ = cfit(ft_,cv_{:});
      8
 81
 82
      % Plot this fit
 83
      h_ = plot(cf_,'fit',0.95);
 84
      set(h (1), 'Color', [1 0 0], ...
          'LineStyle','-', 'LineWidth',2,...
'Marker','none', 'MarkerSize',6);
 85
 86
      \% Turn off legend created by plot method.
 87
 88
      legend off;
 89
      % Store line handle and fit name for legend.
 90
      legh_(end+1) = h_(1);
      legt_{end+1} = 'fit 1';
 91
 92
 93
      % --- Finished fitting and plotting data. Clean up.
 94
      hold off;
 95
      % Display legend
 96
      leginfo_ = {'Orientation', 'vertical', 'Location', 'NorthEast'};
 97
      h_ = legend(ax_,legh_,legt_,leginfo_{:});
 98
      set(h_,'Interpreter','none');
 99
      % labels x- and y-axes.
100
      xlabel(ax_, 'relative player integration');
101
      ylabel(ax_, 'alpha')
```

Curve fitting code: in/out ratio data

```
1
     function createFit(inout j,alph j)
 2
     %CREATEFIT Create plot of data sets and fits
 3
         CREATEFIT (INOUT_J,ALPH_J)
     8
 4
     8
         Creates a plot, similar to the plot in the main Curve Fitting Tool
 5
     % Number of data sets: 2
 6
         Number of fits: 1
     8
 7
 8
     % Data from data set "alph j vs. inout j":
 9
           X = inout_j:
     8
10
     8
           Y = alph_j:
11
     00
           Unweighted
12
13
     % Data from data set "alph_j vs. inout_j (smooth)":
14
     8
           X = inout j:
15
           Y = alph_j:
     8
16
     8
           Unweighted
17
18
     % Set up figure to receive data sets and fits
19
     f = clf;
20
     figure(f );
21
     set(f_, 'Units', 'Pixels', 'Position', [520 49 830 592]);
22
     % Line handles and text for the legend.
23
     legh_ = [];
24
     legt = {};
25
     % Limits of the x-axis.
26
     xlim_ = [Inf -Inf];
27
     % Axes for the plot.
28
     ax = axes;
29
     set(ax_,'Units','normalized','OuterPosition',[0 0 1 1]);
     set(ax_, 'Box', 'on');
30
31
     axes(ax );
32
     hold on;
33
34
     % --- Plot data that was originally in data set "alph j vs. inout j"
35
     inout_j = inout_j(:);
36
     alph j = alph j(:);
     h_ = line(inout_j,alph_j,'Parent',ax_,'Color',[0.333333 0 0.666667],...
37
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
38
39
40
     xlim (1) = min(xlim (1),min(inout j));
41
     xlim (2) = max(xlim_(2), max(inout_j));
42
     legh_(end+1) = h_;
43
     legt_{end+1} = 'alph_j vs. inout_j';
44
45
     % --- Plot data that was originally in data set "alph_j vs. inout_j (smooth)"
46
     sm_.y2 = smooth(inout_j,alph_j,5,'moving',0);
47
     h_ = line(inout_j,sm_.y2,'Parent',ax_,'Color',[0.333333 0.6666667 0],...
         'LineStyle', 'none', 'LineWidth',1,...
'Marker','.', 'MarkerSize',1);
48
49
50
     xlim (1) = min(xlim (1), min(inout j));
51
     xlim(2) = max(xlim(2), max(inoutj));
     legh(end+1) = h;
52
53
     legt_{end+1} = 'alph_j vs. inout_j (smooth)';
54
55
     % Nudge axis limits beyond data limits
56
     if all(isfinite(xlim ))
57
         xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
58
          set(ax ,'XLim',xlim )
59
     else
          set(ax_, 'XLim', [-0.02583333333333333, 11.109166666666667]);
60
61
     end
62
63
     % --- Create fit "fit 1"
64
     ok = isfinite(inout j) & isfinite(alph j);
65
     if ~all( ok_ )
66
         warning( 'GenerateMFile:IgnoringNansAndInfs',...
```

```
67
                'Ignoring NaNs and Infs in data.' );
 68
       end
       st_ = [0.035711678574189554 0.84912930586877711 0.93399324775755055 ];
ft_ = fittype('rat11');
 69
 70
 71
 72
       % Fit this model using new data
 73
       cf_ = fit(inout_j(ok_),alph_j(ok_),ft_,'Startpoint',st_);
 74
       % Alternatively uncomment the following lines to use coefficients from the
 75
       % original fit. You can use this choice to plot the original fit against new
 76
       % data.
 77
           cv = { 0.53776958232266614, -0.050338886556504217, 0.097944630730329385};
       8
 78
            cf_ = cfit(ft_,cv_{:});
       8
 79
 80
       % Plot this fit
       h_ = plot(cf_,'fit',0.95);
 81
 82
       set(h_(1), 'Color', [1 0 0], ...
           'LineStyle','-', 'LineWidth',2,...
'Marker','none', 'MarkerSize',6);
 83
 84
 85
       % Turn off legend created by plot method.
 86
       legend off;
 87
       % Store line handle and fit name for legend.
 88
       legh (end+1) = h (1);
       legt_{end+1} = 'fit 1';
 89
 90
 91
       % --- Finished fitting and plotting data. Clean up.
 92
       hold off;
 93
       % Display legend
 94
       leginfo_ = {'Orientation', 'vertical'};
 95
       h_ = legend(ax_,legh_,legt_,leginfo_{{:}});
 96
       set(h_,'Units','normalized');
 97
       t_ = get(h_, 'Position');
t_(1:2) = [0.591968, 0.251973];
98
 99
       set(h_,'Interpreter','none','Position',t_);
100
       % labels x- and y-axes.
      xlabel(ax_,'in/out ratio');
ylabel(ax_,'alpha')
101
102
```