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Pricing of Idiosyncratic Risk in the Nordics

- An empirical investigation of the idiosyncratic risk-reward relationship in the Nordic equity markets -

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Abstract

We examine the Nordic equity markets during 1992-2011 for the pricing of idiosyncratic risk relative to the CAPM and the Fama-French three factor model. Classical financial theory predicts irrelevance of idiosyncratic volatility (IVOL) for expected returns, while contending theories of undiversified investors and theories from the field of behavioural finance predict a positive relationship. Recent empirical findings from international equity markets however indicate a negative relationship and our analysis support these findings. We find that a zero cost portfolio long in stocks with low IVOL and short stocks with high IVOL earns a positive and statistically significant alpha versus the FF-3 factor model of 1.14 per cent per month, implying a negative return towards holding idiosyncratic risk. Contrary to other low-volatility investment strategies, our results indicate that the low IVOL strategy is negatively related to the value premium. In addition, a positive relationship between IVOL and market beta is identified and evaluated.

Keywords: Idiosyncratic risk, CAPM, Fama-French three factor model, Nordic equity markets

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Idiosyncrasy

a tendency, action, or form of behaviour specific to one person or group., from ***Idiosunkrasia*** (Ancient Greek); means “ones own temperament”; ***idios*** (ones own) + ***sun*** (together) + ***krasis*** (temperament)

I Background

1. Introduction

A traditional cornerstone within financial theory is the positive risk-return relationship; that greater assumed risk should be compensated with higher expected returns. Academics as well as practitioners within finance have since long sought to identify, explain and exploit any existing aberrations to the risk-return relationship.

The risk-return relationship is commonly defined by the Capital Asset Pricing Model (hereafter CAPM) developed by Sharpe (1964) and Lintner (1965). The CAPM assumes a positive linear relationship between market risk and the expected return of a security. Hence, the CAPM distinguishes between systematic[1] and idiosyncratic risk[2]. The model predicts that idiosyncratic risk should not be compensated for by higher expected returns as such risk can in theory be diversified away by holding the market portfolio. The irrelevance of idiosyncratic risk has been one of the areas of discussion relating to the CAPM, and some of the assumptions behind the model have been questioned. Merton (1987) argues that market frictions make diversification costly, and therefore stocks with higher firm-specific risk should be rewarded with higher expected returns. Questioning of the irrelevance of idiosyncratic risk has also come from behavioural finance scholars such as Barberis and Huang (2001), who argue that mental accounting leads to narrow framing and that loss aversion makes investors demand compensation for assuming idiosyncratic risk.

Contrary to classical portfolio theory which assumes a flat risk-return relationship for idiosyncratic risk, and behavioural theory that argues in favour of a positive risk-return relationship, recent findings by Ang et al. (2006 and 2008) identify a negative relationship between idiosyncratic volatility (hereafter IVOL) and returns. This relationship is documented to exist in the US as well as in international equity markets, and stand in stark contrast to classical and behavioural finance theory. Ang et al. (2006) quotes this relationship as a “puzzle”.

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[1] Also referred to as undiversifiable or market risk
[2] Also referred to as diversifiable or firm-specific risk

The inverted risk-return relationship of the idiosyncratic volatility puzzle remains an unexplored field and its proof of life contradicts an intuitive cornerstone of asset pricing. This paper provides further insight to the pricing of securities and tests basic assumptions of theoretical models such as the CAPM, namely the irrelevance of IVOL on stock returns.

By exclusively examining the Nordic equity market, any implications on the pricing of IVOL stemming from different investor bases across geographies, which are often considered to have differing investment preferences (Ferreira and Matos (2008)), are likely reduced. It is, for instance, well documented that the US has a larger institutional ownership as per cent of market capitalisation than many European countries. Although Ang et al. (2008) include the Nordic countries as part of their international survey on pricing of IVOL, the Nordic equity market is not a main focus for the analysis and they do not report any results from the Nordics separately.

Furthermore, the increasing attention for the pricing of idiosyncratic risk following the counterintuitive research of scholars such as Ang et al. (2006) and the non-existence of a thorough study of these dynamics from the Nordic markets puts the results from our study in the frontier of the asset pricing academia, and provides further evidence of asset pricing dynamics that contradicts some of the most acknowledged economic theory within the field.

2. Related research

This thesis is related to several strands of research within asset pricing, including papers that examine the relationship between volatility (total, systematic and idiosyncratic) and returns from both (i) a classical and (ii) a behavioural asset pricing perspective. This section briefly discusses relevant papers from each of these strands.

2.1 Classical research

Our reference points are the classical models of Markowitz (1959), Sharpe (1964), Lintner (1965) and Fama and French (1992), according to which higher systematic risk should be rewarded with higher expected returns while the idiosyncratic risk component should not be priced. Since the earliest tests of the CAPM, researchers have shown that the empirical relation between risk and return is too flat, Fama and MacBeth (1973) among others reach such conclusion. Similarly, others such as Black et al. (1972) report that low beta stocks contain positive alpha. In their seminal paper, Fama and French (1992) show that beta does not predict returns during 1963-1990, the results are more pronounced after controlling for size and value. This was the starting point for the size and value factors popularity as a complement to the standard CAPM.

Fama and Macbeth (1973) were early to dismiss the relevance of IVOL by providing cross-sectional tests which supported the theory that only systematic and undiversifiable risk is priced. Although such findings have been challenged later on by scholars such as Levy (1978), Merton (1987) and Malkiel and Xu (2001), the theory and empirical findings from the challengers point of view have tended to be in favour of, if anything, a *positive* risk-return relationship of holding idiosyncratic risk.

2.2 Recent findings on pricing of idiosyncratic risk

Research and findings leaning towards a *negative* relationship between idiosyncratic risk and returns have been more pronounced during the 21st century. Notably, Ang et al. (2006) investigate the cross-sectional relationship between IVOL and expected returns. IVOL is defined as the volatility of the residuals of the Fama-French three factor model (hereafter FF-3). One hypothesis put forth states that if investors are not able to diversify risk, they will demand a premium for holding stocks with high IVOL. The findings of Ang et al. (2006) point to the opposite direction; stocks with high IVOL have low average returns. The difference in returns from the least volatile decile towards the most volatile decile is 1,06 per cent per month. The results are strong and robust after controlling for factors such as value, size, momentum, liquidity and dispersion in analyst forecasts. Furthermore, the effect persists in bull as well as bear markets, recessions and expansions, and in volatile and stable periods. The authors view this pattern and its persistence as “a puzzle”.

Two years later, Ang et al. (2008) provide further out-of sample evidence by expanding the universe to include an international dataset and identify a negative spread between stocks with high and low IVOL to be significant in the G7 countries and visible across 23 developed countries. The paper also concludes that this negative spread co-moves with the same spread between US stocks with high and low idiosyncratic volatilities, and that the commonality in co-movement across countries suggest that “broad, not easily diversifiable, factors may lie behind this effect”. Sweden, Denmark, Finland and Norway are included in the sample, but the paper does not present results on pricing of IVOL for the Nordics specifically. However, it does conclude the presence of a negative IVOL-return relationship for Europe as a whole, albeit with a smaller coefficient in real terms than for the US. The authors stress that they do not define the underlying factor causing the co-movement as a risk factor, as there is not yet a theoretical framework to understand why investors demand should be higher for high IVOL stocks than for low IVOL stocks. “Further research must investigate if there are true economic sources of risk behind the IVOL phenomenon causing stocks with high volatility to have low expected returns”, they conclude.

Fu (2009) argues that the puzzling findings of Ang et al. (2006) is achieved because they relate lagged IVOL with future returns: “since idiosyncratic volatilities are time-varying, the one month lagged IVOL may not be an appropriate proxy for the expected IVOL of this month”. Instead, Fu (2009) measures IVOL with a different method by using exponential GARCH models to estimate the expected idiosyncratic volatilities. Contrary to the findings of Ang et al. (2006), Fu (2009) finds a positive relationship between IVOL and returns by using his own preferred method, although concluding that the value-weighted portfolios formed based on sorted IVOL do not have significant alphas.

Goyal and Santa-Clara (2003), considered the relevance of idiosyncratic risk for stock market returns. They find a significant positive relationship between the average idiosyncratic risk and market returns (they do not consider cross sectional pricing of IVOL). Subsequent research by Bali et al. (2005) finds that this relationship is weaker in an extended sample.

2.3 Other volatility anomalies

Haugen and Baker (1991) were among early discoverers of the relative outperformance of low volatility portfolios compared to value-weighted indices. Baker et al. (2011) find that during the last 40 years, low volatility and low beta stocks have substantially outperformed high volatility and high beta stocks. They use a sample of U.S. equities between 1968 and 2008 and refer to the observed pattern as the “Low volatility anomaly”. They conclude that the empirical predictions from the theory of efficient markets, where above average returns should only be realised by taking on above average risk, are weak; when risk is measured as either total volatility or systematic risk, the evidence actually points towards a negative relationship.

Another observation that have come to question disqualify the predictions of the CAPM is the “Low beta anomaly”. Frazzini and Pedersen (2010) provide a theoretical model based on leverage constrained investors as explanation as to why, for instance, the security market line for US stocks is too flat relative to the CAPM prediction. The paper commences by asserting that one basic premise of the CAPM is that all agents invest in the portfolio with the highest expected excess return per unit of risk, and then lever or de-lever this portfolio in order to obtain their preferred risk profile. According to their model of leverage constrained investors, margin constrained investors tilt towards risky assets instead of using leverage in order to obtain the preferred portfolio risk, which results in an increased demand and lowers returns for high beta securities. They find further support to their model within global equities, treasury bonds, corporate bonds, and the futures market, where low beta securities are found to give higher risk adjusted returns than high beta securities, and also make the empirical observation that during times of funding liquidity constraints, the outperformance of low beta securities tends to get larger.

The findings of a flat or an inverted risk-return relationship has triggered an increasing interest in low volatility investing from both academics and practitioners. One investment strategy seeking to exploit mispricing of low volatility stocks is the minimum-variance portfolio. Clarke et al. (2007) conclude that by using econometric optimisation procedure, which also takes into account the correlation between assets in constructing minimum variance portfolio, outperforms the value weighted indices. The merits of low volatility strategies have been linked to the value premium by for instance Scherer (2010), who concludes that most of the excess returns of minimum variance portfolios are attributed to the value factor. While there are today strategies seeking to exploit mispricing of volatility employed in practice, as advocated for instance by Blitz and van Vliet (2007), limited or no research has been conducted on developing a strategy specifically aimed at exploiting mispricing of IVOL in practice, although we would expect to see such in the future if further findings in the line of Ang et al. (2006) were to be presented.

3. Purpose

1. Motivation

The purpose of this thesis is to identify or reject the existence of the IVOL puzzle as defined by Ang et al. (2006) in the Nordic equity markets. As the academic research on the IVOL puzzle is still in its cradle, we aim to provide further understanding on the existence of this anomaly by concluding or rejecting a presence within these markets. The results of the thesis should be of interest to academics and practitioners as it will further the understanding of what types of risk that investors can expect to be rewarded for.

Differentiations of this paper versus that of Ang et al. (2006) are our inclusion of CAPM-based residuals and ambition to further study the link between portfolios sorted on IVOL and market beta for the same time period. This selected approach aims at bridging the gap between the research conducted on the risk-return relationship of IVOL and betas. This differentiator compared to other research provides an opportunity to identify any co-movement which could point to a limitation of the selected asset pricing models ability to capture all systematic risk.

2. Hypothesis

Our a priori view is that the, in line with classical portfolio theory, portfolios sorted on idiosyncratic risk should not achieve abnormal returns. Hence, our hypothesis outlined below is based on this.

I. H_0: The resulting zero cost portfolio of taking a long position in the portfolio with the stocks that have the lowest IVOL and a short position in the portfolio with stocks that have the highest IVOL does not earn statistically significant abnormal returns

H_A: The resulting zero cost portfolio of taking a long position in the portfolio with the stocks that have the lowest IVOL and a short position in the portfolio with stocks that have the highest IVOL does earn a statistically significant abnormal returns

3. Delimitations

This study only pertains to the Nordic region as defined by Denmark, Finland, Norway and Sweden. The study therefore excludes Iceland from the sample which could have some effects on our results applicability to the entire Nordic region. We have limited our sample to solely include stocks that are listed on the NASDAQ OMX Nordic exchange and Oslo Børs (the major Norwegian Stock Exchange). Only common stocks have been included.

The purpose of this essay is not to conclude a comprehensive overview of the actual returns one would expect if one were to implement the corresponding methodology to a trading strategy but to identify a possible deviation from the classical risk-return relationship. Hence, adjustments for trading costs will not be included, however, we will comment on the movement among the portfolios which in itself may be considered a proxy for actual trading costs if a corresponding strategy would be applied.

4. Disposition

The thesis is structured as follows; Section II presents the theoretical framework of relevant theories, Section III describes the dataset in detail, Section IV covers the methodology selected, Section V presents and analyses the results, Section VI concludes the study and its findings and provides some suggestions for future research within this field.

II Theoretical framework

This section reviews the classical concepts of risk, the relevant factor models within asset pricing and describes the idiosyncratic risk component with suggested implications for asset pricing.

1. Efficient Market Hypothesis

The theory of an efficient market, that stock prices reflect all readily available information, thereby rejecting an existence of any abnormal returns, was introduced in the academia by Fama (1970). Malkiel (2003) defines this concept in the following manner "I will use a definition of efficient financial markets that they do not allow investors to earn-above average returns, without accepting above-average risks".

Fama (1970) conceptualised the idea of efficient markets in the Efficient Market Hypothesis (hereafter EMH). The EMH distinguishes among three levels: the weak, semi-strong and strong form of the hypothesis. The weak form asserts that the current stock price incorporates all available historical prices in the information. This implies that analysis based on historical returns, trends or the likes will not render any information that can produce consistent above-average returns without assuming above-average risk. The semi-strong form also encompasses all public information that contains any forward-looking statements or prospects of the firm. If the semi-strong form is fulfilled, it is not possible to earn abnormal returns on such information. Finally, the strong form of the EMH includes all relevant information of the firm. This form also includes non-disclosed information to the public, i.e. insider information.

This study will only use historical available information and should, according to the weak form of the EMH, not be able to render any significant excess return if applied to post the first publication uncovering a historical relationship between IVOL and returns rendering above-average returns without above-average risks.

2. CAPM

In 1952, Harry Markowitz set the foundation to modern portfolio theory by presenting that investing in a certain combination of several risky assets, i.e. diversifying, an investor could lower the risk while maintaining an equivalent expected rate of return of the portfolio of assets, thereby optimising the mean-variance trade-off. According to Sharpe (1964) and Lintner (1965) a consequence of optimal diversification is that all investors are assumed to hold the market portfolio. In such a world, the only risk to be priced is undiversifiable market risk. A certain index consisting of a chosen set of equities is often used as a proxy for the market portfolio, and market risk is then defined as the co-movement between an asset and the index. The assumption that all investors hold the market portfolio has been considered too strong and the concept of an identifiable and investable market portfolio has been questioned, i.e. that the identifiable indices are poor proxies for aggregate wealth.

3. Size and Value

The discovery of the size effect is often attributed to Banz (1981). Banz examines the relationship between size, defined as the market value, and returns and found that smaller firms had higher risk adjusted returns within the selected sample. The size effect was mainly prevalent in very small firms. The size effect received further attention when Fama and French (1992) presented their three factor model, where portfolios sorted on size and book-to-market values were shown to have been significantly mispriced according to the standard CAPM-model. Fama and French (1992) argue that small firms in general tend to suffer longer periods of earnings depressions than big firms, making size exposure a common risk factor that might explain the negative relation between size and returns. In later periods, it has been contended that the size effect has diminished, and may even have disappeared.

Chan et al. (2000) and Amihud (2002) find no size premium. Schwert (2003) confirms the results and concludes that the size effect has vanished since papers on the subject were published, possibly assigning this change to the financial community picking up on this. Furthermore, illiquidity has been suggested as an explanation behind the size effect, i.e. that it is not really size that is priced but illiquidity, and since smaller firms tend to be less liquid, smaller firms earn higher expected returns (Amihud (2002)). Despite this, the size factor remains a solid foundation as a potential systematic risk factor within asset pricing academia.

4. Value effect

Fama and French (1992) argue that the CAPM is not able to explain returns from portfolios sorted on the book-to-market equity ratio. Firms with higher ratios seem to deliver abnormally high returns and vice versa. It is still not obvious the book-to-market effect is a compensation for higher risk within high book-to-market stocks, or simply a persistent stock market miss-pricing. Suggestions as why high book-to-market effect would be a compensation for risk includes the theory that high book-to-market firms are in general distressed stocks with low valuations that perform more poorly than other stocks in bad times, which, according to some theory, is an undesirable behaviour which make investors demand a premium for holding the asset.

5. Idiosyncratic risk

Idiosyncratic risk is defined as the unique risk of a specific security. Within equities, this is also referred to as *firm-specific risk*. Therefore, idiosyncratic risk is the risk that is independent of co-movement of the market, according to some asset pricing model, and is possible to avoid by holding a diversified portfolio of enough non-perfectly correlated assets.

According to the classical models of Markowitz, Fama and French, systematic undiversifiable risk should be rewarded with higher returns while the idiosyncratic risk component should not be priced according to Fama and French (1992). However, if the strong assumption of full diversification among investors does not hold, this does not need to be the case.

Among the first to question the irrelevance of idiosyncratic risk for asset pricing was Levy (1978). According to Levy, the CAPM implies two properties: that all investors hold all risky securities in the market and that investors hold risky assets in the same proportions, independent of investors preferences. He then concludes that these properties contradict all market experience established in empirical research. He argues that in a world where investors as a result of transaction costs, indivisibility of investment due to costs of keeping track of new financial development of all securities, some investors who decide to invest in a number of securities smaller than the total investment universe will not only consider the systematic risk. Levy predicts that if one assumes that investors hold undiversified portfolios, the "residual variance should have a strong impact on the risk-return relationship". Levy adds that "the classical CAPM may be the approximate equilibrium model for stocks of firms which are held by many investors, but not for small firms whose stocks are held by a relatively small group of investors".

Another scholar to criticise the implications for firm-specific volatility from CAPM was Merton (1987), who questions the "basic finance model with its frictionless markets, complete information, and rational, optimising economic behaviour", and develops a theoretical model where information efficiencies will lead to segmented markets where less known stocks with smaller investor bases will have larger expected returns than in a comparable complete-information model. Merton predicts that market frictions make it costly to achieve full diversification, expected returns will tend to be higher in firms with larger "firm specific", i.e. idiosyncratic variance.

Malkiel and Xu (2001) extend the CAPM to also account for an idiosyncratic risk-reward. They argue that if one group of investors, due to exogenous reasons, fails to hold the market portfolio, this would prevent all remaining investors from holding the market portfolio as well. Therefore, idiosyncratic risk should be priced to compensate rational investors for not being able to hold the market portfolio, which the CAPM assumes. They derive a variation of the CAPM to account for this.

6. Behavioural perspective

Predictions of the pricing of idiosyncratic risk have also come from the behavioural field of finance. Barberis and Huang (2001) argue that mental accounting make investors demand an extra return premium for holding idiosyncratic risk. The authors conclude that numerous experimental studies suggest that an important feature of mental accounting is *narrow framing*, the idea that people derive utility from narrowly defined gains and losses rather than absolute wealth or consumption. If this is true, investors are not only concerned about the performance of their aggregate portfolio, but also of fluctuations of each individual stock. Using this way of arguing, the authors separate between *portfolio accounting,* as the classical portfolio theory would predict, and *individual stock accounting*, which would be a result of narrow framing.

Behavioural theorists have also offered various explanations to the findings that low volatility stocks outperform high volatility stocks. Baker et al. (2011) mention the preference for lotteries hypothesis which contends that investors have a preference for lottery like payoffs i.e. payoffs characterised by a positive skew. Another explanation offered by the same authors is that investors due to overconfidence regarding their ability to value stocks deliberately choose the most volatile stocks; the disagreement between the confident investor and the market consensus about the future performance of a certain stock will be larger for more volatile stocks which is why the demand for volatile stocks will be higher. This hypothesis hinges on the assumption that confident investors act more aggressively in the markets than pessimists resulting in reluctance towards shorting stocks relative to buying them. If this assumption holds, stocks with high volatility are subject to a greater dispersion between the confident investor and the market consensus will have more optimists among its shareholders, resulting in higher prices and lower subsequent returns. Given these behavioural explanations of why individual investors prefer volatile stocks, the authors still find it a challenge to explain why institutional investors do not exploit these behavioural biases in the markets. According to Baker et al. (2011), this is because "a benchmark makes institutional investors less likely to exploit the low volatility anomaly". The authors claim that a common incentive of investment managers is to stay close to benchmarks in order to maximize information ratios rather than benchmark-free Sharpe ratios. This incentive scheme discourages institutional investors from exploiting mispricing of volatility and therefore provides a possible explanation to why this anomaly persists.

III Data

1. Sample selection

The original dataset has been downloaded from Thomson Datastream and consists of data of 750 stocks listed on the Nordic stock exchanges NASDAQ OMX Stockholm, Copenhagen and Helsinki and the Oslo Børs. The Nordic country Iceland was excluded since only 7 equities were listed in Iceland at the end of our sample period. The downloaded sample consists of 20 years of data with daily stock total return indices, market cap, and market-to-book values, from January 1, 1992 to December 31, 2012. We exclude securities with non-available or negative book values, secondary listings (if the primary listing is included) and preference stocks, leading to a final sample of 687 stocks listed at the end of the period. The number of stocks in the beginning of the sample period was 215, and the average number of included stocks throughout the sample was 469. The time period has been limited to 20 years going back to 1992, in order to include a time period longer than two business cycles while still having sufficient number of observations to achieve at least 40 stocks in each of the five sorted portfolios.

The share prices and market capitalisations are all exchanged into the currency SEK (Swedish Enkronor) as we performed the study on the Nordic market on an aggregated level. The reason for selecting SEK as the common currency is because SEK is the currency in which the largest part of our sample was traded in.

The selected proxy for the risk-free rate has been the Swedish 30 day government bill, which was the shortest yield available that could be considered “risk free”. As there is no official Nordic Total Return index available for our sample period, we construct a value-weighted index from the companies that we have included in the sample as a proxy for the market portfolio.

Ideally, one might want to study markets in isolation in order to draw conclusions specific for a particular market, like for instance the Swedish equity market. However, in order to achieve a sample size we regard as adequate to allow for valid statistical inference, we choose to include several of the Nordic countries in our sample. Selecting to include all four major Nordic equity markets, compared to limiting the study to only Sweden, more than doubled our total sample size. One alternative would be to include all equities in the world to get the perspective from a "global investor". This would, however, have been beyond the scope of this thesis. Furthermore, scholars such as French and Poterba (1992) have highlighted significant home biases among investors, meaning that most investors do not utilize the option to diversify globally optimally but rather stick to investments close to their geographical home, why the study of a local market is a more true market reflection for most investors. The Nordic countries are relatively homogeneous from a political, social and economic point of view where also many of the listed firms are operationally integrated across the Nordics to a large extent. Furthermore, as Haavisto and Hansson (1992) point out, since the Nordic countries are relatively homogeneous, they are sometimes regarded as a unified market with common legislation, low trade barriers etc., which imply small transaction costs. Therefore, due to the lack of information, fear of expropriation, discriminatory taxation, different legislation, i.e. higher transaction costs, and more official restrictions in non-Nordic markets, Nordic investors might prefer the Nordic markets as opposed to global diversification. Finally, the fraction of institutional ownership of total market cap is similar for all four included Nordic countries (Ferreira and Matos (2008)), which we hypothesize could be one of the key determinants for pricing of idiosyncratic risk and therefore is crucial to our study. Therefore, we think our Nordic market definition offers good trade-off between sample size and market relevance.

2. Data evaluation

This section covers a selection of biases and their possible implications for the study.

Data mining

According to Hand et al. (2000), data mining is defined as "the process of seeking interesting or valuable information within large in datasets". The problem arises when a present relationship is spurious and is present in the dataset due to chance. As this thesis is restrictive in reducing the sample size due to the fulfilment of various conditions and as we are applying an established methodology to the dataset, the possibility of such presence is not considered an issue.

Data snooping

White (2000) defines data snooping as something which “occurs when a given set of data is used more than once for purposes of inference or model selection”. As we are conducting this study on a new dataset within a market segment which has received relatively little attention within this field, we do not view this issue as possible bias contaminating our results.

Model mining

Model mining implies making amendments to the model or models in order to achieve satisfactory and significant results. As our study is based on the model of Ang et al. (2006), this study could be viewed as an out-of-sample analysis on a different dataset, thereby mitigating the risk of finding spurious patterns due to model mining.

Selection bias

Selection bias refers to when observations are selected so that they are not independent of the outcome variables in a study, thereby possibly leading to biased inferences. By making reductions to the data set by applying specific criteria may reduce randomness in the remaining sample. We have limited our sample to stocks listed on NASDAQ OMX and Oslo Børs. We motivate such a selection by (i) these market places have more rigorous listing requirements than other smaller Nordic market places wherefore the available financial information should be of greater quality and (ii) selected market places and the stocks included represent an overwhelming portion of the traded volume in Nordics and is therefore the best proxy of a Nordic market portfolio. In addition to this, we have opted to keep the sample as unadjusted as possible in order to remove the possibility of selection bias.

Survivorship bias

The selected universe is based on a freezing of the portfolio at January 1, 2012, and only includes stocks listed on that date. An implication of this is that companies that have been are no longer listed due to buyouts, mergers or bankruptcies are excluded. This could result in some survivorship bias which could cause some distortions to the returns of the portfolios, if firms due to the above mentioned events experience larger IVOL ahead of the events. Although we believe such events are rare enough to not have a significant impact on our conclusions, such biases cannot be ruled out.

IV Methodology

1. Time series regression analysis

The presented methodology below is inspired by previous studies, mainly the one conducted by Ang et al. (2006).

CAPM

In order to run CAPM regressions, we need a proxy for the risk-free rate, the return of the stock and the return of the proxy for the market portfolio. We construct a value-weighted index based on the universe of stocks in the sample. We construct the value weighted Nordic Excess Total Return Index by backtracking the excess total returns of the constituent equities in our sample, which is used as the proxy for the market return.

$$PR_{i,t} = \sum_{i=1}^{n} w_{i,t} \left[\frac{Div_{i,t} + (P_{i,t+1} - P_{i,t})}{P_{i,t}} - rf_{t+1} \right] \quad (1)$$

$For\ t = o, \ldots, T$

where:

$PR_{i,t} =$ *the excess return of the portfolio for one day*

$w_{i,t}$*the weight of the security at closing as defined of the security's market capitalisation divided by total market capitalisation at closing*

$Div_{i,t} =$ *dividend proceeds of security i at the time t*

$P_{i,t} =$ *the amount invested in the beginning of time t in security i*

$P_{i,t+1} =$ *the amount in the following day t for security i*

$rf_t =$ *the risk − free rate at t + 1 days*

Fama-French three factor model

We construct Nordic FF-3 factors Small-Minus-Big (SMB) and High-Minus-Low (HML), inspired by the methodology of Fama and French (1992), where stocks are sorted into six groups according to their size and their book to market value of equity. We divide the sample into two equal parts dependent on their market capitalisation into a Small and a Big group. The Small and Big groups are sequentially split into three groups respectively, with an equal number of equities based on their book-to-market value.

Figure 1: FF-3 portfolios SMB and HML illustrative methodology description

		33%	34%	33%
	B/M / Size	H	M	L
50%	S	Small value	Small neutral	Small growth
50%	B	Big value	Big neutral	Big growth

$$SMB = \frac{1}{3}[SM + SH + SL] - \frac{1}{3}[BM + BH + BL] \tag{2}$$

$$HML = \frac{1}{2}[SH + BH] - \frac{1}{2}[SL + BL] \tag{3}$$

As we are interested in the return dynamics on a monthly basis, the 2x3 size/book-to-market portfolios are rebalanced on a monthly basis. This is a slight deviation from the method of Fama and French (1992), who rebalance the portfolios on a yearly basis. Although we consider that our method better fits the purpose for this particular study, we believe our choice has a limited effect on the results. The value-weighted (daily and monthly) returns of the Big and Small portfolio are calculated, respectively. The return of the small portfolio is then subtracted by the returns of the Big portfolio. A similar method is used to calculate the HML factor, where the value-weighted (daily and monthly) return of the Low book-to-market portfolio is subtracted from the value-weighted return of the High book-to-market portfolio.

2. Estimation of IVOL

We calculate the daily and monthly (log) excess return r_t^i of included stocks, according to the following:

$$r_t^i = \ln(\frac{p_t^i + d_t^i}{p_{t-1}^i}) - r_t^f \tag{4}$$

Where p_t^i is the price of stock *I* at time *t, d* is the dividend during time *t*, and r_t^f is the risk free rate during time *t* as defined earlier.

Following the closing of the last day in the month, from January 1992 until December 2011, the realised monthly IVOL, relative to one of the factor models for each listed stock is calculated. The factor models are used to run monthly linear OLS time series regressions on the daily excess return of stock *i* relative to the CAPM:

$$r_t^i = \alpha^i + B_{mkt}^i MKT_t + \varepsilon_t^i \tag{5}$$

and the FF-3 model:

$$r_t^i = \alpha^i + B_{mkt}^i MKT_t + B_{SMB}^i SMB_t + B_{HML}^i HML_t + \varepsilon_t^i \tag{6}$$

Where $MKT_t$the excess return of the market is, SMB_t is the excess return of the Small portfolio relative to the Big portfolio, and SMB_t is the excess return of the High book-to-market stocks relative to the Low book-to-market stocks (all during time *t*). B_{factor}^i is the estimated factor loading on one of the included factors during in the time series regression. The residual ε_t^f is the return during *t* that is left unexplained by the factor model, and is hence the idiosyncratic part of the return*,* i.e. the non-systematic part of the return in *i* during *t*. We perform these time series regressions for each security for the whole sample period of 20 x 12 = 240 months.

The IVOL, relative to one of the factor models, is defined as the standard deviation in the residuals ε_t^i in the above equations (5) and (6). We sort the stocks into quintiles according to their estimated IVOL during the previous month. The quintiles form the ranked portfolios P1 to P5, where P1 is the portfolio consisting of the stocks with the lowest estimated IVOL in period *T-1.*These portfolios are held for one month during T, i.e. the sequential month after the regressions are performed and the IVOL is estimated. We estimate the value weighted monthly excess returns of these sorted portfolios, before they are rebalanced ahead of the next month in an equal manner. This procedure is repeated for the whole sample period, implying a total of 20*12-1= 239 holding periods, (minus one since we need one starting month to estimate IVOL before we can rank the portfolios). We create a zero-cost portfolio long in high IVOL and short in low IVOL, and measure the excess return as evidence of existence of pricing of idiosyncratic risk.

Lo and MacKinlay (1990) study the possible implications of sorting stocks into portfolios. Benefits of sorting stocks into portfolios may be the reduction of measurement error and often increases the power of the tests. However, as the criteria for sorting seldom are random, but instead often based on some empirical characteristic possibly creating a bias in the selection.

In addition, Berk (2000) states that the explanatory power of an asset pricing model will be reduced when dividing the sample. Berk goes on to discuss the effect of sorting based on certain criteria that may have a relationship to stock returns will render portfolios which are very different but result in the characteristics within the portfolio to possibly be similar, e.g. through similar return variation.

Increasing the number of portfolios by screening on several variables is likely to result in portfolios which show significant bias. In order to account for this fact, we have limited the number of portfolios to five. This results in a range between 43 to 139 securities in each quintile portfolio.

In order to control for market risk and size and value premiums, we evaluate the performance of the sorted portfolios by estimating the CAPM and the FF-3 alphas.

$$r^{i} = \alpha^{i} + B_{mkt}^{i} MKT_{t} + \varepsilon_{t}^{i} \tag{7}$$

$$r_{t}^{i} = \alpha^{i} + B_{mkt}^{i} MKT_{t} + B_{SMB}^{i} SMB_{t} + B_{HML}^{i} HML_{t} + \varepsilon_{t}^{i} \tag{8}$$

Note the difference between the application of the CAPM and FF-3 regressions in equations (5) and (6) and in equations (7) and (8) ; in the prior two, they were used on daily data to perform monthly regressions to capture IVOL in individual stocks, while in the latter two, they were performed to evaluate the returns generated by the IVOL ranked portfolios P1 to P5.

We report the results of the performed regressions with Newey-West (1987) robust standard errors.[3]

[3] The Newey-West robust standard errors corrects the t-statistic for the prevalence of serial correlation and hetereoskedasticity in the residuals.

V Results and analysis

The excess return of the market over the risk-free rate in our sample is 1.0 per cent per month, and the constructed Nordic Index has slightly outperformed the OMXS TR over our sample period (see Figure 4, in the Appendix). A large value premium is present throughout the selected sample period as the High book-to-market has outperformed Low book-to-market by on average 1.1 per cent per month. The results indicate the presence of a minor Size premium in our sample as the Small portfolio has outperformed the Big portfolio by on average 0.1 per cent per month. However, we cannot conclusively confirm any Size effect as the difference between the portfolios is not statistically significant different from zero (see Table 9, in Appendix).

An ocular inspection of Figure 2 below suggests the presence of an inverted relationship between IVOL (measured relative to the CAPM and to the FF-3 model) and (CAPM and FF-3) regression alphas. This counter-intuitive pattern will be the focus of the analysis that is to follow.

Figure 2: CAPM and FF-3 alphas of portfolios P1 to P5 ranked on IVOL relative to the CAPM and FF-3 model

The chart below shows the CAPM and FF-3 regression alphas of the IVOL ranked portfolios with IVOL measured relative to the CAPM and the FF-3 model. As we use two-factor models to measure IVOL and two factor models to evaluate the portfolio performances, a total of four sets of portfolio alphas are seen in the below chart portfolios ranked on IVOL relative to the CAPM and to the FF-3 model. In the notation, the first word denotes which factor model has been used to measure risk, the second denotes what is ranked, and the third denotes which factor model that was used to calculate the alpha. I.e." FF-3IVOLCAPM", means portfolios ranked on IVOL relative to the FF-3 model with alphas calculated against the CAPM.

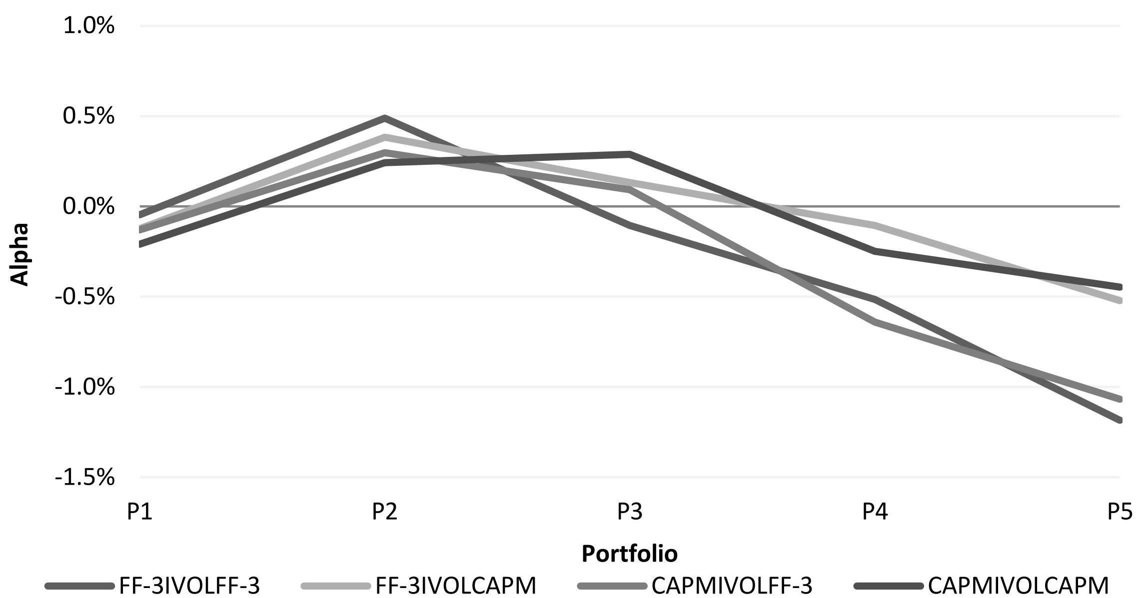


Table 1: Descriptive statistics and regression results on portfolios P1-P5 sorted on IVOL relative to CAPM

The table below provides descriptive statistics and regression results of the value-weighted returns of the ranked portfolio quintiles P1 to P5, based on estimated IVOL during the previous month relative to the CAPM according to equation (5), where P1 is the portfolio with the lowest IVOL and P5 is the portfolio with the highest IVOL. The P1-P5 portfolio presents the excess return of a zero cost portfolio long in P1 and short in P5. The arithmetic and geometric means are the average monthly excess returns of the portfolios. Volatility is the realised monthly standard deviation of the portfolio. Alphas and factor loadings from the CAPM and FF-3 regressions are reported separately with Newey-West (1987) robust t-statistics reported in square brackets below each coefficient.

	Low		Ranking on idiosyncratic volatility		High	
	P1	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Arithmetic mean	0.64%	1.11%	1.21%	0.83%	0.76%	-0.12%
Geometric mean	0.44%	0.92%	0.99%	0.50%	0.31%	0.13%
Median	1.03%	1.41%	1.42%	0.66%	0.87%	-0.02%
Skewness	-0.11	-0.02	0.22	0.34	0.19	-0.16
Kurtosis	1.44	1.56	1.48	2.62	2.76	2.47
Volatility	6.38%	6.14%	6.73%	8.18%	9.49%	7.25%
CAPM						
Alpha	-0.21%	0.24%	0.29%	-0.25%	-0.45%	0.24%
	-[0.91]	[1.29]	[1.33]	-[0.85]	-[1.20]	[0.53]
MKT	0.86	0.88	0.94	1.08	1.22	-0.36
	[17.66]	[22.22]	[21.28]	[18.02]	[16.19]	-[3.71]
FF-3						
Alpha	-0.13%	0.30%	0.09%	-0.64%	-1.07%	0.94%
	-[0.55]	[1.42]	[0.42]	-[2.26]	-[2.99]	[2.20]
MKT	0.84	0.86	0.97	1.17	1.43	-0.59
	[18.24]	[18.41]	[25.00]	[20.00]	[18.60]	-[6.50]
SMB	-0.07	-0.06	0.05	0.17	0.62	-0.69
	-[0.89]	-[0.72]	[0.52]	[1.68]	[4.65]	-[4.27]
HML	-0.05	-0.03	0.14	0.27	0.34	-0.38
	-[0.63]	-[0.59]	[3.07]	[4.05]	[3.86]	-[3.12]

Table 2: Descriptive statistics and regression results of portfolios P1-P5 sorted on IVOL relative to FF-3

The table below provides descriptive statistics and regression results on the value-weighted returns of the ranked portfolio quintiles P1 to P5, based on estimated IVOL during the previous month relative to the FF-3 according to equation (6), where P1 is the portfolio with the lowest IVOL and P5 is the portfolio with the highest IVOL. The P1-P5 portfolio shows the excess return of a zero cost portfolio long in P1 and short in P5. The arithmetic and geometric means are the average monthly excess returns of the portfolios. Volatility is the realized monthly standard deviation of the portfolio. Alphas and factor loadings from the CAPM and FF-3 regressions are reported respectively, with Newey-West (1987) robust t-statistics reported in square brackets below each coefficient.

	Low		Ranking on idiosyncratic volatility		High	
	P1	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Arithmetic mean	0.74%	1.29%	1.03%	0.95%	0.70%	0.04%
Geometric mean	0.54%	1.09%	0.82%	0.62%	0.25%	0.28%
Median	1.00%	1.44%	1.63%	0.71%	0.60%	0.02%
Skewness	-0.07	0.31	-0.20	0.22	0.22	-0.27
Kurtosis	1.48	1.63	1.30	2.22	3.48	2.76
Volatility	6.34%	6.30%	6.54%	8.16%	9.46%	6.84%
CAPM						
Alpha	-0.12%	0.38%	0.13%	-0.10%	-0.52%	0.40%
	-[0.60]	[2.13]	[0.61]	-[0.35]	-[1.37]	[0.92]
MKT	0.87	0.91	0.91	1.06	1.23	-0.36
	[17.65]	[26.39]	[20.41]	[14.87]	[14.70]	-[3.43]
FF-3						
Alpha	-0.05%	0.49%	-0.11%	-0.51%	-1.19%	1.14%
	-[0.21]	[2.67]	-[0.50]	-[1.85]	-[3.39]	[2.85]
MKT	0.85	0.87	0.96	1.16	1.45	-0.60
	[16.84]	[23.96]	[23.48]	[17.62]	[17.89]	-[5.85]
SMB	-0.05	-0.12	0.06	0.20	0.63	-0.68
	-[0.56]	-[1.62]	[0.85]	[1.61]	[5.20]	-[4.38]
HML	-0.05	-0.05	0.17	0.28	0.37	-0.42
	-[0.65]	-[1.43]	[3.62]	[4.11]	[4.36]	-[3.43]

A comparison of Table 1 and Table 2 suggest that the portfolios ranked on IVOL measured relative to the CAPM (reported in Table 1) produce very similar results as when IVOL is measured relative to the FF-3 (reported in Table 2), with similar return distributions, alphas and factor loadings between the corresponding portfolios. The correlation between the monthly returns of the quintile portfolio of IVOL relative to the FF-3 model and its corresponding quintile counterpart when IVOL is measured relative to the CAPM is above 95 per cent for all five twin portfolios. This is in line with what is reported by Ang et al. (2006), who claim correlations of above 99 per cent between the corresponding portfolios. We assume our somewhat lower correlation is explained by the more ample sample size of Ang et al. (2006), which means less noise within the ranked portfolios. As our correlations are still always above 95 per cent and the alphas versus the CAPM and the FF-3 factor models do not deviate significantly between the two methods, we view these results as largely equivalent. The following analysis will predominantly refer to the IVOL results measured relative to FF-3, but the same analysis would also hold for IVOL ranked relative to the CAPM.

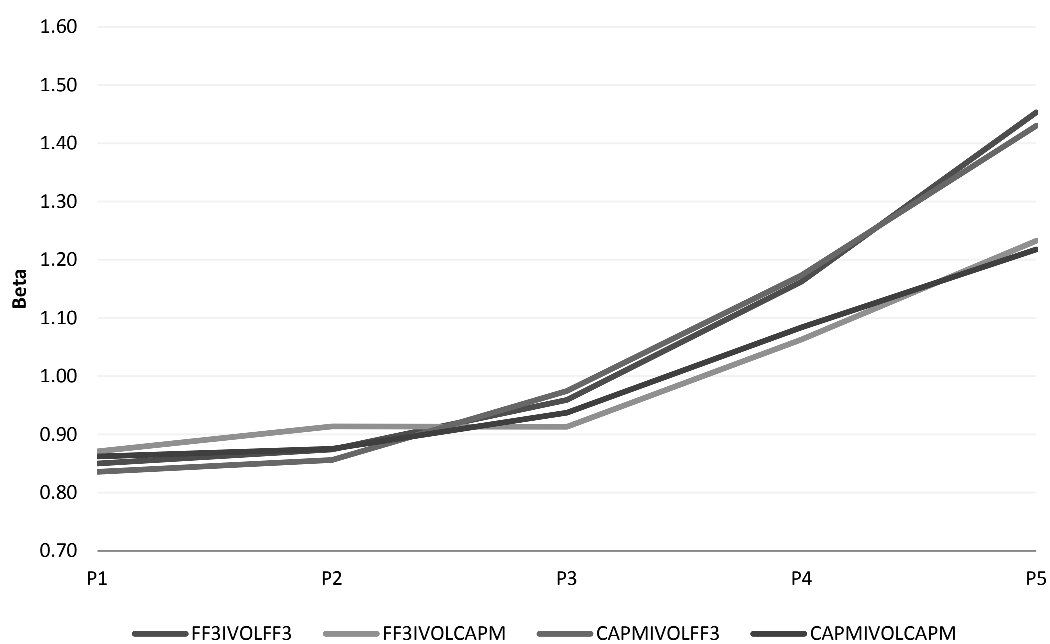
The descriptive statistics in Table 2 suggest an inverted U-shape in the arithmetic returns as we move from P1-P5, implying that the middle IVOL stocks have the highest arithmetic returns. The geometric mean decreases in general as the increased volatility within the higher portfolio numbers decreases the compounded returns. The portfolio volatility is strictly increasing from P2 to P5.

The regression output summarized in Table 2 shows that the zero cost portfolio P1-P5 has an economically large CAPM alpha of 0.4 per cent per month, and an even larger FF-3 alpha of 1.14 per cent per month. The positive FF-3 alpha of the P1-P5 portfolio has a high robust t-statistic of 2.85, meaning that we reject our null hypothesis at the 1% significance level. From P2 to P5, the CAPM and FF-3 alphas and the FF-3 factor model are monotonically decreasing (also visualised in Figure 2 on page 22), meaning that the abnormal returns relative to these factor models decreases as the IVOL increases. Studying the alphas of P1 to P5 individually, we conclude that the P5 portfolios, consisting of the highest IVOL stocks from the previous month, produce the most negative alphas (statistically significant in the FF-3 regression at the 1% level). Since P1 has alphas very close to zero, it is the short leg of the zero cost portfolio which is the main contributor to the abnormal returns from the P1-P5 strategy, suggesting that the mispricing of IVOL in our data relative to the factor model is driven by an excess demand for highest IVOL stocks rather than too low demand for the stocks with the lowest IVOL. Our finding that most of the alpha from the P1-P5 strategy come from the short side of the portfolio is a common phenomenon when mispricings are found (e.g. Finn et al. (1999)), and shorting limitation has been suggested as an explanation to why some mispricings of securities are not exploited (Lamont and Thaler (2003)).

From the reported factor loadings, we can conclude virtually consistent monotonic increases in loadings on MKT and HML as we move from P1 to P5, and P1-P5 portfolio has negative loadings on all factors. The heavy HML loading of P5, combined with the large and significant value premium found in our sample is the main reason why the FF-3 alphas of the P1-P5 strategy is substantially higher than the CAPM alphas of the same strategy. These findings also imply that the highest IVOL stocks consist of relatively high share of high book book-to-market stocks with non-outperforming returns, despite the established large and significant value premium in our sample.

Figure 3: Market beta of the IVOL ranked portfolios

The chart below shows the parameter estimate of the market factor according to the CAPM and the FF-3 factor model for the portfolios ranked on IVOL relative to the CAPM and to the FF-3 model. In the notation, the first word denotes which factor model has been used to measure risk, the second denotes what is ranked, and the third denotes which factor model that was used to calculate the alpha. I.e." FF-3IVOLCAPM", means portfolios ranked on IVOL relative to the FF-3 model with alphas calculated against the CAPM.



The above chart highlights how the market beta increases in the IVOL ranked portfolios from P1 to P5, suggesting that IVOL is postively related to market beta. The identified relationship could cause a spurious relationship between IVOL and alpha if market risk is mispriced i.e. that mispricing of IVOL is due to irrational demand for market risk rather than idiosyncratic risk. For robustness purposes, we evaluate if a ranking on beta would also generate mispriced portfolios. We apply a similar portfolio ranking procedure as previously conducted but instead of ranking according to past months IVOL, we now rank on systematic beta risk estimated by equation (5), and compare the portfolios against the CAPM and FF-3 factor models.

Table 3: Descriptive statistics and regression results on portfolios P1-P5 sorted on market beta

The table below provides descriptive statistics and regression results on the value-weighted returns of the ranked portfolio quintiles P1 to P5, based on the previous months estimated market beta, where P1 is the portfolio with the lowest IVOL and P5 is the portfolio with the highest IVOL. The P1-P5 portfolio shows the excess return of a zero cost portfolio long in P1 and short in P5. The arithmetic and geometric means are the average monthly excess returns of the portfolios. Volatility is the realized monthly standard deviation of the portfolio. Alphas and factor loadings from the CAPM and FF-3 regressions are reported respectively, with Newey-West (1987) robust t-statistics are reported in square brackets below each coefficient.

	Low		**Ranking on beta**		High	
	P1	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Arithmetic mean	1.01%	1.07%	0.88%	0.83%	1.01%	0.00%
Geometric mean	0.87%	0.98%	0.76%	0.68%	0.64%	0.23%
Median	1.34%	1.61%	1.21%	0.88%	1.29%	0.10%
Kurtosis	8.63	0.63	2.34	0.99	1.15	1.53
Skewness	0.87	-0.46	-0.69	-0.42	0.22	-0.01
Volatility	5.30%	4.33%	4.85%	5.43%	8.61%	7.13%
CAPM						
Alpha	0.47%	0.62%	0.31%	0.10%	-0.26%	0.73%
	[1.80]	[2.92]	[1.38]	[0.54]	-[1.20]	[1.99]
MKT	0.55	0.46	0.58	0.73	1.28	-0.74
	[6.83]	[11.45]	[11.31]	[18.08]	[32.41]	-[8.13]
FF-3						
Alpha	-0.09%	0.34%	-0.04%	-0.14%	-0.05%	-0.04%
	-[0.44]	[1.70]	-[0.18]	-[0.69]	-[0.21]	-[0.14]
MKT	0.70	0.54	0.64	0.76	1.23	-0.53
	[8.55]	[11.83]	[14.12]	[18.35]	[25.32]	-[5.88]
SMB	0.38	0.19	0.06	-0.04	-0.11	0.49
	[3.28]	[2.30]	[0.70]	-[0.64]	-[1.37]	[3.56]
HML	0.35	0.17	0.26	0.20	-0.14	0.49
	[4.22]	[3.64]	[5.26]	[5.01]	-[2.77]	[4.83]

The arithmetic mean in value-weighted portfolios remains flat at 1.0 per cent from low past beta P1 to P5, while the geometric mean decreases as we increase the past months estimated beta. As the alpha on the P1-P5 versus the CAPM are statistically significant positive, CAPM fails to price the market risk, which is in line with what has been found early by Fama and Macbeth (1973) and recently predicted and found by Frazzini and Pedersen (2010). The alpha is, however, removed as we apply the FF-3 factor model on the same ranked portfolios.

Although the regression analysis indicate different kinds of mispricings between IVOL and market beta, (since the beta ranked portfolio P1-P5 portfolio only have a significant alpha versus the CAPM alpha whereas the IVOL ranked P1-P5 portfolio only has significant alpha versus the FF-3 model), we consider a closer control of the relationship is warranted. We apply a control for the market risk by applying a double sorting method on both beta and IVOL, where each month the sample is split into three terciles by their estimated market betas from the regression in (5). We then, within each tercile, sort the stocks in the same way as to past months IVOL. By applying such methodology, it is possible to study and possibly identify whether the results are robust to market risk, or whether the effect disappears once the stocks are compared to other stocks of a similar stock market beta.

Table 4: FF-3 alphas and market betas on IVOL sorted portfolios within the terciles sorted on market beta

The table below provides descriptive statistics and regression results on the value-weighted returns of the double ranked portfolio quintiles P1 to P5 within the beta ranked portfolios CAPMBETALOW to CAPMBETAHIGH, based on the previous months estimated market beta estimated with monthly CAPM regressions on daily return data, where P1 is the portfolio with the lowest IVOL and P5 is the portfolio with the highest IVOL. The P1-P5 portfolio shows the excess return of a zero cost portfolio long in P1 and short in P5. The arithmetic and geometric means are the average monthly excess returns of the portfolios. Volatility is the realized monthly standard deviation of the portfolio. FF-3 alphas and market factor loadings from the FF-3 regressions are reported with Newey-West (1987) robust t-statistics are reported in square brackets below each coefficient

	Low		**Ranking on beta and IVOL**		High	
	P1	**P2**	**P3**	**P4**	**P5**	**P1-P5**
CAPMBETALOWFF3						
Alpha	-0.02%	0.49%	0.22%	-0.33%	-0.38%	0.35%
	-[0.10]	[1.76]	[0.80]	-[1.09]	-[0.92]	[0.78]
MKT	0.41	0.55	0.59	0.82	1.12	-0.70
	[8.49]	[7.91]	[7.32]	[8.27]	[8.70]	-[5.06]
CAPMBETAMIDFF3						
Alpha	-0.04%	-0.18%	0.45%	-0.18%	-0.9%	0.9%
	-[0.13]	-[0.67]	[1.59]	-[0.50]	-[2.03]	[1.63]
MKT	0.53	0.61	0.67	0.81	1.15	-0.61
	[7.20]	[12.59]	[12.34]	[10.19]	[13.51]	-[5.45]
CAPMBETAHIGHFF3						
Alpha	0.1%	0.1%	0.0%	-1.4%	-1.5%	1.6%
	[0.30]	[0.27]	[0.02]	-[3.59]	-[2.33]	[2.34]
MKT	1.03	1.10	1.20	1.38	1.97	-0.94
	[17.41]	[21.94]	[16.77]	[16.31]	[7.62]	-[3.56]

We compare the results in Table 4 with the FF-3 regression results in Table 1, with IVOL measured relative to the CAPM in both regressions. It is evident that the FF-3 alphas of P1-P5 decrease for the low and medium beta terciles, where also the statistical significance of the alphas disappears. For the high beta subgroup, the economic size of the alpha actually increases, and the statistical significance remains. This implies that the mispricing of IVOL is to some extent driven by the fact that IVOL is related to mispricing of systematic market risk, and that the abnormal returns of IVOL is largest among high beta stocks. However, the mispricing of IVOL is still economically significant in all three beta subgroups, with monthly alphas ranging from 0.35 per cent for the low beta stocks to 1.6 per cent for the high beta stocks. Furthermore, the lack of statistical significance for pricing of IVOL for low and mid beta terciles have limited interpretational value as the applied methodology significantly reduces the number of securities in each portfolio. A reduction of the portfolio size is likely to result in greater noise in the portfolios and decreases the ability to provide any statistical significance in our limited total sample, i.e. as discussed earlier with reference to Berk (2000). Nonetheless, the significant beta movements from P1 to P5 within the ranked beta portfolios imply a failure to fully control for beta in the analysis of pricing of IVOL.

Table 5: Descriptive statistics and regression results on portfolios P1-P5 sorted by volatility

The table below provides descriptive statistics and regression results on the value-weighted returns of the ranked portfolio quintiles P1 to P5, based on the previous months estimated market volatility, where P1 is the portfolio with the lowest IVOL and P5 is the portfolio with the highest IVOL. The P1-P5 portfolio shows the excess return of a zero cost portfolio long in P1 and short in P5. The arithmetic and geometric means are the average monthly excess returns of the portfolios. Volatility is the realized monthly standard deviation of the portfolio. Alphas and factor loadings from the CAPM and FF-3 regressions are reported, respectively, with Newey-West (1987) robust t-statistics are reported in square brackets below each coefficient.

	Low		**Ranking on volatility**		High	
	P1	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Arithmetic mean	1.11%	0.99%	1.09%	1.12%	1.01%	0.10%
Geometric mean	1.00%	0.86%	0.86%	0.78%	0.34%	0.66%
Median	1.44%	1.48%	1.16%	1.17%	0.52%	0.78%
Kurtosis	1.21	0.97	1.20	1.82	5.97	10.86
Skewness	-0.47	-0.44	0.13	0.30	1.01	-1.58
Volatility	4.66%	5.10%	6.72%	8.31%	11.85%	10.31%
CAPM						
Alpha	0.60%	0.35%	0.15%	-0.02%	-0.48%	1.08%
	[2.47]	[1.57]	[0.63]	-[0.07]	-[1.09]	[2.08]
MKT	0.52	0.64	0.94	1.15	1.50	-0.98
	[10.15]	[13.28]	[18.56]	[19.55]	[16.82]	-[9.59]
FF-3						
Alpha	0.31%	0.16%	0.04%	-0.06%	-0.84%	1.15%
	[1.37]	[0.71]	[0.17]	-[0.19]	-[1.89]	[2.23]
MKT	0.57	0.67	0.95	1.16	1.61	-1.04
	[11.98]	[12.51]	[20.04]	[18.30]	[18.68]	-[9.58]
SMB	0.05	-0.02	-0.03	0.02	0.30	-0.25
	[0.59]	-[0.22]	-[0.41]	[0.16]	[2.10]	-[1.46]
HML	0.22	0.16	0.10	0.03	0.22	0.01
	[4.15]	[3.02]	[1.32]	[0.34]	[1.82]	[0.06]

As a final test for comparison of pricing of various risk measures, we also report performance of portfolios based on past months realised total volatility, which can be considered as the aggregate of the idiosyncratic and systematic risks we have considered. The results, reported in Table 5 above is the most obvious manifestation of the poor risk-return relationship in our sample; the monthly return decreases from 1.0 per cent in the lowest volatility portfolio P1 to 0.3 per cent from the highest volatility portfolio P5. The alphas are economically large and statistically significant versus both CAPM and FF-3 model. Since IVOL and market risk have been concluded to be positively related while both being mispriced individually, the finding that total volatility also is mispriced comes as a logical consequence. It should also be noted that the mispricing of total volatility is not explained by value in our sample (as been concluded in earlier studies, e.g. Scherer (2010)), as the P1-P5 portfolio alpha is not smaller in the FF-3 regression than in the CAPM regression, and has an insignificant loading on the HML factor.

One potential explanation for different HML loadings between high IVOL stocks and stocks with high systematic and total risk could be that the high IVOL stocks are to a large extent financially distressed value stocks which are sensitive to firm-specific news that are crucial for the short-term prospects of the firm, while the stocks with high systematic and total risk are to a larger extent stocks of high growth firms, for which the value of is more sensitive to the macro-economic development and changes in discount rates.

The failure of the systematic risk factors applied in this study to correctly price the ranked portfolios is challenging to explain using theories of efficient markets and risk averse investors, where higher returns would on average only be earned by taking on additional non-diversifiable risk. The fact that we find a significant risk premium for the asset class as a whole of 1.0% per month over the risk-free rate implies that investors are risk averse in the asset class investment decision, which makes the evidence of a poor risk-return trade-off within the asset class in our sample puzzling.

As the theories of efficient markets have yet to come up with such satisfactory risk factors as to why investors would accept lower returns for holding high IVOL stocks, we consider the best available explanation for our findings from the behavioural finance school, such as the theory mentioned by Baker et al. (2011) where overconfident investors create an excessive demand for stocks that exhibit high volatility. The fact that the mispricing of IVOL seems to come from excess demand for high IVOL stocks and that the mispricings are largest within high beta stocks give further support to this theory. To the extent theories of efficient markets and behavioural finance represent competing views within asset pricing, our findings in this study thus supports the latter.

Although this study applies the standardly accepted market, size and value risk factors, there could be other risk factors we have not considered that could, in theory, make our results compatible with efficient markets. Furthermore, according to the theory of “Benchmarks as limits to arbitrage” by Baker et al. (2011), our factor models are poor proxies of risk for the institutional fund managers who are more concerned by deviating from their delegated benchmark.

While we have been able to conclude the mispricing of IVOL relative to the FF-3 factor model and mispricing of market beta and volatility versus the CAPM, we have not elaborated further on whether these mispricings constitute opportunity for arbitrage in the market as potential limits to arbitrage, such as shorting and liquidity constraints, and transaction costs are not considered explicitly. We can conclude a significant movement among the portfolios ranked on IVOL (for details see Table 8 and Figure 6 and 7 in the Appendix). Assuming one applies the screening methodology based on the sorting of IVOL, our results imply a need for rebalancing 60 per cent of the stocks of the portfolio each month. To illustrate the implied cost of an active trading strategy based on the IVOL screening analysis and a long-short equally weighted zero-cost portfolio, we conduct a simplified analysis based on the research by Barber and Odean (2000). According to the authors, trading costs estimates should include such items as the bid-ask spread, possible market impact costs, i.e. the cost of adding additional sell or buy pressure to a stock and commissions charged. Naturally, such costs are considered higher for smaller and less liquid stocks compared to large and very liquid ones. Disregarding such differentiation, Barber and Odean (2000) present an average cost estimate of a round-trip transaction cost (i.e. a completion of one buy and sell order) to 4 per cent. This may be a rather high estimate, especially considering that technological innovation has reduced trading costs since the publication of the Barber and Odean (2000) paper. However, assuming that 60 per cent of the stocks included in the P1-P5 zero cost portfolio would have to be bought and sold, this would imply a 2.4 per cent rebalancing cost for the aggregated portfolio each month. Needless to say, implementing this strategy with a monthly rebalancing would be very costly. However, no further examination has been done on the possibility to create an economically viable trading strategy using other holding periods as this is beyond the designated scope of this essay.

Table 6: Summary of the P1-P5 portfolio characteristics and regression results for the various ranking methods

The table below provides descriptive statistics and regression results on the value-weighted returns of the zero cost portfolio P1-P5, ranked on IVOL, market beta and volatility during the previous month y, where P1 is the portfolio with the lowest IVOL and P5 is the portfolio with the highest IVO. The arithmetic and geometric means are the average monthly excess returns of the portfolios. Volatility is the realized monthly standard deviation of the portfolio. Alphas and factor loadings from the CAPM and FF-3 regressions are reported respectively; with Newey-West (1987) robust t-statistics are reported in square brackets below each coefficient.

Summary of P1-P5 portfolios

				Beta sorted IVOL			
	FF-3IVOL	CAPMIVOL	Beta	Low	Mid	High	VOL
Arithmetic mean	0.04%	-0.12%	0.00%	-0.52%	0.16%	-0.48%	0.10%
Geometric mean	0.28%	0.13%	0.23%	-0.21%	0.48%	0.29%	0.66%
Median	0.02%	-0.02%	0.10%	0.41%	-0.12%	-0.01%	0.78%
Skewness	-0.27	-0.16	1.53	-1.46	1.51	-4.08	10.86
Kurtosis	2.76	2.47	-0.01	6.06	13.03	39.52	-1.58
Volatility	6.84%	7.25%	7.13%	8.63%	8.43%	13.89%	10.31%
CAPM							
Alpha	0.40%	0.24%	0.73%	0.09%	0.66%	-0.06%	1.08%
	[0.92]	[0.53]	[1.99]	[0.19]	[1.23]	-[0.08]	[2.08]
MKT	-0.36	-0.36	-0.74	-0.62	-0.51	-0.42	-0.98
	-[3.43]	-[3.71]	-[8.13]	-[5.15]	-[4.66]	-[2.01]	-[9.59]
FF-3							
Alpha	1.14%	0.94%	-0.04%	0.35%	0.86%	1.56%	1.15%
	[2.85]	[2.20]	-[0.14]	[0.78]	[1.63]	[2.34]	[2.23]
MKT	-0.60	-0.59	-0.53	-0.70	-0.61	-0.94	-1.04
	-[5.85]	-[6.50]	-[5.88]	-[5.06]	-[5.45]	-[3.56]	-[9.58]
SMB	-0.68	-0.69	0.49	-0.25	-0.38	-1.42	-0.25
	-[4.38]	-[4.27]	[3.56]	-[1.24]	-[2.52]	[0.00]	-[1.46]
HML	-0.42	-0.38	0.49	-0.15	-0.06	-0.93	0.01
	-[3.43]	-[3.12]	[4.83]	-[1.09]	-[0.40]	-[4.26]	[0.06]

VI Concluding remarks and suggestions for future research

This study adds to the growing body of evidence of low volatility anomalies by examining the pricing of IVOL in the Nordic equity markets, which is complemented with an analysis of the relationship between pricing of total volatility and systematic market risk. We find a virtually flat relationship between the average returns among the portfolios ranked on previous months IVOL, but that a zero cost portfolio long in low IVOL stocks and short in high IVOL stocks earns positive alpha when evaluated against the FF-3 model, meaning that we reject our null hypothesis. We also conclude that the mispricing of IVOL is largest in high IVOL stocks, which supports behavioural theories of overconfident investors as explanation of why high volatility securities sometimes are found to underperform low volatility securities.

We find a positive relationship between IVOL and market beta, and conclude that portfolios ranked on market beta are mispriced against the CAPM. Although the two measures of risks are mispriced against different risk factors, we have not been able to fully disentangle the effect and conclude whether it is an irrational demand for IVOL or IVOLs relationship to market beta which is driving the mispricing of the IVOL ranked portfolios.

Our findings point toward a weak, flat or even inverted risk-return relationship on the Nordic equity markets as the difference in mean returns in portfolios sorted on volatility, measured as total, systematic or idiosyncratic risk, appear to be non-existent or even decreasing in our sample period as we consider the returns from portfolios sorted on past volatility according to these measures. The lack of risk-reward according to the systematic risk factors is evident from our regression analysis, where the FF-3 factor model fails to price portfolios sorted on IVOL while the CAPM fails to price total volatility and systematic volatility.

The presented results have different implications depending on the type of risk preferences the investor has. For the diversified mean-variance investor, the results prescribe to invest in low total volatility and low beta stocks as a mean to improve portfolio utility, as these portfolios have given significant alphas against the CAPM in our regression analysis. For an investor who dislikes the exposure from value stocks/high book to market stocks, i.e. behaves according to the risk-based explanations of the value premium, our results subscribes towards investing in low IVOL while consequently avoiding high IVOL stocks. Another way of interpreting the results from the IVOL ranked FF-3 regressions is to conclude that a portfolio long low IVOL and short high IVOL results in creating a hedge against high book-to-market exposure, without having resulted in lower returns, which should have increased investor utility according to the risk-based theory of the value premium.

An interesting topic for future research would be to further evaluate how value investing can be combined with low-volatility investing. Value investing has for some time been appraised by scholars and practitioners due to its long and consistent history of generating excess returns not explained by the CAPM, while low volatility investing is currently gaining credit among academics due to its attractive features of yielding equal or above-average returns without exposing the investor to above-average risk. Low-volatility strategies have however sometimes been claimed to derive their above average returns from exposure to the value factor (Scherer 2010). We therefore find the indications in our study from the Nordics that low IVOL strategies relate negatively to the value factor (the P1-P5 IVOL strategy loading on HML is negative and clearly statistically significant) worth to evaluate further within the practice of low volatility investing. The finding that the high IVOL portfolio loads heavily on value without generating the return on value implies there are value stocks with high IVOL which underperform other value peers, and we would encourage studies on methods using the IVOL anomaly to enhance the performance on value investment strategies. Furthermore, as the trading strategy tested in this thesis would result in a too high turnover in order to be profitable after trading costs, it would be interesting to see whether a similar trading strategy but with a longer holding period would generate similar results and therefore could be profitable in practice.

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VIII Appendix

Figure 4: Presentation of computed indices

The chart below presents the development of the Nordic Index Gross TR, the Nordic Index Excess TR and OMX Stockholm All Share over the selected sample period.

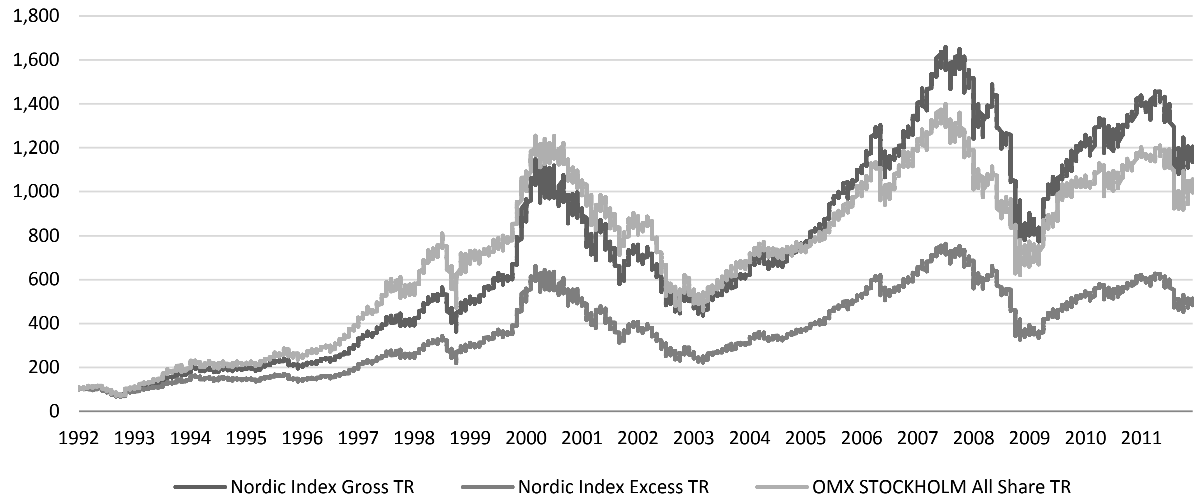

Figure 5: Presentation of the average cross-sectional idiosyncratic volatility

Below chart shows the cross-sectional average idiosyncratic volatility relative to the CAPM and FF-3 measured as the equally weighted 12 month rolling annualized standard deviation.

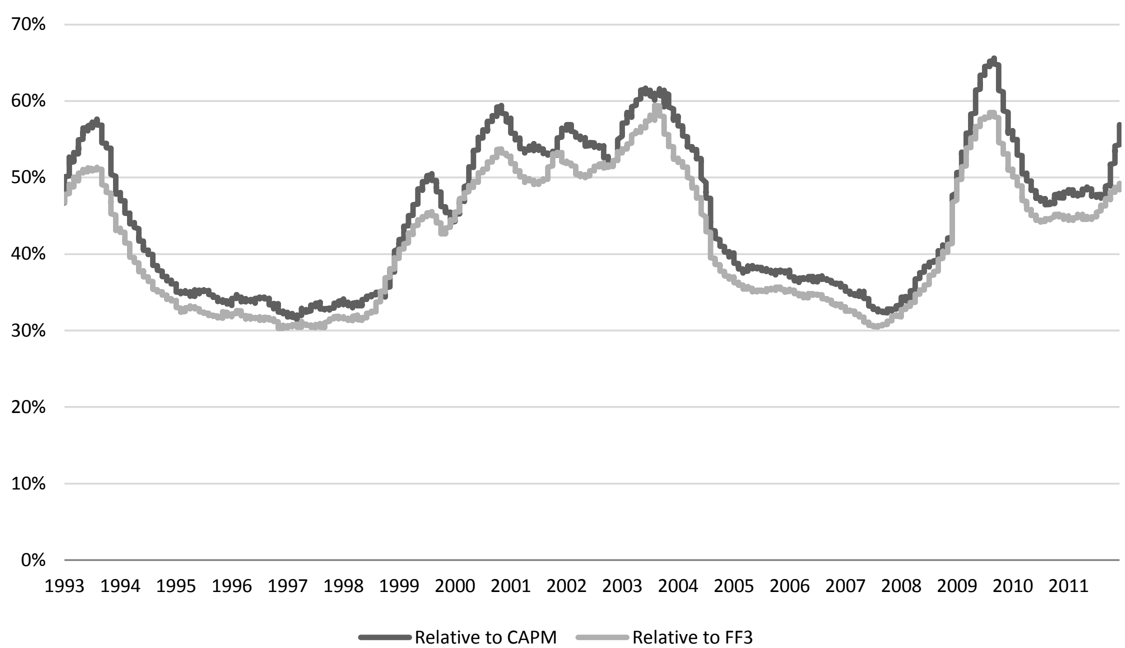

Table 7: Summary of the number of observations per year and month

The table below summarises the number of observations for each month and year of the sample.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	
1992	215	221	219	221	222	222	219	224	230	231	232	233	1992
1993	235	239	239	239	240	241	241	241	241	241	243	243	1993
1994	245	245	247	247	251	255	256	269	269	273	274	276	1994
1995	276	276	278	278	280	281	285	285	285	285	286	288	1995
1996	289	291	291	294	298	303	306	313	314	314	316	316	1996
1997	316	317	319	320	323	329	341	342	343	346	350	358	1997
1998	361	363	365	366	368	375	382	384	384	390	391	394	1998
1999	399	401	401	405	411	412	424	426	426	430	436	440	1999
2000	443	443	444	447	449	457	463	465	465	465	469	470	2000
2001	475	476	477	478	480	482	490	491	491	496	498	499	2001
2002	497	497	498	500	500	502	506	507	510	506	508	508	2002
2003	505	507	508	510	508	512	511	511	510	513	513	515	2003
2004	516	516	516	519	520	523	527	525	525	530	530	529	2004
2005	533	536	535	535	537	543	546	548	551	551	555	560	2005
2006	565	566	566	570	574	578	581	583	584	587	590	599	2006
2007	609	613	614	617	618	626	636	638	638	637	645	654	2007
2008	657	658	658	657	656	657	660	663	663	665	667	667	2008
2009	670	670	670	670	670	670	667	670	669	671	671	671	2009
2010	672	671	671	675	675	675	681	681	681	680	682	682	2010
2011	685	685	684	685	687	687	687	687	687	687	687	687	2011

Table 8: Summary of the movement between portfolios per annum

The table below summarises the movement of the stocks between the FF-3 IVOL portfolios for the entire time series. The values refer to the aggregate number of moves for the 12 rebalances that occur every year, i.e. at the end of every month.

	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	
Remains	1,028	1,116	1,129	1,209	1,361	1,454	1,731	1,926	2,243	2,570	2,709	2,561	2,520	2,492	2,432	2,573	2,711	2,913	3,143	3,472	Remains
% of total	*41.5*	*38.8*	*36.7*	*35.9*	*37.6*	*36.7*	*38.6*	*38.8*	*41.1*	*44.3*	*44.8*	*41.7*	*40.2*	*38.3*	*35.2*	*34.3*	*34.2*	*36.2*	*38.7*	*42.1*	*% of total*
Moves 1	870	1,127	1,190	1,335	1,370	1,467	1,650	1,932	2,147	2,256	2,355	2,407	2,431	2,482	2,627	2,769	2,934	3,080	3,140	3,152	Moves 1
% of total	*35.2*	*39.2*	*38.7*	*39.6*	*37.9*	*37.0*	*36.8*	*38.9*	*39.4*	*38.9*	*39.0*	*39.2*	*38.8*	*38.1*	*38.0*	*36.9*	*37.0*	*38.3*	*38.7*	*38.3*	*% of total*
Moves 2	323	403	489	506	565	716	725	726	765	701	677	829	933	1,073	1,241	1,352	1,450	1,391	1,327	1,167	Moves 2
% of total	*13.1*	*14.0*	*15.9*	*15.0*	*15.6*	*18.1*	*16.2*	*14.6*	*14.0*	*12.1*	*11.2*	*13.5*	*14.9*	*16.5*	*17.9*	*18.0*	*18.3*	*17.3*	*16.3*	*14.2*	*% of total*
Moves 3	148	137	179	203	222	230	286	259	220	200	203	246	262	351	474	610	645	509	400	357	Moves 3
% of total	*6.0*	*4.8*	*5.8*	*6.0*	*6.1*	*5.8*	*6.4*	*5.2*	*4.0*	*3.4*	*3.4*	*4.0*	*4.2*	*5.4*	*6.9*	*8.1*	*8.1*	*6.3*	*4.9*	*4.3*	*% of total*
Moves 4	106	92	87	118	99	95	95	123	76	79	102	96	121	117	141	195	189	146	110	90	Moves 4
% of total	*4.3*	*3.2*	*2.8*	*3.5*	*2.7*	*2.4*	*2.1*	*2.5*	*1.4*	*1.4*	*1.7*	*1.6*	*1.9*	*1.8*	*2.0*	*2.6*	*2.4*	*1.8*	*1.4*	*1.1*	*% of total*
Total	2,475	2,875	3,074	3,371	3,617	3,962	4,487	4,966	5,451	5,806	6,046	6,139	6,267	6,515	6,915	7,499	7,929	8,039	8,120	8,238	Total

Figure 6: Distribution of the monthly movement among the observations

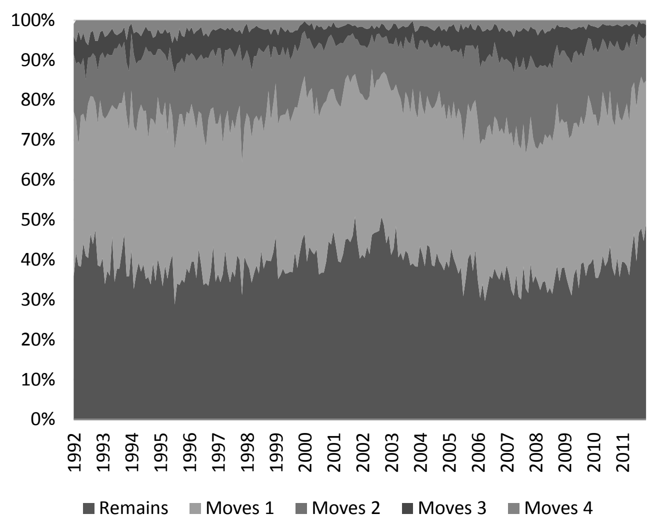

Figure 7: Summary of the monthly movement between portfolios

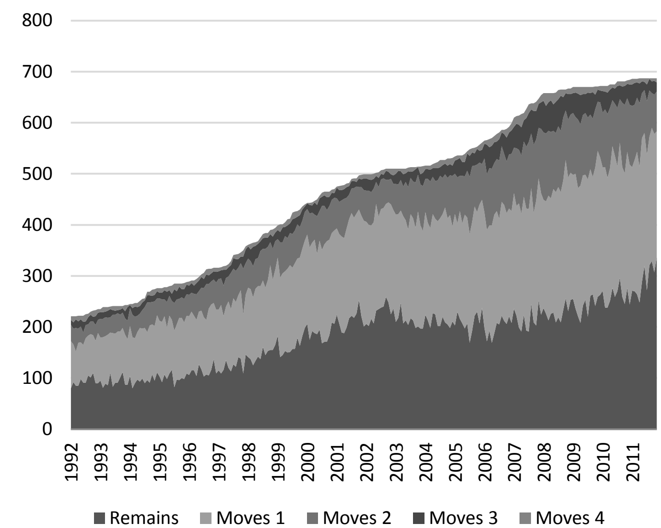

Table 9: Statistics of portfolio returns of MKT, SMB and HML

The table below presents descriptive statistics of the monthly excess return of the market and the zero cost portfolios Small-minus-big and High-minus-low.

Returns	Market	SMB	HML
Arithmetic mean	1.0%	0.1%	1.1%
Median	1.5%	0.0%	0.9%
1st quartile	-2.6%	-1.9%	-1.6%
3rd quartile	4.4%	2.0%	2.9%
Volatility	6.2%	3.5%	5.3%
Kurtosis	1.22	1.75	4.90
Skewness	0.02	0.21	0.74
Year			
1992	-1.3%	0.3%	-0.7%
1993	4.4%	-0.6%	6.8%
1995	-0.1%	0.5%	0.3%
1996	2.7%	-0.2%	0.2%
1997	2.8%	-0.4%	0.7%
1998	1.9%	-2.2%	-2.7%
1999	5.5%	-0.9%	-4.1%
2000	0.2%	0.7%	2.6%
2001	-1.8%	-0.1%	5.0%
2002	-3.1%	-0.1%	6.8%
2003	1.4%	1.9%	1.5%
2004	1.7%	1.3%	1.5%
2005	3.3%	1.6%	-0.1%
2006	1.6%	0.3%	1.2%
2007	0.8%	-0.8%	-0.8%
2008	-4.4%	-1.1%	-0.5%
2009	3.2%	1.4%	3.4%
2010	1.5%	-0.6%	0.5%
2011	-1.2%	-0.1%	-0.9%
Month			
January	1.6%	0.6%	1.7%
February	1.0%	-0.1%	2.1%
March	4.1%	-2.3%	3.4%
April	0.2%	0.0%	2.1%
May	-0.7%	0.1%	0.2%
June	0.2%	0.1%	1.9%
July	-1.3%	0.9%	0.7%
August	-2.1%	0.9%	-1.6%
September	2.3%	-1.5%	0.6%
October	3.6%	-1.1%	-0.3%
November	1.7%	-0.2%	0.3%
December	1.4%	3.5%	1.5%

Table 10: Summary of CAPM regressions of the included ranked portfolios

The table below presents the CAPM regression outcome on all ranking methods, where the first word in the left column denotes the ranking method, and the second regression method. Alphas and factor loadings are reported respectively, with Newey-West (1987) robust t-statistics are reported in square brackets below each coefficient

	Low		**Alpha**		High	
	P1	P2	P3	P4	P5	P1-P5
FF-3IVOLCAPM	-0.12%	0.38%	0.13%	-0.10%	-0.52%	0.40%
	-[0.60]	[2.13]	[0.61]	-[0.35]	-[1.37]	[0.92]
CAPMIVOLCAPM	-0.21%	0.24%	0.29%	-0.25%	-0.45%	0.24%
	-[0.91]	[1.29]	[1.33]	-[0.85]	-[1.20]	[0.53]
CAPMBETACAPM	0.47%	0.62%	0.31%	0.10%	-0.26%	0.73%
	[1.80]	[2.92]	[1.38]	[0.54]	-[1.20]	[1.99]
CAPMBETALOWCAPM	0.43%	1.00%	0.75%	0.16%	0.34%	0.09%
	[1.69]	[3.15]	[2.38]	[0.47]	[0.74]	[0.19]
CAPMBETAMIDCAPM	0.21%	0.04%	0.76%	0.39%	-0.46%	0.66%
	[0.74]	[0.14]	[2.65]	[1.07]	-[1.01]	[1.23]
CAPMBETAHIGHCAPM	-0.19%	0.12%	0.28%	-1.09%	-0.13%	-0.06%
	-[0.84]	[0.49]	[0.77]	-[2.90]	-[0.18]	-[0.08]
VOLCAPM	0.60%	0.35%	0.15%	-0.02%	-0.48%	1.08%
	[2.47]	[1.57]	[0.63]	-[0.07]	-[1.09]	[2.08]

	Low		**MKT**		High	
	P1	P2	P3	P4	P5	P1-P5
FF-3IVOLCAPM	0.87	0.91	0.91	1.06	1.23	-0.36
	[17.65]	[26.39]	[20.41]	[14.87]	[14.70]	-[3.43]
CAPMIVOLCAPM	0.86	0.88	0.94	1.08	1.22	-0.36
	[17.66]	[22.22]	[21.28]	[18.02]	[16.19]	-[3.71]
CAPMBETACAPM	0.55	0.46	0.58	0.73	1.28	-0.74
	[6.83]	[11.45]	[11.31]	[18.08]	[32.41]	-[8.13]
CAPMBETALOWCAPM	0.25	0.42	0.46	0.67	0.87	-0.62
	[5.14]	[6.27]	[6.07]	[7.93]	[6.88]	-[5.15]
CAPMBETAMIDCAPM	0.49	0.56	0.61	0.69	1.00	-0.51
	[7.21]	[11.37]	[10.82]	[8.89]	[11.96]	-[4.66]
CAPMBETAHIGHCAPM	1.10	1.10	1.12	1.29	1.53	-0.42
	[22.32]	[25.92]	[17.32]	[18.06]	[7.42]	-[2.01]
VOLCAPM	0.52	0.64	0.94	1.15	1.50	-0.98

Table 11: Summary of the FF-3 regressions of the included ranked portfolios

The table below presents the FF-3 regression outcome on all ranked portfolios, where the first word in the left column denotes the ranking method, and the second regression method. Alphas and factor loadings are reported respectively, with Newey-West (1987) robust t-statistics are reported in square brackets below each coefficient

	Low		**Alpha**		High	
	P1	P2	P3	P4	P5	P1-P5
FF-3IVOLFF-3	-0.05%	0.49%	-0.11%	-0.51%	-1.19%	1.14%
	-[0.21]	[2.67]	-[0.50]	-[1.85]	-[3.39]	[2.85]
CAPMIVOLFF-3	-0.13%	0.30%	0.09%	-0.64%	-1.07%	0.94%
	-[0.55]	[1.42]	[0.42]	-[2.26]	-[2.99]	[2.20]
CAPMBETAFF-3	-0.09%	0.34%	-0.04%	-0.14%	-0.05%	-0.04%
	-[0.44]	[1.70]	-[0.18]	-[0.69]	-[0.21]	-[0.14]
CAPMBETALOWFF-3	-0.02%	0.49%	0.22%	-0.33%	-0.38%	0.35%
	-[0.10]	[1.76]	[0.80]	-[1.09]	-[0.92]	[0.78]
CAPMBETAMIDFF-3	-0.04%	-0.18%	0.45%	-0.18%	-0.90%	0.86%
	-[0.13]	-[0.67]	[1.59]	-[0.50]	-[2.03]	[1.63]
CAPMBETAHIGHFF-3	0.07%	0.07%	0.01%	-1.38%	-1.48%	1.56%
	[0.30]	[0.27]	[0.02]	-[3.59]	-[2.33]	[2.34]
VOLFF3	0.31%	0.16%	0.04%	-0.06%	-0.84%	1.15%
	[1.37]	[0.71]	[0.17]	-[0.19]	-[1.89]	[2.23]

	Low		**MKT**		High	
	P1	P2	P3	P4	P5	P1-P5
FF-3IVOLFF-3	0.85	0.87	0.96	1.16	1.45	-0.60
	[16.84]	[23.96]	[23.48]	[17.62]	[17.89]	-[5.85]
CAPMIVOLFF-3	0.84	0.86	0.97	1.17	1.43	-0.59
	[18.24]	[18.41]	[25.00]	[20.00]	[18.60]	-[6.50]
CAPMBETAFF-3	0.70	0.54	0.64	0.76	1.23	-0.53
	[8.55]	[11.83]	[14.12]	[18.35]	[25.32]	-[5.88]
CAPMBETALOWFF-3	0.41	0.55	0.59	0.82	1.12	-0.70
	[8.49]	[7.91]	[7.32]	[8.27]	[8.70]	-[5.06]
CAPMBETAMIDFF-3	0.53	0.61	0.67	0.81	1.15	-0.61
	[7.20]	[12.59]	[12.34]	[10.19]	[13.51]	-[5.45]
CAPMBETAHIGHFF-3	1.03	1.10	1.20	1.38	1.97	-0.94
	[17.41]	[21.94]	[16.77]	[16.31]	[7.62]	-[3.56]
VOLFF3	0.57	0.67	0.95	1.16	1.61	-1.04
	11.98	12.51	20.04	18.30	18.68	-9.58

	Low		**SMB**		High	
	P1	P2	P3	P4	P5	P1-P5
FF-3IVOLFF-3	-0.05	-0.12	0.06	0.20	0.63	-0.68
	-[0.56]	-[1.62]	[0.85]	[1.61]	[5.20]	-[4.38]
CAPMIVOLFF-3	-0.07	-0.06	0.05	0.17	0.62	-0.69
	-[0.89]	-[0.72]	[0.52]	[1.68]	[4.65]	-[4.27]
CAPMBETAFF-3	0.38	0.19	0.06	-0.04	-0.11	0.49
	[3.28]	[2.30]	[0.70]	-[0.64]	-[1.37]	[3.56]
CAPMBETALOWFF-3	0.47	0.26	0.24	0.39	0.71	-0.25
	[4.28]	[1.99]	[1.77]	[3.03]	[4.61]	-[1.24]
CAPMBETAMIDFF-3	0.04	0.11	0.08	0.20	0.42	-0.38
	[0.58]	[1.00]	[0.82]	[1.41]	[3.16]	-[2.52]
CAPMBETAHIGHFF-3	-0.17	-0.03	0.18	0.25	1.25	-1.42
	-[2.00]	-[0.25]	[1.24]	[1.97]	[3.60]	-[4.03]
VOLFF3	0.05	-0.02	-0.03	0.02	0.30	-0.25
	[0.59]	-[0.22]	-[0.41]	[0.16]	[2.10]	-[1.46]

	Low		**HML**		High	
	P1	P2	P3	P4	P5	P1-P5
FF-3IVOLFF-3	-0.05	-0.05	0.17	0.28	0.37	-0.42
	-[0.65]	-[1.43]	[3.62]	[4.11]	[4.36]	-[3.43]
CAPMIVOLFF-3	-0.05	-0.03	0.14	0.27	0.34	-0.38
	-[0.63]	-[0.59]	[3.07]	[4.05]	[3.86]	-[3.12]
CAPMBETAFF-3	0.35	0.17	0.26	0.20	-0.14	0.49
	[4.22]	[3.64]	[5.26]	[5.01]	-[2.77]	[4.83]
CAPMBETALOWFF-3	0.24	0.34	0.37	0.28	0.39	-0.15
	[5.59]	[4.76]	[4.17]	[3.17]	[2.91]	-[1.09]
CAPMBETAMIDFF-3	0.19	0.15	0.23	0.40	0.24	-0.06
	[2.33]	[2.71]	[3.52]	[4.98]	[2.13]	-[0.40]
CAPMBETAHIGHFF-3	-0.17	0.05	0.17	0.17	0.76	-0.93
	-[3.19]	[0.94]	[2.28]	[2.18]	[3.52]	-[4.26]
VOLFF3	0.22	0.16	0.10	0.03	0.22	0.01
	[4.15]	[3.02]	[1.32]	[0.34]	[1.82]	[0.06]

Figure 8: Actual versus expected return of P1 and P5 portfolios according to the CAPM

The chart below presents actual returns against the expected returns of the ranked P1 and P5 portfolios according to their exposure to the FF-3 risk factors and their respective factor loadings

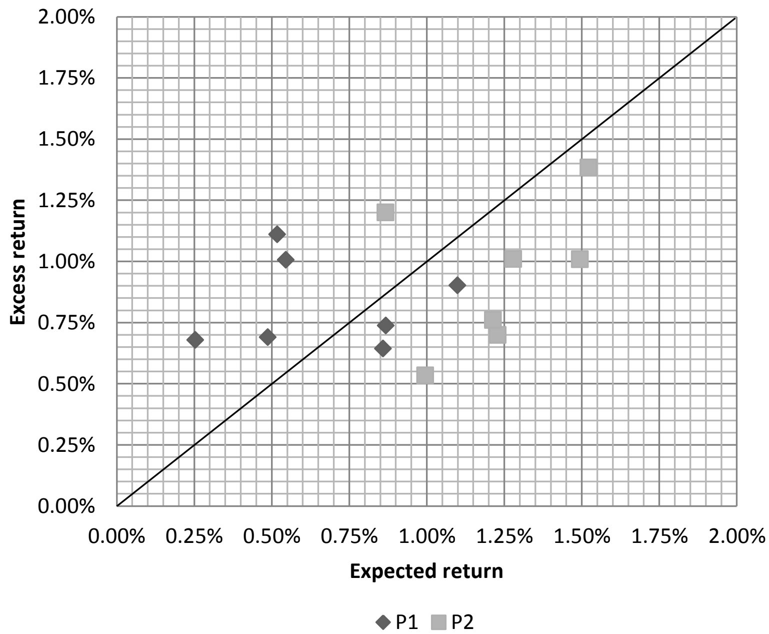

Figure 9: Actual versus expected of P1 and P5 portfolios return according to FF-3

The chart below presents actual returns against the expected returns of the ranked P1 and P5 portfolios according to their exposure to the FF-3 risk factors and their respective factor loadings.

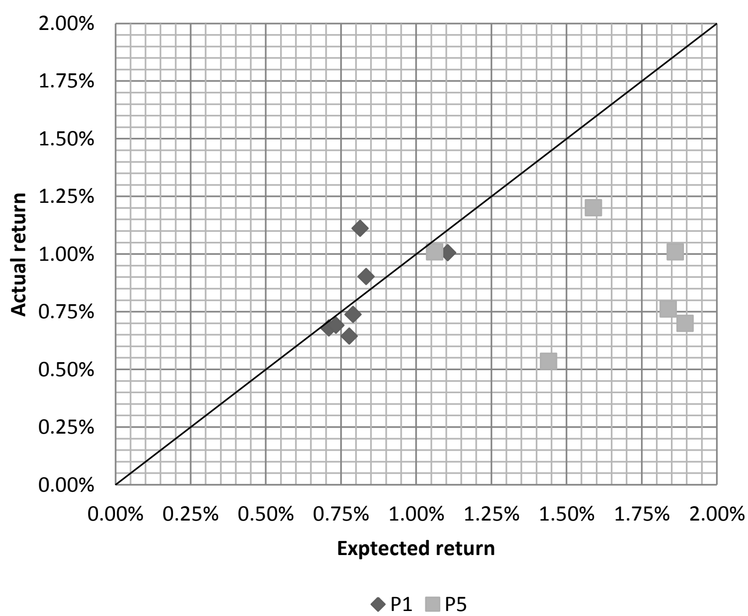

Figure 10: Alphas of the IVOL ranked portfolios versus market beta

The chart below shows the parameter estimate of the alphas against the market factor according to the CAPM and the FF-3 factor model for the portfolios ranked on IVOL relative to the CAPM and to the FF-3 model. In the notation, the first word denotes which factor model has been used to measure risk, the second denotes what is ranked, and the third denotes which factor model that was used to calculate the alpha. I.e. FF-3IVOLCAPM, means portfolios ranked on IVOL relative to the FF-3 factor model with alphas calculated against the CAPM.