

Does The Investor Get Compensated for Investing in Risky Corporate Bonds?

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This study investigates if the investor in US corporate bonds gets sufficiently compensated for default risk. This is done by examining the expected excess return for corporate bonds across varying levels of default probability. By discounting expected cash flows and using three different bankruptcy prediction models to estimate default probabilities, namely Hillegeist et al. (2004), Ohlson (1980) and Shumway (2001), we find that the investor does in fact not get compensated for investing in the riskiest segment of corporate bonds. The results are also consistent when applying historical default probabilities implied by credit ratings.

Keywords: Corporate Bonds, Default Risk, Expected Return, Yield Spread and Credit Ratings

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Over the past few years there has been an increased interest for high yield, or speculative¹, bonds among investors. Speculative bonds are riskier than investment grade bonds and hence the investor is granted a higher promised return, or yield, and thus the name “high yield”. If a company that has issued a high yield bond survives, the high promised return turns into a high realized return for the investor. But there are also cases where the company defaults and the realized return is substantially smaller than the promised. The higher the default probability, the lower expected value of promised payments.

A reason for the raised interest in corporate bonds might be the increasing requirements on banks regarding risk coverage and capital conservation (Basel Committee of Banking Supervision). We believe this will enlarge the market for high yield corporate bonds. With banks becoming more hesitant and adding restrictions to both the amount of funds provided to companies and which companies to provide funding to, we believe that more companies will turn to issuing bonds. The increase in risk consciousness from banks will probably, according to us, further expand the market for especially high yield bonds since the companies with poorer performance will be forced to resort to issuing bonds to raise capital for investments. The general increase in attention for the segment of high yield bonds and the assumed progression of this attention due to new legislation affecting banks is the reason why we choose to investigate if the investor holding these bonds gets sufficiently compensated.

The aim of this study is to examine whether the higher promised return on riskier bonds is sufficient to compensate the investor for the additional default risk when investing in high yield bonds. This is done by converting promised cash flows to expected by using calculated default probabilities from three different bankruptcy prediction models (Hillegeist, Keating, Cram and Lundstedt (2004), Ohlson (1980) and Shumway (2001)), as well as implied default probabilities from credit ratings which are included for reference. The study is performed on US data. Using discounted cash flows as the pricing method the expected return on the bonds is solved for by discounting expected cash flows to reach the market price. The discount rate required to end up at the market price equals the expected return for each bond. When deducting a risk free rate from the expected return an expected excess return is generated. The relation between expected excess return and default probability is examined across levels of riskiness. To facilitate presentation and interpretation, the bonds are grouped into four categories with increasing riskiness based on the level of default risk. Finally expected excess returns are plotted against default risk to illustrate the relation and investigate the trend across default probability categories.

Previous studies conclude that there are several types of risks associated with holding a corporate bond compared to holding a Treasury bond that is assumed to be risk free. The risks are expressed in the yield spread, which is the difference between the yield on a corporate bond and the yield on a corresponding Treasury bond (same maturity, face value, coupons etc.). Riskier corporate bonds have wider yield spreads since the investor requires higher promised yields as the amount of risk increases. Many studies have focused on the determinants of the yield spread, i.e. the factors to consider when

¹ Using S&P notations, bonds rated AAA-BBB are considered investment grade while bonds rated BB-C is classified as speculative

pricing a bond. Default risk is often the starting point and the most intuitive and generally accepted one. Default risk is also the central factor in this study and will be elaborated on later.

Another commonly mentioned factor besides default risk is liquidity, or rather illiquidity (Chacko (2005), Delianedis and Geske (2001), Downing, Underwood and Xing (2006), Driessen (2005) and Schultz (2001)). Studies performed by Delianedis and Geske (2001) and Fleming (2003) conclude that the Treasury bond market is more liquid than the corporate bond market. Chen, Lesmond and Wei (2007) argue that liquidity differs among corporate bonds and indeed can explain part of the yield spread. Studies based on US corporate bond data, including this study, should also consider a tax effect since interest payments from corporate bonds are subject to state taxes while payments from Treasury bonds are not (Delianedis and Geske (2001) and Elton, Gruber, Agrawal and Mann (2001)). Hence the investor requires a higher pre-tax return on corporate bonds.

However, there is no strong theoretic support for the impact of liquidity risk or any other risk but default on corporate bond prices. Theoretically, when buying a corporate bond, the investor buys the right to future cash flows connected to the bond. Future cash flows are only dependent on whether the company survives or defaults on its payments, and in that case the recovery amount. Hence default risk should be the only significant risk. The investor's ability to resell the bond does not affect cash flows. One can argue that taxes have an impact on cash flows and hence should be included in the pricing formula. However the tax exposure is constant across bonds. Income from investment grade bonds are subject to the same taxes as speculative bonds, and in this study this is overlooked. Since we are looking for trends in expected excess return across levels of riskiness, incorporating taxes would not change the behavior of the trend, only shift the trend downwards.

Even if default risk is the only risk with strong theoretic support, the level of its impact on the yield spread has been debated. Many have made efforts to examine to what extent default risk can explain corporate yield spreads. Driessen (2005), Elton et al. (2001) and Delianedis and Geske (2001) claim that default risk only accounts for a limited part of the yield spread but they all conclude that default risk is an important factor, and often the starting point. Worth noticing is their choices of default probability prediction. Driessen (2005) and Delianedis and Geske (2001) model default as a jump process with stochastic intensity while Elton et al. (2001) use default probabilities implied by credit ratings and transition matrixes. The method used by Delianedis and Geske (2001) and Driessen (2005) is derived from the Black-Scholes-Merton framework, which will be described in more detail when discussing corporate bond pricing. The Black-Scholes-Merton framework is often criticized for underestimating default probability (Delianedis and Geske (2001)). The findings and method used by Elton et al. (2001) are however interesting. We also use default probabilities implied by credit ratings in our study even if it is for reference purposes. Besides default risk, Elton et al. (2001) find tax and systematic risk² factors in the yield spread, factors whose implications will be reflected upon in the discussion of the results.

² Systematic risk is risk factors common to the whole economy, nondiversifiable risk; also called market risk

The remaining part of the yield spread, besides default risk, is often referred to as the credit risk puzzle (Amato and Remolona (2003) and Kozhemiakin (2007)). For example, one puzzling thing about the yield spread is the fact that AAA bonds generate returns in excess of the risk free rate when they are assumed to be risk free. In addition to Elton et al. (2001), studies performed by Driessen (2005), Kozhemiakin (2007), Liu, Shi, Wang and Wu (2009), Thorsell (2008) and Zhang (2006) also find support for a systematic risk factor in the yield spread, which can explain parts of this credit puzzle. They draw parallels to the systematic risk that appears on the stock market and argue that if the stock investor gets compensated for systematic risk, so should the bond investor. Intuitively some kind of systematic risk factor makes sense since it is more common that companies default in recessions than in booms. Elton et al. (2001) and Thorsell (2008) also claim that the systematic risk premium is larger for lower rated bonds implying that investors are risk averse³. Elton et al. (2001) claim that a systematic factor accounts for somewhere between 19-41% of the yield spread. Even if empirical evidence indicate that there is some kind of systematic risk factor it will be disregarded in the calculations, primarily since it is hard to quantify in our setting. However, we will consider the impact of systematic risk on our results and conclusions in the discussion part of Section IV.

When examining expected or promised returns, one must reflect upon the pricing method since expected return is a function of price. In literature and practice, two methods have gained the most attention; discounting future cash flows and contingent claims analysis⁴. The discounted cash flow method is the generally accepted theoretic way to value not only bonds but also stocks, companies, individual projects etc. There are however some practical limitations associated with the discounted cash flow method when it comes to accounting for risk. Risk can be accounted for in the numerator by adjusting expected cash flows or the denominator by adjusting the discount rate. A riskier bond should have lower expected cash flows (in relation to the promised) and/or be priced using a higher discount rate. The problems arise when estimating these adjustments. If the risks are expressed in probabilities, expected cash flows can be adjusted. If they are not, a risk premium is often added to the discount rate. The size of the premium needs to be estimated and is often the core of the problem. The edge of the contingent claims approach is that it provides a solution to this issue by solely using variables that are observable and possible to estimate. Black and Scholes (1973) and Merton (1974) presented a contingent claims framework, which has been the ground for the majority of the literature published on risky debt pricing. Using risk-neutral probabilities, where all traded securities have an expected return equal to the risk-free rate, the value of the debt claim is calculated by discounting future cash flows (under the risk-neutral measure) at the risk-free rate. In this way, the problems with determining the discount rate is partly avoided. Only partly though, since the problem is shifted to finding a risk adjusted discount rate for the underlying company (asset) instead. However, given the value of the underlying company, the Black-

³ A risk averse investor will consider risky portfolios only if they provide compensation for risk via a risk premium. Given two investments with the same expected return, a risk averse investor will always choose the less risky one.

⁴ Similar to option pricing theory but modified to be applicable on corporate liabilities.

Scholes-Merton framework provides a closed end formula for determining the value of the bond. This is probably the reason why it is so often referred to in the literature. The framework is criticized for its lack of empirical validity (Jones, Mason and Rosenfeld (1984)) but it has been developed in several studies to fit actual observations better (Black and Cox (1976) and Longstaff and Schwarz (1995) among others). Yet, the improved framework and different extensions still underpredict yield spreads (Covitz and Downing (2007)). In the framework, default occurs when the value of a company's assets, which is a function of a Wiener or Brownian motion process⁵, falls below a pre-set threshold, where all assets are exhausted. However this is an unrealistic simplification since most companies default before all their assets are exhausted.

Even if the Black-Scholes-Merton framework dominates the literature on bond pricing, the basic pricing method with discounted future cash flows is the better alternative for this study since our data allows us to sidestep the drawbacks. This is made possible by using exogenously calculated default probabilities, i.e. adjustments for default risk do not have to be made endogenously by the pricing model. We use four different measures of default probability. Three of the selected measures are calculated using bankruptcy prediction models presented by Hillegeist et al. (2004), Ohlson (1980) and Shumway (2001). One of them, the one by Hillegeist et al. (2004), is derived from the Black-Scholes-Merton framework. This is interesting since most corporate bond studies are based on that framework. The forth measure is historical default rates for each credit rating provided by Standard & Poor's rating institute. The historical default probabilities connected to credit rating classes are included for reference purposes. One of the key contributions of our study is that we do not rely solely on historical default rates or stochastic processes to determine the probability of default. In addition, we use the Ohlson (1980) model which is based on accounting data and the Shumway (2001) model which predicts default probability using accounting and market data as input variables. Using several models we decrease the impact from the choice of bankruptcy prediction model on our findings. We describe and discuss the models in detail in Section I. Previous studies examining corporate yield spreads have, to our knowledge, used either historical default rates provided by rating institutes (Elton et al. (2001)) or default rates implied by the Black-Scholes-Merton framework (Collin-Dufresne, Goldstein and Martin (2001), Delianedis and Geske (2001), Driessen (2005), Huang and Huang (2003) and Kim, Ramaswamy and Sundaresan (1993)). This study is differentiated by also including default risk measures, solely or partly, based on accounting data.

Given that we disregard taxes, systematic risk and risk aversion in the calculations, we use the cash flows in the numerator of the discounted cash flow valuation method to account for default risk. We do this by converting promised cash flows to expected by adjusting for default probabilities calculated by using the selected bankruptcy prediction models. Standard & Poor's credit ratings and their implied default probabilities are also used, as reference. Applying discounted cash flow valuation and using

⁵ A stochastic process where the change in a variable during each short period of time of length Δt has a normal distribution with a mean equal to zero and a variance equal to Δt

expected cash flows and the market price as inputs we solve for an expected return on each bond. Deducting the risk free rate from the calculated expected return generates an expected excess return for each bond. The bonds are grouped into four categories, Category I-IV, with increasing riskiness based on the level of default risk. The expected excess returns are plotted against default risk to illustrate the relation and investigate the trend across default probability categories.

Our findings suggest that the expected excess return increases as the level of default probability rises until it reaches a turning point. The turning point differs between the bankruptcy prediction models and is Category III (0.5-2% default probability) for Ohlson (1980) and Hillegeist et al. (2004) and Category II (0.1-0.5% default probability) for Shumway (2001). This implies that the investor does not get compensated when investing in riskier bonds. These results are consistent also for credit ratings. For B rated bond, the expected excess return is lower than for BB rated.

The remains of this thesis are divided into sections. In Section I we elaborate on bankruptcy prediction models, motivate our choices and explain the methodology of calculating default probabilities using these models. The next section, Section II, includes the sources and description of the dataset and an explanation of the criteria used in designing the dataset and the exclusions made. In the third section we describe the methodology of our study, the rationale and calculations behind expected return and expected excess return. The fourth section contains the results of the study. We plot the results in graphs and explain how the graphs should behave if the market accounts for default risk in a way consistent with the selected bankruptcy prediction models. We also discuss interpretations and possible implications on the results. The final section, Section V states our conclusion.

I. Bankruptcy Prediction Models

Many models for estimating business failure have been developed over the years since ratio analysis was initiated in the beginning of the 20th century. One of the very first authors to use ratio analysis to estimate business failure was Merwin (1942). But the author who is credited with the spark of ratio analysis to predict business failure, or at least is the mostly referred to as the first, is Beaver (1966). He took a univariate⁶ approach to determine if financial ratios were useful for financial decision making. Beaver (1966) found that a number of ratios did have significant explanatory power when it comes to predicting business failure. But univariate approaches can be misleading since only one variable is investigated at a time instead of looking at their joint effect. The univariate approach quickly became outdated within research and was replaced by multivariate analysis⁷. Altman (1968) developed a multiple discriminant analysis⁸ model for predicting business failure that relied on five variables instead of one.

⁶ Univariate analysis is carried out by estimating the effect variables have *individually* on some dependant variable.

⁷ Multivariate analysis is carried out by estimating the effect a number of variables have *jointly* on some dependant variable

⁸ Multiple discriminant analysis is closely related to multiple regression analysis with the difference that the dependant variable is not continuous. It instead takes on specific values as 0 and 1 or Fail and non-Fail

Altman's ratios span across company solvency, profitability and liquidity to give a better description of a company's risk of default. The works of Altman (1968) were a significant contribution to the field of business failure prediction and led the way for similar models developed by for example Deakin (1972) and Edmister (1972). However there are certain limitations to the multiple discriminant analysis approach. Multiple discriminant analysis imposes assumptions on the data.⁹ The output of the model is also an arbitrary number that has to be related to some cut off value classify the company as failing or non-failing, instead of generating a probability of default expressed in percentage points.

Santomero & Visno (1977) and Ohlson (1980) mitigated the above mentioned problems regarding both the data assumptions and the output by developing a logit model using maximum likelihood estimation. A logit model is a way of transforming outputs of maximum likelihood estimation from values into probabilities. In a sample with two groups, failed and non-failed companies, they estimated models consisting of multiple financial ratios as variables. The coefficients for all variables are estimated so the probability of default is as high as possible for failed firms and as low as possible for non-failed firms. By using this approach the output can be presented in terms of probabilities of default using the logistic function rather than just being an arbitrary number (see Equation 2).

A similar model to the one developed by Ohlson (1980) was developed by Skogsvik (1988). He also uses maximum likelihood estimation but instead of using the logistic function he uses the probit function to transform the output into a probability of default. The probit function generates very similar results as the logit function.

The trend in more recent studies has been to include more market data than accounting data in bankruptcy prediction models. This is done to move away from the disadvantages of accounting data such as differing accounting methods and timing issues like reports being released at times when they describe a past state of the company. Using market data allows for bankruptcy prediction continuously over the year for publically traded companies since it is constantly updated. A drawback is that market data only is available for listed companies.

Theodossiou (1993) and Shumway (2001) argue that the previously mentioned models are flawed since they are static¹⁰ and do not take the fact that a company's risk of default varies with time and the age of the company into account. Theodossiou (1993) develops a model that signals a company's deterioration towards default when a selection of financial ratios start to move away from the average of the same financial ratios for a group of non-failed companies. Therefore his model signals if a company is about to fail rather than waiting for it to fail as the static models do in their estimation processes. The

⁹ i) There are certain statistical requirements imposed on the distributional properties of the predictors. For example, the variance-covariance matrices of the predictors should be the same for both groups (failed and non-failed companies); moreover, a requirement of normally distributed predictors certainly mitigates against the use of dummy independent variables. ii) The output of the application of an multiple discriminant analysis model is a score which has little intuitive interpretation, since it is basically an ordinal ranking (discriminatory) device.

¹⁰ They can only observe one set of variables of the failing company and that is usually the year before bankruptcy

model generates scores and not probabilities of default. Shumway (2001) estimates a hazard model¹¹ that can be described as a multi period maximum likelihood estimation. The maximum likelihood estimation presented by Ohlson (1980) is limited in the sense that it can only estimate model coefficients for a single year of company variables. What Shumway (2001) develops is a way of including multiple years of data prior to bankruptcy for each company.

Hillegeist et al. (2004) constructed a model applying the option pricing framework developed by Black and Scholes (1973) and Merton (1974), the Black-Scholes-Merton framework. Hillegeist et al. (2004) claim that their model is general and can be used across industries while the studies based on accounting information are only “accurate” for a specific sample and not consistent over time, however they only use industrial companies in their estimation process. The Black-Scholes-Merton framework makes many simplifications and assumptions which do not reflect reality and these assumptions are also adopted by Hillegeist et al. (2004). The assumptions include that the underlying asset follows a random walk, no commissions are charged in the transaction, interest rates remain constant and stock returns are normally distributed with constant volatility over time. These assumptions might lead to bias in the predictors. Furthermore the Black-Scholes-Merton framework is criticized for underestimating default probability, this critique might also be applicable on the Hillegeist et al. (2004) model.

Many market participants rely on credit ratings to determine the probability of default of a certain obligor. A credit rating is a “*forward-looking opinion about the creditworthiness of an obligor with respect to a specific financial obligation, a specific class of financial obligations, or a specific financial program*”¹². There are three big rating institutions in the western world; Standard & Poor’s, Moody’s, and Fitch. Since these credit ratings are widely used we will take these credit ratings as given. We will not engage in a detailed discussion regarding the process of how these credit ratings are set since the rating institutions do not provide the required transparency for such a discussion.

A. Choice of Bankruptcy Prediction Models

Available bankruptcy prediction models can be divided into those based on accounting data, those based on market data and those, that are a mix between the two. We do not aim to investigate which bankruptcy prediction model is the best at predicting default, our purpose is to use the bankruptcy prediction models to find out whether the investor gets compensated for substantial default risk. However, we wish to reduce the impact of the choice of prediction model by performing the study with several bankruptcy prediction models. Due to restriction of time and resources a choice is made on which bankruptcy prediction models to include. The selection is based on three criteria;

¹¹ A hazard model includes a survivor function (probability of surviving up to time t) and a hazard function (probability of default at time t , given no default prior to t). This separates it from static models that do not consider time spent prior to bankruptcy. For a detailed description of the advantages of this model and how it is estimated see Shumway (2001).

¹² Standard & Poor’s; 2011 Annual U.S. corporate Default Study And Rating Transitions

- i. Applicability
- ii. The general acceptance of the model
- iii. What data the model bases its bankruptcy predictions on

The first criterion is the crucial one, in the design of the study we need bankruptcy predictions that can be turned into probabilities of default. This disqualifies many of the accounting based models such as Altman (1968), Beaver (1966), Deakin (1972) and Edmister (1972). The reason for the exclusion of these models is that their output is an arbitrary number that needs to be related to a model specific cut-off value to divide companies into failing or not failing. Theodossiou (1993) also fails to meet this criterion. The second criterion is one of the keys to raise interest. In the case of our study, where bankruptcy prediction and default probabilities are an important factor, well known bankruptcy prediction models contribute to the legitimatization of our results. All models presented in the introduction to this chapter are frequently mentioned within academia and hence meet this criterion. The third criterion is not really a criterion to meet but rather something to consider when designing the mix of selected models. Data from both accounting and market sources adds another dimension to the analysis. Accounting data is criticized for being time lagged. When the accounting reports become public they describe a past state of the company. New reports are also only available at a limited number of times per year. Accounting based models are also sensitive to the development of accounting standards. Looking at Ohlson (1980) as an example, the changes imposed in US GAAP since the 1970's may have an effect on the size and significance of the coefficients. Ohlson (1980) reflects on changing times and includes an inflation component. This limits the downside of the fact that the model is old but the accounting issues are still present. Worth noticing is that accounting based bankruptcy prediction models are often designed based on samples from one specific industry and time period, industrial companies in Ohlson's (1980) case. Applicability of these models on other industries or types of companies is questioned. But advocates of accounting data often highlight that accounting data is fundamental facts about the company while market data is distorted by expectations and hence we wish to include an accounting based model in the study. Since we require a model that presents estimates of default that can be turned into probabilities, the two models that are relevant are Ohlson (1980) and Skogsvik (1988). Since Ohlson (1980) is more widely mentioned and referred to, justifying criteria ii, we have chosen to include his model in our paper. Also, Ohlson (1980) bases his model on US data rather than Swedish data as Skogsvik (1988) and since we also use US data, Ohlson is a better fit to our study.

The time factor is important to consider in bankruptcy prediction models, both in terms of outdated coefficients and also the fact the publicly made accounting reports describe a past state. Bankruptcy prediction is rather about the future than the past and expectations about the future is incorporated in market data, which makes it suitable for bankruptcy prediction according to Hillegeist et al. (2004) and Queen and Roll (1997). However, relying solely on market data comes with the approval of the assumption of market efficiency. The market efficiency assumption claims that all market participants are aware of, and can accurately analyze, all relevant data regarding a company, as well as there being no

information asymmetry between the company and market participants. This assumption is common but widely debated (Fama (1991), Himmelmann, Schiereck, Simpson and Zschoche (2012) and Sloan (1996)). We choose to rely on this assumption which is necessary to be able to include market based bankruptcy prediction models among the selected models. Another issue about time, raised by Shumway (2001), is that signs of bankruptcy may be present in a company several years prior to default. The Ohlson (1980) model, which is the selected accounting based bankruptcy prediction model, is estimated based on data only one or two years prior to bankruptcy. The model by Shumway (2001) incorporates data for many years prior to bankruptcy when estimating the coefficients of his model. We choose to include Shumway (2001) among the selected models based on this feature since the model also meets criteria i and ii.

The last bankruptcy prediction model we include is the one presented by Hillegeist et al. (2004). It is derived from contingent claims analysis which is the most popular pricing method, in academia, of corporate liabilities and certainly fulfills criterion ii. It accounts for volatility in equity but also in assets, which can not be seen in any of the other models.

Finally we include credit ratings due to their general acceptance among market participants and that they are claimed to be based on both accounting and market factors. There are also probabilities of default connected to each credit rating provided by the rating institutes. Due to limited data access our choice is Standard & Poor's. It has been shown that at least Moody's and Standard & Poor's credit ratings seem to follow each other almost perfectly in the past, hence the choice of rating institution becomes arbitrary (Andersson (2010) and Elton et al. (2001)). However in our setting, critique is raised towards using historic probabilities of default implied by credit rating classes as a proxy for an individual company's probability of default. These historic probabilities are based on the average defaults within a credit rating class, not individual company specifics. While the historic levels of default in a credit rating class might be a good representation of the mean probability of default of the companies included in it, it might say little about the individual company. Regardless of this critique we choose to include credit ratings as a measure of default probability in our study due to the market's acceptance of Standard & Poor's credit ratings. They are used as reference and as a sanity check. If the results using other models diverge too much from the result implied by credit ratings, it is a warning sign.

To summarize, we choose to include three bankruptcy prediction models and credit ratings to generate default probabilities. The selected models and a more in depth description of them can be found in the following subsections. The selected models are;

- i. Ohlson (1980)
- ii. Shumway (2001)
- iii. Hillegeist et al. (2004)

A1. Ohlson (1980)

Ohlson's (1980) developed one of the first bankruptcy prediction models that presents actual probabilities of default. He compares the predictive ability of his model primarily with Altman (1968) but can not distinguish which model performs better. However later studies by Begley, Minh and Watts (1996) and Hillegeist et al. (2004) show that for more recent data the Ohlson (1980) model outperforms Altman (1968). Ohlson (1980) basis his data on companies that have fulfilled all of the three following conditions:

- i. Active or defaulted during the time period from 1970 to 1976
- ii. The equity of the company must have been traded on some stock exchange or over-the-counter (OTC) market
- ii. The company must be classified as a US industrial

With this data, Ohlson (1980) develops a model for predicting bankruptcy. During the time period of 1970 to 1976, Ohlson (1980) gathers a sample of 2,163 companies (105 failed and 2,058 non-failed). He develops a maximum likelihood estimation model using company values the year prior to bankruptcy for multiple variables that generate a y-value. For simplicity reasons, he chooses the logistic function in Equation 2 to convert these y-values into probabilities of default. He includes nine variables in his model to estimate the y-value. The coefficients to all nine variables are estimated so that when the y-values are transformed into probabilities of failure, using Equation 2, the probability of default for failed companies will be as high as possible while simultaneously the probability of default will be as low as possible for the non-failed companies. The Ohlson (1980) formula is expressed in Equation 1.

$$\begin{aligned} y_i = & -1.32 - .407 * SIZE_i + 6.03 * TLTA_i - 1.43 * WCTA_i + 0.0757 * CLCA_i \\ & - 2.37 * NITA_i - 1.83 * FUTL_i + .285 * INTWO_i - 1.72 * OENEG_i \\ & - .521 * CHIN_i \end{aligned} \quad (1)$$

Where;

$SIZE$ = log(total assets/GNP price-level index (1968 as basis year))

$TLTA$ = total liabilities/total assets

$WCTA$ = working capital/total assets

$CLCA$ = current liabilities/current assets

$NITA$ = net income/total assets

$FUTL$ = funds provided by operations/total liabilities

$INTWO$ = (one if net income was negative for the last two years, zero otherwise)

$OENEG$ = (one if total liabilities exceeds total assets, zero otherwise)

$CHIN$ = $(NI_t - NI_{t-1})/(|NI_t| + |NI_{t-1}|)$

NI = net income

This y-value can then be converted to a probability of default (P) using a logistic function expressed in Equation 2.

$$P_i = (1 + \exp\{-y_i\})^{-1} \quad (2)$$

A2. Shumway (2001)

Shumway (2001) claims that all static models, including the ones by Altman (1968) and Ohlson (1980), produce biased and inconsistent estimations. His intent is to develop a model that is not biased or inconsistent in estimation of default probabilities. He compares his model to the models by Altman (1968) and Zmijewski (1984) and finds that his model outperforms both these in out of sample testing. He also finds that about half of the variables included in the two previous studies are no longer statistically significant for bankruptcy prediction when applying his new statistical approach. Shumway's (2001) model is estimated using the following requirements on the company data:

- i. The equity of the company must have been traded on the New York Stock Exchange (NYSE) or American Stock Exchange (AMEX) after 1962 but before 1992, or fail in that period
- ii. The company is not classified as a financial company

Shumway (2001) estimates a hazard model that allows variables to vary with time, but the model can be explained as a multiple period maximum likelihood estimation model. By developing this model, Shumway (2001) allows the included coefficients to be estimated based on multiple years of data for each company. Ohlson (1980) also uses maximum likelihood estimation but is limited to one year of company characteristics, and in his most popular model that year is the year prior to bankruptcy. By allowing for multiple years of data, Shumway (2001) estimates his coefficients on a sample that is ten times as large for each company, compared to static models, since in his sample the average age of each company is ten years. This improvement allows variables to vary with time providing additional information to the model. This statistical approach, according to Shumway (2001), generates more accurate and consistent estimations of default probabilities.

The values presented from the Shumway (2001) model can also be transformed into probabilities of default using the logistic function (Equation 2). Shumway (2001) develops a model with a mix of accounting and market data consisting of five variables. The model by Shumway (2001) is presented in Equation 3.

$$y_{i,t} = -13.303 - 1.982 * NITA_{i,t} + 3.593 * TLTA_{i,t} - .467 * SIZE_{i,t} - 1.809 * (r_{i,t-1} - r_{m,t-1}) + 5.791 * \sigma_{i,t} \quad (3)$$

Where;

$NITA$ = net income/total assets

$TLTA$ = Total liabilities/total assets

$SIZE$ = log(market capitalization/size of NYSE and AMEX),

$(r_{i,t-1} - r_{m,t-1})$ = return for the company last year subtracted by the return for the market last year

σ = idiosyncratic standard deviation of company's stock returns (σ is the standard deviation of the residual in a regression of a company's monthly stock returns on the weighted average of the return from the NYSE/AMEX).

Many of the variables included in Shumway (2001) are directly observable, but not the idiosyncratic standard deviation of a company's stock returns. This variable is computed by regressing a company's monthly stock return on the weighted average of the return from the stock exchange markets NYSE and AMEX as shown in Equation 4. The reason for using the weighted average for NYSE/AMEX is to use the broadest index available. This is due to the fact that Shumway (2001) does not limit his company data to a specific industry as Ohlson (1980) and Hillegeist et al. (2004) do, namely industrial companies.

$$r_i = \beta_0 + \beta_1 * r_m + e \quad (4)$$

Where;

r_i = monthly stock return for the company

β_0 = constant

β_1 = coefficient (fraction of the monthly stock return that can be explained by market return)

r_m = weighted average of the return from the NYSE/AMEX

e = residual

Equation 4 is executed for all bonds in the sample and the idiosyncratic standard deviation of the company's stock returns, σ , is the standard deviation in the residual, e , of the regression. σ is calculated by regressing the monthly returns for each company on the weighted average return from the NYSE/AMEX for the 12 months prior to the date at which the probability of default is estimated.

The more volatility there is in a company's returns, less of the return can be explained by the return of the market. If less of the return of a company can be explained by the market return then more is included in the residual. And if a company has large variations in return compared to the market then the standard deviation in the residual will increase and the model will estimate a higher probability of default for that company. This makes sense since a company with larger variations should have a higher probability of default than a company generating steady returns. However one can argue that it is general volatility in returns rather than volatility in comparison to the market return that is significant for bankruptcy prediction. Bankruptcies occur to a larger extent in recessions rather than in booms, hence if a stock's return follows the market return well and market returns are very low then that might as well be an indication of impending default, even if the idiosyncratic standard deviation is low. But apparently, when the model was estimated, this variable had explanatory power.

A3. Hillegeist, Keating, Cram and Lundstedt (2004)

Hillegeist et al. (2004) developed a bankruptcy prediction model based on the Black-Scholes-Merton framework and market driven variables. They argue that the stock market is a superior source of information regarding bankruptcy prediction variables since it aggregates data from multiple sources into one. They also stress the impact of asset volatility on bankruptcy prediction and claim that this influence is often lost in accounting based bankruptcy prediction models. Hillegeist et al. (2004) compare their results both with the original models by Altman (1968) and Ohlson (1980) but also with updated coefficients to these models, estimated by themselves. They find that their model contains more information¹³ about bankruptcy prediction than both Altman (1968) and Ohlson (1980) even with updated coefficients. Hillegeist et al. (2004) base their estimations with the following requirements on the company data:

- i. The equity of the company must have been traded on the New York Stock Exchange (NYSE) or American Stock Exchange (AMEX) between the years 1980 and 2000, or fail in that period
- ii. The company must be classified as a US industrial.

The model by Hillegeist et al. (2004) determines the probability of default by estimating a function based on the relationship between a company's asset and its liabilities, adjusted for the expected growth in assets as well as the volatility in assets. The probability of default is equal to the probability that the market value of the company's assets at time T is less than the face value of liabilities maturing at time T and is expressed in Equation 5.

$$p_{default} = N \left(- \frac{\ln(V_A/X) + (\mu - \delta - (\frac{\sigma_A^2}{2}))T}{\sigma_A * \sqrt{T}} \right) \quad (5)$$

Where;

V_A = market value of assets

σ_A = asset volatility

μ = expected return on assets

X = face value of debt maturing at time T (assumed to equal book value of total liabilities)

δ = dividend rate (sum of prior year's common and preferred dividends divided by the sum of market value of equity and total liabilities)

T = time (like Hillegeist et al. (2004) we assume T=1)

¹³ More information rather than better predictions since Hillegeist et al. (2004) do not perform a prediction oriented test but a relative information tests to compare the three models. They claim that a prediction oriented test is an inferior test to relative information tests when comparing bankruptcy prediction models when the output is presented in terms of probabilities of default rather than classifications of companies as failing or not failing. For more details see Hillegeist et al. (2004)

Equation 5 shows that the probability of default is a function of the distance between the current value of the company's assets and the face value of its liabilities (V_A/X) adjusted for the expected growth in asset values ($\mu - \delta - (\frac{\sigma_A^2}{2})$) relative to asset volatility σ_A . To empirically apply Equation 5, market value of assets V_A , asset volatility σ_A and expected return on assets μ need to be estimated while face value of debt maturing at time T (X) and the dividend rate δ are directly observable.

The first step is to estimate V_A and σ_A by simultaneously solving Equation 6 and Equation 7.

$$\sigma_E = \frac{V_A e^{-\delta T} N(d_1) \sigma_A}{V_E} \quad (6)$$

Where;

V_E = total market value of equity based on the closing price at the end of the company's fiscal year

σ_E = volatility in equity computed using CRSP daily returns over fiscal year

$$V_E = V_A e^{-\delta T} N(d_1) - X e^{-\delta T} N(d_2) + (1 - e^{-\delta T}) V_A \quad (7)$$

Where $N(d_1)$ and $N(d_2)$ are the standard cumulative normal of d_1 and d_2 respectively and

$$d_1 = \frac{\ln[V_A/X] + \left(r - \delta + \left(\frac{\sigma_A^2}{2}\right)\right)T}{\sigma_A \sqrt{T}} \quad (8)$$

$$d_2 = d_1 - \sigma_A \sqrt{T} = \frac{\ln[V_A/X] + \left(r - \delta - \left(\frac{\sigma_A^2}{2}\right)\right)T}{\sigma_A \sqrt{T}} \quad (9)$$

Where;

r = Treasury bill rate with maturity T.

V_A is set equal to the sum of market value of equity and book value of liabilities. σ_A is given by Equation 10.

$$\sigma_A = \frac{\sigma_E V_E}{V_E + X} \quad (10)$$

In the next step the expected return on assets, μ , is estimated in Equation 11 using actual return on assets during the previous year and V_A as estimated in Equation 6.

$$\mu(t) = \max \left[\frac{V_A(t) + Dividends - V_A(t-1)}{V_A(t-1)}, r \right] \quad (11)$$

Where;

Dividends = sum of common and preferred dividends declared during the year

Finally, all estimated and observed variables are computed in Equation 5 to generate default probabilities for each single company.

The framework has however been criticized especially for relying on the assumptions stated in the beginning of Section I (Black-Scholes-Merton assumptions). Also, the model actually measures the probability of default as the probability that the market value of assets is smaller than the face value of loans maturing at time T ($V_A(T) < X$), a simplification that can be discussed. In reality, companies can file for bankruptcy before this inequality is met ($V_A(T) > X$) or after this inequality is met due to friction in the bankruptcy process and the extension of loans. As Hillegeist et al. (2004) we assume T to equal one and that all liabilities mature at time T . This is also a simplification, or rather a needed falsification, of reality.

II. Data

To calculate the expected excess return for corporate bonds the data required is split up into corporate bond specific data and company specific data. The company specific data is needed to calculate the default probabilities described in Section I and can further be divided into accounting data and market data. To get a dataset as large as possible there is no requirement that a certain company's data should be complete in the sense that it can provide default probabilities for all the selected bankruptcy prediction models. However there are certain criteria that the data needs to meet:

Corporate bond data criteria:

- i. The bond must be traded on the US bond market
- ii. The bond must have a maturity date beyond December 31st 2011
- iii. The bond must have traded in the interval of December 26th 2010 and January 5th 2011
- iv. The bond must have fixed coupon payments

Company data criteria:

- v. The company must be a US company
- vi. The company must use the calendar year as reporting year
- vii. The company must have data that satisfies at least one of the bankruptcy prediction models.

Before describing the data gathering process and the final dataset in detail we want to stress two important underlying assumption. First, we choose the valuation date on the basis that we want recent

and relevant data. During a year there are two possible valuation dates; i) The last date of the reporting year for a company or ii) the day the financial reports are presented to the investor. The choice is important when using accounting data as Ohlson (1980) and partly Shumway (2001) do. But due to time restrictions all bonds need to be valued at the same date. Since the day when the financial reports are released differs for all companies we choose to use the last date of the financial year. The majority of all companies use the calendar year as their financial year and hence December 31st is the chosen valuation date. The advantage of using the last date of the reporting year is that the company characteristics are in fact the same as those presented in the report, which is not true on the day of publication.

We choose year 2010 as the year of valuation since we wish to include as relevant and updated data as possible. The data for 2011 is less complete then for 2010 and is therefore excluded as an option. One can argue that the recent financial crisis has an impact on the results, and this is most certainly true but we claim that the market has stabilized enough by 2010. An alternative might be year 2007 before the crises, but then the market experienced a boom which also dilute the results. Choosing an average and representative year is thus hard and the relevant and updated data factor is prioritized. The valuation date is thus December 31st 2010.

Second, we assume a constant probability of default calculated on a one year time horizon. The reasons for this assumption are based on the design of the bankruptcy prediction models and our intent to make the dataset as large as possible. None of the selected bankruptcy prediction models offer reliable methods to calculate default probabilities for more than one year into the future. If not assuming constant default probabilities we would have to exclude bonds with a maturity date beyond one year into the future. This would have a substantial impact on the size of the dataset and hence the validity of our conclusions. Also, if the time to maturity of the bonds would be very short then the default probabilities would not affect prices to the same degree since the impact on expected cash flows would be limited. Making this assumption implies that the selected bonds must have a maturity date one year beyond the valuation date December 31st 2010.

Basic descriptive statistics of the dataset is shown in Table I.

Table I
Descriptive Statistics

This table reports descriptive statistics of our dataset. Due to limited data on some bonds and companies we are not able to calculate default probabilities for all models. Hence the number of bonds differs across the bankruptcy prediction models and Standard & Poor’s credit ratings. Average coupon, maturity and recovery rates are calculated based on the bonds valid for that default measure.

	Total sample	Ohlson	Shumway	Hillegeist et al.	Rating
Number of bonds	242	215	206	198	225
Average coupon (%)	6.57	6.57	6.44	6.46	6.55
Average time to maturity (years)	8.50	8.63	8.66	8.54	8.56
Average recovery rate	0.44	0.44	0.44	0.44	0.44

A. Bond Specific Data

A1. Selection of Bonds

Data on bonds that fulfill our criteria is gathered from the TRACE database. Since we are aiming for the largest possible dataset the US market is selected as it is the largest market for corporate bonds. The US market is also the most active one which is important to be able to access updated prices at the valuation date and reduce the potential presence of liquidity premiums. Additionally, all the selected bankruptcy prediction models in our paper are based on US company data. The effects on the performance of the bankruptcy prediction models when using data from other countries is not explored either by the authors themselves or supporting literature and therefore we refrain from including company data from other countries. Lastly, data on both US corporate bonds and US companies is easy to access.

Since we calculate default probabilities with a one year time horizon we limit our bond data to bonds with maturity date beyond December 31st 2011 (criteria ii.). An overview of maturities can be found in Figure 1 and more detailed maturity and coupon statistics in Table II. We find that the majority of bonds in the dataset have a maturity of 1-10 years and a trend of slightly decreasing coupons as maturity increases.

In the case where a company has issued multiple bonds that all match our stated criteria we choose to use the bond that is the most liquid, i.e. the most traded bond. This selection is based on the amount of trades and volume of trades within the interval of December 26th 2010 and January 5th 2011. We make this selection to make sure we get as accurate prices on the bonds as possible.

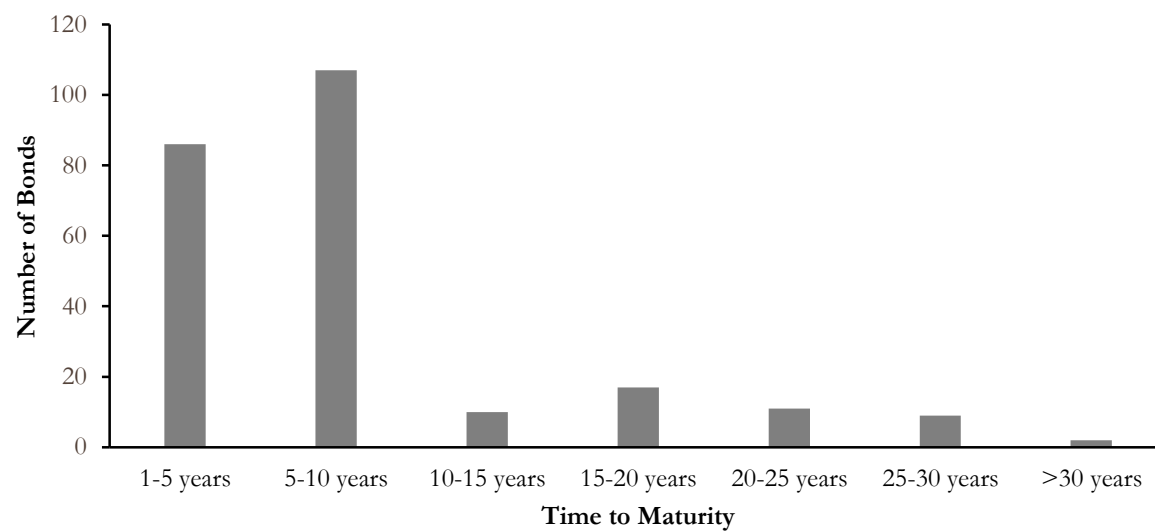


Figure 1. Maturity Distribution

The figure plots the distribution of maturities of bonds included in the sample. A majority of the bonds have 1-10 years to maturity.

Table II
**Number of Bonds and Average Coupon Divided by Bankruptcy Prediction Models
and Time to Maturity**

The table shows the distribution of maturity and average coupons for each bankruptcy prediction model, the Standard and Poor's Ratings and the total sample. Due to limited data on some bonds and companies we are not able to calculate default probabilities for all models. Hence the number of bonds may will across the bankruptcy prediction models and Standard and Poor's ratings. Average coupon is calculated based on the bonds valid for that default measure.

Total sample		Ohlson		Shumway		Hillegeist et al.		Rating
Time to Maturity	Number of Bonds	Average Coupon (%)	Number of Bonds	Average Coupon (%)	Number of Bonds	Average Coupon (%)	Number of Bonds	Average Coupon (%)
1-5 years	86	6.30	76	6.33	75	6.24	74	6.17
5-10 years	107	7.13	96	7.04	84	6.83	91	6.87
10-15 years	10	6.45	10	6.45	10	6.45	10	6.45
15-20 years	17	5.81	12	6.31	10	6.91	10	6.38
20-25 years	11	5.80	11	5.80	10	5.78	11	5.80
25-30 years	9	5.23	9	5.23	8	5.03	9	5.23
>30 years	2	5.73	2	5.73	2	5.73	2	5.73
Total	242		216		199		207	
							226	

A2. Bond Prices

To be able to perform the analysis bond prices need to be expressed as *dirty prices*. Bonds can be quoted either in terms of *clean* or *dirty* prices. The clean price is the price of the bond less the accrued interest on the next upcoming coupon payment, while the dirty price includes this accrued interest. The clean price might not change as the coupon payment date of a bond approaches but the dirty price should increase as you approach a fixed cash flow, assuming that everything else affecting the bond price does not change. In our bond price calculations we take the date of coupon payments into account. Therefore the market price of the bond must be expressed in terms of the dirty price. Given that dirty prices are a requirement it also becomes a requirement that the bonds are traded frequently, or at least in close proximity to the date at which we choose to value the bond. If we include a dirty price observed much earlier than December 31st 2010 that price would be too low since the price would not include all accrued interest up to our chosen measurement date. The opposite is true for an observed dirty price way beyond December 31st 2010. Therefore we include criteria iii., as assurance that the bond prices are relevant. A dirty price observed within 5 days from December 31st 2010 is recorded for all bonds except for seven in our dataset. For the seven remaining bonds we widen the trading interval to within 25 days of December 31st to expand the sample with bonds that are of particular interest since they are classified as risky.

A3. Excluding Bonds

In order to make the bond price calculations manageable we exclude all bonds that do not have fixed coupon payments. The method used to determine the expected return of a bond in this paper requires that the coupon payments are fixed in size and payout day. Thus criteria iv. must be satisfied for the bond data.

A4. Recovery Rates

In order to calculate expected returns we also require data on recovery rates for each bond. These are collected from Mora (2012) where recovery rates are summarized from Moody's statistics for defaults during the time period 1970-2008. The recovery rates are reported either according to industry or level of seniority¹⁴. Each corporate bond in our sample is linked to a certain industry and has a specific level of seniority. Therefore we choose to take the average of the recovery rates for a company's industry and the seniority of the corporate bond as the recovery rate included in the dataset. The summarized recovery rates are shown in table III. For further transparency Table IV and Table V show the bonds included in our sample divided by industry and seniority in that order.

¹⁴ Seniority levels on debt determine in what order debt is repaid in case of bankruptcy. The seniority levels range from (paid back first to paid out last): senior secured debt, senior unsecured debt, senior subordinated debt, subordinated debt, junior subordinated debt and preferred stock.

Table III
Recovery Rates

This table reports recovery rates across categories and bankruptcy prediction models as well as for Standard and Poor's credit ratings. The bonds allocated to each category, and hence the number of bonds allocated to each category, differ between the bankruptcy prediction models as well as ratings. As in Table X the credit rating scale is abandoned and instead divide the rated bonds in accordance with the specified categories. AAA-A belongs to Category I, BBB to Category II, BB to Category III and B to Category IV.

	Total sample	Ohlson	Shumway	Hillegeist et al.	Rating
Categories	Recovery Rate	Recovery Rate	Recovery Rate	Recovery Rate	Recovery Rate
Category I	0.44	0.43	0.43	0.44	0.45
Category II	0.46	0.44	0.47	0.46	0.46
Category III	0.44	0.45	0.44	0.43	0.43
Category IV	0.41	0.43	0.40	0.42	0.41
Average	0.44	0.44	0.44	0.44	0.44

Table IV
Distribution of Bonds Across Industries and Recovery Rates by Industry

This table reports the bonds within our sample divided by industry along with the historic recovery rate for that industry.

	Number of Bonds	Recovery Rate
Agriculture, Forestry and Fishing	2	39.9
Mining	36	50.8
Construction	46	28.7
Manufacturing	49	43.7
Transportation	16	32.7
Communications	22	39.6
Utilities	25	57.5
Wholesale Trade	14	43.2
Retail Trade	6	43.3
Finance, Insurance and Real Estate	18	24.6
Services	8	49.3
Total	242	

Table V
Distribution of Bonds Across Seniorities and Recovery Rates by Seniority

This table reports the bonds within our sample divided by seniority along with the historic recovery rate for that seniority. Unspecified are the bonds where a specific seniority level could not be found. For those bonds, the average recovery rate for the years 1970-2008, summarized by Moody's, was used.

Seniority	Number of Bonds	Recovery Rate
Senior Secured	200	0.55
Senior Subordinated	13	0.23
Subordinated	4	0.29
Junior Subordinated	3	0.153
Unspecified	22	0.305
Total	242	

B. Company Specific Data

B1. Selection of Companies

The selection of companies is based on the selection of bonds. Since bonds that fulfill the criteria are crucial and scarcer than companies we choose to design our dataset based on available bonds and then match company data to these bonds.

B2. Data Gathering

The specific company data needed is the data required by the bankruptcy prediction models listed in Section I.¹⁵ All accounting data on the selected companies is extracted from COMPUSTAT, while all market data is gathered from the CRSP database. To the furthest possible extent, we use the same databases as the authors of the bankruptcy prediction models have in developing their models. Credit ratings for each company is collected from the COMPUSTAT database. Implied default probabilities for the credit ratings are gathered from Standard & Poor's 2011 Annual U.S. Corporate Default Study and Rating Transitions. The reason for choosing implied default rates for 2011 is that membership to Standard & Poor's website is required for the 2010 figures. The impact of this is however deemed to be limited since we use historical default rates. Using the 2011 numbers implies using average default rates from 1981-2011 instead of 1980-2010, hence the difference should be negligible.

¹⁵ That is the data included to generate all variables for Ohlson (1980), Shumway (2001) and Hillegeist et al. (2004)

B3. Excluding Companies

To simplify the calculations we exclude companies that do not use the calendar year as reporting year (criteria vi.). This is a requirement since our available data is only updated, for some companies, at the end of their fiscal year. Including companies that do not use the calendar year as their fiscal year increases the risk of calculating default probabilities based on wrong, outdated numbers.

Our final criterion, criterion vii, is that the company should provide data that satisfies at least one of the bankruptcy prediction models. This is of course a natural requirement since it is a prerequisite for us to perform our analysis. This has implications for the selected models primarily regarding what periods of time the company must have been active. For all models historic data is needed and thus historic data must be readily available for years before 2010 which is the year when the analysis is performed.

C. Additional Data

To calculate the expected excess return (see Equation 13), we need data on the risk free rate to be subtracted from the expected rate of return. We use the US Department of the Treasury yield curve rates for Treasury securities as a proxy for the risk free rate and match the maturity date on these to each corporate bonds maturity. However these rates are reported with specific maturity dates that for the most part do not correspond exactly to the maturity date of the corporate bonds included in our dataset. To mitigate this problem the risk free rates are weighted to match the maturity of the bonds. For example, a bond that matures in 16 years will in our data be allocated a risk free rate that is 4/10 times the 10 year risk free rate and 6/10 times the 20 year risk free rate. Bonds with maturity dates beyond 30 years from December 31st 2010 are allocated the 30 year risk free rate.

GNP data is gathered from the U.S. Bureau of Economic Analysis and recalculated to use 1968 as basis year which is a prerequisite in the Ohlson (1980) model.

III. Method

The aim of this study is to investigate whether the investor gets compensated in terms of expected excess return when investing in bonds with a high probability of default. Compensation is examined by analyzing how expected excess return develops across levels of default probability. Expected excess return is defined as the expected return on a bond less the risk free rate. The expected return is solved for by discounting expected cash flows, which are adjusted for default risk and recovery rates, to the market price.

A. Calculating Probability of Default

Probabilities of default from the selected bankruptcy prediction models are calculated according to the methods explained in Section I. Probabilities implied by credit ratings are taken as exogenously given from Standard & Poor's and not calculated by us.

B. Converting Promised Cash Flows into Expected Cash Flows

It is important to distinguish between *promised* and *expected* cash flows. Promised cash flows are those the issuer assigns to the bond, and if capable promise to pay until maturity, at the time of issuance. Expected cash flows are adjusted for risk and are what the investor can expect to receive, i.e. the expected capability of the issuer to meet its financial obligation is considered.

We argue that default risk and recovery rates are the only factors affecting future cash flows. Hence promised cash flows are only adjusted for risk of default and recovery rates when calculating expected cash flows.

Expected Cash Flows

$$\begin{aligned} &= \left[\sum_{t=1}^T (1 - p_{\text{default}})^t * c * FV \right] + [(1 - p_{\text{default}})^T * FV] \\ &+ \left[\sum_{t=1}^T (1 - p_{\text{default}})^{t-1} * p_{\text{default}} * \text{recr} * FV \right] \end{aligned} \quad (12)$$

Where;

p_{default} = probability of default

c = promised coupon rate

FV = promised face value

recr = expected recovery rate

T = time at maturity

Equation 12 is a simplified version of how we convert promised cash flows into expected cash flows. Equation 12 generates annual expected cash flows while we use semi-annual cash flows in the study to match the payout scheme of corporate bonds.

C. Calculating Expected Excess Return

We use the following construction as definition of expected excess return;

$$\text{Expected Excess Return} = \text{Expected Return} - \text{Risk Free Rate} \quad (13)$$

The *expected return* is the discount rate used to discount expected future cash flows in order to reach the market price. The probability of default, the coupon rate, the face value, the expected recovery rate as well as the market price are all known parameters and we use Equation 14 to solve for the expected return.

*Market Price Bond*₀

$$= \sum_{t=1}^T \frac{(1 - p_{\text{default}})^t * c * FV}{(1 + \text{expected return})^t} + \frac{(1 - p_{\text{default}})^T * FV}{(1 + \text{expected return})^T} \quad (14)$$

$$+ \sum_{t=1}^T \frac{(1 - p_{\text{default}})^{t-1} * p_{\text{default}} * \text{recr} * FV}{(1 + \text{expected return})^t}$$

Where;

p_{default} = probability of default

c = coupon rate

FV = face value

recr = expected recovery rate

T = time at maturity

Equation 14 is a simplification in the same way as Equation 12.

D. Reasoning Behind the Risk Free Rate

The reason for choosing the risk free rate to deduct from the expected return to calculate expected excess return is inferred by the design of this study. Due to quantification issues, default risk is the only risk considered in the calculations even if previous research conclude that systematic risk and risk averse investors are factors that impact bond prices. This implies that these factors must be adjusted for in the results after the calculations have been performed. Expected return is a function of price and is the discount rate required in order for the present value of expected cash flows to equal the market price. In the calculations of expected excess return, we take the market price as given and solve for expected return. If instead taking a discount rate as given the price can be solved for. Given that default risk is the only risk considered in the calculations, and this risk is adjusted for in the expected cash flows, the risk free rate is the appropriate discount rate for pricing the bond in our setting. Hence, to calculate what the market rewards the investor in terms of expected excess return, the appropriate pricing rate, the risk free rate, is deducted from expected return.

If we would include also systematic risk and risk aversion in our calculations, the risk free rate would not be the appropriate pricing rate. Since systematic risk and risk aversion can not be adjusted for in the expected cash flows, there must be an adjustment of the discount rate. This adjustment is made by imposing a premium on the risk free rate, a systematic risk premium. The logic behind increasing the discount rate is that the investor is willing to pay a lower price if accounting for systematic risk and not just default risk. An increased discount rate implies a lower suggested price. This is also intuitively correct if looking at the discount rate as the return, if the investor is to be compensated also for systematic risk, he requires a higher return. Hence, when calculating expected excess return also considering systematic risk, the risk free rate and the systematic risk premium should be deducted from the expected return.

E. Dividing the Bonds into Categories

For the three bankruptcy prediction models we divide the results into four categories; Category I-IV with increasing levels of riskiness. The bonds included in the categories differ between each bankruptcy prediction model. Table VI shows the default probabilities assigned to each category. The results are then examined across categories where the average expected excess return is reported. The reasoning behind the categorization is to facilitate the interpretation of the results. By reporting the average expected excess return from each category one can in an intuitive way see how expected excess return develops as default probabilities increase.

Table VI
Default Probability Categories

This table shows the assigned probabilities of default to each category.

Category	Default probabilities
I	< 0.1%
II	0.1% - 0.5%
III	0.5% - 2.0%
IV	> 2.0%

IV. Results

The aim of this study is to examine whether the investor gets compensated for taking on a high risk of default. Our main finding is that when investing in bonds assigned to the category with highest default risk, the investor does not get sufficiently compensated for additional default risk.

A. Default Probabilities

Starting with default probability, we find that the Ohlson (1980) model estimates considerably higher default probabilities than the other models for all credit rating classes as presented in Table VII. For the entire sample, the difference between Ohlson (1980) and the market based models is about four percentage points. The performance of the bankruptcy prediction models will be discussed in more detail in Subsection C4.

Default probabilities calculated using the bankruptcy prediction models increase for lower rated bonds confirming that default risk is considered in credit rating classifications, which is an important

sanity check. Default probabilities for the different credit ratings are taken as exogenously given from Standard & Poor's.

Table VII
Default Probabilities

This table reports calculated default probabilities for all default probability models divided by credit rating class. The Rating column reports the implied default probabilities by Standard & Poor's for each credit rating. Entire sample, Investment grade and Speculative are average of the bonds included in each group. Worth noticing is that Ohlson (1980) is an accounting based estimation model while Hillegeist et al. (2004) only use market data and Shumway (2001) is a mix of the two. Due to few observations the AAA and AA bonds are pooled.

(%)	Ohlson	Shumway	Hillegeist et al.	Rating
Entire sample	4.08	0.44	0.26	1.35
Investment grade	1.71	0.17	0.01	0.20
Speculative	6.85	0.79	0.59	2.65
AAA-AA	0.52	0.04	0.00	0.03
A	1.86	0.19	0.00	0.08
BBB	1.73	0.17	0.02	0.27
BB	3.83	0.23	0.09	0.96
B	10.32	1.45	1.25	4.59

B. Expected Excess Return

The results from the expected excess return calculations are presented in graphs with probability of default on the x-axis and expected excess return on the y-axis in Figures 2-5. For each measure of default probability the results are categorized into Category I-IV with increasing levels of riskiness. For Standard & Poor's credit ratings the y-axis consists of the official rating scale, AAA and AA bonds are pooled due to few observations. Table VIII contains descriptives of the categories. The reasoning behind the categorization is to facilitate the interpretation and presentation of the results. By reporting the average expected excess return from each category one can in an intuitive way see how expected excess return develops as default probabilities increase.

The aim of this study is to investigate if the investor gets compensated for the higher default risk associated with high yield bonds. Assuming that the bankruptcy prediction models give the same picture as the market, the relation between probability of default and expected excess return should be positive in order for the investor to be sufficiently compensated. A positive relationship means that the graphs must slope upwards. This will be explained in detail in the following sections.

Table VIII
Category Descriptives

This table shows descriptive statistics for each category of default probability and credit rating class. Probabilities of default implied by credit ratings are not divided into categories since it is used mainly for reference. The number of observations for each category is presented by bankruptcy prediction model.

Panel A: By Category				
Category	Default probabilities	Number of observations		
		Ohlson	Shumway	Hillegeist et al.
I	< 0.1%	4	110	176
II	0.1% - 0.5%	33	58	16
III	0.5% - 2.0%	85	22	7
IV	> 2.0%	93	8	7

Panel B: By Rating class	
Rating	Number of observations
AAA-AA	5
A	37
BBB	79
BB	56
B	49

B1. Implication of Risk Aversion, Systematic Risk and Taxes

Assuming that the investor is risk averse, two bonds with the same expected cash flows but different default probabilities should have differing expected excess returns. A simple example; assume that in one year the investor is guaranteed to receive \$100, i.e. expected cash flow is \$100 and no risk of default, then the discount rate used to get the present value should be the risk free rate and expected excess return would be zero. If the investor instead receives \$50 with a 20% probability and \$112.5 with a 80% probability the expected cash flow is still \$100 but since this cash flow is more risky the discount rate should be higher. Therefore the investor would be willing to pay less for the second cash flow than the first. The second alternative would then yield an expected excess return above zero. The expected excess return should however only compensate the investor for systematic risk.

Systematic risk is the kind of risk that can not be diversified away. A systematic risk affects the entire market and not just one specific asset. For instance, the investor can not protect himself from default by diversifying to the same extent when the market suffers from a recession as in booms.

A tax effect is present since US bonds are subject to different tax rates depending on whether they are corporate or Treasury bonds. Corporate bonds are subject to state taxes while Treasury bonds are not and hence the investor requires a higher pre-tax return when investing in corporate bonds compared to Treasury bonds. When considering taxes, the expected cash flows are lower than in a tax free setting.

As previously discussed, none of the above mentioned factors are incorporated into the calculations. This implies that the graphs presented do not contain the effects of these factors but the factors will have an impact. If we would include all factors in the calculations of expected excess return, the risk free rate would not be the appropriate rate to deduct. Instead, to calculate expected excess return, the risk free rate plus a premium should be deducted from the expected return. This premium should contain compensation for risk aversion and systematic risk. The size of the premium is left beyond the scope of this study but is examined by Driessen (2005) and Elton et al. (2001) among others. We will however discuss the impact of the premium on our results. If accounting for taxes the expected cash flows would be lower since parts of them would go to tax payments.

The impact of the premium and taxes is illustrated in Figure 3 in a simplified manner. Line A represents the expected excess return calculated according to the method chosen for this study, i.e. only default risk is considered. Line B shows the impact of decreased cash flows due to taxes and Line C illustrates how an imposed systematic risk premium affect the expected excess return. Worth noticing is that in the simplified Figure 3, Line B and Line C have the same intercept. In reality, there would be a gap between them. This gap is the core of the credit risk puzzle discussed in the introduction. It is puzzling since it should, according to theory, look like it does in Figure 3 but in reality the investor in AAA bonds, that are assumed to be risk free, gets compensated for systematic risk. Since this conflicts with theory and the impact on the results and interpretations is limited, we will disregard this effect in the impending discussions. Also in reality, assuming that default risk, systematic risk and taxes are the only factors affecting bond prices, the expected excess return as illustrated in Line C should be zero, otherwise there would be arbitrage opportunities. If all risks are accounted for in the market price in a correct way, there would be no expected excess returns for any bond.

Starting with taxes, lower expected cash flows means a decreased expected return and by definition a lower expected excess return. Taxes are assumed not to differ across bonds and hence the implications posed by taxes are constant across all default probabilities. This implies that if accounting for taxes, the expected excess return would be decreased with the same amount for any given bond or default probability and result in Line B.

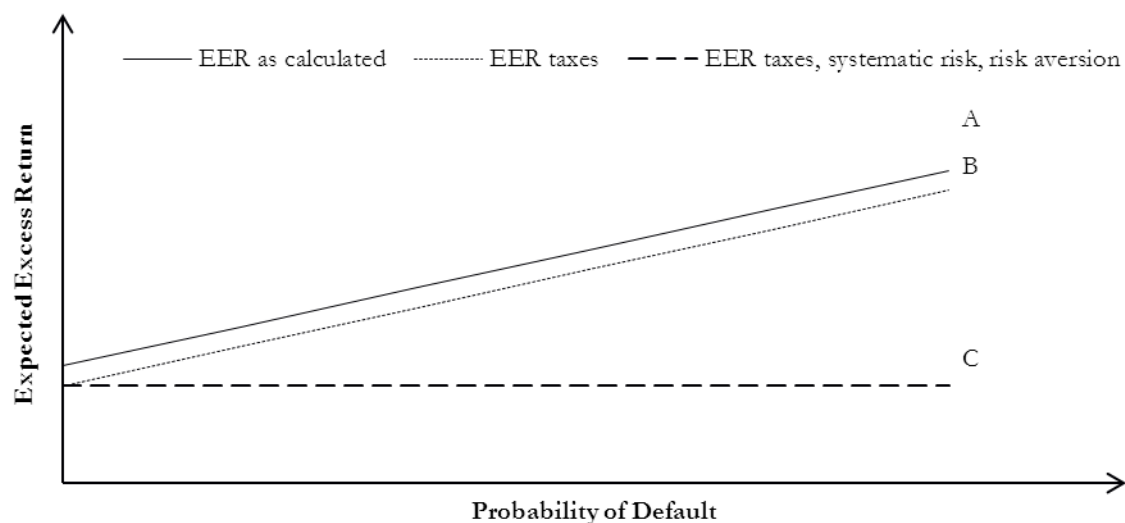


Figure 2. Impact of Systematic Risk, Risk Aversion and Taxes

This figure plots the impact of systematic risk, risk aversion and taxes on the results in a simplified manner. The implications of taxes are assumed to be constant for all bonds and hence the expected excess return is decreased with the same amount for all bonds. Risk aversion on the other hand implies that the systematic risk premium is higher for riskier bonds. Line C represent the expected excess return after accounting for default risk, systematic risk and taxes, assuming that those factors are the only affecting the bond price. Worth noticing is that if those factors are the only important and that they are reflected in the market price in a proper way, the expected excess return should be zero for all bonds, otherwise there would be arbitrage opportunities.

Systematic risk and risk aversion have a more dramatic impact on expected excess return. In our setting, incorporating systematic risk means deducting not only the risk free rate but also a risk premium from the expected return. The logic behind this is that a bond subject to systematic risk should be priced using the risk free rate plus a risk premium as the discount rate. Given that the calculated price is the market price, the expected return of the bond equals the risk free rate plus the premium. But if the market price diverges from the calculated price the actual expected return is different from the calculated. Since the investor should be rewarded the risk free rate plus a premium the expected excess return is calculated by deducting the risk free rate and the premium from the actual expected return. As a risk averse investor requires more compensation when investing in risky bonds compared to risk free, even if the expected return is the same, the effect is not constant for all probabilities of default. Risk aversion implies that this premium must be larger for bonds with higher probability of default and hence the slope is affected. In the setting of this study, this means a horizontal line (Line C). If the investor is compensated for default risk and systematic risk as well as taxes, the expected excess return, when deducting the risk free rate and a premium from expected return, is the same for all probabilities of default, otherwise there would be arbitrage opportunities.

B2. A Positive Relation Indicates a Compensated Investor

Since the results are presented in graphs calculated as described in Section III, only considering default risk, we expect to find a line similar to Line A in Figure 2 in order for the investor to be compensated for the additional risk in a proper way. Since systematic risk, risk aversion and taxes will

shift the intercept and the slope downwards the results from our calculations, including only default risk, must have a positive slope to end up at the horizontal line when combining all factors. However, it is important to keep in mind that Figure 2 is a simplification and the slope coefficients are purely arbitrary.

Naturally, the question is then what is a proper slope of Line A? What is too steep and what is not steep enough? The risk premium will not be quantified and it is nearly impossible to specify. Elton et al. (2001) however claim that 19-41% of the yield spread consists of a systematic risk factor. This means that the slope of our result graphs (and Line A) should be enough to cover a decrease implied by adding a premium corresponding to 19-41% of the yield spread.

B3. Too Steep – the Investor is Overcompensated

If the slope of the graph is too steep, the market overestimates the probability of default and the investor is overcompensated. Overcompensated since even when including all risk factors the slope would be positive and not horizontal and the investor receives too much compensation when investing in riskier bonds.

Given the chosen setting, if the market overestimates the probability of default, this means that the expected future cash flows will be underestimated and the market price is lower than the price suggested by applying the bankruptcy prediction models default probability. If we apply the default probability suggested by the bankruptcy prediction models, and this probability is lower than the default probability estimated by the market, then a lower market price can only be reached if the expected cash flows are discounted with a discount rate higher than the one suggested by the market. Higher required discount rates lead to higher expected excess returns.

B4. Not Steep Enough – the Investor is Undercompensated

Any other case, i.e. the slope is not steep enough means that the market underestimates default risk and the investor is undercompensated. Including all factors would result in a downward slope and expected excess return would decrease and the investor gets less compensated when increasing the default risk. By the same logic as presented in Subsection B3, if the market underestimates default risk the price is too high and expected excess return is too low. A horizontal or downward sloping graph is also a possible scenario. The rule of thumb is that the flatter the slope, the further away from a horizontal line when including all factors and the less compensation.

The results from our calculations are presented in Figure 3-6.

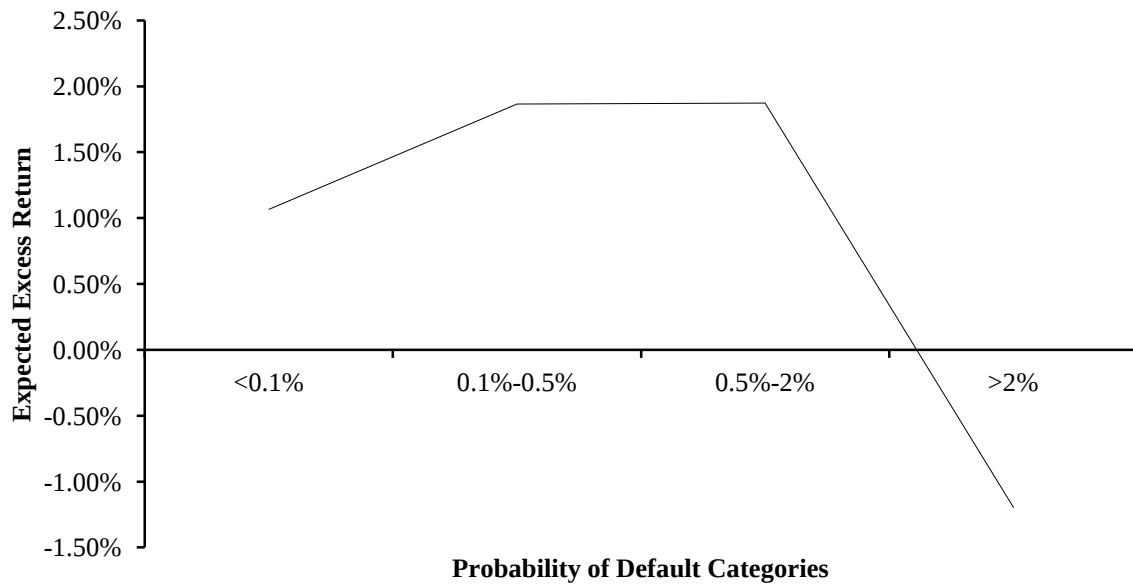


Figure 3. Expected Excess Return – Ohlson (1980)

The figure plots the expected excess return for Default Categories I-IV when using the bankruptcy prediction model developed by Ohlson (1980). Category I (<0.1% default probability), Category II (0.1-0.5% default probability), Category III (0.5-2.0% default probability) and Category IV (>2.0% default probability) correspond the default intervals on the x-axis.

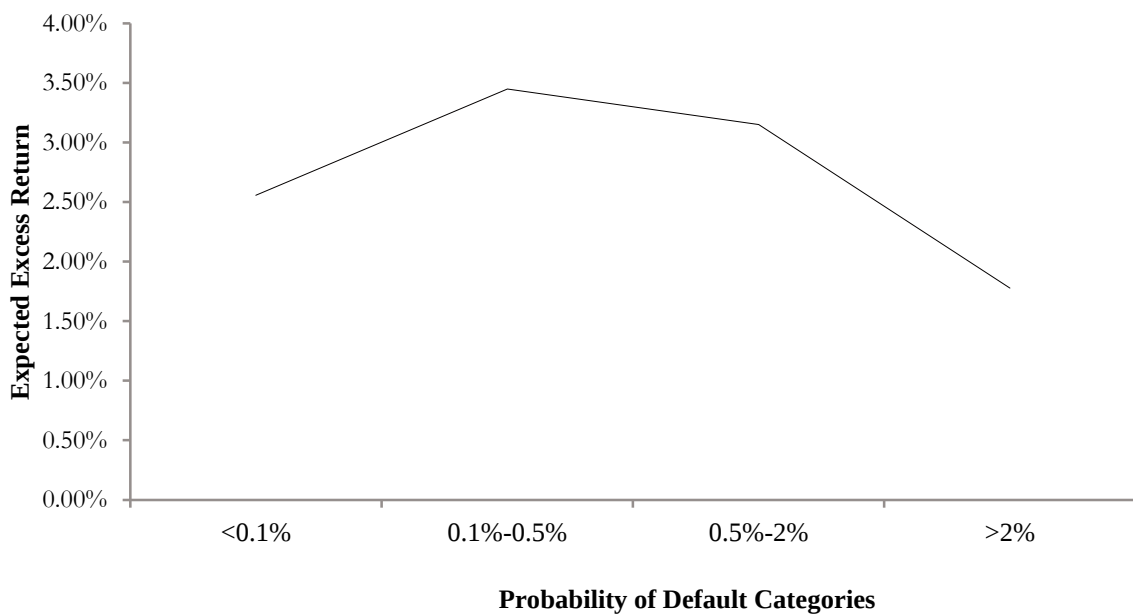


Figure 4. Expected Excess Return – Shumway (2001)

The figure plots the expected excess return for Default Categories I-IV when using the bankruptcy prediction model developed by Shumway (2001). Category I (<0.1% default probability), Category II (0.1-0.5% default probability), Category III (0.5-2.0% default probability) and Category IV (>2.0% default probability) correspond the default intervals on the x-axis.

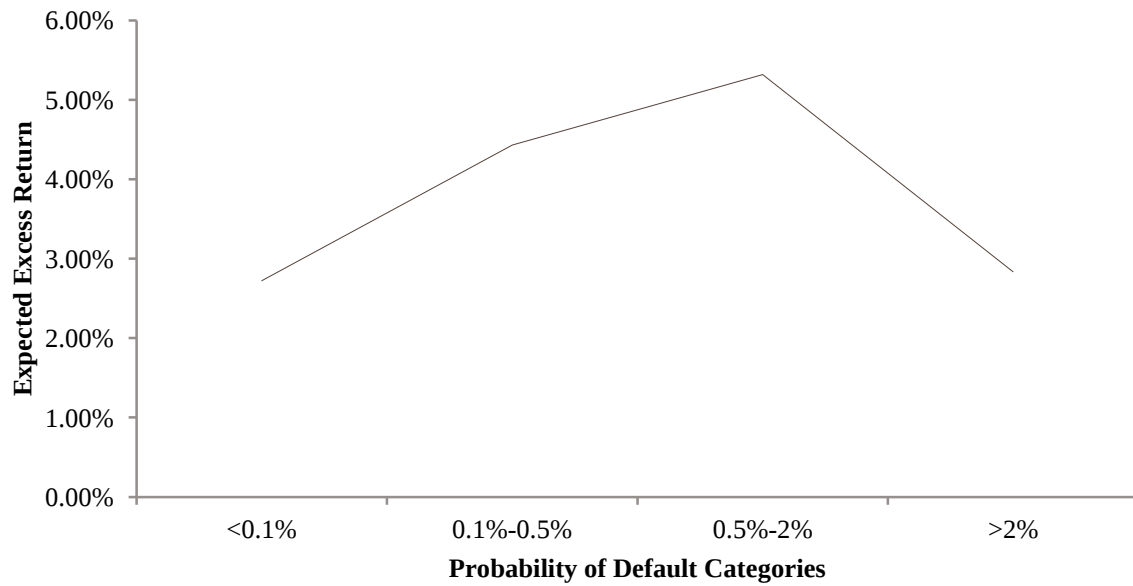


Figure 5. Expected Excess Return –Hillegeist et al. (2004)

The figure plots the expected excess return for Default Categories I-IV when using the bankruptcy prediction model developed by Hillegeist et al. (2004). Category I (<0.1% default probability), Category II (0.1-0.5% default probability), Category III (0.5-2.0% default probability) and Category IV (>2.0% default probability) correspond the default intervals on the x-axis.

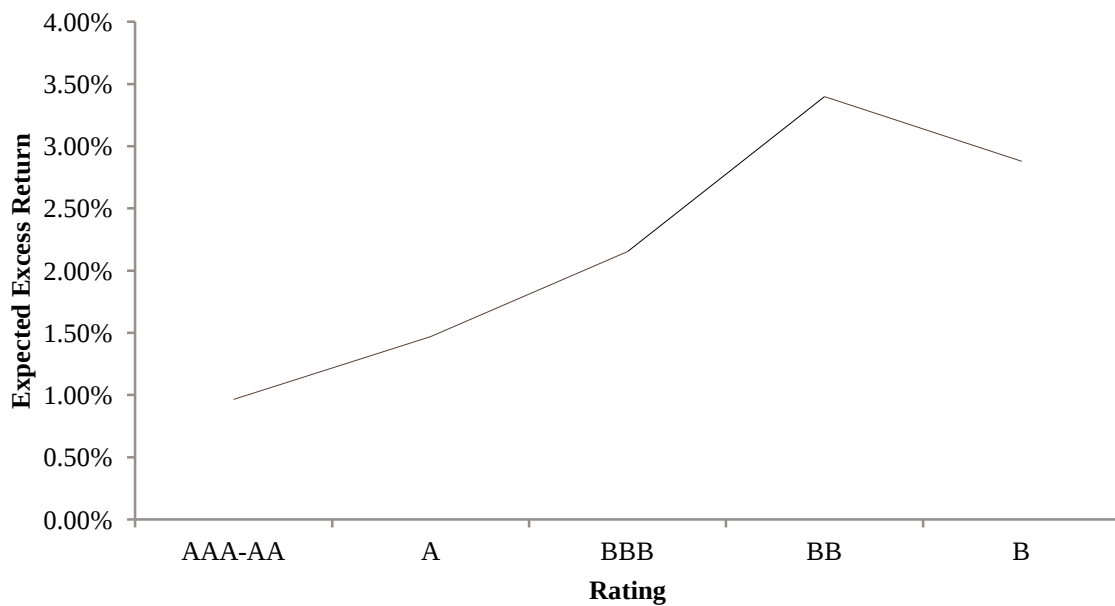


Figure 6. Expected Excess Return - Rating

The figure plots the expected excess return for each rating class when using historical default rates for each rating class. The historical default rates are 0.03% for AAA-AA, 0.08% for A, 0.27% for BBB, 0.96% for BB and 4.59% for B rated bonds.

C. Discussion

C1. Interpretation of Results

As described in Figure 2, if the investor is to get compensated for investing in risky corporate bonds the trend in expected excess return would slope upward as probability of default increases (Similar to Line A in Figure 2). However this is not the trend plotted in Figure 3, 4 and 5 above. Instead a concave trend is plotted for all bankruptcy prediction models. Also, expected excess return across default probabilities implied by credit ratings, Figure 6, show an exponential trend for ratings AAA-AA to BB before declining for B rated bonds. This indicates that the investor does not get compensated for investing in risky bonds. The fact that the results from the bankruptcy prediction models and credit ratings show the same pattern strengthens the conclusions.

Figure 4 plots the results using Ohlson (1980) default probabilities. These results stand out with very low levels of expected excess return. This is due to the fact that the model appears to overestimate the probability of default compared to the other measures. The potential problems of this model is discussed in Section C4 but with a sample average of over 4.1% risk of default it is by far the model that estimates the highest probability of default. This affects the expected cash flows from the bonds negatively. When the expected cash flows decrease, then the discount rate, or expected return, must also decrease if the bond price calculation should reach the same market price as with the models that estimate a lower probability of default. With a lower expected return, the expected excess return is naturally also lower since the same risk free rate is subtracted. We will not read anything into the absolute numbers of the expected excess return when using the Ohlson (1980) model but it is noted that the trend shows the same characteristics as the other models, a concave form. The maximum levels of expected excess return seem to be prevalent in Category II-III rather than Category IV. This indicates that the bonds that Ohlson (1980) classifies as the most risky ones does not generate an expected excess return that exceeds that for less risky bonds.

The patterns of Figure 4 and Figure 5, which show the plotted results for Shumway (2001) and Hillegeist (2004) respectively, are similar and in trends and levels of expected excess return. These two bankruptcy prediction models follow each other more closely than Ohlson (1980) when it comes to absolute levels of default probabilities and therefore also in absolute levels of expected excess return. Again the same trend is identified with the maximum levels of expected excess return in Category II-III rather than Category IV.

There is a similar trend for all bankruptcy prediction models as well as the Standard & Poor's credit ratings for expected excess return as probability of default increases. Figure 6 shows how expected excess return develops along the scale of credit ratings. The trend seems exponential from AAA-AA to BB. Where for AAA-AA bonds the expected excess return is very low, only 1.0%, and rising to the maximum notation of 3.4% for BB rated bonds until turning down for B rated bonds to about 2.9%. This is counter intuitive since one would expect that expected excess return would rise when the risk of default increases.

However, this is not ground breaking research. Kozhemiakin (2007) shows the same trend for corporate bonds when evaluating expected excess return using default probabilities implied by credit ratings. In his study, expected excess return reaches maximum levels for BB rated bonds, with the same pattern as our results show. There are differing ideas as to why the expected excess return declines for the riskiest bonds. Kozhemiakin (2007) mentions three reasons for the observed pattern. The first reason why return is so high for BB bonds is likely due to the “fallen angel effect”. The fallen angel effect refers to institutional investors being forced to sell BB bonds at low prices since they are no longer allowed to hold them after a downgrade (only allowed to hold investment grade bonds). At the same time the active investor in the high yield segment might look for B or CCC rated bonds as the potential upside to the investor is larger. The second reason mentioned is that the investor, in the search for high realized returns, overestimate his bond selection skills, bidding up the entire high yield segment. This can have a substantial effect on credit risk premiums for B and CCC rated bonds where default risk is higher. The third reason mentioned by Kozhemiakin (2007) is that B and CCC rated bonds default more often in recessions than in booms and not many recessions have been observed since the current credit rating system was set in place. This would indicate that market participants in fact underestimate the probability of default inherent in the B and CCC credit rating classes and therefore they are willing to pay a higher price than what historical default probabilities would suggest. This third reason can however be debated in our case. The date chosen in our study is shortly after a financial crisis but the same trend is observed as in Kozhemiakin (2007), which was prior to this crisis. This indicates that the trend can be better explained by the two previous reasons.

We believe that the second reason, the investor overestimating his bond selection skills, is the best explanation to the trend observed in expected excess return when using bankruptcy prediction models to estimate the probability of default. In the pursuit of large realized returns we believe that the investor does not take caution to financial information but rather relies on some form of “gut feeling”. Figure 7 shows the average coupon rate across Category I-IV for each bankruptcy prediction model as well as rating. For both Ohlson (1980) and Shumway (2001) as well as credit ratings, the possible realized return is largest for bonds categorized in Category IV. This is due to the fact that coupon rates are highest while recovery rates are stable over each category as shown in Table III. However, this is not the case for Hillegeist et al. (2004) and that will be discussed in Subsection C4. This overconfidence in their own ability to pick risky bonds that will survive leads the investors to bid up prices in the whole segment of risky bonds, resulting in lower expected excess returns.

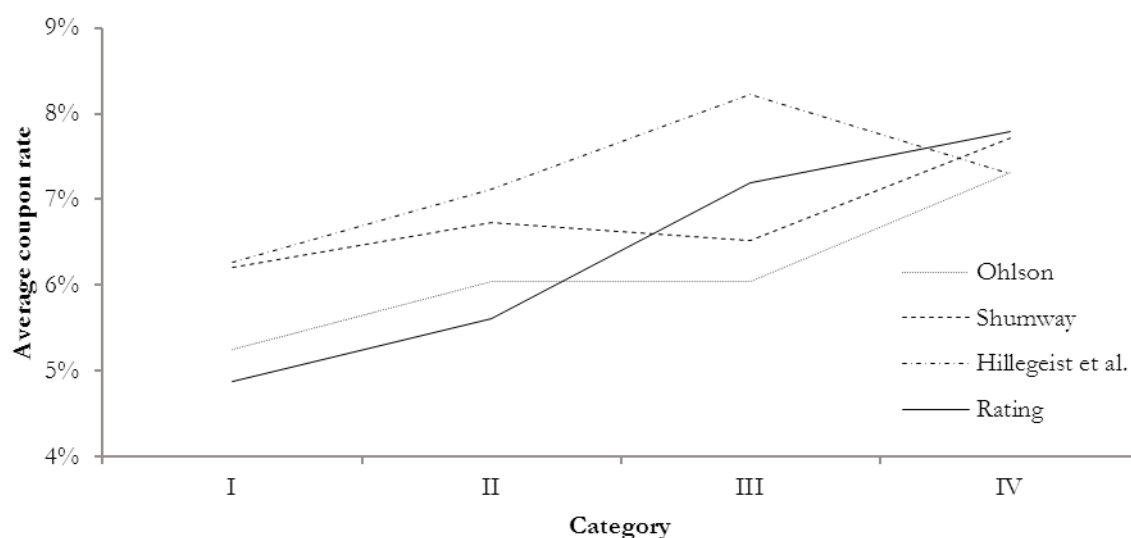


Figure 7. Average Coupon Rates Across Default Categories

The figure illustrates the average coupon rate for each default category divided by default measure. Average coupon rates are calculated based on the bonds assigned to each category by each default measure. Ratings are grouped to categories based on implied default probabilities to enable comparisons. Category I includes bonds rated AA-AAA, Category II A-BBB, Category III BB and Category IV B.

The expected excess return plotted in Figure 3, 4 and 5 is, as stated in Subsection B1 of this chapter, without considering systematic risk, risk aversion or taxes. These factors do however have an impact like the illustrations in Figure 2. When transferring that impact to the results in Figure 3, 4 and 5 both the intercept and the slope of the lines are decreased. In order for the investor to get sufficiently compensated for additional default risk, the graphs in Figure 3, 4 and 5 should slope upwards similarly to Line A in Figure 2. Since they do not, adding implications by the other factors would make the trends even more negative. Hence, incorporating the effect of systematic risk, risk aversion and taxes strengthens the interpretation and conclusion that the investor does not get compensated when default risk increases beyond 2%.

Given the way the study is designed, the results will always show decreasing expected excess return for bonds with higher probabilities of default unless the investor are compensated by large coupons and recovery rates. If a bankruptcy prediction model mispredicts the probability of default this might not be the case. However we are aware of this limitation in our design and thus include several models and also exogenously given credit ratings as reference. The fact that all models show basically the same trend of increasing expected excess return for moderately risky bonds but declining expected excess return for the riskiest segment of bonds is to us a clear indication that the investor does in fact not get compensated for investing in bonds with high probabilities of default.

There is no valid explanation to why the results would indicate that the investor on average does get sufficiently compensated for investing in risky corporate bonds. For the investor to be compensated, given the results of this study, all the selected ways of estimating probability of default must be flawed

and none give a remotely correct picture of true default probabilities. We believe however that the chances are slim to none that all of the included bankruptcy prediction models and credit ratings have no predictive ability at all and therefore with some certainty we can conclude that the investor does in fact not get sufficiently compensated for investing in risky bonds. This is not to say that all investments in risky bonds are bad. An investor with superior talent in predicting business default can in fact earn higher realized returns by investing in the riskier bonds that survive than he could if he invested in the moderately risky bonds. A discussion regarding the coherence of the four selected ways of estimating probability of failure is included in Section C4 in this chapter.

C2. Implications

As mentioned in Section II our choice of valuation date has implications on our result. By choosing December 31st 2010 the market has not yet had time to react to the financial information presented by the company. However we believe that this is a necessary assumption. A detailed analysis of the effect that this time discrepancy has on our results is almost impossible to carry out and beyond the scope of this study. The assumption regarding efficient markets is debated heavily and is again beyond the scope of this study. For Hillegeist et al. (2004) and Shumway (2001), we believe that the effect is limited. As we use market data and bond data from the same date, the fact that a company's financial statements are presented at a later date should not have an effect on our results given the assumption of efficient markets. So the discussion is mainly about the results presented using the Ohlson (1980) bankruptcy prediction model. We believe that even if a company's stock price can react to unexpected figures in financial statements, it is unlikely that the market will be surprised to the same extent about the financial health of a company and therefore the effects on the bond price should be limited. Most of the literature concerning jumps in corporate bond prices is regarding price changes when the company's credit rating is changed. A changed credit rating may be a result of surprisingly strong or weak financial reports, but previous literature diverge in their opinion on whether a rating change is expected by the bond investor or not. One way to examine if the investor in corporate bonds reacts to financial information is to investigate if he can foresee a rating change, and therefore there should be no effect on the price of the bond when a rating is in fact changed. If the investor can not accurately process the financial information the rating change will come as a shock and have an immediate effect on the bond price. In the matter of bond price reactions to rating changes previous literature diverge. Katz (1974) claims that there is no anticipation of changes in a company's credit rating shown by a price change in bonds at the time of the rating change. However, Weinstein (1977) finds that the investor anticipates the change up to 18 months prior to the rating change. The study by Weinstein (1977) indicates that the investor does in fact take financial information into account and that the release date of financial reports can affect bond prices. Given the divergence in previous research and the scope of this study, this will be overlooked.

C3. Statistical Shortcomings

Due to data gathering issues our sample is quite small in Category III and Category IV for both Hillegeist et al. (2004) and Shumway (2001). This means that our results in these categories for both these models have low statistical significance. However, we find that the trend prevalent in all models where expected excess return decreases for bonds included in Category IV and B-rated bonds compensate for the low statistical significance.

C4. Problems Related to Bankruptcy Prediction Models

Although we have included several bankruptcy prediction models in an attempt to limit the effect of specific model characteristics on our results, some issues in these models need to be discussed.

There is little coherence between the selected bankruptcy prediction models when it comes to classifying companies into a certain category, as can be seen in Table IX. This raises doubts as to how these models have in fact been developed. Since few companies get categorized in the same category with multiple bankruptcy prediction models, it is obvious that two or all of the bankruptcy prediction models wrongly estimate default probabilities. Although this problem is present it is impossible to determine the true probability of default and thus it is also challenging to determine which model is the most accurate. However, we will during this section discuss potential problems with the bankruptcy prediction models and why we believe Shumway (2001) presents the best model for estimating probability of failure. And regardless of our belief that Shumway (2001) has developed the most accurate bankruptcy prediction model, all models show the same trend, and it is enough that one of the models predicts the “true” default probability in a sufficient way to make our conclusions valid. That is, no matter what model is the most accurate they all show that the investor does in fact not get sufficiently compensated for investing in risky bonds.

Table IX
Bankruptcy Prediction Model Coherence

This table shows the coherence of the predictions posed by the models divided by category. Each panel shows a specific category. The fraction of a models predictions that are also captured by the other models are reported for each category. To illustrate, 100% of the bonds Ohlson (1980) assigns to Category I is also assigned to Category I by Shumway (2001) and Hillegeist et al. (2004). At the same time, only 3.6% of the bonds assigned to Category I by Shumway (2001) is also assigned to Category I by Ohlson (1980). 3.6% times the 110 bonds that Shumway (2001) assigns to Category I equals 4, hence 4 of the bonds assigned Category I by Shumway (2001) belongs to Category I according to Ohlson (1980). In this table we forego the traditional rating scale and instead divide the rated bonds in accordance with the specified categories. AAA-A belongs to Category I, BBB to Category II, BB to Category III and B to Category IV.

Panel A: Category I (<0.1% probability of default)				
	Ohlson	Shumway	Hillegeist et al.	Rating
<i>Total amount in category</i>	4	110	177	42
Ohlson (%)		100.0	100.0	75.0
Shumway (%)	3.6		89.1	23.6
Hillegeist et al. (%)	2.3	55.4		23.1
Rating (%)	7.1	61.9	97.6	
Panel B: Category II (0.1-0.5% probability of default)				
	Ohlson	Shumway	Hillegeist et al.	Rating
<i>Total amount in category</i>	33	59	16	79
Ohlson (%)		3.0	0.0	45.5
Shumway (%)	1.7		8.5	35.6
Hillegeist et al. (%)	6.3	31.3		18.8
Rating (%)	19.0	26.6	3.8	
Panel C: Category III (0.5-2.0% probability of default)				
	Ohlson	Shumway	Hillegeist et al.	Rating
<i>Total amount in category</i>	86	22	7	56
Ohlson (%)		4.7	2.3	26.7
Shumway (%)	18.2		4.5	13.7
Hillegeist et al. (%)	28.6	14.3		42.9
Rating (%)	41.1	5.4	5.4	
Panel D: Category IV (>2% probability of default)				
	Ohlson	Shumway	Hillegeist et al.	Rating
<i>Total amount in category</i>	93	8	7	49
Ohlson (%)		5.4	5.4	40.9
Shumway (%)	62.5		12.5	87.5
Hillegeist et al. (%)	71.4	14.3		100.0
Rating (%)	77.6	14.3	14.3	

Ohlson (1980) and Hillegeist et al. (2004) only use industrial companies in the development of their models, although Hillegeist et al. (2004) claim that their model is more general and can be applied over a wider sample. To generate our results there is a need to deviate from this limitation. Including only industrial companies would make our sample too small and general conclusions would be impossible to reach. However the effects of the application of the models on data from other industries is beyond the scope of this study but it is a possibility that the default probabilities may be less accurate than for a sample consisting solely of industrial companies. Shumway (2001) does not make the same limitations as Ohlson (1980) and Hillegeist (2004), instead he includes all industries except financial and service companies. Table IV shows the industries represented among the bonds included in our sample.

As mentioned in the beginning of this discussion Ohlson (1980) estimates very high probabilities of default, which affects the expected excess return negatively. Similar results with high probabilities of default using the Ohlson (1980) model have been estimated previously in literature (Begley, Minh and Watts (1996) and Hillegeist et al. (2004)). Hillegeist et al. (2004) estimate an average probability of default of 29.3% for a sample consisting of non-failing companies for the time period 1980 – 2000, which is extremely high. We are aware that this model has limitations when applied to more recent accounting data and little can therefore be said about the absolute levels of the expected excess return for this model. Hillegeist et al. (2004) estimate new coefficients for the Ohlson (1980) and using those coefficients on our sample the average probability of default decreases from 4.1% to 0.6% for the entire sample. But as mentioned earlier the trend using this model follows the same pattern as for all selected bankruptcy prediction models.

Hillegeist et al. (2004) derive their bankruptcy prediction model from the Black-Scholes-Merton framework. The Black-Scholes-Merton framework is criticized for underestimating yield spreads (Delianedis and Geske (2001)), which can explain the low default probabilities estimated by the Hillegeist et al. (2004) model. As discussed previously, the Black-Scholes-Merton framework relies on several assumptions, some criticized for being unrealistic, and this can dilute the results further.

An insight on which model is the best at predicting "true" probability of default is given by Figure 7. Since high promised coupon rates often is an indicator of riskier bonds we investigate the average coupon rates across categories. Figure 7 shows the average coupon rate for all models divided by category. Hillegeist et al. (2004) shows a pattern of decreasing average coupon rate for the riskiest category. This raises serious questions about the validity of that model, especially given the trend of the two other models and ratings. If a bond is risky this would require a higher coupon rate, a higher recovery rate or both for the investor to be willing to invest in it. As presented in Table III, recovery rates are stable over each category for each model. For example, imagine two portfolios, one consisting of all Category III bonds and the other all Category IV bonds as estimated by Hillegeist et al. (2004). The Category III portfolio would then have higher promised returns while having lower estimated default probabilities. This makes no sense and therefore we question the validity of this model. However it should be noted

that these results are based on few estimations in these categories so the statistical significance is low. Ohlson (1980) and Shumway (2001) show more expected trends in the average coupon rate.

Even though the aim of this study is not to reach a conclusion regarding which bankruptcy prediction model is the most accurate, our data suggests that Ohlson (1980) substantially overpredicts default probabilities while Hillegeist et al. (2004) shows counter intuitive results. This leaves Shumway (2001) as the model that seems to generate the most accurate default probabilities.

V. Conclusion

Our main conclusion is that the investor does not get sufficiently compensated when investing in bonds with high default risk. In our setting, this means that the expected excess return on bonds with higher default probability is lower than for less risky bonds. The results are consistent across all bankruptcy prediction models as well as default probabilities implied by credit ratings. We divide all bonds into four categories based on the predicted probability of default by each default measure. The category where the expected excess return starts to decline differs between the models. Using the Ohlson (1980) or Hillegeist et al. (2004) bankruptcy prediction models, the turning point is Category III (0.5-2.0% probability of default), i.e. the investor holding bonds with a probability of default above 2.0% does not get sufficiently compensated for default risk. The corresponding turning point for Shumway (2001) is Category II (0.1-0.5% probability of default). Default probabilities implied by credit ratings are also included in the study as reference besides the bankruptcy prediction models. We find that expected excess return decreases as we move from BB (1.0% probability of default) to B (4.6% probability of default) rated bonds.

The results are even stronger when incorporating risk aversion among investors and systematic risk. A risk averse investor requires a higher expected excess return as risk increases and this decreases the compensation for risky bonds further.

We find two main explanations to the results implied by our study. Firstly, the market seems to underestimate the default risk on the riskier bonds. Even if the bankruptcy prediction models do not provide predictions consistent with each other we can draw this conclusion since they all show similar results. It is enough if one of the prediction models predicts default in a satisfactory way. The conclusion is also strengthened as the default probabilities implied by credit ratings, included for reference, show the same pattern. Secondly, we believe that the investor has an over belief in his ability to select non-failing corporate bonds in the risky segment. This would explain why the risky segment, on average, seems overpriced since the potential realized return is higher for a non-failing risky bond than a safer alternative.

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