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Think on the Downside

Multifactor asset pricing models based on downside risk and their performance relative to the CAPM, FF3F and Momentum

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Abstract

This paper introduces two new measures of asset performance in a downside risk-reward framework. The first measure, Omega-H, is an extension of the Omega ratio from Keating and Shadwick (2002a) that captures an asset's *idiosyncratic* downside risk and upside potential. The second measure, the Vega, captures the asset's *systematic* downside risk and upside potential. The two measures are used to construct two factor portfolios which act as an extension of the CAPM, thus forming a three-factor model incorporating downside risk. As a control, an alternative three-factor model is introduced that extends the CAPM to include factors formed on asset co-skewness and co-kurtosis with the market. The two models, called MOV (Market-Omega-Vega) and MCC (Market-Co-skewness-Co-kurtosis), are tested in a time series and in the cross-section together with the CAPM, the Fama-French three-factor model, and the Fama-French three-factor model extended by Momentum (Carhart, 1997). The Co-skewness and Co-kurtosis portfolios are priced by the other factor models, and the MCC model performs poorly in the cross-section. At the same time, the MOV model outperforms the FF3F model in the cross-section and prices the Momentum factor. Further tests show that the performance of all factor models in the cross-section is conditional on the market return for the period, consistent with conditionally varying risk premia. All factor models perform well in bull markets and poorly in bear markets. Unlike the other models, the MOV model outperforms the CAPM in extreme bear markets. A simple portfolio allocation test shows that including the Co-skewness, Co-kurtosis, Omega and Vega factors into an international index portfolio mitigates the home bias of US investors in portfolio allocation, consistent with Guidolin and Timmermann (2008).

Keywords: *downside risk, MLPM, Omega, co-skewness, co-kurtosis, multifactor model, home bias*

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Introduction

The significance of downside risk is underestimated in traditional asset pricing theory. For example, Butler and Joaquin (2002) show that the correlations between international market indices is higher during bear markets than would be suggested by the standard return distribution models. While the standard models fail to capture the full extent of risk in bear markets, investors' decisions seem to be dictated by downside risk preferences. As shown by Guidolin and Timmermann (2008), the tendency of US investors to allocate excessive proportions of capital to the US markets and underdiversify internationally is explained to a great extent by incorporating higher moment preferences and market phase shifting into the investor's portfolio allocation problem.

At this point there exists a branch of literature that attempts to model asymmetric risk preferences of investors. This is done by either estimating risk by skewness and kurtosis in addition to variance (higher moment models), or by modeling risk as the probability and potential implication of a portfolio loss below a certain threshold (downside risk models). The main problem with the models of both types is their computational complexity. Both the downside risk models and the higher moment models rebuild the CAPM from the ground up, suggesting a different framework altogether for making investment decisions. As practice shows, although these alternative models seem to be superior to the traditional framework, the costs for practitioners to facilitate the change are prohibitive.

This paper's primary purpose is to suggest a downside risk investment framework that is theoretically justified and yet practically plausible. The approach that is taken here is to build on the traditional CAPM model to incorporate downside risk considerations, rather than developing an alternative pricing model altogether. This is done by extending the CAPM and forming a multifactor model where the additional factors explicitly account for downside risk. In this way, the traditional mean-variance approach to investment decision-making can be modified to incorporate downside risk considerations without the accompanying paradigm shift.

We develop two downside risk measures that are used to rank stocks and form factor portfolios. Much like the SMB, HML and Momentum portfolios, our downside risk portfolios represent investable trading strategies. By including these strategies in a multifactor model we treat the downside risk preferences of investors as exotic risk sources that can be priced by traditional methods. The merits of this approach include simplicity, practicality and comparability with the existing factor models of asset returns.

The paper proceeds as follows. The next section gives an overview of the most relevant downside risk and higher moment literature. Some of the articles and topics we consider are not directly relevant to the current research and are included for the purpose of familiarizing the reader with the progress of this branch of the asset pricing literature to date. The most relevant sections are those called *Higher Moments*, *The Omega ratio*, *Some alternative asymmetric risk measures*, *Factor pricing through asymmetric betas*, and *International portfolio diversification with asymmetric risk preferences.*

The subsequent section introduces the *Omega-H* and *Vega* measures of downside risk-reward trade-off. These measures are based on prior downside risk literature but incorporate extensions introduced for the first time in this paper. After that, we describe the process through which the factor portfolios are constructed. We construct for new factors based on asymmetric risk, while also replicating the Fama-French factors and Momentum. The section includes a description of the data and some of the preliminary findings. The next three sections present the empirical results, starting with the time series tests, proceeding with cross-sectional asset pricing regressions, and concluding with a simple portfolio allocation exercise. The time series tests are aimed at examining the risk loadings of the factor portfolios and their risk premia. The cross-sectional tests examine the performance of the different factor combinations (models) in the cross-section of stocks. The dataset is split into several subperiods, which allows to capture some of the time variation in the cross-sectional risk premia and in the overall performance of the pricing models. The subsequent portfolio allocation exercise examines the effect of controlling for the asymmetric risk preference factors on the home bias of US investors in international portfolio allocation. The concluding section sums up the results.

Asymmetric risk preferences in prior asset pricing literature

In the classical risk-return models investors only care about the variance and the expected return of an asset when making wealth allocation decisions. This so-called *mean-variance* framework is the foundation of such well-known concepts as Markowitz's portfolio theorem (Markowitz, 1952 and 1959), the CAPM (Sharpe, 1964; Lintner, 1964; Black, 1972), the Sharpe ratio (Sharpe, 1966) and others.

At the same time, when we talk about investors caring about *risk*, we normally imply the risk of a loss, rather than the risk of an extra gain. The extention from *risk* to *loss* rather than from *risk* to *variance* also dates back several decades, with one of the early references being traceable to Roy (1952). Also, Markowitz (1959) suggests a trade-off between downside variance and expected return as a possible extention to his (traditional) portfolio optimization problem.

Since then, the theory of *asymmetric risk* preferences for gains against losses has drawn some attention in the asset pricing literature. In several prominent papers asymmetric risk preferences have been addressed as a utility issue. In 1979, Kahneman and Tversky show experimentally that risk aversion is asymmetric for a representative agent with respect to gains and losses, implying an asymmetric *S*-shaped utility schedule. They call it the *value function* and sketch it in the following way:

Figure 1: Value function. Source: Kahneman&Tversky, 1979

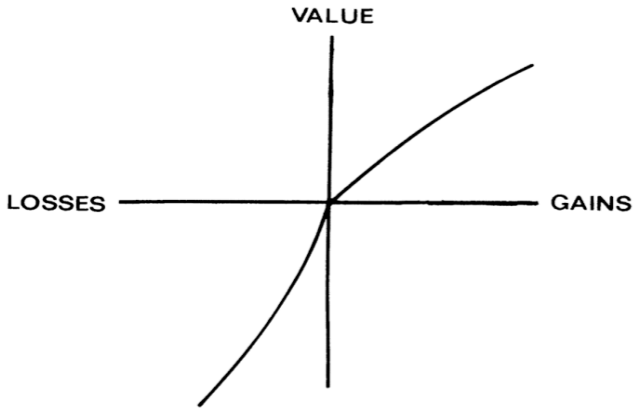


FIGURE 3.—A hypothetical value function.

Another approach to the same idea is taken by Gul (1991) who introduces the concept of *Disappointment Aversion* (DA). Gul's Disappointment aversion theory is basically an algebraic analogy of the concept of the Value function, where investor marginal utility is modeled to be lower for gains than for losses in absolute terms.

Meanwhile, asymmetric risk preferences also become a subject in risk modeling. Here, we can identify two main directions in academia: i) incorporating skewness (and in some cases kurtosis) into the existing asset pricing models, and ii) estimating risk based on partial moments, mostly calculated from

a portion of the return distribution that falls below a certain threshold (e.g., zero, the expected return on the market, the risk-free rate, etc.). We are going to start with a brief overview of the literature from category i), proceeding with a more thorough examination of the literature from category ii). After that, we give an overview of the paper by Ang, Chen and Xing (2006) who present an application of downside risk to the traditional factor pricing framework and whose approach is partially similar to the approach to downside risk taken in this paper. The concluding chapter of our literature review considers asymmetric risk in a setting where agents make international portfolio allocation decisions.

Higher moments

This section presents a brief overview of several academic articles that extend the basic asset pricing framework to incorporate higher distribution moments. Some of the papers considered below are quite technical and we only outline the main ideas and the points that have some relevance to this study. However, the reader is welcome to take a closer look at these articles at their own discretion.

In one of the early papers on alternative risk models, Rubinstein (1973) presents a portfolio optimization problem for a one-period risk-return optimal investment where risk is measured by a (co)moment of an arbitrary order, the mean-variance framework being a special case for order *2*. Rubinstein's model is essentially one of the early extensions of the CAPM providing an abstract general equilibrium for an arbitrary utility function and a variety of risk-return pairs. Several years later, Kraus and Litzenberger (1976) also extend the CAPM framework to incorporate (co)skewness[2].

In the more recent literature, Harvey and Siddique (1999) develop a maximum likelihood model for stock returns that they call GARCHS(1,1,1) to incorporate conditional skewness in addition to the conditional variance of a return time series. The target density is assumed to be a *two-parameter non-central t-distribution* (a skewed fat-tailed bell-curve), rather than a normal, to account for the fact that empirical returns tend to be skewed and fat-tailed. The estimation results suggest that conditional skewness is an important parameter in modeling asset return distributions.

Dittmar (2002) suggests a non-linear model for the stochastic discount factor that extends the (covariance-based) beta-factor framework to incorporate coskewness and cokurtosis. Dittmar's model is based on a third-order Taylor expansion[3] of the (Euler) APT equation under restrictive assumptions about the shape of the utility function[4]. The result is a cubic, rather than a linear, factor representation

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[2] Kraus and Litzenberger's approach is then tested by Friend and Westerfield (1980) who find evidence validating the co-skewness extension

[3] The traditional CAPM model can be derived with a second-order Taylor expansion of the APT equation under the assumption of quadratic utility, or with a first-order Taylor expansion of the APT equation under the assumption of normality.

[4] The assumptions about the utility function made in Dittmar's paper are still much less restrictive than the assumptions in the traditional CAPM framework as in Sharpe (1964), Lintner (1964), Black (1972), etc. While

of systematic risk as a function of future wealth which Dittmar approximates through the return on the market as in the traditional CAPM framework. Empirical tests show that the cubic extention of the CAPM is indeed a better fit to the data and even a feasible, though more complex, alternative to the Fama-French three-factor model (as in Fama&French, 1993).

Feunou, Jahan-Parvar and Tedongap (2011) develop a maximum likelihood model for estimating conditional skewness using the bi-normal distribution. Their application of what they call the BiN-GARCH model to several international equity index returns shows that conditional skewness is associated with a higher conditional mode in the returns, implying that conditional skewness is a priced risk factor in the market. Some additional findings of the paper include empirical evidence of a time-varying risk premium, as well as evidence of a time-variation in investor disappointment aversion.

Downside risk[5]

While skewness and kurtosis incorporate part of the information about the asymmetry in an asset's returns, some researchers argue that these measures do not explicitly capture the asymmetric risk preferences of investors. As Ang, Chen and Xing (2006) put it, *„...the coskewness statistic does not explicitly emphasize asymmetries across down and up markets, even in settings where coskewness may vary over time (as in Harvey and Siddique, 1999)'* (page 3).

This critique of the higher-moment CAPM models has led to the development of an alternative approach to estimating asymmetric risk. The underlying idea is to partition the return distribution of the risky asset and make assumptions about which distribution portions are potential sources of systematic risk for an investor. Of course, the common choices are the distribution portions that fall *below* a certain return threshold, such as the risk-free rate, the sample average, or *zero*. In this respect, the downside risk literature can be considered as intimately related to Value-at-Risk, and as we will shortly see VaR estimates are even included in some of the downside risk papers as a possible downside risk measure.

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the traditional CAPM model is based on quadratic utility or normal returns or both, Dittmar's restrictions are merely assumptions about the *signs* of the first, second and third derivatives of the utility function, which leaves a much larger family of possible functional forms to choose from.

[5] Under this title, academic articles that consider downside risk are briefly described in a mostly chronological fashion. As a complementary source of information, we recommend the book *Managing Downside Risk in Financial Markets* by Sortino and Satchell (2001). It is a diversified collection of academic articles on the topic of downside risk, including the paper that introduces the Sortino ratio that appears in some of the articles considered in this chapter. However, not all of the topics addressed in the book are directly relevant to our paper, and hence the book's contents are not explicitly considered here. The reader is welcome to review this resource independently.

We start with the basic concept of a *partial moment* and its relation to variance as a source of risk in the risk-return trade-off. After that, we consider the Omega ratio as a popular performance measure in the downside risk-return framework, together with its extensions and possible alternatives. The concluding chapters present some existing applications of the downside risk-return framework to international portfolio diversification and to factor models of asset returns.

Partial Moments

In general, a dominant part of the mean-downside risk literature is based on the so-called *Mean Lower Partial Moments (MLPM)*. One of the early uses of the MLPM framework in risk-return analysis can be traced to Bawa and Lindenberg (1977), from now on B&L, who define the *n'th Mean Lower Partial Moment*, i.e. the (partial) moment of an asset's return distribution below a certain threshold τ, as follows:

1: Mean-Lower Partial Moment of order *N* (a)

$$MLPM_n(r;\ \tau) = \int_{r=-\infty}^{\tau} (\tau - r)^n dF(r),$$

where $F(r)$ is the cumulative distribution of the asset's return r. In a more general setting that includes discrete-time cases, formula 1 can be rewritten as:

2: Mean-Lower Partial Moment of order *N* (b)

$$MLPM_n(r,;\tau) = \mathrm{E}\left[(\tau - r)^n 1_{\{r\leq\tau\}}\right],$$

where $1_{\{r\leq\tau\}}$ is an indicator function taking the value of 1 if $r \leq \tau$, 0 otherwise. Next, the *Co-Lower Partial Moment* of asset i on asset j is defined as:

3 Co-Lower Partial Moment of order *N* (a)

$$CLPM_n(r_i, r_j;\ \tau) = \int_{r_i=-\infty}^{\tau} \int_{r_j=-\infty}^{\infty} (\tau - r_i)(\tau - r_j)^{n-1} dF(r_i, r_j),$$

or in a more intuitive formulation,

4: Co-Lower Partial Moment of order *N* (b)

$$CLPM_n(r_i, r_j;\ \tau) = \mathrm{E}\left[(\tau - r_i)(\tau - r_j)^{n-1} 1_{\{r\leq\tau\}}\right].$$

Naturally, *Mean-Higher Partial Moments (MHPM)* and *Co-Higher Partial Moments (CHPM)* can be defined in a very similar manner, namely:

5: Mean-Higher Partial Moment of order *N*

$$MHPM_n(r;\ \tau) = \int_{r=\tau}^{\infty} (r-\tau)^n dF(r) = \mathrm{E}\left[(r-\tau)^n 1_{\{r>\tau\}}\right],$$

6: Co-Higher Partial Moment of order *N*

$$CHPM_n(r_i, r_j;\ \tau) = \int_{r_i=\tau}^{\infty}\int_{r_j=-\infty}^{\infty} (r_i-\tau)(r_j-\tau)^{n-1} dF(r_i, r_j) = \mathrm{E}\left[(r_i-\tau)(r_j-\tau)^{n-1} 1_{\{r>\tau\}}\right].$$

B&L extend the risk-return framework to optimize the $MLPM_n$-return trade-off, rather than the variance-return trade-off as in the traditional CAPM model, subsequently developing a generalized capital asset pricing model with partial moments. Later Harlow and Rao (1989), among others[6], also present a version of the generalized capital asset pricing model based on mean-lower partial moments[7]. These generalized capital asset pricing models, call them the MLPM-CAPM models, are paid special attention to under the following title as a cornerstone in the comparison between the MLPM and MV approaches.

The MLPM framework vs. the MV framework: a closer look at the MLPM-CAPM

Since its inception, the mean-variance framework has been the dominant form of risk-return analysis. As a result, any alternative framework, including MLPM, needs to be proved a superior alternative to MV analysis if it is to become the new dominant frame of reference for academics and practitioners. As Price, Price and Nantell (1982) put it, *„...interest in abandoning the more familiar variance as a portfolio risk measure depends on how significantly different the inferences are from applying the two alternative risk measures'* [page 3 (844)].

One could argue that the discussion of the link between the MV framework and the MLPM framework dates back to Markowitz (1959) who shows that for a special case of an investor utility function, investors optimize according to the *mean-semivariance* trade-off. In this way, Markowitz's derivations foresee the advancement of the downside risk literature, as semivariance later came to be called $MLPM_2$ in the downside risk vinacular.

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[6] Another early derivation of an MLPM-CAPM model can be found in Hogan and Warren (1974). Hogan and Warren's model predeces Bawa and Lindenberg (1977) chronologically, but the B&L model is, in our opinion, much more explicit, and thus more complete, in the derivation methodology. Also, Lee and Rao (1988) develop a $MLPM_2$-CAPM model where different thresholds are used for the returns on a single asset and the return on the market portfolio.

[7] Harlow and Rao further show that the premium on the $MLPM_n$ risk cannot be explained by the CAPM in the case of the CRSP equal-weighted index. The result is robust under a range of arbitrary values of the benchmark τ.

In a more structured context, the main similarities and differences between the MV framework and the MLPM framework are first considered in the B&L paper. As previously mentioned, B&L develop an early MLPM-CAPM model and compare it to the traditional MV-CAPM. In B&L's general $MLPM_n$-CAPM, the beta-representation of an asset's expected return is given by:

7: Downside beta-representation of an asset's expected return

$$\mathrm{E}(R_i) = R_f + \beta_i^{MLPM_n}\big[\mathrm{E}(R_M) - R_f\big], \qquad \text{where}$$

$$\beta_i^{MLPM_n} = \frac{CLPM_n\big(R_i, R_M;\ R_f\big)}{MLPM_n\big(R_M;\ R_f\big)},$$

with R_M being the return on the tangency portfolio on the Capital Market Line[8]. In other words, B&L define the risk-return trade-off in terms of an arbitrary partial moment, with the downside variance being a special case for $n = 2$. Also, the partitioning threshold, denoted as τ in formulas 1-4, is set at the risk-free rate of return, R_f.[9]

In the downside risk-return framework, the B&L MLPM-CAPM is considered to be an example of a better (more general) model than the traditional CAPM (we will subsequently refer to the traditional CAPM as the MV-CAPM due to the fact that it is based on a Mean-Variance trade-off). First of all, B&L's model is consistent with two-fund separation between the market portfolio and a risk-free asset for $n = 1$ and 2,[10] and a theoretical SML also exists in an equilibrium setting. Second, B&L show that their MLPM-CAPM model reduces to the MV-CAPM under several sets of restrictive assumptions about the distribution of the asset returns.

Although the MLPM-CAPM is a more general model that includes the MV-CAPM as a special case and requires much fewer assumptions about the underlying assets, its feasibility would still depend on how much of an improvement it would be compared to the simpler MV-CAPM when it came to valuing assets. Several downside risk papers have attempted to examine how different the MLPM-CAPM predictions are from the MV-CAPM predictions.

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[8] Please note that in the traditional CAPM model the tangency portfolio is the value-weighted portfolio of all assets in the market. However, in the MLPM-CAPM framework this need no longer be the case.

[9] In a subsequent paper, Lee and Rao (1988) examine the $MLPM_2$-CAPM model with the benchmark set at the market expected return $\mathrm{E}(R_M)$ and returns are assumed to be lognormal. The risk of an asset is defined only for the cases when both the asset and the market are below their expected returns.

[10] An important feature of MLPM-CAPM is that its results are identical to the results of the MV-CAPM under the assumption of normality. This has been shown to hold by Bawa and Lindenberg (1977) and by Nantell and Price (1979).

For example, Price, Price and Nantell (1982) consider the case for which MLPM-CAPM is expected to disagree with MV-CAPM based on the example of lognormal returns. Only the case of the $MLPM_2$-CAPM is examined. The paper concludes that MV-CAPM betas *underestimate* MLPM-CAPM betas for lognormal returns with below-average risk, and *overestimate* MLPM-CAPM betas for lognormal returns with above-average risk. The results are mostly statistically significant, and the authors suggest that the insignificant and inconsistent results are attributable to abnormal skewness in the empirical observations.

At the same time, Jarrow and Zhao (2006) study the differences between downside risk-based valuation and mean-variance valuation when optimizing portfolios of stocks and high-yield bonds. They come to the conclusion that the portfolios of low-risk stocks do not benefit from the extension to downside risk-based optimization, while the optimal portfolios of high-yield bonds differ significantly across the two measures, consistent with the application of traditional mean-variance analysis to equities but not to debt.

The Omega ratio

The downside risk framework has seen a number of extensions in the academic literature beyond the MLPM-CAPM models. However, one that deserves special attention is the Omega ratio, introduced by Keating and Shadwick (2002a). In recent years, the Omega ratio has become a dominant performance measure within the downside risk framework, in part due to its relative simplicity and its complete non-reliance on any assumptions about the distribution of the empirical returns or the shape of the utility function. It is originally defined as:

8: The Omega ratio (a)

$$\Omega(\tau) = \frac{\int_{\tau}^{\infty}[1 - F(r)]dr}{\int_{-\infty}^{\tau} F(r)dr},$$

for a reservation return threshold τ and an empirical cumulative return distribution $F(r)$. However, it can also be expressed in a way that is more consistent with the MLPM framework:

9: The Omega ratio (b)

$$\Omega(r;\ \tau) = \frac{\mathrm{E}(r) - \tau}{MLPM_1(r;\ \tau)} + 1.$$

So, the Omega ratio is essentially proportional to the expected return on an asset in excess of the threshold τ divided by the expected loss below τ (the derivation of 7 from 6 is presented in Kaplan and Knowles, 2004). When applied to empirical return observations, the Omega ratio is straightforward to compute because it implies:

10: The Omega ratio (c)

$$\Omega(r;\ \tau) = \frac{\sum r - \tau T}{\sum\left[(\tau - r)1_{\{r \le \tau\}}\right]} + 1,$$

given a sample of T observations.

Keating and Shadwick argue that the Omega ratio is a superior measure compared to the Sharpe ratio even when the returns are distributed normally, because it can incorporate an individual investor's arbitrary loss threshold τ into risk-return analysis. When returns are non-normal, the Omega ratio performs better than the Sharpe ratio and the other measures that only consider some of the distribution moments, because Omega by its design incorporates information about *all* of the distribution moments of the underlying asset. The importance of this property is hard to overestimate because there are infinitely many statistical distributions with equal moments of some order(s) yet different densities altogether, as examplified in the figure below[11].

Figure 2: Two return distributions with identical mean and variance. Source: Keating&Shadwick (2002a).

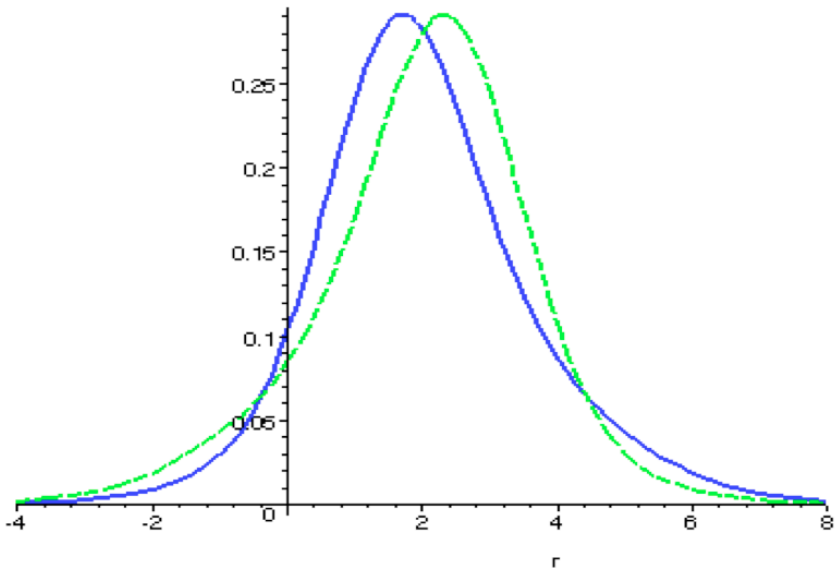


Diagram 1.2 Returns distributions for assets C (dashed) and D (solid) with mean of 2 and standard deviation of 1.6 .

Keating and Shadwick (2002b) further show that with complex distributions one needs to consider moments of up to order 10 in order to be able to capture all of the relevant information about the underlying risk. The Omega ratio is thus a useful measure in the performance evaluation of assets with non-normal empirical distributions (e.g. hedge funds). Omega can also be used as an objective function

[11] Another useful property of Omega is its robustness to affine transformations of the underlying variable $f: r \mapsto Ar + B,\ A > 0$. In the case where $A < 0$, $\Omega(Ar + B;\ \tau) = \frac{1}{\Omega(r;\ \tau)}$.

in portfolio optimization, although the corresponding maximization problem is quite non-trivial for the reasons considered under the next title.

Portfolio optimization with Omega

The natural extension of the MLPM risk-return framework to asset pricing is the family of the MLPM-CAPM models that have been considered earlier. The MLPM-based risk models have also found their way into portfolio optimization. A popular subject in contemporary downside risk literature is portfolio optimization against the Omega ratio. However, the estimation technique for an Omega-optimal portfolio is quite problematic. The objective function is defined as:

11: Objective function for an Omega-optimal portfolio

$$\boldsymbol{\varpi}^* = \underset{\boldsymbol{\varpi}}{\operatorname{argmax}} \frac{\boldsymbol{\varpi}^T \mathrm{E}(\boldsymbol{R}) - \tau}{\mathrm{E}\{[\tau - \boldsymbol{\varpi}^T \mathrm{E}(\boldsymbol{R})] 1_{\{\boldsymbol{\varpi}^T \mathrm{E}(\boldsymbol{R}) \leq \tau\}}\}},$$

for a column vector of portfolio weights $\boldsymbol{\varpi}$, a column vector of expected returns $\mathrm{E}(\boldsymbol{R})$ and the arbitrary threshold τ.

There is no simple solution to Equation 9 because the objective function is discontinuous. The figure below presents the Omega ratio of a portfolio of three assets as a function of the asset weights:

Figure 3: Portfolio Omega as a function of portfolio weights (three risky assets). Source: Gilli&Schumann (2010).

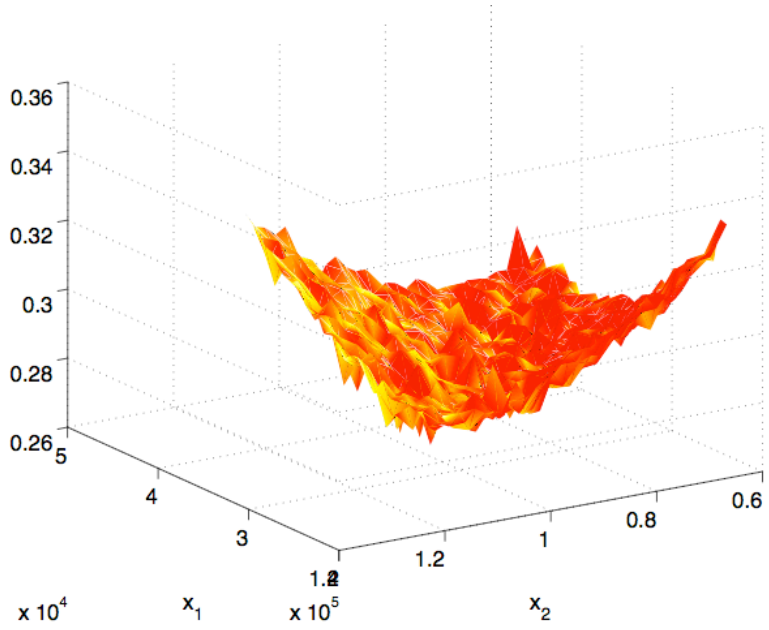


Fig. 1. Shape of Omega for a portfolio of 3 assets.

Gilli et al. (2006), Gilli et al. (2008) and Gilli and Schuman (2010) consider this optimization problem. They apply the Threshold Accepting (TA) algorithm that attempts to solve Equation 9 by searching random regions of the objective function for local extrema under prespecified constraints and robustness settings. Gilli et al. (2008) consider long-short Omega-optimal portfolios, Gilli et al. (2006) apply the algorithm to several alternative downside risk measures (Expected Shortfall, Maximum Loss,

Value-at-Risk and Omega), and Gilli and Schumann (2010) examine how the efficiency of the TA algorithm relates to the amount of computational resources assigned to it. In general, all three papers are quite technical and concerned with the subject of non-linear programming, rather than the financial implications of the Omega concept.

Some alternative asymmetric risk measures

While Omega is arguably one of the most popular measures of downside risk, some academics consider several alternative downside risk measures in their papers. Farinelli et al. (2008) consider a total of 11 alternative downside risk measures and present a possible algorithm for choosing the optimal risk-return measure in portfolio choice. The table below presents the 11 alternative measures from Farinelli et al. (2008).

Table 1: A summary of 11 alternative risk-return measures. Source: Farinelli et al. (2008).

Table 1
Performance ratios, reward and risk measures

Performance	Reward: $r(Y)$	Risk: $\rho(Y)$	Parameters
Sharpe	$\mathbf{E}[Y]$	$\mathbf{E}[(Y-\mathbf{E}[Y])^2]^{\frac{1}{2}}$	–
MiniMax	$\mathbf{E}[Y]$	$\inf_{\omega\in\Omega} Y(\omega)$	–
Stable[a]	$\mathbf{E}[Y]$	$A(p;\alpha)^{\frac{1}{p}}\mathbf{E}[\lvert Y\rvert^p]^{\frac{1}{p}}$	$\alpha\in(0,2)$, $p\in(0,\alpha)$
MAD	$\mathbf{E}[Y]$	$\mathbf{E}[\lvert Y-\mathbf{E}[Y]\rvert]$	–
Gini	$\mathbf{E}[Y]$	$\mathbf{E}[F_Y\circ Y]$	–
CVaR	$\mathbf{E}[Y]$	$\mathbf{E}[-Y\lvert Y\leqslant -\mathrm{VaR}(Y;\alpha)]$	$\alpha\in(0,1)$
Sortino–Satchell	$\mathbf{E}[Y]$	$\mathbf{E}[Y_-^q]^{1/q}$	$q\in(0,\infty)$
VaR	$\mathbf{E}[Y]$	$\mathrm{VaR}(Y-\mathbf{E}[Y];\alpha)$	$\alpha\in(0,1)$
Farinelli–Tibiletti	$\mathbf{E}[Y_+^p]^{1/p}$	$\mathbf{E}[Y_-^q]^{1/q}$	$p,q\in(0,\infty)$
Rachev	$\mathbf{E}[Y\lvert Y\geqslant -\mathrm{VaR}(Y;1-\alpha)]$	$\mathbf{E}[-Y\lvert Y\leqslant -\mathrm{VaR}(Y;\beta)]$	$\alpha,\beta\in(0,1)$
Generalized Rachev	$\mathbf{E}[Y_+^\gamma\lvert Y\geqslant -\mathrm{VaR}(Y;1-\alpha)]^{\frac{1}{\gamma}}$	$\mathbf{E}[Y_-^\delta\lvert Y\leqslant -\mathrm{VaR}(Y;\beta)]^{\frac{1}{\delta}}$	$\alpha,\beta\in(0,1)$, $\gamma,\delta\in(0,\infty)$

After applying their decision algorithm, Farinelli et al. come to the conclusion that the Sharpe ratio is always picked as inferior to the other risk measures in the cases of non-normally distributed asset returns, while the asymmetric risk measures are robustly picked as superior measures.

In another paper, Favre-Bulle and Pache (2003) apply several downside risk measures to portfolio optimization with hedge funds. Mostly the simpler and more popular measures are considered compared to the selection in the Farinelli et al (2008) paper, including VaR, adjusted VaR, downside deviation and Omega. The table summarizing the ratios used in Favre-Bulle and Pache is presented below[12].

[12] The main purpose of the two tables presented under this title is to give the reader a general idea about the downside risk-return ratios presently considered in the downside risk literature. Most of these ratios are not used in the subsequent sections of this paper.

Table 2: Ratios used by Favre-Bulle and Pache (2003) in hedge fund performance evluation.

Framework	Risk	Reward	Performance
Mean – Variance	$\sigma_R = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mu_R - R_i)^2}$	$\mu_R = \frac{1}{n}\sum_{i=1}^{n} R_i$	$Sharpe\ ratio = \frac{\mu_R - R_f}{\sigma_R}$
Mean – Value-at-Risk	$VaR_{99\%} = \lvert\mu_R - 2.33 \times \sigma_R\rvert$	μ_R	$Modified\ Sharpe = \frac{\mu_R - R_f}{R_f - VaR_{99\%}}$
Mean – Adjusted Value-at-Risk	$adjusted\ VaR_{99\%} = \lvert\mu_R - z_{CF} \times \sigma_R\rvert$	μ_R	$Adj.VaR\ Modif.\ Sharpe = \frac{\mu_R - R_f}{R_f - VaR_{99\%}}$
Mean – Downside Deviation	$downside\ \ dev. = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\tau - R_i)^2 \frac{1}{R_i < \tau}}$	μ_R	$Sortino\ ratio = \frac{\mu_R - R_f}{downside\ deviation}$
Omega	$I_1(r) = \int_a^r F(x)\,dx$	$I_2(r) = \int_r^b (1 - F(x))\,dx$	$\Omega(r) = \frac{I_2(r)}{I_1(r)}$

One interesting result that can be observed in the paper is that the mean-variance and the mean-VaR frameworks present nearly identical results in the optimal portfolio choice problem. Another result is that the Omega-based portfolio *'produces markedly different results'* (page 34) for the several return thresholds the authors consider, which is not surprising because it is the only measure that directly incorporates all distribution moments of the returns. In general, it is shown that the asymmetric risk-return measures produce superior portfolio frontiers compared to the mean-variance framework.

Additionally, Roland and Xiang (2004) develop what they call the Modified Omega. They attack the Omega-based portfolio optimization problem from a geometric perspective and redefine the risk measure in the Omega ratio under the Euclidean metric. The resulting performance measure becomes:

12: Omega ratio with an alternative risk metric

$$\widetilde{\Omega}(\tau) = \frac{\int_{\tau}^{\infty}[1 - F(r)]dr}{\left\{\int_{-\infty}^{\tau}[F(r)]^2 dr\right\}^{\frac{1}{2}}}.$$

Roland and Xiang (2004) show that their Modified Omega, as well as two other downside risk measures that they consider, have closed-form portfolio optimization solutions. However, the performance of these alternative risk measures in portfolio choice is not explicitly compared to the performance of the original Omega ratio.

In a more recent paper, Bali, Cakici and Whitelaw (2011) present a hybrid measure of tail covariance risk, defined as:

13: Hybrid tail covariance risk measure (HTCR)

$$\mathrm{HTCR}_i = \mathrm{E}[(R_i - h_i)(R_m - h_m)|R_i < h_i],$$

where h_i and h_m are tail event thresholds for asset i and the market, respectively. The HTCR measure is based on the MLPM$_2$-CAPM beta as in Bawa and Lindenberg (1977) (formula 5) which can be rewritten as:

14: Asset beta in the MLPM framework (a)

$$\beta_i^{MLPM_2} = \frac{\mathrm{E}\left[(R_i - h)(R_m - h)1_{\{R_m<h\}}\right]}{\mathrm{E}\left[(R_m - h)^2 1_{\{R_m<h\}}\right]}.$$

Under the assumption that the partial moments are uncorrelated with the indicator $1_{\{R_m<h\}}$, formula 12 further simplifies to:

15: Asset beta in the MLPM framework (b)

$$\beta_i^{MLPM_2} = \frac{\mathrm{E}[(R_i - h)(R_m - h)|R_m < h]}{\mathrm{E}[(R_m - h)^2|R_m < h]}.$$

The essential modification by Bali et al. (formula 11) compared to the MLPM$_2$-CAPM beta (formula 13) is that the Bali et al. condition the asset's tail risk on the asset itself, rather than the market. The rationale for excluding the denominator $\mathrm{E}[(R_m - h)^2|R_m < h]$ from the expression is that it is identical across all assets and thus irrelevant for measuring the asset's downside risk. The thresholds in the HTCR measure are set to differ between the asset and the market, and both are fixed at the 10th percentiles of the respective distributions. The application of the measure to an underdiversified portfolio of a stock and the market shows that HTCR is a priced factor and even a predictor of future returns.

Kappa as an extension of Omega

One extension to the Omega ratio worth noting separately comes from Kaplan and Knowles (2004) who define what they call the Kappa measure as:

16: The Kappa measure

$$\mathrm{K}_n(r;\ \tau) = \frac{\mathrm{E}(r) - \tau}{\sqrt[n]{MLPM_n(r;\ \tau)}}$$

The significance of the Kappa measure is that it includes the Omega ratio as a special case. As was mentioned before, Kaplan and Knowles show, after some derivation, that:

17: The Kappa-Omega identity

$$\Omega(r;\ \tau) = \frac{\mathrm{E}(r) - \tau}{MLPM_1(r;\ \tau)} + 1 = \mathrm{K}_1(r;\ \tau) + 1.$$

Kaplan and Knowles (2004) further apply K_1, K_2 and K_3 to return data[13]. They show that the risk-return ranks differ significantly depending on the order of the lower partial moment (the n in K_n), suggesting that the Kappa measure is superior to Omega by being a more general formulation of the downside risk-return trade-off.

Factor pricing through asymmetric betas

Ang, Chen and Xing (2006), from now on ACX, present a paper in which they examine downside risk as a factor in asset pricing in the traditional MV-CAPM framework. They model an asset's downside risk through exposure to conditional betas. The risk measures are essentially derived by running a dummy variable regression that conditions the response (beta) coefficient on market performance relative to the risk-free rate. The resulting coefficients are called the *downside beta* and the *upside beta*, defined as follows:

18: Downside and Upside Beta

$$\beta_{rf}^{-} = \frac{\mathrm{cov}(r_i, r_m | r_m \leq r_f)}{\mathrm{var}(r_m | r_m \leq r_f)}, \qquad \beta_{rf}^{+} = \frac{\mathrm{cov}(r_i, r_m | r_m > r_f)}{\mathrm{var}(r_m | r_m > r_f)}.$$

Please recall that the traditional beta estimate is defined as:

19: CAPM Beta

$$\beta = \frac{\mathrm{cov}(r_i, r_m)}{\mathrm{var}(r_m)}.$$

ACX also compute what they call the *relative downside beta* (from now on, *rdb*) and the *relative upside beta* (from now on, *rub*) defined as $(\beta^- - \beta)$ and $(\beta^+ - \beta)$, respectively, as well as a measure that we may call the up-minus-down difference in betas, or the *umdb* for short, defined as $(\beta^+ - \beta^-)$. The realized measures are computed using one-year samples of daily return data and tested as cross-sectional determinants of contemporaneous returns. The hypothesis is that, if investors are more risk-averse towards downside risk than to upside/average risk, we should observe that high *rdb* and low *umdb* coefficients are associated with high asset returns, and vice versa.

After an examination of the properties of the stock portfolios formed from the different coefficients described above (upside beta, downside beta, relative upside and downside betas, and difference in betas), ACX restrict their cross-sectional tests to *rdb* and *rub*. They run Fama-MacBeth regressions with *rdb* and *rub* controlling for different risk factors, such as the market beta, size and book-to-

[13] In a related note, the paper also shows that K_2 is equivalent to the Sortino ratio, defined in Sortino (2001) and presented in Tables 1 and 2.

market as in Fama and French (1992), momentum as in Jagadeesh and Titman (1993), standard deviation, coskewness, cokurtosis, and the liquidity beta as in Pástor and Stambaugh (2003).

In the initial regression with *rub* and *rdb* with no controls, the coefficient on the *rdb* factor is positive, while the coefficient on the *rub* factor is negative (both highly significant), consistent with a premium on downside risk and a discount on upside potential. The addition of controls to the cross-sectional regression eventually drives away the significance of the *rub* factor but not the *rdb* factor, which implies that the premium on (contemporaneous) downside risk is not explained by the other factor loadings in the cross-section. It is further explicitly shown that downside risk as measured by the *rdb* coefficient is not explained by coskewness and is robust to various portfolio weighting strategies and the exclusion of outliers.

One possible drawback of the downside risk measure presented by ACX is that it is not persistent. It is shown that the autocorrelation coefficient of the *rdb* measure in their sample is only 0.082, and hence the *rdb* measure is not a good basis for an implementable trading strategy.

International portfolio diversification with asymmetric risk preferences

The downside risk premium has been recently shown to be directly relevant to international portfolio diversification and the phenomenon known as the *home bias*. From the perspective of a US investor, *home bias* means holding a higher share of US securities in an international portfolio than would be advised by the traditional mean-variance framework. For example, Lewis (1999) estimates that the mean-variance framework suggests a 60% weight in US stocks for a US-based portfolio, while Thomas et al (2006) estimate that an average US investor had an over 85% position in the US market in 2003.

Butler and Joaquin (2002) study cross country market correlations by partitioning the market returns into three regimes: *bull*, *calm* and *bear*. They then apply three standard models (bivariate normal, *RiskMetrics* GARCH(1,1), and Student's t with 4 df) to estimate the confidence intervals for conditional cross-market correlations during the three regimes. Figure 4 below is an extract from Butler and Joaquin's graphical results:

Figure 4: Market correlation corridors under changing market regimes

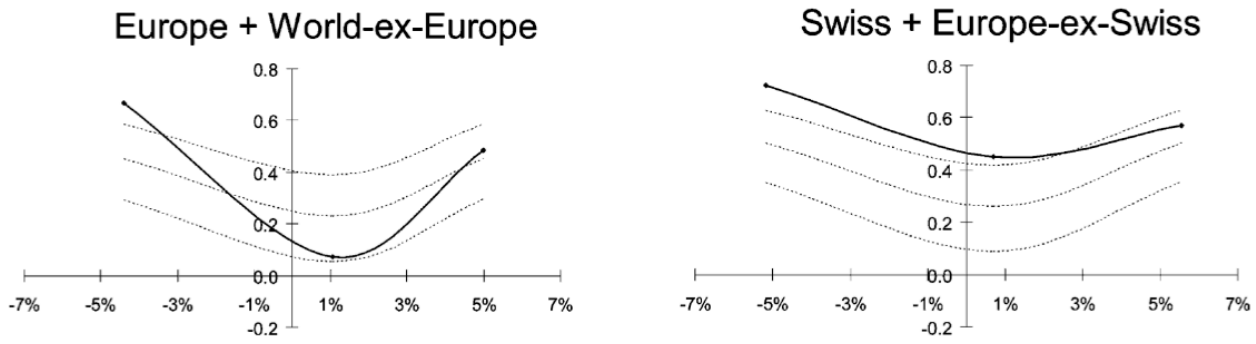


Fig. 2. Correlation profiles: International diversification from a domestic perspective with 2.5 and 97.5 percent confidence limits from the bivariate normal distribution.

Butler and Joaquin show that, as a general rule, cross-market correlations during bear markets are higher than predicted by the traditional models of return distributions.

The same idea is later extended by Guidolin and Timmermann (2008) who show that the exceptionally high correlations across markets in bear regimes are related to the skewness and kurtosis preferences of investors. Incorporating such preferences to a portfolio choice model leads to a substantial increase in the normative estimate for a representative investor's position in domestic stocks, which is a significant step towards solving the home bias puzzle. It is also shown that the unusually high position of a representative US investor in the US equity market may be driven by the US market's favourable coskewness and cokurtosis properties compared to the markets of other countries.

Omega-H and Vega as Measures of Downside Risk-Return Trade-off

Bali et al (2011) identify three sources of downside risk for an asset: an *idiosyncratic* component, a *systematic* component and a *co-tail risk* component. In this section we introduce two downside risk-return trade-off measures aimed at incorporating these three components of downside risk. The first measure is a modification of the Omega measure as in Keating and Shadwick (2002a), we call it the Omega-H measure. The second measure is based on the difference-in-betas as in Ang et al. (2006) and the HTCR measure as in Bali et al. (2011), and we call it the Vega measure.

The Omega-H and Vega measures are designed to capture both the *systematic downside risk* of an asset, as in Bawa and Lindenberg (1977), Harlow and Rao (1989), and Ang et al. (2006), and the *idiosyncratic downside risk*, as in Keating and Shadwick (2002a, 2002b), Roland and Xiang (2004), Sortino (2001), and Kaplan and Knowles (2004). The risk-return space on which our measures are defined is related to, but not the same as, the risk-return space defined in Lee and Rao (1988).

Market assumptions

The economic setting that justifies our measures can be characterized by *six market assumptions*. These assumptions are as follows:

1. The MV-CAPM (value-weighted) market portfolio is a reasonable proxy for an average investor's majority investment position; consistent with Sharpe (1964) Lintner (1964), and Black (1972).
2. Investors prefer assets with higher upside potential and lower downside risk; consistent with Kahnemann and Tversky (1979), and Gul (1991)
3. Downside risk is restricted to the situation where both the market and the asset produce returns below their respective arbitrary thresholds; consistent with Lee and Rao (1988)
4. Upside potential is restricted to the situation when the asset produces a return above its arbitrary threshold, regardless of the movement of the market; consistent with Lee and Rao (1988), and Keating and Shadwick (2002a, 2002b)
5. An investor seeks assets that increase the systematic risk of their portfolio in good times and avoids assets that increase the systematic risk of their portfolio in bad times; consistent with Ang et al. (2006), Guidolin and Timmermann (2008)
6. An investor cares about the probability of a downside/upside event, as well as its expected magnitude; consistent with Keating and Shadwick (2002a, 2002b)

The Hybrid Omega measure

The original Omega measure in Keating and Shadwick (2002a) can be rewritten as:

20: The original Omega measure revisited

$$\Omega(r_i;\ \tau) = \frac{\mathrm{E}\left[(r_i - \tau)1_{\{r_i > \tau\}}\right]}{\mathrm{E}\left[(\tau - r_i)1_{\{r_i \leq \tau\}}\right]}.$$

The vector representation of the sample estimate of $\Omega(r_i;\ \tau_i)$ with a sample of T observations of the return vector R_i is then given by:

21: Sample estimate of the original Omega measure

$$\hat{\Omega}(\boldsymbol{R}_i;\ \tau) = \frac{\mathbf{1}^T{}_{\{\boldsymbol{R}_i > \tau\}}(\boldsymbol{R}_i - \tau)}{\mathbf{1}^T{}_{\{\boldsymbol{R}_i \leq \tau\}}(\tau - \boldsymbol{R}_i)}.$$

This ratio is implicitly defined in a risk-return framework where the source of risk for an investor is given by $\mathrm{E}\left[(\tau - r_i)1_{\{r_i \leq \tau\}}\right]$ in the risk space defined by the set $\{r_i : r_i \leq \tau\}$.

The above specification of risk does not capture the systematic component that arises due to the asset's correlation with the market in bad times. Because we are considering a risk-return space where risk is given by both the market *and* the asset falling below their thresholds (assumption 3), we need to modify the denominator of the original Omega ratio.[14] The corresponding indicator function is then given by the product $1_{\{r_i \leq \tau\}}1_{\{r_m \leq \tau_m\}}$, which gives rise to the *Hybrid Omega measure*:

22: The Hybrid Omega measure

$$\Omega^H(r_i, r_m;\ \tau, \tau_m) = \frac{\mathrm{E}\left[(r_i - \tau)1_{\{r_i > \tau\}}\right]}{\mathrm{E}\left[(\tau - r_i)1_{\{r_i \leq \tau\}}1_{\{r_m \leq \tau_m\}}\right]},$$

with its sample estimate given by:

23: The Hybrid Omega sample estimate

$$\hat{\Omega}^H(\boldsymbol{R}_i, \boldsymbol{R}_m;\ \tau, \tau_m) = \frac{\mathbf{1}^T{}_{\{\boldsymbol{R}_i > \tau\}}(\boldsymbol{R}_i - \tau)}{\mathrm{diag}\left(\mathbf{1}_{\{\boldsymbol{R}_i \leq \tau\}}\mathbf{1}^T_{\{\boldsymbol{R}_m \leq \tau_m\}}\right)^T(\tau - \boldsymbol{R}_i)},$$

where $\boldsymbol{R}_i$ is a column vector of time series return observations of asset i, $\boldsymbol{R}_m$ is a column vector of return observations of the market, and $\mathbf{1}_{\{\cdot\}}$ is a column vector of indicator values giving *1* when condition $\{\cdot\}$ is fulfilled, *0* otherwise.

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[14] More formally, we could say that risk is defined over the set of ordered pairs $\{(r_i, r_m) : r_i \leq \tau_i,\ r_m \leq \tau_m\}$.

The rationale behind this formulation is as follows. When an asset gives a positive return, the investor does not care about the state of the market (assumption 4). Conversely, because the investor holds a well-enough diversified portfolio (assumption 1), they do not care about idiosyncratic risk when the market is growing. At the same time, when the market is falling, the investor would like to avoid cyclical risky assets.

The meaning of Ω^H is thus two-fold. *Firstly*, it captures the *upside potential* that investors seek by assumption 2, taking into account the expected *magnitude* of the return if it falls above its threshold τ_i and the actual *probability* that the stock breaks the threshold. *Secondly*, it captures *downside risk* by taking into account the magnitude of the return if it falls below its threshold and probability that this happens *given that the market is down*. Hence, the Omega-H measure can be interpreted as a possible counterpart of the Sharpe ratio in the risk-return space where return is defined as $\mathrm{E}\big[(r_i - \tau)1_{\{r_i > \tau\}}\big]$, while risk is defined as $\mathrm{E}\big[(\tau - r_i)1_{\{r_i \leq \tau\}}1_{\{r_m \leq \tau_m\}}\big]$.

The Vega Measure

In this section we introduce the Vega measure that is aimed at capturing an asset's *systematic downside risk and upside potential*. The Vega measure can be interpreted from two different perspectives. First of all, it follows directly from assumptions 3, 5 and 6. Second, it is an extension of the *umd beta* as in Ang et al. (2006) and the HTCR as in Bali et al. (2011). We will now derive the Vega measure starting with the formulas for the *upside* and *downside beta* (please recall the section called *Factor Pricing through Asymmetric Betas* in the literature review section):

24: Upside and Downside Betas, revisited

$$\beta^{+}_{i;\tau} = \frac{\mathrm{cov}(r_i, r_m | r_m > \tau_m)}{\mathrm{var}(r_m | r_m > \tau_m)},$$

$$\beta^{-}_{i;\tau} = \frac{\mathrm{cov}(r_i, r_m | r_m \leq \tau_m)}{\mathrm{var}(r_m | r_m \leq \tau_m)}.$$

First of all, this formula does not incorporate the probability that the market return will break the threshold τ. In general, there is a connection between a conditional expectation and a partial moment that can be expressed as follows[15]:

25: Taking the indicator function out of the expectation

$$\mathrm{E}\big[(\tau - r_i)1_{\{r_i \leq \tau\}}\big] = \mathrm{E}[(\tau - r_i)|r_i \leq \tau]\mathrm{P}[r_i \leq \tau],$$

[15] Here, and throughout the rest of this paper, an assumption is being made that the indicator function $1_{\{\}}$ is uncorrelated with the return r.

where $\mathrm{P}[r_i \leq \tau_i]$ is the probability of observing $r_i \leq \tau_i$.[16] We can thus multiply the numerator and the denominator of the upside and downside beta formulas by the probability factors:

$$\beta_{i;\tau}^{+} = \frac{\mathrm{cov}(r_i, r_m | r_m > \tau_m)\mathrm{P}[r_m > \tau_m]}{\mathrm{var}(r_m | r_m > \tau_m)\mathrm{P}[r_m > \tau_m]},$$

$$\beta_{i;\tau}^{-} = \frac{\mathrm{cov}(r_i, r_m | r_m \leq \tau_m)\mathrm{P}[r_m \leq \tau_m]}{\mathrm{var}(r_m | r_m \leq \tau_m)\mathrm{P}[r_m \leq \tau_m]},$$

and then rewrite the variance and covariance functions in terms of expectations, putting the probabilities inside the brackets[17] as indicator functions:

$$\beta_{i;\tau}^{+} = \frac{\mathrm{E}\big([r_i - \mathrm{E}(r_i | r_m > \tau_m)][r_m - \mathrm{E}(r_m | r_m > \tau_m)]1_{\{r_m > \tau_m\}}\big)}{\mathrm{E}\big([r_m - \mathrm{E}(r_m | r_m > \tau_m)]^2 1_{\{r_m > \tau_m\}}\big)},$$

$$\beta_{i;\tau}^{-} = \frac{\mathrm{E}\big([r_i - \mathrm{E}(r_i | r_m \leq \tau_m)][r_m - \mathrm{E}(r_m | r_m \leq \tau_m)]1_{\{r_m \leq \tau_m\}}\big)}{\mathrm{E}\big([r_m - \mathrm{E}(r_m | r_m \leq \tau_m)]^2 1_{\{r_m \leq \tau_m\}}\big)}.$$

To simplify matters even further, we can disregard the denominators because they will be constant and identical for every asset in the universe[18]. Additionally, let us denote a few terms in a more intuitive way:

26: Return thresholds implied by the upside and downside beta

$$\tau_i^d = \mathrm{E}(r_i | r_m \leq \tau_m), \qquad \tau_i^u = \mathrm{E}(r_i | r_m > \tau_m),$$

$$\tau_m^d = \mathrm{E}(r_m | r_m \leq \tau_m), \qquad \tau_m^u = \mathrm{E}(r_m | r_m > \tau_m).$$

This brings us to the familiar MLPM framework, because the upside and downside betas now reduce to:

$$\beta_{i;\tau,\tau_i^u,\tau_m^u}^{*+} = \mathrm{E}\big([r_i - \tau_i^u][r_m - \tau_m^u]1_{\{r_m > \tau_m\}}\big),$$

$$\beta_{i;\tau,\tau_i^d,\tau_m^d}^{*-} = \mathrm{E}\big([r_i - \tau_i^d][r_m - \tau_m^d]1_{\{r_m \leq \tau_m\}}\big).$$

In other words, we have just shown that the downside and upside beta-implied risk measures are just a special case of the generalized MLPM framework where the return thresholds are defined as

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[16] An equivalent relationship holds, of course, for the complement (the higher partial moment):

$$\mathrm{E}\big[(r_i - \tau)1_{\{r_i > \tau\}}\big] = \mathrm{E}[(r_i - \tau) | r_i > \tau]\mathrm{P}[r_i > \tau]$$

[17] Putting the probability terms inside the brackets is in this case futile because it does not change the results in any meaningful way. However, it is a useful step in establishing a link with the other downside risk measures.

[18] Recall that a similar simplification is made by Bali et al (2011) to derive the HTCR measure.

conditional return expectations conditional on the market. These thresholds are indeed computationally convenient because they follow directly from the conditional variance and covariance formulas. However, no explicit justification is provided by Ang et al. (2006) for using these exact thresholds instead of those considered in the other downside risk papers, such as *zero*, the risk-free rate, or the unconditional expected return[19]. This provides us with an excuse to redefine the implied thresholds τ_i^d, τ_i^u, τ_m^d and τ_m^u to simplify the upside and downside beta measures even further. Consider the following setting:

$$\tau_i^d = \tau_i^u = \tau, \qquad \tau_m = \tau_m^d = \tau_m^u.$$

In this case the upside and downside measures reduce to simply:

$$\beta_{i;\tau,\tau_m}^{*+} = \mathrm{E}\big([r_i - \tau][r_m - \tau_m]1_{\{r_m > \tau_m\}}\big) = CHPM_2(r_i, r_m; \tau, \tau_m),$$

$$\beta_{i;\tau,\tau_m}^{*-} = \mathrm{E}\big([r_i - \tau][r_m - \tau_m]1_{\{r_m \leq \tau_m\}}\big) = CLPM_2(r_i, r_m; \tau, \tau_m).$$

In other words, after some derivation and a simplifying assumption, the downside and upside beta reduce to a co-lower partial moment and a co-higher partial moment[20]. This brings us to the *umd beta*-implied asymmetric risk measure, which can be written as:

27: The difference in betas-implied downside risk-return trade-off

$$ARM_{i,umd} = CHPM_2(r_i, r_m; \tau, \tau_m) - CLPM_2(r_i, r_m; \tau, \tau_m),$$

and is defined over a risk-return space covered by the ordered pairs $\{(r_i, r_m): r_m \leq \tau_m\}$.

Next, we project the $ARM_{i,umd}$ measure onto our risk-return space to get the Vega measure. Recall that this paper defines the risk space through the multitude of outcomes characterized by the ordered pairs $\{(r_i, r_m): r_i \leq \tau_i,\ r_m \leq \tau_m\}$ and derived with the indicator function $1_{\{r_i \leq \tau_i\}}1_{\{r_m \leq \tau_m\}}$. This gives the following expression for the *Vega measure*:

28: The Vega measure

$$\mathrm{V}_i = \mathrm{E}\big([r_i - \tau][r_m - \tau_m]1_{\{r_m > \tau_m\}}\big) - \mathrm{E}\big([r_i - \tau][r_m - \tau_m]1_{\{r_m \leq \tau_m\}}1_{\{r_i \leq \tau\}}\big),$$

with a corresponding sample estimate[21]:

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[19] As a reminder, the original Bawa and Lindenberg (1977) formula for a downside beta is defined in terms of an arbitrary threshold altogether, and then the threshold is set at risk-free rate for empirical tests; Lee and Rao (1988) use expected returns; Bali et al. (2011) use the 10th percentiles of the return distributions. None of the papers provides an explicit justification for using these specific thresholds.

[20] In a related note, the HTCR measure of Bali et al. (2011) reduces to $\frac{CLPM_2(r_i, r_m; \tau, \tau_m)}{\mathrm{P}[r_m \leq \tau_m]}$ if we set $\tau = h_i, \tau_m = h_m$.

29: The Vega sample estimate

$$\widehat{\mathrm{V}}_i = \mathbf{1}_{\{\boldsymbol{R}_m>\tau_m\}}^{T}\mathrm{diag}([\boldsymbol{R}_i-\tau][\boldsymbol{R}_m-\tau_m]^T) - \mathrm{diag}\big(\mathbf{1}_{\{\boldsymbol{R}_i\le\tau\}}\mathbf{1}_{\{\boldsymbol{R}_m\le\tau_m\}}^{T}\big)^T\mathrm{diag}([\boldsymbol{R}_i-\tau][\boldsymbol{R}_m-\tau_m]^T).$$

Although the above specifications look somewhat complicated, the underlying idea is quite simple and can be easily conveyed in words. In effect, we define the benefit from holding the asset as the extra upside potential that the asset adds to the investor's portfolio during bull markets (by assumption 1, an average investor's portfolio can be approximated by the market portfolio):

$$\mathrm{V}_i^u = \mathrm{E}\big([r_i-\tau][r_m-\tau_m]1_{\{r_m>\tau_m\}}\big).$$

Next, we define risk as the systematic downside risk that the asset adds to the investor's portfolio during bear markets:

$$\mathrm{V}_i^d = \mathrm{E}\big([r_i-\tau][r_m-\tau_m]1_{\{r_m\le\tau_m\}}1_{\{r_i\le\tau\}}\big).$$

Given these two components, the former defining a form of return and the latter a form of risk, the Vega is essentially a measure of the risk-return trade-off of an asset, defined as the difference between the reward and the risk[22]:

$$\mathrm{V}_i = \mathrm{V}_i^u - \mathrm{V}_i^d.$$

A note on portfolio optimization with the Vega measure

In principle, like any other risk-reward measure, Vega can be used as an objective function in a portfolio optimization problem. A portfolio optimization problem has at this point been defined in the Omega risk-return space (please recall the section *Portfolio optimization with Omega* in the literature review section). The Vega space is, to the best of our knowledge, a new concept in the asset pricing literature, and so we introduce the Vega-implied portfolio optimization problem in *Appendix 1* for potential use in future downside risk research.

By assigning arbitrary weights to the original portfolio and its derivative Vega portfolio, an investor can incorporate their asymmetric risk preferences into the portfolio allocation decision. Alternatively,

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[21] In principle, the proper way to define the sample estimate for Vega would be to normalize by the number of sample observations. However, because we intend to use the same sample size for all comparable stocks, each $\widehat{\mathrm{V}}_i$ should be normalized by the same value, which means that this value can be dropped.

[22] It should be noted that a difference between risk and reward is used instead of a ratio of reward to risk for two reasons. First of all, this makes the Vega measure comparable to the previously derived $ARM_{i,umd}$ measure. Second, it makes the Vega more robust towards alternative risk specifications because a ratio estimate is not comparable across all assets in the cases when the denominator can change signs.

the Vega-implied portfolio weights can be used as a shrinkage target in the Bayesian-type portfolio optimization routines.

The unique benefit of using Vega optimization is that a Vega-optimal portfolio can allow an investor to maintain the desired underlying risk exposure, while minimizing the downside risk and maximizing the upside potential of their risky portfolio. In this way, Vega optimization can prove to be a useful tool for hedging against market phases and adjusting value-at-risk without changing the primary risk-return space.

The significance of Omega-H and Vega in the downside risk framework

The Ω^H and Vega measures represent a marginal contribution to the downside risk-return literature by incorporating several previously established risk-return frameworks in a condensed manner. As previously mentioned, they do so by attempting to explicitly account for all three types of downside risk identified by Bali et al (2011).

Taken together, Ω^H and Vega can be interpreted as the *idiosyncratic* risk-reward trade-off measure and the *systematic* risk-reward trade-off measure, respectively. The Ω^H measure is non-parametric the same as the original Omega measure, allowing it to incorporate all moments of the asset's return distribution. At the same time, just like the original Omega, Ω^H as a measure of a single asset's performance does not explicitly consider the effect of adding the asset to a portfolio. At the same time, the Vega is a systematic risk measure that explicitly models the asset's contribution to the performance of a portfolio across bull and bear phases.

In terms of the MLPM-CAPM framework, Ω^H and Vega are intended as a simpler alternative compared to reformulating the market portfolio in terms of asymmetric risk preferences in the way that is suggested in the traditional MLPM-CAPM literature. Ω^H and Vega are essentially a build-up on the standard MV-CAPM market portfolio and are modeled to account for its downside risk. It is shown in the subsequent sections that equity portfolios constructed based on pre-formation Ω^H and Vega estimates provide a hedge against downside risk for an arbitrary downside risk-averse investor in the MV-CAPM framework. Moreover, Ω^H and Vega can be easily altered to build up downside risk hedges for an arbitrary portfolio with any type of risk exposure without requiring the MLPM-CAPM adjustment of the weights in the original portfolio constituents. The empirical tests of the two measures follow in the subsequent sections.

Factor Portfolios

From a practitioner's perspective, a risk measure is approximately as relevant as the investable portfolio with the corresponding risk loading. In order for this study to be useful for practitioners, we do not consider the Omega-H and Vega measures at face value, but instead study the effects of asymmetric risk through the behaviour of the investable factor portfolios formed on the basis of Omega-H and Vega ranks. For robustness tests and further regression analysis, we replicate the Market portfolio, the Fama-French SMB and HML portfolios, and the Momentum portfolio in the way that is consistent with our construction of what we refer to as the Omega and Vega factors. Also, two additional portfolios are formed by ranking stocks on their co-skewness and co-kurtosis with the market[23]. All of our portfolios are investable within the same universe of stocks. We further use the terms Omega and Vega to refer to the Omega and Vega factor portfolios, unless otherwise specified.

Formation Rules

The factors are constructed as zero-cost portfolios, formed by taking a long position in one portfolio of stocks and an equal short position in another portfolio. The exception is the Market portfolio that is formed as a long-only portfolio of stocks and an identical short position in the risk-free asset. All factors are constructed based on pre-formation data to ensure real-life replicability.

In total, *eight* factor portfolios are formed from the same universe of stocks. We further classify these portfolios into two categories. The *performance-based portfolios* are constructed based on the stocks' past performance relative to the corresponding performance measure. These portfolios include *Momentum* (past returns), *Omega* (average pre-formation Omega-H measure), *Vega* (average pre-formation Vega measure), *Co-skewness* (average pre-formation co-skewness with the market portfolio) and *Co-kurtosis* (average pre-formation co-kurtosis with the market portfolio)[24].

The *property-based portfolios* are constructed based on the last available data on the stocks' properties. These include the *Market* (market capitalization in proportion to the total market capitalization of the stock universe), *SMB* (market value of equity), and *HML* (the ratio of book equity to market equity).

[23] The portfolios formed based on co-skewness and co-kurtosis are intended to capture the main idea behind the higher-moment CAPM framework.

[24] Please note that we use the terms *Co-skewness* and *Co-kurtosis* (capital *C*) to denote the factors and the terms *co-skewness* and *co-kurtosis* (small *c*) to denote the measures.

We construct the factor portfolios assuming monthly rebalancing. Performance-based portfolios[25] are formed based on one year of stock return history for congruence with the formation rules for the HML, SMB and Momentum portfolios that all use no more than one year of historical data. The thresholds for the Omega and Vega portfolios are set at the risk-free rate[26]. Only those stocks that have one year of stock return history are considered for the portfolios, regardless of whether all of that history is necessary for portfolio-related calculations[27]. No adjustment is made to account for the costs of rebalancing, and the monthly change in portfolio weights is thus assumed to be instantaneous and costless[28]. More detailed information on how each portfolio is constructed is presented below.

The Market portfolio.

Given a universe of stocks, an MV-CAPM market portfolio is constructed as a value-weighted average return. At the beginning of each rebalancing period, we select all stocks with non-missing data on the last quoted stock price P_i and number of shares outstanding N_i. The market values of equity are then calculated as $ME_i = P_i N_i$, and the weight on each stock i is given by $w_i^M = \frac{ME_i}{\sum_i ME_i}$. These weights are assigned to all stocks and maintained until the next rebalancing period, at which time the weight estimation is repeated.

The Omega and Vega portfolios

The Omega portfolio is based on ranking each stock in accordance with the Omega-H measure, while the Vega portfolio is formed based on the Vega measure[29]. The weight allocation is based on the

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[25] Please recall that we refer to the portfolios where stocks are selected based on past performance (Momentum, Omega, Vega, Co-skewness, Co-kurtosis) as performance-based portfolios. We refer to portfolios where stocks are selected based on their properties (book-to-market, market cap/weight) as property-based portfolios.

[26] A more detailed description of the data used for portfolio formation follows in the *Data* section.

[27] We eliminate stocks with less than one year of return history to achieve some consistency with Banz and Breen's (1986) findings concerning the data bias in the stocks that are newly included in databases.

[28] Although the assumption of costless rebalancing is unrealistic, our main purpose is to ensure that all of our factor portfolios are comparable. Comparability is achieved by applying the same rules of rebalancing to all eight factors, ensuring that any bias that arises from assuming costless and instantaneous rebalancing is not a major factor in the subsequent comparative study.

[29] It must be noted that the market return used to calculate the Omega-H and Vega measures is not the return on the market portfolio as described in the previous section (monthly rebalancing), but the return on the theoretical market portfolio for which we use daily rebalancing. We use the theoretical market portfolio because for the purpose of the subsequent tests Omega and Vega are designed to perform in relation to the market and not one specific type of the market-replicating portfolio that a theoretical investor is holding.

assumption that investors tend to avoid low-Omega and low-Vega stocks, while investing in high-Omega and high-Vega stocks.

As was mentioned before, we use one year of historical return data to estimate the pre-formation Omega-H and Vega ranks. We only consider the stocks for which all the necessary data is available. The Omega and Vega factors are then formed as a long position in a value-weighted portfolio of the stocks that rank above the 75th percentile, and an equal short position in a value-weighted portfolio of the stocks that rank below the 25th percentile according to the respective measure. The weights are maintained for one month, at which point the annual historical sample is rolled one month forward and the pre-formation Omega-H and Vega ranks are re-estimated.

The Coskewness and Cokurtosis portfolios

Although the capital asset pricing models that incorporate skewness and kurtosis are considered here to be an alternative to the downside risk framework, the two approaches are intimately linked. Indeed, both frameworks have been developed to recognize the asymmetries in investors' risk perceptions. Feunou, Jahan-Parvar and Tedongap (2011), among others, acknowledge the connection between the skewness preferences of investors and downside risk premia and argue that a skewness investor is downside risk-averse.

We consider co-skewness and co-kurtosis as of a stock with the market portfolio as sources of priced risk that could be argued to serve as an alternative to the two downside risk measures presented above. However, once again, we try to go around the complexity inherent in the higher-moment CAPM models by forming simple investable strategies on the basis of the co-skewness and co-kurtosis without modifying the underlying MV-CAPM risk-return space. The two measures are defined as in Ang et al. (2006):

30: The Coskewness and Cokurtosis measures

$$CS_i = \frac{\mathrm{E}\{[r_i - \mathrm{E}(r_i)][r_m - \mathrm{E}(r_m)]^2\}}{(\mathrm{E}\{[r_i - \mathrm{E}(r_i)]^2\})^{\frac{1}{2}}\,\mathrm{E}\{[r_m - \mathrm{E}(r_m)]^2\}},$$

$$CK_i = \frac{\mathrm{E}\{[r_i - \mathrm{E}(r_i)][r_m - \mathrm{E}(r_m)]^3\}}{(\mathrm{E}\{[r_i - \mathrm{E}(r_i)]^2\})^{\frac{1}{2}}\,(\mathrm{E}\{[r_m - \mathrm{E}(r_m)]^2\})^{\frac{3}{2}}}.$$

In terms of portfolio formation, it is crucial to note that co-skewness and co-kurtosis are *risk* measures, unlike Omega-H and Vega that are measures of the *risk-return trade-off*. Hence, when forming a zero-cost portfolio, the co-skewness and co-kurtosis ranks need to be treated differently from the Omega-H and Vega ranks. When forming portfolios based on a risk-return trade-off, an investor would take a long position in the assets that rank *the best* and a short position in the assets that rank *the worst*. At the same time, when forming a portfolio based on risk alone, an investor that seeks risk exposure

would choose a long position in the assets that are *the riskiest* and a short position in the assets that are *the least risky* to capture the corresponding risk premium.

In this respect, the co-skewness measure, as defined above, is a measure of the dependence between the asset's return and the market's volatility. The riskiness of the asset will come from low returns in volatile markets because higher market volatility is associated with high uncertainty. Hence, the co-skewness portfolio needs to have a long position in the stocks with *low* co-skewness and a short position in the stocks with *high* co-skewness (e.g. Harvey&Siddique, 2000).

The opposite is true for the co-kurtosis measure which is a measure of the dependence between the asset's return and market skewness. The riskier assets have lower returns when the market is negatively skewed, which means that exposure to co-kurtosis risk comes from taking a long position in the assets with a *higher* co-kurtosis estimate.

Accordingly, the Co-skewness portfolio is formed by a long position in a value-weighted portfolio of the assets ranking below the 25th percentile, and a short position in a value-weighted portfolio of the assets above the 75th percentile according to the co-skewness measure. The Co-kurtosis portfolio is formed by a long position in a value-weighted portfolio of the assets ranking above the 75th percentile, and a short position in a value-weighted portfolio of the assets below the 25th percentile according to the co-kurtosis measure.

As in Ang et al (2006), our intention with calculating these two portfolios is to see whether they serve as a legitimate alternative to Omega-H and Vega portfolios, i.e. whether coskewness and cokurtosis describe the same risk sources as our downside risk measures.

Size, Value and Momentum

Our proposed factor model is constructed in a somewhat similar fashion as the Fama-French three-factor model. For consistency with prior literature (e.g. Ang et al, 2006), we assess the new multifactor model against the Fama-French framework (Fama&French, 1992) and Momentum (Jagadeesh&Titman, 1993). We replicate the SMB, HML and MOM portfolios based on the same universe of stocks that we use to form the other portfolios.

The SMB factor is formed as a long position in a value-weighted portfolio of all stocks that fall below the median pre-formation market capitalization (ME, the last available price times the last available number of shares), and a corresponding short position in a value-weighted portfolio of all the stocks above the median market capitalization[30].

[30] One important distinction from the method used by Fama and French (1993) is that the overall median market capitalization is used instead of the NYSE median market capitalization to split the stocks into two groups. There is no justification provided in the F&F paper for using the NYSE median or any other arbitrary threshold for that matter. Moreover, the authors point out that *most AMEX and NASDAQ stocks are smaller than the NYSE median* (page 8).

The HML factor is formed by taking a long position in a value-weighted portfolio of the stocks above the 70th percentile of the book-to-market equity (BE/ME) and a short position in a value-weighted portfolio of the stocks in the bottom 30th percentile of the book-to-market equity[31].

The Momentum factor is formed based on the average return on the assets during the period between -12 months and -1 month to avoid the effect of short-term reversals (e.g. Lo&MacKinlay, 1990, and Lehmann, 1990). A long position is taken in a value-weighted portfolio of the assets above the 75th percentile, with a corresponding short position in a value-weighted portfolio of the assets below the 25th percentile.

The four portfolios are then tested for correlation with the portfolios found in the Kenneth French data library, to determine the consistency of our results with the original findings in Fama and French (1993) and Jagadeesh and Titman (1993). After that, a decision is made about whether our portfolio or the portfolio in the Kenneth French library is more suitable to use for further tests.

Data

Daily historical data on the risk-free rate, stock prices, dividends, splits and shares outstanding is downloaded from the CRSP database. We use the period between January 1, 1950 and December 31, 2011.

Prices, dividends, splits and shares outstanding are downloaded on all available stocks ever traded on NASDAQ, NYSE or AMEX (consistent with Fama&French, 1993). Individual companies are identified by their CRSP permanent number (*permno*). Only data from primary exchange listings is considered. Further, we only keep the data on common equity and exclude *americus trust companies*, *closed-end funds* and *real estate investment trusts*[32].

It is a slightly different stock selection process compared to Fama and French (1993) who exclude real estate investment trusts but keep ordinary shares of closed-end funds. The reason that closed-end funds are excluded in this research is two-fold. Firstly, closed-end funds are nothing more than porftolios of other shares (possibly with an added skill of the portfolio management team and a

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Another important difference is that Fama and French (1993) further split the above-median and below-median groups by book-to-market equity and make six value-weighted portfolios. The resulting size portfolio is an equal-weighted portfolio of the three below-the-median portfolios less an equal-weighted portfolio of the three above-the-median portfolios. Our approach is simpler and more straightforward, as simple value-weighted averages are formed below and above the median market capitalization.

[31] Once again, we take simple value-weighted averages, while Fama and French (1993) use equal weights in the two top-BE/ME portfolios and equal weights in the two bottom-BE/ME portfolios taken from the six size- and book-to-market sorted portfolios.

[32] In the CRSP database this selection process corresponds to only keeping the shares with a share code (*shrcd*) of 10, 11 or 12 in exchanges with codes (*exchcd*) 1 (for NYSE), 2 (for AMEX) and 3 (for NASDAQ).

subtracted management fee), which means that including them in the analysis implies duplicating some of the stock observations. Secondly, many closed-end funds use investment strategies closely related to the Fama-French factors, i.e. tracking the market or specializing in small or value stocks, which means that including them may distort the results of our replication of the same strategies.

The data has two types of recurring errors: *a)* several observations under the same date, or *b)* an active company has no listed observation for a given date. If more than one observation is listed under the same date, only one of the observations is kept (it is impossible to determine which observation is the correct one, so the choice of which one to keep is arbitrary). If a company has a date missing in its time series, the prices and shares outstanding from the previous date are replicated.

Stock returns are calculated as follows:

$$R_t = \frac{P_t(1 + F_t) + D_t}{P_{t-1}} - 1, \qquad \text{where}$$

P is the quoted stock price (CRSP data item prc),

D_t is the amount of $cash$ dividends distributed at time t, including liquidating dividends ($divamt$),

F_t is the adjustment factor, adjusting for $stock$ dividends, splits, reverse splits and liquidations ($facpr$).

In simple terms, R_t gives a simple net return adjusted for dividends and splits. In case of reorganizations, mergers, bankruptcy or another event where the stock is delisted, F_t is set to -1 and D_t is the liquidating dividend amount, which means that R_t gives the redemption value that an investor gets when their position in the asset is terminated. In terms of calculating the basic return data, we make no further adjustments for transaction costs or other expenses associated with the investment decision.

For the risk-free rate, we use the yield on the one-month US treasury bill for consistency with monthly rebalancing and with Fama and French (1992 and 1993). Because the yields in the CRSP database are annualized and given in percent, the data is further adjusted to get the daily yield based on an average number of 253 trading days per year:

$$R_f = \left(1 + \frac{R_{\text{t-bill}}}{100}\right)^{\frac{1}{253}} - 1.$$

Book value data

Annual data on company fundamentals is downloaded from the CRSP/COMPUSTAT merged database on WRDS. Once again, all data on NASDAQ, NYSE and AMEX stocks between 1950 and 2011 is downloaded. The data is matched to the CRSP data on the basis of the CRSP permanent number link to the COMPUSTAT record (*lpermno*). Companies with missing links are deleted.

As in Fama and French (1993), the book value of equity is calculated as total shareholder equity (COMPUSTAT item *seq*), plus deferred taxes and investment tax credit (COMPUSTAT item *txditc*), less preferred stock taken at redemption value (*pstkrv*), liquidating value (*pstkl*) or carrying value (*upstk*), in this order, depending on data availability. In case of unavailable data, the observation is set at *zero*. The calculated book value of equity is assumed to become available to an investor at the COMPUSTAT data date (*datadate*).

Portfolio properties and some preliminary results

One year of data is used to calculate the pre-formation ranks of the stocks and is lost, and the first month with estimated daily factor observations is the January of 1951, giving a total of 61 years of estimated daily time series for each factor portfolio. The annualized statistical properties of the eight portfolios and the risk-free asset are outlined below. The returns are transformed into log-returns.

Table 3: Annualized statistical properties of the factor portfolios and the risk-free asset

Statistic	OMEGA	VEGA	Rm	SMB	HML	MOM	CS	CK	Rf
Mean	8.04%	3.12%	6.50%	5.23%	1.83%	9.50%	1.94%	-1.64%	4.32%
Std	13.67%	13.56%	15.13%	9.30%	28.93%	15.36%	11.00%	13.46%	0.17%
Downside std[33]	10.32%	9.54%	11.07%	6.83%	20.59%	11.41%	8.36%	9.35%	0.00%
Skewness	-2.11	0.18	-0.82	-0.91	-0.59	-1.06	-1.81	0.89	0.86
Kurtosis	43.24	19.25	24.25	22.68	60.51	19.05	35.98	24.98	4.08
Sharpe ratio	0.27	-0.09	0.14	0.10	-0.09	0.34	-0.22	-0.44	0.00
Sortino ratio[34]	0.25	-0.09	0.13	0.09	-0.09	0.31	-0.20	-0.46	NaN

It can be seen that the best-performing factor in terms of the average return (9.5%), the Sharpe ratio (0.34) and the Sortino ratio (0.31) is the Momentum factor. The second-best performing factor is the Omega factor. The worst-performing factor is the Co-kurtosis factor whose average return is negative at -1.64% annually over the observed period. An anomalous result is observed with the HML factor that performs below the risk-free rate. We come back to assessing our HML portfolio shortly.

Persistence in portfolio composition

To approximate the practical feasibility of our factors, we estimate the persistence of the stock properties based on which the factors are formed. The persistence of the stock property is an indicator of how often and how extensively the corresponding portfolio needs to be rebalanced to maintain its risk loading. If the stock property is fixed over time, the portfolio formed based on that property will maintain the same characteristics indefinitely and need not be rebalanced. At the same time, if a

[33] Downside deviation is calculated with the risk-free rate as the benchmark.

[34] Here, the Sortino ratio is calculated as the ratio of the expected return in excess of the risk-free rate, divided by the downside deviation with respect to the risk-free rate.

stock's property changes rapidly, the corresponding portfolio needs to be rebalanced often to maintain the same risk loading.

We estimate the first-order autocorrelations in the monthly estimates of the market weights, market values of equity, market-to-book ratios, past returns, as well as the Omega-H and Vega measures, co-skewness and co-kurtosis of the stocks. Equally weighted averages are taken across all of the stock-specific autocorrelation estimates. The average autocorrelations are first estimated over non-overlapping five-year periods and put on a timeline (to see the possible time-variations), and then overall averages are taken across all periods.

The results are presented below. In order to avoid data overload, we omit the Co-skewness and Co-kurtosis autocorrelations in the graphical representation. The overall averages for Co-skewness and Co-kurtosis are, however, presented in the table that follows.

Figure 5: The dynamics of the average persistence of firm characteristics

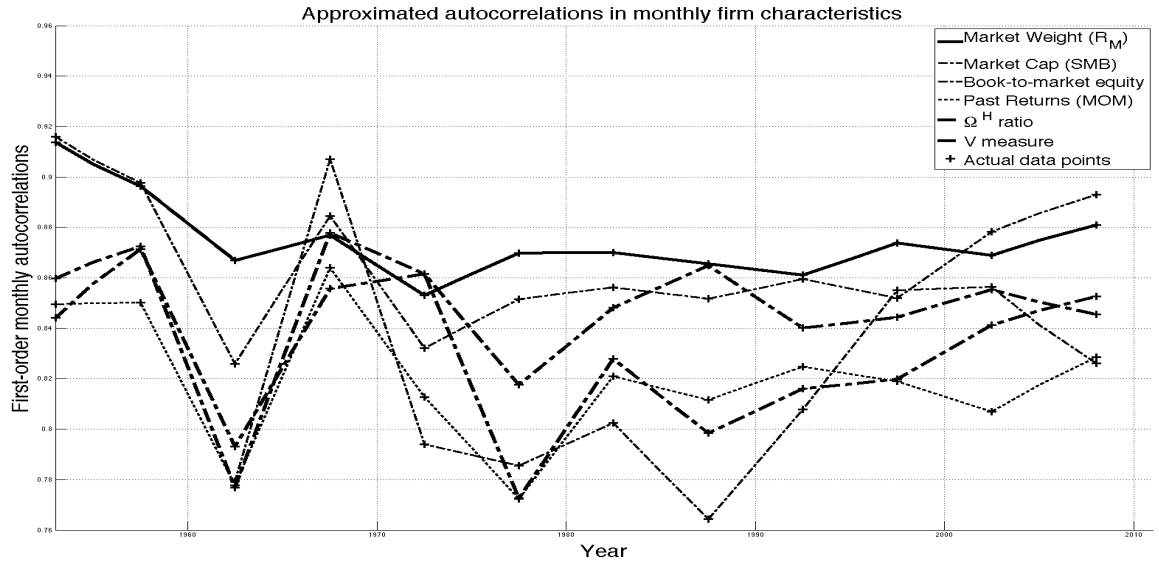


Table 4: Overall average autocorrelations in firm characteristics

Market Weight	Market Cap	Book-to-market	Past Returns	Omega-H	Vega	Co-skewness	Co-kurtosis
87.48%	86.66%	81.78%	81.99%	83.14%	84.51%	82.08%	81.86%

The autocorrelations in all properties are relatively high (above 80% on average) and persistent through time. The lowest autocorrelation estimate is obtained for the HML factor around the year 1990. The corresponding autocorrelation value is at 76.43%. The average persistence is the highest with the Market portfolio, with the overall average autocorrelation estimate of nearly 87.5%, followed by the SMB portfolio with an estimate of 86.7%. The lowest persistence is demonstrated by the HML portfolio (81.78%) and the Co-kurtosis portfolio (81.86%). Given these autocorrelation estimates, we can approximate that the half-life is 7 months for the Market loading, 6 months for the SMB and Vega

loadings, and 5 months for the other property loadings[35]. In general, we observe similar levels of persistence between the factors replicated from previous literature and the factors introduced in this paper, implying that all of the factors are comparable and the costs of rebalancing should fall in the same range for all eight portfolios.

Correlation with the Kenneth French database factors

As a feasibility test, the four portfolios constructed to replicate the established factors (the Market, SMB, HML and MOM) are checked for correlation with the corresponding factor portfolios provided on the Kenneth French website. After that, an assessment is made about whether this paper's replicas should be replaced in favour of the original factors found in the Kenneth French data library.

The estimated daily correlations are as follows:

Rm	SMB	HML	MOM
98.72%	77.77%	11.50%	84.12%

Three of the replicated factors (the Market, SMB and MOM) seem to correlate highly with the original factors from the Kenneth French database. The relatively lower correlation between the two SMB factors compared to the Market and Momentum factors is probably attributable to the slight differences in the portfolio formation rules. In general, the overall high correlations in the factors demonstrate that the risk loadings in the this paper's replicas are consistent with the risk loadings of the original portfolios from Fama and French (1993) and Jagadeesh and Titman (1993). Given that this paper's replicas of the Market, SMB and Momentum are constructed to be directly comparable with the other factor portfolios, we do not replace our Market, SMB and Momentum replicas with their Kenneth French database counterparts.

The exception is the HML factor whose correlation with the Kenneth French library's HML portfolio is only 11.5%. The likely reason is the difference in the book equity datasets used. In the book value dataset retrieved for this paper, the maximum number of companies with available non-missing data at a given point in time is 675, while the minimum number of companies for one given period period is 0. This number of companies is not enough to construct a book-to-market portfolio with consistently meaningful properties because it enforces that most of the stocks in the universe be eliminated for lack of data. This book value dataset is likely different from the book value dataset used by Fama and French (1993), although the same data source was used in both cases. A possible reason is that some type of data reconstitution took place in the database following the Fama and French research paper, but we would prefer not to speculate on this topic further. From now on, we substitute our HML factor replica for the HML portfolio obtained from the Kenneth French website.

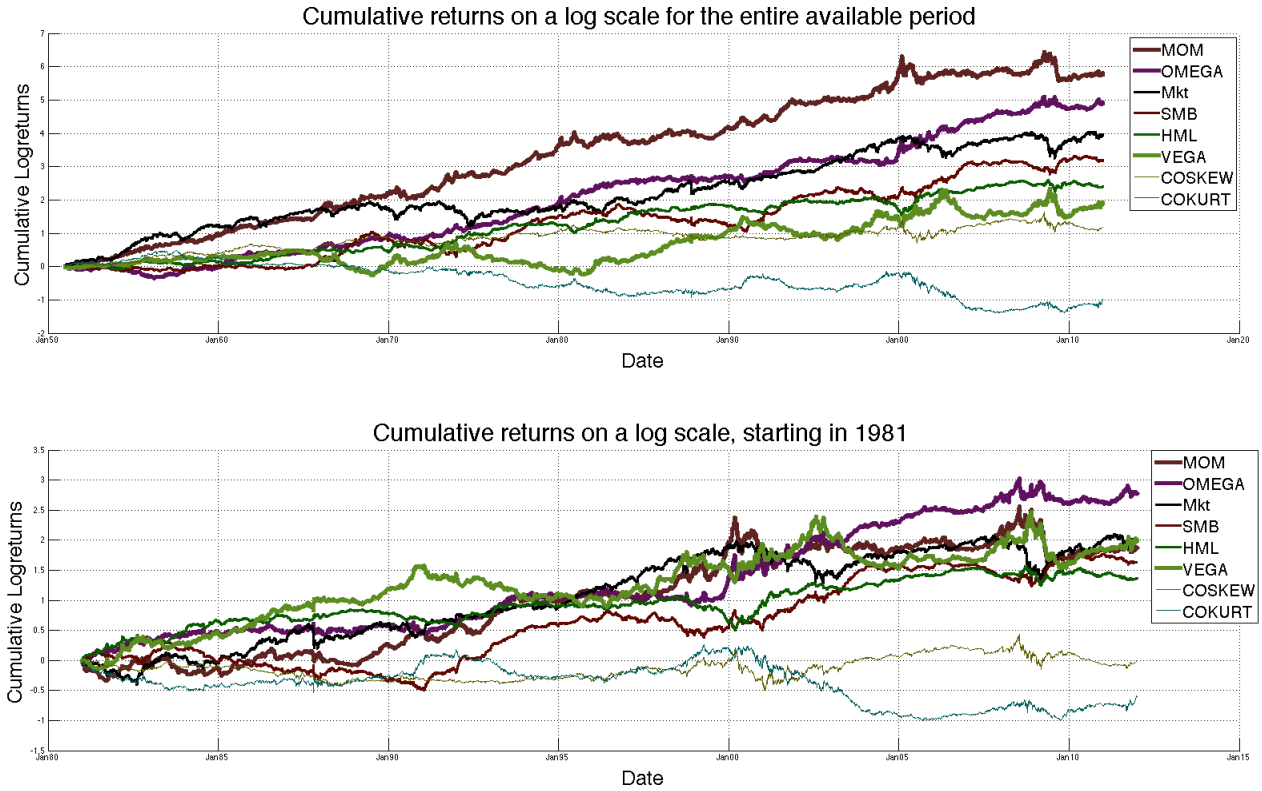
[35] In this setting, the half-life estimate can be interpreted as the approximate amount of time it takes an asset to lose half of its corresponding property loading.

The properties of the finalized factor set

The finalized set of factor portfolios includes all of the factors estimated in this paper, with the exception of the HML factor that is replaced with the HML portfolio from the Kenneth French website. An additional time series is the risk-free rate approximated with the one-month US treasury bill yield.

The figure below presents the cumulative logreturns on the eight factor portfolios over the entire period, as well as the subperiod between 1981 and 2011 to give a closer look at a recent data subsample. The legend presents the factors in the order from the highest to the lowest cumulative return in the top subplot.

Figure 6: Cumulative logreturns of the eight factor portfolios



Over the entire sample period, the Momentum factor appears to be the most profitable, followed by the Omega factor. The Co-kurtosis factor is trading at a discount compared to its original value. However, starting around the year 2000, the Omega portfolio sees a period of rapid growth. This can be seen in the subsample plot that shows that Omega outperforms Momentum over the past 30 years.

The Vega portfolio stalls for approximately thirty years and starts rapid growth in the 80's. By its design, the Vega is countercyclical during bearish markets and cyclical during bullish markets. This can be seen clearly when considering the subsample plot. The market downturn in 1982, the market crash of 1987, the downturn in the beginning of the 1990's, the dotcom crisis and the subprime mortgage crisis all see the Vega portfolio spike upwards and against the market, while the overall return on the Vega portfolio throughout the subperiod is nearly identical to the market return.

Next, we examine the raw correlation estimates of the factor portfolio returns. The corresponding correlation matrix is as follows:

Table 5: Factor portfolio correlation matrix

	OMEGA	VEGA	Rm	SMB	HML	MOM	CS	CK	Rf
OMEGA	100.00%								
VEGA	-2.48%	100.00%							
Rm	-48.72%	-15.44%	100.00%						
SMB	39.25%	-41.96%	-39.05%	100.00%					
HML	6.62%	0.49%	-24.96%	28.02%	100.00%				
MOM	43.13%	18.64%	6.26%	-10.66%	-27.29%	100.00%			
CS	31.42%	-28.24%	5.98%	7.29%	3.37%	38.00%	100.00%		
CK	-67.24%	15.79%	71.07%	-57.51%	-30.25%	1.10%	-13.04%	100.00%	
Rf	0.96%	1.84%	-2.01%	-1.70%	1.36%	0.67%	-0.79%	-0.04%	100.00%

In the table above, it is worth noting that only the Co-kurtosis factor exhibits a statistically significant cyclicality (given a standard significance threshold of 20% for asset correlations), with a market correlation of 71.07%. Momentum and Co-skewness appear to be slightly cyclical, although the market correlation estimates are insignificant.

It is surprising that the correlation between the Omega and Vega factors is negative (and insignificant) at -2.48%, although the two factors are designed to capture risk loadings from similar risk spaces. At the same time, Momentum appears to be strongly correlated with the Omega, Vega and Co-skewness factors, which indicates that the Momentum factor has positive downside risk and market phase loadings. We proceed with a set of time series tests of the factor portfolio models, going over to the cross-sectional (beta-lambda) asset pricing tests and concluding with a short portfolio exercise to examine the implications of our downside risk and higher moment factors for portfolio allocation in an international setting.

Time series regression tests[36]

Under this section, various factor combinations are tested against each other in a set of time series OLS regressions. The entire observation period between 1951 and 2011 is used to derive the time series regression coefficients. This test procedure does not account for time-varying risk premia[37] but it should give the aggregate picture of the risk loadings of the different factor portfolios and the relevance of these risk loadings in the risk-return framework.

The current set of tests can be interpreted as comparative time series analysis of several CAPM extensions based on downside risk factors. The primary purpose of this methodology is to test how well each of the eight individual factor portfolios can be replicated using other factors. This is done by testing the null hypothesis of an insignificant *alpha* (regression intercept) against the alternative hypothesis of a significant and positive alpha.

If factor portfolio Y can be replicated by a combination of other factor portfolios X_i, a simple OLS regression of Y on X_i will produce an insignificant alpha coefficient, and we can thus conclude that the systematic risk premium on Y is identical to the risk premium captured by the set of X_i. If the alpha coefficient is positive and significant, the conventional interpretation is that portfolio Y produces a premium on a risk source that is not captured by the asset pricing model given by a combination of X_i. In such a case, it can be concluded that the X_i pricing model is incomplete and Y is a potential extension that covers an additional risk loading.[38]

In the asset pricing tests to follow, we consider the traditional CAPM, FF3F (Market, SMB, HML) and FF3F-MOM (Market, SMB, HML, Momentum) models, as well as the two downside risk-based factor models that we call MOV (Market, Omega, Vega) and MCC (Market, Co-skewness, Co-kurtosis). Each of these factor models is tested as a replicating strategy for all other factor portfolios (the portfolios not included in the model being considered) by running a set of time series regressions. After that, each of the factor portfolios is regressed on a combination of all other factor portfolios.

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[36] For a detailed description of the time series and cross-sectional procedures, please refer to Cochrane, John H. 2005, Asset Pricing, Revised Edition (Princeton University Press), pp 229-251.

[37] The purpose of the time series tests is to determine whether different factor portfolios load on the same underlying risk sources. Hence, even in the world with varying risk premia, the time series tests will not provide a good intuition about the dynamics of the price of risk, because if two factors load on the same risk source, their risk premia will covary perfectly in time, while if the risk soucres are different a comparison between the risk premia is meaningless. The time dimension in the risk premia is considered in more detail in the cross-sectional (*beta-lambda risk representation*) tests.

[38] Please note that this type of asset pricing tests only works with zero-cost portfolio or excess returns. In our case, all portfolios are constructed as zero-cost portfolios.

The comprehensive set of outputs for all of the time series regression tests is presented in *Appendix 3*. We consider the test results for the factor models on a case-by-case basis below, starting with the CAPM regression outputs. The last set of regressions (each factor against all other factors) is not analyzed here and is left at the discretion of the reader to look at.

The CAPM

Table 6: Time series regression tests against the CAPM[39]

$$R_t^{ei} = \alpha_i + \beta_i^M R_t^{eM} + \varepsilon_{i,t}$$

Dependent	Alpha	Rm	R2
OMEGA	**12.35%**	**-0.44**	24.15%
t-stat	8.15	-70.04	
VEGA	**5.10%**	**-0.14**	2.41%
t-stat	2.97	-19.52	
SMB	**7.51%**	**-0.24**	15.36%
t-stat	6.86	-52.89	
HML	**5.12%**	**-0.12**	6.29%
t-stat	5.68	-32.17	
MOM	**10.20%**	**0.06**	0.38%
t-stat	5.21	7.76	
COSKEW	2.21%	**0.04**	0.37%
t-stat	1.58	7.60	
COKURT	**-5.59%**	**0.64**	50.38%
t-stat	-4.58	125.08	

As can be seen from the table above, the only portfolio that is priced by the CAPM throghout the observation period is the Co-skewness portfolio, with an annualized alpha of 2.21% insignificant at the 5% level (a t-statistic of 1.58). The Co-kurtosis factor produces a statistically significant discount of 5.6% compared to the market portfolio. This result may indicate that an asset's co-kurtosis with the market is not a consistently priced risk factor throughout the observation period.

Both Co-skewness and Co-kurtosis are cyclical (have positive and significant market betas), and so is the Momentum factor. However, the Momentum factor also produces a positive and highly significant alpha of 10.2%. This result is consistent with prior research.

The remaining four factor portfolios are countercyclical (have negative market betas). The lowest beta is estimated for the Omega factor (-0.44, highly significant). The Omega portfolio also generates the highest alpha (12.35%, highly significant) with respect to the CAPM. The SMB portfolio produces a significant alpha of 7.51% and has the second-lowest negative factor loading with the CAPM (-0.24, highly significant). The Vega portfolio and the HML portfolio produce similar results against the CAPM,

[39] In this and the following tables the statistically significant coefficients are marked in bold. Where appropriate, the coefficients are annualized from the daily data assuming 253 trading days in a year.

Vega being marginally more countercyclical than HML with a market beta of -0.14 against -0.12, respectively. However, the HML portfolio produces a marginally higher alpha of 5.12% annually, against 5.10% annually with Vega. The next set of regression tests is run for the Fama-French thee-factor model.

The FF3F model

Table 7: Time series regression tests against the FF3F model

$$R_t^{ei} = \alpha_i + \beta_i^M R_t^{eM} + \beta_i^{SMB} R_t^{SMB} + \beta_i^{HML} R_t^{HML} + \varepsilon_{i,t}$$

Dependent	Alpha	Rm	SMB	HML	R2
MOM	**13.59%**	**-0.02**	**-0.07**	**-0.56**	7.59%
t-stat	7.19	-2.43	-4.78	-32.66	
OMEGA	**10.57%**	**-0.38**	**0.38**	**-0.21**	29.91%
t-stat	7.24	-56.24	34.54	-15.52	
VEGA	**10.78%**	**-0.33**	**-0.86**	**0.15**	30.30%
t-stat	7.41	-49.32	-78.36	10.98	
COSKEW	1.02%	**0.08**	**0.13**	**0.05**	1.53%
t-stat	0.73	12.60	12.00	3.79	
COKURT	-1.29%	**0.50**	**-0.49**	**-0.12**	61.15%
t-stat	-1.19	101.55	-60.24	-12.47	

The Co-skewness and Co-kurtosis factors produce statistically insignificant alphas. The loadings on SMB and HML are negative for the Co-kurtosis factor but positive for the Co-skewness factor. This generally implies that stocks with higher co-skewness risk would tend to be small value stocks, while the stocks with higher co-kurtosis risk would tend to be large growth stocks.

The Momentum factor has negative loadings on the SMB and HML factors. The risk loading on the market is now also reversed in sign compared to the CAPM regression and is now significant at -0.02. The alpha for Momentum is now 3.39 percentage points higher. These results are consistent with prior literature on the Momentum factor.

The alphas of the Omega and Vega factors are still positie and highly significant at 10.57% and 10.78%, respectively. The Omega factor is negatively loaded on value risk (a highly significant coefficient of -0.21) and positively loaded on size risk (a highly significant coefficient of 0.38). The Omega portfolio's loading on the SMB factor contributes to the reduction in the portfolio's alpha by 1.78 percentage points compared to the CAPM regression. The Omega portfolio remains countercyclical.

With the Vega factor the loadings on the SMB and HML factors have opposite signs compared to Omega (significant at -0.86 and 0.15, respectively). Vega's significant negative loading on SMB results in a dramatic increase in the factor's alpha from 5.20% to 10.78%. Like the Omega portfolio, Vega remains countercyclical when controlling for size and value risk.

We can conclude that Momentum, Omega and Vega are not priced by the FF3F model, while Momentum and Vega even see their alphas increase compared to the CAPM test due to significant negative factor loadings. The following section considers the extension of the FF3F model that includes Momentum (as in Carhart, e.g. 1997).

FF3F and Momentum

Table 8: Time series regression tests against the FF3F model and Momentum

$$R_t^{ei} = \alpha_i + \beta_i^M R_t^{eM} + \beta_i^{SMB} R_t^{SMB} + \beta_i^{HML} R_t^{HML} + \beta_i^{MOM} R_t^{MOM} + \varepsilon_{i,t}$$

Dependent	Alpha	Rm	SMB	HML	MOM	R2
OMEGA	**4.66%**	**-0.37**	**0.41**	**0.04**	**0.43**	52.12%
t-stat	3.86	-66.38	45.02	3.36	84.54	
VEGA	**8.60%**	**-0.33**	**-0.85**	**0.24**	**0.16**	33.33%
t-stat	6.03	-49.90	-79.04	17.58	26.45	
COSKEW	**-3.08%**	**0.09**	**0.15**	**0.22**	**0.30**	18.02%
t-stat	-2.42	14.90	15.28	18.18	55.65	
COKURT	-0.28%	**0.50**	**-0.49**	**-0.16**	**-0.07**	61.79%
t-stat	-0.26	102.07	-61.32	-16.27	-16.13	

The addition of Momentum to the FF3F model reduces the alpha of the Co-skewness factor to a negative and significant value of -3.08%. The Co-skewness factor is now positively loaded on all four model parameters. In the case of the Momentum coefficient, this means that stocks with low co-skewness also exhibit high contemporaneous returns. In a reverse argument, this indicates that past-winner stocks perform poorly when market volatility increases. The Co-kurtosis factor is negatively loaded on all factor portfolios except the market. Its alpha is negative and insignificant in both statistical and economic terms (-0.28% annually).

The alphas of the Omega and Vega portfolios are reduced due to a positive loading on Momentum. The estimates are now at 4.66% and 8.60%, respectively. However, both of these alphas are still economically and statistically significant. Controlling for Momentum also gives Omega a slight positive loading on the HML factor (0.04) which is statistically significant at the conventional levels. Both Omega and Vega remain countercyclical when Momentum risk is controlled for.

In general, over half of the Omega risk seems to be explained by SMB, HML and Momentum, as the alpha of the Omega portfolio is reduced from 12.35% in the CAPM regression to 4.66% when the three control factors are added. However, the remaining alpha of the Omega portfolio is still a considerable value at nearly 4.7% annually. The Vega portfolio still sees an improvement in its alpha compared to the CAPM model, although approximately one-fifth of the Vega risk seems to be accounted for by the Momentum loading (the alpha of the Vega portfolio drops from 10.78% in the FF3F regression to 8.60% in the current test). We next consider whether the MOV (Market, Omega, Vega) model succeeds in explaining the underlying risk loadings of any of the other factors in our set.

The MOV model

Table 9: Time series regression tests against the MOV model

$$R_t^{ei} = \alpha_i + \beta_i^M R_t^{eM} + \beta_i^{OMEGA} R_t^{OMEGA} + \beta_i^{VEGA} R_t^{VEGA} + \varepsilon_{i,t}$$

Dependent	Alpha	Rm	OMEGA	VEGA	R2
SMB	**7.48%**	**-0.23**	**0.14**	**-0.32**	42.03%
t-stat	8.23	-51.24	27.98	-75.52	
HML	**5.77%**	**-0.14**	**-0.04**	**-0.02**	6.87%
t-stat	6.41	-32.63	-8.90	-5.42	
MOM	*-0.29%*	**0.43**	**0.72**	**0.30**	35.18%
t-stat	***-0.18***	55.30	85.65	40.56	
COSKEW	-0.94%	**0.17**	**0.33**	**-0.19**	20.90%
t-stat	-0.75	27.18	50.02	-32.49	
COKURT	**-1.90%**	**0.49**	**-0.40**	**0.23**	69.62%
t-stat	-1.98	106.07	-77.30	51.87	

As in the previous tests, the Co-skewness and Co-kurtosis factors are generally priced by the current factor model (the Co-kurtosis factor has a discount that is marginally significant at -1.90%). The Co-skewness factor is loaded positively on the Omega factor, implying that stocks with low market co-skewness also tend to be Omega-H optimal. Conversely, this result is an indication that the Omega portfolio may perform poorly in periods of high market volatility. The loading of the Co-skewness factor on Vega is negative, which is consistent with the definition of the Vega measure, as stocks that perform well in volatile bearish markets obtain higher Vegas. At the same time, the Co-kurtosis portfolio is loaded negatively on the Omega factor and positively on the Vega factor.

An important result is that the Momentum risk is completely captured by the MOV model. The alpha of the Momentum portfolio is now negative at -0.29% and is statistically and economically insignificant. Also, when controlled for Omega and Vega risk, the Momentum factor proves to be strongly cyclical with a highly significant market beta of 0.43 (the CAPM beta of Momentum is at 0.06, while the FF3F-implied market beta of Momentum is negative). A large portion of the Momentum risk is attributable to Omega, with a corresponding beta of 0.72. The Vega loading of Momentum is also strong, with a highly significant beta coefficient of 0.30. The result implies that the higher returns on the Momentum factor compared to the Market, Omega and Vega are simply the result of Momentum's high loadings on Market, Omega and Vega risks.

The SMB and HML portfolios are not priced by the MOV model. The alpha of the SMB factor is significant at 7.48%, while the alpha of the HML factor is significant at 5.77%. The HML factor seems to be negatively loaded on all three portfolios in the MOV model, with small but significant negative response coefficients on Omega and Vega (-0.04 and 0.02, respectively) and a significant market coefficient of -0.14. The SMB factor is positively loaded on the Omega portfolio but negatively loaded

on the other two factors[40]. We next consider the MCC (Market, Co-skewness, Co-kurtosis) model as a possible alternative to the MOV model to see whether the two models are interchangeable (i.e., whether downside risk and higher-moment risk capture the same premium).

The MCC model

Table 10: Time series regression tests against the MCC model

$$R_t^{ei} = \alpha_i + \beta_i^M R_t^{eM} + \beta_i^{COSKEW} R_t^{COSKEW} + \beta_i^{COKURT} R_t^{COKURT} + \varepsilon_{i,t}$$

Dependent	Alpha	Rm	COSKEW	COKURT	R2
SMB	**5.22%**	**0.02**	-0.01	**-0.41**	33.06%
t-stat	5.36	3.59	-1.18	-62.15	
HML	**4.38%**	**-0.04**	0.00	**-0.13**	9.34%
t-stat	4.93	-6.87	0.86	-21.88	
MOM	**9.45%**	-0.01	**0.54**	**0.08**	14.58%
t-stat	5.21	-1.03	50.24	6.44	
OMEGA	**8.49%**	**-0.09**	**0.30**	**-0.57**	50.86%
t-stat	6.96	-12.25	41.21	-69.05	
VEGA	**8.31%**	**-0.43**	**-0.24**	**0.48**	20.52%
t-stat	5.35	-46.11	-25.88	45.30	

Based on the table above, the MCC model seems to replicate none of the other factors. All dependent variables have significant positive alphas, the lowest being 4.38% for the HML portfolio and the highest 9.45% for the Momentum portfolio. The SMB and HML factors have insignificant loadings on the Co-Skewness factor (-0.01 and 0.00, respectively), while the Momentum factor has an insignificant loading on the market portfolio with a response coefficient of -0.01.

When controlled for co-skewness and co-kurtosis, the SMB factor becomes marginally cyclical with a significant market response coefficient of 0.02 (the CAPM response coefficient on SMB is -0.24). The Omega and Vega factors are still countercyclical with market response coefficients of -0.09 and -0.43, respectively. The loadings of Omega and Vega on the Co-skewness and Co-kurtosis factors are consistent with the reverse loadings in the MOV tests.

[40] It should be noted that some of the robustness tests run on these regressions have yielded insignificant alphas for the SMB and HML factors with the MOV model. In particular, the MOV model tends to perform much better at pricing SMB and HML when longer history periods are used to estimate the pre-formation Omega-H and Vega measures. The choice of the history period does not otherwise dramatically affect the performance of the Omega and Vega portfolios. However, these results are omitted here because in our robustness tests the choice of the history period was not based solely on historical data but rather the performance of the factors throughout the observable timeline. The entire timeline would not be available to an investor at the pre-formation period, which means that the choice of an empirically optimal history length would not be meaningful from a practical perspective and hence these test results are omitted in this paper.

The results are consistent with Ang et al's (2006) claim that the higher moments fail to capture downside risk because they are not explicitly conditioned on market cycles. In this respect, our findings are consistent with prior literature. It is explicitly shown that while co-skewness and co-kurtosis risk is captured by the downside risk factors, the reverse is not true, indicating that explicit downside risk is a more appropriate risk source for factor pricing than are higher co-moments. As the next step, we apply our risk factors to test cross-sections of stock returns at a number of time periods.

Beta-Lambda representation tests and a time variation in cross-sectional risk premia[41,42]

This section is devoted to the cross-sectional tests of the risk premia on individual stocks. The general procedure has two major steps. First, the risk loading of each stock $i = 1, \dots, N$ on the different factors $k = 1, \dots, K$ is estimated by running a set of time series regressions of excess stock returns R_t^{ei} on (excess) factor returns R_t^k over the entire sample period spanning $t = 1, \dots, T$:

31: Time series regression for the beta-lambda representation test

$$R_t^{ei} = \alpha_i + \boldsymbol{R}_t^f \boldsymbol{\beta}_i^f + \varepsilon_{i,t}, \qquad i = 1, \dots, N$$

$$\boldsymbol{R}_t^f = (R_t^1, R_t^2, \dots, R_t^K), \qquad \boldsymbol{\beta}_i^f = \left(\beta_i^1, \beta_i^2, \dots, \beta_i^K\right)^T.$$

After that, the beta coefficients $\boldsymbol{\beta}_i$ are collected for each stock to form the matrix:

$$\mathrm{B}_f = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \dots, \boldsymbol{\beta}_N)^T.$$

This matrix is then used as a set of explanatory variables in the following cross-sectional regression:

32: Cross-sectional regression for the beta-lambda representation test

$$\mathrm{E}[\boldsymbol{R}^e] = \mathrm{B}_f \boldsymbol{\lambda} + \boldsymbol{\upsilon},$$

$$\boldsymbol{\lambda} = (\lambda^1, \lambda^2, \dots, \lambda^K)^T, \qquad \mathrm{E}(\boldsymbol{R}^e) = (\mathrm{E}[R^{e1}], \mathrm{E}[R^{e2}], \dots, \mathrm{E}[R^{eN}])^T, \qquad \boldsymbol{\upsilon} = (\upsilon_1, \upsilon_2, \dots, \upsilon_N)^T.$$

In the cross-sectional regression, the coefficient λ^k is sometimes referred to as the *price* of the risk source *k*. Mathematically speaking, it is the expected return on the risk loading *k* implied by the cross-section. With a perfect model, we would have $\lambda^k = \mathrm{E}[R^k]$. Similarly, υ_i represents the cross-sectional

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[41] For a detailed description of the time series and cross-sectional procedures, please refer to Cochrane, John H. 2005, Asset Pricing, Revised Edition (Princeton University Press), pp 229-251.

[42] In the following section, such notations as β_i^2 mean β_i upper-indexed 2, not β_i squared. This applies to all notations at the top-right corner of a symbol. Also, when expectations (and covariances) are taken of matrices, it is implicitly meant across the vertical dimension. The expectation of a $T \times N$ matrix is thus a $1 \times N$ row vector and its covariance is an $N \times N$ matrix.

regression-implied pricing error of the model with respect to asset i, and in a perfect factor model we would have $\upsilon_i = \alpha_i = 0 \; \forall \; i$.[43]

To see whether and to which extent each factor model given by $\boldsymbol{R}_t^f$ is powerful, we test whether the coefficients $\boldsymbol{\lambda}$ are statistically significant and whether the cross-sectional pricing errors $\boldsymbol{\upsilon}$ are jointly *zero*. To account for the fact that the cross-sectional regression uses prior regression estimates, rather than the raw data, as inputs, the sample variance-covariance matrices are defined as follows:

33: The variance-covariance matrices for cross-sectional errors and response coefficients

$$\mathrm{COV}(\widehat{\boldsymbol{\upsilon}}) = \frac{1}{T}\left\{I_N - \mathrm{B}_f\left(\mathrm{B}_f^T\mathrm{B}_f\right)^{-1}\mathrm{B}_f^T\right\}\mathrm{COV}(\boldsymbol{\varepsilon})\left\{I_N - \mathrm{B}_f\left(\mathrm{B}_f^T\mathrm{B}_f\right)^{-1}\mathrm{B}_f^T\right\}\{1 + \boldsymbol{\lambda}^T\mathrm{COV}(\boldsymbol{R}^f)\boldsymbol{\lambda}\},$$

$$\boldsymbol{\varepsilon}_i = \left(\varepsilon_{i,1}, \varepsilon_{i,2}, \dots, \varepsilon_{i,T}\right)^T, \qquad \boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \dots, \boldsymbol{\varepsilon}_N).$$

$$\mathrm{COV}(\boldsymbol{\lambda}) = \frac{1}{T}\left\{\left(\mathrm{B}_f^T\mathrm{B}_f\right)^{-1}\mathrm{B}_f^T\mathrm{COV}(\boldsymbol{\varepsilon})\mathrm{B}_f\left(\mathrm{B}_f^T\mathrm{B}_f\right)^{-1}[1 + \boldsymbol{\lambda}^T\mathrm{COV}(\boldsymbol{R}^f)\boldsymbol{\lambda}] + \mathrm{COV}(\boldsymbol{R}^f)\right\}.$$

The corresponding test statistics are:

34: Coefficient test statistics in the cross-sectional regression[44]

$$\frac{\lambda^k}{\mathrm{COV}(\boldsymbol{\lambda})_{k,k}} \sim t_{N-K}, \qquad \widehat{\boldsymbol{\upsilon}}^T\mathrm{COV}(\widehat{\boldsymbol{\upsilon}})^{-1}\widehat{\boldsymbol{\upsilon}} \sim \chi^2_{N-K}.$$

The paper by Ang et al. (2006) uses an alternative method called the Fama-MacBeth procedure (Fama&MacBeth, 1973). This procedure is slightly simpler and requires running T different cross-sectional regressions in te second step for each time observation of the stock and factor returns. The estimates and standard errors of $\boldsymbol{\lambda}$ and $\boldsymbol{\upsilon}$ are then derived as the sample mean and sample standard

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[43] The intuition behind the relationship between the time series results and the cross-sectional results can be seen by taking expectations from the time series regression:

$$\mathrm{E}[R^{ei}] = \alpha_i + \mathrm{E}[\boldsymbol{R}^f]\boldsymbol{\beta}_i^f, \qquad i = 1, \dots, N, \qquad \boldsymbol{R}^f = \begin{pmatrix} \boldsymbol{R}_1^f \\ \boldsymbol{R}_2^f \\ \vdots \\ \boldsymbol{R}_T^f \end{pmatrix}$$

[44] The lower index k,k in $\mathrm{COV}(\hat{\boldsymbol{\lambda}})_{k,k}$ denotes the k'th element on the diagonal of $\mathrm{COV}(\hat{\boldsymbol{\lambda}})$. Also, the inverse of the matrix $\mathrm{COV}(\boldsymbol{\upsilon})$ is calculated using singular value decomposition, and hence $\mathrm{COV}(\boldsymbol{\upsilon})^{-1}$ denotes a pseudo-inverse, rather than an inverse, of $\mathrm{COV}(\boldsymbol{\upsilon})$. This methodology follows from the necessity of simplifying the numeric procedures required to produce the inverse of a large matrix. Although this method may make the final results somewhat unreliable, this is the only alternative we have available.

deviation (or sample covariance matrix for $\boldsymbol{v}$) across the T estimates from T different cross-sectional regression outputs.

While relatively more straightforward computationally, we believe that the Fama-MacBeth procedure is less powerful because it uses an extra estimation step. Also, a regression R-squared coefficient has less meaning meaning in the Fama-MacBeth procedure. Hence, our subsequent analysis is based on the cross-sectional method outlined above. We do, however, also run the Fama-MacBeth procedure on the same observation samples and present the outputs in the appendix together with the main cross-sectional tests.

As the next step, we collect the adjusted R-squared coefficients from cross-sectional regression tests with annual subsample. The adjusted R-squared coefficient estimates how well the returns on a cross-section of stocks can be explained by the stocks' loadings on the factor portfolios. A time series of adjusted R-squared coefficients for a given model is thus an estimate of the relevance of the corresponding risk loadings in the market through time.

The results are presented in the next sections. We start by examining a selection of the regression output summaries over several five-year periods picked based on a graphical representation of the pricing errors. After that, we take a closer look at the dynamics of the adjusted R-squared coefficients of the models in time and conditionally on market performance.

Cross-sectional tests with varying risk premia

In this section, the returns on the stocks and factors are split into twelve periods. We produce the test results for eleven five-year periods between 1951 and 2005 and one six-year period between 2006 and 2011. The returns on the stocks are Winsorized at the 1st and 99th percentiles[45]. We only use the firms for which all observations are available throughout the period to avoid the effect of bankruptcies, mergers and other extraordinary events on the regression coefficients. The number of firms thus ranges between 355 for the period 1956-1960 and 3952 for the period 1996-2000. We compare the performance of the following factor models:

- CAPM (Market return)
- FF3F (Market, SMB, HML)
- FF3F+MOM (FF3F, Momentum)
- MOV (Market, Omega, Vega)
- MCC (Market, Co-skewness, Co-kurtosis)
- FF3F+OV (FF3F, Omega, Vega)

[45] Winsorization is a procedure that consists in setting the outliers that break a top or a bottom percentile equal to the corresponding percentile value. It is used in, among others, Ang et al. (2002), Knez and Ready (1997) and Basu (1997).

- FF3F+CC (FF3F, Co-skewness, Co-kurtosis)
- FF3F+OV+CC (FF3F, Omega, Vega, Co-skewness, Co-kurtosis)
- FF3F+MOM+OV (FF3F, Momentum, Omega, Vega)
- FF3F+MOM+CC (FF3F, Momentum, Co-skewness, Co-kurtosis)
- All variables (FF3F, Momentum, Omega, Vega, Co-skewness, Co-kurtosis)

This is essentially the list of all plausible factor combinations, comprising eleven different hybrid models. We concentrate on some outputs more than the others, while the rest are considered as robustness tests. In particular, it is has been shown in the *Time Series Regression Tests* that Momentum, Co-skewness and Co-kurtosis are priced by the MOV model. Hence, for example, the extension of the FF3F+OV model that includes Momentum could be considered futile. However, we still include the FF3F+MOM+OV combination to be thorough.

We present the regression outputs in a series of tables available in *Appendix 4a*, with the outputs of the Fama-MacBeth regressions for the corresponding periods available in *Appendix 4b*[46]. The Fama-MacBeth regression coefficients are (quite expectedly) identical to the cross-sectional test coefficients, but the statistical significance of the Fama-MacBeth coefficients is much lower. We attribute the statistical insignificance of the Fama-MacBeth results to time-variation in risk premia, which makes the standard error estimation method in the Fama-MacBeth procedure inefficient when dealing with high-frequency data over non-overlapping time periods. While the Fama-MacBeth outputs provided in *Appendix 4b* are based on daily data, a robustness check that uses monthly returns (as in Fama and French, 1992) produces similar results.

Additionally to the regression output tables, we produce a series of plots of the realized stock returns against predicted stock returns for a selection of models (*Appendix 4c*). The diagonal black line in each graph is the identity line. The vertical distance between the data points and the line represents the pricing errors for the individual stocks. The closer the realized return observations are to the identity line, the smaller the pricing errors and the better the performance of the model. The test of all of the pricing errors being zero (formula 33, right) is essentially the test of all of the vertical distances from

[46] Our method differs from the method proposed in Ang et al. (2002) who consider overlapping 12-month periods of stock returns with monthly roll-overs. Ang et al. Use Fama-MacBeth regressions with Newey-West (1987) standard errors to produce a single output, while we use non-overlapping five-year periods and produce several outputs. In this way, we hope to capture some of the dynamics in the time-varying risk premia found, among others, by Fenou et al (2011).

Our method also differs from Fama and French (1992) who use one sample of monthly return observations covering the entire period. We use daily data and shorter time periods, which we believe will provide more precise results by considering a more comprehensive dataset, even if some of the outputs lose in significance due to the added information base.

the points to the line being zero. In a world where a model perfectly describes the data in the cross-section, all of the data points lie on the identity line.

Graphical results

Considering all of the regression outputs in detail would be a long and, to our opinion, vain endeavour. We further select three selection outputs to concentrate on based on the cross-sectional error plots in *Appendix 4c*. It is clear that the CAPM performs poorly during all sub-periods, while failing completely in some such as 1966-1970 (Figure 15), 1971-1975 (Figure 16), 1981-1985 (Figure 18), 1986-1990 (Figure 19) and, to an extent, 2006-2011 (Figure 23). Further, we can identify the periods when none of the models is a clearly visible improvement on the CAPM. These would be the periods 1976-1980 (Figure 17), 1991-1995 (Figure 20), 1996-2000 (Figure 21), and 2001-2005 (Figure 22).

Already at this stage there appears to be pattern, as the periods that can be characterized by a complete failure of the CAPM do not overlap with the periods that can be characterized by the failure of the multifactor models to clearly outperform CAPM. In other words, there is graphical evidence that when the CAPM tends to fail the multifactor models tend to perform much better than the CAPM. At the same time, when the CAPM performs decently (though still poorly) the multifactor models tend to only be a slight improvement, if any.

If we compare the different multifactor models, it seems that the MOV model tends to perform either as well as or better than the FF3F model in the cross-sectional tests, with the exception of the period 1986-1990 (Figure 19). The exceptional result for this period may be explained by the emergence of the small hi-tech firms in the mid-1980's that went on to outperform the large corporations by a considerable margin. This peculiarity of the period would lead to a boost in the explanatory power of the SMB factor, and hence of the FF3F model.

The MOV model does not seem to consistently outperform the FF3F+MOM model. However, the FF3F+OV extension tends to perform as well as or better than the FF3F+MOM model, with the exception of the period 1961-1965 (Figure 14). The result is generally consistent with the two observations made previously, namely, that the Momentum factor is priced by the MOV model and that the downside risk factors have seen an increase in their risk premia in the more recent history. In a related note, the model that incorporates all factors including Co-skewness and Co-kurtosis generally performs better than the other models.

An examination of the selected regression outputs

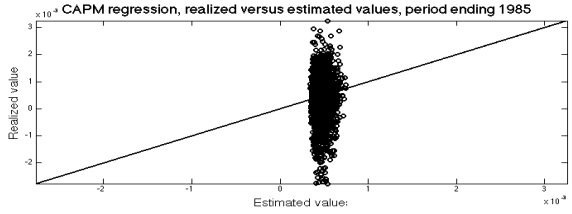
We now select three regression outputs for thorough examination. We do not consider all test results in detail in order to avoid excessive analytical jargon, but the reader is welcome to interpret the outputs for those time periods that they are personally interested in.

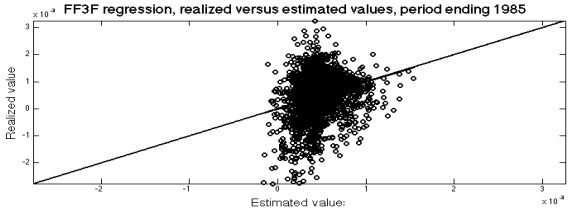
In this section, we analyze the periods 1981-1985 (Figure 18) as an example of a situation when the market fails. Another option would be the period 1971-1975 (Figure 16) but we consider the graphical

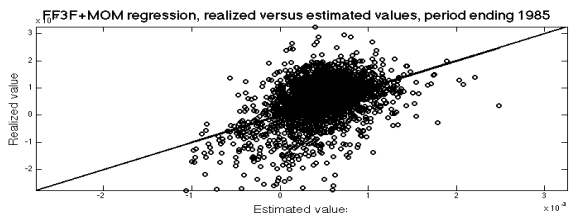
result for the CAPM to be somewhat anomalous because the pricing errors produce a nearly perfect vertical line. Further, we consider the period 1991-1995 (Figure 20) as an example of a period when the CAPM does not fail and the multifactor models are not a visible improvement. And lastly, we consider the period 2006-2011 as the most recent, and thus the most relevant, time period examined in this paper. We start with the period 1981-1985 and the results are as follows:

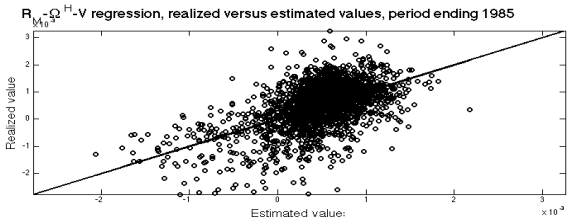
Table 11: Cross-sectional test results for the period 1981-1985

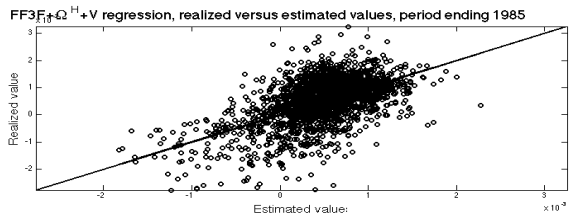
1981-1985 (2410 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	4.47%								1.81%
Chi2/t-stat	1885.30	0.70								
FF3F	-	9.99%	**-8.83%**	**10.08%**						7.88%
Chi2/t-stat	1825.76	1.61	-2.25	2.62						
FF3F+MOM	-	11.85%	**-10.98%**	**10.75%**	**72.84%**					23.05%
Chi2/t-stat	1651.74	1.88	-2.79	2.75	6.16					
MOV	-	8.78%				**17.66%**	**63.55%**			31.62%
Chi2/t-stat	1662.96	1.41				2.62	6.89			
MCC	-	4.81%						**-27.35%**	-2.23%	19.19%
Chi2/t-stat	1663.68	0.79						-6.02	-0.39	
FF3F+OV	-	9.48%	**-11.11%**	6.46%		**26.56%**	**50.79%**			34.11%
Chi2/t-stat	1624.40	1.52	-2.83	1.70		3.94	5.77			
FF3F+CC	-	5.64%	**-10.57%**	4.02%				**-14.36%**	**-12.48%**	29.34%
Chi2/t-stat	1544.22	0.92	-2.70	1.06				-3.08	-2.24	
FF3F+OV+CC	-	7.06%	**-11.08%**	4.51%		**22.74%**	**41.83%**	**-10.38%**	-4.88%	36.76%
Chi2/t-stat	1582.33	1.15	-2.84	1.20		3.45	5.09	-2.31	-0.87	
FF3F+MOM+OV	-	8.68%	**-10.83%**	5.25%	**31.80%**	**25.65%**	**49.76%**			34.57%
Chi2/t-stat	1622.56	1.40	-2.75	1.40	3.74	3.84	5.67			
FF3f+MOM+CC	-	7.50%	**-11.49%**	5.57%	**39.00%**			**-10.45%**	-8.47%	33.78%
Chi2/t-stat	1565.46	1.22	-2.94	1.48	4.36			-2.34	-1.51	
All Variables	-	7.00%	**-11.05%**	4.29%	**29.08%**	**22.63%**	**42.18%**	**-10.54%**	-4.63%	36.74%
Chi2/t-stat	1583.66	1.14	-2.83	1.15	3.51	3.44	5.16	-2.37	-0.83	

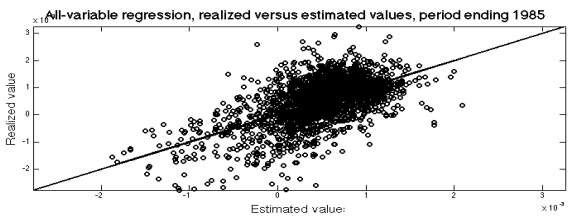








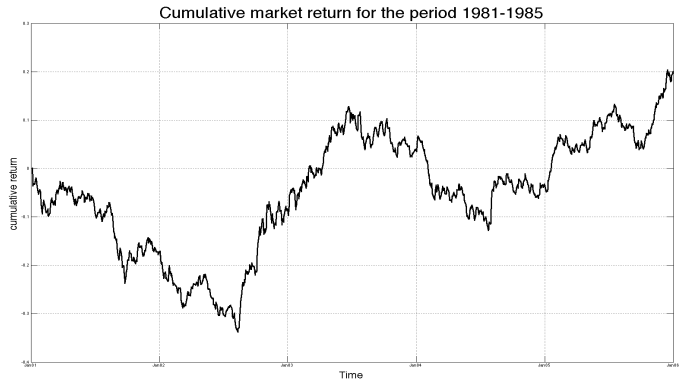




Quite unexpectedly, all of the Chi-square suggest that all alphas are jointly *zero* (the lowest 5% threshold across all the models is 2517 at 2402 degrees of freedom for the all-variable model). This result is inconsistent with prior literature (e.g. Ang et al, 2006, Find that at least some of their models produce significant intercept tests) and possibly due to the fact that the pseudo-inverse of the covariance matrix is used to derive the Chi-square statistics. In a related note, the other regression tests (including the Fama-MacBeth tests) also produce similar results for the intercepts. We do not give the intercept tests any further mention because of their seeming unreliability.

The regression output above is characterized by the failure of the CAPM. All of the market portfolio coefficients are statistically insignificant (the significant coefficients are marked in bold). The corresponding market dynamic for the period is as follows:

Figure 7: Market dynamic for the period 1981-1985



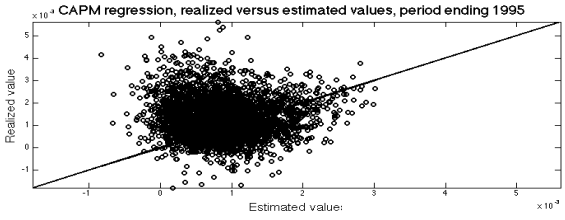
The period is characterized by a market downturn between 1981 and late 1982, corresponding to an over-30% loss in market capitalization, with a subsequent recovery resulting in an overall positive dynamic. During the period, the premium on the SMB factor is negative and signficant in all cases, also lower when the downside risk factors and/or Momentum are added to the model. The premium on the HML factor is positive and significant in the FF3F and FF3F+MOM models, but the significance is driven away when the higher-moment and/or the downside risk factors are added.

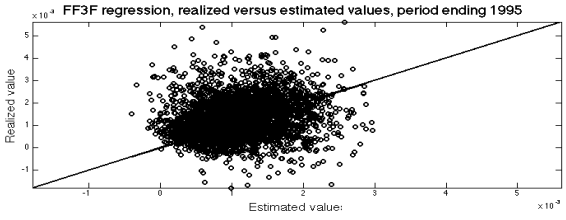
The premium on the Co-skewness factor is negative and significant in all cases, while the premium on the Co-kurtosis factor is negative and insignificant except in the FF3F+CC model. At the same time, the premia on the Momentum, Omega and Vega factors are all positive and significant, with the estimates ranging between 18% and 73% annually. This result is consistent with the intended counter-cyclicality of the Omega and Vega factors. Combining the Omega and Vega factors with Momentum in the FF3F+MOM+OV model reduces the premium on Momentum from 72.84% to 32.80%, which is a substantial decrease. However, the Momentum premium is still statistically and economically significant. It is also worth noting that the explanatory power of the models that include Omega and Vega is on average much higher than that of the other models. For example, the MOV model produces an adjusted R-squared of 31.62% compared to an R-squared of 7.88% for the FF3F model and 23.05% for the FF3F+MOM model.

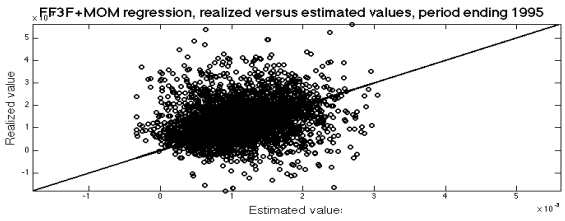
In general, we can conclude that the period between 1981-1985 is characterized by a premium on large companies over small companies which could be explained by a possible liquidity crisis that is normally associated with substantial market downturns. Additionally, we observe a flight to downside risk-efficient stocks and the stocks that performed well in the recent past. The premium on the co-skewness risk is negative and significant, implying that the demand for co-skewness risk premium has dropped substantially. We next consider the period 1991-1995.

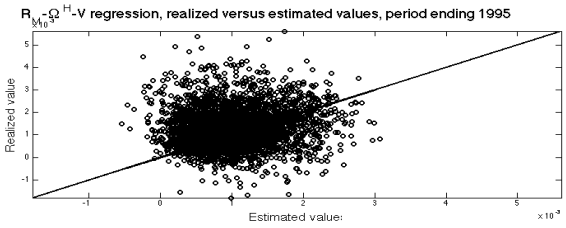
Table 12: Cross-sectional test results for the period 1991-1995

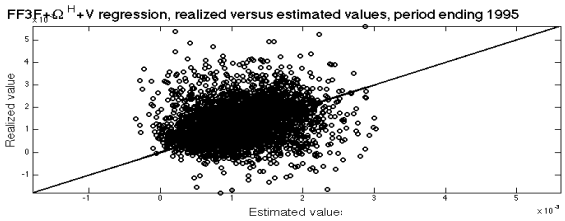
1991-1995 (3701 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	**32.24%**								59.23%
Chi2/t-stat	1062.17	5.63								
FF3F	-	**13.19%**	**21.42%**	-0.55%						42.75%
Chi2/t-stat	956.52	2.68	4.86	-0.19						
FF3F+MOM	-	**12.52%**	**21.67%**	0.60%	**22.80%**					45.45%
Chi2/t-stat	944.43	2.56	4.92	0.20	2.99					
MOV	-	**16.39%**				**14.14%**	**-26.11%**			47.99%
Chi2/t-stat	991.45	3.25				2.86	-5.03			
MCC	-	**22.77%**						**15.99%**	-10.70%	51.15%
Chi2/t-stat	1012.02	4.28						3.95	-1.83	
FF3F+OV	-	**12.95%**	**21.69%**	-0.56%		**10.99%**	**-15.92%**			43.43%
Chi2/t-stat	931.74	2.64	4.92	-0.19		2.25	-2.91			
FF3F+CC	-	**12.91%**	**21.53%**	-0.26%				**9.81%**	-5.65%	44.09%
Chi2/t-stat	952.92	2.63	4.91	-0.09				2.55	-0.94	
FF3F+OV+CC	-	**13.06%**	**21.42%**	-0.81%		9.09%	**-13.96%**	**10.77%**	-6.48%	44.48%
Chi2/t-stat	917.62	2.67	4.88	-0.27		1.90	-2.55	2.78	-1.08	
FF3F+MOM+OV	-	**12.48%**	**21.98%**	0.48%	**24.28%**	**11.22%**	**-15.87%**			46.23%
Chi2/t-stat	925.73	2.55	4.99	0.16	3.16	2.29	-2.90			
FF3f+MOM+CC	-	**12.51%**	**21.65%**	0.50%	**21.88%**			**8.54%**	-5.98%	45.59%
Chi2/t-stat	943.92	2.56	4.94	0.17	2.89			2.28	-1.00	
All Variables	-	**12.85%**	**21.49%**	-0.20%	**23.86%**	9.05%	**-13.75%**	**9.71%**	-6.76%	45.78%
Chi2/t-stat	915.41	2.63	4.90	-0.07	3.12	1.89	-2.51	2.57	-1.13	

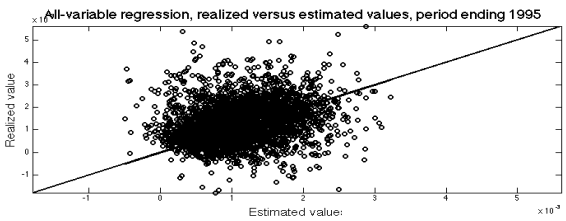












The current period is characterized by a relatively good performance of the CAPM. All of the market risk premia are positive and statistically significant, ranging between 32.24% in the CAPM model and 12.48% in the FF3F+MOM+OV model. The CAPM model also produces the highest adjusted R-squared among the models, estimated at 59.23%. The corresponding market dynamic is presented below:

Figure 8: Market dynamic for the period 1991-1995



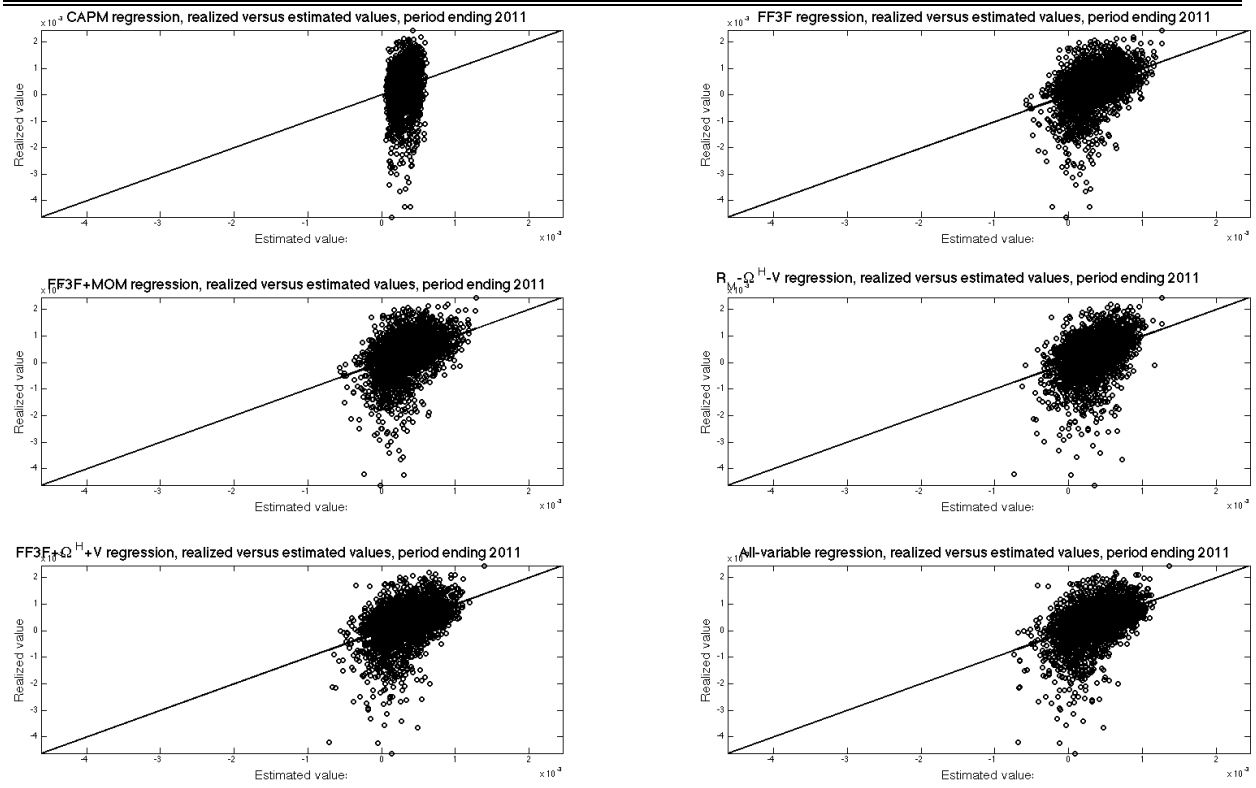
As can be seen from the plot above, the period 1991-1995 is a period of stable market growth, resulting in a nearly 100% increase in market capitalization over the five-year period. The SMB premium is now positive and statistically significant in all cases, suggesting that small firms tend to outperform large firms during the observed period. The estimated coefficients on HML are all statistically insignificant, implying no premium on value stocks over growth stocks.

The premium on co-skewness risk is now positive and significant in all cases. It also drives away the significance of the otherwise positive and significant risk premium on the Omega factor. At the same time, the premium on Vega is negative and significant in all cases. The premium on co-kurtosis risk is insignificant in all models. The Momentum factor earns a significant and positive risk premium. However, the premium is now much lower than in the previous regression (24.28% annually or lower compared to 29.08% or higher in the period 1981-1985), which is consistent with the evidence on the positive loading of the Momentum on the downside risk factors.

To summarize, the current period of stable market growth is characterized by the outperformance of the CAPM compared to all of the multifactor models. However, we still observe significant evidence that in this period small stocks tend to earn a premium on large stocks, while Omega risk and co-skewness risk tend to load on a similar underlying positively priced risk source. The demand on the downside covariance-efficient stocks seems to be down, which is logically consistent with a period of stable market growth. We now consider the period 2006-2011.

Table 13: Cross-sectional test results for the period 2006-2011

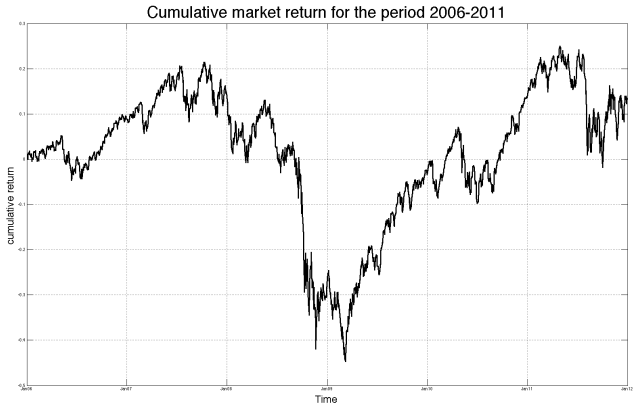
2006-2011 (3039 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	7.53%								2.68%
Chi2/t-stat	1514.91	0.67								
FF3F	-	14.05%	-8.54%	-9.89%						14.05%
Chi2/t-stat	1492.21	1.24	-1.69	-1.91						
FF3F+MOM	-	13.94%	-8.49%	**-9.45%**	12.90%					14.07%
Chi2/t-stat	1485.89	1.24	-1.71	-1.97	0.71					
MOV	-	12.72%				**25.27%**	**42.78%**			13.37%
Chi2/t-stat	1446.57	1.10				2.00	2.55			
MCC	-	8.50%						14.50%	6.63%	5.24%
Chi2/t-stat	1510.47	0.75						1.20	0.60	
FF3F+OV	-	16.45%	**-11.63%**	-8.40%		14.20%	**30.06%**			16.90%
Chi2/t-stat	1454.91	1.45	-2.51	-1.75		1.21	1.99			
FF3F+CC	-	14.56%	**-9.14%**	**-10.69%**				5.07%	13.10%	15.59%
Chi2/t-stat	1487.80	1.29	-1.98	-2.23				0.46	1.17	
FF3F+OV+CC	-	16.34%	**-11.36%**	-8.41%		12.24%	**29.56%**	3.49%	11.93%	17.52%
Chi2/t-stat	1454.30	1.44	-2.50	-1.75		1.08	2.02	0.34	1.08	
FF3F+MOM+OV	-	16.35%	**-11.71%**	-8.24%	**32.01%**	13.13%	**30.23%**			17.02%
Chi2/t-stat	1455.24	1.44	-2.53	-1.71	2.10	1.15	2.00			
FF3f+MOM+CC	-	14.81%	**-10.35%**	-8.74%	29.49%			1.85%	17.46%	16.75%
Chi2/t-stat	1474.65	1.31	-2.26	-1.81	1.94			0.17	1.53	
All Variables	-	16.39%	**-11.32%**	-8.47%	**30.47%**	12.53%	**29.53%**	3.53%	11.70%	17.49%
Chi2/t-stat	1454.05	1.45	-2.50	-1.76	2.04	1.11	2.02	0.35	1.07	



Once again, the period is associated with a seeming failure of the CAPM. All market risk premia are statistically insignificant. The MOV model underperforms marginally compared to FF3F, with the adjusted R-squared coefficient of 13.37% against 14.05%. The FF3F+OV model outperforms the FF3F+MOM model. The corresponding market dynamic presented below is characterized by the

subprime mortgage crisis of 2007-2009 and the European financial crisis of 2011. However, the aggregate market return for the period is positive at around 12%.

Figure 9: Market dynamic for the period 2006-2011



The premium on the SMB factor is negative and significant once controlled for downside risk. This result resembles the result for period 1981-1985 when the SMB risk premium is negative and drops even lower when introducing the downside risk factors. The premium on the HML factor is negative but only significant in the FF3F+MOM and FF3F+CC models. The premia on co-skewness and co-kurtosis risk are negative and insignificant in all models. The premium on Omega is positive but only significant in the MOV model. At the same time, the premium on Vega is positive and significant in all cases.

To summarize the section, the MOV and FF3F+OV models tend to perform well compared to the FF3F and FF3F+MOM models, respectively. The MCC model and the co-skewness and co-kurtosis extensions perform poorly and rarely produce consistent results, which is congruent with our conclusions from the time series tests. The Momentum factor tends to earn a positive premium that is significant in the majority of the cases, while the performance of the SMB, Omega and Vega factors seems to depend largely on the underlying market conditions. In the three sub-periods considered above, the premium on the HML factor is either negative or insignificant. This is probably an issue of the selection of the time periods and partly a consequence of extending the FF3F model. When the other risk factors are accounted for, the significance of the HML risk premium is often driven away.

An important conclusion is that the cross-sectional estimates of the risk premia seem to vary significantly depending on the market phase. Reversals in the risk premia are observed in a large number of cases. It is fair to conclude that an aggregate estimate of the risk premia taken over such varying market conditions would have quite little practical value. At the same time, it may pay off to examine the conditions that drive the time variations in the explanatory powers of the models over the cross-sections of stock returns. One such test is performed in the section that follows.

A closer look at the conditional performance of the factor models

The previous section has demonstrated a significant time-variation in the performance of the factor models in a cross-section. In order to examine the underlying forces of this variation, we split the

returns on stocks and factor portfolios into one-year periods. Each year a cross-sectional test procedure is run and the (adjusted) R-squared coefficients recorded, comprising a total of 61 observations for each model's R-squared estimate[47].

The recorded explanatory powers are tested as a time series. We investigate the absolute explanatory power estimates, as well as their performance relative to the explanatory power of the CAPM model. The explanatory power of the models is examined through time[48], as well as conditionally on the market return for the corresponding year[49]. First, a graphical representation is analyzed, and then two OLS regressions are estimated, one against a time trend and the other against the deviation of the market return from its overall average.

The dynamics of the explanatory power in time can tell us whether and to which extent some of the risk loadings become more or less important to investors. Alternatively, if the explanatory power of a factor becomes lower with time, it might mean that the factor is capturing a market anomaly that is being corrected in the market.

The purpose of conditioning on the market return is to see how well the different models price stocks across market cycles. It is generally easier to price assets in calm and bullish markets than in bearish markets, so we would expect the absolute explanatory powers of the models to be lower in bearish markets than in the other market phases. At the same time, we expect the downside risk factor models (MOV, MCC and the +OV and +CC extensions) to outperform the CAPM in bearish markets because in bearish markets downside risk should become a more relevant risk source for an investor.

The figure below presents the explanatory powers of the MOV and FF3F+OV models, as well as the explanatory powers of the already established CAPM, FF3F and FF3F+MOM models. The first subplot is a timeline, while the second subplot is a plot against market return.

There is no apparent trend in the explanatory power through time, although there appear to be spikes and cycles in the series. At the same time, as predicted, the plot against market return shows an upward trend. It also seems that the the MOV, FF3F+OV and FF3F+MOM models see an increase in explanatory power in bear markets, but not the CAPM or the FF3F model. The result is predicted for

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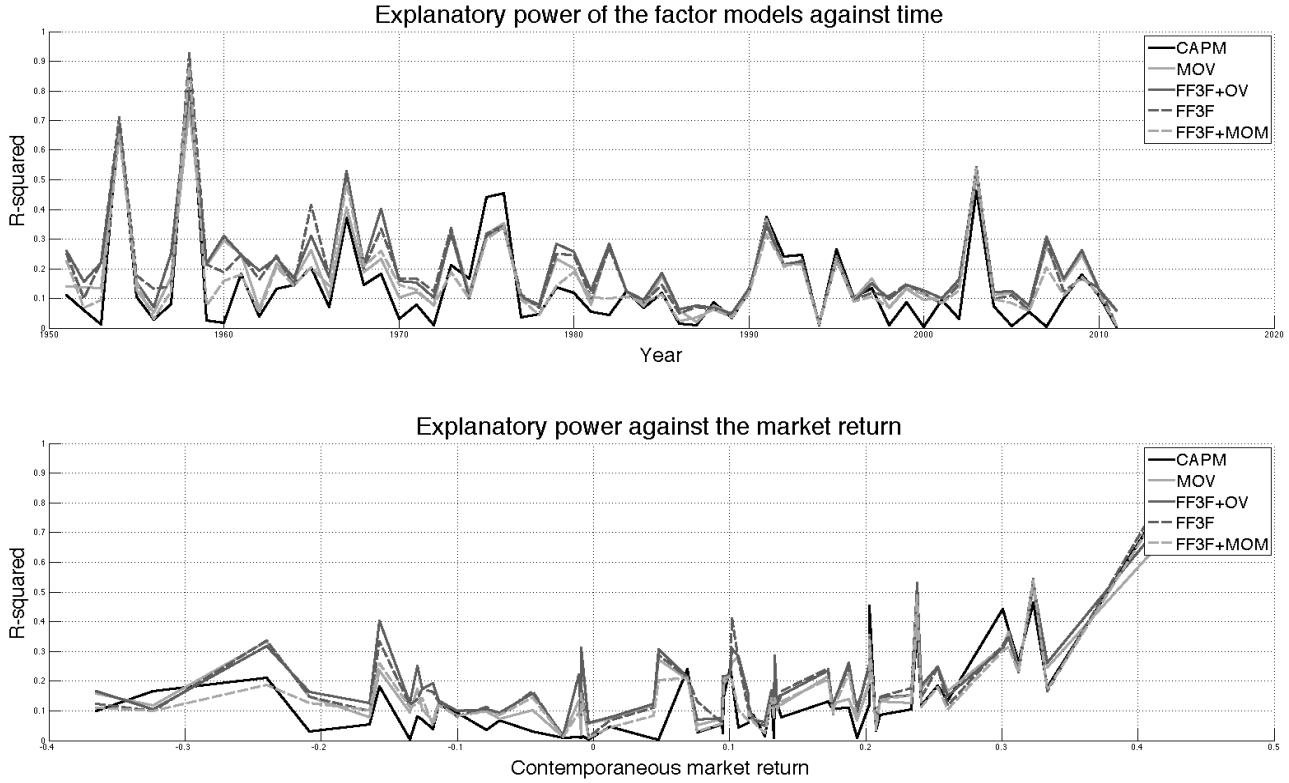
[47] To our best knowledge, the R-squared coefficient is the most straightforward estimate of a model's explanatory power in an OLS-type regression. We use the adjusted R-squared coefficient to penalize for the inclusion of extra explanatory variables. From now on, we use the term *explanatory power* to refer to the cross-sectional R-squared estimates.

[48] A similar test of the R-squared coefficients is performed in Lev (1989) who regresses a time series of R-squared coefficients on time to discover that the explanatory power of accounting earnings with respect to stock returns has a negative trend over twenty years of annual data.

[49] We also perform the tests against market volatility and market skewness, but the results are insignificant and are omitted here.

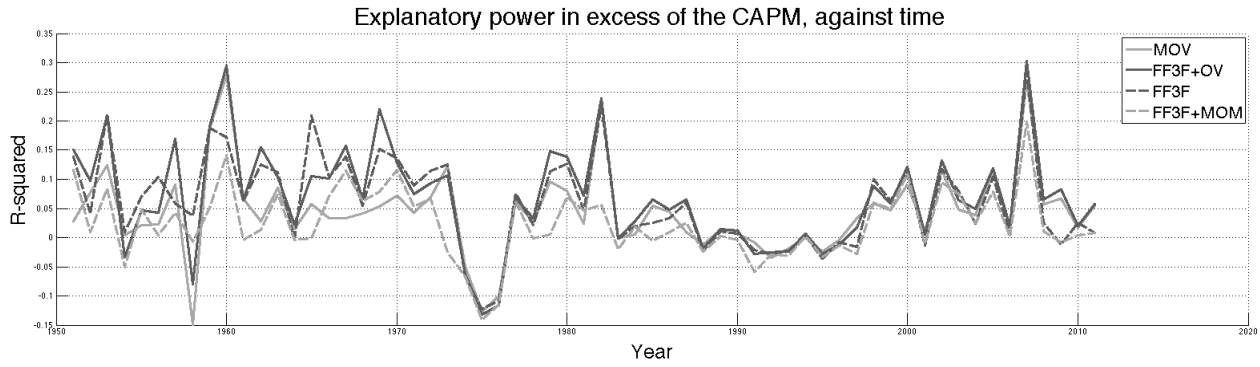
the MOV and FF3F+OV models. The probable explanation of the spike in the FF3F+MOM model in bear markets is the significant positive risk loading of the Momentum factor on Omega and Vega (please recall that the Momentum factor is priced by the MOV model). This result is consistent with the prediction that the downside risk-based models should perform relatively well in bear markets.

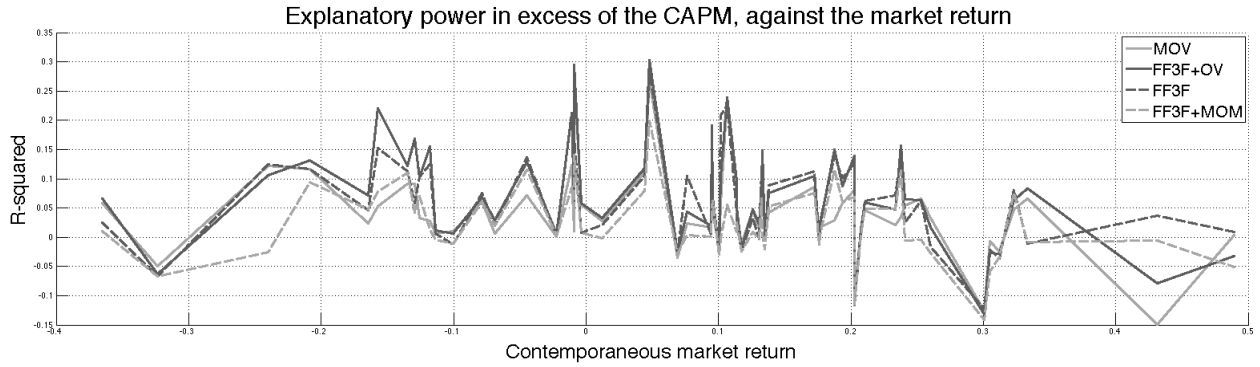
Figure 10: Absolute explanatory power of the models



The following figure presents the explanatory powers of the MOV, FF3F+OV, FF3F and FF3F+MOM models in excess of the CAPM (defined as a simple difference between the model's explanatory power and the explanatory power of the CAPM). There is no apparent time trend in the data. However, the models generally seem to underperform compared the CAPM in extreme bull markets. The exception is the FF3F model, which points towards the possibility that the downside risk-based factor models are inferior during the periods of rapid market expansion. At the same time, the FF3F model seems to perform worse than the other models when the market returns are low, indicating that the bear markets benefit from downside risk pricing. Market returns around the expected market return of 8.23% are associated with the four models generally outperforming the CAPM in terms of explanatory power.

Figure 11: Explanatory power of the models in excess of the CAPM





Next we examine the OLS regressions of the models' explanatory powers on a time trend and on the expected return. The comprehensive regression summary is available in Appendix 3a. Here, we consider part of the output in detail. The table below presents the regressions of the explanatory powers on time and on the contemporaneous market return in excess of its expected value.

Table 14: Absolute explanatory powers against time and the market return

$$Expl.Power_{i,t} = \alpha_i^t + \beta_i^t(t-1) + \varepsilon_{i,t}, \qquad t = 1, \dots, 61$$

$$Expl.Power_{i,t} = \alpha_i^M + \beta_i^M[R_t^M - \mathrm{E}(R^M)] + u_{i,t}, \qquad t = 1, \dots, 61$$

		Regression against time			Regression against the market return		
Absolute Explanatory Power	**Average**	**alpha**	**beta**	**R-squared**	**alpha**	**beta**	**R-squared**
CAPM	**13.95%**	**19.50%**	-0.0019	3.86%	**13.95%**	**0.52**	29.38%
t-stat	6.51	4.70	-1.55		7.75	5.00	
MOV	**18.41%**	**24.11%**	-0.0019	5.65%	**18.41%**	**0.41**	25.43%
t-stat	10.14	6.92	-1.90		11.74	4.52	
FF3F+OV	**20.81%**	**29.11%**	**-0.0028**	11.25%	**20.81%**	**0.37**	19.30%
t-stat	11.11	8.35	-2.76		12.37	3.79	
FF3F	**16.36%**	**22.59%**	-0.0021	5.91%	**16.36%**	**0.44**	26.08%
t-stat	8.43	6.08	-1.94		9.81	4.60	
FF3F+MOM	**19.99%**	**29.40%**	**-0.0031**	12.14%	**19.99%**	**0.43**	22.35%
t-stat	9.77	7.76	-2.88		11.09	4.16	

We first consider the regression against the market return. Please note that the intercept of the regression is the same as the sample average estimate. This result comes from the design of the regression. In a related note, the explanatory powers of all of the models are significant from zero.

We see that the MOV model seems to outperform the FF3F model on average, while the FF3F+OV model on average outperforms the FF3F+MOM model. The CAPM seems to have the lowest average explanatory power of the five alternatives. All of the response coefficients in the regression against the market. All of the response coefficients are positive and statistically significant, which supports our theory that all factor models tend to perform better in up-markets than in down-markets.

The alpha coefficients of the regression against time are designed to represent the estimated explanatory powers of the models in the first observation year, i.e. 1951. The response coefficients with time are negative for all models and marginally insignificant for the MOV and FF3F models but significant for the FF3F+OV and FF3F+MOM models.

The response coefficients in the regression against the trend can be interpreted as the average percentage-point change in the given model's explanatory power per year. The explanatory power of the FF3F+MOM model is thus estimated to decline by an average of 31 basis points every year over the observed time interval, while the explanatory power of the FF3F+OV model is estimated to declne by 28 basis points. This result can be explained by a change in the composition of the data (some companies list themselves on the market, while other companies delist). Another possible explanation is that there is a behavioural aspect to some of the factor risk premia (it has been argued by some researchers to be the case for Momentum) that is diminishing with time.

The regressions of other models on the time trend and the expected market return (results in *Appendix 4d*) are consistent with the results discussed here. All of the response coefficients with the time trend are negative and significant, with the exception of the MCC model with a t-statistic of -1.52. All of the intercepts in the regression against the market return are positive and significant, and so are all of the response coefficients with the market return series.

Next we consider the performance of the models relative to the performance of the CAPM. The table below summarizes the results.

Table 15: Absolute explanatory powers against time and the market return

$$Expl.Power_{i,t} - Expl.Power_{\mathrm{CAPM},t} = \alpha_i^t + \beta_i^t(t-1) + \varepsilon_{i,t}, \qquad t = 1, \dots, 61$$

$$Expl.Power_{i,t} - Expl.Power_{\mathrm{CAPM},t} = \alpha_i^M + \beta_i^M[R_t^M - \mathrm{E}(R^M)] + u_{i,t}, \qquad t = 1, \dots, 61$$

		Regression against time			Regression against the market return		
Difference with the CAPM	**Average difference**	**alpha**	**beta**	**R-squared**	**alpha**	**beta**	**R-squared**
FF3F	**2.41%**	**3.09%**	-0.0002	0.46%	**2.41%**	-0.08	5.07%
t-stat	3.19	2.07	-0.52		3.27	-1.79	
MOV	**4.46%**	**4.60%**	0.0000	0.01%	**4.46%**	**-0.11**	6.58%
t-stat	4.66	2.43	-0.09		4.82	-2.06	
FF3F+OV	**6.86%**	**9.60%**	-0.0009	3.41%	**6.86%**	**-0.15**	9.05%
t-stat	6.10	4.40	-1.46		6.40	-2.44	
FF3F+MOM	**6.04%**	**9.89%**	**-0.0013**	8.09%	**6.04%**	-0.09	3.59%
t-stat	5.88	5.09	-2.30		5.99	-1.49	

Based on this test, all of the models significantly outperform the CAPM (the exception is the MCC model whose output is not presented here but is available in *Appendix 4d*). In the regressions against time, we see that the difference between the four multifactor models and the CAPM does not seem to diminish significantly with time, with the exception of the FF3F+MOM model that appears to lose 13 basis points per year compared to the CAPM. None of the other models including the ones only presented in the appendix have significant negative response coefficients in the time regression. This is a further indication that there is a behavioural aspect to Momentum that is gradually corrected in the market. At the same time, we can not make the same conclusion for the two downside risk factor models whose response coefficients with time are insignificant.

The regressions against the market return are consistent with our prediction. The explanatory power of the downside risk models seems to increase during the periods when the market returns are low and decrease during the periods when the market returns are high. The same is not true for the two factor models that do not include the downside risk factors. The results for the other models that include the downside risk factors and the higher moment factors (presented in *Appendix 4d*) are consistent with this finding.

To summarize, it appears to be the case that all factor pricing models perform better in up-markets than in down-markets. All of the models show some evidence of a decline in explanatory power with time, which can be explained in several ways including changes data composition, the appearance of new underlying risk sources, or diminishing behavioural biases in the market.

As expected, the performance of the downside risk and higher moment models relative to the CAPM is better when the markets are down and worse when the markets are up. On average, however, all models with the exception of MCC outperform the CAPM. There is no apparent time-trend in this outperformance, with the exception of the models that include Momentum (particularly FF3F+MOM) whose explanatory power shows a statistically significant decline with time. We attribute this result to the diminishing behavioural bias in the stock market that is associated with picking past winners and selling past losers. This conclusion would be consistent with what Kahnemann and Tversky (2000) call the *misconceptions of regression*.

Implications of the asymmetric risk factors for international portfolio diversification

It has been established in prior literature that incorporating co-skewness, co-kurtosis and market phases into portfolio optimization explains a significant portion of the home bias effect in international portfolio allocation (Guidolin&Timmermann, 2008). In this section we perform a short study of the effects of our downside risk and higher moment factors on international portfolio diversification decisions. A portfolio optimization problem is approached from the viewpoint of an investor holding a portfolio of funds replicating major equity indices. We further test how the optimal weights in this investor's portfolio change once the Omega, Vega, Co-skewness and Co-kurtosis portfolios become an investment option (we assume that an ETF or another investment vehicle that replicates the four strategies is introduced in the market).

We consider the implications for *three* portfolios with a 130-30 fund constraint (a maximum 130% long and 30% short position in the assets)[50]: *a)* the minimum variance portfolio, *b)* the minimum downside variance portfolio, and *c)* the maximum skewness portfolio. The investor is currently holding the funds replicating the following equity indices:

- S&P500 (representing the domestic investment in the US)
- DAX30 (Germany)
- CAC40 (France)
- FTSE100 (UK)
- SMI (Switzerland)
- OMXS30 (Sweden)
- Nikkei225 (Japan)
- HSI (Hong Kong)

We download the index data from *finance.yahoo.com*. The corresponding return series are based on adjusted closing prices and synchronized with the observations for our four factors. The final dataset comprises 4490 daily return observations between 10 July 1991 and 30 December 2011. The summary statistics for the 8 indices are as follows (the data is annualized where appropriate):

[50] As a robustness check we also solve portfolio optimization problems without short-sales constraints and with a long-only constraint. The results are presented in *Appendix 5*. The conclusions to be drawn from the three alternative formulations are consistent. We do not perform any other robustness tests because this section is just a simple illustration of the performance of our factors in an international portfolio and their relation to the home bias effect.

Table 16: Summary statistics for 8 international equity indices

Summary	Mean	St.Dev.	Downside Dev.	Skewness	Kurtosis	Sharpe ratio	Sortino Ratio
S&P500	9.26%	20.14%	14.24%	-0.05	10.53	0.46	0.65
DAX30	9.61%	25.28%	17.73%	0.13	8.90	0.38	0.54
CAC40	5.70%	24.64%	17.21%	0.26	8.78	0.23	0.33
FTSE100	6.26%	19.93%	13.90%	0.20	10.63	0.31	0.45
SMI	8.40%	20.58%	14.46%	0.30	15.14	0.41	0.58
OMXS30	7.50%	31.61%	25.20%	-11.75	454.47	0.24	0.30
Nikkei225	-2.23%	25.78%	18.43%	-0.01	8.31	-0.09	-0.12
HSI	13.69%	29.38%	20.27%	0.31	12.74	0.47	0.68

Quite expectedly, the index that has the highest average return is the HSI, representing the stellar GDP growth in China over the past years. The HSI index also has the highest Sharpe and Sotrino ratios. The Nikkei225 index performs the worst, producing an average return of -2.23%.

Now, given that the investor decides to form a minimum variance portfolio based on the indices only, a portfolio based on the indices and the higher moment factors, a portfolio based on the indices and the downside risk factors, or a portfolio based on the indices and both types of factors, the corresponding portfolios under the 130-30 constraint would have the following weights:

Table 17: Minimum variance portfolio weights with a 130-30 constraint

	Indices Only	Indices+CS+CK	Indices+Omega+Vega	Indices+CS+CK+Omega+Vega
S&P500	38.46%	0.00%	27.56%	3.73%
DAX30	0.00%	0.03%	0.00%	0.11%
CAC40	0.00%	0.00%	-1.84%	0.00%
FTSE100	18.08%	4.13%	5.11%	2.16%
SMI	17.48%	6.22%	3.07%	3.67%
OMXS30	0.00%	0.00%	1.81%	0.11%
Nikkei225	20.28%	9.08%	5.27%	4.59%
HSI	5.70%	1.85%	2.16%	1.74%
Co-skewness		45.59%		15.08%
Co-kurtosis		33.10%		28.91%
Omega			32.76%	27.28%
Vega			24.11%	12.61%
Foreign/S&P500[51]	1.60	-44176.49	0.57	3.32
abs(Foreign)/S&P500[52]	1.60	44176.49	0.70	3.32

[51] This entry gives the ratio of the sum of the weights in the indices from outside the US to the weight in the S&P500 index. It is thus a ratio of the net capital allocated to an international equity holding to the capital allocated to the S&P500 index.

[52] Unlike the entry above, this entry gives the ratio of the sum of the absolute weights in the international indices to the absolute weight in the S&P500 index. It thus treats any short positions as capital allocated *to* the corresponding investment alternative, rather than *away* from it.

As can be seen in the table above, the portfolio that only includes the indices is heavily invested in the S&P500 index, with a weight of 38.46%, corresponding to only 61.54% invested abroad. There is no investment in the DAX30, CAC40 or OMXS30 indices.

According to columns 2 to 4, the introduction of the factor portfolios draws a considerable investment weight towards them even in the mean-variance framework. In the portfolio in column 2 that only includes the Co-skewness and Co-kurtosis factors (i.e. accounts for higher moment preferences), the two factors together attract an investment of 78.69%. The investor no longer holds a position in the S&P500 index and is still not holding a position in the CAC40 or the OMXS30 index. At the same time, the Omega and Vega portfoios (that account for downside risk preferences and market phase shifting) draw 56.87% of the capital and lead to a negative position in the CAC40 index, while reducing the position in the S&P500 index by 10.90% and increasing the home bias.

When both the higher moment factors and the downside risk factors are added to the portfolio, the majority of the capital is attracted to the four factors, comprising a total of 83.88%. At the same time, the home bias in equity index investments is mitigated as the optimal investment in the S&P500 index is now at 3.73%, while the optimal holding abroad is at 12.39%, which means that the US index holding of the investor is now at 23.14% of the total index holding. A similar result is observed with the minimum downside risk portfolio weights presented in the table below.

Table 18: Minimum downside variance portfolio weights with a 130-30 constraint

	Indices Only	Ind+CS+CK	Ind+Omega+Vega	All
S&P500	39.55%	-4.64%	28.45%	4.91%
DAX30	-12.28%	-1.50%	0.00%	1.51%
CAC40	-0.03%	-4.75%	0.00%	-4.80%
FTSE100	26.65%	5.73%	4.01%	5.79%
SMI	17.10%	11.54%	2.86%	3.63%
OMXS30	0.00%	0.00%	1.57%	2.82%
Nikkei225	20.26%	7.50%	4.49%	3.16%
HSI	8.76%	3.20%	2.60%	1.10%
Co-skewness		42.71%		13.21%
Co-kurtosis		40.21%		26.10%
Omega			30.93%	27.91%
Vega			25.09%	14.65%
Foreign/S&P500	1.53	-4.68	0.55	2.69
abs(Foreign)/S&P500	2.15	7.37	0.55	4.64

With indices only, the minimum downside variance portfolio is slightly more invested in the S&P500 index than is the minimum variance portfolio (39.55% against 38.46%, respectively). This result is consistent with the findings by Guidolin and Timmermann (2008). Once again, we see a significant reduction in the S&P500 holding when the higher moment factors are introduced. This time the weight on the S&P500 index becomes -4.64%. The higher moment factors attract a total of 82.92% of the capital. The downside risk factors take up 56.02% of the capitaland reduce the holding in the S&P500 index to 28.45%, while once again increasing the home bias.

Including both the downside risk factors and the higher moment factors leads to both a positive weight in the S&P500 of 4.91% and a net weight in the international indices of 13.22%, which implies that the holding in the S&P500 index is at 27.08% of the total index holding of the investor. The capital attracted by the four factors is at 81.87% of the total investment. The results for the maximum skewness portfolio are as follows:

Table 19: Maximum skewness portfolio weights with a 130-30 constraint

	Indices Only	Ind+CS+CK	Ind+Omega+Vega	All
S&P500	28.60%	0.00%	29.79%	9.00%
DAX30	0.00%	0.00%	1.72%	1.22%
CAC40	0.02%	0.00%	0.43%	0.52%
FTSE100	0.03%	9.45%	5.60%	9.55%
SMI	63.87%	15.73%	18.69%	12.97%
OMXS30	-14.74%	-14.99%	-14.94%	-14.99%
Nikkei225	21.56%	9.31%	9.43%	6.28%
HSI	0.67%	0.08%	0.00%	0.00%
Co-skewness		38.67%		11.44%
Co-kurtosis		41.74%		24.48%
Omega			26.64%	24.36%
Vega			22.64%	15.18%
Foreign/S&P500	2.50	11390167517.63	0.70	1.73
abs(Foreign)/S&P500	3.53	28822420027.42	1.71	5.06

The results for the maximum skewness portfolio are generally consistent with the results for the minimum variance and minimum downside variance portfolios. One difference is that the strong negative weight in the OMXS30 index in all four portfolios brings down the estimate of the net weight allocated to the international indices. Also, the portfolio weight in the S&P500 index is now higher when the Omega and Vega factors are introduced.

To conclude, there is evidence that introducing the Co-skewness and Co-kurtosis factors that account for higher moment preferences, as well as the Omega and Vega factors that account for downside risk preferences mitigates the home bias of the US investors. Consistent with Guidolin and Timmermann (2008) who argue that market phase shifting and higher moment preferences lead to a higher optimal weight of the US market in an internationally diversified portfolio, our results suggest that cobntrolling for the downside risk and higher moment factors in a portfolio problem reduces the optimal proportion of the US market in an internationally diversified portfolio. At the same time, we must note that the current test methodology is by no means comprehensive and would benefit from additional attention in subsequent asset pricing literature.

Summary and Conclusion

This paper presents a new framework for capturing downside risk and the asymmetric risk preferences of investors. Unlike the preceding downside risk and higher moment frameworks, the current approach does not require the development of an alternative capital pricing model or a complex computational procedure.

Two straightforward downside risk-reward trade-off measures are introduced and are used to construct two factor portfolios called Omega and Vega. The two factors are used as an extension to the CAPM model, forming the Market-Omega-Vega, or MOV, multifactor model. Additionally, two factor portfolios are developed that sort stocks based on co-skewness and co-kurtosis with the market and the MCC multifactor model is formed. The portfolios are constructed with monthly rebalancing using 62 years of daily stock data on all NYSE, NASDAQ and AMEX stocks. For further tests, the Market, SMB, HML and Momentum portfolios are replicated using the same dataset and comparable formation rules. Due to a data issue, the HML portfolio is subsequently replaced with the corresponding portfolio obtained from the Kenneth French website. Preliminary tests show that all of the constructed portfolios are comparable and investable in a similar fashion.

The Omega portfolio performs second-best after Momentum according to the entire period's cumulative return, while outperforming Momentum over the period 1981-2011. The Vega portfolio correlates negatively with the market during market declines, while producing a positive risk premium throughout the entire period and performing equally well as the market between 1981 and 2011. In aggregate, the Co-skewness and Co-kurtosis portfolios do not produce risk premia throughout the observation period.

Time series tests show that the MOV model prices the Momentum portfolio. This is a new accomplishment, as to the authors' best knowledge Momentum has not been successfully priced by a multifactor model in prior literature. Additionally, the Omega and Vega factors are not priced by the FF3F model or FF3F+Momentum, while the Co-skewness and Co-kurtosis factors are priced by the other models. The time series tests that replicate prior research produce results that are consistent with previous findings.

Cross-sectional tests over five-year subsamples show that risk premia vary substantially depending on sample selection. For some of the factor portfolios, cross-sectional risk premia change signs depending on the sample, while the CAPM fails completely in some of the tests. The Fama-MacBeth procedure produces consistent, though statistically insignificant results. The MOV model outperforms the FF3F model in the cross-section, while the extension of the FF3F model that includes Omega and Vega (FF3F+OV) outperforms the FF3F+Momentum model.

A further test of the explanatory power of the models through time reveals that the multifactor models have higher explanatory power in bull markets than in bear markets. We find some evidence of a

decline in the explanatory power of every model over the 61-year period. The evidence is even more pronounced when considering the Momentum factor, which leads us to concur that picking past winners and selling past losers may be the result of a behavioural bias in the market that is slowly disappearing. In a further test, we find that the FF3F and FF3F+Momentum models outperform the CAPM unconditionally on the market return, while the models that include the Omega and Vega and/or Co-skewness and Co-kurtosis outperform the CAPM by a higher margin in bear markets. This proves the intuitive argument that downside and asymmetric risks become more relevant to investors in bad times.

The introduction of the downside risk and higher moment factors into an international portfolio allocation problem mitigates the home bias. At the same time, when introduced into the portfolio the factors draw a substantial amount of investment capital, implying that the control for higher moments and market phase preferences has a considerable effect on portfolio choice, even in the traditional mean-variance optimization framework. Both results are consistent with the findings by Guidolin and Timmermann (2008).

All in all, this paper has developed and presented an alternative methodology for dealing with, and investing in, downside risk. The downside risk strategies suggested here seem to perform well relative to the popular investment strategies developed in prior literature, both in terms of profitability and in terms of theoretical explanatory power, making this paper's findings useful for both academics and practitioners.

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http://wrds-web.wharton.upenn.edu/wrds/ - the Wharton Research Data Servises database (including CRSP and COMPUSTAT), last visited on 9 December 2011

Appendix 1: A theoretical Vega-efficient portfolio

Consider an investor holding a portfolio in a universe of N assets. The weights of the portfolio are given by an $N\times1$ column vector $\boldsymbol{w}_P$ that captures the desired risk exposure (e.g. market risk, size risk, or any other risk source). The purpose of a Vega-efficient portfolio is to maximize the risk exposure of the original portfolio when the portfolio is profitable and minimize the risk exposure otherwise.

Define the following downside covariance estimate for asset i:

$$\sigma_{i,P}^{d} \equiv \mathrm{E}\left([r_i-\tau][\boldsymbol{w}_P^T\boldsymbol{R}-\tau_P]1_{\{r_i\le\tau\}}1_{\{\boldsymbol{w}_P^T\boldsymbol{R}\le\tau_P\}}\right)=\boldsymbol{w}_P^T\boldsymbol{\sigma}_{i,P}^{d},$$

where τ is the threshold separating asset-specific downside risk from asset-specific upside potential, $\boldsymbol{R}$ is a $N\times1$ column vector of asset returns, and τ_p is the threshold separating systematic downside risk from systematic upside potential, and $\boldsymbol{\sigma}_{i,P}^{d}$ is given by:

$$\boldsymbol{\sigma}_{i,P}^{d} \equiv \mathrm{E}\left[\begin{pmatrix}[r_i-\tau]\left[r_1-\dfrac{\tau_P}{Nw_{1,P}}\right]\\ [r_i-\tau]\left[r_2-\dfrac{\tau_P}{Nw_{2,P}}\right]\\ \vdots\\ [r_i-\tau]\left[r_N-\dfrac{\tau_P}{Nw_{N,P}}\right]\end{pmatrix}1_{\{r_i\le\tau\}}1_{\{\boldsymbol{w}_P^T\boldsymbol{R}\le\tau_P\}}\right].$$

The corresponding upside covariance estimate will be:

$$\sigma_{i,P}^{u} \equiv \mathrm{E}\left([r_i-\tau]\left[\boldsymbol{w}_P^T\boldsymbol{R}-\tau_p\right]1_{\{r_i>\tau\}}1_{\{\boldsymbol{w}_P^T\boldsymbol{R}>\tau_p\}}\right)=\boldsymbol{w}_P^T\boldsymbol{\sigma}_{i,P}^{u},$$

$$\boldsymbol{\sigma}_{i,P}^{u} \equiv \mathrm{E}\left[\begin{pmatrix}[r_i-\tau]\left[r_1-\dfrac{\tau_P}{Nw_{1,P}}\right]\\ [r_i-\tau]\left[r_2-\dfrac{\tau_P}{Nw_{2,P}}\right]\\ \vdots\\ [r_i-\tau]\left[r_N-\dfrac{\tau_P}{Nw_{N,P}}\right]\end{pmatrix}1_{\{\boldsymbol{w}_P^T\boldsymbol{R}>\tau_P\}}\right].$$

The vector containing all asset-specific upside/downside covariance estimate will then be given by:

$$\boldsymbol{\sigma}_{P}^{u/d} \equiv \begin{pmatrix}\sigma_{1,P}^{u/d}\\ \sigma_{2,P}^{u/d}\\ \vdots\\ \sigma_{N,P}^{u/d}\end{pmatrix}=\begin{pmatrix}\boldsymbol{w}_P^T\boldsymbol{\sigma}_{1,P}^{u/d}\\ \boldsymbol{w}_P^T\boldsymbol{\sigma}_{2,P}^{u/d}\\ \vdots\\ \boldsymbol{w}_P^T\boldsymbol{\sigma}_{N,P}^{u/d}\end{pmatrix}=\mathrm{COV}_{u/d}^{P}\boldsymbol{w}_P, \qquad \text{where}$$

$$\mathrm{COV}_{u/d}^{P}=\left(\boldsymbol{\sigma}_{1,P}^{u/d},\quad \boldsymbol{\sigma}_{2,P}^{u/d},\quad \dots,\quad \boldsymbol{\sigma}_{N,P}^{u/d}\right)^T.$$

This brings us to the Vega-implied portfolio optimization problem, where the optimal weights $\boldsymbol{w}_{\mathrm{V}}^{*}$ solve:

35: The Vega-implied portfolio optimization problem

$$\boldsymbol{w}_{\mathrm{V}}^{*} = \arg\max_{\boldsymbol{w}_{\mathrm{V}}} \frac{\boldsymbol{w}_{\mathrm{V}}^{T}\boldsymbol{\sigma}_{P}^{u}}{\boldsymbol{w}_{\mathrm{V}}^{T}\boldsymbol{\sigma}_{P}^{d}} = \arg\max_{\boldsymbol{w}_{\mathrm{V}}} \frac{\boldsymbol{w}_{\mathrm{V}}^{T}\mathrm{COV}_{u}^{P}\boldsymbol{w}_{P}}{\boldsymbol{w}_{\mathrm{V}}^{T}\mathrm{COV}_{d}^{P}\boldsymbol{w}_{P}},$$

$$s.t.\,\mathbf{1}^{T}\boldsymbol{w}_{\mathrm{V}} = 1.$$

The current optimization problem is similar to the Omega portfolio problem (please see the section called *Portfolio optimization with Omega* in the literature review section), which means that similar numeric algorithms should apply[53].

[53] We must also note that the existence of a meaningful solution depends on the definition of the risk space. The values of the objective function will only be comparable across the different possible solutions if the denominator (the risk estimate) does not change sign. Otherwise, an asset with a positive return and a negative risk estimate would be deemed a worse alternative than an asset with a positive return and a positive risk estimate. The current risk space given by $1_{\{r_i\leq\tau\}}1_{\{\boldsymbol{w}_P^T\boldsymbol{R}\leq\tau_P\}}$ assures that the risk estimate is always positive because $[r_i-\tau][\boldsymbol{w}_P^T\boldsymbol{R}-\tau_P]1_{\{r_i\leq\tau\}}1_{\{\boldsymbol{w}_P^T\boldsymbol{R}\leq\tau_P\}}$ is always positive. However, the same no longer applies, for example in the risk space of Ang et al. (2002), given by $1_{\{\boldsymbol{w}_P^T\boldsymbol{R}\leq\tau_P\}}$, because $[r_i-\tau][\boldsymbol{w}_P^T\boldsymbol{R}-\tau_P]1_{\{\boldsymbol{w}_P^T\boldsymbol{R}\leq\tau_P\}}$ can take negative values. The original Vega measure is defined as a difference estimator to make it robust to such changes in the sign. However, in terms of portfolio optimization, a ratio estimator is more appropriate, and thus the range of applicable risk spaces needs to be restricted.

Appendix 2: List of Data Variables

CRSP stock exchange database search criteria and data variables

Database section: *CRSP Daily Stock* (searching the entire database by CUSIP)

Downloaded history periods: 1950-1955, 1956-1960, 1961-1965. 1966-1970, 1971-1975, 1976-1980, 1981-1985, 1986-1990, 1991-1995, 1996-2000, 2001-2005, 2006-2011 (January 1 to December 31)

Conditional statements

- *Exchange Code (exchcd) >= 1 AND Exchange Code (exchcd) <= 3 (NYSE, AMEX and NASDAQ stocks only)*

Identifying information

- *CUSIP* – company identifier
- *Share code (shrcd)* – security type identifier
- *Exchange code (exchcd)* – stock exchange identifier
- *Primary exchange (primexch)* – primary stock exchange identifier

Time series information

- *Price (prc)* – Closing price or bid/ask average

Share information

- *Number of shares outstanding (shrout)*

Distribution informaton

- *Distribution code (distcd)* – distribution event type and payment method identifier
- *Dividend cash amount (divamt)* – the amount of cash dividend distributed
- *Factor to adjust price (facpr)* – stock price adjustment factor for stock dividends, splits and liquidations

Market information

- *Value-weighted return including distributions (vwretd)* – value-weighted stock return to verify the adjusted value-weighted portfolio return calculations

Data format: stata file (*.dta) compressed into zip (*.zip) format

CRSP treasury database search criteria and data variables

Database section: *CRSP Monthly Treasury - Fama Risk Free Rates*

Downloaded history period: January 1950 – December 2011

Rate information

- *CRSPID for the 1 Month Issue Used*
- *Yield Based on 1 Month Average*

Data format: stata file (*.dta)

COMPUSTAT database search criteria and data variables

Database section: *CRSP/COMPUSTAT Merged – Fundamentals Annual* (the entire database by CUSIP)

Downloaded history period: January 1950 – December 2011 by *data date* with standard screening variables and all (8 types of) unique links to GVKEY. Fiscal period and link date requirements: *Fiscal period end date must be within link date range*.

Conditional statements

- *Exchange Code (exchcd) >= 1 AND Exchange Code (exchcd) <= 3 (NYSE, AMEX and NASDAQ stocks only)*

Variable types

- *Data Items*

Link information

- *Historical CRSP PERMNO Link to COMPUSTAT (lpermno)*

Identifying information

- *CUSIP*
- *Stock exchange code (exchcd)*
- *Fiscal year-end (fyear)*

Balance sheet items

- *Stockholder equity total (seq)*
- *Deferred taxes and investment tax credit (txditc)*
- *Preferred stock at redemption value (pstkrv)*
- *Preferred stock at liquidating value (pstkl)*
- *Preferred stock at carrying value (upstk)* – preferred stock at par value

Data format: stata file (*.dta) compressed into zip (*.zip) format

Kenneth French data library items

- *Fama/French Factors [Daily] (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors_daily.zip)*
- *Momentum Factor [Daily] (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Momentum_Factor_daily.zip)*

Yahoo database items

Symbols (all available daily history downloaded):

- *^GSPC (http://finance.yahoo.com/q/hp?s=%5EGSPC+Historical+Prices)* – S&P500
- *^GDAXI (http://finance.yahoo.com/q/hp?s=%5EGDAXI+Historical+Prices)* – DAX30
- *^FCHI (http://finance.yahoo.com/q/hp?s=%5EFCHI+Historical+Prices)* – CAC40
- *^FTSE (http://finance.yahoo.com/q/hp?s=%5EFTSE+Historical+Prices)* – FTSE100
- *^SSMI (http://finance.yahoo.com/q/hp?s=%5ESSMI+Historical+Prices)* – SMI
- *^OMX (http://finance.yahoo.com/q/hp?s=%5EOMX+Historical+Prices)* – OMXS30
- *^N225 (http://finance.yahoo.com/q/hp?s=%5EN225+Historical+Prices)* – Nikkei225
- *^HSI (http://finance.yahoo.com/q/hp?s=%5EHSI+Historical+Prices)* – HSI

Appendix 3: All Time Series Test Results

Table 20: Comprehensive time series tests results over the entire firmservation period

Dependent	Alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	COSKEW	COKURT	R2
OMEGA	**12.35%**	**-0.44**								24.15%
t-stat	8.15	-70.04								
VEGA	**5.10%**	**-0.14**								2.41%
t-stat	2.97	-19.52								
SMB	**7.51%**	**-0.24**								15.36%
t-stat	6.86	-52.89								
HML	**5.12%**	**-0.12**								6.29%
t-stat	5.68	-32.17								
MOM	**10.20%**	**0.06**								0.38%
t-stat	5.21	7.76								
COSKEW	2.21%	**0.04**								0.37%
t-stat	1.58	7.60								
COKURT	**-5.59%**	**0.64**								50.38%
t-stat	-4.58	125.08								
MOM	**13.59%**	**-0.02**	**-0.07**	**-0.56**						7.59%
t-stat	7.19	-2.43	-4.78	-32.66						
OMEGA	**10.57%**	**-0.38**	**0.38**	**-0.21**						29.91%
t-stat	7.24	-56.24	34.54	-15.52						
VEGA	**10.78%**	**-0.33**	**-0.86**	**0.15**						30.30%
t-stat	7.41	-49.32	-78.36	10.98						
COSKEW	1.02%	**0.08**	**0.13**	**0.05**						1.53%
t-stat	0.73	12.60	12.00	3.79						
COKURT	-1.29%	**0.50**	**-0.49**	**-0.12**						61.15%
t-stat	-1.19	101.55	-60.24	-12.47						
OMEGA	**4.66%**	**-0.37**	**0.41**	**0.04**	**0.43**					52.12%
t-stat	3.86	-66.38	45.02	3.36	84.54					
VEGA	**8.60%**	**-0.33**	**-0.85**	**0.24**	**0.16**					33.33%
t-stat	6.03	-49.90	-79.04	17.58	26.45					
COSKEW	**-3.08%**	**0.09**	**0.15**	**0.22**	**0.30**					18.02%
t-stat	-2.42	14.90	15.28	18.18	55.65					
COKURT	-0.28%	**0.50**	**-0.49**	**-0.16**	**-0.07**					61.79%
t-stat	-0.26	102.07	-61.32	-16.27	-16.13					
SMB	**7.48%**	**-0.23**				**0.14**	**-0.32**			42.03%
t-stat	8.23	-51.24				27.98	-75.52			
HML	**5.77%**	**-0.14**				**-0.04**	**-0.02**			6.87%
t-stat	6.41	-32.63				-8.90	-5.42			
MOM	*-0.29%*	**0.43**				**0.72**	**0.30**			35.18%
t-stat	***-0.18***	55.30				85.65	40.56			
COSKEW	-0.94%	**0.17**				**0.33**	**-0.19**			20.90%
t-stat	-0.75	27.18				50.02	-32.49			
COKURT	**-1.90%**	**0.49**				**-0.40**	**0.23**			69.62%
t-stat	-1.98	106.07				-77.30	51.87			
SMB	**5.22%**	**0.02**						-0.01	**-0.41**	33.06%
t-stat	5.36	3.59						-1.18	-62.15	
HML	**4.38%**	**-0.04**						0.00	**-0.13**	9.34%
t-stat	4.93	-6.87						0.86	-21.88	
MOM	**9.45%**	-0.01						**0.54**	**0.08**	14.58%
t-stat	5.21	-1.03						50.24	6.44	
OMEGA	**8.49%**	**-0.09**						**0.30**	**-0.57**	50.86%
t-stat	6.96	-12.25						41.21	-69.05	
VEGA	**8.31%**	**-0.43**						**-0.24**	**0.48**	20.52%
t-stat	5.35	-46.11						-25.88	45.30	
Rm	**9.93%**		**-0.19**	**0.04**	**0.13**	**-0.10**	**-0.36**	**0.05**	**0.72**	60.21%
t-stat	8.11		-16.56	3.00	18.66	-10.57	-50.38	6.39	76.12	
SMB	**5.81%**	**-0.09**		**0.21**	**0.01**	**0.10**	**-0.30**	**-0.12**	**-0.18**	48.94%
t-stat	6.80	-16.56		25.69	2.30	15.55	-63.26	-19.82	-23.99	
HML	**4.13%**	**0.02**	**0.19**		**-0.16**	**-0.06**	**0.15**	**0.14**	**-0.15**	23.42%
t-stat	5.04	3.00	25.69		-34.63	-9.65	29.89	25.80	-20.17	
MOM	**3.03%**	**0.17**	**0.03**	**-0.46**		**0.66**	**0.32**	**0.42**	**0.25**	48.96%
t-stat	2.15	18.66	2.30	-34.63		67.48	38.34	45.89	20.39	
OMEGA	**4.43%**	**-0.07**	**0.15**	**-0.10**	**0.35**		**0.05**	**0.13**	**-0.58**	65.97%
t-stat	4.35	-10.57	15.55	-9.65	67.48		8.25	17.77	-73.11	
VEGA	**7.00%**	**-0.39**	**-0.68**	**0.37**	**0.27**	**0.08**		**-0.42**	**0.27**	45.83%
t-stat	5.44	-50.38	-63.26	29.89	38.34	8.25		-49.83	23.91	
COSKEW	-0.97%	**0.05**	**-0.21**	**0.29**	**0.28**	**0.16**	**-0.33**		**-0.02**	32.97%
t-stat	-0.84	6.39	-19.82	25.80	45.89	17.77	-49.83		-2.21	
COKURT	0.62%	**0.38**	**-0.20**	**-0.18**	**0.10**	**-0.45**	**0.13**	**-0.01**		73.53%
t-stat	0.69	76.12	-23.99	-20.17	20.39	-73.11	23.91	-2.21		

Appendix 4a: Main Cross-Sectional Test Results, five-year periods

Table 21: Comprehensive cross-sectional test results over five-year periods

1951-1955 (407 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	16.41%								31.95%
Chi2/t-stat	360.77	3.60								
FF3F	-	22.84%	-7.11%	-5.75%						40.15%
Chi2/t-stat	334.49	4.97	-3.46	-2.02						
FF3F+MOM	-	23.36%	-7.38%	-4.85%	38.70%					50.57%
Chi2/t-stat	315.36	5.06	-3.57	-1.69	5.24					
MOV	-	19.93%				5.88%	24.88%			30.38%
Chi2/t-stat	302.30	4.40				1.32	4.39			
MCC	-	17.12%						13.31%	7.86%	26.98%
Chi2/t-stat	345.49	3.79						3.19	1.92	
FF3F+OV	-	22.84%	-6.91%	-0.95%		7.53%	12.84%			41.78%
Chi2/t-stat	285.97	4.98	-3.37	-0.33		1.72	2.38			
FF3F+CC	-	23.52%	-7.75%	-3.07%				4.61%	5.53%	33.16%
Chi2/t-stat	310.71	5.11	-3.76	-1.08				1.12	1.33	
FF3F+OV+CC	-	22.91%	-6.97%	-0.92%		7.19%	12.66%	6.34%	8.90%	41.14%
Chi2/t-stat	285.36	5.00	-3.41	-0.31		1.65	2.34	1.51	2.13	
FF3F+MOM+OV	-	22.76%	-6.90%	-1.31%	35.56%	8.00%	13.43%			45.92%
Chi2/t-stat	283.93	4.96	-3.36	-0.45	4.88	1.82	2.50			
FF3f+MOM+CC	-	23.85%	-7.96%	-2.43%	37.58%			4.64%	6.50%	43.63%
Chi2/t-stat	293.52	5.16	-3.84	-0.84	5.09			1.12	1.55	
All Variables	-	22.93%	-7.15%	-1.39%	35.85%	6.95%	13.26%	5.83%	7.91%	44.98%
Chi2/t-stat	282.95	5.00	-3.49	-0.48	4.90	1.59	2.45	1.39	1.88	
1956-1960 (355 firms)	**alpha**	**Rm**	**SMB**	**HML**	**MOM**	**OMEGA**	**VEGA**	**CS**	**CK**	**R-squared**
CAPM	-	7.29%								8.05%
Chi2/t-stat	402.79	1.44								
FF3F	-	9.79%	-2.12%	-13.99%						26.53%
Chi2/t-stat	365.03	1.94	-0.83	-5.04						
FF3F+MOM	-	11.16%	-4.72%	-10.09%	52.60%					42.78%
Chi2/t-stat	302.85	2.19	-1.85	-3.45	6.11					
MOV	-	9.02%				30.87%	21.93%			34.82%
Chi2/t-stat	278.80	1.77				5.22	3.65			
MCC	-	6.03%						17.29%	-5.76%	8.04%
Chi2/t-stat	362.81	1.20						2.59	-1.24	
FF3F+OV	-	11.21%	-5.06%	-7.76%		28.18%	16.56%			39.89%
Chi2/t-stat	277.31	2.20	-2.03	-2.54		4.82	2.69			
FF3F+CC	-	11.24%	-3.34%	-11.55%				6.32%	-5.16%	25.90%
Chi2/t-stat	323.76	2.21	-1.30	-4.00				1.00	-1.10	
FF3F+OV+CC	-	11.36%	-5.05%	-8.04%		26.63%	16.86%	3.59%	-1.37%	38.82%
Chi2/t-stat	273.39	2.23	-2.02	-2.67		4.64	2.74	0.56	-0.28	
FF3F+MOM+OV	-	11.24%	-5.26%	-7.34%	44.01%	28.69%	16.07%			42.62%
Chi2/t-stat	274.22	2.21	-2.10	-2.40	5.13	4.93	2.61			
FF3f+MOM+CC	-	11.81%	-5.24%	-8.57%	48.23%			4.59%	-4.51%	41.21%
Chi2/t-stat	271.61	2.31	-2.05	-2.87	5.59			0.71	-0.94	
All Variables	-	11.46%	-5.31%	-7.65%	44.70%	25.84%	16.44%	3.70%	-2.80%	42.26%
Chi2/t-stat	265.31	2.25	-2.12	-2.53	5.21	4.48	2.67	0.58	-0.59	
1961-1965 (470 firms)	**alpha**	**Rm**	**SMB**	**HML**	**MOM**	**OMEGA**	**VEGA**	**CS**	**CK**	**R-squared**
CAPM	-	13.14%								13.51%
Chi2/t-stat	469.26	2.62								
FF3F	-	12.43%	-0.99%	4.93%						18.44%
Chi2/t-stat	456.42	2.53	-0.36	1.72						
FF3F+MOM	-	14.71%	-4.32%	5.42%	49.43%					43.57%
Chi2/t-stat	381.04	2.95	-1.57	1.84	6.59					
MOV	-	15.89%				8.19%	17.23%			12.54%
Chi2/t-stat	418.46	3.15				1.64	2.73			
MCC	-	12.28%						1.12%	-0.78%	7.09%
Chi2/t-stat	466.88	2.44						0.21	-0.18	
FF3F+OV	-	15.57%	-3.42%	5.97%		11.80%	8.62%			16.03%
Chi2/t-stat	407.18	3.17	-1.26	2.04		2.30	1.46			
FF3F+CC	-	13.86%	-1.86%	6.71%				10.60%	0.56%	13.61%
Chi2/t-stat	442.13	2.84	-0.68	2.31				2.05	0.13	
FF3F+OV+CC	-	15.54%	-4.08%	6.54%		17.12%	12.15%	6.20%	7.44%	24.31%
Chi2/t-stat	387.40	3.16	-1.49	2.22		3.27	2.08	1.22	1.71	
FF3F+MOM+OV	-	15.92%	-5.02%	6.50%	48.68%	15.16%	13.63%			37.31%
Chi2/t-stat	366.74	3.22	-1.84	2.18	6.53	2.87	2.28			
FF3f+MOM+CC	-	16.08%	-4.92%	5.64%	50.06%			5.25%	3.17%	36.49%
Chi2/t-stat	370.72	3.25	-1.80	1.91	6.63			1.03	0.71	
All Variables	-	15.77%	-4.83%	6.07%	49.31%	13.38%	12.33%	4.63%	3.27%	36.63%
Chi2/t-stat	364.99	3.20	-1.77	2.05	6.59	2.62	2.09	0.91	0.76	

1966-1970 (1278 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	3.31%								1.37%
Chi2/t-stat	9676.07	0.59								
FF3F	-	3.80%	-0.06%	-10.02%						14.06%
Chi2/t-stat	9549.33	0.69	-0.01	-2.76						
FF3F+MOM	-	4.86%	-0.85%	-10.51%	-10.06%					15.04%
Chi2/t-stat	9499.19	0.88	-0.22	-2.92	-1.14					
MOV	-	7.20%				4.63%	19.60%			11.08%
Chi2/t-stat	9396.00	1.27				0.79	2.72			
MCC	-	5.27%						-11.53%	4.12%	8.56%
Chi2/t-stat	9553.51	0.93						-2.47	0.78	
FF3F+OV	-	6.00%	-2.63%	-11.79%		9.61%	20.52%			27.57%
Chi2/t-stat	9136.95	1.08	-0.68	-3.33		1.72	2.77			
FF3F+CC	-	5.95%	-1.74%	-12.63%				-6.08%	-0.39%	19.71%
Chi2/t-stat	9359.55	1.08	-0.45	-3.79				-1.26	-0.08	
FF3F+OV+CC	-	6.48%	-2.74%	-12.39%		10.68%	16.41%	-7.04%	2.54%	28.46%
Chi2/t-stat	9127.14	1.17	-0.71	-3.69		2.01	2.33	-1.47	0.48	
FF3F+MOM+OV	-	6.43%	-2.70%	-12.35%	7.21%	10.14%	17.56%			27.95%
Chi2/t-stat	9138.03	1.16	-0.70	-3.56	0.78	1.80	2.45			
FF3f+MOM+CC	-	5.78%	-2.00%	-12.11%	8.31%			-8.80%	-2.17%	23.35%
Chi2/t-stat	9166.27	1.05	-0.52	-3.60	0.93			-1.84	-0.42	
All Variables	-	6.44%	-2.75%	-12.35%	11.36%	10.70%	16.48%	-7.39%	2.03%	28.53%
Chi2/t-stat	9111.00	1.16	-0.71	-3.68	1.25	2.01	2.34	-1.56	0.39	

1971-1975 (1498 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	0.11%								0.22%
Chi2/t-stat	2073.06	0.02								
FF3F	-	4.44%	-7.93%	-5.72%						12.10%
Chi2/t-stat	2059.06	0.65	-1.76	-1.84						
FF3F+MOM	-	5.08%	-9.21%	-4.48%	23.67%					19.12%
Chi2/t-stat	1992.84	0.74	-2.06	-1.42	3.23					
MOV	-	4.02%				4.53%	22.55%			11.89%
Chi2/t-stat	1998.87	0.58				0.72	3.05			
MCC	-	3.05%						-8.60%	8.77%	6.51%
Chi2/t-stat	2061.23	0.45						-1.66	1.23	
FF3F+OV	-	4.68%	-9.00%	-5.56%		8.14%	13.24%			18.43%
Chi2/t-stat	1990.86	0.68	-2.01	-1.78		1.25	2.03			
FF3F+CC	-	4.88%	-8.59%	-6.08%				-2.05%	4.61%	13.85%
Chi2/t-stat	2053.05	0.71	-1.93	-1.98				-0.41	0.65	
FF3F+OV+CC	-	4.92%	-9.20%	-5.65%		7.25%	13.42%	-4.50%	7.22%	18.53%
Chi2/t-stat	1991.75	0.72	-2.09	-1.82		1.23	2.12	-0.92	1.01	
FF3F+MOM+OV	-	4.77%	-9.26%	-4.76%	22.88%	7.33%	13.67%			20.70%
Chi2/t-stat	1975.20	0.69	-2.07	-1.53	3.19	1.13	2.10			
FF3f+MOM+CC	-	5.29%	-9.49%	-4.76%	22.24%			-2.86%	6.72%	19.66%
Chi2/t-stat	1994.72	0.77	-2.14	-1.54	3.21			-0.57	0.93	
All Variables	-	4.68%	-9.17%	-4.71%	23.21%	7.77%	13.74%	-4.06%	7.59%	20.73%
Chi2/t-stat	1971.27	0.69	-2.08	-1.54	3.31	1.32	2.17	-0.83	1.06	

1976-1980 (2241 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	21.99%								28.18%
Chi2/t-stat	1571.51	3.45								
FF3F	-	14.36%	8.97%	-10.61%						31.51%
Chi2/t-stat	1510.80	2.42	2.18	-4.24						
FF3F+MOM	-	14.60%	9.02%	-7.30%	42.21%					43.20%
Chi2/t-stat	1459.67	2.45	2.19	-3.00	5.59					
MOV	-	17.52%				27.16%	-1.43%			32.72%
Chi2/t-stat	1488.19	2.92				4.76	-0.25			
MCC	-	16.84%						24.30%	2.95%	28.04%
Chi2/t-stat	1502.40	2.79						4.85	0.51	
FF3F+OV	-	17.24%	5.63%	-8.62%		24.14%	0.22%			37.80%
Chi2/t-stat	1465.51	2.88	1.41	-3.48		4.25	0.04			
FF3F+CC	-	15.56%	7.26%	-9.85%				19.61%	4.49%	34.37%
Chi2/t-stat	1484.59	2.62	1.81	-3.99				4.09	0.76	
FF3F+OV+CC	-	17.69%	5.20%	-8.79%		22.16%	-0.96%	11.35%	7.40%	37.78%
Chi2/t-stat	1457.49	2.96	1.33	-3.66		3.97	-0.20	2.68	1.25	
FF3F+MOM+OV	-	16.42%	6.83%	-7.67%	43.08%	24.38%	0.00%			41.55%
Chi2/t-stat	1449.13	2.76	1.74	-3.18	6.06	4.29	0.00			
FF3f+MOM+CC	-	15.49%	7.60%	-7.73%	38.53%			13.97%	3.29%	41.82%
Chi2/t-stat	1457.53	2.61	1.90	-3.21	5.31			3.12	0.56	
All Variables	-	17.31%	5.86%	-8.22%	44.81%	20.57%	-0.02%	12.74%	4.51%	41.26%
Chi2/t-stat	1426.62	2.90	1.50	-3.42	6.29	3.71	0.00	2.98	0.78	

1981-1985 (2410 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	4.47%								1.81%
Chi2/t-stat	1885.30	0.70								
FF3F	-	9.99%	-8.83%	10.08%						7.88%
Chi2/t-stat	1825.76	1.61	-2.25	2.62						
FF3F+MOM	-	11.85%	-10.98%	10.75%	72.84%					23.05%
Chi2/t-stat	1651.74	1.88	-2.79	2.75	6.16					
MOV	-	8.78%				17.66%	63.55%			31.62%
Chi2/t-stat	1662.96	1.41				2.62	6.89			
MCC	-	4.81%						-27.35%	-2.23%	19.19%
Chi2/t-stat	1663.68	0.79						-6.02	-0.39	
FF3F+OV	-	9.48%	-11.11%	6.46%		26.56%	50.79%			34.11%
Chi2/t-stat	1624.40	1.52	-2.83	1.70		3.94	5.77			
FF3F+CC	-	5.64%	-10.57%	4.02%				-14.36%	-12.48%	29.34%
Chi2/t-stat	1544.22	0.92	-2.70	1.06				-3.08	-2.24	
FF3F+OV+CC	-	7.06%	-11.08%	4.51%		22.74%	41.83%	-10.38%	-4.88%	36.76%
Chi2/t-stat	1582.33	1.15	-2.84	1.20		3.45	5.09	-2.31	-0.87	
FF3F+MOM+OV	-	8.68%	-10.83%	5.25%	31.80%	25.65%	49.76%			34.57%
Chi2/t-stat	1622.56	1.40	-2.75	1.40	3.74	3.84	5.67			
FF3f+MOM+CC	-	7.50%	-11.49%	5.57%	39.00%			-10.45%	-8.47%	33.78%
Chi2/t-stat	1565.46	1.22	-2.94	1.48	4.36			-2.34	-1.51	
All Variables	-	7.00%	-11.05%	4.29%	29.08%	22.63%	42.18%	-10.54%	-4.63%	36.74%
Chi2/t-stat	1583.66	1.14	-2.83	1.15	3.51	3.44	5.16	-2.37	-0.83	

1986-1990 (3366 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	3.45%								0.81%
Chi2/t-stat	958.98	0.39								
FF3F	-	6.28%	-8.83%	-5.64%						6.06%
Chi2/t-stat	937.95	0.75	-1.56	-2.01						
FF3F+MOM	-	7.15%	-9.55%	-5.93%	14.38%					6.80%
Chi2/t-stat	921.80	0.86	-1.69	-2.12	2.34					
MOV	-	7.83%				-8.16%	10.76%			2.67%
Chi2/t-stat	952.30	0.93				-1.14	1.46			
MCC	-	6.33%						4.98%	12.69%	2.74%
Chi2/t-stat	956.19	0.75						0.86	1.43	
FF3F+OV	-	7.36%	-10.01%	-5.84%		-0.20%	6.79%			7.33%
Chi2/t-stat	920.55	0.88	-1.78	-2.08		-0.03	0.96			
FF3F+CC	-	7.57%	-9.89%	-5.96%				1.60%	6.73%	6.84%
Chi2/t-stat	931.82	0.91	-1.76	-2.13				0.28	0.79	
FF3F+OV+CC	-	7.45%	-10.11%	-5.74%		-0.15%	6.57%	2.24%	7.77%	7.39%
Chi2/t-stat	919.22	0.89	-1.80	-2.06		-0.02	0.93	0.39	0.90	
FF3F+MOM+OV	-	7.47%	-10.09%	-5.88%	14.09%	-0.04%	6.39%			7.52%
Chi2/t-stat	914.40	0.89	-1.80	-2.10	2.29	-0.01	0.90			
FF3f+MOM+CC	-	8.10%	-10.32%	-6.15%	14.49%			1.93%	7.06%	7.35%
Chi2/t-stat	918.49	0.97	-1.84	-2.21	2.35			0.34	0.83	
All Variables	-	7.74%	-10.29%	-5.83%	14.21%	-0.34%	6.21%	2.19%	7.68%	7.60%
Chi2/t-stat	913.11	0.93	-1.84	-2.10	2.32	-0.05	0.88	0.38	0.89	

1991-1995 (3701 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	32.24%								59.23%
Chi2/t-stat	1062.17	5.63								
FF3F	-	13.19%	21.42%	-0.55%						42.75%
Chi2/t-stat	956.52	2.68	4.86	-0.19						
FF3F+MOM	-	12.52%	21.67%	0.60%	22.80%					45.45%
Chi2/t-stat	944.43	2.56	4.92	0.20	2.99					
MOV	-	16.39%				14.14%	-26.11%			47.99%
Chi2/t-stat	991.45	3.25				2.86	-5.03			
MCC	-	22.77%						15.99%	-10.70%	51.15%
Chi2/t-stat	1012.02	4.28						3.95	-1.83	
FF3F+OV	-	12.95%	21.69%	-0.56%		10.99%	-15.92%			43.43%
Chi2/t-stat	931.74	2.64	4.92	-0.19		2.25	-2.91			
FF3F+CC	-	12.91%	21.53%	-0.26%				9.81%	-5.65%	44.09%
Chi2/t-stat	952.92	2.63	4.91	-0.09				2.55	-0.94	
FF3F+OV+CC	-	13.06%	21.42%	-0.81%		9.09%	-13.96%	10.77%	-6.48%	44.48%
Chi2/t-stat	917.62	2.67	4.88	-0.27		1.90	-2.55	2.78	-1.08	
FF3F+MOM+OV	-	12.48%	21.98%	0.48%	24.28%	11.22%	-15.87%			46.23%
Chi2/t-stat	925.73	2.55	4.99	0.16	3.16	2.29	-2.90			
FF3f+MOM+CC	-	12.51%	21.65%	0.50%	21.88%			8.54%	-5.98%	45.59%
Chi2/t-stat	943.92	2.56	4.94	0.17	2.89			2.28	-1.00	
All Variables	-	12.85%	21.49%	-0.20%	23.86%	9.05%	-13.75%	9.71%	-6.76%	45.78%
Chi2/t-stat	915.41	2.63	4.90	-0.07	3.12	1.89	-2.51	2.57	-1.13	

1996-2000 (3952 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	21.02%								9.93%
Chi2/t-stat	704.37	2.02								
FF3F	-	31.61%	-21.84%	-11.63%						19.84%
Chi2/t-stat	693.14	3.09	-3.52	-2.08						
FF3F+MOM	-	33.63%	-22.92%	-9.98%	41.28%					20.58%
Chi2/t-stat	685.78	3.28	-3.76	-1.80	2.52					
MOV	-	31.85%				2.76%	25.45%			16.62%
Chi2/t-stat	687.59	3.14				0.23	2.22			
MCC	-	24.64%						-0.82%	30.16%	13.91%
Chi2/t-stat	700.15	2.42						-0.07	2.62	
FF3F+OV	-	31.85%	-22.27%	-10.77%		4.61%	14.92%			20.96%
Chi2/t-stat	684.68	3.14	-3.65	-1.91		0.41	1.39			
FF3F+CC	-	31.42%	-21.95%	-11.85%				9.27%	21.54%	20.23%
Chi2/t-stat	688.77	3.11	-3.60	-2.13				1.02	2.03	
FF3F+OV+CC	-	30.83%	-21.74%	-10.89%		5.64%	14.62%	7.14%	22.30%	20.97%
Chi2/t-stat	684.02	3.07	-3.57	-1.94		0.50	1.37	0.80	2.10	
FF3F+MOM+OV	-	32.88%	-22.79%	-9.80%	40.35%	4.00%	16.23%			21.22%
Chi2/t-stat	681.93	3.24	-3.76	-1.77	2.47	0.36	1.51			
FF3f+MOM+CC	-	32.58%	-22.55%	-10.09%	41.27%			7.43%	20.75%	20.91%
Chi2/t-stat	681.79	3.21	-3.72	-1.83	2.53			0.83	1.97	
All Variables	-	31.80%	-22.25%	-9.68%	39.55%	4.99%	16.66%	6.40%	21.41%	21.32%
Chi2/t-stat	679.78	3.16	-3.67	-1.75	2.44	0.44	1.56	0.72	2.03	

2001-2005 (3828 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	15.76%								20.16%
Chi2/t-stat	1094.13	1.65								
FF3F	-	13.41%	4.48%	17.26%						18.76%
Chi2/t-stat	1051.08	1.48	0.74	3.07						
FF3F+MOM	-	11.94%	4.32%	13.47%	57.13%					22.60%
Chi2/t-stat	1036.47	1.33	0.72	2.54	3.68					
MOV	-	17.59%				30.59%	6.73%			22.90%
Chi2/t-stat	1034.66	1.87				2.25	0.50			
MCC	-	16.10%						31.13%	-13.64%	21.97%
Chi2/t-stat	1059.57	1.70						2.52	-1.39	
FF3F+OV	-	12.65%	4.01%	9.29%		31.15%	20.64%			21.79%
Chi2/t-stat	1019.23	1.40	0.67	1.89		2.35	1.54			
FF3F+CC	-	11.28%	5.76%	11.92%				30.94%	-9.38%	20.74%
Chi2/t-stat	1048.29	1.27	0.97	2.37				2.73	-0.94	
FF3F+OV+CC	-	11.62%	4.68%	9.82%		34.82%	19.13%	21.77%	-5.44%	23.72%
Chi2/t-stat	1004.53	1.30	0.79	2.00		2.60	1.47	2.05	-0.54	
FF3F+MOM+OV	-	12.46%	4.11%	9.64%	44.17%	31.16%	20.43%			22.22%
Chi2/t-stat	1019.05	1.39	0.69	2.00	3.21	2.35	1.53			
FF3f+MOM+CC	-	10.84%	5.44%	13.11%	56.76%			27.91%	-8.63%	23.61%
Chi2/t-stat	1034.19	1.22	0.92	2.60	3.91			2.51	-0.86	
All Variables	-	11.61%	4.68%	9.86%	45.08%	34.82%	19.12%	21.78%	-5.46%	23.75%
Chi2/t-stat	1004.50	1.30	0.79	2.06	3.30	2.60	1.47	2.05	-0.54	

2006-2011 (3039 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK	R-squared
CAPM	-	7.53%								2.68%
Chi2/t-stat	1514.91	0.67								
FF3F	-	14.05%	-8.54%	-9.89%						14.05%
Chi2/t-stat	1492.21	1.24	-1.69	-1.91						
FF3F+MOM	-	13.94%	-8.49%	-9.45%	12.90%					14.07%
Chi2/t-stat	1485.89	1.24	-1.71	-1.97	0.71					
MOV	-	12.72%				25.27%	42.78%			13.37%
Chi2/t-stat	1446.57	1.10				2.00	2.55			
MCC	-	8.50%						14.50%	6.63%	5.24%
Chi2/t-stat	1510.47	0.75						1.20	0.60	
FF3F+OV	-	16.45%	-11.63%	-8.40%		14.20%	30.06%			16.90%
Chi2/t-stat	1454.91	1.45	-2.51	-1.75		1.21	1.99			
FF3F+CC	-	14.56%	-9.14%	-10.69%				5.07%	13.10%	15.59%
Chi2/t-stat	1487.80	1.29	-1.98	-2.23				0.46	1.17	
FF3F+OV+CC	-	16.34%	-11.36%	-8.41%		12.24%	29.56%	3.49%	11.93%	17.52%
Chi2/t-stat	1454.30	1.44	-2.50	-1.75		1.08	2.02	0.34	1.08	
FF3F+MOM+OV	-	16.35%	-11.71%	-8.24%	32.01%	13.13%	30.23%			17.02%
Chi2/t-stat	1455.24	1.44	-2.53	-1.71	2.10	1.15	2.00			
FF3f+MOM+CC	-	14.81%	-10.35%	-8.74%	29.49%			1.85%	17.46%	16.75%
Chi2/t-stat	1474.65	1.31	-2.26	-1.81	1.94			0.17	1.53	
All Variables	-	16.39%	-11.32%	-8.47%	30.47%	12.53%	29.53%	3.53%	11.70%	17.49%
Chi2/t-stat	1454.05	1.45	-2.50	-1.76	2.04	1.11	2.02	0.35	1.07	

Appendix 4b: Fama-MacBeth Procedure Results, five-year periods

Table 22: Comprehensive Fama-MacBeth procedure results over five-year periods

1951-1955 (407 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	16.41%							
Chi2/t-stat	364.74	0.10							
FF3F	-	22.84%	-7.11%	-5.75%					
Chi2/t-stat	351.44	0.14	-0.10	-0.06					
FF3F+MOM	-	23.36%	-7.38%	-4.85%	38.70%				
Chi2/t-stat	343.04	0.14	-0.10	-0.05	0.15				
MOV	-	19.93%				5.88%	24.88%		
Chi2/t-stat	328.63	0.12				0.04	0.12		
MCC	-	17.12%						13.31%	7.86%
Chi2/t-stat	358.07	0.10						0.09	0.05
FF3F+OV	-	22.84%	-6.91%	-0.95%		7.53%	12.84%		
Chi2/t-stat	315.70	0.14	-0.09	-0.01		0.05	0.07		
FF3F+CC	-	23.52%	-7.75%	-3.07%				4.61%	5.53%
Chi2/t-stat	336.55	0.14	-0.11	-0.03				0.03	0.04
FF3F+OV+CC	-	22.91%	-6.97%	-0.92%		7.19%	12.66%	6.34%	8.90%
Chi2/t-stat	314.63	0.14	-0.10	-0.01		0.05	0.07	0.04	0.06
FF3F+MOM+OV	-	22.76%	-6.90%	-1.31%	35.56%	8.00%	13.43%		
Chi2/t-stat	315.62	0.14	-0.09	-0.01	0.14	0.05	0.07		
FF3f+MOM+CC	-	23.85%	-7.96%	-2.43%	37.58%			4.64%	6.50%
Chi2/t-stat	327.08	0.14	-0.11	-0.02	0.15			0.03	0.04
All Variables	-	22.93%	-7.15%	-1.39%	35.85%	6.95%	13.26%	5.83%	7.91%
Chi2/t-stat	314.47	0.14	-0.10	-0.01	0.14	0.05	0.07	0.04	0.05

1956-1960 (355 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	7.29%							
Chi2/t-stat	403.53	0.04							
FF3F	-	9.79%	-2.12%	-13.99%					
Chi2/t-stat	382.28	0.05	-0.02	-0.14					
FF3F+MOM	-	11.16%	-4.72%	-10.09%	52.60%				
Chi2/t-stat	339.83	0.06	-0.05	-0.10	0.18				
MOV	-	9.02%				30.87%	21.93%		
Chi2/t-stat	316.91	0.05				0.15	0.11		
MCC	-	6.03%						17.29%	-5.76%
Chi2/t-stat	378.02	0.03						0.07	-0.04
FF3F+OV	-	11.21%	-5.06%	-7.76%		28.18%	16.56%		
Chi2/t-stat	312.03	0.06	-0.06	-0.07		0.14	0.08		
FF3F+CC	-	11.24%	-3.34%	-11.55%				6.32%	-5.16%
Chi2/t-stat	348.56	0.06	-0.04	-0.12				0.03	-0.03
FF3F+OV+CC	-	11.36%	-5.05%	-8.04%		26.63%	16.86%	3.59%	-1.37%
Chi2/t-stat	306.49	0.06	-0.06	-0.08		0.13	0.08	0.02	-0.01
FF3F+MOM+OV	-	11.24%	-5.26%	-7.34%	44.01%	28.69%	16.07%		
Chi2/t-stat	310.96	0.06	-0.06	-0.07	0.15	0.14	0.08		
FF3f+MOM+CC	-	11.81%	-5.24%	-8.57%	48.23%			4.59%	-4.51%
Chi2/t-stat	311.34	0.07	-0.06	-0.08	0.17			0.02	-0.03
All Variables	-	11.46%	-5.31%	-7.65%	44.70%	25.84%	16.44%	3.70%	-2.80%
Chi2/t-stat	301.79	0.06	-0.06	-0.07	0.15	0.13	0.08	0.02	-0.02

1961-1965 (470 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	13.14%							
Chi2/t-stat	472.24	0.07							
FF3F	-	12.43%	-0.99%	4.93%					
Chi2/t-stat	463.36	0.07	-0.01	0.05					
FF3F+MOM	-	14.71%	-4.32%	5.42%	49.43%				
Chi2/t-stat	427.89	0.08	-0.05	0.05	0.19				
MOV	-	15.89%				8.19%	17.23%		
Chi2/t-stat	441.06	0.09				0.05	0.08		
MCC	-	12.28%						1.12%	-0.78%
Chi2/t-stat	471.80	0.07						0.01	-0.01
FF3F+OV	-	15.57%	-3.42%	5.97%		11.80%	8.62%		
Chi2/t-stat	433.18	0.09	-0.04	0.06		0.07	0.04		
FF3F+CC	-	13.86%	-1.86%	6.71%				10.60%	0.56%
Chi2/t-stat	461.35	0.08	-0.02	0.07				0.06	0.00
FF3F+OV+CC	-	15.54%	-4.08%	6.54%		17.12%	12.15%	6.20%	7.44%
Chi2/t-stat	428.32	0.09	-0.04	0.06		0.09	0.06	0.04	0.05
FF3F+MOM+OV	-	15.92%	-5.02%	6.50%	48.68%	15.16%	13.63%		
Chi2/t-stat	417.68	0.09	-0.05	0.06	0.19	0.08	0.07		
FF3f+MOM+CC	-	16.08%	-4.92%	5.64%	50.06%			5.25%	3.17%
Chi2/t-stat	422.71	0.09	-0.05	0.06	0.20			0.03	0.02
All Variables	-	15.77%	-4.83%	6.07%	49.31%	13.38%	12.33%	4.63%	3.27%
Chi2/t-stat	416.04	0.09	-0.05	0.06	0.19	0.08	0.06	0.03	0.02

1966-1970 (1278 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	3.31%							
Chi2/t-stat	9679.12	0.02							
FF3F	-	3.80%	-0.06%	-10.02%					
Chi2/t-stat	9673.76	0.02	0.00	-0.08					
FF3F+MOM	-	4.86%	-0.85%	-10.51%	-10.06%				
Chi2/t-stat	9660.47	0.03	-0.01	-0.08	-0.03				
MOV	-	7.20%				4.63%	19.60%		
Chi2/t-stat	9658.61	0.04				0.02	0.08		
MCC	-	5.27%						-11.53%	4.12%
Chi2/t-stat	9675.86	0.03						-0.07	0.02
FF3F+OV	-	6.00%	-2.63%	-11.79%		9.61%	20.52%		
Chi2/t-stat	9651.42	0.03	-0.02	-0.10		0.05	0.08		
FF3F+CC	-	5.95%	-1.74%	-12.63%				-6.08%	-0.39%
Chi2/t-stat	9666.76	0.03	-0.01	-0.11				-0.04	0.00
FF3F+OV+CC	-	6.48%	-2.74%	-12.39%		10.68%	16.41%	-7.04%	2.54%
Chi2/t-stat	9648.33	0.03	-0.02	-0.11		0.06	0.07	-0.04	0.01
FF3F+MOM+OV	-	6.43%	-2.70%	-12.35%	7.21%	10.14%	17.56%		
Chi2/t-stat	9648.49	0.03	-0.02	-0.10	0.02	0.05	0.07		
FF3f+MOM+CC	-	5.78%	-2.00%	-12.11%	8.31%			-8.80%	-2.17%
Chi2/t-stat	9649.80	0.03	-0.01	-0.10	0.03			-0.05	-0.01
All Variables	-	6.44%	-2.75%	-12.35%	11.36%	10.70%	16.48%	-7.39%	2.03%
Chi2/t-stat	9642.99	0.03	-0.02	-0.11	0.04	0.06	0.07	-0.05	0.01

1971-1975 (1498 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	0.11%							
Chi2/t-stat	2073.07	0.00							
FF3F	-	4.44%	-7.93%	-5.72%					
Chi2/t-stat	2070.25	0.02	-0.05	-0.05					
FF3F+MOM	-	5.08%	-9.21%	-4.48%	23.67%				
Chi2/t-stat	2042.65	0.02	-0.06	-0.04	0.09				
MOV	-	4.02%				4.53%	22.55%		
Chi2/t-stat	2042.73	0.02				0.02	0.09		
MCC	-	3.05%						-8.60%	8.77%
Chi2/t-stat	2071.94	0.01						-0.05	0.03
FF3F+OV	-	4.68%	-9.00%	-5.56%		8.14%	13.24%		
Chi2/t-stat	2035.60	0.02	-0.06	-0.05		0.04	0.06		
FF3F+CC	-	4.88%	-8.59%	-6.08%				-2.05%	4.61%
Chi2/t-stat	2070.25	0.02	-0.05	-0.06				-0.01	0.02
FF3F+OV+CC	-	4.92%	-9.20%	-5.65%		7.25%	13.42%	-4.50%	7.22%
Chi2/t-stat	2034.62	0.02	-0.06	-0.05		0.03	0.06	-0.03	0.03
FF3F+MOM+OV	-	4.77%	-9.26%	-4.76%	22.88%	7.33%	13.67%		
Chi2/t-stat	2028.93	0.02	-0.06	-0.04	0.09	0.03	0.06		
FF3f+MOM+CC	-	5.29%	-9.49%	-4.76%	22.24%			-2.86%	6.72%
Chi2/t-stat	2042.56	0.02	-0.06	-0.04	0.09			-0.02	0.03
All Variables	-	4.68%	-9.17%	-4.71%	23.21%	7.77%	13.74%	-4.06%	7.59%
Chi2/t-stat	2026.51	0.02	-0.06	-0.04	0.09	0.04	0.06	-0.02	0.03

1976-1980 (2241 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	21.99%							
Chi2/t-stat	1588.88	0.10							
FF3F	-	14.36%	8.97%	-10.61%					
Chi2/t-stat	1568.82	0.07	0.06	-0.12					
FF3F+MOM	-	14.60%	9.02%	-7.30%	42.21%				
Chi2/t-stat	1532.17	0.07	0.06	-0.09	0.16				
MOV	-	17.52%				27.16%	-1.43%		
Chi2/t-stat	1557.85	0.08				0.13	-0.01		
MCC	-	16.84%						24.30%	2.95%
Chi2/t-stat	1573.87	0.08						0.14	0.01
FF3F+OV	-	17.24%	5.63%	-8.62%		24.14%	0.22%		
Chi2/t-stat	1545.71	0.08	0.04	-0.10		0.12	0.00		
FF3F+CC	-	15.56%	7.26%	-9.85%				19.61%	4.49%
Chi2/t-stat	1560.99	0.07	0.05	-0.11				0.12	0.02
FF3F+OV+CC	-	17.69%	5.20%	-8.79%		22.16%	-0.96%	11.35%	7.40%
Chi2/t-stat	1537.52	0.08	0.04	-0.10		0.11	-0.01	0.08	0.04
FF3F+MOM+OV	-	16.42%	6.83%	-7.67%	43.08%	24.38%	0.00%		
Chi2/t-stat	1530.85	0.08	0.05	-0.09	0.17	0.12	0.00		
FF3f+MOM+CC	-	15.49%	7.60%	-7.73%	38.53%			13.97%	3.29%
Chi2/t-stat	1529.01	0.07	0.05	-0.09	0.15			0.09	0.02
All Variables	-	17.31%	5.86%	-8.22%	44.81%	20.57%	-0.02%	12.74%	4.51%
Chi2/t-stat	1523.82	0.08	0.04	-0.10	0.18	0.11	0.00	0.08	0.02

1981-1985 (2410 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	4.47%							
Chi2/t-stat	1886.17	0.02							
FF3F	-	9.99%	-8.83%	10.08%					
Chi2/t-stat	1880.44	0.05	-0.06	0.07					
FF3F+MOM	-	11.85%	-10.98%	10.75%	72.84%				
Chi2/t-stat	1866.86	0.05	-0.08	0.08	0.18				
MOV	-	8.78%				17.66%	63.55%		
Chi2/t-stat	1851.40	0.04				0.08	0.20		
MCC	-	4.81%						-27.35%	-2.23%
Chi2/t-stat	1868.02	0.02						-0.17	-0.01
FF3F+OV	-	9.48%	-11.11%	6.46%		26.56%	50.79%		
Chi2/t-stat	1833.27	0.04	-0.08	0.05		0.11	0.17		
FF3F+CC	-	5.64%	-10.57%	4.02%				-14.36%	-12.48%
Chi2/t-stat	1841.60	0.03	-0.08	0.03				-0.09	-0.06
FF3F+OV+CC	-	7.06%	-11.08%	4.51%		22.74%	41.83%	-10.38%	-4.88%
Chi2/t-stat	1807.59	0.03	-0.08	0.03		0.10	0.15	-0.07	-0.02
FF3F+MOM+OV	-	8.68%	-10.83%	5.25%	31.80%	25.65%	49.76%		
Chi2/t-stat	1832.52	0.04	-0.08	0.04	0.11	0.11	0.16		
FF3f+MOM+CC	-	7.50%	-11.49%	5.57%	39.00%			-10.45%	-8.47%
Chi2/t-stat	1829.54	0.03	-0.08	0.04	0.13			-0.07	-0.04
All Variables	-	7.00%	-11.05%	4.29%	29.08%	22.63%	42.18%	-10.54%	-4.63%
Chi2/t-stat	1806.91	0.03	-0.08	0.03	0.10	0.10	0.15	-0.07	-0.02

1986-1990 (3366 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	3.45%							
Chi2/t-stat	959.12	0.01							
FF3F	-	6.28%	-8.83%	-5.64%					
Chi2/t-stat	945.40	0.02	-0.04	-0.06					
FF3F+MOM	-	7.15%	-9.55%	-5.93%	14.38%				
Chi2/t-stat	933.00	0.02	-0.05	-0.06	0.07				
MOV	-	7.83%				-8.16%	10.76%		
Chi2/t-stat	955.19	0.03				-0.03	0.04		
MCC	-	6.33%						4.98%	12.69%
Chi2/t-stat	958.24	0.02						0.02	0.04
FF3F+OV	-	7.36%	-10.01%	-5.84%		-0.20%	6.79%		
Chi2/t-stat	932.77	0.02	-0.05	-0.06		0.00	0.03		
FF3F+CC	-	7.57%	-9.89%	-5.96%				1.60%	6.73%
Chi2/t-stat	942.40	0.03	-0.05	-0.06				0.01	0.02
FF3F+OV+CC	-	7.45%	-10.11%	-5.74%		-0.15%	6.57%	2.24%	7.77%
Chi2/t-stat	931.61	0.03	-0.05	-0.06		0.00	0.03	0.01	0.03
FF3F+MOM+OV	-	7.47%	-10.09%	-5.88%	14.09%	-0.04%	6.39%		
Chi2/t-stat	927.63	0.03	-0.05	-0.06	0.06	0.00	0.03		
FF3f+MOM+CC	-	8.10%	-10.32%	-6.15%	14.49%			1.93%	7.06%
Chi2/t-stat	931.72	0.03	-0.05	-0.06	0.07			0.01	0.02
All Variables	-	7.74%	-10.29%	-5.83%	14.21%	-0.34%	6.21%	2.19%	7.68%
Chi2/t-stat	926.59	0.03	-0.05	-0.06	0.07	0.00	0.02	0.01	0.03

1991-1995 (3701 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	32.24%							
Chi2/t-stat	1097.36	0.16							
FF3F	-	13.19%	21.42%	-0.55%					
Chi2/t-stat	1024.83	0.08	0.14	-0.01					
FF3F+MOM	-	12.52%	21.67%	0.60%	22.80%				
Chi2/t-stat	1015.31	0.07	0.14	0.01	0.09				
MOV	-	16.39%				14.14%	-26.11%		
Chi2/t-stat	1055.23	0.09				0.08	-0.14		
MCC	-	22.77%						15.99%	-10.70%
Chi2/t-stat	1083.07	0.12						0.11	-0.05
FF3F+OV	-	12.95%	21.69%	-0.56%		10.99%	-15.92%		
Chi2/t-stat	1002.99	0.07	0.14	-0.01		0.06	-0.08		
FF3F+CC	-	12.91%	21.53%	-0.26%				9.81%	-5.65%
Chi2/t-stat	1024.11	0.07	0.14	0.00				0.07	-0.03
FF3F+OV+CC	-	13.06%	21.42%	-0.81%		9.09%	-13.96%	10.77%	-6.48%
Chi2/t-stat	999.70	0.08	0.14	-0.01		0.05	-0.07	0.08	-0.03
FF3F+MOM+OV	-	12.48%	21.98%	0.48%	24.28%	11.22%	-15.87%		
Chi2/t-stat	999.86	0.07	0.14	0.00	0.09	0.07	-0.08		
FF3f+MOM+CC	-	12.51%	21.65%	0.50%	21.88%			8.54%	-5.98%
Chi2/t-stat	1015.25	0.07	0.14	0.00	0.08			0.06	-0.03
All Variables	-	12.85%	21.49%	-0.20%	23.86%	9.05%	-13.75%	9.71%	-6.76%
Chi2/t-stat	997.81	0.07	0.14	0.00	0.09	0.05	-0.07	0.07	-0.03

1996-2000 (3952 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	21.02%							
Chi2/t-stat	707.30	0.06							
FF3F	-	31.61%	-21.84%	-11.63%					
Chi2/t-stat	701.92	0.09	-0.10	-0.06					
FF3F+MOM	-	33.63%	-22.92%	-9.98%	41.28%				
Chi2/t-stat	697.36	0.09	-0.11	-0.05	0.07				
MOV	-	31.85%				2.76%	25.45%		
Chi2/t-stat	698.07	0.09				0.01	0.06		
MCC	-	24.64%						-0.82%	30.16%
Chi2/t-stat	706.93	0.07						0.00	0.07
FF3F+OV	-	31.85%	-22.27%	-10.77%		4.61%	14.92%		
Chi2/t-stat	695.64	0.09	-0.10	-0.05		0.01	0.04		
FF3F+CC	-	31.42%	-21.95%	-11.85%				9.27%	21.54%
Chi2/t-stat	698.81	0.09	-0.10	-0.06				0.03	0.06
FF3F+OV+CC	-	30.83%	-21.74%	-10.89%		5.64%	14.62%	7.14%	22.30%
Chi2/t-stat	695.14	0.09	-0.10	-0.05		0.01	0.04	0.02	0.06
FF3F+MOM+OV	-	32.88%	-22.79%	-9.80%	40.35%	4.00%	16.23%		
Chi2/t-stat	694.13	0.09	-0.11	-0.05	0.07	0.01	0.04		
FF3f+MOM+CC	-	32.58%	-22.55%	-10.09%	41.27%			7.43%	20.75%
Chi2/t-stat	694.52	0.09	-0.10	-0.05	0.07			0.02	0.06
All Variables	-	31.80%	-22.25%	-9.68%	39.55%	4.99%	16.66%	6.40%	21.41%
Chi2/t-stat	692.62	0.09	-0.10	-0.05	0.07	0.01	0.04	0.02	0.06

2001-2005 (3828 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	15.76%							
Chi2/t-stat	1097.00	0.05							
FF3F	-	13.41%	4.48%	17.26%					
Chi2/t-stat	1081.77	0.04	0.02	0.09					
FF3F+MOM	-	11.94%	4.32%	13.47%	57.13%				
Chi2/t-stat	1071.93	0.04	0.02	0.07	0.10				
MOV	-	17.59%				30.59%	6.73%		
Chi2/t-stat	1065.63	0.05				0.06	0.01		
MCC	-	16.10%						31.13%	-13.64%
Chi2/t-stat	1086.08	0.05						0.07	-0.04
FF3F+OV	-	12.65%	4.01%	9.29%		31.15%	20.64%		
Chi2/t-stat	1055.18	0.04	0.02	0.05		0.07	0.04		
FF3F+CC	-	11.28%	5.76%	11.92%				30.94%	-9.38%
Chi2/t-stat	1077.76	0.04	0.03	0.07				0.08	-0.03
FF3F+OV+CC	-	11.62%	4.68%	9.82%		34.82%	19.13%	21.77%	-5.44%
Chi2/t-stat	1049.77	0.04	0.02	0.06		0.07	0.04	0.06	-0.02
FF3F+MOM+OV	-	12.46%	4.11%	9.64%	44.17%	31.16%	20.43%		
Chi2/t-stat	1055.12	0.04	0.02	0.06	0.09	0.07	0.04		
FF3f+MOM+CC	-	10.84%	5.44%	13.11%	56.76%			27.91%	-8.63%
Chi2/t-stat	1071.54	0.03	0.03	0.07	0.11			0.07	-0.02
All Variables	-	11.61%	4.68%	9.86%	45.08%	34.82%	19.12%	21.78%	-5.46%
Chi2/t-stat	1049.69	0.04	0.02	0.06	0.09	0.07	0.04	0.06	-0.02

2007-2011 (3039 firms)	alpha	Rm	SMB	HML	MOM	OMEGA	VEGA	CS	CK
CAPM	-	7.53%							
Chi2/t-stat	1515.40	0.02							
FF3F	-	14.05%	-8.54%	-9.89%					
Chi2/t-stat	1506.98	0.03	-0.04	-0.05					
FF3F+MOM	-	13.94%	-8.49%	-9.45%	12.90%				
Chi2/t-stat	1499.88	0.03	-0.04	-0.05	0.02				
MOV	-	12.72%				25.27%	42.78%		
Chi2/t-stat	1487.85	0.03				0.05	0.07		
MCC	-	8.50%						14.50%	6.63%
Chi2/t-stat	1514.83	0.02						0.03	0.02
FF3F+OV	-	16.45%	-11.63%	-8.40%		14.20%	30.06%		
Chi2/t-stat	1486.73	0.04	-0.06	-0.05		0.03	0.05		
FF3F+CC	-	14.56%	-9.14%	-10.69%				5.07%	13.10%
Chi2/t-stat	1506.27	0.03	-0.05	-0.06				0.01	0.03
FF3F+OV+CC	-	16.34%	-11.36%	-8.41%		12.24%	29.56%	3.49%	11.93%
Chi2/t-stat	1486.14	0.04	-0.06	-0.05		0.03	0.05	0.01	0.03
FF3F+MOM+OV	-	16.35%	-11.71%	-8.24%	32.01%	13.13%	30.23%		
Chi2/t-stat	1486.73	0.04	-0.07	-0.04	0.05	0.03	0.05		
FF3f+MOM+CC	-	14.81%	-10.35%	-8.74%	29.49%			1.85%	17.46%
Chi2/t-stat	1498.01	0.03	-0.06	-0.05	0.05			0.00	0.04
All Variables	-	16.39%	-11.32%	-8.47%	30.47%	12.53%	29.53%	3.53%	11.70%
Chi2/t-stat	1486.11	0.04	-0.06	-0.05	0.05	0.03	0.05	0.01	0.03

Appendix 4c: Graphical representation of the pricing errors, five-year periods

Figure 12: Cross-sectional pricing errors, period 1951-1955

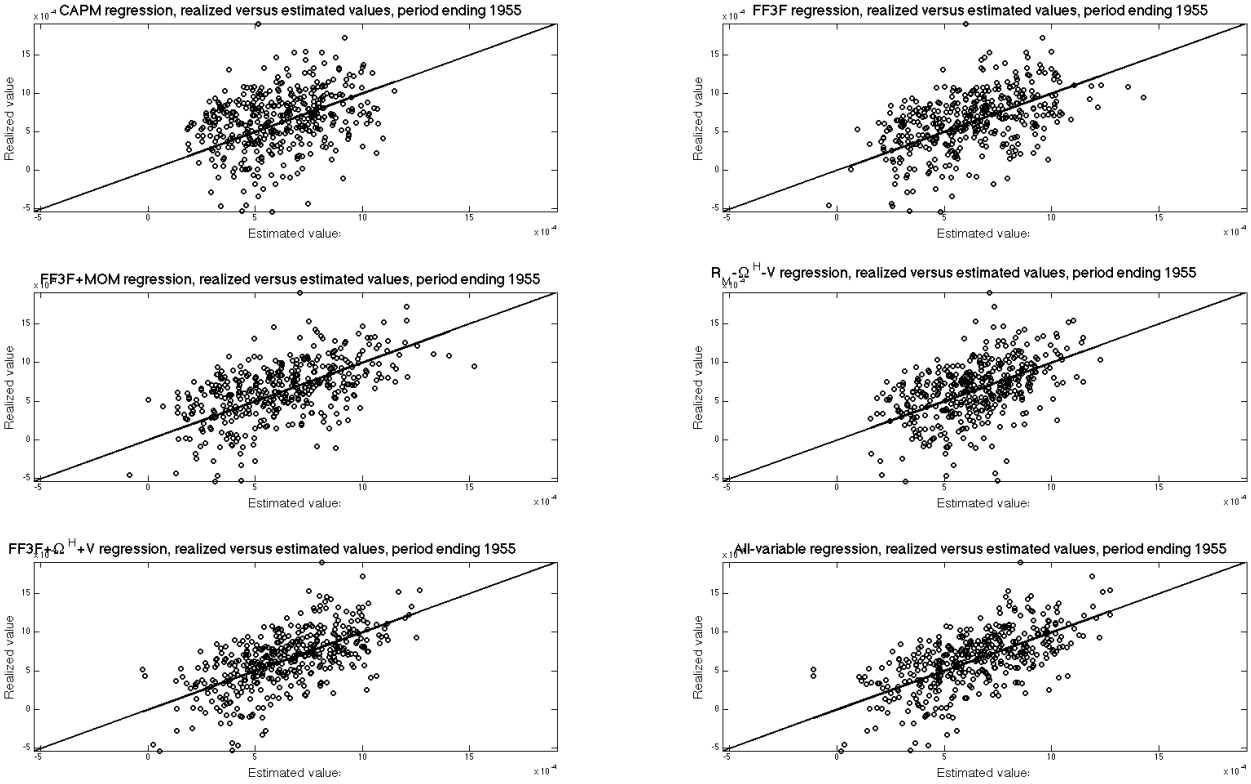


Figure 13: Cross-sectional pricing errors, period 1956-1960

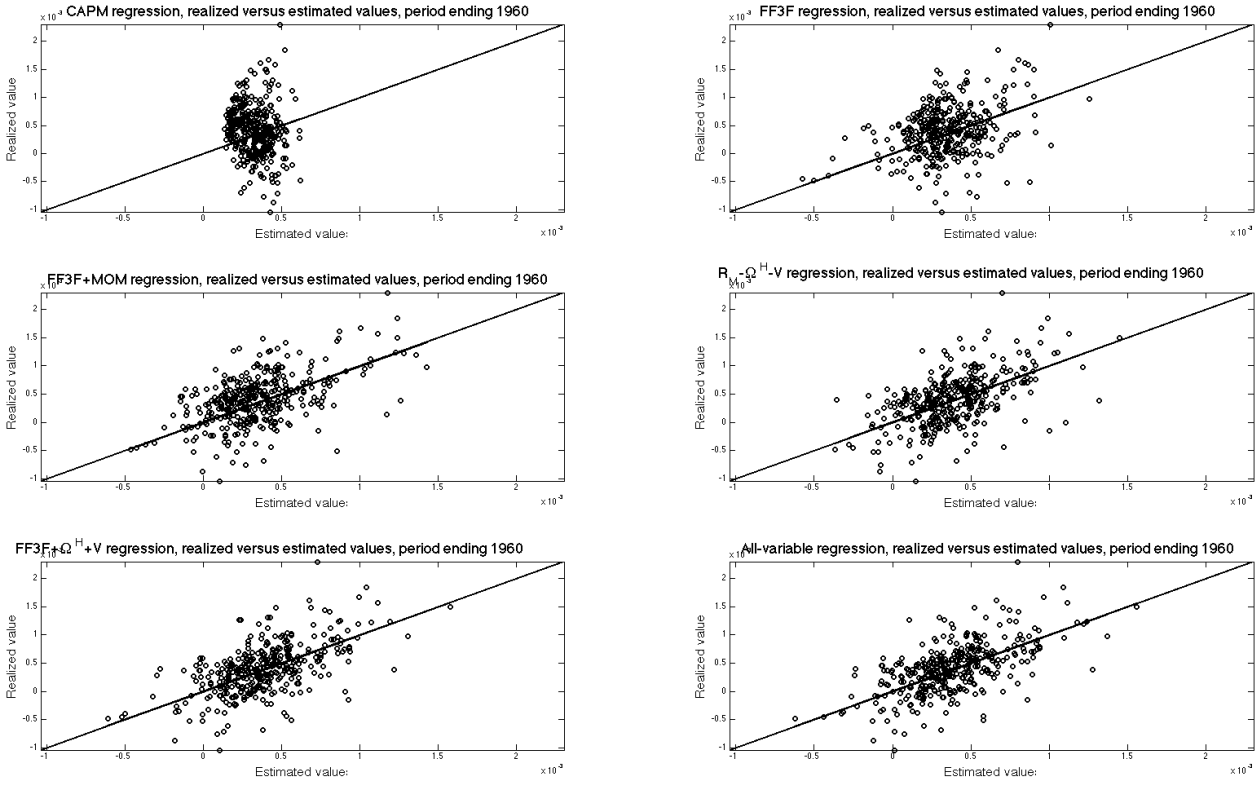


Figure 14: Cross-sectional pricing errors, period 1961-1965

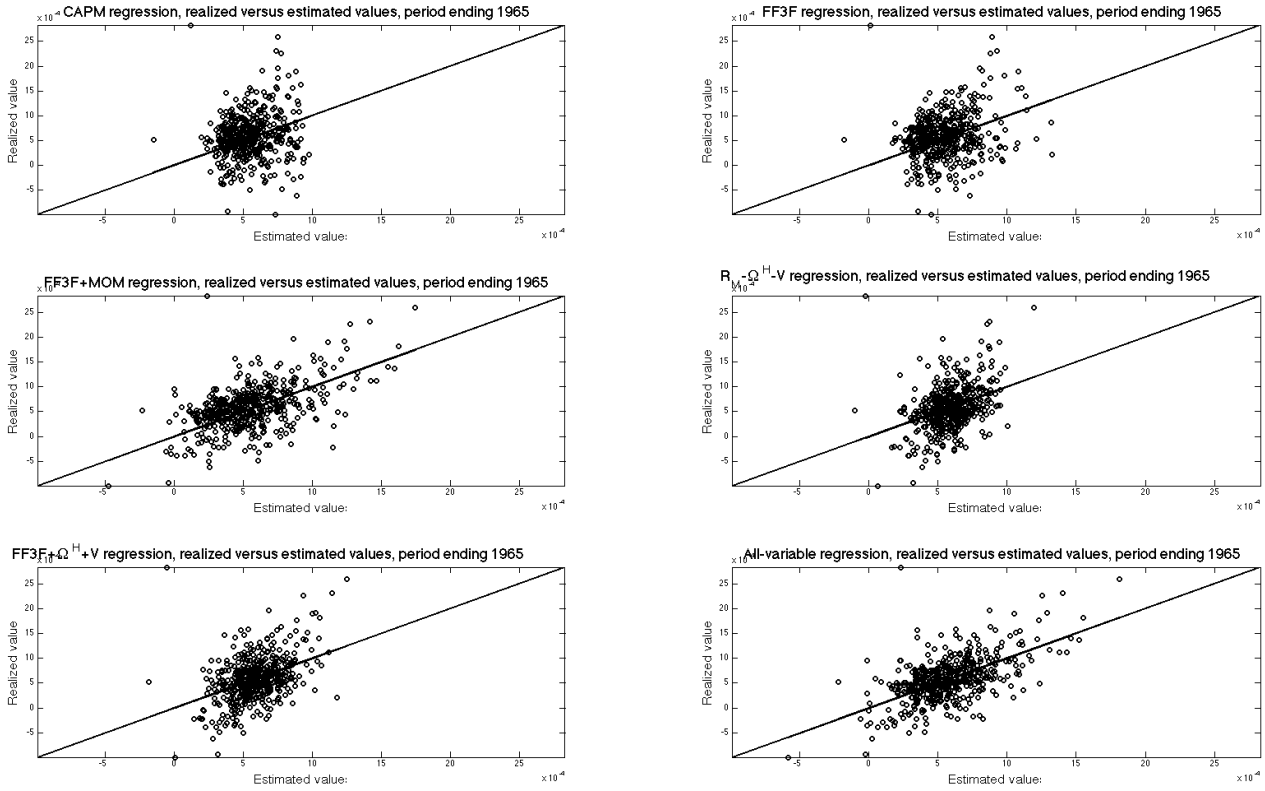


Figure 15: Cross-sectional pricing errors, period 1966-1970

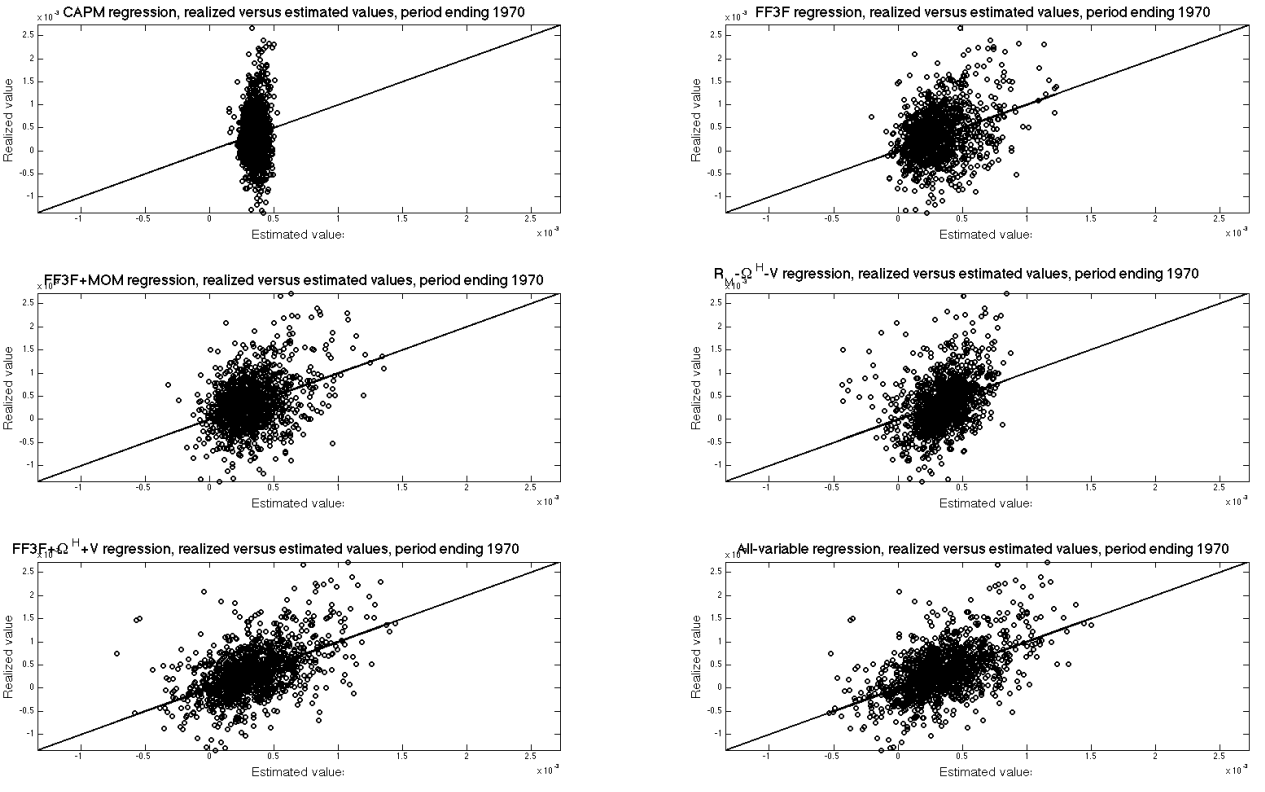


Figure 16: Cross-sectional pricing errors, period 1971-1975

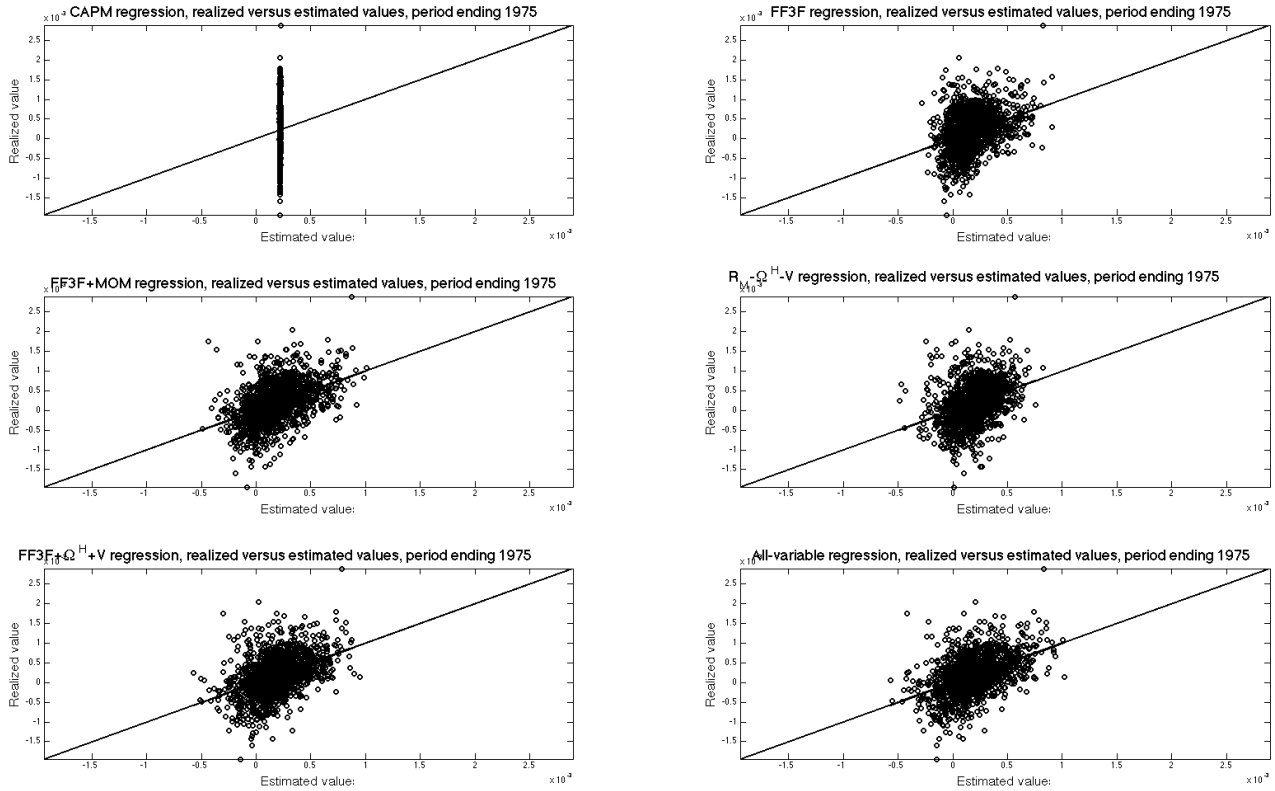


Figure 17: Cross-sectional pricing errors, period 1976-1980

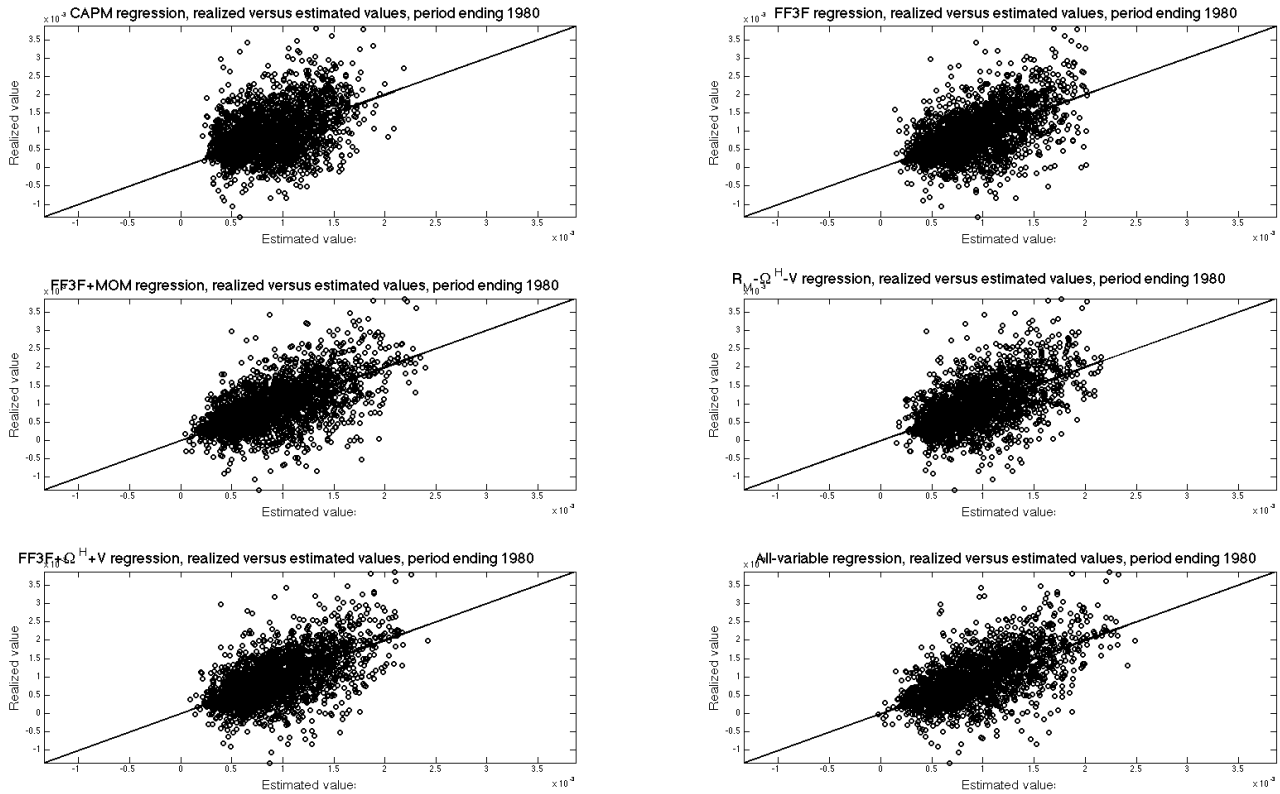
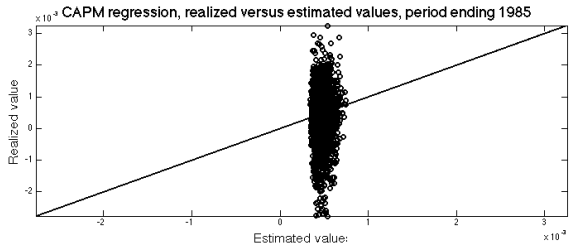
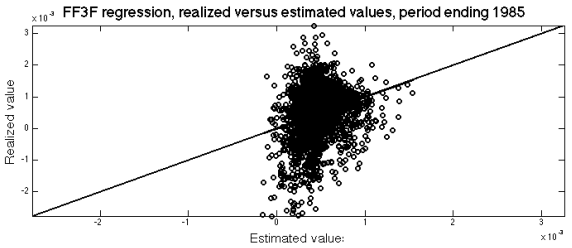
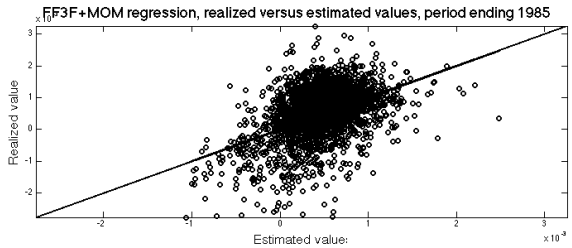
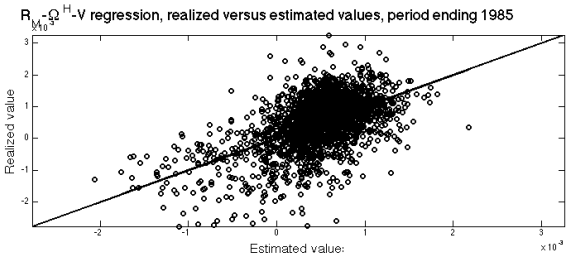


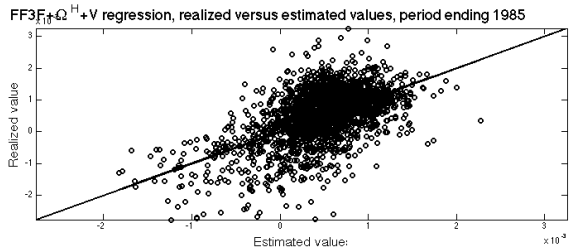
Figure 18: Cross-sectional pricing errors, period 1981-1985











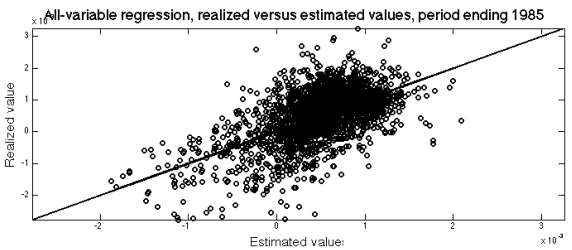
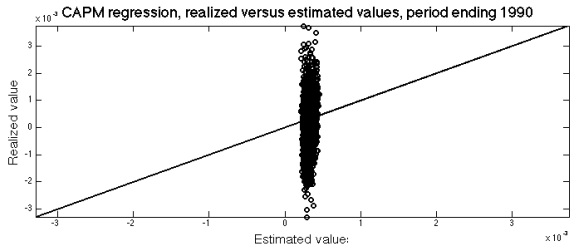
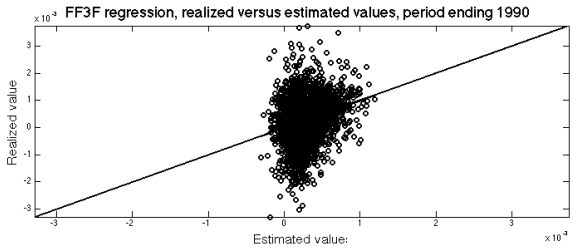
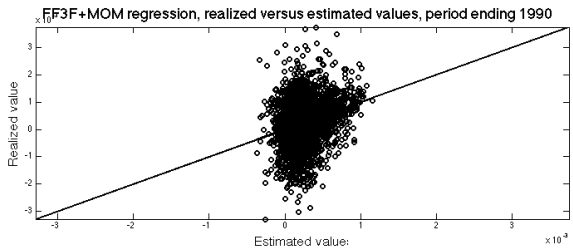
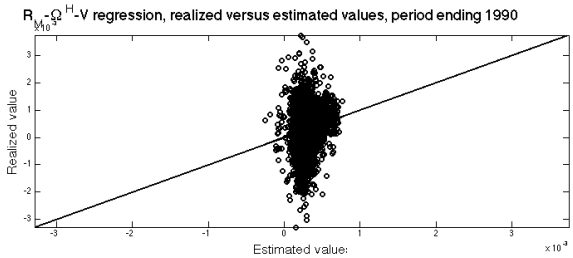


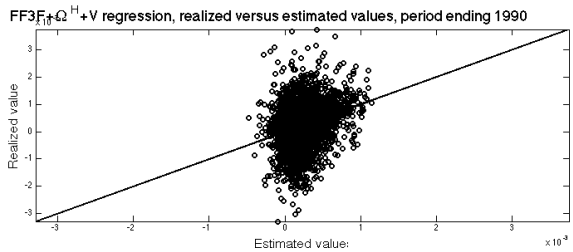
Figure 19: Cross-sectional pricing errors, period 1986-1990











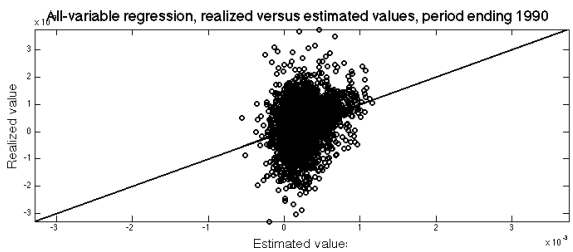


Figure 20: Cross-sectional pricing errors, period 1991-1995

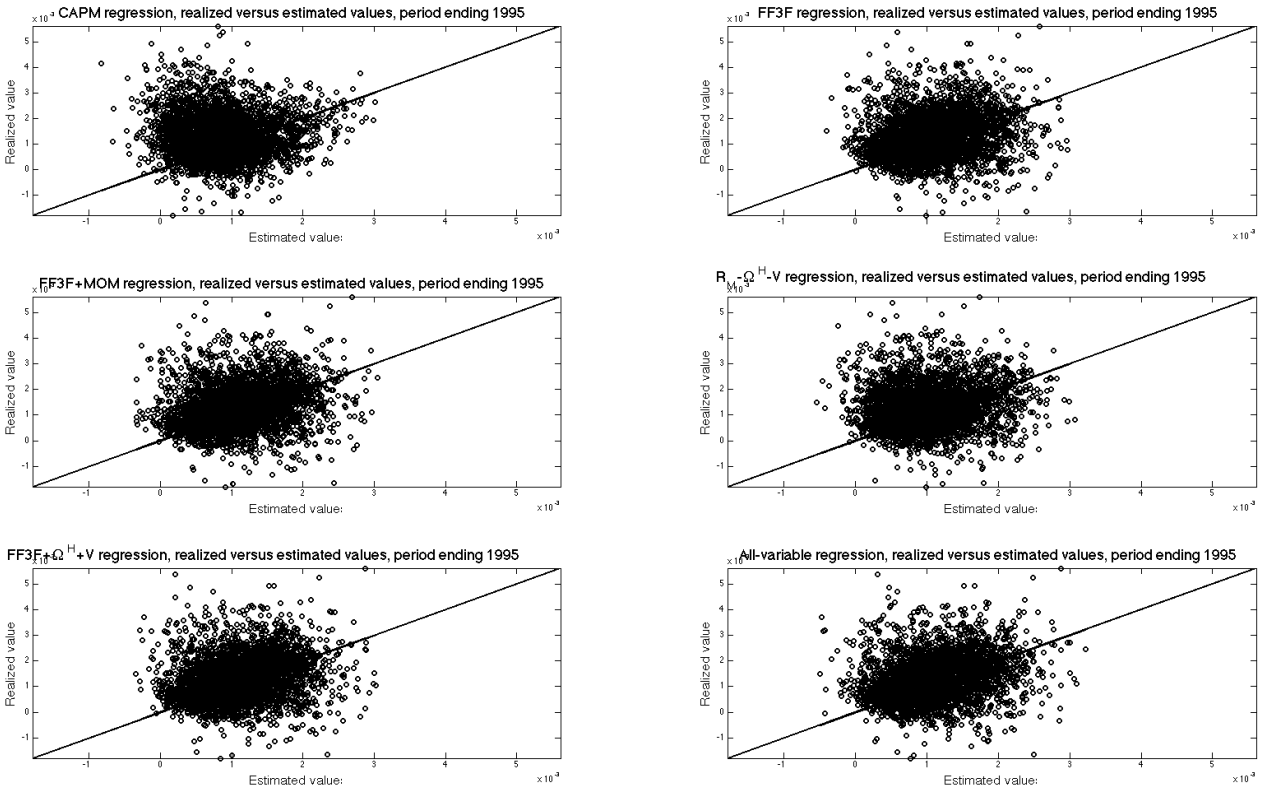


Figure 21: Cross-sectional pricing errors, period 1996-2000

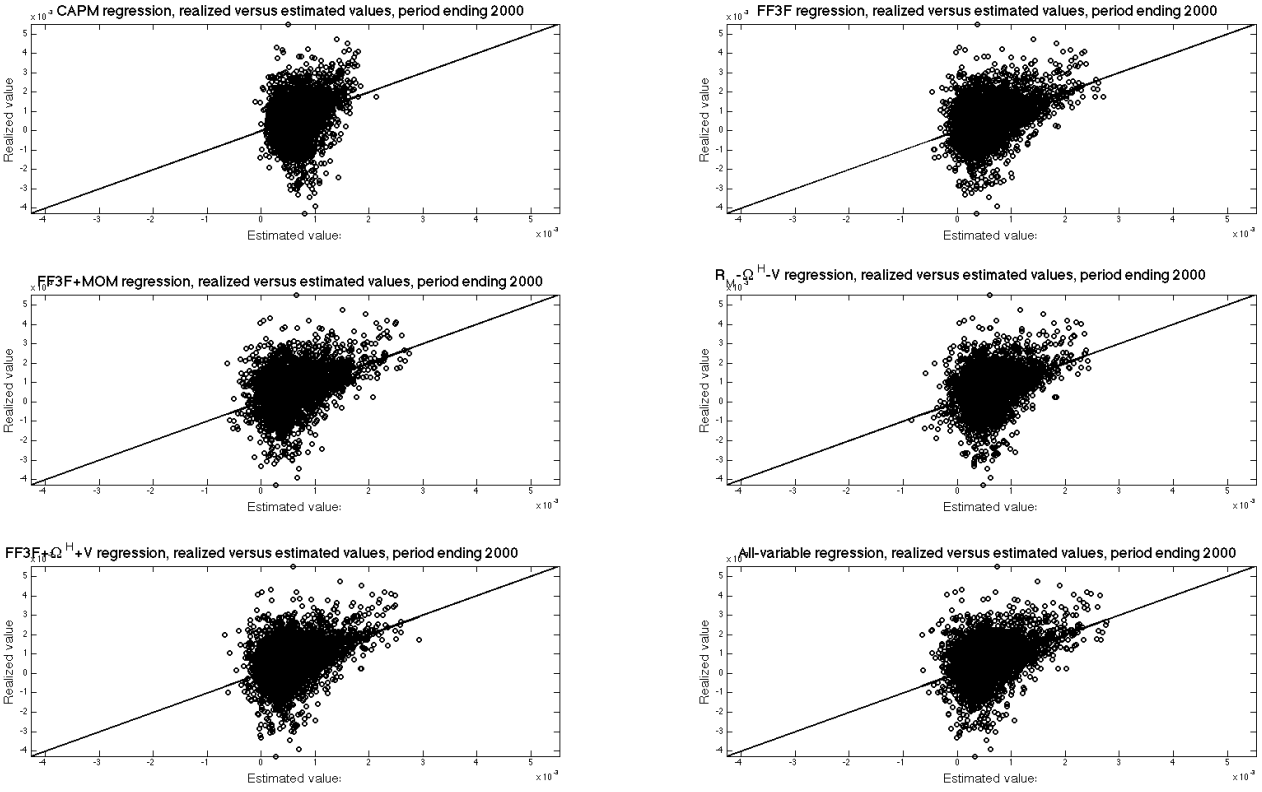


Figure 22: Cross-sectional pricing errors, period 2001-2005

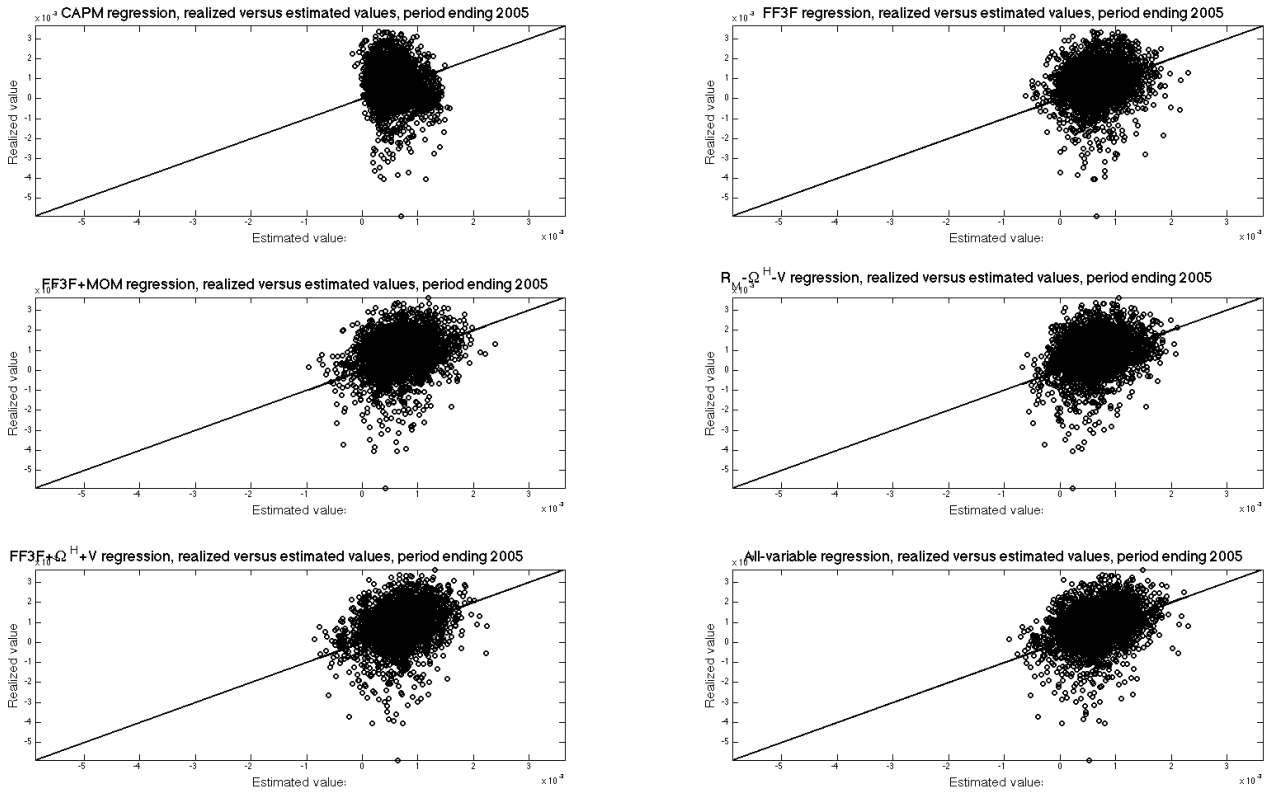
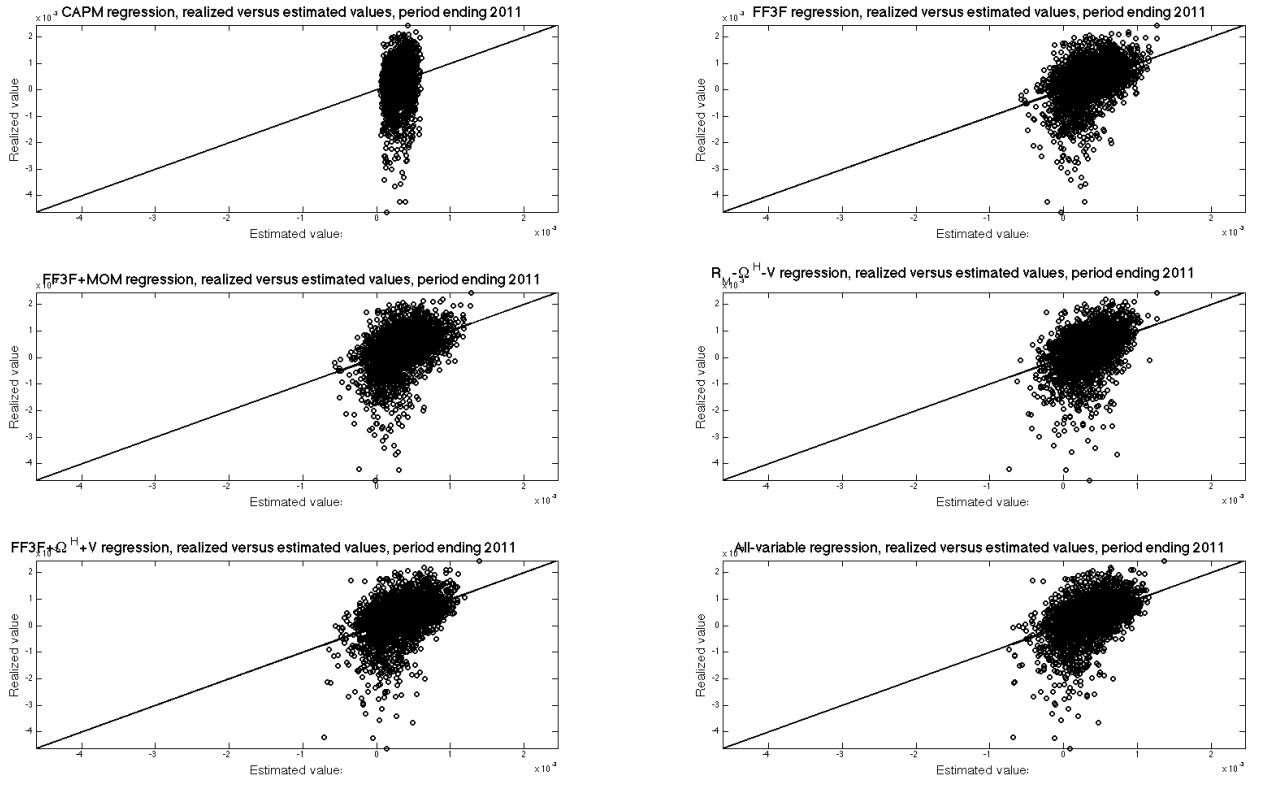


Figure 23: Cross-sectional pricing errors, period 2006-2011



Appendix 3d: Regression tests of the cross-sectional explanatory powers

Table 23: All test result for the cross-sectional explanatory power estimates

		Regression against time			Regression against the market return		
Absolute Explanatory Power	**Average**	**alpha**	**beta**	**R-squared**	**alpha**	**beta**	**R-squared**
CAPM	**13.95%**	**19.50%**	-0.0019	3.86%	**13.95%**	**0.52**	29.38%
t-stat	6.51	4.70	-1.55		7.75	5.00	
FF3F	**16.36%**	**22.59%**	-0.0021	5.91%	**16.36%**	**0.44**	26.08%
t-stat	8.43	6.08	-1.94		9.81	4.60	
FF3F+MOM	**19.99%**	**29.40%**	**-0.0031**	12.14%	**19.99%**	**0.43**	22.35%
t-stat	9.77	7.76	-2.88		11.09	4.16	
MOV	**18.41%**	**24.11%**	-0.0019	5.65%	**18.41%**	**0.41**	25.43%
t-stat	10.14	6.92	-1.90		11.74	4.52	
MCC	**14.62%**	**19.15%**	-0.0015	3.69%	**14.62%**	**0.43**	29.94%
t-stat	8.18	5.53	-1.52		9.78	5.06	
FF3F+OV	**20.81%**	**29.11%**	**-0.0028**	11.25%	**20.81%**	**0.37**	19.30%
t-stat	11.11	8.35	-2.76		12.37	3.79	
FF3F+CC	**18.16%**	**25.06%**	**-0.0023**	8.87%	**18.16%**	**0.35**	19.89%
t-stat	10.35	7.57	-2.42		11.56	3.86	
FF3F+OV+CC	**21.75%**	**30.91%**	**-0.0031**	13.92%	**21.75%**	**0.34**	16.91%
t-stat	11.69	9.06	-3.11		12.83	3.49	
FF3F+MOM+OV	**21.92%**	**31.02%**	**-0.0030**	12.90%	**21.92%**	**0.37**	18.39%
t-stat	11.42	8.77	-2.98		12.65	3.68	
FF3f+MOM+CC	**21.14%**	**30.20%**	**-0.0030**	13.19%	**21.14%**	**0.34**	16.80%
t-stat	11.18	8.68	-3.02		12.25	3.48	
FF3f+MOM+OV+CC	**22.60%**	**32.11%**	**-0.0032**	14.50%	**22.60%**	**0.33**	15.80%
t-stat	11.96	9.30	-3.1897		13.03	3.36	
Difference with the CAPM	**Average difference**	**alpha**	**beta**	**R-squared**	**alpha**	**beta**	**R-squared**
FF3F	**2.41%**	**3.09%**	-0.0002	0.46%	**2.41%**	-0.08	5.07%
t-stat	3.19	2.07	-0.52		3.27	-1.79	
FF3F+MOM	**6.04%**	**9.89%**	**-0.0013**	8.09%	**6.04%**	-0.09	3.59%
t-stat	5.88	5.09	-2.30		5.99	-1.49	
MOV	**4.46%**	**4.60%**	0.0000	0.01%	**4.46%**	**-0.11**	6.58%
t-stat	4.66	2.43	-0.09		4.82	-2.06	
MCC	0.67%	-0.35%	0.0003	1.38%	0.67%	**-0.08**	7.78%
t-stat	1.02	-0.27	0.92		1.06	-2.25	
FF3F+OV	**6.86%**	**9.60%**	-0.0009	3.41%	**6.86%**	**-0.15**	9.05%
t-stat	6.10	4.40	-1.46		6.40	-2.44	
FF3F+CC	**4.21%**	**5.56%**	-0.0004	1.07%	**4.21%**	**-0.17**	14.65%
t-stat	4.26	2.86	-0.81		4.61	-3.21	
FF3F+OV+CC	**7.80%**	**11.40%**	-0.0012	4.92%	**7.80%**	**-0.18**	10.35%
t-stat	6.33	4.81	-1.76		6.68	-2.63	
FF3F+MOM+OV	**7.97%**	**11.52%**	-0.0012	5.33%	**7.97%**	**-0.15**	8.46%
t-stat	6.85	5.15	-1.84		7.16	-2.36	
FF3f+MOM+CC	**7.18%**	**10.70%**	-0.0012	5.09%	**7.18%**	**-0.17**	10.74%
t-stat	6.09	4.71	-1.79		6.45	-2.69	
FF3f+MOM+OV+CC	**8.65%**	**12.60%**	-0.0013	5.70%	**8.65%**	**-0.18**	10.69%
t-stat	6.91	5.24	-1.90		7.31	-2.68	

Appendix 5: All Results for International Portfolio Optimization

Table 24: Minimum variance portfolio weights

Unconstrained	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	42.47%	-10.18%	27.63%	3.15%
DAX30	-13.22%	-3.98%	-0.34%	-0.42%
CAC40	-26.23%	-11.10%	-5.67%	-4.83%
FTSE100	39.44%	13.23%	7.86%	5.96%
SMI	30.75%	16.83%	5.06%	6.70%
OMXS30	0.60%	-0.86%	1.94%	0.57%
Nikkei225	20.19%	8.88%	5.47%	4.39%
HSI	6.00%	2.61%	1.83%	1.59%
Co-skewness		43.56%		12.92%
Co-kurtosis		41.01%		29.83%
Omega			32.57%	27.54%
Vega			23.66%	12.61%
Foreign/S&P500	1.35	-2.52	0.58	4.43
abs(Foreign)/S&P500	3.21	5.65	1.02	7.77

130-30	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	38.46%	0.00%	27.56%	3.73%
DAX30	0.00%	0.03%	0.00%	0.11%
CAC40	0.00%	0.00%	-1.84%	0.00%
FTSE100	18.08%	4.13%	5.11%	2.16%
SMI	17.48%	6.22%	3.07%	3.67%
OMXS30	0.00%	0.00%	1.81%	0.11%
Nikkei225	20.28%	9.08%	5.27%	4.59%
HSI	5.70%	1.85%	2.16%	1.74%
Co-skewness		45.59%		15.08%
Co-kurtosis		33.10%		28.91%
Omega			32.76%	27.28%
Vega			24.11%	12.61%
Foreign/S&P500	1.60	-44176.49	0.57	3.32
abs(Foreign)/S&P500	1.60	44176.49	0.70	3.32

Long Only	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	39.55%	0.00%	27.28%	2.77%
DAX30	0.00%	0.00%	0.00%	0.00%
CAC40	0.00%	0.00%	0.00%	0.00%
FTSE100	17.92%	0.48%	4.33%	2.88%
SMI	15.97%	8.97%	2.92%	4.83%
OMXS30	0.00%	0.00%	1.37%	0.07%
Nikkei225	21.00%	9.15%	5.38%	4.31%
HSI	5.56%	2.08%	1.81%	1.57%
Co-skewness		44.61%		13.07%
Co-kurtosis		34.70%		29.83%
Omega			32.97%	27.80%
Vega			23.94%	12.86%
Foreign/S&P500	1.53	140026.20	0.58	4.92
abs(Foreign)/S&P500	1.53	140026.20	0.58	4.92

Table 25: Minimum downside variance portfolio weights

Unconstrained	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	41.03%	-11.02%	28.58%	4.91%
DAX30	-11.05%	-3.61%	0.98%	1.51%
CAC40	-29.40%	-12.45%	-7.52%	-4.80%
FTSE100	43.39%	14.53%	8.42%	5.79%
SMI	27.32%	16.79%	5.26%	3.63%
OMXS30	-0.43%	-1.14%	1.88%	2.82%
Nikkei225	20.33%	7.61%	4.76%	3.16%
HSI	8.83%	3.21%	2.45%	1.10%
Co-skewness		42.10%		13.21%
Co-kurtosis		43.99%		26.10%
Omega			30.47%	27.91%
Vega			24.73%	14.65%
Foreign/S&P500	1.44	-2.26	0.57	2.69
abs(Foreign)/S&P500	3.43	5.38	1.09	4.64

130-30	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	39.55%	-4.64%	28.45%	4.91%
DAX30	-12.28%	-1.50%	0.00%	1.51%
CAC40	-0.03%	-4.75%	0.00%	-4.80%
FTSE100	26.65%	5.73%	4.01%	5.79%
SMI	17.10%	11.54%	2.86%	3.63%
OMXS30	0.00%	0.00%	1.57%	2.82%
Nikkei225	20.26%	7.50%	4.49%	3.16%
HSI	8.76%	3.20%	2.60%	1.10%
Co-skewness		42.71%		13.21%
Co-kurtosis		40.21%		26.10%
Omega			30.93%	27.91%
Vega			25.09%	14.65%
Foreign/S&P500	1.53	-4.68	0.55	2.69
abs(Foreign)/S&P500	2.15	7.37	0.55	4.64

Long Only	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	37.55%	0.00%	28.45%	4.80%
DAX30	0.00%	0.00%	0.00%	0.00%
CAC40	0.00%	0.00%	0.00%	0.00%
FTSE100	22.04%	0.04%	4.01%	3.80%
SMI	11.62%	8.79%	2.86%	2.73%
OMXS30	0.00%	0.00%	1.57%	2.18%
Nikkei225	20.65%	7.47%	4.49%	3.04%
HSI	8.14%	3.17%	2.60%	1.12%
Co-skewness		43.25%		13.30%
Co-kurtosis		37.29%		26.18%
Omega			30.93%	28.07%
Vega			25.09%	14.78%
Foreign/S&P500	1.66	625657.74	0.55	2.68
abs(Foreign)/S&P500	1.66	625657.74	0.55	2.68

Table 26: Maximum skewness portfolio weights

Unconstrained	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	128.10%	18798.31%	419653619.14%	405.36%
DAX30	400.01%	58987.00%	1318478620.73%	1984.64%
CAC40	604.82%	89625.18%	1668069442.03%	2987.45%
FTSE100	459.66%	63019.22%	1452047667.66%	2270.94%
SMI	308.35%	41429.45%	985142418.88%	1684.67%
OMXS30	-1891.24%	-273103.04%	-5793906508.89%	-9550.27%
Nikkei225	105.27%	13462.27%	312212141.41%	519.35%
HSI	-14.97%	-2765.17%	-62072922.57%	-119.25%
Co-skewness		-3286.18%		-124.40%
Co-kurtosis		-6067.05%		588.03%
Omega			112651408.45%	356.57%
Vega			-412275786.84%	-903.09%
Foreign/S&P500	-0.22	-0.50	-0.29	-0.55
abs(Foreign)/S&P500	29.54	28.85	27.62	47.16

130-30	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	28.60%	0.00%	29.79%	9.00%
DAX30	0.00%	0.00%	1.72%	1.22%
CAC40	0.02%	0.00%	0.43%	0.52%
FTSE100	0.03%	9.45%	5.60%	9.55%
SMI	63.87%	15.73%	18.69%	12.97%
OMXS30	-14.74%	-14.99%	-14.94%	-14.99%
Nikkei225	21.56%	9.31%	9.43%	6.28%
HSI	0.67%	0.08%	0.00%	0.00%
Co-skewness		38.67%		11.44%
Co-kurtosis		41.74%		24.48%
Omega			26.64%	24.36%
Vega			22.64%	15.18%
Foreign/S&P500	2.50	11390167517.63	0.70	1.73
abs(Foreign)/S&P500	3.53	28822420027.42	1.71	5.06

Long Only	**Indices Only**	**Ind+CS+CK**	**Ind+Omega+Vega**	**All**
S&P500	41.05%	41.05%	20.44%	20.44%
DAX30	0.00%	0.00%	0.00%	0.00%
CAC40	0.00%	0.00%	0.00%	0.00%
FTSE100	0.00%	0.00%	0.00%	0.00%
SMI	0.00%	0.00%	36.58%	36.58%
OMXS30	0.00%	0.00%	0.00%	0.00%
Nikkei225	0.00%	0.00%	6.40%	6.40%
HSI	58.95%	58.95%	0.00%	0.00%
Co-skewness		0.00%		0.00%
Co-kurtosis		0.00%		0.00%
Omega			11.96%	11.96%
Vega			24.62%	24.62%
Foreign/S&P500	1.44	1.44	2.10	2.10
abs(Foreign)/S&P500	1.44	1.44	2.10	2.10