STOCKHOLM SCHOOL OF ECONOMICS

Risk-based Indexation on the Nordic Equity Market

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Abstract: By convention and with support of CAPM's theoretical framework, stock indexes have for a long time been market-capitalization weighted. Among the alternative approaches that have gained in popularity are risk-weighted indexation methods, as it has been shown that they offer superior risk-return trade-off compared to the traditional market-capitalization weighted indexes. In this study the performance of four risk-weighted indexes are tested on Nordic stock data: *Equally weighted*, *Minimum variance*, *Equal risk contribution* and *Most diversified*. We show that the minimum variance, equal risk contribution and most diversified indexes all beat the market-capitalization weighted index in terms of higher annualized Sharpe ratio. When determining the source of the performance through the CAPM model, Fama-French Three-factor Model and Carhart Four-factor Model, it is shown that the risk-weighted index portfolios have negative size (SML) betas, positive book-to-market (HML) betas and momentum (WML) betas close to zero. In addition, economically and statistically significant alphas are obtained mainly on the Minimum variance and Equal risk contribution indexes.

Key words: passive investment strategy, market-capitalization weighted index, risk-weighted indexation

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Foreword

Why do we do academic research? Why do we write a thesis? To get a diploma, and put it into your CV? Yes, but I believe it should be more than that. A thesis should have the ambition to produce findings and results that have an impact in real life. It should be more than a ticket to a career and then forgotten to collect dust in a drawer, or today rather occupy mega-bytes on a disk drive. Without such ambition, the academics will be stuck in the ivory tower. After having spent 25 years in the Swedish financial industry I feel strongly that we must use the bright minds of the young generation to see new aspects of our industry. It is the responsibility of my generation to listen, learn and use the academic findings of the upcoming generation and strive to implement them in real life, to improve how we do things, and encourage questioning accepted beliefs.

What is real life in this case? Real life simply means to be able to put more money in the pockets of the future generation when they retire. Real life also means to vitalize the equity market. The basic idea of public equity markets with limited liability, shared risk and secondary liquidity has been a vital component for the creation of wealth in the western world over the last 150 years. In the current economic environment, with nominal, and likely real returns, offered in the bond market at historical low levels, it is even more important that we seek every way of maximizing the possible returns without exposing our principals to unbearable risks. As I see it from a practitioners viewpoint: We need to study how we can put as high equity portion as possible in a broad diversified asset portfolio, without exposing ourselves to risk we cannot handle.

Over the last 30 years the asset management industry has experienced a radical shift towards indexation. We all know the background, starting with academic research more than 50 years back. Today the vast majority of long term assets are managed passively with a market-capitalization weighted index as benchmark, or semi-passively with an ambition to have a minimal tracking error to such indexes. Considering the academic debate that has been on-going ever since the first criticism of the over-simplifications built into CAPM and similar models, it is somewhat surprising to realize how dominant the market-capitalization weighted approach to equity portfolio management still is. Why is this the case? The answer is obviously a mix of reasons. But in my mind two basic forces are behind this, firstly simplicity, a market-capitalization weighted index is very easy to understand, and implement, secondly the famous “hoarding”. As this has become the preferred approach, and accepted benchmark, it is a low risk decision to go down that road, rather than risking (on a personal level) your career and reputation by adopting another model for portfolio optimizing. Therefore the academic world has a responsibility to help the asset manager of today and tomorrow, to give them research and tools to dare to question the common view, which in this case means to question the market-capitalization weighted index approach. I therefore see this thesis as very inspirational and addressing some very important questions for the future of the asset management industry in Sweden as well as globally.

Mathias Liljefors - (Head of Special Situations, SEB Equities)

1. Introduction

Until recently market-capitalization weighted indexes have played a dominant role when investing passively. The global financial crises during the last years have highlighted the importance of treating risk properly. Portfolio risk is too often treated as a consequence of asset allocation rather than as a critical input. As a response, alternative approaches to the conventional market-capitalization weighted index, have gained in popularity by claiming to offer superior risk-return trade-off.

The mean-variance optimization based on Sharpe's (1964) Capital asset pricing model (CAPM), which forms the market-capitalization weighted index, has been regarded by many as the most suitable for passive investment. This can be motivated by the many index funds and exchange traded funds on the market that resembles a market-capitalization weighted index. The mean-variance (CAPM) portfolio's construction is dependent on the estimation of the covariance matrix and the means of the assets' returns. Traditionally, the sample mean and the covariance matrix have been used for this purpose. The popularity of the mean-variance portfolio has been undermined as the portfolio weights can fluctuate substantially over time, due to the fact that the estimator contains estimation errors (Demiguel and Nogales (2009)). In the literature, the instability of the mean-variance portfolio, and thereby its undermined popularity, is mainly explained by the well-documented difficulties associated with estimating the mean of the asset returns (see Merton (1980)).

As a result, alternative methods beside the market-capitalization weighted are now offered to overcome the obstacle associated with forecasting returns. The alternative weighting schemes have slowly started to be accepted and have increased in popularity since several prominent researchers have examined properties and the risk-adjusted performance of the alternative methods.

One of the alternative ways is risk-weighted indexation where the only required estimated input is the covariance matrix and which treat risk as a critical input rather a consequence. This method has gained in popularity as the variance and the covariance are much more accurate than the corresponding expected return estimates (Merton (1980)). The improved accuracy of the estimated variance and covariance is among others determined by the number of observations (daily or monthly) and the length of the time period (Merton (1980)). In short, Merton (1980) shows how variance depends upon the number of observations t, for a fixed time period T, to order 1/t. By choosing finer observation intervals, the accuracy of the variance estimator can be improved, which is not the case for expected return. Hence using daily returns when estimating the variance, instead of monthly, improves the accuracy of the estimated variance. The accuracy of the estimated return will only depend on the length of the total time period T, and even with a very long time period, the estimate can vary significantly depending on the start date and end date of period T.

As the alternative weighting schemes, risk-weighted indexes, are highly dependent on the estimation of the covariance matrix, researchers have focused on improving the estimation of the covariance matrix. Although the covariance matrix is estimated with more accuracy than the expected return, it is still vulnerable for the impact of the estimation errors (DeMiguel and Nogales (2009)). One of the most popular methods to improve the estimated covariance matrix is to use the *shrinkage* method presented by Ledoit and Wolf (2003, 2004b). According to Ledoit and Wolf (2003, 2004b), the shrinkage method improves the estimation of the covariance matrix as it adjusts for the extreme variance and covariance values, which contains the most estimation errors. The practical implications of the shrinkage method is that optimization methods based on the covariance matrix will not to the same extent use variances and covariances that contain estimation errors. Hence, output weights will contain fewer errors as well.

Given the prior studies' attractive results of the risk-weighted indexation methods based on US data, we were asked by SEB (one of the major banks in Sweden) to determine the attractiveness of introducing a risk-weighted index in the Nordic countries. We analyze the attractiveness of four risk-weighted methods applied on an index mimicking the NASDAQ OMX Nordic 120 Index, but where data from all Nordic countries are included. All results are benchmarked against the conventional market-capitalization weighted index.

The four risk-weighted indexes reviewed are: *Equally weighted*, *Minimum variance*, *Equal risk contribution* and *Most diversified*. Talking about risk, in this context, refers to three aspects: the variance, the covariance and the correlation. Depending on what risk-weighted indexation method is analyzed, different risk aspects are emphasized. The equally weighted index is based on the idea of forming a portfolio independently from the estimated statistics and properties of the returns (Windcliff and Boyle (2004)). A minimum variance index attempts to offer the lowest possible portfolio variance. In the equal risk contribution index calculation the contribution of each constituent to overall portfolio variance is equalized in order to avoid concentration in specific risk. Lastly, the most diversified index is based on a *diversification ratio,* which aims for minimal correlations across all index constituents.

The study's main contribution is to determine if the risk-weighted indexes offer a better risk-return trade-off compared to the conventional market-capitalization weighted index. The study is based on Nordic data, reaching from 1988, and is structured in the following way: In section two we give a literature review, in order to present the hypothesis of the study, which is presented in section three. In section four we describe the data. In section five the model is described, beginning with a description how liquidity constraints and the weight rebalancing process are handled, thereafter presenting the estimation method for the covariance matrix and finally each index is described and its calculation method is presented. In section six the performance measures are described and in the following section, section seven, the results are presented. We conclude the study with a brief summary in section eight.

2. Literature Review

The literature review within the area of risk-weighted indexation is based on prior studies comparing risk-weighted indexes to the conventional market-capitalization weighted. In this section first the theory behind the conventional market-capitalization weighted index is reviewed as it is the main comparison to the risk-weighted indexes, followed by prior studies based on the different risk-weighted indexes.

2.1 Market-capitalization weighted index

Over 50 years ago Markowitz (1952) formalized a risk-return relationship in a mean-variance framework with the assumption that the investor aims to maximize the expected return, given a level of volatility. Throughout the years the most effort to develop the portfolio theory has been within the CAPM framework, developed by Sharpe (1964). According to CAPM the efficient fund of risky assets will be the market portfolio, which is one reason why the investors have until recently only weighted assets according to market capitalization values. CAPM is based on estimating an expected return vector as well as a covariance matrix to represent the valued properties of all securities. The investor minimizes the ex-ante risk for a given level of expected return by adjusting the weights, where the risk is measured as variance. In the study by Hsu (2006), he stresses the main benefits of a market-capitalization weighted approach, although he later on argues against it. The main benefits stated are the following:

i. It is a passive strategy, thereby it requires no active management why there is no active management fees.
ii. The re-balancing process occurs nearly automatically, following the security price fluctuations. Thereby the rebalancing cost is mostly due to the changing of index constituents.
iii. The transaction costs are limited as the strategy assigns the largest weights to larger companies which are highly liquid due to the trading volume.
iv. Given the CAPM interpretation the market-capitalization weighted index is the market portfolio and thereby offers the maximum Sharpe ratio, hence it is mean-variance optimal.

The last stated benefit above is also the main drawback of the market-capitalization weighted index, as the index is heavily dependent on the success of CAPM. Since CAPM is heavily dependent on the estimation of returns, the market-capitalization weighted index has been proven to underperform, mainly because of the estimation errors when estimating returns. This finding has undermined the popularity of the mean-variance portfolio, the CAPM portfolio, among the portfolio managers (DeMiguel and Nogales (2009)). Both Windcliff and Boyle (2004) and Chopra and Ziemba (1993) look at the relative impact of the estimation errors in means, variances and covariance and they find that the misspecification of the expected returns has the most impact on the performance of the mean-variance optimized portfolios. The estimation errors when estimating return implies that the mean-variance optimization overweight securities that have a large expected return, negative correlation and small variance. These securities are also the ones that most likely have estimation errors (Michaud (1989)). As a result, the performance of a mean-variance portfolio may be significantly reduced.

As a response to the estimation errors when estimating returns, alternative-weighted indexes are now offered based on either a fundamental-weighted approach or a risk-based approach, where the latter is relevant for this study.

2.2 Equally weighted index

The equally weighted index is the result of a heuristic approach. Unlike the optimization calculations conducted when obtaining constituent weights in other risk-weighted indexes presented in this study, the heuristic approach is not affected by the estimation errors.

Windcliff and Boyle (2004) show that a 1/N strategy has an advantage in terms of robustness compared to a market-capitalization weighted strategy as the 1/N strategy does not depend on the estimation of expected returns. They show that when taking into account the impact of the input parameters' estimation risk, the mean-variance optimization (the market-capitalization weighted index) no longer dominates the equally weighted.

Similar results are found by Demiguel, Garlappi and Uppal (2009). They examine 1/N as a benchmark asset-allocation strategy to assess various portfolio constructions, and then compare 14 different portfolio strategies' performance, market-capitalization weighted among others, to the 1/N strategy across seven empirical datasets of monthly returns. They determine that in terms of Sharpe ratio none of the 14 models that were evaluated performed better than the 1/N. Only when extending the estimation window to 3000 month with 25 assets, the out-of-sample mean-variance strategies outperformed the 1/N strategy, showing that these models need very long estimation windows before they can be expected to outperform the 1/N policy.

As mentioned the equally-weighted index is a heuristic approach, thus it is highly dependent on the number of stocks chosen to be included in the index. This is also stressed in the study by Chow, Hsu, Kalesnik, Little, (2011). They show that the risk-return characteristics differed dramatically between an index based on 1000 securities compared to an index based on 500 securities, where the former was more volatile which can be explained by the greater exposure towards small-cap securities. Hence the choice of the underlying stock universe determines by large the 1/N portfolio performance.

2.3 Minimum variance index

The turmoil market conditions over the last years have led to a re-evaluation of many investment paradigms and several findings support the fact that classical relationships does not hold, e.g. low-volatility stocks which shall offer lower returns actually do better than expected (Gabudean (2011)). Departing from the classical mean-variance efficiency implies that the risk-based indexation aims to maximize the diversification benefits by giving more weight to assets that reduce the overall portfolio risk or increase by less (Gabudean (2011)).

The minimum variance portfolio offers an alternative way to minimize the risk without being concerned with the hardship of estimating expected return. It is the only mean-variance portfolio that does not incorporate information on the expected return, why it is seen as robust according to Demey, Maillard and Roncalli (2010). Its independence of the many assumption of the expected return is also the primary motivation when Clarke, De Silva and Thorley (2006) in their study test the attractiveness of the minimum variance index. They show that the minimum variance portfolio offers both a higher Sharpe ratio and a lower standard deviation compared to the market-capitalization weighted portfolio. Moreover, they show that the minimum-variance portfolio has both a value and small-size bias. But when constraining the minimum-variance portfolio to have the same factor exposure as the market-capitalization, it offers the same realized return but lower standard deviation. Their results are consistent with the studies by Haugen and Baker's (1991) and Ang et al. (2006). In addition to the small-size bias that Clarke, De Silva and Thorley (2006) point out, Maillard, Roncalli and Teiletche (2008) stress the minimum variance index's drawback in terms of portfolio concentration, since a few stocks may have a significant weight of the portfolio. The most straightforward solution to any concentration problem is to impose weight constraints. When imposing lower and/or upper bounds on weights, it limits the optimization but helps to obtain more reasonable portfolios.

2.4 Equal risk contribution index

The equal risk contribution index is similar to the minimum variance index, but an equal risk contribution index is subject to a diversification constraint on the constituents' weights. The equal risk contribution index, like the minimum variance index, is not dependent on estimating expected returns. Instead of focusing on equal pairwise correlations across all portfolio constituents as in the most diversified portfolio described below, the construction of the equal risk contribution is based on the idea of equalizing the impact all individual components have on the total portfolio variance.

Maillard, Roncalli and Teiletche (2008) show how an equal risk contribution portfolio experiences volatility within the boundaries of the equally weighted portfolio and the minimum variance portfolio. They argue that investing according to equal risk contribution is interesting because it mimics the diversification effect of an equally weighted portfolio while taking into account single and joint risk contribution of the assets. The study by Maillard, Roncalli and Teiletche (2008) is a discussion on the properties of the equal risk contribution method, hence they do not present any performance or results based on empirical data, but show a numerical example of the features of the equal risk contribution methodology. In a later study by Demey, Maillard and Roncalli (2010) the performance of the market-capitalization weighted index is compared with the equal risk contribution index, among others. In this study they show that the equal risk contribution portfolio historically has, in terms of risk-adjusted return, performed better than a market-capitalization weighted portfolio.

2.5 Most diversified index

The most diversified portfolio is based on a *diversification ratio*. While the minimum variance strategy accounts for both variance and correlation between single assets, Choueifaty and Coignard (2008) suggest a focus on diversification. They introduced the diversification ratio, which is the ratio of the weighted average of all the constituent volatilities divided by the portfolio's total volatility. Hence while the minimum variance portfolio accounts for both variance and correlations between single assets, the most diversified portfolio focuses on diversification, considered as minimal correlations across all portfolio constituents. Thereby, instead of minimizing total portfolio variance, the portfolio diversification, here defined as the diversification ratio, is maximized to construct the most diversified portfolio. Choueifaty and Coignard (2008) show that the most diversified portfolio has a higher Sharpe ratio than the market-capitalization weighted.

Chow, Hsu, Kalesnik and Little (2011) high-light the fact the diversification ratio not only contains systematic risk, but also individual risk. Thereby it departs from standard finance theory, which says that only the systematic risk should earn premium.

2.6 Estimating the covariance matrix

Although the risk-weighted indexes are independent of any estimation of expected returns, they are heavily reliant on the estimation of the covariance matrix. Thereby, how the estimation of the covariance matrix is conducted is of great importance for this study. In accordance to prior studies on risk-weighted indexation methods we estimate the covariance matrix in several ways as it is the critical input for the weighting procedures.

When estimating the parameters based on a long return series the length of the estimation window is important. Using shorter estimation windows may entail increased estimation errors, while using longer estimation window could imply using observed data that might not tell a lot about the return structure of the future. Thus, there is a trade-off between estimation error and stationarity when choosing the length of the estimation window (Broadie 1993).

When estimating the covariance matrix the practitioner wants the estimated covariance matrix to be unbiased, invertible and well-conditioned, i.e. not suffer from estimation errors. All these aspects are important in order to have a good estimate. The sample covariance matrix is viewed as the base case when stating how the covariance matrix should be estimated. The sample covariance matrix is unbiased, typically invertible but not well-conditioned i.e. suffers from estimation errors (Ledoit and Wolf (2004)).

Due to the estimation errors in the sample covariance matrix, alternative ways are now offered to improve the estimated covariance matrix. There exists a variety of ways to determine the covariance matrix such as simple window, decayed weighting, GARCH and Bayesian updated methodologies (Choueifaty and Coignard (2008)). As mentioned in the introduction, among the most popular is the shrinkage method by Ledoit and Wolf (2003, 2004b). They suggest using this method as it pulls the extreme coefficients towards more central values. By pulling the extreme values toward the center the estimation becomes more accurate. For details see Ledoit and Wolf (2003, 2004b).

In the study Ledoit and Wolf (2004b) they show that in most cases the shrinkage estimator yields the highest average mean excess return, compared to the sample covariance matrix. But also the lowest average standard deviation. These attractive results have been found by other studies as well, showing the advantages of the shrinkage method.

The reduction of estimation errors in a covariance matrix has an importance that can best be described as an extreme weight reduction. Common optimization routines, used when obtaining constituent weights, pick up very fast on extreme variance and covariance values. If we assume that estimation errors are randomly distributed, structure unknown, among the matrix elements, and that the size of the errors has a mean of zero, it has serious implications for the optimization routine. These routines often value the size of variance and covariance values in a non-linear way. It means that a small change in an extreme covariance value has a greater effect on optimization output, in this case weights, than has a small change in a non-extreme covariance value. Thus, it is more important to reduce estimation errors in extreme covariance estimations. Because only constituents with extremely high or low estimations will obtain extreme portfolio weights, using the sample covariance matrix may lead to sub-optimal weighting of portfolio constituents, and the portfolio can hold unnecessary high risk, both idiosyncratic and systematic. The shrinkage method tries to reduce the estimation errors, and thus indirectly errors in the weighting procedure.

3. Study Hypothesis

The purpose of this study is to examine the attractiveness of risk-weighted indexes based on Nordic stock data. Inspired by previous research within this subject and the fact that risk-weighted indexes are offered in both Europe and USA, our main hypothesis is:

(1) *The risk-weighted indexes can offer a better risk-return trade-off compared to the market-capitalization weighted index.*

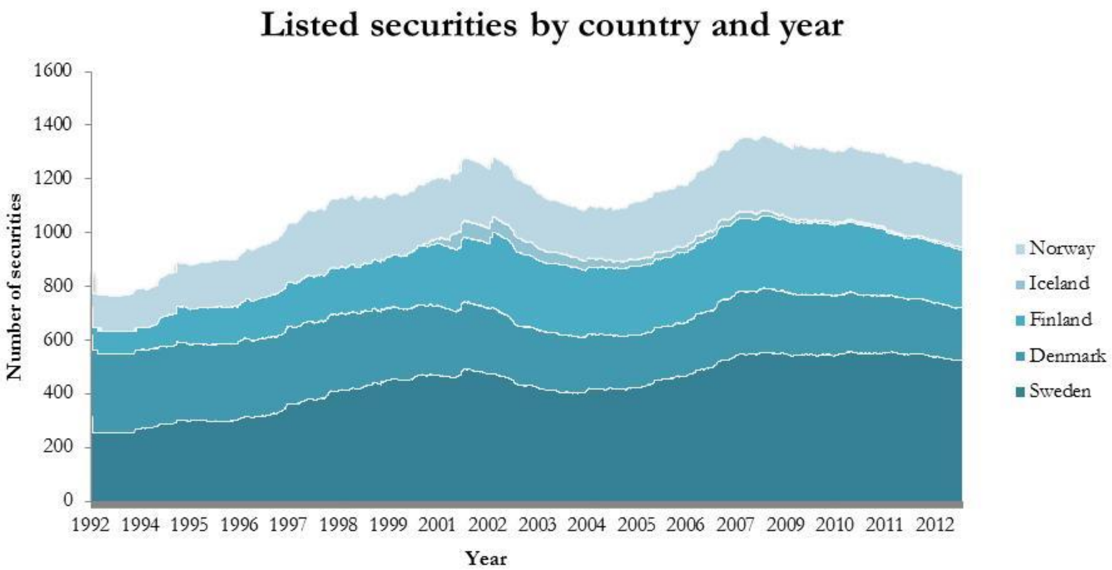
The main motivation behind this better performance is due to their independence of the estimation of return, as they only rely on the estimation of the covariance matrix when assigning weights to the constituents of the indexes. Based on the prior studies how the covariance matrix can be estimated with less estimation errors with the shrinkage method, our second hypothesis is:

(2) *Compared to the sample covariance matrix, the shrinkage covariance matrix can offer a better index risk-return trade off and lower index turnover due to less estimation errors in its elements.*

4. Data

The main data in this study consists of the daily return index (split- and dividend adjusted return) for all Nordic stocks. The exchanges included are NASDAQ OMX Stockholm, NASDAQ OMX Helsinki, NASDAQ OMX Copenhagen, NASDAQ OMX Iceland, Oslo Børs, as well as all smaller market counterparts of each country. In other words, all Nordic stocks are included in the research. Looking at Figure 1, we see how the number of listed securities has gradually increased over time, except for the major dip after the tech-bubble.

Figure 1: Number of listed securities on the Nordic stock exchanges, measured over time.



The main period examined is 20 years, hence our time period reaches from 1992-12-14 to 2012-12-14. Data has been collected from 1973, but due to the limited fundamental data until late 1988, we have chosen a time span of 20 years. Hence before December 1988 the data was not complete as it is first that year the indexes have constituents from all the Nordic countries. Thereby the time span has been chosen in order to assure that the analysis is objectively based on Nordic data, with companies from all countries represented. All stocks that were listed through this period or over a sub period are included in the sample. All data was obtained from Thomson Reuters Datastream.

Observations for Saturdays and Sundays were removed. Apart from return index data, market value data, unadjusted stock price data as well as unadjusted volume data were gathered for the index creation process. Some of the performance methodologies require excess return, rather than return. Thereby, the data also includes the Swedish short-run risk-free interest rate, measured as the Swedish 30-day Treasury bills (Statsskuldsväxlar)[1]. For the *value* risk factor used in both the Fama-

[1] www.riksbank.se

French three-factor model (Fama and French (1993)) and the Carhart four-factor model (Carhart (1997)), price-to-book data had to be gathered as well.

Since our defined stock universe consists of all stocks listed in the Nordic markets, the indexes will be calculated in a common currency, the Swedish krona. The local currencies DKK, NOK, ISK and EUR are thus converted to SEK with their respective exchange rate each day.

As for the risk-free interest rate, all currency data was obtained from Riksbanken[2], the Swedish central bank. Up until 4th of January 1993 a monthly currency rate was used and thereafter a daily currency rate was used. From 1st of February 2002 the Finnish currency was denoted in Euro.

[2] www.riksbank.se

5. The Model – Index Creation Process

5.1 General description

The study consists of a comparison between four risk-weighted indexes and the market-capitalization weighted index. All index portfolios are rebalanced semi-annually and all stocks listed on any of the Nordic stock exchanges are eligible for inclusion. The semi-annual rebalancing is a balance between statistical significance and practicality. Quarterly rebalancing would probably provide more significant and reliable results, however, since rebalancing brings costs to the indexation process in terms of more trades and a larger workforce put to the task, we decided to follow the methodology[3] that NASDAQ OMX uses on most of its indexes. Hence semi-annually re-balancing is used by practitioners due to the cost of rebalancing. As this study emphasizes the importance that practitioners could use our results, it is convenient to do like practitioners do in real life. The periodic review of the indexes will take effect on the third Friday in December and the third Friday in June, respectively.

The constituents' market values are converted to the Swedish krona on a daily basis, in order to create indexes that can be implemented in practice. Converting on a daily basis implies that the return is somewhat effected by a currency effect. By this we mean that a single country's monetary policies can affect the return of the index and thus somewhat pollute the results so that the true effect of an indexation method is hidden. On the other hand, if one would only have converted to a common currency on the rebalancing day or just not converted at all, one would have to assume the investors could obtain a perfect hedge against currency risk, which is a too stark assumption during the observed time period.

The weight on one constituent is limited to 5% and 10% respectively in order to avoid too much concentration, and no short sells are allowed. The fact that short sales are ruled out is beneficial in terms of weights stability as it limits somewhat the estimation errors of the covariance matrixes (Jagannathan and Ma (2002)). As the market-capitalization weighted index should be compared to the other indexes' performance, the constituents need to have data 500 historical weekdays prior to rebalancing day. This is due to the fact that the risk-weighted indexes (equally weighted excluded) require historical return series in order to estimate the covariance matrix, which is the base on how to determine the weights.

For comparison reasons, all indexes will make use of the same constituent universe, where the market-capitalization weighted index' constituents form this constituent universe. This implies that all risk-weighted indexation methods may only pick securities that currently are included in the market-capitalization weighted index. The risk-weighted indexes is because of this limitation

[3] www.nasdaqomxnordic.com

ensured to only invest in large, highly liquid companies, and the relative performance versus the classical market-capitalization weighted index is thus both comparable and relevant.

5.1.2 Liquidity criteria

By convention practitioners adjust the constituents of the indexes based on a free float factor in order to assure that the index is based on the tradable part of the constituents. In this study we have not included the free float factor as it was only available from 2002 and our data reaches from 1988, this implies that the investability adjustments are somewhat different in this study compared to the conventional. Thereby we prioritized having a longer time period of data. But our liquidity constraint in the index creation process, described in section 5.1.4, captures most of this free float factor adjustment.

We include a liquidity constraint to assure that the index constituents meet investability requirement. Thereby, the liquidity constraint is added to assure that the indexes can be implemented in practice, since the constraint assures that only the tradable part of the stock universe is actually traded in the index simulations. The liquidity is measured based on unadjusted price and unadjusted trading volume, which is used to calculate the unadjusted daily turnover in SEK. The detailed process of how the liquidity constraint is applied is found in section 5.1.4.

5.1.3 Estimating the covariance matrix

5.1.3.1 Sample covariance matrix

The risk-based indexes' construction focuses on risk information rather than relying on estimated expected returns. Thus, the construction is based on the estimation of a covariance matrix, which forms the only input that needs to be supplied when determining the constituent weights. Today it is commonly accepted that the estimation errors in the expected returns is much larger than in a sample covariance matrix (DeMiguel and Nogales (2009)). For this reason the risk-weighted indexation has become popular as it solely relies on the estimation of the covariance matrix.

As this study focuses on the risk-weighted indexes, how the covariance is estimated is of great importance. In the literature review, presented above, the effect of the length of the estimation window is described, as well as the estimation errors when using the sample covariance matrix. The calculation of the sample covariance matrix is presented below.

$\mathrm{i} = 1,2, \dots, \mathrm{N}$ is the number of stock constituents in the index.

$\mathrm{t} = 1,2, \dots, \mathrm{T}$ is the number of historical return observations, where T alters between either 250 days and 500 days, in order to test for the output sensitivity to window length.

$r_{i,t}$ is the ith constituent's return at time t. The return is defined by $r_{i,t} = \frac{P_t - P_{t-1}}{P_{t-1}}$, where P_t is the constituent return index value, adjusted for splits and dividends, at time t.

Thus the set of observations for the constituents' returns is a $T \times N$ matrix with each element (t, i) being $r_{i,t}$ for $\mathrm{i} = 1,2, \dots, \mathrm{N}$ and $\mathrm{t} = 1,2, \dots, \mathrm{T}$. The sample variance σ_i^2 of constituent i and the sample covariance $\sigma_{i,j}$ of constituent i and j are,

(1)

$$\sigma_i^2 = \frac{1}{T-1} \sum_{t=1}^{T} (r_{i,t} - \bar{r}_i)^2$$

and

(2)

$$\sigma_{i,j} = \frac{1}{T-1} \sum_{t=1}^{T} \left(r_{i,t} - \bar{r}_i\right) * \left(r_{j,t} - \bar{r}_j\right)$$

where $\bar{r}_i = \frac{1}{T} \sum_{t=1}^{T} r_{i,t}$ and $\bar{r}_j = \frac{1}{T} \sum_{t=1}^{T} r_{j,t}$ are the corresponding sample means. The $N \times N$ matrix, where each element (i, j) being $\sigma_{i,j}$ for $\mathrm{i}, \mathrm{j} = 1, 2, \dots, \mathrm{N}$, is the sample covariance matrix labeled as $\sum$ in the index calculations for each index.

5.1.3.2 Shrinkage estimator of the covariance matrix

As mentioned in the literature review in section 2.6, Ledoit and Wolf (2004b) offer an alternative way of estimating the covariance matrix called shrinkage, in order to overcome the estimation errors that a sample covariance matrix contains. The calculation is in brief described below, which focuses on the intuition of the calculation (for more details see Wolf and Ledoit (2004b)).

Consider the sample covariance matrix, here denoted S, and a *structured estimator*, denoted F. The structured estimator is used as it contains less estimation errors (for detailed explanation why this is the case see Ledoit and Wolf, 2004b). Comprise the sample covariance matrix S and the structured estimator F, in a the following relationship,

(3)

$$\delta * F + (1 - \delta) * S$$

,where δ is a value between 0 and 1 and is called the *shrinkage constant,* and intuitively it measures the weight given to the structured estimator F. Hence a shrinking constant of zero corresponds to the sample covariance matrix. Combining the two, S and F, is called shrinkage, as the sample covariance matrix is shrunk towards the structured estimator. Hence, looking at the linear combination above, *the shrinkage estimator* has three components: an estimator with no structure (in this context the sample covariance matrix), an estimator with a lot of structure (in this study the *constant correlation matrix*) and a shrinkage constant. In the study by Ledoit and Wolf (2004b) the constant correlation model is used as the structured estimator F. The structured estimator needs to meet two requirements: it has to have a small number of parameters with a lot of structure and it has to reflect important characteristics of the unknown that is being estimated (Ledoit and Wolf (2004)). For a detailed description on what these two requirements for the structured estimator imply, see Ledoit and Wolf (2004).

The average of all the sample correlations becomes the estimator of the common constant correlation. The sample correlation between constituent i and j is given by,

(4)

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sqrt{\sigma_i^2 * \sigma_j^2}}$$

Hence the average sample correlation becomes,

(5)

$$\bar{\rho}_{i,j} = \frac{2}{(N-1) * N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{i,j}$$

Combining the average sample correlation with a vector of the sample variances gives the constant correlation matrix, hence the structured estimator F.

(6)

$$F_{i,j} = \bar{\rho}_{i,j} * \sqrt{\sigma_i^2 * \sigma_j^2}$$

Looking at equation (3), which states the linear combination of the sample covariance matrix and the structured estimator, one can determine that the practical problem is to choose the shrinkage constant δ. Intuitively the optimal shrinkage constant δ^* is the one that minimizes the distance between the estimated and the true covariance matrix, where the true covariance matrix is an estimated covariance matrix without any estimation errors. The estimated optimal shrinkage constant is denoted $\hat{\delta}^*$. A detailed version of how the optimal shrinkage constant is found is a very technically challenging and outside the scope of this study, why a detailed version is provided in Ledoit and Wolf (2004b). The shrinkage estimator of the covariance matrix is

(7)

$$\hat{\sum_{shrink}} = \hat{\delta}^* * F + (1 - \hat{\delta}^*) * S$$

Ledoit and Wolf (2004b) offer to the public the Matlab code for the shrinkage process. Thereby this was used when determining the shrinkage covariance matrix.

In conclusion, we estimate the covariance matrix in four ways: using the sample covariance matrix altering the estimation window length between 500 days and 250 days, and using the shrinkage covariance matrix altering the estimation window length between 500 days and 250 days.

The sample covariance matrix and the shrinkage covariance matrix are both labeled $\sum$ in the index calculations for each index. Since the weighting procedures are the same independently of covariance estimation method, only the input in terms of which covariance matrix is used, differs.

5.1.4 Selection process of the indexes' constituents

The selection process is based on conventional market-capitalization as this is used as the main comparison with the alternative indexes. It is based on the selection process of the NASDAQ OMX Nordic 120 Index, which is in the tradable index family and thus encourages mimicking financial products like index funds and exchange traded funds. The main difference is that we also include Norwegian stocks in our sample, but do not adjust for a free float factor. Our selection process can be summarized as,

i. All securities are ranked according to market-capitalization, denoted in SEK.
ii. All securities are ranked according to six months cumulative turnover, denoted in SEK.
iii. If all of the 120 securities with greatest market capitalization are among the top 140 most liquid securities they are all selected as index constituents. If not, the non-liquid security will be replaced by the liquid security that is closest to qualify in terms of market-capitalization.
iv. A company may only be represented by one index constituent. If two constituent securities are representing the same company, the security with the highest six months cumulative turnover in SEK will be used, while the other is replaced by the security that is closest to qualify in terms of market-capitalization and turnover.
v. All indexes have the same constituent universe as that of the market-capitalization weighted. The only difference is how the different weighting schemes assign weights to the constituents. The decision to make the stock universe of all indexes the same was done to be able to better compare the performance of the different index portfolios.

Based on the selection process of the indexes' constituents the indexes assign weights to the constituents differently. This processes are below described in depth, starting with the market-capitalization weighted index.

5.2 General index calculation

All indexes are calculated using the following formula:

$$I_t = \frac{\sum_{i=1}^{N} q_{i,t} * p_{i,t} * e_{i,t}}{\sum_{i=1}^{N} q_{i,t} * p_{i,t-1} * e_{i,t-1} * j_{i,t}} * I_{t-1}$$

Where,

I_t is the index value at time t.

$i = 1, 2, \ldots, N$

N is the number of stock constituents in the index.

$q_{i,t}$ is the number of shares of constituent i applied to the index at time t. This number is determined at the last rebalancing date prior to time t and is also directly dependent on the indexation method.

$p_{i,t}$ is the price in quote currency of constituent i at time t.

$e_{i,t}$ is the foreign exchange rate of index quote currency to quote currency of the constituent at time t.

$j_{i,t}$ is the adjustment factor for adjusting the share price of a constituent security due to corporate actions by the issuing company at time t.

The above presented calculation is shown in order to clarify how a market-capitalization weighted, or any other type of index, is created, hence it is for a pedagogical purpose. But as Datastream offers already calculated market-capitalization values and dividend and split adjusted return index values, those parameters were used when creating the indexes (see the market-capitalization weighted index example).[4]

[4] $I_t = I_{t-1}\left(1 + \sum_{i=1}^{N}\left[\left(\frac{MV_{i,t-1}*e_{i,t-1}}{\sum_{i=1}^{N} MV_{i,t-1}*e_{i,t-1}}\right) * \left(\left(\frac{RI_{i,t}}{RI_{i,t-1}} - 1\right) + \left(\frac{e_{i,t}}{e_{i,t-1}} - 1\right) * \left(\frac{RI_{i,t}}{RI_{i,t-1}}\right)\right)\right]\right)$, where I_t is the index value at time t, $MV_{i,t}$ the market-capitalization value of constituent i at time t, $RI_{i,t}$ is the dividend and split adjusted return index value of constituent i at time t, $e_{i,t}$ is the foreign exchange rate of index quote currency to quote currency of the constituent at time t.

5.3 Market-capitalization weighted index

The market-capitalization weighted index is used as the main comparison throughout the study and consists of the 120 largest and most liquid market-capitalized companies listed on the Nordic stock exchanges.

5.3.1 Index calculation

The market-capitalization weighted index is created with a method, which adjusts constituent weights according to the market capitalization of each constituent in relation to the total market capitalization of all the index constituents. The daily index value is then derived from the daily weighted return of all the constituents. Thus, the price movement of a larger stock (say, representing five percent of the value of the index) will, therefore, have a larger effect on the change in index value than would a smaller stock (say, representing one percent of the value of the index).

5.4 Equally weighted index - 1/N

The equally weighted (1/N) index is an equally weighted version of the market-capitalization weighted index. The index contains the same constituents as the market-capitalization weighted index, yet allocates capital equally to these, rather than based on market-capitalization. There are mainly two reasons for using the equally-weighted indexation method. First, it is easy to implement because it does not rely either on estimation of the moments of asset returns or on optimization (it is a heuristic approach). Secondly, despite the advanced methods developed throughout the years, the equally-weighted approach is still commonly used by investor due to its simplicity. The equally-weighted portfolio is the unique portfolio on the efficient frontier if all assets have the same correlation coefficient and identical means and variances.

5.4.1 Index calculation

The index is constructed so that exactly the same amount of capital is allocated to each index constituent at the re-balancing date.

5.5 Minimum variance index

The minimum variance method is defined so that the ex-ante volatility of the minimum variance index is minimized. The covariance matrix stated below, refers to the four different estimation procedures described in section 5.1.3. Hence the index calculation of the minimum variance index is conducted eight times based on the four different versions of the estimated covariance matrix and two different versions weight constraints.

5.5.1 Index calculation

Consider a portfolio $x = (x_1, x_2, \dots, x_N)$ of N risky assets. Let $\sum$ be the covariance matrix and $\sigma^2(x) = x^T \sum x$, be the variance of the portfolio. T represents the transpose procedure.

Let Γ be a set of constraints applied to the weights (with two weight limit constraints: 5% and 10%) of portfolio x.

$$\Gamma: \mathbf{1}^T x = 1; 0 \leq x \leq 0.05 \; and \; 0.10$$

The portfolio x, which under Γ minimizes the variance $\sigma^2(x)$, is the minimum variance portfolio. Thus $\arg\min \sigma^2(x) | \Gamma \rightarrow x_{MV} = (x_1, x_2, \dots, x_N)$.

A constraint on the weight of a single stock ensures the index is investable and diversified. In addition, short-selling is forbidden and the total weight of all constituents must sum to 1.

5.6 Equal risk contribution index

In the equal risk contribution index, the contribution of each constituent to overall portfolio risk is equalized in order to avoid concentration of specific risk. The constituent's risk contribution is the share of the total portfolio risk attributable to that constituent. The marginal risk contribution is the change in the total risk induced by a small increase of constituent weight. Furthermore risk-equal weights create the most diversified portfolio in terms of individual risk contributions.

The weight of a stock decreases as its volatility or its correlation with other constituents increases, while the weight increases as its volatility and its correlation with other constituents decreases. The risk contribution rises directly with asset weight and the assets volatility, but the net effect of an increase in weight can still be negative (falling risk) if the asset shows sufficiently negative covariance with the other assets.

As this approach takes into account both the single (idiosyncratic) and joint (systematic) risk contribution, each constituent has the same risk contribution. The approach implies that the higher volatility the constituent has, the lower its weight will be in the equal risk contribution portfolio. Moreover, the constituent's weight is inversely related to the beta, which implies the higher the beta the lower the weight.

The covariance matrix stated below, refers to the four different estimation procedures described in section 5.1.3. Hence the equal risk contribution index calculation is conducted eight times based on the four different version of the estimated covariance matrix and two different versions of weight constraint.

5.6.1 Index calculation

Consider a portfolio $x = (x_1, x_2, \dots, x_N)$ of N risky assets. Let σ_i^2 be the variance of asset i, σ_{ij} be the covariance between asset i and asset j, and $\sum$ be the covariance matrix. Let,

$$\sigma(x) = \sqrt{x^T \sum x} = \sqrt{\sum_i x_i^2 \sigma_i^2 + \sum_i \sum_{i \neq j} x_i x_j \sigma_{ij}}$$

,be the risk of the portfolio. T represents transpose. The marginal risk contribution of asset i on the portfolio, $\partial_{x_i}\sigma(x)$, is then defined as,

$$\partial_{x_i}\sigma(x) = \frac{\partial\sigma(x)}{\partial_{x_i}} = \frac{x_i\sigma_i^2 + \sum_{i\neq j} x_j\sigma_{ij}}{\sqrt{\sum_i x_i^2\sigma_i^2 + \sum_i\sum_{i\neq j} x_i x_j\sigma_{ij}}} = \frac{x_i\sigma_i^2 + \sum_{i\neq j} x_j\sigma_{ij}}{\sigma(x)}.$$

If $\sigma_i(x) = x_i * \partial_{x_i}\sigma(x)$, then $\sigma(x) = \sum_{i=1}^{N}\sigma_i(x)$. The risk of the portfolio can thus be described as the weighted sum of the marginal risk contribution.

Let Γ be a set of constraints applied to the weights of portfolio x.

$$\Gamma\colon \mathbf{1}^T x = 1;\ 0 \leq x \leq 0.05\ and\ 0.10$$

The portfolio x, which under Γ equalizes the risk contributions of the assets in portfolio x, is the equal risk contribution portfolio. Thus,

$$\left(\left(x_1 * \partial_{x_1}\sigma(x) = x_2 * \partial_{x_2}\sigma(x) = \cdots = x_N * \partial_{x_N}\sigma(x)\right)\middle|\Gamma\right) \rightarrow x_{ERC} = (x_1, x_2, \ldots, x_N)\ .$$

A constraint on the weight of a single stock ensures the index is investable and diversified. To achieve an acceptable investability of the index, short-selling is forbidden, and the total weight of all constituents must sum to 1.

5.7 Most diversified index

The most diversified index assumes that the risk-premium of a constituent is proportional to its volatility. The construction of this index is governed by the maximization of a diversification ratio. The numerator of the ratio is the average volatility of the portfolio's constituent stocks, while the denominator is the volatility of the portfolio itself. Thereby, there is a tension between the numerator and denominator since the numerator pushes the optimizer towards risky stocks, while the denominator pushes the optimizer toward stocks that have low risk in combination. The optimizer resolves this tension by looking for stocks with low correlations, so that the resulting portfolio exhibits the greatest difference between the volatility of constituent stocks on their own and their total volatility in combination.

The covariance matrix stated below, refers to the four different estimation procedures described in section 5.1.3. Hence the index calculation of the most diversifiedindex is conducted eight times based on the four different version of the estimated covariance matrix and two different versions of weight constraints.

5.7.1 Index calculation

Consider a portfolio $x = (x_1, x_2, \ldots, x_N)$ of N risky assets. Let $\sigma^2 = (\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2)$ be the volatility vector of a N risky assets and $\sum$ be the covariance matrix. The diversification ratio, $D(x)$, of portfolio x, is defined as,

$$D(x) = \frac{x^T \sigma^2}{\sqrt{x^T \sum x}}$$

T represents transpose. Let Γ be a set of constraints applied to the weights of portfolio x.

$$\Gamma\text{: } \mathbf{1}^T x = 1;\, 0 \leq x \leq 0.05 \; and \; 0.10$$

The portfolio x, which under Γ maximizes the diversification ratio $D(x)$, is the most-diversified portfolio. Thus,

$$(\arg\max D(x)\,|\Gamma) \rightarrow x_{MDP} = (x_1, x_2, \ldots, x_N)\ .$$

A constraint on the weight of a single stock ensures the index is investable and diversified. To achieve an acceptable investability of the index, short-selling is forbidden, and the total weight of all constituents must sum to 1.

6. Performance Measures and Characteristics of the Indexes

For each index annualized return, annualized volatility, annualized Sharpe ratio and *Maximum Drawdown* is presented. The 20 year time period is used for the main results, but the results for shorter time periods are presented in the appendix (section 9). A long observation period is preferred as any random component of the results probably diminishes as the number of years, and thus the number of index rebalancing periods, increases. One can argue about the relevance of index performance decades ago, and that is why the appendix presents the results of more recent time periods.

The annualized return r_{annual} is defined as,

$$r_{annual} = \left(\frac{Ending\ Value}{Starting\ Value}\right)^{\frac{1}{\#\ of\ years}} - 1$$

The annualized volatility, based on the standard deviation of daily arithmetic returns (σ_{SD}), is defined as,

$$\sigma_{annual} = \sigma_{SD}/\sqrt{P}$$

,where P represents the chosen time period of returns. As mentioned in the introduction, the term risk throughout the study refers to volatility, why we have chosen to present the annualized volatility for each index.

The Sharpe ratio is used as a performance measurement as it is well-established and well-known by investors, although drawbacks with the measurement exist. The ratio represents the risk-adjusted returns as it quantifies the excess return per risk unit. The measurement considers the total risk, both the systematic and the idiosyncratic, as it is based on the volatility. One commonly known drawback is the fact that measurement uses volatility as the risk measure, which can be problematic if the underlying return series are not normally distributed. Although drawbacks exist, the Sharpe ratio is the most appropriate risk-return measure to use in this study, as it is most widely used by practitioners. The annualized Sharpe ratio is defined as,

$$SR = \frac{r_{annual} - rf_{annual}}{\sigma_{annual}}$$

,where rf_{annual} is the annualized risk free rate.

Maximum drawdown measures the maximum loss from the prior peak period. Hence it is the accumulated loss of buying an investment at its highest local maximum price and selling it at its lowest local minimum price. It can be seen as a measurement of the "expected worst case" risk.

The maximum drawdown is chosen as it complements the Sharpe ratio and is used by many practitioners when looking at indexes' performance over time.

6.1 Regression models

In addition, we have chosen to evaluate the indexes based on three regression models: The Capital asset pricing model (Sharpe (1964)), the Fama-French three-factor model (Fama French (1993)), and the Carhart four-factor model (Carhart (1997)). This is of importance in order to determine the source of the performance. In a prior study by Chow, Hsu, Kalesnik, Little (2011), they examine several risk-based weighting indexes such as minimum variance, equally weighted, equal risk contribution and maximum diversification. They find that the alternative methods examined outperform the market-capitalization index, but the outperformance is entirely explained by a positive exposure to the value and size factors, hence the alternative betas show no Fama-French alpha. Thereby from the factor analysis they find that the reduced volatility in the alternative-weighted indexes is mainly driven by a reduction in the exposure to the market factor, rather than an actual reduction in idiosyncratic risk. Given prior studies' findings that the higher risk-return trade-off is entirely explained by positive factor exposure, it highlights the importance that we should also in this study determine the source of the indexes' performance.

CAPM gives us the following regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$

Where $r_{i,t} - r_{f,t}$ is the excess return at time t, α_i is the over performance if it is positive and underperformance if it is negative, compared to what CAPM expects, β_i is the sensitivity towards the market factor and $\varepsilon_{i,t}$ is the residual, that is unexplained by the regression.

As the main conclusion for CAPM is that the model works poorly when tested empirically, when determining the source of the indexes' performance it is important to also use other conventional factor models like the Fama-French three-factor model presented in 1992-1993, that has had greater success when tested empirically. Fama and French (1993) estimated a three-factor model consisting of a market factor, a size factor and a value factor. The three-factor model is considered to capture co-movements in returns missed by the market return. It picks up much of the size and value effects in returns left unexplained by the CAPM.

Fama and French (1993) grouped the portfolio in order to estimate a characteristic-based factor model. The excess return to a value-weighted market index is used as a proxy for the market factor. The difference in return between a portfolio of low-capitalization stocks and a portfolio of high-capitalization stocks is used as a proxy for the size factor. The difference in return between a portfolio of high-book-to-price stocks and a portfolio of low-book-to-price stocks is used as a proxy for the value factor. Fama and French (1993) conduct a time-series regression to estimate the factor betas. This model has played a significant role in the explanation of co-movements of individual stock returns, and logically it is a pedagogical way of evaluating the different indexation methods. When regressing based on the three factors; market, size and value, the alpha shows the relationship between an investment's historical betas and its current performance. Thereby an alpha of zero indicates that the investment performed as expected, a positive alpha indicates that the investment performed more than its betas indicated, and a negative alpha implies that it performed less. Hence the alpha measures an added value, if it is positive.

The Fama-French three factor model is given by the following regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}(r_{m,t} - r_{f,t}) + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \varepsilon_{i,t}$$

Where $r_{i,t} - r_{f,t}$ is the excess return at time t, α_i is the over performancee if it is positive and underperformance if it is negative, compared to what the Fama-French three-factor model expects, $\beta_{1,i}$ is the sensitivity towards the market, $\beta_{2,i}$ is the exposure towards small stocks, SMB_t is the size factor return at time t, and represents Small minus Big in terms of market-capitalization, $\beta_{3,i}$ is the exposure towards value stocks, HML_t is the value factor return at time t, and represents High minus Low in terms of book-to-market ratio, $\varepsilon_{i,t}$ is the residual, that is unexplained by the regression.

Carhart (1997) extended Fama and French' (1993) three-factor model with a momentum factor, based on the one-year momentum effects discovered by Jegadeesh and Titman (1993). Carhart (1997) stresses that the factor model may be interpreted as a performance attribution model, which suits this study well. Adding the momentum factor, adds another aspect when determining the source of performance.

Carhart four-factor model is given by the following regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}(r_{m,t} - r_{f,t}) + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \beta_{4,i}WML_t + \varepsilon_{i,t}$$

Where $r_{i,t} - r_{f,t}$ is the excess return at time t, α_i is the over performance if it is positive and underperformance if it is negative, compared to what the Carhart model expects, $\beta_{1,i}$ is the sensitivity towards the market, $\beta_{2,i}$ is the exposure towards small stocks, SMB_t is the size factor return at time t, and represents Small minus Big in terms of market-capitalization, $\beta_{3,i}$ is the exposure towards value stocks, HML_t is the value factor return at time t, and represents High minus Low in terms of book-to-market ratio, $\beta_{4,i}$ is the exposure towards the momentum effect, WML_t is the momentum factor return at time t, and represents Winners minus Losers, in terms of prior return, $\varepsilon_{i,t}$ is the residual, that is unexplained by the regression.

We will thus use all three factor models to determine the source of the indexes' performance, and leave the reader to look into other studies why these factors are commonly used (as this is outside the scope of this study).

6.2 Creating risk factors

The three factor models, presented above, used for performance evaluation of the indexes, consist in total of four factors. When creating the factors we have chosen the longest possible time period with good data quality. Hence the creation of the factors is based on a 24 year time period, as we wanted to provide good factors in the regression. Below is a description of how the factors were constructed.

1. Mkt (Excess market return) was obtained by subtracting the risk free rate of return from the returns of our constructed market-capitalization weighted Nordic 120 index, with a 10% weight constraint. Continuing to the other factors, we have used the entire universe (hence not the only constituents of the market-capitalization weighted index, in order to avoid biases such as survivorship bias etc.). For the market factor on the other hand, the index was used as the investability aspect (discussed in section 5.1.2) is of greater importance than the potential biases which can occur when using the index rather than the total universe when creating the factor.

2. SMB (Small minus Big, in terms of market-capitalization), with annual rebalancing, was obtained by creating two equally weighted indices conditional on a binary variable that has the following properties:

$$Small_{i,t} = 0 \; if \; MV_{i,t} > percentile_{75}\left(MV_{i=1 \; to \; N,t}\right)$$

$$Small_{i,t} = 1 \; if \; MV_{i,t} \leq percentile_{75}\left(MV_{i=1 \; to \; N,t}\right)$$

,where $Small_{i,t}$ is the binary value of security i at time t, $MV_{i,t}$ is security i's market capitalization value measured in SEK, at the last annual rebalancing date t, and N is the number of securities listed at the last annual rebalancing date t. Hence,

$$SMB_t = \left(\frac{1}{M_{1,t}}\sum_{i=1}^{M_{1,t}}\left(r_{i,t} \middle| Small_{i,t} = 1\right)\right) - \left(\frac{1}{M_{0,t}}\sum_{i=1}^{M_{0,t}}\left(r_{i,t} \middle| Small_{i,t} = 0\right)\right)$$

,where SMB_t is the factor return at time t, $M_{1,t}$ and $M_{0,t}$ is the number of securities at time t with $Small_{i,t} = 1$ and $Small_{i,t} = 0$ respectively, and $r_{i,t}$ is the return of security i at time t adjusted for any currency exchange rate change.

3. HML (High minus Low, in terms of book-to-market ratio) was obtained by creating two equally weighted indices conditional on a binary variable that has the following properties:

$$High_{i,t} = 0 \; if \; PTBV_{i,t} > \widetilde{PTBV}_t$$

$High_{i,t} = 1 \; if \; PTBV_{i,t} \leq \widetilde{PTBV}_t$

where $High_{i,t}$ is the binary value of security i at time t, $PTBV_{i,t}$ is the price-to-book ratio of security i at the last annual rebalancing date t, and $\widetilde{PTBV}_t$ is the median price-to-book ratio at the last annual rebalancing date t. Hence,

$$HML_t = \left(\frac{1}{L_{1,t}} \sum_{i=1}^{L_{1,t}} \left(r_{i,t} \middle| High_{i,t} = 1 \right) \right) - \left(\frac{1}{L_{0,t}} \sum_{i=1}^{L_{0,t}} \left(r_{i,t} \middle| High_{i,t} = 0 \right) \right)$$

,where HML_t is the factor return at time t, $L_{1,t}$ and $L_{0,t}$ is the number of securities at time t with $High_{i,t} = 1$ and $High_{i,t} = 0$ respectively, and $r_{i,t}$ is the return of security i at time t adjusted for any currency exchange rate change.

4. WML (Winners minus Losers, in terms of prior return) was obtained by creating two equally weighted indices conditional on a binary variable that has the following properties:

$Winner_{i,t} = 0 \; if \; Ret_{i,t-12 \; to \; t-2} \leq percentile_{30}\left(Ret_{i=1 \; to \; N, \;\; t-12 \; to \; t-2}\right)$

$Winner_{i,t} = 1 \; if \; Ret_{i,t-12 \; to \; t-2} > percentile_{70}\left(Ret_{i=1 \; to \; N, \;\; t-12 \; to \; t-2}\right)$

,where $Winner_{i,t}$ is the binary value of security i at time t, $Ret_{i,t-12 \; to \; t-2}$ is the prior 12 month to 2 month return of security i at the last annual rebalancing date t, and N is the number of securities listed at the last annual rebalancing date t. This is according to methodology presented by Carhart (1997). Hence,

$$WML_t = \left(\frac{1}{K_{1,t}} \sum_{i=1}^{L_{1,t}} \left(r_{i,t} \middle| Winner_{i,t} = 1 \right) \right) - \left(\frac{1}{K_{0,t}} \sum_{i=1}^{L_{0,t}} \left(r_{i,t} \middle| Winner_{i,t} = 0 \right) \right)$$

,where WML_t is the factor return at time t, $K_{1,t}$ and $K_{0,t}$ is the number of securities at time t with $Winner_{i,t} = 1$ and $Winner_{i,t} = 0$ respectively, and $r_{i,t}$ is the return of security i at time t adjusted for any currency exchange rate change.

6.3 Re-allocation costs

As the indexes are rebalanced semi-annually, it is important to determine the cost of rebalancing. The turnover is an important indication of the different indexes transaction costs. The explicit transaction cost is represented by the bid-ask spread, whereas the implicit transaction cost is based on the effect on the market one has when trading. Thereby, there are significant transaction costs with those strategies that have higher turnover. Annual turnover is here defined as the average percentage of the index portfolio that is traded each year. To keep comparability between the turnover of the indexation methods, only those transactions that are not a result of delisting or constituent replacement (i.e. a security falls out of the market capitalized index portfolio due to a too low market capitalization or liquidity) are taken into account. Hence,

$$\overline{TO}_i = \frac{1}{T}\sum_{t=1}^{T-1}\sum_{j=1}^{N_t}\left(abs\left|\frac{w_{j,t+1}^{i} * I_{i,t+1}}{P_{j,t+1} * e_{j,t+1}} - \frac{w_{j,t}^{i} * I_{i,t}}{P_{j,t} * e_{j,t}}\right| * \frac{P_{j,t+1} * e_{j,t+1}}{I_{i,t+1}}\right)$$

,where $\overline{TO}_i$ is the average annual turnover for index i, T is the total time-span in term of years, t is a cumulative number of re-allocation dates, N_t is the number of constituents in the index at re-allocation time t, abs is the absolute value, $w_{j,t}^{i}$ is the weight of constituent j in index i at at re-allocation time t, $I_{i,t}$ is the portfolio value of index i, denoted in SEK, at re-allocation time t, $P_{j,t}$ is the price in quote currency of constituent j at re-allocation time t, and $e_{j,t}$ is the foreign exchange rate of index quote currency to quote currency of constituent j at re-allocation time t.

7. Results

The results are presented in the following way. First the performances of the different indexes are presented in terms of annualized return, annualized volatility, annualized Sharpe ratio and Maximum drawdown. Thereafter the source of the performances is determined through CAPM (Sharpe (1964)), the Fama-French three factor model (Fama French (1993)) as well as the Carhart four-factor model (Carhart (1997)). Lastly, the overall results are discussed and related to the two hypotheses of the study.

7.1 Performance of the indexes

The prior studies referred to in this study present results based on US data. Hence this study contributes with interesting results based on Nordic data. The main results presented are based on a 20 year time period. In the appendix shorter time spans are presented due to the fact that we aim to combine the academic point of view and the preferences of the practitioners. From an academic point of view it is often preferred to use the longest time span possible, while some practitioners prefer a shorter time span when determining the attractiveness of a strategy as they argue that historical data becomes less relevant as time passes.

Looking at table 1 with the weight constraint set to maximum 10% one can determine that for 250 days estimation window when estimating the covariance matrix, the annualized return over a 20 year time period is generally higher in the risk-weighted indexes minimum variance and equal risk contribution, compared to the market-capitalization weighted. The most diversified index shows more mixed results, but had generally higher annualized returns, whereas the annualized return for the equally weighted index was equal to the market-capitalization weighted. On the other hand when using 500 days estimation window when estimating the covariance matrix the results vary somewhat. Only when using the shrinkage method for estimating the covariance matrix for the equal risk contribution index and the most diversified index a higher annualized return was obtained, compared to the market-capitalization weighted. Overall the highest annualized return is offered by the minimum variance index, with up to 2.9 percentage points' higher annualized return than the market-capitalization weighted.

What is somewhat more striking is that for all the risk-weighted indexes, the annualized volatility lies clearly below the market-capitalization weighted index. The lowest annualized volatility is offered by the minimum variance index. Given the lower annualized volatilities and in general the higher annualized returns, higher annualized Sharpe ratios are expected for the risk-weighted indexes. No matter if using 500 days estimation window when estimating the covariance matrix, or using the sample covariance matrix or the shrinkage method, the annualized Sharpe ratios offered by the minimum variance index, the equal risk contribution index and the most diversified index are all higher compared to the market-capitalization weighted. Looking at table 1 we can determine that the equally weighted index also offers a higher annualized Sharpe ratio compared to the market-capitalization weighted. Just like the case for the annualized return, the best annualized Sharpe ratio is offered by the minimum variance index.

No absolute conclusion can be given when looking at the shrinkage method results compared to the sample covariance matrix results. For the minimum variance index, the equal risk contribution index and the most diversified index, the annualized return has a higher average for the shrinkage method using 500 days of estimation window or 250 days of estimation window, compared to the sample covariance matrix. The annualized volatility is lower or the same level for the shrinkage, when comparing the results of sample covariance matrix. Looking at the average of the annualized Sharpe ratio, using the shrinkage offers a higher annualized Sharpe ratio compared to the sample covariance matrix, when looking at equal risk contribution and most diversified index. The opposite is true for the minimum variance index. Hence looking at the performance, there is no clear advantage of using the shrinkage method, in terms of better index performance.

Continuing the comparison with the weight constraint 5% we can determine the following from table 2. Similar to the observation in table 1, the annualized return offered by the risk-weighted indexes varies depending on the estimation method of the covariance matrix. But in contrast to when having a weight constraint of 10%, using the 250 days estimation window length does not give a higher annualized return for the risk-weighted indexes compared to the market-capitalization weighted. In the case of the minimum variance index and the equal risk contribution index, they offer a higher annualized return than the market-capitalization weighted for all estimation methods except when looking at the sample covariance matrix estimated with 500 days.

Comparing the annualized volatility gives a much clearer overall result, as all risk-weighted indexes offer a lower annualized volatility no matter which estimation method that is used. Just like the case when having 10% weight constraint, the minimum variance index offers the lowest annualized return. Looking at the annualized Sharpe ratio, all risk-weighted indexes except the equally-weighted index offer a higher annualized Sharpe ratio, compared to the market-capitalization weighted. When comparing the results of the shrinkage method to the sample covariance matrix, just like the case when having a weight constraint of 10%, no clear observation can be given. The average of the annualized return in using the sample covariance matrix with 500 days of estimation window and 250 days is lower in the minimum variance index compared to the shrinkage average. The opposite is true when looking at the most diversified index, where the sample covariance index offers a higher annualized return average between 500 days estimation window and 250 days. On the other hand, the average of the annualized volatility is lower using the shrinkage for all three: the minimum variance, the equal risk contribution and the most diversified. Comparing the average of the annualized Sharpe ratio, one can determine that the average of the shrinkage method is higher for the minimum variance index and the equal risk contribution, while the opposite is true for the most diversified.

Based on the overall results presented in table 1 and 2, one can determine that the annualized Sharpe ratios are higher for minimum variance, equal risk contribution and most diversified. A clear observation is also the fact that these indexes offer a higher annualized return, while no distinct trend can be determined when comparing the performance when using the sample covariance matrix and the shrinkage method.

Table 1 - Index Summary Statistics (10% constituent weight constraint): 1992-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	**Annualized return**	**Annualized volatility**	**Annualized Sharpe ratio**	**Maximum drawdown**
weights ≤ 10%	13,7%	20,1%	0,50	-54,1%
Equally weighted index				
weights ≤ 10%	13,7%	19,3%	0,52	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	16,6%	13,4%	0,97	-51,6%
250 days estimation window weights ≤ 10%	15,9%	13,8%	0,89	-50,6%
Shrinkage				
500 days estimation window weights ≤ 10%	15,6%	13,6%	0,87	-51,6%
250 days estimation window weights ≤ 10%	16,2%	12,8%	0,98	-51,4%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	12,6%	15,7%	0,56	-57,8%
250 days estimation window weights ≤ 10%	15,1%	15,3%	0,75	-51,2%
Shrinkage				
500 days estimation window weights ≤ 10%	15,6%	15,3%	0,78	-52,4%
250 days estimation window weights ≤ 10%	14,2%	15,2%	0,69	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	13,2%	16,7%	0,57	-61,1%
250 days estimation window weights ≤ 10%	14,5%	16,4%	0,66	-56,4%
Shrinkage				
500 days estimation window weights ≤ 10%	13,9%	16,0%	0,63	-57,7%
250 days estimation window weights ≤ 10%	13,6%	15,7%	0,63	-56,6%

Table 2 - Index Summary Statistics (5% constituent weight constraint): 1992-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	**Annualized return**	**Annualized volatility**	**Annualized Sharpe ratio**	**Maximum drawdown**
weights ≤ 5%	14,3%	19,4%	0,54	-54,5%
Equally weighted index				
weights ≤ 5%	13,7%	19,3%	0,52	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	13,3%	15,7%	0,61	-58,1%
250 days estimation window weights ≤ 5%	15,5%	14,0%	0,84	-54,3%
Shrinkage				
500 days estimation window weights ≤ 5%	16,1%	14,1%	0,88	-52,3%
250 days estimation window weights ≤ 5%	15,4%	13,5%	0,87	-50,7%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	13,3%	15,7%	0,61	-58,1%
250 days estimation window weights ≤ 5%	16,1%	15,3%	0,81	-49,6%
Shrinkage				
500 days estimation window weights ≤ 5%	16,2%	15,4%	0,81	-52,9%
250 days estimation window weights ≤ 5%	14,7%	15,1%	0,72	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	14,0%	16,9%	0,61	-59,5%
250 days estimation window weights ≤ 5%	14,3%	16,4%	0,64	-55,7%
Shrinkage				
500 days estimation window weights ≤ 5%	13,8%	16,3%	0,62	-58,1%
250 days estimation window weights ≤ 5%	13,0%	16,1%	0,58	-56,1%

7.2 Capital asset pricing model regression

Given the positive results of the risk-weighted indexes, presented in table 1 and 2, it is of great importance to determine the sources of the performance.

Looking at table 3 with the weight constrain of 10% we can determine that the equally weighted index, the equal risk contribution index and the most diversified index have alphas close to zero that are not significantly different from zero. The intercept, the alpha, can be interpreted as excess return not explained by any risk factor, as it measures the difference between an investment's actual returns and its expected return, given its level of risk (measured by beta). This implies the indexes have earned a return almost adequate for the risk taken according to how CAPM prices risk. Only the minimum variance index has larger alphas that are significantly different from zero, at a 5% and 10% significance level, which indicates that the minimum variance index has over performed what CAPM predicted by these percentages on a yearly basis. For all alphas that are not significantly different from zero we can determine that these indexes do not perform better, according to the risk taken. Hence CAPM prices the risk-return trade-off correctly.

What is important to clarify early in this section is the difference between statistical significance and economic significance, with regards to alpha especially. In table 3 we notice that some alphas that can be considered quite large and important, although they were not statistically significant. This, however, does not mean they are to be considered unimportant. A high alpha, though not statistically significant, may indeed indicate the strategy is outperforming benchmark strategies. The regression models are tested under strict but conventional assumptions that may not hold in reality. Hence considering both the statistical significance and the economic significance is important in order to have both an academic and practitioner's point of view.

Continuing looking at beta of the risk-weighted index, we observe that all are positive which implies that all has a positive exposure against the market factor. Beta measures the systematic risk, which is the component of risk that is correlated to market movements and cannot be eliminated through diversification The minimum variance index has the lowest market beta loading, whereas the equally weighted has the highest. A low beta implies a low exposure against the market risk, which is often deemed desirable since investors want to be sure the value of their investments are not fully correlated with the market movements, such as a period of higher-than-normal unemployment, or bad economic times.

The coefficient of determination, R-squared, can be interpreted as how much the movements of daily excess index returns that can be explained by movements of daily factor returns. The equally weighted index, with its high beta, also has the highest R-squared. The risk-weighted indexes all have lower R-squared, which indicates that the excess returns of these is rather explained by something else than a market factor. CAPM's failure when tested empirically, which was pointed out earlier, is here shown by the lower R-squared for the risk-weighted indexes, as it indicates CAPM's failure to explain their excess return.

Continuing the analysis looking at table 4 with the weight constraint of 5% we can determine that just like the case when the weight constraint was 10%, the only alphas observed that are considerably larger than zero are the ones of the minimum variance index. Although one of the alphas of the equal risk contribution index is 2.77%, it is significantly different from zero only at a 10% level. All the alphas that are not significantly different from zero, implies that adequate risk is taken given the return. Like the case with 10% weight constraint, the minimum variance index has the lowest market loading, as its beta is the lowest. All the betas are significant at a 5% significance level. There is no significant change in the R-squared values, compared to the values obtained with a 10% weight constraint.

Given table 3 and table 4 the overall observation is that there is a reduction of exposure to a market factor, when the risk-weighted index strategy is used. This is a good property of these indexes, as it implies less co-movement with the market, described above. The minimum variance index is the only index with consistently large and significant alphas. Together with the low betas, the index seems to have many good properties.

Table 3 - Capital Asset Pricing Model Regression (10% constituent weight constraint): 1988-12-16 - 2012-12-15

Regression results of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified daily index portfolio returns. Alpha is scaled of with a factor of 260, the approximate amount of working days each year. One * implies that the value is significant at a 10% significance level, whereas two ** implies that the value is significant at a 5% significance level. Numbers within parentheses implies negative numbers.

Regression model: $r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$

Equally weighted index	α	t	β	t	R2
weights ≤ 10%	(0,19)%	-0,15	0,90	125,49**	0,89
Minimum variance index					
Sample covariance matrix					
500 days estimation window weights ≤ 10%	3,44%	2,69**	0,52	56,38**	0,58
250 days estimation window weights ≤ 10%	3,67%	2,20**	0,54	51,97**	0,63
Shrinkage					
500 days estimation window weights ≤ 10%	4,44%	1,93*	0,53	47,32**	0,63
250 days estimation window weights ≤ 10%	3,64%	2,26**	0,51	51,20**	0,62
Equal risk contribution index					
Sample covariance matrix					
500 days estimation window weights ≤ 10%	0,32%	0,20	0,67	73,83**	0,75
250 days estimation window weights ≤ 10%	1,52%	0,92	0,65	66,53**	0,71
Shrinkage					
500 days estimation window weights ≤ 10%	2,52%	1,67*	0,66	80,66**	0,76
250 days estimation window weights ≤ 10%	0,63%	0,41	0,67	74,63**	0,75
Most diversified index					
Sample covariance matrix					
500 days estimation window weights ≤ 10%	0,63%	0,35	0,68	65,16**	0,70
250 days estimation window weights ≤ 10%	1,67%	0,97	0,68	66,23**	0,72
Shrinkage					
500 days estimation window weights ≤ 10%	1,11%	0,69	0,68	71,21**	0,74
250 days estimation window weights ≤ 10%	0,82%	0,50	0,66	66,90**	0,73

Table 4 - Capital Asset Pricing Model Regression (5% constituent weight constraint): 1988-12-16 - 2012-12-15

Regression results of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified daily index portfolio returns. Alpha is scaled of with a factor of 260, the approximate amount of working days each year. One * implies that the value is significant at a 10% significance level, whereas two ** implies that the value is significant at a 5% significance level. Numbers within parentheses implies negative numbers.

Regression model: $r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$

Equally weighted index	α	**t**	β	**t**	**R2**
weights ≤ 5%	(0,19)%	-0,15	0,90	125,49**	0,89
Minimum variance index					
Sample covariance matrix					
500 days estimation window weights ≤ 5%	3,68%	2,34**	0,56	58,61**	0,67
250 days estimation window weights ≤ 5%	3,07%	1,91*	0,57	55,78**	0,67
Shrinkage					
500 days estimation window weights ≤ 5%	3,75%	2,27**	0,57	55,27**	0,66
250 days estimation window weights ≤ 5%	2,81%	1,83*	0,56	60,96**	0,68
Equal risk contribution index					
Sample covariance matrix					
500 days estimation window weights ≤ 5%	0,69%	0,44	0,67	74,93**	0,75
250 days estimation window weights ≤ 5%	2,37%	1,49	0,65	67,86**	0,73
Shrinkage					
500 days estimation window weights ≤ 5%	2,77%	1,90*	0,67	81,71**	0,78
250 days estimation window weights ≤ 5%	1,13%	0,75	0,66	74,23**	0,76
Most diversified index					
Sample covariance matrix					
500 days estimation window weights ≤ 5%	1,15%	0,64	0,70	68,47**	0,71
250 days estimation window weights ≤ 5%	1,45%	0,86	0,69	66,20**	0,74
Shrinkage					
500 days estimation window weights ≤ 5%	0,96%	0,59	0,69	72,48**	0,75
250 days estimation window weights ≤ 5%	0,31%	0,19	0,68	68,74**	0,74

7.3 Fama-French three-factor model regression

Looking at table 5 we can determine that none of the risk-weighted indexes have an alpha that is significantly different from zero on a 5% level. The equally weighted portfolio and one of the minimum variance portfolios have an alpha significantly different from zero on a 10% level, which implies that this index slightly over performs what the Fama-French three factor model expects.

Looking at the market betas, just like in the market factor regression presented above, the minimum variance index has the smallest market loading. Looking at the size beta, small-minus-big, one can determine that both the equal risk contribution index and the most diversified index have size betas that are negative and significant. Looking at the value beta, the high-minus-low factor, one can determine that all the three risk-weighted indexes have positive and significant betas. Hence their over performance compared to the market-capitalization weighted can partly be explained by their exposure toward large companies and companies with a high book-to-market ratio. These findings are aligned with the results found by Chow, Hsu, Kalesnik and Little (2011), which also found that the risk-weighted indexes had exposure toward the size and value factor. Looking at the betas for the equally weighted index, there are all significant. Hence the equally weighted index underperforms compared to the market-capitalization weighted indexes, presented in table 1 and 2, but has exposure towards the market, size and value factor. The R-squared is generally not increasing from the earlier tested one-factor model. This is a drawback in the sense that the three-factor model does not explain the co-movements of the dependent and independent variables better than the one-factor model. Hence it is questionable if it is worth adding the extra risk-factors (size and value) when evaluating the different index methods.

Looking at table 6 with the Fama-French three-factor regression and 5% weight constraint, we can determine that similar results are found compared to those with 10% weight constraint. But the main difference is that the equally weighted index and all three of the risk-weighted indexes have both significant size and value betas. Hence the performance of the risk-weighted indexes can be even more explained by their exposure towards the size and value factors. Also here the R-squared does not increase compared to the one-factor regression.

Table 5 - Fama-French Three-Factor Model Regression (10% constituent weight constraint): 1988-12-16 - 2012-12-15

Regression results of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified daily index portfolio returns. Alpha is scaled of with a factor of 260, the approximate amount of working days each year. One * implies that the value is significant at a 10% significance level, whereas two ** implies that the value is significant at a 5% significance level. Numbers within parentheses implies negative numbers.

Regression model: $r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}(r_{m,t} - r_{f,t}) + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \varepsilon_{i,t}$

Equally weighted index	α	t	β_Mkt	t	β_SMB	t	β_HML	t	R2
weights ≤ 10%	2,45%	1,88*	0,82	52,57**	(0,23)	(6,85)**	0,03	2,06**	0,90
Minimum variance index									
Sample covariance matrix									
500 days estimation window weights ≤ 10%	3,24%	1,91*	0,55	36,07**	(0,02)	(0,93)	0,09	5,78**	0,63
250 days estimation window weights ≤ 10%	2,88%	1,61	0,55	44,54**	(0,05)	(1,70)*	0,09	6,16**	0,64
Shrinkage									
500 days estimation window weights ≤ 10%	2,00%	1,08	0,54	34,94**	(0,02)	(0,62)	0,10	6,15**	0,58
250 days estimation window weights ≤ 10%	2,53%	1,52	0,52	38,81**	(0,04)	(1,65)*	0,10	6,85**	0,62
Equal risk contribution index									
Sample covariance matrix									
500 days estimation window weights ≤ 10%	0,29%	0,18	0,65	42,52**	(0,16)	(5,77)**	0,13	7,91**	0,76
250 days estimation window weights ≤ 10%	1,73%	1,03	0,62	39,62**	(0,14)	(4,75)**	0,10	6,36**	0,72
Shrinkage									
500 days estimation window weights ≤ 10%	2,03%	1,33	0,66	51,96**	(0,05)	(2,66)**	0,07	5,35**	0,76
250 days estimation window weights ≤ 10%	0,94%	0,59	0,63	44,53**	(0,17)	(5,72)**	0,12	8,02**	0,76
Most diversified index									
Sample covariance matrix									
500 days estimation window weights ≤ 10%	0,60%	0,32	0,68	45,41**	(0,04)	(1,54)	0,03	(1,93)*	0,70
250 days estimation window weights ≤ 10%	2,12%	1,21	0,65	44,64**	(0,15)	(5,72)**	0,09	6,25**	0,73
Shrinkage									
500 days estimation window weights ≤ 10%	1,10%	0,67	0,67	49,68**	(0,07)	(3,35)**	0,06	4,25**	0,75
250 days estimation window weights ≤ 10%	0,57%	0,34	0,65	50,04**	(0,07)	(3,38)**	0,07	5,52**	0,74

Table 6 - Fama-French Three-Factor Model Regression (5% constituent weight constraint): 1988-12-16 - 2012-12-15

Regression results of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified daily index portfolio returns. Alpha is scaled of with a factor of 260, the approximate amount of working days each year. One * implies that the value is significant at a 10% significance level, whereas two ** implies that the value is significant at a 5% significance level. Numbers within parentheses implies negative numbers.

Regression model: $r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}(r_{m,t} - r_{f,t}) + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \varepsilon_{i,t}$

Equally weighted index	α	t	β_Mkt	t	β_SMB	t	β_HML	t	R2
weights ≤ 5%	2,45%	1,88*	0,82	52,57**	(0,23)	(6,85)**	0,03	2,06**	0,90
Minimum variance index									
Sample covariance matrix									
500 days estimation window weights ≤ 5%	2,94%	1,83*	0,56	35,8**	(0,07)	(3,05)**	0,11	6,65**	0,68
250 days estimation window weights ≤ 5%	2,67%	1,56	0,56	44,63**	(0,09)	(3,28)**	0,10	6,98**	0,68
Shrinkage									
500 days estimation window weights ≤ 5%	3,14%	1,81*	0,56	37,67**	(0,07)	(2,65)**	0,10	6,30**	0,66
250 days estimation window weights ≤ 5%	2,34%	1,48	0,55	42,93**	(0,10)	(4,19)**	0,11	8,03**	0,69
Equal risk contribution index									
Sample covariance matrix									
500 days estimation window weights ≤ 5%	0,98%	0,62	0,64	41,52**	(0,17)	(6,28)**	0,12	7,63**	0,77
250 days estimation window weights ≤ 5%	2,74%	1,70*	0,62	39,61**	(0,15)	(5,37)**	0,10	6,69**	0,74
Shrinkage									
500 days estimation window weights ≤ 5%	2,85%	1,95*	0,65	50,47**	(0,11)	(5,07)**	0,08	5,67**	0,78
250 days estimation window weights ≤ 5%	1,55%	1,02	0,62	43,67**	(0,18)	(6,14)**	0,12	8,15**	0,77
Most diversified index									
Sample covariance matrix									
500 days estimation window weights ≤ 5%	1,26%	0,68	0,69	45,45**	(0,05)	(2,08)**	0,03	2,09**	0,71
250 days estimation window weights ≤ 5%	1,98%	1,16	0,66	44,95**	(0,15)	(5,90)**	0,09	6,41**	0,75
Shrinkage									
500 days estimation window weights ≤ 5%	1,07%	0,65	0,67	49,58**	(0,09)	(4,19)**	0,07	4,82**	0,75
250 days estimation window weights ≤ 5%	0,32%	0,19	0,66	49,34**	(0,11)	(4,70)**	0,09	6,28**	0,75

7.4 Carhart four-factor model regression

Looking at table 7 we can determine that the majority of the alphas are not significantly different from zero, hence adequate return is achieved given the risk taken, according to the model. The only risk-weighted index that has a significant alpha is the minimum variance index, which also shows the highest over performance of 3.27% per year. Looking at the market betas, as determined in the other tables, the minimum variance index has the lowest market beta. As in the former section, all indexes have negative size betas, though not all of these are statistically significant. At the same time, all value betas are positive and very significant. The new momentum factor does not add a lot to performance understanding as the size of the momentum betas generally are very low, and both positive and negative. Thus, the returns are better explained by Fama and French's (1993) three factors, rather than Carhart's (1997) new momentum factor. The R-squared is barely affected by the extra factor, which further implies that the momentum factor is not needed.

Continuing with table 8, we determine that some more alphas are statistically significant on a 10% level. Notably, it is the equally weighted index, the minimum variance index and the equal risk contribution index show positive alphas. From an economic point of view, the results show high alphas, where the best index show an over performance of 3.15% per year. The risk-weighted indexes again show generally low, positive market betas, while having negative size betas and positive value betas. Also here the momentum betas are generally insignificant, while the R-squared show no improvement.

Table 7 - Carhart Four-Factor Model Regression (10% constituent weight constraint): 1988-12-16 - 2012-12-15

Regression results of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified daily index portfolio returns. Alpha is scaled of with a factor of 260, the approximate amount of working days each year. One * implies that the value is significant at a 10% significance level, whereas two ** implies that the value is significant at a 5% significance level. Numbers within parentheses implies negative numbers.

Regression model: $r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}(r_{m,t} - r_{f,t}) + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \beta_{4,i}WML_t + \varepsilon_{i,t}$

Equally weighted index	α	t	β_Mkt	t	β_SMB	t	β_HML	t	β_WML	t	R2
weights ≤ 10%	2,38%	1,83*	0,81	48,66**	(0,24)	(6,49)**	0,02	1,72*	(0,03)	(3,13)**	0,90
Minimum variance index											
Sample covariance matrix											
500 days estimation window weights ≤ 10%	3,27%	1,93*	0,55	35,52**	(0,02)	(0,64)	0,09	5,81**	0,01	1,22	0,63
250 days estimation window weights ≤ 10%	2,87%	1,60	0,55	43,66**	(0,05)	(1,71)*	0,09	6,18**	(0,01)	(0,74)	0,64
Shrinkage											
500 days estimation window weights ≤ 10%	2,04%	1,10	0,54	34,14**	(0,01)	(0,35)**	0,10	6,24**	0,01	1,51	0,58
250 days estimation window weights ≤ 10%	2,54%	1,53	0,52	38,22**	(0,04)	(1,47)	0,10	6,85**	0,01	0,55	0,62
Equal risk contribution index											
Sample covariance matrix											
500 days estimation window weights ≤ 10%	0,29%	0,18	0,65	41,25**	(0,16)	(5,49)**	0,13	7,79**	0,00	(0,27)	0,76
250 days estimation window weights ≤ 10%	1,69%	1,00	0,62	37,92**	(0,15)	(4,64)**	0,10	6,17**	(0,02)	(1,73)*	0,72
Shrinkage											
500 days estimation window weights ≤ 10%	2,04%	1,34	0,66	50,98**	(0,05)	(2,47)**	0,07	5,34**	0,00	0,36	0,76
250 days estimation window weights ≤ 10%	0,89%	0,56	0,63	41,98**	(0,18)	(5,53)**	0,11	7,83**	(0,02)	(2,12)**	0,76
Most diversified index											
Sample covariance matrix											
500 days estimation window weights ≤ 10%	0,60%	0,33	0,68	45,30**	(0,04)	(1,42)	0,03	1,96**	0,00	0,35	0,70
250 days estimation window weights ≤ 10%	2,08%	1,19	0,65	43,37**	(0,16)	(5,62)**	0,09	6,11**	(0,02)	(1,65)**	0,73
Shrinkage											
500 days estimation window weights ≤ 10%	1,12%	0,68	0,67	49,88**	(0,07)	(3,10)**	0,06	4,31**	0,01	0,74	0,75
250 days estimation window weights ≤ 10%	0,55%	0,33	0,65	49,75**	(0,08)	(3,46)**	0,07	5,43**	(0,01)	(1,05)	0,74

Table 8 - Carhart Four-Factor Model Regression (5% constituent weight constraint): 1988-12-16 - 2012-12-15

Regression results of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified daily index portfolio returns. Alpha is scaled of with a factor of 260, the approximate amount of working days each year. One * implies that the value is significant at a 10% significance level, whereas two ** implies that the value is significant at a 5% significance level. Numbers within parentheses implies negative numbers.

Regression model: $r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}(r_{m,t} - r_{f,t}) + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \beta_{4,i}WML_t + \varepsilon_{i,t}$

Equally weighted index	α	t	β_Mkt	t	β_SMB	t	β_HML	t	β_WML	t	R2
weights ≤ 5%	2,38%	1,83*	0,81	48,66**	(0,24)	(6,49)**	0,02	1,72*	(0,03)	(3,13)**	0,90
Minimum variance index											
Sample covariance matrix											
500 days estimation window weights ≤ 5%	2,96%	1,85*	0,56	35,32**	(0,07)	(2,71)**	0,11	6,62**	0,01	1,03	0,68
250 days estimation window weights ≤ 5%	2,65%	1,56	0,56	43,56**	(0,10)	(3,19)**	0,10	6,96**	(0,01)	(0,76)	0,68
Shrinkage											
500 days estimation window weights ≤ 5%	3,15%	1,83*	0,57	37,04**	(0,07)	(2,38)**	0,10	6,32**	0,01	0,79	0,66
250 days estimation window weights ≤ 5%	2,37%	1,50	0,55	42,79**	(0,10)	(3,80)**	0,11	8,04**	0,01	1,3	0,69
Equal risk contribution index											
Sample covariance matrix											
500 days estimation window weights ≤ 5%	0,96%	0,61	0,64	39,98**	(0,18)	(5,98)**	0,12	7,48**	(0,01)	(0,81)	0,77
250 days estimation window weights ≤ 5%	2,70%	1,68*	0,62	37,94**	(0,17)	(5,19)**	0,10	6,52**	(0,01)	(1,52)	0,74
Shrinkage											
500 days estimation window weights ≤ 5%	2,84%	1,95*	0,65	49,34**	(0,11)	(4,92)**	0,08	5,61**	(0,00)	(0,47)	0,78
250 days estimation window weights ≤ 5%	1,50%	0,99	0,62	41,25**	(0,19)	(5,91)**	0,11	7,98**	(0,02)	(2,01)**	0,77
Most diversified index											
Sample covariance matrix											
500 days estimation window weights ≤ 5%	1,27%	0,69	0,69	45,41**	(-0,05)	(1,95)*	0,04	2,12**	0,00	0,36	0,71
250 days estimation window weights ≤ 5%	1,95%	1,14	0,66	44,01**	(-0,16)	(5,75)**	0,09	6,31**	(0,01)	(1,16)	0,75
Shrinkage											
500 days estimation window weights ≤ 5%	1,08%	0,65	0,67	49,70**	(-0,09)	(3,96)**	0,07	4,85**	0,01	0,54	0,75
250 days estimation window weights ≤ 5%	0,29%	0,17	0,66	48,51**	(-0,11)	(4,72)**	0,08	6,16**	(0,01)	(1,45)	0,75

7.5 Transaction costs

The main advantages of passive investment strategies have long been their diversification to a relatively low cost. This index property becomes even more important as practitioners explore new ways of constructing diversified portfolios. The market-capitalization weighted index is by construction a low-cost index, where constituent weight changes adjust more or less automatically as prices of securities increase or decrease. Thus, leaving this conventional passive investment strategy demands a thorough analysis of the transaction costs that most surely will rise. With the calculations stated in section 6.2, we estimated turnover values as shown in table 9 below:

Table 9 - Average Annual Index Turnover (10% constituent weight constraint, 250 days estimation window): 1988-12-16 - 2012-12-15

Average annual turnover of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. The values represents the average share of the index portfolios that is traded each year.

	Market capitalization index	Equally weighted index	Minimum variance index	Equal risk contribution index	Most diversified index
Normal covariance estimation method	0,14	0,30	1,20	1,21	1,14
Shrinkage covariance estimation method			1,33	1,15	1,03

Looking at table 9 we can determine that the market-capitalization weighted index has the lowest turnover, which implies the lowest transaction costs. Hence our results are aligned with the prior study by Hsu (2006, as the main advantage of the market-capitalization weighted index is its low turnover. Approximately 14% of our market-capitalization weighted portfolio is rebalanced every year. However, the real number is probably higher since this measured turnover does not take into account securities that fell out of the index due to too low market capitalization or liquidity. Hence, the results in table 9 are better interpreted relative to each other, as they represent the value of an index transaction cost property, rather than a value that directly can be used for historical transaction cost calculation.

Comparing the risk-weighted indexes, one can determine that their annual turnovers are about equally high, all over 100% of the portfolio value. Hence all the risk-weighted index portfolios have relatively high transaction costs. This is one of the largest drawbacks of optimization routines used for determining constituent weights. The equally weighted index suffers less from a high annual turnover, but as we go to the other three indexation methods that need optimization routines to determine constituent weights, we see a sharp increase in annual turnover. Practitioners have to be able to restrain this implied increase in costs somehow if they want to capture most of the Sharpe ratio increase of the risk-weighted indexation methods. This study proposes further research in this field, to better quantify the relationship higher Sharpe ratios versus higher transaction costs.

Comparing the transaction costs of the sample covariance method and the shrinkage method, no notable difference can be observed. The transaction costs are lowered for the equal risk contribution index and the most diversified index when using the shrinkage method, but at the same time increases for the minimum variance index. This unclear result is somewhat surprising as the shrinkage method ought to decrease the number of extreme weights, and thus reduce the risk of too many constituents with extreme weights needing to be re-balanced.

7.6 Overall results

Given the results in table 1 and table 2, one can determine that these results thereby support this study's first hypothesis:

(1) *The risk-weighted indexes can offer a better risk-return trade off compared to the market-capitalization weighted index.*

As described above, looking at table 1 and 2, one can determine that the risk-weighted indexes have a higher risk-return trade-off, measured in terms of their higher annualized Sharpe ratio. Hence the main differentiator of the risk-weighted indexes is the fact that it does not rely on the estimation of returns when assigning weights to the constituents of the indexes, and thus leads to the higher risk-return trade-off.

What is somewhat less clear is the advantage of using the shrinkage method when estimating the covariance matrix, compared to use the sample covariance matrix. In terms of both the performance and the turnover no general results can be determined when comparing the covariance matrix based on shrinkage method and the sample covariance matrix. The shrinkage covariance matrix offer higher annualized Sharpe ratios compared to the sample covariance matrix for the minimum variance index using 5% weight constraint, the equal risk contribution index using 10% and 5% weight constraint and the most diversified index using 10% weight constraint. Hence only for the equal risk contribution index, the shrinkage method offers a higher annualized Sharpe ratio, no matter what maximum weight constraint is set. As mentioned in the section 7.5 the constituents' turnover are lower for the equal risk contribution and the most diversified, but which is not the case for the minimum variance. Thereby, one can determine that the study's second hypothesis is not fully supported by the overall results:

(2) *Compared to the sample covariance matrix, the shrinkage covariance matrix can offer a better index risk-return trade off and lower index turnover due to less estimation errors in its elements.*

Comparing our results to prior studies one can determine the following. Our results stand in contrast to the results found by Demiguel, Garlappi and Uppal (2009) as in our data the equally weighted index does not offer a higher annualized return when having the weight constraint of 5% of the market-capitalization weighted index. In addition, the equally weighted index also offers a lower annualized return. When having 10% weight constraint on the market-capitalization weighted index, the equally weighted index offers a slightly higher annualized Sharpe ratio.

Comparing our results to the one found by Clark, De Silva and Thorley (2006), we can determine that we also found the minimum variance index to offer higher annualized Sharpe ratio and a lower annualized volatility. In addition, the annualized volatility of the equal risk contribution lies within the boundaries of the equally weighted and the minimum variance which was also found by Maillard, Roncalli and Teiletche (2008).

Looking at the performance of the equal risk contribution index, Demey, Maillard and Roncalli (2010) determine that this index offers the higher Sharpe ratio, whereas this is not the case for our results. Here the minimum variance index offers the most attractive results.

Furthermore, one can determine that the most diversified index performed less good compared to the other risk-weighted indexes, minimum variance index and equal risk contribution. But just like Choueifaty and Coignard (2008) found, the most diversified beats the market-capitalization weighted index.

Our results are in contrast with the prior study by Chow, Hsu, Kalesnik, Little (2011), as we did not found that the risk-weighted indexes had a momentum biases. In contrast to Choueifaty and Coignard (2008), we do not find a statistically significant alpha for the most diversified index.

Just like Maillard, Roncalli and Teiletche (2008) argue that although the most diversified and equal risk contribution approach have the idea of a well-diversified portfolio in common, the resulting portfolio is be completely different except the case when correlation across all assets is constant. This is shown in our results, both in terms of the higher Sharpe ratio by the equal risk contribution but also how the different indexes differ in terms of factor exposure.

Looking at our results when using the shrinkage covariance matrix, compared to the sample covariance matrix, one can determined that they are in some cases aligned with the findings by Ledoit and Wolf (2004b). We found, just like they did, that when using the shrinkage covariance matrix the average annualized returns were higher compared to the average annualized returns when using the sample covariance matrix. In addition, the average annualized volatility when using shrinkage was lower or the same level compared to the sample covariance matrix. Similar results were found by Ledoit and Wolf (20004b). As mentioned, even though some of our results indicate the advantages of shrinkage, we found that they were not enough to fully support our second hypothesis.

Although some of our results stand in contrast to prior studies, overall, our results are consistent with prior studies' conclusion that the risk-weighted indexes offer a better performance. However, most of the prior studies only compare one of the risk-weighted indexes to the market-capitalization weighted, such as minimum variance vs. market-capitalization. Thereby it is hard to determine if our results, where the minimum variance index offers the best performance compared to the other risk-weighted indexes, is consistent with prior studies.

Looking at the results from the Fama-French three factor regression model and Carhart four-factor model, one can determine that although not all alphas of the risk-weighted indexes were statistically significant they were economically significant due to their sizes. The most striking alphas can be found in the minimum variance index and the equal risk contribution. The importance of the high alphas is also supported by the fact that the risk-weighted indexes also offer a lower annualized volatility. Hence the over performance measured by alpha are achieved, although the risk level of the risk-weighted indexes are lower than the conventional market-capitalization weighted, which is a finding of great importance.

Although great annualized Sharpe ratios and economically significant alphas are achieved by the risk-weighted indexes, their higher transaction cost measured by the higher turnover needs to be considered. The higher turnover indicated in table 9, shows that the risk-weighted indexes implies higher transaction costs, which is indicated by their higher annual turnover. Thereby, although they have superior annualized Sharpe ratios compared to the market-capitalization weighted index, these drawbacks need to be considered when determining how well risk-weighted indexes actually perform.

Given the results of the different risk-weighted indexes one can determine that they entail a model selection risk and a performance evaluation risk. The model selection risk refers to the fact that the different risk-weighted indexes favors different aspects of the risk, since their index calculation reflects a set of assumptions one has under which the optimal portfolio is created. Hence under the 20 year time period that has been examined, the minimum variance index performed the best in general. But this can be due to the fact that the minimum variance model is favored by the prevailing market conditions over this time period. Hence one can expect the different risk-weighted indexes to favor different market conditions, which explains why the indexes perform differently depending on the market conditions. Looking at the results in the appendix, which displays the results for shorter time periods, we can determine that the indexes performed differently relative to each other depending on what time span one looks at. Thereby the best performing strategy in one sub-period can be the worst performing in another period. This implies that no index is uniquely superior. As the indexes favor different market conditions, it is a risk to choose a model that may not yield an attractive performance in a given period. This is reflected in the performances that vary depending on what time span one looks at.

The second risk is the performance evaluation risk, which refers the fact that evaluation model (such as Fama-French three factor model and Carhart four-factor model) and measurements (such as the annualized Shape ratio) are not representing investors' true risk-return trade-off considerations. Hence there is a risk that the performance and the attractiveness of the different indexes are based on inappropriate models and measurements.

Furthermore one can briefly mention that although the proven success of the risk-weighted indexes, it is likely that the Chief Investment Officer that is about to introduce a risk-weighted index will need to take a reputational risk since it is a risk to deviate from the common capitalization-weighted index as it represents the common reference in the market. This was also pointed out in the foreword. Thereby, although the number of studies conducted on this area have become considerably much higher compared to a couple of years ago, it still remains a lot of work in order to convince the market to leave the conventional market-capitalization weighted index, as humans by nature are reluctant to changing what has for long been regarded as the optimal method. In order for the risk-weighted indexes to gain interest in the Nordic countries there is a need for collaboration between academia and practitioners in order to show the market that the risk-weighted indexes is great alternative when choosing a passive investment strategy.

8. Conclusion

We present a detailed analysis of four alternative-weighted indexes, compared to the conventional market-capitalization weighted index; namely equally-weighted, minimum variance, equal risk contribution and most diversified. What these have in common is that they do not require any estimation of expected returns.

Initially, based on prior studies' findings, we stated two hypotheses:

(1) *The risk-weighted indexes can offer a better risk-return trade-off compared to the market-capitalization weighted index.*
(2) *Compared to the sample covariance matrix, the shrinkage covariance matrix can offer a better index risk-return trade off and lower index turnover due to less estimation errors in its elements.*

Overall, our results support the main findings of prior studies conducted on US data that show that the risk-weighted indexes outperform the conventional market-capitalization weighted. But some variation exists, especially when looking at our results within a short time span (found in appendix). Hence, our results support our first hypothesis that the risk-weighted indexes over perform compared to the market-capitalization weighted. On the other hand the advantage of the shrinkage covariance matrix is less conclusive, as the advantages are less clear when comparing the shrinkage covariance matrix to the sample covariance matrix, hence our second hypothesis were not fully supported by our results.

Determining the source of the performance of the indexes was conducted by using three factor models: CAPM, the Fama-French three-factor model and the Carhart four-factor model. We conclude that the performance of the risk-weighted indexes can partly be explained by their exposure toward a market portfolio, toward large companies and companies with a high book-to-market ratio. Hence the risk-weighted indexes had a negative size beta and a positive value beta. The momentum betas were generally not significant and did not improve the ability to determine the source of the indexes' performances.

In conclusion, our study show that the risk-weighted indexes offer a better risk-return trade-off, measured in terms of the annualized Sharpe ratio, compared to the market-capitalization weighted index, when transaction costs are excluded. Comparing the different risk-weighted indexes, one can conclude that the minimum variance index offered the most attractive results, both in terms of highest annualized Sharpe ratio, lowest market factor exposure and significant alphas.

8.1 Future research

For future studies it would be interesting to set constraints on the risk-weighted indexes to have the same market, size and value exposure as the market-capitalization weighted. Hence, the investors need to have a process in place, which measures and manage the different exposure to risk factors between the alternative-weighted indexes and the market-capitalization weighted in order to relate the over performance. This constraint was set by Clarke R., de Silva H. and Thorley S. (2006) and would permit an analysis between the different indexes performance when assuring that the differences in performance are not due to different factor exposures. It would also be of interest to change the frequency of rebalancing, for example from semi-annually to quarterly, although this implies higher transaction costs.

As the risk-weighted indexes are heavily dependent on the estimation of the covariance matrix, it would be of interest to estimate this matrix using more alternative methods, besides the sample covariance matrix and the shrinkage covariance matrix.

Maybe the most important field for future studies, when determining the attractiveness of risk-weighted indexes, is to determine the actual transaction cost that is an effect of their higher turnover compared to the conventional market-capitalization weighted. Hence in order for practitioners to start implementing these risk-weighted indexes, they need to related the over performance with the increased transaction costs. A thorough cost analysis would give the answer to if the advantages of creating risk-weighted indexes without the dependency on the estimation of the expected returns, but only the covariance matrix, provides high enough risk-return performance given the higher transaction costs. After that conclusion, one can finally determine if the risk-weighted indexes are worth implementing.

9. Appendix

In this appendix, results of shorter time periods are presented in the same way as in table 1 and 2.

Table 10 - Index Summary Statistics (10% constituent weight constraint): 1997-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	**Annualized return**	**Annualized volatility**	**Annualized Sharpe ratio**	**Maximum drawdown**
weights ≤ 10%	8,9%	21,9%	0,28	-54,1%
Equally weighted index				
weights ≤ 10%	8,6%	21,2%	0,28	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	13,6%	14,0%	0,78	-51,6%
250 days estimation window weights ≤ 10%	12,0%	13,6%	0,69	-50,6%
Shrinkage				
500 days estimation window weights ≤ 10%	12,1%	13,7%	0,69	-51,6%
250 days estimation window weights ≤ 10%	12,9%	13,1%	0,78	-51,4%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	8,3%	17,0%	0,33	-57,8%
250 days estimation window weights ≤ 10%	11,3%	16,3%	0,53	-49,5%
Shrinkage				
500 days estimation window weights ≤ 10%	12,7%	16,4%	0,61	-52,4%
250 days estimation window weights ≤ 10%	10,7%	16,3%	0,49	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	8,6%	18,1%	0,33	-61,1%
250 days estimation window weights ≤ 10%	9,7%	17,7%	0,40	-56,4%
Shrinkage				
500 days estimation window weights ≤ 10%	9,5%	17,3%	0,40	-57,7%
250 days estimation window weights ≤ 10%	8,6%	16,9%	0,35	-56,6%

Table 11 - Index Summary Statistics (5% constituent weight constraint): 1997-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 5%	9,6%	21,2%	0,33	-54,5%
Equally weighted index				
weights ≤ 5%	8,6%	21,2%	0,28	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	9,1%	17,0%	0,38	-58,1%
250 days estimation window weights ≤ 5%	11,4%	14,2%	0,62	-54,3%
Shrinkage				
500 days estimation window weights ≤ 5%	12,8%	14,4%	0,70	-52,3%
250 days estimation window weights ≤ 5%	11,8%	14,0%	0,65	-50,7%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	9,1%	17,0%	0,38	-58,1%
250 days estimation window weights ≤ 5%	12,4%	16,4%	0,59	-49,6%
Shrinkage				
500 days estimation window weights ≤ 5%	13,3%	16,6%	0,64	-52,9%
250 days estimation window weights ≤ 5%	11,2%	16,2%	0,52	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	9,6%	18,4%	0,38	-59,5%
250 days estimation window weights ≤ 5%	9,5%	17,7%	0,38	-55,7%
Shrinkage				
500 days estimation window weights ≤ 5%	9,3%	17,6%	0,38	-58,1%
250 days estimation window weights ≤ 5%	7,9%	17,4%	0,30	-56,1%

Table 12 - Index Summary Statistics (10% constituent weight constraint): 2002-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 10%	10,4%	20,8%	0,40	-54,1%
Equally weighted index				
weights ≤ 10%	8,6%	21,2%	0,28	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	14,9%	14,8%	0,87	-51,6%
250 days estimation window weights ≤ 10%	13,4%	14,1%	0,80	-50,6%
Shrinkage				
500 days estimation window weights ≤ 10%	13,7%	14,5%	0,80	-51,6%
250 days estimation window weights ≤ 10%	14,1%	13,7%	0,88	-51,4%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	11,9%	18,1%	0,55	-57,8%
250 days estimation window weights ≤ 10%	13,3%	17,3%	0,65	-49,5%
Shrinkage				
500 days estimation window weights ≤ 10%	14,0%	16,8%	0,71	-52,4%
250 days estimation window weights ≤ 10%	11,1%	17,2%	0,52	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	9,8%	18,6%	0,41	-61,1%
250 days estimation window weights ≤ 10%	10,1%	18,9%	0,42	-56,4%
Shrinkage				
500 days estimation window weights ≤ 10%	10,4%	18,2%	0,46	-57,7%
250 days estimation window weights ≤ 10%	9,8%	17,9%	0,43	-56,6%

Table 13 - Index Summary Statistics (5% constituent weight constraint): 2002-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 5%	11,8%	20,8%	0,46	-54,5%
Equally weighted index				
weights ≤ 5%	8,6%	21,2%	0,28	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	12,3%	18,2%	0,56	-58,1%
250 days estimation window weights ≤ 5%	12,5%	14,9%	0,70	-54,3%
Shrinkage				
500 days estimation window weights ≤ 5%	14,5%	15,4%	0,81	-52,3%
250 days estimation window weights ≤ 5%	13,2%	14,9%	0,75	-50,7%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	12,3%	18,2%	0,56	-58,1%
250 days estimation window weights ≤ 5%	14,6%	17,5%	0,72	-49,6%
Shrinkage				
500 days estimation window weights ≤ 5%	14,9%	17,3%	0,74	-52,9%
250 days estimation window weights ≤ 5%	11,9%	17,2%	0,57	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	10,5%	19,0%	0,44	-59,5%
250 days estimation window weights ≤ 5%	10,0%	19,1%	0,41	-55,7%
Shrinkage				
500 days estimation window weights ≤ 5%	10,2%	18,7%	0,43	-58,1%
250 days estimation window weights ≤ 5%	8,7%	18,6%	0,36	-56,1%

Table 14 - Index Summary Statistics (10% constituent weight constraint): 2007-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 10%	-0,6%	25,2%	-0,09	-54,1%
Equally weighted index				
weights ≤ 10%	-0,5%	27,6%	-0,08	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	4,0%	17,7%	0,13	-51,6%
250 days estimation window weights ≤ 10%	1,9%	16,6%	0,02	-50,6%
Shrinkage				
500 days estimation window weights ≤ 10%	1,1%	17,0%	-0,03	-51,6%
250 days estimation window weights ≤ 10%	2,1%	16,3%	0,03	-51,4%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	-0,4%	22,4%	-0,09	-57,8%
250 days estimation window weights ≤ 10%	2,1%	21,2%	0,02	-49,5%
Shrinkage				
500 days estimation window weights ≤ 10%	2,9%	20,4%	0,06	-52,4%
250 days estimation window weights ≤ 10%	-0,2%	21,3%	-0,09	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	-6,9%	23,0%	-0,37	-61,1%
250 days estimation window weights ≤ 10%	-6,4%	23,7%	-0,34	-56,4%
Shrinkage				
500 days estimation window weights ≤ 10%	-5,6%	22,5%	-0,32	-57,7%
250 days estimation window weights ≤ 10%	-6,8%	22,0%	-0,38	-56,6%

Table 15 - Index Summary Statistics (5% constituent weight constraint): 2007-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 5%	0,3%	25,4%	-0,05	-54,5%
Equally weighted index				
weights ≤ 5%	-0,5%	27,6%	-0,08	-60,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	-0,2%	22,5%	-0,08	-58,1%
250 days estimation window weights ≤ 5%	-0,3%	17,9%	-0,11	-54,3%
Shrinkage				
500 days estimation window weights ≤ 5%	3,0%	18,4%	0,07	-52,3%
250 days estimation window weights ≤ 5%	0,3%	18,2%	-0,07	-50,7%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	-0,2%	22,5%	-0,08	-58,1%
250 days estimation window weights ≤ 5%	2,6%	21,4%	0,05	-49,6%
Shrinkage				
500 days estimation window weights ≤ 5%	3,9%	21,0%	0,11	-52,9%
250 days estimation window weights ≤ 5%	0,4%	21,3%	-0,06	-54,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	-6,1%	23,6%	-0,33	-59,5%
250 days estimation window weights ≤ 5%	-6,7%	24,0%	-0,35	-55,7%
Shrinkage				
500 days estimation window weights ≤ 5%	-6,3%	23,1%	-0,34	-58,1%
250 days estimation window weights ≤ 5%	-8,7%	23,1%	-0,45	-56,1%

Table 16 - Index Summary Statistics (10% constituent weight constraint): 2009-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 10%	5,7%	19,3%	0,23	-31,7%
Equally weighted index				
weights ≤ 5%	2,9%	22,1%	0,08	-35,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	9,3%	13,3%	0,61	-19,2%
250 days estimation window weights ≤ 10%	8,0%	13,7%	0,50	-24,8%
Shrinkage				
500 days estimation window weights ≤ 10%	9,1%	12,6%	0,62	-28,2%
250 days estimation window weights ≤ 10%	9,0%	12,7%	0,61	-25,2%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	8,0%	18,0%	0,38	-32,4%
250 days estimation window weights ≤ 10%	5,9%	16,8%	0,28	-23,2%
Shrinkage				
500 days estimation window weights ≤ 10%	9,2%	17,2%	0,47	-21,8%
250 days estimation window weights ≤ 10%	6,3%	17,5%	0,29	-28,5%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	-3,1%	18,2%	-0,24	-44,6%
250 days estimation window weights ≤ 10%	-5,5%	18,9%	-0,36	-44,5%
Shrinkage				
500 days estimation window weights ≤ 10%	-3,5%	17,4%	-0,27	-40,0%
250 days estimation window weights ≤ 10%	-5,5%	16,7%	-0,40	-43,9%

Table 17 - Index Summary Statistics (10% constituent weight constraint): 2009-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 5%	5,9%	19,5%	0,24	-29,6%
Equally weighted index				
weights ≤ 5%	2,9%	22,1%	0,08	-35,1%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	7,5%	18,5%	0,34	-33,3%
250 days estimation window weights ≤ 5%	5,8%	14,8%	0,31	-30,3%
Shrinkage				
500 days estimation window weights ≤ 5%	8,3%	14,2%	0,50	-21,0%
250 days estimation window weights ≤ 5%	4,6%	15,5%	0,22	-26,5%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	7,5%	18,5%	0,34	-33,3%
250 days estimation window weights ≤ 5%	6,0%	17,1%	0,28	-24,0%
Shrinkage				
500 days estimation window weights ≤ 5%	8,7%	17,5%	0,43	-21,6%
250 days estimation window weights ≤ 5%	6,7%	17,9%	0,30	-29,1%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	-3,5%	18,9%	-0,25	-42,5%
250 days estimation window weights ≤ 5%	-6,3%	19,4%	-0,39	-44,9%
Shrinkage				
500 days estimation window weights ≤ 5%	-4,7%	18,3%	-0,32	-42,6%
250 days estimation window weights ≤ 5%	-9,2%	18,4%	-0,57	-48,7%

Table 18 - Index Summary Statistics (10% constituent weight constraint): 2011-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	Annualized return	Annualized volatility	Annualized Sharpe ratio	Maximum drawdown
weights ≤ 10%	21,0%	16,3%	1,20	-26,5%
Equally weighted index				
weights ≤ 10%	22,9%	19,1%	1,12	-30,2%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	15,3%	12,1%	1,14	-9,1%
250 days estimation window weights ≤ 10%	17,0%	12,1%	1,28	-16,9%
Shrinkage				
500 days estimation window weights ≤ 10%	15,8%	11,8%	1,21	-18,3%
250 days estimation window weights ≤ 10%	14,9%	12,0%	1,12	-15,1%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	22,2%	15,5%	1,34	-27,7%
250 days estimation window weights ≤ 10%	17,4%	11,5%	1,39	-18,1%
Shrinkage				
500 days estimation window weights ≤ 10%	22,0%	14,8%	1,39	-15,3%
250 days estimation window weights ≤ 10%	18,0%	13,8%	1,20	-23,4%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 10%	8,5%	13,1%	0,54	-43,7%
250 days estimation window weights ≤ 10%	13,4%	16,2%	0,74	-43,5%
Shrinkage				
500 days estimation window weights ≤ 10%	8,8%	13,4%	0,55	-39,6%
250 days estimation window weights ≤ 10%	9,4%	14,7%	0,54	-42,7%

Table 19 - Index Summary Statistics (5% constituent weight constraint): 2011-12-14 - 2012-12-15

Summary statistics of the market-capitalization weighted, equally weighted, minimum variance, equal risk contribution and most diversified index portfolios. Four different properties are presented: annualized return, annualized volatility, annualized Sharpe ratio and maximum drawdown.

Market capitalization index	**Annualized return**	**Annualized volatility**	**Annualized Sharpe ratio**	**Maximum drawdown**
weights ≤ 5%	21,7%	16,5%	1,22	-24,3%
Equally weighted index				
weights ≤ 5%	22,9%	19,1%	1,12	-30,2%
Minimum variance index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	22,2%	16,3%	1,28	-28,5%
250 days estimation window weights ≤ 5%	18,3%	12,1%	1,38	-25,9%
Shrinkage				
500 days estimation window weights ≤ 5%	17,7%	12,0%	1,35	-12,2%
250 days estimation window weights ≤ 5%	18,6%	13,9%	1,24	-23,5%
Equal risk contribution index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	22,2%	16,3%	1,28	-28,5%
250 days estimation window weights ≤ 5%	19,1%	11,8%	1,50	-18,9%
Shrinkage				
500 days estimation window weights ≤ 5%	21,6%	15,1%	1,33	-16,0%
250 days estimation window weights ≤ 5%	20,7%	14,3%	1,34	-24,0%
Most diversified index				
Sample covariance matrix				
500 days estimation window weights ≤ 5%	9,1%	14,4%	0,53	-41,2%
250 days estimation window weights ≤ 5%	12,4%	16,7%	0,65	-43,7%
Shrinkage				
500 days estimation window weights ≤ 5%	8,7%	14,3%	0,51	-42,2%
250 days estimation window weights ≤ 5%	8,3%	15,5%	0,44	-48,0%

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