Stockholm School of Economics

Master's Thesis in Finance

Optimal Hedging Strategies for OMX Option Portfolios

-A Study of Different Strategies for Option Hedging Using the Greeks-

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Abstract:

The option markets have developed substantially during the past decades ever since the construction of a pricing model for options. After the model had been constructed tools for hedging were gradually developed, usually collectively referred to as the Greeks. This thesis empirically studies hedging using the Greeks and expands the research field by studying discrete hedging of OMX index options, while taking transaction costs into consideration. From the perspective of a market maker who manages a portfolio, hedging strategies for Gamma, Vega and Delta are examined. All portfolios and strategies are generated using a computer program constructed for this sole purpose. Each option which is included in a hedge is characterised according to its volatility, moneyness, call/put and maturity to analyse which characteristics are the most efficient in a hedge. In order to evaluate the different strategies the expected return, risk and the reward to variability ratio are evaluated in five time periods. The results indicate that no single strategy examined is optimal for all portfolios, but there is a single option type that is very efficient for all portfolios when combined wisely. The separate characteristics lead to the conclusion that maturity and moneyness are the pivotal attributes for all performance measurements while volatility is only vital for the risk.

Tutor: Assistant Professor Joel Reneby

Presentation: 10.15, June 16, 2006

Location: 550

Discussants: Georges Mansourati and Anders Olsson

"**Because options are specialized and relatively unimportant financial securities, the amount of time and space devoted to the development of a pricing theory might be questioned**."
-Robert Merton (1973), page 141.

We would like to thank our tutor Joel Reneby for his support and valuable opinions. We would also like to express our gratitude towards the interviewees who have contributed with helpful insights. Last but not least we would like to thank Magnus Petersson, whose remarkable knowledge of computer programming has been greatly appreciated and significantly facilitated our work with this thesis.

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1. Introduction

The current situation on the financial markets is diametric to the description of options expressed in Merton's quote on the previous page. In the financial markets of today, options are a very important investment vehicle for financial institutions, companies and individuals. This thesis will, however, not focus on options in general but one in particular; the OMXS30[1] index option. Index options lend themselves particularly well to study since options based on an index are usually the most liquid. It is also the case that a market maker often manages a portfolio consisting of several different assets, on several different markets. If the portfolio approaches the risk limits of the financial institution the market maker will choose to hedge on the easiest market. The option which is chosen for the hedge is often an index option and therefore one can argue that it is more relevant to study hedging strategies for this particular family of options.

OMX options are primarily issued and traded by large financial institutions, but they can be issued and traded by anyone. The institutions issuing OMX options face numerous risks such as changes in volatility and in the price of the underlying asset. The trading with clients leaves the institution with a portfolio of options on which it has already earned a profit due to the bid-ask spread. In an attempt to avoid excessive risk taking the financial institutions often hedge these portfolios. The purpose of the hedge is dependant on the overall portfolio policy and the risk management vision within each independent institution.

A hedge can be constructed in several different ways and the choice affects not only the efficiency of the hedge, but also the cost that can be attributed to the chosen approach. According to fundamental arbitrage theory a portfolio that is perfectly hedged should yield a return equal to the risk free interest rate. This assumes, however, that the underlying option pricing model is accurate. In reality this is seldom the case and the return on a hedged portfolio will rarely be exactly the risk free interest rate. Transaction costs, discrete trading and indivisibility of contracts, must, for example, be taken into consideration, something which is not done in the standard pricing model. This implies that the portfolio no longer is self financing and that the market maker will be forced to add money to the position. The consequence is that the true cost of hedging a portfolio depends on the strategy used. Hence it is important to have an explicit strategy for hedging seeking to minimize the transaction costs. The aim of this thesis is therefore

[1] OMXS30 has had several different names over the years and will, for clarity, be referred to as OMX within this thesis.

to investigate different hedging strategies using the Greeks, taking the transaction costs and discrete trading, but not indivisibility of contracts into account.

The study is conducted through an analysis of several stylized portfolios and strategies in order to find an optimal hedging strategy, depending on the market maker's portfolio. The hedging strategies are defined with certain characteristics and the same Greeks are hedged throughout the study. The method used involves considering the transaction costs in the form of bid-ask spreads. This is done by constructing theoretical portfolios using options that are traded on a particular day. These are then hedged using other options that are also traded on that particular day. The hypothetical portfolios and hedges required for the study are generated through a computer program, constructed using the programming language *Python*. The strategies are evaluated with respect to their expected return, risk and reward to variability.

The thesis is subdivided into seven sections. Next, Section 2 reviews some previous research which is considered relevant for the theoretical approach. Section 3 continues with explanations of all variables and a description of the methodology. Thereafter the data and its limitations are discussed in Section 4. Next, Section 5 includes the results from the investigation as well as some relevant comments. The conclusions are found in Section 6, while Section 7 finishes with some suggestions for further research based on the findings in this thesis. In the Appendix a description of Black 76, figures of volatility smiles and all of the relevant tables are presented.

2. Previous Research

There are numerous papers investigating pricing of options, hedging and market makers. However, it has not been possible to find a study that investigates the optimal characteristics of the options included in a hedging strategy for a portfolio of index options.

2.1 Discretely Rebalanced Option Hedges

In the original Black and Scholes model it is assumed that it is possible to hedge continuously, something which is not possible in reality. Therefore there exists extensive research that investigates what happens when the rebalancing interval is discrete rather than continuous. Gilster (1990) finds that the risk free hedge in Black and Scholes becomes subject to systematic risk when it is rebalanced discretely. The notion that discrete rebalancing can introduce substantial risk in option trading is further supported by Mello and Neuhaus (1998). They find that the hedging errors are correlated which implies that they could be significantly decreased by constructing portfolios rather than investing in individual options. Further Carverhill et al. (2002) finds that a *Delta* and *Vega* neutral portfolio which is long in Out of the money (In the money) puts and short in OTM (ITM) calls yields less (more) than the risk free interest rate. The authors argue that this is due to a market imperfection which means that OTM puts are too dear relative to OTM calls, possibly because they in themselves are a good portfolio insurance.

Robins and Schachter (1994) perform another study of discretely rebalanced *Delta* hedges using Black and Scholes. They find that this is not the minimum variance strategy. Systematic departures from the *Delta* hedge can yield a significantly lower variance even if the rebalancing interval is as short as one day. Their model for the minimum variance is an extension of the model developed in Cox and Rubenstein (1985). Robins and Schachter (1994) take both systematic and total risk into account, but when only the market risk is considered they find that the *Delta* based techniques perform rather well.

In Galai (1983) the return of a hedging strategy is determined to consist of the risk free interest rate, the return from discretely rebalancing the hedge and the return arising from differences between the model price and the actual price. The latter one is found to be the dominating one, while the other two are quite small for both ex post and ex ante tests. The return from the risk arising from the market's volatility is investigated by Bakshi and Kapadia (2001) and is found to be negative. They study a *Delta* hedged option portfolio which is found to yield less than zero and this underperformance is greater when the volatility is high and for options that are ITM. The return of this portfolio is determined by the volatility risk premium and the *Vega* of the option.

The research regarding discretely rebalanced option hedges has provided evidence that the hedge no longer should be expected to yield the risk free interest rate as is expected from arbitrage theory. It is therefore important to study different hedging strategies since it is desirable to have a positive rather than negative return.

2.2 Portfolios of Options

In their study of different call option portfolio strategies Merton et al. (1978) investigates covered calls and options/paper-buying strategies. A covered call is constructed by holding short calls together with long stocks, equivalent to holding a short call and a future, and options/paper-buying strategies are constructed by holding a specific proportional blend of call options and fixed income securities. When issuing a covered call, the least risky of the two strategies, the authors find that the return can not be expected to be as high as the return on the stock itself. A market maker might, however, expect to receive a significantly higher yield from these strategies than from holding low-risk, fixed income securities. It is also the case that the returns from these strategies can not be replicated by holding a portfolio of stocks and fixed income securities. This was the traditional method for changing the return of a portfolio at the time when this article was written. The existence of options has therefore, through its insurance characteristic, significantly extended the range of returns that are available to an investor. Merton et al. (1978) note that both covered calls and options/paper-buying strategies are bullish on the underlying although the underlying mechanism works in slightly different ways.

Merton et al. (1982) note that a protective-put buying strategy is bullish on the underlying. A protective-put buying strategy is constructed by holding a specific number of shares and then buying puts on the same number of shares. Their finding implies that it is important to study hedging on different market developments since different portfolios perform well in bear and bull markets. They also conclude that a single best strategy for all investors can not be determined. Investors should choose the strategy that is optimal for them taking their personal preferences into account.

3. Methodology

The framework used in this paper is the Black 76 model, which is presented in Appendix A.1. This model enables the estimation of a number of variables and measurements essential to option analysis. The main reason to use the Black 76 model instead of the “normal” Black and Scholes model is that the trading stop for options and futures on the Stockholm Stock Exchange is 17.20, while the closing price for the index is recorded at 17.30. During the ten minutes difference in closing time the index may change and hence the estimated volatilities and the Greeks would suffer from estimation errors. Using the Black 76 model also implies that the dividend yield is removed from the calculations since the options and futures are cash settled. This is an additional advantage since the index pays dividends in discrete and irregular intervals, mainly clustered in April and May. Hence to be able to use Black and Scholes it is necessary to include a variable that specifies the present value of the exact dividend paid out each day and it is not possible to assume a constant dividend yield.

3.1 The Greeks

In the Black 76 universe there are several different Greeks, all representing an individual risk. Hull (2003) specifies the Greeks as *Delta*, *Gamma*, *Vega*, *Theta* and *Rho*. *Delta* measures the price change related to small movements in the underlying asset while *Gamma* measures price changes associated with changes in *Delta*. *Vega* is the elasticity between the price and the volatility. *Rho* represents the elasticity between the value of the option and the interest rate while *Theta* measures the change in the price caused by the passing of time.

In the market almost all participants hedge *Delta* and to some extent *Gamma* and *Vega* while *Theta* and *Rho* are less important for index options. *Theta* can be regarded as a cost since the value of an option decreases as time passes if everything else is held constant unless it is a deep ITM put. The passing of time can hardly be characterized as a risk since everyone knows that tomorrow will be a new day. Hence *Theta* is included in the reasoning of a trader as a cost but it is rarely hedged. Black 76 assumes a constant interest rate of zero when the options and futures are cash settled and therefore *Rho* will not be applicable. As a result the focus of this thesis is on *Delta*, *Gamma* and *Vega*. In the constructed computer program *Delta* is always set to zero by buying/selling OMX futures on the market after *Gamma* and *Vega* have been hedged. This means a smaller upfront payment for the trader compared to buying/selling the index as well as a faster execution. Buying the index would induce buying all the stocks included in the index while buying the future only requires buying one security.

In reality *Gamma* and *Vega* are set to a desired target level determined by the market maker, taking the risk limits into account. In this study *Gamma* and *Vega* are set to zero with a hedge using two different OMX options. The portfolios used in this analysis are small compared to those market makers manage in reality. It would therefore be difficult to set the limits for *Delta*, *Gamma* and *Vega* to something other than zero for the investigated portfolios without risking that several of the portfolios will remain unhedged since they do not break the limits. To use options on the same underlying means that the correlation between different assets does not have to be taken into account.

Any estimation of the Greeks is affected by the volatility smile. The regime within this thesis is to adjust for this by using the sticky strike rule. This is one of the rules of thumb that were first discussed by Derman (1999) and later by Daglish et al. (2002). The assumption made when using the sticky strike rule is that the implied volatility is constant between two days. Hence the Greeks are correct when calculated using the implied volatilities. Daglish et al. (2002) found that the sticky strike rule is not the most efficient method, but since there are no formulas available for *Gamma* and *Vega* for the alternative method the sticky strike rule is chosen.

As is pointed out in Green and Figlewski (1999) hedging *Gamma* and *Vega* will require trading options with other market makers. This might prove problematic if all market makers want to hedge the same risks. Therefore they do not completely neutralize any risks except *Delta*, since *Delta* can easily be hedged using futures, but use risk limits instead for *Gamma* and *Vega*.

3.2 The Variables Used in the Calculations

In order to calculate the Greeks for the hypothetical portfolios it is necessary to define the variables included in the Black 76 model. The variables defined are the volatility, the time to maturity and the price of the underlying. The exercise price and the option's price are taken from the data.

3.2.1 The Volatility

When seeking to estimate the volatility it is possible to study for example the historical development of the underlying index or the actual option price. Within this thesis only the implied volatility is used. From the market maker's point of view it is useful to use implied volatilities rather than historical estimations; this is something that is pointed out by Green and Figlewski (1999). Often an option trader thinks about the price in terms of volatility rather than in monetary terms, and therefore the current implied volatility is of greater importance than the historical volatility. Jordan et al. (1987) found that the Black 76 model works best when implied volatilities from ATM options are used. Christensen and Prabhala (1998) further found that the implied volatility is best for forecasting future volatility. Hence the implied volatility seems to be the more useful of the two.

The implied volatility is determined with the same methodology as Jameson and Wilhelm (1992) and Pan (2002) use. Each day the implied volatility is calculated by setting the model price equal to the quoted price. An implied volatility has been established for each of the quoted bid and ask prices since the market maker can have both long and short positions in an option. The development of the average implied volatility over time is found in *Figure 1*.

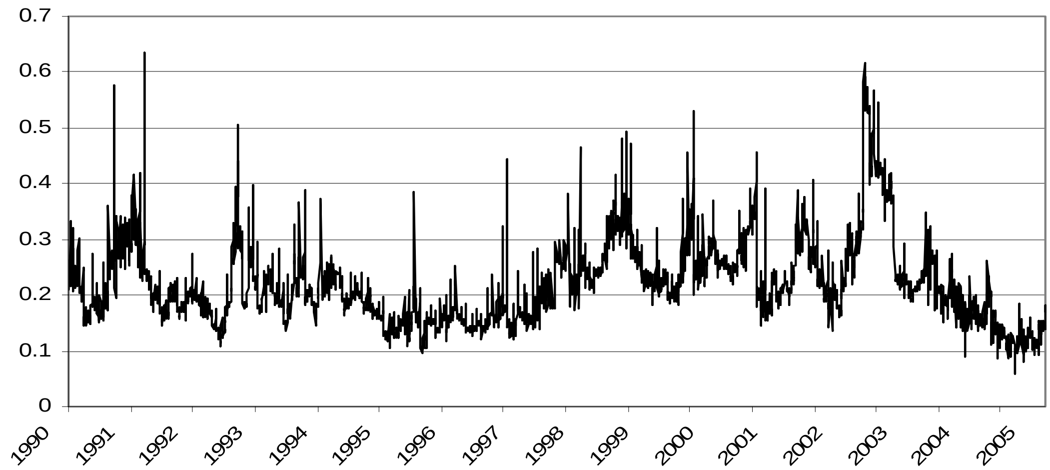

Figure 1: Average Daily Implied Volatility, 1990-2005

The average implied volatility does not give the entire picture of the volatility on a particular day. Option prices are characterized by volatility smiles which can have different shapes and inherent variability. Four graphs of different volatility smiles that are found in the data are included in Appendix A.2 to illustrate this phenomenon.

Hull and White (1987) find that the implied volatility decreases with maturity for Near the money options. They also found that the correlation between the volatility and the underlying asset affects the pricing efficiency of Black 76. By deriving the expression for the instantaneous variance they found that if the volatility is stochastic At the money (ATM)[2] options will be underpriced compared to OTM and ITM options. This, however, assumes that the correlation between the volatility and the underlying is zero. When the correlation is positive the formula instead underprices OTM options and overprices ITM options. The opposite is true for when the correlation is negative.

3.2.2 The Time to Maturity

Natenberg (1988) argues that the time to maturity of an option should be measured in trading days rather than calendar days. This is also the methodology used to calculate the time to maturity in this thesis. When establishing the time to maturity the number of trading days to the last Friday in the expiry month is divided by 251. The number 251 refers to, of course, the total number of

[2] The label ATM does not only cover the options exactly At the money but also those some strikes away on both sides; hence the exact term is rather that these options are Near the money. ATM is, however, used as the term for these Near the money options within this thesis.

yearly trading days. This value is the mean as well as the median of the number of total trading days during the years 1990-2005.

3.2.3 The Price of the Underlying

This paper takes the perspective of a market maker and as for all traders both the bid and ask prices are relevant. When market makers' portfolios change due to their clients' trades, they are price givers and buy the options to the bid and sell them to the ask price. However, when they hedge their portfolios they are price takers and hence buy to the ask and sell to the bid price. If a trader wants to buy future contracts the only price where a trade will be guaranteed is at the ask price. A trade initiated by the same trader to buy a call means that the call is bought at the ask price. Therefore the price of the underlying when valuing a long call option is the future's ask price while a short call option is evaluated using the future's bid price. Hence when the formula calculates the bid of a call option, the bid price of the future is used as the current price of the underlying. This pattern is of course also applicable for put options.

3.3 The Portfolios

The market maker makes money on the spread when the portfolios are constructed while the hedge is bought from other participants and therefore the spread is a loss instead. In this paper 32 different portfolios are investigated with respect to the optimal hedging strategy when owning a portfolio with these particular proportions. The portfolios and strategies can be found in Appendix A.3 together with an interpretation key for the individual options included in the hedging strategies. The portfolios have been divided into reference portfolios and main portfolios. The main portfolios are supposed to resemble a portfolio similar to those of a market maker, whose portfolio consists of several hundred different options at any point in time. The 24 reference portfolios only consist of one contract on an option specified according to its moneyness and time to maturity. These are similar to for example the portfolios of financial institutions that sell foreign institutions issues to Swedish clients with an obligation to buy them back, placing the issues on their books.

When choosing the eight main portfolios effort has been made to make them as realistic as possible and also to make them diverge so they become clearly distinguishable from each other. In these portfolios one contract of each of the five options is included. They have been validated by participants in the option market and were found to be realistic, although one has to keep in mind that a market maker's portfolio is ever evolving and much larger than the proxy portfolios in this study. The portfolios are hedged daily using different strategies. The hedge and the portfolio are settled every day and a new hedge is entered into the next day for a new portfolio.

3.4 The Hedging Strategies

Each individual hedging strategy that is used consists of two different options. This study takes a broad perspective and several different aspects are taken into account for each strategy. The options are categorized according to four separate criteria: volatility, moneyness, call/put and maturity. These four are included since it is interesting to study their joint effect and they represent different parts of the pricing formula using Black 76.

For the analysis of the volatility it has been decided to use fixed boundaries rather than flexible boundaries, these were set to between 0 and 25 percent and larger than 25 percent but smaller or equal to 50 percent. The upper limit was set to its level based on the notion among the market participants interviewed that volatilities above 50 percent often are a result of mispricings in the market rather than an accurate picture of the actual volatility at this time. Fixed boundaries were used following discussions with the interviewees because it would lead to an interesting result for the participants in the market.

Moneyness is determined using the absolute values of the option's *Delta*. This criterion has been used to eliminate the options that are deep ITM or OTM since they have been found to be mispriced in most cases; there are even problems with finding any volatility which makes the formula price equal to the actual price. The option can be either a call or a put and there is no classification of whether the call or put is long or short since this is determined within the formula itself.

The maturity of the options might affect the transaction costs and therefore it was included as a criterion. This is based on a study by Swidler and Diltz (1992) which found that transaction costs are inversely related to the maturity. The maturity category has been determined with one closed interval of 0-30 days and one open interval of more than 30 days. In total, this leads to 24 different types of options. Considering that a strategy consists of two options there are 300 different strategies to handle.

3.5 Performance and Risk Measurements

Within this thesis the epithet "best strategy" is defined in three different ways: the largest expected return, the lowest risk and the highest positive reward to variability ratio. The choice is to use the expected return as the performance measurement while the standard deviation is used as the risk measurement. The expected return is defined as the mean return of the strategy. The standard deviation is one of many risk measurements, but since they often yield very similar results only the standard deviation is chosen as a risk measurement within this study. The reward to variability ratio is used as a joint measurement of the two to find out which strategy is optimal when the objective is to receive the best expected return given the risk or vice versa. These three measurements are chosen to cover several different aspects that a market maker is interested in.

The returns are in SEK since the portfolio value is constant each day and therefore the price and size differences between different hedging strategies will be incorporated into the monetary return. A hedge which requires large outlays receives a lower return since the loss due to the bid-ask spread increases.

To distinguish the best strategy from the others confidence intervals are used. The Black 76 formula uses the normal distribution in its pricing function. Therefore the normality assumption can be used when calculating the confidence intervals for the expected return. The confidence intervals for the expected return are calculated using the following formula, given by for example Gujarati (2003):

$$\overline{X} - Z \cdot \frac{\sigma}{\sqrt{N}} \leq \mu \leq \overline{X} + Z \cdot \frac{\sigma}{\sqrt{N}}$$

Confidence intervals are calculated for every strategy and from these how many of the strategies the best strategy can be distinguished from are determined. The significance level of the confidence interval is set to 95 percent. If an expected return is within the critical regions of another strategy it is not possible to reject the null hypotheses that two strategies have the same true expected return.

In addition to the return a market maker is concerned with the variability in the portfolio combined with the hedge and therefore the same analysis is performed for the risk. The formula for the confidence interval for the risk is defined as:

$$\sqrt{(n-2)\cdot\frac{\hat{\sigma}^2}{\chi^2_{\frac{\alpha}{2}}}} \leq \sigma \leq \sqrt{(n-2)\cdot\frac{\hat{\sigma}^2}{\chi^2_{1-\frac{\alpha}{2}}}}$$

The significance level is set to 95 percent for these intervals as well. The hypotheses for the two tests are:

H_0: The risk/expected return of two strategies are equal.
H_1: The risk/expected return of two strategies are not equal.

Often a market maker does not only care about the expected return or the risk but rather the combination of the two. To investigate which strategy is optimal when investigating the expected return and the risk jointly a measurement similar to the Sharpe ratio is used. The Sharpe ratio, first developed in Sharpe (1966), measures the portfolio return given the risk that the portfolio manager has taken. There has been a lot of research related to this subject and a summary of this is found in Sharpe (1994) where several different definitions of the ratio are explained. The formula for the Sharpe ratio is $S = \frac{r_P - r_f}{\sigma_P}$, but the ratio which is used in this thesis does not include the risk free interest rate since it is the only assumption which is consistent with the specific pricing formula used for the calculations of the implied volatilities and the Greeks. Therefore the formula for the reward to variability ratio is $R/V = \frac{r_P}{\sigma_P}$. The best strategy with respect to both the risk and the expected return is the strategy with the highest reward to variability ratio. To make an analysis to be compared to the indistinguishable strategies for the expected return and the risk, the reward to variability ratios that are lower than the largest ratio by no more than 25 percent are analyzed.

The entire analysis is repeated for four different time periods as well as for the total time period. The first period is January 2, 1990 to October 9, 1998 and the second period is October 12, 1998 to March 3, 2000. The third period is March 6, 2000 to March 14, 2003 and the fourth period is

March 17, 2003 to September 30, 2005. The time periods are included to investigate if there is a difference in which strategies perform best depending on how the underlying asset moves. The choice of time periods has been determined by the development of the OMX index. As can be seen in *Figure 2*, the chosen time periods correspond to different states of nature in the market.

There is also a separate analysis of the 30 strategies with the largest number of observations for each portfolio in every time period. The limit of 30 strategies is chosen since it represents one tenth of the total number of strategies and gives a representative image of the best among those with many occurrences. A limit which is stated as an absolute number of observations is rejected since the number of days varies within the periods and also within the portfolios for a certain period. There are days within a period when a certain portfolio has not been able to be constructed at all. These days have been deleted from the sample and therefore it does not give a fair and representative view to set a fixed limit. Market makers can always find the options they want even if the particular strategy has few observations within this sample. One way for a market maker to create a rare option position is to construct a basket of the included stocks' options that has the desired characteristics. However, in some instances there might not be time to construct this or the market maker just wants a sure bet, something which has a high probability of being on the market without any searching costs. It is therefore still important to study the strategies with many observations.

4. Data

4.1 The OMX Index and OMX Options

The OMX index is a stock index which consists of the 30 most traded stocks on the Stockholm Stock Exchange. The index is rebalanced every six months; hence it is not necessarily the stocks that were included in the index in the early nineties that are included today. The original index was created on September 30, 1986 with an initial value of 500 and OMX index options have been traded since 1988. On April 27, 1998 there was a 4:1 split of the index and the new value on September 30, 1986 became 125. After this adjustment the development of the OMX index between 1990 and 2005 is visible in *Figure 2*.

Figure 2: OMX' historical development

The naming of options is based on a standardized system where the first part symbolizes the underlying, the second the year, the third the last trading month and the fourth the strike price of the option. Call options are named A to L, where A is January and L is December while put options are named with the same procedure but with M to X. Futures are named in the same way, but lack strike prices. An example of a name is OMX1D900, which is a call option on OMX that matures in April 2001 and has a strike price of 900 SEK. The year can always be inferred from the name since there are no ten year index options traded.

The last trading day for an index future or an index option is the last Friday in the month when it matures, or the day before if the Friday is a holiday. An OMX index option is cash settled on the last trading day while an OMX future is cash settled everyday with its value at the beginning of trading being set to zero.[3] In every trade OMX is the counterparty thereby ensuring that every investor can issue options.[4] The stated price is the price for 1/100 of a contract, i.e. the price represents one unit of the underlying and the contract represents 100 units of the underlying.

4.2 The Data Sample

This paper is based on daily closing prices for OMX index options and futures received from the OMX Group. The original data cover the period January 1, 1988 to September 30, 2005. The data include bid, ask, high, low and volume figures for all options and futures that were given a quote on a particular day. In the study only the data from 1990 and onwards are used since there were no future prices available in 1988 and 1989. For practical reasons the observations where the

[3] ”10 frågor och svar om options- och terminshandel”
[4] Swedish Financial Markets 2005, p 21

equivalent future price is missing have been removed. If there is not a future price available an implied volatility, and as a consequence also the Greeks, can not be calculated. It is therefore not possible to use the option for hedging, at least not using the strategies chosen within this thesis. The quantity of such occurrences is, however, very limited.

An adjustment has also been made to remove all options where prices do not exist at least two days in a row. The portfolios in the study are bought one day and then sold the following day; hence there must be prices on two executive days. The final data set consists of around 300 000 option prices, divided between approximately 8 000 different options. The data set also includes some 15 000 future prices.

It is assumed that if a bid-ask spread exists it is possible to trade a reasonable quantity at the quoted bid-ask spread. On the market when quoting a spread a market maker should always be prepared to trade at least 50 contracts on each side without affecting the prices. This is an important assumption, especially since Ho and Macris (1984) among others found that the bid-ask spread has a large effect on the transaction return. If the spread were to change when trading occurred it would affect the return and this effect is difficult to estimate using the available data. It is also important since this thesis has decided to focus only on the spread component of the transaction costs and not the commissions, since the large financial institutions encourage their traders to act without taking the fixed commissions into account. The situation is very different for private individuals owning a portfolio of options though. For them the commission fees are large and can not be disregarded.

5. Empirical Findings

Before going through the results it is noticeable that the hedging strategy considered as being the best sometimes has relatively few observations which make it potentially difficult to use. An alternative approach is to use the best strategy considering the strategies with the most number of observations. Market makers can, however, always contact another market maker and ask for a quote on the particular option they want to trade. Once the market makers receive a quote it is up to them to make the decision whether or not it is a fair price and if the hedge will be the most efficient choice given the price.

There is one pattern which stands out for all measurements and time periods irrespectively of the number of observations: calls are usually most optimally hedged with calls while puts are usually best hedged with puts. An explanation for this is that a long put plus a short put with the same characteristics will be neutral or close to neutral. Hence, if one owns a portfolio consisting of only puts or calls, take the best option and just adjust call/put to receive an efficient hedge. It can be concluded from the study that calls and puts are used equally frequent, and they should therefore be quite interchangeable for a well diversified portfolio.

5.1 All Strategies

In this section there is a joint analysis of all the different time periods for all of the strategies. All of the tables of the best strategies can be found in Appendix A.4. Firstly it is important to note that there are no complete strategies that are best for all portfolios in all time periods for any of the expected return, the risk and the reward to variability ratio. This is the finding that was expected since the portfolios are very different. With the diversity among strategies that are present it is difficult to see the reason why a single strategy should be best for all portfolios, after all each strategy has to adhere to four different criteria for two separate options. Secondly, in more than half of the cases the same strategy is best for the period 1990-2005 as well as for the period 1990-1998 for all three of the performance measurements. When studying the development of the OMX index during these two periods it is found that the periods have a similar development and it is therefore not surprising that the best strategies for these two often are equal. The strategies that are identical for different time periods for a particular portfolio in the tables in Appendix A.4 and A.8 are in *italics*.

5.1.1 The Expected Return

The best strategy with regards to this criterion has been defined as the strategy with the highest expected return for the portfolio and the hedge combined. The single best strategy for each portfolio is depicted in *Table A.4.1*.

To shed light over the optimal strategy the individual options will be investigated since no strategy is found to be optimal for all portfolios. When seeking to determine which option that is the best to use at least once in a hedge, both the number of occurrences among the two categories, the best strategies and the indistinguishable strategies, are taken into account. The option type "Hocs"[5], i.e. an option with the characteristics ***H***igh volatility, ***O***TM, ***C***all and ***S***hort time to maturity, is the best option in all time periods except 2003-2005 where it is the second best. The

[5] For a complete list of all abbreviations of the options' characteristics, please consult *Table A.3.4*

conclusion from this is that it is recommended to include a Hocs when forming the hedge of an option portfolio when concerned about the expected return.

Once this is established it is of course important to know whether or not Hocs is efficient to combine with all of the other option types. It does seem like it is efficient together with most other options except with the four option types Hicl, Licl, Lipl and Lopl. The options that are most efficient to combine with Hocs are Locs, Lics and Lacs. If a market maker manages a portfolio that only consists of puts it could, however, be more optimal to use a Hops instead of a Hocs, the only difference being a p(ut) instead of a c(all).

5.1.2 The Risk

The individual option that is established to give the lowest risk most frequently is Lops, see *Table A.4.2*. It is the most efficient strategy for all periods except 2000-2003 when Lips is the optimal strategy. Lips is hence the best option type in a bearish market environment.

Lops works well with most strategies but it should be avoided in combination with Hopl since that combination is rarely found among the indistinguishable strategies. If market makers have several options to choose from they should choose either of Lacs, Laps, Haps and Hacs to combine with their Lops.

5.1.3 The Reward to Variability Ratio

The option most frequently used when considering the reward to variability ratio is Hocs. For the efficient strategies for this performance measurement see *Table A.4.3*. Hocs is the best option both when taking the best reward to variability ratio and the indistinguishable strategies into account in all periods except 2003-2005. This is the same as for the expected return. Hence it can be said that the expected return is the dominating variable in the ratio since it is the same strategy that is optimal for the expected return and the reward to variability ratio. It is consequently fair to say that Hocs is the preferred choice if a market maker only chooses one option to include in a hedge. The volatility component should, however, be taken into account since it is the differing component in 2003-2005.

A market maker has to choose which option to combine Hocs with, something which appears to be more important for the reward to variability ratio than for the expected return and the risk since Hocs is combined with fewer strategies than before. The indistinguishable strategies are much fewer than for the other two performance measurements since confidence intervals are not used. The four options that are most often used together with Hocs among the indistinguishable strategies are Locs, Lacs, Lics and Haps. It is interesting to note that Locs, Lacs and Lics also are among the most favourable alternatives when studying the expected return. This further strengthens the argument about the expected return's dominating impact on the reward to variability ratio.

5.1.4 The Individual Components

In order to disentangle the result drivers of the strategies the individual characteristics are analyzed. The individual characteristics are those that were used to specify the options included in each strategy i.e. volatility, moneyness, call/put and maturity. The characteristics have similar patterns for all of the performance measurements and therefore the analysis will be made jointly. The relevant tables for this analysis can be found in Appendix A.5-A.7.

The volatility has a fairly equal division for the expected return and the reward to variability ratio but there is a clear tendency toward more low than high volatility for the risk. The overall pattern of more low than high volatilities for the risk is expected since a low implied volatility should lead to a lower risk.

When seeking to analyse the moneyness it is found that OTM seems to be the most efficient moneyness to include in a hedge except in 2000-2003 for all performance measurements, and in 1990-1998 if being concerned about the expected return or the risk. In the period 2000-2003 ATM is instead the optimal moneyness for the expected return and the reward to variability ratio. The conclusion from this is that ATM options are best in a hedge when the market trend is downward sloping while OTM options are the most efficient when the market is in a steady upturn. This could be explained by the positive correlation found between the underlying asset's price and the implied volatility except in the period 2000-2003. In accordance with Hull and White (1987), this would lead to underpriced OTM options with Black 76 as well as with Black and Scholes. Hence OTM options are relatively cheaper than the other options which yield a relatively higher volatility, when the Black 76 price is set equal to the actual price. This will in turn affect the Greeks. Their behaviour are, however, mixed and holding all other things equal they increase up to a certain point and then decrease when the volatility is increased. Since OTM options are most efficient it seems like they increase rather than decrease. This would yield a higher *Gamma* and *Vega* per SEK which is beneficial for the cost of the hedge. The mixed behaviour arises since the derivatives with respect to the volatility do not give a clear sign indication. Overall the pattern for the moneyness is more dispersed for the risk than for the expected return and the reward to variability ratio. Therefore the conclusions are less robust and a market maker has to be more careful when constructing the hedge.

The call/put characteristic is evenly dispersed for almost all performance measurements and time periods. Hence it is possible to see the choice between calls and puts as fairly irrelevant and that it does not matter significantly which is chosen as long as it matches the other criteria desired for a well diversified portfolio. The underlying mechanisms are that calls and puts are each others mirror images, put-call parity should always hold and a put with a given set of criteria should have the same *Gamma* and *Vega* as a call which adheres to the same criteria. The absolute value of the *Delta* will also be the same; a long put always has a negative *Delta* while a long call always has a positive *Delta*. The *Gamma* and *Vega* are always positive for both calls and puts.

In the maturity category short maturity is the most numerous in all periods except in 2000-2003 for the risk and the difference between short and long maturity is often rather large. This is in line with what Swidler and Diltz (1992) found; that transactions costs increase with maturity. An explanation for this could be that short maturity options are more frequently traded which leads to an increased possibility to use them in a hedge. See *Figure 3* for an illustration of the relationship between average trading volume and time to maturity.

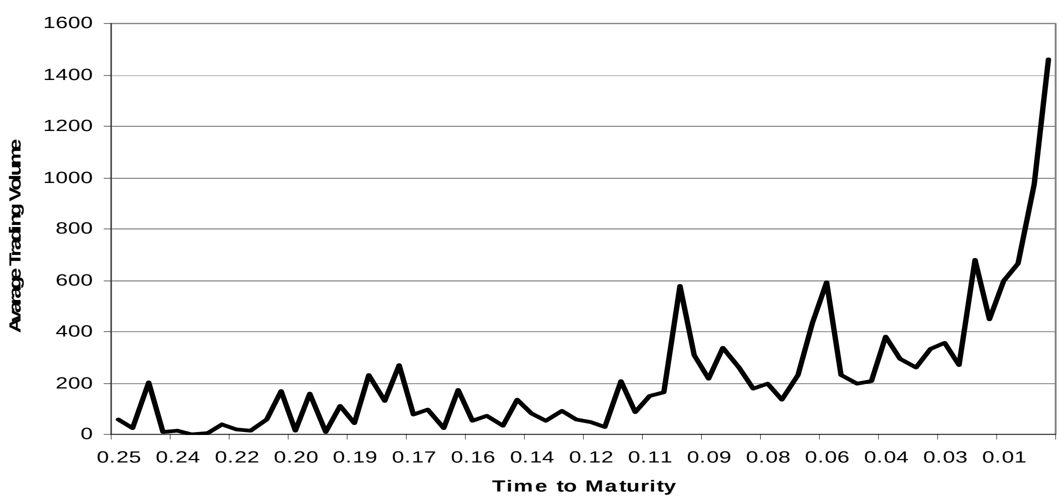

Figure 3: The Average Liquidity Compared to the Time to Maturity for OMX9T

The increased trading activity may then result in smaller bid-ask spreads which would reduce the cost for a hedging strategy. Because the liquidity is inversely related to the maturity the shorter the maturity the more efficiently priced the options will be. This effect leads to larger spreads for the long maturity options and since the market maker loses the spread on the hedge this will make them relatively more expensive to use. The difference for the maturity in 2000-2003 for the risk depends on the period's high volatility. Since the price of an option with a long maturity is less affected by the movements in the market today than an option with a short maturity, options with a long maturity will result in a lower risk when the market movements are large.

5.2 The Strategies with the Largest Number of Observations

In this section there is a joint analysis of all the different time periods and performance measurements for the 30 strategies with the largest number of observations. In the analysis of the performance measurements for all strategies regardless of the number of observations a single option type stood out as being optimal for several of the available portfolios. This is no longer the case. It is the expected result since the sample has been diminished and the conclusions should no longer be exactly the same, especially since it was previously noted that the best strategy quite often had rather few observations. The relevant tables for the strategies with the largest number of observations for all performance measurements are found in Appendix A.8.

A difference from the analysis of all strategies is found in the strategies that are common for one or more of the time periods. Earlier they were equal for 1990-2005 and 1990-1998, now they are instead equal for 1990-2005 and 1998-2000 and/or 2000-2003. 1998-2000 and 2000-2003 exhibit the largest amount of trading and the largest number of available options and since the option already included in the portfolio can not be included in the hedge this affects the number of observations. A large number of the observations can therefore be attributed to these two periods which is reflected in this particular subsample of the strategies. The reward to variability ratio has a characteristic which is not present anywhere else, there are portfolios where a best strategy is not available, see *Table A.8.*3. This occurs because there is no strategy among those with the largest number of observations that has a positive expected return. It is not possible to compare two negative reward to variability ratios and determine that the least negative ratio is the best. The same ratio is for example received for -100/100 and -10/10, however, -10/10 is clearly better since it has a lower risk and a higher expected return. Hence it is not possible to use this ratio for negative returns without adjustments.

5.2.1 The Individual Components

The patterns for the individual components are similar to those found when all of the strategies were investigated; this is visible when comparing the tables in Appendix A.9-A.11 with the tables in Appendix A.5-A.7. There are, of course, some differences but the overall conclusions are consistent except for the volatility and maturity. The moneyness characteristic illuminates that OTM is still optimal for the highest expected return but ATM might be optimal under certain conditions for the risk and the reward to variability ratio. However, ITM is still the least optimal moneyness to include in a hedge. It is found, yet again, that the call/put characteristic is rather irrelevant for a well diversified portfolio.

The volatility characteristic is a lot more dispersed than before and low volatility is more frequent in 1990-1998 and 2003-2005 for the expected return, the risk and the reward to variability ratio and for the risk in 1990-2005. This is explained by the overall volatility in the market being lower during these periods and therefore the strategies with the largest number of observations include at least one option with a low volatility. An option with a certain level of volatility is almost always found on a particular day but it might be difficult to find two with that level of volatility.

The pattern for the maturity is clearly different from the analysis of the entire sample, here there are as many short as long maturities while before there was a clear tendency towards a short maturity. This can be explained by the fact that it is easier to find one of each maturity and therefore they become the most frequently found strategies.

5.3 Domination Analysis

Earlier in this thesis it has been established that the moneyness and maturity seem to be the most crucial characteristics of the option in determining the efficiency of a strategy. This is based on the finding that there have been an overweight of these components among the best and indistinguishable options. To further examine this, *Tables A.12.1-A.12.3* have been constructed. These tables are matrixes where the different components have been matched with each other and then ranked to analyse whether or not any of the two components tends to dominate the other. This has been done for all time periods, but since they gave the same results only the tables for 1990-2005 are included.

In the matrix representing the analysis when focusing on the expected return it can be concluded that moneyness dominates volatility and call/put. Maturity dominates volatility and call/put while volatility only dominates call/put. Hence the call/put characteristic is being dominated by all the other and can therefore be concluded to be irrelevant in this context. In the case between moneyness and maturity there is no clear cut result to be found since they are more dispersed.

In the risk matrix the volatility is, as expected, dominating all of the other attributes. More surprising is that there are no noticeable patterns for the other characteristics; these have a more mixed behaviour. It is found in the study that the options that are OTM dominate all other criteria while the pattern is mixed for ATM and ITM. When studying the reward to variability ratio matrix it is suspected that the biased risk results to some extent have contaminated the result, leading to mixed results for volatility/moneyness and volatility/maturity.

The most apparent conclusion from the domination analysis is that the call/put characteristic is dominated by all the others. A noteworthy remark is that the only intersection where it can not be concluded which characteristic who dominates the other in all three matrixes is moneyness/maturity. A further conclusion from the domination analysis is that the maturity and the moneyness are of vital importance for the expected return and the reward to variability ratio. Logically the risk analysis gives that the volatility is the most dominating characteristic.

6. Conclusion

This thesis has found strong evidence for the proposition that no single hedging strategy is best for all of the portfolios regardless of which performance measurement the market maker is interested in. However, in all the explored dimensions Out of the money options tend to have a superior hedging return for OMX index options except in the bearish market of 2000 to 2003. This exception might be explained by the negative correlation found between the volatility and the underlying asset's price in that specific period.

It is possible to find an option that outperforms the other options when utilizing the entire sample although this is not possible when only the 30 strategies which have the maximum number of observations are studied. Hence it could be worth the effort to search for the less common options that actually perform best when constructing a hedge. A suggestion to market makers in need of a hedging strategy is to consult the relevant table in the Appendix in order to find out which strategy is the most efficient for their particular portfolio. They must then keep in mind to use the strategy from the time period which best reflects the suitable market situation.

With regards to the expected return it is determined that the option type Hocs is the most efficient to include when hedging *Gamma*, *Vega* and *Delta*, given that it is combined wisely to create an effective hedge. Hocs implies the use of a call, but if the portfolio to be hedged consists of puts it is more efficient to use Hops, the only difference being a put instead of a call. This is only true for hedging a reference portfolio though. In a situation where Hocs can not be found and a substitute is needed the most important characteristics to take into consideration are the moneyness and the maturity.

Market makers whose main concern is to minimize the risk of their position should turn their attention to the relevant table for the risk. As a rule of thumb, Lops is the most efficient option to include in a hedge when combined wisely. The most important characteristic when being concerned about the risk, no matter whether a Lops is included or not, is of course primarily the volatility, even though the moneyness and maturity aspects are still pivotal.

Perhaps the most important performance measurement is the reward to variability ratio since it is a joint measurement of the expected return and the risk. If this is the case market makers are better of using a Hocs in their hedge. However, here it is more crucial which option Hocs is combined with since more than half of the other option types are not present among the preferred strategies together with Hocs. If Hocs is not used, ideal individual characteristics to search for when concerned about the reward to variability ratio are generally Out of the money and a short maturity, except during major downturns in the market.

7. Suggestions for Further Research

The conclusions of this thesis open up several innovative topics for further financial research. An obvious topic would be to examine the four characteristics separately and jointly in different combinations, perhaps excluding the call/put dimension. It could also be interesting to study different combinations of the Greeks. Within this thesis moneyness is found to be a fundamental component when constructing a hedge. Therefore a fascinating development of the results would be to study narrower and a greater number of intervals in an attempt to find out where the limits of efficiency are drawn. The same procedure could also be applied to the maturity characteristic. Investigating the volatility aspect there is several different approaches available for further examination. A first would be to analyse the volatility per SEK, a second to have several and flexible limits and a third to focus on whether the bid implied volatility might be better suited to use than the ask implied volatility and vice versa.

There are no previous academic studies on this particular subject, therefore comparison studies on different markets and underlying assets could add valuable insights. A study commenced on for example the S&P 500 would benefit from a higher liquidity, a longer timeframe and less dependency on individual stocks within the index.

This thesis' data sample consists of daily closing prices. It would, of course, be interesting to perform this study with data that is on a minute or hour basis. This would imply that the rebalancing interval would be different from the one within this thesis, but it would also be possible to add an extra dimension in the form of the optimal rebalancing frequency. It would then, however, not be possible to use such a long time period as 15 years, since this would result in a data sample which would be overwhelming to work with.

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8.2 Interviews

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Structured Derivatives Trader, Major Swedish Investment Bank, January 26, 2006

Head of Structured Products, Major Swedish Fund Commissioner, February 27, 2006

Head of Equities, Major Swedish Investment Bank, March 7, 2006

8.3 Other Publications

“10 frågor och svar om options- och terminshandel”, Information Booklet from OMX

”Swedish Financial Markets 2005”, Sveriges Riksbank

Appendix

A.1 The Black 76 Model

The theory of option pricing developed in Black and Scholes (1973), and simultaneously in Merton (1973), was developed to price equity options and corporate liabilities. A formula which takes into consideration the effect of dividends on the stock price in the valuation of options was presented in Merton (1973). The most important extension of the Black and Scholes model for this thesis is, however, the Black 76 model which was presented in Black (1976). This paper extends the original model to also cover options on future contracts. The value of a future contract is, due to its construction, always zero at the beginning of each day and therefore the equity in an option position on a future is zero. A consequence of the zero equity is that the interest rate factor is no longer included in the formula. When pricing a future the expected dividends are taken into consideration, hence the dividend yield is zero in the option formula. Black 76 therefore gives the following formulas for valuing a call and a put option on a future:

Call: $C(S_t, T) = S_t \cdot N(d_1) - K \cdot N\left(d_1 - \sigma \cdot \sqrt{T}\right)$

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right)}{\sigma \cdot \sqrt{T}} + \frac{\sigma \cdot \sqrt{T}}{2}$$

Put: $P(S_t, T) = K \cdot N\left(\sigma \cdot \sqrt{T} - d_1\right) - S_t \cdot N(-d_1)$

S: Price of the future today K: Exercise price

T: Time to maturity σ: Volatility of the underlying

The assumptions which underlie this model are that the option is European and the index price follows a Geometric Brownian Motion. It is also assumed that markets are perfect, hence there are no restrictions on short selling, assets are divisible and there are no transaction costs or taxes. Finally, there is no arbitrage, trading is continuous and the interest rate is constant.

As follows from Natenberg (1988) the Greeks using the Black 76 formula are:

Call *Delta*: $\Delta = N(d_1)$ Call/Put *Gamma*: $\Gamma = \frac{N'(d_1)}{S_t \cdot \sigma \cdot \sqrt{T}}$

Put *Delta*: $\Delta = N(d_1) - 1$ Call/Put *Vega*: $V = S_t \cdot \sqrt{T} \cdot N'(d_1)$

A.2 Volatility Smiles

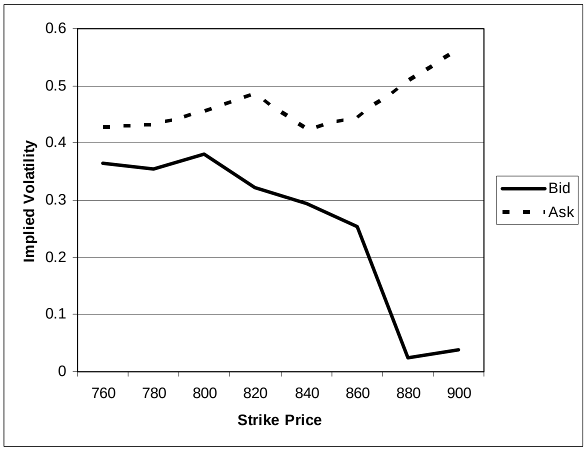

Figure A.1: Volatility Smile 1990-01-24

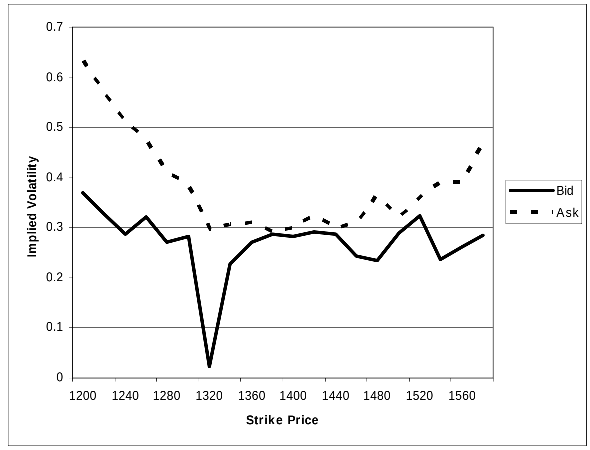

Figure A.2: Volatility Smile 2000-06-14

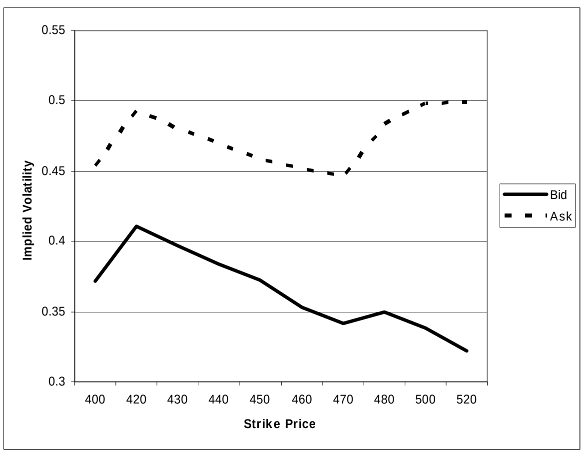

Figure A.3: Volatility Smile 2000-10-01

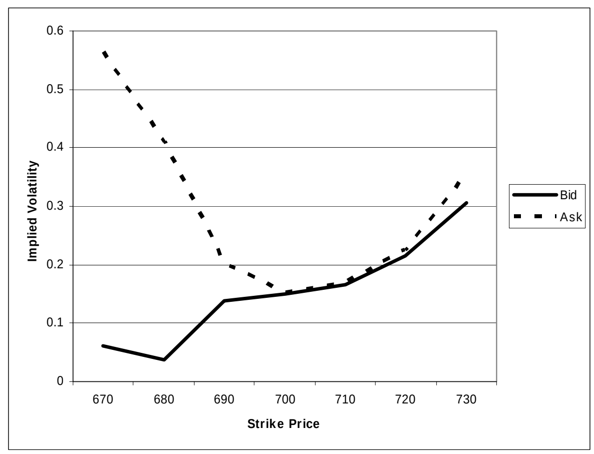

Figure A.4: Volatility Smile 2004-02-26

A.3 Portfolios and Strategies

Portfolio	Call/Put	Position	Moneyness	Maturity
1	Call	Long	ATM	Short
2	Call	Long	ATM	Long
3	Call	Long	ITM	Short
4	Call	Long	ITM	Long
5	Call	Long	OTM	Short
6	Call	Long	OTM	Long
7	Call	Short	ATM	Short
8	Call	Short	ATM	Long
9	Call	Short	ITM	Short
10	Call	Short	ITM	Long
11	Call	Short	OTM	Short
12	Call	Short	OTM	Long
13	Put	Long	ATM	Short
14	Put	Long	ATM	Long
15	Put	Long	ITM	Short
16	Put	Long	ITM	Long
17	Put	Long	OTM	Short
18	Put	Long	OTM	Long
19	Put	Short	ATM	Short
20	Put	Short	ATM	Long
21	Put	Short	ITM	Short
22	Put	Short	ITM	Long
23	Put	Short	OTM	Short
24	Put	Short	OTM	Long

Table A.3.1: Reference Portfolios

Portfolio	Call/Put	Position	Moneyness	Maturity	Portfolio	Call/Put	Position	Moneyness	Maturity
25	Call	Short	ATM	Long	29	Call	Short	ATM	Long
	Call	Short	OTM	Short		Call	Short	ITM	Short
	Call	Long	ITM	Short		Put	Short	ATM	Long
	Put	Short	ATM	Short		Put	Short	OTM	Short
	Put	Short	ITM	Long		Put	Long	ATM	Short
26	Call	Short	OTM	Short	30	Call	Long	OTM	Long
	Put	Short	ATM	Short		Call	Long	ATM	Short
	Put	Short	ITM	Short		Call	Short	ITM	Short
	Put	Long	ITM	Long		Put	Short	OTM	Short
	Put	Long	OTM	Long		Put	Long	ATM	Long
27	Call	Long	ITM	Short	31	Call	Short	ATM	Long
	Call	Long	ATM	Long		Call	Short	OTM	Short
	Call	Long	ATM	Long		Put	Long	ITM	Short
	Put	Short	ATM	Long		Put	Long	ATM	Long
	Put	Short	OTM	Short		Put	Long	ATM	Short
28	Call	Short	ATM	Long	32	Call	Short	ITM	Short
	Call	Short	OTM	Long		Call	Long	ATM	Long
	Call	Long	ITM	Short		Put	Long	ITM	Short
	Call	Long	OTM	Short		Put	Long	ATM	Long
	Put	Short	ITM	Short		Put	Short	OTM	Long

Table A.3.2: Main Portfolios

Low Volatility	0-25%
High Volatility	25-50%
ATM (*Delta*)	0.4-0.6
OTM (*Delta*)	0.1-0.4
ITM (*Delta*)	0.6-0.9
Short Maturity	0-30 days
Long Maturity	>30 days

Table A.3.3: Definitions of the Strategies' Characteristics

Abbreviation	Volatility	Moneyness	Call/Put	Maturity
Lacl	Low	ATM	Call	Long
Lacs	Low	ATM	Call	Short
Lapl	Low	ATM	Put	Long
Laps	Low	ATM	Put	Short
Licl	Low	ITM	Call	Long
Lics	Low	ITM	Call	Short
Lipl	Low	ITM	Put	Long
Lips	Low	ITM	Put	Short
Locl	Low	OTM	Call	Long
Locs	Low	OTM	Call	Short
Lopl	Low	OTM	Put	Long
Lops	Low	OTM	Put	Short
Hacl	High	ATM	Call	Long
Hacs	High	ATM	Call	Short
Hapl	High	ATM	Put	Long
Haps	High	ATM	Put	Short
Hicl	High	ITM	Call	Long
Hics	High	ITM	Call	Short
Hipl	High	ITM	Put	Long
Hips	High	ITM	Put	Short
Hocl	High	OTM	Call	Long
Hocs	High	OTM	Call	Short
Hopl	High	OTM	Put	Long
Hops	High	OTM	Put	Short

Table A.3.4: Translation of the Abbreviations of the Options used in the Strategies

A.4 The Best Strategies

Portfolio	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
1	*HocsHocs*	HocsHocl	LoplHocs	*HocsHocs*	*HocsHocs*
2	*LicsHaps*	*LicsHaps*	LicsHacs	LicsHapl	*LicsHaps*
3	*LicsHocs*	*LicsHocs*	LicsHaps	HocsLopl	HacsHipl
4	*LicsHaps*	*LicsHaps*	LopsHips	LocsHocl	LopsHaps
5	*HocsHocs*	LacsHocs	LicsHocs	*HocsHocs*	*HocsHocs*
6	*LicsHaps*	*LicsHaps*	*LicsHaps*	LipsHacl	LopsHaps
7	LicsHocs	HocsHocs	HocsHops	LaplHocs	LocsHocs
8	LoclHocs	LaclHocs	LoplHaps	LaclHacs	HacsHacs
9	*LacsHocs*	*LacsHocs*	*LacsHocs*	LiclHapl	HacsHipl
10	*LopsHocs*	*LopsHocs*	LoplHocs	LapsHapl	LipsHips
11	*LacsHocs*	LoplHocs	HocsHocs	*LacsHocs*	LocsHocs
12	LoclHocs	LaclHocs	LoplHocs	LapsHapl	LopsHaps
13	LopsHaps	HopsHops	LaplLopl	HapsHops	HocsHocs
14	*LopsHaps*	LopsHocs	LopsHocl	LapsHicl	*LopsHaps*
15	*LipsHops*	*LipsHops*	LapsHacs	HapsHocs	HacsHipl
16	*LapsHocs*	*LapsHocs*	LicsHaps	LapsHicl	LopsHops
17	*HopsHops*	*HopsHops*	LopsHops	*HopsHops*	HocsHocs
18	LocsHops	LapsHacl	LicsHocl	*LopsHaps*	*LopsHaps*
19	*LicsHocs*	*LicsHocs*	LocsHocs	LocsHicl	LicsHaps
20	*LoplHaps*	*LoplHaps*	*LoplHaps*	LoclHaps	LopsHaps
21	*LicsHocs*	*LicsHocs*	LoclLopl	LapsHapl	HipsHops
22	*LicsHocs*	*LicsHocs*	LaclLopl	LocsHapl	LocsHaps
23	LicsHocs	LocsHips	LocsHocs	LaplHaps	LopsHaps
24	*LoplHaps*	*LoplHaps*	*LoplHaps*	LaclHops	LocsHips
25	LaplHips	LiplHaps	LoplHops	LiclHapl	LocsHaps
26	HipsHips	LapsHocs	HapsHops	LapsHicl	LapsHacs
27	*LicsHaps*	*LicsHaps*	*LicsHaps*	LaclHics	LopsHaps
28	LoplHips	LoplHacs	LaplHocs	LapsHopl	HacsHipl
29	*LaclHocs*	*LaclHocs*	LoplHacs	LaclHics	HacsHacs
30	LicsHaps	LocsHaps	LicsHacs	HopsHops	HocsHocs
31	LapsHocs	LapsHocl	HapsHaps	HapsHops	LipsHips
32	*LipsHocs*	*LipsHocs*	HacsHacs	HipsHips	LopsHaps

Table A.4.1: The Strategies with the Highest Expected Return within the Different Time Periods

Portfolio	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
1	*LocsLocs*	*LocsLocs*	LacsLops	LipsHocl	HacsHacs
2	HoclHocl	LoclHocl	LipsHips	LiplHicl	LocsLocs
3	*LacsLics*	*LacsLics*	LopsLops	LaplHicl	LacsHips
4	LoclHocl	LiclLicl	*LicsLics*	LipsHacl	*LicsLics*
5	*LacsLops*	*LacsLops*	LacsHocs	LipsHicl	HocsHocs
6	LicsLocl	LopsHocl	LocsHocs	LaplHocl	LoclHacs
7	*LocsLocs*	*LocsLocs*	LocsLopl	LiplHacs	LopsHocl
8	LaclLacl	LiplLipl	LicsLics	LaplHicl	LacsLocs
9	*LacsLics*	*LacsLics*	LopsHips	HacsHics	LacsLacs
10	LaclLics	LiplLipl	LopsLops	LiplHicl	LiclHocs
11	LacsLacs	LacsLops	LacsHacs	LocsHicl	LacsHocs
12	*LoclLocl*	*LoclLocl*	HoclHocl	LaclHicl	LoclHocl
13	*LopsHops*	*LopsHops*	LopsLops	LipsHocl	LopsHips
14	LiplLipl	LaplHapl	LaplLipl	LapsHopl	LipsHaps
15	*LapsLaps*	LopsHipl	LapsHocs	LipsHocl	*LapsLaps*
16	LopsHipl	LiplLipl	LaplHipl	LaclHipl	LocsHocl
17	*LopsLops*	*LopsLops*	*LopsHaps*	LaplHaps	*LopsHaps*
18	LiplLipl	LiplHipl	LicsHacs	LaplHapl	LopsLops
19	*LapsLaps*	*LapsLaps*	LoplHops	LoclHaps	LopsHaps
20	*LiplLipl*	*LiplLipl*	LaplLipl	LipsHopl	LicsHaps
21	LapsLops	LapsLips	HiplHocl	LaclHips	LipsHaps
22	LiplLops	HiplHipl	LaplLipl	LiplHopl	LaplHapl
23	LopsLops	LaplHips	LopsHacs	LipsHipl	LapsHips
24	LaplLapl	HiplHipl	LopsHaps	LipsLipl	LaplHips
25	LopsLopl	LoclHipl	LopsHacs	LaclHips	LipsHaps
26	LiplLipl	LopsHics	LipsLops	*LopsLops*	*LopsLops*
27	LipsHocl	HiplHipl	LiclLopl	LapsHicl	LiclHaps
28	*LicsLocl*	*LicsLocl*	LopsHacs	LapsHicl	LocsHips
29	HoclHocl	LiplLipl	LopsHics	LaplHacl	HiplHipl
30	LiplLipl	LoclHocl	LoclHocs	LipsHocl	LaclHocl
31	LopsLopl	LoplHops	LipsHocs	LiclHips	LocsLocs
32	LiplLipl	LopsHocl	HipsHips	LipsHopl	LopsHaps

Table A.4.2: The Strategies with the Lowest Risk within the Different Time Periods

Portfolio	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
1	LocsHocs	*LocsHocs*	LapsHocs	*HocsHocs*	*HocsHocs*
2	*LopsHocs*	*LopsHocs*	*LicsHaps*	LipsHapl	*LicsHaps*
3	*LocsHocs*	LicsHocs	*LocsHocs*	HocsHocl	*LocsLocs*
4	LicsHocl	*LicsHocl*	LopsHaps	LocsHicl	*LocsLocs*
5	HocsHocs	LacsHocs	LacsLacs	HocsHocs	*HocsHocs*
6	LocsHocs	LocsHops	LopsHips	LipsHacl	LicsHaps
7	LacsHocs	*LacsHocs*	LocsLopl	LocsHapl	*LocsLocs*
8	LoclHocs	HoclHocl	LoplHaps	LaclHacs	LacsLocs
9	*LacsHocs*	*LacsHocs*	*LacsHocs*	LapsHapl	*LicsHocs*
10	LopsHocs	*LopsHocs*	LoplHacs	LapsHopl	*LacsHocs*
11	LacsHocs	*LacsHocs*	HocsHocs	LaclHocs	*LocsHocs*
12	LoclHocs	HiplHipl	LoplHaps	LapsHopl	HocsHocs
13	LapsHops	*LapsHops*	LopsLops	HopsHops	*HocsHocs*
14	LopsHaps	HiplHipl	LaplHipl	LapsHicl	LipsHaps
15	LapsLaps	*LapsLaps*	LoplHips	HacsHaps	*HipsHips*
16	*LapsHocs*	*LapsHocs*	*LapsHocs*	LapsHicl	*LopsLops*
17	HopsHops	*HopsHops*	LopsHaps	*HopsHops*	*HocsHocs*
18	LocsHops	LocsHocs	LicsHaps	LopsHops	LopsHaps
19	*HopsHops*	*HopsHops*	*HopsHops*	LocsHipl	*LopsHaps*
20	LopsLops	*LopsLops*	LoplHips	LicsHopl	*LopsHaps*
21	LapsHaps	*LapsHaps*	LapsHocs	LicsHipl	*LacsHips*
22	LicsHocs	*LicsHocs*	HiplHipl	LocsHicl	*LocsHaps*
23	*LicsHocs*	LiplHicl	*LoplHaps*	LiclHops	LopsHaps
24	*LoplHaps*	LoplLopl	*LoplHaps*	LocsHacl	*LacsHaps*
25	LoplHaps	LicsHocl	LopsHacs	LocsHicl	LocsLops
26	LacsHops	LopsHaps	LapsHocs	LopsLops	LapsHacs
27	HocsHocl	LicsHocl	LopsHaps	HocsHocl	LiclHaps
28	HocsHopl	*HocsHopl*	LiplLopl	LiplLipl	*LocsHips*
29	HoclHocl	LoclHocl	LoplHips	LicsHacl	LicsHops
30	HacsHapl	LoclHocl	LoclHocs	LipsLipl	LacsHops
31	LapsHocs	LocsHaps	LipsHocs	LipsHicl	LocsHips
32	*LopsHocl*	*LopsHocl*	*LiplLipl*	*LipsLipl*	*LopsHaps*

Table A.4.3: The Strategies with Highest Reward to Variability Ratio within the Different Time Periods

A.5 The Individual Characteristics for the Expected Return

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.44	0.44	0.47	0.38	0.31
High Volatility	0.56	0.56	0.53	0.63	0.69
OTM	0.50	0.52	0.53	0.36	0.45
ATM	0.23	0.30	0.33	0.45	0.36
ITM	0.27	0.19	0.14	0.19	0.19
Call	0.56	0.58	0.50	0.45	0.44
Put	0.44	0.42	0.50	0.55	0.56
Short Maturity	0.89	0.83	0.73	0.63	0.94
Long Maturity	0.11	0.17	0.27	0.38	0.06

Table A.5.1: The Dispersion of the Individual Characteristics for the Highest Expected Returns

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.47	0.49	0.48	0.34	0.44
High Volatility	0.53	0.51	0.52	0.66	0.56
OTM	0.45	0.35	0.44	0.34	0.47
ATM	0.35	0.47	0.37	0.39	0.36
ITM	0.21	0.18	0.19	0.28	0.17
Call	0.49	0.51	0.49	0.55	0.47
Put	0.51	0.49	0.51	0.45	0.53
Short Maturity	0.65	0.67	0.63	0.58	0.83
Long Maturity	0.35	0.33	0.37	0.42	0.17

Table A.5.2: The Dispersion of the Individual Characteristics for the Indistinguishable Expected Returns

A.6 The Individual Characteristics for the Risk

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.88	0.72	0.64	0.52	0.58
High Volatility	0.13	0.28	0.36	0.48	0.42
OTM	0.35	0.41	0.47	0.20	0.41
ATM	0.28	0.44	0.23	0.31	0.36
ITM	0.37	0.16	0.30	0.48	0.23
Call	0.44	0.38	0.41	0.41	0.48
Put	0.56	0.63	0.59	0.59	0.52
Short Maturity	0.50	0.56	0.73	0.38	0.78
Long Maturity	0.50	0.44	0.27	0.63	0.22

Table A.6.1: The Dispersion of the Individual Characteristics for the Lowest Risk

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.87	0.81	0.67	0.41	0.66
High Volatility	0.13	0.19	0.33	0.59	0.34
OTM	0.45	0.29	0.36	0.29	0.41
ATM	0.27	0.45	0.34	0.32	0.34
ITM	0.28	0.27	0.29	0.40	0.25
Call	0.40	0.52	0.46	0.54	0.48
Put	0.60	0.48	0.54	0.46	0.52
Short Maturity	0.50	0.55	0.69	0.44	0.78
Long Maturity	0.50	0.45	0.31	0.56	0.22

Table A.6.2: The Dispersion of the Individual Characteristics for the Indistinguishable Risk

A.7 The Individual Characteristics for the Reward to Variability Ratio

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.42	0.45	0.53	0.45	0.50
High Volatility	0.58	0.55	0.47	0.55	0.50
OTM	0.70	0.64	0.50	0.42	0.55
ATM	0.25	0.19	0.30	0.25	0.28
ITM	0.05	0.17	0.20	0.33	0.17
Call	0.58	0.56	0.31	0.48	0.56
Put	0.42	0.44	0.69	0.52	0.44
Short Maturity	0.83	0.70	0.72	0.41	0.98
Long Maturity	0.17	0.30	0.28	0.59	0.02

Table A.7.1: The Dispersion of the Individual Characteristics for the Highest Reward to Variability Ratios

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.39	0.45	0.66	0.37	0.62
High Volatility	0.61	0.55	0.34	0.63	0.38
OTM	0.55	0.61	0.57	0.36	0.54
ATM	0.30	0.21	0.29	0.40	0.32
ITM	0.15	0.18	0.15	0.24	0.14
Call	0.49	0.48	0.42	0.51	0.50
Put	0.51	0.52	0.58	0.49	0.50
Short Maturity	0.77	0.77	0.71	0.44	1.00
Long Maturity	0.23	0.23	0.29	0.56	0.00

Table A.7.2: The Dispersion of the Individual Characteristics for the Indistinguishable Reward to Variability Ratios

A.8 The Best Strategies with the Largest Number of Observations

Portfolio	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
1	*HocsHopl*	*LocsLopl*	*HocsHopl*	*HocsHopl*	*LocsLopl*
2	HacsHacl	LacsLacl	HopsHopl	HocsHocl	LicsLocl
3	*HocsHopl*	LacsLapl	*HocsHopl*	*HocsHopl*	HopsHopl
4	HacsHacl	*LocsLocl*	HocsHopl	HacsHocl	*LocsLocl*
5	*HocsHopl*	LocsLopl	HacsHacl	*HocsHopl*	LacsLocl
6	*HacsHacl*	LocsHopl	*HacsHacl*	HaclHocs	LocsLocl
7	*HocsHopl*	LocsLocl	*HocsHopl*	*HocsHopl*	LocsLopl
8	*LocsLocl*	*LocsLocl*	HocsHopl	HacsHacl	*LocsLocl*
9	*HocsHopl*	LacsLapl	*HocsHopl*	*HocsHopl*	HopsHopl
10	*HocsHopl*	*LocsLocl*	*HocsHopl*	HocsHocl	*LocsLocl*
11	*HocsHopl*	LacsHocs	LocsHapl	*HocsHopl*	LacsLopl
12	*HocsHopl*	LocsLocl	*HocsHopl*	HaclHocs	LapsLocl
13	*HopsHopl*	LopsLopl	*HopsHopl*	*HopsHopl*	LocsLopl
14	*HapsHapl*	LopsLopl	*HopsHopl*	*HapsHapl*	*HopsHopl*
15	HapsHocl	LopsLopl	HacsHopl	HocsHocl	LocsLopl
16	HapsHopl	LocsLopl	HopsHopl	HapsHapl	LacsLopl
17	*HopsHopl*	*HopsHopl*	*HopsHopl*	*HopsHopl*	LocsLopl
18	*HopsHopl*	LapsHopl	HapsHapl	LocsHopl	*HopsHopl*
19	*HopsHopl*	LopsLopl	*HopsHopl*	*HocsHopl*	LocsLocl
20	*HocsHopl*	LopsLopl	*HocsHopl*	*HopsHopl*	*HopsHopl*
21	*HocsHopl*	LapsLapl	LocsHopl	*HocsHopl*	HacsHopl
22	*HocsHopl*	LacsLopl	LocsHopl	*HocsHopl*	LocsLopl
23	*HopsHopl*	LopsLopl	*HopsHopl*	HocsHopl	LapsLocl
24	HopsHopl	LocsLopl	HocsHopl	HacsHapl	LocsLocl
25	*HopsHopl*	*HopsHopl*	HapsHopl	HaplHops	*HopsHopl*
26	*HocsHocl*	LacsHopl	*HocsHocl*	*HocsHocl*	HocsHopl
27	*HocsHocl*	LocsHopl	*HocsHopl*	*HocsHocl*	*HopsHopl*
28	*HocsHopl*	HopsHopl	*HocsHopl*	*HocsHopl*	*HopsHopl*
29	*HocsHopl*	HacsHopl	*HocsHopl*	HocsHocl	LocsLocl
30	HacsHopl	*HopsHopl*	HocsHopl	HacsHapl	*HopsHopl*
31	*HocsHopl*	LapsLopl	*HocsHopl*	*HocsHopl*	HopsHopl
32	*HocsHopl*	HapsHopl	*HocsHopl*	*HocsHopl*	HacsHopl

Table A.8.1: The Strategies with the Highest Expected Return for the Strategies with the Largest Number of Observations

Portfolio	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
1	LaclLocs	LocsLopl	HicsHocl	HapsHapl	LopsLopl
2	*HapsHocl*	LaclLics	HaclHaps	*HapsHocl*	LacsLapl
3	LacsLacl	*LopsLopl*	HicsHopl	HacsHapl	*LopsLopl*
4	LacsLacl	LicsLocl	LocsHopl	HacsHacl	LipsLopl
5	LacsLacl	*LacsLocl*	HacsHacl	HacsHipl	*LacsLocl*
6	*HapsHocl*	LocsLocl	HaclHocs	*HapsHocl*	LapsLocl
7	LocsLocl	*LocsLopl*	LocsHapl	HocsHopl	*LocsLopl*
8	LaclLocs	LaclLics	LocsHopl	HaclHocs	LacsLacl
9	LacsLapl	LopsLopl	LocsHopl	HacsHipl	LacsLopl
10	LacsLacl	LaclLics	LocsHacl	HocsHocl	LapsLocl
11	LacsLapl	LacsLops	LocsHapl	HocsHocl	LapsLocl
12	LacsLacl	*LipsLocl*	HoclHops	HapsHocl	*LipsLocl*
13	LacsLacl	LopsLopl	HoclHops	HipsHocl	LacsLopl
14	HicsHopl	LapsLopl	HaplHopl	HacsHapl	LacsLapl
15	HapsHocl	LapsLapl	HacsHapl	HapsHicl	LocsLopl
16	HapsHapl	LopsLopl	HaplHopl	HacsHapl	LipsLopl
17	LacsLacl	*LocsLopl*	HicsHocl	HapsHicl	*LocsLopl*
18	*HacsHopl*	LapsLopl	*HacsHopl*	*LopsLopl*	*LopsLopl*
19	LocsLocl	LopsLopl	LocsHacl	HapsHocl	LacsLopl
20	LacsLacl	LapsLapl	HaplHops	HacsHapl	LacsLapl
21	LacsLacl	LapsLopl	LocsHapl	HapsHicl	LocsLocl
22	HacsHipl	LopsLopl	HapsHapl	HacsHapl	LapsLapl
23	LocsLocl	LopsLopl	LocsHapl	HopsHopl	LapsLicl
24	LapsLapl	LopsLopl	LocsHapl	HocsHopl	LocsLopl
25	HapsHicl	LopsLopl	LocsHapl	HapsHapl	LaplLops
26	LacsLacl	HaplHops	HacsHapl	LopsLopl	LacsLopl
27	HocsHocl	LacsLocl	LocsHacl	HacsHacl	LicsLopl
28	HapsHocl	LopsLopl	LocsHapl	HacsHacl	LapsLopl
29	LaclLocs	LacsLapl	LocsHopl	HacsHapl	LacsLacl
30	HapsHocl	LapsLopl	HacsHapl	HapsHapl	LipsLopl
31	LacsLacl	HicsHopl	HapsHopl	HapsHocl	LopsLopl
32	LacsLacl	LopsLopl	LocsHopl	HapsHocl	LocsLopl

Table A.8.2: The Strategies with the Lowest Risk for the Strategies with the Largest Number of Observations

Portfolio	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
1	*HocsHopl*	N/A	N/A	*HocsHopl*	N/A
2	N/A	N/A	HopsHopl	N/A	LicsLocl
3	HacsHocl	LacsLapl	HopsHopl	HocsHocl	LocsLopl
4	LacsLacl	LocsLocl	LocsHopl	HacsHacl	LicsLocl
5	N/A	N/A	N/A	N/A	N/A
6	*HacsHacl*	LocsHopl	*HacsHacl*	HaclHocs	LocsLocl
7	*HocsHopl*	N/A	LocsHapl	*HocsHopl*	LocsLopl
8	*LocsLocl*	*LocsLocl*	HocsHopl	N/A	LapsLopl
9	*HocsHopl*	LacsLapl	*HocsHopl*	*HocsHopl*	LocsLopl
10	*LacsLacl*	*LacsLacl*	HapsHopl	HocsHocl	LapsLocl
11	N/A	LacsHocs	N/A	HocsHopl	N/A
12	N/A	N/A	HocsHopl	HaclHocs	N/A
13	N/A	N/A	N/A	N/A	N/A
14	*HapsHapl*	LopsLopl	HopsHopl	*HapsHapl*	LacsLopl
15	HapsHocl	LapsLopl	HacsHacl	HapsHicl	LocsLopl
16	*HapsHapl*	LopsLopl	HopsHopl	*HapsHapl*	LacsLopl
17	N/A	N/A	N/A	HopsHopl	N/A
18	*HopsHopl*	N/A	HapsHapl	LocsHopl	*HopsHopl*
19	N/A	LopsLopl	HopsHopl	HocsHopl	N/A
20	*HocsHopl*	LopsLopl	*HocsHopl*	HopsHopl	LocsLocl
21	HapsHocl	LapsLopl	LocsHapl	HocsHopl	LocsLocl
22	HocsHopl	LopsLopl	LocsHopl	HacsHapl	LacsLopl
23	N/A	N/A	N/A	HopsHopl	N/A
24	N/A	LocsLopl	HocsHopl	HacsHapl	LocsLocl
25	HocsHopl	LopsLopl	LocsHopl	HaplHops	LacsLopl
26	*HocsHocl*	LacsHopl	HocsHopl	*HocsHocl*	HopsHopl
27	*HocsHocl*	LapsLocl	LocsHacl	*HocsHocl*	LicsLopl
28	HocsHopl	LopsLopl	HipsHopl	HacsHapl	LocsLopl
29	HocsHocl	HacsHopl	HocsHopl	HacsHacl	LocsLocl
30	*HacsHapl*	HapsHopl	*HacsHapl*	*HacsHapl*	LocsLopl
31	*HapsHocl*	HicsHopl	HapsHopl	*HapsHocl*	LocsLocl
32	HocsHopl	HapsHopl	LocsHapl	HapsHapl	LocsLopl

Table A.8.3: The Strategies with the Highest Reward to Variability for the Strategies with the Largest Number of Observations

A.9 The Individual Characteristics for the Strategies with the Largest Number of Observations for the Expected Return

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.03	0.73	0.05	0.02	0.59
High Volatility	0.97	0.27	0.95	0.98	0.41
OTM	0.83	0.77	0.86	0.78	0.88
ATM	0.17	0.23	0.14	0.22	0.11
ITM	0.00	0.00	0.00	0.00	0.02
Call	0.45	0.39	0.41	0.56	0.48
Put	0.55	0.61	0.59	0.44	0.52
Short Maturity	0.50	0.52	0.50	0.50	0.50
Long Maturity	0.50	0.48	0.50	0.50	0.50

Table A.9.1: The Dispersion of the Individual Characteristics for the Highest Expected Returns

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.09	0.84	0.07	0.03	0.82
High Volatility	0.91	0.16	0.93	0.98	0.18
OTM	0.56	0.60	0.56	0.59	0.70
ATM	0.42	0.33	0.39	0.38	0.23
ITM	0.02	0.07	0.05	0.03	0.06
Call	0.49	0.47	0.44	0.49	0.51
Put	0.51	0.53	0.56	0.51	0.49
Short Maturity	0.50	0.49	0.50	0.49	0.50
Long Maturity	0.50	0.51	0.50	0.51	0.50

Table A.9.2: The Dispersion of the Individual Characteristics for the Indistinguishable Expected Returns

A.10 The Individual Characteristics for the Strategies with the Largest Number of Observations for the Risk

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.66	0.94	0.23	0.06	1.00
High Volatility	0.34	0.06	0.77	0.94	0.00
OTM	0.28	0.64	0.52	0.34	0.55
ATM	0.67	0.27	0.44	0.56	0.36
ITM	0.05	0.09	0.05	0.09	0.09
Call	0.77	0.33	0.53	0.50	0.41
Put	0.23	0.67	0.47	0.50	0.59
Short Maturity	0.50	0.52	0.47	0.50	0.50
Long Maturity	0.50	0.48	0.53	0.50	0.50

Table A.10.1: The Dispersion of the Individual Characteristics for the Lowest Risk

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.52	0.97	0.15	0.02	1.00
High Volatility	0.48	0.03	0.85	0.98	0.00
OTM	0.33	0.56	0.41	0.30	0.47
ATM	0.61	0.31	0.47	0.51	0.39
ITM	0.05	0.13	0.12	0.19	0.13
Call	0.69	0.38	0.44	0.55	0.40
Put	0.31	0.63	0.56	0.45	0.60
Short Maturity	0.50	0.52	0.49	0.50	0.50
Long Maturity	0.50	0.48	0.51	0.50	0.50

Table A.10.2: The Dispersion of the Individual Characteristics for the Indistinguishable Risk

A.11 The Individual Characteristics for the Strategies with the Largest Number of Observations for the Reward to Variability Ratio

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.13	0.65	0.17	0.02	0.92
High Volatility	0.87	0.35	0.83	0.98	0.08
OTM	0.65	0.72	0.69	0.57	0.81
ATM	0.35	0.28	0.29	0.41	0.13
ITM	0.00	0.00	0.02	0.02	0.06
Call	0.61	0.39	0.27	0.52	0.60
Put	0.39	0.61	0.73	0.48	0.40
Short Maturity	0.50	0.33	0.17	0.50	0.50
Long Maturity	0.50	0.67	0.83	0.50	0.50

Table A.11.1: The Dispersion of the Individual Characteristics for the Highest Reward to Variability Ratios

	1990-2005	1990-1998	1998-2000	2000-2003	2003-2005
Low Volatility	0.06	0.82	0.09	0.00	0.92
High Volatility	0.94	0.18	0.91	1.00	0.08
OTM	0.50	0.64	0.58	0.43	0.66
ATM	0.45	0.32	0.38	0.42	0.26
ITM	0.06	0.04	0.04	0.15	0.08
Call	0.48	0.57	0.59	0.52	0.53
Put	0.52	0.43	0.41	0.48	0.47
Short Maturity	0.50	0.51	0.50	0.50	0.50
Long Maturity	0.50	0.49	0.50	0.50	0.50

Table A.11.2: The Dispersion of the Individual Characteristics for the Indistinguishable Reward to Variability Ratios

A.12 Domination Matrixes

	Volatility	Moneyness	Call/Put	Maturity
Volatility	*x*	Moneyness	Volatility	Maturity
Moneyness		*x*	Moneyness	Mixed
Call/Put			*x*	Maturity
Maturity				*x*

Table A.12.1 Domination Matrix for the Expected Return

	Volatility	Moneyness	Call/Put	Maturity
Volatility	*x*	Volatility	Volatility	Volatility
Moneyness		*x*	Mixed	Mixed
Call/Put			*x*	Mixed
Maturity				*x*

Table A.12.2 Domination Matrix for the Risk

	Volatility	Moneyness	Call/Put	Maturity
Volatility	*x*	Mixed	Volatilty	Mixed
Moneyness		*x*	Moneyness	Mixed
Call/Put			*x*	Maturity
Maturity				*x*

Table A.12.3 Domination Matrix for the Reward to Variability Ratio