Bitter Roots that Prevent the Sweet Fruits of Education

Demographic change, tuition subsidies and borrowing constraints in an OLG model with endogenous human capital accumulation

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ABSTRACT In nearly all developed countries, the fraction of elderly in the population will rise considerably during the next four decades. This rise in old-age dependency will have significant ramifications for welfare. Additional investments in education induced by factor price changes could counteract some of the negative welfare effects if markets are efficient. To evaluate this human capital adjustment mechanism, we develop an overlapping generations (OLG) model with endogenous labor supply and Ben-Porath human capital accumulation calibrated for the case of Sweden. In our model, education is not only financed by firms, but also by agents themselves. The government subsidizes tuition and pays out a pay-as-you-go financed pension benefit to the retired. We show that the existence of borrowing constraints can significantly hamper the welfare improving adjustments of investments in education in response to higher old-age dependency. Increasing tuition subsidies does not solve the problem of a binding constraint on credit. Rather than alleviating the borrowing constraint, higher subsidies make it more binding.

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The title of this thesis is a reference to a phrase attributed to the Greek philosopher Aristotle (384BC–322BC). He said that "the roots of education are bitter, but its fruits are sweet"*. This phrase can be taken to mean that although the process of being educated can be painful at times, the result is more than worth it. In this thesis, we model a market imperfection that can prevent the sweet fruits of education from materializing. Hence the choice of title.*

ACKNOWLEDGEMENTS We would like to express our gratitude to Johanna Wallenius and Thomas Eisensee who, despite the high opportunity costs of their time, stimulated the human capital development of the authors of this thesis through continuous support, helpful comments and good suggestions.

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Introduction

"Education: the inculcation of the incomprehensible
into the indifferent by the incompetent."

- Lord John Maynard Keynes (1883-1946)

Lord Keynes' famous phrase is beside the mark. Education is one of the engines of economic growth. Already during the 1960s, Schultz (1961) was singing the praises of education. He showed that it raised productivity considerably during the first half of the 20th century. Others have pointed out that more education reduces crime rates and improves health[1]; both allow for lower public expenditures. Education even has non-economic benefits such as improved happiness[2] and more civic engagement[3]. In this thesis, we show that yet another benefit can be added to this list of positive effects of education: it can mitigate the negative welfare consequences of a rising fraction of elderly in the population.

We develop an overlapping generations (OLG) model with Ben-Porath type human capital accumulation in which agents choose each period how much to save, how much to work, and how much time to invest in education. Agents enter the model at age 19, when they have completed secondary education. To be able to study the economic effects of rising old-age dependency, we include population distributions into our model. Following a higher fraction of elderly in the population, macro-economic conditions change. To allow for these changing conditions to affect the decisions of agents in our model, wages and interest rates can adjust.

Our model builds on the literature that aims to estimate the welfare effects of rising old-age dependency and examines what policy responses would be optimal to overcome them. In a recent paper, Ludwig, Schelkle and Vogel (2012) show that increases in educational time investment could counteract some of the negative welfare consequences of a higher fraction of elderly in the population of the United States. We take a simplified version of Ludwig et al. (2012)'s model as starting point in this thesis.

We add to the literature by extending Ludwig et al. (2012) analysis in two ways. First of all, Ludwig et al. (2012) model perfectly functioning capital markets. This means that young agents who spend much of their time in training can borrow unlimitedly to smooth consumption and leisure over their lifecycle. In reality, there often is a limit to the amount students can borrow. We introduce borrowing constraints into Ludwig et al. (2012)'s set-up and show how this affects their conclusion that investments in education can counteract the welfare losses from higher old-age dependency.

Secondly, Ludwig et al. (2012) assume educational costs are fully paid for by firms. We include tuition costs into our model, which are partly paid for by economic agents themselves

[1] See, for instance, Mazumder (2008).
[2] See, for instance, Gerdtham and Johannesson (2001).
[3] See chapter VI of Friedman (1962).

and partly through taxes. This allows us to find out whether endogenous human capital accumulation can still counteract the welfare effects of higher old-age dependency when the costs of education enter the budget constraints of agents and the government.

We calibrate our model on data for Sweden. This choice has two reasons. Firstly, this thesis was written in cooperation with the Swedish Ministry of Finance. Secondly, subsidies to tertiary education are quite high in Sweden, making a study of the effects of public policy on educational time investment when the fraction of elderly in the population is rising both more interesting and more relevant.

This thesis is organized as follows. In chapter 1, we present an overview of key statistics on adult education in Europe to show how people structure investments in education over their lifecycle and how the government is involved in the market for education in different European countries. This chapter puts the case of Sweden in a European context. In chapter 2, we will give an overview of the literature on three central topics in this thesis: borrowing constraints, tuition subsidies and increasing old-age dependency. In chapter 3, we will describe our model and then present the results we obtain with it in chapter 4. Chapter 5 contains a discussion of the policy implications of our results and the limitations of our model. Chapter 6 concludes the thesis.

Chapter 1

Human capital accumulation in Europe

"The mere thought of investment in human beings is offensive to some among us."

- Theodore William Schultz (1902-1998)

In this chapter we will start by putting human capital investments in European countries into perspective. We will use the approach of lifelong learning to encompass all aspects of human capital development. With this concept, the Organization for Economic Co-operation and Development (OECD) refers to all learning endeavours over the lifespan, including primary, secondary and tertiary education, as well as on-the-job training efforts. We focus on learning after the age of 18, which typically includes: (1) the last stage of (formal) education at school, the tertiary level, and (2) (non-formal) training on-the-job.

Over the last decade, the attainment of tertiary education has soared in Europe. The amount of time spent on training on-the-job still varies widely across countries. In some countries, on-the-job training is yet in the early stage of development.

Whereas broad consensus exists that a large degree of government involvement is needed in the provision of primary and secondary education, the degree of public involvement in tertiary education differs. Consequently, the ways tertiary education is financed diverge. As on-the-job training typically includes training in job-specific skills, financing this kind of investment is usually seen as a duty of the employer.

So far, much of academic research on human capital has focused on the United States. In tertiary education, the U.S. system is characterized by large private investments and relatively low government involvement. As public investment in tertiary education is generally much higher in Europe, the role for the government and the impact of policy is of greater importance. This is why we focus on European countries in our analysis.

In this chapter we will firstly present trends in attainment and timing of education and training efforts. Secondly, we will analyze the way in which countries finance tertiary education and the role of the government in the financing structure. Lastly, we will look at the case of Sweden and formulate the research goals of this thesis.

1.1 Attainment

As indicated by the OECD, a number of socio-economic factors require learning that is no longer centered at the early ages of the lifecycle, but as a continuous part of life. These factors include the pace of globalization and technological change[1], the changing nature of the labor market, and the aging of populations (Borkowsky, 2013, p. 6). For these reasons, human capital accumulation can nowadays be thought of as a process of lifelong learning, which has both formal (in-school training, resulting in a formal degree), and non-formal (training on-the-job, usually sponsored by the employer) components[2]. Figure 1.1 shows participation in formal education and non-formal on-the-job training for the average of OECD countries. Almost all children attain primary and secondary education. Tertiary education is attended for some years by a majority of the population, but enrollment rates decrease sharply after the age of 20. At the age of 25, about more than half of the population is enrolled in on-the-job training, a participation rate that decreases slowly until retirement[3].

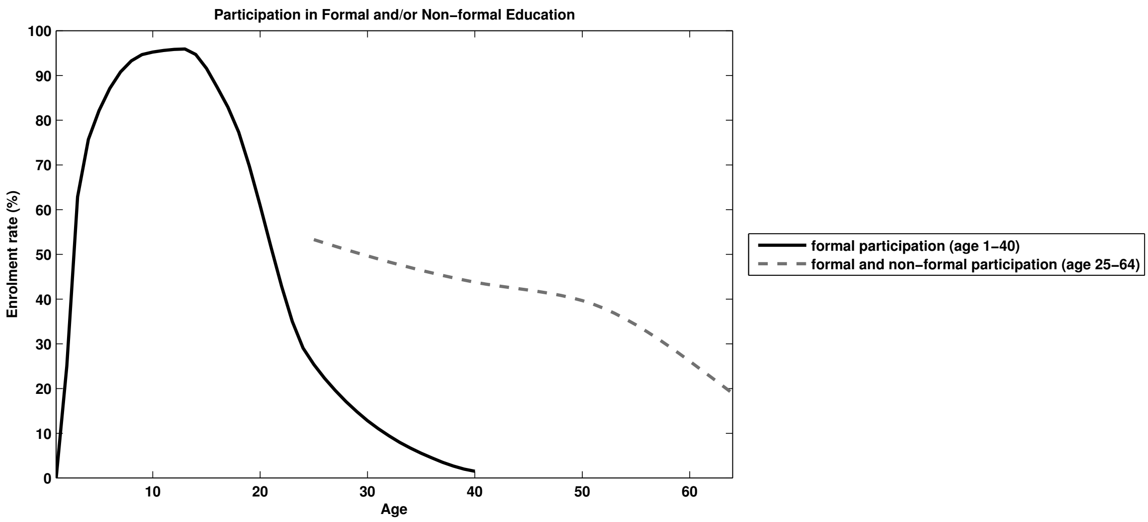

Figure 1.1: Enrollment in public and private formal education institutions of age group 1-40 and participation in formal and/or non-formal education[4] of age group 25-64. Year: 2010.
Source: OECD Education at a Glance 2012: OECD Indicators, table C1.1a and C6.4a.

1.1.1 Rising attainment of tertiary education

The attainment of tertiary education is rapidly increasing in Europe. As part of the Europe 2020 goals, a target is set by the European Union to increase the share of the 30-34 years old having completed tertiary education to at least 40%[5]. As shown in figure 1.2, the European Union is on its way in achieving its goal: during the last decade, the attainment of tertiary education has increased by more than 10 percentage points[6]. The increase is both due to young EU members

[1] Due to globalization, which induces more competition on low skill-intensive goods, and because of skill-biased technological change, production will shift to more skill-intensive goods (Bekman, Bound and Machin, 1998).

[2] The OECD defines **formal education** as "education provided in the system of schools, colleges, universities and other formal educational institutions", and **non-formal education** as "organised and sustained educational activity not fulfilling the definition of formal education, [taking place] both within and outside educational institutions and cater[ed] to individuals of all ages" (OECD, 2012, p. 412).

[3] The dashed line represents both formal training in school and non-formal on-the-job training. However, as enrollment in formal institutions reduces sharply after the age of 20, the largest part of the dashed line represents participation in non-formal training on-the-job.

[4] Interpolated over the 10 year interval data.

[5] European Commission, 2013, retrieved from `http://ec.europa.eu/europe2020/targets/eu-targets/`.

[6] Or 54.5 percent growth on average for the EU27 countries between 2000 and 2011. Especially for the developed countries, the growth in attainment rates is exceptionally high and might in-part result from changes in classification over this period.

such as Poland, where attainment rapidly increased from 14.4 to 36.9 percent, as well as due to established members, such as the Netherlands, in which attainment increased from 26.5 to 41.1 percent over this period.

The increasing trend in tertiary education attainment in European countries will continue. As shown in table 1.1, the intake of students in tertiary education has increased rapidly, which will result in a further rise in the share of the population with a tertiary degree.

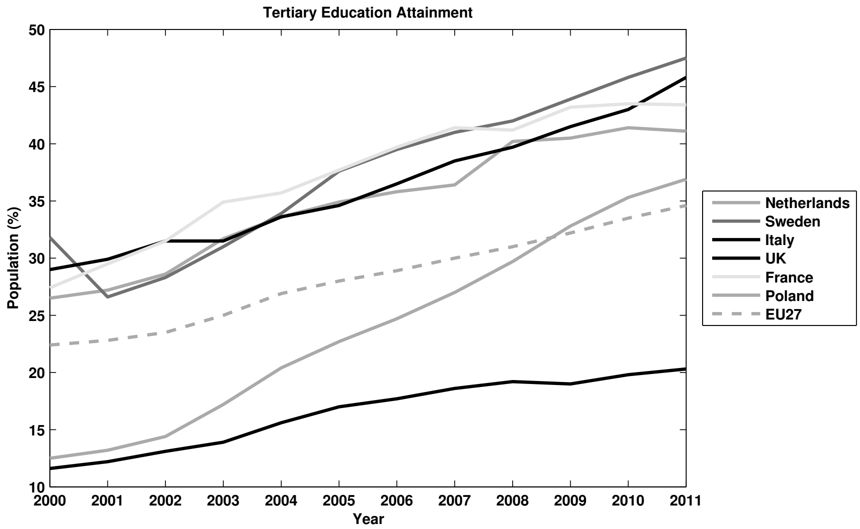

Figure 1.2: Tertiary education attainment of age group 30-34 (ISCED[7] levels 5 and 6). Source: Eurostat, 2013, table code: t2020_41.

	2000	2010	Change
Czech Republic	38,022	84,353	+122%
Austria	32,836	66,459	+102%
United Kingdom	350,172	521,869	+49%
Germany	284,658	417,218	+47%
Sweden	73,471	94,199	+28%
Netherlands	104,978	134,422	+28%
Poland	413,70	477,120	+15%
Italy	278,379	303,533	+9%

Table 1.1: Number of new entrants into tertiary education (ISCED level 5A). Source: Eurostat 2013, table code: educ_entr2tl.

[7]The OECD and Eurostat use the ISCED97 (International Standard Classification of Education) framework to separate education levels. Level 5 is the first stage of tertiary education, where level 5A refers to programs with academic orientation (leading to a BSc or MSc degree), and 5B to typically shorter programs with occupational or vocational orientation (leading to nonacademic Bachelor or Master degrees). Level 6 is the second stage of tertiary education, leading to advanced research qualifications (Ph.D. or doctorate).

1.1.2 Attainment of on-the-job training

On average, 37% of all employees between 25 and 34 years old engage in some form of non-formal on-the-job training (OECD, 2012, p. 420). For the people aged 55 to 64, this participation rate is 23%.

There are large differences between European countries in how common it is to participate in some form of training after education in school. In Sweden, 72% of the people aged 25-34 participate; this is 60% among people aged 55-64. In the Netherlands and Germany, around half of the working-age population aged 25-34 attends regular on-the-job training; this decreases to one third for the group aged 55-64. In countries such as Italy, Greece and Turkey, the percentage of people participating is less than 25% for the young and 12% or lower for the older part of the working-age population.

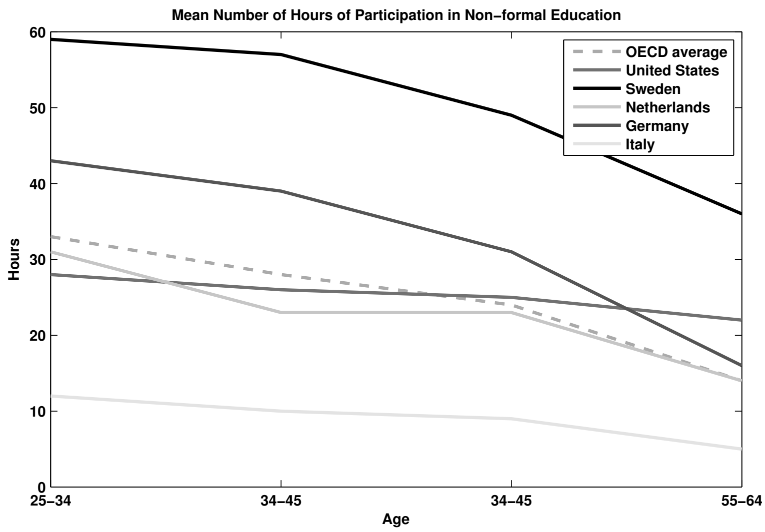

Figure 1.3: Mean number of hours of participation in non-formal education per adult per year. Year: 2007 (United States, Sweden: 2005; Netherlands: 2008).
Source: OECD Education at a Glance 2012: OECD Indicators, table C6.8.

As shown in figure 1.3, the number of hours spent in training is 33 per year for the OECD average adult between age 25 and 34. This average declines to 14 hours for people aged 55 to 64. On average, 25 hours per year are spent on on-the-job training throughout the work-life of workers in OECD countries. Counting a full-time working year as 411 hours, this represents 6.1% of Full-Time Equivalent (FTE) hours. The figure confirms that there are large differences in investments in on-the-job training across countries. Swedes spend 50 hours per year on training during their work-life, whereas Italians spent only 9 hours.

From the hours invested in non-formal training, it is unclear how many hours are spent during the leisure time of the agent and how many within the contractual time. From the training hours, more than 70% is specific to the job. The remainder is general education, for example to update knowledge of the profession.

Given global trends such as globalization and skill-biased technological change that increase the need for lifelong learning, the intensity of on-the-job training can be expected to rise in the future. The OECD sees increasing investment in adult learning and reducing inequality of participation as policy goals for its member states (Borkowsky, 2013, p. 8).

1.2 Timing

While the share of the population graduating from tertiary education is increasing across countries, the timing of studying in the lifecycle varies considerably internationally. Becker (1964, p. 94) points out that returns to human capital investment depend on the age at which they are made. The older agents are, the lower the returns as agents benefit from higher wages for a smaller number of years. Also public returns decrease with later timing of education. Tax revenues decline as there are less working years over which agents pay taxes. If students take longer to finish their studies, subsidy costs to education will be higher as well.

Explaining student median age differences

Figure 1.4 shows that the student body of especially Scandinavian countries consists of very old students, with a median age[8] of over 25. Irish, French and Belgian students attain university at an earlier stage in life, resulting in a low median age of the student body. Between 2003 and 2010, there are no major changes in the median age of students with some exceptions, most notably Greece[9].

Various reasons could account for a different timing of tertiary education. One of them is a different entry age in tertiary education. Figure 1.5 shows the most recent OECD statistics on the median age at which students enroll in tertiary education. The figure makes clear that Scandinavians (with the exception of Norwegians) start their tertiary studies approximately 2 years later than the EU average. This could be the reason why students from Scandinavian countries are so much older. There are various explanations for delayed entry into degree programs. Secondary schooling could take more years in these countries (see figure A.1, appendix), a military or voluntary service has to be fulfilled prior to entering university (see table A.1, appendix), gap years to travel or work are common practice (see Holmlund, Liu and Skans (2008) for an analysis), students might incur a delay when dropping out of university but re-enter within a year (so-called *stop-outs*, see Stratton, O'Toole and Wetzel (2008)), and so on.

Another factor that could affect the median age of students is the duration of degree programs. In 1999, the Bologna Process was started with the aim of reaching a more comparable structure of higher education in Europe and to provide guidelines as to how long the cycle of obtaining a degree in a certain field should take. Still, as shown in figure 1.6, actual duration varies considerably across countries. Partially, this variation can be explained by remaining differences in the prescribed duration of degree programs outside of the Bologna Accords. However, for many students, the time to obtain a degree exceeds the prescribed completion time largely. To our knowledge, no data is available that decomposes study time for various countries. Such data would help to establish what part of time-to-degree is a delay from prescribed program duration. Nevertheless, there is a small collection of studies providing empirical evidence showing that delays are large[10].

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[8] Figure 1.4 presents the median rather than average age of students in order to avoid upward bias from adults (re)entering into degree programs.

[9] A possible explanation for the rise in median age in Greece is the fact that the country has the highest unemployment rate among graduates (aged 20-29) with a tertiary degree in Europe (13% in 2009, Source: OECD Education at a Glance 2011: OECD Indicators, table C4.3). Over the last decade, there has been a steep increase in attainment of tertiary education in Greece, but as noted by the OECD, the Greek economy has not yet shifted towards a more knowledge-based model. This means that opportunities for graduates are low. As studying is free in Greece and comes with student benefits, a large number of almost finished students is likely to stay in education while looking for work, which explains the substantial increase in the median age of students.

[10] Garibaldi, Giavazzi, Ichino and Rettore (2006) report that of all Italian students in the year 1999-2000, 41.1% already exceeded the official duration of their degree. Of all students graduating in that year, even 83.3% can be defined as *fuoricorso* –out of track– students with a delay from the nominal study duration. Using 1998 data, the paper reports an average effective time-to-degree of around 7.41 years for an average prescribed duration

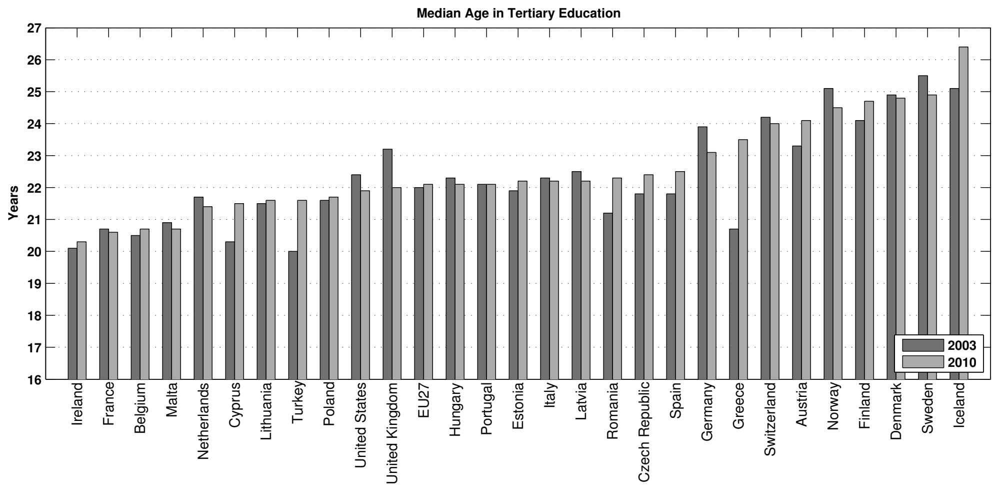


Figure 1.4: Median age of students in tertiary education (ISCED levels 5 and 6) in 2005 and 2010. Source: Eurostat, 2013, table code: tps00061.

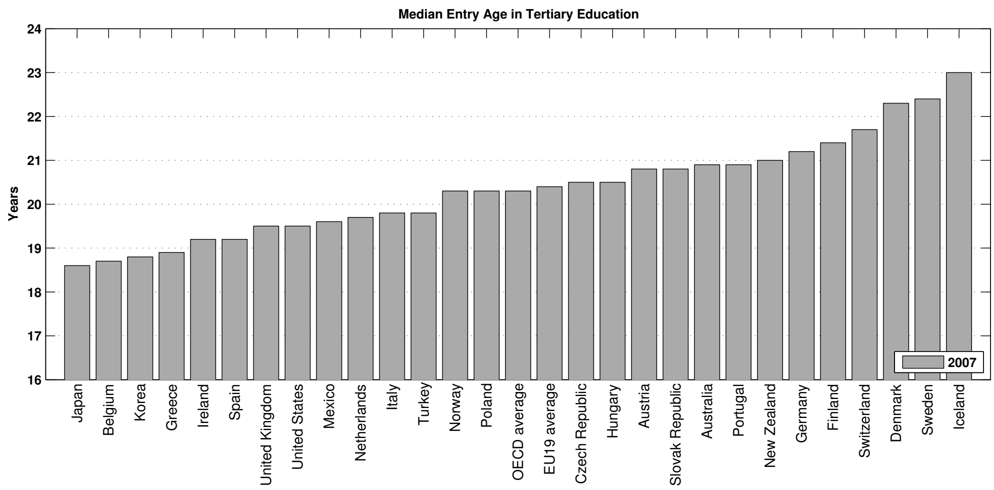


Figure 1.5: Median age of entry into tertiary education (5A level) in 2007.
Source: OECD Education at a Glance 2009: OECD Indicators, table A2.4.

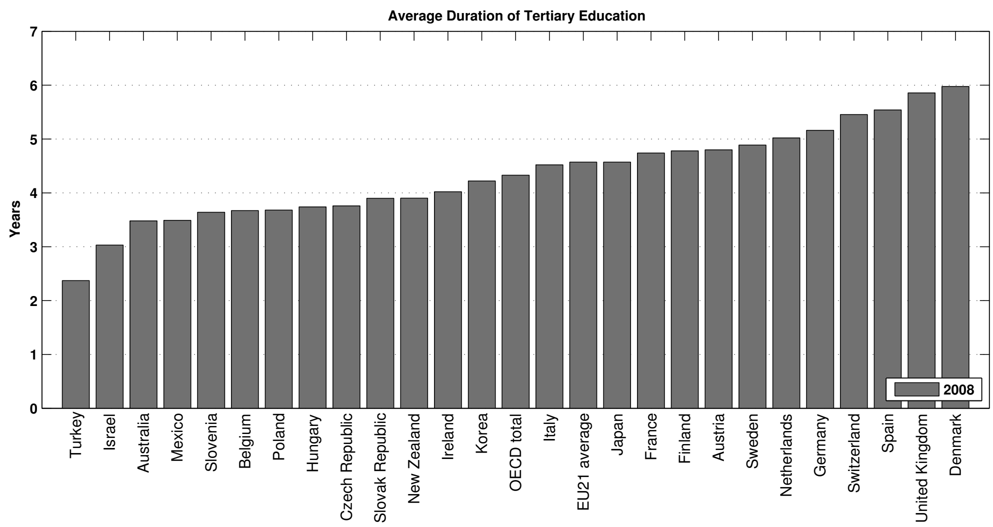


Figure 1.6: Average duration of tertiary studies (5A level) in 2008.
Source: OECD Education at a Glance 2012: OECD Indicators, table B1.3a.

1.3 Financial aspects

The key driver of the decision to obtain education is the prospective of an increase in future earnings. Especially a tertiary degree generates significant benefits (see figures 1.7 and 1.8), which motivates individuals to postpone employment today for future rewards. The two main costs of education are (1) tuition and (2) forgone wage. The overall return to a degree is generally found to be of importance to the share of the population that enters tertiary education (e.g. Flannery and Odonoghue, 2009; Lauer, 2002; Heckman, Lochner and Todd, 2006).

Benefits are not exclusive to the individual: there are social gains from tertiary education. Becker (1964, p. 209-211) distinguishes two social gains and two social costs to education; the gains being (1) increased tax revenue and (2) higher growth in output because knowledge can increase productivity, and the costs (1) educational subsidies and (2) forgone tax revenue from people enrolled in education programs.

1.3.1 The returns to education

"In addition to monetary returns, it is also possible that education creates non-pecuniary benefits in terms of more pleasant or interesting jobs, lower unemployment risk, higher social prestige or even better chances at the marriage market."

- Uusitalo (2011, p. 205)

The returns to a tertiary education degree vary largely across countries. The OECD uses the method of net present value (NPV) to estimate the returns of a degree to the individual. This method subtracts the costs of education and forgone earnings from the net earnings benefit of an agent. The net earnings benefit consists of the earnings premium minus additional taxes paid in a (progressive) tax system, and higher contributions to social security. The benefit of reduced unemployment risk is also taken into account. A similar method is applied to estimate the public returns. Direct cost of education and foregone taxes on government revenues are subtracted from additional income tax revenues and social contributions.

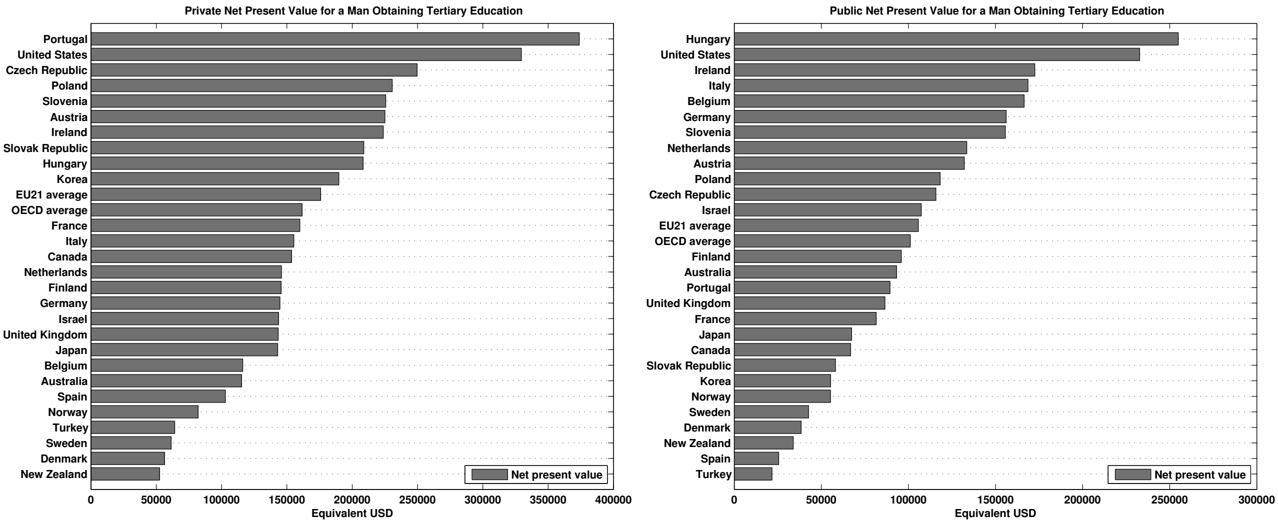
The OECD estimates of the private and the social returns to schooling show that with the current level of educational investment, the private NPV of a tertiary education degree is about 3 times higher than the public NPV. However, one should keep in mind that the NPV method underestimates the true social returns. The productivity growth resulting from education (see Schultz, 1961, p. 13) is not included in the public NPV.

of 4.39 years.

The Statistics Agency of the Netherlands reports an average study time of university (academic) students of about 5 years to obtain a bachelor's degree (source: Statistics Agency of the Netherlands (CBS), 2013, table "Average study duration of graduates, WO"). Taking into account that bachelor programs take either 3 or 4 years in the Netherlands, the delay is significant.

By using a survey method rather than university entry and exit data, Brunello and Winter-Ebmer (2003) analyzed students' delay in obtaining a degree in 10 European countries. They asked whether students were expecting to complete their first degree at least 1 year later than the official curriculum duration. In Italy and Sweden, over 30% of students expected a delay of more than one year. 17.1% of French and 10% of the German students expected to be delayed and of the Swiss and Portuguese students, 3.5% and 4.6% were expecting this delay, respectively.

Private and public net present value



(a) Private NPV

(b) Public NPV

Figure 1.7: The private and public net present value for a man obtaining a tertiary education degree as part of initial education across OECD countries (2008 or latest available year)[11].
Source: OECD Education at a Glance 2012: OECD Indicators, table A9.3 and table A9.4.

In figures 1.7a and 1.7b the private and public net present value for a man obtaining tertiary education is given (in equivalent U.S. dollars) as estimated by the OECD[12]. Generally, the benefits of tertiary education are large. The sum of the private and public returns of a man who completes a tertiary education degree is \$388,300 (€239,037) on average. For a woman, this return is \$250,700 (€193,068). Gross of taxes, the earnings premium of a tertiary diploma exceeds \$340,000 (€261,841) for men and \$235,000 (€180,978) for women (OECD, 2012, p. 162).

The differences between countries are striking. Private returns are above \$300,000 (€231,035) in the U.S. and Portugal. The list is closed by Turkey, Sweden, Denmark and New Zealand who have private returns below \$75,000 (€57,759). Public NPV is above \$200,000 (€154,024) in Hungary and the United States and below \$50,000 (€38,506) in Sweden, Denmark, New Zealand, Spain and Turkey. A decomposition of the elements of public and private NPV is given in appendix A.3 to this thesis.

Figure 1.8 shows the relative earnings of someone with a tertiary degree compared to someone with only secondary or non-tertiary post-secondary education. From the figure, it becomes clear that there are large differences in the earnings premium of tertiary education across countries. Typically, the countries with an extensive welfare system, such as the Scandinavian countries, have a low earnings premium. Countries that recently were still developing countries, such as Hungary and Slovenia, show very high earnings premia of tertiary education. A liberal economy such as the U.S. has a very high earnings premium too, which corresponds to the large private returns to a degree as shown in figure 1.7a.

[11] 2005: Belgium, Australia, Turkey; 2006: Portugal; 2007: Japan, Slovenia.

[12] An explanation of the NPV calculation method used by the OECD can be found in: OECD Education at a Glance 2012: OECD Indicators, box A9.1, p. 167.

[13] 2009: Australia, Canada, Finland, Portugal, Spain; 2008: France, Italy, the Netherlands.

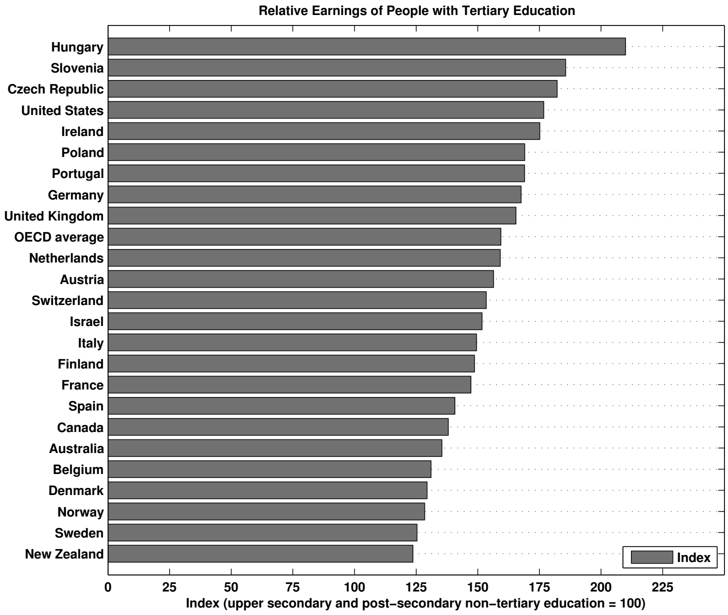


Figure 1.8: Index of earnings of people (25-64 year-olds) with a tertiary education degree relative to people with an upper secondary or post-secondary non-tertiary degree (2010 or latest available year[13]). Source: OECD Education at a Glance 2012: OECD Indicators, table A8.2a.

1.3.2 The case for government involvement

The government is heavily involved in the market for education in Europe and the United States. Primary and secondary education are compulsory. Consequently, governments in these countries make them available free - or very low - of charge. Tertiary education is not compulsory, however. Therefore, government intervention is not imperative. Still, nearly every government in the world does intervene in the tertiary education market. The reason is that individual decisions on tertiary education are not socially optimal absent government intervention. Uusitalo (2011, p. 206) explains that government intervention in the market for tertiary education is *not* necessary if three conditions hold. First of all, schooling decisions should be made rationally and solely based on "expected monetary returns". Secondly, agents should receive all benefits of education and pay all costs themselves, there should be no social returns or costs. Thirdly, credit constraints that affect the enrollment decision of agents should be absent. In reality, these three conditions do not hold. As a result, government intervention in the market for tertiary education is required.

The public share in the cost of higher education

In an ideal world, the government would invest in education until the marginal social returns to education are equal to the marginal social costs of it. Yet, as the true social costs and returns are hard to assess we can only conjecture about the optimal subsidy. According to table 1.2, large differences exist across countries in the extent public money is used to finance tertiary education. The Scandinavian countries are one extreme, financing a high share of the cost of tertiary education publicly. At the other end, the Anglo-Saxon countries have large private investments but limited public funding. In general, it is more common in Europe than in developed countries in other parts of the world to fund tertiary education publicly as can be inferred from comparing the OECD average public share in the cost of education of 68% to the EU21 average of 81% (OECD, 2011, p. 230).

	Public	Private	Public share		Public	Private	Public share
Norway	1.62	0.05	97%	Czech Rep.	0.94	0.24	80%
Finland	1.62	0.07	96%	Italy	0.75	0.21	78%
Denmark	1.61	0.07	96%	Netherlands	1.11	0.40	73%
Belgium	1.28	0.09	94%	Poland	1.03	0.45	70%
Austria	1.23	0.09	93%	Portugal	0.85	0.49	63%
Iceland	1.16	0.10	92%	U.K.	0.63	0.56	53%
Sweden	1.42	0.17	89%	Australia	0.67	0.82	45%
Estonia	1.14	0.17	87%	U.S.	1.00	1.68	37%
Germany	1.00	0.17	86%				
France	1.20	0.22	84%	**OECD avg.**	1.04	0.49	68%
Spain	0.98	0.24	81%	**EU21 avg.**	1.08	0.24	81%

Table 1.2: Public and private investment in tertiary education as % of GDP (2008). The right columns display public investment as a share of total investment[14].
Source: OECD Education at a Glance 2011: OECD Indicators, table B2.3.

Rising public investment

With increasing numbers of students, costs of tertiary education for the government rise. In the European Union countries, the average public expenditure on tertiary education has increased from 1.08 to 1.22 percent of GDP between 2001 and 2009, a rise of 13% (Eurostat, 2013, table code educ_figdp). Several countries recently took steps to decrease the public cost of tertiary education. In 2012, the U.K. increased the upper tier on tuition fees from £3,290 (€3,896) to a maximum of £9000 (€10,659) per year[15]. Sweden does not have fees for EU students, but does charge non-EU students fees of up to 140,000 SEK (€16,400) since the academic year 2011[16].

A potential way to reduce costs of tertiary education would be to increase the efficiency with which students finish their degree. As indicated in section 1.2, reducing the time-to-degree of students could increase the social return of studies. The Netherlands has tried to implement a fine for students who were delayed with more than one year over the prescribed duration of their program. However, it was believed that this measure disproportionally harmed students who had good reasons to delay disproportionally. Soon after its introduction, the measure was withdrawn. Now, the Netherlands aims to increase the pace in which students finish their degree by abolishing the student grant system altogether[17].

Although many countries are implementing austerity measures on educational subsidies, large differences remain in the amount of tuition, the availability of grants and the size of other subsidies to education across countries in Europe. Figure 1.9 shows the average tuition fees and access to grants in the academic year 2008/2009 at public tertiary institutions. The Scandinavian countries do not ask tuition fees and provide extensive access to grants and subsidized loans for students. The Anglo-Saxon countries are characterized by very high fees, but generally decent

[14] Calculated as: public investment divided by the sum of public and private investment in tertiary education.
[15] Source: BBC, *Students face tuition fees rising to £9,000*, 3 November 2010, retrieved from http://www.bbc.co.uk/news/education-11677862.
[16] Source: Stockholm University, *Tuition fees for non-EU/EEA students*, 18 December 2012, retrieved from http://www.su.se/english/study/2.604/tuition-fees-for-non-eu-eea-students-1.2113.
[17] Source: NRC Handelsblad, *Dag langstudeerboete, welkom sociaal leenstelsel - 4 vragen en antwoorden*, 10 October 2012, retrieved from http://www.nrc.nl/nieuws/2012/10/10/dag-langstudeerboete-welkom-sociaal-leenstelsel-vier-vragen/.

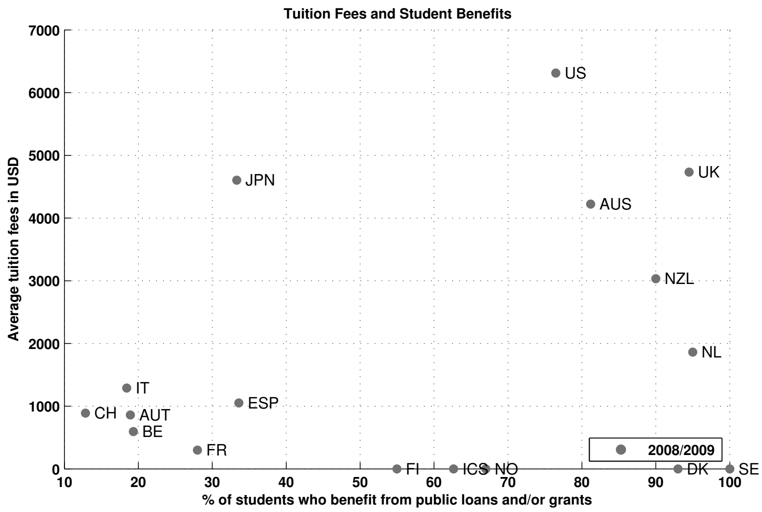


Figure 1.9: Average tuition fees charged by public institutions and proportion of students who benefit from public loans and/or grants (academic year 2008-09).
Source: Education at a Glance 2012: OECD Indicators, chart B5.1.

access to support mechanisms. The Netherlands asks moderate fees but provides basic grants to almost all students. The central and southern European countries subsidize institutions in order to maintain low tuition fees, but generally provide only low levels of financial support to students.

1.3.3 To make ends meet

The investment in tertiary education not subsidized by the government should come from private sources. Typically, those are (1) transfers from parents to their children, (2) a part-time job while studying, and (3) borrowing on the private market. In practice, borrowing on the private market to finance education is difficult. Human capital cannot function as collateral to back loans for investments in education (Becker, 1965, p. 93). Moreover, the returns on human capital investment are generally uncertain (Becker, 1965, p. 91).

As borrowing on the private market is often difficult for students and limited data on student borrowing in the private market in Europe is available, we will exclusively deal with the degree to which parents and jobs contribute to funding education costs here.

Parental transfers

In the United States, where governmental contributions to tertiary education are relatively small, there is a large role for parents in financing college for their children. Keane and Wolpin (2001, p. 1076-1077) estimate the cost of college to $3,673 (€ 2,829) per semester of 4,8 months. Their projections show that even the least wealthy parents in the U.S., as measured by their schooling level, fund at lease half of the college costs incurred by their children. Parents who attainted the highest level of education themselves even transfer almost twice the cost of college to their children. On average, the additional parental transfer contingent on the college attendance of

Total income	Austria		France		Finland		Germany	
Parental transfers	€380	36.0%	€538	51.6%	€86	7.4%	€416	49.0%
Government	€113	10.6%	€257	24.7%	€387	33.0%	€129	15.2%
Job	€447	42.3%	€247	34.7%	€583	49.7%	€236	27.7%
Other	€116	11.0%	€0	0%	€116	9.9%	€69	8.1%
Total	**€1,056**	100%	**€1,042**	100%	**€1,172**	100%	**€850**	100%

Total income	Netherlands		Norway		Sweden		England	
Parental transfers	€184	23.3%	1,523 kr	10.7%	736 kr	7.3%	£ 235	18.5%
Government	€360	45.5%	4,536 kr	31.7%	5,700 kr	56.6%	£ 547	43.0%
Job	€192	24.3%	7,084 kr	49.6%	2,027 kr	20.1%	£ 356	28.0%
Other	€55	7.0%	1,152 kr	8.1%	1,609 kr	16.0%	£ 133	10.5%
Total	**€791**	100%	**14,296 kr**[1]	100%	**10,072 kr**[2]	100%	**£ 1,271**[3]	100%

Table 1.3: Composition of monthly income of students not living with their parents (rounded averages, local currency). Source: Eurostudent IV (2008-2011), HIS – Institute for Research on Higher Education. [1]€1,895; [2]€1,180; [3]€1,505

their offspring is $3,673 (€2,829) per semester, which is $765 (€589) per month[18,19]. As shown in table 1.3, there is no European country in which parents contribute such a large amount to the tertiary education of their children[20]. In the Scandinavian countries parents contribute 10% or less to the budget of students. In the Netherlands, Austria and the U.K., parental transfers make up between 18.5% and 36% of the income of students. In Germany and France, where education is heavily subsidized but no basic grants are given, this is about 50%.

Part-time employment while studying

Another important source of income for students is performing part-time work while studying. In countries with low public subsidies to education, having a part-time job could be necessary in order to fund tuition costs and maintain a minimum standard of living while studying.

Table 1.4 shows that paying tuition fees or covering living costs is most probably not the only reason for students to work while studying, especially in Europe. In countries that have generous grant and subsidized loan systems (in particular the Netherlands, the Scandinavian countries, Austria and Switzerland), a high share of students work during their studies. A possible explanation for this part-time employment could be that students want to obtain useful professional experience, which will have a positive effect on their after study wage. Häkkinen (2006) performs one of the few quantitative analyses on work while enrolled in tertiary education. Using his longitudinal data set of Finnish students, he finds that work while studying increases

[18] Parents with children not attending college transfer on average $1,940 (€1,494) per semester. Parents whose children do attend college transfer an additional $3,670 (€2,826) on average. The additional transfer varies between $2,037 (€1,569) and $6,276 (€4,833) depending on the schooling level of the parents.

[19] It is difficult to estimate the size of parental transfers in the United States. Keane and Wolpin (2001, p. 1054) infer the size of parental transfers to students in the United States from the 1979 National Longitudinal Survey of Youth cohort. The National Center for Education Statistics (NCES) in the United States also reports that parental transfers to students can be expected to be large. The Expected Family Contribution, a statistic used to determine the eligibility of students for financial aid in the U.S. (see `http://www.fafsa.ed.gov/help/fftoc01g.htm`) was on average $10,200 (€7,855) in the United States for the academic year 2008-2009 (see NCES, 2010, `http://nces.ed.gov/pubs2010/2010162.pdf`). This statistic can be interpreted as how much a student's family can be expected to contribute to their child's college education.

[20] Thanks to the survey Eurostudent IV (2008-2011), it is easier to determine the size of parental transfers in European countries compared to the U.S. estimates.

Slovenia	77.7%	Estonia	54.7%
Netherlands	71.9%	Turkey	52.7%
Iceland	71.1%	Ireland	48.4%
Finland	69.8%	Norway	47.6%
Denmark	68.2%	Sweden	41.1%
Austria	68.1%	Spain	35.6%
Switzerland	65.7%	Hungary	35.6%
United States	59.0%	Portugal	34.9%
Germany	58.8%	France	26.3%
United Kingdom	57.4%		
Poland	56.0%	**OECD average**	52.3%
Canada	54.7%	**EU21 average**	47.2%

Table 1.4: Percentage of students (tertiary level 5/6) employed while enrolled.
Source: own calculations based on Education at a Glance 2012: OECD Indicators, table C5.2.

earnings one year after graduation, but the positive effect vanishes quickly in subsequent years[21]. Given that the students in his sample who worked next to studying typically took longer to finish their degree, he does not find a significant return to student employment.

If we assume returns to student jobs to be low, what is it that makes students work in such high numbers? One possibility is that students want to increase consumption during the study period. Another potential reason, noted by Uusitalo (2011) could be that highly progressive tax systems make it beneficial to work and study in parallel for many years to remain in low tax brackets. It is also possible that countries with high education subsidies are also the countries with good employment opportunities for students, making it easier to find a job. The study of Messer and Wolter (2010) provide empirical evidence for this claim by showing that low unemployment rates in Switzerland increased the probability that students engaged in paid work between 1981 and 2001.

1.3.4 Who bears the cost of on-the-job training?

The sections above have focused on returns and costs of tertiary education. Of course, adult learning by means of on-the-job training also costs tuition, as well as the opportunity costs of forgone work and leisure time. At present, the OECD does not have data on direct expenditures on adult education (OECD, 2012, p. 414), neither on the opportunity costs of leisure time devoted to training.

The OECD does estimate total cost of working time devoted to non-formal education[22]. Across OECD countries, time worth 0.4% of GDP was invested in this kind of training in 2007, ranging from 0.5% or more in Denmark, Germany, Norway and the United Kingdom to less than 0.2% in Greece, Hungary and Italy (OECD, 2012, p. 414). As indicated previously, the majority of on-the-job training is specific to a particular job, which is why most training time takes place during working time. There is no evidence for large government subsidies to on-the-job training, nor for large spending of individuals on this type of training. For these reasons, it can be assumed that the majority of on-the-job training costs are borne by the employer.

[21] Häkkinen (2006) uses local unemployment rate during enrollment in university as an instrument for work experience. With this instrument, he eliminates bias in post-graduation wages from individual ability characteristics, isolating the income effect of work experience while studying.

[22] The OECD refers to training investments executed in the working time of the employer as employer-sponsored non-formal training (OECD, 2012, p. 415).

1.4 The case of Sweden

This thesis focuses on Sweden. With respect to the data presented in this section, Sweden is a particular and therefore interesting example in several ways.

Firstly, Sweden,together with Denmark and New Zealand, belongs to the countries with the lowest private net present value of a tertiary degree, amounting to only $61,454 (€47,327) (see figure 1.7a). With 6.4%, the rate of return to a degree investment is less than half of the OECD average. Also, public NPV is among the lowest of the OECD countries, amounting to $42,683 (€32,871) (see figure 1.7b) or a mere 5.1% rate of return.

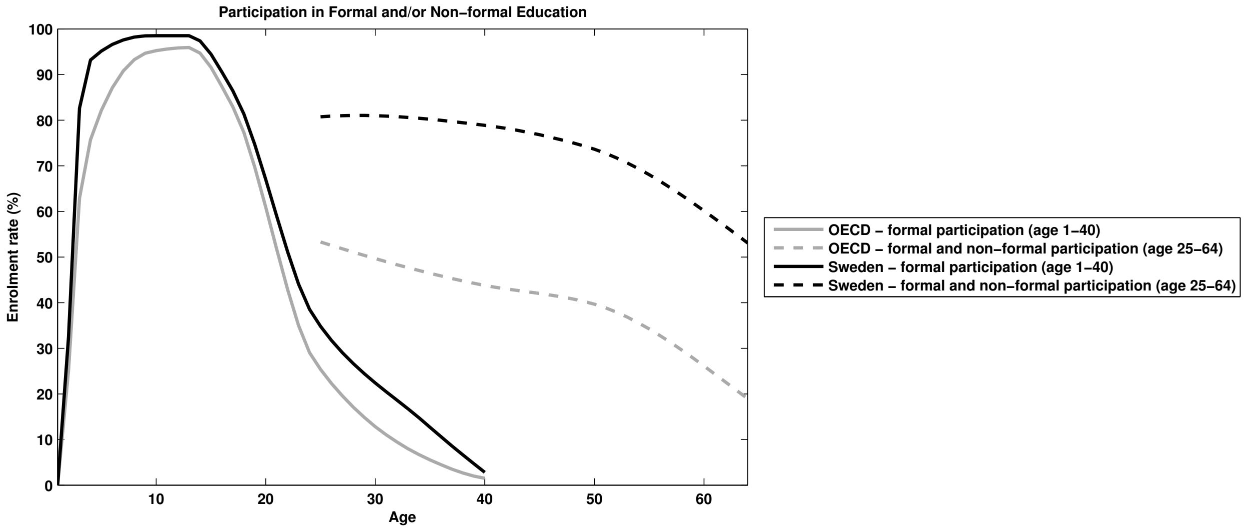


Figure 1.10: Enrollment in formal education of age group 1-40 (2010) and enrollment in formal and/or non-formal education[23] of age group 25-64.
Source: Education at a Glance 2012: OECD Indicators, table C1.1a and C6.4a

Secondly, as shown in figure 1.10, the participation in education is generally much higher in Sweden than the OECD average. Almost all children attend primary and secondary education. More than 50% of Swedes participate in formal education until the age of 22. Involvement in on-the-job training is high as well. With a participation rate of over 80%, enrollment in on-the-job training in Sweden even is the highest of all OECD countries. On average, 12.2% of FTE is spent on training, double the OECD average (see figure 1.3). The Swedish population is highly educated and, consequently, the stock of human capital in the country is sizable.

Thirdly, the Swedish system of tertiary education is characterized by very high government involvement. The public share in the total investment in tertiary education is 89%. All education is free of tuition. In addition, students - who can make use of the financial aid system until they are 54 years old - benefit from a monthly grant of about 2,950 SEK (€345) and can borrow 5,890 SEK (€690) from the government each month during 10 months of the year[24]. Last year, 900 000 students received financial aid, which includes virtually all students (see figure 1.9). All in all, this study support system is costly. Total costs are estimated to equal 22 billion SEK (€2.6 billion) in 2012[24].

Fourthly, parental transfers are low in Sweden compared to other European countries (see table 1.3). Income of students not coming from government grants is earned in part-time jobs or borrowed with subsidized loans.

[23]Interpolated over the 10 year interval data.
[24] Source: Regeringskansliet, 2013, retrieved from `http://www.government.se/sb/d/2098/a/69849`.

Lastly, Swedish students attend education late in life. Only in Iceland the median age of enrolled students is higher than in Sweden. The age at which Swedish students enter tertiary education is, with 22 years, far above the European average. Before entering, taking a few gap years is common in Sweden (Holmlund et al., 2008).

Potentially, there are relations between the high participation in education, the generous education subsidy of the Swedish government, the late timing of education, and the very low earnings premium of a tertiary degree in Sweden. As empirical analyses mostly deliver correlations which are no evidence of causation[25], we will use a simplified economic model approach to isolate the specific economic mechanisms. In this thesis, we calibrate our model to match the particular case of Sweden with low returns to education, a rather high public share in the cost of education, and a large educational time investment. We will abstract away from parental transfers, as they are low in Sweden.

We will show how variations in the high public share in the cost of education change macroeconomic conditions in the country and affect individual time and resource allocation decisions. Moreover, we will show how human capital accumulation in Sweden is affected by aging, as rigorous demographic changes are expected for the next decades. Lastly, we will analyze how borrowing constraints could impact this adjustment channel.

[25]As noted by Rogerson (2006, p. 369-370).

Chapter 2

Theoretical framework

"I love to talk about nothing. It is the only thing I know about."

- Oscar Wilde (1854-1900)

A myriad of factors determines how much education high school graduates obtain. To give just some examples, the education choice of high school graduates depends on the amount of subsidy they get to pay for their education, the extent in which they are borrowing constrained, the wage they receive when working next to their studies, and the wage premium they get from education. With empirical analysis alone, it is difficult to disentangle the puzzle of potential determinants of human capital accumulation. Models can help.

This chapter has three aims. Firstly, to define human capital and explain how it can be modeled economically with the Ben-Porath framework. Secondly, to give a brief overview of previous work on borrowing constraints, fiscal policy and rising old-age dependency; the three topics that are the core of this thesis. Thirdly, to explain the contribution of our work to the literature.

2.1 Defining Human Capital

Human capital is "any stock of knowledge or characteristics [a] worker has (either innate or acquired) that contributes to his or her productivity" (Acemoglu and Autor, 2012, p. 3). Human capital is thus much more than only schooling; it includes learning on-the-job, or indeed all kinds of learning that raise productivity. Investments in human capital, then, can be defined as "the accumulation of a future income stream through the investment of time [and usually money] to provide for higher income in the future" (Davies and Whalley, 1991, p. 165).

Mincer (1958), Schultz (1961) and Becker (1964) are pioneers in the neoclassical literature on human capital. Especially Becker (1964)'s contribution was controversial at the time. The reason? He argued that human capital closely resembles physical capital[1], a revolutionary thought back then. As a capital good in its own right, human capital could account for much of the unexplained part of increases in income in the United States[2].

Human capital evolved into an integral part of many models in labor economics. To understand the research our model builds on, two points require elaboration. First of all, what sets

[1]Becker (1964) doubted whether he should give his book the title "Human Capital" as it could significantly decrease book sales. In the end, he kept the title and used a very long subtitle with the word "education" in it to compensate.

[2]Schultz said during his presidential address at an annual meeting of the American Economic Assocation in 1961 that between 36 and 70 percent of this unexplained change in income could be interpreted as returns on investments in human capital (Schultz, 1961, p. 13).

human capital apart from physical capital? If human capital is a capital good, why would it require a different treatment in models than physical capital at all? Secondly, human capital is built up through both training at school and at work. How can this lifelong learning be approximated in models?

Human versus physical capital

The skills of human beings can be thought of as a capital good. Despite similarities, human capital cannot simply be studied as physical capital. Three distinctions are important. Firstly, investments in human capital do not only cost money[3], but also time. Investments in physical capital usually do not require time investments. Secondly, human capital is one of the most illiquid investments imaginable. It is not transferable across individuals, whereas physical capital can generally be sold. Thirdly, human capital is "synergistic" (Carneiro and Heckman, 2003, p. 90). Human capital already obtained increases returns to future human capital investments; learning facilitates learning. Previous investments in physical capital do not necessarily increase the returns to future investments.

Schooling + on-the-job training = on-the-job training

Human capital is developed at school and on-the-job. Schooling is the extensive margin of human capital investment; it affects wage level after graduation. On-the-job training is the intensive margin. It determines the shape of wage profiles (Heckman, Lochner and Taber, 1998, p. 2). The two are obviously linked. More schooling generally leads to higher returns to on-the-job training. Often, both are needed: almost every profession requires a mix of education received in school and on-the-job. Think of the finance student who cannot suddenly start trading derivatives on her own without having gone through a traineeship to get to know the ropes of the job.

Becker (1964, p. 53) argues that schooling and on-the-job training are economically similar activities and therefore do not need to be modeled separately. On-the-job training requires an investment of time and money in exchange for wage-increasing raises in human capital. So does schooling. Since the important dynamics that govern investment decisions in on-the-job training pertain to schooling as well, results from studying on-the-job training can be generalized to schooling. No need to study both.

A separate schooling period

Becker's idea to approximate lifelong learning by on-the-job training found a good hearing in future research. Ben-Porath (1967)'s seminal model of on-the-job training[4] has been used in many studies and is still regarded as a good way to endogenize human capital accumulation in economic models (see Davies and Whalley, 1991; Heckman et al., 1998, 1999; Ludwig et al., 2012). Nevertheless, some recent papers specifically model a schooling period to do policy analysis (see Keane and Wolpin, 2001; Garriga and Keightley, 2006; Kindermann, 2009).

Explicitly modeling a schooling periods allows one to study factors that specifically affect schooling decisions and are only of minor importance to decisions on on-the-job training when on-the-job training is fully paid for by firms. Some examples are parental transfers (Keane and Wolpin, 2001) and uncertainty about returns to a degree (Kindermann, 2009) or college ability (Garriga and Keightley, 2006). As none of these topics is the focus of this thesis, we do not include a schooling period in our model.

[3] At the very least the economic cost of forgone earnings.

[4] See section 2.3.1.

2.2 Simulation analysis and human capital

Policy analysis in economics was for a long time based on a comparison of analytically derived steady states. Auerbach and Kotlikoff (1987)'s book on *dynamic* fiscal policy changed this reliance on static equilibria. They introduced simulation analysis; the analysis of changes in economic variables over time following policy changes (Auerbach and Kotlikoff, 1987, see p. 4-13)[5].

The basic set-up in Auerbach and Kotlikoff (1987) is well-known. Households optimize their consumption according to Modigliani and Brumberg (2005)'s[6] lifecycle hypothesis of savings rule - smoothing their consumption over their lifecycle and borrowing to do so if required (Auerbach and Kotlikoff, 1987, p. 10). The private sector is modeled as one representative firm which uses capital and labor as inputs to production. The government levies taxes to "provide general public services" and to fund social security (1987, p. 11): it has to intertemporally balance its budget.

Auerbach and Kotlikoff (1987, p. 10) assume agents to be rational. Agents know what the economic future will bring and act accordingly. Assuming rational expectations is essential in the calculation of welfare effects of policy changes as otherwise "one cannot separate the effects of the policy itself from the effect of irrational household behavior" (Auerbach and Kotlikoff, 1987, p. 9). Auerbach and Kotlikoff find this rationality assumption realistic, as deviations from the rational expectations hypothesis are likely to balance out.

Davies and Whalley (1991) were the first to incorporate endogenous human capital accumulation into the Auerbach and Kotlikoff (1987) framework. We will now move on to explain how human capital is typically modeled in the labor markets literature.

2.3 Modeling human capital accumulation

There are two main ways to incorporate human capital accumulation in economic models. Either it is modeled as learning by doing, or as Ben-Porath type on-the-job training. The learning by doing approach is simple, growth in human capital constitutes a concomitant of labor supply. Ben-Porath type on-the-job training, on the other hand, requires the investment of time that could have been allocated to labor supply[7] to achieve raises in productivity. We use a Ben-Porath type human capital accumulation function in our model and thus deem it worthwhile to present the basic dynamics in the Ben-Porath framework.

2.3.1 Ben-Porath on-the-job training

Ben-Porath (1967) developed his influential model of on-the-job training to match observed patterns of lifecycle earnings for white males in the United States. From age 18 on, earnings in this group are typically close to zero during a short period, then rise consistently, to finally decline again some years before retirement (Ben-Porath, 1967).

[5]An important advantage of being able to analyze transition dynamics is that through numerical simulations, intergenerational redistribution can be analyzed; this allows for a much broader range of policy analysis involving time. In chapter 5 of their book for instance, Auerbach and Kotlikoff explain that pre-announcing (1, 5, 10 or 20 years in advance) a switch to consumption and wage tax has more negative consequences on the current young through a crowding out of capital, than a direct implementation of the tax change.

[6]Originally published in 1954.

[7]Or, in models that also include leisure, the enjoyment of free time.

Set-up

In the set-up of Ben-Porath, agents derive utility only from consumption. Their wage depends on their level of human capital. This level is determined by: (1) their initial amount of human capital, (2) their past investments in on-the job-training, and (3) the rate at which human capital depreciates. Agents have a fixed time endowment each period, to be allocated between work and on-the-job training. How much time to spend on training is determined by solving a simple cost-benefit problem; balancing the benefit of increased future earnings resulting from additional training to the costs of (1) forgone earnings and (2) market inputs into human capital production. Note that these market inputs often do not enter the budget constraint of economic agents in models in the labor economics literature because they are assumed to be inter firm transactions.

Intuition

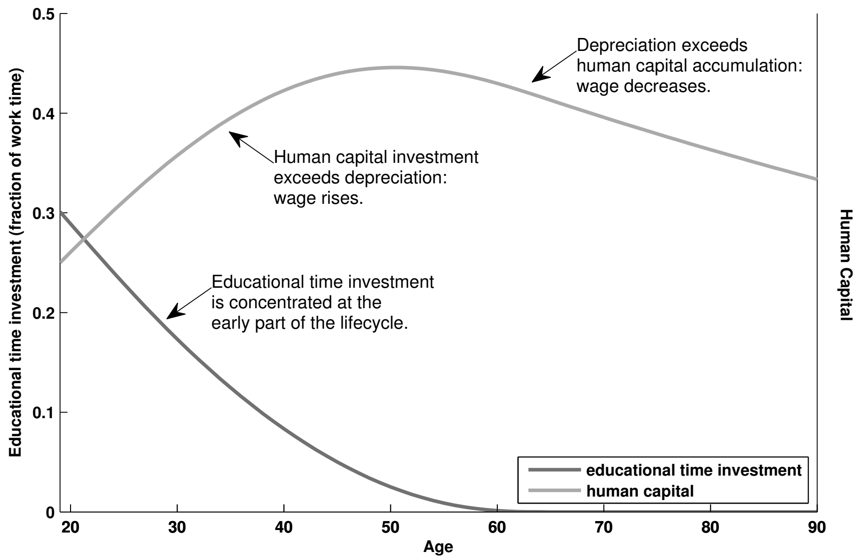


Figure 2.1: The evolution of educational time investment (left-axis) and human capital (right-axis) over the lifecycle of agents in the Ben-Porath model of human capital accumulation.

The Ben-Porath framework matches lifecycle earnings profiles well. There are two forces at play. When agents are young, they can benefit from the increased wages resulting from on-the-job training for a longer period of time and thus invest much of their time endowment to training, as is clear in figure 2.1. As the human capital stock of agents increases, the forgone earnings cost of investing time in training increases. This increased opportunity cost of time, and the increasingly short period during which agents can benefit from augmentations in their human capital stock lead to decreased investment during later years of their lifecycle. At some point, investment in human capital is too low to offset depreciation of human capital and wages start to decline again.

Assumptions

Ben-Porath (1967) makes three key assumptions. Firstly, he assumes that agents can perfectly smooth consumption over their lifecycle through unlimited borrowing and lending at market interest rates - i.e. there are no borrowing constraints. Secondly, Ben-Porath (1967) maintains that the human capital of one agent is perfectly substitutable for that of other agents. Agents only differ in ability, which determines their initial level of human capital and their returns to investments in on-the-job training. Thirdly, he specifies utility to be solely dependent on consumption, meaning that agents do not value free time. Consequently, they always work or study. All these assumptions have been relaxed in later work and turn out to be quite important for the results of policy analysis done with the Ben-Porath framework.

We will now focus in more detail on the literature on the three topics on which our thesis builds: borrowing constraints, tuition subsidies, and demographics. Afterwards, we will explain what our model adds to this literature. An overview of the most important papers and how this thesis relates to them is given in table 2.1.

2.4 Borrowing constraints

"Throughout this paper, we will use "rich", "college educated" and "skilled" interchangeably, as we will "poor", "school educated" and "unskilled"."

- Caucutt and Kumar (2003, p. 1463)

There is a tight limit to the amount students can borrow in the private market to finance their studies. Banks are hesitant to lend to students for two main reasons. First of all, human capital is not a good collateral for loans because it is specific to its owner (Becker, 1965, p. 93)[8]. Moreover, the returns on human capital investment are very uncertain because they are collected over a long period, the work life of students. During this long period, family circumstances, illness and unemployment could all affect returns (Becker, 1964, p.91).

It is quite intuitive that banks do not shower students with loans. The important question is, however, whether the tight limit on how much students can borrow in the private market affects their decision to enroll in tertiary education with the system of grants, subsidized loans and public subsidies to universities in place.

Empirical evidence

> *"[A] common failing of review articles: the judging of a certain number of studies to have shown one result, a certain number to have shown the opposite and the author concluding that no clear answer is possible. "*
>
> - Leslie and Brinkman (1987, p. 182)

The traditional evidence for borrowing constraints is based on tuition fee variation. Leslie and Brinkman (1987) compare the results of twenty-five "student demand studies", which estimate enrollment responses to changes in the cost of education (1987, p. 182). The modal result they find, is that a $100 increase in tuition leads to a 1.8% decline in enrollment rates of 18-24 year olds (1987, p. 189). This is no evidence of borrowing constraints yet, as an increase in tuition fees could simply render college attendance a non-worthwhile investment for students with lower ability.

[8] A bank can sell the house of someone who defaults on her mortgage. Selling the knowledge of an unemployed professor who cannot pay back her loan is more of a challenge.

Kane (1994) decomposes enrollment rate responses to tuition fee changes by parental income[9]. He finds that enrollment decisions of students from low-income households are disproportionally affected by tuition fee increases, which does constitute evidence that borrowing constraints have an effect on college enrollment in the United States[10].

A second, more indirect indication that borrowing constraints could impact enrollment comes from studies that aim to establish returns to schooling[11]. A major problem of estimating returns to schooling is that ability affects both earnings and completed schooling level, creating an upward bias in regression estimates. Researchers have used instruments such as distance to school, or variation in the number of years of compulsory schooling due to regulation[12] to eliminate the ability effect.

Surprisingly, after controlling for ability, estimated returns to schooling turn out to be *higher* (Card, 2001, p. 1146-1148). One interpretation for these higher estimates of returns to schooling is that the instruments used disproportionally affect those who are constrained in the means they have to finance a college education, providing evidence for the effect of borrowing constraints on human capital investment.

Arguments against borrowing constraints

The empirical evidence in favor of borrowing constraints can be challenged. When controlling for ability, much or all of the differences in enrollment between children from parents with different income levels disappear (see Cameron and Heckman, 1998; Carneiro and Heckman, 2002; Cameron and Taber, 2004). Carneiro and Heckman (2002, p. 706) also argue that the evidence for borrowing constraints based on instrumental variable regressions that aim to estimate returns to schooling is invalid. The instruments commonly used are flawed because they are either "uncorrelated with schooling", or "correlated with omitted variables".

Without controlling for ability, Shea (2000) also shows that borrowing constraints are unimportant in an analysis of Panel Study of Income Dynamics (PSID) data for children aged less than 18 before 1968. He uses "luck" factors such as "father's union [membership], industry and job loss experience" that cause exogenous variation in income. These factors can be utilized to isolate the causal effect of parents' (to be more precise, father's) income on the educational attainment of their offspring (p. 180). The effect turns out insignificant[13].

Alternative channels

Even if borrowing constraints do not affect college enrollment, they could affect other decisions of students. Keane and Wolpin (2001)[14] show in a rich model of college attendance that making

[9] He uses data from the Current Population Survey during the 1980s, a decade in which tuition fees increased significantly in many states in the U.S.

[10] Similar reasoning can be applied to international differences in enrollment rates. Björklund and Jäntti (1997) provide evidence that the intergenerational correlation of income is higher in the United States than in Sweden. As tuition fees are substantial in the United States, but non-existent in Sweden, this could be an argument in favor of the impact of borrowing constraints on education decisions. Prudence in interpretation is required though, they warn that their estimates are not precise enough to reach definitive conclusions.

[11] Mincer (1958) laid the groundwork for research in this area. He showed that differences in pay across professions are "a function of differences in training" (1958, p. 301). Also, he pointed out that "increases in productivity with age are more pronounced, and declines are less pronounced, in jobs requiring greater amounts of training" (Mincer, 1958, p. 301); Card (2001) provides an overview of the literature.

[12] Angrist and Kreuger (1991) is a famous example in which quarter of birth is used as instrument.

[13] Except in a subset of the data with fathers with less than 12 years of schooling. Shea (2000) shows, however, that the effect is not significant for low income families *an sich*.

[14] They develop a model to capture the effects of borrowing constraints and parental transfers on tertiary education attendance. They calibrate the model using data from the 1979 cohort of the National Longitudinal Survey of

borrowing constraints less binding does not increase human capital investment. Instead, agents use the additional resources they can borrow to work less and consume more. Garriga and Keightley (2006) confirm this finding in a model of college attendance with uncertainty about college ability. In empirical work, credit constrained students have been found to enter college later (see Kane, 1996), and drop out of college more often (see Stinebrickner and Stinebrickner, 2008).

2.4.1 And then there were the nineties

Evidence that borrowing constraints affect enrollment in tertiary education in the United States is shaky at best. After controlling for ability and family background, most of the differences in college attendance between children from different parental income quartiles disappear. It is worthwhile to note, however, that most of the papers on borrowing constraints presented here use data from the 1970s and the 1980s[15]. This matters.

During the 1990s, both the costs of and the returns to college have increased in the United States. At the same time, limits on government loans to students failed to keep up with inflation and rising tuition costs (Garriga and Keightley, 2006; Belley and Lochner, 2007). Using cohort data from the late 1990s and early 2000s[16], Belley and Lochner (2007, p. 37) find "a dramatic increase in the effect of family income on college attendance rates [...] after controlling for family background and ability" compared to data from the 1970s and 1980s. At age 21, college attendance of students with the same ability is now 16 percentage points higher for those from the highest parental income quartile compared to those from the lowest income quartile (Belley and Lochner, 2007, p. 52). This suggests that borrowing constraints have become a much more important determinant of college attendance in the United States recently.

2.4.2 Borrowing constraints in Europe

There is little literature on borrowing constraints in Europe. Winter-Ebmer and Wirz (2002) provide some indirect evidence when they find in a sample of 14 European countries that if a country has tuition fees, enrollment into college is significantly lower than if it does not have fees. It remains to be seen whether this drop in enrollment is due to borrowing constraints, and even more fundamentally, whether the effect is causal at all[17].

On the level of individual countries, Nielsen, Sørensen and Taber (2008) show that borrowing constraints barely affect enrollment in Denmark, a result that is not surprising given the country's high public share in the cost of education and the generous financial aid available to students[18]. Dearden, McGranahan and Sianesi (2004) find evidence for some effect of borrowing on enrollment in the United Kingdom, but this effect manifests itself around the mandatory school leaving age, rather than the age at which high school graduates make a college enrollment decision.

Youth (NLSY). A key feature in their set-up is that agents can work while attending college. Each semester, agents choose how much time to allocate to work, and how much to studying.

[15] Except for Garriga and Keightley (2006) who use data from the early 1990s.

[16] They use the 1997 NLSY cohort. The 1979 cohort of the NLSY was used by Keane and Wolpin (2001) and Carneiro and Heckman (2002). They find little to no evidence of an effect of borrowing constraints on enrollment.

[17] Winter-Ebmer and Wirz (2002) explain that it is hard to control for the endogeneity of public funding. When enrollment rates increase, there will be more students, who could vote for political parties that aim to increase public funding to education, further increasing enrollment.

[18] See table 1.2 in section 1.3.2.

2.4.3 Conclusion

To sum up this section on borrowing constraints, there used to be no evidence that borrowing constraints have a significant impact on enrollment in tertiary education in the United States. This no longer holds. Models based on data for the 1990s and 2000s find correlations between parental income and enrollment that cannot be explained fully by ability differences and family background alone. Borrowing constraints can account for the remaining correlation.

An explanation of the increased importance of borrowing constraints for enrollment is that during the 1990s, both the costs of tertiary education and the returns to it increased while the amount of loans available to students did not increase proportionally. In Europe, little literature exists on the effect of borrowing constraints on enrollment in tertiary education.

2.5 Tuition subsidies

Borrowing constraints are the primary reason[19] for government involvement in the market for tertiary education. To stimulate equality of opportunity for students from all socio-economic backgrounds[20], the public share in the cost of tertiary education in European countries is high[21]. Usually, governments contribute to tertiary education in two ways; through (1) subsidies to universities which allow them to charge low - or in the case of some countries, no - tuition fees, and (2) student financial aid programs, which usually consist of grants and the availability of subsidized loans.

2.5.1 The equality and equity tradeoff

A subsidy to tertiary education stimulates equal access to tertiary education, but also has implications for efficiency and welfare. García-Peñalosa and Wälde (2000) develop a simple model of higher education with unskilled and skilled workers to study these welfare effects. When agents can borrow indefinitely in this model, wages of unskilled and skilled workers will adjust until the net present value of lifetime income of unskilled workers equals that of skilled workers[22]. This is the point where output and thus "social welfare" are maximized (2000, p. 706).

When borrowing on the private market is not possible, only students who receive high transfers from their parents will obtain tertiary education and become skilled workers. Government subsidies can overcome this inequality, but at a cost. The costs of education have decreased due to the subsidy, which is paid out of general tax revenue. As a result, the number of skilled workers at which the net present value of lifetime income of skilled and unskilled workers is equal, increases. This leads to a too high unskilled labor wage and a too low skilled labor wage in the economy, which is inefficient; output and welfare are lower than in the case without borrowing constraints and subsidies.

[19] Another often-mentioned reason for subsidies is that tertiary education has a positive effect on economic growth (Friedman, 1962; Lucas, 1988). Note that Milton Friedman changed his mind later on (Hall, 2006).

[20] Many have suggested that the current system of providing high subsidies to universities and generous financial aid to students involves reverse redistribution of income, from low-income households to students who will be the high-earners of the future. García-Peñalosa and Wälde (2000) give a list of international empirical evidence in favor of reverse redistribution. Others show that reverse distribution can be an equilibrium outcome in voting procedures; see Creedy and Francois (1990), Creedy (1994), and Fernandez and Rogerson (1995).

[21] See table 1.2 in section 1.3.2.

[22] When there are no taxes, the net present value of lifetime income of skilled workers consists of the present value of lifetime income minus the costs of education. For unskilled workers, the net present value of income simply consists of the present value of lifetime income.

Caucutt and Kumar (2003) confirm the existence of a tradeoff between equity and efficiency of subsidies to tertiary education in a two-period dynamic optimization model of educational choice without asset markets. In this model, total welfare decreases if subsidies are increased to a level at which educational opportunity is equal for everyone[23].

2.5.2 The equality and equity tradeoff - or not?

Tuition subsidy increases do not have a negative effect on welfare *per se.* Heckman, Lochner and Taber (1999) shows that a \$500 increase in tuition subsidy would increase college attendance in the United States by 0.46% in general equilibrium with only minor, positive effects on welfare. Abbott, Gallipoli, Meghir and Violante (2013) show that a \$1,000 tuition grant would increase college attendance in the United States by 2.3%[24], while increasing welfare slightly by 0.15%.

The welfare effects one finds when evaluating tuition subsidy changes depend on the elements included in the model. Progressive taxes on labor supply can be a disincentive to invest in education. Subsidies can overcome this disincentive, thereby having a less negative effect on welfare (see Heckman et al., 1999; Dur and Teulings, 2002; Bovenberg and Jacobs, 2005; Krueger and Ludwig, 2013). Subsidies could also have a positive effect on welfare in case of uncertainty about future labor income (see Keane and Wolpin, 1997; García-Peñalosa and Wälde, 2000; Kindermann, 2009; Abbott et al., 2013), but note that some have found no effect (e.g. Hogan and Walker, 2007). And of course, subsidies could positively affect welfare by decreasing the importance of borrowing constraints in the tertiary education enrollment decision of students (see Kindermann, 2009; Abbott et al., 2013).

2.5.3 Partial and general equilibrium analysis

Decisions of agents affect returns to factors of production, the wage and interest rate in the economy. In partial equilibrium, these effects on factor prices are not considered. In general equilibrium, they are. The impact of policy changes on welfare and the time and resource allocation of agents varies greatly depending on whether one considers partial or general equilibrium effects[25].

As pointed out already, Heckman et al. (1999) show that a \$500 increase in tuition subsidy increases college enrollment in the United States by 0.46% in general equilibrium. In partial equilibrium, the effect is much larger: an enrollment increase of 5.3%. The reason for the difference is that the number of high skilled workers increases when subsidies go up, which decreases the wage of high-skilled labor. The same effect is found by Abbott et al. (2013). When they increase the tuition subsidy in their model by \$1.000, enrollment increases by 2.3% in general equilibrium. In partial equilibrium, this increase is as much as 13.0%.

Also welfare effects of policy changes differ between partial and general equilibrium. Kindermann (2009) finds that a complete privatization of tertiary education in Germany decreases welfare of agents from the top third socio-economic background by 7.57% in partial equilibrium. In general equilibrium, this welfare decrease is only 2.83% because the college wage premium shoots up[26].

[23] Welfare is proxied for by consumption equivalence of utility changes of parents who are either high school graduates or college graduates. The set-up puts the role of the parent at the heart of the model. Parents choose the education level of their children; the utility of their children enters their own utility function.

[24] The fact that students from low-income families receive higher subsidies in their baseline model could be one of the reasons for the higher increase in college attendance compared to Heckman et al. (1999).

[25] As was already pointed out by Auerbach and Kotlikoff (1987).

[26] See Table 7 in Kindermann (2009, p. 21).

2.5.4 Financial aid

In this thesis, we focus exclusively on tuition subsidies to education, not on the design of optimal student financial aid policies. This means that we do not evaluate systems such as subsidized borrowing (García-Peñalosa and Wälde, 2000; Kindermann, 2009), graduate taxes (García-Peñalosa and Wälde, 2000; Kindermann, 2009), ability-based grants (Caucutt and Kumar, 2003; Abbott et al., 2013), means-tested grants (Bénabou, 2002; Abbott et al., 2013) and the many other financial aid policies that have been suggested and evaluated in the literature.

2.5.5 Conclusion

In conclusion, the stimulation of equal opportunities for students from all socio-economic backgrounds to enroll in tertiary education is an important reason for the high public share in the cost of education in most European countries. By reducing tuition fees, these subsidies make borrowing constraints less of an inhibition to enrollment. Some have suggested that public subsidies can potentially decrease welfare. Whether subsidies indeed have a negative effect on welfare depends on the presence of market imperfections, such as imperfect capital markets and uninsurable labor earnings risk, or of a distortionary progressive labor tax system.

2.6 Demographics

Demography seems a peculiar topic to follow an overview of literature on borrowing constraints and tuition subsidies. In this thesis, we set ourselves the task to show that these three topics are linked, rather much so. The single most important projected demographic development in developed economies during the next fifty years is a rise in old-age dependency. An increase in the share of retirees in the total population has significant welfare costs when the public pension system in a country is not fully funded[27]. There are three ways to keep the pension system sustainable: increase contribution rates, decrease retirement benefits, and/or cut other government expenditures. Any of these three options seriously affects the welfare of certain groups of the population. Consequently, studying what economic mechanisms cause the welfare losses of aging and what policies are best to minimize these losses is important.

2.6.1 The economic consequences of increased old-age dependency

Auerbach, Kotlikoff, Hagemann and Nicoletti (1989) develop a dynamic model to evaluate the economic effects of increased old-age dependency and policies that could mitigate them. They report that a higher fraction of elderly in the population has three consequences. It will (1) reduce the national saving rate[28], (2) increase the social security contribution rate significantly[29] and (3) have negative effects on lifetime consumption when the social security system is left intact in the four OECD countries[30] they consider.

An interesting finding of Auerbach et al. (1989) is that a greater fraction of elderly in the population will raise the capital to labor ratio. A higher capital to labor ratio means that in

[27]Public pension systems are typically not fully funded (Hagemann and Nicoletti, 1989, p. 60-61).

[28]Auerbach et al. (1989, p. 15) assume closed economies in most of their results and thus define the national saving rate as "the level of public and private saving (GDP less public and private consumption and depreciation) divided by the net domestic product (GDP less depreciation)".

[29]In Sweden, the projected change is moderate. Auerbach et al. (1989) report an increase of about 4 percentage points from a contribution rate of 17.6% in 1985 to 21.7% in 2050. Hagemann and Nicoletti (1989) predict a slightly larger rise in contribute rates in Sweden of 5 percentage points, from a rate of 15.8% in 1990 to a rate of 20.5% in 2050.

[30]They consider the following countries: Germany, Japan, Sweden, and the United States.

general equilibrium, wages will rise and interest rates will go down. Hviding and Mérette (1998), Ludwig and Vogel (2010), and Ludwig et al. (2012) also find that higher old-age dependency causes the capital to labor ratio in the economy to increase.

Pension system reforms

If countries do not reform their pension system, lifetime consumption of workers decreases the most; their contributions to the pension system need to increase considerably to keep pension benefits at the same level. Retirees also suffer from higher old-age dependency, but to a lesser extent: the return on their savings goes down because of the higher capital to labor ratio in the economy.

Auerbach et al. (1989) model three policies that could distribute the welfare effects of aging more equally among workers and retirees: (1) no growth in government expenditure per capita, (2) a two-year rise in the retirement age, and (3) a twenty percent cut in retirement benefits (see Auerbach et al., 1989, p. 18-19). They find that these three policies do indeed reduce the large loss in lifetime consumption of workers caused by higher old-age dependency as they decrease the required raise in contribution rates to keep the pension system sustainable.

Hviding and Mérette (1998) use the framework of Auerbach et al. (1989) to evaluate four pension system reforms[31] that could alleviate the cost of aging for seven OECD countries[32]. They conclude that in the long-run, the most effective reform of pension systems is their gradual abolishment. When public pension benefits are abolished, the wage tax rate in a country will decrease and private saving will increase. Both have a favorable effect on lifetime consumption per capita. In the short-run, however, raising the retirement age is the best option.

Hviding and Mérette (1998)'s finding that abolishing pension systems is the most effective reform when old-age dependency rises could turn out differently if there would be uncertainty in their model. Huang, Imrohoroglu and Sargent (1997) use a model with uncertain returns to labor supply and uncertain lifespan to evaluate two extreme social security reforms: (1) a full privatization of the social security system and (2) keeping social security benefits at their current level but making the social security system independent of tax revenue by giving it a high amount of own capital; the returns on this capital could then be used to pay out pension benefits. They find that a government-run scheme is preferable to full privatization because it provides insurance against income and lifespan risk.

Hubbard and Judd (1987, p. 642-643) also point out that publicly provided social security can considerably improve "lifetime consumption and welfare" when there is uncertainty about lifespan. They show, however, that this welfare improving effect of social security disappears when there are binding credit constraints in the economy. If tax rates are proportional, the tax that is necessary to fund the social security system increases the severity of the borrowing constraint and thereby decreases lifetime consumption and thus welfare.

Human capital

Fougère and Mérette (1999) extend the model of Hviding and Mérette (1998) to include human capital accumulation and endogenous growth. They show that higher old-age dependency leads to an increase in wages, which in turn increases human capital accumulation. Higher human capital has a positive effect on economic growth in their set-up. If economic growth is higher and the replacement rates of the pension system are fixed, the contribution rates to fund the

[31] They analyze (1) a "gradual removal of public pensions", (2) a "20 percent cut in the replacement rate", (3) "fiscal consolidation", (4) "raising the retirement age" (Hviding and Mérette, 1998, p. 16-17).

[32] They consider the following countries: the United States, Japan, France, Canada Italy, the United Kingdom and Sweden.

pension system do not need to increase as much, decreasing the negative welfare effects of higher old-age dependency.

Fougère, Harvey, Mercenier and Mérette (2009) reaffirm that endogenous human capital accumulation can alleviate the negative welfare effects of higher old-age dependency in a detailed model calibrated on Canadian data, but add a qualification to this finding. Initially, increased investment in human capital aggravates the welfare effects of rising old-age dependency as young agents spend more time in school. It is only when these more educated agents get older that the welfare effects of aging are lessened. Moreover, they show that higher levels of human capital can increase labor supply of older workers, which further mitigates the negative welfare effects of aging.

Ludwig and Vogel (2010) show in an overlapping generations model that falling birth rates have a positive effect on educational attainment. The reason is that the rise in old-age dependency caused by lower birth rates results in higher wages, increasing the payoff to education. They do not find the same effect for increasing survival rates because there are "interactions between annuity markets, the pension system, and productivity of education" when people live longer (Ludwig and Vogel, 2010, p. 704).

Ludwig et al. (2012) use a rich framework with endogenous human capital accumulation, survival rates, and a population transition model[33] to study how investments in education affect the welfare consequences of rising old-age dependency. They consider two extreme social security scenarios: (1) constant replacement rates, which mean that benefits to retirees are kept constant, and (2) constant contribution rates, which imply that the tax rate to fund the pension system is kept constant.

Ludwig et al. (2012) compute the welfare losses for all birth cohorts alive in 2005. They find that the maximum losses in consumption-equivalent variation[34] are large in both pension policy scenarios they model. Furthermore, they show that endogenous human capital can reduce the loss in consumption-equivalent variation.

In case of constant replacement rates, the birth cohort that suffers the worst effects of aging experiences a loss in consumption-equivalent variation of 5.6% when no adjustments in educational time investment can be made. This loss decreases to 4.4% when agents can adjust their investment in education. In case of constant contribution rates, the maximum welfare loss is 12.5% when human capital is exogenous; this decreases to 8.7% when human capital is endogenous.

2.6.2 Open versus closed economy

Many papers on the welfare effects of aging assume a closed economy (see Auerbach et al., 1989; Hviding and Mérette, 1998; Fougère and Mérette, 1999; Fougère et al., 2009; Ludwig and Vogel, 2010; Ludwig et al., 2012), a situation in which a country's wage, interest and savings rate do not impact its balance of payments (Auerbach et al., 1989, p. 20). The welfare effects of aging could turn out differently if one assumes an open economy. Then, wages and interest rates are "pegged from abroad" (Auerbach et al., 1989, p. 20). This means that excess capital flows out of a country when old-age dependency rises, preventing increases in the capital to labor ratio

[33]This model describes the predicted evolution of the population distribution in the United States between 1950 and 2200.

[34]Defined as "the percentage increase in consumption that needs to be given to an agent [...] at each date in her remaining lifetime at fixed prices to make her as well off as she would be in the situation with changing prices" (Ludwig et al., 2012, p. 103).

and the accompanying decline in interest rates and rise in wages[35].

As Fougère and Mérette (1999, p. 426) point out, the closed economy assumption is often not realistic, especially not for smaller countries for which a large share of their GDP depends on trade. Auerbach et al. (1989) explain, however, that one should consider that many developed countries are experiencing the same demographic transition. Then, it is difficult to maintain the assumption that the outside world functions as a "sink" for capital outflows (Auerbach et al., 1989, p. 21).

Auerbach et al. (1989) show that the projected rise in contribution rates in their model barely changes when they assume there are no adjustments in interest and wage rates. The reason is that in their set-up, pension benefits are indexed for past earnings. Moreover, they find that when the economy is assumed to be open, the national savings rate rises compared to a closed economy scenario because interest rates do not adjust downwards. This rise in national savings has a positive effect on lifetime consumption.

Attanasio et al. (2007) reach similar conclusions. They show in a two-region model that if one is interested in the evolution of policy variables such as tax or replacement rates following changes in the population distribution, the open and closed economy assumption yield similar results. If, however, one aims to evaluate the effect of changing old-age dependency on macro-economic variables and welfare, the results differ substantially between the open and closed economy assumption. Welfare effects are much larger when one works with the closed economy assumption because interest rates can adjust downwards.

2.6.3 Conclusion

To wrap up this section on demographics, the fraction of elderly in the total population will rise rapidly in developed countries during the next four decades. The projected costs of this rise in old-age dependency are significant. In absence of reforms to the pension system, contribution rates will have to rise - decreasing the welfare of workers. Moreover, a higher fraction of elderly in the population causes the capital to labor ratio to rise, which lowers interest rates. A lower interest rate decreases the welfare of especially the elderly.

The higher capital to labor ratio brought about by higher old-age dependency does have a serendipitous concomitant, however. It increases wage. In case of endogenous human capital, therefore, human capital investments will rise. A higher average level of human capital in the economy raises tax revenue, thereby reducing the need for increases in the contribution rate to fund the pension system. As a result, the welfare effects of aging can be mitigated partially by increased investment in education. This positive effect is even stronger when economic growth is increasing with the level of human capital in the economy.

[35] Note that the distinction between partial and general equilibrium is not the same as the distinction between an open and closed economy. In partial equilibrium, no effects of the decisions of agents on macro-economic variables such as wages and interest rates are considered. In an open economy model, a country has a balance of payments that allows for capital inflows and outflows. Prices do not adjust, usually due to the existence of an international asset on which interest rates are fixed (McCandless, 2008, p. 370).

Paper	Aging	(Policy) Analysis
Heckman et al. (1998)	No	Explaining rising wage inequality in the United States since the 1960s
Heckman et al. (1999)	No	Tax system (progressive/flat), tuition subsidy on enrollment
Fougère and Mérette (1999)	Yes	Constant contribution rate to fund pension system
Keane and Wolpin (2001)	No	Borrowing constraints, parental transfers on schooling attainment
Garriga and Keightley (2006)	No	Tuition subsidies, grants, and loan limit restrictions on enrollment
Fougère et al. (2009)	Yes	Welfare effects of aging; constant contribution rate to fund pension system
Kindermann (2009)	No	Tuition subsidy, graduate tax on enrollment and welfare
Ludwig and Vogel (2010)	Yes	Welfare effects of aging; constant contribution or replacement rate to fund pension system
Ludwig et al. (2012)	Yes	Welfare effects of aging; constant contribution or replacement rate to fund pension system
Johnson (2012)	No	Borrowing constraints, tuition subsidies on enrollment
Abbott et al. (2013)	No	Grants, and government loans on enrollment
Krueger and Ludwig (2013)	No	Tuition subsidy and the design of the tax system on schooling attainment
This paper	**Yes**	**Welfare effects of aging; constant contribution or replacement rate to fund pension system; borrowing constraints, tuition subsidy**

Paper	Study period	Borrowing constraints	On-the-job training	Labor supply	Pension system
Heckman et al. (1998)	No	No	Ben-Porath	Exogenous	-
Heckman et al. (1999)	No	No	Ben-Porath	Exogenous	-
Fougère and Mérette (1999)	No	No	Ben-Porath	Exogenous	PAYG
Keane and Wolpin (2001)	Yes	Yes	Learning by doing	Endogenous (study period), Exogenous (after studies)	-
Garriga and Keightley (2006)	Yes	Yes	No	Endogenous	-
Fougère et al. (2009)	No	No	Ben-Porath	Endogenous	PAYG
Kindermann (2009)	Yes	Yes	Ben-Porath	Endogenous	PAYG
Ludwig and Vogel (2010)	No	No	Ben-Porath	Exogenous	PAYG
Ludwig et al. (2012)	No	No	Ben-Porath	Endogenous	PAYG
Johnson (2012)	Yes	Yes	No	Endogenous (study period), Exogenous (after studies)	-
Abbott et al. (2013)	Yes	Yes	No	Endogenous	PAYG
Krueger and Ludwig (2013)	Yes	Yes	No	Endogenous	PAYG
This paper	**No**	**Yes**	**Ben-Porath**	**Endogenous**	**PAYG**

Table 2.1: Overview literature OLG models with endogenous education decisions.

2.7 Addition to literature

The model we use is an extension of the model used by Ludwig et al. (2012). They show that the negative welfare effects of higher old-age dependency can be mitigated by rises in educational time investment. This welfare-improving adjustment channel is automatic. Higher old-age dependency increases the capital to labor ratio and thereby increases wage. As a result, the payoffs to human capital investments increase, rendering more educational time investment worthwhile.

In the conclusion to their paper, Ludwig et al. (2012, p. 106) write that they:

> "have operated in a frictionless environment, where all endogenous human capital adjustments are driven by relative price changes. If, instead, human capital formation is affected by market imperfections, such as borrowing constraints, then these automatic adjustments will be inhibited."

Besides the absence of market imperfections, Ludwig et al. (2012) assume that all training is paid for by firms. The purpose of this thesis is to show how borrowing constraints and government involvement in the market for education would alter their analysis. To realize this aim, we combine elements from models used in the literature on borrowing constraints, tuition subsidies and demographics, see table 2.1.

We set out to answer two main questions:

1. Does endogenous human capital accumulation still decrease the negative welfare effects of rising old-age dependency when the cost of education is partly borne by agents and the government?

2. How do capital markets imperfections - i.e. borrowing constraints - affect the welfare consequences of rising old-age dependency?

We calibrate our model on data for the Swedish economy. This is for two reasons. The first one is practical; we wrote this thesis in cooperation with the Swedish Ministry of Finance. The second reason is more academic. In Sweden, the state finances a rather high share of the total costs of tertiary education, much higher than in the United States[36]. This makes an analysis of the effect of public involvement in the education market in the context of aging quite relevant.

[36] See table 1.2 for an overview of public shares in the cost of tertiary education in European countries.

Chapter 3

Model description

This section gives an overview of the model we use. We first explain the structure of the model, then show how we calibrate it to match stylized facts for the Swedish economy. Afterwards, we give a short overview of the computational algorithm we use to solve the model and finally, we compare our model to the model used by Ludwig et al. (2012).

3.1 The model

Agents maximize lifetime utility, which is a concave function of consumption and leisure. Human capital accumulation is endogenous and occurs through Ben-Porath on-the-job training. Time is in years. Agents enter the model at age 19 ($j = 1$), retire at age 65 ($j = 46$) and die when they turn 91 ($J = 72$).

Agents go through two stages in their lives:

1. <u>Work and Ben-Porath on-the-job training</u>
   (a) *Time* can be allocated to on-the-job training ($e_{t,j}$), labor supply ($l_{t,j}$) and leisure $(1 - l_{t,j} - e_{t,j})$
   (b) *Sources of income*: (1) labor income, (2) borrowing on the private market
   (c) *Sources of expenditure*: (1) consumption ($c_{t,j}$), (2) costs of on-the-job training ($e_{t,j}(1-\mu)T$), (3) income tax (τ_t) to finance the public share in the costs of on-the-job training and the lump-sum pension
2. <u>Retirement</u>
   (a) *Time* can only be allocated to leisure ($e_{t,j} = l_{t,j} = 0$)
   (b) *Sources of income*: (1) pension benefits ($p_{t,j}$), (2) savings
   (c) *Sources of expenditure*: (1) consumption

3.1.1 Household preferences and budget

Agents maximize life-time utility according to a Cobb-Douglass utility function often used in the macro-economics literature[1]:

$$
\begin{aligned}
u(c_{t,j}, l_{t,j}, e_{t,j}) &= \frac{1}{1-\sigma}\left\{c_{t,j}^{\phi}(1-l_{t,j}-e_{t,j})^{1-\phi}\right\}^{1-\sigma} \\
&\phi \in (0,1), \sigma > 0 \\
&e_j \in [0,1],\ l_j \in [0,1]
\end{aligned}
\tag{3.1}
$$

In which $c_{t,j}$ denotes consumption, $l_{t,j}$ labor supply, $e_{t,j}$ time spent on education, and $(1 - l_{t,j} - e_{t,j})$ leisure. t reflects the time in which the agent lives, and j the age of the agent. The parameter ϕ determines the weight of consumption in utility and σ is the inverse of the intertemporal elasticity of substitution.

The budget constraints are:

$$
a_{t+1,j+1} = \begin{cases} a_{t,j}(1+r_t) + w_{t,j}^n - e_{t,j}(1-\mu)T - c_{t,j} & \text{if } j < jr \\ a_{t,j}(1+r_t) + p_{t,j} - c_{t,j} & \text{if } j \geq jr \end{cases} \tag{3.2}
$$

$$
a_{t,J+1} = 0
$$

$$
a_{t,1} = 0
$$

In which $a_{t,j}$ denotes assets accumulated, r_t the interest rate, and $c_{t,j}$ consumption. $w_{t,j}^n$ is net wage, given by $w_{t,j}^n = l_{t,j}h_{t,j}w_t(1-\tau_t)$, with w_t denoting gross wage per unit of human capital and τ_t the tax rate on labor income. μ is the public share in the the costs of training T. $p_{t,j}$ are lump-sum pension benefits provided by the government. Agents have zero assets when they are born ($a_{t,1} = 0$) and use up all their assets before they die (boundary condition: $a_{t,J+1} = 0$).

3.1.2 Ben-Porath type on-the-job training

We use Ben-Porath on-the-job training to approximate lifelong learning. This means that in the spirit of Becker (1964), we abstract away from modeling a schooling period. Recent work[2] on schooling periods allows agents to divide their time when enrolled in tertiary education between schooling and work. Consequently, the schooling period is becoming increasingly similar to the period in Ben-Porath models of human capital accumulation during which agents devote the majority of their non-leisure time to education.

Davies and Whalley (1991), Heckman et al. (1998, 1999) and Ludwig et al. (2012) also do not model a schooling period[3]. They assume on-the-job training is fully financed by firms. Then, the costs of on-the-job training are foregone wage and a loss in leisure time. Because lifelong learning includes schooling which is not paid for by firms, we introduce tuition into our model. Part of the tuition costs are paid by the agent ($e_{t,j}(1-\mu)T$), and part by the government ($e_{t,j}\mu T$).

Human capital accumulates according to the following Ben-Porath human capital technology function:

$$
\begin{aligned}
h_{t+1,j+1} &= h_{t,j}(1-\delta^h) + \xi(h_{t,j}e_{t,j})^{\psi}, \psi \in (0,1), \xi > 0, \delta^h \geq 0 \\
h_{t,1} &= h_1
\end{aligned}
\tag{3.3}
$$

[1] In this utility function, consumption and leisure are multiplied with each other. Domeij and Flodén (2006, p. 249) point out that the assumption that preferences are not separable in consumption and leisure is more realistic than assuming utility from consumption and from leisure is separable.

[2] See Keane and Wolpin (2001), Garriga and Keightley (2006) and Kindermann (2009).

[3] Note that Heckman et al. (1998, 1999) do allow for a schooling choice which enables them to model an economy with heterogenous human capital. They do not model a schooling period, however.

in which ξ is a scaling factor, ψ reflects the curvature of human capital technology, and δ^h is the depreciation rate of human capital. When agents enter the model at age 19, they get an initial level of human capital of h_1.

3.1.3 Demographics

We load an exogenous population distribution for Sweden in our model to include demographics. We then transform the distribution in aggregation weights, denoted $N_{t,j}$, which assign the correct weight of each age group to the calculation of aggregate variables in our model. To model aging, we use population distribution predictions by Statistics Sweden[4] for the years between 2010 and 2050. As these predictions are published for 5-year intervals, we interpolate them to get yearly estimates[5].

We do not adjust for changes in life expectancy[6]. In our model, a higher life expectancy would increase lifetime utility of agents because they live longer. As we want to isolate the pure effects of a changing population distribution on welfare, we keep life expectancy at age 90.

3.1.4 Production function

Cobb-Douglas technology is assumed:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \ \alpha \in (0,1) \tag{3.4}$$

with:

$$K_t = \sum_{j=1}^{J} N_{t,j} a_{t,j}$$
$$L_t = \sum_{j=1}^{jr-1} N_{t,j} l_{t,j} h_{t,j}$$

From the production function, the wage and interest rate can be derived:

$$\begin{aligned} w_t &= \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} \\ r_t &= \frac{\partial Y_t}{\partial K_t} = \alpha\left(\frac{L_t}{K_t}\right)^{(1-\alpha)} \end{aligned} \tag{3.5}$$

We work with the closed economy assumption and thus solve for general equilibrium. In equilibrium, factors of production K_t and L_t are paid their marginal products r_t and w_t.

Although Sweden is a small open economy, we do not deem the closed economy assumption unrealistic. Old-age dependency is rising in developed economies around the world. As Auerbach et al. (1989, p. 21) point out, it is difficult to maintain that the outside world functions as a "sink" for capital outflows in this case.

[4]Source: Statistics Sweden, 2013, "The future population of Sweden 2013-2060", table: 9.

[5]Due to the interpolation, the yearly estimates of old-age dependency become slightly imprecise. We correct for this imprecision and match the old-age dependency ratios as predicted by Statistics Sweden, 2013, in "The future population of Sweden 2013-2060", table: 9. For a detailed description of how we incorporate old-age dependency ratios in the model, see appendix D.3.

[6]For Sweden, a rise in life expectancy is expected from 80.9 in 2010 to 86.2 in 2050. Source: Statistics Sweden, 2013, "The future population of Sweden 2013-2060", table: 1.

3.1.5 Pension system

Agents receive a pay-as-you-go financed lump-sum transfer from the government when retired. The sum of paid out pension benefits is equal to $\sum_{j=jr}^{J} N_{t,j}p_{t,j}$, in which $p_{t,j}$ denotes the pension each retiree receives. This yearly pension benefit is equal to $p_{t,j} = \rho w_t$, in which ρ reflects the replacement rate - defined as the share of the gross wage per unit of human capital that is paid out to the retired.

3.1.6 Government budget

τ_t is the sum of the tax rates required for financing the public share in the costs of training (τ_t^e), and the pension benefits (τ_t^p). The government has two options to balance its budget: (1) through adjustments in τ_t, keeping the public share in the cost of education $(1-\mu)$ and the replacement rate for retirees ρ fixed or (2) through adjustments in the replacement rate ρ, keeping the public share $(1-\mu)$ and the tax rate to fund pensions (τ_t^p) fixed:

$$\tau_t w_t \sum_{j=1}^{jr-1} N_{t,j} l_{t,j} h_{t,j} = \sum_{j=1}^{jr-1} N_{t,j}(1-\mu)e_{t,j}T + \sum_{j=jr}^{J} N_{t,j}\rho w_t \tag{3.6}$$

3.1.7 Household problem

Agents maximize the present discounted value of their lifetime utility by choosing consumption, labor supply, and educational time investment given the set of budget constraints in 3.2 and the human capital accumulation function in 3.3:

$$\begin{gathered}\max_{c,l,e,a',h'} \sum_{j=1}^{J} V(c_{t,j}, 1-l_{t,j}-e_{t,j}) = \sum_{j=1}^{J} \beta^j \frac{1}{1-\sigma} \left\{ c_{t,j}^{\phi} (1-l_{t,j}-e_{t,j})^{1-\phi} \right\}^{1-\sigma}, \\ \sigma > 0,\ \phi \in (0,1)\end{gathered} \tag{3.7}$$

3.2 Calibration

We calibrate our model to match stylized facts for Sweden. Table 3.1 provides an overview of the calibration targets and table 3.2 summarizes the parameter values we use in order to achieve these targets.

	Baseline model	Calibration target
Capital-output ratio	3.09	3.00
Investment-output ratio	18.4%	18%[7]
Pension benefits (% of GDP)	7.94%	7.9%[8]
Total cost of education (% of GDP)	1.64%	1.59%[9]
Labor-income share	65.1%	65%[10]
Hours worked (share of total time)	0.33	0.33[11]
Earnings variability	1.78	1.78[12]
Interest rate	3%	-

Table 3.1: Baseline model and calibration targets.

[7]Source: Worldbank, 2012, "Gross Fixed Capital Formation as percentage of GDP", retrieved from `http://data.worldbank.org/indicator/NE.GDI.FTOT.ZS`

Preferences	σ	Inverse of inter-temporal elasticity of substitution	2.00
	β	Pure time discount factor	0.983
	ϕ	Weight of consumption in utility	0.34
Human capital	ξ	Ability scaling factor	0.11
	ψ	Ability curvature parameter	0.64
	δ^h	Depreciation rate of human capital	0.85%
	h_1	Initial human capital endowment	1.00
Production	α	Share of physical capital in production	0.277
	δ	Depreciation rate of physical capital	6%

Table 3.2: Overview of parameter settings.

3.2.1 Household preferences

We follow Ludwig et al. (2012) by setting σ, the inverse of the intertemporal elasticity of substitution, to 2. We calibrate the time-discount factor β to get an interest rate (net of depreciation) of 3%, resulting in $\beta = 0.983$. We follow common macroeconomic practice[13] by matching hours worked to one third of the time endowment. This requires a weight of consumption in utility ϕ of 0.34 in our model.

3.2.2 Human capital

We assume a unisex model. Consequently, we set the ability scaling factor, ξ, the ability curvature parameter, ψ, and the depreciation rate of human capital, δ^h to match the 2010 average productivity profile of men and women in Sweden as closely as possible[14]. To do so, we set $\xi = 0.11$, $\psi = 0.64$ and $\delta^h = 0.0085$.

As a measure for earnings variability over the lifecycle, we compute the ratio of the maximum h^{max} and the starting value of human capital h_1. We match this to the ratio of the maximum and minimum hourly wage data of 1.78 for Sweden in 2010.

3.2.3 Production

We set the capital share in production α to match the commonly assumed capital-output ratio of 3[15], which requires $\alpha = 0.277$. Although this value of α is lower than the usual 0.33, we believe our lower value is justifiable as it results in a labor income-share of 0.65, which is exactly

[8]Source: OECD, 2012, Social Expenditure Tables, table "Public and private expenditure on pension".

[9]Source: Education at a Glance 2011: OECD Indicators, table: B2.3.

[10]OECD, 2010, Labor Statistics Database, table "Unit Labour Costs: Labour Income Share Ratios, annual indicators"

[11]We normalize data of hours worked of Statistics Sweden by 72 hours. This might seem a low number of available hours per week, but dividing by this number is required to match the standard assumption in macroeconomic models that agents spend around one third of their time working. When we would divide hours worked by 100 instead, we find that in Sweden agents work a share 0.245 of their time. Rogerson (2006, p. 367) shows that aggregate hours are between 5 and 10 percentage points lower in Sweden than in the United States.

[12]Calculated as the ratio of the maximum and minimum hourly wage during the life cycle. Source of wage profile for Sweden: Statistics Sweden, 2010, Productivity profiles. Available at the Swedish Ministry of Finance.

[13]See Ludwig et al. (2012, footnote 20).

[14]Source: Statistics Sweden, 2010, productivity profiles. Hourly wage rate in SEK, based on full-time salary divided over 172 hours per month. Available at the Swedish Ministry of Finance.

[15]See Domeij and Flodén (2005, p. 7) for a table of capital-output ratios from various sources.

equal to the 2010 labor-income share for Sweden[16]. We calibrate the depreciation rate of capital $\delta = 0.06$ such that the investment to output ratio of our model is around 18%[17,18].

3.2.4 Government Expenditure

The government budget consists of two parts. Firstly, expenditure on education. We set the public share μ in the financing of education costs per unit of time (T) to the OECD estimate for Sweden of 0.89. The cost of education per unit of time (T) is set to approximate the average expenditure on tertiary education as share of GDP for Sweden of 1.59% in 2011.

The government also finances a fixed transfer to the elderly. This pension benefit is equal to $p_{t,j} = \rho w_t$ each year after retirement. We set ρ to match the ratio of public expenditure on old-age pensions to GDP in Sweden. According to OECD[19], the expenditure was 7.6% of GDP in 2008 and 8.2% in 2009. As there is a large difference between the two years, we approximate the average of the two (7.9%) by setting $\rho = 0.205$.

3.3 Equilibrium and computations

Given the set of model parameters, we solve for a general equilibrium as defined in appendix D.1.1. We compute this general equilibrium through outer and inner loop iterations using first-order conditions derived in appendix D.1.

The household problem is solved within the inner loop. We use a nonlinear solver to find the terminal guesses of assets, consumption and human capital (a_J, c_J, and h_J) that maximize the household problem given the initial condition ($a_1 = 0$) and the boundary condition ($a_J = 0$) on assets, and the initial level of human capital (h_1).

The outer loop is used to find the equilibrium wage, interest rate and budget balancing tax rates and replacement rate. A detailed explanation of the Matlab algorithm we use can be found in appendix D.2.

3.4 Comparison to the model of Ludwig et al. (2012)

Model elements	Our model	Ludwig et al. (2012)
1. Population distribution	Exogenous	Population model
2. Pension benefits	Fixed	Indexed for past earnings
3. Survival rates	No	Yes
4. Growth in total factor productivity	No	Exogenous at 2%
5. Borrowing constraints	Yes	No
6. Training cost paid by:	Agents, firms and public sector	Firms

Table 3.3: A comparison between our model and that of Ludwig et al. (2012).

Our model is based on that of Ludwig et al. (2012). Some important differences between our and their model remain, however. This renders a direct comparison of the results in our

[16]Source: OECD Labor Statistics Database, 2013, table: "Unit Labour Costs: Labour Income Share Ratios, annual indicators".

[17]A similar approach is used in Ludwig et al. (2012).

[18]The investment to output ratio is calculated as the sum of depreciated capital over total output.

[19]Data from the OECD Social Expenditure Database, 2012, table "Social Expenditure: Aggregated Data. Public and private expenditure on pension".

work and the results in Ludwig et al. (2012) ill-advised. Table 3.3 gives an overview of the most important differences between the two models.

Deviations

Ludwig et al. (2012) construct a population model that starts in the year 1950 and reaches equilibrium by the year 2200. This allows for the calculation of the welfare effects of a transition towards a population with a higher old-age dependency for cohorts alive in a certain year. We use exogenous population distributions and assume these population distributions are in steady state. This simplifying assumption renders it impossible to calculate the welfare effects of the transition of the population distribution for specific birth cohorts; instead, we compare welfare across different population distributions.

The pension system in Ludwig et al. (2012) adjusts pension benefits for average indexed past yearly earnings. We assume pension benefits paid out by the government are equal for everyone. Our simplified pension system could create a slight bias in our results because it creates an incentive for agents to underinvest in human capital to prevent paying high taxes for a benefit that is equal for everyone if the replacement rate of the pension system is constant. Because the replacement rate is only 20.5% of the wage per unit of human capital and average human capital in our baseline model is 1.5, however, we think that this potential bias is small.

Ludwig et al. (2012) include survival probability in their model, but keep survival rates constant in the welfare comparisons they do. Although survival rates do lead to higher discounting at the late stage of the lifecycle and create some redistribution through accidental bequests, we do not think comparisons between models with different population distributions or subsidy policies will be significantly affected by the fact that we do not include survival rates because their influence would be constant in each run[20]. Also, Ludwig et al. (2012) does and we do not include growth in total factor productivity. The inclusion of exogenous growth in factor productivity is unlikely to change our results as productivity growth would be constant between different runs of the model.

Extensions

We extend Ludwig et al. (2012)'s model in two ways. Firstly, we include borrowing constraints, which allow us to study the impact of market imperfections on the welfare mitigating effects of endogenous human capital. Also, we include tuition costs because we aim to capture lifelong learning, which typically includes tertiary education. Ludwig et al. (2012) assume the cost of training are paid for by firms and therefore these costs do not enter the budget constraint of agents and the government.

[20]In an online appendix to their paper, Ludwig et al. (2012) point out that even varying survival rates barely affects the welfare changes they report in the paper.

Chapter 4

Results

We first present our baseline model and briefly explain the dynamics in it. Then, we replicate Ludwig et al. (2012) for the case of Sweden and show that the welfare effects of an increasing old-age dependency ratio are lower if one allows for endogenous human capital accumulation. After that, we introduce borrowing constraints into the model and calculate the welfare effects of aging when agents are constrained in the amount of credit they can obtain. We end with a policy analysis in which we vary the public share in the cost of education in an economy with and without borrowing constraints.

In appendix C to this thesis, we perform a robustness check on our results by varying the inter temporal substitution elasticity.

4.1 Baseline model

In this section, we evaluate the fit of our baseline model to asset, consumption, labor supply and wage profiles for Sweden. The life time optimal profiles can be found in figure 4.1.

First, we take a look at the asset profile. Agents borrow during the first part of their life to be able to smooth consumption and leisure over their lifecycle. They save until retirement at age 65 and after that deplete their savings until their death at age 90. The asset profile is broadly consistent with net worth data for Sweden[1], but suffers from three discrepancies[2]: (1) agents in the model borrow too much, (2) the peak of assets is too high, and (3) asset profiles for Sweden do not end at zero, i.e. there are bequests. The first discrepancy can be explained by the absence of borrowing constraints from our baseline model[3], the third by the absence of a bequest motive.

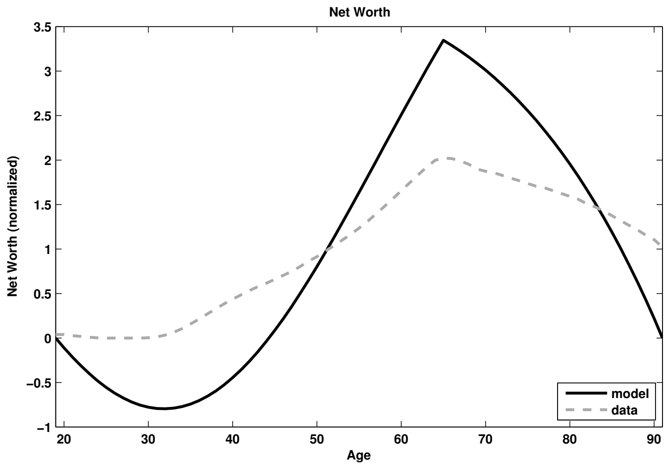
Next, we examine the consumption profile. In the model, consumption is continuously increasing over the lifecycle[4], drops significantly when agents retire and then increases again until age 90. The large drop in consumption can be attributed to the sudden increase in leisure that ensues

[1] Asset data from Förmögenhetsregistret, Statistics Sweden, 2007. Available at the Swedish Ministry of Finance.

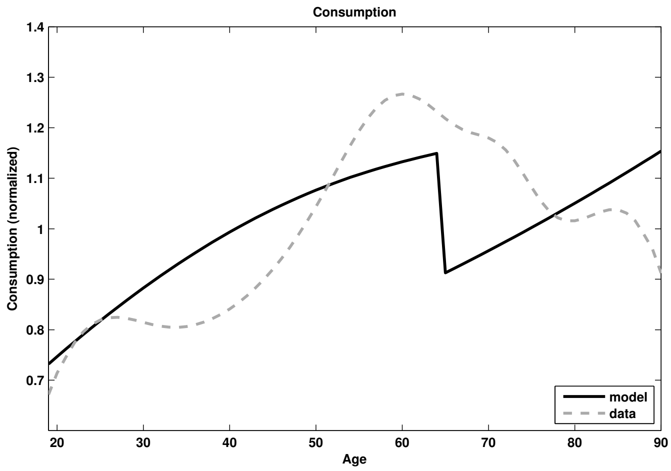
[2] The main discrepancies with net-worth data are the same as the ones Ludwig et al. (2012) point out when comparing the asset vector in their model to net-worth profiles for the United States.

[3] As pointed out in Section 2.4.2 the high public share in the cost of tertiary education in Sweden renders it unlikely that borrowing constraints impact enrollment into tertiary education. Nevertheless, young agents could be constrained in the amount they can borrow for consumption, or might not want to borrow due to uncertainty about future labor earnings.

[4] This is because the time discount factor ($\beta = 0.983$) in the model is higher than the interest rate discount factor ($\frac{1}{1+r} = 0.971$).

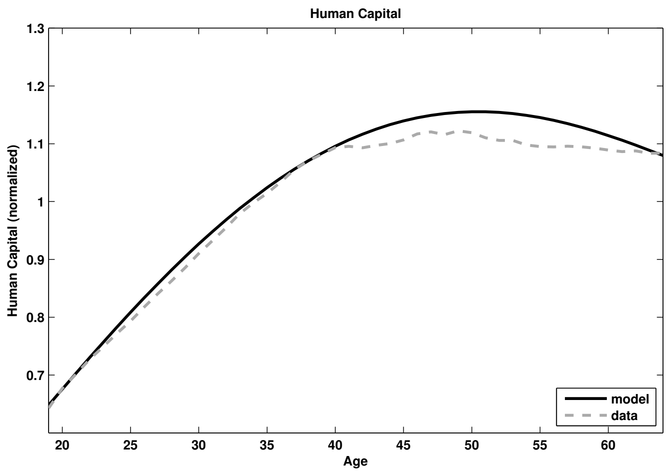


(a) Assets



(b) Consumption

(c) Labor Supply and Educational Time Investment



(d) Human Capital

Figure 4.1: Life time optimal profiles of (a) asset accumulation; (b) consumption; (c) labor supply and educational time investment; and (d) human capital accumulation of the agent. All variables (except labor supply and educational time investment) are normalized by their means.

when agents retire; the utility gain from increased leisure makes that agents attain the same level of utility with less consumption.

The consumption profile is not consistent with empirical evidence for Sweden[5]. In the data, consumption rises discontinuously, peeks at an age of 60 instead of 65 and declines from age 60 onwards. This mismatch between the consumption profile generated by the model and the profile in the data is no reason for concern. We base our results on differences between consumption profiles. Ludwig et al. (2012, p. 100) point out that these differences between profiles are unlikely to be affected by the shape of the profile.

Now, we consider the labor supply profile. Agents invest more time in education than in work at first. As the period during which returns to training can be collected becomes shorter and forgone wage costs increase, they decrease their educational time investment and increase la-

[5]The consumption profile is obtained through spline interpolation on estimations by Domeij and Johannesson (2006, table 1). We compute a profile for the latest birth-year available: 1970 or older.

bor supply until both curves intersect at age 26.4. Labor supply increases to a maximum of 0.44 at age 52, after which it declines again as a result of the decreasing level of human capital which lowers wage per hour worked. Educational time investment gets close to zero when retirement is imminent. The model approximates hours worked data for Sweden[6], but the fit is not perfect. In the model, hours worked peak at age 52; in the data, the peak occurs at age 47[7].

We assume a unisex model[8]. This choice for a model that applies to men and women means that the labor supply elasticity in our model is relatively high[9,10]. We use the Frisch labor supply elasticity[11] as measure of the elasticity of labor supply. This measure captures the response of labor supply to a change in income holding the marginal utility of wealth and time invested in human capital formation constant. In our base model, the Frisch labor supply elasticity is 0.95 for agents aged between 30 and 50. Amongst younger and older ages, the elasticity is higher. The average elasticity for agents aged between 20 and 60 is 1.37. Across all agents, the average Frisch elasticity is 1.5. The reason for the difference in elasticity between the age groups 30-50 and 20-60 is that workers enter into and exit from the labor market - the extensive margin of labor supply[12]. Browning, Hansen and Heckman (1999) point out that traditional estimates of the labor supply elasticity ignore labor supply responses at the extensive margin. We set out to match the labor supply profile for ages 19-65, ages that include significant amounts of entry into and exit from the labor market. These movements in and out of the labor market explain the low hours at the early and late part of work life. Consequently, the Frisch labor supply elasticity in our model is higher for young and old agents[13].

Several biases can arise when estimating the intertemporal labor supply elasticity[14]. Failure to account for endogenous human capital accumulation is one of them. Imai and Keane (2004) point out that agents have two returns to work: their wage and returns to learning by doing on the job, which increase future wage. The returns to learning by doing are high during the early

[6] Source: Statistics Sweden, 2011, Arbetskraftsundersökningarna (AKU). Available at the Swedish Ministry of Finance.

[7] The standard assumption in labor economics models is that training occurs on the job. As such, the sum of labor supply and time investments in on-the-job training is usually matched to fit hours worked data. We cannot do this as we assume educational time investment to include investments in tertiary education, which do not occur on-the-job.

[8] As do Ludwig et al. (2012).

[9] Estimates of the intertemporal labor supply elasticity based on hours worked data for men typically range between 0.1 and 0.4 (Browning et al., 1999, p. 552). For female labor supply, estimates are much higher; Browning et al. (1999, p. 552) report an estimate of 1.61.

[10] MaCurdy (1981) was the first to use an empirical model to estimate the intertemporal labor supply elasticity. He used data for prime-aged males and found low estimates of the intertemporal elasticity of labor supply; ranging between 0.1 and 0.5.

[11] We use the standard Frisch elasticity calculation provided by Ludwig et al. (2012) on page 5 of the online appendix to their paper. ϵ denotes the elasticity. The calculation is:

$$\epsilon^j = \frac{1-\phi(1-\sigma)}{\sigma}\frac{1-l_j-e_j}{l_j} \tag{4.1}$$

[12] The intensive margin is hours worked when working.

[13] Rogerson and Wallenius (2009) construct a model of endogenous retirement through a non-linear mapping of hours worked to labor supply. This model enables them to consider responses on both the intensive and the extensive margin of labor supply when income changes. They point out that micro-elasticity estimates only capture an inconstant - it depends on the micro elasticity - part of the adjustment in hours, because these elasticities do not account for adjustments at the extensive margin. The higher Frisch elasticities at the beginning and end of the worklife of agents in our model implicitly capture these adjustments at the extensive margin.

[14] Domeij and Flodén (2006, p. 245) mention borrowing constraints, human capital accumulation, home production, progressive taxes, and failure to account for changes in labor supply on the extensive margin as potential sources of bias in labor supply elasticity estimates.

part of the lifecycle and decline during later years. Not accounting for this type of human capital accumulation leads to a downward bias in labor supply elasticity estimates as wages increase substantially over the lifecycle whereas hours worked are relatively stable if one considers data for prime-aged males who have finished school[15]. When accounting for human capital accumulation, total returns to labor supply are more constant over the lifecycle. Consequently, labor supply elasticity estimates will go up[16].

When accounting for human capital accumulation in the calculation of our Frisch labor supply elasticity estimate[17], we find that elasticity for ages 30-50 increases by a factor 1.7 to 1.66. Elasticity for ages 20-60 increases by a factor 2.1 to 2.88. Across all agents, the average Frisch elasticity increases by a factor 2.1 to 3.25. The increase in the labor supply elasticity estimates we get when accounting for human capital is slightly smaller than the factor 2.6 increase Wallenius (2011) finds when accounting for human capital formation through Ben-Porath on-the-job training.

Finally, we evaluate the human capital profile, which shows the hourly wage of agents in our model. Human capital increases rapidly during the early part of the lifecycle as educational time investments are high. From age 51 on, educational time investment is not high enough anymore to offset depreciation in human capital and human capital starts to decline. Our model fits wage data for Sweden very well[18].

One additional graph of our baseline model can be found in Appendix B.1; it shows the evolution of leisure, labor supply and educational time investment during the work-life of agents.

4.2 Increasing old-age dependency

Demographic developments will put significant pressure on the finances of the Swedish state. Figure 4.2a shows that the ratio of retirees to the total population in Sweden will rise by 13 percentage points from 28.4% in 2010 to 41.0% in 2050. Figure 4.2b also shows that the number of persons alive aged over 70 in Sweden will be much higher in 2050 than it was in 2010.

The rise in old-age dependency can be explained by a rise in life expectancy, the retirement of the baby boom generation, and a decrease in fertility. In reference to the paper on which our model is based, Ludwig et al. (2012), we also show the development of the old-age dependency

[15]This is typically the case. Many estimates of the labor supply elasticity are based on hours worked data for prime-aged males, who have finished school, and are employed during the whole sample period (Browning et al., 1999)

[16]Imai and Keane (2004) report an estimate of the intertemporal labor supply elasticity of 3.820 for white males. Wallenius (2011) shows that the out of sample performance of Imai and Keane (2004) is bad. Imai and Keane (2004) base their estimate on data for ages 20-36. When Wallenius (2011, p. 578) only uses this age range, her estimates of the intertemporal elasticity of substitution of labor supply increase by a factor 5.3 when accounting for human capital in the elasticity calculation in a model with human capital accumulation through learning by doing. When considering ages 20-62, this factor decreases to 2.1.

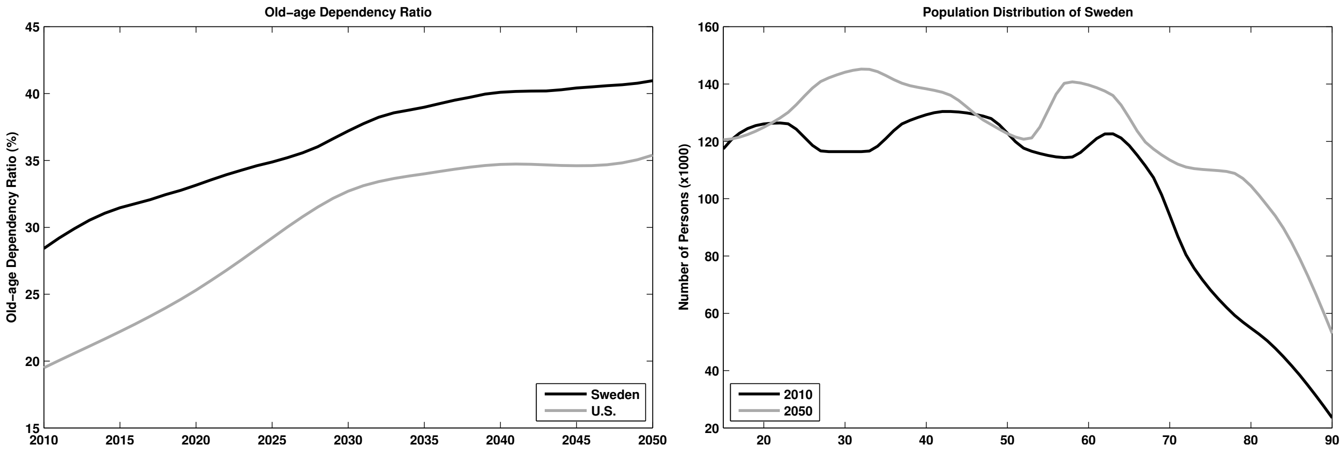
[17]Now, the calculation becomes:

$$\epsilon^j = \frac{1-\phi(1-\sigma)}{\sigma}\frac{1-l_j-e_j}{l_j} + \frac{1}{1-\psi}\frac{e_j}{l_j} \tag{4.2}$$

[18]Source wage data: Statistics Sweden (SCB), 2010, Hourly wage rate in SEK, based on full-time salary divided over 172 hours per month. Available at the Swedish Ministry of Finance.

[19]Source: United Nations, World Population Prospects, the 2010 Revision, table: 'Old-Age Dependency Ratio (Age 65+ / Age 15-64)'. Interpolated on the 5 year interval of the data.

[20]Source: Statistics Sweden, 2013, own calculations based on *Sveriges framtida befolkning 2013-2060 (The future population of Sweden 2013-2060).* Table: 9. "Population by age 1960-2012 and forecast 2013-2060".



(a) Old-age dependency ratio **(b)** Population distribution

Figure 4.2: (a) Old-age dependency ratios for Sweden[19] and the U.S.[20] Period: 2010-2050.; (b) Population distribution by age (age range 15-90) for Sweden in 2010 and 2050[20] .

ratio in the United States. There, old-age dependency rises even more; it increases by 16 percentage points, from 19.5% in 2010 to 35.4% in 2050.

Funding the costs of aging

Public pension benefit systems are typically not fully funded[21]. Therefore, increased old-age dependency means that an increasingly small group of workers will need to pay for the retirement benefits of an increasingly large group of retirees[22]. To ensure sustainability of government finances, the government can choose between three options if it wants to keep expenditures in other areas constant: (1) decrease pension benefits to keep the costs of the pension system constant, (2) increase tax rates to keep the replacement rate constant, (3) a mix of the two options.

For the sake of clarity we only analyze the extreme cases of (1) constant contributions to the pension system and (2) a constant replacement rate. Either option will lead to significant welfare losses: if contributions to the pension system are kept constant, retirees will suffer from a decrease in their pension. If replacement rates are kept constant, workers will suffer from a higher tax rate. We compute the welfare effects of each of the two policy options for two cases, one in which agents can adjust their human capital investment and one in which they cannot adjust it. Based on Ludwig et al. (2012) we expect that endogenous human capital can mitigate some of the negative welfare consequences of aging.

For the sake of intuition, we will first present graphs that show changes in agent decisions and macro-economic variables when moving from an old-age dependency ratio of 28.4% in 2010 to an old-age dependency ratio of 41.0% in 2050 in the case of exogenous and endogenous human capital. For the sake of brevity, we only present these transition dynamics for case (2), constant replacement rates[23]. After that, we explain why we cannot compare our results to the ones

[21] Total contributions to the pension system in Sweden are 18.5% of "pensionable income" of workers. Around 85% of the total contributions goes to current retirees - the pay-as-you-go part of the pension system. Source: Swedish Ministry of Health and Social Affairs, 2000, retrieved from `http://www.globalaging.org/pension/world/sweden.pdf`.

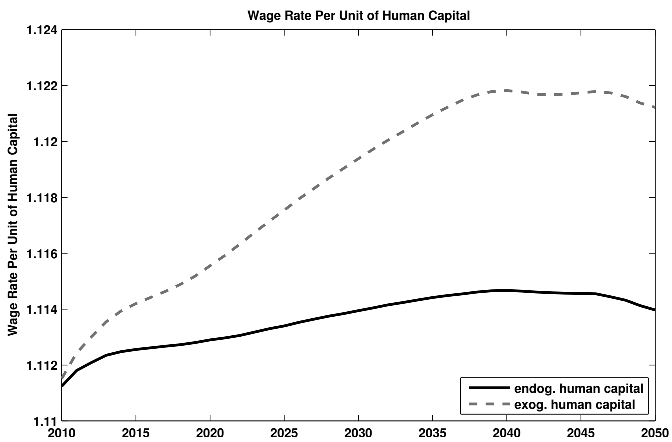
[22] Of course, state expenditures on health care will increase as well when the share of retirees in the total population increases. Health care, however, is not the focus of this thesis.

[23] The transition dynamics are similar for the case of constant contribution rates. Graphs are available from the

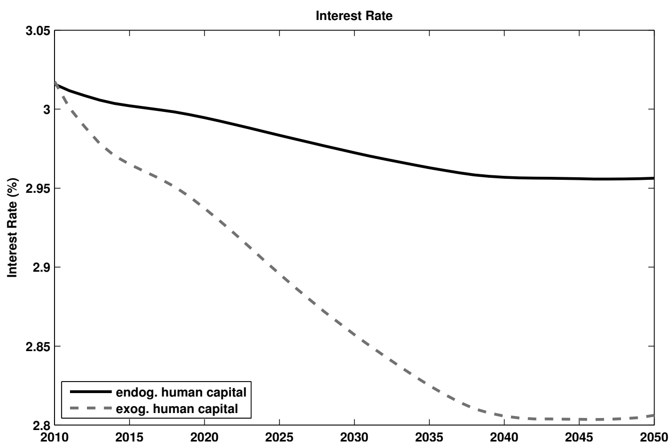
presented in Ludwig et al. (2012). Then, we present the welfare effects we get for a constant contribution and a constant replacement rate scheme with endogenous and exogenous human capital.

4.2.1 Intuition: the transition from 2010 - 2050

It is 2010, the old-age dependency ratio of Sweden is at 28.4%. During the coming 40 years, old-age dependency will increase each year. When old-age dependency rises, the fraction of the population that is retired increases, while the fraction that is of working-age declines. Retirees generally hold high levels of capital[24], but they do not work. As a result, the amount of capital relative to labor in the economy will increase. This affects wages and interest rates. As figure 4.3 shows, wage - the marginal product of labor - will increase, whereas interest rates - the marginal product of capital - will decrease. The effects are more pronounced in the case of exogenous human capital.



(a) Wage rate



(b) Interest rate

Figure 4.3: The development of (a) Wage rate per unit of human capital; (b) Interest rate (%) over time. Period: 2010-2050.

When human capital is exogenous, agents cannot adjust their investment in education. This is subobtimal if wages increase and interest rates decrease. After all, when wages go up, returns to investment in education are higher than before. Moreover, it becomes cheaper to borrow to finance the costs of investment in education when interest rates decline. When human capital is endogenous, then, agents will increase their educational investment, as can be seen in figure 4.4. The increase is substantial. In 2050, agents invest 8.7% more time in education than in 2010.

Increased investment in education also means that the period in which agents spend most of their non-leisure time in training becomes longer. In 2010, agents start to work more than they study at age 26.4. In 2050, this age increases by one year to 27.4.

authors upon request.

[24]See the asset profile in Figure 4.1a.

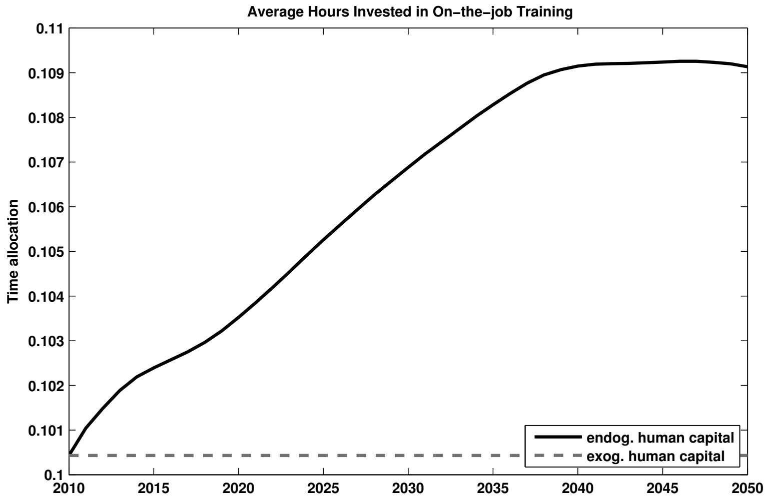


Figure 4.4: Average educational time investment in the case of exogenous and endogenous human capital. Period: 2010-2050.

Increased investment in human capital has general equilibrium effects: wage per unit of human capital decreases when the level of human capital increases. Interest rates increase when agents borrow more to finance their studies. This explains the lower increase in wage and the lower decrease in interest rates in case of endogenous human capital in figure 4.3.

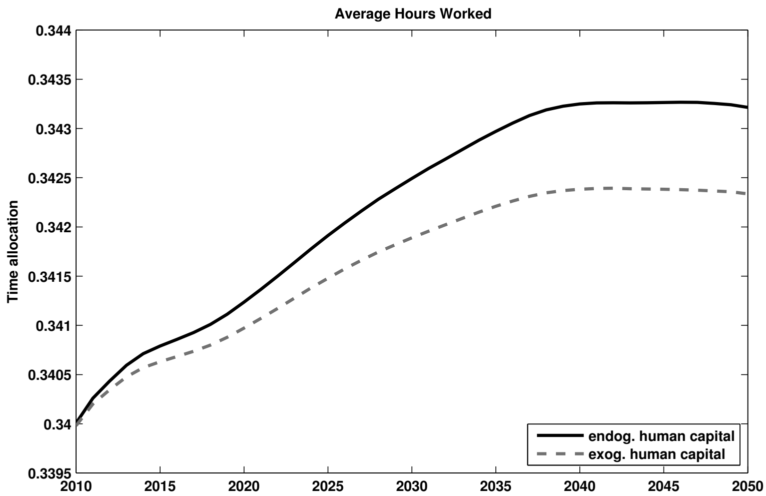


Figure 4.5: Average hours worked in the case of exogenous and endogenous human capital. Period: 2010-2050.

Higher wage has a positive effect on labor supply. Figure 4.5 shows that average hours worked increases most in the case of endogenous human capital. This might seem remarkable at first sight. Wage increases are larger in the case of exogenous human capital in figure 4.11a. The explanation is that in the case of endogenous human capital accumulation, the increases in human capital following higher educational time investment more than offset the wage decreasing effect of a higher stock of human capital on labor supply.

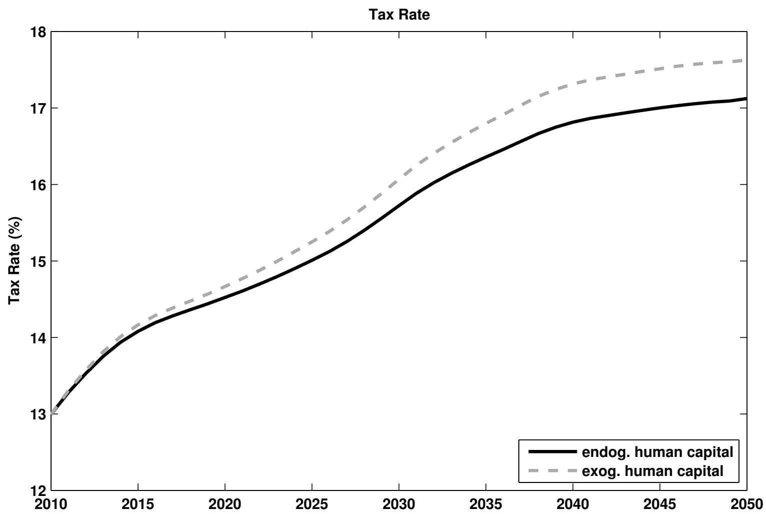


Figure 4.6: Tax rate in the case of exogenous and endogenous human capital. Period: 2010-2050.

Higher levels of human capital are good news for a government that tries to keep replacement rates in the pension system constant. More investment in human capital means, *ceteris paribus*, that tax revenue will increase without adjustments in the tax rate. Consequently, a government that has to deal with growing costs of the pension system does not need to increase tax rates as much when agents are free to choose their investment in education. Figure 4.6 shows this effect: the tax rate increases to 17.6% in 2050 when human capital is exogenous; this is 17.1% in case of endogenous human capital.

We all know that free lunch does not exist. The government finances a large share of investments in education in Sweden. Increased investment in education means that more tax needs to be collected to pay for the higher cost of education. Ludwig et al. (2012) do not have this counteracting effect of rising educational costs on tax rates in their model, as they assume training costs are inter firm transactions.

As shown in figure 4.7, aging has a negligible effect on GDP per capita when educational time investment is allowed to adjust[25]. Many effects are at work here. In essence, the decrease in total assets per capita over the lifecycle caused by lower interest rates and increased borrowing is offset by an increase in labor supply and human capital caused by higher wage.

When human capital is exogenous, total assets per capita decline further following a stronger interest rate decrease, labor supply increases less[26] and human capital cannot adjust. Consequently, GDP per capita drops. In 2050, GDP per capita is 3.4% lower compared to 2010 in case human capital is exogenous.

Finally, we evaluate the welfare effects of a higher old-age dependency ratio. We measure welfare by consumption-equivalent variation, which shows by how much consumption would need to change to make an agent equally well of in a new situation, compared to a benchmark case[27]. If consumption needs to increase, welfare in the new situation is better than in the benchmark case, and vice versa.

There are two main channels through which increased old-age dependency affects welfare. Firstly, fewer workers have to pay for the retirement benefits of more retirees. In case retirees

[25]Note that we do not include growth in total factor productivity into our model. As a result, GDP per capita does not grow over time.

[26]See figure 4.5.

[27]See Appendix D.4 for an explanation of our calculation of consumption-equivalent variation.

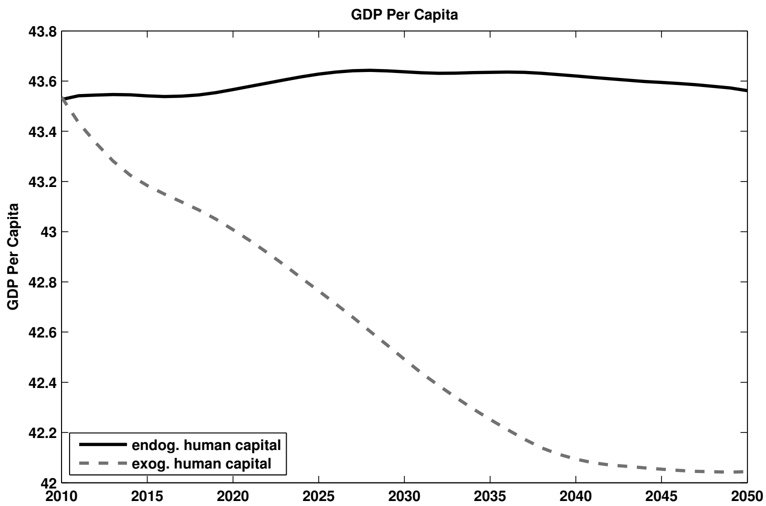


Figure 4.7: GDP per capita in the case of exogenous and endogenous human capital. Period: 2010-2050.

are spared a cut in their benefits - i.e. replacement rates are kept constant - the government will need to increase taxes to balance its budget. More social security is a distortion to the economy as it does not have an insurance effect in our model[28]. Secondly, a higher old-age dependency means that the stock of capital relative to the stock of labor in the economy will increase. A higher capital to labor ratio means that wages will increase and interest rates decrease. Lower interest rates decrease welfare because the return on savings goes down.

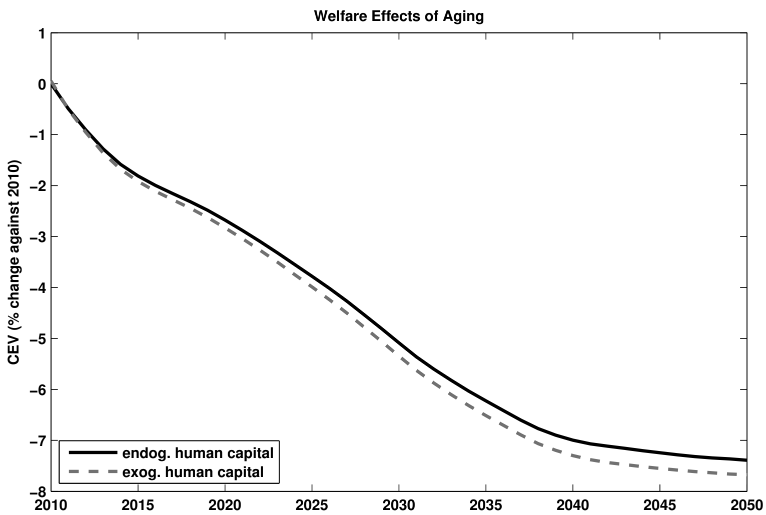


Figure 4.8: Change in consumption-equivalent variation (CEV) in the case of exogenous and endogenous human capital. Period: 2010-2050.

Figure 4.8 shows that welfare decreases sharply when old-age dependency rises. As expected, the welfare decrease is slightly smaller when agents can adjust their educational time investment. Welfare decreases by 7.38% compared to 2050 when human capital is endogenous; it decreases by 7.68% when human capital is exogenous.

[28]See section 4.2.2 under "Welfare effects".

4.2.2 Policy experiments - two social security scenarios

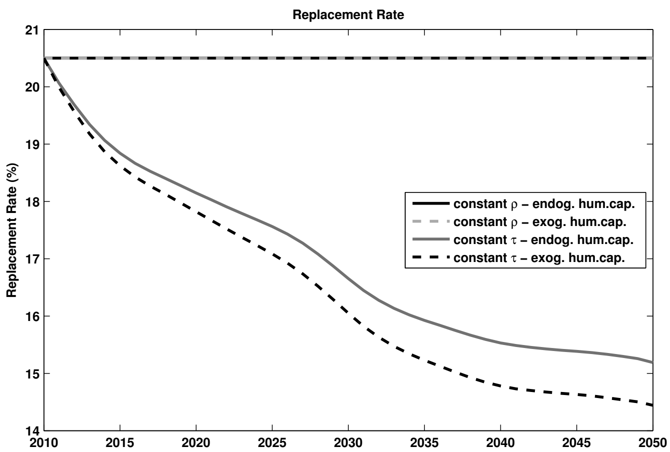
Up till now, we presented the results of one social security policy option, that of constant replacement rates. Ludwig et al. (2012) consider two options: a constant replacement rate and a constant tax rate. They show that the *maximum* welfare loss from aging for generations (birth cohorts) alive in 2005 under both scenarios is substantially mitigated when agents can adjust their educational time investment.

In case of constant replacement rates, Ludwig et al. (2012) find a maximum loss in consumption-equivalent variation of 5.6% when human capital is exogenous; this loss is highest for young agents as they have to pay higher taxes to fund the pension system during the rest of their work life. The maximum loss is reduced to 4.4% when human capital is endogenous.

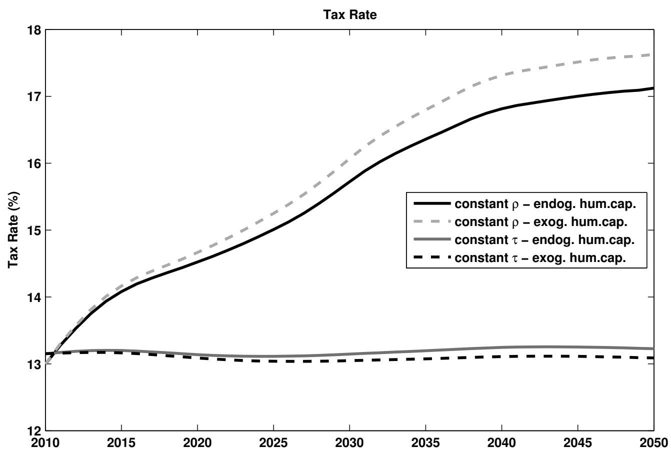
In case of constant tax rates, they find a much larger maximum loss in consumption-equivalent variation of 12.5% when human capital is exogenous. This time, losses are concentrated amongst agents aged between 45 and 65. These agents suffer both from a loss in interest income and lower pension benefits than expected when they retire. The maximum loss is reduced substantially to 8.7% when human capital is endogenous.

Two social security extremes

We perform the same social security experiment as Ludwig et al. (2012): a constant replacement rate and a constant contribution rate. This allows us to show how borrowing constraints would alter their analysis. A one for one comparison between our results and those obtained by Ludwig et al. (2012) is not possible, however. The most important reason is that Ludwig et al. (2012) have an endogenous population model. We use exogenous population distributions and assume population is in steady state. Also, Ludwig et al. (2012) assume education costs are paid for by firms. In our model, the government and agents themselves also pay for the cost of education[29].



(a) Replacement rate



(b) Tax rate

Figure 4.9: The development of (a) the replacement rate and (b) the tax rate and over time in both social security scenarios in the case of exogenous and endogenous human capital. Period: 2010-2050.

Figure 4.9 shows the evolution of replacement and tax rates for both social security scenarios when human capital is fixed and when agents can adjust their human capital investment. Endogenous human capital reduces the severity of both social security policy options.

[29]Section 3.4 gives a complete comparison between our model and that of Ludwig et al. (2012).

In the case of fixed contributions to the pension system, τ is fixed at $\bar{\tau} = 0.13$. When we fix the level of human capital agents have, the replacement rate falls from 20.5% of wage per unit of human capital in 2010 to 14.4% in 2050. The fall in replacement rates is smaller when agents can adjust their educational time investment. Then, the replacement rate drops to only 15.2% in 2050.

In case of fixed replacement rates, ρ is fixed at $\bar{\rho} = 0.205$. In case of exogenous human capital, the tax rate increases from 13.0% in 2010 to 17.6% in 2050. The increase is smaller when human capital adjustments are allowed for. Then, the tax rate increases to 17.1% in 2050.

Detailed results of the constant replacement rate and constant contribution rate scenarios can be found in table B.1 and B.2 in the appendix to this thesis.

Welfare effects

	Constant replacement rate ($\rho_t = \bar{\rho}$)		Constant contribution rate ($\tau_t = \bar{\tau}$)	
	Average	Maximum	Average	Maximum
Endog. human capital	-7.38%	-10.50%	-4.62%	-8.24%
Exog. human capital	-7.68%	-11.85%	-4.44%	-9.88%

Table 4.1: Welfare effects of aging (consumption-equivalent variation). Differences with respect to baseline case of 2010 ($OAD = 28.4\%$). Differences are percentage changes.

When old-age dependency increases, average welfare decreases significantly both in the case of constant replacement rates and constant contribution rates. Table 4.1 shows that both average and maximum welfare decreases are always larger when the government decides to keep replacement rates constant. This can be explained by the fact that there is no risk in our model.

In this model, agents do not have an uncertain lifespan and there is no uncertainty about labor earnings. This means that publicly provided pension benefits cannot improve efficiency by functioning as insurance[30]. As a result, social security is only distortionary. It disincentives agents to save for retirement and thus artificially increases interest rates in the economy[31]. The relatively lower pension benefits resulting from a constant contribution rate are therefore better than a system in which contribution rates increase to keep replacement rates constant.

Now, when replacement rates are kept constant, average and maximum welfare decrease most if agents cannot adjust their educational time investment. When old-age dependency increases, the capital to labor ratio in the economy increases and as a result, wages go up. Because wages go up, it would be optimal for agents to increase their educational time investment. This explains why the average and maximum welfare decreases are less strong when agents can increase their educational time investment.

When we fix contribution rates, free adjustments in human capital do not lead to lower average welfare losses. The reason is that size of the pension system as a share in GDP remains fixed both in the case of exogenous and endogenous human capital. The higher investment in education when human capital is endogenous now leads to higher tax rates because government expenditures on education rise and the contribution rate to the pension system remains constant. Subsidies to education are a distortion to the economy in absence of market imperfections such

[30] Huang et al. (1997) show that this insurance property of publicly provided social security can make a government social security scheme with its own physical capital - built up through a high tax on labor supply during a transition period - preferable over a situation in which the government does not provide social security at all.

[31] Auerbach et al. (1989, p. 61) already pointed out this distortionary effect of a pay-as-you-go pension system.

as borrowing constraints. Consequently, the rise in expenditure on education when agents can freely choose their educational time investment leads to a welfare loss.

4.2.3 Conclusion

By comparing population distributions in 2010 and 2050, we show that a rising old-age dependency ratio will have large negative welfare effects for Sweden both when replacement rates and when contribution rates are kept constant. The welfare consequences of demographic change are much larger in the case of constant replacement rates. If instead contribution rates are fixed and the replacement rate adjusts, negative welfare effects are lower.

Increased investment in human capital can to some extent mitigate the welfare loss when replacement rates are constant, even when education is costly to agents and the government. Increased investment in human capital does not lead to lower welfare losses when contribution rates are kept constant, because additional subsidies to education would raise the tax rate.

4.3 Borrowing constraints

In this section, we extend Ludwig et al. (2012)'s model to include borrowing constraints. We show that a credit constraint that is not binding in 2010 can become binding over time when old-age dependency increases. It turns out that this mildly binding borrowing constraint nullifies the welfare mitigating effects that endogenous human capital accumulation has when the old-age dependency ratio in a country is rising.

Before moving on to this policy experiment, we will use an example of a very binding borrowing constraint to show graphically how credit constraints affect the time and resource allocation decisions of agents in our model. The method we use to implement borrowing constraints is described in appendix D.2.2.

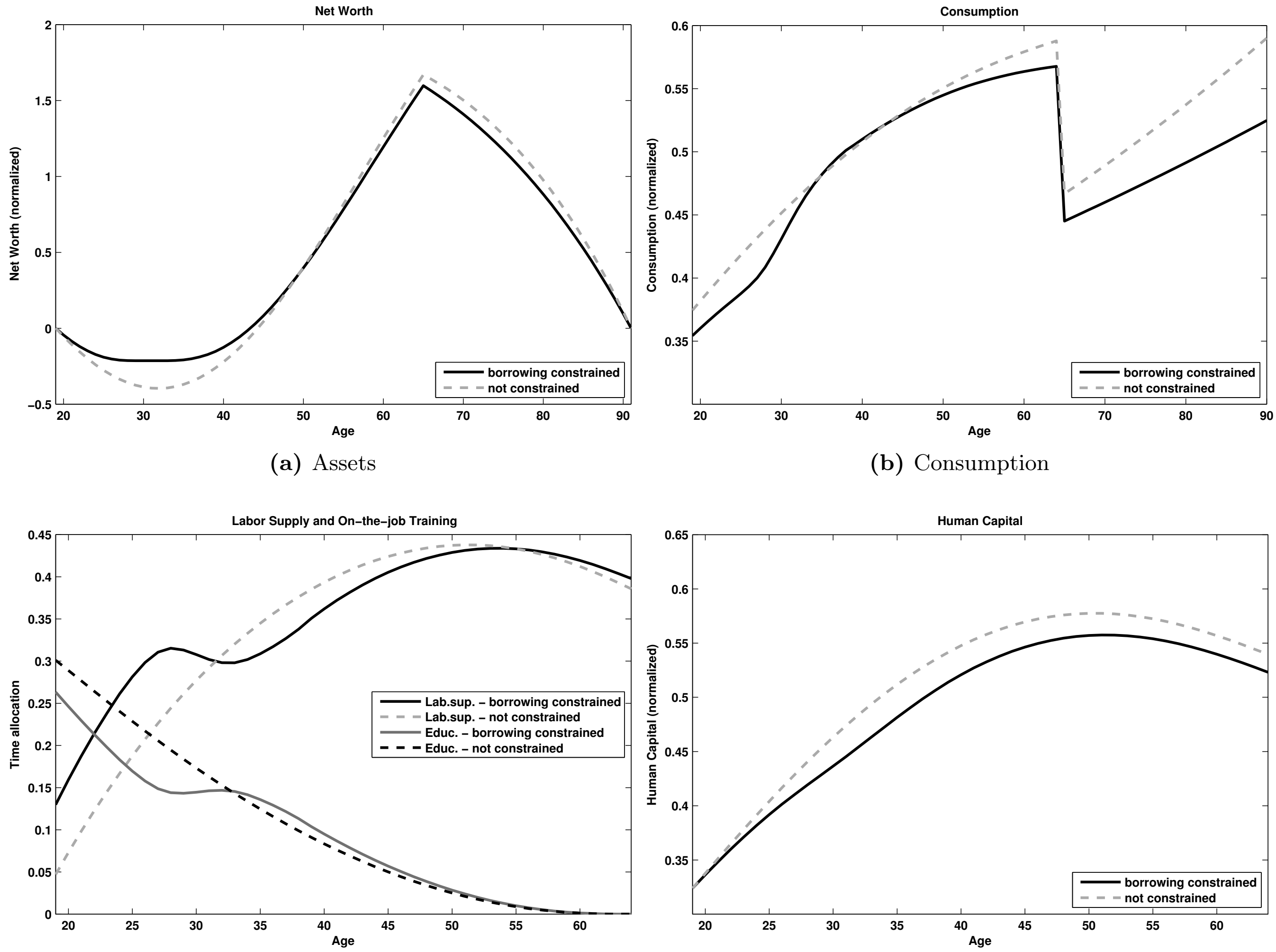
4.3.1 Intuition

Figure 4.10 shows how a negative, but quite binding borrowing constraint affects the time and resource allocation decisions of agents in our model in the year 2010. Clearly, agents cannot borrow as much as would be optimal when borrowing constraints are binding. This leads to the higher minimum of assets in figure 4.10a compared to a situation without borrowing constraints. Less borrowing means that interest rates in the economy will decrease. This lowers asset accumulation.

When agents cannot borrow as much as would be optimal, it becomes difficult for them to smooth consumption over the lifecycle. Figure 4.10b shows that consumption is lower during the period in which agents are borrowing constrained compared to the amount consumed by agents who do not experience constraints on credit. At the same time, figure 4.10c shows that labor supply is higher during the early part of the lifecycle in order to finance consumption and educational time investment.

When borrowing constraints are binding, agents cannot borrow enough to do the same educational time investment as agents who are not borrowing constrained. Consequently, they postpone their training[32]. This delay of educational time investment affects the age from which agents spend more time at work than in training. With borrowing constraints, the transition

[32]The finding that agents postpone their investment in education is in line with Kane (1996, p. 185, 188). He shows that borrowing constraints cause a delay in enrollment into tertiary education in a comparison of enrollment rates across states with different tuition levels in the U.S.



(a) Assets

(b) Consumption

(c) Labor Supply and Educational Time Investment

(d) Human Capital

Figure 4.10: Lifetime optimal profiles of (a) asset accumulation; (b) consumption; (c) labor supply and educational time investment; and (d) human capital accumulation of agents in case of no borrowing constraints and a tight constraint on credit. All variables (except labor supply and educational time investment) are normalized.

from school bench to desk already takes place at age 22.1; in the absence of borrowing constraints, this age is 26.4.

From age 33 on, credit constrained agents increase their investments in education to a level above that of agents who are not credit constrained. This catch-up in training is not sufficient, however, to ensure that the level of human capital of credit constrained agents becomes equal to that of agents who are not credit constrained. As their level of human capital is lower, labor supply is lower as well. Consumption increases when borrowing constraints are not binding anymore[33]. When agents get closer to retirement, the catch-up effect in educational time investment slowly diminishes and labor supply increases with the rising level of human capital of agents.

With the basic response of agents to credit constraints in mind, it becomes easier to understand the combined effects of borrowing constraints and aging in our policy experiment.

[33] Note that the slope of the consumption graph is different in case of borrowing constraints because the interest rate is lower.

4.3.2 Policy experiment - borrowing constraint fixed at 2010 asset minimum

Ludwig et al. (2012, p. 106) ended by remarking that the welfare improving adjustment of human capital in response to higher old-age dependency could be hampered by "market imperfections, such as borrowing constraints". They went on to write that these market imperfections and policies to overcome them are an important topic for future research. We perform an experiment to show how borrowing constraints can inhibit human capital adjustments following aging.

Suppose the maximum amount that students can borrow is fixed at the 2010 minimum of assets. When old-age dependency increases, wages will go up and interest rates will go down. Higher wages raise the returns to education. Lower interest rates decrease the costs of borrowing to finance training. Consequently, young agents would like to invest more of their time and money in education. Despite the lower interest rate, they need to borrow more to smooth consumption and leisure over their lifecycle and finance the additional tuition costs. This is not possible when credit constraints are binding. As a result, credit constrained agents cannot increase their educational time investment as much as would be optimal given the higher returns to training. This lack of adjustment in educational time investment has negative welfare consequences.

Welfare effects

	Constant replacement rate ($\rho_t = \bar{\rho}$)		Constant contribution rate ($\tau_t = \bar{\tau}$)	
	Average	Maximum	Average	Maximum
No borrowing constraint				
Endogenous human capital	-7.38%	-10.50%	-4.62%	-8.24%
Exogenous human capital	-7.68%	-11.85%	-4.44%	-9.88%
Borrowing constraint (2010 asset min.)				
Endogenous human capital	-8.23%	-12.00%	-6.02%	-10.93%
Exogenous human capital	-8.77%	-13.11%	-7.33%	-17.44%

Table 4.2: Welfare effects of aging (consumption-equivalent variation). Year: 2050 (OAD=41.0%). Differences with respect to baseline case of 2010 ($OAD = 28.4\%$). Differences are percentage changes.

Table 4.2 shows that welfare decreases due to borrowing constraints are substantial[34]. A mild borrowing constraint[35] is even worse for welfare than keeping human capital exogenous. Consider the case of endogenous human capital. When the replacement rate is kept constant, the welfare decrease worsens by an additional 0.85 percentage points when agents are credit constrained. When the contribution rate is kept constant, this additional decrease is 1.4 percentage points. Additional decreases are slightly higher when human capital is exogenous.

To understand what is driving these extra decreases in welfare when credit constraints are binding, we compare changes in agents' savings and time investment decisions and changes in macro-economic variables both in the case of no borrowing constraints and the case of a

[34] See Appendix B.4 for a plot of the development of welfare between 2010 and 2050 in case borrowing constraints are binding and in case there is no borrowing constraint. The policy considered in the graph is a constant replacement rate.

[35] We fix the borrowing constraint at the 2010 minimum of assets, which is 18% higher than the minimum of assets in 2050 when there are no credit constraints and the replacement rate is kept constant. This borrowing constraint is 29% higher than the 2010 minimum of assets in case of a constant contribution rate.

borrowing constraint fixed at the 2010 minimum asset level in our baseline model. To avoid repetition, we only show the case of endogenous human capital.

Detailed results

Borrowing constraints inhibit educational time investment. Table 4.3 shows that the increase in educational time investment following higher old-age dependency is about half as large when borrowing constraints are binding in both social security scenarios. Although human capital increases by less in a world with credit constraints, average labor supply increases more strongly because agents work more when they are borrowing at the credit limit[36]. These two factors, suboptimal educational time investment and higher than optimal labor supply, are responsible for the large drop in welfare from aging when credit constraints are binding.

At the macro-economic level, factor prices change more when borrowing constraints are binding compared to a case without credit constraints. The reason is again the suboptimal investment in education. Wage per unit of human capital would rise by less when investments in human capital are higher; interest rates would decrease by less if agents would be able to borrow more than the credit constraint. The effects on tax and replacement rates are intuitive. Because income per capita decreases when credit constraints are binding, tax rates increase by slightly more and replacement rates decrease by slightly more.

	Constant replacement rate ($\rho_t = \bar{\rho}$)		Constant contribution rate ($\tau_t = \bar{\tau}$)	
	No constraint	Borrowing constraint	No constraint	Borrowing constraint
Educational time investment	8.66%	4.29%	11.53%	5.06%
Labor supply	0.94%	1.74%	2.02%	3.55%
Wage per unit of human capital (w)	0.25%	0.91%	0.16%	1.21%
Interest rate (r)	-0.06	-0.20	-0.04	-0.26
Tax rate (τ)	4.13	4.29	0.08	-0.03
Replacement rate (ρ)	-	-	-5.31	-5.70
National income per capita	0.08%	-0.99%	2.40%	1.09%

Table 4.3: Expected consequences of aging on agents' decisions and macro-economic conditions. Year: 2050 (OAD = 41.0%). Borrowing constraint fixed at the 2010 minimum asset level. Human capital and labor supply are endogenous. Differences with respect to baseline case of 2010 (OAD = 28.4%). Differences are percentage changes, with the exception of interest, replacement, and tax rate differences which are in percentage points.

4.3.3 Conclusion

Ludwig et al. (2012) conclude that endogenous human capital can alleviate some of the negative welfare effects of higher old-age dependency. They warned in the conclusion to their paper that this alleviating effect could be reduced if borrowing constraints prevent agents from investing more in education. When we introduce a borrowing constraint fixed at the 2010 minimum asset level in a model based on Ludwig et al. (2012), we find that this constraint becomes binding when old-age dependency increases. In the year 2050, the welfare effects of aging compared to

[36]Note that this effect is also, to a small extent, caused by the stronger increase in wages when educational time investment is subobtimal.

the year 2010 worsen by at least 0.85 percentage points when we introduce a credit constraint. This means that binding credit constraints can severely hamper the welfare improving effect of increases in human capital investment when old-age dependency is rising.

4.4 Educational subsidies

We now present a policy analysis in which we vary the public share in the cost of education. The public share in the baseline cases is set at $\mu = 0.89$, the current proportion of tertiary education financed publicly in Sweden[37]. We analyze varying the share by 10 percentage points, to $\mu = 0.79$ and $\mu = 0.99$. Results are presented in table 4.4 for the population distribution of the year 2010.

	Partial equilibrium		General equilibrium	
	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)
Educational time investment	-7.59%	7.85%	-2.44%	2.46%
Labor supply	-0.80%	0.71%	-0.04%	0.03%
Wage per unit of human capital (w)	-	-	0.17%	-0.14%
Interest rate (r)	-	-	-0.03	0.03
Tax rate (r)	0.20	-0.16	-0.13	0.14
National income per capita	-2.30%	2.21%	-1.04%	1.07%

Table 4.4: Varying the public share in the cost of education from the baseline case in 2010 ($\mu = 0.89$, $OAD = 28.4\%$). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

There is a positive relationship between the public share and the time investment in education. The table shows that the effect of changing the share is largest in partial equilibrium when prices are assumed to stay constant. In general equilibrium, the increase in educational time investment from a higher public share affects the wage and interest rate. As the stock of human capital increases[38] with more educational time investment, the wage per unit of human capital declines. At the same time, the interest rate increases as agents borrow more to finance their additional educational time investment[39]. Both adjustments provide a counterincentive to invest in education as the return declines and the cost of borrowing rises. This explains the smaller effect on educational time investment in general equilibrium. In case of a reduction in the public share, the signs of the effects are reversed.

In similar fashion, labor supply adjusts more strongly when prices are kept constant. Because human capital increases and wages do not adjust downwards, labor supply is increased by 0.71% in partial equilibrium. The effect is largely mitigated when wage and the interest rates have adjusted, resulting in a labor supply increase of only 0.03%[40].

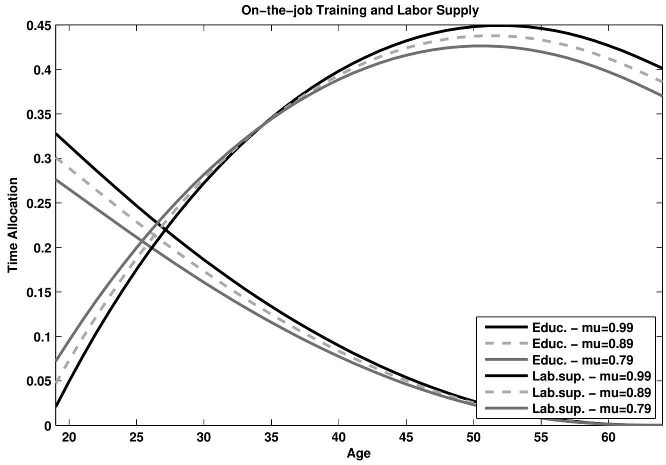
[37] See table 1.2 which gives an overview of the public shares in the cost of tertiary education in European countries.

[38] The stock of human capital in the economy increases by +1% when $\mu = 0.99$ and reduces by -1% when $\mu = 0.79$.

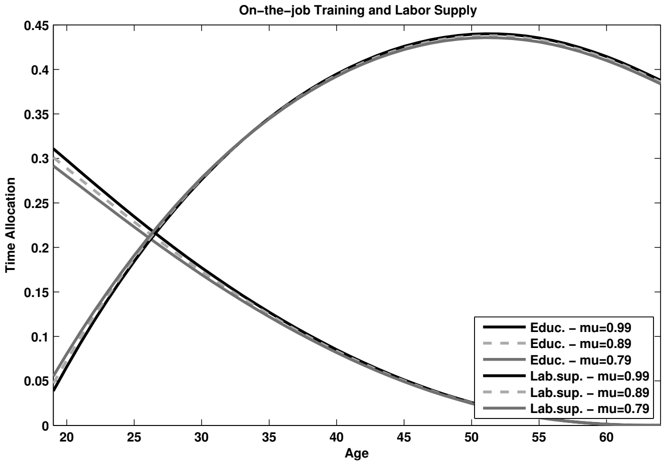
[39] Since no borrowing constraints are assumed in the basic case, the minimum of assets can move freely. Due to the subsidy increase, the minimum reduces by 2.37% as agents want to borrow additionally for consumption to compensate for the loss in utility from reduced leisure time when they invest more time in education.

[40] In table B.3 in the appendix to this thesis, we compare the situation of a tax increase to augment educational subsidies and a tax increase of which proceeds are thrown in the sea. As indicated by Rogerson (2007, p. 75), hours worked should be independent of the tax rate τ. The table shows that there is only a very small effect

Figure 4.11 shows the magnitudes of the partial and general equilibrium effects on educational time investment and labor supply graphically. In general equilibrium, the intersection of the labor supply and educational time investment curves shifts from 26.4 to 26.7 years when the public share is increased, and from 26.4 to 26.2 years when the public share is decreased.



(a) Partial equilibrium



(b) General equilibrium

Figure 4.11: The educational time investment when varying the public share in the cost of education in (a) partial equilibrium and (b) general equilibrium. Year: 2010.

The effect of changing the public share on the tax rate requires further clarification. Interestingly, the tax rate is reduced when the public share is increased in partial equilibrium. This can be explained by the development of national income per capita: the large increase in income (+2.21%) reduces the tax burden as the cost of the subsidy can be divided over a larger income stock. When prices adjust in general equilibrium, national income per capita increases much less (+1.07%). The positive effect on national income from more human capital in the economy no longer outweighs the increased tax revenues needed to finance the additional subsidies. As a result, the tax rate increases in general equilibrium. Once again, signs of the results are reversed when a reduction of μ is considered.

In table B.4 in the appendix to this thesis, the results of a change in the public share are presented for the population distribution of 2050. The results are very similar to the case of 2010. As the initial educational time investment is higher in the situation with aging, effects of the subsidy on educational time investment and other variables are somewhat stronger.

Table B.5 shows the results of varying the public share in the cost of education in 2050 with a borrowing constraint at the 2010 minimum asset level. The effects of a change in the public share on educational time investment are smaller when a borrowing constraint is present. The reason is that agents are limited in the amount they can borrow additionally to smooth consumption and leisure while spending more time on education. As a result, they cannot adjust their time investment as much. The subsidy is less effective.

from the tax increase *per se* on the labor supply of agents in our model. It is the way the tax revenues are used to subsidize education that accounts for the changes in labor supply in our model.

4.4.1 Welfare effects

	Subsidy decrease ($\mu = 0.79$)	Subsidy increase ($\mu = 0.99$)
Endog. human capital	0.24%	-0.22%

Table 4.5: Welfare effects of changing the public share in the cost of tuition (consumption-equivalent variation). Differences with respect to baseline case of 2010 ($\mu = 0.89$, $OAD = 28.4\%$) in general equilibrium. Differences are percentage changes.

Table 4.5 shows that the welfare consequence of an increase in tuition subsidy is negative[41]. This means that, in a situation without borrowing constraints, an increase in subsidies works mainly as a distortion in the market. By increasing the subsidy, the government artificially augments the stock of human capital, which reduces the wage per unit of human capital that people get.

García-Peñalosa and Wälde (2000, p. 709) point out that subsidies have positive equity effects for agents with heterogeneous ability[42]. An increase in subsidies decreases inequality in the net present value of lifetime income of unskilled and skilled workers at an efficiency cost. Governments are typically not only concerned with efficiency arguments when formulating policy. For this reason, implementation of a policy such as increasing the public share is conceivable. Our results show that the macroeconomic effects of such a policy are suboptimal when no market imperfections are present.

Several papers, notably Heckman et al. (1999), Abbott et al. (2013) and Kindermann (2009) find that, in the case of market imperfections such as borrowing constraints and uncertainty about lifetime earnings, tuition subsidies do not necessarily have negative welfare consequences. This raises the question of whether our conclusion that tuition subsidies are distortionary would change in the case of borrowing constraints.

	Subsidy decrease ($\mu = 0.79$)	Subsidy increase ($\mu = 0.99$)
Endog. human capital	0.36%	-0.49%

Table 4.6: Welfare effects of changing the public share in the cost of tuition (consumption-equivalent variation). Differences with respect to the case of 2050 ($\mu = 0.89$, $OAD = 41\%$) in general equilibrium. Borrowing constraint fixed at the 2010 minimum asset level. Differences are percentage changes.

Table 4.6 shows the results of a change in the public share when a borrowing constraint is present. Interestingly, increasing the tuition subsidy is not welfare improving in the case of a borrowing constraint either. Due to the subsidy, the educational time investment increases. As a result, agents need to borrow more to be able to smooth consumption and leisure over their lifecycle. Although they would like to borrow more, they can not. Effectively, the subsidy increase makes the borrowing constraint more binding. As a result, the welfare effect of a higher subsidy is negative.

4.4.2 Conclusion

In line with Heckman et al. (1999) we find much stronger responses of educational time investment to changes in public subsidies in partial equilibrium than in general equilibrium. As

[41] Changing the public share in the year 2050 results in similar welfare changes of 0.32 and -0.25, respectively.

[42] Under the assumption that agents cannot borrow and receive different amounts of parental transfers.

subsidies to education artificially raise the level of human capital in absence of market imperfections, increasing public subsidies in our baseline model decreases welfare. We show that the negative welfare effect of a higher subsidy persists when agents are credit constrained. When subsidies are higher, agents invest more in time in education. However, to be able to smooth consumption and leisure, borrowing would need to be increased. This is impossible in case of binding credit constraints. Hence, consumption decreases further and labor supply increases further, decreasing the welfare of agents.

4.5 Key findings

In the paper of Ludwig et al. (2012), the welfare costs of demographic change are mitigated by rising educational time investment. In an extension of their model, we introduced human capital accumulation that is costly to the agent and the government. We find that, even when education is partially financed by the government and partially by the agent, the negative welfare consequences of demographic change can still be reduced by endogenous human capital accumulation when replacement rates are fixed.

Secondly, we introduced a borrowing constraint into the model of Ludwig et al. (2012). We showed that the welfare effects of a higher old-age dependency ratio rise significantly when borrowing is constrained at the 2010 minimum asset level. When old-age dependency is rising, it becomes optimal to increase educational time investment. Consequently, agents need to borrow more to smooth consumption and leisure over their lifecycle, which makes the borrowing constraint more binding. As a result, welfare costs of the demographic change rise. Welfare is reduced even more strongly in the presence of a borrowing constraint than in the case of no constraint and no adjustments in human capital investment[43].

Thirdly, we find that a higher tuition subsidy increases the educational time investment of agents. The effect is much larger in partial than in general equilibrium. When factor prices can adjust, a rising interest rate and declining wage rate lower the return to additional human capital investment. Therefore, there is a dampening effect of changing factor prices on educational time investment. Heckman et al. (1999) and Abbott et al. (2013) reach similar conclusions on the different magnitude of adjustments in partial and general equilibrium.

Fourthly, we find that the welfare consequences of a higher tuition subsidy are negative. In absence of market imperfections, agents obtain the optimal amount of education without any government subsidies. Any increase in subsidies is therefore distortionary, hence reduces welfare.

Lastly, we showed that an increase in tuition subsidies still does not improve welfare when agents are borrowing constrained. As before, agents increase their educational time investment in response to the higher subsidy. However, as they are now limited in the amount they can borrow to finance consumption while studying, they face an additional utility loss from the constraint. Consequently, the constraint becomes more binding and the subsidy increase results in a welfare loss.

[43]Note that this effect only holds when the replacement rate of the public pension system is kept constant. If the replacement rate can adjust, endogenous human capital will lead to lower welfare as government expenditure on education rises when agents spend more time educating themselves. In absence of market imperfections, subsidies are a distortion to the economy and therefore decrease welfare.

Chapter 5

Discussion

"Once upon a time, before we had quite the high status that we enjoy today, it was common for economists to be harassed by scientists who wanted to know whether we had ever come up with anything that was neither trivial nor obvious."

- Angus Deaton, March 2005[1]

Our results demonstrate that there is a link between borrowing constraints and demographic change. A constraint on credit can become more binding due to a rising old-age dependency ratio. Making education (virtually) free of charge by means of subsidies cannot solve this problem: agents attempt to smooth consumption and leisure over their lifecycle and as a result need to borrow while spending large fractions of their time in training. If they cannot borrow enough, they will underinvest in education. Even though our results are mostly analytical, there are practical implications to be derived from them.

5.1 Policy implications

When a higher fraction of the population gets older, government expenditures on social security and health will rise. To fund these costs of rising old-age dependency, austerity measures will need to be implemented. Cuts on subsidies to universities and cuts on student financial aid have already been implemented in various European counties[2].

We show that to not inhibit the welfare mitigating effects of endogenous human capital adjustment, governments should not reduce or keep constant the maximum amount that students can borrow. If they do, borrowing constraints can become binding, chocking off automatic adjustment mechanisms that would otherwise smooth the transition towards a population with a higher old-age dependency.

We have not analyzed student financial aid systems in this thesis. Although in the discussion so far it might seem that ensuring students can borrow sufficiently is the best way to resolve imperfections in the market of education, student loans could be an suboptimal policy option because of uncertainty (see García-Peñalosa and Wälde, 2000; Kindermann, 2009). If agents are risk-averse and the returns to a degree are uncertain, increasing the amount that can be borrowed in a pure loan scheme would not be optimal as agents would not use the opportunity to borrow more.

An alternative would be to provide a grant to students. This would reduce their need to borrow against an uncertain stream of future income. However, a traditional grant financed

[1]Source: Deaton, 2005, retrieved from `http://www.princeton.edu/~deaton/downloads/romelecture.pdf`.
[2]See section 1.3.2 for a description of the austerity measures taken in different European countries.

from general tax revenue and given to all university students involves reverse redistribution, from lower-income levels to higher income-levels. Also, such a grant has been shown to be inefficient in comparison to more targeted grants. To overcome reversed distribution, introducing a special tax on the income of university graduates has been suggested (see García-Peñalosa and Wälde, 2000; Kindermann, 2009). Examples of targeted grants are ability-based grants and means-tested grants (Abbott et al., 2013).

Currently, there is no strong evidence of credit constraints preventing students from enrollment into tertiary education in Sweden. However, Swedish students may become constrained in how much they can borrow for consumption while studying if the limits to the government loans available to them are not increased. As we have shown, with demographic change, a borrowing constraint can become more binding. This may hold true especially for Sweden, where the fraction of elderly in the population is expected to reach a level over 40% in 2050. If the loan limit does not increase in proportion to growing educational time investment, borrowing constraints can become a problem, also in Sweden.

5.2 Limitations

Models are simplified representations of reality. As such, every model has limitations. We will discuss the most important limitations of our model in this section.

First of all, we use exogenous population distributions and assume these population distributions are in steady state. As a consequence, we are unable to calculate the welfare effects of demographic change for a specific birth cohort. In practice, this means that no generation will be affected by the exact welfare effects of a rise in the old-age dependency ratio that we have reported. To be able to calculate the welfare effects of aging for specific birth cohorts, a population model such as the one used by Ludwig et al. (2012) would be necessary.

We assume a proportional tax on labor. In reality, labor taxes are progressive in many countries (Bovenberg and Jacobs, 2005). A progressive tax on labor income could potentially lower the welfare mitigating effect of endogenous human capital accumulation we find[3]. Under progressive tax systems, investing more in education means that a higher fraction of one's future income is to be paid to the government. This discourages human capital investment. Also, progressive taxes could have implications for the welfare consequences of increases in subsidies. If investment in education is suboptimal under progressive taxes, subsidies could counteract this negative effect, thereby having less negative effects on welfare than in the case of proportional taxes on labor.

There is no uncertainty about lifespan or future earnings in our model. If agents are risk-averse, uncertainty could have similar implications for the positive welfare effects of endogenous human capital we find as in the case of progressive taxes: the effect would be lower. Also, government subsidies to education can function as insurance against earnings risk[4]. As a result, increases in subsidies might not necessarily decrease welfare in case of uncertainty.

The pension system we model does not index pension benefits for past earnings[5]. This creates a slight downward bias in the results when replacement rates in the system are kept constant. In that case, there is an incentive for each agent to underinvest in human capital in order to avoid contributing more to a pension system that is paying out fixed benefits.

[3]See Heckman et al. (1999), Dur and Teulings (2002), Bovenberg and Jacobs (2005), Krueger and Ludwig (2013).
[4]See Keane and Wolpin (1997), García-Peñalosa and Wälde (2000), Kindermann (2009), Abbott et al. (2013).
[5]In Sweden, public pension benefits are indexed for past earnings. See, Swedish Ministry of Health and Social Affairs, 2000, retrieved from `http://www.globalaging.org/pension/world/sweden.pdf`.

Chapter 6

Conclusion

This thesis began with some simple observations on education in Sweden: a high rate of participation and a long study phase, large tuition subsidies and relatively low returns on investments in education. We have calibrated a rich overlapping generations model for the case of Sweden. Our model includes endogenous educational time investment of Ben-Porath type as well as endogenous labor supply. In our set-up, education is financed not only by firms but also by agents and the government.

Main findings

Using this model, we have been able to analyze the driving forces of human capital investment decisions. We show that a high governmental tuition subsidy stimulates investment in human capital. We find, however, that in absence of market imperfections, subsidies to tuition costs work distortionary. As agents obtain the optimal amount of education without any government subsidies, any increase in subsidies distorts factor prices, hence reduces welfare.

Moreover, by including exogenous population distributions, our model allowed us to research the macroeconomic effects of demographic change for Sweden. We find that the Swedish population will suffer from considerable welfare losses over the next 40 years due to a steep rise in the old-age dependency ratio, which makes the social security system more expensive and reduces the return on savings.

We show that increased investment in human capital may significantly mitigate the macroeconomic impact of demographic change. In a society in which the fraction of elderly is growing, capital will become relatively abundant and labor relatively scarce. This results not only in a decrease in the interest rate, but also an increase in the wage rate. If we allow for the endogenous formation of human capital, the rise in wages will cause educational time investment to increase. Following increased investment in education, the effects on factor prices will be much smaller and income will rise, reducing the welfare costs of rising old-age dependency.

Cuts on subsidies to education have already been proposed in various European countries. We argue that governments should implement austerity on education with care. Most importantly, the adjustment mechanism of human capital to higher old-age dependency should not be hampered. To allow this adjustment mechanism to work properly, governments must see to it that students are not constrained in the amount they can borrow to finance tuition and living cost while investing the majority of their time in education. As we show, a borrowing constraint can become more binding when old-age dependency is rising, inducing underinvestment in human capital.

Contribution to the literature

Our thesis contributes to existing literature on human capital accumulation in the following ways. Firstly, we show that rising old-age dependency will have significant negative welfare effects in Sweden; there is a considerable difference between the welfare of the Swedish population in 2010 and 2050 due to a higher fraction of elderly in the population. Secondly, we demonstrate that borrowing constraints can become more binding with demographic change. In order to not hamper the automatic adjustment mechanism of increased investment in human capital, governments should find ways to raise the loan limit proportionally to educational time investment. Thirdly, we show that even if there are large subsidies to education in countries, students may underinvest in education when they are constrained in financing their desired level of consumption while enrolled.

Further research

Our main suggestion for further research is to investigate which student financial aid policies would be optimal in countries in which old-age dependency is rising using models with borrowing constraints, progressive taxes and uncertainty about future labor earnings. This would also allow for the assessment of which system, or mix of systems, of student financial aid is most efficient in reducing underinvestment in education that results from these market imperfections. An interesting experiment would be to capture the current system of student financial aid in Sweden with grants and subsidized loans in a model and evaluate how this system could be improved to minimize the welfare costs of aging.

Another important extension would be to include parental altruism and model agents with heterogeneous ability levels. This would allow for an analysis of potentially more efficient student aid policies, such as ability-based and means-tested grants. Moreover, with heterogeneous agents from families with different levels of wealth, one could study what the welfare consequences of demographic change and education policies are to specific socio-economic groups. In this way, the equity effects of rising old-age dependency and the policy responses to it could be considered.

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Appendix

Appendix A

Additional data on human capital accumulation in Europe

A.1 Graduation age from secondary education

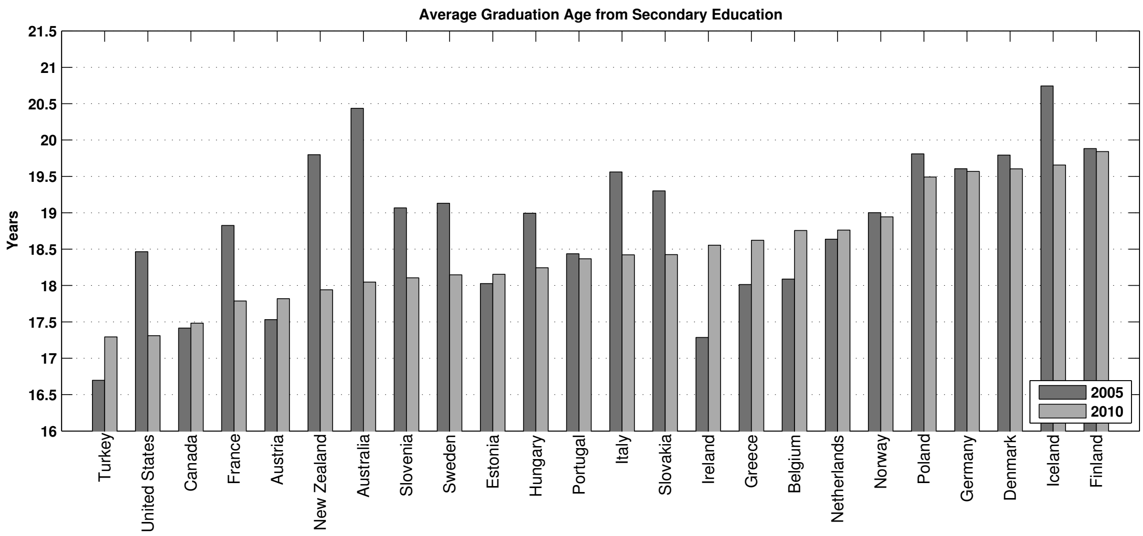


Figure A.1: Average graduation age from upper secondary education. Years: 2005 and 2010. Source: Own calculations based on Eurostat, 2013, table "Graduates by Age, upper secondary education"[1]

[1]Note: the average graduation age has been calculated based on the number of graduates of each age between 14 and 25 years old. Graduates above the age of 25 have been neglected in order to prevent bias from adults in secondary education. For Austria, Estonia, France and the Netherlands, the 2007 instead of 2005 values are presented. For New Zealand, the 2009 instead of 2010 value is used.

A.2 Military conscription

Selection of OECD countries with compulsory military conscription:	
Country	Remarks
Austria	Men only. Duration 6 months. Could be replaced by 12 month civil service.
Denmark	Men only. Duration 4 months. Some people excluded based on randomization.
Finland	Men 6-12 months. Women minimum 45 days. Possibility of switching to 12 month civil service.
Greece	Men only. Duration 9-12 months.
Norway	Men only. Duration 18 months. Only 27% of men were actually completing the service in 2011.
Turkey	Men only. Duration 6 months for university students, can be postponed till after graduation of university.
Selection of OECD countries with no military conscription:	
Australia, Belgium, Canada, Estonia, France, Germany[2], Hungary, Iceland, Ireland, Italy, Netherlands, New Zealand, Poland, Portugal, Slovakia, Slovenia, Sweden, U.S, U.K.	

Table A.1: Compulsory military service or civil service in OECD countries.
Source: The Economist, *Does your country need you?*, 4 July 2010, retrieved from `http://www.economist.com/blogs/dailychart/2011/07/military-conscription` and national sources.

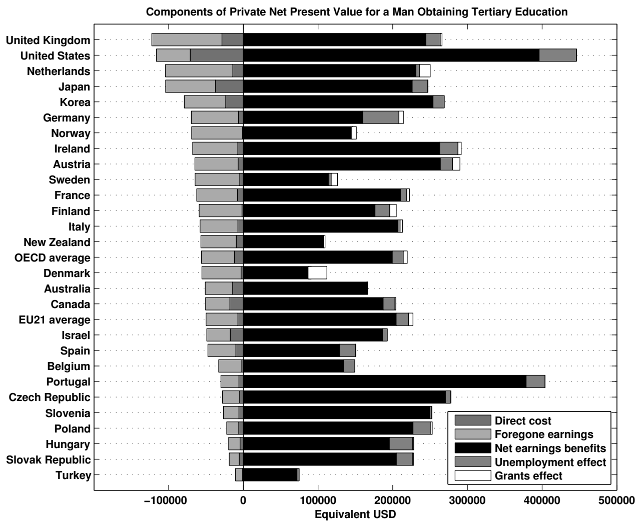
A.3 Decomposition of public and private NPV

Figures A.2a and A.2b give further insight in the components of the private returns to tertiary education. Private costs of tertiary education constitute direct costs (tuition) and foregone earnings. Direct costs are highest in the United States, reflecting the high tuition fees of college that have to be paid by students. Some countries, such as Norway, Germany, Sweden and Austria, have nearly zero direct private costs, because no or very small tuition fees are asked by public institutions. The Netherlands and the U.K. have relatively large foregone earnings to individuals. Looking at the private revenue side, the net wage gain of a tertiary degree is highest in the U.S. and Portugal, more than $400,000 (€308,000). Only minor net wage gains are obtained in Sweden, Denmark, New Zealand and Turkey. The lifetime benefit of a reduced risk of unemployment is reflected in the unemployment effect, which is a large private gain in for example the U.S. and Germany.

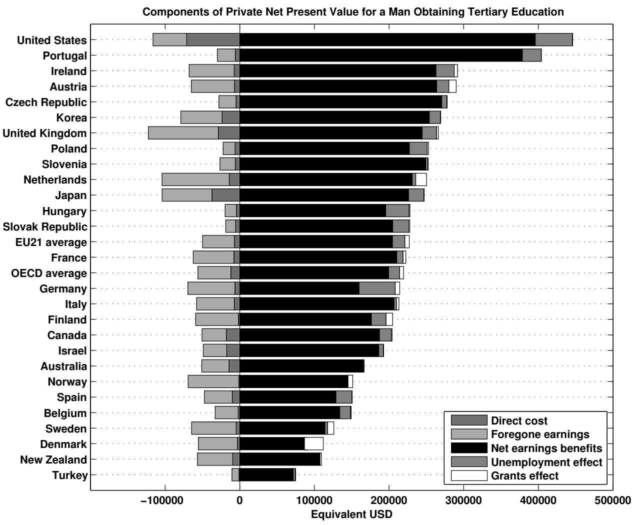
Figures A.2c and A.2d show the various component of the public costs and benefits of tertiary education. Public costs are the sum of foregone taxes on earnings, and the direct costs of education. Denmark has the highest direct public cost of a degree, which is in line with the high public share in the cost of tertiary education (table 1.2). The Netherlands is the country with the highest foregone taxes of people who study, which is in line with large foregone private earnings. The U.S. and Spain have very low foregone tax revenues. In the U.K., Portugal and Turkey, the direct costs of a degree to the government are low. Looking at the public benefits of

[2] Compulsory military service was suspended in Germany on July 1 2011. It used to be 9 months. An alternative was a 9 month civil service.

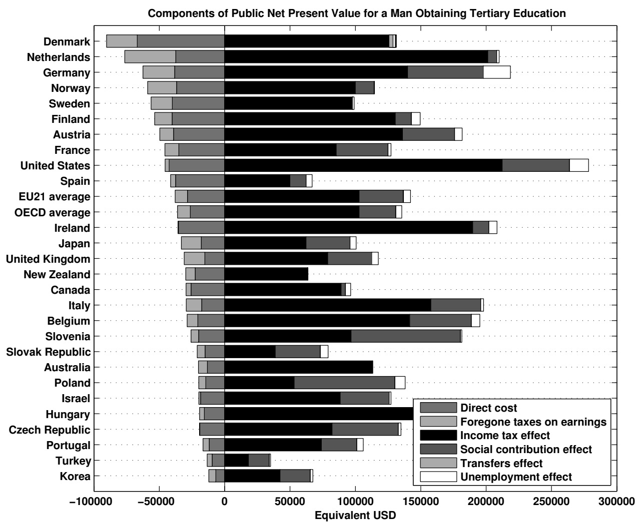
tertiary education, it is mainly additional tax revenues of the higher educated plus their additional social contributions that drives the public return. The positive effect of tertiary education on income tax is highest in the Netherlands and the U.S.. Increased social contributions provide the majority of benefits only in Poland.



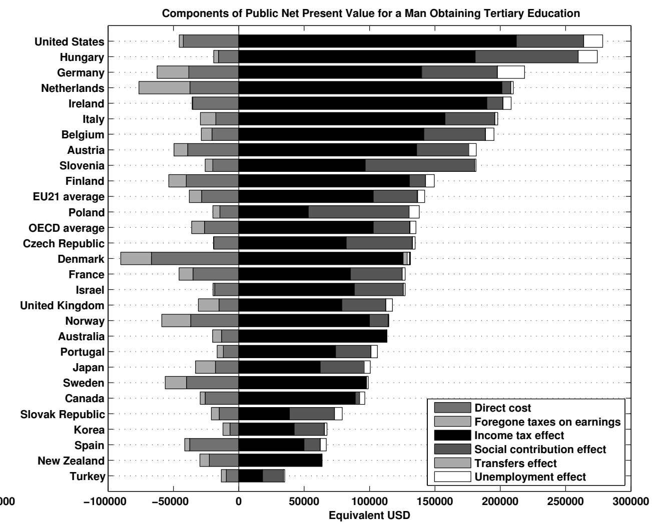
(a) Components of private NPV, sorted by total costs



(b) Components of private NPV, sorted by total benefits



(c) Components of public NPV, sorted by total costs



(d) Components of public NPV, sorted by total benefits

Figure A.2: The public and private net present value for a man obtaining a tertiary education degree as part of initial education across OECD countries (2008 or latest available year[3])[4,5]
Source: OECD Education at a Glance 2012: OECD Indicators, table A9.3 and table A9.4.

[3] 2005: Belgium, Australia, Turkey; 2006: Portugal; 2007: Japan, Slovenia.
[4] In figure (a) and (b), the net earnings benefit is calculated as the gross earnings benefit minus the income tax and social contribution effect.
[5] In figure (c) and (d), the social contribution effect is the sum of the social contribution minus grants for studying.

Appendix B

Additional results

B.1 Time allocation of agents in baseline model

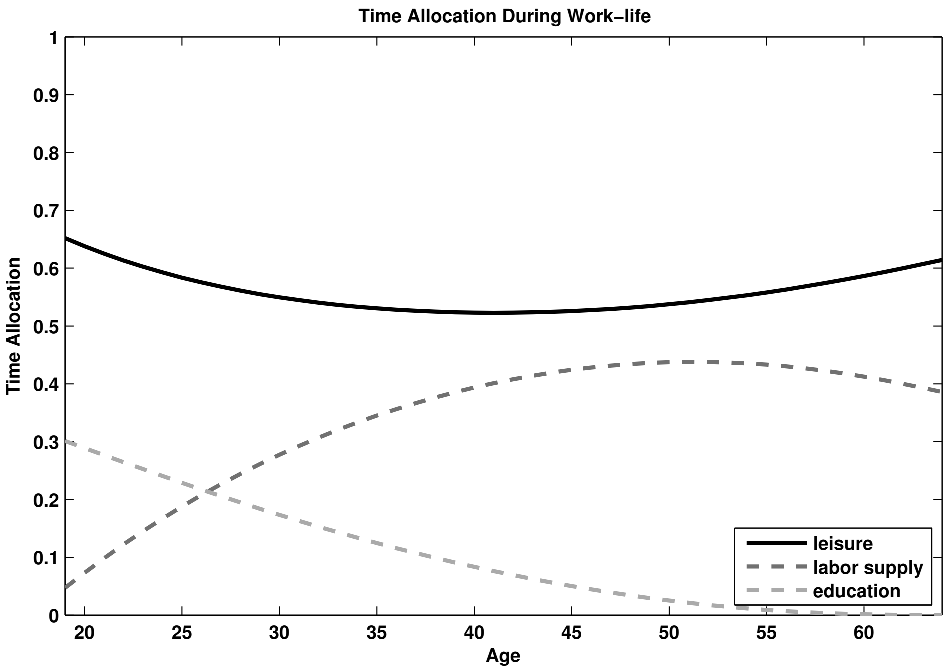


Figure B.1: The allocation of time during the work life (age 19-65) of agents in the baseline model.

B.2 Detailed results: constant ρ

	Human capital	
	Endogenous	Exogenous
Educational time investment	8.66%	-
Labor supply	0.94%	0.68%
Wage per unit of human capital (w)	0.25%	0.90%
Interest rate (r)	-0.06	-0.21
Tax rate (τ)	4.13	4.63
Replacement rate (ρ)	-	-
National income per capita	0.08%	-3.41%

Table B.1: Expected welfare consequences of aging in 2050 ($OAD = 41.0\%$). Differences with respect to baseline case of 2010 ($OAD = 28.4\%$). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

B.3 Detailed results: constant τ

	Human capital	
	Endogenous	Exogenous
Educational time investment	11.53%	-
Labor supply	2.02%	1.95%
Wage per unit of human capital (w)	0.16%	1.11%
Interest rate (r)	-0.04	-0.25
Tax rate (τ)	0.08	-0.06
Replacement rate (ρ)	-5.31	-5.75
National income per capita	2.40%	-1.95%

Table B.2: Expected welfare consequences of aging in 2050 ($OAD = 41.0\%$). Differences with respect to baseline case of 2010 ($OAD = 28.4\%$). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

B.4 Welfare effects of a borrowing constraint

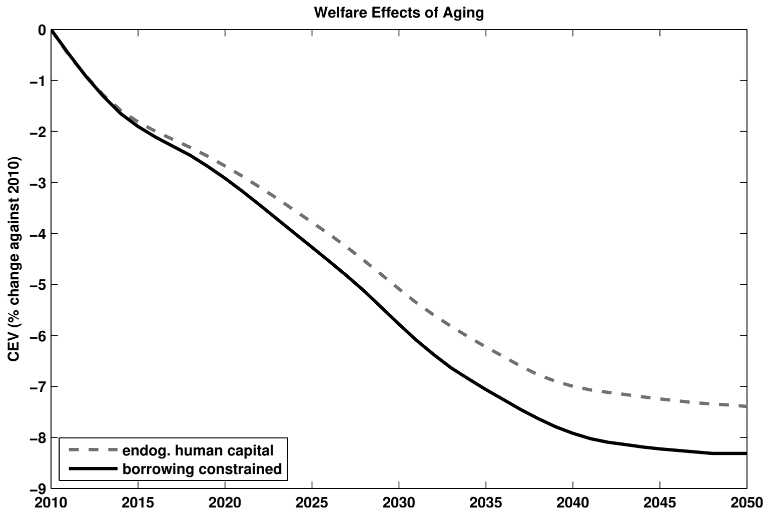


Figure B.2: Change in consumption-equivalent variation (CEV) in the case of endogenous human capital with and without borrowing constraints. Period: 2010-2050.

B.5 Wasteful spending

	Increase in subsidies ($\mu = 0.99$)	Wasteful spending
Educational time investment	2.46%	-0.09%
Labor supply	0.03%	-0.01%
Wage per unit of human capital (w)	-0.14%	-0.01%
Interest rate (r)	0.03	0.01
Tax rate (τ)	0.14	0.14
National income per capita	1.07%	-0.07%

Table B.3: A tax increase to augment educational subsidies and a tax increase of which proceeds are thrown in the sea. Differences with respect to the baseline case in 2010 ($\mu = 0.89$). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

B.6 Varying the public share in 2050

	Partial equilibrium		General equilibrium	
	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)
Educational time investment	-7.59%	7.24%	-2.64%	2.58%
Labor supply	-0.70%	0.55%	-0.05%	0.04%
Wage per unit of human capital (w)	-	-	0.19%	-0.12%
Interest rate (r)	-	-	-0.03	0.03
Tax rate (r)	0.39	-0.29	-0.06	0.08
National income per capita	-2.23%	1.97 %	-1.16%	1.19%

Table B.4: Varying the public share in the cost of education from the baseline case of 2050 ($\mu = 0.89$, $OAD = 41\%$, endogenous human capital accumulation). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

B.7 Varying the public share in 2050 with a borrowing constraint

	Partial equilibrium		General equilibrium	
	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)
Educational time investment	-1.84%	2.14%	-1.87%	1.42%
Labor supply	-0.21%	0.23%	-0.15%	0.29%
Wage per unit of human capital (w)	-	-	0.03%	0.04%
Interest rate (r)	-	-	-0.01	-0.01
Tax rate (r)	0.09	0.08	-0.10	0.11
National income per capita	-0.91%	0.96 %	-0.93%	0.89%

Table B.5: Varying the public share in the cost of education from the baseline case of 2050 with a borrowing constraint ($\mu = 0.89$, $OAD = 41\%$, endogenous human capital accumulation). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

Appendix C

Sensitivity analysis

We vary the inverse of the intertemporal elasticity of substitution σ as robustness check. Our sensitivity analysis is based on the one carried out by Ludwig et al. (2012, see pages 10-12 in the online appendix).

C.1 The Frisch Labor Supply Elasticity

The intertemporal elasticity of substitution affects the Frisch labor supply elasticities in our model. Table C.1 shows that these elasticities vary significantly with our choice of σ[1]. We report both the elasticity when assuming the time investment in human capital is constant over the lifecycle of agents and when it is assumed to be variable[2].

	Constant time inv. in Hum. Cap.			Variable time inv. in Hum. Cap.		
	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$
Age 30-50	1.03	0.95	0.90	1.84	1.66	1.56
Age 21-60	1.73	1.25	1.10	3.80	2.53	2.12
All ages	2.74	1.30	1.11	6.69[3]	2.66	2.14

Table C.1: Change in Frisch labor supply elasticity following changes in the intertemporal elasticity of substitution.

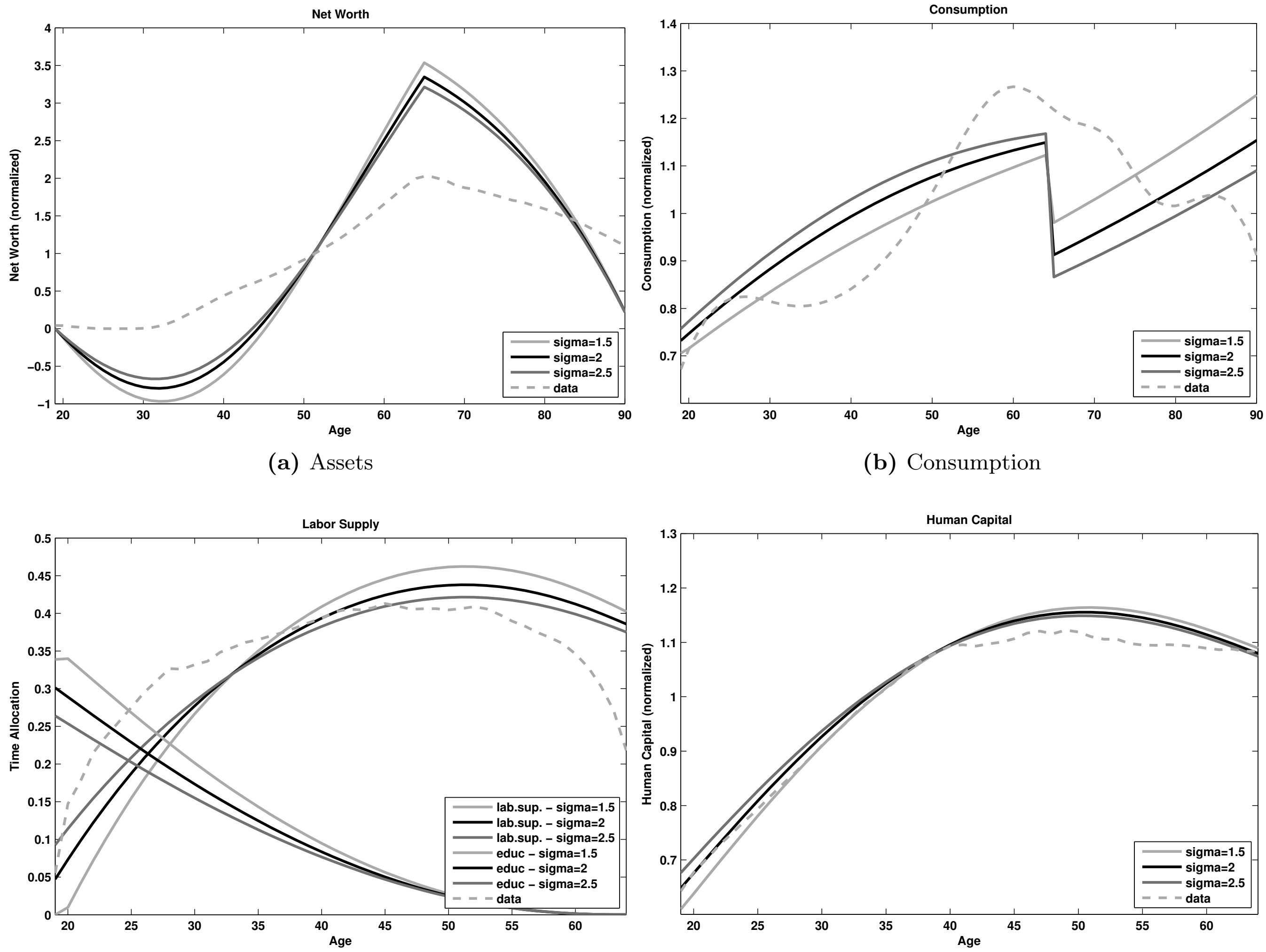
The strong variation in labor supply elasticities displayed in table C.1 could significantly affect the resource and time allocation decisions of the agents in our model. Consequently, we show in this section how variation of σ affects (1) the fit of the model with asset, consumption, labor supply and hours worked profiles in the data; (2) welfare calculations done with the model; (3) the detailed results from varying the public share in the cost of education.

[1] Note that in this table, contrary to the estimates reported in the main text of the thesis, we exclude age 19 from the calculations because labor supply is zero at this age when $\sigma = 1.5$.

[2] As is to be expected, Ludwig et al. (2012) find even stronger variation in Frisch labor supply elasticities when they change σ to $\sigma = 1$ and $\sigma = 3$.

[3] This average Frisch labor supply elasticity is extremely high because labor supply is almost zero at age 20.

C.2 Resource and time allocation when σ varies



(a) Assets

(b) Consumption

(c) Labor Supply

(d) Human Capital

Figure C.1: Life time optimal profiles of (a) asset accumulation; (b) consumption; (c) labor supply and educational time investment; and (d) human capital accumulation of the agent. All variables (except labor supply and on-the-job training) are normalized[4].

We can be brief about the change in fit of our baseline model to profiles in the data when the intertemporal elasticity of substitution varies. Figure C.1 shows that this fit is not affected much by a change in σ. The strongest effects are on labor supply and educational time investment. The intersection of the educational time investment and labor supply graphs varies from age 24.9 in case σ equals 2.5 to age 28.1 in case σ equals 1.5.

Note, for intuition purposes, that the labor supply graph clearly shows that hours worked become more sensitive to income when σ decreases - i.e. the intertemporal labor supply elasticity goes up - and vice versa. It is also important to see that educational time investment varies more when the intertemporal labor supply elasticity is higher. This means that the welfare mitigating effects of endogenous human capital can be expected to be higher when σ is lower.

[4] We normalize each human capital profile by its own mean; this explains why they have different starting points.

C.3 Welfare effects of aging when σ varies

	Constant replacement rate ($\rho_t = \bar{\rho}$)			Constant contribution rate ($\tau_t = \bar{\tau}$)		
	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$
Average loss in CEV						
Endog. human capital	-7.36%	-7.38%	-7.43%	-4.95%	-4.62%	-4.34%

Table C.2: Welfare effects of aging in 2050 ($OAD = 41.0\%$). Change in consumption-equivalent variation (CEV) with respect to the 2010 baseline case (OAD=28.4%). Differences are percentage changes.

We first consider the case of constant replacement rates. Table C.2 shows that when the labor supply elasticity is higher - i.e. the inverse of the intertemporal elasticity of substitution is lower - aging is slightly less damaging to welfare. This is because both labor supply and educational time investment are more responsive to changes in income.

When contribution rates are constant, increased educational time investment following higher old-age dependency decreases welfare because of the higher tax rate that is necessary to fund more training. When the labor supply elasticity is higher, increases in educational time investment in response to higher old-age dependency are stronger. This means that taxes increase more as well, explaining the stronger decrease in welfare when the intertemporal labor supply elasticity is higher.

As the welfare effects of aging are not strongly affected by changes in the intertemporal substitution elasticity, we conclude our results are robust to the specification of this parameter.

C.4 Varying the public share with different values for σ

	$\sigma = 1.5$		$\sigma = 2$		$\sigma = 2.5$	
	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)	Decrease ($\mu = 0.79$)	Increase ($\mu = 0.99$)
Educational time investment	-2.31%	2.43%	-2.44%	2.46%	-2.33%	1.59%
Labor supply	0.02%	-0.01%	-0.04%	0.03%	-0.05%	0.10%
Wage per unit of human capital (w)	0.1%	-0.12%	0.17%	-0.14%	0.15%	-0.01%
Interest rate (r)	-0.02	0.02	-0.03	0.03	-0.03	0.02
Tax rate (τ)	-0.15	0.15	-0.13	0.14	-0.12	0.13
National income per capita	-1.10%	1.14%	-1.04%	1.07%	-0.94%	0.86%

Table C.3: Sensitivity of subsidy variation results to a change in the intertemporal elasticity (baseline case 2010, $\mu = 0.89$). Differences are percentage changes, with the exception of interest and tax rate differences which are in percentage points.

Table C.3 shows that the change in resource and time allocation of agents as well as the change in macro-economic variables following variations in the public share in the cost of education are barely affected by the choice of σ[5].

[5]The only remarkable change in the table is the decrease in labor supply when educational subsidies increase and the increase in labor supply when these subsidies decrease. The effects are too small to give a clear explanation; we think it could have to do with the fact that labor supply responds very strongly to changes in subsidies during the early part of the lifecycle of agents when the labor supply elasticity is high.

Appendix D

Computations

The problem to be solved is the maximization of D.1. We drop the t subscripts because we analyze the model in steady state. The computations presented largely follow the approach used in Ludwig et al. (2012).

D.1 The problem

Agents maximize the present discounted value of their lifetime utility by choosing consumption, labor supply, and educational time investment given a set of budget constraints and the human capital accumulation function:

$$\begin{aligned} \max_{c,l,e,a',h'} \sum_{j=1}^{J} V(c_j, 1-l_j-e_j) &= \sum_{j=1}^{J} \beta^j \frac{1}{1-\sigma} \left\{ c_j^{\phi} (1-l_j-e_j)^{1-\phi} \right\}^{1-\sigma}, \\ & \sigma > 0,\ \phi \in (0,1), \\ & e_j \in [0,1],\ l_j \in [0,1] \end{aligned} \tag{D.1}$$

with:

$$a_{j+1} = a_j(1+r) + y_j - c_j$$

$$y_j = \begin{cases} l_j h_j w(1-\tau) - e_t(1-\mu)T & \text{if } j < jr \\ \rho w & \text{if } j \geq jr \end{cases} \tag{D.2}$$

$$\begin{aligned} a_{J+1} &= 0 \\ a_1 &= 0 \end{aligned}$$

and:

$$h_{j+1} = g(e_j, h_j) = h_j(1-\delta_h) + \xi(e_j h_j)^{\psi} \tag{D.3}$$

As Ludwig et al. (2012) point out, the upper constraints on educational time investment ($e_j = 1$) and labor supply ($l_j = 1$) will never be binding due to Inada conditions on the utility function. Similarly, the lower constraint on educational time investment ($e_j = 0$) will never be binding as that would mean a divestment of human capital and thus a decrease in the wage of agents.

D.1.1 Equilibrium

An equilibrium occurs when the following conditions hold simultaneously:

1. Households solve the maximization problem in D.1 given the interest rate (r) and wage rate (w) per unit of human capital
2. Capital and labor receive their marginal products; $r = \alpha \frac{Y}{K}$ and $w = (1-\alpha)\frac{Y}{L}$
3. Government budget is balanced

$$\tau w \sum_{j=1}^{jr-1} N_j l_j h_j = \sum_{j=jr}^{J} N_j \rho w + \sum_{j=1}^{jr} N_j (1-\mu) e_j T$$

4. Markets for capital, labor and goods clear

$$K_j = \sum_{j=1}^{J} N_j a_j \tag{D.4a}$$

$$L_j = \sum_{j=1}^{J} N_j l_j h_j \tag{D.4b}$$

$$Y = \sum_{j=1}^{J} N_j a_j^{\alpha} (l_j h_j)^{1-\alpha} \tag{D.4c}$$

D.1.2 Karush-Kuhn-Tucker-Conditions

The maximisation problem in D.1 can be rewritten as:

$$\begin{aligned}\max_{c,l,e,a',h'} \mathcal{L} = \sum_{j=1}^{J} \Big(& \beta^j \frac{1}{1-\sigma} \big(c_j^{\phi} (1 - l_j - e_j)^{(1-\phi)}\big)^{(1-\sigma)} \\ & + \lambda_j^a [a_{j+1} - a_j(1+r) - y_j + c_j] \\ & + \lambda_j^h [h_{j+1} - h_j(1-\delta_h) - \xi (e_j h_j)^{\psi}] \\ & + \mu_j l_j \Big)\end{aligned} \tag{D.5}$$

Now, computing the Karush-Kuhn-Tucker conditions gives:

<u>For $j < jr$: Work and Ben-Porath on-the-job training</u>

$$\frac{\partial \mathcal{L}}{\partial c_j} = \beta^j u_c + \lambda_j^a = 0 \tag{D.6a}$$

$$\frac{\partial \mathcal{L}}{\partial l_j} = -\beta^j u_l - \lambda_j^a h_j (1-\tau) w + \mu_j = 0 \tag{D.6b}$$

$$\frac{\partial \mathcal{L}}{\partial e_j} = -\beta^j u_e + \lambda_j^a (1-\mu) T - \lambda_j^h \xi \psi e_j^{\psi-1} h_j^{\psi} = 0 \tag{D.6c}$$

$$\frac{\partial \mathcal{L}}{\partial a_{j+1}} = -\lambda^a_{j+1}(1+r) + \lambda^a_j = 0 \tag{D.6d}$$

$$\frac{\partial \mathcal{L}}{\partial h_{j+1}} = -\lambda^a_{j+1} l_{j+1}(1-\tau)w - \lambda^h_{j+1}((1-\delta_h) + \xi\psi e^{\psi}_{j+1} h^{\psi-1}_{j+1}) + \lambda^h_j = 0 \tag{D.6e}$$

$$\mu_j l_j = 0 \tag{D.6f}$$

$$u_j \geq 0 \tag{D.6g}$$

with:

$$u_c = \phi c_j^{\phi(1-\sigma)-1}(1-l_j-e_j)^{(1-\phi)(1-\sigma)} \tag{D.6h}$$

$$u_l = u_e = (1-\phi)c_j^{\phi(1-\sigma)}(1-l_j-e_j)^{\phi(\sigma-1)-\sigma} \tag{D.6i}$$

For $j \geq jr$: Pension

During retirement, agents do neither study nor work ($e_j = l_j = 0$). This means that h_{j+1} is given and that we can solve directly for c_j using the intertemporal Euler equation in D.8.

D.1.3 Obtaining the intertemporal Euler equation for consumption

Updating D.6a for one period and using D.6d gives:

$$\frac{\partial \mathcal{L}}{\partial c_{j+1}} = \beta^{j+1} u_{c'} + \frac{\lambda^a_j}{(1+r)} = 0 \tag{D.7}$$

To create the intertemporal Euler equation for consumption, we equate D.7 and D.6a and get:

$$u_c = \beta(1+r)u_{c'} \tag{D.8}$$

If we substitute D.6h for age j and $j+1$ in D.7, and enter the budget constraints in D.2 through isolation of c_j, we get:

For $j < jr$: Work and Ben-Porath on-the-job training

$$\begin{aligned} a_j(1+r) = {} & (a_{j+1}(1+r) + w^n_{j+1} - e_{j+1}(1-\mu)T - a_{j+2})\frac{\gamma_2}{\gamma_1}(\beta(1+r))^{\frac{1}{\phi(1-\sigma)-1}} \\ & - w^n_j + e_j(1-\mu)T + a_{j+1} \end{aligned} \tag{D.9a}$$

For $j \geq jr$: Pension

$$\begin{aligned} a_j(1+r) = {} & (a_{j+1}(1+r) + p_{j+1} - a_{j+2})\frac{\gamma_2}{\gamma_1}(\beta(1+r))^{\frac{1}{\phi(1-\sigma)-1}} \\ & - p_j + a_{j+1} \end{aligned} \tag{D.9b}$$

in which:

$$\gamma_1 = (1-l_j-e_j)^{\frac{(1-\phi)(1-\sigma)}{\phi(1-\sigma)-1}} \tag{D.10a}$$

$$\gamma_2 = (1-l_{j+1}-e_{j+1})^{\frac{(1-\phi)(1-\sigma)}{\phi(1-\sigma)-1}} \tag{D.10b}$$

D.1.4 Obtaining the intratemporal Euler equation for leisure

Equating D.6a and D.6b gives the intratemporal Euler equation for leisure. If we assume that $\mu_j = 0$, i.e. the constraint on labor supply is not binding, we get the following expression:

$$u_l = u_c h_j (1-\tau) w \tag{D.11}$$

Note that equating D.6a and D.6b is the same as equating D.6a and D.6c. Since $u_l = u_e$, combining D.6b and D.6c gives:

$$\begin{aligned} \lambda_j^h &= \frac{(1-\mu)T + h_j(1-\tau)w}{\xi\psi e_j^{\psi-1} h_j^{\psi}} \lambda_j^a \\ \lambda_j^a &= \frac{\xi\psi h_j^{\psi} e_j^{\psi-1}}{h_j(1-\tau)w} \lambda_j^h - \frac{(1-\mu)T}{h_j(1-\tau)w} \lambda_j^a \end{aligned} \tag{D.12}$$

D.1.5 Solving for h_j in terms of next period variables

The single constraint to the optimization problem in D.5 is that labor supply has to be non-negative. If this constraint is not binding ($\mu_j = 0$), one can compute the current period level of human capital by solving a simple one variable non-linear problem with a solver. If the constraint is binding ($\mu_j > 0$), a non-linear solving algorithm with three equations and three unknowns needs to be used to solve for h_j.

Solving for h_j when $\mu_j = 0$

From the human capital accumulation function D.3 we define

$$\frac{\partial h_{j+1}}{\partial e_j} = g_e = \xi\psi(e_j h_j)^{\psi-1} h_j \tag{D.13}$$

$$\frac{\partial h_{j+1}}{\partial h_j} = g_h = (1-\delta^h) + \xi\psi(e_j h_j)^{\psi-1} e_j \tag{D.14}$$

Isolating λ_{j+1}^h in D.6e gives

$$-\lambda_{j+1}^h = \frac{-\lambda_j^h + \lambda_{j+1}^a (l_{j+1}(1-\tau)w)}{g_{h'}}$$

Substituting in D.6c expressed for age $j+1$ gives:

$$\frac{\beta^{j+1} u_{e'} - \lambda_{j+1}^a (1-\mu)T}{g_{e'}} = \frac{-\lambda_j^h + \lambda_{j+1}^a (l_{j+1}(1-\tau)w)}{g_{h'}}$$

$$\iff$$

$$-\frac{g_{e'}}{g_{h'}} \lambda_j^h = \beta^{j+1} u_{e'} - \lambda_{j+1}^a \Big((1-\mu)T + \frac{g_{e'}}{g_{h'}} l_{j+1}(1-\tau)w\Big)$$

Entering the expression for λ_{j+1}^a from D.7 and dividing by $-\frac{g_{e'}}{g_{h'}}$

$$\lambda_j^h = -\beta^{j+1} \left(\frac{g_{h'}}{g_{e'}} u_{e'} + \frac{g_{h'}}{g_{e'}} (1-\mu) T u_{c'} + l_{j+1}(1-\tau) w u_{c'} \right) \tag{D.15}$$

Now, substituting in the expression for λ_j^h from D.12

$$\lambda_j^a = -\beta^{j+1}\Big(\frac{g_{h'}}{g_{e'}}u_{e'} + \big(\frac{g_{h'}}{g_{e'}}(1-\mu)T + l_{j+1}(1-\tau)w\big)u_{c'}\Big)\frac{\xi\psi e_j^{\psi-1}h_j^{\psi}}{(1-\mu)T + h_j(1-\tau)w}$$

Equating this to D.7 through the isolation of λ_j^a gives

$$\begin{aligned}
-\beta^{j+1}(1+r)u_{c'} &= -\beta^{j+1}\left(\frac{g_{h'}}{g_{e'}}u_{e'} + \big(\frac{g_{h'}}{g_{e'}}(1-\mu)T + l_{j+1}(1-\tau)w\big)u_{c'}\right) \\
&\quad \times \frac{\xi\psi e_j^{\psi-1}h_j^{\psi}}{(1-\mu)T + h_j(1-\tau)w} \\
&\iff \\
e_j &= \left(\frac{\Big(\frac{g_{h'}}{g_{e'}}u_{e'} + \big(\frac{g_{h'}}{g_{e'}}(1-\mu)T + l_{j+1}(1-\tau)w\big)u_{c'}\Big)\xi\psi h_j^{\psi}}{(1+r)u_{c'}((1-\mu)T + h_j(1-\tau)w)}\right)^{\frac{1}{1-\psi}}
\end{aligned} \tag{D.16}$$

Entering D.3 expressed for e_j gives a distance function that can be set to zero by solving for h_j with a non-linear solver:

$$f = \frac{1}{h_j}\left(\frac{h_{j+1} - h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}} - \left(\frac{\Big(\frac{g_{h'}}{g_{e'}}u_{e'} + \big(\frac{g_{h'}}{g_{e'}}(1-\mu)T + l_{j+1}(1-\tau)w\big)u_{c'}\Big)\xi\psi h_j^{\psi}}{(1+r)u_{c'}((1-\mu)T + h_j(1-\tau)w)}\right)^{\frac{1}{1-\psi}} \tag{D.17}$$

Solving for h_j when $\mu_j > 0$

Equating D.6b and D.6c gives

$$\lambda_j^h = \frac{\lambda_j^a(h_j(1-\tau)w + (1-\mu)T) - \mu_j}{g_e} \tag{D.18}$$

Entering this expression into D.15 gives

$$\begin{aligned}
&\frac{\lambda_j^a(h_j(1-\tau)w + (1-\mu)T) - \mu_j}{g_e} = \\
&-\beta^{j+1}\left(\frac{g_{h'}}{g_{e'}}u_{e'} + \frac{g_{h'}}{g_{e'}}(1-\mu)Tu_{c'} + l_{j+1}(1-\tau)wu_{c'}\right) \\
\iff& \\
&\frac{-\beta^{j+1}(1+r)u_{c'}(h_j(1-\tau)w + (1-\mu)T) - \mu_j}{g_e} = \\
&-\beta^{j+1}\left(\frac{g_{h'}}{g_{e'}}u_{e'} + \frac{g_{h'}}{g_{e'}}(1-\mu)Tu_{c'} + l_{j+1}(1-\tau)wu_{c'}\right) \quad \text{(from D.7)} \\
\iff& \\
&g_e = \frac{(1+r)u_{c'}(h_j(1-\tau)w + (1-\mu)T) + \frac{1}{\beta^{j+1}}\mu_j}{\frac{g_{h'}}{g_{e'}}u_{e'} + \frac{g_{h'}}{g_{e'}}(1-\mu)Tu_{c'} + l_{j+1}(1-\tau)wu_{c'}}
\end{aligned}$$

Isolating e_j allows for the formulation of a distance function f that needs to equal zero if the first-order conditions hold.

$$f = e_j - \left(\frac{\frac{g_{h'}}{g_{e'}}u_{e'} + \frac{g_{h'}}{g_{e'}}(1-\mu)Tu_{c'} + l_{j+1}(1-\tau)wu_{c'}}{(1+r)u_{c'}(h_j(1-\tau)w + (1-\mu)T) + \frac{1}{\beta^{j+1}}\mu_j}\right)^{\frac{1}{1-\psi}} \tag{D.19}$$

A non-linear solver can be used to find the e_j and h_j that sets f to zero.

D.2 Computational algorithm in Matlab

We use Matlab to compute the above model. We split the computations in two loops: an inner loop to solve the household problem and an outer loop to find the equilibrium wage, interest rate, and budget balancing tax rate.

D.2.1 Inner loop

Since we know that assets are zero at birth ($a_1 = 0$) and must be zero at death given the boundary condition ($a_{J+1} = 0$), the model can be solved through backwards iteration starting from initial guesses of consumption and human capital in the last period (c_J, h_J). After going through the backwards iteration steps, we update these terminal value guesses until convergence.

Backwards iteration

We solve by standard backward shooting making use of our initial guesses of last period consumption c_J and human capital h_J. Our guess of c_J is used to compute a guess of terminal period assets using the fact that consumption in the last period equals the remaining assets in that period plus the pension benefit: $c_J = (1+r)a_J + p_j$.

The detailed steps in the inner loop are as follows:

<u>For $j \geq jr$: Pension</u>

1. In this phase, $l_j = e_j = 0$ and human capital depreciates at the rate of depreciation of human capital, δ^h. Compute h_j using:

$$h_j = \frac{h_J}{(1-\delta^h)^{J-j}} \tag{D.20}$$

2. Solve for c_j by substituting the expression for u_c from D.8 in D.6h and rearranging:

$$c_j = \left(\frac{\beta(1+r)u_{c'}}{\phi}\right)^{\frac{1}{\phi(1-\sigma)-1}} \tag{D.21}$$

3. Compute a_j using the budget constraint in D.9b.

<u>For $j < jr$: Work and Ben-Porath on-the-job training</u>

1. Assume $l \geq 0$, which implies that $\mu_j = 0$ in D.6b. Now, the value of h_j can be solved for numerically in a simple problem with one equation and one unknown[1].

[1]Note that if μ and T are both zero, the problem can in fact be solved analytically. h_j can be computed with:

$$h_j = \frac{1}{1-\delta^h}\left(h_{j+1} - \xi\left(\frac{\xi\psi(\frac{g_{e'}}{g_{h'}}u_{e'} + l_{j+1}(1-\tau)wu_{c'})}{(1-\tau)w(1+r)u_{c'}}\right)^{\frac{1}{1-\psi}}\right)$$

2. Compute h_j using D.17 in the non-linear solver `fsolve` in Matlab:

$$f = \frac{1}{h_j}\left(\frac{h_{j+1} - h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}} - \left(\frac{\left(\frac{g_{h'}}{g_{e'}}u_{e'} + \left(\frac{g_{h'}}{g_{e'}}(1-\mu)T + l_{j+1}(1-\tau)w\right)u_{c'}\right)\xi\psi h_j^{\psi}}{(1+r)u_{c'}((1-\mu)T + h_j(1-\tau)w)}\right)^{\frac{1}{1-\psi}}$$

iterate on the guess of h_j until the distance function f equals zero.

3. Compute the educational time investment e_j from D.3 as

$$e_j = \frac{1}{h_j}\left(\frac{h_{j+1} - h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}} \tag{D.22}$$

4. Compute the leisure-to-consumption ratio lcr_j using the derivatives of the utility function in D.11:

$$lcr_j = \frac{1 - l_j - e_j}{c_j} = \frac{1-\phi}{\phi}\frac{1}{h_j(1-\tau)w} \tag{D.23}$$

5. In order to calculate c_j, we rewrite marginal utility of consumption as:

$$u_c = \phi c_j^{\phi(1-\sigma)-1}(1 - l_j - e_j)^{(1-\sigma)(1-\phi)} \tag{D.24}$$

Using lcr_j we then obtain:

$$\begin{aligned} u_c &= \phi c_j^{\phi(1-\sigma)-1}(lcr_j \times c_j)^{(1-\sigma)(1-\phi)} \\ &= \phi c_j^{-\sigma} lcr_j^{(1-\sigma)(1-\phi)} \end{aligned} \tag{D.25}$$

Because u_c is given from D.8 we can now compute consumption c_j as

$$c_j = \left(\frac{u_c}{\phi lcr_j^{(1-\sigma)(1-\phi)}}\right)^{-\frac{1}{\sigma}} \tag{D.26}$$

6. Lastly, we compute labor supply l_j using:

$$l_j = 1 - lcr_j \times c_j - e_j \tag{D.27}$$

7. If $l_j < 0$ we set $l_j = 0$. Now, the constraint in D.6b is binding and $\mu_j > 0$. We proceed as follows:

   (a) Since h_j cannot be calculated directly from D.17, guess h_j

   (b) Compute e_j as in D.22 using the guess of h_j

   (c) Since $l_j = 0$, recalculate c_j as

$$c_j = \left(\frac{\beta(1+r)u_{c'}}{\phi(1-e_j)^{(1-\sigma)(1-\phi)}}\right)^{\frac{1}{\phi(1-\sigma)-1}} \tag{D.28}$$

   (d) Compute μ_j from D.6b as

$$\mu_j = u_l - \beta_j h_j w(1-\tau)(1+r)u_{c'} \tag{D.29}$$

(e) As pointed out in D.1.5, in case the constraint on labor supply is binding, we can solve for the root of D.19 to find h_j

$$f = e_j - \left(\frac{\frac{g_{h'}}{g_{e'}} u_{e'} + \frac{g_{h'}}{g_{e'}}(1-\mu)Tu_{c'} + l_{j+1}(1-\tau)wu_{c'}}{(1+r)u_{c'}(h_j(1-\tau)w + (1-\mu)T) + \frac{1}{\tilde{\beta}_{j+1}}\mu_j} \right)^{\frac{1}{1-\psi}}$$

where e_j is given from step (b). Let the nonlinear solver `fsolve` in Matlab iterate on steps (a) to (e) until f equals zero. This gives us the correct value of h_j.

8. Compute a_j using the budget constraint in D.9a.

Updating guesses of a_J, c_J and h_J until convergence

We now have an optimal life cycle pattern for consumption, labor supply, educational time investment, assets and human capital based on the initial guesses (a_J, c_J, h_J). We proceed by updating these guesses simultaneously within the non-linear solver `fsolve` in Matlab. The convergence mechanisms are as follows:

1. *Convergence of a_J*
   We update our guess of terminal period asset holdings (and thus terminal period consumption) until the model converges given the boundary condition ($a_{J+1} = 0$, agents die broke) and the condition that first period asset holdings must be zero ($a_1 = 0$).

2. *Convergence of h_J*
   The level of human capital that agents have when they enter the model should equal the predetermined level h_1. To ensure this, we update our guess of h_J until $h_1^n = h_1$.

3. *Convergence of c_J*
   We update our guess of final period consumption as follows:

   (a) We calculate the sum of the present value of consumption and income at each age:

$$PVC = \sum_{j=1}^{J} \left(\beta \frac{1}{(1+r)} \right)^j * c_j$$

$$PVI = \sum_{j=1}^{J} \left(\beta \frac{1}{(1+r)} \right)^j * income_j$$

   where $income_j$ denotes the sum of income that can be freely spent by the agent.

   (b) Compute a new guess of initial period consumption c_1^n by using

$$c_1^n = c_1 \times \frac{PVI}{PVC}$$

   If $\frac{PVI}{PVC} = 1$ agents perfectly smooth consumption over the life cycle, meaning that our guess of c_1^n has converged[2].

   (c) Starting with c_1^n and using the previously computed values $\{l_j\}_{j=1}^J$ and $\{e_j\}_{j=1}^J$, iterate forward using the Euler equation for consumption D.8 to form an update of the guess of final period consumption, denoted c_J^n:

$$c_{j+1} = \left(\frac{c_j^{\phi(1-\sigma)-1}(1-l_j-e_j)^{(1-\phi)(1-\sigma)}}{\beta(1+r)(1-l_{j+1}-e_{j+1})^{(1-\phi)(1-\sigma)}} \right)^{\frac{1}{\phi(1-\sigma)-1}}$$

[2] We follow Ludwig et al. (2012) in using this criterium for convergence of consumption.

(d) Rerun the model using this new guess. Continue updating the terminal guess until $c_1^n = c_1$.

D.2.2 The implementation of borrowing constraints

We use a simple, yet effective[3] method to implement borrowing constraints in our model. When a borrowing constraint is active, an additional loop is executed after the inner loop has converged. The detailed procedure is as follows:

1. After finishing the loop described in section D.2.1, we check whether the minimum of the generated asset vector is lower than the borrowing constraint (bc).

2. If $\min\{a\}_{j=1}^{J} < bc$, we add $\{\lambda\}_{j=1}^{J}$ to the intertemporal Euler equation of consumption (D.8) in the constrained periods using:

$$
\begin{aligned}
u_c &= \beta(1+r)u_{c'} + \lambda_j \\
\text{with} \quad \lambda_j &= (bc - a_j) \times update_{bc}
\end{aligned}
$$

   In which $update_{bc} < 1$ is an update rule used for stability of the convergence process. By adding $\{\lambda\}_{j=1}^{J}$ to the intertemporal Euler equation, we decrease the utility of consumption, causing agents to consume less and work more during the periods in which they are borrowing constrained. Lower consumption and more labor income will reduce the borrowing of the agents, therefore simulating a world in which a borrowing constraint is present.

3. We rerun the model with the borrowing constraint going through the inner loop of section D.2.1. We check again whether $\min\{a\}_{j=1}^{J} < bc$ and update λ if necessary. We stop when $\min\{a\}_{j=1}^{J} = bc$.

D.2.3 Outer loop

Whereas the inner loop serves to solve the agent's problem given the total tax rates $\tau = \tau^e + \tau^p$, the replacement rate ρ and factor prices w and r, the outer loop is used to: (1) solve for the budget balancing tax rate τ (in case of constant replacement rates) or the budget balancing replacement rate ρ and tax to finance consumption τ^e (in case of constant contribution rates), in an aggregate world, and (2) solve for the general equilibrium factor prices w and r. We choose to converge the government budget and w and r using a manual iterative algorithm, rather than a non-linear solver. This is for the sake of stability of the process.

1. (a) Based on the optimal paths of $\{l_j\}_{j=1}^{J}$, $\{c_j\}_{j=1}^{J}$, $\{e_j\}_{j=1}^{J}$ and $\{h_j\}_{j=1}^{J}$, calculate the tax revenue from all agents using τ, which consists of the contribution rate to the pension system τ^p and the tax rate required to fund tuition subsidies τ^e.
   (b) Sum yearly governmental subsidies to education ($\mu T e_j$) and transfers (ρw).
   (c) *In case of constant replacement rates:* calculate the budget balancing total tax rate τ by dividing the sum of total government expenditures over the sum of total gross wage.

   In case of constant contribution rates to the pension system:

   i. Compute the pension budget balancing replacement rate ρ by dividing total tax revenue for the pension system by the number of retirees times the gross wage.

[3] It was recommended to us by one of the authors of Ludwig et al. (2012), Alexander Ludwig, in a meeting with him on 14 November 2012.

ii. Compute the education budget balancing tax rate for the tuition subsidy τ^e by dividing the sum of government expenditures on education by the sum of gross wages earned in the economy.

(d) *In case of constant replacement rates:* Use the update rule $\tau^n = \tau^{n_{previous}} - update_\tau(\tau^{n_{previous}} - \tau)$ to formulate a new guess τ^n and rerun the model. $update_\tau < 1$ is used to dampen the update process.

In case of constant contribution rates: update guesses of ρ and τ^e with similar dampening update rules.

(e) Repeat steps (a) to (d) until the budget is balanced according to equation 3.6.

2. (a) Based on the optimal paths of $\{l_j\}_{j=1}^J$, $\{c_j\}_{j=1}^J$, $\{e_j\}_{j=1}^J$ and $\{h_j\}_{j=1}^J$, and the initial guesses of w and r, calculate aggregate production Y and the stock of factors of production K and L using equation 3.4.

(b) Using K and L, calculate the implied wage per unit of human capital w and the implied interest rate r using equation 3.5.

(c) Update the guess of w and r alternatively, using the update rules $w^{new} = w - update_w(w^{new_{previous}} - w)$ and $r^{new} = r - update_r(r^{new_{previous}} - r)$. $update_w < 1$ and $update_r < 1$ are used to dampen the update process.
Rerun the model with the updated guess of w^{new} or r^{new}.

(d) Repeat steps (a) to (c) until both the the wage rate per unit of human capital w and the interest rate r have converged to their general equilibrium values specified by equation 3.5.

D.3 Demographics

We implement rising old-age dependency by loading population distribution predictions of Statistics Sweden[4] for the years between 2013 and 2050. As these predictions are published for five year age intervals, we interpolate on the data to get yearly estimates. Due to various demographic developments, the old-age dependency ratio, $OAD = \frac{\sum_{j=jr}^{J} N_{t,j}}{\sum_{j=1}^{jr-1} N_{t,j}}$ will rise. Due to the interpolation, the estimates of old-age dependency in the population become slightly imprecise when computed using the population weights. Therefore, we apply a method to match the true old-age dependency ratios as predicted by Statistics Sweden[4].

The old-age dependency ratio is matched as follows:

1. Calculate the additional weight that needs to be given to retired agents at each age:

$$OAD_{weight} = \frac{OAD_{data} \sum_{j=1}^{jr-1} N_{t,j} - \sum_{j=jr}^{J} N_{t,j}}{J - jr} \tag{D.30}$$

2. Add this weight to each aggregation weight during the pension period, $\{N_{t,j}\}_{j=jr}^J$.

Note that we do not include survival rates in the model and therefore cannot adjust for changes in life expectancy. Increased life expectancy would increase utility of agents because they live longer. As we want to isolate the pure effects of changing population distribution on aggregate variables, rather than the utility effects of increased life expectancy, we keep survival rates constant at 1 throughout our analyses.

[4]Source: Statistics Sweden, 2013, "The future population of Sweden 2013-2060", table: 9.

D.4 Consumption-equivalent variation

An important measure in the results section of our model is consumption-equivalent variation (CEV), denoted g_j. This measure shows the increase in consumption that needs to be given to an agent to make her equally well off as in a baseline case. It is computed by isolating the c_j term in the utility function and then entering the labor supply and educational time investment of the *current run*, and the utility of the *baseline run* of the model (denoted u_j^o):

$$g_j = \Big(\frac{u_j^o * (1-\sigma)}{\beta^j (1 - l_j - e_j)^{(1-\phi)(1-\sigma)}}\Big)^{\frac{1}{\phi(1-\sigma)}} \tag{D.31}$$

Summing this consumption-equivalence measure for all periods and comparing it to the consumption of the current run gives the change in consumption-equivalence (G) between the two runs:

$$G = \frac{\sum_{j=1}^{J} c_j - \sum_{j=1}^{J} g_j}{\sum_{j=1}^{J} g_j} * 100 \tag{D.32}$$

Positive numbers indicate that households obtain a net welfare gain and negative numbers indicate a welfare loss.

Appendix E

Matlab program files – readme

The algorithm of the model in this thesis is programmed in Matlab. The authors are happy to share the code upon request. Please find a description of the program files here. To make configuration easy, the files have been extensively commented.

E.1 Scripts and functions

The package with the program files consists of the following files and folders:

1. **v1_9.m** [the main script file]
   This is the starting point to run the model. From this file the loops of the model are called upon. The script is structured as follows:
   (a) **Settings:** the specifications of the model run as well as convergence criteria and maximum iterations per loop can be configured here.
   (b) **Parameters:** the parameter settings of the model can be set.
   (c) **Header:** generates some general output to the command window.
   (d) **Demographics:** loads the population distribution of the desired year for Sweden and calculates the population weights to match the old-age dependency ratio.
   (e) **Main program:** loads data from the 'Input' folder and calls upon the 'mainloop.m' function file to run the model. After the model is run, output is normalized.
   (f) **Graphs:** in this section, graphs are generated (if set) using the 'graphs_standard.m' function file and saved in the 'Output' folder.
   (g) **Footer:** saves the output of the command window (if set) in the 'Output' folder and closes the model.
2. **Functions** This folder contains the functions used in the algorithm:
   - **mainloop.m** The outer loop (1) solves for the budget balancing tax rate τ, or replacement rate ρ and education tax rate τ^e, (2) solves for the general equilibrium factor prices w and r (if general equilibrium is activated), and (3) calls upon 'innerloop.m' using the non-linear solver `fsolve` to solve for the final guesses of consumption, human capital and assets given a borrowing constraint bc (if set). See section D.2.3 for a description of the algorithm.

- **innerloop.m** Calls upon 'lifecycle.m' to optimize lifecycle assets, consumption, labor supply, educational investment and human capital. Within the inner loop, the final guesses of consumption, human capital and assets are updated.
- **lifecycle.m**
  Optimizes assets, consumption, labor supply, educational investment and human capital for the life cycle of the agent. The function takes as input (1) the parameters, (2) the tax rate, replacement rate, interest rate, and wage rate per unit of human capital, and (3) the guesses of human capital, assets and consumption in the last period. See section D.2.1 for a description of the algorithm[1]. The following additional functions are called upon:
  (a) **UcFunc.m** – the utility of consumption function as in D.6h.
  (b) **UlFunc.m** – the utility of leisure function as in D.6i
  (c) **HumCap.m** – the human capital accumulation function as in D.17. This function is used when $\mu = 0$ and $T = 0$ and h_j can be solved analytically.
  (d) **HumCap2.m** – the human capital accumulation function as in D.19 when the constraint on labor supply is binding. e_j, h_j and μ_j are solved for using the non-linear solver `fsolve`.
  (e) **HumCap3.m** – the human capital accumulation function as in D.17 when the constraint on labor supply is not binding. This function is used when $\mu > 0$ and $T > 0$. h_j is solved for using the non-linear solver `fsolve`.

3. **Input**
   Consists of data for Sweden used in the model and for the graphs.
4. **Output**
   In this folder, the output of the command window and generated variables will be saved. If multiple years are run, a separate folder will be created in which the results for each year will be saved. The output files will be created with a unique file name stating the time and date of the run. Note that graphs will be saved in the main folder and will be overwritten with each new run.
5. **Auxiliary**
   Some auxiliary files and functions used within the script:
   - **graph_standard.m:** to generate nice looking line charts in the standard format.
   - **smooth.m:** the standard smooth function of Matlab
   - **CompareResults.m:** can be used to calculate the change in key variables between two runs. Call upon the function in the following way:
     `[results_change results_change_percent] = CompareResults('filename1','filename2');`
     Replace 'filename1' and 'filename2' with the respective filenames of the .mat output files generated after a run of the algorithm.

[1] The lifecycle program also allows for modeling a separate schooling period. As our thesis assumes lifelong learning, this period has not been modeled separately in the results presented in this thesis. Therefore, this period is skipped in the standard configuration.

E.2 Configuration

In the main program file under `Settings` and `Handles`, a couple of setting can be configured. For details on these settings, see the comments in the Matlab code. The following is an easy example configuration:

`%Handles`	
`info.balance=1;`	Balance the budget
`info.general_eq=1;`	Solve for factor prices in general equilibrium
`info.bc=0;`	Do not apply a borrowing constraint
`info.runprecise=1;`	Generate precise estimates rather than approximations (reduces speed)
`info.constantTauP=0;`	Model the constant replacement rate scenario (tax rate adjusts)
`info.OAD=1;`	Adjust for the old-age dependency ratio of the specified year
`info.population=1;`	Load the population distribution of the specified year
`info.Save=0;`	Do not save output

E.3 Dashboard

The Matlab command window will display the following information during the iteration process:

- `Borrowing constraints: currently processing iteration...`
  Within the outer loop, the script is updating to match the borrowing constraint (if set).
- `Budget: processing iteration...; guess tax rate = ...`
  Within the outer loop, the script is solving for the budget balancing τ (in case of constant replacement rates) or ρ and τ^e (in case of constant contribution rates).
- `Gen. Eq: processing iteration...`
  Within the outer loop, the script is solving for the general equilibrium factor prices and the budget balancing tax rate (in case of constant replacement rates); or the budget balancing replacement rate and tax rate to fund tuition subsidies (in case of constant replacement rates).

You could encounter the following error messages:

- `Complex numbers generated. Set different starting values.`
  Within the lifecycle, complex numbers are generated. Change parameters or starting values to make sure the algorithm runs smoothly.
- `General equilibrium: maximum number of iterations reached.`
  The true general equilibrium wage and interest rate could not be found. Increase the maximum number of iterations, or change the initial guesses to reach the true value of the prices more quickly.