Abstract

We extend the theory of uncovered interest rate parity to explain the black market premium. The theory is applied to different economic-monetary regimes in Belarus and tested through the estimation of a vector error-correction model (VECM) for the cointegrating relationship between black market prices, interest rate differentials and inflation differentials, using a newly created dataset. In support of the theory, our findings suggest that the black market premium is highly affected by the inflation differential, whereas the impact from the nominal interest rate differential is not straightforward. We do not find support for the presence of a constant risk premium prevailing on the market.

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I. INTRODUCTION

Black market exchange rates, or parallel foreign exchange systems, is a common feature of transition and development economies. They develop mainly as a result of capital controls, which induce the population to seek new sources of foreign currency (Yuk & Wong 1995). These dual exchange rate regimes cause problems for policymakers in their attempts to regulate balance of payments. For example, the black market exchange rate affects the level of international reserves and the official exchange rate in the long run (Olgun, 1984); (Caporale & Cerrato, 2008). Therefore, understanding the determinants of the black market exchange rate is crucial not only for good policy making but also for investors trying to manage exchange rate risk.

Quirk (1987) showed that parallel exchange rates reduced the effectiveness of stabilization policy as the monetary instruments lose power. Similar results were presented by Gulati (1988), who argued that black market exchange rates increase the cost of defending the official exchange rate. Panel cointegration studies also point at black markets as a cause for central banker headache. Bahmani-Oskooee et al. (2002) showed that in the long run, the parallel exchange rates are cointegrated, thus implicating that any foreign exchange controls only have short-run effects on the official rate. In the long run, the official rate will adjust toward the black market rate such that the Black Market Premium (BMP)\(^1\) decreases. Put differently, if there is a significant difference between black market and official market exchange rates, the central bank will eventually close that gap by devaluing (in most cases) the official currency at some future point in time.\(^2\) While the problems associated with black market exchange rates are many, they play an important economic role in foreseeing shifts in the exchange rate or, following the results from parallel exchange rates cointegration, even correcting mispricing in the official exchange rate. In that sense, a black market is not the problem, but rather a symptom of the problem (and, following the implications from long-run equilibrium, perhaps even a remedy).

The relation between black market rates and official exchange rates is relatively well-documented, but there is a lack of understanding of what economic parameters actually causes black market exchange rates to differ from their official equivalents, and also what pushes the official rate to follow black market prices. Our contribution is an attempt to fill this gap and enhance the understanding of the BMP determinants. With a modified theory of black market behaviour based on Uncovered Interest Rate Parity (UIP) as well as with a newly created dataset of black market exchange rates in Belarus, we hope to shed light on the question of whether the black market exchange rate is caused by failure to compensate for currency risks arising from interest rate differential and inflation differential. For this purpose we estimate a VECM model. While the specific details of this study are chosen with the Belarus case in mind, the theory might be applicable to other countries as well.

The rest of this paper is organised as follows: Section II presents previous research on black market exchange rates. Section III provides a short history of the Belarusian economic development and should be read briefly. Section IV presents the theoretical framework and shows how the black market exchange rate can be brought into the UIP framework. Section

\(^1\) The black market premium is defined as the percentage difference between the black market rate and the official rate.

\(^2\) Of course the underlying assumption is that we are examining a fixed (or semi-fixed) exchange rate regime.
V presents remarks and considerations of the data. Section VI provides a tentative analysis of black market determinants which motivate the use of the econometric framework presented in section VII. The empirical results from cointegration analysis are presented in section VIII. Section IX summarises.

II. PREVIOUS RESEARCH

Today, the idea that the official and black market exchange rates have a long-run relationship is a well-established fact (see for example Baghestani & Noer (1993), Bahmani-Oskooee et al. (2002) and Caporale & Cerrato (2006)). In a sense, the enigma of the black market exchange rate determinant is thereby solved. Since the black market exchange rate is cointegrated with the official rate, the long-run determinant of the black market exchange rate has already been disclosed. However, while the parallel rates are cointegrated, most research point towards the black market exchange rate as being the dominant variable. The long run equilibrium between them is therefore maintained by corrections in the official exchange rate rather than in the black market rate (see especially Bahmani-Oskooee et al. (2002)). This suggests that factors, other than the official exchange rate, contribute to changes in the black market rate.

During the past thirty years, a number of attempts to explain the phenomena of black currency markets have been made, presenting different views. Dornbusch et al. (1983) developed an analytical framework based on the current account in Brazil to understand the determinants of the BMP. They showed that the current expectation of a future maxi-devaluation leads to an immediate rise in the BMP. Dornbusch et al. did not however allow for any effects from the BMP back into macro variables.

From an equilibrium point of view, the research on black market exchange rates presents different views on what characteristics it should be awarded. In his argumentation for financial liberation, Fry (1997) implicitly assumes that black market prices are efficient equilibrium prices. Others question the idea of black market efficiency. Emran & Shilpi (2010) present empirical evidence from India and Sri Lanka for which the null of black market exchange rate as the equilibrium exchange rate in the specification of an aggregate import demand function is strongly rejected. In fact Lizondo showed already in 1987 that black market exchange rate can miss the equilibrium level, either by undershooting or overshooting the target. Yet the idea that black market exchange rate tend to be closer to the equilibrium price, as formulated by Dornbusch et al. (1983), has survived the wear and tear of three decades of research.

Glen (1988) analysed black market currency as an equilibrium asset, which allowed investors to invest in foreign currency based assets. In his work the black market exchange rate is determined by the interest rate differential between countries and the expected future black market exchange rate. While Glens’ approach has a certain appeal from an efficient markets perspective, it failed in empirical testing to provide a convincing explanation of the movement in the black market exchange rate over time, and did not provide an explanation for the black-market premium. It also left out the connection between the official exchange rate and the black market rate, purely focusing on the no-arbitrage conditions within current and expected (ex post) black exchange rates.

Some have suggested routes to explain black markets that lie outside the domains of portfolio balance or speculation. For example, Sylwester (2003) approached the problem from a different angle looking at income inequality effects on the BMP. The findings indeed confirmed that the Gini coefficient is associated with the level of BMP, but the way

\[ \text{A low Gini coefficient indicates a more equal distribution, with 0 corresponding to complete equality, while higher Gini coefficients indicate more unequal distribution, with 1 corresponding to complete inequality.} \]
inequality would cause the BMP is not straightforward to explain. Speculators and arbitrageurs who gain from interest differentials might very well explain why income equality would co-move with BMP, rather than the inequality itself. This idea also gets support by Sylwester, who conclude that interest differentials probably are more relevant in explaining the BMP.

The notion of interest differentials as a driver of a currency “shadow price” is also supported by the work of Flood & Marion (1998). Although their work is not aimed toward the BMP in particular they conclude that the shadow price, which is calculated with the interest rate differentials as fundamental driver, pushes the official currency to devaluation. Thereby the Flood & Marion shadow price shares the property of black market exchange rate in the black-official cointegration studies (Bahmani-Oskooee et al. (2002) and Caproale & Cerrato (2006) among others) as a driver for official exchange rate adjustments.

III. HISTORICAL DEVELOPMENT

In this section, we provide an overview of the Belarusian economic development since the dissolution of the Soviet Union, which will serve as a background for our numerical exercises. The section can be read briefly.

We find that the period can be split into six distinguishable economic-monetary phases:⁴ Turbulent Years (1990-1996), In Search for a Nominal Anchor (1996-2001), Crawling Peg (2001-2004), USD de facto Hard Peg (2004-2009), The Currency Basket Peg (2009-2012) and The Free Float Regime (2012 – present⁵). For an overview of relevant variables, please see the graphs at the end of this section.

_Turbulent Years (1990-1996)_

In July 1990, the Declaration of the State Sovereignty was passed and on August 25, 1991, Belarus declared independence. In addition to a period of political turbulence, all sectors of the national economy were affected by the consecutive economic crisis. Belarus, with imports and exports constituting a half the GDP, experienced a twin shock: a sharp increase in cost of critical inputs and a collapse of traditional markets (The World Bank, 1997). Additionally, the poor quality of its products and low productivity made it difficult for Belarus to redirect its exports to international markets.

Throughout the first years of independence, old Soviet ruble combined with newly issued Russian ruble was considered as the main means of exchange. In May 1992, the first issue of Belarusian money (BYB) was introduced, though it took about two years before the Belarusian ruble became the official currency. In May 1994, by the decision of the National Bank of the Republic of Belarus (NBRB), the Belarusian ruble became the only means of payment officially recognized within the country. However, many people still preferred to pay with strong currencies like the U.S. dollar.

The same year, 1994, a newly elected president, Aleksander Lukashenko, trying to deal with economic problems caused by the collapse of the USSR, launched the country on a very different route of “market socialism”. The government tried to maintain living standards and employment through expansionary monetary and credit policies. As a result, nominal prices spiked – GDP inflation was nearly 2000% in 1994 and 660% during 1995 while GDP, per capita and total consumption fell sharply. Overall, prices increased by over 50,000 times during 1990-1995 (The World Bank, 1997).

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⁴ _IMF Staff Reports_ and yearly _Country Reports_ on Belarus have been very helpful in this part.
⁵ April 2013.
In 1996 the first signs of economic recovery appeared. For the first time since 1991, GDP per capita exhibited positive growth and inflation was brought down to relatively moderate levels (The World Bank, 1997). An official and fairly recognized currency was now in place and relative political stability prevailed. During this period, the central bank tried to achieve exchange rate stability using both the US dollar and the Russian ruble in attempts to establish a nominal anchor. However, the exchange rate became subject to heavy influence from initiatives to stimulate the economy through monetary expansion. Added to these problems, the high rate of dollarization restricted the room for adequate central bank manoeuvring. The share of dollar deposits in deposits of commercial banks increased from 40% at end of 1995 to over 60% in April 1999 (International Monetary Fund, 1999).

In 1998, Belarus was hit by the aftermath of the Russian financial crisis (International Monetary Fund, 1999). Due to its extreme dependence on the Russian economy, GDP growth declined and exports and imports fell heavily. This caused the government to further loosen the monetary policy, resulting in even more pressure on the exchange rate. As a result, net domestic credit increased by 95% and broad money (M3) grew by 130%. During 1999 the inflation had again reached high levels and was nearly 300% despite diligent attempts to impose price controls on the markets - a quite reasonable “inconsistency” as the printing press was steaming hot from all the hard labour.

In order to maintain the exchange rate level, the inflationary monetary policy was combined with severe restrictions on the exchange market such as limiting daily conversion of BYR to USD to $300 per day (International Monetary Fund, 1999). During this time, the BMP soared, reaching 260% in January 2000. Another potentially important restriction during this regime was the 30% export surrender requirement. This regulation forced companies to declare their export profits converted at the official exchange rate, which was equivalent to an implicit export tax between 15 - 20% depending on the spread between official and black markets during 1999 (International Monetary Fund, 2000).

The pressure on the exchange markets, finally, caused the BYB to depreciate sharply: by 200% against the US dollar and even 50% against Russian ruble. As banknote digits became extensively large, a new ruble (BYR) was introduced in 2000. It replaced the old one at a rate of 1 new = 1000 old rubles (see Table 1 for a compilation of banknote development). This marked the beginning of a new, more liberal, currency regime with a crawling peg against the Russian ruble in which the BMP disappeared.

Crawling Peg Regime (2001-2004)

The intention to establish a monetary union with Russia dictated the optimal exchange rate system in the years to come. Belarus adopted a crawling peg vis-à-vis Russian ruble in the beginning of 2001 (International Monetary Fund, 2003). A newly established trade union with Russia and liberalization of the foreign market improved current account sufficiently and set a new period of economic recovery. The National Bank of Belarus merged the main and additional trading sessions and announced that the official exchange rates, although pegged, would be determined by supply and demand (International Monetary Fund, 2000). This kept the BMP at low levels (3% in January 2001 compared to 260% one year earlier). According to International Monetary Fund (2002), during this period a number of capital restrictions were removed both in private and commercial sector. An important change for

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6 Exports to Russia accounted for more than 50% of total exports.
7 Black market/official market.
general public was the rescindment of the $300 exchange per day limit for households. For authorized commercial entities, the removal of over-the-counter restrictions on foreign exchange transactions between banks and corporations as well as between resident banks and non-resident banks was important.

However, the amount of liberal reforms was overall limited. For example, the 30% export surrender requirements were still at place putting pressure on the exports. According to IMF’s Republic of Belarus: Selected Issues (2002), dollarization of the Belarusian economy continued due to lack of financial instruments that preserved the real value of portfolios in Belarusian ruble. In September 2001, dollarization was again more than 50% which made the peg towards Russia seem more and more like a political ambition rather than a monetary reality. The crawling peg towards the Russian ruble continued to remain the official standpoint; however the credibility of the regime did not sustain.

USD de facto Hard Peg (2004-2009)

From 2004, Belarus and many other economies experienced a tranquil period of steady growth, which lasted until the beginning of the financial crisis in late 2008. The inflation was relatively steady and kept at, by Belarusian standards, a low average level of 12% annually (World bank). While the de jure exchange regime remained to be the Russian crawling peg, this period was de facto a hard peg against the US dollar (SITE, 2007). During this period the black market exchange rate and the official exchange rate had no significant difference.

An important factor that helped to stabilize economy was additional income coming from cheap Russian oil and gas. While providing Belarus with cheap access to natural resources, the great economic dependence on Russia proved to be problematic. Already in 2004, negotiations had started regarding the price of resources. By 2007 the price of imported oil and natural gas doubled putting pressure on the competitiveness of Belarusian goods and current account balance (SITE, 2007). However, the USD/BYR exchange rate remained relatively stable until the end of 2008, when the financial crisis hit the markets.

The Currency Basket Peg: A Period of Distress (2009 -2012)

Throughout previous years, national currency exchange rate had remained stable. During the previous regime, the annual inflation had been kept at moderate 12%. In January 2009, in the aftermath of the financial and economic crisis, Russia devaluated the Russian ruble against hard currencies. Due to its dependency on Russian exports, the Belarusian central bank followed. In June 2009, a new anchor in a form of currency basket peg with equal

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8 General public was still required to present identification documents (passport) when purchasing hard currency.

9 Exporters were forced to buy and sell currency for the official rates at the trading sessions.
weights assigned to the U.S. dollar, the Euro, and the Russian ruble, was introduced (KPMG in Belarus, 2011).

In 2010 Belarus experienced several external shocks. Russia reduced the discount on oil while prices for natural gas rose to new all time highs. Despite the shortage in means to payment, the authorities pushed for higher economic growth in an attempt to honour the President's promises to increase average wages in the economy to $500 a month\(^\text{10}\). As a result, public sector wages rose by 50% and current account deficit widened and public debt increased by $3.8 billion\(^\text{11}\). Borrowings to finance these efforts were mostly conducted in foreign currency.

During this time, political repressions against the opposition put Belarus in severe confrontation with EU and USA. Loans from IMF became impossible and credit risk skyrocketed (Preiherman, 2011). This led to severe economic misalignments. Rumours about possible devaluation were spreading so people started to convert their savings into hard currencies. In an attempt to signal stability, as of January 1 2011, the currency band was narrowed from ±10% to ±8% (International Monetary Fund, 2011). Markets ignored the gesture and the central bank was forced to spend $1 billion of the foreign reserves to balance the supply and demand of currency.

In March, the government banned sales of the foreign currency to the banks. Foreign exchange offices dried out in a matter of days and Belarus found itself in a full-scale currency crisis (Lenta.ru, 2011). On the black market, BYR now was traded at half the official value. As the situation became impossible, the central bank declared a 50% devaluation against the currency basket. This was the largest devaluation in 20 years worldwide (KPMG in Belarus, 2011).

A great portion of the savings in BYR disappeared over night and the public panicked in an attempt to protect what remained. Queues to the exchange offices became monstrous and store shelves were empty (Karmanau, 2011). The central bank, despite the devaluation, had difficulties to secure external founding and could not support newly established exchange. On August 30, 2011, President Lukashenko therefore announced that from mid September, the Belarusian ruble would be allowed to float freely against foreign currencies (RIA Novosti, 2011)\(^\text{12}\), which led to a unification of exchange rates at the end of 2011.

**The Free Float Regime (2012 -present)**

The 2011 crisis revealed weaknesses of the Belarusian economy and called for a number of structural changes. However, Belarus found its own way to deal with the problems. During the first five months of 2012, the resurgence of oil exports allowed government to avoid the necessary reforms (Firsava, 2012). This led to a trade surplus of 3 billion dollars, which was two times more than forecasted. Because of this development, new promises of high wages, low inflation and interest rates could be made somewhat credible. However there was a big problem with the promises made by Lukashenko, the pleasant figures came from evasion of the Russia-Belarus oil agreement\(^\text{13}\). Belarus was actually exporting oil under false label such as solvents and thinners (Firsava, 2012). Russia did not leave the “success” unnoticed and already in October current balance turned to deficit.

While in the first half of the year supply of foreign currency by households exceeded demand, in the second half demand was running high. However, hoarding of foreign cash became less profitable than deposits in BYR since real interest were positive again (BEROC, 2012).

\(^{10}\) Increase in real wages was not accompanied by changes in productivity level.

\(^{11}\) $2.3 billion where borrowed in the final quarter of the year.

\(^{12}\) Some interventions would still be optional though – for example a separate rate was applied to government gas and electricity payments (Exclusive Analysis, 2011).

\(^{13}\) The two countries agreed that Belarus would get access to duty free Russian oil. In return, Belarus committed to transfer all export duties stemming from exports of this oil.
During 2013 Belarus will start to repay foreign debt accumulated during the 2011 crises. High debt burden in combination with the possibility of Belarusian producers facing higher competition both at home and abroad\textsuperscript{14} will ensure high pressure on the current account.

Even though authorities claim that there is little room for one-time devaluation, the public is careful with what is left of their savings. Around thousand Belarusians received an SMS text on January 2, 2013 that stated that the central bank was preparing a new devaluation of up to 60%. The message turned out to be a provocation, and, in the end, the currency market didn’t move. However, it shows the high level of tension and peoples’ distrust in the officials.

\textsuperscript{14} When Russia was granted membership in the WTO, due to free trade agreements, Belarus became \textit{a de facto} member.

Graph 1: Black Market & Official Exchange Rates

![Graph 1: Black Market & Official Exchange Rates](image)

Source: Own calculations based on IPM Research Center Database and NBRB’s \textit{Bulletin of Banking Statistics}
Graph 2: Interest Rates & Inflation

Source: Own calculations based on IPM Research Center Database and NBRB's Bulletin of Banking Statistics

Graph 3: Trade Balance

Source: Own calculations based on IPM Research Center Database and NBRB's Bulletin of Banking Statistics
In this section, we present the theoretical framework that connects black market exchange rates, black market premium (BMP) and uncovered interest rate parity (UIP). The analysis is based on a two agents (central bank and investors) problem where the former “signals” what a future exchange rate should be and the latter “accepts” or “rejects” the proposition. We argue that in case both agents’ beliefs coincide no black-market is possible; otherwise the differential determines magnitude of the BMP. Further, the analysis assumes that the country being studied is a small open economy in sense that changes in the domestic interest rates and inflation do not impact the foreign. This is consistent with the topic at hand. At the end of this section, a more concrete theoretical discussion about driving forces behind black market exchange rates is presented.

We start by looking at a very basic UIP. As already mentioned, it is a no-arbitrage condition representing an equilibrium state under which investors are indifferent to interest rates available on bank deposits in two countries. The main two assumptions are capital mobility and perfect substitutability of the assets. Investors can readily exchange domestic assets for foreign and no additional interest is required because the assets are somewhat identical (Mishkin, 2006). According to Froot and Thaler (1990), if investors are risk neutral and have rational expectations, then the market’s forecast of the future exchange rate is implicit in the interest differential. To illustrate the principle, suppose that the one-year USD interest rate is 5% and that comparable interest rate in BYR is 10%. In this case, a risk neutral, rational investor must expect a 5% depreciation of Belarusian ruble against the US dollar. This amount should be just enough to equalize returns from the investment alternatives. On the other hand, if investors expect BYR to appreciate by, say, 4% they would wish to borrow in BYR and lend in USD. Consequently, interest in BYR would rise whereas interests in USD would fall until the difference between the rates is exactly 4%. Thus, UIP
implies that the interest differential is a proxy for future exchange rate. If expectations are rational, this estimate should be unbiased. The equilibrium relationship is the following

\[(1 + i_t) = (1 + i_t^*) \frac{E_t}{E_{t+1}}\]

or

\[E_{t+1} \approx E_{t+1}^e = \frac{(1 + i_t^*)}{(1 + \pi_t^e)} E_t\]

where \(i_t\) is domestic nominal interest rate in time \(t\), \(i_t^*\) foreign nominal interest rate, \(E_t\) is the spot exchange rate, \(E_{t+1}^e\) expected exchange rate in period \(t + 1\) and \(E_{t+1}\) is the future exchange rate. Both exchange rates are expressed as amount of foreign currency for one unit of domestic. We should also note that, so far, no distinction between floating and fixed exchange rate arrangements has been made.

Gulde (2008) argued that the best currency regime should ensure credibility of monetary and exchange rate policies in transition economies. Consequently, one of the main arguments in favour of not only fixed, but possibly very hard exchange rate regimes is the desire to gain credibility and send strong “anti-inflationary” signals. Theoretically, putting monetary policy under strict rules would eliminate wilful mischief or policy mistakes. Applying this argument on UIP, this means that the main aim of the central bank under a fixed exchange rate is to conduct monetary policy that persuades investors that the target exchange rate is the one that will prevail on the market. This condition can be expressed as \(E_{t+1}^e = E_{t+1} = E_{t+1}^*, \) where \(E_{t+1}^*\) is the desired exchange rate by the government in period \(t + 1\). Kamin (1993) suggested that most developing countries initiating stabilization programs have extensive exchange controls in place, which, in case the central bank fails to meet the condition, lead to a BMP due to anticipation about future devaluation/revaluation and excess demand/supply for the foreign currency.

So, the central bank decision is somewhat trivial. It “signals” future exchange rate by choosing appropriate \(i_t \)

\[E_{t+1}^e = \frac{(1 + i_t^*)}{(1 + i_t)} E_t\]

Now, the important question is whether investors are convinced by the actions of the central bank. To see that, we need to investigate how \(E_{t+1}^e\) evolves. Since investors are only interested in real return on investment, letting \(r_t\) and \(r_t^*\) to denote domestic and foreign real interest rate in time \(t\), \(\pi_t\) and \(\pi_t^*\) denote the domestic and foreign inflation rate, we use an expectations augmented Fisher equation\(^ {15}\) to decompose nominal interest rates into real interest rates and inflation

\[i_t^* \approx r_t^* - \pi_t^*\]

and

\[i_t \approx r_t - \pi_t^e,\]

a more precise version

\[1 + i_t^* = (1 + r_t^*)(1 + \pi_t^e)\]

and

\[1 + i_t = (1 + r_t)(1 + \pi_t^e),\]

\(^ {15}\) Irving Fisher, in his article, “Appreciation and interest” (1896), showed the relation between the nominal interest rate, the real interest rate and inflation. The Fisher equation can be used in either \( \text{ex ante} \) or \( \text{ex post} \) analysis. The main difference is that \( \text{ex post} \) describes the real return the investor actually received on an investment, and \( \text{ex post} \) describes how an investor decide upon the nominal rate that should be charged for the loan based on his inflation expectation and risk profile for a given period.
substituting into UIP yields following result

\[ E_{t+1}^e = \frac{(1 + r_1^e)(1 + \pi_t^e)}{(1 + r_t)(1 + \pi_t^e)} E_t \]

and

\[ E_{t+1}^e = E_t (1 + r_t) \phi \]

or

\[ E_{t+1}^e = E_t \phi : \phi > 0, \]

where \((1 + r_1^e)/(1 + r_t) = 1\) due to perfect mobility and substitutability of the assets; \((1 + \pi_t^e)/(1 + \pi_t^e) = \phi\), which should be constant over time given monetary policy commitment to hold chosen exchange rate arrangement\(^{16}\). The constant can also be interpreted as a degree of flexibility of the fixed exchange rate chosen by the central bank. If \(\phi < 1\) a depreciation and \(\phi > 1\) an appreciation is expected. Moreover, \(\phi = 1\) under “hard regimes” such as Currency Boards\(^{17}\) or outright “dollarization”, which implies that \(E_{t+1}^e = E_t\). In other words, if investors believe in good faith of the central bank, there is no reason for them to expect deviations from the established monetary policy and targeted domestic inflation\(^{18}\). Expectations of exchange rate movement purely depend on the chosen exchange rate arrangement \(\phi\), meaning that, \(E_{t+1}^d = E_{t+1}^e\), no black market is possible.

We should also note that the way we defined flexibility of the exchange rate \(\phi\) builds on assumptions of Relative Purchasing Power Parity. If two countries have different rates of inflation, then the relative prices of goods in two countries change. This change leads to a flow of goods across borders until inflation differential is fully covered by appreciation/depreciation of the exchange rate. However, in reality, there is a mixed evidence of the exchange rates converging to what PPP predicts\(^{19}\). The deviations are attributed to differences (among countries) in: (1) consumer baskets, (2) trade restrictions, (3) transaction costs and location. Further, those factors can vary over time (have dynamic nature). Even though rigidities exist, some of them are determined by the policymakers whereas others are somewhat constant over time. Regulatory systems changes slowly and defines international relations, trade restrictions, taxes, etc. Geographic location is constant over time and consumer preferences are slow to change. Therefore, the central bank can cover some deviations from PPP. For simplicity we will not touch the stochastic component of the rigidities for now, and reintroduce it at the end of the section. So, interpretation of the degree of flexibility \(\phi\) is unchanged and implications described in the previous paragraph are still valid.

So far, we have assumed perfect substitutability between domestic and foreign assets. Froot and Thaler (1990) state that if the investors on foreign exchange markets are risk averse and, if foreign exchange rate is not fully diversifiable, then the interest differential or forward discount can no longer be interpreted as a pure estimate of the expected change in the future exchange rates. Rather, the interest differential is the sum of the expected change

\[^{16}\]Some of the exchange rate arrangement allow for constant appreciation/depreciation. More detailed classification of the exchange rate regimes is presented in Appendix I.

\[^{17}\]A Currency Board Arrangement (CBA) is a monetary regime based on an explicit legislative commitment to exchange domestic currency for foreign at a fixed exchange rate.

\[^{18}\]When monetary rules are in place, the policy makers have temptations to cheat, creating inflationary shocks. However, due to repeated nature of the “game” the benefits from deviant behavior rapidly disappear and inflationary costs rise. Thus, it is reasonable to argue that potential loss of reputation is much more costly compared to the benefits. Thus, from a “repeated games”-perspective, policymakers should never cheat under normal settings.

\[^{19}\]For general discussion and overview of empirical studies see Taylor and Taylor (2004). For empirical studies of PPP in East European countries see Sideris (2005).
in the exchange rate plus a risk premium. In other words, when it comes to economies in transition, it is reasonable to believe that transaction costs, taxation, differences in liquidity, and other factors that determine country risk profile and reflect perfect substitutability of the assets are much higher compared to developed countries. Investors need to be compensated for those risks. Thus, BYR interest rates should be higher, even if the exchange rate is not expected to change. Further, assuming stability of political and legal systems and public debt targeting, we introduce a constant \( \theta \) to describe this type of time-invariant compensation

\[
E^{r}_{t+1} = E_t \frac{(1 + r^*_t)}{(1 + r_t + \theta \epsilon^*_t)} \phi.
\]

Real interest rates can also differ due to changes in the risk profile that come from external shocks (economical, political or even purely speculative). For example, suppose that investors observe a large-scale crisis in Russia. Because of the trade dependency, a reasonable expectation is that part of the crisis will transfer into Belarusian economy. Therefore real risk of investment increases and the real value of the loans in Belarus decreases compared to one in USA, which is less linked to the Russian economy. To compensate for the risks, the Belarusian real interest rate must follow in each period where such expectation exists. Consequently, elaborating on work of Flood and Marion (1998), we introduce a time-variant stochastic risk premium \( \theta_t = \theta e^{\epsilon_t} \), where \( \epsilon_t \sim N(0, \sigma^2) \), in the parity condition

\[
E^{\theta}_{t+1} = E_t \frac{(1 + r^*_t)}{(1 + r_t + \theta \epsilon^*_t)} \phi.
\]

We should also note that there is strong empirical evidence that supports the notion of time-variant risk premium in explaining the deviations from UIP\(^{21}\). For example, Fama (1984), Hansen and Hodrick (1980), Hodrick and Srivastava (1984), Korajczyk (1985), Cumby (1988), Mark (1985, 1988), Kaminsky and Peruga (1990) all conclude that forward rates differ from expected future spot rates by a time-varying risk premium.

Going back to the model, an important implication of the equation above is that, now, expectations about exchange rate movement are not only determined by the regime at hand but are allowed to deviate even under a very hard currency arrangement and credible monetary policy. Therefore, in order for the central bank to keep the foreign exchange rate at a chosen level, they should allow for full real compensation in each period where the deviation of the risk profile exists. In this case, the compensation is such that \( (1 + r^*_t) = (1 + r_t + \theta \epsilon^*_t) \) and \( E^{\theta}_{t+1} = E^{\phi}_{t+1} = E_t \phi \).

Conclusively, we argue that there is another channel through which economical, political or even purely speculative shocks can enter into the parity, namely, through an uncertainty about the central bank’s commitment. In fact real interest, inflation and monetary policy are closely related to real sector activities. Consider a model by Barro and Gordon (1983), in which policymakers have two objectives (output and inflation), but their interest in former tempts them to raise output above the equilibrium level, creating an inflationary bias. With rational expectations, this bias builds into the private sector’s inflationary expectations and inflation occurs. In our case, the model can be interpreted in the following way. Developing countries or countries in transition, being poorer, attach higher weight on the output.

---

\(^{20}\) The way we present a time variant stochastic risk premium differs from Flood and Marion (1998). Their approach is to endogenize the risk premium, while we present it as a pure stochastic process that drives compensation for the risks. Our approach is inspired by the way McCandeld in *The ABCs of RBCs* (2008) presented a stochastic process for technology.

\(^{21}\) A good overview of previous empirical studies on the topic is presented by Li, Ghoshray and Morley (2012)
objective compared to the developed. So, it is reasonable to believe that, in case of external shocks policymakers can be tempted to abandon the monetary goal, leading to higher inflation and expected devaluation. Accordingly, investors’ expectations about future exchange rate are

\[ E_{t+1}^f = E_t \frac{(1 + r_t^*)}{(1 + r_t + \theta e^{\varepsilon_t})} \varphi e^{-\beta \epsilon_t}, \]

where \( \beta \) is the parameter that describes the degree to which shocks to the risk premium effects investor’s inflationary belief. The parameter has a negative sign by the definition of \( \varphi \), where expectation about domestic inflation is in the denominator. In case of a positive shock, we should observe an increase in the demanded real interest and a belief that central bank might abandon the target leading to a higher inflation. Note that the definition allows the channel to go in both ways. There is a possibility of pure inflationary shocks that can transfer to the risk premium. A good example would be pre-election monetary expansions driven by candidates in power; however the effect might take time.

Now, given that we know determinants of \( E_{t+1}^g \) and \( E_{t+1}^f \), equilibrium, where no BMP is possible can be expressed as

\[ \frac{(1 + r_t^*)}{(1 + r_t + \theta e^{\varepsilon_t})} \varphi e^{-\beta \epsilon_t} = \frac{(1 + i_t^*)}{(1 + i_t)}. \]

Therefore, to avoid black market exchange, the central bank should set nominal interest (\( i_t^* \)) rates such that

\[ (1 + i_t) = (1 + i_t^*) \frac{(1 + r_t + \theta e^{\varepsilon_t})}{(1 + r_t^*)} e^{\beta \epsilon_t}, \]

using natural logarithms

\[ \ln[1 + i_t] = \ln[1 + \pi_{t+1}^c] + \ln[1 + r_t + \theta e^{\varepsilon_t}] - \ln[\varphi] + \beta \epsilon_t \]

and rules for first-order approximation we get

\[ i_t = \pi_{t+1}^c + r_t^* + \theta - \xi + (\theta + \beta) \epsilon_t, \]

where the degree of flexibility can be written as \( \varphi = 1 + \zeta \). Using this definition, \( \zeta < 0 \Rightarrow \) depreciation, \( \zeta > 0 \Rightarrow \) appreciation (even though it is rarely seen in reality), and \( \zeta = 0 \Rightarrow \) “hard” regimes. In the context of a small open economy \( r_t = r_t^* \) by the definition we used in the beginning of the analysis. Reapplying Fisher equation, we get that

\[ i_t = i_t^* + \theta - \zeta + (\theta + \beta) \epsilon_t, \]

and the BMP (defined in percent) can be expressed in the following way

\[ BMP_t = i_t^* - i_t + \theta - \zeta + (\theta + \beta) \epsilon_t, \]

and

\[ (\theta + \beta) \epsilon_t = \eta_t \sim N(0, (\theta + \beta)^2 \sigma^2), \]

where \( \epsilon_t > 0 \) represent positive inflationary shocks and positive shocks to the risk profile (higher risk), and vice versa.

\[ \text{In our case a positive shock actually means a negative shock to the real economy or positive speculative shock.} \]
So, in a small open economy under a fixed exchange rate in a presence of capital restrictions and extensive exchange rate controls, the level of BMP should be determined by the nominal interest rates, country risk premium, “softness” of the exchange rate regime arrangement (inflation differential), and time varying risks (variability of the fundamentals). The model also implies that countries with relatively low country risk will need less adjustment to the domestic nominal exchange rate in case of shocks, leading to a lower BMP. Note that BMP is independent of the level of today’s exchange rate.

More generally, we would expect that a politically unstable country with bad infrastructure (communication), undeveloped or complex legal system, which is dependent on a single trade partner, and is a subject to conflicting macroeconomic targets to have a large BMP under longer periods of time.

V. DATA REMARKS AND CONSIDERATIONS

Following the theoretical framework proposed in section IV, we proceed with selecting variables appropriate for the analysis of determinants of the BMP. Revisiting the last equation of the previous chapter, and re-grouping the components

\[ BMP_t = (i_t^* - i_t) + (1 + \epsilon_t)\theta - (\zeta - \beta\epsilon_t), \]

the following data should be considered: (1) parallel exchange rates, (2) nominal interest rates, (3) time variant risk premium and (4) inflationary components. We should mention that we are considering a structural system framework to represent the relationship. In this case, one should be careful with degrees of freedom since there is a trade-off between number of explanatory variables and estimation power. Therefore, only “strong” components of the theoretical expression will be considered. We find that BMP, nominal interest differential and inflation differential could be considered for this purpose. As an approximation of the theoretical relationship, we use the above-mentioned variables to estimate following the relationship

\[ a_t = v_0 - v_1b_t + v_2c_t + e_t : e_t \sim N(0, \sigma^2) \]

and

\[ \text{Cov}(e_t, \epsilon_t) = 0, \]

where \( a_t \) is the BMP; \( b_t \) is the interest rate differential; \( c_t \) is the inflation differential, \( v_0, v_1, v_2 \) are the parameters to be estimated and \( e_t \) is an error term. Note that the signs of the coefficients are what we expect drawing upon the theoretical framework. Further in this section, the variables are described and discussed in terms of construction, underlying data, its origin, and reliability. For a graphical overview, see Graph 5 at the end of the section.

**BMP (USD/BYR): Official and Black Market Exchange Rates**

The choice of US dollars as the reference currency demands a further explanation. Since Russia has remained the main trading partner during the years, the Russian ruble could be considered for this purpose. However, from 1991-2013, the dollar has been widely used as a store of value due to its stability. A great portion of bank deposits in Belarus consists of dollars, ranging from 30-70% (see Graph 4). Moreover, despite the large fraction of trade with Russia, 50% of all import transactions are valued in dollars. Therefore the dollar, despite initiatives to peg it towards the Russian ruble and later towards the currency basket,
has maintained the foreign currency of choice of Belarusians (see IMF country reports 1998-2012).

For our analysis, we use monthly averages of exchange rates. The official exchange rate is provided by the National Bank of the Republic of Belarus (NBRB). NBRB follows IMF standards for reporting and its data is deemed to be reliable in the sense that it truly reflects the official exchange rate. The black market exchange rate data for period 1994-2010 is provided by the IPM Research Center and complemented with data for period 2010-2013 from a medium size Belarusian enterprise. Since black market exchange rates are at the core of our analysis, a brief discussion of the validity of this data is required.

The IPM Research Center is a member of the CASE research network (Center for Social and Economic Research Foundation), and cooperates with institutions such as SITE (Stockholm Institute of Transition Economics) and BEROC (the Belarusian Economic Research and Outreach Center). While we can never be certain that the black market data is free from errors, the data from IPM Research Center is likely to be as certain as it gets when it comes to black markets.

During the subsequent period 2010-2013, we use the black market exchange rate obtained from a medium size enterprise in Belarus whose name cannot be revealed due to legal circumstances. It is simply the rate to which the enterprise buys and sells goods on the international market. Clearly, when using different sources like this, problems of bias might occur due to differences in the measurement technique. To account for this potential problem we check for exchange rate deviations by letting the rates overlap during certain time periods. We find that the fit is close to exact between IPM Research Center and the enterprise data during the period of overlap 2004-2010. Additionally, we validate the enterprise data during 2010-2011 with the help from Prokopovi.ch, an independent currency trade platform operating in the Czech Republic during the crisis years 2010-2011. We find that the enterprise data during the overlapping periods fits very well with the data from Prokopovi.ch. We thereby conclude that merging the IPM Research Center data with the enterprise data should not result in any major bias in our analysis.

We construct the black market premium as a percent difference between the black market exchange rate and the official exchange rate so that

$$ BMP = \frac{\text{black market exchange rate} - \text{official market exchange rate}}{\text{official market exchange rate}} $$

**Interest Rate Differential**

For the purpose of our study, we use the nominal interest given to deposit holders in US dollars and Belarusian ruble. The data is provided by the National Bank of the Republic of Belarus. The main reason for looking at the deposit market is that it allows us to circumvent the problem of finding proper proxies for the time varying risk component \((\theta \epsilon_t)\) of the stochastic risk premium\(^{25}\). Since currency is deposited within the same country, shocks to the risk premium affecting the risk profile should equalize, leading to no large movements on the currency market through this channel. Therefore, \((1 + \epsilon_t)\theta = \nu_0\) becomes a constant, which reflects Belarusian investors’ preferences toward dollar.

There are potentially many deposit rates that can be considered, referring to different maturities and investors. Since we need representative interest rates reflecting a composite

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\(^{23}\) Yes, a play with the name of the former Belarusian finance minister Prokopovich.

\(^{24}\) We should note that their dataset is only available in form of a graphic representation.

\(^{25}\) There is no clear consensus on how to proxy the stochastic part. The simplest one is to follow prices of a single long term Eurobond. In our case, Eurobond quotes are only available from 2010. Other techniques will involve conversion of the variables using exchange rates. Since the topic is on BMP, no “easy” conversion is possible.
of short and long-term compensation, we use the weighted average yearly deposit rate based on deposit volumes. This composite includes both interests quoted to legal entities as well as private persons since decisions in both groups are believed to affect the exchange rate. Since the composite interest is quoted on an annual basis, we transform it to make it compatible under the parity conditions (other variables included are all on monthly basis). We use the geometric average of the composite annual rate to convert the interest rates to a monthly base, that is

$$i_m = (1 + i_y)^{1/12} - 1,$$

where \(i_m\) = nominal interest rates on a monthly basis and \(i_y\) = interest rates on an annual basis. Finally, to reduce the number of variables, we construct the nominal interest rate differential such that

$$\text{nominal interest differential} = i_m - i_m^* = b_t.$$

This transformation requires that adjustments in the system is made by changing the domestic rate on BYR\(^26\), rather than the USD interest, which is a reasonable assumption supported by the data (for a graphical overview, see Graph 5).

One could also consider use of the refinancing rate which is what the Government pays to roll over short-term debt and for internal financing. However, in our case, the rate is somewhat static and is changed only occasionally\(^27\) and the liquidity is very low, which makes it unfit for the necessary statistical exercise.

**Inflation Differential**

We approximate the inflationary components \((\gamma - \beta\epsilon_i)\), using the differential between inflation based on consumer price index in Belarus and U.S. We believe that, in comparison with other price indexes such as PPI, it better captures phenomena, which are desirable to include such as "shop runs"\(^28\). We construct the measure as

$$\text{inflation differential} = \pi_m - \pi_m^* = \epsilon_t,$$

where \(\pi_m\) refers to domestic monthly inflation and \(\pi_m^*\) is foreign monthly inflation. The Belarusian data is gathered from the NBRB and US data is obtained from the FED.

(Continued on next page)

\(^{26}\) Lowering rates on the USD deposits can increase the interest rates differential. Even though the possibility exists, it has not been exercised.

\(^{27}\) This is likely to reflect the fact that bonds denoted in the domestic currency are rare due to lack of trust in the ability to maintain the value of the currency and is mainly driven by political relationship with neighbouring countries, mainly Russia, rather than market forces.

\(^{28}\) In case of no access to the black market, people try to purchase of goods to secure value of their savings. As the panic spreads prices consumer prices follow.
VI. A TENTATIVE ANALYSIS OF DETERMINANTS

Before we can continue with a proper statistical analysis of black market determinants, a correct selection of time periods and an econometric model is needed. Simply including data for the whole time period from 1991-2013 would be inappropriate since it includes a number of structural changes in economic-monetary regimes and money transmission mechanism.

The statistical analysis should be based on: (1) relevant time periods and (2) possible relationship between key variables and also take into account if (3) they exhibit any trends.

**Time Periods Selection**

The selection of proper time periods should not be made solely on a numerical basis, but should also make sense in light of the historical background provided in section III. Based on historical facts we draw the conclusion that there are two main periods that are interesting in trying to explain the black market premium; those are 1996-2001 and 2010-2012. The reasons why these are particularly suitable for the analysis are: (1) there is a significant difference between the black and official exchange rates; (2) the periods are subject to severe capital restrictions and (3) they have enough observations that are contained within distinct exchange rate regimes in the Belarusian economic history. These requirements do not hold for the remaining periods. During the first regime, 1990-1996 (Turbulent Years), there was no official currency until 1994 and data quality is questionable. During 2001-2004 (Crawling Peg Regime), and 2004-2009 (USD de facto Hard Peg Regime)
there was virtually no BMP. Finally, we do not have a satisfactory number of observations during 2012 –present (The Free Float Regime).

**Key Variable Relationships**

We begin with visual inspection to determine the possible relationship between the key variables. As described in the graphs below, we do not see any clear-cut relation between the nominal interest rate spread and the black market premium in any of the periods. If any connection, an increase in the nominal interest rate differential seem to increase the BMP.

However, at the core of the theory lies the idea that inflation risk has a potentially large role to play in this relationship. If this is true, then trying to explain the BMP using only nominal interest rates would be an incorrect analysis since inflationary movements affects the real return on the currency.

When looking at how the inflation co-moves with the black market premium (Graph 7), we see that it has strong positive relationship with BMP. This means that high levels of inflation in Belarus compared to the US are associated with a high black market premium. This can be compared to the predictions of the theory in which people seek black market alternatives when there is high inflation risk, for example due to lack of commitment from the central bank.

---

29 This is likely because of the relative economic stability and the liberal reforms on the exchange system where capital controls were removed. The theory cannot be properly evaluated during these periods since the framework presupposes capital restrictions and variation in the BMP.
By combining nominal rates and inflation we obtain the real rate differential and its co-movement with the black market premium (see Graph 8). The pattern is not surprising since agents will seek alternative currency markets to transfer savings when the real return in Belarusian ruble is low compared to the US. We can also see that when the real interest rate differential becomes positive the BMP disappears. This could represent the requirement from investors to be somewhat compensated for all the risks.
All in all, the above results suggest that the series are related and that a movement in the inflation differential or/and movement in nominal interest differential lead to a change in the BMP, which is in line with predictions from theory. However, these are only tentative conclusions, which will be followed by VECM estimation.

Further, Graph 9 shows that the variables do not seem to have any trend, which is important for model estimation and unit root tests.

**Graph 9: Variables During Selected Periods**

![Graph 9](image.png)

**VII. ECONOMETRIC PROCEDURE**

In this section, we briefly describe the econometric procedure to obtain the suitable VECM for the variables and also define outcomes needed to confirm the theory.

Interrelation of processes integrated of order one, or $I(1)$, could be modelled using first differences of each series and including them in a VAR (vector autoregressive model). However, this approach would be incorrect if it was determined that the series are indeed cointegrated. In that case, the VAR would only express the short-run responses of these series. This implies that the VAR in first-differences is misspecified. It is quite possible for there to be a linear combination of integrated variables that is stationary; such variables are said to be cointegrated.\(^\text{30}\) A VAR in first-differences, although properly specified in terms of covariance-stationary series, would not capture those long-run tendencies.

\[^{30}\text{Formally, Engle and Granger (1987) defined the components of the vector } x \text{ as cointegrated of order } d, b \text{ denoted } x_i \sim C(d,b), \text{ if (i) all components of } x_i \text{ are } I(d) \text{ and (ii) there exists a vector } a \neq 0 \text{ so that } x_i = a' x_i - I(d - b), b > 0. \text{ The vector } a \text{ is called the cointegrating vector.}\]

23.
Cointegration & Vector Error-Correction Model (VECM)

We estimate the VECM following the method proposed by Johansen (1991). The estimation is done in five steps:

(1) Since by definition, cointegration necessitates that the variables are integrated of the same order, the first step is to pre-test each variable. Therefore, unit root tests for each period are performed using the DF-GLS method as proposed by ERS (1996). These tests are done for all reasonable amount of lags (up to 12). Since the variables do not exhibit any trend (see section VI), we set the null to be a random walk with no drift, with a stationary process being the alternative hypothesis.

(2) Second, to test for cointegration, we specify the number augmentation lags. We estimate the appropriate lag length of the augmentations using Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC). Since our dataset is limited, especially in the second estimation period (25 observations) choosing short lag length is desirable since it enables lower degrees of freedom, although this must be balanced against the potential benefits of reduced residual autocorrelation.

(3) Next step is to determine the number of cointegrating vectors. If the variables contained in matrix $Y_t = [a_t \ b_t \ c_t]$, where $a_t$ is the BMP, $b_t$ is the interest rate differential and $c_t$ is the inflation differential, are cointegrated – the rank of the companion matrix ($\Pi$) is $0 < r < K$, where $K$ represents the number of variables. On the other hand, if the system has a $\Pi$ matrix with rank 0, the variables are independent random walks. Alternatively, suppose that $\Pi$ matrix is of full rank ($r = 3$) then the variables are stationary and a VAR would be the proper estimation method. In case of cointegration, the general VECM(p-1) has the following representation

$$\Delta Y_t = A_0 + \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \epsilon_t,$$

where $\Delta Y_t = Y_t - Y_{t-1}$; $A_0$ is a $K \times 1$ vector of parameters; $\Pi = \sum_{j=1}^{j=p} A_j - I$, $I$ is $K \times 1$ identity matrix; $\Gamma_i = -\sum_{j=1}^{j=p} A_j$, $A_1$ to $A_p$ is a $K \times K$ matrix of parameters and $\epsilon_t$ is a $K \times 1$ matrix of disturbances such that $\epsilon_t$ has a mean of zero and a variance covariance matrix $\Sigma_{\epsilon_t}$ and is i.i.d over time. All parameters denoted by $A$ are parameters of a VAR (see Appendix C for a full description of VAR and VECM derivation).

If we assume that the rank of $\Pi$ matrix fulfills the criteria $0 < r < K$, $\Pi$ can be expressed as reduced rank decomposition; where the column space of $\alpha$ and $\beta$ equals $r$. As described in section VI, variables do not seem to exhibit any trend therefore $A_0$ is a vector of constants and can be rewritten as $A_0 = \alpha \mu$. Using the transformation, $\mu$ is absorbed into long-run relationship. In case of single cointegration ($r = 1$), the VECM then has the following structural representation

$$\Delta a_t = \alpha_t(a_{t-1} - \beta_2 b_{t-1} - \beta_3 c_{t-1} - \mu) + \sum_{i=1}^{i=n-1} (\gamma_{i,11} \Delta a_{t-i} + \gamma_{i,12} \Delta b_{t-i} + \gamma_{i,13} \Delta c_{t-i}) + \epsilon_{at}$$

$$\Delta b_t = \alpha_2(a_{t-1} - \beta_2 b_{t-1} - \beta_3 c_{t-1} - \mu) + \sum_{i=1}^{i=n-1} (\gamma_{i,21} \Delta a_{t-i} + \gamma_{i,22} \Delta b_{t-i} + \gamma_{i,23} \Delta c_{t-i}) + \epsilon_{bt}$$

$$\Delta b_t = \alpha_3(a_{t-1} - \beta_2 b_{t-1} - \beta_3 c_{t-1} - \mu) + \sum_{i=1}^{i=n-1} (\gamma_{i,31} \Delta a_{t-i} + \gamma_{i,32} \Delta b_{t-i} + \gamma_{i,33} \Delta c_{t-i}) + \epsilon_{ct}$$
where $\alpha_t$ is the speed of adjustment towards the long run equilibrium, $\beta = [1 - \beta_2 - \beta_3 - \mu]$ is the cointegrating vector that describes the long-run equilibrium. The augmentations are determined by the lag length estimation and describe the short-run dynamics of the model.

In order to confirm the theory, we need the BMP, the interest differential and the inflation differential to exhibit a single cointegrating relationship as described above. Besides, VECM estimation should produce the following relationship in the long-run equilibrium: (1) high domestic nominal interest rates relative to the foreign should decrease the black market premium, whereas (2) high level of domestic inflation relative to foreign should increase the black market premium. We also expect a positive constant to be present due to preferential differences (a constant risk factor) towards investments in US dollars. In other words, the following cointegrating vector should be present

$$\beta = [1 \quad \beta_2 \quad -\beta_3 \quad -\mu] ,$$

by the definition $\beta' Y_{t-1} = e_t$ and can be written as

$$a_{t-1} + \beta_2 b_{t-1} - \beta_3 c_{t-1} - \mu = e_{t-1} : e_t \sim N(0, \sigma^2) \Rightarrow I(0) ,$$

rewriting and iterating one step forward

$$a_t = \mu - \beta_2 b_t + \beta_3 c_t + e_t ,$$

which is identical to the expression in section V.

By the construction of our variables, we expect the parameters of adjustment towards equilibrium to be fairly low. Further, for our theory to be valid we need that $\alpha \neq 0$ (is not a matrix of zeros). Finally, we need to confirm that either $b_t \rightarrow c_t \rightarrow a_t$, or $b_t \rightarrow a_t$, or $c_t \rightarrow a_t$, meaning that the coefficients of $\Delta b_{t-1}$ and $\Delta c_{t-1}$ in equation for $\Delta a_t$, are jointly different from zero and BMP is endogenous. In other words, we need short-term adjustments in the BMP to be dependent on laged changes of interest rate differential and inflation differential.

(4) The next step is to estimate the VECM model. Although there are several different procedures for this, we use the maximum likelihood (ML) method developed by Johansen (1991).

(5) The final step is to perform post-estimation diagnostic checks. We test residuals for remaining autocorrelation as proposed by Johansen (1991). In case residuals exhibit autocorrelation additional lags should be added to VECM. The procedure is continued until the null of no serial correlation cannot be rejected. Even though this procedure is appropriate for time period 1, we must limit the number of lags in period 2, due to small sample size.

Additionally, we should confirm that the number of cointegrating relationships is specified correctly using the eigenvalue stability condition. In our case the companion matrix $\Pi$ of a VECM with 3 endogenous variables and one cointegrating relationship should have 2 unit eigenvalues.

Furthermore, we check that the errors are i.i.d. and normally distributed with zero mean and finite variance. The Jarque-Bera test is used for this purpose. If the errors do not come from a normal distribution but are just i.i.d. with zero mean and finite variance, the parameters estimates are still consistent but not efficient.
VIII. EMPIRICAL RESULTS

In this section, we present the results from the VECM estimations of the black market determinants. It is worth noting that the low number of observations in period 2 (25 obs.) implies that results from this period are to be viewed as mainly indicative.

The results from unit root (DF-GLS) tests show that we cannot reject the null of a random walk at the 5% significance level for up to 12 lags.

Table 2: DF-GLS Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>H0</th>
<th>H1</th>
<th>Result</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996m1-2001m1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black market premium</td>
<td>R.W. with no drift</td>
<td>Stationary process</td>
<td>Cannot reject</td>
<td>5%</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>R.W. with no drift</td>
<td>Stationary process</td>
<td>Cannot reject</td>
<td>5%</td>
</tr>
<tr>
<td>Inflation differential</td>
<td>R.W. with no drift</td>
<td>Stationary process</td>
<td>Cannot reject</td>
<td>5%</td>
</tr>
<tr>
<td>2010m1-2012m1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black market premium</td>
<td>R.W. with no drift</td>
<td>Stationary process</td>
<td>Cannot reject</td>
<td>5%</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>R.W. with no drift</td>
<td>Stationary process</td>
<td>Cannot reject</td>
<td>5%</td>
</tr>
<tr>
<td>Inflation differential</td>
<td>R.W. with no drift</td>
<td>Stationary process</td>
<td>Cannot reject</td>
<td>5%</td>
</tr>
</tbody>
</table>

We estimate two vector error correction models, one for each relevant period (1996-2001) – period 1 and (2010-2012) – period 2, and find that the BMP, the nominal interest rate and the real interest rate are cointegrated in a single long-run relationship, $r(II) = 1^{31}$, although the relationship differs between the two periods.

Looking at Table 3, the most striking fact about the long run relationship during period 1 is that the inflation differential has a great impact on the BMP ($\beta_2 = 26.7$). The effect is also apparent during period 2, for which the inflation differential shows a positive relation toward the BMP, although in a slightly less dramatic manner ($\beta_3 = 3.1$). For both these periods, the inflation differential is significant at the 1% level and confirms the predictions. During both periods, there were exchange restrictions in order to maintain the exchange rate – for example the $300 exchange per day restriction during period 1 and the banning of banks to trade with foreign currency during period 2. Such restrictions combined with high inflation might cause people to seek alternative markets to exchange their money, which imply a higher black market premium.

31 This is important since, in case interest rates differentials and inflation differentials are cointegrated, we would get spurious results, however eigenvalue stability test shows that we have a single cointegrating relationship between the variables (see appendix A and B for details)
**Table 3: VECM (1) Excerpt**

<table>
<thead>
<tr>
<th>Period 1</th>
<th>BMP = -1.22** + 3.1 Int. Dif + 26.7 Inf. Dif***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.54) (17.3) (3.9)</td>
</tr>
<tr>
<td>Period 2</td>
<td>BMP = 0.15 - 6.1 Int. Dif + 3.1 Inf. Dif***</td>
</tr>
<tr>
<td></td>
<td>(0.02) (2.68) (0.52)</td>
</tr>
</tbody>
</table>

*Note: Significance levels reported (*) at 10 percent (**) at 5 percent, at 1 percent (***)) levels*

Drawing upon theory, we would expect the interest differential to exhibit a negative relation to the BMP, which is also the case for the second period. During these years the interest differential exhibits a negative sign ($\beta_2 = -6.1$), significant at the 5% level. This is likely to reflect the fact that increased domestic interest lowers the black market premium since it increases the attractiveness of keeping deposits in Belarusian ruble.

The interest differential in the first period exhibits the opposite sign compared to the predicted result ($\beta_2 = 3.1$); however this parameter is highly insignificant. The reason for this could potentially be found in the inflation. Looking at the remarkably high impact from the inflation differential during period 1, it is likely to reflect that hyperinflation was an important problem during this period, reaching as high as 300% during 1999 (see section III). During such extreme conditions, it is possible that inflation distorts the normal mechanisms so that the potential effects from interest rate policy on BMP vanish.

Unexpectedly, the intercept is significantly negative during period 1 ($\mu = -1.22$, significant at 5% level). This parameter was expected to be slightly positive, since it reflects a constant risk premium in the economy. Revisiting the theory, one could potentially think of the parameter as containing a threshold for entering black markets. Black market trade in Belarus is illegal and not an option for every citizen. Therefore, one can imagine that the inflationary pressure has to rise to some critical level before people seek alternative sources for managing their transactions, which is something we have not accounted for in the theoretical framework. Such a threshold might trump the effects of a constant risk premium, leading to a negative intercept.

As a reference to the long-run results from VECM, we can estimate the simple OLS regression for each of the periods. Comparing the result in Table 3 with Table 4, we conclude that the signs of the variables in the long-run relationship estimated in the VECM match those using the OLS in both periods. The OLS regression does not however assign the inflation differential as high an importance as the VECM during the first period. The OLS also confirms that the interest rate differential in period 1 has a positive sign.

**Table 4: Reference Regression (OLS)**

<table>
<thead>
<tr>
<th>Period 1</th>
<th>BMP = -0.238** + 7.2 Int. Dif** + 8.97 Inf. Dif***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2.1) (2.1) (11.8)</td>
</tr>
<tr>
<td>Period 2</td>
<td>BMP = 0.005 - 3.2 Int. Dif + 4.134 Inf. Dif***</td>
</tr>
<tr>
<td></td>
<td>(.017) (-1.53) (9.06)</td>
</tr>
</tbody>
</table>

*Note: Significance levels reported (*) at 10 percent (**) at 5 percent, at 1 percent (***)) levels,
Table 5: VECM (1)

<table>
<thead>
<tr>
<th></th>
<th>Δa_t</th>
<th>Δb_t</th>
<th>Δc_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>βγ_t-1</td>
<td>.0324319 (.0291618)</td>
<td>-.0018675*** (.0024801)</td>
<td>.0130884*** (.0038884)</td>
</tr>
<tr>
<td>Δa_t-1</td>
<td>.124675 (.124675)</td>
<td>.000123 (.0005801)</td>
<td>.0155553 (.00166238)</td>
</tr>
<tr>
<td>Δb_t-1</td>
<td>-.4927393 (6.404643)</td>
<td>-.331938*** (.1274059)</td>
<td>.0919921 (.65979579)</td>
</tr>
<tr>
<td>Δc_t-1</td>
<td>3.893681*** (1.073354)</td>
<td>-.0227282 (.0213519)</td>
<td>.333721** (.02020)</td>
</tr>
</tbody>
</table>

| R²   | 0.3366 | 0.1939 | 0.2324 |
| p > χ² | (0.0000) | (0.0092) | (0.0020) |

Cointegrating equations (βγ_t-1) χ² = 47.78, p=0.0000

<table>
<thead>
<tr>
<th>a_{t-1}</th>
<th>b_{t-1}</th>
<th>c_{t-1}</th>
<th>Cons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-26.68863 (17.27789)</td>
<td>1.222654 (3.874857)***</td>
<td>.5366944**</td>
</tr>
</tbody>
</table>

Note: (1) Significantly different from zero at the 10 percent (*), at 5 percent (**), at 1 percent (***) levels.
(2) The model has the following specification: ΔY_t = a_0(βγ_{t-1}) + b_{t-1}ΔY_{t-1} + ε_t, where Y_t = [a_1, b_1, c_1]; a_0 is the BMP; b_1 is the interest rate differential and ε_{t-1} is the inflation differential; cointegrating vector β = [1 β_2 β_3 μ].

Table 6: VECM (1)

<table>
<thead>
<tr>
<th></th>
<th>Δa_t</th>
<th>Δb_t</th>
<th>Δc_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>βγ_t-1</td>
<td>-.0052605 (.3891683)</td>
<td>.0165779*** (.003721)</td>
<td>.1569498*** (.0795713)</td>
</tr>
<tr>
<td>Δa_t-1</td>
<td>.4418907 (.3439453)</td>
<td>-.0151104*** (.0032886)</td>
<td>.1272364* (.0703248)</td>
</tr>
<tr>
<td>Δb_t-1</td>
<td>-18.32278 (17.72344)</td>
<td>.0987274 (.1694589)</td>
<td>-2.987461** (3.623821)</td>
</tr>
<tr>
<td>Δc_t-1</td>
<td>-2.366645** (1.9819099)</td>
<td>.0384885*** (.0093883)</td>
<td>-.5418723*** (2.007662)</td>
</tr>
</tbody>
</table>

| R²   | 0.3245 | 0.7968 | 0.5746 |
| p > χ² | (0.0476) | (0.0000) | (0.0000) |

Cointegrating equations (βγ_{t-1}) χ² = 47.78, p=0.0000

<table>
<thead>
<tr>
<th>a_{t-1}</th>
<th>b_{t-1}</th>
<th>c_{t-1}</th>
<th>Cons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.141609 (2.681417)</td>
<td>-.098292 (.5292376)***</td>
<td>-.0154818 (.0215437)</td>
</tr>
</tbody>
</table>

Note: (1) Significantly different from zero at the 10 percent (*), at 5 percent (**), at 1 percent (***) levels.
(2) Model has the same specification as for 1996m1-2001m1 time period.
In Tables 5 and 6 we see that the equilibrium is maintained, i.e. adjustments towards equilibrium, is made through corrections in the interest differential (1% significance level for both periods) and the inflation differential (1% and 5% significance level respectively for period 1 and 2). The $\chi^2$ for the long-run equation validate that we can reject exclusion of these in the VECM for any reasonable significance level ($p=0.0000$). The black market premium does not participate in the adjustments toward the equilibrium at any reasonable significance level, which supports the idea that deviations from the long run relationship indeed have an effect on important macroeconomic factors.

An interesting fact is that inflationary changes, $\Delta c_{t-1}$, subsequently affects changes in all variables (significant at the 5% level in five cases out of six). So we can conclude that effects on BMP during both periods; both in the long-run equilibrium, the adjustments towards equilibrium and the short-run adjustments, are heavily dominated by the inflation variable. This also confirms that BMP is endogenous since $c_t \rightarrow a_t$. However, normality tests of the residuals in period 1 reject the null hypothesis of normality at 1% significance level, so the $p$-values should be interpreted with prudence.

In Table 6, we see that the variables in period 2 overall exhibit high values of $R^2$, in which the interest differential has the best fit ($R^2 = 0.8$). The $R^2$ for variables during period 1, shown in table 5, is overall smaller, ranging from 0.19-0.34. As a validity check, we look at the $p > \chi^2$ values and conclude that for all equations in the VECM, we reject the null that the model would fare better without the equation in question at 5% significance level.

When comparing the VECM for the two periods in table 5 and 6, two striking facts emerge. Inflation is a relatively more important factor in the first period whereas the nominal interest rate is important mainly in the second period, both in terms of describing the long run relationship and the adjustments. Drawing on the historical background in section III, the reason for this might be due to lack of trust in the central bank’s ability to counter inflation. During the years preceding period 1, yearly inflation was incredibly high, reaching 2000% in 1994, so the overall risk from inflation was likely to be very present, potentially leading to higher effects on the BMP. During the years preceding period 2, inflation had been rather low – an average of 12% during the de facto US peg (2004-2008) and as little as 6% in 2009. So, entering period 2, the overall risks of inflation might not have been perceived to be as severe as in the previous case. This might contribute to why the interest differentials are relatively more important during this period.

Although most results in table 5 and 6 point in the direction anticipated by theory, the above highlights the fact that, although similar in many respects, the cointegrating relationship differs between the two periods. How can we explain this fact? The first possible explanation is that the true relationship is more complex and that we should include other variables to properly account for factors that explain the stochastic component. This could be considered; although one has to be ware that including more variables further decreases the statistical power.

Another potential explanation could be that the small number of observations during especially the second period, obstruct the possibilities to properly estimate the same true relationship.

The third possibility, which we have already touched upon, is that we estimate the relationship in two distinctly separate macroeconomic environments in which the true relation differs. In the first period, the Belarusian ruble was fairly new and the central bank had problems in establishing a nominal anchor for the currency. Central bank credibility was low and expectations on inflation were likely very high subsequent to the hyperinflation of 1992-1995. This can be compared to the second period where a stable currency peg against the Russian ruble and the US dollar was in place, which set the agenda for monetary and political policy. The different regimes and expectations might therefore affect the true

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32 See Appendix A & B for details.
mechanisms of the black market determinants and cause structural breaks in the cointegrating relationship, which we do not account for.

Conclusively, the evidence of the theory is mixed. Indeed, the results validate the predictions regarding inflation, whereas the negative constant in period 1, no matter how plausible explanation there might be, cannot be explained by this particular theory. Moreover, the effects from the interest rate differential on the BMP support the theory in the second period, whereas the results from period 1 are ambiguous. Our predictions implicitly assumed that the central bank would try to compensate for risk by raising nominal rates in order to generate positive real returns. This might not be the case. Therefore, the inability to connect high nominal interest rates with lowered BMP does not speak against the theory per se, but would call for a revised method of testing it.

IX. SUMMARY

We have suggested that the black market premium should increase during periods of exchange restrictions in which the central bank does not cover for the risks that the holding of the currency implies. We test this theory during two economic-monetary regimes in Belarus history marked by a high black market premium and severe exchange restrictions. In support of the theory, the findings from both periods suggest that the difference in domestic and foreign inflation is an important factor for explaining the black market premium movements.

We would have expected the interest rate differential to have a significant negative effect on the BMP since we assumed the central bank to prioritise positive real returns. We find support for this during the second period although these results are only indicative due to the small number of observations. The lack of impact in the first period might be due to the central bank not having the ability or priority to ensure positive real returns during hyperinflation. To properly evaluate the empirical support for the theory, analysis of other markets exhibiting capital restrictions and black market premium could be considered as well as an alternative way of defining the testable model.

Connecting our results with earlier research, the importance of inflation in determining the BMP might be resourceful in explaining why official and black markets exhibit long-run relationships. The connection between depreciation and high inflation is a well-established fact. If we add the finding that inflation is an important determinant also for the black market premium, inflation differentials might be an important link to explain why the official rates follow black market rates in the long run.
Table 7: Selection-order Criteria

<table>
<thead>
<tr>
<th>Lag</th>
<th>Ll</th>
<th>LR</th>
<th>df</th>
<th>p</th>
<th>FPE</th>
<th>AIC</th>
<th>HQIC</th>
<th>SBIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>254.56</td>
<td>2.20*</td>
<td>9</td>
<td>0.00</td>
<td>7.0e-10</td>
<td>-12.571</td>
<td>-12.4086</td>
<td>-12.1561*</td>
</tr>
<tr>
<td>1</td>
<td>409.72</td>
<td>281.24</td>
<td>9</td>
<td>0.001</td>
<td>5.9e-10*</td>
<td>-12.7474*</td>
<td>-12.4626*</td>
<td>-12.0207</td>
</tr>
<tr>
<td>2</td>
<td>416.06</td>
<td>13.618</td>
<td>9</td>
<td>0.137</td>
<td>6.5e-10</td>
<td>-12.6756</td>
<td>-12.2687</td>
<td>-11.6375</td>
</tr>
<tr>
<td>3</td>
<td>421.83</td>
<td>10.495</td>
<td>9</td>
<td>0.312</td>
<td>7.2e-10</td>
<td>-12.5526</td>
<td>-12.0237</td>
<td>-11.2031</td>
</tr>
<tr>
<td>4</td>
<td>429.25</td>
<td>14.789</td>
<td>9</td>
<td>0.097</td>
<td>5.9e-10*</td>
<td>-12.4999</td>
<td>-11.8489</td>
<td>-10.8389</td>
</tr>
<tr>
<td>5</td>
<td>435.50</td>
<td>12.507</td>
<td>9</td>
<td>0.186</td>
<td>8.7e-10</td>
<td>-12.4099</td>
<td>-11.6368</td>
<td>-10.4374</td>
</tr>
<tr>
<td>6</td>
<td>440.50</td>
<td>10.001</td>
<td>9</td>
<td>0.350</td>
<td>1.0e-09</td>
<td>-12.2787</td>
<td>-11.3836</td>
<td>-9.94983</td>
</tr>
<tr>
<td>7</td>
<td>445.57</td>
<td>10.141</td>
<td>9</td>
<td>0.339</td>
<td>1.0e-09</td>
<td>-12.1499</td>
<td>-11.1327</td>
<td>-9.54854</td>
</tr>
</tbody>
</table>

Note: (1) Endogenous: BMP, Nominal rates differential, Inflation differential; (2) Exogenous: constant

Table 8: Johansen Tests for Cointegration

<table>
<thead>
<tr>
<th>Trend constant</th>
<th>Sample: 1996m1 - 2001m1</th>
<th>Maximum rank</th>
<th>Lags = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lags</td>
<td></td>
<td>parms</td>
<td>LL</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>393.79479</td>
<td>32.045</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>403.91169</td>
<td>0.28230</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>408.79199</td>
<td>0.14786</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>409.79704</td>
<td>0.03242</td>
</tr>
</tbody>
</table>

Note: * at 10% level

Table 9: Lagrange-multiplier Test

<table>
<thead>
<tr>
<th>Lag</th>
<th>χ²</th>
<th>df</th>
<th>p &gt; χ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2669</td>
<td>9</td>
<td>0.89299</td>
</tr>
<tr>
<td>2</td>
<td>14.9661</td>
<td>9</td>
<td>0.09187</td>
</tr>
<tr>
<td>3</td>
<td>7.8309</td>
<td>9</td>
<td>0.54842</td>
</tr>
<tr>
<td>4</td>
<td>6.2589</td>
<td>9</td>
<td>0.71755</td>
</tr>
</tbody>
</table>

Note: null hypothesis of no autocorrelation at lag order. The test was conducted for 12 lags, there is strong evidence of no serial autocorrelation after lag 4.

Table 10: Jarque-Bera test

<table>
<thead>
<tr>
<th>Equation</th>
<th>χ²</th>
<th>df</th>
<th>p &gt; χ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δb0</td>
<td>86.823</td>
<td>2</td>
<td>0.00000</td>
</tr>
<tr>
<td>Δb1</td>
<td>79.271</td>
<td>2</td>
<td>0.00000</td>
</tr>
<tr>
<td>Δc0</td>
<td>148.940</td>
<td>2</td>
<td>0.00000</td>
</tr>
<tr>
<td>all</td>
<td>315.054</td>
<td>6</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Note: Δb0 is the BMP, Δb1 is the interest rate differential and Δc0 is the inflation differential

Table 11: Eigenvalue Stability Condition

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.616005 + .255723i</td>
<td>.666914</td>
</tr>
<tr>
<td>.616005 - .255723i</td>
<td>.666914</td>
</tr>
<tr>
<td>.3295201 - .0035869i</td>
<td>.32952</td>
</tr>
<tr>
<td>.0035869</td>
<td>.003584</td>
</tr>
</tbody>
</table>

Note: The VECM specification imposes 2 unit moduli, implying that a single cointegrating vector is correct.
### Table 12: Selection-order Criteria

<table>
<thead>
<tr>
<th>lag</th>
<th>Sample: 2010m1 - 2012m1</th>
<th>Number of obs. = 63</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L.L.</td>
<td>LR</td>
</tr>
<tr>
<td>0</td>
<td>158.401</td>
<td>8.0e-10</td>
</tr>
<tr>
<td>1</td>
<td>232.316</td>
<td>147.83</td>
</tr>
<tr>
<td>2</td>
<td>240.121</td>
<td>15.611</td>
</tr>
<tr>
<td>3</td>
<td>256.509</td>
<td>32.777</td>
</tr>
<tr>
<td>4</td>
<td>275.821</td>
<td>38.624*</td>
</tr>
</tbody>
</table>

Note: (1) Endogenous: BMP, Nominal rates differential, Inflation differential; (2) Exogenous: constant

### Table 13: Johansen Tests for Cointegration

<table>
<thead>
<tr>
<th>Trend: rconstant</th>
<th>Sample: 2010m1 - 2012m1</th>
<th>Number of obs. = 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lags = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>maximum rank</td>
<td>pars</td>
<td>L.L.</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>218.9612</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>230.71793</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>239.49037</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>240.12086</td>
</tr>
</tbody>
</table>

| maximum rank | pars | L.L. | eigenvalue | max statistic | % critical value | 5% critical value |
| 0    | 9    | 218.9612 | 23.5135 | 22.00 | 26.81 |
| 1    | 15   | 230.71793 | 0.60958 | 17.5449 *1 15.67 | 20.20 |
| 2    | 19   | 239.49037 | 0.50431 | 1.2610 | 9.24 | 12.97 |
| 3    | 21   | 240.12086 | 0.04919 | 0.05119 | 9.24 | 12.97 |

Note: *1 at 1% and *5 at 5% level.

### Table 14: Lagrange-multiplier Test

<table>
<thead>
<tr>
<th>Lag</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p &gt; \chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.388</td>
<td>9</td>
<td>0.10917</td>
</tr>
<tr>
<td>2</td>
<td>13.289</td>
<td>9</td>
<td>0.15009</td>
</tr>
<tr>
<td>3</td>
<td>19.8545</td>
<td>9</td>
<td>0.01883</td>
</tr>
<tr>
<td>4</td>
<td>14.0583</td>
<td>9</td>
<td>0.12027</td>
</tr>
</tbody>
</table>

Note: null hypothesis of no autocorrelation at lag order

### Table 15: Jarque-Bera test

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p &gt; \chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta a_t$</td>
<td>5.472</td>
<td>2</td>
<td>0.06484</td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>0.520</td>
<td>2</td>
<td>0.73120</td>
</tr>
<tr>
<td>$\Delta c_t$</td>
<td>1.359</td>
<td>2</td>
<td>0.05086</td>
</tr>
<tr>
<td>$\Delta \phi$</td>
<td>7.350</td>
<td>6</td>
<td>0.28966</td>
</tr>
</tbody>
</table>

Note: $a_t$ is the BMP, $b_t$ is the interest rate differential and $c_{t-1}$ is the inflation differential

### Table 16: Eigenvalue Stability Condition

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.882851</td>
<td>1</td>
</tr>
<tr>
<td>0.0242388</td>
<td>0.541051</td>
</tr>
<tr>
<td>0.0242388</td>
<td>0.541051</td>
</tr>
<tr>
<td>-0.3223051</td>
<td>-0.3223051</td>
</tr>
</tbody>
</table>

Note: The VECM specification imposes 2 unit modulus, implying that a single cointegrating vector is correct.
APPENDIX C: VAR, VAR IDENTIFICATION & VECM

In the three-variable case, (1) we can let the time path of \( \{a_t\} \) be affected by current and past realizations of the sequences \( \{b_t\}, \{c_t\} \) and (2) the time path of the \( \{b_t\} \) be affected by current and past realizations of the \( \{a_t\}, \{c_t\} \) and (3) the time path of the \( \{c_t\} \) be affected by current and past realizations of the \( \{a_t\}, \{b_t\} \). The structural model with \( n = 3 \) and \( p \) lags is

\[
a_t = d_{10} - d_{12} b_t - d_{13} c_t + \sum_{i=1}^{p} (\phi_{i,11} a_{t-i} + \phi_{i,12} b_{t-i} + \phi_{i,13} c_{t-i}) + \eta_{at}
\]

\[
b_t = d_{20} - d_{21} a_t - d_{23} c_t + \sum_{i=1}^{p} (\phi_{i,21} a_{t-i} + \phi_{i,22} b_{t-i} + \phi_{i,23} c_{t-i}) + \eta_{bt}
\]

\[
c_t = d_{30} - d_{31} a_t - d_{32} b_t + \sum_{i=1}^{p} (\phi_{i,31} a_{t-i} + \phi_{i,32} b_{t-i} + \phi_{i,33} c_{t-i}) + \eta_{ct}
\]

where \( \{a_t\} \) is the BMP, \( \{b_t\} \) is the interest rate differential and \( \{c_t\} \) is the inflation differential; \( \eta_{at}, \eta_{bt}, \eta_{ct} \) are the white-noise disturbance terms that are uncorrelated for all \( s \) and \( t \). The structural system above can be written in reduced form, noticing

\[
a_t + d_{12} b_t + d_{13} c_t = d_{10} + \sum_{i=1}^{p} (\phi_{i,11} a_{t-i} + \phi_{i,12} b_{t-i} + \phi_{i,13} c_{t-i}) + \eta_{at}
\]

\[
d_{21} a_t + b_t + d_{23} c_t = d_{20} + \sum_{i=1}^{p} (\phi_{i,21} a_{t-i} + \phi_{i,22} b_{t-i} + \phi_{i,23} c_{t-i}) + \eta_{bt}
\]

\[
d_{31} a_t + d_{32} b_t + c_t = d_{30} + \sum_{i=1}^{p} (\phi_{i,31} a_{t-i} + \phi_{i,32} b_{t-i} + \phi_{i,33} c_{t-i}) + \eta_{ct}
\]

\[
\begin{bmatrix}
1 & d_{12} & d_{13} \\
-d_{21} & 1 & d_{23} \\
d_{31} & d_{32} & 1
\end{bmatrix}
\begin{bmatrix}
a_t \\
b_t \\
c_t
\end{bmatrix}
= 
\begin{bmatrix}
d_{10} \\
d_{20} \\
d_{30}
\end{bmatrix} + 
\sum_{i=1}^{p} 
\begin{bmatrix}
\phi_{i,11} & \phi_{i,12} & \phi_{i,13} \\
\phi_{i,21} & \phi_{i,22} & \phi_{i,23} \\
\phi_{i,31} & \phi_{i,32} & \phi_{i,33}
\end{bmatrix}
\begin{bmatrix}
a_{t-i} \\
b_{t-i} \\
c_{t-i}
\end{bmatrix} + 
\begin{bmatrix}
\eta_{at} \\
\eta_{bt} \\
\eta_{ct}
\end{bmatrix}
\]

\[
\equiv
\]

\[
D_1 Y_t = D_0 + \sum_{i=1}^{p} \phi_{i} Y_{t-i} + \eta_t
\]

\[
\equiv
\]

\[
D_1^{-1} D_1 Y_t = D_1^{-1} D_0 + \sum_{i=1}^{p} D_1^{-1} \phi_i Y_{t-i} + D_1^{-1} \eta_t
\]

\[
\equiv
\]
\[ Y_t = D_1^{-1}D_0 + \sum_{i=1}^{p} D_1^{-1} \Phi_i Y_{t-i} + D_1^{-1} \eta_t \]

\[ \implies \]

\[ Y_t = A_0 + \sum_{i=1}^{p} A_i Y_{t-i} + \epsilon_t, \]

where \( D_1 = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & 1 \end{bmatrix}_{3 \times 3} \), \( Y_t = \begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix}_{3 \times 1} \), \( D_0 = \begin{bmatrix} d_{10} \\ d_{20} \\ d_{30} \end{bmatrix}_{3 \times 1} \), \( \Phi_i = \begin{bmatrix} \phi_{i,11} & \phi_{i,12} & \phi_{i,13} \\ \phi_{i,21} & \phi_{i,22} & \phi_{i,23} \\ \phi_{i,31} & \phi_{i,32} & \phi_{i,33} \end{bmatrix}_{3 \times 3} \),

\[ Y_{t-1} = \begin{bmatrix} a_{t-1} \\ b_{t-1} \\ c_{t-1} \end{bmatrix}_{3 \times 1} \], \( \eta_t = \begin{bmatrix} \eta_{at} \\ \eta_{bt} \\ \eta_{ct} \end{bmatrix}_{3 \times 1} \); \( A_0 = D_1^{-1}D_0 \), \( A_i = D_1^{-1} \Phi_i \), \( \epsilon_t = D_1^{-1} \eta_t \),

and

\[ D_1^{-1} = \frac{1}{\det(D_1)} \begin{bmatrix} 1 - d_{23}d_{32} & d_{13}d_{23} - d_{12} & d_{12}d_{23} - d_{13} \\ d_{23}d_{31} - d_{21} & 1 - d_{13}d_{31} & d_{13}d_{31} - d_{23} \\ d_{21}d_{32} - d_{31} & d_{12}d_{31} - d_{32} & 1 - d_{12}d_{21} \end{bmatrix}, \]

\[ \det(D_1) = (1 - d_{23}d_{32}) - d_{12}(d_{21} - d_{23}d_{32}) + d_{13}(d_{21}d_{32} - d_{31}). \]

The properties of the new disturbance terms \( \epsilon_t \) can be summarized as follows

\[ E(\epsilon_t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{3 \times 1}, \]

\[ E(\epsilon_t \epsilon_t') = \begin{bmatrix} \epsilon_{at}^2 & \epsilon_{at}\epsilon_{bt} & \epsilon_{at}\epsilon_{ct} \\ \epsilon_{bt}^2 & \epsilon_{bt}\epsilon_{ct} & \epsilon_{bt}\epsilon_{ct} \\ \epsilon_{ct}^2 & \epsilon_{ct}\epsilon_{ct} & \epsilon_{ct}\epsilon_{ct} \end{bmatrix} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} & \sigma_{ac} \\ \sigma_{ab} & \sigma_b^2 & \sigma_{bc} \\ \sigma_{ac} & \sigma_{bc} & \sigma_c^2 \end{bmatrix} \equiv \Sigma_\epsilon \]

and properties of the \( \eta_t \) disturbances are

\[ E(\eta_t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{3 \times 1}, \]

\[ E(\eta_t \eta_t') = \begin{bmatrix} \eta_{at}^2 & \eta_{at}\eta_{bt} & \eta_{at}\eta_{ct} \\ \eta_{bt}^2 & \eta_{bt}\eta_{ct} & \eta_{bt}\eta_{ct} \\ \eta_{ct}^2 & \eta_{ct}\eta_{ct} & \eta_{ct}\eta_{ct} \end{bmatrix} = \begin{bmatrix} \sigma_a^2 & 0 & 0 \\ 0 & \sigma_b^2 & 0 \\ 0 & 0 & \sigma_c^2 \end{bmatrix} \equiv \Sigma_\eta \]

**Suggested VAR Identification**

I order to recover all the information present in the primitive system from the estimated VAR system, we need to appropriately restrict the primitive system, \( D_1 \) (matrix with
contemporaneous coefficients). In our case, we need to make 3 restrictions. It is reasonable to argue that interest rate differential and inflation do not have any contemporaneous effect on BMP. Since it takes time for investors to react on changes to interest rates, and the inflationary expectations or expectation about inflationary differential are built on the lagged values rather than todays. Further, it is commonly accepted to think that it takes time for the changes in interest rates to go through the monetary transmission mechanism and effect inflation. Therefore, we argue that interest rates have no contemporaneous effect on inflation. Following decomposition can be used to identify reduced form VAR

$$D_1 = \begin{bmatrix} 1 & 0 & 0 \\ d_{21} & 1 & d_{23} \\ d_{31} & 0 & 1 \end{bmatrix}_{3 \times 3}.$$

From VAR(p) to VECM(p-1)

Start by rewriting VAR(p)

$$Y_t = A_0 + \sum_{i=1}^{p} A_i Y_{t-i} + \varepsilon_t$$

as

$$Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t,$$

taking the first difference get following

$$\Delta Y_t = A_0 + (A_1 - I)Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t.$$

Now add and subtract $(A_1 - I)Y_{t-2}$ from right hand side

$$\Delta Y_t = A_0 + (A_1 - I)\Delta Y_{t-1} + (A_1 + A_2 - I)Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t,$$

then add and subtract $(A_1 + A_2 - I)Y_{t-3}$

$$\Delta Y_t = A_0 + (A_1 - I)\Delta Y_{t-1} + (A_1 + A_2 - I)\Delta Y_{t-2} + (A_1 + A_2 + A_3 - I)Y_{t-3} + \cdots + A_p Y_{t-p} + \varepsilon_t.$$

Continuing in this fashion we obtain VECM(p-1)

$$\Delta Y_t = A_0 + \Pi \gamma Y_{t-p} + \sum_{i=1}^{p-1} \gamma_i Y_{t-i} + \varepsilon_t.$$