Parametric Estimation of Value at Risk

Master's Thesis in Finance Fall 2013

Abstract:

Previous research has shown that the non-parametric Value at Risk methods normally used by financial institutions are conservative and that violations often happen in clusters. While earlier research has found efficiency gains from using parametric methods over non-parametric methods, a majority of the studies have been carried out on equity indices. Parametric modelling of variance requires underlying distributional assumptions that depend on return characteristics. This thesis evaluates the use of parametric Value at Risk methods for different asset classes. The empirical analysis tests six different parametric methods; a standard variance model, RiskMetrics, a normal symmetric GARCH, a normal asymmetric GARCH, a symmetric t-distributed GARCH and an asymmetric t-distributed GARCH on 17 different assets. The data consists of daily indices and exchange rates from 2000 to 2013. The models are backtested by Christoffersen's (1998) conditional coverage test and by the backtest from the Basel Committee on Banking Supervision that determines market risk capital requirements from Value at Risk estimates. The results show that asset classes matter for the choice of parametric method. While an asymmetric t-distributed GARCH fulfils the criterion of conditional coverage for equity, different parametric models work well for different assets. One implication from this is that earlier research on equity cannot be generalized. The analysis also tests the models on an equally weighted portfolio of all assets and finds that parametric methods in general work well for a well-diversified portfolio. The results also show that there is a tension between the most accurate model and the model giving the lowest capital charge.

Author: Lisa Wimmerstedt*
Supervisor: Michael Halling
Stockholm School of Economics

(*) 21479@student.hhs.se

1. Introduction

During the last 20 years, risk management has grown to be one of the most important areas in financial management. Recent financial disasters as well as the growth of the derivatives market has forced the development of increased supervision and management of financial risk. With this development, a conceptually simple measure called Value at Risk has grown to become an industry standard used for internal risk control as well as by regulators. Value at Risk, or VaR, is defined as the maximum loss over a horizon that will not be exceeded with a given level of confidence.

Jorion (2007) reports that Value at Risk was developed as a remedy after the financial disasters in the beginning of the 1990s. These disasters showed that poor supervision could lead to serious outcomes. By adopting Value at Risk, financial institutions as well as regulators got a simple and easy-to-understand measure of financial risk. Value at Risk can summarize the market exposure of a large financial institution into one single number in a manner that is comparable across portfolios and institutions. Furthermore, this number is expressed in the same units as the original holding, making it easy to understand.

Jorion (2007) argues that three recent economic changes have driven the development of Value at Risk. The first is that regulators need better control of financial risks. Secondly, as financial markets have become more global, there are many sources to financial risk. Finally, technological advances have made risk management a possibility.

In Basel II, regulators forced the adoption of VaR further by basing capital requirements on banks internal Value at Risk measures. The market risk charge is the capital required to maintain a certain level of market risk. At first, a standardized method was presented where a bank had to calculate a market risk charge based on risk computed from exposure to interest-rates, exchange rates, equity and commodity. The standard approach was criticized of being too conservative and in 1995 an alternative was introduced; see Basel Committee on Banking Supervision (1995, 1996). This alternative allowed banks to use an internal model to calculate a VaR estimate. From this estimate, the market risk charge was then determined by the maximum value of yesterday's internally calculated VaR and an average of the last 60 day's VaR multiplied by a factor k. To avoid that banks underestimated their VaR to keep the capital requirements down, the factor k was implemented to reward banks with good models and punish those with less accurate measures. The factor k took a minimum value of 3, and any extra factor was determined by a backtest on historical data. In other words, a test of how many times the loss exceeded the VaR forecast during the last 250 days. For a 1-day 99% VaR, the expected number of exceptions in a period should be 2.5. The Basel Accord could accept a model that had up to four violations during the last period. If the violations were more than four then a penalty term would be added to k, giving the bank a higher market capital charge. The penalty zones are shown in table 1.

Basel Accord Penalty Zones		
Zone	Number of exceptions	Increase (n)
Green	0	0
	1	0
	2	0
	3	0
	4	0
Yellow	5	0.4
	6	0.5
	7	0.65
	8	0.75
	9	0.85
Red	10 or more	1

Table 1. The table shows the penalty added from the Basle back testing

The market risk capital charge is determined by (1).

$$MRC_t = \max\left(k \frac{1}{60} \sum_{i=1}^{60} VaR_{t-1} \ , \ VaR_{t-1}\right) \tag{1}$$

$$k = 3 + n$$

This implies that calculating an accurate estimate is not only important for internal control but also for regulatory purposes. Being too conservative will lead to high capital requirements, while too many violations also will result in high capital requirements through the penalty function.

Many different methods can be used for calculating Value at Risk. The methods are generally divided into four different categories; parametric methods, non-parametric methods, semi-parametric methods and hybrid methods. Parametric methods assume an underlying return distribution and estimate the VaR from the threshold value given by the distribution and the standard deviation of the portfolio. The non-parametric methods do not assume any distribution but instead estimate the Value at Risk through simulations based on historical data. Hendricks (1996) highlights one of the contradictions with risk management measurement; extreme events occur more often and are larger than predicted by the normal distribution and the size of market movements is not constant. When volatility varies over time, historical simulations are not able to capture the rapid changes. On the other hand, parametric approaches need distributional assumptions. Fama (1965) was among the first to suggest that returns have high kurtosis and are skewed implying that returns are not normally distributed.

Many attempts have been made to get around these issues. Among the semi-parametric methods is Extreme Value Theory, proposed by Danielsson and de Vries (2000). This theory is a combination of historical simulation and parametric estimation of the tails of the return distribution. A popular hybrid method is the exponentially weighting of historical observations to put more weight on recent observations as proposed by McNeil and Frey (2000). Another approach is to look at the quantile directly like Engle and Manganelli (2004).

In general, many banks use conservative non-parametric models; see Gizycki and Hereford (1998), Berkowitz et al. (2005) and Pérignon et al. (2008). With high estimates follow high capital charges. Research on banks has also shown that violations are often clustered together (Berkowitz and O'Brien, 2002). Clustering is particularly common when using non-parametric methods such as historical simulations due to the difficulty to quickly incorporate higher volatility levels in the models. This suggests that efficiency gains can be achieved from using parametric models. Jorion (1996) uses a different approach and argues that standard errors in VaR estimates are smaller for parametric methods and hence, those models are more efficient.

The use of parametric methods requires precision in the choice of distribution and to account for characteristics in returns. From this it follows that assets with different return patterns and volatilities may need different underlying distributional assumptions as well as different calibration of the parameters. I believe that this possibility to choose between different parametric VaR methods is an interesting area to investigate. Furthermore, it is evident that by using more efficient models, financial institutions could lower their capital requirements. Therefore, the purpose of this thesis is to evaluate the use of parametric Value at Risk methods for different asset classes.

The remainder of this thesis will divided as follows; the next section will present an overview of previous literature. Section 3 will state the research questions followed by section 4 and 5 that describe methodology and data. The results are presented in section 6 and conclusions are drawn in section 7.

2. Review of literature

2.1 Choosing between Value at Risk methods

Previous literature gives guidance on how to choose an appropriate method to model Value at Risk. McAleer (2009) puts the decision of which VaR methodology to choose into Ten Commandments. (i) The use of conditional, stochastic or realized volatility; (ii) accounting for symmetry, asymmetry and leverage; (iii) choosing between dynamic covariances and dynamic correlations; (iv) modelling single index or portfolio models; (v) choosing between parametric, non-parametric and semi-parametric models; (vi) doing estimation, simulation or calibration of parameters; (vii) the assumptions, regularity conditions and statistical properties; (viii) the accuracy in calculating moments and forecasts; (ix) optimizations of threshold violations and economic benefits and; (x) optimizations of private and public benefits.

Giot and Laurent (2004) suggest that using realized volatility, defined as the sum of squared intra day returns, does not improve the accuracy of a VaR estimate compared to using the conditional daily variance.

Knowing characteristics of the underlying volatility becomes particularly important when using parametric methods in order to use the correct distributional assumptions. Mandelbrot (1963) showed that there is correlation in volatility. Black (1976) was among the first to find that stock returns are negatively correlated to changes in volatility. This is stressed even further in Glosten et al. (1993), who highlight the importance in asymmetries when modeling volatility. When Sener et al. (2012) rank the performance of different VaR methods they confirm all of these observations. They use emerging market equity indices and seven developed market equity indices. They find that the CAViaR asymmetric method developed by Engle & Maganelli (2004) and the EGARCH by Nelson (1991) are best ranked. The CAViaR method proposes a way to model the quantiles autoregressively. They argue that asymmetric parametric methods perform best, supporting the view that equity returns react differently to positive and negative shocks.

McAleer and Da Veiga (2008) test different VaR approaches for single-index and portfolio models for four different stock indices. They find that the worst performing model is the standard approach with a constant equally weighted variance. This leads to a daily capital charge of almost 13 %. The best performing model is a single-index EGARCH or PGARCH by Ding et al. (1993) with a daily capital charge of around 8 %. Both methods are asymmetric to shocks. In general, they find that the single-index methods lead to lower capital charges.

Among many new approaches is one presented by McAleer et al. (2013a). They suggest a strategy that is robust to the Global Financial Crisis as calculating VaR with different parametric methods and then take the median of these estimates. They claim that this method works well before, during and after the financial crisis.

2.2 Empirical Testing of Different Asset Classes

The main focus of previous research has been to investigate methods for equity indices, while some research has been made on commodity prices, such as Hung et al. (2008). They estimate VaR for five different energy commodities and then backtest the models to test the accuracy and efficiency. They use GARCH-modelling with normal distribution, student's t distribution and a heavy-tailed distribution proposed by Politis (2004). They find that the heavy-tailed GARCH gives the most accurate and the most efficient VaR estimate.

Hammoudeh et al. (2011) investigate the volatility of precious metals. They test different conditional volatility models, calculate VaR and backtest the models with Kupiec's (1995) unconditional coverage test and Christoffersen's (1998) conditional coverage test. Their results show that a t-distributed GARCH performs best in the backtests. They also find that this model leads to fewest violations but has the highest average capital requirement.

Value at Risk estimations for the oil market has been investigated by Marimoutou et al. (2009). They test the performance of historical simulations, fully parametric models and Extreme Value Theory. They find that the GARCH model with a normal distribution underestimates the risk. They suggest that using a generalized Pareto distribution, as in Extreme Value Theory, and the filtered historical simulation give the most accurate results. They argue that the t-distributed GARCH gives equally good results for the left tail but overestimates the right tail.

Hammoudeh et al. (2013) investigate different VaR methods for precious metals, oil and S&P500 separately as well as in a portfolio. They use a variety of methods such as standard GARCH models and Extreme Value Theory. They rank the models based on statistical backtests as well as under the Basle capital charges. They get different results depending on the asset used or the portfolio of assets. When doing the analysis independently for each asset they find that the conditional EVT or the median strategy performs best for several assets while J.P. Morgan's RiskMetrics (1996) underperforms. However, when they look at capital charges in a portfolio setting, they find that RiskMetrics is the method yielding lowest capital charge.

Brooks and Persand (2002) test VaR methods for six assets. They use three UK indices on stocks, commodities and bonds and an equally weighted portfolio of the assets. They also test US stocks, a T-bill, a government bond index and an equally weighted portfolio. Their work suggest that using the wrong parametric method can lead to a worse outcome than using historical simulation. They claim that a VaR calculated with a parametric approach with threshold values from a normal distribution can give very inaccurate VaR estimates if the returns have fat tails. In that case, a non-parametric method would be preferred.

The current literature is focused on testing classical models, investigating how well banks succeed with their models as well as the development of new methods. The research has mainly been carried out on different equity indices, while some previous publications looked at Value at Risk estimations for commodity prices. There is a substantial amount of research claiming that parametric methods could improve the efficiency for financial institutions by decreasing their capital requirements. However, it also becomes clear that return characteristics such as asymmetries or fat tails become important for finding a correct parametric method. Little effort has been made on finding differences between asset classes and how the best parametric Value at Risk method depends on the asset class used.

3. Delimitations

The aim of this thesis is to evaluate the use of parametric Value at Risk methods for different asset classes. Reviewing previous literature showed that no study had been made across different asset classes but rather that the focus has been on equities. The thesis will try to answer the following questions:

(i) For the calculation of a 99% VaR, what is the best choice of parametric method according to statistical backtests, for each of the different assets?

This is a key question. If the same models work well for all asset classes, then the results of previous research on equity can be generalized to other asset classes.

(ii) What can be said about the underlying volatility of the assets given which parametric models that work well for each asset?

For a parametric method to work, the conditional variance modelling needs to take into account fat tails, asymmetries or other characteristics in returns. Interpreting the results will give information on the volatility of different assets.

(iii) For each of the assets, which parametric model gives the lowest average daily capital charge?

One can assume that a bank would prefer a model that gives the lowest possible capital charge. This will give some guidance on the trade-off between taking on penalties with a low Value at Risk or having a high Value at Risk without being penalized.

(iv) Do parametric VaR models in general work well for a well-diversified portfolio?

This question will evaluate a well-diversified portfolio and see what model returns most accurate results.

4. Methodology

The empirical analysis was carried out by first calculating a Value at Risk using six different parametric methods for every asset class as well as for an equally weighted portfolio of all assets. Each day's calculated VaR was then backtested to find the observations when the loss exceeded the estimated value. The violations were used to calculate the daily capital charges and to test the statistical properties of the VaR estimates.

4.1 The Independent Value at Risk

The Value at Risk of a parametric method, assuming a mean of zero, is calculated according to (2).

$$VaR_t = z_\alpha \sigma_{t-1} V \tag{2}$$

Where z_α is the one-day threshold value calculated from an assumed return distribution. For a one day 99% VaR with a normal distribution, this threshold value is 2.33. The standard deviation σ_t is estimated from historical data according to the models below. V is the value of the asset.

4.1.1 The Standard Approach

The first model that was used is an equally weighted variance model where the variance is estimated as the standard equally weighted squared return from the last 250 observations. Any change in volatility will be incorporated into the model very slowly, and this method does not have any advantages compared to a non-parametric method since it does not capture time-varying volatility, fat tails or asymmetries in returns. I will refer to this as the standard approach.

4.1.2 RiskMetrics

The second model comes from J.P. Morgan's RiskMetrics (1996) and is one of the most frequently used parametric methods. The formula for the conditional volatility is (3), where ϵ_t^2 is the squared return.

$$\sigma_t^2 = (1-\lambda)\epsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \tag{3}$$

$$\lambda = 0.94$$

The model is simple and still time-varying allowing for the incorporation of new volatility levels. The method does not require any optimizations and is therefore easy to use. The underlying assumption is that the returns are normally distributed giving a threshold value of 2.33 for a 99 % VaR.

4.1.3 Normal GARCH

The third model is a standard GARCH(1,1) with normally distributed errors as first proposed by Engle (1982) and Bollerslev (1986). The variance process is the following:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4}$$

$$\omega \geq 0;\ \alpha > 0\ ;\ \alpha + \beta < 1;$$

There are two major differences between using a GARCH model and the previous RiskMetrics model. The first characteristic is that it now allows for a long run variance (5).

$$V = \frac{\omega}{1 - \alpha - \beta} \tag{5}$$

For RiskMetrics, the long run variance is zero. The second difference is that the parameters ω, α and β are calculated using maximum likelihood estimation. This allows each asset class to have its own variance process with different persistence in volatility shocks and different long run variances. This becomes particularly important when analyzing differences between asset classes.

4.1.4 Student's t GARCH

The fourth model is similar to the GARCH (4) presented above with the exception that the returns are now assumed to follow a student's t distribution. This distribution will capture the fatter tails in the return series. For the Value at Risk estimations, this has two implications. The first being that the maximum likelihood estimation will be different, giving new values on ω, α and β as well as the conditional variance. Furthermore, the maximum likelihood estimation will return the degrees of freedom of the distribution as well. The degrees of freedom will be used to find the t-distributed threshold value.

4.1.5 Normal Asymmetric GARCH

The fifth model incorporates asymmetries into the GARCH modeling. To do this an asymmetric term γ is added that takes the value one when the shock in returns is negative.

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma I(\epsilon_{t-1})\epsilon_{t-1}^2 \tag{6}$$

$$\omega \geq 0;\ \alpha > 0\,;\gamma > 0;\ \alpha + \beta + \frac{\gamma}{2} < 1;$$

$$I(\epsilon_{t-1}) = \begin{cases} 1 & if\ \epsilon_{t-1} < 0 \\ 0 & else \end{cases} \tag{7}$$

Since a symmetric GARCH does not distinguish between positive and negative shocks, the parameter alpha adjusts to both shocks. If there is asymmetries in the return series, then using this model will lead to a lower VaR when shocks are positive, hence lower capital requirements. At the same time, negative shocks will increase the VaR when needed and avoid violations.

4.1.6 Student's t Asymmetric GARCH

The last model is the asymmetric GARCH model (6) with the exception that the returns are assumed be t-distributed. As above, using a t-distribution will give a different threshold value and a new conditional variance from the maximum likelihood estimation.

4.1.7 General Discussion of Methodologies

Except for the standard approach, all models use all past observations to calculate the standard deviation of the parametric model. This will decrease the complexity of each day's estimation, since each parametric model takes the previous day's variance as input value and no recalculation of past variance is needed. One drawback from this could be that the models are less truthful in the beginning of the sample since little information has been incorporated into the conditional variance. In appendix A is a deeper analysis of where the violations happen in the sample. The analysis shows that violations do not happen more often in the beginning of the sample. For all asset classes, the violations are centred in the middle or tilted towards the end of the period.

Another potential issue here is in the four models where the parameters are estimated with maximum likelihood. Due to the complexity of each estimation, the parameters of the GARCH models were only estimated once and this estimation included all observations. I see two potential issues with this. The first being that the volatility is in one sense estimated with observations from the future. To evaluate this, I compared the parameters of the analysis using a normal GARCH for equity with the maximum likelihood parameters from a previous sample. This is not possible to do for all assets due to lack of data. However, equity should give some guidance on how the parameters vary. Table 2 shows the maximum likelihood parameters from two different sampling periods. It turns out that the results are very similar for the two periods. This means that the analysis could have been made with maximum likelihood estimations from the past giving almost the same results.

	Difference in ML-parameters for a normal GARCH	
Time period	20000103-20130917	19850103-19991231
ω	0.00000	0.00000
α	0.08018	0.08708
β	0.90997	0.90171

Table 2. The table compares the maximum likelihood parameters for a normal symmetric GARCH(1,1) from two different time periods with daily data

The second issue I see is that a shorter estimation window for the maximum likelihood estimations could have given more accurate and time varying GARCH parameters. It is not unlikely that the parameters change in times of market distress. However, this would become a complex issue. If there is an equilibrium level of the parameters, as suggested by table 2, then a period of high persistence would have to be followed by a period of low persistence, and then forecasting in the low persistence period would be based on parameters from the high persistence period. This is why on average; the long run GARCH process should give the most accurate volatility modelling.

4.2 The portfolio Value at Risk

The same six models presented above were estimated for an equally weighted portfolio of the assets. I do not claim that this portfolio is representative of a bank or any market portfolio. Adding all assets together gives a portfolio that is well diversified and that is my only intention.

The VaR for a portfolio is calculated as:

$$VaR_t = z_\alpha w \Sigma_{t-1} w^T V \tag{8}$$

where Σ is the variance covariance matrix and *w* is the matrix of portfolio weights. This will change the focus of the analysis from conditional variance to conditional variance covariance modelling.

I chose to model the covariances between each pair of assets independently instead of estimating a multivariate GARCH. Using a multivariate GARCH would have needed some simplifying assumptions in order for the number of parameters to be manageable. The potential issue with modelling covariances directly is that this method does not guarantee positive semi-definite matrices. This means that for a few observations for some models, I had to estimate the VaR with previous day's value if the variance-covariance matrix was not positive semi-definite. For 3576 observations, the number of matrices not positive semi-definite was never above 15 for any model. This approximation was only needed for the two t-distributed models.

For the standard approach, the variance covariance matrix is the equally weighted variance or covariance over the last 250 days following the standard definition of variance and covariance.

The RiskMetrics approach can quite easily be adjusted to a covariance setting where λ equals 0.94.

$$\sigma_{ij,t} = (1-\lambda)\epsilon_{i,t-1}\epsilon_{j,t-1} + \lambda\sigma_{ij,t} \tag{9}$$

For the symmetric GARCH(1,1), I used the normal variance approach but for covariances instead.

$$\sigma_{ij,t} = \omega + \alpha\epsilon_{i,t-1}\epsilon_{j,t-1} + \beta\sigma_{ij,t} \tag{10}$$

One of the issues with this approach is that ω is normally restricted to be above zero in order for the variance to be positive. However, it is important to allow the long run covariance to be negative. Hence, this restriction does not hold for covariances. To make the optimizations manageable I used variance targeting and set the long run covariance to the sample mean covariance V_L. That means that ω does not have to be estimated in the maximum likelihood estimation but is set as

$$\omega = V_L(1-\alpha-\beta) \tag{11}$$

The procedure is similar for the GARCH model with the student's t distribution. The problem here arises when trying to choose the degrees of freedom for the threshold value. Since every covariance is estimated independently, I got 153 different degrees of freedom for 17 assets, when the threshold value only takes one input. From previous research I know that t-distributions usually return low failure rates, so to make the model perform better I used the maximum number of degrees of freedom to estimate the VaR. An alternative would have been to use a weighted average, but due to the overly conservative estimates of the t-distributed models, taking the maximum degrees of freedom gives better estimates.

The asymmetric GARCH becomes less intuitive when modelling covariances. In a variance setting, the return shock is either positive or negative. However, for covariances there could be two positive shocks, two negative shocks or a positive shock for one asset and a negative for the other. The two positive shocks and the two negative shocks should imply the same thing for the covariance, an increase with a factor α. While one negative and one positive will decrease the covariance with a factor α. Intuitively, it is difficult to construct an asymmetric covariance process and hence, I chose to model the variances with asymmetries and the covariances without asymmetries.

4.3 Backtests

From previous literature, I have chosen four of the methods most commonly used to backtest Value at Risk estimations. For all frameworks, the essence of the evaluation is based on violations of the VaR estimates. A violation if defined as follows:

$$I_t = \begin{cases} 1, & if\ R_t < VaR_{t-1} \\ 0, & else \end{cases} \tag{12}$$

The first evaluation method is the daily capital charge from the Basel Accord. This method looks at the average capital requirement and finds the average daily capital charges. The number of violations during the last 250 days determines if the capital charge should be higher than a factor three of the VaR estimate, according to (1). This procedure is relatively simple and does not guarantee that banks with accurate models are rewarded. Since only four exceptions per year is allowed for a 99% VaR, a bank with a correct model still has a 10.8% probability of ending up in the yellow zone (Jorion, 2007). The Basle backtest has faced a lot of criticism. Stahl (1997) and Danielsson and Hartmann (1998) argue that the multiplicative factor of three is too high to impose the incentives intended for banks to implement a good model. If the minimum value of k instead was one then inaccurate models ending up in the red zone would be punished with a capital charge twice the size of an accurate model. They argue that a simple model giving the lowest Value at Risk estimate is optimal when it comes to capital requirements. However, minimizing this capital requirement is not the only concern for banks. Too many violations in one period can lead to bad publicity and intervention by authorities. This means that the best model cannot be determined just by looking at the capital charge, but that the number of violations in one period also has to be considered.

To look at the statistical properties of each model, the rest of the analysis will follow the procedure presented by Christoffersen (1998), commonly used within the VaR literature. The intuition behind the analysis is that an accurate Value at Risk model has to fulfill two criteria, the first being that the coverage rate is close to the VaR percentile. For a 99 % Value at Risk, this means that the failure rate should be approximately one per cent. The second criterion is that violations should be independent. This means that clusters of violations signals that there is something wrong with the volatility modeling.

The first part of the backtest tests the hypothesis that the observed violation rate is the same as the VaR percentile. This is called the unconditional coverage test since it implicitly assumes that violations are independent of each other. This is similar to tests done by Kupiec (1995) and McNees (1995). The null hypothesis being

$$H_0: \pi = p \tag{13}$$

The likelihood under the null hypothesis is

$$L(I, p) = p^N (1 - p)^{T-N} \tag{14}$$

The likelihood in the observed sample is

$$L(I, \pi_1) = \pi_1^N (1 - \pi_1)^{T-N} \tag{15}$$

Giving the likelihood test

$$LR_{uc} = 2\ln(L(I, \hat{\pi}_1)) - 2\ln\,(L(I, p)) \sim \chi^2(1) \tag{16}$$

Where p is the VaR percentile of 0.01, π is the observed failure rate, N is number of violations, T is number of observations, and $\pi_1 = \frac{N}{T}$. LR_{uc} is Chi-square distributed with one degree of freedom.

Christoffersen (1998) argues that finding unconditional coverage is not enough to conclude that the VaR forecasting is accurate. He proves that for the coverage rate to be equal to the VaR percentile conditional on previous violations, the sequence I_t has to be identically and independently distributed Bernoulli with parameter p. Furthermore; large losses in a sequence can lead to serious outcomes for a financial institution. Hence, a model that fails to take this into account does not accurately capture the risk. Christoffersen (1998) performs a test where he as an alternative assumes that the violations follow a Markov sequence with switching probability matrix

$$\Pi = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \tag{17}$$

π_{01} is the probability of a day with 0 (no violation) followed by a day with 1 (violation) and π_{11} is the probability of a 1 followed by a 1. He tests the null of independence against the alternative giving the test of independence:

$$H_0\colon \pi_{01} = \pi_{11} \tag{18}$$

The likelihood function for this being:

$$L(I, \hat{\pi}_{01}, \hat{\pi}_{11}) = (1-\pi_{01})^{T_0 - T_{01}} \pi_{01}^{T_{01}} (1-\pi_{11})^{T_1 - T_{11}} \pi_{11}^{T_{11}} \tag{19}$$

As above, giving the likelihood test:

$$LR_{ind} = 2\ln\big(L(I, \pi_{01}, \pi_{11})\big) - 2\ln\left(L(I, \hat{\pi}_1)\right) \sim \chi^2(1) \tag{20}$$

T_1 is the number if violations, T_0 is the number of days without violations. T_{01} is the number of days with a violation followed by a day without violation, while T_{11} is the number of days with a violation followed by a second violation. $\hat{\pi}_{01} = \frac{T_{01}}{T_0}$ and $\hat{\pi}_{11} = \frac{T_{11}}{T_1}$. LR_{ind} is also Chi-square distributed with one degree of freedom.

The unconditional coverage rate does not account for independence, and the independence test does not account for coverage. In order to get a complete test of the VaR model, the two tests are added in a conditional coverage test, comparing the conditional coverage with the coverage rate. This gives the null hypothesis:

$$H_0\colon \pi_{01} = \pi_{11} = p \tag{21}$$

The likelihood test becomes:

$$LR_{cc} = 2\ln\big(L(I, \hat{\pi}_{01}, \hat{\pi}_{11})\big) - 2\ln\big(L(I, p)\big) = LR_{uc} + LR_{ind} \sim \chi^2(2) \qquad (22)$$

In this case, LR_{cc} is Chi-square distributed with two degrees of freedom.

Another backtest frequently used in the literature is the duration-based test of independence, proposed by Christoffersen and Pelletier (2004). The intuition behind their test is that a model that is not correctly specified will face clustered violations. An accurate model should have an average duration of 1/p between each violation while an inaccurate model with clustered violations returns few long durations and many short durations. This test can improve statistical power in small samples. However, due to the relatively large sample used, the method will not be evaluated in this thesis.

5. Data

Below is a description of the data used. All prices are daily closing prices in USD from Thomson Reuters Datastream ranging from 2000/01/03 to 2013/09/17. The time period is interesting because it contains periods of extreme market movements, such as the financial crisis between 2008 and 2009. Furthermore, the period includes the unusual movements in commodity prices during the last years. The periods of turmoil become particularly important for risk management purposes.

Returns are defined as the natural logarithm of the price change.

$$R_t = \ln(P_t) - \ln\,(P_{t-1}) \qquad (23)$$

5.1 Asset Classes

The three standard asset classes are normally said to be equity, fixed income and cash. However, recently commodities, currencies and real estate have become more important. For the purpose of the thesis, it is important to distinguish asset classes with different return characteristics from each other. Furthermore, adding several assets in the same standard asset class makes it possible to discuss whether the results varies within the asset class or if they can be generalized. To deepen the analysis further, an equity volatility index was also included since investing in volatility through different structured products is becoming more popular. This is also a highly volatile asset compared to the others giving an additional perspective to the analysis. An overview of the assets used is shown in table 3. A detailed description of the price indices and exchange rates used can be found in appendix B.

Data Sources	
Asset class	**Index**
Equity	S&P500
Fixed Income	
High Yield bonds	Barclays United States Corporate High Yield
Emerging debt	J.P. Morgan Emerging Markets Bond Index
Corporate bonds-high grade	Dow Jones Equal Weight U.S. Issued Corporate Bond Index
Government bonds	JP Morgan GBI Global all Maturities
FX	
GBP	World Market Close
EUR	World Market Close
YEN	World Market Close
CAD	World Market Close
SEK	World Market Close
CHF	World Market Close
Commodities	
Precious metals	S&P GSCI Precious metals
Industrial metals	S&P GSCI Industrial Metals
Agriculture and livestock	S&P GSCI Agriculture & Livestock
Energy	S&P GSCI Energy
Real Estate	GPR 250 PSI Americas
Volatility	CBOE VIX S&P 500

Table 3. The table shows the data used for the analysis. The observations are daily and ranging from 2000/01/03-2013/09/17

One could argue that three asset classes are excluded; cash, hedge funds and private equity. Cash is not included because there is basically no risk in holding cash. The only risk would be inflation, and on a daily basis, inflation risk becomes very small. Hedge funds and private equity are both excluded due to lack of daily reliable data with exposure to these asset classes. For a bank, the three important asset classes are generally equity, fixed income and commodities while the remaining part represents a small fraction of the holdings. The analysis will still be valid without the inclusion of hedge funds and private equity. However, the independent analysis of hedge funds and private equity would be an interesting area to investigate further.

All indices are well known, widely accepted and frequently used in research and for benchmarking and could therefore be regarded as reliable. However, one problem arises when deciding what market to track. My reference point has been to calculate Value at Risk for an American bank. From this I have chosen a local equity index since the majority of equity investments are likely to be invested in U.S. equity. The same reasoning holds for corporate bonds, high yield bonds and real estate. Emerging debt is by definition a global market. I have chosen to stay with a global perspective for government bonds as well since investments in foreign government bonds are common. This will possibly drive up volatility in this asset class since U.S. Treasuries are safer than many other government bonds. For commodities and exchange rates, the market is global so no problem arises there.

A different issue with the data is that even though all indices are well known and frequently used, they are still approximations. While some asset classes, such as precious metals, are easy to measure, others are more complex. The real estate market is an example of a market with few daily benchmarks that is difficult to track. The index value also depends on weighing of the assets in the portfolio and how the index is composed. For this reason, the thesis will only give guidance on how the different parametric methods work for each asset class. For more general results of each asset class, several indices would have to be used to guarantee robustness. Another potential concern is how the sample period affects the results. McAleer et al. (2013b) found that in short periods of time, such as before, during and after the financial crisis, different VaR methods may be optimal. While my findings are intended to find what is optimal in the long run, the short run results of each asset class would be an interesting extension.

5.2 Descriptive Statistics

Table 4 presents the descriptive statistics of each asset used in the analysis.

Descriptive Statistics								
	Mean	Variance	Std	Skewness	Kurtosis	Min	Max	Obs.
Equity	0.0044%	0.00017	1.30%	-0.174	10.971	-9.47%	10.96%	3576
Corporate Bonds	0.0048%	0.00001	0.37%	-0.302	6.614	-2.99%	2.13%	3576
High Yield Bonds	0.0057%	0.00001	0.37%	-1.341	45.420	-5.30%	4.86%	3576
Emerging debt	0.0358%	0.00002	0.46%	-1.528	31.725	-6.31%	4.74%	3576
Government bonds	0.0207%	0.00002	0.43%	0.172	6.120	-2.12%	3.71%	3576
Precious Metals	0.0423%	0.00015	1.23%	-0.440	8.863	-10.11%	8.76%	3576
Industrial Metals	0.0189%	0.00023	1.51%	-0.250	5.847	-9.11%	7.58%	3576
Energy	0.0382%	0.00040	2.00%	-0.268	5.685	-14.40%	9.81%	3576
Agriculture	0.0209%	0.00011	1.04%	-0.202	5.940	-5.81%	5.72%	3576
Real Estate	0.0236%	0.00036	1.90%	-0.333	21.615	-21.26%	16.49%	3576
EUR	0.0077%	0.00004	0.64%	0.139	5.464	-3.84%	4.62%	3576
GBP	-0.0006%	0.00004	0.59%	-0.054	7.188	-3.92%	4.47%	3576
YEN	0.0006%	0.00004	0.66%	0.270	6.714	-3.71%	4.61%	3576
CAD	0.0095%	0.00004	0.60%	0.067	8.539	-4.34%	5.05%	3576
SEK	0.0074%	0.00006	0.79%	0.158	6.213	-3.54%	5.55%	3576
CHF	0.0149%	0.00005	0.70%	-0.367	11.848	-8.47%	5.45%	3576
VIX	-0.0143%	0.00387	6.22%	0.665	7.515	-35.06%	49.60%	3576

Table 4. Descriptive Statistics of the data used. Ranging from 2000/01/03-2013/09/17

All assets have low average daily returns. The standard deviation is high for VIX, while it is relatively high also for commodities, equity and real estate, lower for exchange rates and low for fixed income. One interesting observation is that the standard deviation of government bonds is higher than for corporate bonds. This can possibly be explained by the fact that the government bonds index measures the world market of bonds while the corporate bonds index captures the U.S. market. Some markets, especially European, have experienced great volatility in government bonds during the last years. It is also important to note that government bonds have the highest minimum value implying that they are still safest among the assets. Many of the assets are negatively skewed. This is particularly evident for emerging debt and high yield bonds, while government bonds are positively skewed. High Yield Bonds and Emerging debt also have a high kurtosis.

The issue of autocorrelation is normally treated in volatility modelling. In appendix C, the LjungBox test statistics of the return series are presented. They show that for some of the asset returns there is autocorrelation. However, throughout the analysis I have used the return series rather than residuals. Risk management is concerned with returns, which means that to use residuals in the parametric modelling, a time-varying mean would have to be added. The number of autocorrelation lags would differ between assets and possibly also through time. It is unlikely to assume that daily VaR calculations would start with an analysis of each time series to determine the number of autocorrelation lags to be removed. This would add yet another step to the already complex Value at Risk calculations.

6. Results

6.1 The Independent Value at Risk

6.1.1 Equity

For equity, the results are shown in table 5. The only model that is not rejected by the backtests is the asymmetric GARCH with t-distribution. This is in line with previous research that equity experiences asymmetric returns with fat tails and supports the earlier results of McAleer and Da Veiga (2008) and Sener et al. (2012). The daily capital charge becomes lower with asymmetric modelling. The normal asymmetric GARCH gives the lowest average capital requirement but at the cost of nine violations in one period. Being one violation away from ending up in the red zone could give a bank bad publicity and problems with authorities.

Equity	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	35.797	27.283	14.868	12.326	12.770	2.967
p-value	*0.00%*	*0.00%*	*0.01%*	*0.04%*	*0.04%*	*8.50%*
Independence test statistic	31.016	20.149	20.335	0.162	8.122	0.381
p-value	*0.00%*	*0.00%*	*0.00%*	*68.69%*	*0.44%*	*53.71%*
Conditional test statistic	66.813	47.432	35.203	12.488	20.892	3.348
p-value	*0.00%*	*0.00%*	*0.00%*	*0.19%*	*0.00%*	*18.75%*
Failure rate	2.20%	1.99%	1.71%	0.48%	1.65%	0.73%
Avg. daily capital charge	9.46%	8.83%	8.79%	10.65%	8.67%	10.08%
Min. daily capital charge	3.20%	2.93%	3.93%	4.98%	3.82%	4.54%
Max. daily capital charge	26.63%	38.48%	37.57%	39.17%	35.42%	39.00%
Max. no. violations in 250 days	23	12	12	5	9	5
Avg. no. violations in 250 days	5.38	4.71	4.07	1.14	3.93	1.72

Table 5. The table shows the results for the independent analysis of six parametric methods on equity

6.1.2 Fixed Income

The results are shown in table 6-9. The descriptive statistics showed that different fixed income assets had different return characteristics. This seems to matter for the choice of parametric method. For corporate bonds, the backtest rejects neither of the GARCH models with a student's t distribution. However, the asymmetric model gives a lower capital charge and can therefore be regarded as the better of the two. RiskMetrics is the model that gives the lowest capital charge, but with ten violations in one period, the bank would end up in the red zone. The descriptive statistics showed that corporate bonds have relatively high kurtosis, which is in favour of the t-distribution. The skewness of the series is negative which may contribute to explain why asymmetries add value to the model.

None of the models for high yield bonds return acceptable results. Both the criterion of independence and unconditional coverage are rejected. The data had kurtosis of around 45 while the normal distribution has kurtosis of three, making it obvious why the normal distribution is a bad fit. The t-distribution needs approximately 4.1 degrees of freedom to have a kurtosis of 45. The low number of degrees of freedom will give a high threshold value and may explain the low failure rate of the t-distributed models. The normal asymmetric GARCH gives the lowest capital charge.

Emerging debt has similar return characteristics as high yield bonds with high kurtosis and negative skewness. In the same manner, none of the models for emerging debt give adequate results. The t-distributed GARCH models pass the test of unconditional coverage but fail the test of independence. The kurtosis of 32 is lower than for high yield bonds and gives a failure rate closer to one. The normal asymmetric GARCH gives the lowest capital charge.

For government bonds, both the normally distributed GARCH models pass the statistical backtests. The capital charges of the symmetric and asymmetric models are almost the same, implying that there is no gain to adding an asymmetric term. Using a t-distribution gives the model independence but too few violations. The normal symmetric GARCH also gives the lowest capital charge.

Corporate Bonds	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	31.133	21.948	9.882	2.363	9.882	0.988
p-value	*0.00%*	*0.00%*	*0.17%*	*12.42%*	*0.17%*	*32.02%*
Independence test statistic	25.553	14.315	14.281	0.411	14.281	0.508
p-value	*0.00%*	*0.02%*	*0.02%*	*52.15%*	*0.02%*	*47.61%*
Conditional test statistic	56.687	36.263	24.163	2.774	24.163	1.496
p-value	*0.00%*	*0.00%*	*0.00%*	*24.98%*	*0.00%*	*47.33%*
Failure rate	2.11%	1.87%	1.57%	0.76%	1.57%	0.84%
Avg. daily capital charge	2.80%	2.63%	2.66%	2.95%	2.66%	2.95%
Min. daily capital charge	0.61%	0.83%	0.83%	0.99%	0.83%	0.99%
Max. daily capital charge	5.63%	7.72%	7.23%	6.87%	7.38%	7.00%
Max. no. violations in 250 days	15	10	10	7	10	7
Avg. no. violations in 250 days	4.66	4.35	3.60	1.76	3.60	1.93

Table 6. The table shows the results for the independent analysis of six parametric methods on corporate bonds

High Yield Bonds	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	88.086	104.951	34.595	10.886	17.100	13.883
p-value	*0.00%*	*0.00%*	*0.00%*	*0.10%*	*0.00%*	*0.02%*
Independence test statistic	137.414	141.025	65.013	17.305	62.244	17.742
p-value	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Conditional test statistic	225.500	245.976	99.609	28.191	79.344	31.625
p-value	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Failure rate	3.01%	3.13%	2.13%	0.50%	1.76%	0.45%
Avg. daily capital charge	2.55%	2.35%	2.36%	3.56%	2.22%	3.54%
Min. daily capital charge	0.74%	0.55%	0.53%	1.00%	0.52%	0.97%
Max. daily capital charge	8.85%	11.88%	12.82%	17.03%	12.78%	18.05%
Max. no. violations in 250 days	23	17	11	4	9	4
Avg. no. violations in 250 days	7.14	7.43	5.17	1.21	4.26	1.07

Table 7. The table shows the results for the independent analysis of six parametric methods on high yield bonds

Emerging debt	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	53.110	44.255	28.689	1.834	17.100	3.649
p-value	*0.00%*	*0.00%*	*0.00%*	*17.57%*	*0.00%*	*5.61%*
Independence test statistic	135.338	96.383	88.671	38.722	74.189	31.907
p-value	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Conditional test statistic	188.448	140.637	117.360	40.556	91.289	35.556
p-value	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Failure rate	2.50%	2.29%	2.01%	0.78%	1.76%	0.70%
Avg. daily capital charge	3.29%	3.09%	3.05%	3.86%	3.00%	3.73%
Min. daily capital charge	1.09%	0.88%	1.07%	1.48%	1.07%	1.44%
Max. daily capital charge	9.18%	16.17%	14.13%	18.20%	14.33%	16.39%
Max. no. violations in 250 days	18	12	12	7	11	7
Avg. no. violations in 250 days	5.57	5.42	4.66	1.91	4.12	1.70

Table 8. The table shows the results for the independent analysis of six parametric methods on emerging debt

Government bonds	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	5.090	5.105	1.395	19.344	1.045	19.344
p-value	*2.41%*	*2.39%*	*23.76%*	*0.00%*	*30.66%*	*0.00%*
Independence test statistic	7.810	7.902	1.047	0.095	0.999	0.095
p-value	*0.52%*	*0.49%*	*30.62%*	*75.80%*	*31.76%*	*75.80%*
Conditional test statistic	12.900	13.007	2.442	19.439	2.044	19.439
p-value	*0.16%*	*0.15%*	*29.50%*	*0.01%*	*35.99%*	*0.01%*
Failure rate	1.41%	1.40%	1.20%	0.36%	1.17%	0.36%
Avg. daily capital charge	3.13%	3.02%	3.02%	3.80%	3.02%	3.80%
Min. daily capital charge	0.87%	0.98%	0.64%	0.82%	0.64%	0.81%
Max. daily capital charge	5.45%	6.00%	5.60%	6.32%	5.61%	6.33%
Max. no. violations in 250 days	9	8	6	3	6	3
Avg. no. violations in 250 days	3.11	3.29	2.86	0.81	2.79	0.81

Table 9. The table shows the results for the independent analysis of six parametric methods on government bonds

6.1.3 Commodities

The results are shown in table 10-13. The results show that parametric methods are not always a good fit for commodities. For industrial metals, both normal GARCH models return adequate results and there seems to be no gain in adding the asymmetric term. The normal symmetric GARCH also gives the lowest capital requirement. For the three remaining commodity classes, none of the models work well. The t-distributed models have low failure rates while the normally distributed models either have high failure rates or fail the independence test. For all of the three asset classes, RiskMetrics gives the lowest capital charge. From the results it seems that commodities are better modelled by the normal distribution and that the t-distribution is too conservative. The descriptive statistics showed that among all assets, commodities had the lowest kurtosis, closest to the normal distribution.

Precious Metals	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	37.408	24.556	10.809	17.384	10.809	17.384
p-value	*0.00%*	*0.00%*	*0.10%*	*0.00%*	*0.10%*	*0.00%*
Independence test statistic	36.482	37.609	26.695	9.189	26.695	9.189
p-value	*0.00%*	*0.00%*	*0.00%*	*0.24%*	*0.00%*	*0.24%*
Conditional test statistic	73.890	62.165	37.504	26.572	37.504	26.572
p-value	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Failure rate	2.23%	1.93%	1.59%	0.39%	1.59%	0.39%
Avg. daily capital charge	9.35%	8.79%	8.89%	12.50%	8.89%	12.47%
Min. daily capital charge	2.06%	2.78%	2.66%	3.99%	2.66%	3.99%
Max. daily capital charge	18.90%	19.00%	17.06%	27.08%	17.10%	27.76%
Max. no. violations in 250 days	16	10	10	3	10	3
Avg. no. violations in 250 days	5.31	4.62	3.85	0.85	3.85	0.85

Table 10. The table shows the results for the independent analysis of six parametric methods on precious metals

Industrial metals	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	13.043	7.321	0.744	13.883	0.744	13.883
p-value	*0.03%*	*0.68%*	*38.85%*	*0.02%*	*38.85%*	*0.02%*
Independence test statistic	8.019	20.682	7.843	0.144	7.843	0.144
p-value	*0.46%*	*0.00%*	*0.51%*	*70.45%*	*0.51%*	*70.45%*
Conditional test statistic	21.062	28.003	8.586	14.027	8.586	14.027
p-value	*0.00%*	*0.00%*	*1.37%*	*0.09%*	*1.37%*	*0.09%*
Failure rate	1.68%	1.48%	1.15%	0.45%	1.15%	0.45%
Avg. daily capital charge	11.32%	10.40%	10.34%	12.42%	10.34%	12.43%
Min. daily capital charge	2.09%	3.41%	1.96%	2.45%	1.96%	2.45%
Max. daily capital charge	24.64%	28.56%	26.07%	27.89%	26.16%	28.40%
Max. no. violations in 250 days	12	8	7	4	7	4
Avg. no. violations in 250 days	4.06	3.60	2.76	1.12	2.76	1.12

Table 11. The table shows the results for the independent analysis of six parametric methods on industrial metals

Energy	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	9.115	12.770	8.991	13.883	10.809	13.883
p-value	*0.25%*	*0.04%*	*0.27%*	*0.02%*	*0.10%*	*0.02%*
Independence test statistic	26.658	32.681	26.861	0.144	26.695	8.951
p-value	*0.00%*	*0.00%*	*0.00%*	*70.45%*	*0.00%*	*0.28%*
Conditional test statistic	35.773	45.450	35.852	14.027	37.504	22.834
p-value	*0.00%*	*0.00%*	*0.00%*	*0.09%*	*0.00%*	*0.00%*
Failure rate	1.56%	1.65%	1.54%	0.45%	1.59%	0.45%
Avg. daily capital charge	14.69%	13.98%	14.23%	16.64%	14.30%	16.62%
Min. daily capital charge	4.92%	4.51%	4.25%	5.24%	4.25%	5.23%
Max. daily capital charge	32.89%	35.75%	33.69%	36.91%	34.23%	37.34%
Max. no. violations in 250 days	18	11	8	4	8	3
Avg. no. violations in 250 days	3.85	3.94	3.75	1.03	3.89	1.03

Table 12. The table shows the results for the independent analysis of six parametric methods on energy

Agriculture	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	9.115	9.882	3.829	13.883	4.446	17.384
p-value	*0.25%*	*0.17%*	*5.04%*	*0.02%*	*3.50%*	*0.00%*
Independence test statistic	20.409	1.782	7.874	0.144	7.887	0.110
p-value	*0.00%*	*18.19%*	*0.50%*	*70.45%*	*0.50%*	*74.01%*
Conditional test statistic	29.524	11.664	11.703	14.027	12.333	17.494
p-value	*0.00%*	*0.29%*	*0.29%*	*0.09%*	*0.21%*	*0.02%*
Failure rate	1.56%	1.57%	1.34%	0.45%	1.37%	0.39%
Avg. daily capital charge	7.62%	7.33%	7.36%	8.36%	7.40%	8.36%
Min. daily capital charge	1.50%	2.35%	1.52%	1.86%	1.52%	1.85%
Max. daily capital charge	17.64%	19.99%	19.86%	18.98%	20.33%	18.65%
Max. no. violations in 250 days	16	8	9	4	10	3
Avg. no. violations in 250 days	3.83	3.74	3.25	1.05	3.32	0.91

Table 13. The table shows the results for the independent analysis of six parametric methods on agriculture and livestock

6.1.4 Currencies

The results are presented in table 14-19. For currencies in general, the results are very varying. For EUR, YEN and CHF, the two normal GARCH models pass the backtests and give low capital requirements. For GBP and CAD, neither of the models are a good fit. The results from SEK show that both asymmetric models work at a 1 % significance level. A change in one exchange rate means a positive return for one of the countries and a negative return for the other and there should not be any asymmetries. However, in this case the two countries are very different in size and economic power. While a change in the USD/SEK exchange rate may not matter much for the U.S., it can have large effects on the Swedish economy and hence explain the asymmetries. Another aspect to take into account when modelling currencies is whether exchange rates are controlled by monetary policy. This is not an issue in this case but may have to be considered for an in-depth analysis of other exchange rates. For currencies, independently of the statistical backtest, the normal symmetric GARCH, the normal asymmetric GARCH and RiskMetrics always return low capital requirements.

EUR	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	3.187	5.804	0.140	21.460	0.491	23.742
p-value	*7.42%*	*1.60%*	*70.81%*	*0.00%*	*48.33%*	*0.00%*
Independence test statistic	1.180	7.919	0.817	0.081	7.847	0.068
p-value	*27.73%*	*0.49%*	*36.62%*	*77.62%*	*0.51%*	*79.44%*
Conditional test statistic	4.367	13.723	0.957	21.540	8.338	23.810
p-value	*11.26%*	*0.10%*	*61.98%*	*0.00%*	*1.55%*	*0.00%*
Failure rate	1.32%	1.43%	1.06%	0.34%	1.12%	0.31%
Avg. daily capital charge	4.67%	4.51%	4.55%	5.20%	4.52%	5.18%
Min. daily capital charge	1.66%	1.44%	1.59%	1.89%	1.59%	1.89%
Max. daily capital charge	9.68%	10.52%	11.14%	10.27%	10.49%	10.19%
Max. no. violations in 250 days	18	7	10	4	8	4
Avg. no. violations in 250 days	3.25	3.50	2.59	0.79	2.73	0.72

Table 14. The table shows the results for the independent analysis of six parametric methods on EUR

GBP	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	13.043	15.968	3.829	9.559	5.105	9.559
p-value	*0.03%*	*0.01%*	*5.04%*	*0.20%*	*2.39%*	*0.20%*
Independence test statistic	20.216	26.342	14.440	0.203	7.902	0.203
p-value	*0.00%*	*0.00%*	*0.01%*	*65.23%*	*0.49%*	*65.23%*
Conditional test statistic	33.259	42.309	18.269	9.762	13.007	9.762
p-value	*0.00%*	*0.00%*	*0.01%*	*0.76%*	*0.15%*	*0.76%*
Failure rate	1.68%	1.73%	1.34%	0.53%	1.40%	0.53%
Avg. daily capital charge	4.34%	4.18%	4.13%	4.61%	4.15%	4.58%
Min. daily capital charge	1.25%	1.34%	1.15%	1.32%	1.15%	1.30%
Max. daily capital charge	10.94%	12.76%	12.06%	12.61%	12.31%	13.32%
Max. no. violations in 250 days	22	10	10	7	9	8
Avg. no. violations in 250 days	3.95	4.11	3.17	1.30	3.36	1.33

Table 15. The table shows the results for the independent analysis of six parametric methods on GBP

YEN	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	10.040	8.991	0.744	21.460	0.290	19.344
p-value	*0.15%*	*0.27%*	*38.85%*	*0.00%*	*59.03%*	*0.00%*
Independence test statistic	7.931	14.292	0.951	0.081	0.860	0.095
p-value	*0.49%*	*0.02%*	*32.94%*	*77.62%*	*35.37%*	*75.80%*
Conditional test statistic	17.971	23.283	1.695	21.540	1.150	19.439
p-value	*0.01%*	*0.00%*	*42.85%*	*0.00%*	*56.27%*	*0.01%*
Failure rate	1.59%	1.54%	1.15%	0.34%	1.09%	0.36%
Avg. daily capital charge	4.63%	4.60%	4.58%	6.07%	4.57%	6.05%
Min. daily capital charge	1.45%	1.48%	1.85%	2.51%	1.85%	2.51%
Max. daily capital charge	8.37%	9.07%	7.95%	10.75%	8.41%	11.16%
Max. no. violations in 250 days	11	9	6	4	6	4
Avg. no. violations in 250 days	3.60	3.66	2.69	0.75	2.55	0.78

Table 16. The table shows the results for the independent analysis of six parametric methods on YEN

CAD	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	10.040	5.804	1.045	10.886	1.395	8.337
p-value	*0.15%*	*1.60%*	*30.66%*	*0.10%*	*23.76%*	*0.39%*
Independence test statistic	51.370	20.801	14.680	8.750	14.632	8.579
p-value	*0.00%*	*0.00%*	*0.01%*	*0.31%*	*0.01%*	*0.34%*
Conditional test statistic	61.410	26.605	15.725	19.637	16.027	16.916
p-value	*0.00%*	*0.00%*	*0.04%*	*0.01%*	*0.03%*	*0.02%*
Failure rate	1.59%	1.43%	1.17%	0.50%	1.20%	0.56%
Avg. daily capital charge	4.37%	4.00%	4.01%	4.55%	4.01%	4.54%
Min. daily capital charge	0.72%	1.35%	0.63%	0.73%	0.64%	0.73%
Max. daily capital charge	10.50%	13.69%	12.86%	14.80%	12.99%	14.87%
Max. no. violations in 250 days	12	7	7	6	7	6
Avg. no. violations in 250 days	3.78	3.30	2.68	1.16	2.75	1.30

Table 17. The table shows the results for the independent analysis of six parametric methods on CAD

SEK	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	3.777	6.543	0.140	13.883	0.290	8.337
p-value	*5.20%*	*1.05%*	*70.81%*	*0.02%*	*59.03%*	*0.39%*
Independence test statistic	20.884	14.339	14.907	0.144	7.854	0.225
p-value	*0.00%*	*0.02%*	*0.01%*	*70.45%*	*0.51%*	*63.52%*
Conditional test statistic	24.660	20.882	15.047	14.027	8.144	8.562
p-value	*0.00%*	*0.00%*	*0.05%*	*0.09%*	*1.70%*	*1.38%*
Failure rate	1.35%	1.45%	1.06%	0.45%	1.09%	0.56%
Avg. daily capital charge	5.60%	5.42%	5.36%	6.06%	5.36%	6.07%
Min. daily capital charge	1.50%	1.78%	1.30%	1.51%	1.30%	1.50%
Max. daily capital charge	14.26%	14.28%	13.90%	13.83%	13.43%	13.31%
Max. no. violations in 250 days	18	8	7	4	8	6
Avg. no. violations in 250 days	3.20	3.49	2.54	1.00	2.58	1.28

Table 18. The table shows the results for the independent analysis of six parametric methods on SEK

CHF	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	0.091	4.446	0.016	26.208	0.044	26.208
p-value	*76.20%*	*3.50%*	*89.93%*	*0.00%*	*83.45%*	*0.00%*
Independence test statistic	0.745	1.362	0.692	0.056	0.774	0.056
p-value	*38.80%*	*24.32%*	*40.55%*	*81.28%*	*37.90%*	*81.28%*
Conditional test statistic	0.836	5.808	0.708	26.264	0.818	26.264
p-value	*65.80%*	*5.48%*	*70.18%*	*0.00%*	*66.45%*	*0.00%*
Failure rate	1.05%	1.37%	0.98%	0.28%	1.03%	0.28%
Avg. daily capital charge	5.00%	4.82%	4.85%	5.86%	4.87%	5.84%
Min. daily capital charge	1.57%	1.57%	1.71%	2.12%	1.71%	2.12%
Max. daily capital charge	9.23%	12.07%	11.02%	11.08%	11.38%	10.68%
Max. no. violations in 250 days	10	8	7	3	8	3
Avg. no. violations in 250 days	2.40	3.31	2.36	0.66	2.45	0.66

Table 19. The table shows the results for the independent analysis of six parametric methods on CHF

6.1.5 Real Estate and Volatility

The results for real estate are presented in table 20 and they show that none of the models work well. Like emerging debt, real estate has very high kurtosis that can lead to issues with the parametric modelling. RiskMetric gives the lowest capital charge.

For VIX both the standard approach and RiskMetrics work, but RiskMetrics gives the lowest capital charge. All six methods fulfil the criterion of independence, but the GARCH models have too low unconditional coverage. All failure rates are low and the average capital charge is substantially higher than above. This asset is interesting because the volatility is much higher than for the other assets and this leads to a rejection of the more sophisticated GARCH methods. The investment in equity volatility shows different patterns from the standard asset classes and this may contribute to explain why these investment strategies are becoming so popular. Even though one could argue that investing in volatility is more complex than investing in other asset classes, the variance modelling becomes less complex and simple methods are preferred over more sophisticated ones.

Real Estate	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	54.983	15.968	17.100	1.834	15.968	1.376
p-value	*0.00%*	*0.01%*	*0.00%*	*17.57%*	*0.01%*	*24.08%*
Independence test statistic	62.105	38.410	26.281	15.776	32.377	15.666
p-value	*0.00%*	*0.00%*	*0.00%*	*0.01%*	*0.00%*	*0.01%*
Conditional test statistic	117.088	54.377	43.381	17.610	48.344	17.042
p-value	*0.00%*	*0.00%*	*0.00%*	*0.02%*	*0.00%*	*0.02%*
Failure rate	2.53%	1.73%	1.76%	0.78%	1.73%	0.81%
Avg. daily capital charge	12.27%	10.40%	10.52%	12.00%	10.62%	11.90%
Min. daily capital charge	1.47%	3.14%	2.59%	3.18%	2.56%	3.14%
Max. daily capital charge	52.60%	58.68%	60.89%	66.41%	63.25%	66.86%
Max. no. violations in 250 days	19	9	10	5	9	5
Avg. no. violations in 250 days	5.76	3.95	4.12	1.79	4.05	1.86

Table 20. The table shows the results for the independent analysis of six parametric methods on real estate

VIX	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	1.726	1.834	13.883	46.262	19.344	50.934
p-value	*18.89%*	*17.57%*	*0.02%*	*0.00%*	*0.00%*	*0.00%*
Independence test statistic	0.410	0.442	0.144	0.009	0.095	0.005
p-value	*52.21%*	*50.61%*	*70.45%*	*92.46%*	*75.80%*	*94.34%*
Conditional test statistic	2.136	2.276	14.027	46.271	19.439	50.940
p-value	*34.37%*	*32.05%*	*0.09%*	*0.00%*	*0.01%*	*0.00%*
Failure rate	0.78%	0.78%	0.45%	0.11%	0.36%	0.08%
Avg. daily capital charge	43.12%	41.37%	42.27%	59.21%	41.91%	58.47%
Min. daily capital charge	12.55%	14.04%	16.79%	23.54%	16.78%	23.30%
Max. daily capital charge	65.38%	77.84%	68.64%	98.94%	71.49%	100.80%
Max. no. violations in 250 days	6	5	5	2	3	2
Avg. no. violations in 250 days	1.87	1.88	1.06	0.22	0.83	0.15

Table 21. The table shows the results for the independent analysis of six parametric methods on VIX

6.1.6 Discussion

All of the six models work well for at least one asset class. The standard approach works well for VIX, while it causes too many violations for almost all the other models. It is never the model with lowest capital charge. The backtests for all asset classes except for CHF and VIX rejected RiskMetrics. However, RiskMetrics is often the model giving the lowest capital charge, in line with the results of Hammoudeh et al. (2013). The t-distributed GARCH models are generally too conservative and always give failure rates lower than one per cent. The normally distributed GARCH models face the opposite problem and usually give high failure rates and many violations. Another interesting observation is that using a normal GARCH when the return distribution is clearly not normal, such as for high yield bonds and emerging debt can lead to an excessive number of violations. This supports the findings of Brooks and Persand (2002).

When a normal GARCH model fulfils the criterion of conditional coverage, then this method also gives the lowest capital charge. On the other hand, if a t-distributed model is the statistically most accurate method then there is a tension between the best model and the model with the lowest capital charge. From this it is evident that the Basel Accord does not always reward financial institutions with good Value at Risk models since more accurate models may give higher capital requirements. From my results it seems that some of the critique on the Basle backtest of Value at Risk methods, for example Stahl (1997), Danielsson et al. (1998) and Jorion (2007), is justified also in this setting.

6.2 The Portfolio Value at Risk

The results from the portfolio of asset are shown in table 22 with the number of matrices not positive semi-definite presented in table 23. The standard approach is rejected. The t-distributed symmetric GARCH is rejected on not being independent. Other than that, all models pass the backtest. RiskMetrics gives the lowest capital requirement. As with the independent models above, using t-distributions give somewhat higher capital requirement due to overestimation of Value at Risk.

The Equally Weighted Portfolio	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
Unconditonal test statistic	5.090	2.234	0.140	0.219	0.219	2.363
p-value	*0.024*	*0.135*	*0.708*	*0.640*	*0.640*	*0.124*
Independence test statistic	7.810	1.147	0.817	15.281	0.615	0.411
p-value	*0.005*	*0.284*	*0.366*	*0.000*	*0.433*	*0.521*
Conditional test statistic	12.900	3.382	0.957	15.500	0.834	2.774
p-value	*0.002*	*0.184*	*0.620*	*0.000*	*0.659*	*0.250*
Failure rate	1.41%	1.26%	1.06%	0.92%	0.92%	0.76%
Avg. daily capital charge	3.28%	3.07%	3.13%	3.50%	3.11%	3.44%
Min. daily capital charge	0.82%	1.62%	1.01%	1.25%	1.12%	1.25%
Max. daily capital charge	6.95%	6.98%	7.07%	6.84%	7.08%	5.72%
Max. no. violations in 250 days	11	6	7	7	7	5
Avg. no. violations in 250 days	3.31	2.98	2.49	2.15	2.12	1.72

Table 22. The table shows the results for the independent analysis of six parametric methods on an equally weighted portfolio

	Number of matrices not positive semi-definite					
	Standard Approach	RiskMetrics	Normal GARCH	Student's t GARCH	Normal asym GARCH	Student's t asym GARCH
No. Matrices not positive semi-definite	0	0	0	15	0	12

Table 23. The table shows the number of matrices that were not positive semi-definite

This implies that parametric models in general work well for a well-diversified portfolio. One striking result is that RiskMetrics never exceeds six violations in 250 days, fulfils the criterion of conditional independence and gives the lowest capital requirement. This method does not suffer from any complex optimization problems using maximum likelihood nor any of the problems with the parameters as discussed above.

7. Conclusion

The purpose of this thesis has been to investigate the use of parametric Value at Risk methods for different asset classes. The results indicate that asset characteristics matter when choosing what parametric Value at Risk method to implement. One important lesson from this is that results from previous research on equity cannot be generalized without further analysis.

This has several implications. One is that the efficiency gains from parametric methods found in previous research do not directly hold for every asset class based on an analysis of equity. This finding becomes particularly important when banks increase their investments in other asset classes. Another insight is that since different parametric methods work well for different asset classes, parametric modelling becomes much more complex than suggested by previous literature. This may contribute to explain why financial institutions choose to stay with the non-parametric methods.

The analysis showed that some assets, such as currencies, are best modelled by assuming a normal distribution, while the same distributional assumption for other asset classes may lead to excessive violations in a short period. Equity and corporate bonds are examples of assets that are best modelled with a t-distribution as underlying assumption. For assets such as equity, asymmetries also turned out to be a key to estimating a good model. For other assets, none of the parametric methods presented here fulfilled the criterion of conditional coverage. Table 24 shows the best model for every asset class according to three different criteria used throughout the thesis. The table highlights how the best model varies across assets as well as well as with the capital charge, statistical backtests and the need to be conservative.

	Summary results		
	Model with lowest capital charge	Best model from backtest	Safe models
Equity	Normal asymmetric GARCH	Student's t asymmetric GARCH	Student's t GARCH
Corporate Bonds	RiskMetrics	Student's t asymmetric GARCH	Student's t GARCH
High Yield Bonds	Normal asymmetric GARCH	-	Student's t GARCH
Emerging debt	Normal asymmetric GARCH	-	Student's t GARCH
Government bonds	Normal symmetric GARCH	Normal asymmetric GARCH	All GARCH
Precious Metals	RiskMetrics	-	Student's t GARCH
Industrial Metals	Normal symmetric GARCH	Normal symmetric GARCH	All GARCH
Energy	RiskMetrics	-	Student's t GARCH
Agriculture	RiskMetrics	-	Student's t GARCH
EUR	RiskMetrics	Normal symmetric GARCH	Student's t GARCH+RiskMetrics
GBP	Normal symmetric GARCH	-	Student's t symmetric GARCH
YEN	Normal asymmetric GARCH	Normal asymmetric GARCH	All GARCH
CAD	RiskMetrics	-	All GARCH+RiskMetrics
SEK	Normal asymmetric GARCH	Normal asymmetric GARCH	Student's t +Normal symmetric GARCH
CHF	RiskMetrics	RiskMetrics	Student's t GARCH
Real Estate	RiskMetrics	-	Student's t GARCH
VIX	RiskMetrics	RiskMetrics	All six models

Table 24. The table shows the best model according to capital charge and backtesting. Best model from backtest is defined as the model that fulfils the criterion of conditial coverage and returns the lowest capital charge. A safe model is defined as a model that never exceeds seven violations in a period of 250 days.

The findings suggest that the use of the t-distribution usually returns very conservative estimates. The t-distribution is safe in the sense that it is unlikely that the models would give enough violations to face problems with bad publicity or intervention by authorities. However, this distributional assumption is never optimal for minimizing capital charges. Berkowitz and O'Brien (2002) were among the first to find that banks use very conservative models. My results show that banks that are very concerned with violations can always choose a t-distributed GARCH and feel confident that it is conservative enough. However, using a normal GARCH modeling for assets such as high yield bonds or other assets with high kurtosis can lead to excessive violations.

These conclusions may contribute to explain why banks are more confident with historical simulations than with parametric methods. The evaluation of parametric methods needs an underlying distributional assumption, and the most frequently used one is the normal distribution. This may be intimidating for banks since assets with high kurtosis modelled by a normal distribution fail the Basle backtest frequently and as a consequence, will end up in the red zone.

Another important finding is that the design of the Basle backtest sometimes rewards inaccurate models by imposing lower capital charges on models with many violations due to having low Value at Risk estimates. This criticism has been brought up in the Value at Risk literature before and is particularly evident in this analysis across asset classes. Changing the design of the Basle backtest could impose the correct incentives on banks to choose more accurate models. One problem with the Basle backtest is that the evaluation period is a short sample of 250 observations, with only 2.5 expected violations. An alternative could be to base capital requirements on a different VaR percentile with more violations, such as a 95% VaR with 12.5 expected exceptions or to increase the sample size to a longer evaluation period. A different approach would be to change the multiplicative factor relative to the penalty factor, so that the penalty becomes a larger part of the capital charge. The backtest also fails to take the criterion of independence into account. This could be imposed in the test by adding an extra penalty factor if violations happened in a sequence.

My findings also suggest that parametric methods for Value at Risk work well for a well diversified portfolio and that the choice of parametric method becomes less important. Even a method as simple as RiskMetrics gives low capital requirements.

These results are supposed to give guidance on how future research should think about asset classes in parametric modelling of Value at Risk. It is evident that generalizations from earlier results on equity cannot to be made without further analysis. Suggestions for future research are in-depth analyses of each asset class. The assets such as emerging debt, high yield bonds and real estate that could not be modelled properly with any of the parametric methods suggested in this thesis should be investigated further with other distributional assumptions, such as the heavy tailed distribution proposed by Politis (2004).

A next step could be to construct portfolios with different assets and change the composition in a sensitivity analysis. Since my findings suggest that different parametric methods are best suited for different asset classes, portfolio composition becomes particularly interesting for the Value at Risk estimations.

Comparing capital requirements from historical simulations and parametric models using t-distributions would also be of interest. Since banks prefer conservative models where the risk of ending up in the red zones is very low, it would be interesting to compare how t-distributed GARCH models perform compared to historical simulations in terms of capital requirements. If banks were to change from historical simulations to a parametric GARCH modelling using t-distributions then the models would still be conservative but in many cases, the clustering of violations would disappear. Clustering of violations means substantial losses in a sequence, so even if capital requirements would remain the same, avoiding clusters would add value to the internal risk control.

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Appendix A. Weighted Average of Violations

To analyse if the models are inaccurate in the beginning of the sample, I did a weighing of each violation. Each observation got a number from 1 to 3576 with the date 2000/01/03 being observation one and 2013/09/17 being observation 3576.

The weighted average of violations was defined as:

$$w = \frac{1}{T_1}\sum_{n=1}^{3576} n * I_n$$

$$I_t = \begin{cases} 1, & if\ R_t < VaR_{t-1} \\ 0, & else \end{cases} \tag{A1}$$

$$T_1 = \sum_{n=1}^{3576} I_n$$

This gives the average point of the violations in the sample. The results are shown in table A1. Only for two of the assets are the violations weighted towards the first half of the sample. For the rest of the assets the weight is tilted towards the end. This is possibly explained by the fact that violations happened more often during the financial crisis, and that the financial crisis is in the second part of the sample.

	Weight of violations	
	Wegihted Average	In percentile of 3576 obs
Equity	2107	58.92%
Corporate Bonds	1822	50.95%
High Yield Bonds	1671	46.74%
Emerging debt	1579	44.15%
Government bonds	1864	52.14%
Precious Metals	2087	58.36%
Industrial Metals	2085	58.30%
Energy	1926	53.87%
Agriculture	2067	57.80%
Real Estate	1947	54.44%
EUR	1966	54.98%
GBP	1850	51.73%
YEN	2050	57.33%
CAD	2033	56.85%
SEK	1947	54.43%
CHF	1794	50.16%
VIX	2466	68.96%

Table A1. The table shows where violations happen on average

Appendix B. Detailed Description of Data

S&P 500 from Standard and Poor's is an index consisting of the 500 leading U.S. companies, weighted by market capitalization. It is the most commonly used market benchmark.

Barclays United States Corporate High Yield is a market value weighted index that captures the non-investment grade debt market in the U.S. with maturity larger than a year.

Dow Jones Equal Weight U.S. Issued Corporate Bond Index is an equally weighted index consisting of the 96 latest issued investment-grade corporate bonds.

J.P. Morgan Emerging Markets Bond Index tracks returns for external debt instruments in emerging markets. The index is market weighted.

JP Morgan Government Bond Index Global is an index that measures returns on world government bonds across all maturities weighted by market value. The index only includes assets available to international investors.

The six exchange rates come from *the U.S. Dollar Index* and is the value in USD of one unit of foreign currency. All exchange rates are the world market closing prices. The six currencies have had an historical importance for the U.S. While other markets have gained in importance, the currencies in the U.S. Dollar Index are still representable.

S&P GSCI is the most commonly used index to track the commodity market. The indices represent a broad categorization of the commodity market into precious metals, industrial metals, agriculture and livestock and energy, all weighted by production.

The Global Property Research Index 250 Americas tracks the 250 most liquid property securities and is weighted by market capitalization. The index represents the real estate market in America and is provided by Thomson Reuters.

The most commonly used index for equity volatility is the *CBOE VIX Volatility Index*. The index is derived from S&P500 stock index option prices.

Appendix C. LjungBox

The LjungBox test statistics with p-values are shown in table C1.

	Ljung Box	
	LB test statistic	p-value
Equity	48.13	0.00%
Corporate Bonds	9.30	31.76%
High Yield Bonds	1173.07	0.00%
Emerging debt	351.06	0.00%
Government bonds	16.18	3.99%
Precious Metals	14.22	7.62%
Industrial Metals	16.70	3.34%
Energy	13.91	8.42%
Agriculture	2.62	95.57%
Real Estate	128.93	0.00%
EUR	8.08	42.53%
GBP	13.14	10.72%
YEN	8.47	38.86%
CAD	12.18	14.32%
SEK	25.00	0.16%
CHF	9.84	27.62%
VIX	73.06	0.00%

Table C1. The table shows the LjungBox test statistics of the data