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Information Acquisition and Conflict Outbreak

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Abstract: Using a game-theoretic framework, we investigate the role of uncertainty of information for states that make decisions about whether or not to go to war to acquire resources. In our model, a player may receive a signal about the strength of the opponent with some probability, which makes this piece of information come with a degree of uncertainty. We find that the impact of uncertainty in the acquisition of information depends on the ex-ante resource incentives players face. In situations where peace just barely prevails, uncertainty increases the risk of war. On the other hand, where war with certainty occurs without information about player types, more uncertainty lowers the probability of conflict outbreak.

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Contents

"War is the province of chance. In no other sphere of human activity must such a margin be left for this intruder. It increases the uncertainty of every circumstance and deranges the course of events."

General Karl von Clausewitz (1780 - 1831)

1 Introduction

Engaged in a conflict and deciding whether or not to go to war, a state has to act on information it has about itself and other nations. Of course, this information need not always be accurate, which introduces an element of uncertainty for leaders before declaring war. How this added uncertainty of information affects decision makers has not yet been fully explained by scholars in the field. In this essay, we investigate the role of uncertainty in information acquisition by constructing a game-theoretic model of conflict between two players. We find two possible effects. On the one hand, added uncertainty increases the likelihood of war between states in tense situations where peace nonetheless prevails. On the other hand, added uncertainty can have a deterrent effect on aggression between states that would otherwise have attacked each other with certainty, given large enough resource incentives to do so.

To understand the role of uncertainty, we first have to investigate the problem of why wars break out. If we assume there is an overall welfare cost of waging war between two states, in the form of resource destruction, why is there armed conflict at all? Would it not be more beneficial for two states engaged in a dispute to resolve the disagreement by a form of compromise in which the two states both gain more than their expected return in case of a violent conflict? If it could be possible for states to "share the pie" instead of shrinking it due to destruction of resources, what prevents this from happening?

Wars are inefficient and wasteful, and as such undesirable for worldwide economic prosperity. Research in this field could potentially save many human lives, as well as reduce future suffering. Could it be possible to gain a better understanding of why states choose to go to war - and through that, adopt policies that minimize their occurrence?

Despite a decline in the number of wars breaking out per year during the latter half of the 20th century, potential contemporary applications of our research can still be found. Considering the current developments in Eastern Europe, with the Russian invasions first of Georgia in 2008, then of the Crimean peninsula earlier this year following the revolution in Ukraine, as well as the growing Chinese expansionary interests in South East Asia, we would argue that the field of conflict theory has maintained its relevance in the 21st century.

The text is organized as follows. Section 2 provides a background on the subject matter, where we examine relevant previous work in the field and describe the War's Inefficiency Puzzle in more detail. In section 3, we present our research questions and the aims of this thesis. Section 4 sets up the framework of the model we use to study our research problem, which is then solved in section 5. Section 6 offers an additional perspective in which our primary model is extended. In section 7, we present some concluding remarks and wider implications of our findings. Finally, section 8 contains our bibliography, while section 9 serves as an appendix with some probability distributions that were best left out of the main body of text.

2 Background and the current state of knowledge

Within the traditional literature on economics, scholars have used three main lines of reasoning to explain the War's Inefficiency Puzzle, or, why wars break out despite the costs involved (Fearon, 1995).

Firstly, it could be argued that leaders are irrational and subject to various biases. They either neglect the costs of war, which leads to bad decisions, or fail to understand the consequences of their actions. They may also become overconfident, either through inflated personal beliefs in the military prowess of their own country, or through various groupthink biases when decisions are made collectively, as in the case of the American invasion of Cuba at the Bay of Pigs in the 1960s (Whyte, 1998; Janis, 1982). Any of these cognitive biases in leaders could contribute to erroneous decision making.

Secondly, it could be that the costs of war are borne by peasants and the lower classes while the benefits are enjoyed by leaders and decision makers. In many societies throughout history, those making the decisions about conflicts and those fighting in them have not been the same. In other words, those in power lack "skin in the game." It could thus be utility maximizing for decision makers to fight a war, but not for the nation overall.

Finally, it is claimed that even fully rational leaders concerned with maximizing the welfare of their nation can decide to go to war. Other than purely for signaling and reputational purposes, suggested economic arguments of this sort have been of three different types.

Consider two countries, A and B, engaged in a disagreement over a particular land area. The probability of A winning a war between the two would be p_a and the probability of B winning would be p_b. $p_i \in (0, 1)$ for A and B respectively. If the countries had symmetric information, they could agree to a compromise in which the land area would be divided among them so that A received X_a and B received X_b, both a function of p_a and p_b respectively. Fearon argues that such a compromise could be impossible for the following three reasons.

Asymmetric information, leading to optimistic beliefs
One or more sides have an inflated idea of its probability of winning, making

$(p_a + p_b)$ add up to more than 1 and thus making compromise through negotiation impossible. In general, this stems from asymmetric information about military strength. This could be explained either through overestimation of one's own strength (perhaps more likely in terms of the morale and combat readiness of troops than the number of soldiers and weapons available), underestimation of the strength of the opponent, or both.

Shifting relative powers and commitment problems
The risk of a shift in relative power in the future makes immediate conflict an attractive option for one of the actors rather than having to accept a less favorable compromise later on (Powell, 2006). In this case, the two nations A and B share a belief that one of them, say A, will become more powerful relative to the other in the future. The rising power A could in theory promise to divide the resources in a way acceptable to the currently dominant power B, but would have difficulties making this promise credible. Nothing would stop A from going back on its promise and claiming more resources for itself in the future. Therefore, it would be rational for B to opportunistically attack before A had time to increase its military strength.

Issue indivisibility
Not all resources can be divided in a meaningful way. In the Iliad, the Trojan War is said to have started after Helen of Sparta was taken from her husband by the Trojans. As beautiful women are hard to divide fairly among rival suitors, compromise becomes difficult. Perhaps more relevant would be the example of a holy city, over which control is valued almost infinitely highly, and the lack of control over it (meaning another nation, possibly of a different religion, is instead controlling it) is perceived to be undesirable. If the indivisible object is valued highly enough, both nations A and B may have a positive expected utility of fighting despite different relative strengths.

Rather than studying these three explanations broadly, we are interested in the origins of asymmetric information, and, more specifically, how uncertainty of information affects the decision to go to war. Below is a map of the field as it currently stands, and what we aim to investigate further.

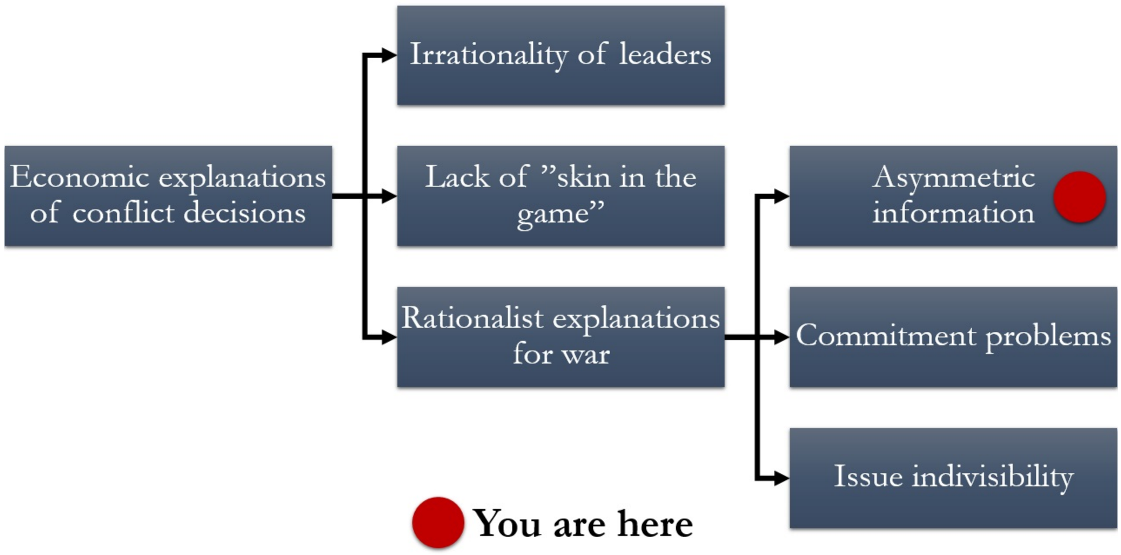

Figure 1. A map over the economic explanations for conflict decisions, and where we aim to conduct further research.

In the real world, countries can often be observed to attempt to create ambiguity around their actual military capabilities, sometimes regarding conventional weaponry (Meirowitz & Sartori, 2008), and sometimes regarding their possession of weapons of mass destruction, as in the case of Saddam Hussein in the 1990s (Baliga & Sjöström, 2008). The general idea is that uncertainty about strength is a powerful deterrent to a potential aggressor, who could not with certainty estimate their chances of winning (or, at the very least, be able to get an accurate forecast of casualties) in advance due to incomplete data, and may therefore prefer peace to war. However, how this effect works, and in which types of conflict it can be useful, has not yet been fully investigated.

Note that this outcome is not necessarily the result of risk aversion. For a potential attacker, the added uncertainty in these models affects the expected payoff of a conflict by adding information that has a risk of being incorrect. Here, it is therefore the possibility that one may act on inaccurate information that deters aggressive behavior, rather than risk aversion.

Several compelling attempts have been made to study the equilibrium investment in weapons in situations of anarchy (Skaperdas, 1992; Grossman & Kim, 1995;

Hirshleifer, 1995), but these are primarily concerned with modeling how much of a given pool of resources should be allocated to defending productive endeavors or acquiring new resources. We are more interested in investigating the decision to go to war, given what information is available to a state, than how to allocate resources in the period in time leading up to the conflict.

Wärneryd (2003) studies the effect of asymmetric information in two-player all-pay auctions (that is, contests where you expend resources to win a prize, and end up having to pay the cost regardless of whether you win or not). Applied to warfare, the all-pay auction can be thought of as expenditures on military strength before the outbreak of a conflict. He finds that having more information about the true value of the prize being competed for can, in some cases, decrease the likelihood of winning, compared to only knowing the underlying distribution of its value. Moreover, in a lottery contest, player expenditures to win the prize are lower under asymmetric information than if either both agents are informed or neither agent is informed.

In a more recent paper, Wärneryd (2013) investigates all-pay auctions with asymmetric information further and finds cases where, in equilibrium, both players win with equal probability despite the asymmetry. Bester & Wärneryd (2006) have also studied the role of social contracts in resolving conflict. Unfortunately, none of these approaches capture the effect of uncertainty of information, making this an area that could benefit from further research.

Hurley & Shogren (1998) examine how asymmetric information affects contest behavior, noting the difficulties of modeling contests with two-sided asymmetric information (the private information here is, for example, the private valuation of each side of some good being exerted effort to obtain). They find that one-sided asymmetric information models may be sufficient to capture contest behavior under uncertainty. However, we are instead concerned with the possibility of receiving a signal about the opponent's strength, rather than having a private valuation of the resources in the contest.

Perhaps closest to what we are interested in studying, Bernard (2008), who investigated the economics of spying, finds that the hiring of a spy increases the risk of war breaking out, as the information made available by a spy makes it easier to detect power asymmetries between countries - in turn leading to more opportunistic aggression by the stronger player. In his model, the employment of a spy is a dominant strategy, although the outcome where none of the rival countries hire a spy is Pareto-efficient, which renders the game similar to a Prisoner's Dilemma situation. Here, more information about the other player strictly increases the risk of war.

Despite the efforts of these excellent researchers, the role of uncertainty of information when states make the decision of whether or not to go to war has not yet been fully investigated.

3 Research questions

With this background in mind, we wish to investigate the role of uncertainty in the conflict decision making of rational states. More specifically, we aim to clarify whether increased uncertainty of information increases or decreases the risk of war between states of this type.

We limit ourselves to conflicts that do not involve nuclear weapons, as the destructiveness of these would require a different approach to modeling. Moreover, we additionally choose to limit ourselves to situations of uncertainty where only strength can be detected with certainty. That is, we are primarily interested in how uncertainty of information in confirmational signals works, such as a satellite taking pictures to ascertain the military capabilities of a rival nation. Further discussion on both of these limitations of scope follows in section 7.

Our aim is to study the distribution of information at the point in time when a war breaks out. We are therefore more interested in how the setup of the conflict decision affects incentives to fight than how the build-up to war occurs, or how the war plays out after it has been started. While the two latter factors are certainly relevant, we would argue that the uncertainty of information at the time of the conflict decision is currently more in need of further research.

With new insights into how states deal with uncertainty, it could be easier to understand which types of international rules and agreements are likely to promote peace between nations. International organizations that aim to minimize outbreaks of conflict could benefit from ways to understand how states deal with uncertainty. Should we intentionally attempt to create ambiguity to deter countries from attacking one another, or are spy satellites and military intelligence agencies instrumental in keeping peace?

4 Setup of the model framework

In order to investigate our research questions, we first need a framework to examine how rational actors deal with uncertainty of information. For this, we have determined that the use of a game-theoretic approach is the best way to understand the core issue, as we can solve it in equilibrium to figure out when wars will break out. More concretely, we propose a simple game of incomplete information, also known as a Bayesian game due to the application of Bayes' rule as part of the solution concept. This can, in turn, help us understand how the incentives for countries are structured, which is useful if we want to reduce the benefits of fighting. We therefore introduce a new model of information acquisition and conflict outbreak between states as follows.

First, we assume that there are two risk-neutral players that can be thought of as countries. These players, while not necessarily geographically adjacent to each other, have the technological capabilities of reaching each other with military force, and are considering whether or not to go to war over some valuable resources.

Since we are only interested in the specific moment at the brink of war, each player possesses an exogenously determined military strength, $R_i > 0$, that can be either $Weak$ or $Strong$, with $R_i \in \{W, S\}$ such that $S > W > 0$. Also, let r_i denote the value R_i did not assume. The military strength encompasses all aspects of warfare that could be useful in a conflict between nations: military personnel, materials such as tanks or ships, and supplies - but also less tangible assets such as troop morale and combat readiness.

Further, each player possesses a stock of valuable resources, $Y_i > 0$. The set of valuable resources is comprised both of tangible, material ones such as access to metals, food sources, and energy, but also of more intangible resources that make a nation more wealthy, such as access to capital, knowledge, productivity, and working institutions. Each player has a utility function $u_i(Y_i) = Y_i$. That is, only consumption of resources is valued, and there is no intrinsic value to having a large military. We assume that players are rational in that they aim to maximize their expected utility given what information they have about themselves and the opponent.

Players can choose to either play peace, and do nothing, or play war, and attack the other player hoping to claim her resources. Peace occurs if both players play peace, otherwise war breaks out. It is therefore sufficient that one player decides to attack for war to break out, but both of them playing war does not affect the conflict in terms of destructiveness compared to only one player being aggressive.

If war breaks out, the winner receives $\theta(Y_a + Y_b)$ with $1 > \theta > 0$, while the loser gets nothing. This implies there is an overall cost of war, as less resources remain after the war than were there before. θ can be thought of as a factor of conflict destructiveness; the lower it is, the less resources remain after the war is fought.

In the real world, wars normally do not result in the complete takeover of resources by the winner from the loser. However, there are clearly large incentives to avoid losing wars. We therefore allow ourselves to make this simplifying assumption to make the model easier to solve and easier to understand.

The success probability of player i is a function of the military strength of player i as well as the military strength of player j, such that $f_i(R_i, R_j)$. Thus, in order to estimate her chances of winning a conflict, a player needs to consider both her own strength and that of the opponent. To quantify our results we assume that $f_i(S, W) = a$, $f_i(W, S) = b$. $1 \geq a > b \geq 0$ and $a + b = 1$. A strong player in a conflict with a weak player will have a strictly larger chance of winning than the weak player. Note that the strong player's chance of winning the war can take the value of 1. Furthermore, $f_i(S, S) = f_i(W, W) = \frac{1}{2}$. That is, if both players are of the same type, they both have an equal chance of winning.

Initially, each player draws her type of military strength from nature with probability γ of being S and $1 - \gamma$ of being W. Both players here have the same probability of being strong, and these probabilities are known by each player.

After drawing her own strength, a player may receive a signal about the strength of the other player. If player j is strong, player i receives a true signal about her strength with probability β. If player j is weak, player i will never receive a signal. The signal can be thought of as a satellite taking pictures of the amount of aircraft lined up on the other player's airfields. This type of technology is limited by the fact that it does not detect weakness directly; the information received on weakness is simply through the absence of evidence, but never through evidence of absence. Even if the opposing player is strong, there is a possibility of missing vital information and failing to notice, for example, the true amount of tanks in use.

Uncertainty, in the context of the model, is at its peak when $\beta = 0, \gamma = \frac{1}{2}$. It therefore decreases as $\gamma \to 0$ or 1, or as β increases.

Finally, in an extension of the model we change the properties of the signal so that it is only received at a cost of $c > 0$. In other words, players who choose to have a chance of receiving the signal will have c less utility. This can be thought of as the opportunity cost of military intelligence; a country buying a satellite will have less resources available to spend on leisure or tanks, for example.

Events

In order to maintain a clear and concise framework, we define the following events for i, j:

$$S_i : \text{player } i \text{ is strong.}$$

$$W_i : \text{player } i \text{ is weak.}$$

$$s_i : \text{player } i \text{ receives the signal.}$$

$$ns_i : \text{player } i \text{ does not receive the signal.}$$

$$X_i : \text{player } i\text{'s best response is war, i.e (1) holds for player } i.$$

We allow ourselves to drop the subscript if the notation is clear. Essentially, this means that it holds for any country i, j and the distinction only really becomes necessary when we deal with conditional probabilities to avoid ambiguity in meaning.

5 Solving the model

On expectations

From our assumptions it is clear that the player strengths, R_i, R_j, are Bernoulli random variables.

It is known that f_i is a random variable since it depends on the random variables R_i, R_j. Moreover, the conditional expectation of f_i, denoted $E(f_i) = \hat{f}_i$ will also depend indirectly on the signal through Bayesian inference. It should be noted that f_i is only known to a player that receives information. $\hat{f}_i$ is only known after the signal (or atleast after player strengths have been assigned, in the case of $\beta = 0$). Therefore, in our decision rule, players will use their conditional expectations to assess their probability to win.

Equilibrium

Since players are risk neutral, their equilibrium decision is simply that which maximizes expected utility. It follows that a player's best response is war if

$$\hat{f}_i \theta(Y_i + Y_j) \geq Y_i$$

or equivalently

$$\hat{f}_i \geq \frac{Y_i}{\theta(Y_i + Y_j)}. \tag{1}$$

For convenience, we assume that in the case of equality of (1), peace is the best response. This eliminates the case where mixed strategies with arbitrary probability assignment are dominant. To be more precise, this case is not of interest to our research question since not even the probabilities are deterministic. Moreover, the measure of the subset of parameter values which yield this case is zero (that is, the fraction of these cases, out of all cases, tends to zero), implying that the exact conditions for this to hold are not likely to occur. We now define the normed lower profitability bound of war

$$\Psi_i = \frac{Y_i}{\theta(Y_i + Y_j)}.$$

As can be seen from equation (1), our assumptions dictate that war dominates peace for a player i, whenever her conditional expectation to win is greater than Ψ_i. Notice also that player i's best response is completely independent of player j's.

Before our first proposition we introduce some notation. We denote any player i's probability to win by $\hat{f}_i$ with her strength as capitalized superscript and whether or not she received the signal as a lower case superscript. In addition, we denote the probability of war when both players are behind the veil of uncertainty ($\beta = 0$), $P(X_i \cup X_j)^*$. Moreover, we adopt the notational convention that the probability of war is zero whenever outside of the defined intervals of the probability function.

Proposition 1. Suppose that there is no signal. Then the ex-ante probability of war is

$$P(X_i \cup X_j)^* =$$

$$\begin{cases} \gamma & (1-\gamma)a + \frac{\gamma}{2} \leq \Psi_i, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \\ \gamma & (1-\gamma)a + \frac{\gamma}{2} > \Psi_i, (1-\gamma)a + \frac{\gamma}{2} \leq \Psi_j \\ 2\gamma - \gamma^2 & (1-\gamma)a + \frac{\gamma}{2} > \Psi_i, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \\ 1 & \text{else, if any weak player is willing to attack.} \end{cases}$$

We skip the proof as it is the trivial case of proposition 5 with $\beta = 0$ which is proven later. Notice that the probability of war is simply the probability of some player with significantly large resource incentives to be of strong type, or in the case of both having sufficient incentives, the union of these events. Now, before we proceed with our second proposition, we present a small lemma related to the Nash equilibria of the game.

Lemma. For each player there exists only one unique pure strategy equilibrium.

Proof. By contradiction, assume that there exists a mixed strategy equilibria. Since the best response is independent of the opponents decisions, for $\lambda \in [0, 1]$ the expected utility of such a strategy is then

$$E(Y_i) = \lambda \hat{f}_i \theta (Y_i + Y_j) + P(X_j')(1 - \lambda) Y_i$$

Note that we do not need to know $P(X_j')$ as it is exogenous to our decision; it suffices to realize that $P(X_j') \geq 0$. Since the maximum of this occurs either at $\lambda = 1$ or $\lambda = 0$ it follows that there exists no mixed strategy equilibria and the pure strategy equilibrium is unique by assumption (see (1)). Existence of at least one equilibrium (which according to above must be pure) is guaranteed by Nash's theorem on the existence of Nash equilibria for finite games (Fudenberg & Tirole, 1991). $\square$

Proposition 2.
a) In the event $S_i \cup ns_i$ player i's best response is war if and only if

$$\hat{f_i}^{Sns} = a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i$$

b) In the event $S_i \cup s_i$ player i's best response is war if and only if

$$\hat{f_i}^{Ss} = \frac{1}{2} > \Psi_i$$

c) In the event $W_i \cup ns_i$ player i's best response is war if and only if

$$\hat{f_i}^{Wns} = b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i$$

d) In the event $W_i \cup s_i$ player i's best response is war if and only if

$$\hat{f_i}^{Ws} = b > \Psi_i.$$

Proof. Using the results on the conditional probability distribution of (R_i, R_j) in the appendix, first assume player i is strong and that she receives no signal. She then assesses her probability to win, $\hat{f_i}^{Sns}$, as

$$\begin{aligned} E(f_i(R_i,R_j)|S_i,ns_i) &= f_i(S,W)P(S,W|S_i,ns_i) + f_i(S,S)P(S,S|S_i,ns_i) \\ &= a\frac{1-\gamma}{1-\gamma\beta} + \frac{1}{2}\left(1 - \frac{1-\gamma}{1-\gamma\beta}\right) = a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \frac{1}{2} \end{aligned} \quad (2)$$

Now assume that she receives a signal. It is then clear that her opponent is strong. Therefore she assesses her probability to win, $\hat{f_i}^{Ss}$ as

$$E(f_i(R_i,R_j)|S_i,s_i) = f_i(S,S) = \frac{1}{2}. \quad (3)$$

Assume player i is weak and that she receives no signal. She then assesses her probability to win, $\hat{f_i}^{Wns}$, as

$$\begin{aligned} E(f_i(R_i,R_j)|W_i,ns_i) &= f_i(W,S)P(W,S|W_i,ns_i) + f_i(W,W)P(W,W|W_i,ns_i) \\ &= b\left(1 - \frac{1-\gamma}{1-\gamma\beta}\right) + \frac{1}{2}\left(\frac{1-\gamma}{1-\gamma\beta}\right) = b + \left(\frac{1}{2} - b\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) \quad (4) \\ &= b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) < \frac{1}{2} \end{aligned}$$

Now assume that she receives a signal. It is then clear that her opponent is strong. Therefore she assesses her probability to win, $\hat{f}_i^{Ws}$, as

$$E(f_i(R_i, R_j)|i = W, s) = f_i(W, S) = b < b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right). \tag{5}$$

Applying (1) to (2)-(5), the result follows. The "only if" part follows from the lemma above. □

It is interesting to observe that both types of player receive the same benefit to their probability to win given that they do not receive a signal, namely $\left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right)$. This phenomenom occurs because both weigh their probability of facing a weak opponent independently of their own wartime strength.

Corollary

$$\hat{f}_i^{Sns} > \hat{f}_i^{Ss} > \hat{f}_i^{Wns} > \hat{f}_i^{Ws} \tag{6}$$

and both $\hat{f}_i^{Sns}$ and $\hat{f}_i^{Wns}$ are monotonically increasing in β.

This corollary follows immediately from equation (2)-(5). The implication of the first part is that through Bayesian updating, not receiving the signal implies that there is a non-zero probability of facing a weak opponent. This establishes a pecking order for when the different player types will choose to play the war strategy. This will be used in later propositions to establish, for instance, that if a strong player with signal plays war, so will a strong player without the signal since her probability to win is greater. C.f. the normed lower probability bound. The second part yields that, given that no signal is received, the higher the probability of receiving a signal, the higher is the probability of facing a weak opponent. It is through these mechanisms that players form their expectations given the signaling event.

Proposition 3. Suppose $Y_i \in (\frac{Y_j}{2}, 2Y_j), i \neq j$. It then follows that a weak player i always plays peace.

Proof. By the previous corollary we know that $E(f_i|W_i) \leq \frac{1}{2}$. That is, the probability of a weak player winning is bounded above by one half. This is the case when a weak player with absolute certainty faces another weak player. The result then follows from the fact that $\Psi_i > \frac{Y_i}{Y_i+Y_j} > \frac{1}{2}$ whenever $Y_i \in (\frac{Y_j}{2}, 2Y_j)$ for $i \neq j$. $\square$

The result is to be expected, and simply states that players with low military capacity do not wish to engage in warfare unless faced with disproportionately large resource incentives to do so. Before we proceed, we briefly present the distribution of $\hat{f}_i$.

Probability distribution of $\hat{f}_i$

We now establish the distribution of $\hat{f}_i$ ex-ante and denote this F_i. Note that this is essentially equivalent to the distribution of any player i's type, since her probability to win the contest is unique to her type.

$$F_i = \begin{cases} \hat{f}_i^{Sns} & \gamma(1-\beta) \\ \hat{f}_i^{Ss} & \gamma\beta \\ \hat{f}_i^{Wns} & (1-\gamma)(1-\beta) \\ \hat{f}_i^{Ws} & (1-\gamma)\beta \end{cases}$$

In addition, we also present the joint probability distribution of player expectations.

Table 1. Joint probability distribution of $\hat{f}_i, \hat{f}_i$

	$\hat{f}_i^{Sns}$	$\hat{f}_i^{Ss}$	$\hat{f}_i^{Wns}$	$\hat{f}_i^{Ws}$
$\hat{f}_i^{Sns}$	$\gamma^2(1-\beta)^2$	$\gamma^2\beta(1-\beta)$	$\gamma(1-\gamma)(1-\beta)^2$	$\gamma(1-\gamma)\beta(1-\beta)$
$\hat{f}_i^{Ss}$	$\gamma^2\beta(1-\beta)$	$\gamma^2\beta^2$	$\gamma(1-\gamma)\beta(1-\beta)$	$\gamma(1-\gamma)\beta^2$
$\hat{f}_i^{Wns}$	$\gamma(1-\gamma)(1-\beta)^2$	$\gamma(1-\gamma)\beta(1-\beta)$	$(1-\gamma)^2(1-\beta)^2$	$(1-\gamma)^2\beta(1-\beta)$
$\hat{f}_i^{Ws}$	$\gamma(1-\gamma)\beta(1-\beta)$	$\gamma(1-\gamma)\beta^2$	$(1-\gamma)^2\beta(1-\beta)$	$(1-\gamma)^2\beta^2$

Since by proposition 2 and its lemma, there exists only one unique best reply for each player, we can now proceed to study the probability of war, given that actors play in equilibrium strategies.

Proposition 4. For all players i such that $\Psi_i \in \left(a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), \frac{\gamma}{2} + (1-\gamma)a\right]$ or $\left(b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), \frac{1-\gamma}{2} + \gamma b\right]$ the ex-ante probability of starting a war increases by $\gamma(1-\beta)$ and $(1-\gamma)(1-\beta)$ respectively. On the other hand, for all players i such that $\Psi_i \in \left(\frac{\gamma}{2} + (1-\gamma)a, \frac{1}{2}\right]$ or $\left(\frac{1-\gamma}{2} + \gamma b, b\right]$ the ex-ante probability of starting a war decreases by $\gamma\beta$ and $(1-\gamma)\beta$ respectively. Moreover, the probability of any player i attacking as a function β and γ is

$$P(X_i) = \begin{cases} \gamma(1-\beta), & a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2} \\ \gamma, & \frac{1}{2} > \Psi_i \geq b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) \\ \gamma + (1-\gamma)(1-\beta), & b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b \\ 1, & b > \Psi_i \end{cases} \tag{7}$$

Before we commence with the proof, we illustrate this graphically using two plots with parameters $a = 0.7$, $\Psi_i = 0.35$ and $\Psi_i = 0.75$ respectively.

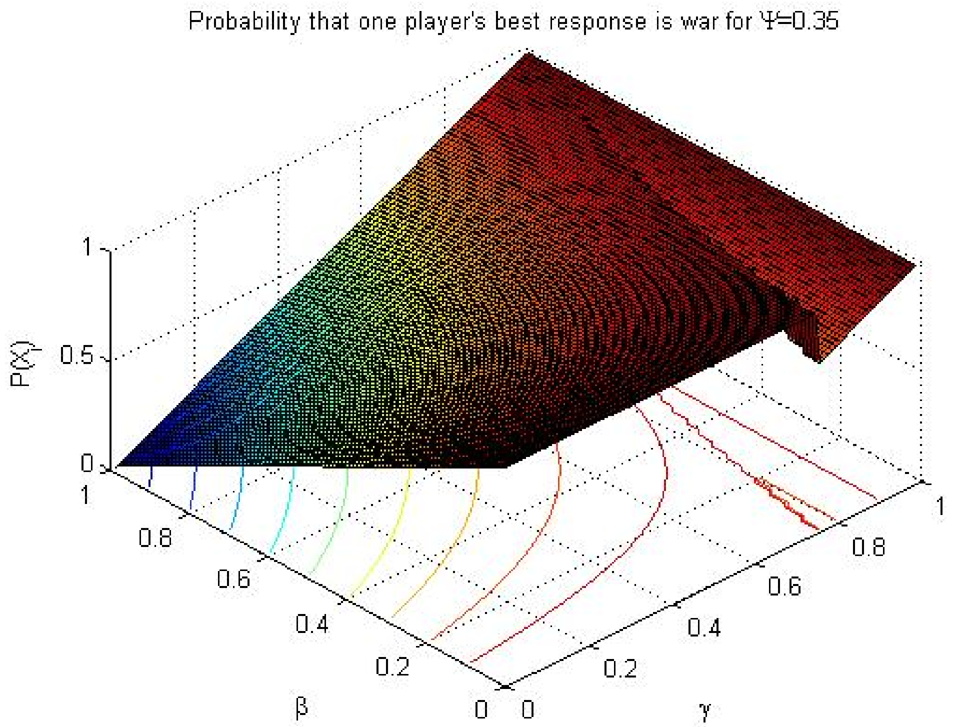

Figure 2. In the case of perfect information, $\beta = 1$, we notice that as the probability of strength increases, so does the probability of any player starting a war. However, under the veil of uncertainty, $\beta < 1, \gamma < 1$, there exists a downward discontinuity as γ increases and an upward discontinuity as β increases.

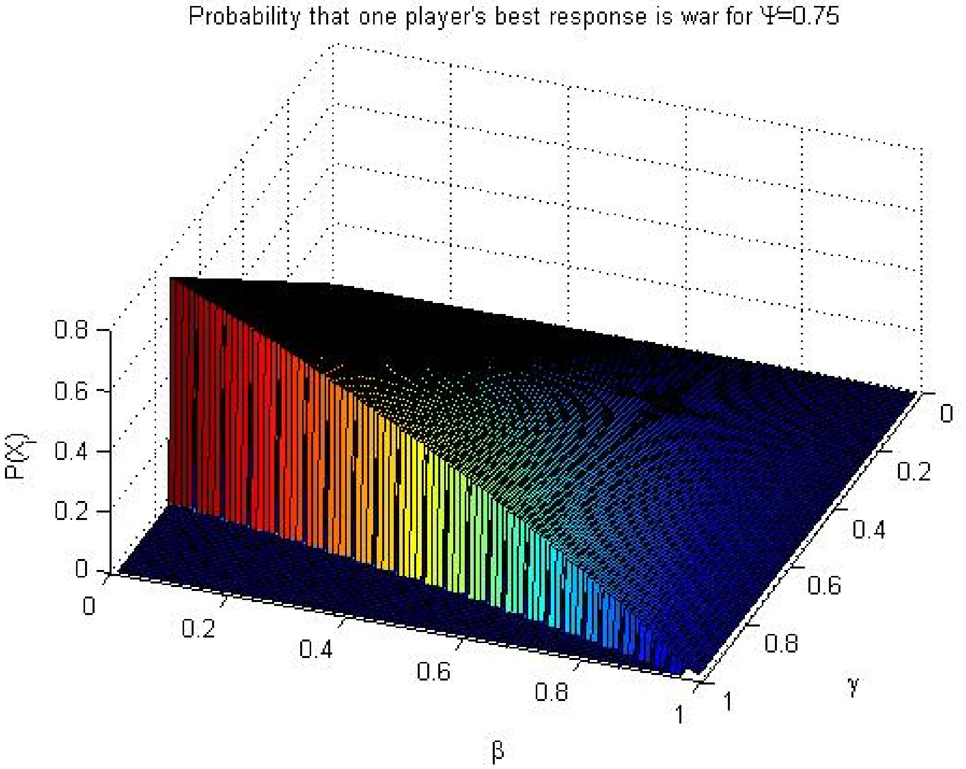

Figure 3. Notice that for high probabilities of any player drawing a strong military (high γ), an increase in the probability of receiving information of strength increases the individual probability to attack by an upwards discontinuity from zero to $\gamma(1-\beta)$, that is, to the probability of her drawing strong, without signal.

Proof. Define $\zeta_i = \{x \in \{\hat{f}_i^{Sns}, \hat{f}_i^{Ss}, \hat{f}_i^{Wns}, \hat{f}_i^{Ws}\} | x > \Psi_i\}$, the set of all possible expectations. Then

$$P(X_i) = P(\hat{f}_i > \Psi_i) = \sum_{x \in \zeta_i} P(x).$$

Or equivalently, and by use of (6), the pecking order, we find

$$P(X_i) = \begin{cases} \gamma(1-\beta), & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{Ss} \\ \gamma\beta + \gamma(1-\beta), & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns} \\ \gamma\beta + \gamma(1-\beta) + (1-\gamma)(1-\beta), & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{Ws} \\ 1, & \hat{f}_i^{Ws} > \Psi_i \end{cases}$$

$$
= \begin{cases} \gamma(1-\beta), & a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2} \\ \gamma, & \frac{1}{2} > \Psi_i \geq b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) \\ \gamma + (1-\gamma)(1-\beta), & b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b \\ 1, & b > \Psi_i \end{cases}
$$

□

As β increases, more players are willing to consider war in the absence of information about strength; i.e. it is interpreted as an uncertain signal about weakness and they become willing to attack if they receive information on weakness. Conversely, notice that as β increases, so does the probability of players knowing that they with certainty are up against a strong opponent. Those who may be willing to attack in the absence of a signal all in all have a probability of receiving information about their opponent's strength and therefore re-assess their probability to win, and the probability of these cases occuring therefore has an opposing effect which decreases the ex-ante probability of war.

Proposition 5a.

$$
P(X_i \cup X_j) =
$$

$$
\begin{cases} 2\gamma(1-\beta) - \gamma^2(1-\beta)^2 & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{Ss} \\ \gamma(2-\beta) - \gamma^2(1-\beta) & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\ \gamma(2-\beta) + (1-\gamma)(1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma) & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{Ws} \\ \gamma(2-\beta) - \gamma^2(1-\beta) & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{Ss} \\ 2\gamma - \gamma^2 & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\ 2\gamma + (1-\gamma)(1-\beta) - \gamma(\gamma + (1-\beta)(1-\gamma)) & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{Ws} \\ \gamma + (1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma)) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{Ss} \\ \gamma(1+\beta) + (1-\beta) - \gamma(\gamma + (1-\beta)(1-\gamma)) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\ 2(\gamma + (1-\beta)(1-\gamma)) - (\gamma + (1-\beta)(1-\gamma))^2 & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{Ws} \\ 1 & \hat{f_i}^{Ws} > \Psi_i \text{ or } \hat{f_i}^{Ws} > \Psi_j \end{cases}
$$

Before we proceed with the proof we need some supplementary results. This is since the occurence of wars is not driven by the sole decision of one player, but by the decisions of both. Hence, when determining the aggregate probability of a war breaking out, we need to adjust for both players wanting to start the war. By the probability axioms, we can write the ex-ante probability of war breaking out, for

$i \neq j$, as

$$P(X_i \cup X_j) = P(X_i) + P(X_j) - P(X_i \cap X_j). \tag{8}$$

since for given parameter values we know the $P(X_i)$ from proposition 4, for any i. We now need to compute $P(X_i \cap X_j)$, the probability of the event that both players decide to play war at the same time. I.e. we need to find the probability of the event

$$X_i \cap X_j : \hat{f}_i > \Psi_i, \hat{f}_i > \Psi_j.$$

Lemma.

$$P(X_i \cap X_j) = \begin{cases} \gamma^2(1-\beta)^2 & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{Ss} \\ \gamma^2(1-\beta) & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\ \gamma(1-\beta)(\gamma+(1-\beta)(1-\gamma)) & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{Ws} \\ \gamma^2(1-\beta) & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{Ss} \\ \gamma^2 & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\ \gamma(\gamma+(1-\beta)(1-\gamma)) & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{Ws} \\ \gamma(1-\beta)(\gamma+(1-\beta)(1-\gamma)) & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{Ss} \\ \gamma(\gamma+(1-\beta)(1-\gamma)) & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\ (\gamma+(1-\beta)(1-\gamma))^2 & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{Ws} \end{cases}$$

Proof. As before, we define a set consisting of all possible expectations. That is, we define the product set $\zeta_i \times \zeta_j$, as

$$\eta = \{(x_i, x_j) | x_i \in \{\hat{f}_i^{Sns}, \hat{f}_i^{Ss}, \hat{f}_i^{Wns}, \hat{f}_i^{Ws}\}, x_j \in \{\hat{f}_i^{Sns}, \hat{f}_i^{Ss}, \hat{f}_i^{Wns}, \hat{f}_i^{Ws}\}, x_i > \Psi_i, x_j > \Psi_j\}$$

Which leads to

$$P(X_i \cap X_j) = P(\hat{f}_i > \Psi_i, \hat{f}_i > \Psi_j) = \sum_{(x_i, x_j) \in \eta} P(\hat{f}_i^k, \hat{f}_i^k) \tag{9}$$

Writing this explicitly leads to the same sort of mapping (of γ, β) as in proposition 3. The equivalence of (9) and proposition 4 is an immediate consequence of the joint probability distribution of $\hat{f}_i, \hat{f}_j$ and another applyication of the pecking order presented in the corollary to our first proposition.

Notice also that we have only presented 9 of 16 possible combinations of player types. In fact, we have omitted the cases where weak players who receive the signal attack. In order to justify this, we use some intuition. We realize that, whenever the equillibrium for a weak player who receives the signal is war, the probability of war is unity. These cases make up for 7 of the 16 player combinations possible and therefore we only require explicitly the components of (10) for the remaining 9 cases to prove proposition 4. □

Proof of proposition 5a. First considering $P(X_i) + P(X_j)$ restricted to the 9 non-trivial combinations we gather

$$P(X_i)+P(X_j) = \begin{cases} 2\gamma(1-\beta) & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{Ss} \\ \gamma(2-\beta) & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\ \gamma(2-\beta)+(1-\gamma)(1-\beta) & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{Ws} \\ \gamma(2-\beta) & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{Ss} \\ 2\gamma & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\ 2\gamma+(1-\gamma)(1-\beta) & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{Ws} \\ \gamma+(1-\beta) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{Ss} \\ \gamma(1+\beta)+(1-\beta) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\ 2(\gamma+(1-\beta)(1-\gamma)) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{Ws} \end{cases}$$

Then, subtracting $P(X_i \cap X_j)$ from the former and applying the fact that the probability of war is unity whenever a weak player without signal attacks, proposition 5 follows. □

Proposition 5b. Suppose only player i has probability β of receiving the signal. The ex-ante probability of war is then

$$P(X_i \cup X_j | B_i, B_j') =$$

$$\begin{cases} \gamma+\gamma(1-\beta)-\gamma^2(1-\beta), & a+\left(a-\frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2}, (1-\gamma)a+\frac{\gamma}{2} > \Psi_j \geq \gamma b+\frac{1-\gamma}{2} \\ 2\gamma-\gamma^2, & \frac{1}{2} > \Psi_i \geq b+\left(a-\frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), (1-\gamma)a+\frac{\gamma}{2} > \Psi_j \geq \gamma b+\frac{1-\gamma}{2} \\ 2\gamma-\gamma^2+(1-\gamma)^2(1-\beta), & b+\left(a-\frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b, (1-\gamma)a+\frac{\gamma}{2} > \Psi_j \geq \gamma b+\frac{1-\gamma}{2} \\ 1, & b > \Psi_i \text{ or } \gamma b+\frac{1-\gamma}{2} > \Psi_j \end{cases}$$

Proof. Since $P(X_i)$ is the same as above we first determine the probability that player j, not receiving intelligence, has war as best response. In a similar manner to previous propositions this is found as

$$P(X_j) = \begin{cases} \gamma & (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ 1 & \gamma b + \frac{1-\gamma}{2} > \Psi_j \end{cases}$$

Next, we determine

$$P(X_i \cap X_j) =$$

$$\begin{cases} \gamma^2(1-\beta), & a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2}, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ \gamma^2, & \frac{1}{2} > \Psi_i \geq b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ \gamma^2 + \gamma(1-\gamma)(1-\beta), & b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ \gamma, & b > \Psi_i, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ \gamma(1-\beta), & a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2}, \gamma b + \frac{1-\gamma}{2} > \Psi_j \\ \gamma, & \frac{1}{2} > \Psi_i \geq b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), \gamma b + \frac{1-\gamma}{2} > \Psi_j \\ \gamma + (1-\gamma)(1-\beta), & b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b, \gamma b + \frac{1-\gamma}{2} > \Psi_j \\ 1, & b > \Psi_i, \gamma b + \frac{1-\gamma}{2} > \Psi_j \end{cases}$$

Then, applying (8), it follows that

$$P(X_i \cup X_j) =$$

$$\begin{cases} \gamma + \gamma(1-\beta) - \gamma^2(1-\beta), & a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2}, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ 2\gamma - \gamma^2, & \frac{1}{2} > \Psi_i \geq b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ 2\gamma - \gamma^2 + (1-\gamma)^2(1-\beta), & b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ 1, & b > \Psi_i, (1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \gamma b + \frac{1-\gamma}{2} \\ 1, & a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq \frac{1}{2}, \gamma b + \frac{1-\gamma}{2} > \Psi_j \\ 1, & \frac{1}{2} > \Psi_i \geq b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right), \gamma b + \frac{1-\gamma}{2} > \Psi_j \\ 1, & b + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq b, \gamma b + \frac{1-\gamma}{2} > \Psi_j \\ 1, & b > \Psi_i, \gamma b + \frac{1-\gamma}{2} > \Psi_j \end{cases}$$

Which is equivalent to the result stated in the proposition. □

Proposition 6. $P(X_i \cap X_j) > 0$ implies overconfidence.

Proof. The event $X_i \cap X_j$ is defined by the inequalities (1) for i, j

$$\hat{f}_i > \frac{Y_i}{\theta(Y_i + Y_j)} \text{ and } \hat{f}_i > \frac{Y_j}{\theta(Y_j + Y_i)}$$

both holding. This implies that

$$\hat{f}_i + \hat{f}_i > \frac{Y_i + Y_j}{\theta(Y_i + Y_j)} = \frac{1}{\theta} > 1. \tag{10}$$

This means that, in order for both players to want to play war, at least one of them must be overconfident. □

We are now ready to present our main results.

Proposition 7. Given assumptions:
a) The conditions of proposition 3 are binding. That is $Y_i \in (\frac{Y_j}{2}, 2Y_j), i \neq j, \forall i$.
b) That $\beta < \frac{1}{2}$.
c) That $\gamma < \frac{1-2\beta}{(1-\beta)^2}$.
d) That $\Psi_i \in (\hat{f}_i^{S}, \hat{f}_i^{Ss}]$ holds for at most one player i, j.

i) If a), b), c), and d) hold $P(X_i \cup X_j) - P(X_i \cup X_j)^* > 0$. Explicitly, this means that, for a signal of this type, the probability of war is strictly heightened by any non-zero probability of receiving a signal under this restricted class of games.

ii) If only a), b) and c), but not d) hold the opposite is true. I.e. $P(X_i \cup X_j) - P(X_i \cup X_j)^* < 0$.

Proof. Directly by subtracting $P(X_i \cup X_j)^*$ (proposition 1) from $P(X_i \cup X_j)$ (proposition 5) and applying the inequalities a)-d). As the details of the algebra are rather tedious and uninformative, this is best left to the appendix. □

We focus on the implications of assumptions b) through d). First, it should again be noted that not receiving the signal is also a transmission of information. Players who do not acquire it are more likely to believe their opponent is weak than others. Hence, the effect of low values of β essentially means that the probability of receiving information about your opponent's weakness is high. However, it also entails that this information is unreliable. Second, the restriction on γ means that in a given scenario where war may occur there is a sufficient probability of facing a weak opponent, such that the signal renders sufficient confidence of either player's own militiary superiority. Lastly, the assumption d) removes the extreme case were both players have sufficiently high expectations to win without the signal to engage in warfare. If d) does not hold, the signal operates in the opposite direction and has a probabilistic effect of lowering the expectations of players.

6 Extending the model: costly information

Actors in the real world do not face static military and spying capacity but rather face a sequential set of decisions on intelligence and warfare. It is therefore of interest to investigate Nash equilibria where rational actors first face a decision to invest in intelligence or not, and then decide whether or not to wage war.

We propose the added complication that instead of receiving information with probability β players face the decision of paying $c > 0$ in order to have probability β to receive the signal or to do nothing and remain entirely behind the veil of uncertainty. Moreover, we assume that the investment decision is not observable to the opponent. This has the great advantage that we then need not investigate the signaling effect of investing, which is outside of the scope of this thesis.

Notice that this is now a two-stage game. Players' choices in the first round will very much depend on their preferred outcomes in the second. We begin illustrating this below.

Proposition 8. Suppose $Y_i \in (\frac{Y_j}{2}, 2Y_j), i \neq j, \forall i \in \{1, 2\}$ in the context of the extended model. Then, in addition to always playing peace, weak players never purchase information as part of any equilibrium strategy.

Proof. By proposition 2, with given initial strength, weak players will choose not to attack regardless of information. Hence they will strictly prefer not to invest for any $c > 0$, as the alternative is worthless to them. □

For simplicity we make the assumption that the conditions of proposition 2 are binding. I.e. suppose $Y_i \in (\frac{Y_j}{2}, 2Y_j), i \neq j$ throughout the extension. Furthermore, we assume that any player i does not receive any information about the type of player j after learning the strategy of player j in the first round. With this in mind, it is now of interest to investigate under what conditions a strong player will consider the trade-off between information and cost.

Proposition 9. Suppose player i is strong and $(1-\gamma)a + \frac{\gamma}{2} > \Psi_i \geq \frac{1}{2}, \forall i \in \{1, 2\}$ such that both players will attack if there is no signal available at all. Then

i) For $\beta \in (1 - \delta, 1]$ there exists a pure strategy Nash equilibrium where neither player acquires information in the first round.

ii) In addition, if we assume that $c \in \left(0, \gamma\left(Y_i - \frac{\theta(Y_i+Y_j)}{2}\right)\right)$ then for $\beta \in (1-\delta, 1]$ there also exists a pure strategy Nash equilibrium where either player purchases information in the first round if she draws type strong and remains behind the fog of war if weak.

Proof. Since the decision of acquiring information is not observable to the opponent, the value of information must be positive definite. Hence, we must analyze the benefits of acquiring information and compare it with c. First, suppose that player i purchases information and that $\beta = 1$, i.e. perfect information may be purchased. Then $i's$ expected utility if she plays her best response given j's best response and type is

$$\max(a\theta(Y_i + Y_j), Y_i) - c = a\theta(Y_i + Y_j) - c \text{ if } j \text{ is weak,}$$

$$\max\left(\frac{\theta(Y_i + Y_j)}{2}, Y_i)\right) - c = Y_i - c \text{ if } j \text{ is strong and acquires the signal,}$$

or

$$\frac{\theta(Y_i + Y_j)}{2} - c \text{ if } j \text{ is strong and does not acquire the signal.}$$

Now suppose i does not acquire the signal. Then i's expected utility given j's best response and type is

$$\left((1-\gamma)a + \frac{\gamma}{2}\right)\theta(Y_i + Y_j) \text{ regardless of player } j\text{'s properties.}$$

Suppose instead that neither player acquires the signal. As a weak player always plays no information and peace, any strong player i compares

$$\gamma\frac{\theta(Y_i + Y_j)}{2} + (1-\gamma)a\theta(Y_i + Y_j) - c$$

$$= \theta(Y_i + Y_j)\left(\frac{\gamma}{2} + a(1-\gamma)\right) - c$$

with her ex-ante probability of winning less the expected value of committing to the strategy with lower payoff

$$\left((1-\gamma)a + \frac{\gamma}{2}\right)\theta(Y_i + Y_j) - 0$$

which is strictly greater since the benefit of acquiring information against a veiled opponent whose best response is war if she is strong is 0. Moreover, by proposition 8, weak players in equilibrium never acquire information. Hence, there exists a pure strategy Nash equilibrium of the Bayesian game where neither player invests in information.

For the second pure strategy equilibrium, suppose that player i acquires information and that j does so if and only if she is strong. Player i then similarily compares

$$\begin{aligned}&\gamma\frac{\theta(Y_i+Y_j)}{2}+(1-\gamma)a\theta(Y_i+Y_j)-c\\&=\theta(Y_i+Y_j)\left(\frac{\gamma}{2}+a(1-\gamma)\right)-c\end{aligned}$$

with her ex-ante probability of winning less the expected value of "falsely" committing to the strategy with lower payoff

$$\left((1-\gamma)a+\frac{\gamma}{2}\right)\theta(Y_i+Y_j)-\gamma\left(Y_i-\frac{\theta(Y_i+Y_j)}{2}\right).$$

Thus, player i acquiring information, and j doing so conditioned on her type, is a pure strategy Bayesian Nash equilibrium if and only if it holds $\forall i\neq j$ that

$$c\in\left(0,\gamma\left(Y_i-\frac{\theta(Y_i+Y_j)}{2}\right)\right). \tag{11}$$

In addition, if we appeal to the continuity of the Von-Neumann Morgenstern utility function, it follows that this also holds for all sufficiently small deviations from $\beta=1$. $\square$

Part i) means that, under this restricted class of games, in equilibrium players anticipating their opponent not to invest in intelligence may result in a self-fulfilling prophecy where neither player invests in information and player i chooses to start the war.

Proposition 10. Suppose player i is strong and $a+\left(a-\frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right)>\Psi_i\geq(1-\gamma)a+\frac{\gamma}{2},\forall i\in\{1,2\}$ such that neither player attacks unless she receives some signal of weakness. Further, suppose $\beta\in(1-\delta,1]$ for some sufficiently small $\delta>0$. Then

i) For $c > (1-\gamma)(2a\theta(Y_i + Y_j) + Y_i)$ there exists a unique pure strategy Nash equilibrium where neither player acquires information in the first round.

ii) For $c < (1-\gamma)(2a\theta(Y_i + Y_j) + Y_i)$ there exists a unique pure strategy Nash equilibrium where strong players obtain information in the first round and weak players do not.

Proof. As before, first suppose neither player acquires the signal. As neither player attacks if behind the veil of uncertainty, any strong player i then compares the expected utility from acquiring the signal,

$$\gamma Y_i + (1-\gamma)a\theta(Y_i + Y_j) - c,$$

with her utility if peace occurs, less the expected loss from committing to the wrong strategy if the opponent is weak,

$$Y_i - (1-\gamma)(a\theta(Y_i + Y_j) - Y_i).$$

Hence, neither player acquiring the signal is an equilibrium if and only if it holds $\forall i \neq j$ that

$$\gamma Y_i + (1-\gamma)a\theta(Y_i + Y_j) - c < (2-\gamma)Y_i - (1-\gamma)(a\theta(Y_i + Y_j)$$

which is equivalent to

$$c > (1-\gamma)(2a\theta(Y_i + Y_j) + 2Y_i). \tag{12}$$

Second, consider the case where both players obtain information if strong, but otherwise do not. The comparison for a strong player i is the same, since he again knows that a rational opponent will not attack. Therefore, we find that purchasing information is an equilibrium if and only if

$$c < (1-\gamma)(2a\theta(Y_i + Y_j) + 2Y_i). \tag{13}$$

The two equilibria are mutually exclusive in terms of the cost. As before (c.f. proof of proposition 9), these also hold for small deviations from $\beta = 1$. □

Proposition 11. Suppose player i is strong and $a + \left(a - \frac{1}{2}\right)\left(\frac{1-\gamma}{1-\gamma\beta}\right) > \Psi_i \geq (1-\gamma)a + \frac{\gamma}{2}$ and that $(1-\gamma)a + \frac{\gamma}{2} > \Psi_j \geq \frac{1}{2}$. In addition, let $\beta \in (1-\delta, 1]$ for sufficiently small $\delta > 0$. Then

i) For $c > (1-\gamma)(a\theta(Y_i + Y_j) - Y_i)$ there exists a unique pure strategy Nash equilibrium where neither player acquires information in the first round.

ii) For $c < \max\left(\gamma Y_j, (1-2\gamma)Y_i + \gamma\left(\frac{\theta(Y_i+Y_j)}{2}\right) + (1-\gamma)a\theta(Y_i + Y_j)\right)$ there exists a unique pure strategy Nash equilibrium where strong players obtain information in the first round and weak players do not.

Proof. Since the reasoning is so similar to the previous two propositions, we present the details without much motivation. Suppose neither player acquires information. Player i then compares

$$(1-\gamma)Y_i + \gamma\frac{\theta(Y_i + Y_j)}{2} - c$$

with

$$(1-\gamma)Y_i + \gamma\frac{\theta(Y_i + Y_j)}{2} - (1-\gamma)(a\theta(Y_i + Y_j) - Y_i).$$

Using proposition 9, it follows that there exists a pure strategy equilibria where neither player acquires information if and only if

$$c > (1-\gamma)(a\theta(Y_i + Y_j) - Y_i). \tag{14}$$

Once more, assume instead that both players acquire information if they are strong and refrain from doing so otherwise. Player i then contrasts

$$(1-\gamma)Y_i + \gamma\frac{\theta(Y_i + Y_j)}{2} - c$$

with

$$Y_i - (1-\gamma)(a\theta(Y_i + Y_j) - Y_i).$$

Player j, if strong, compares

$$a(1-\gamma)\theta(Y_i + Y_j) - c$$

with

$$\left((1-\gamma)a + \frac{\gamma}{2}\right)\theta(Y_i + Y_j) - \gamma\left(Y_j - \frac{\theta(Y_i + Y_j)}{2}\right).$$

Therefore, there exists a pure strategy Bayesian Nash equilibrium in which both players choose to obtain information in the first round if and only if

$$c < \max\left(\gamma Y_j, (1-2\gamma)Y_i + \gamma\left(\frac{\theta(Y_i+Y_j)}{2}\right) + (1-\gamma)a\theta(Y_i+Y_j)\right). \tag{15}$$

□

Conjecture. The results from proposition 9 through 11 continue to hold for all $\beta > 0$. However, the cut-off cost of the signal must be adjusted downward.

Motivation. Previously we argued by continuity of the Von Neumann-Morgerstern utility function that the results also hold for small deviations from $\beta = 1$. As mathematical objects, there is not much that separates any game with $0 < \beta < 1$ from another. In equilibrium, they should therefore behave similarily. However, the probability of correcting a "wrong" move decreases; actors are less certain about the type of their opponent. Since the expected payoff from such a correction is the maximum cost and we are less likely to make the correction, the maximum cost must be lower.

The significance of the addendum becomes apparent if brought together with proposition 7. We note that there exists in equilibrium situations where the signal is never acquired (depending on whether equations (11)-(15) hold), which mitigates the direction of any previous results if such an equilibrium comes into play. Moreover, we have not considered mixed strategy equilibria in this extension, which may well exist under conditions where at least two pure strategy equilibria arise. In this case we refer to proposition 5b where the ex-ante probability of war given that only one player receives the signal is presented.

7 Conclusions and further implications

As we have shown in propositions 1 through 11, increasing uncertainty of information, that is, a decrease in β, may have different effects depending on which types of players are present.

Unreliable satellites start wars

In close cases where, without the chance of a signal, peace barely prevails, the addition of a signal increases the probability of war by making some players overconfident after their Bayesian updating. After not having received a signal, players will see their belief in the probability of facing a strong opponent decrease. If they then, despite this, are indeed up against a strong opponent, they will unknowingly be overconfident in the sense that the sum of both players' perceived probability to win is greater than unity. This can lead to attacks that are not probabilistically profitable, and would have been avoided had there been perfect information.

When tensions are high between countries, but war has not yet broken out, the possibility of receiving a signal increases the risk of conflict. The reliability of the signal thus becomes vital for peace to last.

Uncertainty as a deterrent to opportunistic behavior

However, in cases where, ceteris paribus, war would with certainty break out, adding the possibility of receiving the signal decreases the risk of war due to the possibility of acting on incorrect information. We discussed in section 2 the properties of uncertainty for rational players; it is not risk aversion of the players that dampens the incentives to fight, but the added possibility of going to war on information that risks being incorrect. This could have disastrous consequences for the aggressor if a player expected to be weak turns out to be strong.

We see that some ambiguity regarding military capability between states that are already very keen on going to war with each other may be good for overall world welfare. Uncertainty here deters aggression from, for example, stronger players who look to opportunistically acquire resources.

In extension

When we include a first round in the game, where players initially have to decide whether or not to acquire information, our general conclusions do not change.

However, for sufficiently large costs of information, there exists equilibria in which players never obtain the probability of receiving the signal. This is not surprising, as any economic decision has an opportunity cost which need be bounded for the decision to be profitable. Nevertheless, these findings are relevant seeing as most contemporary conflicts arise in areas with few resources and limited military intelligence.

Where our model breaks down

An implication of the parameters in our model is that players with a relatively low Y_i (that is, poor players in terms of valuable resources assigned) are more likely to engage in aggressive behavior, as the potential upside of waging war is greater to them. A possible critique against our model emerges here: in the real world, we can observe that a large stock of valuable resources often, but not always, correlates with high military strength; rich countries often have correspondingly large armies to defend their interests.

A conceivable retort we would like to address is that some resource-rich nations in, for example, Africa, traditionally have been exploited through colonialism, due to low military strength. However, in our model we define valuable resources not only as natural resources, but also access to capital, knowledge, productivity, and working institutions. In this view, these states (while certainly rich in natural resources) can be seen as relatively poor.

How can the correlation between resources and military strength be understood within the parameters of our model? One interpretation is that the world we see today is the result of many runs over a long period of time of this model. This means that multiple conflicts between many pairs of states have taken place, which has shifted the balance of resources. Nations with large armies have, over time, been able to attack their neighbors, claiming more resources at the expense of others until their relative share of resources in a local cluster corresponded to their relative strength.

This is more in line with what we see empirically, but of course there are exceptions. We would however argue that some of these exceptions, such as the resource-rich but militarily weak Sweden, for example, have been protected from opportunistic behavior by other nations through implied alliances, with the silent understanding that help would come in the event of an attack (which, in the case of Sweden, would be from NATO). Be that as it may, the intricacies of military alliances in deterring aggressive behavior are beyond the scope of this essay. Here, further research is needed in the future.

The distribution of resources and weapons brings us to another, more relevant, aspect for consideration. As our model stands, the military strength of a player is implicitly assumed to be exogenous, in that her type is drawn from nature separate from her distribution of natural resources. We would expect this variable to show some measure of endogeneity in the real world; rich nations with much resources will be able to afford building larger and better armies. Perhaps it is no coincidence that the world's currently greatest economic power, the United States, is also its currently greatest military power. Numerous examples of corresponding relationships can be found in history.

In order to construct a more accurate model, the variable of military strength would need to be modified to allow for some endogeneity. This, too, we leave for future research.

Another point of divergence between our model and the reality we can empirically observe is that military strength in the real world tends to correlate with the military strength of potential aggressors. That is, strong neighbors may scare other states into strengthening their own forces to protect against aggression, thereby creating a local arms race. Perhaps it could be possible to create a future model where the army size is a function of nearby states' army sizes to incorporate this effect. Otherwise, unprepared neighbors of players arming themselves would quickly be exploited for their resources.

Moreover, as mentioned in the specification of our scope and our research questions, we have only considered conventional weapons in our framework. A conflict between states involving nuclear weapons could play out very differently from what we model, and there may be no clear winner at all; if most resources get destroyed, θ will be close to 0 and it may not be profitable to attack regardless of resource distribution and military strengths. But we can still get some insights from our model: in nuclear warfare, $\theta = 0$ represents the case of mutually assured destruction, which would explain how a rational player would be deterred from attacking. A scenario where one player has nuclear weapons and the other does not becomes very different to model, however. This would, yet again, be another possible area for future research.

Note that we, in our model, have only considered a limited set of the intelligence technology available. We modeled a signal that is able to give evidence of strength, but not evidence of weakness. In the real world, most military decision-makers have tools available which, with some certainty, may be able to determine whether or not a potential opponent is weak. While it could be argued that spies are able to find evidence of weakness in an opponent, we believe our results can be applicable to satellite type situations where soley strength can be detected.

The difficulty of modeling cultural differences

In our model, we make an implicit assumption about the lack of differences between states. As all players are risk-neutral and rational in the framework, and considering the utility preferences are identical, no room is left for diverging cultures and preferences.

Is this really realistic? Not quite. We can observe that some cultures and nations are more aggressive than others. They can be thought of not as risk-seeking, but as having an additional utility variable, w_i, from going to war. This could, for example, be a strengthening of national pride experienced during wartime. With this additional benefit of fighting, the new best response of a player in equilibrium is war if

$$\hat{f}_i\theta(Y_i + Y_j) + w_i > Y_i$$

or, equivalently,

$$\hat{f}_i > \frac{Y_i - w_i}{\theta(Y_i + Y_j)}$$

holds.

As the incentive constraint becomes less demanding, this implies that more players will be willing to go to war, and the frequency of wars should therefore increase. In our framework, wartime nationalism becomes deterimental to world peace.

On rationality, and how Prospect Theory would change our results

We have assumed in our model that players are perfectly rational in that they are risk-neutral economic agents when making decisions. Relaxing the main assumption of rational behavior is not our intention in this section, but it could be valuable to note the implications of bounded rationality on opportunistic wartime behavior.

In Prospect Theory, an agent sets a frame of reference and is then risk-averse in the domain of gains and risk-seeking in the domain of losses (Kahneman & Tversky, 1979). This implies that the utility function is concave for gains and convex for losses.

If this were to be applied in our model, a reasonable assumption to make would be that the framing reference point would be the status quo; that is, a player i would use what resources she is already in possession of, Y_i, as the frame of reference. Any losses beyond this would "hurt" more than the gains above the reference point would feel advantageous. We can think of this as states being less willing to gamble with their current territory than with land acquired in a campaign to conquer another nation. Since our model involves losing all resources upon losing a war, an application of Prospect Theory to our model would dampen incentives for players to seek out conflict, and therefore make it less probable for wars to break out.

Probability, uncertainty, and risk, or: how I learned to stop throwing dice and love the bomb

We present for the interested reader a note on terminology. We have consistently used the word "uncertainty" in our framework, where it means the uncertainty for a given player of the information reaching her being correct. We have not dealt with Knightian uncertainty. That is, the uncertainty facing a player in our model is best thought of as "risk" in the sense that it is possible to calculate it. To paraphrase Donald Rumsfeld, the uncertainty in our model is a "known unknown" rather than an "unknown unknown."

This distinction may at first seem minor, but it is an important one for our conclusions. In the event of war, history has shown that the unknown unknowns can have large and unforseen consequences for the states involved. Towards the end of the second World War, the Japanese could not in advance estimate the chances of nuclear weapons hitting their cities because the technology did not exist prior to the war. Loss of life from American troops landing on the shores of Japan was a known unknown in that casualties could be estimated. This was a risk. Loss of life to nuclear weapons was something else entirely; the uncertainty of the existence of a new weapon made it impossible to model.

Going into a conflict, a state would do well to consider the possibility of being hit by something impossible to estimate using today's models. Introducing Knightian uncertainty therefore becomes a noteworthy argument for being cautious about going to war, and for focusing on improving robustness instead of calculating risk. Unfortunately, it is in its nature impossible to model, and we therefore have no choice but to exclude it from our main model.

Why we may yet need James Bond

What about the other uses of military intelligence? Looking back at Powell's argument for why wars break out between rational states (that is, the inability of a rising power to credibly commit in the event of shifting relative powers), there is another role for intelligence agencies to play. In this framework, they can attempt to detect power surges early on from another state, to enable the currently dominant power to attack before the challenger has amassed enough of an army to pose a serious threat. The argument of states having a propensity to exploit temporary advantages where military power fluctuates has also been examined by Morrow (1985).

For a real-world example of this, the United States has shown great interest in the nuclear capabilities of Iran, and there has been discussion among policymakers whether or not to attack uranium enrichment centers before a nuclear weapon can be developed. While data on this is understandably kept a secret, it would not be an unreasonable assumption to make that the United States has attempted extensive intelligence gathering on the nuclear program of Iran in order to determine if the relative strengths of the states are going to shift.

To provide some historical context, Great Britain, led by Neville Chamberlain, pursued a foreign policy strategy of "appeasement" against Adolf Hitler's Nazi Germany in the 1930s. This involved concessions to Germany's expansive strategy, letting them invade Czechoslovakia in 1938 after which a peace treaty was signed between Great Britain and Germany. Chamberlain then famously returned to England to proclaim he had secured "peace for our time." If Britain had had access to more information about the intentions of Hitler and the ever-increasing production of military equipment, perhaps they would have preferred to enter into a conflict sooner, before Germany could shift the relative power between the states further.

The central point to be understood here is that there are other uses of keeping a competent intelligence agency than to detect whether an enemy is strong or weak. Perhaps an immediate war can stop several other ones from breaking out in the future. Would World War II have been less severe if the major European powers had been less keen on appeasing Germany and declared war sooner? How should NATO react to the recent expansionary strategy of Russia, after seeing parts of Georgia and the Ukraine invaded? These questions are difficult to answer, but monitoring and acting on future aspirations of neighbors is clearly rational for a state. This falls outside of the scope of our model as we only consider one period in time in the first version (and a short-term two-period model in the extension), but it is a relevant argument in favor of intelligence gathering.

Concluding remarks

With previous research in the field in mind, this thesis offers some new insights on how the informational asymmetry between two players can arise, and what consequences it can have for contest behavior under measurable uncertainty. It also raises new questions for further research, such as the role of uncertainty of military alliances for deterrence and the role of uncertain behavior in situations involving nuclear weapons.

While our model rests on some assumptions of varying strength, such as the rationality of leaders and the possibility to completely take over the resources of another country, we have found two distinct effects. In conflict decisions, the impact of uncertainty in the acquisition of information depends on the ex-ante resource incentives players face. In situations where peace just barely prevails, uncertainty increases the risk of war. On the other hand, where war with certainty occurs without information about player types, more uncertainty lowers the probability of conflict outbreak.

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9 Appendix

Conditional probability distribution of R_i

Since we can only receive a signal if the opponent is strong and since the events s and ns are mutually exclusive and collectively exhaustive we find that

$$P(s) = \gamma\beta$$

and

$$P(ns) = 1 - \gamma\beta.$$

We can now assess the conditional probability distribution of the ordered pair (R_i, R_j) by Bayesian inference. First,

$$P(W_j|ns_i) = \frac{P(W)P(ns_i|W_j)}{P(ns)} = \frac{(1-\gamma)(1)}{\gamma(1-\beta)+(1-\gamma)(1)} = \frac{1-\gamma}{1-\gamma\beta}$$

$$P(S_j|ns_i) = \frac{P(S)P(ns_i|S_j)}{P(ns)} = 1 - P(W|ns) = 1 - \frac{1-\gamma}{1-\gamma\beta}$$

$$P(W_j|s_i) = 0$$

$$P(S_j|s_i) = 1$$

Assume player i gets no signal, then (R_i, R_j) has distribution

$$P(S_i \cap W_j|ns_i) = P(S)P(W_j|ns_i) = \gamma\frac{1-\gamma}{1-\gamma\beta}$$

$$P(S_i \cap S_j|ns_i) = P(S)P(S_j|ns_i) = \gamma\left(1 - \frac{1-\gamma}{1-\gamma\beta}\right)$$

$$P(W_i \cap W_j|ns_i) = P(W)P(W_j|ns_i) = (1-\gamma)\frac{1-\gamma}{1-\gamma\beta}$$

$$P(W_i \cap S_j|ns_i) = P(W)P(S_j|ns_i) = (1-\gamma)\left(1 - \frac{1-\gamma}{1-\gamma\beta}\right)$$

Assume player i gets a signal, then (R_i, R_j) has distribution

$$P(S_i \cap W_j|s_i) = 0$$

$$P(S_i \cap S_j|s_i) = P(S) = \gamma$$

$$P(W_i \cap W_j|s_i) = 0$$

$$P(W_i \cap S_j|s_i) = P(W) = 1 - \gamma$$

Algebraic results of proposition 7

Using the results acheived above from the Bayesian framework we can now establish f_i in the different cases. These are essentially the weightings each player uses to assess the situation exactly before the decision is made.

$$P(X_i \cup X_j) - P(X_i \cup X_j)^* =$$

$$\begin{cases}
2\gamma(1-\beta) - \gamma^2(1-\beta)^2 & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
2\gamma(1-\beta) - \gamma^2(1-\beta)^2 - \gamma & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
2\gamma(1-\beta) - \gamma^2(1-\beta)^2 - \gamma & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
2\gamma(2-\beta) - \gamma^2(1-\beta)^2 - 2\gamma + \gamma^2 & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
(\gamma - \gamma^2)(1-\beta) + \gamma & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
(\gamma - \gamma^2)(1-\beta) & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
\gamma(2-\beta) + (1-\gamma)(1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma) & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{W} \\
\gamma(2-\beta) + (1-\gamma)(1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma) - 1 & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{W} > \Psi_j \geq \hat{f}_i^{Ws} \\
\gamma(2-\beta) + (1-\gamma)(1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma) - \gamma & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{W} \\
\gamma(2-\beta) + (1-\gamma)(1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma) - 1 & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{W} > \Psi_j \geq \hat{f}_i^{Ws} \\
\gamma(2-\beta) - \gamma^2(1-\beta) & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
\gamma(2-\beta) - \gamma^2(1-\beta) - \gamma & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
0 & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
2\gamma + (1-\gamma)(1-\beta) - \gamma(\gamma + (1-\beta)(1-\gamma)) & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{W} \\
2\gamma + (1-\gamma)(1-\beta) - \gamma(\gamma + (1-\beta)(1-\gamma)) - 1 & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{W} > \Psi_j \geq \hat{f}_i^{Ws} \\
\gamma + (1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma)) & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{W}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
\gamma + (1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma)) - 1 & \hat{f}_i^{W} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
\gamma + (1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma)) - \gamma & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{W}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
\gamma + (1-\beta) - \gamma(1-\beta)(\gamma + (1-\beta)(1-\gamma)) - 1 & \hat{f}_i^{W} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
\gamma(1+\beta) + (1-\beta) - \gamma(\gamma + (1-\beta)(1-\gamma)) & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{W}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
\gamma(1+\beta) + (1-\beta) - \gamma(\gamma + (1-\beta)(1-\gamma)) - 1 & \hat{f}_i^{W} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
2(\gamma + (1-\beta)(1-\gamma)) - (\gamma + (1-\beta)(1-\gamma))^2 & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{W}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{W} \\
2(\gamma + (1-\beta)(1-\gamma)) - (\gamma + (1-\beta)(1-\gamma))^2 - 1 & \hat{f}_i^{Wns} > \Psi_i \geq \hat{f}_i^{W}, \hat{f}_i^{W} > \Psi_j \geq \hat{f}_i^{Ws} \\
2(\gamma + (1-\beta)(1-\gamma)) - (\gamma + (1-\beta)(1-\gamma))^2 - 1 & \hat{f}_i^{W} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{Wns} > \Psi_j \geq \hat{f}_i^{W} \\
2(\gamma + (1-\beta)(1-\gamma)) - (\gamma + (1-\beta)(1-\gamma))^2 - 1 & \hat{f}_i^{W} > \Psi_i \geq \hat{f}_i^{Ws}, \hat{f}_i^{W} > \Psi_j \geq \hat{f}_i^{Ws} \\
0 & \hat{f}_i^{Ws} > \Psi_i \text{ or } \hat{f}_i^{Ws} > \Psi_j
\end{cases}$$

$$=$$

$$
\begin{cases}
2\gamma(1-\beta)-\gamma^2(1-\beta)^2 & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{S}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{S} \\
2\gamma(\frac{1}{2}-\beta)-\gamma^2(1-\beta)^2 & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{S}, \hat{f_i}^{S} > \Psi_j \geq \hat{f_i}^{Ss} \\
2\gamma(\frac{1}{2}-\beta)-\gamma^2(1-\beta)^2 & \hat{f_i}^{S} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{S} \\
\gamma^2\beta^2-2\beta(\gamma-\gamma^2) & \hat{f_i}^{S} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{S} > \Psi_j \geq \hat{f_i}^{Ss} \\
(\gamma-\gamma^2)(1-\beta)+\gamma & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{S}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\
(\gamma-\gamma^2)(1-\beta) & \hat{f_i}^{S} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\
\beta^2(\gamma^2-\gamma)-\beta(\gamma^2-2\gamma+1)+1 & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{S}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{W} \\
\beta^2(\gamma^2-\gamma)-\beta(\gamma^2-2\gamma+1) & \hat{f_i}^{Sns} > \Psi_i \geq \hat{f_i}^{S}, \hat{f_i}^{W} > \Psi_j \geq \hat{f_i}^{Ws} \\
\beta^2(\gamma^2-\gamma)-\beta(\gamma^2-2\gamma+1)+(1-\gamma) & \hat{f_i}^{S} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{W} \\
\beta^2(\gamma^2-\gamma)-\beta(\gamma^2-2\gamma+1) & \hat{f_i}^{S} > \Psi_i \geq \hat{f_i}^{Ss}, \hat{f_i}^{W} > \Psi_j \geq \hat{f_i}^{Ws} \\
(\gamma-\gamma^2)(1-\beta)+\gamma & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{S} \\
(\gamma-\gamma^2)(1-\beta) & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{S} > \Psi_j \geq \hat{f_i}^{Ss} \\
0 & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\
2\gamma+(1-\gamma)(1-\beta)-\gamma(\gamma+(1-\beta)(1-\gamma)) & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{W} \\
2\gamma+(1-\gamma)(1-\beta)-\gamma(\gamma+(1-\beta)(1-\gamma))-1 & \hat{f_i}^{Ss} > \Psi_i \geq \hat{f_i}^{Wns}, \hat{f_i}^{W} > \Psi_j \geq \hat{f_i}^{Ws} \\
\gamma+(1-\beta)-\gamma(1-\beta)(\gamma+(1-\beta)(1-\gamma)) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{W}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{S} \\
\gamma+(1-\beta)-\gamma(1-\beta)(\gamma+(1-\beta)(1-\gamma))-1 & \hat{f_i}^{W} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Sns} > \Psi_j \geq \hat{f_i}^{S} \\
(1-\beta)-\gamma(1-\beta)(\gamma+(1-\beta)(1-\gamma)) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{W}, \hat{f_i}^{S} > \Psi_j \geq \hat{f_i}^{Ss} \\
\gamma+(1-\beta)-\gamma(1-\beta)(\gamma+(1-\beta)(1-\gamma))-1 & \hat{f_i}^{W} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{S} > \Psi_j \geq \hat{f_i}^{Ss} \\
\gamma(1+\beta)+(1-\beta)-\gamma(\gamma+(1-\beta)(1-\gamma)) & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{W}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\
\gamma(1+\beta)+(1-\beta)-\gamma(\gamma+(1-\beta)(1-\gamma))-1 & \hat{f_i}^{W} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Ss} > \Psi_j \geq \hat{f_i}^{Wns} \\
2(\gamma+(1-\beta)(1-\gamma))-(\gamma+(1-\beta)(1-\gamma))^2 & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{W}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{W} \\
2(\gamma+(1-\beta)(1-\gamma))-(\gamma+(1-\beta)(1-\gamma))^2-1 & \hat{f_i}^{Wns} > \Psi_i \geq \hat{f_i}^{W}, \hat{f_i}^{W} > \Psi_j \geq \hat{f_i}^{Ws} \\
2(\gamma+(1-\beta)(1-\gamma))-(\gamma+(1-\beta)(1-\gamma))^2-1 & \hat{f_i}^{W} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{Wns} > \Psi_j \geq \hat{f_i}^{W} \\
2(\gamma+(1-\beta)(1-\gamma))-(\gamma+(1-\beta)(1-\gamma))^2-1 & \hat{f_i}^{W} > \Psi_i \geq \hat{f_i}^{Ws}, \hat{f_i}^{W} > \Psi_j \geq \hat{f_i}^{Ws} \\
0 & \hat{f_i}^{Ws} > \Psi_i \text{ or } \hat{f_i}^{Ws} > \Psi_j
\end{cases}
$$

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\begin{cases}
2\gamma(1-\beta)-\gamma^2(1-\beta)^2 & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
2\gamma(\frac{1}{2}-\beta)-\gamma^2(1-\beta)^2 & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
2\gamma(\frac{1}{2}-\beta)-\gamma^2(1-\beta)^2 & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
\gamma\beta(2\gamma-\gamma\beta-2) & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
(\gamma-\gamma^2)(1-\beta)+\gamma & \hat{f}_i^{Sns} > \Psi_i \geq \hat{f}_i^{S}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
(\gamma-\gamma^2)(1-\beta) & \hat{f}_i^{S} > \Psi_i \geq \hat{f}_i^{Ss}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns} \\
(\gamma-\gamma^2)(1-\beta)+\gamma & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Sns} > \Psi_j \geq \hat{f}_i^{S} \\
(\gamma-\gamma^2)(1-\beta) & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{S} > \Psi_j \geq \hat{f}_i^{Ss} \\
0 & \hat{f}_i^{Ss} > \Psi_i \geq \hat{f}_i^{Wns}, \hat{f}_i^{Ss} > \Psi_j \geq \hat{f}_i^{Wns}
\end{cases}
$$