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A MATTER OF CONFIDENCE
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SELF-FULFILLING SOVEREIGN DEBT CRISES AND BAILOUTS IN THE EUROZONE

Johannes Kasinger (40439) and Alexander Schlomberg (40494)

Abstract:

This thesis analyzes the influence of self-fulfilling mechanisms and potential bailouts on the current sovereign debt crisis in the Eurozone. For this purpose we develop a dynamic stochastic general equilibrium model that extends previous self-fulfilling debt crisis models by (i) incorporating two different players, a single country and the aggregated Eurozone countries, and (ii) including the possibility of a bailout after a default. We show that the debt levels of most European countries are inside a critical interval, called the crisis zone, which makes them vulnerable to a self-fulfilling debt crisis. Furthermore, we demonstrate that bailouts may mitigate self-fulfilling mechanisms. However, bailouts also provide incentives, particularly for smaller countries, to raise debt levels up to a certain threshold that eventually leads to a default. Our findings also rationalize why the EU has bailed out Greece and why smaller countries, such as Portugal, Greece and Ireland, continue to raise their debt levels.

Keywords: Self-fulfilling debt crisis, Eurozone, Sovereign debt, Bailout
JEL: F33, F34, F41, G28

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Variable	Explanation
s	State of the economy
B	Country's debt level
b	Debt purchased by international investors
z	Initial default decision (z = 1 if a country defaulted in the past and z=0 if not)
$\boldsymbol{\zeta}$	Sunspot variable ∈ [0,1] (Determines arbitrarily whether or not international lenders lose trust in a country's ability to repay its debts)
$\boldsymbol{\beta}$	Constant discount factor ∈ [0,1], β = 1/(1+r)
$\bar{\boldsymbol{y}}$	"Normal" government output without default penalties
y	Government output
g	Consumption of government
c	Consumption of consumers
x	Consumption of international investors
Z	Default penalty for country P if it defaults
M	Default penalty for country EU if country P defaults
$\boldsymbol{\theta}$	Constant tax rate
V	Value function for governments
W	Value function for international investors
u	Consumers' utility function
q	Bond prices ∈ [0,1], price that international investors are willing to pay for a bond with face value B
w	International investors' endowment (either used for consumption or investment)
$\boldsymbol{\pi}$	∈ [0,1] Probability that a self-fulfilling crisis occurs next period
$\overline{\boldsymbol{B}}$	Upper debt level boundary
$\overline{\boldsymbol{b}}$	Lower debt level boundary
A	Constant that rules out Ponzi schemes
$\boldsymbol{\lambda}$	Lagrange multiplier
$\tilde{\boldsymbol{t}}$	Period in which default decision is made
$\boldsymbol{\kappa}$	Bailout costs
h	Bailout decision by *EU* government
z^*	Realized default outcome
$\widetilde{\boldsymbol{b}}$	Bailout cutoff debt level
B^*	Country's debt level after bailout
q^*	Bond prices after bailout

1. Introduction

"There has been a clear crisis of confidence that has seriously aggravated the situation. Measures need to be taken to ensure that this vicious circle is broken."

(Christine Lagarde, IMF Managing Director, 2011)

In line with Christine Lagarde's recommendation, emergency measures, such as the European Stability Mechanism (ESM)[1], were implemented in an attempt to mitigate negative effects of the recent European sovereign debt crisis. However, these emergency measures, in particular the bailout of Greece, received harsh critique from politicians and economists all around the world (e.g. The Economist, 2012). However, this criticism does often not consider important mechanisms of the recent Eurozone sovereign debt crisis. In particular it does not account for self-fulfilling mechanisms, which seem to aggravate the situation in the Eurozone crisis (De Grauwe, 2012a, 2012b). In general, self-fulfilling mechanisms have the potential to trigger a debt crisis, which is not caused by unhealthy economic fundamentals but rather a consequence of pessimistic expectations of investors. If investors loose trust in a government's ability to repay its debts, rollover costs may rise to a level that makes repaying impossible. Hence negative expectations about a default can become self-fulfilling and force a solvent but illiquid government into default (Chamon, 2007). Since most European countries are highly indebted and rollover their debt on a regular basis, the fundamental conditions for a self-fulfilling debt crises are satisfied in the Eurozone. So far, little is understood about the underlying problems of self-fulfilling features in the Eurozone crisis. Therefore our thesis aims on answering the following research question:

"Are countries in the Eurozone vulnerable to self-fulfilling crises, and if yes, can bailing out troubled countries solve this problem?"

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[1] The ESM is a permanent crisis resolution mechanism for the countries of the euro area. It provides loans and other forms of financial assistance to troubled member states with a maximum lending capacity of €500 billion. (ESM Europa, 2014)

Preceding literature on self-fulfilling debt crises developed useful frameworks of self-fulfilling debt crises (see section 2.3. for details). However, these frameworks are hardly applicable to the Eurozone, as they incorporate only one single country. The strong economic interdependencies between European countries are likely to influence the rationale of self-fulfilling debt crises. Therefore, it is important to consider these interdependencies, when evaluating the self-fulfilling mechanism of the current Eurozone debt crisis. In order to provide a better fit to the current situation in the Eurozone and fill the aforementioned research gap, we extend previous dynamic stochastic general equilibrium models of self-fulfilling debt crises (Cole & Kehoe, 1996, 2000; Conesa & Kehoe, 2012) by (i) incorporating a second country as a separate player, that represents the aggregated Eurozone countries and ii) including the possibility of a cross-country bailout.

We contribute to current literature on self-fulfilling sovereign debt crises by applying and simulating our model. Our results provide a clear answer to the first part of our research question (whether countries in the Eurozone are vulnerable to self-fulfilling crises). In particular we show that most European countries are in the crisis zone[2] and hence vulnerable to self-fulfilling debt crisis. The second part of our research question (whether bailing out troubled countries can solve the problem) cannot be answered that clear. On the one hand, we provide theoretical evidence that bailouts have the potential to prevent debt crises, which are triggered by self-fulfilling mechanisms. On the other hand, however, this does not imply that bailouts lead to fewer defaults in the future. In effect, we show that potential bailouts incentivize countries to raise their debt levels up to a bailout cut-off-level and may choose to default afterwards. This bailout cut-off-level represents the boundary for which it is beneficial for the EU to bailout a defaulting country. This bailout cut-off-level depends on the absolute size of sovereign debt and thus indirectly on the economic size of a single country.

In sum, our results can help to explain current developments in the Eurozone crisis. In particular, our results rationalize why European leaders decided to bailout Greece (small economic size), and why small countries, such as Portugal, Ireland and Greece,

[2] A crisis zone defines the range of the country's debt level in which a self-fulfilling crisis can occur. See Section 4.2 for a precise definition

continue to raise their debts, as seen in Figure 1. Many previous models (e.g. Cole & Kehoe, 2000) are not able to explain this stylized fact[3].

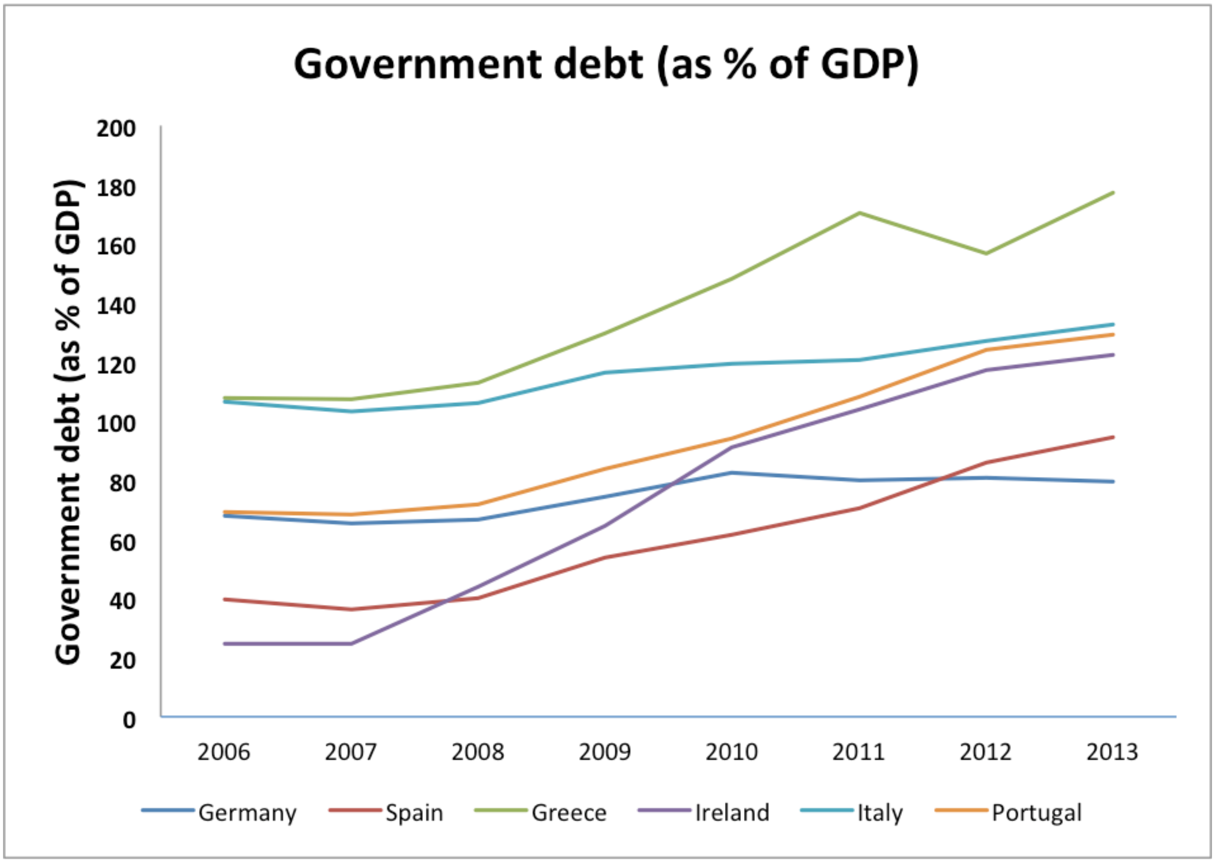

Figure 1: Government debt level development in the Eurozone (Source: ECB)

As the literature of self-fulfilling is rather new, we provide a detailed literature review in the next section to fit our work into current academic literature and to better understand the underlying concepts and structure of our analysis. Building on this, we start our theoretical analysis of our model, by defining the general model features in section 3. Our further theoretical analysis is twofold: In section 4, we derive the equilibrium conditions and crucial concepts in a scenario without bailouts; in section 5, we extend our analysis with a bailout possibility to derive the effect of a bailout on the equilibrium. To bridge our theoretical results to real numerical outcomes we calibrate and simulate the model afterwards (section 6). Before we round our thesis with a brief

[3] Cole & Kehoe (2000) concluded that rational governments have an incentive to decrease their debts to the lower debt level boundary, which makes self-fulfilling crises impossible. In contrast, our model calibration showed that in many cases European countries have an incentive to increase their debts to the bailout cut-off level

conclusion (section 9.), we present our general results and discuss possible implications, limitations and extension of our analysis in sections 7 and 8.

2. Literature Review

This literature review progressively guides through the evolution of current research on self-fulfilling sovereign debt crises by presenting relevant papers about i) sovereign debt crises without self-fulfilling features (e.g. Eaton & Gersovitz, 1981); ii) the emergence of self-fulfilling features (e.g. Obstfeld, 1986); iii) sovereign debt crises with self-fulfilling features (e.g. Cole, Kehoe, 2000) and iv) related literature on government bailouts (e.g. Uhlig, 2013) and financial contagion (e.g. Allen & Gale, 2000).

2.1 Sovereign Debt Crises Without Self-fulfilling Features

Traditional literature on sovereign debt crises without self-fulfilling features emerged in the 1970/80s. This period saw a startling increase in the volume of international loans to less-developed countries. A crucial characteristic of this borrowing is the absence of explicit penalties for non-repayment. The availability of legal remedies against a sovereign borrower is limited by the "doctrine of sovereign immunity", which is based on the intuitive idea that there are no courts where a foreign creditor can seek redress for the non-repayment of a sovereign's debt obligations (Wright, 2011). The fact that sovereign debt contracts cannot be enforced raised an obvious question that laid the foundation for sovereign debt literature: why do sovereigns repay their debts at all? Reviewing academic literature on sovereign debts crises reveals a common answer: sovereigns repay their debts because defaults are costly (Dooley, 2000). Despite the common consensus about the existence of default costs there is much less agreement on what actually causes these costs. Traditionally, literature on sovereign debt has focused on two mechanisms: i) reputational costs, which may result in a permanent exclusion from financial markets and ii) direct sanctions such as legal attachments of property or international trade sanctions (Borenstein & Panizza, 2009).

The first mechanism (reputational costs) was formally established by Eaton & Gersovitz (1981). In their seminal paper they showed that reputational costs and the associated exclusion from financial markets create equilibria, in which repaying sovereign debts is less costly than defaulting. Other researchers such as Eaton et al (1986), Grossmann & van Huyck (1988), Manuelli (1986) and Cole & English (1987) confirmed reputational costs as the main reason for sovereign debt repayment.

However, an influential paper by Bulow & Rogoff (1989) casted doubt on Eaton & Gersovitz's (1981) findings and proposed the second mechanism, direct sanctions, as a valid explanation for sovereign debt repayments. In particular Bulow & Rogoff (1989) established conditions under which the threat of exclusion from financial markets may not be credible. They showed that in certain settings a country could default, keep the debt payments and invest them with foreign financial institutions to generate a higher welfare than they could obtain from future borrowing. Thus, they concluded that an exclusion from financial markets may not be sufficient to enforce repayments of debts. Consequently, Bulow & Rogoff (1989) emphasized the second mechanism (direct sanctions) as a valid explanation for why countries repay their debts. Sachs (1989) and Krugman (1988) support Bulow & Rogoff's (1989) sanction view by establishing their results on similar assumptions.

In light of these two competing explanations, Tomz (2007) provides an empirical analysis of the importance of both mechanisms and finds evidence for reputational costs being more important than direct sanctions. Next to these two mechanisms, Borensztein & Panizza, (2009) identified additional costs associated with a default, namely political costs and costs to the domestic economy through the financial system.

In sum, this section revealed two important findings that are essential for our further analysis: i) defaults are costly (regardless of where these costs are coming from) and ii) a sovereign's decision whether to default or to repay depends on the associated costs of each alternative.

2.2 The Emergence of Self-fulfilling Features

Self-fulfilling features play a crucial role in determining the above-mentioned costs of repaying debts[4] and for our whole thesis. Therefore, it is important to provide a review

[4] Chamon (2007) showed that self-fulfilling features can increase the costs for repaying debts significantly, thus making a default more likely.

of economic literature on the emergence of self-fulfilling features to better understand the mechanisms behind our study.

Early adoptions of self-fulfilling mechanisms can be found in Diamond and Dybvig's (1983) bank run theory. Their model showed that a bank's mix of illiquid assets and liquid liabilities may trigger a bank run, which implies that too many depositors attempt to withdraw money at once. This situation may give rise to self-fulfilling panics. This means that the more people withdraw their deposits, the more likely it is that a bank defaults, thus triggering further cash withdrawals. This self-fulfilling prophecy can destabilize a (healthy) bank to a point where it runs out of cash and is forced to default (Diamond & Dybvig, 1983). Sachs (1984) extended the bank run scenario to international borrowing. His model incorporated self-fulfilling elements in an international setting and showed that investor panic could lead to a dramatic breakdown of sovereign debt supply, in which fundamentally sound economies can be forced into default due to a shortage of new credits. A more detailed analysis of self-fulfilling features began with the emergence of second-generation models of currency crises. A currency crisis is defined as an episode in which the exchange rate depreciates substantially during a short period of time (Burnside et al 2007). Models of currency crises are often categorized as first- second- or third generation models. First-generation models explain the occurrence of currency crises mainly by fundamental inconsistencies, such as inflationary budget deficits or fixed exchange rates (see e.g. Krugman, 1979). Introduced by Obstfeld (1986), second-generation models include self-fulfilling mechanisms as additional triggers for currency crises. Obstfeld (1986) showed that a currency crisis may be a purely self-fulfilling event rather than the inevitable result of unsustainable macroeconomic policies. Furthermore, he emphasized that different expectations about a government's willingness to maintain an exchange rate peg lead to multiple market equilibria; a good equilibrium, where the government survives and a bad equilibrium, where the government defaults. Therefore self-fulfilling elements create situations where a government default becomes a possibility but not an economic necessity. In the aftermath of the Mexican crisis (1994-95), third-generation models of currency crises emerged. These models focused on the interplay between currency crises and other crises, e.g. debt crises. Therefore research on debt crises came to the forefront of academic discussions (see e.g. Graciela et al, 1999; Krugman, 1999).

As a result, self-fulfilling features, which were already modelled in the context of currency crises, whee also integrated into models of sovereign debt crises. Summing up, self-fulfilling features in the context of sovereign debt crises received more academic attention due to the successions of the Mexican crisis and antecedent models of currency crises.

2.3 Sovereign Debt Crises with Self-fulfilling Features

The models and papers presented in this section are most closely related to our work and serves as the foundation of our analysis. A detailed review of this literature is thus inevitable to understand the underlying assumptions and concepts of our work.

In their seminal paper Cole and Kehoe (1996) modelled the Mexican crisis as a self-fulfilling debt crisis and their model became the benchmark for subsequent models of self-fulfilling sovereign debt crises. The basic setting of their model consists of three types of agents: Consumers, Government and International lenders. Consumers maximize their utility from private and public consumption, subject to their budget constraint. The government is benevolent and chooses the level of public consumption, new debts and whether or not to default in order to maximize the utility of the consumers. International lenders are endowed with a consumption good that can be either consumed or invested in government bonds. International lenders maximize their utility based on their private consumption today and in the future. Furthermore there is a sunspot variable, which is realized every period and randomly determines whether or not a crisis takes place. The sunspot variable is random and illustrates the uncertain properties in the beliefs of international lenders. In every period, the government generates income by issuing new bonds and collecting taxations. This income is used for governmental consumption and for repaying old debt obligations. Hence, the government takes new debts in order to repay old debts (debt-rollover). Depending on the sunspot variable, debt-rollovers can become very expensive (due to high interest rates for taking new debts). In this case the government may choose to default on its debts because defaulting is less expensive than repaying old debts.[5] This

[5] This situation is similar to the one described in the previous section about sovereign debt crises without self-fulfilling features. The crucial distinction however is, that this situation was caused by self-fulfilling expectations and could have been avoided otherwise

model showed that if a country's debts lie within a certain interval (the crisis zone), a crisis can occur arbitrarily, depending on the realization of the sunspot variable. This means that self-fulfilling features may trigger the occurrence of a crisis. Further, Cole and Kehoe (1996) demonstrated that governmental debts with short maturity structures incorporate a larger crisis zone compared to longer maturity debts. Applying their model to a different scenario, Cole and Kehoe (2000), defined optimal policy responses if a country is inside the crisis zone. In particular, they demonstrated that if a country is inside the crisis zone, the respective government finds it optimal to reduce debts in order to move out of the crisis zone. This in turn leads to an economic boom and a reduction in the interest rate on new governmental debts. Furthermore they showed that during the Mexican crisis a decisive action of a third party (a bailout issued from the Clinton administration) was sufficient to move Mexico out of the crisis zone and to prevent a possible default.

In sum, Cole and Kehoe's (1996, 2000) model of self-fulfilling debt crises was successful in explaining the circumstances during and after the Mexican debt crisis. After all, follow-up literature on self-fulfilling debt crises was mainly based on their framework. For instance, Chamon (2007) showed that self-fulfilling features could be mitigated by a combination of state-contingent securities and a mechanism that allows investors to promise to lend only if enough other investors do so as well. Rocha et al. (2010) discussed optimal debt policies and argue that the lower the perceived probability of a government default is the longer it will take for a country to leave the crisis zone. In addition, Araujo et al (2011) considered the effects of monetary unions on self-fulfilling debt crises. They showed that monetary unions may increase or decrease self-fulfilling mechanisms depending on the risk of political inflation and other factors.

The emergence of the Eurozone crisis, however, projected limitations of Cole and Kehoe's (1996, 2000) framework. Their model had difficulties in explaining the topology of the Eurozone crisis. In contrast to the Mexican crisis, financial rescue packages (ESM) from a third party (EU / IMF) did not have the same healing properties in Europe. In fact, bond prices and debt levels continued to rise after the announcement and first implementation of the ESM rescue packages. Chamley and Pinto (2011) used Cole and Kehoe's (2000) initial framework to show why rescue packages were not successful in Greece. In particular they illustrate with a numerical example that a bailout is hard to implement if countries have fundamental fiscal insolvency problems, as it was the case in Greece. This indicated that there were more fundamental solvency problems than those presented in previous models of Cole, Kehoe (1996, 2000). Conesa and Kehoe (2012) accommodated this issue with their "gambling for redemption" model. "Gambling for redemption" means that a country has a rational incentive to increase their debt level in times of a recession (even in the crisis zone) in the hope of future economic recovery. The debt level increase can be so high that the country exposes itself to the possibility of a self-fulfilling crisis. This in turn implies that if the recovery does not take place in the near future, it may force the government to default, even if decreasing its debt level earlier could have prevented a default. Conesa and Kehoe (2012) demonstrate that increasing debts and "gambling for redemption" is the optimal rational choice given a country is in a recession and the likelihood of a recovery is large enough. Therefore, Conesa and Kehoe (2012) remedy some shortcomings of the Cole, Kehoe (1996, 2000) model, which concluded that it is always optimal to reduce governmental debts if a country is in the crisis zone.

It thus contradicts with irrational behaviour explanations, such as Reinhart and Rogoff (2009), who concluded that governments increase debts due to irrational behaviour and fooling themselves into the irrational hope that "this time is different" and expectations about bailouts. Hence their model provides sharp predictions of when a third party bailout helps to avoid a crisis, as it was the case in Mexico 1994-95, and when such a strategy would not lead to the desired effects. Arellano et al. (2012) applied the theory of "gambling for redemption" to the Eurozone and concluded that policy interventions taken by the EU and IMF (lowering cost of borrowing and reducing default penalties) have encouraged Eurozone government to gamble for redemption. In line with this reasoning, Jorra (2013) confirms previous findings by pointing out that the ESM rescue package and monetary approach was suboptimal for various reasons.

2.4 Bailouts and Financial Contagion

There is further related literature, which has certain overlaps with research on self-fulfilling sovereign debt crises and the context used in our model. In particular, two relevant concepts (government bailouts and financial contagion) are important for our analysis and will therefore be explained in this section.

First of all, a government bailout refers to a situation, where a country is rescued from a potential or actual insolvency by another economic entity such as another country, the EU or the IMF (Dictionary of Economics, 2014). Jeanne & Zettelmeyer's (2005) model illustrates that bailouts may bring the world closer to a Pareto optimum due to the provision of IMF funds to troubled countries. More negative aspects of bailouts are emphasised by e.g. Ennis & Malik (2005) and Lee & Shin (2008). They conclude that the possibility of a bailout leads to moral hazard problems and excessive risk taking behaviour. Another relevant concept for our further analysis is financial contagion. Financial contagion refers to a scenario, where small shocks, which initially only affected one country, spread to other countries, whose economies where previously healthy. Allen & Gale (2000), Hernández & Valdés (2001) and Van Rijckeghem & Weder (2001), analysed the effects of financial contagion and showed that in certain settings it is possible that a crisis in one country spreads to other countries due to uncertainty and the interdependence in financial and economic markets.

In conclusion, this literature review presented relevant papers that laid the foundation for our analysis. It was shown that current research on self-fulfilling sovereign debt crises builds up on concepts from sovereign debt crises without self-fulfilling features, bank run theory, second-generation models of currency crises, financial contagion and government bailouts. These concepts lay the foundation for our own analysis and the corresponding model development. The next section will present our model in more detail.

3. A Model of Self-fulfilling Debt Crisis

Cole & Kehoe's (1996, 2000) and Conesa & Kehoe's (2012) benchmark models analyze self-fulfilling sovereign debt crises in the context of one country. However, they remain silent about the interactions between countries. Given the strong financial and economical connections within the Eurozone, these models seem therefore hardly applicable to the current debt crisis in Europe. Our model accounts for this with a two-country setting that includes interactions between two countries; country *P (*a single country inside the EU) and "country" *EU* (representing the remaining EU countries). Additionally, our model extends the benchmark models by allowing for a potential bailout provided by "country" EU in case country P defaults. Our framework ultimately leads to endogenous bailout conditions, which enable us to rationalize the recent bailout of Greece as well as current debt level developments in the Eurozone.

In this section we present the general features of our model including the involved players, the underlying mechanisms and the timing of events. After that we apply the model to two scenarios: scenario A, in which no bailouts are possible[6], and scenario B, in which bailouts are possible[7]. For each scenario we will derive the equilibrium[8], which implies that all involved players maximize their utility and the market clears.

Our model incorporates a representative international lender, domestic governments and domestic households for two different countries P and EU $(i = P, EU)$. The only differences between these countries are thereby exogenously given input factors such as the "normal" output, initial debt levels and tax rates. We could also define the model in more general terms, but decided to clearly distinguish between these two countries from the beginning in order to keep our analysis easier to follow.

The state of the economy in period t $(t = 0,1,2, \dots, \infty)$: $s_t(B_{i,t}, z_{i,t-1}, \zeta_{i,t})$ depends on the debt level of both countries at the beginning of period t, $(B_{i,t})$, whether or not the countries have defaulted in the past, $z_{i,t-1}$ and two sunspot variables $\zeta_{i,t} \in [0,1]$ that are drawn randomly from an uniform distribution at the beginning of each period. The sunspot variables arbitrarily determine whether or not the investors lose confidence in the governments' ability to repay their debts in the next period. $z_{i,t}$ is the default decision in period t made by the government on debt at the beginning of period t $(B_{i,t})$ and not on debt newly issued in period t $(B_{i,t+1})$. $z_{i,t-1} = 1$, if country i has not defaulted in the past and $z_{i,t-1} = 0$ otherwise. We assume that if country P has defaulted in the past it cannot return to the financial markets. Thus, z_P stays equal to zero for the future[9].

[6] Scenario A is similar to the model of Cole-Kehoe (1996, 2000) but differs on two dimensions i) we include cross-country interaction, meaning that the output of a country is dependent on the other country's default decision and ii) we exclude private capital since it does not change the qualitative results of our analysis.

[7] Scenario B is similar to scenario A but includes an additional step in the sequential game, namely the decision whether to bail out the other country or not.

[8] See Section 4 for a precise definition of equilibrium

[9] This assumption is derived from Cole, Kehoe (1996, 2000); Empirical evidence does not support this view, but it is very helpful for simplifying the model (Gelos et al ,2010; Borenstein & Panizza, 2008)

To keep things simple, we further assume that the rest of the Eurozone countries (*EU)* is not subject to a self-fulfilling debt crisis. This implies that the "country" *EU* has not and will not default in the future. Hence, $z_{EU,t} = 1 \; \forall \; t \in \mathbb{N}$.[10] This assumption makes $\zeta_{EU,t}$ and $z_{EU,t-1}$ meaningless and we can simplify the state of the economy to: $s_t(B_{i,t}, z_{P,t-1}, \zeta_{P,t})$.

All players discount with the constant discount factor β. The governments are benevolent and maximize the expected, discounted utility of the respective domestic households for the infinite future by choosing government consumption, the amount of newly issued debt and whether or not to default on the current debt.

A crucial feature of our model is that a default in country P not only affects the output of country P but also the output of other Eurozone states. Previous literature[11] provides four applicable explanations for such an effect: (i) a default disrupts a country's ability to engage in international trade. This implies that the value of exports to the defaulting country ceases and hence the output of the other country decreases as well. This is especially applicable to the Eurozone due to strong economic connections between European countries; (ii) a default damages a government's reputation and therefore harms its ability to operate efficiently, which in turn results in an output drop. It is likely that a default in one country does not only harm the reputation of the respective government but also the reputation of other countries with similar characteristics and a close relationship to the defaulting country. These similar characteristics and close relationships can be found all across the Eurozone (e.g. economic union, same currency, connection through European institution)[12]. This feature of the Eurozone makes it likely that a default in one European country also affects the reputation of other European countries; (iii) a default can trigger a financial contagion process, which lead to an output loss as involved banks tighten their investments[13] (Allen & Gale, 2000; Hernández & Valdés, 2001; Van Rijckeghem & Weder, 2001); (iv) a default entails other

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[10] We can motivate this assumption, for instance, by the fact that all aggregated Eurozone countries control the European central bank that can print more money in case of a crisis and thus repay all international lenders. However, it is important to note that we rule out the possibility for the aggregated Eurozone countries to raise its debt level to infinity. The exact threshold is not important here, but will be discussed in more detail later.

[11] Bulow & Rogoff (1989) or Eaton & Gersovitz's (1981)

[12] see Hooge & Marks, (2008), Diez, (2006)

[13] A financial contagion process can be shortly illustrated as follows: First, a default in one country leads to a loss for all European banks that hold its bonds. This in turn may force these banks to decrease their investment and tighten their funding,which ultimately leads to an output loss.

political costs, such as employment losses due to privatization, which are likely to expand to other Eurozone countries due to interconnected labor markets[14].

The motivation behind direct domestic output costs is similar. Borenstein & Panizza (2009) outline four possible sources: reputational costs, international trade exclusion costs, costs to the domestic economy through the financial system, and political costs to the authorities. In addition, Gennaioli, Martin & Rossi (2013) add a possible fifth source by pointing out that sovereign defaults are costly because they destroy the balance sheets of domestic banks and thus have a negative influence on investments.

Incorporating these insights, we define the outputs of country P and country EU as follows:

$$y_{P,t}(z_{P,t}) = Z^{1-z_{P,t}}\overline{y_P}$$
$$y_{EU,t}(z_{P,t}) = M^{1-z_{P,t}}\overline{y_{EU}}$$

where $\overline{y}_i$ is the "normal", constant GDP-level of country i *(i=P,EU).* $Z \in (0,1)$ and $M \in (0,1)$ represent constants that determine the new output level of country P and "country" EU after a default in country P. Therefore, $(1 - Z)$ and $(1 - M)$ illustrate the default penalties. $1 > (1 - Z) > 0$ represent the default penalty for country P and $1 > (1 - M) > 0$, represent the default penalty for the EU if country P defaults. We assume that $(1 - Z) > (1 - M)$, meaning that the penalty following a default of country P is more severe for country P than for the other EU countries[15].

Defining $y_{i,t}(z_{P,t})$ as above implies that the penalty occurs immediately. We could also define it in a way that the penalty occurs from the next period onwards (replace $z_{P,t}$ by $z_{P,t-1}$). However, changing the timing of the default penalty may change our numerical results but not our qualitative results.

The definition of the countries' output $y_{i,t}$ follows Conesa and Kehoe (2012) with two major differences. First, we introduce a new parameter, M, that allows us to account for a negative cross-border effect of a default in the single country. Second, we do not distinguish between a recession and normal times. This is mainly because our evaluation focuses on the interaction between the two countries rather than on the effect of a recession and potential recovery on the country's optimal debt level.

[14] See e.g. Borenstein & Panizza (2009)

[15] We motivate these output costs thoroughly above.

Including such a parameter can easily be done, but would complicate our notation and analysis considerably without providing any new implications for our research question. In addition, please note that the output of both countries depends on the default decision of country *P* and not on the default decision of country "*EU*", as we neglected this possibility earlier.

Similar to Cole & Kehoe (1996, 2000) and Conesa & Kehoe (2012), the timing of the interactions between the different players is as follows:

1. The sunspot variable $\zeta_{P,t}$ is realized and the aggregated state of period t is defined as follows: $s_t(B_{i,t}, z_{P,t-1}, \zeta_{P,t})$. The governments choose how much new debt $B_{i,t+1}$ to sell.
2. The international lenders choose how much debt to purchase $b_{i,t+1}$and to what price q_i, in equilibrium: $b_{i,t+1} = B_{i,t+1}$
3. The government of country *P* makes its default decision $z_{P,t}$. Doing that, private consumption $c_{i,t}$, government consumption $g_{i,t}$, and $y_{i,t}$ of both countries are determined.

The timing of events above shows that the governments act first and can thus anticipate the best reply functions of the households, the international bankers and themselves when making is initial decision.

Moreover, the first two steps hold true for both countries, but only government *P* makes a default decision. This default decision of country *P*, however, influences the outcome in both countries.

Additionally, it is important to notice that in each period the international lenders make their decision on how much debt to buy, $b_{i,t+1}$, before government *P* makes their default decision on the old debt, $B_{i,t}$. Hence, the bond price q_i depends on the newly issued debt, $B_{i,t+1}$, while the default decision/occurrence of a crisis depends on the old debt, $B_{i,t}$. This distinction allows the government to issue new debt before repaying the old debt and is crucial for our later analysis. For a clearer notation we simplify functions in the remainder, such that we do not include all arguments if not necessary. For instance we reduce $s_t(B_{i,t}, z_{i,t-1}, \zeta_{i,t})$ to s_t if applicable.

In sum, this section presented general model features, which are applicable to scenarios with and without bailout possibilities. The next section will focus on the scenario, where no bailout possibilities exist.

4. Scenario A: No Potential Bailout Possibilities Exist

This section focuses on the scenario, where no bailout possibilities exist. We will analyze this scenario by looking at i) the governments' maximization problems, ii) the crisis zone, iii) the international lender's maximization problem and iv) the respective bond prices. Thereafter the findings will be used to define and solve for an equilibrium.

4.1. The Governments' Maximization Problems

We assess two maximization problems: one concerning governments and one concerning international lenders[16]. As a basic principle, the countries in our model differ in some important exogenously given features, namely size of output, tax rate and initial debt level. Nevertheless, we can formulate the maximization problems in a general manner for both countries, as we assume the same maximization process, same preferences and the same general budget constraints across countries. Moreover, since we are particularly interested in the interaction between the countries, we do not look closer at the consumer savings decision and thus do not allow for private saving. This setup neglects the possibility to look closer at the development of the private capital stock and private investment, but allows us to keep the model lucid without jeopardizing the results of our analysis. However, the preferences of the private households are still crucial, as the governments are benevolent and maximize the utility of the country's consumer. Henceforth the governments' maximization problem can be illustrated as follows:

$$V_i(s_t) = \max_{B_{i,t+1}, z_{i,t}, g_{i,t}, c_{i,t}} E \sum_{t=1}^{\infty} \beta^{t-1} \, u\big(c_{i,t}, g_{i,t}\big)$$

$$s.t.$$

[16] We do not have to consider the consumers' maximization problem due to benevolent governments that maximizes the utility for its consumers

$$c_{i,t} = (1-\theta_i) y_{i,t}(z_{P,t})$$
$$g_{i,t} + z_{i,t} B_{i,t} = \theta_i y_{i,t}(z_{P,t}) + q_i(B_{i,t+1}, s_t, \pi_P) B_{i,t+1}$$
$$z_{i,t} = 0 \; if \; z_{i,t-1} = 0$$

The government of country *i (i=P,EU)* chooses private consumption $c_{i,t}$, government consumption $g_{i,t}$, the new debt level $B_{i,t+1}$ and $z_{i,t}$ to maximize its expected utility $V(s_t)$ for an infinite horizon. Doing that, the government faces the budget constraint of the private households and the government budget constraint. It is important to note that "country" *EU* always chooses to repay its debt: $z_{EU,t} = 1 \; \forall \; t \in \mathbb{N}$. θ_i is the constant tax rate in country *i* and $q_i \in [0,1]$ is the price international investors are willing to pay for a bond with a face value of $B_{i,t+1}$.[17] $u(c_{i,t}, g_{i,t})$ represents the utility function of a representative consumer in this economy that depends on private and government consumption[18]. Since we do not allow for private savings, the constraint of the private households is very simple and implies that the private consumption is equal to the untaxed output of each period. Moreover, the government budget constraint signifies that the public capital inflow has to be equal to the public capital outflow. The public capital inflow is composed of newly issued debt, discounted by the bond price $q_i(B_{i,t+1}, s_t, \pi_i) B_{i,t+1}$ and tax revenues $\theta_i y_{i,t}(z_{P,t})$. The public capital outflow consist of government consumption $g_{i,t}$ and repayment of old debt $z_{i,t} B_{i,t}$. If the government decides to default on its debt the repayment is equal to zero, as $z_{i,t} = 0$. $\pi_P \in [0,1]$ can be interpreted as the probability that a self-fulfilling crisis occurs next period in country

[17] Please note that this bond price notation implies that a government issues only zero-coupon or discount bonds

[18] Some possible, commonly used, utility functions:

$$u(c_{i,t}, g_{i,t}) = (1-\vartheta)\log(c_{i,t}) + \vartheta \log(g_{i,t})$$
$$u(c_{i,t}, g_{i,t}) = (1-\vartheta)\log(c_{i,t}) + \vartheta \log(g_{i,t} - \bar{g})$$
$$u(c_{i,t}, g_{i,t}) = \log(c_{i,t} + g_{i,t} - \bar{c} - \bar{g})$$

We can also think about risk neutral consumers (Cole & Kehoe, 2000) or even more concave utility functions. The latter two utility functions represent a case, where customer need or expect a certain level of government consumption or private consumption $(\bar{c}, \bar{g})$ in order to have a positive utility. This could for instance be a minimum level of infrastructure or patient care.

P, given country P is in the crisis zone. The magnitude of π_P depends on the debt level boundaries $\bar{b}$ and $\bar{B}$ that determine the crisis zone (See Figure 2). In the next section we will explain the crisis zone in more detail before moving on to the international lenders' maximization problem.

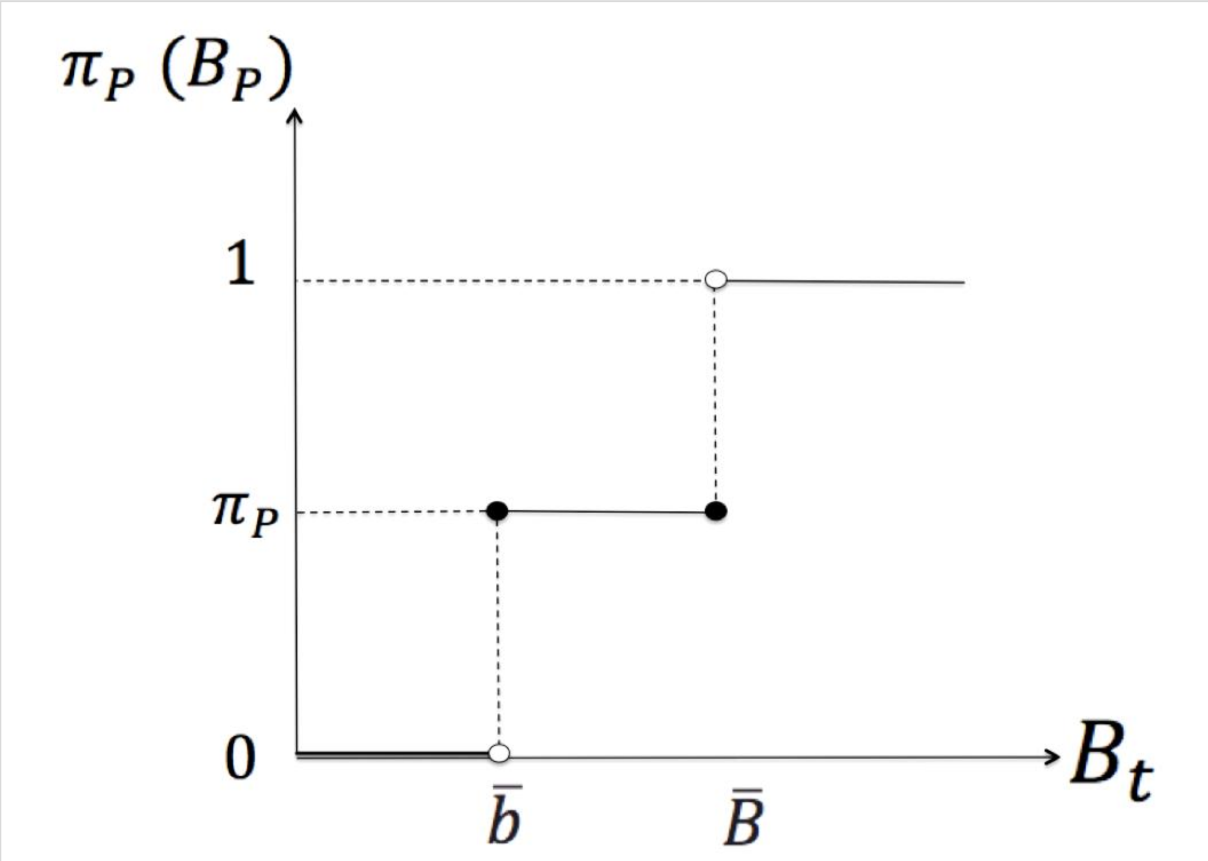

Figure 2: Different values for π_P depending on countries' debt levels

4.2. The Crisis Zone

As described earlier, the crisis zone is a debt level range in which a self-fulfilling debt crisis may occur with positive probability. In our case, the concept of a crisis zone is only applicable to country P, as we assumed above that country EU is not subject to a self-fulfilling crisis. Previous literature, particularly Cole & Kehoe (2000), define two features about the debt level that have to hold if the country is in the crisis zone.

First, the "No-lending condition" has to be satisfied. This means that the government only chooses to repay its debt if it can sell its new debt to the international lenders and default otherwise The debt level of country P is above the lower debt level boundary:

$B_{P,t+1} \geq \bar{b}(\pi_P)$. If the newly issued debt is too low, the government will repay anyways and a self-fulfilling crisis cannot occur.

Second, the "Participation constraint" has to hold, stating that the debt level is small enough that the government wants to issue new debt. The debt level of country P is below the upper debt level boundary: $B_{P,t+1} \leq \bar{B}(\pi_P)$. If the new debt level is too high, the government chooses to default anyways, independent of the decision/confidence of the international bankers.

Therefore, we can distinguish three different cases:

- $B_{P,t+1} < \bar{b}(\pi_P)$: no crisis (default) occurs for all $\zeta_{P,t}$
- $B_{P,t+1} > \bar{B}(\pi_P)$: a crisis (default) always occurs for all $\zeta_{P,t}$
- $\bar{b}(\pi_P) \leq B_{P,t+1} \leq \bar{B}(\pi_P)$: a crisis (default) occurs if $\zeta_{P,t} > 1 - \pi_P$ (crisis zone)

The international bankers lose confidence in the governments ability to repay their debt and do not lend to country P in the next period, if $\zeta_{P,t+1} > 1 - \pi_p$. Intuitively, one can say that with probability π_P, the investor's will lose confidence next period, which in turn triggers a default on this year's issued debt, $B_{i,t+1}$,. This is due to the fact that the government cannot rollover its debt in the next period. This loss in confidence only provokes a self-fulfilling debt crisis if a country is in the crisis zone. Therefore, the different cutoff levels are crucial for our later analysis, both for the optimal decision of the government and for the international investors. Incorporating the characteristics of the crisis zone, we will analyze the maximization problem of the international lenders and the corresponding bond prices in the following.

4.3. The International Lenders' Maximization Problem

The international lenders have deep pockets, are risk neutral, can lend to both countries and are perfectly competitive. Every investor is endowed with good w in every period that can either be consumed (as consumption good x) or invested in government bonds.

The maximization problem of a representative international investor and thereby its value function, $W\left(b_{i,t}, B_{i,t+1}, s_t, \pi_P\right)$, is defined as follows:

$$W(b_{i,t}, B_{i,t+1}, s_t, \pi_P) = \max_{q_i . b_{i,t}} E \sum_{t=1}^{\infty} \beta^{t-1} x_t$$

$$s.t.$$

$$x_t + q_P(B_{P,t+1}, s_t, \pi_P) b_{P,t+1} + q_{EU}(B_{EU,t+1}, s_t, \pi_P) b_{EU,t+1}$$
$$= w_t + z_{P,t}(B_{P,t+1}, s_t, q_P(B_{P,t+1}, s_t, \pi_P), \pi_P) b_{P,t}$$
$$+ \ z_{EU,t}(B_{EU,t+1}, s_t, q_{EU}(B_{EU,t+1}, s_t, \pi_{EU}), \pi_{EU}) b_{EU,t}$$
$$x \geq 0$$
$$b_{i,t} \geq -A$$

where $b_{i,t} \geq -A$ eliminates Ponzi schemes, but $-A$ is assumed to be large enough that it does not otherwise bind in equilibrium. Furthermore, we rule out corner solutions, which mean that the endowment good w_t is assumed to be big enough (deep pockets).

We see that the international investors maximize their consumption over an infinite horizon under the constraint that the investor spends what they receive in each period. Beside the endowment good w_t, they receive the repayment of the government bonds that depend on the default decision of the governments. The expenditures of the international investors on the other side include the consumption of good x_t and the amount paid for newly issued government bonds.

4.4. Deriving Bond Prices

The optimality conditions of the maximization problem above implies the following[19]:

$$q_P\left(B_{P,t+1}, s_t, \pi_P\right) = \beta E(z_{P,t+1}\left(B_{P,t+1}, s_t, q_P\left(B_{P,t+1}, s_t, \pi_P\right), \pi_P\right)$$
$$q_{EU}\left(B_{EU,t+1}, s_t, \pi_P\right) = \beta E(z_{EU,t+1}\left(B_{EU,t+1}, s_t, q_{EU}\left(B_{EU,t+1}, s_t, \pi_P\right)\right)$$

These equations signify that the price of a bond is equal to the expected probability of country *i* repaying its debt in the next period, discounted by the common discount factor β. Intuitively, this makes sense as the international lenders will not invest in something with a negative expected return, but as international lenders are perfectly competitive, the market pushes the prices to equilibrium (market clearing condition: $b_{i,t+1} = B_{i,t+1}$). The price $q_{EU}\left(B_{EU,t+1}, s_t, \pi_P\right)$ is therefore easy to find, as we assumed that the *EU* will always repay its debt in the future: $E(z_{EU,t+1}\left(B_{EU,t+1}, s_t, q_{EU}\left(B_{EU,t+1}, s_t, \pi_P\right)\right) = 1$. This implies the following *EU* bond price:

$$q_{EU}\left(B_{EU,t+1}, s_t, \pi_P\right) = \beta$$

To find the corresponding price for country *P*, we have to distinguish between the different cut-off debt levels we defined above:

- $B_{P,t+1} < \bar{b}(\pi_P)$: no crisis (default) occurs for all $\zeta_{P,t}$
- $B_{P,t+1} > \bar{B}(\pi_P)$: a crisis (default) always occurs for all $\zeta_{P,t}$
- $\bar{b}(\pi_P) \leq B_{P,t+1} \leq \bar{B}(\pi_P)$: a crisis (default) occurs if $\zeta_{P,t} > 1 - \pi_P$ (crisis zone)

If country *P*'s debt level $B_{P,t+1} < \bar{b}(\pi_P)$, no self-fulfilling crisis can occur, as the government always chooses to repay independently of the decision of the international lenders. This amounts to the same bond price as for the aggregated Eurozone countries:

$$q_P = \beta$$

[19] See appendix A.

If country P's debt level $B_{P,t+1} \geq \bar{B}(\pi_P)$, which implies that country i always defaults in the next period. The international banker will anticipate the default and thus will not lend to country P. This case can be represented by a bond price of zero:

$$q_P = 0$$

If country P is in the crisis zone $\bar{b}(\pi_P) \leq B_{P,t+1} \leq \bar{B}(\pi_P)$, which means that country P is subject to a self-fulfilling debt crisis. In this case, the bond price is given by:

$$q_p = \beta(1 - \pi_P)$$

where $(1 - \pi_P)$ represents the probability that the investors do not lose confidence in the government's ability to repay its debt next period. This loss in confidence lead to the case that the countries can not rollover their debt and hence choose to default next period. Hence, country P choose to repay ($z_{P,t+1} = 1$) with probability $(1 - \pi_P)$ and to default ($z_{P,t+1} = 0$) with probability π_P

The equations above imply the following bond price function for the model with no potential bailout, q_P:

$$q_P\left(B_{P,t+1}, s_t, \pi_P\right) = \begin{cases} \beta & if\ B_{P,t+1} < \bar{b}(\pi_P) \\ 0 & if\ B_{P,t+1} \geq \bar{B}(\pi_P) \\ \beta(1 - \pi_P) & if\ \bar{b}(\pi_P) \leq B_{P,t+1} \leq \bar{B}(\pi_P) \end{cases}$$

In sum, the maximization problem of the international lenders determines the bond price schedule above. This price schedule is taken as given by the governments when they make its decisions. In the next section, we put all insights so far together to define an equilibrium in general terms.

4.5. Definition of an Equilibrium

On the side of the governments, an equilibrium is defined with a value function $V_i(s_t)$, a policy function $B_{i,t+1}(s_t)$, the default decision $z_{i,t}(B_{i,t+1}, s_t, q_i, \pi_P)$ and the decision on government spending $g_{i,t}(B_{i,t+1}, s_t, q_i, \pi_P)$ that solve the maximization problem of the governments. The international lenders also maximize their utility in the equilibrium which results in a value function for the lenders, $W(b_{i,t}, B_{i,t+1}, s_t, \pi_P)$, the policy equivalent $b_{i,t+1}(b_{i,t}, B_{i,t+1}, s_t, \pi_P)$ and a bond price function $q_i(B_{i,t+1}, s_t, \pi_P)$. Following the timing of events as described above implies:

1. $B_{i,t+1}(s_t)$ maximizes the utility, $V_i(s_t)$ of both governments at the beginning of the period:

$$V_i(s_t) = \max_{B_{t+1}, z_t, g_t, c_t} E \sum_{t=1}^{\infty} \beta^{t-1} u(c_{i,t}, g_{i,t})$$

$$s.t.$$

$$c_{i,t} = (1 - \theta_i) y_{i,t}(z_{P,t})$$

$$g_{i,t}(B_{i,t+1}, s_t, q_i, \pi_P) + z_{i,t}(B_{i,t+1}, s_t, q_i, \pi_P) B_{i,t} = \theta_i y_{i,t}(z_{P,t}) + q_i(B_{i,t+1}, s_t, \pi_P) B_{i,t+1}(s_t)$$

$$z_{i,t} = 0 \ if \ z_{i,t-1} = 0$$

2. The international investors choose $b_{i,t+1}(b_{i,t}, B_{i,t+1}, s_t, \pi_P)$ and $q_i(B_{i,t+1}, s_t, \pi_P)$, consistent with rational expectation, such that the maximization problem of the investors is solved and the market clears ($b_{i,t+1} = B_{i,t+1}$):

$$q_i(B_{i,t+1}, s_t, \pi_P) = \beta E(z_{i,t+1}(B_{i,t+1}, s_t, q_i(B_{i,t+1}, s_t, \pi_P), \pi_P)$$

$$B_{i,t+1} = b_{i,t+1}(b_{i,t}, B_{i,t+1}, s_t, \pi_P)$$

3. Lastly, the default decision $z_{i,t}(B_{i,t+1}, s_t, q_i, \pi_P)$ and the consumption decision $g_{i,t}(B_{i,t+1}, s_t, q_i, \pi_P)$ of the governments solve their maximization problem at the end of the period, displayed under the first step.

As described above the government solves its maximization problem using backward induction. This means it takes the later optimal decisions of the international lenders, the foreign government and itself into account, when it decides on its debt level, $B_{i,t+1}$, at the beginning of the period. This is illustrated by the government budget constraint above. If the sunspot variable, $\zeta_{i,t}$, signifies that the international investors are losing trust in this period, it is already too late for the government to react. This is because the occurrence of a crisis depends on the debt level brought to the period, $B_{i,t}$, and not to the newly issued debt, $B_{i,t+1}$. Moreover, there is no commitment mechanism that enables the government for country P to commit on repaying its debt later. This is a key assumption as a perfect commitment mechanism would eliminate the occurrence of any self-fulfilling crisis. However, it is nearly impossible to establish a perfect commitment mechanism in reality[20].

It is important to notice that our model has many equilibria and our definition allows for several further possibilities. We could for example, vary the probability of a crisis π_P, include a time rule such that crises can only occur in certain periods[21]. Nevertheless, the extra value of such adaptions are limited and go beyond the scope of this paper. Therefore, we concentrate on equilibria with a simple Markov structure, meaning that the next state only depends on the current state and the decisions in that period. This approach makes it easier to constitute and calculate equilibria. In sum, this section showed how a general equilibrium is defined in our model. The next section will analyze the corresponding optimal government behavior, the value functions and the crucial cutoff debt boundaries.

4.6. Optimal Government Behaviour, Value Functions & Crucial Cutoff Levels

This part will determine equilibrium conditions for the scenario without a potential bailout. In order to make our derivation as comprehensible as possible we divide our analysis into two cases: the restricted case 4.6.1, where no self-fulfilling crises are

[20] The underlying concept behind this problem is the same as behind the question why countries actually choose to repay its debt: there is no authority that enforces the contract. There will be always a extreme case, where it will beneficial for the government to breach the commitment contract, even if the costs of breaching such a contract are very high (Dooley, 2000; Wright, 2011).

[21] This could be useful if we want to replicate business cycles or seasonal differences (given one period represents a quarter or month)

possible ($\pi_P = 0$) and the general case, 4.6.2, where self-fulfilling crises are possible ($\pi_P > 0$). For each case we will determine (a) the optimal government behavior, (b) the value functions and (c) the crucial debt boundaries that specify the crisis zone. After we derive the optimal behavior of the single country P the maximization problem of country EU gets quite straightforward. However, to define the equilibrium completely and to initialize the case with a potential bailout, we shortly illustrate the optimal behavior and value functions of the EU government in section 4.6.3.

4.6.1a Optimal Government Behavior if no Self-fulfilling Crisis is Possible

For this analysis we assume that no default has occurred in country P so far, and that the government will repay its debt in the future: $z_{P,t} = 1 \; \forall \; t \in \mathbb{N}$. For the repayment condition to be credible, we further have to assume that the initial debt level is low enough ($B_{i,0} = B_i \leq \bar{B}(0)$), so that the government has no incentive to default right away.

These assumptions imply that: $y_{P,t}(1) = \overline{y_P}$ and $q_P = \beta$, which corresponds precisely to the conditions of the other country, *EU.*

Given these assumptions the government solves the following maximization problem:

$$\max \sum_{t=1}^{\infty} \beta^{t-1} u\left(c_{P,t}, g_{P,t}\right)$$

$$s.t. \; c_{P,t} = (1-\theta)\,\overline{y_P}$$

$$g_{P,t} + B_{P,t} = \theta \overline{y_P} + \beta B_{P,t+1}$$

$$B_{P,0} \equiv B_P \leq \bar{B}(0)$$

where B_P, is the exogenously given, pre-existing debt level of country P, when "entering" the model at $t = 0$.

The maximization problem of the government can be solved by means of the following Lagrangian function:

$$\mathcal{L}(g_{P,t}, B_{P,t+1}, \lambda_t, \lambda_{t+1}, \dots)$$
$$= \sum_{t=1}^{\infty} \beta^{t-1} u\big((1-\theta)\,\overline{y_P}, g_{P,t}\big) - \lambda_t\big(g_{P,t} + B_{P,t} - \theta\overline{y_P} - \beta B_{P,t+1}\big)$$
$$- \lambda_{t+1}\big(g_{P,t+1} + B_{P,t+1} - \theta\overline{y_P} - \beta B_{P,t+2}\big) - \cdots$$

If we differentiate $\mathcal{L}(g_{P,t}, B_{i,t+1}, \lambda_t, \lambda_{t+1}, \dots)$, with respect to the policy variables in period *t,* $g_{P,t}$ and $B_{P,t+1}$, we get the following first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial g_{P,t}} = \beta^{t-1} \frac{\partial u}{\partial g_{P,t}}\big((1-\theta)\,\overline{y_P}, g_{P,t}\big) - \lambda_t = 0$$
$$\frac{\partial \mathcal{L}}{\partial\, B_{P,t+1}} = \beta\lambda_t - \lambda_{t+1} = 0$$

The transversality condition is given by:

$$\lim_{t \to \infty} \lambda_t B_{P,t+1} \geq 0$$

First and foremost the transversality condition states that the government does not choose to have a negative debt level. Intuitively, this arises from the fact that the marginal utility of one extra unit of government consumption will not be negative and that there is no reason for the government to keep non interest-paying assets.

Knowing that $\beta\lambda_t = \lambda_{t+1}$ and $\beta^t \frac{\partial u}{\partial g_{P,t+1}}\big((1-\theta)\,\overline{y_P}, g_{P,t+1}\big) = \lambda_{t+1}$, we get that:

$$\frac{\partial u}{\partial g_{P,t}}\big((1-\theta)\,\overline{y_P}, g_{P,t}\big) = \frac{\partial u}{\partial g_{P,t+1}}\big((1-\theta)\,\overline{y_P}, g_{P,t+1}\big)$$

which implies a constant government consumption, $\overline{g_P}$:

$$g_{P,t+1} = g_{P,t} = \bar{g}_P$$

Plugging this into the government budget constraint, we get:

$$B_{P,t+1} = \frac{1}{\beta}(\bar{g}_P + B_{P,t} - \theta\overline{y_P})$$

Now we show that, with this condition, the debt level has to be constant as well. In order to have a constant debt level ($B_{P,t+1} = B_{P,t} = B_P$) , the government consumption has to be equal to:

$$\bar{g}_P = \theta\overline{y_P} - B_P(1-\beta)$$

If we choose, $\bar{g}_P$ too large the budget constraint implies that our debt level would be steadily increasing until it reaches the upper debt level boundary $\bar{B}(\pi_P)$. In the period in which the government hits the upper debt level boundary, it would default on the debt. It can be easily shown that this is not optimal, as the utility would be strictly higher if the government keeps the debt level smaller and chooses to repay the debt[22]. If we choose government consumption to be too small, we will ultimately reach the point where the government has a negative debt level, which violates the transversality condition. Therefore we can conclude that it is optimal to keep the debt level constant: $B_{P,t+1} = B_{P,t} = B_P$.

The optimal decisions for the government if a default has already occurred ($z_{P,t-1} = 0$) is trivial, as we know that in this case the international lenders will not lend the government any money, which implies in turn that: $B_{i,t+1} = B_{i,t} = 0$. Hence, the optimal decisions that determine the respective utility is given by the budget constraints:

$$c_{P,t} = (1-\theta)\, Z\overline{y_P}$$
$$g_{P,t} = \theta Z\overline{y_P}$$

The derivation of the optimal government decisions enables to derive the respective value functions.

[22] This is because of the default penalty, see section 4.6.1b.

4.6.1b Value Functions if no Self-fulfilling Crisis is Possible

Knowing the conditions derived above, we can calculate the values of being in different states and thereafter derive the crucial cutoff debt levels, $\bar{B}(\pi_P)$ and $\bar{b}(\pi_P)$. To determine the value $V_i(B_i, z_{P,t}, \zeta_{P,t}, \pi_P)$ for different states of the economy, we have to insert the optimal consumption decisions ($c_{i,t}$ and $g_{i,t}$) in these states into the utility function. As a reminder, the state of the economy is dependent on the debt levels of the countries (constant for both countries as derived above), the preceding default decision of country P and the sunspot variable: $s_t(B_{i,t}, z_{P,t-1}, \zeta_{P,t})$.

So far we can distinguish between two states of the economy and the respective value functions:

i. No default has occurred in country P: $s_t(B_{i,t}, 1, \zeta_{P,t})$. As we derived above, this implies that also no default will occur in country P in the future, as $B_{P,0} = B_P \leq \bar{B}(0)$ and $\pi_P = 0$.The respective value function of this state is given by:

$$V_i(B_i, 1, \zeta_{P,t}, 0) = \frac{u((1-\theta_i)\bar{y}_i, \theta_i\bar{y}_i - B_i(1-\beta))}{1-\beta}$$

Please note that this value function is the same for both "countries". However, the derived values differ in size due to different "normal" output levels and tax rates.

ii. A default has occurred in country P: $s_t(B_{i,t}, 0, \zeta_{P,t})$. If a default has occurred already, the probability of confidence loss does not influence the value of the countries anymore, and is hence excluded in the notation of the value function. The value functions differ for the countries in this case. For country P the value function is given by:

$$V_P(B_P, 0, \zeta_{P,t}) = \frac{u\big((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P}\big)}{1-\beta}$$

The value function for country EU is defined by:

$$V_{EU}\left(B_{EU},0,\zeta_{P,t}\right)=\frac{u\left((1-\theta_{EU})M\overline{y_{EU}},\theta_{EU}M\overline{y_{\iota}}\right)}{1-\beta}$$

We can conclude that a default in country P decreases the value for both countries, which is a consequence from our output definition. Knowing the value functions, we are now able to define the crucial debt levels in the next section.

4.6.1c Crucial Debt Levels if no Self-fulfilling Crisis is Possible

This section will analyze the crucial debt levels (i.e. the upper and lower bound of the crisis zone) if no self-fulfilling crisis is possible ($\pi_P=0$). In order to determine the crucial debt levels for country P we have to find the debt level where the value of not defaulting $\left(V_P^n\left(B_P,z_{P,t-1},\zeta_{P,t},\pi_P,q_P\right)\right)$ is equal to the value of defaulting $(V_P^d\left(B_P,z_{P,t-1},\zeta_{P,t},\pi_P,q_P\right))$ in the respective state. The state of the economy is now supplemented by the bond price q_P that represents the actions of the international lenders. One should note that there is no sense in analyzing the case in which country P already defaulted on its debt ($z_{P,t-1}=0$), as we assumed that the international lenders will not lend to this country anymore. This makes the derivation of the crucial debt levels pointless. Moreover, we do not need to determine the lower debt level boundary, $\bar{b}(\pi_P)$ because it is not relevant in the case where no self-fulfilling crises are possible ($\pi_P=0$). This is due to the fact that a rational government will always choose to repay if the newly issued debt level is smaller or equal to the upper debt level ($B_{P,t+1}\leq\ \bar{B}(\pi_P)$).

Therefore we only need to derive the upper cutoff-level ($\bar{B}(0)$). Above this threshold debt level the government optimally chooses to default even if the international lenders lend money to the government. In contrary to the lower debt level boundary $\bar{b}(\pi_P)$, the upper debt level boundary $\bar{B}(\pi_P)$ depends on the likelihood of a self-fulfilling debt crisis, (π_P). Given that this section excludes the possibility of self-fulfilling crises, we assume that the bond price for which the international investors will lend money to the government is equal to: $q_P=\beta$. This follows from the bond price schedule above ($\pi_P=0$). To preclude that the government will just raise its debt-level to infinity when they know that they can sell the new debt anyways without repaying it later, we limit the amount that the international lenders will buy to the old, pre-existing debt level, $B_{P,\tilde{t}} = B_P$. The optimal government consumption decision is the same as derived above.

Hence, the value of repaying $\left(V_P^n\left(B_P, z_{P,t-1}, \zeta_{P,t}, \pi_P, q_P\right)\right)$ is given by:

$$V_P^n\left(B_P, 1, \zeta_{P,t}, 0, \beta\right) = \frac{u((1-\theta_P)\overline{y_P}, \theta\overline{y_P} - B_P(1-\beta))}{1-\beta}$$

However, if the international lenders still lend to the government and the government chooses to default, the government gets money from the investors, which it will not have to repay later and thus can be consumed. We denote the period, in which the government faces the decision whether or not to default as: $\tilde{t}$. The optimal government consumption in the respective period $\tilde{t}$ will therefore increase by βB_P, the amount received from the international lenders "for free".

The value of defaulting $\left(V_P^d\left(B_P, z_{P,t-1}, \zeta_{P,t}, \pi_P, q_P\right)\right)$ is thus given by:

$$V_P^d\left(B_P, 1, \zeta_{P,t}, 0, \beta\right) = u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P} + \beta B_P\right) + \frac{\beta u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

Accordingly, the upper debt level $\bar{B}(0)$[23] is defined by:

$$V_P^n\left(\bar{B}(0), 1, \zeta_{P,t}, 0, \beta\right) = V_P^d\left(\bar{B}(0), 1, \zeta_{P,t}, 0, \beta\right)$$

$$\frac{u((1-\theta_P)\overline{y_P}, \theta\overline{y_P} - \bar{B}(0)(1-\beta))}{1-\beta}$$
$$= u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P} + \beta\bar{B}(0)\right) + \frac{\beta u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

$$u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P} + \beta\bar{B}(0)\right) - u\left((1-\theta_P)\overline{y_P}, \theta_P\overline{y_P} - \bar{B}(0)(1-\beta)\right)$$

[23] Conesa & Kehoe (2012, p.16) show that $\bar{B}(0)$ is stationary.

$$= \frac{\beta}{1-\beta}\left(u\left((1-\theta_P)\overline{y_P}, \theta_P\overline{y_P} - \bar{B}(0)(1-\beta)\right) - u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P}\right)\right)$$

In this section we derived the optimal government behavior, the value functions and the crucial debt boundaries that specify the crisis zone when no self-fulfilling crises are possible. In a similar approach, the next section analyzes the case in which self-fulfilling crises are possible.

4.6.2a Optimal Government Behavior if a Self-fulfilling Crisis is Possible

When evaluating the optimal government policy functions under the threat of a possible self-fulfilling debt crisis, we have to think about the possible actions that the government has in such a situation. First, the government can run down its debt until it reaches the lower debt-level boundary $\bar{b}(\pi_P)$. This makes the country invulnerable to self-fulfilling debt crises. Second, it can keep the current debt level constant, which leaves the country exposed to the confidence of the international investors. Third, it can raise its debt level even further. However, it can only raise its debt up to the upper debt level $\bar{B}(\pi_P)$, as the government will be unable to sell new debt to the international investors beyond this point. The theoretical case, in which the government increases its debt level in one period but decrease it in another (or vice versa) is not possible in our model, as we assume time-invariant preferences.

As shown in Cole & Kehoe (2000), it cannot be optimal for the government to raise its debt sequentially in our setup with no potential bailouts. Hence, we can discard this possibility here and concentrate on the other two potential actions, decreasing debts or keeping them constant.

Suppose now that the initial debt level $B_P = B_{P,0} > \bar{b}(\pi_P)$ and the government will decrease its debt level B_P to $\bar{b}(\pi_P)$ in T periods. Observing the maximization problem of the government above, we see that the first-order conditions are not subject to π_P, implying that it is still optimal for the government to keep the governmental consumption constant:

$$g_{P,t} = g_{P,T}\left(B_{P,0}, \pi_P\right)$$

Here $g_{P,T}(B_{P,0},\pi_P)$ represent the optimal government consumption, given the government's plans to run down its debt in T periods. As $g_{P,T}(B_{p,0},\pi_P)$ has to be constant over time, the government will reduce its debt in each period by the same fraction. The government budget constraints in each period, up to the point when the lower debt boundary is reached are thus defined as follows:

$$g_{P,T}(B_{P,0},\pi_P) + B_{P,0} = \theta_P\overline{y_P} + \beta(1-\pi_P)B_{P,1}$$
$$g_{P,T}(B_{P,0},\pi_P) + B_{P,1} = \theta_P\overline{y_P} + \beta(1-\pi_P)B_{P,2}$$

.

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$$g_{P,T}(B_{P,0},\pi_P) + B_{P,T-2} = \theta_P\overline{y_P} + \beta(1-\pi_P)B_{P,T-1}$$
$$g_{P,T}(B_{P,0},\pi_P) + B_{P,T-1} = \theta_P\overline{y_P} + \beta\bar{b}(\pi_P)$$

It is important to note that the boundary case $T \rightarrow \infty$ thereby represents the policy where the government keeps the debt level constant. As $T \rightarrow \infty$ the debt level reduction in each period strives to 0.

If we now multiply both sides of all constraints by $(\beta(1-\pi_P))^t$ and sum up all constraints we get:

$$\sum_{t=0}^{T-1}(\beta(1-\pi_P))^t\, g_{P,T}(B_{P,0},\pi_P) + B_{P,0}$$
$$= \sum_{t=0}^{T-1}(\beta(1-\pi_P))^t\;\; \theta_P\overline{y_P} + (\beta(1-\pi_P))^{T-1}\beta(1-\pi_P)\bar{b}(\pi_P))$$
$$g_{P,T}(B_{i,0},\pi_P) = \theta\overline{y_P} - \frac{1-\beta(1-\pi_P)}{1-(\beta(1-\pi_P))^T}(B_{P,0} - (\beta(1-\pi_P))^{T-1}\beta(1-\pi_P)\bar{b}(\pi_P))$$

To find the optimal government consumption when the government keeps its debt level constant, we let T strive to infinity:

$$g_{P,\infty}(B_{i,0},\pi_P) = \lim_{T\rightarrow\infty} g_{P,T}(B_{P,0},\pi_P,0) = \theta_P\overline{y_\iota} - ((1-\beta(1-\pi_P))B_{P,0})$$

As you can see, this government consumption corresponds to the optimal government consumption above, $\bar{g}_P$, with the difference that the discount the government has to pay on its newly issued debt is now equal to $\beta(1-\pi_P)$. This incorporates the probability of a self-fulfilling debt crisis. Summing up, this section derived the optimal government consumption for different policies. The government will choose the policy, which provides the highest value according to the value functions analyzed in the next section.

4.6.2b Value Functions if a Self-fulfilling Crisis is Possible

Since we determined the optimal government consumption $g_{P,T}(B_{P,0},\pi_P)$ for each policy (different values for *T)* we are now also able to calculate the respective value $V_T(B_{i,0},\pi_P)$ for each of these policies. $V^T_{P,m}(B_{i,0},\pi_P)$ thereby represents the value for the policy "running its debt down in T periods" with m periods remaining till we reach the lower debt level boundary:

$$V^T_{P,T}(B_{P,0},\pi_P) = u\left((1-\theta_P)\overline{y_P}, g_{P,T}(B_{P,0},\pi_P)\right) + \beta(1-\pi_P)V^T_{P,T-1}(B_{P,0},\pi_P) + \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

$$V^T_{P,T-1}(B_{P,0},\pi_P) = u\left((1-\theta_P)\overline{y_P}, g_{P,T}(B_{P,0},\pi_P)\right) + \beta(1-\pi_P)V^T_{P,T-2}(B_{P,0},\pi_P) + \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

$$\vdots$$

$$V^T_{P,2}(B_{P,0},\pi_P) = u\left((1-\theta_P)\overline{y_P}, g_{P,T}(B_{P,0},\pi_P)\right) + \beta(1-\pi_P)V^T_{P,1}(B_{P,0},\pi_P) + \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

$$V^T_{P,1}(B_{P,0},\pi_P) = u\left((1-\theta_P)\overline{y_P}, g_{P,T}(B_{P,0},\pi_P)\right) + \frac{\beta u((1-\theta_P)\overline{y_P}, \theta\overline{y_P})}{1-\beta}$$

For a better comprehension of the value function above it helps to understand the value of each strategy as the sum of the value generated in the respective period (first part) plus the expected future utility from this policy (second part). With probability $(1-\pi_P)$

there is no self-fulfilling crisis in the next period and the government continues to run its debt down. With a probability π_P there will be a default in the next period and the output will shrink by the default penalty $(1 - Z)$. The last parts of each of the value functions above represent the expected future value of the respective policy. Using backward induction we can now find $V_{P,T}^{T}(B_{P,0}, \pi_P) = V_P^T(B_{P,0}, \pi_P)$, the value of running its debt down in T periods when the decision is made (see appendix B). $V_P^T(B_{P,0}, \pi_P)$ is given by:

$$V_P^T(B_{P,0}, \pi_P) = \frac{1 - (\beta(1 - \pi_P))^T}{1 + \beta(1 - \pi_P)} u\left((1 - \theta_P)\overline{y_P}, g_{P,T}(B_{P,0}, \pi_P)\right) + \frac{1 - (\beta(1 - \pi_P))^{T-1}}{1 + \beta(1 - \pi_P)} \frac{\beta\pi_P u((1 - \theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1 - \beta} + \beta(1 - \pi_P)^{T-2} \frac{\beta u((1 - \theta)\overline{y_P}, \theta\overline{y_P})}{1 - \beta}$$

The value for keeping the debt constant $V_\infty(B_{P,0}, \pi_P)$is thus given by:

$$V_P^\infty(B_{P,0}, \pi_P) = \frac{u\left((1 - \theta_P)\overline{y_P}, \theta_P\overline{y_P} - (1 - \beta(1 - \pi_P)B_{P,0})\right)}{1 + \beta(1 - \pi_P)} + \frac{\beta\pi_P u((1 - \theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{(1 - \beta)(1 + \beta(1 - \pi_P))}$$

4.6.2c Crucial Debt Levels if a Self-fulfilling Crisis is Possible

In this section we will use a similar approach as in 4.6.1c in order to derive the crucial debt levels that determine the crisis zone. Since self-fulfilling crises are possible ($\pi_P > 0$), we now need to consider both, the upper and the lower bound.

We first consider the lower debt level boundary, $\bar{b}(\pi_P)$, which is the debt level boundary for which the government chooses to repay even if the international investors do not lend any money in the future ($q_P = 0$). It is important to note that the likelihood of a confidence loss by the international investors does not play a role when we determine the lower debt boundary because we deal with the extreme case, in which international investors do not lend to the country in the next period anyways. This implies $\bar{b}(\pi_P) = \bar{b}(0)$. Since $q_P = 0$, the optimal government consumption in period, $\tilde{t}$ (remember $\tilde{t}$ denotes the period in which the government faces the decision whether or not to default) is equal to: $\bar{g}_{P,\tilde{t}} = \theta_P \overline{y_P} - B_P$, if the government decides to repay the debt nonetheless. If the government chooses to default if it cannot lend any money: $\bar{g}_{P,\tilde{t}} = \theta_P Z \overline{y_P}$.

All future consumption decisions are equivalent to the case of a default with the difference that the output stays $\overline{y_P}$ if the government chooses to repay the debt. This implies the following value if the government repays its debt (V_P^n) even if the international lenders will not buy any debt in the future:

$$V_P^n\big(B_P, 1, \zeta_{P,t}, \pi_P, 0\big) = u\big((1-\theta_P)\overline{y_P}, \theta_P \overline{y_P} - B_i\big) + \frac{\beta u((1-\theta_P)\overline{y_P}, \theta_P \overline{y_P})}{1-\beta}$$

The value of defaulting, $V_P^d\big(B_P, z_{P,t}, \zeta_{P,t}, \pi_P, q_P\big)$, on the other hand is given by:

$$V_P^d\big(B_P, 1, \zeta_{P,t}, \pi_P, 0\big) = \frac{u((1-\theta_P)Z\overline{y_P}, \theta_P Z \overline{y_P})}{1-\beta}$$

So the lower cutoff level, $\bar{b}(\pi_P)$, can be found at the debt level where the values above are equal to each other ($V_n = V_d$). Consequently, $\bar{b}(\pi_P)$, is defined by:

$$V_P^n\big(\bar{b}(\pi_P), 1, \zeta_{P,t}, \pi_P, 0\big) = V_P^d\big(\bar{b}(\pi_P), 1, \zeta_{P,t}, \pi_P, 0\big)$$

$$u\Big((1-\theta_P)\overline{y_P}, \theta_P \overline{y_P} - \bar{b}(\pi_P)\Big) + \frac{\beta u((1-\theta_P)\overline{y_P}, \theta_P \overline{y_P})}{1-\beta} = \frac{u((1-\theta_P)Z\overline{y_P}, \theta_P Z \overline{y_P})}{1-\beta}$$

$$(u\big((1-\theta_P)Z\overline{y_P}, \theta Z \overline{y_P}\big) - u\Big((1-\theta_P)\overline{y_P}, \theta \overline{y_P} - \bar{b}(\pi_P)\Big)) =$$

$$\frac{\beta}{1-\beta}(u\big((1-\theta_P)\overline{y_P}, \theta_P \overline{y_P}\big) - u\big((1-\theta_P)Z\overline{y_P}, \theta_P Z \overline{y_P}\big))$$

The left-hand side of the last equation represents the difference in utility between the different policies in the current period and the right-hand side denotes the reversed difference in future utility of both policies.

As a last step, we now determine the upper boundary level, $\bar{B}(\pi_P)$. We are doing this in a similar manner as above. In this case, we have to find the debt level for which the value of the optimal policy (policy that provides the maximal value for the government) is equal to the value of defaulting even if the investors lend money to the government. Please note that in this case the price for which the international lenders are willing to buy the government bonds is equal to $\beta(1-\pi_P)$.[24]

Putting these insights together we can illustrate the condition for the upper debt level boundary $\bar{B}(\pi_P)$ as follows:

$$\max(V_P^1(\bar{B}(\pi_P)), V_P^2(\bar{B}(\pi_P)), \ldots, V_P^{\infty}(\bar{B}(\pi_P))) =$$

$$u\big((1-\theta_P)Z\overline{y_P}, \theta Z\overline{y_P} + \beta(1-\pi_P)\bar{B}(\pi_P)\big) + \frac{\beta u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

The exact cutoff levels $\bar{b}(\pi_i)$ and $\bar{B}(\pi_i)$ depend on the utility function and will be calculated numerically later.

Finally, we can put all the characteristics derived above together and define the general value of being in state $s_t(B_{P,t}, 1, \zeta_{P,t})$ for country *P*:

$$V(B_{i,t}, 1, \zeta_{P,t}, \pi_P, q_P) = \begin{cases} \dfrac{u((1-\theta_P)\overline{y_P}, \theta_P\overline{y_P} - B_P(1-\beta))}{1-\beta} & if\ B_P \leq \bar{b}(\pi_P) \\ \max(V_P^1(\bar{B}(\pi_P)), V_P^2(\bar{B}(\pi_P)), \ldots, V_P^{\infty}(\bar{B}(\pi_P))) & if\ \bar{b}(\pi_P) \leq B_P \leq \bar{B}(\pi_P, 0) \\ & and\ \zeta_{P,t} \leq (1-\pi_P) \\ \dfrac{u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta} & if\ \bar{b}(\pi_P) < B_P \leq \bar{B}(\pi_P, 0) \\ & and\ \zeta_{P,t} > (1-\pi_P) \\ \dfrac{u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta} & if\ B_P > \bar{B}(\pi_P) \end{cases}$$

From this formula we can see that the value of country *P,* is mainly dependent on the crucial debt level boundaries, the current debt level and the likelihood of a self-fulfilling debt crisis. We can further state, that the probability of a self-fulfilling crisis decreases

[24] see bond price schedule in section 4.4.

the value for country *P*, which incentivizes the country in this scenario to decrease its debt level down to the lower debt level boundary $\bar{b}(\pi_P)$[25].
After we derived the optimal decisions, value functions and crucial debt levels for country *P* in sections 4.6.1. and 4.6.2., we focus on the EU in the next period.

4.6.3. Optimal Behavior and Value Functions for the EU zone

We assumed that the EU government will never default $(z_{EU,t} = 1 \forall\, t \in \mathbb{N})$. As a consequence, the EU only has to choose its optimal consumption decisions and its newly issued debt level, given the default decision of the other country.
As we stated above the maximization problem of the EU is similar to the one of country *P*. However, the optimal consumption and debt level decisions depend on the default decision of country *P*. As we derived in appendix C*, t*he optimal consumption decisions for country *EU* if country *P* chooses to repay its debts $(z_{P,t} = 1)$ are given by:

$$g_{EU,t} = \bar{g}_{EU} = \theta_{EU}\overline{y_{EU}} - B_{EU}(1-\beta)$$
$$c_{EU,t} = (1-\theta_{EU})\,\overline{y_{EU}}$$

If country P chooses to default $(z_{P,t} = 0)$ the output of country EU changes and thus its budget constraint and the optimal consumption decisions:

$$g_{EU,t} = \bar{g}_{EU} = \theta_{EU}M\overline{y_{EU}} - B_{EU}(1-\beta)$$
$$c_{EU,t} = (1-\theta_{EU})M\,\overline{y_{EU}}$$

The determination of the value function of the *EU* $(V_{EU}(B_{EU}, z_{P,t}, \zeta_{P,t}))$ for the cases of (i) the initial debt level of country *P* is below the lower debt level boundary $(B_P \leq \bar{b}(\pi_P))$[26] and (ii) a default in country *P* are straightforward (see appendix D):

i. $B_P \leq \bar{b}(\pi_P) : \rightarrow\ V_{EU}(B_{EU}, 1, \zeta_{P,t}) = \frac{u((1-\theta_{EU})\overline{y_{EU}}, \theta_{EU}\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$

ii. $z_{P,t} = 0 : \rightarrow V_{EU}(B_{EU}, 0, \zeta_{P,t}) = \frac{u((1-\theta_{EU})M\overline{y_{EU}}, \theta_{EU}M\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$

[25] This incentive to decrease its debt level is more formally derived in Cole & Kehoe (2000)
[26] $B_P \leq \bar{b}(\pi_P)$ implies that country P will always choose to repay in the future.

In the third case, in which country P is in the crisis zone $\bar{b}(\pi_P) \leq B_P \leq \bar{B}(\pi_P)$, the determination of the value function ($V_{EU}\left(B_{EU}, z_{P,t}, \zeta_{P,t}\right)$) is a little bit more complex as we need to consider the probability of a self-fulfilling crisis (π_P) and the optimal policy of government P (T_P). The value function is derived in the appendix D and is given by:

$$V_{EU}^{T}\left(B_{EU,0}, \pi_P\right) = \frac{1-(\beta(1-\pi_P))^{T_P}}{1-\beta(1-\pi_P)}\, u\left((1-\theta_{EU})\overline{y_{EU}}, \theta_{EU}\overline{y_{EU}} - B_{EU}(1-\beta)\right)$$
$$+ \frac{1-(\beta(1-\pi_P))^{T_P-1}}{1-\beta(1-\pi_P)}\, \frac{\beta\pi_P u((1-\theta_{EU})M\overline{y_{EU}}, \theta_{EU}M\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$$
$$+ \beta(1-\pi_P)^{T_P-2}\, \frac{\beta u((1-\theta_{EU})\overline{y_{EU}}, \theta_{EU}\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$$

These value functions give us some valuable implications, also applicable to later cases. First, the value for country *EU* is strictly higher if country P does not default. Second, the faster country *P* runs down its debt (if in the crisis zone), the higher the value for the *EU.* Country *P* does not take this effect into account when making its decision. This means in turn that the overall value of all countries can be increased if a third benevolent planner, for instance the ECB, makes the debt decisions integrative for both countries for instance through Eurobonds.

Furthermore, a higher probability of a self-fulfilling crisis (π_P) decreases the value of country *EU* as well as it automatically increases the probability of a foreign default, if country *P* is in the crisis zone.

With this section we completed the analysis of scenario A in which bailout possibilities do not exist. In the next part of our thesis we focus on scenario B to evaluate the effects of a potential bailout.

5. *Scenario B: Bailout Possibilities Exist*

This section focuses on the scenario, where bailout possibilities exist. In the following we will explain in more detail how the model is extended. Thereafter we show the corresponding effects on the equilibrium conditions. The findings are then used to define new bond prices and optimal behavior of governments.

5.1 Bailing out the Other Country

Extending our basic setup we now allow the aggregated countries in the Eurozone to bailout the single country in order to avoid a default penalty. This means that the maximization problem of the *EU* government includes one more decision variable, $h_{EU,t}$, which represents the bailout decision of country *EU*. $h_{EU,t} = 1$, if the government decides to bailout the single country and $h_{EU,t} = 0$, otherwise. As the *EU* does not default per assumption, there is no need to consider a bailout decision for country *P*.[27]

The *EU* government makes its bailout decision in each period after all other decisions in the respective period. Again, it is important to notice that the bailout refers to country *P*'s old debt level, $B_{P,t}$. For the timing within each period this implies that we add a fourth step, following the three steps we defined above[28].

The fourth step is:

4. Given a default in country *P* ($z_{P,t} = 0$), the *EU* government decides whether or not to bailout the defaulting country, $h_{EU,t}$.

The timing also implies that the players in the economy will anticipate a potential bailout when making their decisions. Obviously, this will influence the governments' and international lenders' decisions, as we will see later in this section.

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[27] Furthermore, it is nearly impossible that a single country has the funds and financial power to bailout the entire rest of the Eurozone.

[28] The initial three steps from Section 3.1 are:

1. The sunspot variable $\zeta_{P,t}$ is realized and the aggregated state of period t are defined as follows: $s_t(B_{i,t}, z_{P,t-1}, \zeta_{P,t})$. The government chooses how much new debt $B_{i,t+1}$ to sell.
2. The international lenders choose how much debt to purchase $b_{i,t+1}$ and to what price $q_i(B_{i,t+1}, s_t, \pi_P)$, in equilibrium: $b_{i,t+1} = B_{i,t+1}$
3. The government of country *P* makes its default decision $z_{i,t}$. Doing that, private consumption $c_{i,t}$, government consumption $g_{i,t}$, and $y_{i,t}$ of BOTH countries are determined.

Because of a potential bailout the first-hand default decision, $z_{P,t}$, may differ from the realized default decision at the end of the period, denoted as $z^*_{P,t}$. This implies that it is now possible that country P decides to default $(z_{P,t} = 0)$, but no default is actually realized $\left(z^*_{P,t} = 1\right)$ because country EU decided to bailout country P to avoid a default penalty. $z^*_{P,t}$ can be defined as:

$$z^*_{P,t}\left(z_{P,t}\,, h_{EU,t}\right) = \max\left(z_{P,t}\,, h_{EU,t}\right)$$

If country *EU* chooses to bailout country *P*, then $h_{EU,t} = z^*_{P,t} = 1$, meaning that no default occurs. In this case, neither the *EU* nor country *P* suffers from any default penalty.
Hence, the output level of each country changes and is now dependent on the realized default decision $z^*_{P,t}$ of the single country and thus also on the bailout decision of the *EU*:

$$y_{P,t}\left(z^*_{P,t}\left(z_{P,t}\,, h_{EU,t}\right)\right) = Z^{1-z^*_{P,t}}\overline{y_P}$$
$$y_{EU,t}\left(z^*_{P,t}\left(z_{P,t}\,, h_{EU,t}\right)\right) = M^{1-z^*_{P,t}}\overline{y_{EU}}$$

For a clearer notation we will denote $z^*_{P,t}\left(z_{P,t}\,, h_{EU,t}\right)$ as $z^*_{P,t}$ in the remainder. Furthermore, a potential bailout also influences the state of the economy at period t *(*$t =$ 0,1,2, …*)* that is now given as: $s_t\left(B_{i,t}, z^*_{P,t-1}, \zeta_{P,t}\right)$.
The bailout comes at a cost, $\kappa(B_{P.t})$, depending on the current absolute debt level of country P. To keep things simple we assume that if a country bails out the other country it has to repay all its debt to the international lenders. The cost of the bailout is thus given as:

$$\kappa\left(B_{P,t}\right) = B_{P,t}$$

This assumption illustrates an extreme case that may not easily fit to reality[29]. However, it is still valid to replicate the current situation in the Eurozone as the involved parties bear substantial bailout costs in reality as well[30].

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[29] This bailout cost definition implicitly signifies that a bailout is large enough to decrease the debt level of the bailed out country below the lower debt level boundary, which eliminates the possibility of a self-

We further assume that the cost of a bailout can be borrowed "overnight"[31] from a not closer specified third party, lets say the Troika, that may refer to institutions such as the IMF, ESM, ECB etc. The price is equal to 1 and has to be repaid at the beginning of next period. Moreover, there is no other possibility as the bailout of another country to receive such an "overnight credit".

Hence, if government *EU* decides to bailout the other country, the new debt level at the beginning of period *t+1* is given by $B^*_{EU,t+1} = \left(B_{EU,t+1} + B_{P,t}\right)$. The debt level in period *t+1* for country *P* is then equal to zero: $B^*_{P,t+1} = 0$.

We further assume that there will be no second bailout. This rules out the possibility of a never-ending circle of raising debts, defaulting, getting bailed out and raising the debt level again.

We see that the bailout costs depend on the absolute current debt level of country *P* and hence indirectly also on its economic size. This may not only explain why the European Union bailed out Greece (small economic size), but may also explain why other European countries, with a comparably low absolute debt level continue to raise their debt level even in the crisis zone (e.g. Portugal, Ireland, see Figure 1). The respective countries anticipate a potential bailout by the other countries, which is obviously more likely when the cost of a bailout is smaller. This feature of our model offers a further explanation for why countries may "gamble"(increase their debt level) rationally when it comes to public debt (see more details in section 5.4.). Before we take a closer look on country *P,* we have to derive the bailout cutoff debt level that signifies whether or not it is beneficial for the EU to bail out the single country.

5.2 Deriving the Conditions for the Bailout Cutoff Level

In this section we derive the optimality conditions for the country *EU* that determine the bailout cutoff level. In particular, we will analyze optimal government behavior, value functions and crucial debt levels for the *EU* in this scenario. The timing of the actions

fulfilling debt crisis. The public debt of Greece after the recent *EU* bailouts remained too high (IMF, 2013). Hence, our model is not perfectly applicable to the case of Greece.

[30] In order to account for more realistic bailout costs we could for example include the lower debt boundary $\bar{b}(\pi_P)$ into the cost equation. This can be motivated by the fact that the country may lose their reputation completely after the second default and is hence not bailed out in the future

[31] "Overnight" in this respect means that the credit is granted at the end of the period (when bailout decision is made) and has to be repaid at the beginning of next period (step 3.)

described above states that the government's decision whether or not to bailout the other country is the last decision/action within each period. This implies that the government of country *EU* knows all previous decisions by the agents, the resulting price, the consumption and the debt levels when making its decision whether or not to bailout the single country, $h_{EU,t}$. Furthermore, this means that the bailout conditions are not subject to the beliefs of the investors, as the EU government observes the first-hand default decision ($z_{P,t}$) and does not care why the default actually took place.

We derived the optimal decisions for the government of country *EU*, if there is no realized default for country *P*, $\left(z^*_{P,t} = 1\right)$[32]. The respective optimal government behavior is then:

- Keeping its debt constant: $B_{EU,t+1} = B_{EU,t} = B_{EU}$
- Setting government consumption: $g_{EU,t} = \theta_{EU}\overline{y_{EU}} - B_{EU}(1-\beta)$
- Setting private consumption: $c_{EU,t} = (1-\theta_{EU})\overline{y_{EU}}$

These optimal decisions imply the following value function:

$$V_{EU}\left(B_{EU}, 1, \zeta_{P,t}\right) = \frac{u((1-\theta_{EU})\overline{y_{EU}}, \theta_{EU}\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$$

In a similar approach we determine the optimal decisions for the government of country EU, if a realized default has occurred or will occur in country *P* ($z^*_{P,t} = 0$)[33]:

- Keeping its debt constant: $B_{EU,t+1} = B_{EU,t} = B_{EU}$
- Setting government consumption: $g_{EU,t} = \theta_{EU}M\overline{y_{EU}} - B_{EU}(1-\beta)$
- Setting private consumption: $c_{EU,t} = (1-\theta_{EU})M\overline{y_{EU}}$

These optimal decisions imply the following value function:

$$V_{EU}\left(B_{EU}, 0, \zeta_{P,t}\right) = \frac{u((1-\theta_{EU})M\overline{y_{EU}}, \theta_{EU}M\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$$

[32] See Appendix C and D for details
[33] See Appendix C and D for details

If country P has chosen to default in this period ($z_{P,t} = 0$), the government faces the decision whether or not to bailout the other country ($h_{EU,t}$). A rational EU government would hence bailout the other country as long as the future expected value of a bailout $\left(V_{EU}^{b}\left(B_P, z_{P,t-1}^{*}, \zeta_{P,t}, \pi_P, q_P\right)\right)$ is higher than the future expected value of not bailing out the other country $\left(V_{EU}^{k}\left(B_P, z_{P,t-1}^{*}, \zeta_{P,t}, \pi_P, q_P\right)\right)$.

Based on this, we can define a crucial cutoff debt level, $\tilde{b}_P(M, \bar{y}_{EU,t})$, that defines whether or not a bailout is beneficial for country *EU*. $B_{P,t} > \tilde{b}_P(M, \bar{y}_{EU,t})$ implies thereby that the EU will not bailout country *P;* $B_{P,t} \leq \tilde{b}_P(M, \bar{y}_{EU,t})$ means that the *EU* government is going to bailout country P.

Hence, the bailout cutoff level is defined as follows:

$$V_{EU}^{b}\left(\tilde{b}_P(M, y_{EU}), 1, \zeta_{P,t}, \pi_P, q_P\right) = V_{EU}^{k}\left(\tilde{b}_P(M, \bar{y}_{EU}), 1, \zeta_{P,t}, \pi_P, q_P\right)$$

$$\frac{u((1-\theta_{EU})\overline{y_{EU}}, \theta_{EU}\overline{y_{EU}} - (B_{EU,t} - \tilde{b}_P(M, \bar{y}_{EU})(1-\beta))}{1-\beta} = \frac{u((1-\theta_{EU})M\overline{y_{EU}}, \theta_{EU}M\overline{y_{EU}} - B_{EU,t}(1-\beta))}{1-\beta}$$

Given a concave utility function, our assumptions and the definition of the cutoff level above, the partial derivatives of $\tilde{b}_P(M, \bar{y}_{EU,t})$ are: $\frac{\partial \tilde{b}_P}{\partial M} \leq 0$ and $\frac{\partial \tilde{b}_P}{\partial \bar{y}_{EU}} \geq 0$. These derivatives are quite intuitive and imply: (i) a higher default penalty $(1-M)$ increases the incentive to bailout the other country and hence increases the bailout cutoff debt level, $\tilde{b}_P(M, \bar{y}_{EU})$; (ii) the higher the normal GDP of the aggregated Eurozone countries, EU, the lower the relative cost of a bailout and thus the higher the bailout cutoff debt level, $\tilde{b}_P(M, \bar{y}_{EU})$.

This section derived the optimal behavior of the *EU* government, when it faces the decision whether or not to bailout the defaulting country *P*. Knowing the optimal behavior and the respective value functions, we are able to derive the bailout cut-off levels. The bailout decision of the *EU* ($h_{EU,t}$), influences also the outcome for the international lenders and hence the bond prices. In the next section, we take a closer look on the bailout effect on bond prices.

5.3 Bond Prices when Bailouts are Possible

Obviously, as we assume rational international investors, not only the governments take a potential bailout into consideration, but also the international lenders. Hence, the bond price function (q_i^*) changes.

As we derived above, the bond prices reflect the discounted expected probability that a country repays its debt in the next period. In the model with a potential bailout, this means that even if a country chooses to default in the next period, it can be bailed out and the international lenders receive their money. Therefore, the international investors form expectations about the realized default decision, $z_{P,t+1}^*$, rather than expectations about the actual default decision, $z_{P,t+1}$.

The first order conditions imply[34]:

$$q_P^*\big(B_{P,t+1}, s_t, \pi_P\big) = \beta E(z_{P,t+1}^*\big(B_{P,t+1}, s_t, q_P\big(B_{P,t+1}, s_t, \pi_P\big), \pi_P, \tilde{b}_P(M, \bar{y}_{EU,t})\big)$$
$$q_{EU}^*\big(B_{EU,t+1}, s_t, \pi_P\big) = \beta E(z_{EU,t+1}\left(B_{EU,t+1}, s_t, q_{EU}\big(B_{EU,t+1}, s_t, \pi_P\big)\right)$$

First of all we can note that q_{EU} does not change compared to the previous case, as we still assume that the *EU* will repay its debt for sure $z_{EU,t+1} = 1$. Hence:

$$q_{EU}^* = \beta$$

For country P the bond price schedule changes due to a potential bailout and hence depends on bailout cutoff level $\tilde{b}_P(M, \bar{y}_{EU,t})$.

First we consider the case, where the new debt level of country *P,* is smaller than the bailout cutoff debt level, $B_{P,t+1}^* < \tilde{b}_P(M, \bar{y}_{EU,t})$. This implies that even if the country chooses to default ($z_{P,t+1} = 0$), the international investors will receive their funds as the country gets bailed out by the EU government ($h_{EU,t+1} = z_{P,t+1}^* = 1$). Then the bond price just represents the discount factor:

[34] See complete maximization problem in appendix E

$$q_P^* = \beta \qquad if\, B_{P,t+1}^* < \tilde{b}_P(M, \bar{y}_{EU,t})$$

As we see from the bond price function above, the bond price is not subject to the probability of a self-fulfilling crisis (π_P) anymore. This implies that for debt levels below the bailout cut-off, the possibility of "pure" self-fulfilling debt crises is eliminated.

In the case that the new debt level of country *P*, is above the bailout cutoff debt level, $B_{P,t+1}^* > \tilde{b}_P(M, \bar{y}_{EU,t})$, country *P* will not get bailed out if it defaults ($h_{EU,t+1} = 0$). This implies that the investors will only receive their funds in case the first-hand default decision of country *P* is negative ($z_{P,t+1} = 1$). Thus, the bond prices are the same as in the scenario with no potential bailout. So we check again if the newly issued debt level is below the lower debt level boundary $B_{P,t+1} \leq \bar{b}(\pi_P)$, above the upper debt level boundary ($B_{P,t+1} > \bar{B}(\pi_P)$) or in the crisis zone($\bar{b}(\pi_P) \leq B_{P,t+1} \leq \bar{B}(\pi_P)$). The bond price if $B_{P,t+1}^* > \tilde{b}_P(M, \bar{y}_{EU,t})$, is therefore given as:

$$q_P^* = \begin{cases} \beta & if\ B_{P,t+1}^* \leq \tilde{b}_P\left(M, \bar{y}_{EU,t}\right) \vee B_{P,t+1} \leq \bar{b}(\pi_P) \\ 0 & if\ B_{P,t+1}^* > \tilde{b}_P\left(M, \bar{y}_{EU,t}\right) \wedge B_{P,t+1} > \bar{B}(\pi_P) \\ \beta((1-\pi_P) & if\ B_{P,t+1}^* > \tilde{b}_P\left(M, \bar{y}_{EU,t}\right) \wedge \bar{b}(\pi_P) \leq B_{P,t+1} \leq \bar{B}(\pi_P) \end{cases}$$

If the bailout cutoff level $\tilde{b}_P(M, \bar{y}_{EU,t})$ is larger than the upper debt level boundary $\bar{B}(\pi_P)$, the confidence of the investors does not play a role anymore at all[35], as the investors will always receive their funds if $B_{P,t+1}^* \leq \tilde{b}_P(M, \bar{y}_{EU,t})$ but will not lend otherwise (see figure 3; figure at the bottom right).[36] To better illustrate the influence of different bailout cutoff levels on the bond prices, we present the bond price schedule for scenario A (no potential bailout) and for different bailout cut-off levels $\tilde{b}_P\left(M, \bar{y}_{EU,t}\right)$ in the figure below (Figure 3.). We can clearly see that the height of the bailout cutoff level influences the bond prices.

[35] Self-fulfilling features do not have any influence anymore in this case.

[36] In our example we assume that the bailout covers the entire debt of the bailed out country, which does not correspond perfectly to reality. If we assume a more realistic, partial bailout we have to consider the size of the bailout into the price function.

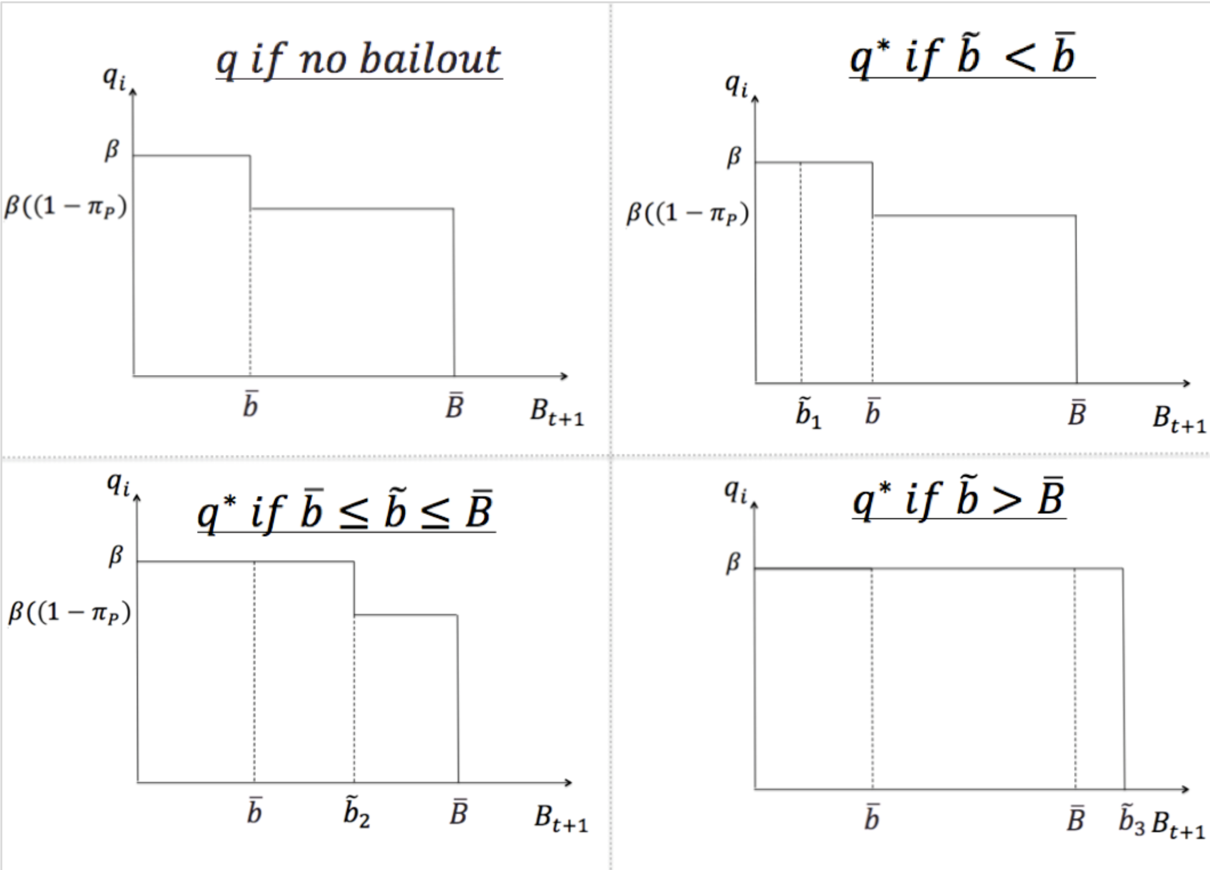

Figure 3: Bond prices for different crucial debt levels

5.4 Optimal Government Behaviour of Country P if Bailouts are Possible

The findings above allow us to investigate the optimal government behavior of country P for the scenario when bailouts are possible. For that we will distinguish between two cases. In the first case, country P's debt level is higher than the bailout cutoff debt level ($B^*_{P,t+1} > \tilde{b}_P(M, \bar{y}_{EU,t})$). This means that country P does not get bailed out after a default. In the second case, country P's debt level is lower than the bailout cutoff debt level $B^*_P \leq \tilde{b}_P(M, \bar{y}_{EU,t})$. This implies that country P will get bailed out if it decides to default.

5.4.1 Debt Level is Higher as Bailout Cutoff

First, we consider the case in which the debt level of country P is higher than the bailout cutoff debt level.

If the debt level of country P is in the "safe zone", $B^*_{P,t+1} < \bar{b}(\pi_P)$, or in the "default zone" $B^*_{P,t+1} \geq \bar{B}(\pi_P)$, the optimal behavior does not change compared to the case with no potential bailout (section 4.6)[37]. However, the optimal behavior for country P may change, if the debt level of country P is in the crisis zone, $\bar{b}(\pi_P) \leq B^*_{P,t+1} \leq \bar{B}(\pi_P)$. In this case, the optimal behavior remains the same, if the cutoff debt level $\tilde{b}_P(M, \bar{y}_{EU,t})$ is smaller than the lower debt level boundary $\bar{b}(\pi_P)$. The only change in the optimal behavior can be observed if $\tilde{b}_P(M, \bar{y}_{EU,t}) \geq \bar{b}(\pi_P)$ and $\tilde{b}_P(M, \bar{y}_{EU,t}) \leq B^*_{P,t+1} \leq \bar{B}(\pi_P)$, which defines the "new crisis zone" (see Figure 4).

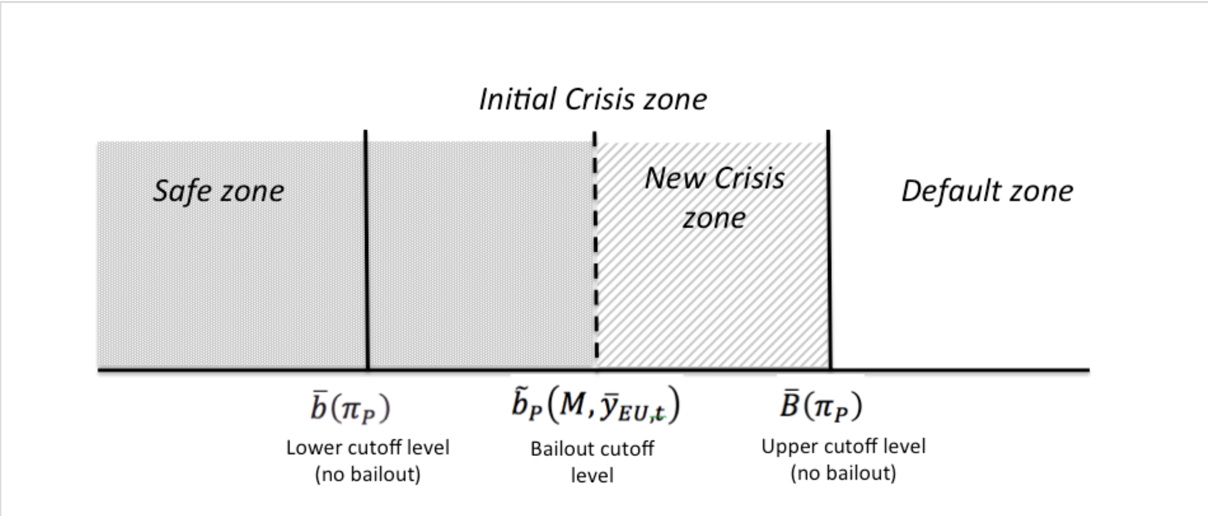

Figure 4: Debt level boundaries with and without bailout

In this case country P is still subject to a self-fulfilling crisis, which implies that the country will default on its debt without being bailed out later[38], if the investors lose confidence in the next period.

Similar as above, the government chooses to run down its debt in T periods[39] or to keep its debt constant. However, it will run down its debt only to $\tilde{b}_P(M, \bar{y}_{EU,t})$ and not to $\bar{b}(\pi_P)$ anymore. This is because the value with debt level $\tilde{b}_P(M, \bar{y}_{EU,t})$ is always larger or equal to the value with debt level $\bar{b}(\pi_P)$:

$$V_P(\tilde{b}_P(M, \bar{y}_{EU,t}), z^*_{P,t-1}, \zeta_{P,t}, \pi_P, q_P) \geq V_P(\bar{b}(\pi_P), z^*_{P,t-1}, \zeta_{P,t}, \pi_P, q_P)$$

[37] However, the crucial debt level boundaries may change, as defaulting offers a "bonus", as the debt level will be zero after a default.

[38] This holds only true if the countries are still above the bailout cut-off level if the confidence loss occurs.

[39] This is the case if the costs of debt reduction is very high (e.g. because of an low β) and the probability of a self-fulfilling crisis are very small, as shown in Conesa & Kehoe (2012). However, the bailout requires even more „extreme" values for this policy to be optimal.

Hence, if the debt level of government P is in the crisis zone, its optimal government decision is equal to[40]:

$$g_{P,T}^{*}\left(B_{i,0},\pi_P\right) = \theta\overline{y_P} - \frac{1-\beta(1-\pi_P)}{1-(\beta(1-\pi_P))^T}\left(B_{P,0}-(\beta(1-\pi_P))^{T-1}\beta\tilde{b}_P\left(M,\bar{y}_{EU,t}\right)\right)$$

And the respective value function is:

$$\begin{aligned}V_P^{T^*}\left(B_{P,0},\pi_P\right) = & \frac{1-(\beta(1-\pi_P))^{T}}{1-\beta(1-\pi_P)}\, u\left((1-\theta_P)\overline{y_P}, g_{P,T}^{*}\left(B_{P,0},\pi_P\right)\right) \\ & + \frac{1-(\beta(1-\pi_P))^{T-1}}{1-\beta(1-\pi_P)}\, \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P},\theta_P Z\overline{y_P})}{1-\beta} \\ & + \beta(1-\pi_P)^{T-2}\frac{\beta u((1-\theta)\overline{y_P},\theta\overline{y_P})}{1-\beta}\end{aligned}$$

This influences also the upper debt level boundary:

$$\begin{aligned}&\max(V_P^1\left(\bar{B}(\pi_P)\right), V_P^2\left(\bar{B}(\pi_P)\right), \dots, V_P^{\infty}\left(\bar{B}(\pi_P)\right)) \\ &\qquad = u\left((1-\theta_P)Z\overline{y_P}, \theta Z\overline{y_P} + \beta(1-\pi_P)\bar{B}(\pi_P)\right) + \frac{\beta u((1-\theta_P)Z\overline{y_P},\theta_P Z\overline{y_P})}{1-\beta}\end{aligned}$$

Observing the upper debt level boundary condition, we can see that $\bar{B}(\pi_P)$ will increase as the value of running down its debt to $\tilde{b}_P\left(M,\bar{y}_{EU,t}\right)$ will increase for all policies (T), as running down its debt includes the "bonus" of defaulting which lowers country P's debt level to zero from the next period onwards.

In the period when the country reaches the bailout cutoff level, the government faces the decision, when its debt level is smaller or equal to the bailout cutoff level $\tilde{b}_P\left(M,\bar{y}_{EU,t}\right)$. We analyze this case in the next section.

[40] This can easily be derived from section 4.6.2.

5.4.2 Debt Level is Lower than Bailout Cutoff

Suppose that the initial debt level of country P, $B_{P,0}$ is below or equal to the bailout cutoff level, $\tilde{b}_P(M, \bar{y}_{EU,t})$, meaning that country P will get bailed out if it chooses to default ($z_{P,t} = 0$).

For this case, we first show that it is better to default than to repay. In a second step we provide theoretical evidence that it can be optimal to increase the debt level up to the bailout cutoff debt level and choose to default afterwards.

If the government chooses to repay its debt and if the initial debt level is smaller than the lower debt level ($B_P \leq \bar{b}(\pi_P)$), the optimal consumption decisions are the same as in the case with no potential bailout:

$$c_{P,t} = (1 - \theta_P)\overline{y_P}$$

$$g_{P,t} = \theta_P \overline{y_P} - B_P(1 - \beta)$$

That implies the following value of repaying:

$$V_P^r\left(B_P, 1, \zeta_{P,t}, \pi_P, \beta\right) = \frac{u((1 - \theta_P)\overline{y_P}, \theta_P \overline{y_P} - B_P(1 - \beta))}{1 - \beta}$$

If, however the country chooses to default on its debt, it will not suffer from the default penalty in this case, as the country gets bailed out at the end of the period. Furthermore, its debt level will decrease to zero after the bailout. The calculation of the optimal consumption path in the case of no debt and thus after a bailout ($B_{P,t} = 0$) are mechanical and are given as:

$$c_{P,t} = (1 - \theta_P)\overline{y_P}$$

$$g_{P,t} = \theta_P \overline{y_P}$$

Implying following value of defaulting:

$$V_P^h\left(B_P, 1, \zeta_{P,t}, \pi_P, \beta\right) = \frac{u((1 - \theta_P)\overline{y_P}, \theta_P \overline{y_P})}{1 - \beta}$$

It is easy to see that, for any prevailing utility function[41], the value of defaulting ($V_P^h(B_P, 1, \zeta_{P,t}, \pi_P, \beta)$) is larger than the value of repaying $V_P^r(B_P, 1, \zeta_{P,t}, \pi_P, \beta)$). Hence, the government will always choose to default if $B_{P,t}^* \leq \tilde{b}_P(M, \bar{y}_{EU,t})$.

Now we can go one step further and suppose that the government will issue new debt in this period ($B_{P,t+1} > 0$) and plans to default next period. The government anticipates two things: a) As long as the newly issued debt level is below the bailout cutoff level ($B_{P,t+1} \leq \tilde{b}_P(M, \bar{y}_{EU,t})$), the international investors will buy country P' s debt for $q_P^* = \beta$; b) Knowing the optimality conditions of the EU, the EU government will bailout country P in a case of $z_{P,t+1} = 0$ as long as ($B_{P,t+1} \leq \tilde{b}_P(M, \bar{y}_{EU,t})$)[42].

So, if the government P i) issues new debt such that $B_{P,t+1} \leq \tilde{b}_P(M, \bar{y}_{EU,t})$), ii) repays the debt brought to the period, but defaults in the next period, it makes following optimal consumption decisions:

$$g_{P,\hat{t}} = \theta\overline{y_P} - \left(B_{P,\hat{t}} - \beta B_{P,\hat{t}+1}\right)$$
$$g_{P,\hat{t}+l} = \theta\overline{y_P} \ , l = 1,2, \dots, \infty$$

$$c_{P,t} = (1 - \theta_P)\overline{y_P}$$

It is easy to show that it is optimal for the government to set $B_{P,\hat{t}+1} = \tilde{b}_P(M, \bar{y}_{EU,t})$[43].

This implies the following value:

$$V_P^e(B_P, 1, \zeta_{P,t}, \pi_P, \beta) = u\left((1 - \theta_P)\overline{y_P}, \theta_P\overline{y_P} - \left(B_{P,\hat{t}} - \beta B_{P,\hat{t}+1}\right)\right) + \frac{\beta u((1 - \theta_P)\overline{y_P}, \theta_P\overline{y_P})}{1 - \beta}$$

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41 A prevailing utility function has a positive marginal utility for government consumption for every government consumption level, $g_{P,t}$.

42 This statement implicitly assumes that the optimality conditions for the EU government will not change next period. This assumption holds true in our model, but might not be true in reality.

43 Suppose to the contrary, that government P will set $B_{P,\hat{t}+1} \leq \tilde{b}_P(M, \bar{y}_{EU,t})$. In this case it can increase its value by increasing its debt level even further but only up to the point, where $B_{P,\hat{t}+1} = \tilde{b}_P(M, \bar{y}_{EU,t})$. If $B_{P,\hat{t}+1} > \tilde{b}_P(M, \bar{y}_{EU,t})$ the government will not be bailed out next period and hence its value decreases as shown above.

The value $V_P^e(B_P, 1, \zeta_{P,t}, \pi_P, \beta)$ is larger than the value of defaulting immediately ($V_P^h(B_P, 1, \zeta_{P,t}, \pi_P, \beta)$), if:

$\beta B_{P,\hat{t}+1} > B_{P,\hat{t}}$.

So we can summarize the optimal strategy of the government as follows:

- If $\beta \tilde{b}_P(M, \bar{y}_{EU,t}) \leq B_{P,\hat{t}}$: Default immediately $(z_{P,t} = 1)$
- If $\beta \tilde{b}_P(M, \bar{y}_{EU,t}) > B_{P,\hat{t}}$: Issue new debt: $\tilde{b}_P(M, \bar{y}_{EU,t}) = B_{P,\hat{t}+1}$, repay debt this period $(z_{P,\hat{t}} = 0)$ and default next period $z_{P,\hat{t}+1} = 0$

After the government gets bailed out we showed above that it is optimal to keep the debt level constant at zero, because we neglect the possibility of a second bailout.
In the figure below, we illustrate possible optimal government debt level paths, derived in the last two sections (5.4.1. & 5.4.2.). Lines I.) and II.) represent the cases if the initial debt level is below the bailout cut-off level: I.) illustrates the case, if $\beta \tilde{b}_P(M, \bar{y}_{EU,t}) \leq B_{P,\hat{t}}$, the country defaults immediately; II.) shows the case in which the government raise its debt level in the first period, but default in the subsequent period ($\beta \tilde{b}_P(M, \bar{y}_{EU,t}) > B_{P,\hat{t}}$).
Lines III.) and IV.) refer to the case, in which the country's debt level is above the bailout cut-off level but below the upper debt level boundary. We derived above that the country chooses either to decrease its debts down to the bailout cut-off level and default afterwards (line III) or to keep debt constant, if the utility costs of debt reduction are very high and the probability of self-fulfilling crisis is low (line IV).

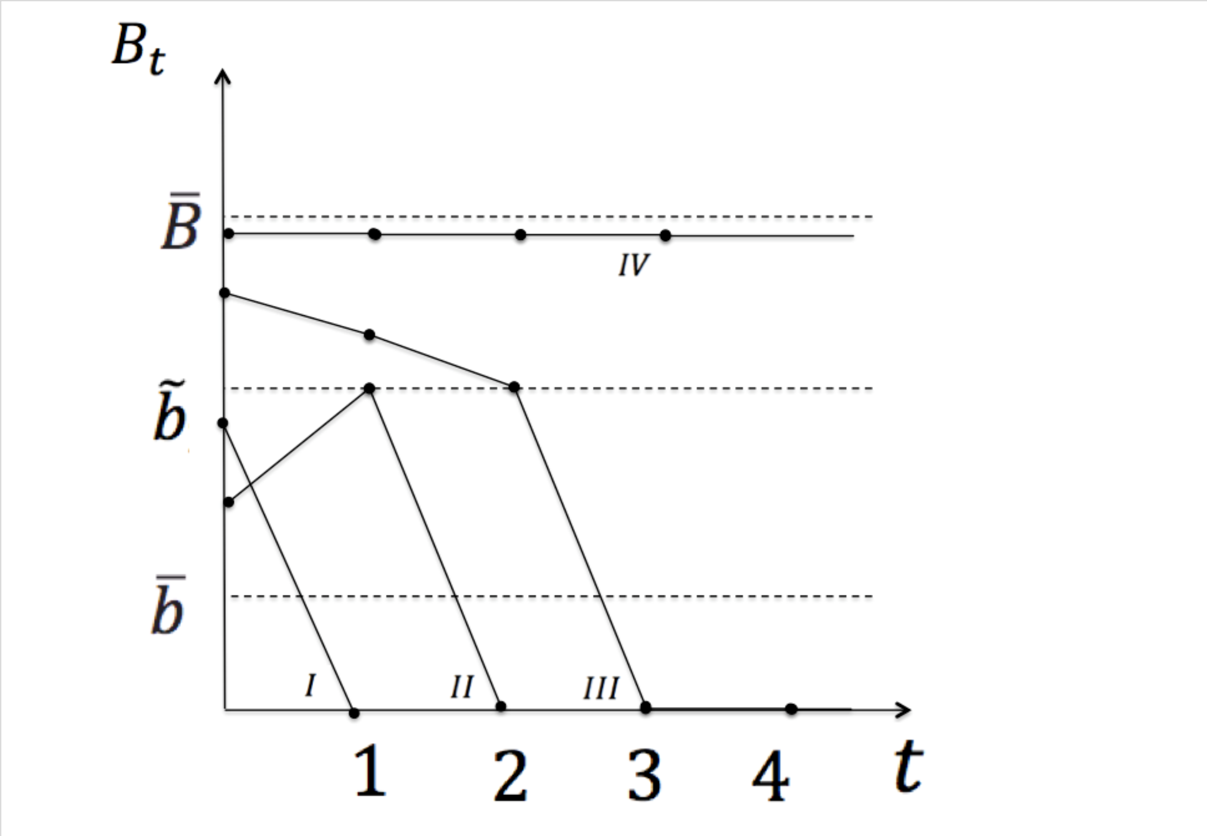

Figure 5: Optimal debt policies

As a conclusion, we can acknowledge that country P can have an incentive to increase its debts and default later on its debt. This moral hazard problem of a potential bailout likely outweighs the positive effect of eliminating a potential self-fulfilling debt crisis. To make further judgments about this point and about other conclusions made above, we calibrate and simulate our model in the next section.

6. Calibration of model

In this section we calibrate our model to several Eurozone countries. We applied recent Eurozone data on our theoretical framework in order to substantiate our theoretical results numerically. We will determine the prevailing crucial debt level boundaries that define the crisis zone ($\bar{B}(\pi_P)$ and $\bar{b}(\pi_P)$) and the bailout cutoff debt level $\tilde{b}_P(M, \bar{y}_{EU,t})$ for selected EU countries. In the following we will describe the calibration approach in more detail. After that we exhibit and interpret actual results that can be used to rationalize recent developments in the Eurozone debt crisis.

6.1 Calibration Approach

In our model we defined the conditions that specify the crucial debt level boundaries: $\bar{B}(\pi_P)$ and $\bar{b}(\pi_P)$ and the bailout cutoff level $\tilde{b}_P(M, \bar{y}_{EU,t})$. The findings are now used to simulate our model for Portugal, Ireland, Italy, Greece, Spain and Germany. In a first step, we need to determine the exogenously given input factors for each country and the entire Eurozone[44]. These inputs are taken from economic data provided by the European Central Bank for the end of year 2013. We include the initial debt level B_i, the "normal" output $\bar{y}_i$ and the constant tax rate θ_i. Table 1, summarizes our input factors for each country.

Input	**Model variable**	**Description**
Debt level	B_i;	Absolute total debt level in 2013
GDP	$\bar{y}_i$,	Absolute GDP in 2013
Tax rate	θ_i	Total government revenues as share of GDP

Table 1 - Exogenous input factors

Furthermore, we need to define the utility function and the general parameters for our model. We do our simulation with two different commonly used log-specifications[45] as our utility function:

1) $$u(c_{i,t}, g_{i,t}) = \vartheta \log(c_{i,t}) + (1-\vartheta) \log(g_{i,t} - \bar{g})$$

2) $$u(c_{i,t}, g_{i,t}) = \vartheta \log(c_{i,t}) + (1-\vartheta) \log(g_{i,t})$$

The general parameters of our model include: the discount factor $\beta = \frac{1}{1+r}$; the default penalty of a default in the own country $(1-Z)$, the default penalty for the entire Eurozone if one country chooses to default $(1-M)$, the likelihood of a confidence loss of the international investors if a country is in the crisis zone π_P, the weight of private consumption vs. government consumption in the consumers utility function ϑ and the minimum required government consumption $\bar{g}$, for the first utility function. The corresponding parameters are presented in Table 2.

[44] For this we limit the Eurozone to the Euro-17 countries.

[45] These two utility functions are commonly used in many economic papers and were also proposed in the study of Conesa and Kehoe (2012).

Parameter	Value	Description
$\boldsymbol{\beta}$	0.98	$\beta = \frac{1}{1+r}$ = 0.98, implies a real interest rate of $r \approx 0.02$, which is equal to the current interest rate for a 12-month risk free bond (AAA) within the Eurozone[46]
$(\mathbf{1}-\boldsymbol{Z})$	0.05	As in Cole & Kehoe (1996)
$\boldsymbol{\pi_P}$	0.03	As in Conesa & Kehoe (2012)
$\boldsymbol{\vartheta}$	*0.75*	Signifies that private consumption is valued 3 times more than government consumption, see Conesa & Kehoe (2012)
$\overline{\boldsymbol{g}}$	$0.25y_{i,t}$	We suppose that 25% of GDP is needed to assure a minimum standard in the Eurozone. Everything below that point will lead to a utility of zero from government consumption.[47] (Conesa & Kehoe, 2014)
$(\mathbf{1}-\boldsymbol{M})$	*0.01*	Foreign default penalty[48]

Table 2: Calibration Parameters

6.2 Model Simulation

To determine the crisis zone $(\bar{b}(\pi_P), \bar{B}(\pi_P))$ of each country and the bailout cutoff-level, $\tilde{b}_P(M, \bar{y}_{EU,t})$, we use the conditions derived above (see sections 4.6 and 5.2) and include the parameters and utility functions defined in this section (see appendix F and G for more details). Our corresponding simulation results are presented in Table 3.

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[46] Source: ECB (http://www.ecb.europa.eu/stats/money/yc/html/index.en.html)
[47] $\bar{g}$ can be interpreted as an essential level of government consumption needed to assure an appropriate infrastructure, social-/health system, save political/economic environment etc.
[48] (1-M) = 0.01 is based on the assumption (1-Z)>(1-M), meaning that the default penalty has to be higher for the actually defaulting country than for the other, only indirectly affected EU countries. The exact estimation of this parameter, however, is difficult and requires further empirical research. Given that the calibration results are quite sensitive to the parameter value, we incorporated a sensitivity analysis for other values in appendix H. Please note that the parameter M does only influences the bailout cut-off level but not the crisis zone.

Country P	Portugal	Ireland	Italy	Greece	Spain	Germany	EU-17
Inputs							
GDP in Mio € ($\overline{\boldsymbol{y_P}}$)	*149.43*	*166.72*	*1,365.23*	*160,98*	*920.95*	*2,482.43*	*8,510.14*
Total Debt in Mio € ($\boldsymbol{B_P}$)	*192.77*	*206.24*	*1,810.29*	*281.88*	*864.77*	*1,946.23*	*7,891.46*
Debt-to-GDP	*1.29*	*1.24*	*1.33*	*1.75*	*0.94*	*0.78*	*0.93*
$\bar{y}_{EU}$ *(EU-GDP ex. country P)*	*8,360.70*	*8,343.42*	*7,144.92*	*8,349.16*	*7,589.19*	*6,027.71*	-
B_{EU} *(EU-Debt, ex. Country P)*	*7,698.69*	*7,685.23*	*6,081.17*	*7,609.58*	*7,026.68*	*5,945.24*	-
θ_P: *avg. tax rate*	*44%*	*36%*	*46%*	*46%*	*38%*	*45%*	*39%*
Utility function 1							
$\bar{\boldsymbol{b}}$ $(abs.)$	*28.31*	*18.26*	*286.65*	*33.81*	*119.60*	*488.95*	-
$\bar{\boldsymbol{b}}$ *(% of total GDP)*	*19%*	*11%*	*21%*	*21%*	*13%*	*20%*	
$\bar{\boldsymbol{B}}$ $(abs.)$	*276.20*	*221.67*	*2,834.72*	*322.38*	*1,365.84*	*4,785.30*	-
$\bar{\boldsymbol{B}}$ $(\%\ of\ total\ GDP)$	*185%*	*133%*	*208%*	*200%*	*148%*	*193%*	
$\tilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(abs.)$	*2,077.28*	*2,073.26*	*1,789.90*	*2,076.89*	*1,884.58*	*1,485.84*	-
$\tilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(\%\ of\ total\ GDP)$	*1390%*	*1243%*	*131%*	*1290%*	*205%*	*59%*	
Utility function 2							
$\bar{\boldsymbol{b}}$ $(abs.)$	*65.56*	*59.76*	*627.88*	*74.06*	*349.59*	*1,109.42*	-
$\bar{\boldsymbol{B}}$ $(abs.)$	*478.80*	*446.59*	*4,692.35*	*553.47*	*2,615.43*	*8,161.00*	-
$\tilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(abs.)$	*6,195.00*	*6,183.09*	*5,308.68*	*6,189.19*	*5,622.54*	*4,454.44*	-

Table 3: Calibration Results

6.3 Interpretation of Results

The results depicted in in table 3 incorporate interesting findings. First of all, it can be seen that all countries have total debt levels (B_P) that are between the upper $\bar{B}(\pi_P)$ and lower $\bar{b}(\pi_P)$ crucial debt level boundaries. In other words, all countries are inside the crisis zone. This implies that, in theory, all countries are prone to self-fulfilling debt crises. Interestingly, not only troubled countries (such as the PIIGS) but also countries, which are perceived as being “safe” (such as Germany) are threatened by the risk that self-fulfilling mechanisms may evolve. These results are robust for different parameters and utility functions (see also appendix H).

This fact gets less surprising in light of the publicly discussed debate about inappropriate borrowing behavior in the EU. In effect, the debt levels of all countries exceed the allowed maximums of the Maastricht treaty since 2010. This was possible because many EU member states were able to circumvent the rules of the Maastricht treaty, which requires a maximum debt level of 60 percent of GDP, by masking deficit and debt levels through a combination of techniques such as inconsistent accounting, off-balance-sheet transactions and the usage of complex currency and credit derivative structures. (Brown & Chambers, 2005; Simkovic 2009). Therefore our model supports the critique that current EU debt levels violate pre-agreed conditions and are in effect too high.

In addition to that, the calibration results show that Greece’s and Ireland’s debt levels (B_P) are close to the upper boundary $\overline{\mathrm{B}}(\pi_{\mathrm{P}})$, where defaults become inevitable (in case of no bailout). In particular, Ireland’s and Greece’s upper crisis zone boundaries are 107% and 114% of total debts. So, if the debt levels of these countries continue to rise (more than 7% and 14%, respectively), both countries will, according to our model, optimally choose to default.

Further interesting results can be derived from the numerical findings associated with the bailout cut-off levels $\tilde{b}_P$ (i.e. the debt level, above which it gets optimal to default, anticipating a bailout). First, it can be seen that smaller countries (such as Portugal and Ireland) have higher bailout cut-off levels. This implies that these countries are more likely to get bailed out if they decide to default given their smaller economic size. Anticipating this effect, smaller countries therefore have a higher incentive to raise their debts and eventually default on them. In addition, it is interesting to see that the crucial bailout cutoff debt levels for Portugal, Ireland, Greece and Spain lie above the upper debt level boundaries. This implies that these countries will get bailed out in case of a default. Therefore a default cannot happen solely because of an investors’ confidence loss, which implies that a “pure” self-fulfilling crisis is not possible for these countries.

However, larger countries such as Germany or Italy are still prone to the threat of self-fulfilling crises because they will not get bailed out in case of a default. This is because their actual debt levels are higher than the bailout cutoff debt levels. Further, it is interesting to note that the bailout cutoff-levels for Germany and Italy are between the lower and the upper debt level boundary. As can be seen in Figure 4, this will reduce the range of the crisis zone to a new, smaller crisis zone. However, self-fulfilling mechanisms are still possible. Consequently, Germany and Italy are still vulnerable to a self-fulfilling debt crisis even in a scenario, where bailouts exist. [49]In sum, the calibration results of our model seem to represent the current situation of the Eurozone not only qualitatively but also quantitatively.[50]

7. General Results and Implications

Our model extended current models of self-fulfilling debt crises by incorporating the effects of bailout possibilities and cross-country influences in a two country setup. The model enabled us to provide a theoretical and empirical analysis of a self-fulfilling debt crisis in the Eurozone. We derived upper and lower debt level boundaries that define the crisis zone, in which self-fulfilling debt crises can occur theoretically. Based on that, we calibrated our model and showed empirically that nearly all European countries, even rather healthy ones such as Germany, are inside the crisis zone and are therefore vulnerable to self-fulfilling debt crises. We empirically illustrated that some European countries, namely Portugal, Greece and Ireland, are quite close to the upper debt level boundary, above which defaulting is the optimal decision.

A possibility for the EU to improve this situation would be to increase the default penalty for a single country. This can be achieved by several policies, for example exclude countries from supporting measures after a default or make bilateral trade dependent on debt repayment.[51]

Moreover, we evaluated the effect of bailouts on the outcome of our model. We showed, both theoretically and empirically in the simulation, that whether or not the Eurozone decides to bailout one of its member states depends on the absolute debt level and therefore indirectly on the economic size of the troubled country. Therefore, the incentive to raise debt levels is much larger for comparably small countries with a low

[49] Changing the foreign default parameter M (see appendix H) leads to a situation, in which a self-fulfilling crisis is still possible for Germany, but in which the current debt level is outside of the crisis zone.

[50] The quantitative part, however, is quite sensitive to the parameterization, see for instance appendix H

[51] Supporting measures include EU subsidies, special taxes for inter-European trade etc. It is important to note that these threats have to be credible in order to work.

absolute debt level, such as Ireland or Greece, because the likelihood of a bailout is larger. Additionally, we demonstrated that the EU might not bail out bigger countries, such as Germany, in case of a default due to too high bailout costs. These results can rationalize current developments in the Eurozone crisis, such as why European leaders decided to bailout Greece and why debt levels of smaller countries (such as Ireland or Portugal) continue to rise (as seen in Figure 1).

It is important to note that the way, in which we modeled a bailout in our thesis, is not perfectly applicable to the case of Greece. We assume that a bailout will be powerful enough to decrease the defaulting country's debt to a level where no self-fulfilling features can occur anymore. However, the recent bailout for Greece was too small and "the public debt remained too high" (IMF, 2013), such that a self-fulfilling crisis is till possible.

In addition, we derived a bailout cut off debt level that determines the maximum debt level of the single country for which it is beneficial for the *EU* to bailout the respective country. According to our model, this bailout cut off level also represents the upper ceiling of the recent debt level hike of European countries.

If the bailout cutoff level is large enough a potential bailout may shrink the range of the crisis zone or even completely eliminate the possibility of default that is solely triggered by self-fulfilling features. Hence, bailing out a country might be suggestive if the threat of a self-fulfilling crisis is very large.

However, a shrinking crisis zone does not automatically imply that there will be fewer defaults in the future. It just implies that the reason why such a default occurs is not self-fulfilling anymore.

This arising moral hazard problem can have severe effects for the Eurozone as it incentivizes countries to raise their debt level in hope of a potential bailout. This holds even if they were initially in the safe zone and consequentially trigger a default, which in turn leads to an increase of the overall EU debt level or to a decline of EU output.

Moreover, in reality it is nearly impossible to certainly predict a future bailout. Hence, the government can be in a situation, in which it raised its debt level in the hope of a future bailout initially, but cannot hope for a bailout anymore when the actual default decision is made. Through such a process a potential bailout can even trigger a self-fulfilling crisis[52].

Ruling out future bailouts has to be credible, which is hard to achieve in reality.[53] Hence, evoking additional output costs for countries that get bailed out, such as the austerity measures for Greece, might be a better solution to tackle this moral hazard problem.

In general, all strategies in which players can perfectly commit to future actions, such as repaying or not bailing out a country, has the potential to eliminate the occurrence of self-fulfilling crisis of any kind. However, this is nearly impossible to achieve in reality[54].

A further interesting result of our model is that the entire Eurozone would benefit from policies that faster decrease the debt level of a single country, if the single country is in the crisis zone. However, the government of the single country does not take this effect on the Eurozone into account when it makes its decision. This implies, that the overall welfare in the Eurozone could be increased if a third benevolent party would make the debt decisions integrated, for example through Eurobonds.[55]

We can conclude that, in theory, the best way to completely erase the possibilities of a self-fulfilling debt crisis in the Eurozone is to decrease the European countries' debt levels below the lower debt level boundary. Since the current debt levels are far away from the lower debt level boundary, the period of a sufficient debt reduction would be long and painful, which make this endeavor politically nearly impossible to achieve.

Hence, thoroughly pondering the pros and cons of future bailouts might be the best feasible way to limit negative outcomes of a self-fulfilling debt crisis to a tolerable level.

8. Possible Extensions & Limitations

Our model provides a framework to analyze self-fulfilling debt crises in situations, in which close economical interactions between countries are important. Hence, further extensions in the context of monetary or economic unions would be interesting to

[52] Such a situation can arise from changes in the underlying assumption, such as political changes, strategy changes because of increasing overall debt levels, or from wrong expectations about future events.

[53] Not bailing out a country, even if it would be beneficial to do so, may be favorable in the long run as it signalizes to other countries that they will not be bailed out in the future as well.

[54] As discussed in section 4.5.

[55] Making the debt decision integrated would have also the advantage that the probability of a confidence loss of the investors' would probably decrease as the economic power of creditor is larger and the ECB can intervene in emergency.

incorporate in future analyses. Relaxing some simplifying assumptions applied in our model would advance the analysis to a more applicable and realistic setup. For instance, incorporating endogenous output functions or varying the private budget constraint would provide interesting insights about the influence of labor and capital factor productivity or the effect of private capital and investments.[56]

Another promising extension would be to evaluate potential limitations of current *EU* policies. For that, the aggregated Eurozone countries could be made subject to a confidence loss of the investors as well, if a certain level of overall debt level is reached. This would negatively influence the incentive to bailout other member states because the subsequent debt level increases, which may make the entire Eurozone vulnerable to self-fulfilling mechanisms. In that respect it may be also interesting to incorporate a default penalty, which is dependent on other factors. These factor could be either the economic importance of the single country or alternatively, the likelihood of a confidence loss subject to the relative debt level of the single country.

We further show that the effect of a potential bailout is twofold. On the one hand, it can counteract self-fulfilling mechanisms. On the other hand, a moral hazard problem that incentivizes countries to raise its debt levels arises. Closer evaluating which of these opposing effects is more favorable in the current situation would provide valuable policy implications for European decision makers.

Moreover, our model incorporates perfectly available information whether or not a bailout will be provided (depending on a country's size and debt level). However, in reality this may not be applicable. Therefore governments and international lenders would have to form expectations. It is very likely that wrong expectations will lead to decisions that differ greatly from the optimal rational behavior derived in our model. Hence, future research on the influence and evolution of diverging expectations on the development of self-fulfilling debt crises would be very promising in our opinion.

The importance of expectations is underlined by the fact that the simulation of our model is quite sensitive with respect to its parameterization. The sensitivity of our model signifies that even slightly wrong expectations can have large effects on the economical outcome. In this regard, it also might also be helpful to dedicate more time to an more precise calibration of the model, to get more meaningful and robust results.

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[56] Further simplifying assumptions that may be worth to relax are: the strict exclusion from financial markets after a default, the cost and effect of a bailout and time invariant preferences (e.g. because of political changes).

In general, our model does not predict whether a self-fulfilling crisis will happen or not, it rather derives conditions under which a self-fulfilling crisis can happen. Extending this strand of literature by the above mentioned extensions would be valuable in understanding and ultimately preventing the negative consequences of self-fulfilling mechanisms.

9. Conclusion

The on-going Eurozone crisis is to a large extent a sovereign debt crisis, in which self-fulfilling mechanisms aggravated the situation. Self-fulfilling mechanisms may cause a crisis, which is not solely caused by unhealthy economic fundamentals but rather a consequence of pessimistic expectations of investors (Chamon, 2004). To better understand these self-fulfilling mechanisms in the context of the Eurozone crisis, this thesis presents a dynamic stochastic equilibrium model to analyse the following research question: *Are countries in the Eurozone vulnerable to self-fulfilling crises, and if yes, can bailing out troubled countries solve this problem?*

To answer this question we distinguished between two scenarios. In scenario A, we derived the condition for self-fulfilling crises in the Eurozone. In scenario B we extended the framework with a bailout possibility to determine the influence of a potential bailout.

In scenario A we derived the conditions to answer the first part of our research question, whether countries in the Eurozone are vulnerable to self-fulfilling debt crisis. We showed that not only troubled countries, such as Portugal, Ireland, Italy, Greece and Spain, but also rather healthy countries such as Germany, are inside this crisis zone, which makes them vulnerable to a self-fulfilling debt crisis.

Scenario B enabled us to approach the second part of our research question concerning the effects of bailouts. However, this part is more difficult to answer because the effect of bailing out troubled countries is ambiguous. On the one side it can prevent defaults that happen solely because of an investor's confidence loss. On the other side it incentivizes countries, which anticipate a potential bailout, to raise their debts up to a certain cut-off-level. Whether or not a country will get bailed out depends on this endogenous bailout cut-off-level and is more likely the lower the absolute debt level of the respective country is. In sum, our model provides a new framework to analyse self-fulfilling debt crises in an environment of close cross-country interdependencies. This framework cannot only explain why the EU decided to bailout Greece, but also why particularly small European countries, such as Ireland, Portugal and Greece, continue to raise their debts.

10. References

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11. Appendices

Appendix A: Maximization Problem International Lenders

The maximization problem of the international lenders can be illustrated as follows:

$$W\left(b_{i,t}, B_{i,t+1}, s_t, \pi_P\right) = \max_{q_i, b_i} E \sum_{t=1}^{\infty} \beta^{t-1} x_t$$

$$s.t.$$

$$\begin{aligned} x_t + q_P\left(B_{P,t+1}, s_t, \pi_P\right) b_{P,t+1} + q_{EU}\left(B_{EU,t+1}, s_t, \pi_P\right) b_{EU,t+1} \\ = w_t + z_{P,t}\left(B_{P,t+1}, s_t, q_P\left(B_{P,t+1}, s_t, \pi_P\right), \pi_P\right) b_{P,t} \\ + z_{EU,t}\left(B_{EU,t+1}, s_t, q_{EU}\left(B_{EU,t+1}, s_t, \pi_{EU}\right), \pi_{EU}\right) b_{EU,t} \end{aligned}$$

$$x \geq 0$$

$$b_{i,t} \geq -A$$

This problem can be rewritten as follow Lagrangian:

$$
\begin{aligned}
&\mathcal{L}\big(q_{i,t}, b_{i+1}, \lambda_t, \lambda_{t+1}, \dots\big) \\
&\qquad = \sum_{t=1}^{\infty} \beta^{t-1} x_t \\
&\qquad - \lambda_t\big(x_t + q_P\big(B_{P,t+1}, s_t, \pi_P\big)b_{P,t+1} + q_{EU}\big(B_{EU,t+1}, s_t, \pi_P\big)b_{EU,t+1} - w_t \\
&\qquad - z_{P,t}\big(B_{P,t+1}, s_t, q_P\big(B_{P,t+1}, s_t, \pi_P\big), \pi_P\big)b_{P,t} \\
&\qquad - \ z_{EU,t}\big(B_{EU,t+1}, s_t, q_{EU}\big(B_{EU,t+1}, s_t, \pi_{EU}\big), \pi_{EU}\big)b_{EU,t}\big) \\
&\qquad - \lambda_{t+1}\big(x_{t+1} + q_P\big(B_{P,t+2}, s_{t+1}, \pi_P\big)b_{P,t+2} + q_{EU}\big(B_{EU,t+2}, s_{t+1}, \pi_P\big)b_{EU,t+2} \\
&\qquad - w_{t+1} - z_{P,t+1}\big(B_{P,t+2}, s_{t+1}, q_P\big(B_{P,t+2}, s_t, \pi_P\big), \pi_P\big)b_{P,t+1} \\
&\qquad - \ z_{EU,t+1}\big(B_{EU,t+2}, s_{t+1}, q_{EU}\big(B_{EU,t+2}, s_{t+1}, \pi_{EU}\big), \pi_{EU}\big)b_{EU,t+1}\big) - \cdots
\end{aligned}
$$

If we differentiate $\mathcal{L}\big(g_{P,t}, B_{i,t+1}, \lambda_t, \lambda_{t+1}, \dots\big)$, with respect to the variables $x_{i,t}$ *and* $b_{i,t+1}$, we see that it is beneficial for the government to buy bonds as long as the expected repayment next period is higher as the respective price. We further assumed perfect competition in the bonds market, meaning that price will be pushed down do its "minimum" (no positive profit possible). This bond price "minimum" is just the discounted expected probability of repayment in the next period. Hence, the bond price conditions can be illustrated as follows:

$$
q_P\big(B_{P,t+1}, s_t, \pi_P\big) = \beta E(z_{P,t+1}\big(B_{P,t+1}, s_t, q_P\big(B_{P,t+1}, s_t, \pi_P\big), \pi_P\big)
$$
$$
q_{EU}\big(B_{EU,t+1}, s_t, \pi_P\big) = \beta E(z_{EU,t+1}\left(B_{EU,t+1}, s_t, q_{EU}\big(B_{EU,t+1}, s_t, \pi_P\big)\right)
$$

Appendix B: Deriving the Value Function by Backward Induction

Plug in $V_{P,1}^T\big(B_{P,0}, \pi_P\big)$ in $V_{P,2}^T\big(B_{P,0}, \pi_P\big)$:

$$
\begin{aligned}
V_{P,2}^T\big(B_{P,0}, \pi_P\big) = u\left((1-\theta_P)\overline{y_P}, g_{P,T}\big(B_{P,0}, \pi_P\big)\right) + \beta(1-\pi_P)u\left((1-\theta_P)\overline{y_P}, g_{P,T}\big(B_{P,0}, \pi_P\big)\right) \\
+ \frac{(1-\pi_P)\beta u((1-\theta_P)\overline{y_P}, \theta\overline{y_P})}{1-\beta} + \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}
\end{aligned}
$$

$$= \left(1 + \beta(1-\pi_P)\right)u\left((1-\theta_P)\overline{y_P}, g_{P,T}\left(B_{P,0},\pi_P\right)\right) + \frac{(1-\pi_P)\beta u((1-\theta_P)\overline{y_P}, \theta\overline{y_P})}{1-\beta} + \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

$V_{P,2}^T\left(B_{P,0},\pi_P\right)$ in $V_{P,3}^T\left(B_{P,0},\pi_P\right)$:

$$V_{P,3}^T\left(B_{P,0},\pi_P\right) = u\left((1-\theta_P)\overline{y_P}, g_{P,T}\left(B_{P,0},\pi_P\right)\right) + \beta(1-\pi_P)(\left(1 + \beta(1-\pi_P)\right)u\left((1-\theta_P)\overline{y_P}, g_{P,T}\left(B_{P,0},\pi_P\right)\right) + \frac{\beta u\left((1-\theta_P)\overline{y_P}, \theta\overline{y_P}\right)}{1-\beta} + \frac{\beta\pi_P u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P}\right)}{1-\beta}) + \frac{\beta\pi_P u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P}\right)}{1-\beta}$$

$$= \left(1 + \beta(1-\pi_P) + \left(\beta(1-\pi_P)\right)^2\right) u\left((1-\theta_P)\overline{y_P}, g_{P,T}\left(B_{P,0},\pi_P\right)\right) + \left(1 + \beta(1-\pi_P)\right)\frac{\beta\pi_P u\left((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P}\right)}{1-\beta} + \left(\beta(1-\pi_P)\right) \frac{\beta u\left((1-\theta_P)\overline{y_P}, \theta\overline{y_P}\right)}{1-\beta}$$

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Until you reach the last step T, which is given (applying geometric series):

$$V_P^T\left(B_{P,0},\pi_P\right) = \frac{1-(\beta(1-\pi_P))^T}{1-\beta(1-\pi_P)} u\left((1-\theta_P)\overline{y_P}, g_{P,T}\left(B_{P,0},\pi_P\right)\right) + \frac{1-(\beta(1-\pi_P))^{T-1}}{1-\beta(1-\pi_P)} \frac{\beta\pi_P u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta} + \beta(1-\pi_P)^{T-2}\frac{\beta u((1-\theta)\overline{y_P}, \theta\overline{y_P})}{1-\beta}$$

Appendix C: Maximization Problem of the EU Government

The EU government solves the following maximization problem:

$$\max_{g_{EU,t}, B_{EU,t}, c_{EU,t}} \sum_{t=1}^{\infty} \beta^{t-1} u\left(c_{EU,t}, g_{EU,t}\right)$$

$$s.t.\ c_{EU,t} = (1-\theta_{EU})\,\overline{y_{EU}}$$

$$g_{EU,t} + B_{EU,t} = \theta\overline{y_{EU}} + \beta B_{EU,t+1}$$

$$B_{EU,0} = B_{EU} \leq \bar{b}(\pi_{EU})$$

, where B_{EU}, is the exogenous given, pre-existing debt level of country EU, when country EU "enters" the model at $t = 0$. This assumption can be represented by $B_{EU,0} = B_{EU} \leq \bar{b}(\pi_{EU})$

$$\mathcal{L}(g_{EU,t}, B_{EU,t+1}, \lambda_t, \lambda_{t+1}, \dots) = \sum_{t=1}^{\infty} \beta^{t-1} u\big((1-\theta)\,\overline{y_{EU}}, g_{EU,t}\big) - \lambda_t\big(g_{EU,t} + B_{EU,t} - \theta\overline{y_{EU}} - \beta B_{EU,t+1}\big) - \lambda_{t+1}\big(g_{EU,t+1} + B_{EU,t+1} - \theta\overline{y_{EU}} - \beta B_{EU,t+2}\big) - \cdots$$

If we differentiate $\mathcal{L}(g_{EU,t}, B_{EU,t+1}, \lambda_t, \lambda_{t+1}, \dots)$, with respect to the policy variables in period t, $g_{EU,t}$ and $B_{EU,t+1}$, we get the following first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial g_{EU,t}} = \beta^{t-1} \frac{\partial u}{\partial g_{EU,t}}\big((1-\theta)\,\overline{y_{EU}}, g_{EU,t}\big) - \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial\, B_{EU,t+1}} = \beta\lambda_t - \lambda_{t+1} = 0$$

The transversality condition is given by:

$$\lim_{t \to \infty} \lambda_t B_{EU,t+1} \geq 0$$

Knowing that $\beta\lambda_t = \lambda_{t+1}$ and $\beta^t \frac{\partial u}{\partial g_{EU,t+1}}\big((1-\theta)\,\overline{y_{EU}}, g_{EU,t+1}\big) = \lambda_{t+1}$, we get that:

$$\frac{\partial u}{\partial g_{EU,t}}\big((1-\theta)\,\overline{y_P}, g_{EU,t}\big) = \frac{\partial u}{\partial g_{EU,t+1}}\big((1-\theta)\,\overline{y_{EU}}, g_{EU,t+1}\big)$$

which implies a constant government consumption, $\overline{g_{EU}}$:

$$g_{EU,t+1} = g_{EU,t} = \bar{g}_{EU}$$

Plugging this into the government budget constraint, we get:

$$B_{EU,t+1} = \frac{1}{\beta}(\bar{g}_{EU} + B_{EU,t} - \theta\overline{y_{EU}})$$

We showed that with this condition the debt level has to be constant as well. In order to have a constant debt level $(B_{P,t+1} = B_{P,t} = B_P)$, the government consumption has to be equal to:

$$\bar{g}_{EU} = \theta\overline{y_{EU}} - B_{EU}(1-\beta)$$

Hence,

$$B_{EU,t+1} = B_{EU,t} = B_{EU}$$

The optimal private consumptions can be derived mechanical:

$$c_{EU,t} = (1-\theta_{EU})\,\overline{y_{EU}}$$

If a default has occurred in country P ($z_{P,t} = 0$). EU's output changes to $y_{EU,t} = M\overline{y_{EU}}$. Thus, the budget constraints but not the actual optimality conditions change. This implies following optimal decisions for the EU government if a default occurs in the other country:

$$g_{EU,t+1} = g_{EU,t} = \bar{g}_{EU} = \theta M\overline{y_{EU}} - B_{EU}(1-\beta)$$
$$c_{EU,t} = (1-\theta_{EU})M\,\overline{y_{EU}}$$

Please note that the optimal consumption levels and debt levels are constant, but may plummet to a lower level if the other country defaults. Therefore, EU's value function depend on decisions in the other country.

Appendix D: EU Value Function if Country P is in the Crisis Zone

If the country P is in the crisis zone it faces the maximization problem illustrated in section 4.6.2. The government can choose between different debt policies, namely different time horizons to decrease its debt level down to the lower debt boundary $\bar{b}(\pi_P)$.

Government *P*'s value of running down its debt in *T* – periods is given by:

$$V_P^T(B_{P,0},\pi_j) = \frac{1-(\beta(1-\pi_P))^{\,T}}{1-\beta(1-\pi_P)}\, u\Big((1-\theta)\overline{y_P}, g_{P,T}(B_{P,0},\pi_P)\Big) + \frac{1-(\beta(1-\pi_P))^{\,T-1}}{1-\beta(1-\pi_P)}\,\frac{\beta\pi_P u((1-\theta)Z\overline{y_P},\theta Z\overline{y_P})}{1-\beta} + (\beta(1-\pi_P))^{T-2}\frac{\beta u((1-\theta)\overline{y_P},\theta\overline{y_P})}{1-\beta}$$

The government chooses the policy (the number of periods *T*) that maximize its value function:

$$T_P = \arg\max_T V_T(B_{P,0},\pi_P)$$

Government *EU* anticipates the optimal debt policy and incorporates it into his maximization problem. Even if the government knows what the other country is doing the outcome is still uncertain, as it is not clear whether a self-fulfilling default will occur or not. In each period, the probability that there will be a loss in investor's confidence and thus a self-fulfilling debt crisis, is equal to π_P. If there is a confidence crisis before country *P's* debt level reaches the safe zone ($B_P \leq \bar{b}(\pi_P)$), there is a default on country *P*'s debt and the output from country *EU* will drop as well.

The value for country i in state: $s_t(B_{i,t}, z_{P,t}, \zeta_{P,t})$, depends on π_P and the policy of country P (T_P). We denote the value of country *EU* as $V_{EU}^T(B_{i,0}, 1, \pi_j)$ for policy T_P. $V_{EU,n}^T(B_{i,0}, 1, \pi_j)$ illustrates country *EU*'s value for the policy country *P* "running its debt down in *T* periods" with *n* periods remaining till we hit the lower debt level boundary. As the outcome in the future is uncertain, $V_{EU,n}^T(B_{EU,0},\pi_j)$ it is an expected value:

$$V_{EU,T}^T(B_{EU,0},\pi_P) = u\big((1-\theta_{EU})\overline{y_{EU}}, \theta\overline{y_{EU}} - B_{EU}(1-\beta)\big) + \beta(1-\pi_P)V_{EU,T-1}^T(B_{EU,0},\pi_P) + \frac{\beta\pi_P u((1-\theta_{EU})M\overline{y_{EU}}, \theta_{EU}M\overline{y_{EU}} - B_{EU}(1-\beta))}{1-\beta}$$

$$V_{EU,T-1}^{T}\left(B_{EU,0},\pi_P\right)$$
$$= u\left((1-\theta_{EU})\overline{y_{EU}},\theta\overline{y_{EU}}-B_{EU}(1-\beta)\right)+\beta(1-\pi_P)V_{EU,T-2}^{T}\left(B_{EU,0},\pi_P\right)$$
$$+\frac{\beta\pi_P u((1-\theta_{EU})M\overline{y_{EU}},\theta_{EU}M\overline{y_{EU}}-B_{EU}(1-\beta))}{1-\beta}$$

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$$V_{EU,2}^{T}\left(B_{EU,0},\pi_P\right)$$
$$= u\left((1-\theta_{EU})\overline{y_{EU}},\theta\overline{y_{EU}}-B_{EU}(1-\beta)\right)+\beta(1-\pi_P)V_{EU,1}^{T}\left(B_{EU,0},\pi_P\right)$$
$$+\frac{\beta\pi_P u((1-\theta_{EU})M\overline{y_{EU}},\theta_{EU}M\overline{y_{EU}}-B_{EU}(1-\beta))}{1-\beta}$$

$$V_{EU,1}^{T}\left(B_{EU,0},\pi_P\right)$$
$$= u\left((1-\theta_{EU})\overline{y_{EU}},\theta_{EU}\overline{y_{EU}}-B_{EU}(1-\beta)\right)$$
$$+\frac{\beta u((1-\theta_{EU})\overline{y_{EU}},\theta_{EU}\overline{y_{EU}}-B_{EU}(1-\beta))}{1-\beta}$$

Again we can use backward induction to get the general value of policy T_P, the number of periods in which country P runs down its debt:

$$V_{EU}^{T}\left(B_{EU,0},\pi_P\right)=\frac{1-(\beta(1-\pi_P))^{T_P}}{1-\beta(1-\pi_P)}\,u\left((1-\theta_{EU})\overline{y_{EU}},\theta_{EU}\overline{y_{EU}}-B_{EU}(1-\beta)\right)$$
$$+\frac{1-(\beta(1-\pi_P))^{T_P-1}}{1-\beta(1-\pi_P)}\,\frac{\beta\pi_{EU}u((1-\theta_{EU})M\overline{y_{EU}},\theta_{EU}M\overline{y_{EU}}-B_{EU}(1-\beta))}{1-\beta}$$
$$+\beta(1-\pi_P)^{T_P-2}\frac{\beta u((1-\theta_{EU})\overline{y_{EU}},\theta_{EU}\overline{y_{EU}}-B_{EU}(1-\beta))}{1-\beta}$$

The value for keeping the debt constant $V_{EU}^{\infty}\left(B_{EU,0},\pi_P\right)$ is then given by:

$$V_{EU}^{\infty}\left(B_{EU,0},\pi_P\right)$$
$$=\frac{u\left((1-\theta_{EU})\overline{y_{EU}},\theta_{EU}\overline{y_{EU}}-B_{EU}(1-\beta)\right)}{1-\beta(1-\pi_P)}$$
$$+\frac{\beta\pi_P u((1-\theta_{EU})M\overline{y_{EU}},\theta_{EU}M\overline{y_{EU}}-B_{EU}(1-\beta))}{(1-\beta)(1-\beta(1-\pi_P))}$$

Appendix E: Maximization Problem of International Lenders with Potential Bailout

Hence, the maximization problem of the international lenders changes to:

$$W\left(b_{i,t}^{*}, B_{i,t+1}^{*}, s_t, \pi_i, \xi_{j,t}^{B}\right) = \max(x_t + \beta EW(b_{i,t+1}^{*}, B_{i,t+2}^{*}, s_{t+1}, \pi_i, \xi_{j,t+1}^{B}))$$

$$s.t.\ x_t + \sum_{i=1}^{2} q_i^{*}\left(B_{i,t+1}^{*}(\xi_{j,t}^{B}), s_t, \pi_{i,}, \xi_{j,t+1}^{B}\right) b_{i,t+1}$$

$$= w_t + \sum_{i=1}^{2} z_{i,t}^{*}\left(B_{i,t+1}^{*}, s_t, q_i^{*}\left(B_{i,t+1}^{*}(\xi_{j,t}^{B}), s_t, \pi_i, \xi_{j,t+1}^{B}\right), \pi_i, \tilde{b}_P(M, \bar{y}_{EU,t})\right) b_{i,t}, i \neq j$$

$$x \geq 0$$

$$b_{i,t} \geq -A$$

Appendix F: Determining the Absolute Bailout Cutoff Debt Level (Calibration)

$\tilde{b}_P(M, \bar{y}_{EU})$ is defined as follows (see section 5.2):

$$V_{EU}^{b}\left(\tilde{b}_P(M, y_{EU}), 1, \zeta_{P,t}, \pi_P, q_P\right) = V_{EU}^{k}\left(\tilde{b}_P(M, \bar{y}_{EU}), 1, \zeta_{P,t}, \pi_P, q_P\right)$$

$$\frac{u((1-\theta_{EU})\overline{y_{EU}}, \theta_{EU}\overline{y_{EU}} - (B_{EU,t} - \tilde{b}_P(M, \bar{y}_{EU}))(1-\beta))}{1-\beta} = \frac{u((1-\theta_{EU})M\overline{y_{EU}}, \theta_i M\overline{y_{EU}} - B_{EU,t}(1-\beta))}{1-\beta}$$

Applying utility function (1) ($u\left(c_{i,t}, g_{i,t}\right) = \vartheta \log\left(c_{i,t}\right) + (1-\vartheta)\log\left(g_{i,t} - \bar{g}\right)$) to this condition we get:

$$\frac{\vartheta \log\left((1-\theta_{EU})\overline{y_{EU}}\right) + (1-\vartheta)\log\left(\theta_{EU}\overline{y_{EU}} - (B_{EU,t} - \tilde{b}_P(M, \bar{y}_{EU}))(1-\beta) - \bar{g}\right)}{1-\beta} = \frac{\vartheta \log\left((1-\theta_{EU})M\overline{y_{EU}}\right) + (1-\vartheta)\log\left(\theta_{EU}M\overline{y_{EU}} - (B_{EU,t} - \tilde{b}_P(M, \bar{y}_{EU}))(1-\beta) - \bar{g}\right)}{1-\beta}$$

For utility function (2), we get following conditions:

$$\frac{\vartheta\log\left((1-\theta_{EU})\overline{y_{EU}}\right)+(1-\vartheta)\log\left(\theta_{EU}\overline{y_{EU}}-(B_{EU,t}-\tilde{b}_P(M,\bar{y}_{EU}))(1-\beta)\right)}{1-\beta}$$

$$=\frac{\vartheta\log\left((1-\theta_{EU})M\overline{y_{EU}}\right)+(1-\vartheta)\log\left(\theta_{EU}M\overline{y_{EU}}-(B_{EU,t}-\tilde{b}_P(M,\bar{y}_{EU}))(1-\beta)\right)}{1-\beta}$$

Using our parameters defined above for the respective countries, we can then easily derive $\tilde{b}_P(M,\bar{y}_{EU})$, which we present above (section 6)

Appendix G: Determining the Crisis Zone (Calibration)

Lower debt level boundary ($\bar{b}(\pi_P)$)

As seen in section 4.6.2 $\bar{b}(\pi_P)$ is given by:

$$u\left((1-\theta_P)\overline{y_P},\theta_P\overline{y_P}-\bar{b}(\pi_P)\right)+\frac{\beta u((1-\theta_P)\overline{y_P},\theta_P\overline{y_P})}{1-\beta}=\frac{u((1-\theta_P)Z\overline{y_P},\theta_P Z\overline{y_P})}{1-\beta}$$

Using our utility function (1) we get following conditions:

$$\vartheta\log\left((1-\theta_P)\overline{y_P}\right)+(1-\vartheta)\log\left(\theta_P\overline{y_P}-\bar{b}(\pi_P))(1-\beta)-\bar{g}\right)$$
$$+\frac{\beta\vartheta\log\left((1-\theta_P)\overline{y_P}\right)+(1-\vartheta)\log(\theta_P\overline{y_P})(1-\beta)-\bar{g})}{1-\beta}$$
$$=\frac{\vartheta\log\left((1-\theta_P)Z\overline{y_P}\right)+(1-\vartheta)\log(\theta_P Z\overline{y_P})(1-\beta)-\bar{g})}{1-\beta}$$

Now, we just have to use our parameter and input factors for each country (defined in section 6). Please note that if we are interested in utility function (2), we can just set: $\bar{g}=0$.

Upper debt level boundary $\bar{B}(\pi_P)$

$\bar{B}(\pi_P)$ is defined as follows:

$$\max(V_P^1(\bar{B}(\pi_P)), V_P^2(\bar{B}(\pi_P)), \dots, V_P^\infty(\bar{B}(\pi_P)))$$
$$= u\big((1-\theta_P)Z\overline{y_P}, \theta Z\overline{y_P} + \beta(1-\pi_P)\bar{B}(\pi_P)\ \big) + \frac{\beta u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

To determine the upper debt level boundary, we first have to find the maximum value of the different policies *T:* $V_P^T\left(\bar{B}(\pi_P)\right)$. $V_P^T\left(\bar{B}(\pi_P)\right)$ is defined as:

$$V_T(B_{P,0}, \pi_P) = \frac{1-(\beta(1-\pi_P))^T}{1-\beta(1-\pi_P)}\, u\left((1-\theta)\overline{y_P}, g_{P,T}(B_{P,0},\pi_P)\right)$$
$$+ \frac{1-(\beta(1-\pi_P))^{T-1}}{1-\beta(1-\pi_P)}\,\frac{\beta\pi_P u((1-\theta)Z\overline{y_P}, \theta Z\overline{y_P})}{1-\beta}$$
$$+ \beta(1-\pi_P)^{T-2}\frac{\beta u((1-\theta)\overline{y_P}, \theta\overline{y_P})}{1-\beta}$$

And the government P choose *T,* such that:

$$T_P = \arg\max_T V_T(B_{P,0}, \pi_P)$$

After we found the optimal policy *T,* by applying our parameters, utility functions and input factors to the value function, $V_T(B_{P,0}, \pi_P)$ above, we can find the absolute value for: $\bar{B}(\pi_P)$:

$$\max(V_P^1(\bar{B}(\pi_P)), V_P^2(\bar{B}(\pi_P)), \dots, V_P^\infty(\bar{B}(\pi_P)))$$
$$= u\big((1-\theta_P)Z\overline{y_P}, \theta Z\overline{y_P} + \beta(1-\pi_P)\bar{B}(\pi_P)\ \big) + \frac{\beta u((1-\theta_P)Z\overline{y_P}, \theta_P Z\overline{y_P})}{1-\beta}$$

$$\max(V_P^1(\bar{B}(\pi_P)), V_P^2(\bar{B}(\pi_P)), \dots, V_P^\infty(\bar{B}(\pi_P)))$$
$$= \vartheta\log\big((1-\theta_P)Z\overline{y_P}\big)$$
$$+ (1-\vartheta)\log(\theta_P Z\overline{y_P} + \beta(1-\pi_P)\bar{B}(\pi_P)\ - \bar{g})$$
$$+ \frac{\beta(\vartheta\log\big((1-\theta_P)Z\overline{y_P}\big) + (1-\vartheta)\log(\theta_P Z\overline{y_P} - \bar{g}))}{1-\beta}$$

Appendix H: Simulation results for different parameters (M=0.98)

Country P	Portugal	Ireland	Italy	Greece	Spain	Germany	EU-17
Inputs							
GDP in Mio € ($\overline{\boldsymbol{y_P}}$)	*149.43*	*166.72*	*1,365.23*	*160,98*	*920.95*	*2,482.43*	*8,510.14*
Total Debt in Mio € ($\boldsymbol{B_P}$)	*192.77*	*206.24*	*1,810.29*	*281.88*	*864.77*	*1,946.23*	*7,891.46*
Debt-to-GDP	*1.29*	*1.24*	*1.33*	*1.75*	*0.94*	*0.78*	*0.93*
$\bar{y}_{EU}$ *(EU-GDP ex. country P)*	*8,360.70*	*8,343.42*	*7,144.92*	*8,349.16*	*7,589.19*	*6,027.71*	-
B_{EU} *(EU-Debt, ex. Country P)*	*7,698.69*	*7,685.23*	*6,081.17*	*7,609.58*	*7,026.68*	*5,945.24*	-
θ_P: *avg. tax rate*	*44%*	*36%*	*46%*	*46%*	*38%*	*45%*	*39%*
Utility function 1							
$\overline{\boldsymbol{b}}$ $(abs.)$	*28.31*	*18.26*	*286.65*	*33.81*	*119.60*	*488.95*	-
$\overline{\boldsymbol{b}}$ *(% of total GDP)*	*19%*	*11%*	*21%*	*21%*	*13%*	*20%*	
$\overline{\boldsymbol{B}}$ $(abs.)$	*276.20*	*221.67*	*2,834.72*	*322.38*	*1,365.84*	*4,785.30*	-
$\overline{\boldsymbol{B}}$ $(\%\ of\ total\ GDP)$	*185%*	*133%*	*208%*	*200%*	*148%*	*193%*	
$\widetilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(abs.)$	*4090.31*	*4,082.38*	*3,524.60*	*4,089.57*	*3,710.85*	*2,925.60*	-
$\widetilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(\%\ of\ total\ GDP)$	*2710%*	*2412%*	*261%*	*2570%*	*408%*	*117%*	
Utility function 2							
$\overline{\boldsymbol{b}}$ $(abs.)$	*65.56*	*59.76*	*627.88*	*74.06*	*349.59*	*1,109.42*	-
$\overline{\boldsymbol{B}}$ $(abs.)$	*478.80*	*446.59*	*4,692.35*	*553.47*	*2,615.43*	*8,161.00*	-
$\widetilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(abs.)$	*6,195.00*	*6,183.09*	*5,308.68*	*6,189.19*	*5,622.54*	*4,454.44*	-

Appendix I: Calibration Results for Different Utility Function

Country P	Portugal	Ireland	Italy	Greece	Spain	Germany	EU-17
Inputs							
GDP in Mio € ($\overline{\boldsymbol{y_P}}$)	*149.43*	*166.72*	*1,365.23*	*160,98*	*920.95*	*2,482.43*	*8,510.14*
Total Debt in Mio € ($\boldsymbol{B_P}$)	*192.77*	*206.24*	*1,810.29*	*281.88*	*864.77*	*1,946.23*	*7,891.46*
Debt-to-GDP	*1.29*	*1.24*	*1.33*	*1.75*	*0.94*	*0.78*	*0.93*
$\bar{y}_{EU}$ *(EU-GDP ex. country P)*	*8,360.70*	*8,343.42*	*7,144.92*	*8,349.16*	*7,589.19*	*6,027.71*	-
B_{EU} *(EU-Debt, ex. Country P)*	*7,698.69*	*7,685.23*	*6,081.17*	*7,609.58*	*7,026.68*	*5,945.24*	-
θ_P: *avg. tax rate*	*44%*	*36%*	*46%*	*46%*	*38%*	*45%*	*39%*
Utility function (2)							
$\overline{\boldsymbol{b}}$ $(abs.)$	*65.56*	*59.76*	*627.88*	*74.06*	*349.59*	*1,109.42*	-
$\overline{\boldsymbol{b}}$ *(% of total debt)*	*34%*	*29%*	*35%*	*26%*	*40%*	*57%*	
$\overline{\boldsymbol{B}}$ $(abs.)$	*478.80*	*446.59*	*4,692.35*	*553.47*	*2,615.43*	*8,161.00*	-
$\overline{\boldsymbol{B}}$ $(\%\ of\ total\ debt)$	*248%*	*217%*	*259%*	*196%*	*302%*	*419%*	
$\widetilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(abs.)$	*6,195.00*	*6,183.09*	*5,308.68*	*6,189.19*	*5,622.54*	*4,454.44*	-
$\widetilde{\boldsymbol{b}}_{\boldsymbol{P}}$ $(\%\ of\ total\ debt)$	*3214%*	*2998%*	*293%*	*2196%*	*650%*	*228%*	