Winning with IVOL*

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Abstract

Idiosyncratic volatility has an increasing effect on returns. This increasing effect is strongest for small stocks, which can be explained by investors' underdiversification. Previous studies that found a decreasing effect did not control for the low-beta anomaly. A factor that captures the premium for idiosyncratic volatility has significant explanatory power for the cross-section of returns.

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1 Introduction

Variations in stock returns can be decomposed into a systematic part and an idiosyncratic part. The latter, the so called idiosyncratic volatility (IVOL), is uncorrelated with risk factors that are used to price assets. Several theoretical arguments including the CAPM (Lintner, 1965) or APT (Ross, 1976) suggest that this idiosyncratic volatility should have absolutely no effect on stock returns. Other papers (Merton, 1987; Malkiel and Xu, 2002) argue that if anything this idiosyncratic volatility should have a positive effect on stock returns, in line with the economic intuition that risk requires a reward.

However, recent empirical observations (Ang et al., 2006, 2009) suggest that idiosyncratic volatility might have a negative effect on stock returns. Ang et al. (2006) call their finding a 'substantive puzzle', because it contradicts standard asset pricing theories.

In our thesis we present a potential answer to this puzzle by arguing that the negative effect of idiosyncratic volatility on stock returns found in previous studies is most likely due to an omitted variable bias: We show that idiosyncratic volatility of a stock is highly correlated with its CAPM-beta. Frazzini and Pedersen (2014) show that stocks with a high CAPM-beta have lower risk adjusted returns and they also provide plausible theoretical arguments why this should be the case. We argue that stocks with high idiosyncratic volatility have on average lower returns not because of higher idiosyncratic volatility, but because of their higher CAPM-beta. When controlling for CAPM-beta, the pure effect of idiosyncratic volatility on stock returns actually turns out to be positive, a phenomenon for which some theoretical explanations exist (Merton, 1987; Malkiel and Xu, 2002).

After we presented empirical evidence for our hypothesis that IVOL has ceteris paribus a positive effect on stock returns, we look at different market capitalizations. We find that the effect of IVOL on stock returns is much larger for small market-cap stocks. We offer some theoretical arguments that this is also an outcome of rational investor behavior under market frictions: Roughly speaking, investors care about idiosyncratic volatility, because they are not perfectly diversified.

We furthermore present evidence for our claim that the negative IVOL effect found in previous studies is due to the failure to control for mispricing of high-beta stocks. We replicate some of the portfolios used in these studies and add the BAB-factor of Frazzini and Pedersen (2014) to the three Fama-French factors when calculating alphas. The BAB-factor is meant to capture the mispricing of high-beta stocks. Since the spread in alphas between high IVOL and low IVOL portfolios becomes insignificant after adding the BAB-factor we conclude that the previously found negative IVOL effect is indeed due to the mispricing of high-beta stocks rather than IVOL itself.

To see whether IVOL is sufficiently important to price stocks across all market caps, we construct a factor to capture the premium on high IVOL stocks and examine its effect on the stochastic discount factor that prices all stocks. We find that this factor has a very significant impact on the stochastic discount factor, which means that it significantly helps to price stocks.

Our findings are robust to the use of different statistical methodologies. We also argue that limits of arbitrage will prevent most investors from taking advantage of the IVOL effect. The IVOL effect is strongest among small stocks, while many large institutional arbitrageurs do not want to invest in small stocks due to their bad performance during liquidity crises. Private investors on the other hand are in general not diversified enough to ignore IVOL in their investment decisions. Therefore, the IVOL effect is likely to persist in the future.

2 Literature Review

In this section we summarize the main empirical findings on the effect of idiosyncratic risk on stock returns. This discussion is followed by a review of theories that aim to explain a potential effect of idiosyncratic risk on returns. We also include a discussion on the low beta anomaly since it turns out that idiosyncratic risk and CAPM-beta are highly correlated. One cannot establish the pure effect of idiosyncratic volatility on returns if one does not separate one anomaly from the other. We conclude this section with a discussion on limits to arbitrage to deal with the question whether market anomalies like the ones discussed in our work should disappear due to arbitrage.

2.1 The idiosyncratic volatility puzzle

A reason why the effect of IVOL on returns is interesting is that CAPM predicts that there is no effect. Tinic and West (1986) tested this and instead found a positive relationship between IVOL and stock returns. The same conclusion was reached in a similar test by Malkiel and Xu (2002). However, perhaps the most famous paper on the subject, Ang et al. (2006), conversely found a negative relation between IVOL and future returns. Both returns and FF3-alphas were found significantly negative for the spread between the highest and lowest IVOL quintiles (H-L IVOL). However, the IVOL anomaly did not stand the robustness checks in Bali and Cakici (2008).

Fu (2009) argues that to make a conclusion about whether IVOL can explain future returns, one needs to look at the expected IVOL, which not necessarily is close to the historical IVOL used in the studies above. Instead, he uses exponential generalized autoregressive conditional heteroskedasticity (EGARCH) models to estimate expected IVOL and finds significantly increasing returns in the cross-section. However, according to Fink et al. (2012) the method in Fu (2009) is flawed since the conditional IVOL estimation includes one forward looking observation. (Guo et al., 2014) shows that this methodology could create a considerable look-ahead bias. In fact, no positive relation between returns and expected IVOL has been found when only information available to investors are included in EGARCH estimations.

The IVOL effect also seems fragile to different control variables. Bodnaruk and Östberg (2009) found positive returns for H-L IVOL portfolios after controlling for other shadow costs. This is also true when return reversals are controlled for (Huang et al., 2010; Bali et al., 2011). Furthermore, Bali et al. (2011) found that when controlling for the maximum return during the last month, the IVOL effect even reverses. The authors interpretation is that while investors demand compensation for IVOL, they also have a preference for so called lottery-type stocks. The demand for stocks with extreme payoffs could thus drive the results in Ang et al. (2006, 2009) and hide an otherwise positive relationship between IVOL and returns. Moreover, Han and Lesmond (2011) show that the effect could stem from illiquidity, consistent with the descriptive statistics in Bali and Cakici (2008) which shows that IVOL is positively related to firm size and illiquidity as measured in Amihud (2002).

2.2 Theoretical explanations

The CAPM equilibrium results of Lintner (1965) where idiosyncratic volatility has no impact on returns hold only if investors have unrestricted access to all securities. Therefore, Levy (1978) proposes a general CAPM framework in which each investor has access to only a limited number of securities. In his model, total variance (and therefore idiosyncratic volatility) of stocks begins to become important and has to be compensated by higher returns. Merton (1987) proposes a similar model in which he also assumes that investors know only a fraction of all the stocks in the market, e.g. because there is a fixed cost to get informed about each stock. This model implies that idiosyncratic volatility matters more for stocks with a smaller investor base. Bodnaruk and Östberg (2009) show empirically that stocks with a smaller investor base indeed have higher returns. Malkiel and Xu (2002) stress the intuition that idiosyncratic risk gets priced in Merton (1987) due to the investors' inability to hold the market portfolio. Malkiel and Xu (2002) also develop an own model in which investors have a less diversified *available* market portfolio than the *actual* market portfolio and idiosyncratic volatility has an increasing effect on returns.

Furthermore, Jones and Rhodes-Kropf (2004) argue that venture capitalists have to be compensated for idiosyncratic risk and Barberis and Huang (2002) offers a behavioral explanation based on mental accounting and loss aversion why idiosyncratic volatility might have positive effects on returns. Berrada and Hugonnier (2013) argues that idiosyncratic risk has an effect on expected returns if investors have biased expectations.

Finally, Stambaugh et al. (2013) offer an explanation why the overall effect of idiosyncratic volatility might be negative: It is more difficult for arbitrageurs to offset mispricing of stocks with high idiosyncratic volatility. Since it is furthermore more easier to buy stocks than to short-sell stocks, arbitrage asymmetry leads on average to an overpricing of stocks with high idiosyncratic volatility.

2.3 The beta anomaly

There seems to be a positive relation between market beta and idiosyncratic volatility (Blitz and van Vliet, 2007; Bali and Cakici, 2008; Fu, 2009; Berrada and Hugonnier, 2013). Regardless of using EGARCH as in Fu (2009) or historical returns as in Bali and Cakici (2008), higher IVOL quintiles also have higher realized market betas. The relationship still holds if return outliers are removed and after adjusting for the Fama and French (1993) size and value factors. Coincidentally, beta has an anomaly of its own. Although CAPM predicts the market beta to have a special positive relationship with stock returns (Sharpe, 1964), multiple studies have found contradicting evidence (Black et al., 1972; Baker et al., 2011; Frazzini and Pedersen, 2014). The reason could lie in borrowing restrictions. CAPM assumes that all investors are free to take long and short positions and borrow or lend any amount at the risk free rate. When this assumption is relaxed, the Capital Market Line flattens (Brennan, 1971; Black, 1972). This was also supported by Black et al. (1972) and later by Fama and French (1992) who found a flat relationship between beta and returns.

As Black (1993) points out, an insignificant slope does not rule out a positive relationship. However, a more recent test was conducted by Baker et al. (2011) and now the relationship was found significantly negative. An intuitive explanation to the phenomenon is based on investor benchmarking: since a large portion of investors are leverage constraint and their performance is evaluated against other investors, they overweight (underweight) high (low) beta stocks, thus making them overpriced (underpriced) (Karceski, 2002). Since beta and volatility are correlated and investors refrain from shorting risky stocks, the mispricing persists.

Frazzini and Pedersen (2014) created a model based on this idea. In the model, the mispricing is positively related to investors' funding constraints, but this was not supported by their empirical tests. However, they found strong evidence for a beta anomaly. They created a betting-against-beta factor (BAB) taking advantage of the anomaly. BAB was found to have significant alphas in the whole sample as well as within size and IVOL deciles, adjusting for the market, size, value, momentum and liquidity. Furthermore, the effect is consistent within different industries which disproves the view that low beta investing is an industry bet which has happened to pay off in a specific sample period (Asness et al., 2014).

2.4 Limits of arbitrage

A recent area of research deals with the question how market anomalies could persist although many arbitrageurs should be able to take advantage of mispricings. Important developments of the limits of arbitrage theory are summarized in Gromb and Vayanos (2010). One limit of arbitrage is risk. Wurgler and Zhuravskaya (2002) model limits to cross-asset arbitrage and Grossman and Miller (1988) model limits to intertemporal arbitrage when arbitrageurs face fundamental risk stemming from the uncertainty of asset payoffs. The slow arrival of new investors in the case of intertemporal arbitrage can be motivated by search costs (Weill, 2007; Duffie et al., 2007; Duffie and Strulovici, 2012; Lagos et al., 2011) or agency frictions (He and Xiong, 2008). Risk can also come from demand shocks by certain traders as in DeLong et al. (1990).

Establishing and financing a trade is often costly. That such holding costs inhibit arbitrage is discussed by Tuckman and Vila (1992, 1993). Dow and Gorton (1994) show that holding costs strongly inhibit short-horizon arbitrageurs. Holding costs related to short positions are often especially severe. D'Avolio (2002) and Lamont and Thaler (2001) estimated significant short-selling costs for Palm stocks in March, 2000 that prevented arbitrageurs from taking advantage of mispricing between Palm and 3Com stocks. Short-selling costs can be endogenously modelled by frictions in the repo market (Duffie, 1996; Krishnamurthy, 2002; Duffie et al., 2002; Vayanos and Weill, 2008). Miller (1977), Harrison and Kreps (1978), Scheinkman and Xiong (2003) and Hong et al. (2006) show how short-selling constraints, i.e. infinite short-selling costs, can lead to overpricing. However, Diamond and Verrecchia (1987) show that short-selling constraints do not necessarily lead to overpricing and Bai et al. (2006) show that constraints sometimes lead to underpricing.

Another source of limits of arbitrage is that arbitrageurs do not have frictionless access to capital. Gromb and Vayanos (2002) show that leverage constraints decrease arbitrageurs' ability to eliminate mispricings. Their analysis is extended by Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2012).

In addition to frictions with margin debt, access to equity also poses a problem to eliminating mispricings. Shleifer and Vishny (1997) claim that arbitrage is ineffective when prices diverge far from fundamental values and investors withdraw equity from arbitrageurs if the performance is negative. In Vayanos (2004), arbitrageurs have a preference for liquid assets in times of high volatility, because it is assumed that their funds get liquidated after poor performance. He and Krishnamurthy (2008, 2011) assume that equity cannot exceed a certain multiple of the fund manager's wealth. Guerrieri and Kondor (2009) introduce reputation concerns and in Vayanos and Woolley (2013) investors rationally observe managers' inefficiency from arbitrageurs underperformance.

2.5 Our contribution to the literature

Our work adds to the existing literature in showing that a previously found negative IVOL effect can be found only if one does not control for the CAPM-beta of stocks. Our work connects research on both the IVOL puzzle and the beta anomaly and leads to a new understanding of the empirical behavior of stock returns. Our work also brings empirical findings back in line with finance theory which mostly predicts a positive IVOL effect. Since we argue that the IVOL effect is strongest for stocks that arbitrageurs might not want to hold, our work also relates to the limits of arbitrage literature.

3 Development of Hypotheses

The model presented in Merton (1987) provides a theoretical framework that motivates our hypotheses. We will therefore briefly review its main implications on asset prices. Under the assumptions that there is a common risk factor and investors have the same initial wealth, same risk aversion and choose mean-variance optimal portfolio weights, Merton derives the following equation for security k's excess return:

$$\bar{R}_k - R = b_k b\delta + \delta x_k \sigma_k^2 / q_k, \tag{1}$$

where the left-hand side denotes the security's expected excess return, b_k denotes the common factor exposure of security k, b denotes the common factor exposure of the market, δ is the investors' risk aversion, x_k is the relative market share of security k, σ_k is the idiosyncratic volatility of security k and q_k is the share of the investors that know security k.

If $q_k = 1$ for all securities and x_k is sufficiently close to zero, this equation converges to the Capital Market Line in Sharpe (1964). If however $q_k << 1$, the second term on the right-hand side of equation (1) can remain significantly positive even if x_k converges to zero. Idiosyncratic risk remains an important return driver although the security's relative market share x_k is very small if the security is known only by a very small fraction of investors q_k, with $q_k \sim x_k$. In such a case, this stock would have to make up a relatively large portion in the respective investors' portfolios compared to the stock's market capitalization. Thus, investors cannot completely diversify away all idiosyncratic risk of the stock and want to be compensated for taking this risk.

However, we find empirically in section 4 that idiosyncratic volatility is positively correlated with the CAPM-beta. Frazzini and Pedersen (2014) show that stocks with a high CAPM-beta have significantly lower Fama-French alphas than stocks with low CAPM-beta. We therefore conclude that we have to control for the beta anomaly in order to get an unbiased estimate of the effect of IVOL on stock returns. Not controlling for the beta anomaly would attribute the negative impact on returns that comes from a higher CAPM-beta to the effect of higher IVOL. Our first hypothesis is:

Hypothesis 1: After controlling for CAPM-beta, IVOL has a positive effect on Fama-French alphas. We control for the beta anomaly by sorting stocks not only according to their IVOL but also according to their CAPM-beta. We indeed see a positive effect of IVOL on returns in our data.

Merton (1987) suggests that the IVOL effect is due to underdiversification of investors. There are reasons to believe that investors in small stocks are less diversified than investors in large stocks, because large institutional investors who can diversify better generally do not invest in small stocks (Ferreira and Matos, 2008). If investors in small stocks are less diversified than investors in large stocks, then the IVOL effect should be stronger among small stocks. This leads to our second hypothesis:

Hypothesis 2: The spread between high-IVOL and low-IVOL portfolios is larger among small stocks. We sort all stocks in a three-dimensional way according to their beta, their market capitalization and their IVOL. We find indeed a significantly stronger effect for small stocks, even though we find a significantly positive effect in most of the portfolios considered.

Our estimated positive IVOL effect leads us to conjecture that previous studies that found a negative effect indeed confused the IVOL effect and the beta anomaly.

Hypothesis 3: If we control for a factor that captures the part of returns due to the beta anomaly, we make the negative IVOL effect found in previous research insignificant. We first replicate results in Ang et al. (2006) then we make the negative IVOL effect insignificant by adding the BAB-factor from Frazzini and Pedersen (2014) to the three Fama-French factors. We see our third hypothesis confirmed and conclude that the negative IVOL effect found in Ang et al. (2006)

can be explained by the beta anomaly.

Finally, our findings of a positive IVOL effect suggest that we can improve the Fama-French model by taking into account a factor that captures the premium attached to idiosyncratic risk. We construct a factor that captures this risk premium and call it 'Betting-On-IVOL' (BOIVOL).

Hypothesis 4: The BOIVOL factor, which captures the risk premium of stocks due to their idiosyncratic risk, has a significant impact on the stochastic discount factor. We estimate different models for the stochastic discount factor (SDF) where the SDF depends linearly on a set of factors. The impact of the BOIVOL factor on the SDF is statistically significant even if we control for the 3 Fama-French factors and the BAB factor from Frazzini and Pedersen (2014). We see our fourth hypothesis confirmed.

4 Data and Methodology

The data we used for our analysis comprises daily and monthly total returns of stocks listed on NYSE, AMEX and NASDAQ from January 2, 1963 until December 31, 2013. We also use the respective prices and market capitalizations. Stocks with prices below 5 USD or above 1000 USD are deleted. We further restrict our analysis to stocks with share codes 10 or 11, which comprise all ordinary stocks. We obtained the data from the CRSP database. Market returns, the risk free rate, HML, SMB and momentum (MOM) factors as well as 25 double sorted portfolios on market capitalization and book/market ratio were downloaded from Kenneth French's website. [1]. Tables in the appendix also use portfolios based on deciles of earnings/price and dividend/price ratios, which can be downloaded from Kenneth French's website, too.

Next we describe a method commonly used in some previous articles (Ang et al., 2006) to sort stocks into portfolios based on their IVOL. This method is based on a one month estimation window and the Fama-French 3-factor model.

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

We argue that a longer estimation window increases precision and leads to a larger ex post spread in idiosyncratic volatility of the resulting portfolios. In the following subsections we describe how we calculated various other measures used in our analysis such as CAPM beta, the Betting-Against-Beta (BAB) factor of Frazzini and Pedersen (2014), how we form the portfolios on which we base our analysis as well as how we calculated our own Betting-On-IVOL (BOIVOL) factor.

4.1 Portfolio formation based on one-month estimation window for IVOL and the Fama-French model

The following method was proposed in Ang et al. (2006). For each stock, returns in a given month are deleted if that month had less than 17 trading days. This should increase precision of the IVOL estimation. In each month, daily excess returns of each stock are regressed on the 3 Fama-French factors. Each stock's IVOL is calculated as the standard deviation of the residuals. The stocks are sorted into 5 quintiles based on their IVOL of the current month. The next month's portfolio returns are the value-weighted average returns in the 5 quintiles.

4.2 Portfolio formation based on longer estimation window and the CAPM

A one-month estimation window that uses 3 factors introduces a tremendous uncertainty in the estimated parameter values. In our analysis we estimate IVOL using only the market factor that is estimated over a much longer horizon. As a result we get a larger ex post spread in both CAPM and Fama-French IVOL for the resulting portfolios.

The CAPM beta is calculated for each month very similar to the approach in Frazzini and Pedersen (2014) with daily excess returns. Using daily excess returns improves the estimates (Merton, 1980). We estimate each stock's correlation with the market over a rolling 1200 trading days-window. Stocks' and the market's standard deviation of returns are calculated over a rolling 120 trading days-window. For correlations we require that stocks have been traded on at least 750 days during the estimation window. For standard deviations we require that stocks have been traded at least on 120 days during the estimation window. If a particular stock does not meet one of the criteria in one particular month, it is excluded from the analysis in that month. the estimated beta of stock i is then determined by

$$\hat{\beta}_i = \hat{\rho}_{i,m} \frac{\hat{\sigma}_i}{\hat{\sigma}_m}.$$

$\hat{\rho}_{i,m}$ denotes the estimated correlation between stock i and the market, $\hat{\sigma}_i$ denotes the estimated standard deviation of stock i and $\hat{\sigma}_m$ denotes the standard deviation of the market.

We determine the exponentially weighted moving average (EWMA) of the squared difference of daily returns and the daily CAPM implied returns at the end of each month for each stock as recursively as follows:

$$EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}.$$

The monthly pre-formation IVOL is the square root of this EWMA estimate at the end of each month. We sort stocks based on this IVOL into five quintiles. The returns of the five portfolios are calculated as the value- or equally-weighted returns of the previous month's quintiles. This procedure ensures that our portfolios are in principle tradable.

4.3 Comparison of ex post IVOL

To support our claim that we get a larger spread in ex post IVOL we summarize some descriptive statistics in tables 1, 2 and 3.

Table 1 and Table 2 show that there is a considerable spread in both CAPM and Fama-French IVOL for both methods. It seems to not matter much that we used the CAPM to estimate the pre-formation IVOL although we are more interested in Fama-French IVOL. Conversely, an approach that uses the Fama-French model to

Table 1: Ex post CAPM IVOL

	Low Ivol	2	3	4	High Ivol
AHXZ	1.28	1.13	1.40	2.45	4.04
EWMA	1.42	1.31	1.58	2.82	6.06

Note: The table shows average realized idiosyncratic volatility (IVOL) as estimated in the CAPM for value weighted quintile portfolios of stocks sorted based on their pre-formation IVOL. In *AHXZ*, IVOL was estimated based on one month of historical daily returns in the Fama-French regression $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. In *EWMA*, IVOL was estimated as the squared exponentially weighted moving average, estimated as $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$ where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month when market beta is estimated based on between 1200 and 750 daily returns for correlation and 120 daily returns for standard deviation. Portfolios were reallocated monthly. The sample contains US stocks for the period 1965:12-2013:12.

determine the pre-formation IVOL yields a considerable spread in ex post IVOL based on CAPM or Fama-French factors for the whole sample period. However, if we take a close look at Table 2, we see that the spread in Fama-French IVOL is more than twice as large if we use a longer estimation window for the pre-formation IVOL. Since we are interested in the effect of IVOL on returns, having assets that show a large spread in IVOL should provide us with more precise estimates.

We would argue that the method used in our analysis has another advantage. As Table 3 shows, portfolios with higher IVOL have higher CAPM betas. Table 32 in the appendix shows that the same holds true for pre-formation estimates of CAPM-beta and IVOL. Since high-beta stocks are generally overpriced (Frazzini and Pedersen, 2014), this can lead to a confusion of two things: the effect of IVOL on returns and the mispricing due to high CAPM-beta. Since our method leads to more than twice the spread in IVOL but only increases the spread in CAPM-betas slightly, we would say that our method is better suited to disentangle these two effects.

Table 2: Ex post Fama-French IVOL

	Low Ivol	2	3	4	High Ivol
AHXZ	1.01	1.06	1.30	1.69	2.66
EWMA	1.07	1.15	1.54	2.26	4.53

Note: The table shows average realized idiosyncratic volatility (IVOL) as estimated in the Fama-French regression $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$ for value weighted quintile portfolios containing stocks sorted based on their pre-formation IVOL. In *AHXZ*, IVOL was estimated based on one month of historical daily returns in the Fama-French model above. In *EWMA*, IVOL was estimated as the squared exponentially weighted moving average, estimated as $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$ where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month when market beta is estimated based on between 1200 and 750 daily returns for correlation and 120 daily returns for standard deviation. Portfolios were reallocated monthly. The sample contains US stocks for the period 1965:12-2013:12.

Table 3: Ex post CAPM-betas

	Low Ivol	2	3	4	High Ivol
AHXZ	0.78	0.99	1.16	1.35	1.51
EWMA	0.75	0.97	1.18	1.42	1.76

Note: The table shows average realized CAPM-betas for value weighted quintile portfolios containing US stocks sorted based on their pre-formation idiosyncratic volatility (IVOL). In *AHXZ*, IVOL was estimated based on one month of historical daily returns in the Fama-French model above. In *EWMA*, IVOL was estimated as the squared exponentially weighted moving average, estimated as $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$ where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month when market beta is estimated based on between 1200 and 750 daily returns for correlation and 120 daily returns for standard deviation. Portfolios were reallocated monthly. The sample period for the test is 1965:12-2013:12.

4.4 Other portfolio formations

Besides sorting stocks just on IVOL, we use a double-sorting process in which we first sort stocks on CAPM-beta and then sort the stocks in each beta quintile according to their IVOL. In this way we get 25 double sorted portfolios sorted on CAPM beta and IVOL. The portfolio returns are the next month's value-weighted or equally-weighted average returns of the stocks in each of the 25 established groups. We also form portfolios from stocks first sorted on IVOL and then sorted according to the CAPM-beta in each IVOL quintile. At last, we perform a three-dimensional sorting: We first divide stocks in market capitalization quintiles. In each of these quintiles we perform the beta-IVOL double-sorting. In total we obtain 125 portfolios based on stocks that are sorted according to market capitalization, beta and IVOL.

The reason to do multi-dimensional sorting is to determine the effect on returns of the variable one uses last in the sorting process and control thereby for the variables one uses first. For example, if we perform a beta-IVOL sorting we want to determine the effect of IVOL on returns controlling for beta.

The standard errors of all our estimates are calculated using a robust Newey and West (1987) covariance matrix.

4.5 Measuring the effect of market illiquidity on returns

We argue that there are limits to arbitrage regarding the IVOL effect. One such limit is bad performance of profitable portfolios during times of high market illiquidity. We compute a measure of market illiquidity according to Amihud (2002). The monthly measure of stock i's illiquidity in month m is:

$$ILLIQ_{i,m} = \frac{1}{D_{i,m}} \sum_{d=1}^{D_{i,m}} |R_{i,m,d}|/VOLD_{i,m,d},$$

where $D_{i,m}$ is the number of trading days of stock i in month m, $R_{i,m,d}$ is the return of stock i in moth m and trading day d. $VOLD_{i,m,d}$ is the daily dollar trading volume of stock i in month m and trading day d. The market-wide illiquidity measure $MILLIQ$ in month m is just the average of stock illiquidity measures in month m. To measure the effect of liquidity on returns we perform the OLS regression according to the model

$$R^e_{i,t} = \beta_0 + \beta_1 \ln MILLIQ_{t-1} + \beta_2 \ln MILLIQ^U_t + \epsilon_t,$$

where $\ln MILLIQ^U$ are the residuals of an ARMA(1,1) model of $\ln MILLIQ$ and should represent unexpected changes in illiquidity. An ARMA(1,1) model of $\ln MILLIQ$ is considered optimal according to AIC and BIC.

4.6 Betting-Against-Beta

In order to control for the beta anomaly in previous research we reconstruct the BAB-factor in Frazzini and Pedersen (2014). We already calculated our CAPM betas in a very similar way as the one proposed in Frazzini and Pedersen (2014). We obtain the BAB-factor as follows.

In each month, stocks are ranked according to their estimated beta. The stock with the highest beta receives the rank $z_{i,t} = 1$. The stock with the second highest beta receives the rank $z_{j,t} = 2$ and so forth until each stock has received a rank. Next we calculate an average rank $\bar{z}_t = \frac{1}{N}\sum_{i=1}^{N} z_{i,t}$, where N is the number of stocks traded in a given month. Next, a low-beta and a high-beta portfolio are formed:

$$r^L_{t+1} = \left(\frac{2}{\sum_{i=1}^{N} |z_{i,t} - \bar{z}_t|}\right) \sum_{z_{i,t} \geq \bar{z}} r_{i,t+1}(z_{i,t} - \bar{z}_t)$$

$$r^H_{t+1} = \left(\frac{2}{\sum_{i=1}^{N} |z_{i,t} - \bar{z}_t|}\right) \sum_{z_{i,t} < \bar{z}} r_{i,t+1}(\bar{z}_t - z_{i,t}).$$

r^L_t denotes the return of the low-beta portfolio in period t. r^H_t denotes the return of the high-beta portfolio in period t. The return on the BAB-factor in period $t+1$ is then:

$$t_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^L - r_{t+1}^f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r_{t+1}^f).$$

β_t^L is the CAPM-beta of the low-beta portfolio in period t. β_t^H is the CAPM-beta of the high-beta portfolio in period t.

Because the BAB-factor is composed of excess returns, it is an excess return itself. It is also ex ante beta neutral, since the weights on high-beta and low-beta portfolio are scaled inversely by their CAPM betas.

4.7 Betting-On-IVOL

We furthermore create a Betting-On-IVOL (BOIVOL) factor that should capture the risk premium that investors put on stocks with high idiosyncratic volatility. This factor is based on value-weighted returns of the beta-IVOL double sorted portfolios. Let r_t^i denote the return difference in period t between high- and low-IVOL portfolios in the i-th beta quintile. Then the return on the BOIVOL-factor is calculated as follows:

$$r_t^{BOIVOL} = \frac{1}{5}\sum_{i=1}^{5} r_t^i$$

Since for all $i, 1 \leq i \leq 5$ the return difference r_t^i is an excess return, the return of the BOIVOL factor is also an excess return. Instead of taking the difference in the returns between single-sorted stocks, we use double-sorted stocks to control for the beta anomaly. If we would use single-sorted stock, short selling low-IVOL stocks and buying high-IVOL stocks would involve implicitly short-selling low-beta stocks and buying high beta stocks. This would be due to the strong correlation between CAPM beta and IVOL as documented in tables 1, 2 and 3. As a consequence the returns on our BOIVOL factor would be biased downwards. Using double-sorted portfolios and creating excess returns for each beta quintile avoids this downward bias.

4.8 The stochastic discount factor

In order to test our fourth hypothesis, whether the BOIVOL factor has explanatory power for the cross-section of returns, we check which factors have a significant contribution to the stochastic discount factor (SDF). We briefly review how a linear factor model implies the existence of a stochastic discount factor m, such that

$$E(mR^e) = 0 \tag{2}$$

holds for a vector of excess returns R^e. Equation (2) means that the risk adjusted excess returns of all assets should on average be zero. Cochrane (2001) is an excellent reference for this topic.

We assume that R^e_t, the vector of excess returns in period t, is dependent on certain factors, e.g. the market excess return, SMB or HML factors:

$$R^e_t = \beta_1 f_{1,t} + ... + \beta_s f_{s,t} + \epsilon_{i,t}, \tag{3}$$

where β_j vector denotes the exposure of each asset to factor j, $f_{j,t}$ denotes the factor return in period t and ϵ_t is a vector of error terms. It follows that

$$E(R^e) = \beta_1 \lambda_1 + ... + \beta_s \lambda_s = \beta' \lambda. \tag{4}$$

$E(R^e)$ is the vector of average excess returns. $\lambda_j = E(f_j)$ denotes the risk premium of factor j, β is a matrix of factor exposures and λ is the vector of risk premia. Let us set

$$\lambda = \mathrm{var}(f)b, \tag{5}$$

where $var(f)$ denotes the variance-covariance matrix of dimensions $s \times s$ of the different factors and b is an $s \times 1$ vector of some parameters. If we combine equation 4 and 5, we get

$$
\begin{aligned}
E(R^e) &= \beta' \lambda \\
&= \beta' \operatorname{var}(f) b \\
&= \operatorname{cov}(R^e, f) \operatorname{var}(f)^{-1} \operatorname{var}(f) b \\
&= \operatorname{cov}(R^e, f) b, \qquad (6)
\end{aligned}
$$

where $cov(R^e, f)$ is a matrix with covariances of each asset's excess returns with the different factor returns f. If we now combine equation (2) with equation (6), we get

$$
\begin{aligned}
0 &= E(mR^e) \\
&= E(R^e) - \operatorname{cov}(R^e, f) b \\
\Leftrightarrow E(mR^e) &= E(R^e) - \operatorname{cov}(R^e, f) b \qquad (7) \\
\Leftrightarrow m_t &= 1 - (f_t - E(f))' b, \qquad (8)
\end{aligned}
$$

where f_t is a $s \times 1$ vector of factor returns at time t and m_t is the corresponding value of the discount factor in that period. We have shown that assuming a factor structure of excess returns as in equation (4) is equivalent to assuming that there exists a discount factor as in (8) that satisfies (2). The next subsection deals with the estimation of the vector b, which is necessary to calculate values of the discount factor. Once we have the parameter vector b, we can calculate risk premia according to equation (5).

4.9 GMM estimation of the stochastic discount factor

To estimate the parameter vector b in (8), we use the Generalized Method of Moments (GMM) examined in Hansen (1982). The assets we want to price with the stochastic discount factor are the Fama-French 25 double sorted portfolios on market capitalization and book/market ratio with the sample period from December 1965 to December 2013. The GMM estimator minimizes a quadratic form that depends on the vector b:

$$\hat{b} = \mathrm{argmin}_b \, g_T(b)' W g_T(b), \tag{9}$$

where

$$g_T(b) = \frac{1}{T} \sum_{t=1}^{T} m_t R_t^e \tag{10}$$

is the vector of sample averages of risk-adjusted discounted excess returns, which should on average be zero and W is the weighting matrix. We set the weighting matrix equal to the inverse of the second moment matrix of portfolio excess returns:

$$W = E(R^e R^{e\prime})^{-1}. \tag{11}$$

Using this weighting matrix minimizes the minimum distance the GMM discount factor estimate to a feasible discount factor, i.e. a discount factor that satisfies $E(mR^e) = 0$ for all excess returns. This weighting matrix has been proposed by Hansen and Jagannathan (1997). We will use the minimum distance to a feasible discount factor as a criterion for how well the estimated discount factor prices assets.

T is the number of observations and R_t^e is a 25×1 vector of excess returns at time t of the 25 Fama-French portfolios. Together with equation (8) we can rewrite (10) as:

$$g_T(b) = \frac{1}{T} \sum_{t=1}^{T} R_t^e - \frac{1}{T} \sum_{t=1}^{T} \left(R_t^e (f_t - E_T(f)' \right) b, \tag{12}$$

where $E_T(f)$ denotes the vector of sample averages of the factor returns.

As a robustness-check to the GMM estimation of the discount factor, we also perform ordinary Fama and MacBeth (1973) regressions. To see whether a factor is priced we would have to look at the implied risk premia when regressing average returns on betas. We account for the fact that betas are estimated by using the Shanken (1992) correction when calculating the variances of the estimated risk premia:

$$\sigma^2(\hat{\lambda}) = \frac{1}{T}\left[(\beta'\beta)^{-1}\beta\Sigma\beta(\beta'\beta)^{-1}\left(1+\lambda'\Sigma_f^{-1}\lambda\right)\right], \tag{13}$$

where T is the sample length, Σ is the variance-covariance matrix of errors from the time-series regression, Σ_f is the variance-covariance matrix of the factors, β is the vector of betas estimated in the time series regressions and λ is the vector of estimated risk premia.

4.10 The minimum distance between the estimated SDF and a feasible SDF

An estimated discount factor m is in general misspecified. This means that

$$E(mR^e) \neq 0.$$

As a result, The discount factor does not price the excess returns correctly, since risk adjusted discounted excess returns have to be zero in expectation. We have done the GMM estimation such that the distance of m to a true discount factor y with $E(yR^e) = 0$ is minimized. Therefore, we use as a measure of model-misspecification the distance δ of m to the nearest feasible y.

$$\delta = \min_{y\in Y} ||m-y||, \qquad Y = \{y \mid E(yR^e) = 0\},$$

where Y denotes the set of feasible discount factors (there are in general many). Hansen and Jagannathan (1997) show that when dealing with excess returns this minimum distance is equal to

$$\delta = \sqrt{E(mR^e)'E(R^eR^{e\prime})E(mR^e)}. \tag{14}$$

This δ has an economic interpretation: It is the absolute value of the maximum Sharpe ratio of all possible pricing errors given asset returns R^e and discount factor m. This Sharpe ratio can often only be attained through a complex dynamic strategy on our available assets. This intuition is stressed in Campbell and Cochrane (2000). Since we are often interested in how well our model explains returns on a specific set of portfolios (such as the Fama-French size-book/market portfolios) we additionally compute a different measure of model misspecification also used in Campbell and Cochrane (2000).

This second measure of misspecification does not assess the maximum possible Sharpe ratio of unpriced returns, but looks at the maximum mispricing within the 25 Fama-French portfolios that we want to price.

To compute pricing errors we need to compute the expected excess returns that our discount factor implies. From $E(mR_i^e) = 0$ it follows that

$$E(m)E(R_i^e) + \operatorname{cov}(m, R_i^e) = 1. \tag{15}$$

From equation (8) it follows that $E(m) = 1$. Therefore, we can write the implied expected return $E^m(R_i^e)$ of portfolio i using equation (15) as

$$E^m(R_i^e) = -\operatorname{cov}(m, R_i^e). \tag{16}$$

We can now calculate the absolute value of the maximum Sharpe ratio among the pricing errors within our 25 Fama-French portfolios as

$$\delta^{FF} = \max_{1 \le i \le 25} \frac{|E^m(R_i^e) - E(R_i^e)|}{\sigma(R_i^e)}, \tag{17}$$

where $\sigma(R_i^e)$ is the standard deviation of portfolio i's excess return, $E(R_i^e)$ is the actual sample average excess return and $E^m(R_i^e)$ is defined in equation (16).

5 Results

This section presents the results from testing the four hypotheses. We concentrate on one hypothesis at a time. To start, we connect our work with previous empirical research.

5.1 Understanding the controversy

To connect our analysis to previous research, Table 4 presents FF3-alphas and t-stats for portfolios sorted on IVOL using the short window estimation method used in Ang et al. (2006) as described in the methodology section. Ang et al. (2006) found a significant negative spread between high- and low-IVOL portfolios when returns were value weighted. We see exactly the same pattern in the lower part of Table 4. What is also consistent with their findings is that a large part of this spread seems to be due to the highest IVOL-quintile.

Table 4: Alphas and t-stats for AHXZ replicated portfolios

	Low Ivol	2	3	4	High Ivol	High -Low
EW	0.34	0.33	0.43	0.45	0.27	-0.07
	(4.77)	(5.42)	(7.57)	(7.80)	(2.81)	(-0.54)
VW	0.08	0.05	0.02	-0.04	-0.39	-0.48
	(1.66)	(1.27)	(0.32)	(-0.53)	(-3.14)	(-3.04)

Note: The table shows monthly alphas and corresponding (t-stats) for quintile portfolios of stocks sorted based on their pre-formation idiosyncratic volatility (IVOL). Alphas were estimated in the Fama-French regression $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$ and t-stats are robust Newey-West (1986) t-statistics. IVOL was estimated as in Ang et al. (2006), based on one month of historical daily returns in the Fama-French model above. The rows *EW* and *VW* report results for equal and value weighted portfolios respectively. The column *High-Low* gives results for the spread between the high and low IVOL quintiles. Portfolios were reallocated monthly. The sample contains US stocks over the period 1965:12-2013:12.

Table 5: Alphas and t-stats for IVOL portfolios

	Low Ivol	2	3	4	High Ivol	High -Low
EW	-0.19	-0.18	-0.18	-0.07	2.04	2.22
	(-2.85)	(-2.55)	(-2.44)	(-1.07)	(14.72)	(12.30)
VW	0.12	0.06	0.00	-0.22	0.12	-0.00
	(2.12)	(1.18)	(0.02)	(-2.12)	(0.41)	(-0.00)

Note: The table shows monthly alphas and corresponding (t-stats) for quintile portfolios of stocks sorted based on their pre-formation idiosyncratic volatility (IVOL). Alphas were estimated in the Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. IVOL was estimated as the squared exponentially weighted moving average calculated as $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$ where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month. The rows *EW* and *VW* report results for equal and value weighted portfolios respectively. T-stats are robust Newey-West (1986) t-statistics. The column *High-Low* gives results for the spread between the high and low IVOL quintiles. Portfolios were reallocated monthly. The sample contains US stocks over the period 1965:12-2013:12.

However, as Bali and Cakici (2008) showed in their empirical research, the IVOL effect is not robust at all to estimation methods or return computations. To present evidence against the robustness of the IVOL effect we show in the first row of Table 4 how the pattern would look like if returns were equally weighted instead of value weighted, holding the pre-formation estimation of IVOL unchanged. One can see that the spread is much less negative and totally insignificant with equal weighted returns. What is also striking is that the alphas seem to be increasing up to the fourth quintile. We get even more striking results if we change the IVOL estimation method.

As laid out in the methodology section, we have reasons to believe that a longer estimation window combined with an EWMA estimation of IVOL lead to more accurate results. We present in Table 5 the patterns in portfolio alphas that result from the by us preferred method. For the value weighted returns, the IVOL effect disappears entirely with the new portfolio formation method. The spread between high-IVOL and low-IVOL portfolio alphas is exactly zero. There seems to be no recognizable pattern in the alphas for the quintiles in the middle. If we use equally weighted returns, there is a clearly positive IVOL effect. Although most of the spread seems to come out of the highest IVOL quintile, the alphas seem to be increasing throughout, as IVOL rises. The spread in alphas between high- and low-IVOL portfolios is significant at all standard significance levels.

5.2 Why the true effect is positive

So far we have documented that the in recent research discussed negative IVOL effect is not robust at all to different methodology. We now want to further analyze why there seems to be such a controversy about the effect of IVOL on returns. To start, we look again at Table 3 in the methodology section. It is obvious that sorting stocks on IVOL involves sorting stocks on CAPM-beta. However, it is well documented in the literature that stocks with a high CAPM-beta are overpriced. Frazzini and Pedersen (2014) examine this phenomenon. The general explanation for mispricing of high-beta stocks is that investors use high-beta stocks to lever-up their portfolio if they do not have access to cheap debt to lever-up their low-beta portfolio. It seems like the research dealing with a negative IVOL effect has not documented a negative effect of IVOL on alphas, but rather has documented a negative effect of CAPM-beta on returns. To examine the true effect of IVOL on portfolio alphas, it is necessary to control for mispricing due to CAPM-beta. For that reason, Table 6 shows alpha-spreads of 25 double-sorted portfolios.[2] Sorting stocks first on CAPM-beta and then divide each beta-quintile into IVOL quintiles allows us to examine portfolios that have different IVOL, but roughly the same CAPM-beta.

We can see in Table 6 that spreads between high-IVOL and low-IVOL portfolios are positive in all 5 beta quintiles. Furthermore, the spreads are statistically

[2]Table 6 shows just the spreads in alphas. The complete tables, containing also alphas on all 25 portfolios for equally weighted and value weighted as well as for independent sorting can be found as Tables 13, 14, 15 and 16 in the appendix.

Table 6: Alphas and t-stats for High-Low IVOL portfolios

	Low Beta	2	3	4	High Beta	All Stocks
High Ivol -Low Ivol	1.37 (7.12)	0.92 (3.23)	0.91 (3.10)	0.54 (1.40)	0.42 (0.87)	-0.00 (-0.00)

Note: The table shows monthly alphas and (t-stats) for the spread between the highest and lowest quintile portfolios of US stocks sorted based on idiosyncratic volatility (IVOL) within CAPM-beta quintiles. The portfolios were value weighted. IVOL was estimated as the squared exponentially weighted moving average calculated as $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$ where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month, when market beta is estimated based on between 1200 and 750 daily returns for correlation and 120 daily returns for standard deviation. The rightmost column reports results for all stocks without sorting on beta. Portfolios were reallocated monthly. The time period for the test is 1965:12-2013:12.

significant in beta quintiles 1-3. As a comparison we show the high-low spread for all stocks in the last column. Even though the spread for all stocks is zero, the spreads for all 5 quintiles are positive. How does this make sense? The explanation is as follows. When we sort stocks only according to IVOL, we get a lot of high beta stocks in the high-IVOL quintile and a lot of low beta-stocks in the low-IVOL quintile. This drags the spread between high- low-IVOL stocks down. If we however look on each beta quintile separately, there is no room for large variation in the CAPM-betas. Therefore, we get a more precise estimate of the IVOL effect on returns. The fact that the spread in alphas for all stocks together is smaller than the spread in each beta quintile really enforces our hypothesis that previously found negative effects are due to other reasons than IVOL, e.g. the beta anomaly in Frazzini and Pedersen (2014).

We do not show the corresponding table for equally weighted returns, since the spread between high-IVOL and low-IVOL stocks was already significantly positive for these portfolios without controlling for betas. Controlling for betas adds little new information in that case.

Interestingly, our results that IVOL has a positive effect on returns are perfectly in line with research done before Ang et al. (2006). Early papers mentioned in the literature review that found a positive relation between IVOL and returns also controlled for the CAPM beta in some way. An example can be found in Tinic and West (1986). Our findings are also perfectly in line with intuition and theoretical papers in asset pricing. Intuition suggests that risk, if priced at all, should be positively priced. Negatively priced risk should only occur where there is some hedging demand, as is the case for options. Ang et al. (2006) estimate plausibly a negative risk premium for exposure to the VIX. Theoretical papers that deal with idiosyncratic volatility also mostly predict a positive relation between IVOL and returns. The main theoretical arguments can be found in the literature review.

We believe that the patterns for value weighted returns in Table 6 as well as the patterns for equally weighted returns in Table 5 are strong evidence for a positive effect of IVOL on returns across all stocks. The fact that we can explain negative pattern in alphas by other generally known and better understood anomalies also confirms our hypothesis of a positive IVOL effect.

5.2.1 A small caveat

We claim that investors care about IVOL because of rational reasons such as their underdiversification and limited information. Despite the quite considerable evidence for a positive IVOL effect, there is an important caveat: IVOL affects returns not only through rational agents, but also inhibits arbitrage. This line of reasoning is fully laid out in Stambaugh et al. (2013). The limits to arbitrage argument says that IVOL makes underpriced stocks more underpriced and overpriced stocks more overpriced because smart arbitrage is riskier for high IVOL stocks. Stambaugh et al. (2013) conjecture that arbitrageurs can not hedge against IVOL and therefore are more inhibited to take advantage of mispricing of high IVOL stocks. Indeed, we found some mild evidence for this limits to arbitrage phenomenon in our data. Table 17 and Table 18 in the appendix each show alphas of 25 double sorted portfolios with value weighted returns and equally weighted returns respectively. The difference between the portfolios in Tables 17 and 18 and the portfolios underlying Table 6 is the order of sorting. In Tables 17 and 18 we first sorted stocks according to IVOL and then for each IVOL quintile according to the CAPM-beta. High-beta stocks are on average overpriced and low-beta stocks are underpriced. Therefore, a limits to arbitrage hypothesis would imply that the spread between high- and low-beta stocks is higher for high IVOL stocks. This is indeed the case.

Stambaugh et al. (2013) further hypothesize that arbitrage asymmetry, i.e. the relatively higher difficulty to short-sell asset than to buy assets would lead to an overall overpricing of high-IVOL stocks. Sorting stocks in a very special way, they could indeed show a negative return pattern as IVOL increases.

However, since the negative IVOL effect disappears, as soon as we control for the low-beta anomaly, we conclude that an effect of IVOL on returns through limits to arbitrage might exist, but that the positive effect obviously dominates.

5.2.2 Robustness of the positive effect

Robustness against portfolio formation: Tables 16 and 15 in the appendix show FF3-alphas and t-stats for portfolios constructed by sorting stocks into beta and IVOL quintiles independently of each other. The results are similar to those presented in Table 5. Alphas are significantly positive in all of the beta quintiles for the equal weighted portfolios and the same is true for the three lowest beta quintiles for the value weighted portfolios. Notably, the alpha for an equal weighted portfolio with low IVOL and low beta stocks is 4.38% monthly.

Robustness against arbitrage: Although impressive, these high returns might not be accessible for an investor since the average firm size in this portfolio was only \$0.12 billion (see Table 20 in appendix). We also calculated relative market capitalizations for the portfolios of stocks sorted first on beta and then on IVOL. The results are shown in Table 19 in the appendix. It is observable that stocks in the high-IVOL quintiles are in general small stocks. This has important implications for the robustness of the positive IVOL effect to arbitrage. Arbitrageurs might not be willing to trade these small stocks to alleviate their mispricing. Therefore, the IVOL effect might persist even in the presence of arbitrageurs. However, it would be to fast to conclude at this point that the IVOL effect exists mostly among small stocks and that therefore arbitrage is inhibited. We will deal with the connection between firm size and IVOL effect in the next section and afterwards consider limits to arbitrage again.

5.3 IVOL effect across market cap

The second hypothesis of our thesis is that the positive IVOL effect is significantly greater for small stocks. Merton (1987) and Malkiel and Xu (2002) provide the economic theory that justifies this hypothesis. Merton (1987) considers a setting where each stock is known only by a small fraction of investors. The smaller the fraction of investors that know a particular stock, the greater the effect of idiosyncratic volatility on returns. This relation can be seen in equation (1). The smaller the fraction q_k, the higher the impact of idiosyncratic volatility σ_k on the expected risk premium $\bar{R}_k - R$.

If it is true that the ratio of the stock's market share to investors informed about this stock $\frac{x_k}{q_k}$ is greater for small stocks, then the return of small stocks should be more strongly driven by IVOL. Intuitively speaking, IVOL is higher priced in small stocks because a given amount of capital has to be provided by fewer investors. This makes IVOL relatively more important for investors in small stocks, since these make up a large fraction of the investors' portfolios. We have some reasons to believe that small stocks are indeed known only by a very limited number of investors. By 'knowing' we mean that an investor is informed about the financial performance about the company and considers the stocks for a potential investment. Many large institutional investors a priori rule out the investment in small stocks due to reasons like illiquidity and higher trading costs. The investors in small stocks are consequently mostly private investors that know the small company well. Another reason to believe that small stocks are known by a very small fraction of investors is that the cost of attaining information is higher for small firms. Large companies publicly disclose information under strict standards. For small firms, disclosure is much less regulated and public disclosure is more limited.

Another, though related theoretical argument, why the positive effect might be stronger for small stocks can be inferred from Malkiel and Xu (2002). If investors are unable to hold the market portfolio, they demand compensation for IVOL. The portfolios of investors that hold small stocks might differ more strongly from the market portfolio. First, because small stocks per definition make up only a small fraction of the market portfolio. Second, because investors in small stocks might be underdiversified private investors, as claimed above. It is fair to say that smaller stocks in general are less known by investors.

Both arguments support our hypothesis of a stronger positive relation between IVOL and returns among small cap stocks on theoretical grounds. In the following we take a closer look at our data to see if we find empirical support.

5.3.1 The evidence

The fact that the IVOL effect in Table 5 is stronger for equally weighted returns than for value weighted returns already suggests that the IVOL effect is stronger for small firms. We now want to examine the relation between firm size and IVOL effect in a more detailed way.

In order to look for empirical support of our hypothesis we perform the previously done beta-IVOL double-sorting for each market cap quintile. First, we sort all the stocks according to their CAPM-beta into five quintiles to control for the low-beta anomaly. Then we create 5 different market cap quintiles for each market beta quintile. In the third step we sort stocks in each beta-market cap group into 5 IVOL quintiles. This leads to 5×25 portfolios which are sorted in a three-dimensional way. [3]

Table 7 shows FF3-alphas and robust t-stats for value weighted H-L IVOL portfolios across beta and market cap quintiles. Due to the large number of port-

[3] All results are to be found in the appendix. Tables 21, 23, 25, 27, 29 show results for equally weighted returns. Tables 22, 24, 26, 28, 30 show the results for value weighted returns.

Table 7: Alphas and t-stats for High-Low IVOL portfolios

	Low MCAP	2	3	4	High MCAP	High -Low
Low Beta	4.87	3.04	2.27	1.59	0.17	-4.7
	(19.97)	(15.81)	(12.03)	(9.02)	(0.93)	(-16.25)
2	5.79	2.55	1.68	0.84	-0.00	-5.79
	(18.53)	(10.07)	(7.63)	(4.37)	(-0.01)	(-17.35)
3	5.64	2.47	1.52	0.47	-0.10	-5.74
	(16.14)	(7.51)	(5.93)	(2.58)	(-0.48)	(-14.96)
4	6.39	2.83	1.15	0.34	-0.79	-7.18
	(14.56)	(8.55)	(3.95)	(1.55)	(-2.86)	(-14.80)
High Beta	8.91	3.77	1.39	0.42	-0.94	-9.85
	(15.57)	(8.77)	(3.59)	(1.24)	(-2.48)	(-16.17)

Note: The table shows monthly alphas and (t-stats) for the spread between the highest and lowest quintile portfolios of US stocks sorted based on idiosyncratic volatility (IVOL) across CAPM-beta and market cap quintiles. Stocks were first sorted on beta then on market cap. Alphas were estimated in the Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. T-stats are robust Newey-West (1986) t-statistics. The portfolios are value weighted. IVOL was estimated as the squared exponentially weighted moving average calculated as $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$ where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month. The rightmost column reports results within beta quintiles without sorting on market cap. Portfolios were reallocated monthly. The time period for the test is 1965:12-2013:12.

folios, the table only shows the spreads between the portfolio with the highest IVOL and the portfolio with the lowest IVOL in each beta quintile for each market cap (MCAP) quintile. The rightmost column in Table 7 shows the difference in IVOL effect between high market cap and low market cap within each beta quintile.

Looking at Table 7, we note that the spreads between high and low IVOL portfolios are significantly positive for all beta quintiles and MCAP quintiles 1-3. For the fourth MCAP quintile, these spreads are throughout positive, but only statistically significant for the lowest three beta quintiles. For the largest stocks, the IVOL effect is mostly insignificant and even turns negative for the two highest beta quintiles. Furthermore, alphas are decreasing with increasing market cap in all beta quintiles. Among stocks with the highest market cap the IVOL effect flips sign and becomes significantly negative in the highest 2 beta quintiles.

Most importantly for our second hypothesis, the rightmost column shows a significant negative difference in the IVOL effect between high market cap and low market cap within each beta quintile. Alphas for these spreads are strongly negative across all beta portfolios, confirming that the IVOL effect is significantly more positive among small cap stocks than among large cap stocks. The results are very similar for equal weighted portfolios which are shown in Tables 22, 24, 26, 28 and 30 the appendix.

5.3.2 Is there a negative IVOL effect?

In Table 7 we see that the H-L IVOL spread is significant for two of the high-market cap portfolios. This should not cause large concerns about our hypothesis that there is in general a positive IVOL effect, since the other spreads are mostly significantly positive. However, these two negative coefficients are striking enough that we want to briefly comment on them.

First, it is possible, that IVOL indeed has a negative effect on alphas for some portfolios. As we described in section 5.2.1, we found some mild evidence that high beta stocks are slightly more overpriced when they have high IVOL, and low

Table 8: Alphas and t-stats for High-Low Beta portfolios

	Low MCAP	2	3	4	High MCAP	High -Low
EW	-0.76	-0.85	-0.74	-0.95	-2.07	-1.32
	(-6.24)	(-6.69)	(-6.33)	(-6.46)	(-8.21)	(-4.77)
VW	-0.67	-0.60	-0.43	-0.59	-1.40	-0.73
	(-5.04)	(-4.84)	(-3.92)	(-4.34)	(-6.94)	(-2.86)

Note: The table shows monthly alphas and (t-stats) for the spread between the highest and lowest quintile portfolios of US stocks sorted based on CAPM-beta. Alphas were estimated in the Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. T-stats are robust Newey-West (1986) t-statistics. CAPM-betas were calculated based on between 1200 and 750 daily returns for correlation and 120 daily returns for standard deviation. *EW* and *VW* report results for equal and value weighted portfolios respectively. The rightmost column shows results for the spread between the highest and lowest market cap quintiles. Portfolios were reallocated monthly. The sample includes US stocks over the period 1965:12-2013:12.

beta stocks are slightly more underpriced if they have high IVOL. We explained this with a limits to arbitrage argument made by Stambaugh et al. (2013). Since the positive effect of high idiosyncratic volatility is decreasing in market capitalization, the negative IVOL effect stemming from limits to arbitrage begins to outweigh for large market cap stocks with high betas.

Second, there might be an explanation for a negative spread in alphas unrelated to IVOL. Although, we try to control for variations in CAPM-beta by sorting stocks into five different quintiles, such a control is imperfect. It is possible that within each quintile there is still some variation in CAPM-betas and that high-IVOL portfolios still have a slightly higher beta than low-IVOL portfolios. Table 8 shows that the beta anomaly is observable among all market cap quintiles and is especially large for the highest MCAP quintile. Thus we might get a negative H-L IVOL spread in alphas for large high-beta firms, where the positive effect of IVOL is very weak.

All in all, it is not quite clear what causes these two negative alphas. We can only speculate about the reasons.

5.3.3 Limits to arbitrage

We now want to turn back to the question whether the positive IVOL effect is robust to arbitrage. We have seen in Table 8 that the IVOL effect is strongest among small stocks. There are reasons to believe that institutional arbitrageurs might not be willing to trade these small stocks. Amihud (2002) showed empirically that in times of general market illiquidity, small stocks are highly negatively affected. Since institutional investors need to be able to liquidate their assets without a large discount, this exposure to illiquidity might be a serious limit to arbitrage.

Furthermore, Table 8 shows that among large stocks, the IVOL effect is strongest among low-beta stocks. Baker et al. (2011) showed theoretically that institutional investors are incentivized to hold high-beta assets. These incentives might be a further reason why institutional investors might not take advantage of the IVOL effect.

Finally, IVOL is itself a limit to arbitrage, as Stambaugh et al. (2013) argues. Institutional investors that care about short-term returns do not like risk of any kind and idiosyncratic volatility is hard to hedge against.

Our hypothesis is that IVOL has an effect on returns because investors have knowledge about only a limited number of stocks.[4] Therefore, we do not expect the average private investor to be willing to take advantage of the IVOL effect either, since their knowledge about different stocks seems to be most limited compared to institutional investors.

5.3.4 An emprical test of limits to arbitrage

In the previous section we hypothesized that one reason why institutional investors do not want to take advantage of the IVOL effect are negative returns during times of market illiquidity. Table 9 shows the dependence of the spreads shown in Table 7 on market illiquidity measured as in Amihud (2002). We see that smaller stocks have more negative coefficients which means that they suffer more when

[4] This is equivalent to saying that each stock is known by only a fraction of investors as in Merton (1987) or investors do not hold the market portfolio as in Malkiel and Xu (2002)

illiquidity goes up unexpectedly. Furthermore, the spread in coefficients between large and small stocks shown in the last column are statistically very significant. Small stocks are exactly the stocks where the IVOL effect is strongest. Thus, we see our hypothesis made in section 5.3.3, that institutional investors might not want to hold small stocks due to their poor performance during market illiquidity, confirmed.

Table 9: Amihud regression coefficients

	Low MCAP	2	3	4	High MCAP	High -Low
Low Beta	-6.82	-5.75	-4.54	-4.29	-2.43	4.39
	(-4.46)	(-4.48)	(-3.72)	(-3.64)	(-2.72)	(3.08)
2	-6.71	-4.62	-4.43	-2.73	-2.39	4.32
	(-3.66)	(-3.40)	(-3.28)	(-2.75)	(-2.94)	(2.79)
3	-6.46	-4.34	-3.75	-4.34	-2.73	3.73
	(-3.95)	(-3.25)	(-3.50)	(-4.62)	(-3.00)	(2.50)
4	-6.82	-4.50	-4.90	-3.74	-3.05	3.77
	(-3.67)	(-3.12)	(-4.57)	(-4.42)	(-3.16)	(2.09)
High Beta	-9.85	-6.57	-5.70	-4.03	-4.55	5.29
	(-4.50)	(-3.55)	(-4.04)	(-2.73)	(-3.96)	(2.93)

Note: Coefficient β_2 and (t-stats) from the regression $R^e_{i,t} = \beta_0 + \beta_1 \ln MILLIQ_{t-1} + \beta_2 \ln MILLIQ^U_t + \epsilon_t$, where $\ln MILLIQ^U$ are the residuals of an ARMA(1,1) model of $\ln MILLIQ$ and should represent unexpected changes in illiquidity. $MILLIQ_t$ is the average illiquidity measure across stocks. Illiquidity was estimated with the Amihud measure in Amihud (2002). The portfolios in the test are value weighted high-low IVOL portfolios within CAPM-beta and market cap quintiles. The time period for the test is 1965:12-2013:12.

5.4 Betting-Against-Beta explains previous research

By now we gathered strong evidence for a positive IVOL effect throughout all kinds of stocks.[5] We argued that previously found negative effects as documented in Ang et al. (2006) arose by not controlling for the beta anomaly. We now want to elaborate on this third hypothesis by regressing portfolio returns not only on the three Fama-French factors, but also on a fourth factor that is supposed to capture the premium of the low-beta anomaly. The resulting alpha will show the part of Fama-French alphas that cannot be explained by the low-beta anomaly.

The factor we use to capture the premium on the low-beta anomaly is the in Frazzini and Pedersen (2014) developed Betting-Against-Beta (BAB) factor. The construction of this factor is described in the methodology section. If negative effects found in previous research are due to the higher CAPM-beta of high IVOL stocks, the high-IVOL portfolios constructed in previous research should be more correlated with the BAB factor. Moreover, we expect the spread in alphas between high-IVOL and low-IVOL portfolios not to be significantly negative anymore after controlling for the BAB factor.

[5] The only two outliers were two high-beta large cap portfolios discussed in section 5.3.2.

Table 10: Alphas and t-stats for value weighted AHXZ replicated portfolios

	Low Ivol	2	3	4	High Ivol	High -Low
FF-alpha	0.08	0.05	0.02	-0.04	-0.39	-0.48
	(1.66)	(1.27)	(0.32)	(-0.53)	(-3.14)	(-3.04)
FF+BAB-alpha	0.03	0.03	0.06	0.06	-0.20	-0.22
	(0.50)	(0.60)	(1.04)	(0.70)	(-1.61)	(-1.45)

Note: The table shows monthly alphas and (t-stats) for value weighted quintile portfolios of US stocks sorted based on their idiosyncratic volatility (IVOL) estimated with daily returns in the CAPM over the previous month. This method follows the one used in Ang et al. (2006). *FF-alpha* shows results from the Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. *FF+BAB-alpha* shows results when the BAB-factor is added to the regression. T-stats are robust Newey-West (1986) t-statistics. The rightmost column shows results for the spread between the highest and lowest IVOL quintiles. Portfolios were reallocated monthly. The sample period is 1965:12-2013:12.

The first row of Table 4 replicates the results of Ang et al. (2006) for our sample period. Although the spread in Fama-French alphas between high-IVOL and low-IVOL stocks is mostly driven by the highest-IVOL quintile, it is statistically significant.

The second row of Table 4 shows alphas for the same portfolios that were used in the first row. However, in the second row we regress returns not only on the three Fama-French factors, but also on the BAB factor to control for the low-beta anomaly. We can see that the previously significant negative alpha of the high-IVOL portfolio becomes insignificant. The spread in the alpha between high-IVOL and low-IVOL portfolios becomes insignificant as well. Moreover, while alphas were previously decreasing in each IVOL quintile, this is no longer the case. The slightly negative alpha of the high-IVOL portfolio is an outlier now.

In Table 11 the above discussed analysis is performed for equally weighted returns. If returns are weighted equally, the spread between high-IVOL and low-

Table 11: Alphas and t-stats for equal weighted AHXZ replicated portfolios

	Low Ivol	2	3	4	High Ivol	High -Low
FF-alpha	0.34	0.33	0.43	0.45	0.27	-0.07
	(4.77)	(5.42)	(7.57)	(7.80)	(2.81)	(-0.54)
FF+BAB-alpha	0.20	0.22	0.37	0.47	0.38	0.18
	(4.71)	(4.58)	(6.62)	(8.02)	(4.06)	(1.92)

Note: The table shows monthly alphas and (t-stats) for equal weighted quintile portfolios of US stocks sorted based on their idiosyncratic volatility (IVOL) estimated with daily returns in the CAPM over the previous month. This method follows the one used in Ang et al. (2006). *FF-alpha* shows results from the Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. *FF+BAB-alpha* shows results when the BAB-factor is added to the regression. T-stats are robust Newey-West (1986) t-statistics. The rightmost column shows results for the spread between the highest and lowest IVOL quintiles. Portfolios were reallocated monthly. The sample period is 1965:12-2013:12.

IVOL stocks even loses its significance if we just regress the returns on the three Fama-French factors. More strikingly, we almost get a significantly positive spread in alphas at the 5% level if we regress the equally weighted returns not only on the three Fama-French factors, but also on the BAB factor.

We interpret these results as further evidence that the previously found negative IVOL effect was really due to the beta anomaly and a high correlation between IVOL and CAPM-beta.

5.5 Betting-On-IVOL

Due to the large unexplained excess returns of IVOL-beta sorted portfolios, we hypothesize that a factor that captures the IVOL risk premium can significantly improve the Fama-French model. Our fourth hypothesis is that this factor has a significant influence on the stochastic discount factor after controlling for the influence of Fama-French factors. To be better able to compare our factor to the Fama-French Factors, we price the original Fama-French portfolios sorted on

Table 12: Factor parameters and t-stats from different discount factor models

Factor set	1	2	3	4	5	6
MKT	-0.22	2.38	0.45	5.60	2.51	3.78
	(-0.16)	(2.19)	(0.38)	(3.33)	(1.81)	(2.01)
SMB	-16.37	6.67		7.62		-5.39
	(-2.45)	(3.13)		(3.14)		(-0.67)
HML	-0.12	13.25		14.80		13.39
	(-0.02)	(4.67)		(3.44)		(1.67)
BAB	7.22		7.33		6.16	0.09
	(2.50)		(4.64)		(3.29)	(0.02)
BOIVOL	10.32		3.70		3.44	6.06
	(3.34)		(3.85)		(3.81)	(1.72)
MOM				17.40	11.35	14.45
				(3.24)	(2.41)	(2.45)
δ	0.33	0.39	0.36	0.32	0.33	0.29
δ^{FF}	0.11	0.17	0.11	0.11	0.10	0.10

Note: The table shows factor b-coefficients in $m = 1 - b'(f - E(f))$ and robust (t-stats), estimated in six different factor models for 25 value weighted portfolios double sorted in CAPM-beta and IVOL. *MKT* = Market factor, *SMB* = Small Minus Big, *HML* = High Minus Low, *BAB* = Betting On Beta, *BOIVOL* = Betting on IVOL and *MOM* = Momentum. Results are left empty for factors excluded from a factor set. The sample includes US stocks over the period 1965:12-2013:12.

market capitalization and book/market ratio.

5.5.1 Model comparison

Table 12 shows different parameter estimations for the vector b in $m = 1 - (f - E(f))b$, where m is the stochastic discount factor.[6] The numbers in parentheses below the parameter estimates are their robust t-values. δ and δ^{FF} are two measures of model misspecification δ refers to the model's ability to price all possible

[6] A significant b_i coefficient means that factor i helps pricing the assets. The risk premium of a factor is strictly speaking something different. A risk premium says whether a factor is priced. Estimates of the by Table 12 implied risk premia can be found in Table 33 in the appendix.

excess returns, even dynamic trading strategies on our portfolios. δ^{FF} measures the ability of the model to price only our 25 Fama-French portfolios. The lower each number is, the closer is the estimated discount factor to a discount factor that prices excess returns correctly, and the better is the model.

Let us now have a more detailed look on the i-th elements of the vector b, which measures the impact of factor i on the discount factor m. The first column of Table 12 shows the b_i estimates for a discount factor model, where the discount factor m depends on the three Fama-French factors, the BAB factor of Frazzini and Pedersen (2014) and our own BOIVOL factor. The estimates in the second column are for a discount factor that only depends on the three Fama-French factors.

Comparing the first two columns, it is striking that the MKT and HML factors become insignificant if we control for BAB and BOIVOL factors. Equally striking is the fact that the coefficient on the SMB factor changes its sign as we go from column 2 to column 1. Also, the BAB and BOIVOL factors are highly significant with robust Newey and West (1987) t-stats. If we look at our measures of model misspecification, we see that δ decreases by about 0.06 if we add BAB and BOIVOL to the Fama-French model. This means that the estimated discount factor in column 1 is closer to the set of feasible discount factors and that the Sharpe ratio of maximum possible pricing errors decreases. Looking at δ^{FF} leads to the same conclusion. The factors in column 1 explain the excess returns better than the factors in column 2, since the respective δ^{FF} is lower in column 1.

In column 3 we consider a model where the discount factor depends only on the MKT, BAB and BOIVOL factors. We see that while the MKT factor has an insignificant coefficient (t-stat 0.38), both BAB and BOIVOL are highly significant with t-stats of 4.64 and 3.85. Thus, these coefficients have a significant impact on the discount factor also when not controlling for the Fama-French factors. We also see that the model in column 3 explains excess returns better than the Fama-French model. Both δ^{FF} and δ are lower in column 3 than in column 2, which means that the model in column 3 can better explain excess returns of

Fama-French portfolios and excess returns of all dynamic trading strategies based on our 25 test assets.

In column 4-6 we add a momentum factor (MOM) to the models examined in columns 1-3. We can see that the momentum factor has a significant impact in each of the three columns. Adding the momentum factor to the Fama-French or the MKT+BAB+BOIVOL model improves their ability to price excess returns as measured by δ and δ^{FF}. The Fama-French + MOM model can price all combinations of excess returns slightly better than the MKT+BAB+BOIVOL model, while the latter can price the Fama-French portfolios slightly better, as is shown by the δ^{FF}. However, differences in δs and δ^{FF}s are quite small for columns 4 and 5.

Column 6 shows the parameter estimates when all factors are included. BOIVOL is only significant at the 10% level now. This decrease in statistical significance is most likely due to the increased standard errors in estimated parameters. Controlling for many variables leaves less variation in the independent variables for the estimation of each parameter. Not surprisingly, the model including all factors can price excess returns best. δ and δ^{FF} are lowest in column 6.

5.5.2 Implications for investor preferences

Since the BOIVOL factor has a significant impact on the estimated stochastic discount factor in all examined model specifications (with the significance level at 10% only in the last column), we see our last hypothesis confirmed: A factor that captures the risk premium on high-IVOL stocks has indeed significant explanatory power for the cross-section of average excess returns.

We explain this significant explanatory power by preferences of the average investor to hold assets with less idiosyncratic volatility. Underdiversification and a limited information set of the average investor would be a good explanation for these investor preferences. Investors that are in principle able to take advantage of the mispricing of high-IVOL stocks seem to be inhibited. Possible limits to arbitrage have already been discussed in section 5.3.3.

Although not related to our four hypotheses we also see that the BAB factor has quite a considerable impact on the discount factor. This is of course another piece of evidence that the average investor has a preference for high beta stocks, whether it is because of leverage constraints as Frazzini and Pedersen (2014) suggest or because of institutional incentives as discussed in Baker et al. (2011).

It is impressive that two market anomalies together with the market factor can explain returns better than the Fama-French model. This suggests that market frictions and their impact on investor behavior have a great impact on expected returns.

Our analysis also suggests an explanation of the small firm effect. The SMB factor had a significant impact on the discount factor before controlling for the BAB and BOIVOL factors and was attached a positive risk premium. After controlling for BAB and BOIVOL, the impact of the SMB factor on the stochastic discount factor changed its sign. It seems as if BAB and BOIVOL explain more than all the impact that SMB had on the stochastic discount factor before.

5.5.3 Robustness of the IVOL effect in the data

As robustness check we also performed Fama and MacBeth (1973) regressions with the same factors and excess returns that were used in Table 12. The results of the Fama-MacBeth regressions are shown in Table 38. With Fama-MacBeth regressions, we check whether a factor can explain the cross-section of average excess returns by testing whether a factor is given a significant risk premium. We see in Table 38 that BAB and BOIVOL have significantly positive risk premia and even make the risk premia attached to HML and SMB insignificant. The risk premium for the SMB factor changes its sign again after controlling for the BAB and BOIVOL factors. It is also interesting to look at the fraction of variation in average excess returns that can be explained by various factor exposures (the R^2 of the cross sectional regression). A factor model with MKT+BAB+BOIVOL explains 84%, while the Fama-French model can only explain 77%. Similar results hold for equally weighted returns as shown in Table 39.

As another type of robustness check we look at different portfolios when pricing excess returns. Table 34 shows GMM estimates of discount factors if we use 10 portfolios sorted on earnings/price ratios. The analysis of Table 12 applies here as well. Table 35 shows the implied risk premia for the different factors. Although irrelevant to determine whether a factor helps pricing returns, we see that BOIVOL is assigned a positive risk premium.

Even if we use portfolios sorted on dividend yield, adding BAB and BOIVOL to the Fama-French model improves the ability of the discount factor to price excess returns. However, it is fair to say that the parameters in Table 36 and risk premia in Table 37 are estimated fairly imprecisely. We guess that the 10 portfolios based on dividend yield provide not enough dispersion in returns to estimate parameters precisely. Therefore, the analysis performed in Table 34 for the 25 size-book/market portfolios should carry larger weight when drawing inferences about the different pricing factors. The Fama-MacBeth regression results for earnings-price and dividend yield portfolios can be found in Table 40 and Table 41 in the appendix. The results of the Fama-MacBeth regressions are overall in line with the results we get by using the stochastic discount factor methodology.

6 Conclusion: Winning with IVOL

We see all our 4 hypotheses confirmed in the data. We gathered evidence that IVOL has a positive effect on expected stock return. This effect seems to be stronger among small firms. Both the positive effect and its stronger prevalence among small firms are in line with theoretical explanations like Merton (1987) and Malkiel and Xu (2002). Finally, we found evidence for our explanations why some previous research reached different conclusions. The fact that a BOIVOL factor has significant explanatory power for the cross section of expected returns has important implications for both practitioners and researchers in finance.

6.1 Winning while investing

We argued in section 5.3.3 that the IVOL effect is robust to arbitrage, because both private and institutional investors might find it difficult to take advantage of this market anomaly. This means that an investor who is not subject to the in section 5.3.3 mentioned market-frictions can find a sustainable source of abnormal returns in high-IVOL stocks.

An investor could take advantage of the positive IVOL effect if he can invest in sufficiently many stocks to diversify away a large part if the idiosyncratic risk. Otherwise, idiosyncratic volatility of stocks is an important determinant of portfolio risk.

Furthermore, he must be willing to hold stocks that might perform especially bad in liquidity crises since small stocks are hurt during these times (Amihud, 2002) This means that the investor should not be pressured to sell assets when market liquidity is low. A wealthy private investor is not subject to this pressure, but many mutual funds are.

If an institutional investor wants to take advantage of the positive IVOL effect, benchmarks should not set the wrong incentives. Baker et al. (2011) argued that benchmarks lead institutional investors to hold more high-beta stocks. An institutional investor who only wants to trade large stocks has to trade low-beta stocks to gain from the positive IVOL effect as talbe 7 shows.

6.2 Winning in research

Not only practitioners but also researchers can benefit from our findings. The BAB factor and the BOIVOL factor derive their theoretical justification purely from market frictions. The fact that our factors can explain the returns of Fama-French portfolios better than the Fama-French model has implications for the importance of market frictions for expected returns.

If we want a theory of expected returns that comes close to explaining the real world, the importance of these market frictions has to be acknowledged. Many papers quoted in our work, especially those related to limits to arbitrage, already acknowledge the importance of market frictions on asset prices. However, it seems like most people either view mispricings as negligible exceptions from the efficient market hypothesis or view markets as irrational and driven mostly by the moods of investors.

Our work might rather support the view that what happens in the markets is in some sense quite rational. Market behavior is however more complex than just the solution to a frictionless lifecycle-consumption optimization problem. The institutional environment is what makes markets really interesting and what is most likely a fruitful area of future research.

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Appendix

Table 13: Alphas for value weighted double sorted portfolios.

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low Beta	-0.03	-0.17	-0.04	0.24	1.34	1.37	0.12
	(-0.35)	(-1.75)	(-0.34)	(1.52)	(7.50)	(7.12)	(1.21)
2	0.07	0.03	0.23	0.22	0.99	0.92	0.12
	(0.81)	(0.31)	(2.09)	(1.62)	(3.75)	(3.23)	(1.47)
3	0.04	0.08	0.12	0.05	0.95	0.91	0.07
	(0.59)	(0.85)	(1.04)	(0.29)	(3.53)	(3.10)	(1.17)
4	-0.05	-0.01	-0.20	-0.24	0.48	0.54	-0.11
	(-0.68)	(-0.14)	(-1.54)	(-1.43)	(1.37)	(1.40)	(-1.60)
High Beta	-0.19	-0.30	-0.32	-0.50	0.23	0.42	-0.35
	(-2.03)	(-2.42)	(-2.02)	(-2.26)	(0.51)	(0.87)	(-3.48)
High-Low	-0.16	-0.13	-0.28	-0.74	-1.11	-0.95	-0.46
	(-1.21)	(-0.68)	(-1.18)	(-2.53)	(-2.34)	(-2.06)	(-2.70)
All Stocks	0.12	0.06	0.00	-0.22	0.12	-0.00	
	(2.12)	(1.18)	(0.02)	(-2.12)	(0.41)	(-0.00)	

Note: The table shows monthly alphas and (t-stats) for portfolios of US stocks double sorted based on pre-formation beta and idiosyncartic volatility (IVOL). Stocks were first sorted based on beta and then on IVOL. Alphas were estimated from the Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$. Portfolios were value weigted. IVOL was estimated as the square root of an exponential weighted moving average as calculated in the equation $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$. where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month. The column (row) *High-Low* shows results for portfolios long the highest IVOL (beta) quintile and short in lowest IVOL (beta) quintile within corresponding beta (IVOL) quintiles. The column (row) *All Stocks* shows results for portfolios long long the highest beta (IVOL) quintile and short in lowest beta (IVOL) quintile. The sample period is 1965:12-2013:12.

Table 14: Alphas for equal weighted double sorted portfolios.

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low Beta	-0.23	-0.21	-0.08	0.19	2.36	2.59	0.40
	(-3.27)	(-2.24)	(-0.73)	(1.73)	(15.02)	(18.03)	(4.43)
2	-0.15	-0.19	-0.05	0.13	2.05	2.20	0.36
	(-1.93)	(-2.19)	(-0.61)	(1.49)	(13.87)	(12.79)	(4.95)
3	-0.17	-0.23	-0.19	-0.04	1.86	2.03	0.24
	(-2.37)	(-2.62)	(-2.26)	(-0.43)	(12.43)	(11.32)	(3.87)
4	-0.20	-0.33	-0.35	-0.17	1.97	2.18	0.18
	(-2.57)	(-3.94)	(-4.15)	(-2.00)	(11.46)	(10.75)	(2.85)
High Beta	-0.45	-0.54	-0.58	-0.17	2.89	3.34	0.23
	(-4.66)	(-4.73)	(-5.28)	(-1.20)	(9.36)	(10.13)	(1.97)
High-Low	-0.22	-0.33	-0.51	-0.36	0.53	0.74	-0.18
	(-1.87)	(-2.21)	(-2.91)	(-1.68)	(1.44)	(2.16)	(-1.01)
All Stocks	-0.19	-0.18	-0.18	-0.07	2.04	2.22	
	(-2.85)	(-2.55)	(-2.44)	(-1.07)	(14.72)	(12.30)	

Note: The table shows monthly alphas and (t-stats) for beta and IVOL double sorted portfolios comprised of US stocks between 1965:12 and 2013:12. Portfolios are equal weighted but otherwise formed as described under Table 13.

Table 15: Alphas for equal weighted independently double sorted portfolios.

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low Beta	-0.19	-0.05	0.21	0.83	4.19	4.38	0.40
	(-2.46)	(-0.44)	(1.81)	(5.56)	(12.36)	(12.55)	(4.43)
2	-0.16	-0.08	0.03	0.45	3.17	3.33	0.36
	(-2.10)	(-0.95)	(0.30)	(4.58)	(14.56)	(13.79)	(4.95)
3	-0.17	-0.22	-0.14	0.06	2.34	2.51	0.24
	(-2.29)	(-2.65)	(-1.61)	(0.76)	(13.08)	(12.09)	(3.87)
4	-0.16	-0.21	-0.31	-0.21	1.82	1.98	0.18
	(-2.07)	(-2.51)	(-4.02)	(-2.43)	(11.25)	(10.36)	(2.85)
High Beta	-0.30	-0.43	-0.43	-0.54	1.42	1.72	0.23
	(-2.57)	(-4.79)	(-4.08)	(-4.63)	(6.78)	(7.24)	(1.97)
High-Low	-0.10	-0.39	-0.64	-1.36	-2.77	-2.67	-0.18
	(-0.75)	(-2.93)	(-3.98)	(-6.20)	(-6.66)	(-6.59)	(-1.01)
All Stocks	-0.19	-0.18	-0.18	-0.07	2.04	2.22	
	(-2.85)	(-2.55)	(-2.44)	(-1.07)	(14.72)	(12.30)	

Note: The table shows Fama-French alphas and (t-stats) for equal weighted portfolios of US stocks independently sorted on idiosyncratic volatility and CAPM-beta between 1965:12 and 2013:12. It is the independently sorted equivalent to Table 13.

Table 16: Alphas for value weighted independently double sorted portfolios.

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low Beta	0.00	0.08	0.11	0.81	2.90	2.89	0.12
	(0.05)	(0.61)	(0.71)	(3.92)	(8.57)	(7.97)	(1.21)
2	0.07	0.21	0.18	0.33	1.74	1.67	0.12
	(0.84)	(2.19)	(1.29)	(1.84)	(4.80)	(4.30)	(1.47)
3	0.00	0.07	0.16	0.09	1.15	1.15	0.07
	(0.03)	(0.89)	(1.30)	(0.56)	(3.58)	(3.27)	(1.17)
4	0.02	-0.02	-0.11	-0.26	0.48	0.46	-0.11
	(0.21)	(-0.26)	(-1.13)	(-1.83)	(1.37)	(1.20)	(-1.60)
High Beta	-0.12	-0.23	-0.07	-0.32	-0.13	-0.00	-0.35
	(-0.93)	(-2.20)	(-0.60)	(-2.27)	(-0.39)	(-0.01)	(-3.48)
High-Low	-0.13	-0.31	-0.18	-1.13	-3.02	-2.89	-0.46
	(-0.73)	(-1.70)	(-0.83)	(-4.10)	(-6.72)	(-6.30)	(-2.70)
All Stocks	0.12	0.06	0.00	-0.22	0.12	-0.00	
	(2.12)	(1.18)	(0.02)	(-2.12)	(0.41)	(-0.00)	

Note: The table shows Fama-French alphas and (t-stats) for value weighted portfolios of US stocks independently sorted on idiosyncratic volatility and CAPM-beta between 1965:12 and 2013:12. It is the independently sorted equivalent to Table 14.

Table 17: Alphas for value weighted portfolios first sorted on IVOL then on beta.

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low Beta	-0.01	0.06	0.04	0.41	1.96	1.97	0.12
	(-0.10)	(0.51)	(0.23)	(2.14)	(7.00)	(6.34)	(1.21)
2	-0.10	0.21	0.24	0.06	0.86	0.96	0.12
	(-1.15)	(2.07)	(1.95)	(0.38)	(2.78)	(2.85)	(1.47)
3	0.04	0.09	0.03	-0.11	0.22	0.18	0.07
	(0.42)	(1.13)	(0.24)	(-0.71)	(0.67)	(0.52)	(1.17)
4	0.03	0.04	-0.08	-0.34	0.05	0.03	-0.11
	(0.32)	(0.44)	(-0.76)	(-2.04)	(0.16)	(0.07)	(-1.60)
High Beta	0.06	-0.10	-0.12	-0.31	-0.26	-0.32	-0.35
	(0.89)	(-1.23)	(-1.06)	(-2.02)	(-0.78)	(-0.89)	(-3.48)
High-Low	0.07	-0.17	-0.16	-0.73	-2.21	-2.28	-0.46
	(0.60)	(-1.03)	(-0.71)	(-2.70)	(-5.40)	(-5.39)	(-2.70)
All Stocks	0.12	0.06	0.00	-0.22	0.12	-0.00	
	(2.12)	(1.18)	(0.02)	(-2.12)	(0.41)	(-0.00)	

Note: The table shows alphas and (t-stats) for value weighted portfolios of US stocks between 1965:12 and 2013:12. Stocks are first sorted on idiosyncratic volatility and then on beta. For more information on portfolio construction see note to Table 13.

Table 18: Alphas for equal weighted portfolios first sorted on IVOL then on Beta.

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low Beta	-0.21	-0.07	0.12	0.57	3.28	3.49	0.40
	(-2.72)	(-0.66)	(1.05)	(4.61)	(14.51)	(14.73)	(4.43)
2	-0.20	-0.12	-0.01	0.16	2.36	2.56	0.36
	(-2.49)	(-1.33)	(-0.16)	(1.72)	(14.26)	(13.24)	(4.95)
3	-0.16	-0.20	-0.23	-0.16	1.71	1.87	0.24
	(-2.07)	(-2.61)	(-2.57)	(-1.87)	(11.03)	(9.68)	(3.87)
4	-0.20	-0.21	-0.32	-0.33	1.59	1.79	0.18
	(-2.86)	(-2.60)	(-4.05)	(-3.68)	(8.91)	(7.96)	(2.85)
High Beta	-0.16	-0.31	-0.47	-0.61	1.27	1.43	0.23
	(-2.13)	(-3.93)	(-4.48)	(-4.68)	(4.86)	(4.87)	(1.97)
High-Low	0.05	-0.25	-0.59	-1.19	-2.02	-2.06	-0.18
	(0.48)	(-2.06)	(-3.98)	(-5.70)	(-5.53)	(-5.78)	(-1.01)
All Stocks	-0.19	-0.18	-0.18	-0.07	2.04	2.22	
	(-2.85)	(-2.55)	(-2.44)	(-1.07)	(14.72)	(12.30)	

Note: The table shows alphas and (t-stats) for equal weighted portfolios of US stocks between 1965:12 and 2013:12. Stocks are first sorted on idiosyncratic volatility and then on beta. For more information on portfolio construction see note to Table 13.

Table 19: Average market cap for double sorted portfolios

	Low Ivol	2	3	4	High Ivol
Low Beta	3.07	1.28	0.83	0.55	0.30
2	8.43	3.71	2.10	1.08	0.47
3	11.98	4.45	2.32	1.26	0.62
4	18.76	5.85	2.66	1.40	0.70
High Beta	17.72	5.27	2.65	1.59	0.98

Note: The table shows average market cap within each double sorted portfolios of US stocks over the time period 1965:12-2013:12.. Stocks were first sorted based on CAPM-beta and then based on IVOL estimated as the square root of an exponential weighted moving average as calculated in the equation $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$. where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month.

Table 20: Market cap and observations for independently double sorted portfolios

	Low Ivol	2	3	4	High Ivol	All Stocks
Low Beta	4.22	0.90	0.40	0.20	0.12	5.85
	(7.24)	(4.00)	(3.24)	(2.78)	(2.75)	(20)
2	10.34	3.21	1.25	0.50	0.21	15.51
	(5.70)	(4.85)	(3.90)	(3.08)	(2.46)	(20)
3	11.43	5.26	2.33	0.99	0.38	20.40
	(3.76)	(4.84)	(4.55)	(3.84)	(3.01)	(20)
4	14.96	8.18	3.87	1.81	0.72	29.54
	(2.49)	(4.11)	(4.64)	(4.66)	(4.11)	(20)
High Beta	7.55	8.63	6.07	3.97	2.48	28.70
	(0.80)	(2.19)	(3.69)	(5.67)	(7.64)	(20)
All Stocks	48.50	26.19	13.92	7.47	3.91	
	(20)	(20)	(20)	(20)	(20)	

Note: The table shows average market cap and (number of observations) for each double sorted portfolio in each month over the period 1965:12-2013:12. Stocks are sorted into quintiles based on IVOL calculated as described under Table 19 and CAPM-beta independently of each other.

Table 21: Equal weighted double sorted portfolios within the 1st beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-0.46	-0.45	-0.01	0.69	4.81	5.27	0.89
	(-5.15)	(-3.80)	(-0.11)	(3.84)	(18.27)	(20.77)	(7.28)
2	-0.43	-0.21	-0.10	0.33	2.67	3.10	0.96
	(-5.02)	(-1.65)	(-0.82)	(2.52)	(12.93)	(15.76)	(8.44)
3	-0.29	-0.29	-0.05	0.29	2.02	2.31	0.81
	(-3.72)	(-2.63)	(-0.38)	(2.16)	(10.08)	(12.30)	(7.63)
4	-0.21	-0.15	-0.08	0.18	1.41	1.62	0.90
	(-2.82)	(-1.44)	(-0.61)	(1.31)	(7.91)	(9.60)	(6.97)
High MCAP	-0.01	0.01	0.03	0.21	0.47	0.48	1.72
	(-0.17)	(0.11)	(0.23)	(1.54)	(2.90)	(3.15)	(6.79)
High-Low	0.45	0.46	0.04	-0.48	-4.34	-4.79	0.83
	(5.12)	(3.71)	(0.30)	(-2.50)	(-15.59)	(-17.39)	(2.80)
All Stocks	-0.23	-0.21	-0.08	0.19	2.36	2.59	
	(-3.27)	(-2.24)	(-0.73)	(1.73)	(15.02)	(18.03)	

Note: Alphas and (t-stats) for portfolios first sorted on market cap then on IVOL for stocks within the lowest Beta quintile. Portfolios are equal weighted. IVOL is the square root of an exponential weighted moving average as calculated in the equation $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$. where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month. Alphas are each double sorted portfolio's a in the regression Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$.

Table 22: Value weighted double sorted portfolios within 1st beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-0.46	-0.45	-0.06	0.60	4.41	4.87	0.71
	(-5.35)	(-3.90)	(-0.45)	(3.45)	(17.34)	(19.97)	(6.13)
2	-0.41	-0.23	-0.10	0.35	2.63	3.04	0.69
	(-4.87)	(-1.77)	(-0.77)	(2.67)	(12.92)	(15.81)	(6.62)
3	-0.29	-0.29	-0.06	0.30	1.99	2.27	0.50
	(-3.58)	(-2.64)	(-0.45)	(2.20)	(9.82)	(12.03)	(5.11)
4	-0.21	-0.16	-0.08	0.14	1.37	1.59	0.46
	(-2.76)	(-1.55)	(-0.66)	(0.99)	(7.55)	(9.02)	(4.17)
High MCAP	0.03	0.03	-0.09	0.02	0.20	0.17	1.01
	(0.31)	(0.23)	(-0.74)	(0.11)	(1.05)	(0.93)	(5.08)
High-Low	0.49	0.48	-0.03	-0.59	-4.21	-4.70	0.30
	(5.37)	(3.43)	(-0.20)	(-2.79)	(-14.27)	(-16.25)	(1.20)
All Stocks	-0.03	-0.17	-0.04	0.24	1.34	1.37	
	(-0.35)	(-1.75)	(-0.34)	(1.52)	(7.50)	(7.12)	

Note: Alphas and (t-stats) for portfolios first sorted on market cap then on IVOL for stocks within the lowest Beta quintile. Portfolios are value weighted. IVOL is the square root of an exponential weighted moving average as calculated in the equation $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$. where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month. Alphas are each double sorted portfolio's a in the regression Fama-French regression, $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$.

Table 23: Equal weighted double sorte portfolios within the 2nd beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.02	-0.65	-0.11	1.18	5.51	6.53	0.44
	(-9.20)	(-4.74)	(-0.88)	(7.11)	(18.58)	(21.42)	(4.36)
2	-0.62	-0.39	-0.11	0.39	2.03	2.65	0.25
	(-6.80)	(-3.23)	(-0.93)	(2.97)	(9.11)	(10.84)	(2.83)
3	-0.36	-0.34	0.15	0.48	1.29	1.65	0.14
	(-3.38)	(-3.31)	(1.33)	(3.52)	(7.04)	(7.45)	(1.81)
4	-0.14	-0.07	0.18	0.46	0.72	0.86	0.03
	(-1.71)	(-0.71)	(1.81)	(4.34)	(4.60)	(4.71)	(0.39)
High MCAP	-0.02	0.08	0.08	0.12	0.31	0.34	0.10
	(-0.26)	(0.82)	(0.80)	(1.00)	(2.00)	(2.07)	(0.69)
High-Low	1.00	0.74	0.19	-1.06	-5.20	-6.19	-0.34
	(8.59)	(5.16)	(1.24)	(-5.88)	(-16.06)	(-19.47)	(-1.62)
All Stocks	-0.15	-0.19	-0.05	0.13	2.05	2.20	
	(-1.93)	(-2.19)	(-0.61)	(1.49)	(13.87)	(12.79)	

Note: Alphas and (t-stats) for portfolios formed as described under Table21 within the second Beta quintile.

Table 24: Value weighted double sorted portfolios within the 2nd beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-0.96	-0.57	-0.07	1.19	4.83	5.79	0.42
	(-8.86)	(-4.03)	(-0.52)	(7.24)	(16.06)	(18.53)	(4.21)
2	-0.60	-0.38	-0.07	0.44	1.95	2.55	0.23
	(-6.38)	(-3.16)	(-0.57)	(3.45)	(8.46)	(10.07)	(2.59)
3	-0.32	-0.28	0.18	0.50	1.36	1.68	0.13
	(-3.11)	(-2.75)	(1.59)	(3.64)	(7.30)	(7.63)	(1.65)
4	-0.12	-0.05	0.22	0.48	0.72	0.84	0.03
	(-1.54)	(-0.53)	(2.17)	(4.30)	(4.34)	(4.37)	(0.39)
High MCAP	0.07	0.07	0.13	0.09	0.07	-0.00	0.06
	(0.81)	(0.61)	(1.32)	(0.73)	(0.44)	(-0.01)	(0.39)
High-Low	1.04	0.64	0.20	-1.10	-4.76	-5.79	-0.37
	(8.72)	(3.90)	(1.29)	(-5.46)	(-14.33)	(-17.35)	(-1.76)
All Stocks	0.07	0.03	0.23	0.22	0.99	0.92	
	(0.81)	(0.31)	(2.09)	(1.62)	(3.75)	(3.23)	

Note: Alphas and (t-stats) for portfolios formed as described under Table22 within the second Beta quintile.

Table 25: Equal weighted double sorted portfolios within the 3rd beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.27	-0.83	-0.22	1.21	5.29	6.56	0.33
	(-10.70)	(-6.57)	(-1.58)	(6.26)	(17.93)	(19.19)	(3.15)
2	-0.75	-0.52	-0.24	0.47	1.81	2.56	0.24
	(-7.25)	(-4.09)	(-1.94)	(3.59)	(7.01)	(8.08)	(2.73)
3	-0.50	-0.34	-0.02	0.34	1.07	1.56	0.10
	(-4.93)	(-3.01)	(-0.16)	(3.02)	(5.16)	(6.25)	(1.25)
4	-0.13	-0.07	0.05	0.23	0.43	0.55	-0.00
	(-1.54)	(-0.66)	(0.43)	(1.86)	(2.78)	(2.99)	(-0.06)
High MCAP	-0.01	0.03	0.09	0.19	0.03	0.03	-0.16
	(-0.08)	(0.32)	(1.07)	(1.91)	(0.17)	(0.18)	(-1.38)
High-Low	1.26	0.86	0.31	-1.01	-5.26	-6.53	-0.49
	(10.48)	(6.35)	(2.04)	(-4.76)	(-16.47)	(-18.08)	(-2.61)
All Stocks	-0.17	-0.23	-0.19	-0.04	1.86	2.03	
	(-2.37)	(-2.62)	(-2.26)	(-0.43)	(12.43)	(11.32)	

Note: Alphas and (t-stats) for portfolios formed as described under Table21 within the third Beta quintile.

Table 26: Value weighted double sorted portfolios within the 3rd beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.12	-0.84	-0.22	1.17	4.52	5.64	0.31
	(-9.48)	(-6.41)	(-1.60)	(6.12)	(14.95)	(16.14)	(3.01)
2	-0.71	-0.48	-0.18	0.49	1.76	2.47	0.26
	(-6.80)	(-3.77)	(-1.42)	(3.75)	(6.53)	(7.51)	(2.90)
3	-0.45	-0.33	0.03	0.37	1.07	1.52	0.11
	(-4.60)	(-2.93)	(0.22)	(3.23)	(4.98)	(5.93)	(1.40)
4	-0.11	-0.06	0.08	0.21	0.37	0.47	-0.01
	(-1.26)	(-0.54)	(0.68)	(1.74)	(2.36)	(2.58)	(-0.10)
High MCAP	-0.02	0.18	0.18	0.14	-0.12	-0.10	-0.17
	(-0.18)	(1.94)	(1.54)	(1.19)	(-0.65)	(-0.48)	(-1.37)
High-Low	1.10	1.01	0.39	-1.02	-4.64	-5.74	-0.48
	(8.79)	(6.87)	(2.20)	(-4.94)	(-13.33)	(-14.96)	(-2.50)
All Stocks	0.04	0.08	0.12	0.05	0.95	0.91	
	(0.59)	(0.85)	(1.04)	(0.29)	(3.53)	(3.10)	

Note: Alphas and (t-stats) for portfolios formed as described under Table22 within the third Beta quintile.

Table 27: Equal weighted double sorted portfolios within the 4th beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.39	-0.95	-0.13	1.18	5.95	7.34	0.22
	(-9.98)	(-6.90)	(-0.81)	(6.40)	(14.75)	(16.43)	(2.22)
2	-0.97	-0.69	-0.32	0.28	1.92	2.88	0.23
	(-9.01)	(-5.01)	(-2.47)	(2.11)	(7.05)	(8.77)	(3.02)
3	-0.56	-0.32	-0.11	0.28	0.71	1.28	0.10
	(-5.37)	(-2.98)	(-0.96)	(2.18)	(3.00)	(4.46)	(1.24)
4	-0.25	-0.10	0.01	0.25	0.22	0.47	0.03
	(-2.66)	(-0.94)	(0.12)	(2.20)	(1.15)	(2.15)	(0.32)
High MCAP	-0.04	0.01	-0.11	0.13	-0.24	-0.20	-0.19
	(-0.48)	(0.15)	(-1.20)	(1.42)	(-1.25)	(-0.94)	(-1.68)
High-Low	1.35	0.97	0.02	-1.04	-6.19	-7.54	-0.41
	(10.39)	(6.39)	(0.11)	(-5.31)	(-13.47)	(-15.79)	(-2.38)
All Stocks	-0.20	-0.33	-0.35	-0.17	1.97	2.18	
	(-2.57)	(-3.94)	(-4.15)	(-2.00)	(11.46)	(10.75)	

Note: Alphas and (t-stats) for portfolios formed as described under Table 21 within the fourth Beta quintile.

Table 28: Value weighted double sorted portfolios within the 4th beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.35	-0.93	-0.11	0.99	5.04	6.39	0.20
	(-9.93)	(-6.62)	(-0.60)	(5.42)	(13.22)	(14.56)	(1.98)
2	-0.92	-0.63	-0.29	0.33	1.91	2.83	0.23
	(-8.81)	(-4.48)	(-2.19)	(2.54)	(6.86)	(8.55)	(3.10)
3	-0.51	-0.28	-0.07	0.31	0.63	1.15	0.08
	(-4.97)	(-2.62)	(-0.62)	(2.33)	(2.58)	(3.95)	(1.01)
4	-0.24	-0.08	0.10	0.25	0.10	0.34	0.02
	(-2.46)	(-0.69)	(0.82)	(2.19)	(0.52)	(1.55)	(0.22)
High MCAP	0.06	-0.07	-0.27	0.09	-0.73	-0.79	-0.18
	(0.63)	(-0.65)	(-2.50)	(0.73)	(-3.00)	(-2.86)	(-1.59)
High-Low	1.41	0.86	-0.16	-0.90	-5.77	-7.18	-0.37
	(9.75)	(5.30)	(-0.83)	(-4.30)	(-12.70)	(-14.80)	(-2.20)
All Stocks	-0.05	-0.01	-0.20	-0.24	0.48	0.54	
	(-0.68)	(-0.14)	(-1.54)	(-1.43)	(1.37)	(1.40)	

Note: Alphas and (t-stats) for portfolios formed as described under Table 22 within the fourth Beta quintile.

Table 29: Equal weighted double sorted portfolios within the 5th beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.58	-1.18	0.28	2.19	9.12	10.70	0.14
	(-10.35)	(-6.49)	(1.22)	(6.47)	(13.56)	(16.00)	(1.40)
2	-1.13	-0.86	-0.32	0.07	2.81	3.94	0.11
	(-7.53)	(-5.23)	(-2.00)	(0.37)	(7.07)	(9.37)	(1.25)
3	-0.78	-0.61	-0.21	0.11	0.71	1.48	0.07
	(-5.81)	(-4.18)	(-1.47)	(0.61)	(2.12)	(3.86)	(0.92)
4	-0.42	-0.28	-0.17	-0.07	0.03	0.45	-0.05
	(-3.72)	(-1.93)	(-1.17)	(-0.47)	(0.10)	(1.32)	(-0.61)
High MCAP	-0.19	-0.18	-0.26	-0.37	-0.75	-0.55	-0.35
	(-1.83)	(-1.73)	(-2.00)	(-2.38)	(-2.57)	(-1.72)	(-3.32)
High-Low	1.39	1.00	-0.54	-2.57	-9.87	-11.25	-0.49
	(9.85)	(5.23)	(-2.28)	(-7.93)	(-13.86)	(-16.19)	(-2.92)
All Stocks	-0.45	-0.54	-0.58	-0.17	2.89	3.34	
	(-4.66)	(-4.73)	(-5.28)	(-1.20)	(9.36)	(10.13)	

Note: Alphas and (t-stats) for portfolios formed as described under Table21 within the fifth Beta quintile.

Table 30: Value weighted double aorted portfolios within the 5st beta quintile

	Low Ivol	2	3	4	High Ivol	High -Low	All Stocks
Low MCAP	-1.54	-1.21	0.13	1.96	7.37	8.91	0.04
	(-10.43)	(-6.65)	(0.62)	(5.93)	(13.27)	(15.57)	(0.37)
2	-1.09	-0.84	-0.29	0.04	2.68	3.77	0.10
	(-7.36)	(-5.08)	(-1.72)	(0.22)	(6.62)	(8.77)	(1.10)
3	-0.74	-0.58	-0.22	0.14	0.65	1.39	0.07
	(-5.46)	(-3.94)	(-1.47)	(0.75)	(1.95)	(3.59)	(1.04)
4	-0.42	-0.24	-0.12	-0.04	0.01	0.42	-0.13
	(-3.74)	(-1.58)	(-0.79)	(-0.27)	(0.02)	(1.24)	(-1.75)
High MCAP	-0.12	-0.17	-0.25	-0.49	-1.06	-0.94	-0.39
	(-1.13)	(-1.43)	(-1.65)	(-2.55)	(-3.03)	(-2.48)	(-3.56)
High-Low	1.41	1.04	-0.38	-2.45	-8.43	-9.85	-0.43
	(9.43)	(5.08)	(-1.49)	(-6.99)	(-13.99)	(-16.17)	(-2.32)
All Stocks	-0.19	-0.30	-0.32	-0.50	0.23	0.42	
	(-2.03)	(-2.42)	(-2.02)	(-2.26)	(0.51)	(0.87)	

Note: Alphas and (t-stats) for portfolios formed as described under Table22 within the fifth Beta quintile.

Table 31: High-Low IVOL spread for equal weighted portfolios

	Low MCAP	2	3	4	High MCAP	High -Low
Low Beta	5.27	3.10	2.31	1.62	0.48	-4.79
	(20.77)	(15.76)	(12.30)	(9.60)	(3.15)	(-17.39)
2	6.53	2.65	1.65	0.86	0.34	-6.19
	(21.42)	(10.84)	(7.45)	(4.71)	(2.07)	(-19.47)
3	6.56	2.56	1.56	0.55	0.03	-6.53
	(19.19)	(8.08)	(6.25)	(2.99)	(0.18)	(-18.08)
4	7.34	2.88	1.28	0.47	-0.20	-7.54
	(16.43)	(8.77)	(4.46)	(2.15)	(-0.94)	(-15.79)
High Beta	10.70	3.94	1.48	0.45	-0.55	-11.25
	(16.00)	(9.37)	(3.86)	(1.32)	(-1.72)	(-16.19)

Note: The table shows monthly alphas and (t-stats) as estimated in $R_{i,t} = a + bS_{t-1} + cMKT_t + dSMB_t + eHML_t + \epsilon_{i,t}$ for equal weighted portfolios long in the quintile of US stocks with high idiosyncratic volatility and short in the quintile of US stocks with low idiosyncratic volatility within quintiles of CAPM-beta and market cap. IVOL was estimated as the square root of an exponential weighted moving average as calculated in the equation $EWMA_t = 0.06 \cdot (r_t - r_{t,CAPM})^2 + 0.94 \cdot EWMA_{t-1}$. where r_t is an asset's realized return in excess of the risk free rate in month t and $r_{t,CAPM}$ is the excess return as predicted by CAPM for the same month. The column *High-Low* shows results for the spread between high market cap and low market cap. The time period for the test is 1965:12-2013:12.

Table 32: Average pre-formation betas across IVOL quintiles

	Low Ivol	2	3	4	High Ivol
VW	0.88	1.04	1.17	1.30	1.44
EW	0.56	0.74	0.85	0.97	1.10

Note: The table shows average pre-formation betas for value (*VW*) and equal weighted (*EW*) portfolios of stocks sorted into quintiles depending on their idiosyncratic volatility. IVOL was estimated as described in the note to Table 13. Betas are CAPM-betas. The sample consists of US stocks from 1965:12 to 2013:12 and portfolios were reallocated monthly.

Table 33: Estimated factor risk premia from 25 double sorted portfolios

Factor set	1	2	3	4	5	6
MKT	0.45	0.25	0.36	0.45	0.50	0.49
SMB	-0.24	0.49		0.66		0.21
HML	0.37	0.90		0.53		0.59
BAB	0.86		0.86		0.76	0.69
BOIVOL	1.24		1.21		1.24	1.26
MOM				2.84	2.09	2.27

Note: The table shows factor risk premia $\lambda = var(f)b$ estimated in six different factor models for 25 value weighted portfolios double sorted on CAPM-beta and IVOL. *MKT* = Market factor, *SMB* = Small Minus Big, *HML* = High Minus Low, *BAB* = Betting On Beta, *BOIVOL* = Betting on IVOL and *MOM* = Momentum. Results are left empty for factors excluded from a factor set. The sample includes US stocks over the period 1965:12-2013:12.

Table 34: Factor parameters and t-stats from different discount factor models, EP

Factor set	1	2	3	4	5	6
MKT	-5.87	3.27	0.34	5.14	0.65	-3.61
	(-0.75)	(2.49)	(0.18)	(2.90)	(0.29)	(-0.49)
SMB	-5.12	2.31		3.46		-19.71
	(-0.16)	(0.66)		(0.87)		(-0.64)
HML	-15.91	6.75		9.77		-2.81
	(-1.33)	(3.22)		(3.95)		(-0.16)
BAB	27.53		9.13		11.32	13.23
	(1.88)		(3.35)		(3.47)	(0.62)
BOIVOL	15.87		5.28		7.19	19.45
	(0.81)		(2.33)		(2.44)	(1.09)
MOM				10.98	8.89	10.31
				(2.33)	(1.78)	(0.90)
δ	0.07	0.13	0.11	0.09	0.06	0.04
δ^{FF}	0.01	0.02	0.02	0.02	0.01	0.01

Note: The table shows factor b-coefficients in $m = 1 - b'(f - E(f))$ and robust (t-stats), estimated in six different factor models for 10 decile portfolios of stocks sorted on earnings-price ratio (EP). *MKT* = Market factor, *SMB* = Small Minus Big, *HML* = High Minus Low, *BAB* = Betting On Beta, *BOIVOL* = Betting on IVOL and *MOM* = Momentum. Results are left empty for factors excluded from a factor set. The sample includes US stocks over the period 1965:12-2013:12.

Table 35: Estimated risk premia from EP decile portfolios

Factor set	1	2	3	4	5	6
MKT	0.48	0.51	0.50	0.54	0.52	0.52
SMB	1.49	0.23		0.35		0.57
HML	-0.03	0.40		0.34		0.13
BAB	2.57		1.05		1.38	1.49
BOIVOL	4.07		1.73		2.25	3.43
MOM				1.74	1.67	1.93

Note: The table shows factor risk premia $\lambda = var(f)b$ estimated in six different factor models for 10 decile portfolios of stocks sorted on earnings-price reatio (EP) . *MKT* = Market factor, *SMB* = Small Minus Big, *HML* = High Minus Low, *BAB* = Betting On Beta, *BOIVOL* = Betting on IVOL and *MOM* = Momentum. Results are left empty for factors excluded from a factor set. The sample includes US stocks over the period 1965:12-2013:12.

Table 36: Factor parameters and t-stats from different discount factor models, DP

Factor set	1	2	3	4	5	6
MKT	6.29	3.25	4.24	2.98	3.78	9.40
	(2.46)	(2.57)	(1.92)	(1.87)	(1.64)	(1.31)
SMB	3.60	-0.98		-0.90		-3.08
	(0.40)	(-0.19)		(-0.17)		(-0.17)
HML	7.40	3.51		2.84		19.11
	(1.16)	(1.32)		(0.77)		(0.76)
BAB	-6.27		2.56		1.09	-19.00
	(-0.69)		(0.71)		(0.28)	(-0.66)
BOIVOL	-6.26		-2.25		-2.34	-7.57
	(-1.20)		(-0.57)		(-0.59)	(-1.07)
MOM				-1.12	-2.58	6.06
				(-0.29)	(-0.87)	(0.48)
δ	0.10	0.11	0.11	0.11	0.11	0.09
δ^{FF}	0.02	0.03	0.03	0.03	0.03	0.02

Note: The table shows factor b-coefficients in $m = 1 - b'(f - E(f))$ and robust (t-stats), estimated in six different factor models for 10 decile portfolios of stocks sorted on dividend-price ratio (DP). *MKT* = Market factor, *SMB* = Small Minus Big, *HML* = High Minus Low, *BAB* = Betting On Beta, *BOIVOL* = Betting on IVOL and *MOM* = Momentum. Results are left empty for factors excluded from a factor set. The sample includes US stocks over the period 1965:12-2013:12.

Table 37: Estimated risk premia from DP decile portfolios

Factor set	1	2	3	4	5	6
MKT	0.45	0.25	0.36	0.45	0.50	0.49
SMB	-0.24	0.49		0.66		0.21
HML	0.37	0.90		0.53		0.59
BAB	0.86		0.86		0.76	0.69
BOIVOL	1.24		1.21		1.24	1.26
MOM				2.84	2.09	2.27

Note: The table shows factor risk premia $\lambda = var(f)b$ estimated in six different factor models for 10 decile portfolios of stocks sorted on earnings-price reatio (EP) . *MKT* = Market factor, *SMB* = Small Minus Big, *HML* = High Minus Low, *BAB* = Betting On Beta, *BOIVOL* = Betting on IVOL and *MOM* = Momentum. Results are left empty for factors excluded from a factor set. The sample includes US stocks over the period 1965:12-2013:12.

Table 38: Fama-MacBeth risk premia and t-stats from VW portfolios

	b1	b2	b3	b4	b5	R^2
FF3	0.54	0.75	1.66			0.77
	(2.68)	(3.78)	(5.69)			
FF3+BAB	0.49	-0.25	-0.36	0.91	1.34	0.87
+BOIVOL	(2.28)	(-0.94)	(-0.82)	(4.55)	(5.18)	
MKT+BAB	0.57			1.10	1.36	0.84
+BOIVOL	(2.90)			(5.77)	(5.25)	

Note: The table shows risk premia and (robust Newey-West (1986)) for factors in three different factor models. The results were derived from Fama-MacBeth regressions of 25 value weighted portfolios formed from double sorting stock first based on pre-formation CAPM-beta and then on pre-formation idiosyncratic volatility. R^2 shows the r-square for each model. The sample included US stocks over the period 1965:12-2013:12.

Table 39: Fama-MacBeth risk premia and t-stats from EW portfolios

	b1	b2	b3	b4	b5	R^2
FF3	-0.57	2.19	1.55			0.80
	(-2.55)	(11.69)	(5.21)			
FF3+BAB	1.91	-1.45	-2.05	0.09	8.49	0.98
+BOIVOL	(4.12)	(-3.53)	(-1.46)	(0.48)	(10.43)	
MKT+BAB	-0.04			0.98	3.95	0.86
+BOIVOL	(-0.19)			(6.04)	(12.59)	

Note: The table shows risk premia and (robust Newey-West (1986)) for factors in three different factor models. The results were derived from Fama-MacBeth regressions of 25 equal weighted portfolios formed from double sorting stock first based on pre-formation CAPM-beta and then on pre-formation idiosyncratic volatility. R^2 shows the r-square for each model. The sample included US stocks over the period 1965:12-2013:12.

Table 40: Fama-MacBeth risk premia and t-stats from EP portfolios

	b1	b2	b3	b4	b5	R^2
FF3	0.40	0.35	0.36			0.89
	(2.09)	(1.01)	(2.29)			
FF3+BAB	-0.50	1.81	-0.11	2.51	4.72	0.98
+BOIVOL	(-2.56)	(1.56)	(-0.33)	(1.66)	(1.99)	
MKT+BAB	0.42			0.88	1.69	0.93
+BOIVOL	(2.18)			(2.77)	(2.44)	

Note: The table shows risk premia and (robust Newey-West (1986)) for factors in three different factor models. The results were derived from Fama-MacBeth regressions of 10 value weighted portfolios formed from sorting stock based on their pre-formation earnings-price ratio (EP). R^2 shows the r-square for each model. The sample included US stocks over the period 1965:12-2013:12.

Table 41: Fama-MacBeth risk premia and t-stats from DP portfolios

	b1	b2	b3	b4	b5	R^2
FF3	-1.07	-0.51	-0.19			0.59
	(-5.36)	(-1.02)	(-1.06)			
FF3+BAB	-1.08	-0.94	-0.16	-0.63	-1.60	0.63
+BOIVOL	(-5.39)	(-1.11)	(-0.84)	(-0.58)	(-1.19)	
MKT+BAB	-0.53			-0.40	-1.47	0.58
+BOIVOL	(-2.67)			(-0.83)	(-1.24)	

Note: The table shows risk premia and (robust Newey-West (1986)) for factors in three different factor models. The results were derived from Fama-MacBeth regressions of 10 value weighted portfolios formed from sorting stock based on their pre-formation dividend-price ratio (DP). R^2 shows the r-square for each model. The sample included US stocks over the period 1965:12-2013:12.