# Commodities as a Diversification Instrument: Implications of the Commodity Financialization

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**Abstract**

This study examines if potential diversification benefits from adding non-energy commodity indices to a portfolio consisting of stocks and bonds have changed over time. This is examined for two types of investors: one that aims to maximize the risk-adjusted return and one who seeks to minimize the portfolio risk. Due to the financialization of commodities, Tang and Xiong (2012) among others have argued that non-energy commodities have become increasingly correlated with equity. This study's findings on how diversification benefits have change over time is mixed. For the minimum variance investor diversification benefits have declined due to the financialization, while for the maximum Sharpe ratio investor diversification benefits have actually increased over time. These findings suggest that the financialization of commodity markets might not be permanent, or at least it has not transformed non-energy commodity indices to an adverse diversification instrument.

**Keywords:** Commodities, Index investment, Diversification, Financialization

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# 1 Introduction

Commodity markets serve as a platform that make it possible for suppliers to hedge for future changes in price, but the markets have also attracted investors seeking to diversify their portfolios (Cheng & Xiong, 2014). Due to their low correlation with stock markets, commodity futures have long been seen as a good instrument for diversification. Some studies have even found commodities to have negative correlation with stock markets, and can thus also be used for hedging objectives (Gorton & Rouwenhorst, 2006). Yet, more recent studies argue that commodity markets have been financialized and that this fact have made it possible for shocks in external markets to be transmitted to commodity markets (Tang and Xiong, 2012). Consequently, correlation between commodity markets and stock markets have risen which in theory should reduce the diversifications benefits typically associated with commodities. These findings are most prominent for commodities included in a commodity index, as an investor seeking to diversify her portfolio often choose to invest in a commodity index.

Commodity indices vary in weighing schemes but includes all different sorts of commodities. One main portion of an commodity index is often energy commodities, which consists of crude oil and other oil-related products. As the oil price is highly correlated with the stock market (Kilian & Park, 2009), an investor seeking a proper diversification instrument for a portfolio consisting of stocks theoretically would prefer a non-energy commodity index.

In this study, we build on the literature regarding the financialization of commodities by reexamining if diversification benefits may still be earned by including non-energy commodity indices in a portfolio consisting of stocks and bonds. This is interesting because it examines whether the increased correlation between stock markets and non-energy commodity indices, caused by financialization, still prevails in the U.S. commodity markets. Hence, this study analyzes the implications the development of financialization have had on investors looking to diversify their portfolios.

On the basis of previous studies suggesting that non-energy commodity indices have been impaired as a diversification instrument, we formulate the hypothesis: The diversification benefits of adding non-energy commodities to a portfolio of equities and bonds have decreased due to the financialization of commodities.

We examine our hypothesis through the perspective of two investors: one trying to minimize the standard deviation, i.e. portfolio risk, and one trying to maximize the risk adjusted return in form of the Sharpe ratio. The hypothesis is tested for both investor for a pre- and a postfinancialization period. We use data from 1995 to 2015, and define prefinancialization as 1995-2004 and postfinancialization as 2004-2015. This is in line with what has been established in previous literature. The findings of this paper suggests that the investor seeking to minimize the portfolio risk would benefit from adding a commodity index to the portfolio in both time periods. For the investor evaluating portfolio performance on the basis of the risk-adjusted return, commodity indices would only lead to improvement during the postfinancialization period. The latter could be suggest that the high correlation between commodity indices and the stock market, caused by financialization, has begun to decline. By that, the financialization of commodity markets experienced in recent years, might not necessarily be a permanent state.

The outline of the remainder of the paper is as follows: Section 2 presents a review of existing literature. Section 3 presents the data used in this study. Section 4 discusses the key theoretical concepts and presents the methodology implemented in the study. Section 5 analyzes the obtained results. Section 6 aims to discuss the implication of the findings with regards to the postulated hypothesis. Section 7, concludes the study and provide suggestions for future research.

# 2 Literature Review

This section reviews the related literature on the topic of the study. First, the existing literature discussing commodities as an asset class from a portfolio theoretical perspective is presented. Then, a review of the literature regarding the financialization of commodities follows.

## 2.1 Commodities as Diversification Instrument

Commodities as a diversification instrument has been widely discussed in previous literature. Bodie and Rosansky (1980) compare commodity futures to common stocks over the period 1950-1976 in the U.S. market and find that the return of their benchmark commodity portfolio is about the same as the return of a portfolio solely consisting of common stocks. The authors find that commodity futures tends to perform well in times when common stocks were performing poorly and vice versa. Bodie and Rosansky (1980) show that by holding a portfolio consisting of both stocks and commodity futures, investors could decrease the variability of returns by one third, and still earn the same return. Thus, the authors conclude commodity futures are an attractive diversification instrument for equity portfolios, due to the low correlation with equity return.

Further, some studies have suggested that commodities could also be used as a hedging instrument. Gorton and Rouwenhorst (2006) use data from 1959 to 2004 and find that the returns of commodity futures are negatively correlated to the returns of equity and bond, meaning that commodities could also serve as a hedging instrument.

Despite commodities popularity for either diversification or hedging purposes, some studies claim that the benefits have been overestimated. Amongst them are Erb and Harvey (2006), who argue that even though commodity indices previously have had the same average annual return as S&P 500, it is naive to believe that this could predict future returns. Taking this view into account, Conover et al. (2010) re-examine the benefits of adding commodity futures to equity portfolios. They find that regardless of investment strategy, commodity futures are a desirable addition to a portfolio, as it reduces portfolio risk through diversification.

Daskalaki and Skiadupoulos (2011) look at diversification benefits from adding commodity indices and individual commodity futures to a traditional portfolio of equity and bonds. Daskalaki and Skiadupoulos (2011) find that an investor would not earn any diversification benefits from adding commodities to their portfolio under a mean-variance framework. Still, a shortcoming with Daskalaki and Skiadupoulos

(2011) approach is the inclusion of energy commodities in the study which Kilian and Park (2009) show is highly correlated with equity markets. This study addresses this shortcoming by examining potential diversification benefits from adding non-energy commodities only.

## 2.2 Financialization of Commodities

In recent years, commodity markets have experienced a large inflow of capital which have increased the correlation between commodity indices and the stock market. Tang and Xiong (2012) refer to this phenomenon as the financialization of commodities and connects the large capital inflow to the increased co-movement between indexed non-energy commodities and oil. The fact that the effects are most prominent for indexed commodities is in line with the theory of identification strategy, brought forward by Barberis, Schleifer and Wurgler (2005) who find that when a stock is included in a index, its correlation with the index increases. Moreover, oil is considered to be the most liquid commodity and also able to explain the variations in the U.S. stock market. Kilian and Park (2009) show that shocks in oil prices can explain 22% of the variations in the U.S. stock market. For that reason, Tang and Xiong (2012) use oil as a benchmark when investigating the financialization through cross-commodity correlation. The authors focus solely on non-energy commodities and exclude energy commodities as they constitute of oil and oil-related products. The authors are able to show that even though the increase in correlation was most apparent after the financial crisis in 2008, the financialization process began long before the crisis. Tang and Xiong (2012) argue that the findings help explain the increase in price volatility of non-energy commodities during the 2008 financial crisis.

Cheng and Xiong (2014) also argue that the large capital inflow has changed the commodity markets. The authors find that sudden price drops in other markets may cause investors to want to unwind their positions in commodity indices, transmitting chocks in external markets to commodity markets. Cheng and Xiong (2014) mainly focus on examining the functions that underpin commodity markets, and how financialization has affected each one of these. The first function is storage, which is the spread between futures price and spot price and serves as an incentive to store the commodity. The second is risk sharing, which refers to the fact that commodity markets serves as a platform where producers and users can hedge commodity price risk. By hedging in commodity markets, agents who have large exposure to price risk in a particular commodity are able to share price risk efficiently. The third and last function examined is information discovery, which refers to the fact that commodity markets provide price signals to market participants. This is crucial for suppliers and buyers as the market experience high level of information frictions, and price signals inform about supply and demand. The authors finds evidence that the financialization of commodities has transformed risk-sharing and information discovery.

This study builds upon the literature regarding the financialization of commodities. In contrast to Tang and Xiong (2012) and others, this paper analyzes the implications of the financialization of commodities from a portfolio allocation perspective. Furthermore, this study focus on commodity indices rather than futures instrument which previous studies have primarily focus on. This paper also excludes energy commodities, which have been included in previous similar studies. Finally, this study contribute to existing literature by analyzing if non-energy commodity indices still remains an attractive asset class for investor interested in diversifying their portfolios.

# 3 Data

The data used in this study is gathered from Thomson Reuters Datastream. The data consists of daily price data for the value weighted S&P 500 index, Barclays US Aggregate Bond index (BABI) , Bloomberg non-energy Commodity Index (BCI) and the S&P Goldman Sachs non-energy Commodity Index (GSCI). Furthermore, the 1-year London Interbank Offered Rate (Libor) is used as a proxy for the risk-free rate and is collected from Federal Reserve Economic Data. All the data series consist of daily data between March 20th 1995 and March 13th 2015, containing a total of 5230 observations for each series.

The S&P 500 index consists of the 500 leading companies listed on NASDAQ or NYSE. It is weighted based on the market capitalization of each company and is commonly used as an approximation of the market portfolio. Similar to the S&P 500, the BABI is weighted based on the market capitalization, but it consists of bonds instead of equity. It is supposed to reflect the U.S. bond market for investment grade securities with a maturity over 1 year (Barclays, 2014).

While both the commodity indices included in this study contain a similar collection of various commodities, they have a different weighting schemes and purposes. The BCI, formerly called Dow Jones non-energy Commodity Index, is equally weighted and aims to be well diversified and to have a high level of liquidity. In contrast, the weighting scheme of the GSCI is based on world production and aims to reflect each commodity's significance for the world economy. Thus, the composition of each index differs to a large extent (Gunzberg, 2014) which may have implications on the findings. Appendix. A displays an example of the different allocation for the respective index. Ex-ante, we believe the fact that the BCI aims to be well diversified may lead to the potential diversification benefits being more pronounced when it is used in the analysis.

# 4 Theory and Methodology

This section presents the key theoretical concepts and the methodology used in this study. First, the hypothesis of this paper is developed and postulated. Next, a description of the study's empirical strategy is presented. This is followed by a description of the evaluation metrics used in this study. Lastly, the mean-variance spanning test which is used for analyzing the robustness of the findings is explained.

## 4.1 Hypothesis Development

The purpose of this study is to examine the diversification benefits of adding commodities to a portfolio of equities and bonds in a mean-variance framework. Diversification is a strategy investors may use to avoid excessive risk exposure. A diversified portfolio contains multiple different assets which allows an investor to reduce her exposure to the idiosyncratic risks of each asset. A perfectly diversified portfolio only contains systematic risk (Bodie, Kane & Marcus, 2014). The extent to which a diversification strategy is successful depends typically on the correlation between the portfolio's assets. The more the assets' behaviors are correlated, the less will the diversification benefits be. Traditionally, commodities have been an attractive asset class for diversifying a portfolio of equity and bonds due to its low correlation with the two assets (Bodie & Rosansky, 1980). Yet, Tang and Xiong (2012) among others have found non-energy commodities, due to a large capital inflow, becoming increasingly correlated to equities during the last decade. This is typically referred to as the financialization of commodities (Tang & Xiong, 2012; Cheng, & Xiong, 2014). With these developments in mind the main hypothesis of the study is postulated:

- The diversification benefits of adding non-energy commodities to a portfolio of equities and bonds have decreased due to the financialization of commodities.

This hypothesis is investigated by a relative comparison of the diversification benefits from adding non-energy commodities to a portfolio of equities and bonds before and after the financialization of commodities. The period before the financialization of commodities is denoted prefinancialization and the period after the financialization is called postfinancialization. Thus, mathematically the hypothesis may be expressed as:

$$\frac{\sigma_{CI}^{Pre}}{\sigma_{Base}^{Pre}} < \frac{\sigma_{CI}^{Post}}{\sigma_{Base}^{Post}} \tag{4.1}$$

Eq.(4.1) may be interpreted as the diversification benefit of adding commodities (CI) to portfolio of equity and bonds (Base), in terms of the standard deviation, is lower in the postfinancialization period than the prefinancialization period. The hypothesis is expressed in terms of the standard deviation since the study operates in a mean-variance framework. Note that $\frac{\sigma_{CI}}{\sigma_{Base}} = 1$ means no diversification benefits may be enjoyed from adding commodities.

Due to limitations in available data, pre- and postfinancialization are defined in congruence with Tang and Xiong's (2012) seminal paper regarding the financialization of commodities. The authors define prefinancialization as the period before January 1st 2004 and postfinancialization as the period after the aforementioned date (Tang & Xiong, 2012). Although the financialization is a process and did not happen over a day, Tang and Xiong (2012) argue the choice of cutoff point is innocuous as the results builds on correlation trends. With this in mind, this study define pre- and postfinancialization as:

- Prefinancialization: The period between March 20th 1995 and December 31st 2003.
- Postfinancialization: The period between January 1st 2004 and March 13th 2015.

## 4.2 Empirical Strategy

The hypothesis is investigated by constructing three portfolios consisting of index instrument for equities, bonds and non-energy commodities for the pre- and the postfinancialization period respectively. The selected instruments are:

- Equity Index: S&P 500
- Bonds Index: BABI
- Commodities Indices[1]: GSCI and BCI

The reason to why index instrument are used in this study is because we argue that a diversification oriented investor invests in indices rather than individual securities. The inclusion of two different commodity indices is motivated by their different weighting scheme and goal, as mentioned in section 3. The three different portfolio constructions are:

- Base Portfolio: S&P 500, BABI

[1] The indices used in this study are non-energy commodities indices. From here on forth, the word "commodities" refers to non-energy commodities.

- GSCI Portfolio: S&P 500, BABI and GSCI
- BCI Portfolio: S&P 500, BABI and BCI

The portfolios are created for two different investors: an investor interested in minimizing the portfolio variance and an investor looking to maximize her risk-adjusted return. The following subsections present how the optimal portfolio is determined for the respective investor.

### 4.2.1 Minimizing the Portfolio Variance

In the first scenario, the portfolios are constructed for an investor interested in minimizing the portfolio variance. Since an investor in the mean-variance framework views standard deviation as a measure of risk, minimizing the portfolio variance is equivalent to minimizing the portfolio risk. The optimal portfolio for an investor interested in minimizing the risk is determined by solving the following quadratic optimization problem[2]:

$$\begin{aligned} \underset{\mathbf{w}}{\text{Minimize}} \quad & \mathbf{w^T \Sigma w} \\ \text{subject to} \quad & \mathbf{w^T 1} = 1, \\ & w_i \geq 0. \end{aligned} \tag{4.2}$$

where $\mathbf{w^T} = (w_1, ..., w_n)$ i.e. a vector of the portfolio weights $w_i$. $\mathbf{\Sigma}$ is the covariance matrix of the $n$ portfolio assets. The first constraint specifies that the portfolio weights must sum to 1 which means no capital may be thrown away. The second constraint may be interpreted as no short selling is allowed. The portfolios are also analyzed in a setting where short selling is allowed, i.e. when the last constraint is removed.

[2] The objective function $\mathbf{w^T \Sigma w}$ is equal to $\sum_{j=1}^{n} \sum_{i=1}^{n} w_i w_j Cov(r_i, r_j)$ with $r_i$ being the return of asset $i$ (Bodie et al., 2014).

### 4.2.2 Maximizing the Risk-Adjusted Return

Although the minimum variance portfolio provides a good illustration of the diversification benefits in terms of risk reduction, it may be more suitable for hedging purposes as it does not considers the portfolio return. In practice, investors are often more interested in maximizing the expected return while lowering risk. For this reason, this paper also analyzes the diversification benefits an investor interested in maximizing the risk-adjusted return may receive by adding non-energy commodities to her portfolio. As the study operates in the mean-variance framework, the risk-adjusted return is measured by the Sharpe ratio. The optimal portfolio for this investor is determined by solving the following problem:

$$
\begin{aligned}
\underset{\mathbf{w}}{\text{Maximize}} \quad & \frac{\mathbf{w}^{\mathbf{T}}\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}^{\mathbf{T}}\boldsymbol{\Sigma}\mathbf{w}}} \\
\text{subject to} \quad & \mathbf{w}^{\mathbf{T}}\mathbf{1} = 1, \\
& w_i \geq 0.
\end{aligned}
\tag{4.3}
$$

where $\mathbf{w}^{\mathbf{T}} = (w_1, ..., w_i)$ i.e. a vector of the portfolio weights $w_i$. $\boldsymbol{\mu}$ is a vector of the expected return and $\boldsymbol{\Sigma}$ is the covariance matrix of the portfolio assets. $r_f$ is the risk-free rate. The first constraint specifies that the portfolio weights must sum to 1 which means no capital may be thrown away. The second constraint says no short selling is allowed. Analogous to the case of the minimum variance investor, an analysis of the maximum Sharpe ratio investor is also made when short selling is allowed.

A disadvantage with the problem formulation in Eq.(4.3) is that it is not a convex optimization problem. Consequently, the solution to this problem may not be the optimal portfolio. However, maximizing the Sharpe ratio can be formulated as a convex optimization problem with the following transformation:

$$
\tilde{\mathbf{w}} = t\mathbf{w}, \qquad t = \frac{1}{\mathbf{w}^{\mathbf{T}}\boldsymbol{\mu} - r_f}. \tag{4.4}
$$

Using Eq.(4.4) the optimal portfolio for an investor interested in maximizing the Sharpe ratio is determined by solving the following problem (proof is provided in Appendix. B):

$$\begin{aligned} \text{Minimize} \quad & \tilde{\mathbf{w}}^{\mathbf{T}} \mathbf{\Sigma} \tilde{\mathbf{w}} \\ \text{subject to} \quad & (\boldsymbol{\mu} - r_f \mathbf{1}) \tilde{\mathbf{w}} = 1 \\ & \tilde{\mathbf{w}}^{\mathbf{T}} \mathbf{1} \geq 0, \\ & \tilde{w}_i \geq 0. \end{aligned} \tag{4.5}$$

The optimal weights $\mathbf{w}$ is the determined using Eq.(4.4). Eq.(4.2) and Eq.(4.5) are used to construct the optimal portfolios for the investor interested in minimizing the risk exposure and maximizing the risk-adjusted return respectively. The specified optimization problems are implemented in MATLAB.

## 4.3 Evaluation Metrics

This section describes the metrics used for evaluating the constructed portfolios. In total, four evaluation metrics are used. The first two are the Sharpe ratio and the standard deviation of the portfolio. These metrics are used as an investor in the mean-variance framework is typically interested in maximizing the risk-adjusted portfolio return and perceive standard deviation as a measure of risk. Furthermore, this study also evaluates the portfolios with higher order of moments risk metrics such as kurtosis and skewness. The inclusion of kurtosis and skewness is motivated by their superior ability, compared to standard deviation, to capture tail risk (Harvey, Liechty, Liechty and Muller, 2010). Moreover, past studies have indicated that only examining two order of moments, i.e. mean and standard deviation, may result in an incomplete assessment if the asset returns are non-normally distributed (Harvey et al., 2010). The diversification benefits of adding a non-energy commodity index to the portfolio is determined by measuring the relative improvement, in terms of the respective metrics. Each metric is described in detailed below.

### 4.3.1 Standard Deviation

In the mean-variance framework investors measure volatility by standard deviation. This study measures a portfolio's volatility by computing the sample standard deviation defined as:

$$\sigma_p = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^2} \tag{4.6}$$

where $N$ is the sample size, $r_i$ and $\bar{r}$ is the portfolio return at time $i$ and mean return of the portfolio respectively. The idea of the standard deviation is that the higher it is the more riskier is the portfolio.

### 4.3.2 Sharpe ratio

The Sharpe ratio is a measure of the risk-adjusted return of a portfolio. It measures the risk premium a portfolio earns in relation to the undertaken risk (Bodie et al., 2014). The risk premium is equal to the portfolio return less the risk-free rate. The Sharpe ratio is defined as:

$$\frac{r_p - r_f}{\sigma_p} \tag{4.7}$$

where $r_p$ is the portfolio return, $r_f$ the risk-free rate and $\sigma_p$ is the standard deviation of portfolio p. The risk-free rate, $r_f$, used in this study is the 1-year Libor. The higher the value of the Sharpe ratio the more attractive is the investment. A negative Sharpe ratio implies the risk-free asset yields a higher return than the portfolio.

### 4.3.3 Skewness

Skewness is a third order of moment measure which describes the asymmetry of a distribution (Bodie et al, 2014). Typically, returns are assumed to be normally distributed which implies they have skewness of 0. An investor prefers assets with positive skewness as it indicates the mean return is higher than the median return, which implies positive extreme outcomes are more likely than negative. Sample skewness is defined as:

$$s = \frac{\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^3}{(\sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^2})^3} \tag{4.8}$$

where $N$ is the sample size, $r_i$ and $\bar{r}$ is the portfolio return at time $i$ and mean return of the portfolio respectively.

### 4.3.4 Kurtosis

Kurtosis is a fourth order of moment metric which describes the peakiness of a distribution. A normal distribution have a kurtosis of 3. A low value of kurtosis suggest an asset's return distribution has a small peak and fat tails. If an asset's distribution has fat tails the likelihood of extreme outcomes are high. A risk-averse investor prefers a low value of the kurtosis as it indicates the likelihood of experiencing extreme outcomes is low. Sample kurtosis is defined as:

$$k = \frac{\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^4}{(\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^2)^2} \tag{4.9}$$

where $N$ is the sample size, $r_i$ and $\bar{r}$ is the portfolio return at time $i$ and mean return of the portfolio respectively.

## 4.4 Variable Construction

This section describes the variable definitions used in this study. In order to implement this study, the asset returns and the risk-free rate must be specified. The data used to construct the portfolios and compute the evaluation metrics are daily asset returns. This study computes the daily return for asset $i$ by:

$$r_{i,t} = log(P_{i,t}) - log(P_{i,t-1}) \tag{4.10}$$

where $P_{i,t}$ is the price of asset $i$ at day $t$. Eq.(4.10) says the daily returns are estimated as difference of the log-normal price at $t$ and the log-normal price the previous day.

As mentioned in section 3, the 1-year Libor is used as the risk-free rate in this study. Since the provided data presents the annualized Libor, it must be transformed in order to match the daily returns of the assets. The transformation is done by assuming daily compounding and that a year consists of 250 trading days, which is the approximately number of trading days on U.S. markets. The transformation to daily Libor is performed with the following formula:

$$Libor_{daily} = (1 + Libor_{yearly})^{\frac{1}{250}} - 1 \tag{4.11}$$

The risk-free rate which is used in estimating the Sharpe ratio is the arithmetic average of the corresponding time period. This means that the risk-free rate for the prefinancialization period differs from the risk-free rate during the postfinancialization period. This difference reflects the overall economic climate. The risk-free rates for the different time periods are displayed in Table. 4.1.

*Table 4.1: The risk-free rate. Prefinancialization is the period between March 13th 1995 - December 31st 2003 and Postfinancialization is the period between January 1st 2004 - March 20th 2015. The risk-free rate used is the daily 1-year Libor.*

| | Risk-free rate |
|---|---|
| **Prefinancialization** | 0.0186% |
| **Postfinancialization** | 0.0089% |

## 4.5 Mean-Variance Spanning Test

This section describes the mean-variance spanning test implemented in this study to evaluate the robustness of the findings. The mean-variance spanning test is a regression based inference test that examines if an investor's efficient frontier may be improved by adding an additional risky asset (Huberman and Kandel, 1987). The additional risky asset is commonly denoted the test asset. The test is developed by Huberman and Kandel (1987) and is performed by regressing the additional risky asset on the base portfolio:

$$r_{k,t} = \alpha + \beta_1 r_{1,t} + \beta_2 r_{2,t} + \varepsilon_t \tag{4.12}$$

where $r_{k,t}$ is the return for additional risky asset $k$ at $t$, $r_{1,t}$ the return of asset 1 at time $t$ in the base portfolio (in this case S&P 500) and $r_{2,t}$ the return of asset 2 at time $t$ in the base portfolio (in this case BABI). $\varepsilon_t$ is the disturbance term which is assumed to be normally distributed with zero mean and variance $V$.

The null hypothesis of the test is:

$$H_0 : \quad \alpha = 0 \quad and \quad \delta = 0 \tag{4.13}$$

where $\delta = 1 - \beta_1 - \beta_2$. The economic implication of $\alpha = 0$ is that the tangency portfolio allocates zero weight to the test asset. Analogously, $\delta = 0$ may be interpreted as the global minimum variance portfolio allocates zero weight to the test asset.

The mean-variance spanning test is performed by computing the likelihood ratio test-statistic[3]:

$$\frac{(\frac{1}{U} - 1)(T - K - N)}{N} \sim F_{2N,2(T-K-N)} \tag{4.14}$$

[3] Readers interested in the mathematical derivation of the test-statistic is referred to Huberman and Kandel (1987)

where $T$ is the number of observations, $N$ is the number of assets in the base portfolio and $K$ is the number of test assets. $U$ is the likelihood ratio, defined as:

$$U = \frac{\tilde{V}}{\hat{V}} \tag{4.15}$$

where $\tilde{V}$ and $\hat{V}$ is the unconstrained and constrained maximum likelihood estimator of the variance V respectively.

# 5 Results and Analysis

In this section, the obtained findings are presented and analyzed. Initially, the descriptive statistics is presented and discussed. This is followed by a presentation of the results and analysis with respect to the respective investor. Lastly, the results from the robustness analysis are discussed.

## 5.1 Descriptive Statistics

This section presents the descriptive statistics of each instrument used in this analysis. The summary statistics are presented in Table. 5.1. and a graph illustrating the rolling correlation between S&P 500 and the other instruments are displayed in Figure. 5.1. Lastly, the efficient frontiers for all portfolios during both the pre- and the postfinancialization periods are presented in Figure. 5.2.

### 5.1.1 Summary Statistics

The summary statistics in Table. 5.1 show how the characteristics for each index have changed between the pre- and the postfinancialization period. Examining the average return it may be noted that the S&P 500 had the highest average return in the prefinancialization period but it has declined over time. Analogous trend in the average return is seen for the BABI. In contrast, both commodity indices have experienced a significant increase in the average return over time. The GSCI have gone from a negative average return in the prefinancialization period to a positive average return, while the BCI have become the index instrument yielding the highest average return.

The significant increase in the average return for the GSCI and the BCI seems to be associated with them becoming more risky over time. Comparing the standard deviation between the pre- and the postfinancialization period, the associated risk for both the GSCI and the BCI have roughly doubled to become on par with the S&P 500. In contrast, the S&P 500 and the BABI have relatively stable standard deviations over time.

The developments in the average return and standard deviation for each instrument have implications on the Sharpe ratio. As displayed in Table. 5.1, the S&P 500 was the only instrument to have positive Sharpe ratio in the prefinancialization period. The other instruments had negative Sharpe ratios, implying the risk-free rate was a more attractive instrument to invest in. In contrast, for the postfinancialization period the Sharpe ratio has become positive for all instruments except the BABI. The significant increase in average return for the BCI have resulted in it having the highest Sharpe ratio, despite the fact it's standard deviation have roughly doubled.

The skewness and kurtosis for each instrument also displays interesting patterns. In the prefinancialization period, the GSCI was the only instrument with a positive skew. It also had the lowest kurtosis. Coupled together, this result suggests it was less exposed to extreme negative outcomes than the other instruments. In fact, the positive skew indicates it was more likely to experience positive extreme outcomes than negative. In contrast, in the postfinancialization period all instruments except the S&P 500 had a similar kurtosis. The kurtosis for the S&P 500 became approximately three times larger than the corresponding kurtosis for the other instruments, implying it was significantly more exposed to tail risk i.e. negative extreme outcomes.

*Table 5.1: Summary Statistics for each asset in the study for the pre- and the postfinancialization period. All metrics are computed based on the daily returns.*

| **Prefinancialization** | Average Returns | Std. dev | Sharpe ratio | Skewness | Kurtosis | Observations |
|---|---|---|---|---|---|---|
| S&P 500 | 0.03521% | 1.17804% | 0.01407 | -0.10670 | 5.99621 | 2292 |
| S&P GSCI Non Energy | -0.00263% | 0.58065% | -0.03661 | 0.01987 | 3.91565 | 2292 |
| BCI Non Energy | 0.00566% | 0.58197% | -0.02228 | -0.21223 | 8.45804 | 2292 |
| Barclays Aggregate Bong Index | 0.00249% | 0.25493% | -0.06332 | -0.36260 | 5.16767 | 2292 |
| **Postfinancialization** | | | | | | |
| S&P 500 | 0.02099% | 1.21873% | 0.00992 | -0.33964 | 15.14090 | 2922 |
| S&P GSCI Non Energy | 0.01653% | 1.01264% | 0.00752 | -0.32908 | 5.95458 | 2922 |
| BCI Non Energy | 0.02384% | 1.02069% | 0.01463 | -0.32729 | 5.96474 | 2922 |
| Barclays Aggregate Bong Index | -0.00003% | 0.23484% | -0.03806 | -0.06141 | 5.24097 | 2922 |

### 5.1.2 Correlation

Figure. 5.1 displays the 1-year rolling correlations for each instrument, respectively, with the S&P 500. The two commodity indices seem to have about the same degree of correlation with the S&P 500 over time. Up until 1998, correlations are mostly at a level below zero, which also is the case during some shorter periods between 1999 to 2001. From 2001, the correlation of respective commodity indices with the S&P 500 have a slow upward trend which continues till the 2008 financial crisis. In the aftermath of the financial crisis the correlations experience a sharp increase, reaching a peak of approximately 0.6 in 2010. This high level of correlation continues until September 2012 when it starts to decrease and eventually return to the low levels observed in the beginning of the examined time period.

When looking at the correlation between the BABI and the S&P 500, we can see large fluctuations and the correlation does not seem to be that constant at any longer period of time. Interestingly, the correlation between the BABI and the S&P 500 has a downward trend and have been mostly negative for the entire time period.

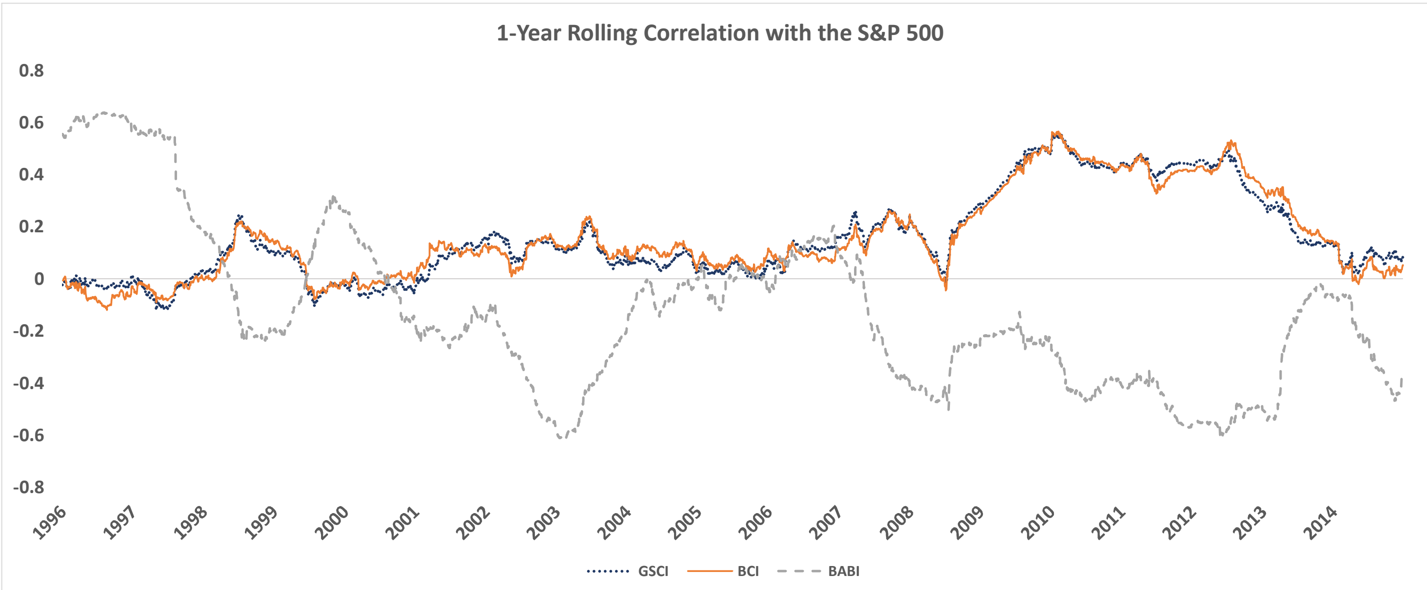


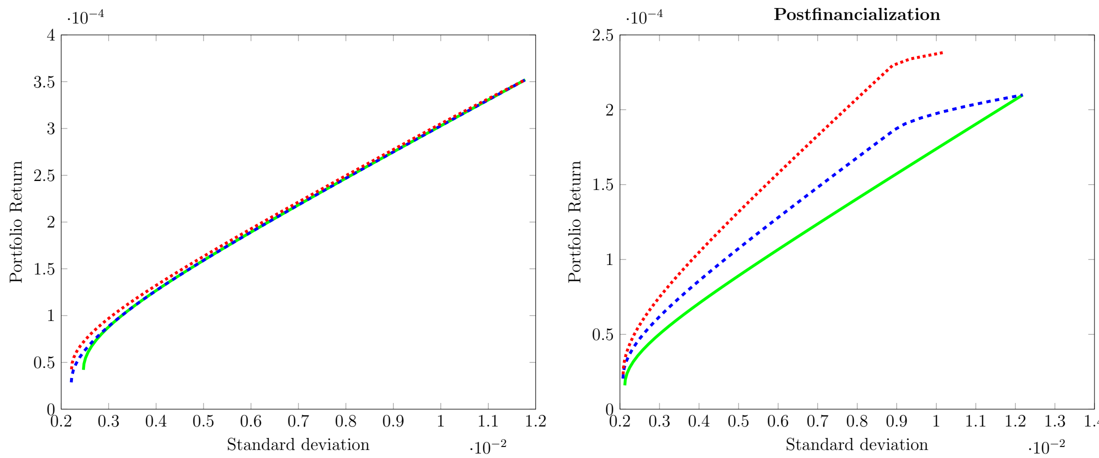

*Figure 5.1: 1-year Rolling Correlation for the GSCI (blue dotted line), the BCI (orange solid line) and the BABI (grey dashed line) with the S&P 500, respectively, over the examined time period.*

### 5.1.3 Efficient Frontiers

Figure. 5.2a displays the efficient frontiers for the three portfolios during the prefinancialization period. The efficient frontiers display the highest expected return given a specific level of risk for each portfolio. The different levels of risk vary depending on the portfolio weight allocated to each instrument included in the portfolio.

For the prefinancialization period the frontiers start at different places and then converge as the portfolio weights changes. Yet, the portfolio including the BCI have slightly higher return for low levels of risk. This implies that a very risk-averse investor may benefit from adding the BCI to her portfolio during the prefinancialization period. In contrast, a less risk averse investor would not earn any benefits from holding a portfolio which includes a commodity index.

In Figure. 5.2b which displays the efficient frontiers during postfinancialization, a more distinct discrepancy between the three portfolios may be observed. Now, both the GSCI portfolio and the BCI portfolio outperforms the Base portfolio in terms of




*(a) Prefinancialization*

*(b) Postfinancialization*

*Figure 5.2: Efficient frontiers for the BCI portfolio (red dotted line), the GSCI portfolio (blue dashed line) and the Base portfolio (green solid line).*

return given a level of risk. This results implies that an investor, regardless of her degree of risk aversion, may benefit from adding a commodity index to her portfolio.

## 5.2 The Maximum Sharpe ratio Investor

This section presents the results for an investor interested in maximizing the Sharpe ratio. First, the results for the prefinancialization period are presented and then the corresponding results for the postfinancialization period are discussed. In each time period, the results are initially analyzed when investor may only take long positions. Next, the results when short selling is allowed are discussed.

### 5.2.1 Prefinancialization

Table. 5.2 shows how an investor seeking to maximize the Sharpe ratio would allocate her portfolio in the prefinancialization period when short selling is not allowed. The results indicate the investor would not want to include the GSCI, the BCI nor the BABI to the portfolio. Hence, the maximum Sharpe ratio investor would hold a portfolio with only the S&P 500. Evaluating the risk measures of higher order of moments, i.e. skewness and kurtosis, the skewness are slightly negative which suggests the investor is more exposed to negative extreme outcomes than positive. Moreover, the kurtosis is slightly leptokurtic which indicates that the likelihood of, for an investor holding this portfolio, extreme outcomes occurring are slightly higher than normal. These results are in line with what can be seen in the descriptive statistics (Table. 5.1), where the S&P 500 has the highest Sharpe ratio during the prefinancialization period. In addition, all efficient frontiers displayed in Figure. 5.2a converge for higher levels of risk, which indicates that they all represent the same portfolio, i.e. a portfolio only consisting of the S&P 500.

*Table 5.2: Optimal portfolio allocation for a maximum Sharpe ratio investor in the prefinancialization period. Short selling is not allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 100.0% | 100.0% | 100.0% |
| S&P GSCI Non Energy | NA | 0.0% | NA |
| BCI Non Energy | NA | NA | 0.0% |
| Barclays Aggregate Bond Index | 0.0% | 0.0% | 0.0% |
| St. Dev | 1.1780% | 1.1780% | 1.1780% |
| Sharpe Ratio | 0.0141 | 0.0141 | 0.0141 |
| Skewness | -0.1066 | -0.1066 | -0.1066 |
| Kurtosis | 5.9831 | 5.9831 | 5.9831 |

The above results rested on the assumption of short selling not being allowed. The result when short positions are allowed is presented in Appendix C.1. While short selling is allowed, this study constrains the maximum amount that can be short sold to 50% of the portfolio value. This is done since some instruments have negative Sharpe ratios which implies it, in theory, would be optimal to short them infinitely. The implication of this assumption is that all portfolios' risks and risk-adjusted returns are underestimated. Yet, the conclusion of the results are not expected to be affected.

In contrast to the case when only long positions are allowed, diversification benefits seems to be existent when short selling is allowed. As seen in Table. C.1 in Appendix C, there exists diversification benefits of adding the GSCI to the Base portfolio but not of adding the BCI. While the inclusion of the GSCI does not improve the standard deviation, the Sharpe ratio, skewness and kurtosis are improved. This implies that adding the GSCI improves the risk-adjusted return and lower the possibility of extreme negative outcomes. It should be noted that all optimal portfolios allocate the maximum weight to a long position in the S&P500 and minimum weight to either the BABI or the GSCI. As displayed in Table. 5.1, the reason why the GSCI provides diversification benefits is due to its negative average return which makes it an attractive instrument to short.

### 5.2.2 Postfinancialization

For the postfinancialization period the results, as displayed in Table. 5.3, differ from the corresponding results for the foregoing period. With the alternative to include the GSCI, an investor interested in maximizing the Sharpe ratio would now allocate roughly 42% of her portfolio to the GSCI and the remaining to the S&P 500. This would lead to a 11% improvement of the Sharpe ratio in comparison to the Sharpe ratio of holding the Base portfolio. Comparing the performance of the GSCI Portfolio during the pre- and the postfinancialization in the summary statistics (Table. 5.1), it is clear that the GSCI has improved in terms of the Sharpe ratio to becoming on par with the S&P 500. Thus, it is expected that the GSCI portfolio has higher a Sharpe ratio in the postfinancialization period than in the prior. Moreover, adding the GSCI to the portfolio does not only increase Sharpe ratio but it also reduces the portfolio risk by approximately 24%. Yet, the skewness of the Base portfolio and the GSCI portfolio suggest that adding the GSCI to the portfolio does lower the skewness. This indicates that the GSCI portfolio is more susceptible to negative outcomes than the Base Portfolio. In contrast, the kurtosis is slightly improved by adding the GSCI to the portfolio which indicates the likelihood of extreme outcomes occurring are lowered. Although the kurtosis is slightly lowered when adding the

GSCI to the portfolio, it is still very high and subsequently so is also the likelihood of extreme outcomes occurring.

If the investor would use the BCI to invest in commodities, an investor would maximize the Sharpe ratio by allocating 71% of the portfolio to the BCI and the remaining to the S&P 500. This would improve the investor's Sharpe ratio by almost 60% compared to the Base portfolio. The drastic change in portfolio allocation between the analyzed time period is not too surprising, as the summary statistics (Table. 5.1) displayed that the BCI is the instrument that has the highest Sharpe ratio in the postfinancialization period. Analogously to the case when the GSCI is added to the Base portfolio, adding the BCI worsens the portfolio skewness. Yet, the kurtosis is significantly reduced implying the investor's exposure to extreme outcomes are lowered when adding the BCI to the portfolio.

These results are in line with what can be seen from the efficient frontiers for the postfinancialization period Figure. 5.2b, where the BCI portfolio has the highest return given any level of risk, and the Base portfolio has the lowest. Thus, an investor seeking to maximize the Sharpe ratio would hold the BCI portfolio in the aftermath of the financialization of commodities.

*Table 5.3: Optimal portfolio allocation for a maximum Sharpe ratio investor in the postfinancialization period. Short selling is not allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 100.0% | 58.1% | 28.9% |
| S&P GSCI Non Energy | NA | 41.9% | NA |
| BCI Non Energy | NA | NA | 71.1% |
| Barclays Aggregate Bond index | 0.0% | 0.0% | 0.0% |
| St. Dev | 1.2187% | 0.9263% | 0.8926% |
| Sharpe Ratio | 0.0099 | 0.0110 | 0.0158 |
| Skewness | -0.3394 | -0.6056 | -0.5530 |
| Kurtosis | 15.1150 | 12.3052 | 7.5823 |

The above results are based on the assumption of short selling not being allowed. The results when short positions are allowed is presented in Appendix C.1. Analogous to the prefinancialization period, the maximum amount that may be short sold is 50% of the portfolio value. As before, this is not expected to affect the validity of the conclusions.

When short selling is allowed, the diversification benefits an investor may enjoy of adding the GSCI or the BCI are attenuated. Due to the BABI having an average return of zero, (Table. 5.1) an investor maximizing the risk-adjusted return would short it as much as possible to acquire more capital to allocate to the other indices. This is also what the obtained findings indicate, with the extra capital gained by shorting the BABI is allocated to the instrument with the highest average return (in the GSCI portfolio it is the S&P500 and in the BCI portfolio it is the BCI). Besides an improvement in the Sharpe ratio, diversification benefits in terms of the standard deviation and kurtosis are also higher when the short selling constraint is relaxed. Analoguously, the skewness is worsened which implies an investor including a commodity index becomes more exposed to negative outcomes when short selling is allowed. Still, the findings when an investor is allowed to short sell is in line with the results when short selling is constrained. The only difference is that the diversification benefits of adding the BCI or the GSCI are attenuated when short selling is allowed.

## 5.3 The Minimum Variance Investor

In the following section the results for the minimum variance investor are analyzed. Below the results when short selling is not allowed are presented. For completeness, an analysis is also made when short selling is allowed. The results when short selling is allowed are presented in Appendix C.2. Note that the results for the minimum variance investor do not change as the short selling constraint is relaxed.

### 5.3.1 Prefinancialization

Table. 5.4 displays the portfolio allocations for the three portfolios under the assumption that the investor is seeking to minimize portfolio risk, i.e standard deviation. The lowest risk is found in the GSCI portfolio, where the standard deviation is approximately 10% lower than in the Base portfolio. For the BCI portfolio, the standard deviation is similar to that of the GSCI portfolio. The estimates for the skewness and the kurtosis indicate that both the GSCI portfolio and the BCI portfolio reduce the investor's risk exposure to negative extreme outcomes. Thus, both the commodity portfolios offer an improvement compared to the Base portfolio during the prefinancialization period, and they are almost equally beneficial. These portfolios are also very similar in terms of the allocation between the BABI, commodity index and the S&P 500. This is also displayed in the prefinancialization efficient frontier, Figure. 5.2a, where it may be observed that both commodity portfolios have tangents further to the left than the Base portfolio. This is also in line with what is seen in the summary statistics (Table. 5.1) for the prefinancialization period, where both commodity indices have a lower standard deviation than the S&P 500. While there seems to exist benefits of adding a commodity index to the base portfolio, the Sharpe ratio is negative for all constructed portfolios during this period. This implies that the minimum variance investor would in practice be better off by investing in the risk-free rate than any of the constructed portfolios.

*Table 5.4: Optimal portfolio allocation for a minimum variance investor in the prefinancialization period. Short selling is not allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 5.3% | 4.0% | 4.0% |
| S&P GSCI Non Energy | NA | 16.9% | NA |
| BCI Non Energy | NA | NA | 16.8% |
| Barclays Aggregate Bond index | 94.7% | 79.1% | 79.2% |
| St. Dev | 0.2468% | 0.2213% | 0.2217% |
| Sharpe Ratio | -0.0582 | -0.0708 | -0.0644 |
| Skewness | -0.3967 | -0.3032 | -0.2908 |
| Kurtosis | 5.7910 | 4.9291 | 5.1281 |

### 5.3.2 Postfinancialization

Table. 5.5 shows the portfolio weights for a risk minimizing investor in the postfinancialization period. Similar to the foregoing period, the GSCI portfolio offers the lowest standard deviation. However, in this period the improvement is merely a 2% lower standard deviation compared to the Base portfolio. Thus, the benefits that can be earned from shifting from a stock and bond portfolio to a portfolio including a commodity index is much smaller in this period. The BCI portfolio also has a lower risk than the Base portfolio, with the magnitude of the improvement, in terms of standard deviation, being slightly smaller than the GSCI portfolio. For this period the standard deviation of the commodity indices are also smaller than the standard deviation of the S&P 500 as displayed in Table. 5.1, which is in line with these results. Analogously to the corresponding investor problem in the prefinancialization period, adding either the BCI or the GSCI improves the skewness and kurtosis of the investor's portfolio. This implies the investor is less exposed to negative extreme outcomes when she includes a commodity index in her portfolio. In contrast to the corresponding result for the prefinancialization period, the benefits of adding the GSCI or the BCI in terms of skewness and kurtosis are not as pronounced for the postfinancialization period. Analogous to the prefinancialization period, all constructed portfolios have a negative Sharpe ratio which implies the minimum variance investor should in practice be better off by investing in the risk-free rate.

*Table 5.5: Optimal portfolio allocation for a minimum variance investor in the postfinancialization period. Short selling is not allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 7.7% | 6.3% | 6.4% |
| S&P GSCI Non Energy | NA | 4.7% | NA |
| BCI Non Energy | NA | NA | 4.5% |
| Barclays Aggregate Bond index | 92.3% | 89.0% | 89.1% |
| St. Dev | 0.2123% | 0.2076% | 0.2081% |
| Sharpe Ratio | -0.0344 | -0.0329 | -0.0313 |
| Skewness | -0.1293 | -0.1030 | -0.1181 |
| Kurtosis | 6.9645 | 6.7050 | 6.7628 |

## 5.4 Robustness Evaluation

In this section, an attempt is made to evaluate the robustness of the results by using the mean-variance spanning test developed by Huberman and Kandel (1987). The results of the test are presented in Table. 5.6.

*Table 5.6: The results from the mean-variance spanning test. (1) refers to prefinancialization period, (2) refers to postfinancialization period. The null hypothesis, $H_0$ is $\alpha = 0, \delta = 0$.*

| | $\alpha$ | $\delta$ | P-value | Degrees of Freedom | Observations |
|---|---|---|---|---|---|
| GSCI (1) | -0.0000300 | 1.1576237 | 0.0000 | 2289 | 2292 |
| BCI (1) | 0.0000520 | 1.1481142 | 0.0000 | 2289 | 2292 |
| GSCI (2) | 0.0001162 | 0.9762151 | 0.0000 | 2919 | 2292 |
| BCI (2) | 0.0001898 | 0.9294296 | 0.0000 | 2919 | 2292 |

As displayed in Table. 5.6 the null hypothesis is rejected at the 1% significance level for each portfolio during both the pre- and the postfinancialization period. This support the study's main results of diversification benefits being existent during both pre- and postfinancialization. Recall that the $\alpha = 0$ and $\delta = 0$ implies the tangency portfolio and minimum variance portfolio allocates zero weight to the commodity indices respectively. Since the mean variance spanning test is joint hypothesis test no statistically supported conclusion may be drawn concerning alpha and delta on an individual basis. Still, the point estimates for each coefficient provide some information regarding the tangency portfolio and minimum variance investor. Examining the coefficients for $\alpha$, the point estimates are more than 200 times larger for the postfinancialization period compared to the prefinancialization period. This indicates an investor interest in maximizing the risk-adjusted return would benefit more of adding the BCI or the GSCI to the portfolio in postfinancialization period, which supports the obtained findings for the maximum Sharpe ratio investor. Indeed, the negative point estimate for the $\alpha$ coefficient when testing potential benefits of adding the GSCI in the prefinancialization period indicate that the maximum Sharpe ratio investor would be worse off by including the GSCI.

Analogously, the point estimates of $\delta$ suggest the benefits of adding commodities to the Base portfolio were greater during the prefinancialization period. Still, the magnitude of the point estimates of the deltas suggest there are significant diversification benefits from adding a commodity index to a portfolio of equity and bonds during both the pre- and the postfinancialization period. Finally, the magnitude and sign of the point estimates for $\delta$ and $\alpha$ suggest that the BCI is a better diversification for the maximum Sharpe ratio investors while GSCI is the better instrument for the minimum variance investor. This holds for both examined time periods. Nevertheless, these interpretations should be taken with caution as it is not statistically validated.

# 6 Discussion

In this section an attempt to discuss the portfolio theoretical implications of the findings is made. First, the results are discussed in relation to the hypothesis and an answer is formulated. Then a discussion regarding the limitations of the study and possible implications of them are presented.

## 6.1 Diversification Benefits of Adding Commodities

Recall that the hypothesis of this study is that the diversification benefits of adding commodities to a portfolio of equity and bonds have decreased due to the financialization of commodities. The hypothesis was examined for two investors: the maximum Sharpe ratio investor and the minimum variance investor.

In contrast to the postulated hypothesis, the results for the maximum Sharpe ratio investor suggest diversification benefits of adding commodities to a portfolio of equity and bonds have increased in over time. During the prefinancialization period, the optimal portfolio for an investor wanting to maximize the risk-adjusted returns (i.e. Sharpe ratio) would only consist of the S&P 500 index. Yet, in the postfinancialization period this investor would improve her Sharpe ratio by adding either the GSCI or the BCI to the Base portfolio. Interestingly, this investor would not invest anything in the BABI which is likely caused by its low average return compared to the other instruments.

The findings also suggest that the diversification benefits, in terms of the study's evaluation metrics, are more prominent when the BCI is added to the portfolio. The relative improvements of adding BCI or GSCI is presented in Table. 6.1. Not only does the Sharpe ratio improve by almost 60%, but the portfolio's overall risk in terms of standard deviation is also reduced. A disadvantage of adding the BCI is the increased exposure to negative outcomes, as evident by the worsened skewness. Still, this effect is somewhat counteracted by the considerably lower portfolio kurtosis. The lowered kurtosis implies the likelihood of extreme outcomes, both negative and positive, of the portfolio return are significantly reduced.

The results when short selling is allowed provide similar results. The only difference is that an investor interested in maximizing the risk-adjusted return would in the prefinancialization period take a short position in the GSCI to free up more capital to invest in the S&P 500. This is explained by the negative average return of the GSCI during the time period. Still as when short selling is constrained, the diversification benefits are more pronounced in the postfinancialization period. The sole difference is that the investor would short the BABI, due to its low average return, and use the freed capital to invest more in the other instruments. Thus, the diversification benefits becomes attenuated when short selling is allowed.

The results for the minimum variance investor provide weak support in favor of the hypothesis. Though diversification benefits are possible to enjoy during both the pre- and the postfinancialization periods, they have declined based on the standard deviation, skewness and kurtosis. The relative improvement of adding the BCI or the GSCI is presented in Table. 6.1. Before the financialization of commodities, including a commodity index in the portfolio would render the overall risk in terms of standard deviation to reduce by roughly 10% and tail risk in terms of kurtosis by 10-15%. In the aftermath of the financialization the benefits in terms of standard deviation and kurtosis are below 3% and 4% respectively. Though the minimum variance investor may lower risk by adding a commodity index to the portfolio, it comes with the cost of a negative Sharpe ratio. This implies that the investor in practice would be better of investing in the risk-free rate rather than any of the constructed portfolios. The results do not change when the short selling constraint is relaxed.

An interesting observation in the results is the role of the BABI. A maximum Sharpe ratio investor would never invest in the BABI and only take a short position in it to free up capital to the other indices. This is due to its low return, compared to the other instruments. In contrast, the minimum variance investor would predominantly invest in the bond index. This, coupled with its negative correlation with the S&P 500, suggest the BABI is able to diversify most of the unsystematic risk of the S&P 500. Lastly, another observation which should be highlighted is the different results for the GSCI and the BCI. The findings for the maximum Sharpe ratio investor provide support of the GSCI being the better commodity index to include in the portfolio in the prefinancialization period. Due to the GSCI negative average return, it was a suitable instrument to short during the prefinancialization period. In contrast, the maximum Sharpe ratio investor would reap the most diversification benefits in the postfinancialization period by taking long position in the commodities indices. This due to their high average return compared to the other instruments. The results for the minimum variance investor are mixed. The findings in the postfinancialization period are in congruence with the stated purpose of the BCI and the GSCI, with the former aiming to be well diversified while the latter aims to reflect each included commodity's significance to the world economy.

To summarize the above discussion, the obtained findings present mixed evidence to the hypothesis. The results indicate diversification benefits have increased for a maximum Sharpe ratio investor after the financialization of commodities, rejecting the hypothesis. In contrast, the diversification benefits for a minimum variance investor mostly decreased after the financialization of commodities, supporting the hypothesis. The results also provide some support to the BCI being a better diversification instrument than the GSCI. Moreover, as evident by Figure. 5.1 it should also be noted that the correlation levels of the GSCI and the BCI with the S&P 500, after a sharp increase between 2010 and 2013, have returned to the low levels observed during the prefinancialization period. This further supports the findings for the maximum Sharpe ratio investor and contradicts the study's hypothesis.

*Table 6.1: Relative improvement in terms of the Sharpe ratio, standard deviation, skewness and kurtosis a minimum variance investor or a maximum Sharpe ratio investor may reap by adding either GSCI or BCI to the portfolio. The table presents result for both examined time periods.*

| | Prefinancialization | | Postfinancialization | |
|---|---|---|---|---|
| Minimum Variance Investor | GSCI Portfolio | BCI Portfolio | GSCI Portfolio | BCI Portfolio |
| St. Dev | 10.35% | 10.16% | 2.21% | 1.97% |
| Sharpe Ratio | -21.59% | -10.55% | 4.45% | 8.99% |
| Skewness | 23.56% | 26.70% | 20.37% | 8.67% |
| Kurtosis | 14.88% | 11.45% | 3.73% | 2.90% |
| Maximum Sharpe ratio Investor | | | | |
| St. Dev | 0.00% | 0.00% | 23.99% | 26.76% |
| Sharpe Ratio | 0.00% | 0.00% | 11.22% | 59.37% |
| Skewness | 0.00% | 0.00% | -78.42% | -62.92% |
| Kurtosis | 0.00% | 0.00% | 18.59% | 49.84% |

## 6.2 Limitations

There are two limitations the reader should be aware of. The first regards the 1-year Libor which is used as a risk-free rate in the analysis. As the data for all the other instruments consists of daily observations, ideally one would use the overnight Libor as a proxy for the risk-free rate. This was not feasible in this study as the overnight Libor was first introduced in 2001 while the analyzed data began in 1995. For this reason, the 1-year Libor has been used and discounted with the assumption of daily compounding in order to reflect the risk-free rate with a maturity of one day. Generally, instruments with longer maturities tend to have higher interest rates, implying that the risk-free rate used in the study might be slightly too high and the use of a risk-free rate with a daily maturity may improve the accuracy of the results.

Second, an implicit assumption prevalent in the analysis is that the portfolio weights are constant. As an investor in practice tend to reallocate her portfolio with a regular interval, allowing for reallocations may lead to result which better reflect an active investor. Still, this is not examined in this study due to the associated algorithmic complexity required to implement this.

# 7 Conclusion

This study examines the diversification benefits that can be earned through adding non-energy commodity indices to a portfolio consisting of a bond index and equity index. Large capital inflows to commodity markets have increased the correlation between commodity indices and stock markets (Tang and Xiong, 2012). As investors traditionally have considered commodities as a good diversification instrument, financialization has brought up the question on whether these diversification benefits still prevail. This is examined for two types of investors; one seeking to minimize the portfolio risk and one trying to maximize the risk-adjusted portfolio return as measured by the Sharpe ratio. This study analyze how diversification benefits in terms of the standard deviation, Sharpe ratio, skewness and kurtosis from adding commodities have changed due to the financialization.

The findings suggest that while a minimum variance investor may earn diversification benefits from adding non-energy commodities both before and after the financialization, the diversification benefits have declined in the aftermath of the financialization. In contrast, the results for the maximum Sharpe ratio investor suggest she may enjoy greater diversification benefits today than she would have done before the financialization of commodities, rejecting the hypothesis. These findings suggest that the financialization of commodity markets might not be permanent, or at least it has not transformed non-energy commodity indices to an adverse diversification instrument as previous studies have indicated (e.g. Tang and Xiong, 2012).

A delimitation of the study is the assumption of no portfolio reallocation. Since investors in practice typically tend to reallocate their portfolio with regular interval, it would be interesting if future studies examines the implication of regular portfolio reallocations. This would be a valuable extension to the study, as it would reflect a more active investor.

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# Appendix A: Data Structure

*Table A.1: Weights in respective commodity for the GSCI Non-Energy and the BCI Non-Energy as of August 2014. Please note that the indices are re-weighted continuously, meaning that this is simply an example of how the weighting of the indices can differ.*

| Commodity Futures | S&P GSCI Non-Energy | BCI Non-Energy |
|---|---|---|
| Chicago Wheat | 2,5% | 0,7% |
| Cocoa | 1,1% | 0,9% |
| Coffee "C" | 2,5% | 1,2% |
| Copper - Grade A | 3,2% | 1,3% |
| Corn | 8,6% | 1,9% |
| Cotton #2 | 2,5% | 2,2% |
| Feeder Cattle | 4,7% | 3,6% |
| Gold | 11,8% | 4,0% |
| High Grade Primary Aluminum | 9,3% | 4,9% |
| Kansas Wheat | 11,8% | 12,3% |
| Lean Hogs | 9,7% | 16,9% |
| Live Cattle | 1,4% | 1,3% |
| Primary Nickel | 1,8% | 2,5% |
| Silver | 2,2% | 2,8% |
| Soybeans | 1,1% | 4,6% |
| Special High Grade Zinc | 6,5% | 7,3% |
| Standard Lead | 8,2% | 15,7% |
| Sugar #11 | 11,1% | 15,5% |

# Appendix B: Problem Formulation Derivation

This section derives a proof for how the optimization problem for the maximum Sharpe ratio may be expressed as Eq. (4.5). Recall that the optimization problem with regards to the Sharpe ratio may be expressed as:

$$\begin{aligned} \underset{\mathbf{w}}{\text{Maximize}} \quad & \frac{\mathbf{w^T}\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w^T\Sigma w}}} \\ \text{subject to} \quad & \mathbf{w^T 1} = 1, \\ & w_i \geq 0. \end{aligned} \tag{B.1}$$

where $\mathbf{w^T} = (w_1, ..., w_i)$ i.e. a vector of the portfolio weights $w_i$. $\boldsymbol{\mu}$ is a vector of the expected return and $\mathbf{\Sigma}$ is the covariance matrix of the portfolio assets. $r_f$ is the risk-free rate. Eq. (B.1) is a concave problem but can be formulated as a convex minimization problem, which typically is easier to compute, as:

$$\begin{aligned} \underset{\mathbf{w}}{\text{Minimize}} \quad & \frac{\sqrt{\mathbf{w^T\Sigma w}}}{\mathbf{w^T}\boldsymbol{\mu} - r_f} \\ \text{subject to} \quad & \mathbf{w^T 1} = 1, \\ & w_i \geq 0. \end{aligned} \tag{B.2}$$

While Eq. (B.2) is a convex problem it is only a quasi-convex problem. The implication of this is that a solution to Eq. (B.2) is not necessarily the global optimal solution. To ensure a global optimal solution is obtained the following transformation is made:

$$\tilde{\mathbf{w}} = t\mathbf{w}, \qquad t = \frac{1}{\mathbf{w^T}\boldsymbol{\mu} - r_f} > 0 \tag{B.3}$$

Using Eq.(B.3) the problem formulation become:

$$\begin{aligned}
\text{Minimize} \quad & \tilde{\mathbf{w}}^{\mathbf{T}} \boldsymbol{\Sigma} \tilde{\mathbf{w}} \\
\text{subject to} \quad & t = \frac{1}{\mathbf{w}^{\mathbf{T}} \boldsymbol{\mu} - r_f} \\
& \tilde{\mathbf{w}}^{\mathbf{T}} \mathbf{1} = t > 0, \\
& \tilde{w}_i \geq 0.
\end{aligned} \tag{B.4}$$

The first constraint $t = \frac{1}{\mathbf{w}^{\mathbf{T}} \boldsymbol{\mu} - r_f}$ may be expressed in terms of $\tilde{\mathbf{w}}$ as $(\boldsymbol{\mu} - r_f \mathbf{1})\tilde{\mathbf{w}} = 1$. Note that this implies that $\tilde{\mathbf{w}}^{\mathbf{T}} \mathbf{1} = t > 0$ is equivalent to $t \geq 0$ since $t = 0$ means that $\tilde{w} = 0$ which cannot happen as the first constraint would not be fulfilled. Thus, the problem may be formulated as:

$$\begin{aligned}
\text{Minimize} \quad & \tilde{\mathbf{w}}^{\mathbf{T}} \boldsymbol{\Sigma} \tilde{\mathbf{w}} \\
\text{subject to} \quad & (\boldsymbol{\mu} - r_f \mathbf{1})\tilde{\mathbf{w}} = 1 \\
& \tilde{\mathbf{w}}^{\mathbf{T}} \mathbf{1} \geq 0, \\
& \tilde{w}_i \geq 0.
\end{aligned} \tag{B.5}$$

Q.E.D.

# Appendix C: Relaxing the Short-Selling Constraints

This section presents the results for the maximum Sharpe ratio investor and minimum variance investor, respectively, when short selling is allowed. First, the results for the maximum Sharpe ratio investor during both the pre- and the postfinancialization period are displayed. Then, the corresponding results for the minimum variance investor are presented.

## C.1 The Maximum Sharpe ratio Investor

The results for the maximum Sharpe ratio investor when short selling is allowed. Note that these results have been obtained numerically using MATLAB rather than solving the optimization problem as specified by Eq. (4.5). This was done due to limited time. Still, the validity of the results remains high and although minor approximation inconsistencies may be present it does not affect the drawn conclusion of the findings.

*Table C.1: Optimal portfolio allocation for a maximum Sharpe ratio investor in the prefinancialization period when short selling is allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 150% | 150% | 150% |
| S&P GSCI Non Energy | NA | -50 | NA |
| BCI Non Energy | NA | NA | 0% |
| Barclays Aggregate Bond Index | -50% | 0% | -50% |
| St. Dev | 1.7800% | 1.7800% | 1.7800% |
| Sharpe Ratio | 0.0186 | 0.0200 | 0.0186 |
| Skewness | -0.1000 | -0.0615 | -0.1000 |
| Kurtosis | 6.1464 | 5.6857 | 6.1464 |

*Table C.2: Optimal portfolio allocation for a maximum Sharpe ratio investor in the postfinancialization period when short selling is allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 150.0% | 79.4% | 43.5% |
| S&P GSCI Non Energy | NA | 70.6% | NA |
| BCI Non Energy | NA | NA | 106.5% |
| Barclays Aggregate Bond Index | -50.0% | -50.0% | -50.0% |
| St. Dev | 1.8600% | 1.4000% | 1.3700% |
| Sharpe Ratio | 0.0121 | 0.0139 | 0.0188 |
| Skewness | -0.3387 | -0.6248 | -0.5627 |
| Kurtosis | 14.6612 | 11.2309 | 7.5558 |

## C.2 The Minimum Variance Investor

The results for the minimum variance investor when short selling is allowed. Note that there is some marginal differences in the results compared to when short selling is not allowed. This is attributed to approximation inconsistencies in MATLAB's built-in function 'quadprog', which is used in this study.

*Table C.3: Optimal portfolio allocation for a minimum variance investor in the prefinancialization period when short selling is allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 5.2% | 3.8% | 3.8% |
| S&P GSCI Non Energy | NA | 17.0% | NA |
| BCI Non Energy | NA | NA | 16.8% |
| Barclays Aggregate Bond Index | 94.8% | 79.2% | 79.4% |
| St. Dev | 0.2468% | 0.2213% | 0.2217% |
| Sharpe Ratio | -0.0583 | -0.0711 | -0.0646 |
| Skewness | -0.3967 | -0.3029 | -0.2907 |
| Kurtosis | 5.7844 | 4.9089 | 5.1130 |

*Table C.4: Optimal portfolio allocation for a minimum variance investor in the postfinancialization period when short selling is allowed. "NA" indicate the asset is not included in the portfolio. The last four rows display the evaluation metrics.*

| Porfolio Allocation | Base Portfolio | GSCI Portfolio | BCI Portfolio |
|---|---|---|---|
| S&P500 | 7.7% | 6.3% | 6.4% |
| S&P GSCI Non Energy | NA | 4.5% | NA |
| BCI Non Energy | NA | NA | 4.2% |
| Barclays Aggregate Bond Index | 92.3% | 89.2% | 89.3% |
| St. Dev | 0.2123% | 0.2076% | 0.2081% |
| Sharpe Ratio | -0.0344 | -0.0330 | -0.0316 |
| Skewness | -0.1293 | -0.1041 | -0.1184 |
| Kurtosis | 6.9620 | 6.7220 | 6.7807 |

## C.3 Relative Improvement

*Table C.5: Relative improvement in terms of Sharpe ratio, standard deviation, skewness and kurtosis a minimum variance investor or a maximum Sharpe ratio investor may reap by adding either GSCI or BCI to the portfolio. Short-selling is allowed. The table presents result for both examined time periods.*

| | Postfinancialization | | Postfinancialization | |
|---|---|---|---|---|
| Minimum Variance Investor | GSCI Portfolio | BCI Portfolio | GSCI Portfolio | BCI Portfolio |
| St. Dev | 10.35% | 10.17% | 2.22% | 1.98% |
| Sharpe Ratio | -21.89% | -10.84% | 4.07% | 8.27% |
| Skewness | 23.63% | 26.72% | 19.47% | 8.40% |
| Kurtosis | 15.14% | 11.61% | 3.45% | 2.60% |
| Maximum Sharpe ratio Investor | | | | |
| St. Dev | 0.00% | 0.00% | 24.73% | 26.34% |
| Sharpe Ratio | 7.53% | 0.00% | 14.88% | 55.37% |
| Skewness | 38.50% | 0.00% | -84.47% | -66.14% |
| Kurtosis | 7.50% | 0.00% | 23.40% | 48.46% |