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Risk Managed Time Series Momentum

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ABSTRACT

This paper aims to investigate the crashes of time series momentum and to explore a systematic approach that mitigates the crashes of this strategy. Similar to cross-sectional momentum, time series momentum is also prone to severe drawdowns subsequent of a market decline when market volatility is high, contemporaneous with market reversals. However, such crash risk and option-like behaviour appear to be statistically predictable. Based on the insight on momentum crashes, we construct a risk-managed time series momentum strategy (RTSMOM) through a dynamic loading on the basic time series momentum (BTSMOM) strategy using in-sample predictions of the strategy's return and volatility. Our findings demonstrate that RTSMOM has a lower crash risk as negative volatility, maximum drawdown, VaR and expected tail loss decrease. Furthermore, RTSMOM has a higher average return and a substantial increased Sortino ratio. These findings are robust in subsample, back-testing, and cross asset analysis.

Keywords: momentum crash, time series momentum, time-varying risk, optionality

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1 Introduction

This paper aims to investigate the crashes of time series momentum and to mitigate such downside risk with a systematic approach that is based on the insight on momentum crashes.

A momentum strategy bets on that past returns will predict return continuation in the future. It can be implemented by buying past winners and selling past losers (WML) cross-sectionally, which is called cross-sectional momentum, or can be based purely on a security's own past return, which is called time series momentum.

Cross-sectional momentum has gained increased popularity since the classical studies of Jegadeesh and Titman (1993) and Asness (1995). The premium generated from buying winners and selling losers has proved to be an asset pricing anomaly. Yet, since the recent study of Moskowitz, Ooi, and Pedersen (2012), time series momentum poses a greater puzzle than cross-sectional momentum. As shown by Moskowitz, Ooi, and Pedersen (2012), time series momentum generates even higher and steadier returns than cross-sectional momentum, while its premium cannot be explained by standard asset pricing risk factors and market indices.

However, the time series momentum strategy is far from impeccable. Time series momentum is prone to large drawdowns subsequent of large market declines as the market reverses quickly, similar to its cross-sectional counterpart as studied by Daniel and Moskowitz (2014). According to the study of Moskowitz, Ooi, and Pedersen (2012), the largest drawdown of the cross asset strategy amounts to 15.2%, and that of an individual asset class portfolio accumulates to 56.2%. However, their sample period ends at December 2009, which means that potential losses could be larger as the market rebounded further after the crisis 2008. This feature of time series momentum produces a risk that can be unacceptable to investors.

1.1 Research Motivation and Focus

The aim of this study is to investigate the crashes of time series momentum, as introduced before, and to explore a systematic approach that mitigates such a drawback of this strategy. There are barely any studies on the crashes of time series momentum and potential mitigation approaches, as time series momentum is an emerging topic within academia. Yet, regarding the crashes of cross-sectional momentum, a popular and credible explanation is the time-varying risk (Kothari and Shanken, 1992, Grundy and Martin, 2001, Barroso and Santa-clara, 2014, Daniel and Moskowitz, 2014). Based on the insight on the crashes of cross-sectional momentum, there are several studies that have used a systematic approach to successfully mitigate the downside of cross-sectional momentum compared to the original strategy, such as Barroso and Santa-Clara (2014), and Daniel and Moskowitz (2014). Due to the close relationship of time series momentum and cross-sectional momentum, we are curious whether the crashes of time series momentum is associated with similar time-varying risks as cross-sectional momentum and thus if it also can be mitigated through a rule-based approcach. Therefore, we aim to answer the following research questions:

Does time-varying risks lead to the crashes of time series momentum?

and

Is there a systemic approach to mitigate the crashes of time series momentum?

1.2 Summary of Study

To explore our research questions, we initially investigate the crashes of time series momentum with a basic strategy, and then explore a mitigation approach using the insight on momentum crashes.

We use 118 futures contracts to construct a basic time series momentum (BTSMOM) strategy using the construction method documented by Moskowitz, Ooi, and Pedersen (2012). Similar to their study, BTSMOM experiences large drawdowns – the largest one being nearly 25% over the period 1985:01 to 2015:02, while for an individual asset class the maximum drawdown is 63.3%.

Through an in-depth analysis of the crash periods, we observe that time series momentum is especially vulnerable when the equity market experiences high volatility, contemporaneous with sharp market reversals. This characteristic is similar to the payoff of a short call option, in line with Daniel and Moskowitz (2014) findings on cross-sectional momentum. Such option-like behaviour is explained by the time-varying market beta of the strategy. Following the specification of Henriksson and Merton (1981), we compute the strategy's up and down betas. Given that the market is in a bear market state, the up beta of the strategy is more than 1.5 times larger than the down beta (-0.68 versus -0.42). This implies that in the condition of a bear market state, the strategy is profitable when the market continues to fall, while the strategy suffers severe losses when market reverses to the upside. Based on the insight on momentum crashes, a model that predicts the strategy's return is constructed.

We construct a systematic approach that dynamically weights the loading on BTSMOM based on the methodology presented by Daniel and Moskowitz (2014). The approach takes into account our findings regarding time series momentum's return predictability together with a common volatility prediction model, which combines GARCH-GJR volatility estimation and the strategy's realised volatility. The dynamic weighting is set to maximise the in-sample Sharpe ratio of the strategy based on the strategy's predicted return and volatility.

The results for the main sample period 1985:01 to 2015:02 demonstrate that, compared to basic strategy, RTSMOM has 0.9% lower monthly negative volatility and the crash risk is mitigated with 16.6% lower maximum drawdown, 1.1% lower Value at Risk, and 1.7% lower expected shortfall. RTSMOM also has an increase in the average monthly return of 0.28%, which consequently raises the annualised Sortino ratio by 79.4%.

As a robustness check, we split the main sample into two evenly divided subsamples, on which the same analysis is conducted. We also conduct a back-test assessment based on the data prior to 1985. The performance of RTSMOM is consistent in all robustness checks, which gives weight to our findings in the main sample. We also categorise all 118 futures contracts into four asset classes and conduct cross asset class analysis. The results support the risk-managing efficacy of the dynamic weighting system, although it fails to increase the average monthly return and Sortino ratio materially for the asset class rates.

1.3 Relationship with Existing Literature

The studies most related to ours are Daniel and Moskowitz (2014) and Barroso and Santa-Clara (2014) in terms of momentum crashes and mitigation methods. However, they study the crashes of cross setional momentum, while our focus is on the crashes of time series momentum. Our study is also related to the research of Moskowitz, Ooi, and Pedersen (2012), as our study adopts their definition of time series momentum and is also conducted on cross asset class futures data.

This study contributes to the existing literature in two aspects. Firstly, it investigates the crashes of time series momentum, and extends the study of Moskowitz, Ooi, and Pedersen (2012) from the perspective of risk management. This study is conducted with a broader range of futures contracts (118 in total) and a more up-to-date time window (1973:09 – 2015:02). Secondly, it explores a rule-based system that mitigates the crashes of time series momentum. Such system can be easily implemented and it improves the basic strategy materially, which leads to a more attractive investment strategy for investors.

1.4 Outline

The structure of this paper is as follows: Section 2 reviews the previous studies in the three related areas: cross-sectional momentum, time series momentum, and time-varying risk of momentum. It is followed by section 3, which includes construction method of the basic strategy (BTSMOM), data preliminaries, and the performance of BTSMOM. We also investigate the momentum crashes, time-varying beta, and the prediction model of BTSMOM's return. Section 4 introduces a risk-managed time series momentum strategy (RTSMOM) that is based on a dynamic weighting system. Furthermore a discussion regarding the performance of RTSMOM in the main sample period, subsample analysis, back-testing, and cross asset analysis is included. The final section includes the conclusion, limitations, and potential areas for further research.

2 Literature Review

This section reviews the relevant previous studies in the areas of cross-sectional momentum, time series momentum, as well as crashes and return predictability for momentum strategies.

2.1 Cross-Sectional Momentum

In the financial literature, Jegadeesh and Titman (1993) and Asness (1995) first document momentum strategies in the U.S common stock returns. They sort stocks on their own past three to twelve months return, taking a long (short) position in the stocks in the top (bottom) 30%. They find that investors can earn positive abnormal returns that show a low correlation to standard risk factors, providing a challenge to the efficient market hypothesis.

Jegadeesh and Titman (1993) motivate their study of momentum with "… a majority of the mutual funds examined by Grinblatt and Titman (1989) show a tendency to buy stocks that have increased in price over the previous quarter." The momentum strategy return is introduced as an extension to the standard risk factors of Fama and French (1992, 1993) by Carhart (1997) and further demonstrates the observation made by Jegadeesh and Titman (1993) that mutual funds on average load positively on momentum. The momentum strategy has been further scrutinized in academia but has consistently shown large positive returns across different time periods and asset classes, justifying its status as a market anomaly.

Israel and Moskowitz (2013) extends the time period, 1965-1989, studied by Jegadeesh and Titman (1993) and Asness (1995) by providing results from 1927-1965 and 1990-2012. Geczy and Samonov (2013) conduct a study with data from 1802-2012, a study which the authors calls "the world's longest back-test". There is even evidence of the momentum effect that dates back to the Victorian age, presented by Chabot, Ghysels, and Jagannathan (2009).

Strong and persistent momentum effects are also present in the international equity markets and other asset classes. Rouwenhorst (1998) finds evidence of momentum in other developed equity markets, and Rouwenhorst (1999) documents momentum in emerging equity markets. Asness, Liew, and Stevens (1997) demonstrate positive momentum in country indices. Among common stocks, there is also evidence that momentum strategies perform well for industry strategies, and for strategies that are based on the firm specific component of returns (Moskowitz and Grinblatt, 1999, Grundy and Martin, 2001, and Asness, Porter, and Stevens, 2000). Outside of equity markets, Okunev and White (2003) find momentum in currencies; Miffre and Rallis (2007) and Erb and Harvey (2006) in commodities; Moskowitz, Ooi, and Pedersen (2012) in futures contracts; and Asness, Moskowitz, and Pedersen (2013) in bonds. Asness, Moskowitz, and Pedersen (2013) also integrate the evidence of intra cross-sectional equity momentum with equity index, fixed income, currency, and commodity momentum.

The momentum discussed previously belongs to the most frequent discussed momentum in the financial literature; cross-sectional momentum. Cross-sectional momentum is based on a security's relative performance to its peers. In contrast, time series momentum focuses on the absolute past return of a security as a predictor of future price continuation.

2.2 Time Series Momentum

Our study emanates from the paper "Time series momentum" by Moskowitz, Ooi, and Pedersen (2012). The authors examine time series momentum returns across 58 futures contracts in a wide variety of asset classes for the years 1985 to 2009. They find significant positive returns in each of the 58 markets; the persistence in returns is strongest when a 12-month look back period and a 1-month hold period is applied. The authors construct a time series momentum strategy, by taking a long (short) position in all contracts that have had a positive (negative) excess return over the previous twelve months and rebalance the portfolio on a monthly basis. The results show that the time series momentum factor has large significant returns with a low exposure to standard risk factors in combination with the benefit of strong performance during market declines.

The authors find that correlations of time series momentum strategies across asset classes are larger than the correlations of the asset classes themselves. This suggests that there is a mutual component that affects time series momentum strategies concurrently across asset classes that is not present in the underlying asset classes themselves. Furthermore, the authors decompose the returns of a time series momentum and a cross-sectional momentum strategy in an auto-covariance component, a cross-covariance component, and a dispersion in mean returns component. The authors find that positive auto-covariance is the main driver of both the time series and cross-sectional momentum effect. There is a small effect from the dispersion in mean present. However, the effect of cross-covariance is negligible for cross-sectional momentum and even negative for time series momentum which implies that time series momentum will have higher return than cross-sectional momentum as it only captures the auto-covariance and the dispersion in mean components.

The authors conclude that the characteristics of time series momentum agree with many of the existing behavioural and rational asset pricing theories. The existence of positive time series momentum and the subsequent reversal hints that it is consistent with theories of sentiment leading to initial under-reaction and later on over-reaction. The fact that different types of investors across different markets are producing the same price patterns at similar time poses a challenge to some theories. In addition, the authors cannot find a link between time series momentum and investor sentiment measures from the literature. In an attempt to better explain the driver of time series momentum the authors investigate the trading activity of hedgers and speculators. They find that speculators on average trade with time series momentum and appear to profit from the strategy at the expense of hedgers. One explanation to this might be that speculators earn a premium through time series momentum by providing liquidity to hedgers. The authors in this way provide an alternative explanation for the existence of time series momentum.

2.3 Momentum Crashes, Time-Varying Risk, and Return Predictability

Our study is related to the time-varying risk of momentum strategies and the predictability of future volatility and return in order to find a dynamic weighting scheme that improves the performance of the time series momentum strategy. Daniel and Moskowitz (2014) focus on the drawback of cross-sectional momentum, which they call “momentum crashes”. They show that cross-sectional momentum is prone to large drawdowns, these drawdowns occur after a period of negative market returns and a subsequent reversal. These results are in line with those of Cooper, Gutierrez, and

Hameed (2004) and Stivers and Sun (2010), as they show that the momentum premium falls to zero when the past three-year market return is negative and that the momentum premium is low when market volatility is high. Daniel and Moskowitz (2014) further show that low ex-ante expected returns in panic states is the consequence of a conditionally high premium linked to the option-like payoff of past losers.

Cross-sectional momentum has proved to generate large returns but with the cost of a very high excess kurtosis. One possible cause for the excess kurtosis could be time-varying risk (see Engle, 1982 and Bollerslev, 1986). Kothari and Shanken (1992), and Grundy and Martin (2001) find that a cross-sectional momentum strategy has a pro-cyclical time-varying beta exposure. In rising markets, the strategy increases its market beta loading making it vulnerable to sudden market reversals. In falling markets the cross-sectional momentum strategy has a negative conditional beta loading. This negative beta loading is what causes the strategy’s large drawdowns when markets reverse to the upside. Grundy and Martin (2001) argue that hedging this time-varying beta exposure reduces the crashes of the momentum strategy and leads to a more stable return series. However, Daniel and Moskowitz (2014) show that when using betas without look-ahead bias, Grundy and Martin (2001) hedged the exposure using forward-looking betas, and, there is no mitigation in the crashes of the momentum strategy.

Barroso and Santa-Clara (2014) find that the volatility of cross-sectional momentum is highly variable over time but also predictable. They propose to scale the exposure to the strategy by its own realized variance, targeting a constant volatility of the strategy, in order to risk-manage the momentum strategy. They find that this weighting scheme nearly eliminates the momentum crashes and almost doubles the Sharpe ratio of the momentum strategy. The reason Barroso and Santa-Clara (2014) find that it is possible to risk-manage momentum with realized variance but not with time-varying beta is due to the composition of momentum volatility. They find that the market component is a minority of the total volatility and that the strategy specific part is the main component. Furthermore, the specific risk is more persistent and predictable than the market component.

Due to the fact that the momentum strategy's volatility itself is predictable and distinct from the predictability in mean returns, Daniel and Moskowitz (2014) design a dynamic momentum strategy that varies its exposure to the momentum strategy in order to maximize the unconditional Sharpe ratio of the strategy. The results show that the dynamic strategy significantly outperforms the standard momentum strategy and other suggested constant volatility variations of momentum strategies (e.g. Barroso and Santa-Clara, 2014). The authors conduct robustness checks across countries and asset classes, and results are consistent. All studies referred to in this section deal with cross-sectional momentum. Therefore, this paper aims to contribute with new insights on the time-varying risk of time series momentum and to explore how suggested improvement mechanisms for cross-sectional momentum affect the performance of a time-series momentum strategy.

3 Basic Time Series Momentum

This section presents our study on the performance and crashes of the basic time series momentum strategy. To start with, the portfolio construction method and data description is described.

3.1 Portfolio Construction and Data Preliminaries

3.1.1 Portfolio Construction Method

We construct the time series momentum strategy by adopting the methodology of Moskowitz, Ooi, and Pedersen (2012). Firstly, the trading signal for each individual asset is decided based on the asset's return over the previous 12 months in excess of the risk free rate. The strategy takes a long (short) position if the asset's past excess return is positive (negative).

Secondly, the position of each individual asset is scaled by its own ex-ante volatility due to the following reasons: (1) it prevents a few assets from driving the majority of the strategy's return, as the return volatility of the assets differs substantially across asset classes but also within an asset class; (2) it is also econometrically beneficial to have a time series with a relative stable volatility so the strategy's return is not dominated by a few extremely volatile periods.

The position size of each asset is scaled by the target volatility and its ex-ante volatility. The target volatility of 40% is kept the same as what Moskowitz, Ooi, and Pedersen (2012) use, as it increases the comparability of our portfolio to others in the previous literature. As all individual assets are aggregated into a single strategy the volatility of the strategy is roughly the same as the volatility of the SMB, HML, WML, and the CSMOM factors that we use in our analysis. The return for an individual asset is therefore given by:

$$r_{s,t+1}^{TSMOM} = sign(r_{s,12m})\frac{40\%}{\sigma_t}r_{s,t+1} \tag{1}$$

Aggregating the individual assets into a strategy gives the following return calculation for the overall strategy:

$$r_{TSMOM,t+1} = \frac{1}{N}\sum_{s=1}^{N}\left[sign(r_{s,12m})\frac{40\%}{\sigma_t}r_{s,t+1}\right] \tag{2}$$

To compute the ex-ante volatility for each individual asset, a simple exponentially weighted moving average model of lagged squared daily returns is used as follows:

$$\sigma_{s,t}^2 = 261 \sum_{i=0}^{\infty} [\delta^i (1-\delta)\left(r_{s,t-1-i} - \bar{r}_{s,t}\right)^2] \tag{3}$$

where the scalar 261 annualizes the estimated daily variance, the weights $\delta^i(1-\delta)$ adds up to one, and, the average return $\bar{r}_{s,t}$ is the exponentially weighted average return. The parameter is set so that the centre of the mass of the weights is 60 days, i.e. $\sum_{i=0}^{\infty} \delta^i (1-\delta) = 60$. To insure that no look-ahead bias occurs, we use one period lagged volatility estimate.

3.1.2 Data Preliminaries

The following subsection will present the futures data that has been used in this thesis and the adjustments made to the futures prices. The data consists of futures prices, exchange rates, index benchmarks, and standard risk factors for evaluation purposes.

a. Futures data

Our data consists of 118 futures prices[1], among them 45 commodities contracts, 14 exchange rates contracts (both single currency indices and cross-currency pairs), 23 government bonds contracts, and 36 equity index contracts, from January 1972 to February 2015. Yet, our main sample starts from January 1985 based on the considerations that (1) a sufficient amount of contracts is needed in order for the strategy to be truly diversified; (2) it allows for consistency with the pioneering study by Moskowitz, Ooi, and Pedersen (2012), but further expands the sample they used with the interesting post financial crisis period 2010-2015; (3) a back-test, in which the omitted data from January 1972 to December 1984 is used, can be conducted as a robustness check of the results from the main sample.

There are several reasons why futures data is chosen for this thesis: (1) it gives us access to a wide range of assets in several asset classes. This is a prerequisite in order for the strategy to be diversified enough so the characteristics are driven by time series momentum rather than sample specific conditions for an asset class or even a single

[1] All futures data was generously provided to us by Andreas Clenow from ACIES Asset Management, an Absolute Return Capital Management firm based in Switzerland. For a complete list of each contract, please refer to Appendix A.

asset; (2) future markets are liquid which insures that the strategy is implementable and not only of theoretical interest; (3) as the strategy includes both long and short positions, future contracts are ideal in the sense that it is just as easy to take a short position as a long position.

However, due to the limited life span and roll over gap of futures contracts, we back adjust the futures data to remove the roll over gap by matching the old contract's closing price with the new contract's closing price on the roll over date. A detailed explanation of the back adjustment method is presented in Appendix B.

b. Exchange rates

To allow us to make meaningful comparisons and to aggregate the returns of the individual positions of the strategy, all futures prices are converted to US dollars. The 14 cross-currency pairs against US Dollars used are as follows: Australian Dollars, Canadian Dollars, Euro, British Pounds, Japanese Yen, Hong Kong Dollars, South African Rand, Swiss Francs, Swedish Krona, Malaysian Ringgit, Singapore Dollars, Norwegian Krone, and South Korean Won. All exchange rates data is downloaded from Datastream.

c. Asset pricing benchmarks – risk factors

In order to evaluate the strategies, we employ commonly used asset class indices and factor returns, namely the *MSCI World equity index, JP Morgan Global Government Bond (JPMGGB) index, Standard & Poors Goldman Sachs Commodity Index (GSCI), MKT-Rf, SMB (Small Minus Big), HML (High Minus Low)* and *WML (Up Minus Down)*[1]. The data for the asset class indices was obtained from Bloomberg data terminal, whilst the data for the factor returns was obtained from Kenneth French's data library[2].

The cross asset class *cross-sectional momentum (CSMOM)* factor[3], constructed by Asness, Moskowitz, and Pedersen (2013), is also used to investigate the relationship

[1] The data used for constructing the factors is stock data from 23 developed countries accessed from the CRSP database. A more detailed account of the factors can be found in Fama and French (2012), "Size, Value and Momentum in International Stock Returns".
[2] Please refer to http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[3] The factor is formed by the 65 futures contracts that are ranked on their past 2-12 month return. A zero-cost portfolio is constructed by going long the group with the best returns and going short the group with the worst returns.

between time series momentum and cross-sectional momentum on an across asset class level. The underlying data for the construction of the *CSMOM* factor is future returns for equity indices, fixed-income, exchange rates, and commodities. This adds another layer of comparability to our study. The *CSMOM* factor is downloaded from Tobias Moskowitz web page (2015)[1].

All the data used in this study are obtained from either well-known databases or credible online data libraries. Therefore, we believe that the data are of sufficient quality for the empirical results to be reliable.

3.2 Momentum Portfolio Performance

Fig.1 plots the cumulative excess monthly returns from 1985:01 to 2015:02 for the (1) basic time series momentum strategy (BTSMOM), (2) the passive long strategy, and (3) the MSCI World index. For comparison, the passive long strategy invests in all possible long position of every futures contract we study, and each contract is scaled by the same amount of constant volatility like BTSMOM.

Though our investment universe is broader than only equity futures, the MSCI World index is still used as the market benchmark based on the following considerations: (1) it simplifies our calculation and also includes information of 23 equity markets, covering a broad geographical area and provides comprehensive information of the global economy; 2) the momentum of each asset classes are significantly correlated, as shown by Asness, Moskowitz, and Pedersen (2013) in the paper "Value and Momentum Everywhere".

As shown in Fig.1, the BTSMOM demonstrates a relatively steady stream of positive returns, compared to the passive long strategy and the MSCI World index, which is consistent with the findings of Moskowitz, Ooi, and Pedersen (2012). In Tab.1 return and risk statistics of the three portfolios are displayed. Compared to the passive long strategy and the MSCI World index, the monthly return volatility of the BTSMOM is 4.5% and 0.9% lower respectively, while it also has a higher average monthly return, 1.1%, and 1.6%, respectively.

[1] Please refer to http://faculty.chicagobooth.edu/tobias.moskowitz/research/data.html

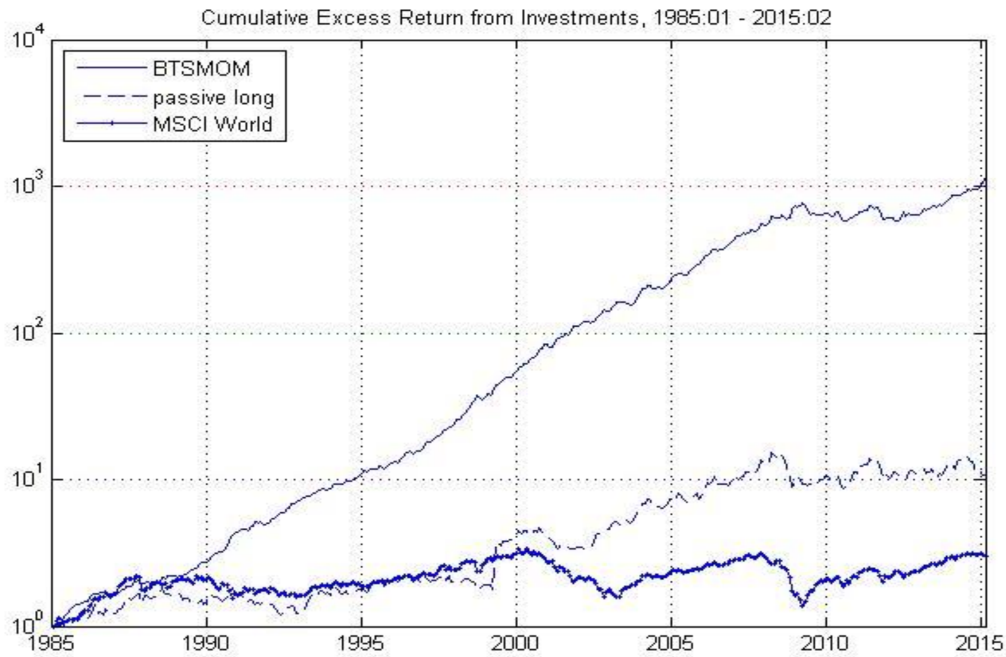

Fig. 1 Cumulative Excess Return of BTSMOM, Passive Long Strategy, and MSCI World
The graph plots the cumulative monthly excess return of basic time series momentum portfolio (BTSMOM), passive long portfolio, and MSCI World, from 1985:01 to 2015:02. BTSMOM invests across all futures contracts we investigate which are weighted by their own volatilities as defined in equation (3). The passive long strategy invests in possible long position in each contract that is also scaled by its own volatility.

We also decompose the return of BTSMOM, and observe significant alpha over a number of benchmark indices and asset pricing factors, as shown in Tab.2. In Panel A, the monthly and the non-overlapping quarterly returns of BTSMOM are decomposed by the classical Fama-French factors and market indices. BTSMOM has a statistical significant positive abnormal return of 1.7% and 4.9% on a monthly and quarterly basis respectively, and only the cross-sectional momentum factor (WML) has a significant loading upon BTSMOM in both regression at the 5% significance level. In Panel B the results when regressing the Value and CSMOM factor of Asness, Moskowitz and Pedersen (2013) is displayed. CSMOM is a cross asset factor that should provide insights into what extent cross-sectional momentum explains time series momentum due its similarities in underlying assets to BTSMOM. In addition to the cross asset value and momentum factor, the MSCI World index is also included in the regression as a proxy for the broader market. Adjusted for the VAL and CSMOM factors, BTSMOM, once again, delivers a highly significant abnormal return of 1.5% and 4.2% on a monthly and quarterly basis.

Tab. 1 Monthly Return Statistics

This table summarises the monthly return statistics of BTSMOM, passive long strategy and the MSCI World index over the time period of 1985:01 – 2015:02. The Sharpe ratio and the Sortino ratio are annualised, while all other statistics are reported on a monthly basis. Volatility represents standard deviation and absolute kurtosis is reported.

	BTSMOM	Passive Long Strategy	MSCI World
Average Return	2.0%	0.9%	0.4%
Minimum Return	-13.1%	-18.2%	-19.1%
Maximum Return	15.1%	110.7%	11.1%
Volatility	3.5%	8.0%	4.4%
Negative Volatility	2.3%	3.5%	3.3%
Skewness	-0.10	7.14	-0.70
Kurtosis	5.09	100.33	4.74
Sharpe Ratio	2.00	0.40	0.33
Sortino Ratio	3.06	0.89	0.43
Maximum Drawdown	23.9%	43.0%	59.7%
95% VaR	-3.4%	-8.5%	-7.2%
Expected Shortfall	-5.8%	-11.4%	-10.7%

Tab. 2 Return decomposition of BTSMOM

Panel A reports the results of time series regression of monthly return and non-overlapping quarterly return of BTSMOM on MSCI World Equity Index, GS Commodity Index, JPM Government Bond Index, and global Fama French factors, SMB, HML, and WML. The Panel A regression sample period is from 1990:01 to 2015:02. Panel B presents the results replacing Fama French factors with Asness, Moskowitz, and Pedersen (2013) Value (VAL) and Momentum (CSMOM) Everywhere factors, which captures the value and cross-sectional momentum globally across asset classes. The Panel B regression sample period is from 1985:01 to 2015:02.

Panel A Regression of BTSMOM on Fama French Global Factors and Indices

		Intercept	MSCI World	SMB	HML	WML	SP GSCI	JPM GBI	R^2
Monthly	Coefficient	1.7%	-0.01	-0.14	0.16	0.40	0.00	-0.08	19%
	(T-stat)	(8.53)	(-0.11)	(-1.47)	(1.86)	(7.65)	(-0.08)	(-0.73)	
Quarterly	Coefficient	4.9%	0.11	-0.11	0.31	0.44	-0.07	-0.10	25%
	(T-stat)	(7.25)	(1.48)	(-0.64)	(2.57)	(5.11)	(-1.28)	(-0.66)	

Panel B Regression of BTSMOM on Asness, Moskowitz, and Pedersen (2013) factors

		Intercept	CSMOM	VAL	MSCI World	R^2
Monthly	Coefficient	1.5%	1.07	0.44	-0.02	28%
	(T-stat)	(8.9)	(10.45)	(3.48)	(-0.59)	
Quarterly	Coefficient	4.2%	1.23	0.74	0.03	35%
	(T-stat)	(7.31)	(7.72)	(3.74)	(0.45)	

3.3 Post Crisis Crashes and Optionality

As shown in Fig. 1, BTSMOM is profitable during the height of the financial crisis, but it suffers from crashes and sustained drawdowns following market declines when the market volatility is high and the market rebounds swiftly. This characteristic of time series momentum is coherent with the findings of Moskowitz, Ooi, and Pedersen (2012) and has been observed in studies on cross-sectional momentum, such as Daniel and Moskowitz (2014).

In the worst case scenario, as shown in Tab. 1, BTSMOM loses 13.1% in one month and the largest drawdown accumulates to 23.9%. For a general investor, such a large loss can be unbearable and demonstrates the demand for a risk management approach for BTSMOM. Our analysis will continue by investigating the periods when BTSMOM performs at its worst to get a thorough understanding of the strategy's drawdowns in order to potentially mitigate the crash risk and improve the performance of BTSMOM.

Fig.2 plots the daily cumulative excess return of BTSMOM in its two largest drawdown periods of the main sample. These two periods are selected with the purpose to investigate the post crisis crashes that we study more generally in this paper.

The first period is from August 2008 to August 2010. As shown in Fig.2 (A), BTSMOM accumulates a 25.7% return that outperforms the market by 65% from August 2008 to October 2008. However, when the market rebounds sharply in October 2008, BTSMOM experiences a severe drawdown and moves inversely to the market. The market then has another drop followed by a rebound to the upside in November 2008. During this rebound BTSMOM struggles and losses 12.9%. The market continues in a state of high volatility from March 2009 until August 2010 with several periods of drawdowns and sudden rebounds. Two such periods are in March 2009 and July 2009, these periods results in a 33.4% loss for the strategy, underperforming the market by 63.8%.

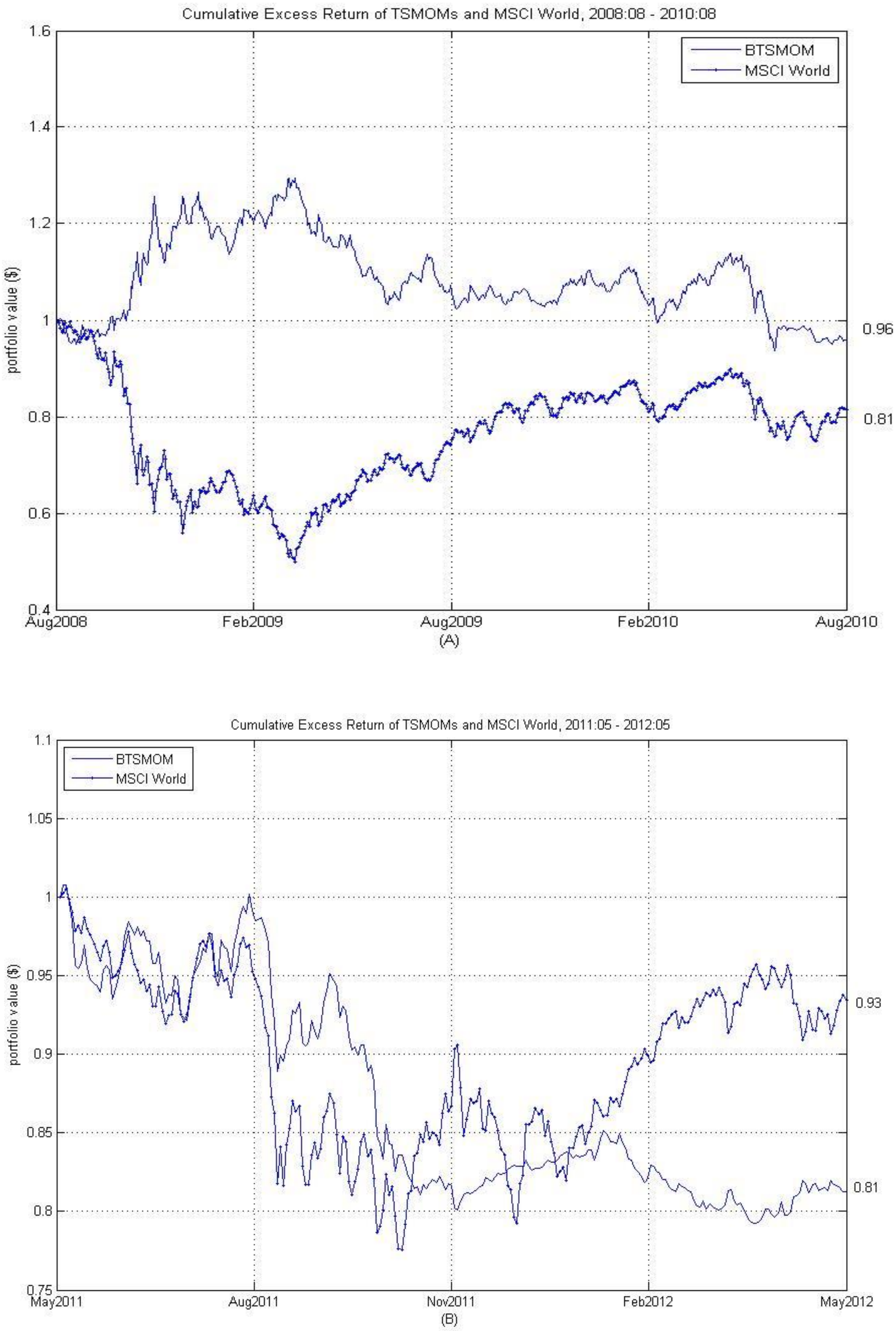

Fig. 2 Crashes of Time Series Momentum

The figure plots the cumulative daily excess return of the basic time series momentum strategy (TSMOM) and MSCI World over two crash periods in the main sample. Fig.2 (A) displays the first crash period 2008:08 to 2010:08, and Fig.2 (B) graphs the second crash period 2011:05 to 2012:05.

A similar pattern is observed when the market rebounds in the second drawdown period 2011:05 to 2012:05, as demonstrated in Fig. 2 (B). From August 2011 to November 2011, the market experienced a very volatile period with a series of dramatic plunges and rebounds. During this period, the return of BTSMOM drops by 20%. A similar period follows although in a milder fashion. Over the entire period, BTSMOM underperforms the market by 12%.

To summarise the key points of post crisis crashes, we observe some option-like behaviour of BTSMOM, similar to what Daniel and Moskowitz (2014) found with regards to cross-sectional momentum. More specifically, the strategy's return pattern behaves like the payoff structure of a short call option during the end of a market decline. When the market has moved close to its bottom position, the strategy will pick up small gains if the market falls further. However, if the market reverses, the strategy will experience large losses. The strategy's vulnerability to sharp market rebounds from a bottom position is due to the fact that the strategy establishes large short positions during the market decline. Combining the information from Fig.2 and Fig.1, we find that most of the largest drawdowns of BTSMOM are associated with this optionality.

We further illustrate BTSMOM's optionality with a set of time series regression on the return of BTSMOM and the market index, as displayed in Tab.3. The explanation is conducted in two steps. We first explain the beta difference in bear markets, and then explain the optionality in beta in post-bear market periods.

To capture the differences in expected return and market beta in market downturns like previous studies (Daniel and Moskowitz, 2014 and Grundy and Martin, 2001), we first construct an ex-ante bear market dummy variable$I_{B,t-1}$, which equals one if the past 12 months market return prior to month t is negative. Compared with the standard CAPM model, equation (4), we observe a striking change in the market beta of BTSMOM and a higher R-square, after adding the bear market indicator $I_{B,t-1}$ to the regression, equation (5), as shown in Tab.3. The market beta of the strategy in a non-bear market state $\beta_{R_m}$is significantly positive, 0.22, meaning it rises with the market's upward trend, while the bear market beta is $\beta_{R_m} + \beta_B = -0.42$, reflecting the positive return generation during negative market conditions.

$$\tilde{R}_{BTSMOM,t} = \alpha_0 + \beta_{R_m}\tilde{R}_{m,t} + \hat{\varepsilon}_t \quad (4)$$

$$\tilde{R}_{BTSMOM,t} = (\alpha_0 + \alpha_B I_{B,t-1}) + (\beta_{R_m} + \beta_B I_{B,t-1})\tilde{R}_{m,t} + \hat{\varepsilon}_t \quad (5)$$

$$\tilde{R}_{BTSMOM,t} = (\alpha_0 + \alpha_B I_{B,t-1}) + (\beta_{R_m} + (\beta_B + \beta_{U,B} I_{U,t}) I_{B,t-1})\tilde{R}_{m,t} + \hat{\varepsilon}_t \quad (6)$$

Equation (6) further introduces an additional term that aims to capture the optionality of time series momentum – the contemporaneous bull market indicator $I_{U,t}$, which equals one if the market return is positive in the month t. Like Henriksson and Merton (1981), we employ a cross term of the ex-ante bear market indicator and the contemporaneous bull market indicator $I_{U,t}I_{B,t-1}\tilde{R}_{m,t}$ in the regression, which attempts to explain the post bottom crashes of the BTSMOM.

The results show that in the condition of a bear market, the down beta (the beta when market falls in the next period) is negative; $\beta_{R_m} + \beta_B = -0.26$. However, given the bear market condition, the up beta (when the market rebounds to the upside in the next period) is even more negative; $\beta_{R_m} + \beta_B + \beta_{U,B} = -0.68$. This means the BTSMOM experiences crashes when the market rebounds, and therefore provides empirical evidence of its option-like behaviours.

Tab. 3 Optionality of Time Series Momentum Return

This table shows the results of three monthly time-series regressions run over the period from 1985:01 – 2015:02. In all regression models, the dependent variable is the monthly return of BTSMOM. The independent variables are a dummy ex-ante indicator for bear markets $I_{B,t-1}$, which equals one if cumulative market excess return over the last 12 months is negative, the excess market return $\tilde{R}_{m,t}$, and a contemporaneous bull market indicator $I_{U,t}$, which equals one if contemporaneous monthly excess return is positive.

	Estimate Coefficients		
	(1)	(2)	(3)
Intercept	1.14	0.80	0.80
	(29.56)	(14.61)	(14.65)
$I_{B,t-1}$		0.60	0.69
		(8.21)	(5.31)
$\tilde{R}_{m,t}$	-0.12	0.22	0.22
	(-3.03)	(4.02)	(4.03)
$I_{B,t-1}\tilde{R}_{m,t}$		-0.60	-0.48
		(-8.25)	(-5.1)
$I_{U,t}I_{B,t-1}\tilde{R}_{m,t}$			-0.42
			(-1.96)
R^2	1.8%	13.8%	14.4%
F-stat	9.16	26.36	20.84
F test P-value	0.00	0.00	0.00

3.4 Market Stress and Momentum Return Prediction

As discussed in 3.3, BTSMOM is subject to crashes when the market volatility is high contemporaneous with market rebounds. However, we find that these extreme returns are not spikes, rather they span over numerous months, such as from March 2009 to June 2009, and September 2011 to Nov 2011, as can be seen in Fig.2. This suggests that there is a possibility to predict the returns of BTSMOM and to optimise the strategy with a timing system.

In terms of predicting the strategy's return, an interpretation of the option-like behaviour, discussed in 3.3, is that the value of such a short call options should be a function of the future market variance. From an empirical aspect, this hypothesis is also coherent with the fact that when crashes occur, the market volatility is usually high.

Tab. 4 Momentum Returns and Estimated Market Variance

This table shows the results of four monthly time-series forecasting models run over the period from 1985:01 – 2015:02. In all models, the dependent variable is the monthly return of BTSMOM. The independent forecast variables are ex-ante estimated market variance$\sigma_{m,t-1}$, constructed by the daily return over the past 6 months before month t, cumulative market excess return over the last 12 months before month t, $\tilde{R}_{m,t-1}$, and the cross term between them $\tilde{R}_{m,t-1}\sigma_{m,t-1}$.

	Estimate Coefficients			
	(1)	(2)	(3)	(4)
Intercept	1.03	1.07	1.04	1.09
	(244.31)	(65.49)	(187.42)	(47.27)
$\sigma_{m,t-1}$	-1.24	-1.96		-4.24
	(-2.75)	(-3.64)		(-2.44)
$\tilde{R}_{m,t-1}$		-0.03		-0.06
		(-2.41)		(-2.5)
$\tilde{R}_{m,t-1}\sigma_{m,t-1}$			-1.82	-0.73
			(-2.96)	(-1.38)
R^2	2.1%	3.6%	2.4%	4.1%
F-stat	7.58	6.73	8.77	5.13
F test P-value	0.01	0.00	0.00	0.00

To assess this hypothesis, we regress BTSMOM's monthly return on the ex-ante estimated market volatility, $\sigma_{m,t-1}$, and the cumulative market return, $\tilde{R}_{m,t-1}$, which replaces the bear market indicator to better capture the return pattern. A cross-term of the ex-ante market volatility and past market return, $\tilde{R}_{m,t-1}\sigma_{m,t-1}$,is employed in the regression as well. The ex-ante estimated market variance is constructed by daily market return data over the last six months, while the cumulative return is calculated as the market return over the past 12months prior to month t. The predicting regression model is shown below:

$$\tilde{R}_{BTSMOM,t} = \alpha_0 + \beta_{R_m}\tilde{R}_{m,t-1} + \beta_{\sigma_m}\sigma_{m,t-1} + \beta_{cross}\tilde{R}_{m,t-1}\sigma_{m,t-1} + \hat{\varepsilon}_t \qquad (7)$$

As presented in Tab. 4, the future market variance and the market state indicator forecast future momentum returns. Columns (1) and (2) show significantly negative coefficients of estimated market variance and past market returns, which implies that BTSMOM generates lower returns in the state of high market stress. Adding the cross term of volatility and the market state indicator, columns (3) and (4) further infer that BTSMOM's returns are extremely poor during highly fluctuating market downturns.

4. Risk-Managed Time Series Momentum

Based on the analysis of BTSMOM in section 3, a risk-managed time series momentum strategy (RTSMOM) is constructed in this section. RTSMOM employs dynamic weights on BTSMOM dependent on the predicted return and volatility of BTSMOM. We observe that RTSMOM has a mitigated crash risk with a lower negative volatility and maximum drawdown. While it has a nearly tripled cumulative return which increases the annualised Sortino ratio by 79.7%.

4.1 Dynamic Weighting System

We start by establishing a dynamic weighting scheme of BTSMOM based on its forecasted return and volatility. The weight changes as the basic strategy is rebalanced on a monthly frequency. Further, we assume no transaction costs and no interest gained from unallocated capital. As explained in detailed in Appendix C, the dynamic strategy aims to maximise the in-sample Sharpe ratio and at time $t-1$ the expected optimal weight in next period, is given by

$$w_{t-1}^{*} = \frac{1}{2\lambda}\frac{\mu_{t-1}}{\sigma_{t-1}^{2}} \tag{8}$$

where w_{t-1}^{*} is estimated at period $t-1$ for the next month, $\mu_{t-1} = \mathbb{E}_{t-1}[\tilde{R}_{BTSMOM,t}]$ is the conditional expected return of BTSMOM for the next month, $\sigma_{t-1}^{2} = \mathbb{E}_{t-1}[(\tilde{R}_{BTSMOM,t} - \mu_{t-1})^{2}]$ is the conditional expected volatility of BTSMOM for the next month, and λ is a time-invariant risk-tolerant scalar that controls the risk exposure of the dynamic portfolio.

As for the return forecasting (μ_{t-1}), equation (7) explained in Section 3.4 enables us to estimate the return for time t at time $t-1$. For each proxy, we regress BTSMOM monthly returns on the market's past 12 months return, the realized market volatility in the past 6 months, and a cross term of them two.

With regards to the volatility prediction, we first fit the well-known GARCH-GJR (Glosten, Jagannathan, and Runkle, 1993) model to the monthly return series of BTSMOM, for the purpose of capturing the asymmetric shocks while maintaining computing simplicity. The process is defined as:

$$R_{BTSMOM,t} = \mu + \varepsilon_t\,, \qquad \varepsilon_t \sim N(0, \sigma_t^2) \tag{9}$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + [\alpha + \gamma I(\varepsilon_{t-1} < 0)]\varepsilon_{t-1}^2 \qquad (10)$$

In order to include the information from the trailing window as well as forward-looking estimates, the final volatility prediction is conducted through a linear combination of the past 6 months realized volatility prior to month t and the predicted volatility from the GARCH-GJR model. A further detailed description is presented in Appendix D.

4.2 Risk-Managed Momentum Performance

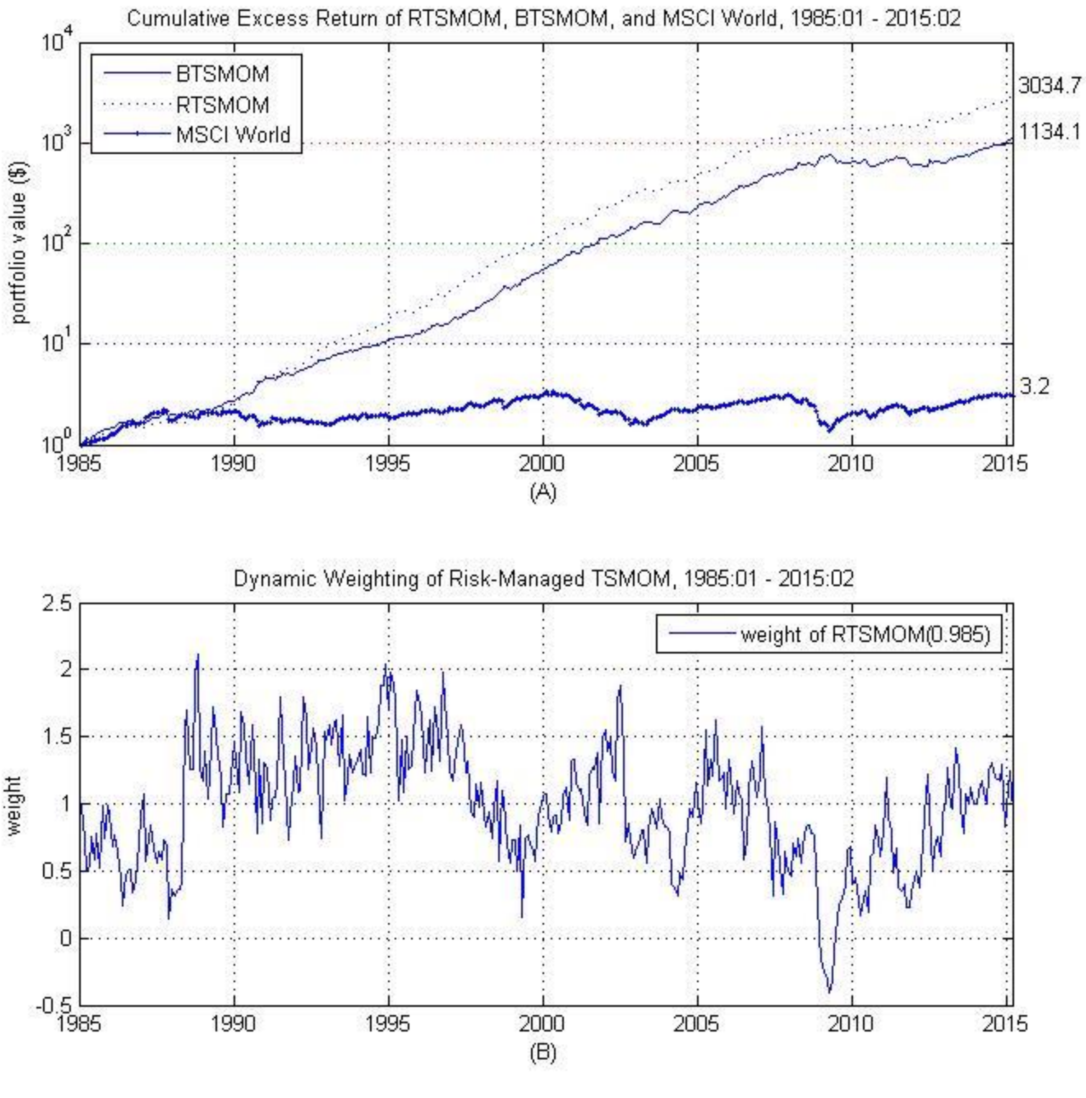

Fig. 3 TSMOMs Performance in Main Sample Period

Fig. 3 (A) plots the cumulative monthly excess return of the risk-managed time series momentum (RTSMOM), basic time series momentum strategy (BTSMOM) and MSCI World in the main sample period, 1985:01 to 2015:02. Fig. 3 (B) displays the weights of RTSMOM over the entire period, with average weight reported in the parentheses. The weight of RTSMOM is changed on a monthly basis, defined by equation (8).

Fig.3 (A) plots the cumulative monthly return of RTSMOM in the time period 1985:01-2015:02. The risk aversion parameter lambda is selected such that the in-sample volatility of RTSMOM is the same as for BTSMOM, 3.5% per month. Over the entire sample period, RTSMOM has a steadier performance compared to BTSMOM, as it experiences smaller drawdowns in the post-crisis periods of 1999, 2009 and 2012, while it nearly triples the profits of BTSMOM over the whole sample period.

Fig. 3 (B) displays the weight loading of RTSMOM over time, which is within the range of -0.4 to 2.1. On average, the weight is 0.985, very close to one. It infers that the high cumulative return of RTSMOM depends on the timing ability rather than employing leverage.

Tab. 5 Monthly Return Statistics of TSMOMs in Main Sample Period

This table summarises the monthly return statistics of BTSMOM, RTSMOM, and the differences between them (changes) over the period of 1985:01 – 2015:02. Sharpe ratio and Sortino ratio are annualised, while all other statistics are reported on a monthly basis. Volatility represents standard deviation and absolute kurtosis is reported. In the third column, all the changes are taken as absolute differences of the two strategies, except for the Sharpe ratio and Sortino ratio which are taken as relative differences.

	BTSMOM	RTSMOM	Changes
Average Return	2.02%	2.30%	0.28%
Minimum Return	-13.1%	-6.1%	7.05%
Maximum Return	15.1%	17.6%	2.52%
Volatility	3.5%	3.5%	0.00%
Negative Volatility	2.3%	1.4%	-0.84%
Skewness	-0.10	0.79	0.90
Kurtosis	5.09	4.71	-0.38
Sharpe ratio	2.00	2.27	13.66%
Sortino ratio	3.06	5.51	79.70%
Maximum Drawdown	23.9%	7.3%	-16.69%
95% VaR	3.4%	2.3%	-1.09%
Expected Shortfall	5.8%	4.1%	-1.70%

To analyse the RTSMOM further than Fig.3 (A), Tab. 5 presents the monthly return statistics of RTSMOM. We observe that the dynamic weighting system effectively mitigates the crashes, while also generating higher return. Compared with BTSMOM, RTSMOM demonstrates:

1) 0.28% higher monthly return on average over the whole sample 362 months, and 2.5% higher maximum return.

2) 0.9% lower negative volatility on average per month, suggesting more positive shocks and skewness given the same amount of volatility by construction.
3) 79.7% higher Sortino ratio, meaning a largely improved negative risk-adjusted return.
4) Mitigated crash risk with 16.6% lower maximum drawdown, 1.1% lower 95% VaR, 1.7% lower expected shortfall, and 7.0% lower minimum monthly return.

Tab. 6 Return decomposition of RTSMOM

Panel A reports the results of time series regression of monthly return and non-overlapping quarterly return of RTSMOM on MSCI World Equity Index, GS Commodity Index, JPM Government Bond Index, and global Fama French factors, SMB, HML, and WML. The Panel A regression sample period is from 1990:01 to 2015:02. Panel B presents the results replacing Fama French factors with Asness, Moskowitz, and Pedersen (2013) Value (VAL) and Momentum (CSMOM) Everywhere factors, which captures the value and cross-sectional momentum globally across asset classes. The Panel B regression sample period is from 1985:01 to 2015:02.

Panel A Regression of RTSMOM on Fama French Global Factors and Indices

		Intercept	MSCI World	SMB	HML	WML	SP GSCI	JPM GBI	R^2
Monthly	Coefficient	2.1%	0.00	-0.14	0.17	0.30	0.00	-0.04	11%
	(T-stat)	(9.91)	(-0.06)	(-1.44)	(1.84)	(5.63)	(-0.11)	(-0.35)	
Quarterly	Coefficient	6.0%	0.10	-0.04	0.31	0.38	-0.06	-0.09	18%
	(T-stat)	(8.31)	(1.16)	(-0.22)	(2.4)	(4.13)	(-1.03)	(-0.53)	

Panel B Regression of RTSMOM on Asness, Moskowitz, and Pedersen (2013) factors

		Intercept	CSMOM	VAL	MSCI World	R^2
Monthly	Coefficient	1.9%	0.89	0.43	-0.04	19%
	(T-stat)	(10.32)	(8.24)	(3.24)	(-0.99)	
Quarterly	Coefficient	5.4%	1.08	0.67	-0.04	26%
	(T-stat)	(8.23)	(5.96)	(2.99)	(-0.58)	

We replicate the return decomposition as in section 3.2, and compare the results to Tab.2, which further proves the outperformance of RTSMOM over BTSMOM. As shown in Tab.6 Panel A, we observe that RTSMOM generates a 2.1% monthly abnormal return on average, 0.4% higher than BTSMOM. On a quarterly basis, RTSMOM outperforms BTSMOM with a 1.1% abnormal return with a 6.0% alpha. As expected, the explanatory ability of all factors decreases, and WML is still the only significant loading in both regression at the 5% significance level. Panel B further investigates the relationship with cross-sectional momentum by using the cross asset factor CSMOM. Once again, the alpha of RTSMOM is significantly positive 1.9% and 5.4% on a monthly and quarterly basis, which is higher than BTSMOM by 0.4% and 1.2% respectively.

4.3 Crash Time Performance

To further investigate the performance of RTSMOM during crash periods, we select the same two periods as in 3.3 to illustrate how the dynamic weighting mitigates the crashes of time series momentum.

As presented in Fig. 4 and Fig. 5, RTSMOM decreases its loading on BTSMOM when the market experiences large volatility with a number of swift rebounds. In the first crash period (Fig. 4), the weight on BTSMOM drops sharply, from 0.78 to -0.4[1] during a seven months period starting from October 2008, while the return of BTSMOM fluctuates. The average weight on BTSMOM is only 26.4% over the whole period. The cumulative return is steady over the time period and outperforms BTSMOM and the market, by 10% and 25% respectively.

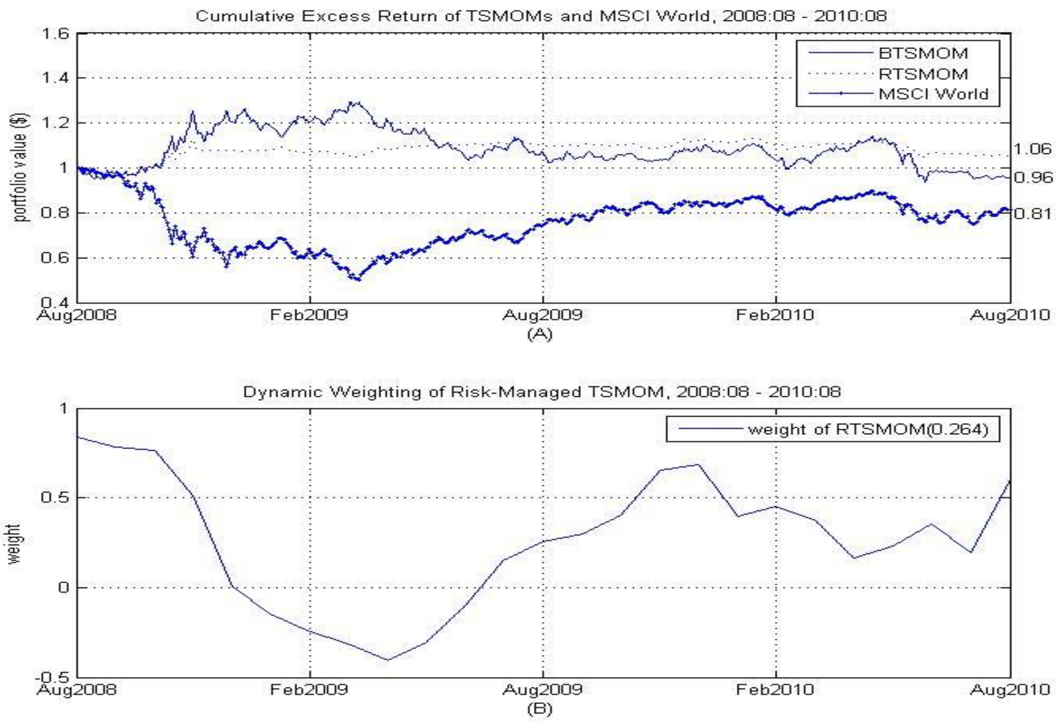

Fig. 4 TSMOMs Performance in First Crash Period

Fig. 4 (A) plots the cumulative daily excess return of the risk-managed time series momentum (RTSMOM), basic time series momentum strategy (BTSMOM) and MSCI World in the first crash period, 2008:08 to 2010:08. Fig. 4 (B) displays the weights of RTSMOM over this period, with average weight reported in the parentheses. The weight of RTSMOM is changed on a monthly basis, defined by equation (8).

[1] A negative weight means that RTSMOM is exposed to a mean reverting strategy, this possibility helps to mitigate downside of the strategy in the crash periods studied. The weight is negative in only 6 out of 362 months over the entire sample, and thus it does not change the strategy significantly.

A similar situation applies to the second crash period, shown in Fig. 5. Over the whole period, from May 2011 to May 2012, the average loading on BTSMOM is only 49%, as the market experiences high volatility and a serial of rebounds. The BTSMOM suffers a loss of 19%, while RTSMOM outperforms its counterpart and the market by 14% and 2% respectively, even though the return of RTSMOM is almost static.

The results above shows that the dynamic weighing system is able to predict the returns and volatility of BTSMOM based on the in-sample historical information. The dynamic weighting reduces crashes and drawdown risks of RTSMOM by keeping a small weight on BTSMOM.

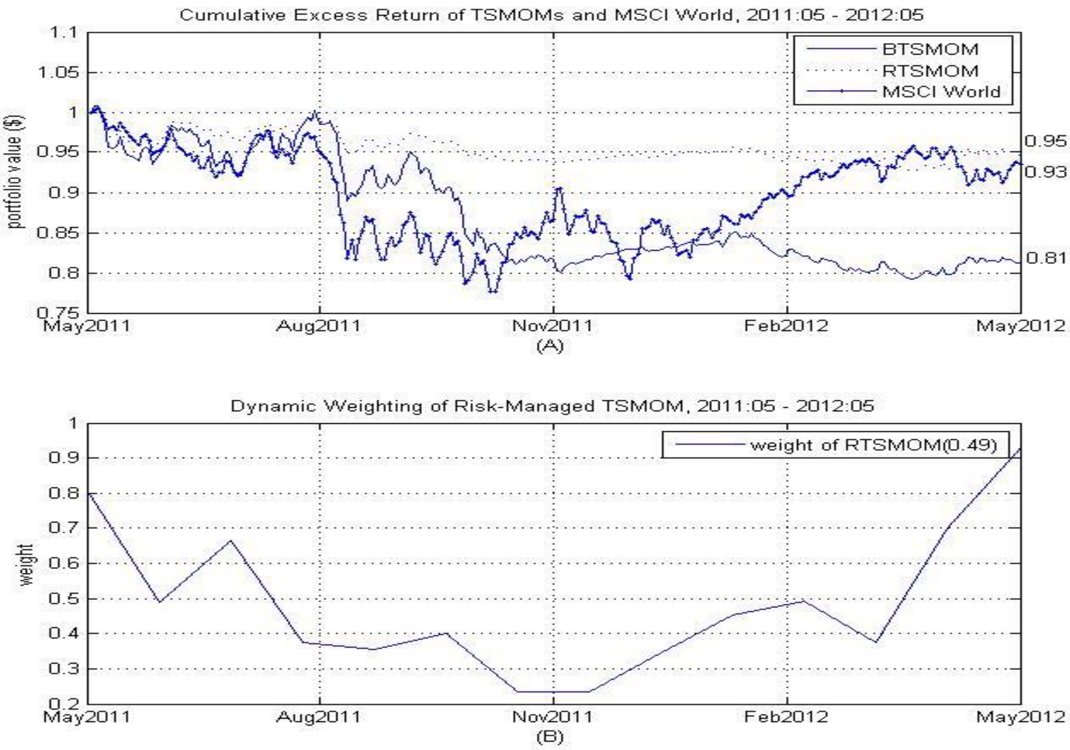

Fig. 5 TSMOMs Performance in Second Crash Period

Fig. 5 (A) plots the cumulative daily excess return of the risk-managed time series momentum (RTSMOM), basic time series momentum strategy (BTSMOM) and MSCI World in the second crash period, 2011:05 to 2012:05. Fig. 5 (B) displays the weights of RTSMOM over this period, with average weight reported in the parentheses. The weight of RTSMOM is changed on a monthly basis, defined by equation (8).

4.4 Sub-Sample Analysis

We split the entire main sample evenly into two sub-periods and conduct the same analysis on each sub-sample as a robustness check. These two sub-sample periods are 1985:01 - 1999:07 and 1999:08 – 2015:02. For each sub-period, the same return and volatility prediction approach is employed, and the risk aversion parameter is selected so that the in-sample volatility of RTSMOM is the same as for BTSMOM in each period.

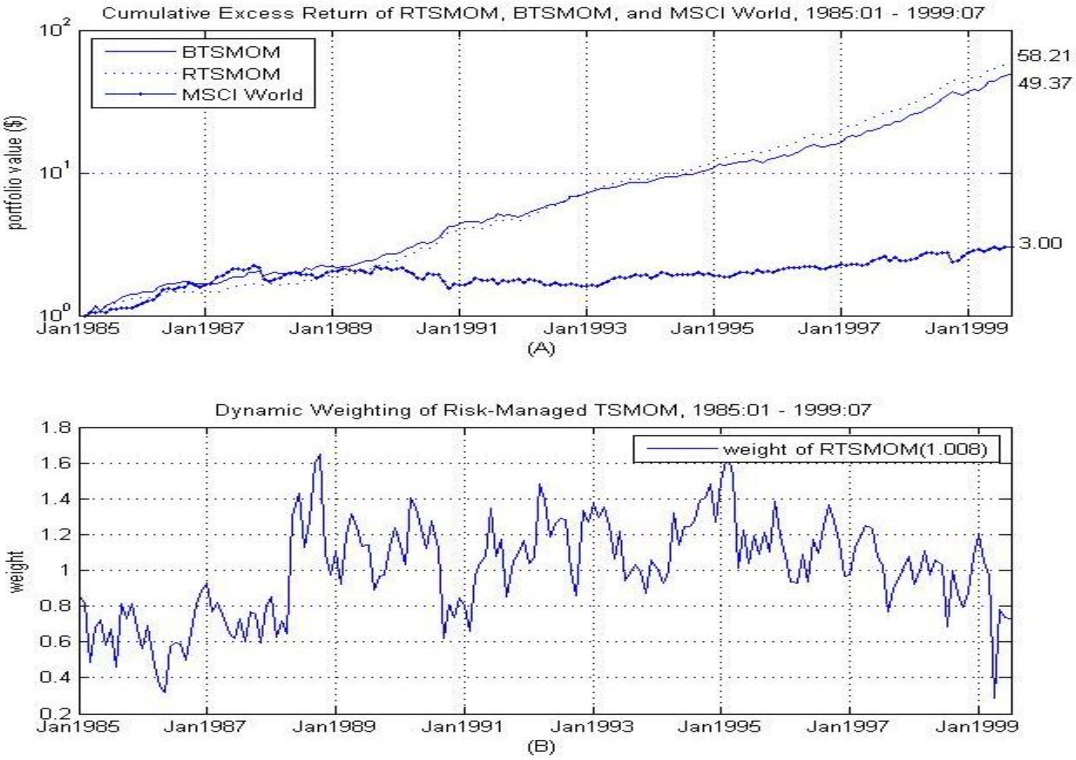

Fig. 6 TSMOMs Performance in First Subsample Period

Fig. 6 (A) plots the cumulative monthly excess return of the risk-managed time series momentum (RTSMOM), basic time series momentum strategy (BTSMOM) and MSCI World in the first subsample, 1985:01 to 1999:07. Fig. 6 (B) displays the weights of RTSMOM over this period, with average weight reported in the parentheses.

Generally, RTSMOM demonstrates similar risk-managing characteristics in each subsample, coherent with the results obtained in the full sample. As shown in Fig.6 and Fig.7, RTSMOM shows a steadier positive return pattern and yields a higher cumulative return, compared with BTSMOM in each subsample. In a further analysis on the monthly return statistics as shown in Tab.7, we observe higher average return, lower negative volatility, more positive skewness in RTSMOM monthly returns compared to BTSMOM. More impressively, the dynamic weighting system mitigates the momentum crashes with a lower maximum drawdown, VaR and expected shortfall, while increases the strategy's annualised Sortino ratio by 37.1% and 62.2% respectively for the first and second subsample.

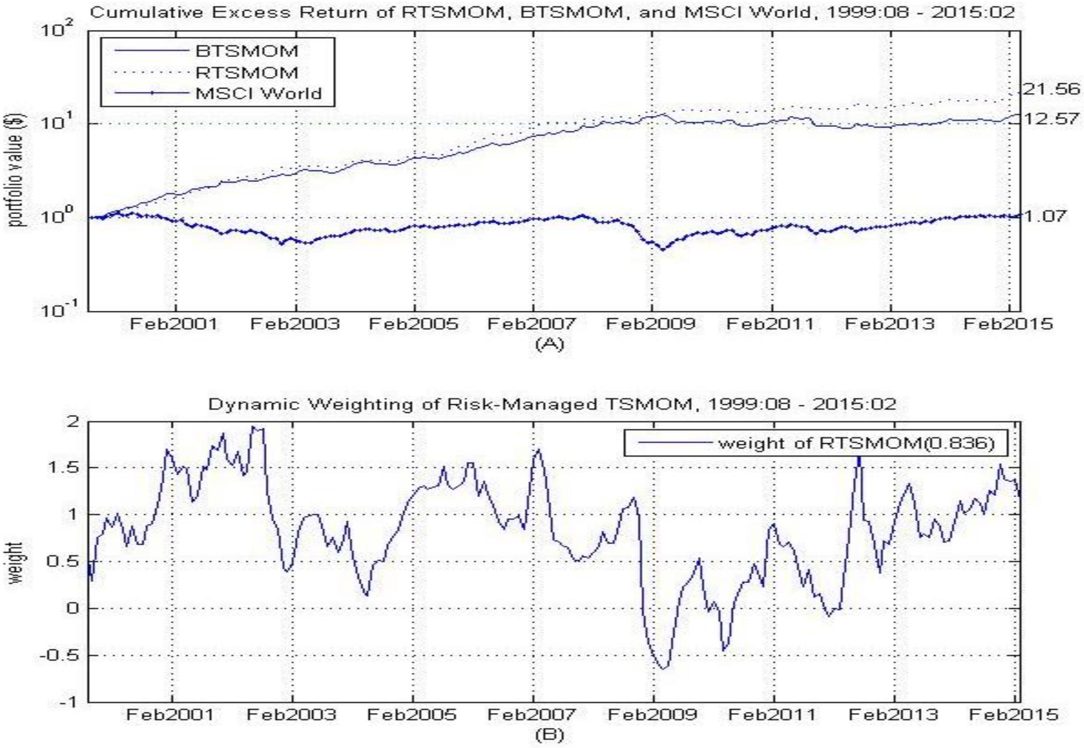

Fig. 7 TSMOMs Performance in Second Subsample Period

Fig. 7 (A) plots the cumulative monthly excess return of the risk-managed time series momentum (RTSMOM), basic time series momentum strategy (BTSMOM) and MSCI World in the second subsample, 1999:08 to 2015:02. Fig. 7 (B) displays the weights of RTSMOM over this period, with average weight reported in the parentheses.

Tab. 7 Monthly Return Statistics of TSMOMs in Subsample Periods

This table summarises the monthly return statistics of BTSMOM, RTSMOM, and the differences between them in the two subsample periods. The two periods are divided evenly; the first subsample is from 1985:01 to 1999:07, and the second subsample is from 1999:08 to 2015:02. The Sharpe ratio and Sortino ratio are annualised, while all other statistics are reported on a monthly basis. Volatility represents standard deviation and absolute kurtosis is reported. In the third column, all the changes are taken as absolute differences of the two strategies, except for the Sharpe ratio and Sortino ratio which are taken as relative differences.

	first half			second half		
	Start 1985:01 End 1999:07			Start 1999:08 End 2015:02		
	BTSMOM	RTSMOM	Changes	BTSMOM	RTSMOM	Changes
Average Return	2.30%	2.40%	0.10%	1.44%	1.73%	0.29%
Minimum Return	-8.2%	-5.8%	2.41%	-14.2%	-11.3%	2.91%
Maximum Return	14.9%	12.7%	-2.18%	13.4%	22.6%	9.22%
Volatility	3.1%	3.1%	0.00%	4.0%	4.0%	0.00%
Negative Volatility	1.9%	1.5%	-0.47%	2.5%	1.8%	-0.64%
Skewness	0.11	0.35	0.24	-0.18	1.27	1.44
Kurtosis	4.98	3.69	-1.29	4.47	7.83	3.35
Sharpe ratio	2.59	2.69	4.2%	1.26	1.52	20.1%
Sortino ratio	4.11	5.64	37.1%	2.03	3.30	62.2%
Maximum Drawdown	8.2%	5.8%	-2.41%	28.5%	12.7%	-15.73%
95% VaR	2.4%	2.4%	-0.04%	4.1%	3.0%	-1.07%
Expected Shortfall	4.4%	3.7%	-0.71%	7.2%	5.1%	-2.06%

4.5 Back-Testing

In this section, the robustness of the dynamic weighting is further checked by conducting a back-test on the data prior to 1985. The same dynamic weighting approach, as used on the main sample 1985-2015, is used on the period 1973:09 to 1984:12. The return and volatility of BTSMOM is predicted, and then used to optimise the in-sample Sharpe ratio with the risk parameter set so that RTSMOM has the same volatility as the basic strategy.

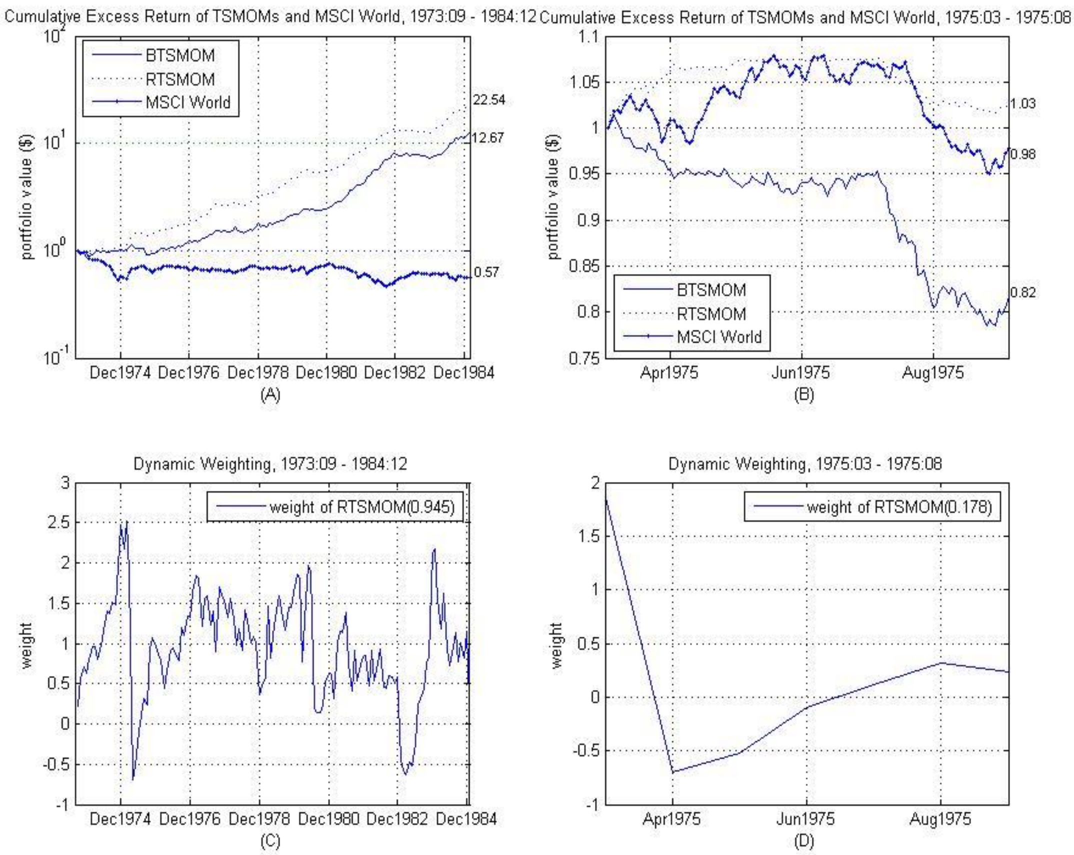

Fig. 8 TSMOMs Performance in Back-Test

Fig. 8 (A) plots the cumulative monthly excess return of the risk-managed time series momentum (RTSMOM), basic time series momentum strategy (BTSMOM) and MSCI World in the back-testing period, 1973:09 to 1984:12, while Fig. 8 (B) graphs the cumulative daily excess return of three portfolio in the back-testing crash period 1975:03 – 1975:08. Fig. 8 (C) and (D) display the weights of RTSMOM over the entire back-testing period and back-testing crash period respectively, with average weight reported in the parentheses of (C) and (D). The weight of RTSMOM is changed on a monthly basis, defined by equation (8).

In this back-test, the RTSMOM strategy performs similarly to its performance in the main sample period. As displayed in Fig.8 (A) (C), RTSMOM demonstrates a robust timing ability and generates a steadier return with a 9.87 higher cumulative return (per one unit of initial capital), compared with BTSMOM.

Tab.8 lists the monthly return statistics in detail: Compared to BTSMOM, RTSMOM monthly return is increased by 0.43%, while the negative volatility is reduced from 2.4% to 1.5%. As a result, the annualised Sortino ratio increases by 92.4%. The dynamic weighting approach effectively lowers the crash risks with a 6.94% lower maximum drawdown, 0.52% VaR at 95% percentile, and 2.3% expected tail loss. The skewness of RTSMOM turns from -0.07 to 0.54, even though the volatility is the same, as the negative tail risk decreases.

Tab. 8 Monthly Return Statistics of TSMOMs in Back-Test

This table summarises the monthly return statistics of BTSMOM, RTSMOM, and the differences between them in the back-testing period from 1973:09 to 1984:12. The Sharpe ratio and Sortino ratio are annualised, while all other statistics are reported on a monthly basis. Volatility represents standard deviation and absolute kurtosis is reported. In the third column, all the changes are taken as absolute differences of the two strategies, except for the Sharpe ratio and Sortino ratio which are taken as relative differences.

	BTSMOM	RTSMOM	Changes
Average Return	1.95%	2.38%	0.43%
Minimum Return	-12.7%	-6.2%	6.49%
Maximum Return	12.8%	12.4%	-0.43%
Volatility	4.1%	4.1%	0.00%
Negative Volatility	2.4%	1.5%	-0.88%
Skewness	-0.07	0.54	0.61
Kurtosis	3.64	2.73	-0.91
Sharpe ratio	1.64	1.99	21.9%
Sortino ratio	2.82	5.43	92.4%
Maximum Drawdown	19.2%	12.2%	-6.94%
95% VaR	4.0%	3.5%	-0.52%
Expected Shortfall	6.9%	4.6%	-2.30%

To further investigate the crash periods, the performances of the two strategies are presented in Fig. 8 (B) and (D). Over the drawdown period from March 1975 to August 1975, BTSMOM cumulative return declines from 1.2% to -18% due to high market volatility and a serial of market rebounds. In the same period, RTSMOM swiftly adjusts its weight on BTSMOM from the leveraged weight of 180% to only a 17.8% weight on average over the crash period. The dynamic weighting approach reduces crashes and tail losses, though its return is quite flat during periods of high market volatility. Over the whole crash period, RTSMOM has 21% higher return than BTSMOM, which empirically supports the risk managing efficacy of dynamic weighting system in the back-test period.

4.6 Cross Asset Analysis

Having shown the robustness in the subsample analysis and back-test, we examine the efficacy of the dynamic weighting approach further in each individual asset class to check its robustness.

First we categorise each contract into four asset classes over the whole main sample and back-test periods, from January 1972 to February 2015. For each asset class, BTSMOM starts once there are ten contracts available to allow for sufficient diversification, and RTSMOM is initiated eight months after the start of BTSMOM.

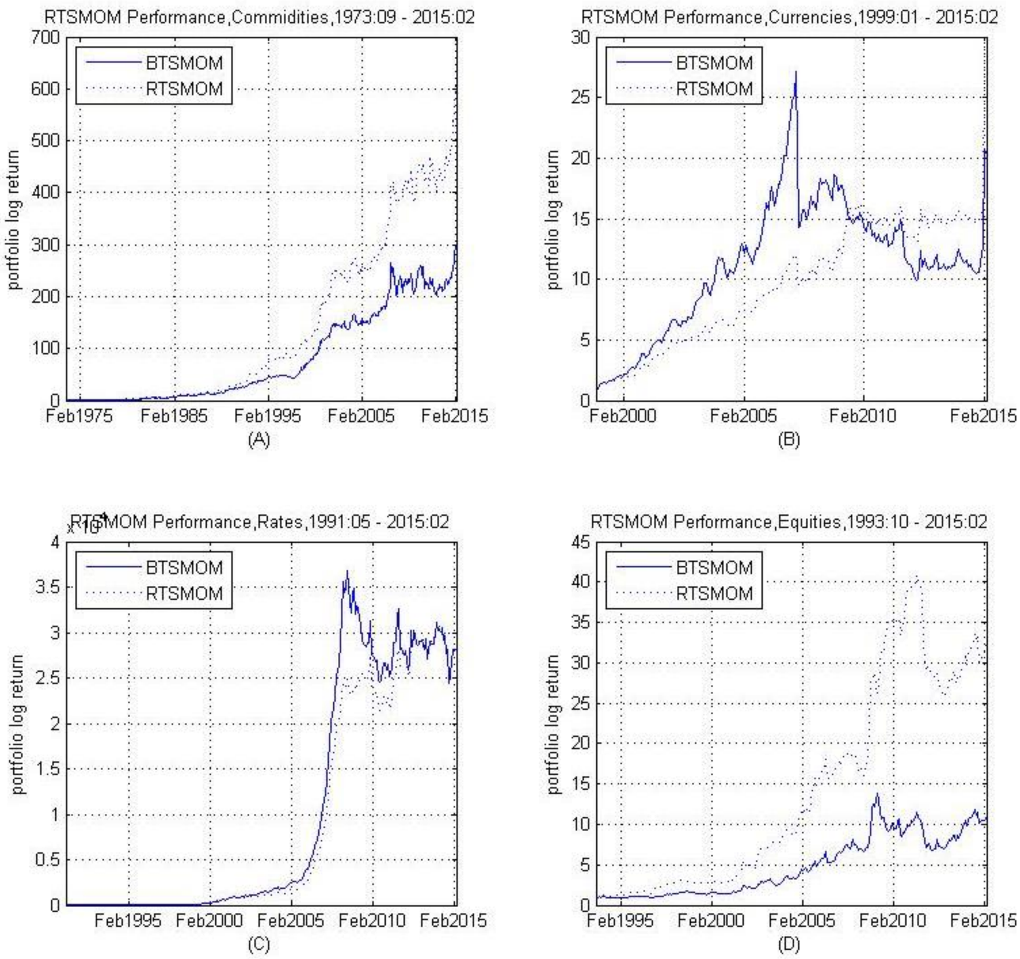

Fig. 9 TSMOMs Performance in Cross Asset Analysis

Fig.9 (A), (B), (C), and (D) plots the cumulative monthly excess return of the risk-managed time series momentum (RTSMOM) and basic time series momentum strategy (BTSMOM) from investments in individual asset classes of commodities, currencies, rates, and equities, respectively. For each asset class, the portfolio initiates on the month when it has ten individual assets - the sample period for each asset class is 1973:09-2015:02, 1999:01-2015:02, 1991:05-2015:02 and 1993:10-2015:02 respectively.

Fig. 9 shows the log return (as the magnitude of simple return for some asset classes is too large and exceeds the limit of our computing program) of the four asset classes. Compared to BTSMOM, RTSMOM shows an improved performance in all asset classes with mitigated crash risk. Even though the return of RTSMOM grows slower than for the basic strategy during some highly volatile time periods for the currency and equity asset classes, the strategy cumulates a higher return over time.

To analyse the return of RTSMOM further, Tab. 9 presents the statistics of monthly return series for four asset classes. For all asset classes except rates, the results is largely consistent with the results from previous section - RTSMOM has a higher average monthly return, a lower negative volatility, which consequently boosts the Sortino ratios by more than 30%. Apart from that, dynamic weighting reduces the crash risks with a lower maximum drawdown, 95% VaR and expected shortfall.

For the asset class rates, RTSMOM still has a mitigated crash risk with a 11.07% lower maximum drawdown, 0.8% lower 95% VaR, and 0.63% lower expected shortfall, even though the dynamic weighting fails to raise the average return and decrease the negative volatility effectively. This is explained by the fact that the basic rates strategy seems to be highly fluctuating but less than 50% (3.1% out of 7.2%) of its volatility is due to negative shocks, consistent with its positive skewness. This means that the maximum Sharpe ratio weighing system will avoid the high volatility periods when the basic strategy actually is more likely to benefit from right tail events. However, the dynamic weighting still mitigates the crash risk of asset class rates as defined, and provides a slightly higher Sortino ratio.

Overall, the findings are consistent from the cross asset analysis, subsample analysis, and back-testing study, and further support the robustness of the risk-managing system of RTSMOM. Compared to BTSMOM, RTSMOM has a mitigated crash risk and a steadier return pattern, which raises the risk-return indicator Sortino ratio.

Tab. 9 Monthly Return Statistics of TSMOMs in Cross Asset Analysis

This table summarises the monthly return statistics of BTSMOM, RTSMOM, and the differences between them for the four asset classes, commodities, currencies, rates, and equities. For each asset class, the monthly return starts on the month when it has ten individual assets. The Sharpe ratio and Sortino ratio are annualised, while all other statistics are reported on a monthly basis. Volatility represents standard deviation and absolute kurtosis is reported. In the third column, all the changes are taken as absolute differences of the two strategies, except for the Sharpe ratio and Sortino ratio which are taken as relative differences.

	Commodities			Currencies		
	Start 1973:09 End 2015:02			Start 1999:01 End 2015:02		
	BTSMOM	RTSMOM	Changes	BTSMOM	RTSMOM	Changes
Average Return	1.22%	1.36%	0.14%	1.90%	2.02%	0.12%
Minimum Return	-16.2%	-14.0%	2.15%	-47.5%	-21.7%	25.76%
Maximum Return	19.4%	19.4%	0.01%	61.2%	87.5%	26.33%
Volatility	4.0%	4.0%	0.00%	8.2%	8.2%	0.00%
Negative Volatility	2.5%	2.1%	-0.38%	5.8%	3.3%	-2.47%
Skewness	-0.04	0.67	0.70	1.11	6.18	5.08
Kurtosis	4.88	5.49	0.61	22.96	64.03	41.07
Sharpe ratio	1.06	1.18	11.58%	0.80	0.85	6.49%
Sortino ratio	1.68	2.21	31.44%	1.14	2.11	85.83%
Maximum Drawdown	26.4%	23.0%	-3.40%	63.3%	21.7%	-41.63%
95% VaR	5.2%	4.4%	-0.80%	6.6%	4.8%	-1.85%
Expected Shortfall	7.4%	6.5%	-0.96%	13.6%	8.5%	-5.16%

	Rates			Equities		
	Start 1991:05 End 2015:02			Start 1993:10 End 2015:02		
	BTSMOM	RTSMOM	Changes	BTSMOM	RTSMOM	Changes
Average Return	3.87%	3.87%	0.00%	1.15%	1.57%	0.42%
Minimum Return	-13.9%	-14.3%	-0.45%	-20.6%	-17.1%	3.48%
Maximum Return	74.8%	79.6%	4.81%	33.2%	49.4%	16.23%
Volatility	7.2%	7.2%	0.00%	6.6%	6.6%	0.00%
Negative Volatility	3.1%	3.1%	-0.07%	4.0%	3.2%	-0.73%
Skewness	3.29	4.11	0.82	0.58	2.88	2.30
Kurtosis	33.82	43.52	9.70	6.20	19.38	13.17
Sharpe ratio	1.85	1.85	-0.11%	0.60	0.82	36.83%
Sortino ratio	4.28	4.38	2.26%	1.00	1.68	67.78%
Maximum Drawdown	33.8%	22.7%	-11.07%	51.1%	36.7%	-14.44%
95% VaR	5.4%	4.6%	-0.80%	9.2%	5.6%	-3.68%
Expected Shortfall	8.6%	8.0%	-0.63%	12.4%	9.7%	-2.68%

5 Conclusion

The purpose of this paper has been to investigate whether time-varying risk leads to the crashes of time series momentum, in similar fashion as for cross-sectional momentum documented by Daniel and Moskowitz (2014). Furthermore, we also set out to explore a systematic approach to mitigate the crashes of time series momentum based on our findings. In order to examine this back-adjusted futures data for 118 individual contracts have been used, all futures data has been received from ACIES Asset Management.

We find that a basic time series momentum strategy (BTSMOM) exhibits abnormal positive returns which provide a challenge to standard asset pricing models. However, BTSMOM experiences several large drawdowns in periods after large market declines, when the volatility is high in combination with sharp market reversals to the upside. Such characteristics are coherent with the findings of Moskowitz, Ooi, and Pedersen (2012) and have also been observed in studies on cross-sectional momentum, for example Daniel and Moskowitz (2014).

In a detailed analysis of the crash periods of BTSMOM we observe an option-like behaviour and, more specifically, during the end of market declines the strategy's return pattern behaves like the payoff structure of a short call option. This characteristic can be explained by the strategy's difference in up and down beta conditioned on bear market state. Such return optionality is statistically significant, and therefore enables a model for predicting the future return of the time series momentum strategy.

In order to curtail the crashes of BTSMOM, a risk-managed time series momentum strategy (RTSMOM) is constructed based on a dynamic weighting system that maximises the in-sample Sharpe ratio using predicted strategy return and volatility. The dynamic weighting mechanism mitigates the crashes and enhances the return of RTSMOM, which leads to a substantial increase in the Sortino ratio.

The robustness checks conducted on subsamples, back-testing and individual asset class analysis indicate that the dynamic weighting approach improves the time series momentum strategy from a risk-management perspective. The results support the risk-managing efficacy of RTSMOM empirically, even though RTSMOM fails to increase the average monthly return and Sortino ratio substantially for the asset class rates.

Our suggestion for further research is based on the limitations and the findings of our study. We have chosen not to account for transaction costs in this study as this would still only be an estimate without further expertise on how real futures trading works. This would obviously be of interest for further research to explore in detail the main challenges, such as capacity and liquidity constraints, and costs for implementing a time series momentum based strategy in a real trading setting.

Given the enhanced performance of RTSMOM through the dynamic weighting mechanism it would be of interest to further study in what way the dynamic weighting affects the performance of time series momentum. An obvious extension would be to try to break done the value added between the prediction of return and volatility.

As discussed in section 4.6, the dynamic weighting mechanism fails to materially improve the performance of the asset class rates. We argue that this is due to rates relativity low amount of negative volatility in relation to total volatility. Optimizing the Sharpe ratio will target total volatility and makes no distinction between positive and negative volatility but from an investor perspective, there is obviously a huge difference. An interesting extension to our study would be to construct the dynamic weighting scheme so it maximizes the Sortino ratio rather than the Sharpe ratio.

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Appendix

A List of Futures Contracts

Equity indices: SPI 200 (Australia), FTSE 100 (UK), Hang Seng (Index and Mini, Hong Kong), S&P 500 (US, Index and E-mini), Nikkei 225 (Japan, index and combined), CAC 40 (France), TOPIX (Index and Mini, Japan), FTSE/JSE Top 40 (South Africa), S&P 400 MidCap (US), SMI (Switzerland), Russell 2000 (Index and Mini, US), DAX (Germany), OMXS30 (Sweden), IBEX 35 (Spain), EOE (Netherlands), MIB FTSE (Index and Mini, Italy), MSCI Taiwan (Taiwan), MSCI Singapore (Singapore), Nasdaq 100 (US), KLSE Composite (Malaysia), Euro STOXX 50 (Europe), OBX (Norway), S&P/TSX 60 (Canada), KOSPI 200 (South Korea), S&P CNX Nifty (India), Dow Jones Industrial Average (US), FTSE/ASE 20 (Greece), S&P 600 SmallCap (Index and E-mini, US), Hang Seng China Enterprise Index (Hong Kong), CBOE Volatility Index (volatility of S&P 500 index).

Rates: US T-bond, US 2 year T-note, US 5 year T-note, US 10 year T-note, Australian 3 month Bill, Australian 3 year Bond, Australian 10 year Bond, Great Britain Long Gilt (8.75-13 years), Great Britain 3 month, Eurodollar 3 month, Canada 3 month Banker's Acceptance Rate, Canadian 10 year Bond, Euroyen 3 month, Japanese 10 year Bond (Index and Mini), Euribor 3 month, Swiss 10 year Bond, German Bund Euro, German Bobl Euro, German Schatz Euro, Korean 3 year T-bond, Italian Long-Term Bond, French 10 year OAT Euro.

Currencies: CAD/USD, YEN/USD, CHF/USD, GBP/USD, EUR/USD, AUD/USD, KRW/USD, EUR/CHF, EUR/YEN, EUR/GBP, Mexican Peso Index, New Zeeland Dollar Index, US Dollar Index.

Commodities: Wheat, Soybean Meal, Soybeans, Corn, Oat, Soybean Oil, Silver, Platinum (COMEX and Tokyo), Frozen Orange Juice, Cattle Live, Cocoa, Kansas City Wheat, Copper HG, Sugar #11, Robusta Coffee, Lumber, Orange Juice, Cotton #2, Lean Hogs, Cocoa LCE, Cattle Feeder, Coffee, Wheat Spring MGE, Gold (COMEX and Tokyo, Rapeseed (WCE and ICE), Palladium, NY Harbour ULSD, Crude Oil Light, Gasoline Reformulated Blendstock, Gas Oil (Combined), Rough Rice, White Sugar #5, Natural Gas Henry Hub (NYMEX), Brent Crude Combined, Rubber, Palm Oil Crude, Natural Gas (ICE), Wheat Milling, White Maize, Gasoline, EUA Emissions, Crude Oil WTI.

B Futures Price Back Adjustment Method

Futures contracts have a limited time span due to its expiry date. So when trading a strategy on futures over a time period you actually trade a number of contracts. When to close a positon in one contract and open a position in a later contract, i.e. when to "roll over", is generally given by a set rule. This can be decided by open interest, volume or a combination of them both. In this thesis all calculations are based on the fact that the strategy is invested in the contract with the highest open interest. The first step to create a return series for a futures contract is to concatenate the prices of the traded contracts.

To illustrate the adjustment process we assume that the live span of a front contract is$1,2 \cdots t, \cdots T+1$, the rollover date is $T+1$, and the closing price of the front contract at time t is $p(t)$. Further assume the closing price of the back contract at time $T+1$ is $q(T+1)$, so at the rollover date $T+1$ the strategy closes its position (i.e. sells) at the price $p(T+1)$ and establish a new position at the price $q(T+1)$. The strategy's actual return between period t and $T+1$ is given by:

$$\frac{\big(p(T+1)-p(t)\big)}{p(t)}$$

However, the return calculation based on the unadjusted time series will mistakenly account for the roll-over gap as well, which shows that the return is:

$$\frac{q(T+1)-p(t)}{p(t)}$$

In order to adjust for the price gap between $q(T+1)$ and $p(T+1)$ to make the return calculations correct, each price $p(t)$ on every date t on and before date T is adjusted as follows:

$$p(t)^{adj} \ = \frac{q(T+1)}{p(T+1)} * p(t)$$

Alternatively, it can be expressed as:

$$p^{adj}(t) \ = rollover\ ratio * p(t) \tag{9}$$

$$rollover\ ratio = \frac{q(T+1)}{p(T+1)}$$

This leads to the return calculation on adjusted time series equals to strategy actual return at time $T+1$:

$$\frac{q(T+1)-\frac{q(T+1)}{p(T+1)}*p(t)}{\frac{q(T+1)}{p(T+1)}*p(T)}=\frac{\big(p(T+1)-p(t)\big)}{p(T)}$$

To back-adjust a contract other than the adjacent contract the process is similar to the one period back-adjustment is shown above. The difference is that the unadjusted price at time t - $p_n(t)$ - is adjusted by the cumulative rollover ratio from contract *2* to contract *n* given by

$$\prod_{i=2}^{n}\frac{q_i(T+1)}{p_i(T+1)}$$

Where q_i and p_i are front contract and new contract, respectively, for contract i, and $p_i(T+1)$ and $q_i(T+1)$ are their unadjusted prices at the roll-over day $T+1$.
Thus, the adjusted price for contract *n* is then given by:

$$p_n^{adj}(t)=p_n(t)\prod_{i=2}^{n}\frac{q_i(T+1)}{p_i(T+1)}$$

C Maximum Sharpe Ratio Optimisation

This appendix explains the maximum Sharpe ratio optimisation approach employed in the dynamic weighting presented in Section 4.1.
The assumed setting is that in the discrete period from 1,2,…,T, investors can allocate capital in two assets, a risky asset, the basic time series strategy in this study, and a risk free asset. No trading cost and no interest for risk free asset are assumed. Further assume over the period t to $t+1$, the excess return on risky asset $\tilde{r}_{t+1}$ is distributed normally, which a conditional mean μ_t and conditional variance σ_t^2. That is:

$$\mu_t=\mathbb{E}_t[\tilde{r}_{t+1}] \quad \text{and} \quad \sigma_t^2=\mathbb{E}_t[(\tilde{r}_{t+1}-\mu_t)^2] \tag{11}$$

Suppose investors know μ_t and σ_t^2 at $t=0$ for $t\in\{0,\ldots,T-1\}$, and investors aim to maximise the full period Sharpe ratio of a managed portfolio by allocating a fraction w_t of portfolio value in the risky asset and a fraction $1-w_t$ in the risk-free asset at the beginning of each period $t\in\{0,\ldots,T-1\}$. Thus, at time t the expected return and variance of the portfolio in next period, $t+1$ is:

$$\tilde{r}_{p,t+1} = w_t \tilde{r}_{t+1} \sim N(w_t \mu_t\,, w_t^2 \sigma_t^2)$$

The Sharpe ratio over T periods is:

$$Sharpe\ Ratio = \frac{\mathbb{E}[\frac{1}{T}\sum_{t=1}^{T} \tilde{r}_{p,t}]}{\sqrt{\mathbb{E}[\frac{1}{T}\sum_{t=1}^{T}(\tilde{r}_{p,t} - \overline{r_p})^2]}}$$

Given assumptions above, to maximise Sharpe ratio is equivalent to solving the optimisation problem as follows:

$$\max_{w_0,\ldots,w_{t-1}} \mathbb{E}[\frac{1}{T}\sum_{t=1}^{T} \tilde{r}_{p,t}]$$

$$s.t.\ \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}\left(\tilde{r}_{p,t} - \overline{r_p}\right)^2\right] = \sigma_p^2$$

If the period length is sufficiently short, then$\mathbb{E}_t\left[\left(\tilde{r}_{p,t} - \bar{r}\right)^2\right] \approx\ \sigma_t^2 = \mathbb{E}_t[(\tilde{r}_{t+1} - \mu_t)^2]$.

With this approximation, it arrives the Lagrangain as below:

$$\max_{w_0,\ldots,w_{t-1}} \mathcal{L} \equiv \max_{w_t}\left(\frac{1}{T}\sum_{t=0}^{T-1} w_t \tilde{r}_{t+1}\right) - \lambda(\frac{1}{T}\sum_{t=0}^{T-1} \sigma_p^2)$$

Further substitute in the conditional expectation from equation (11), it gives the following function:

$$\max_{w_0,\ldots,w_{t-1}} \mathcal{L} \equiv \max_{w_t}\left(\frac{1}{T}\sum_{t=0}^{T-1} w_t \mu_t\right) - \lambda(\frac{1}{T}\sum_{t=0}^{T-1} w_t^2 \sigma_t^2) \quad (12)$$

Take the first order conditions of (12), it arrives the optimising condition as shown below:

$$\frac{\partial \mathcal{L}}{\partial w_t}|_{w_t = w_t^*} = \frac{1}{T}\sum_{t=1}^{T} \mu_t - 2\lambda w_t^* \sigma_t^2 = 0 \quad \forall t\ \in \{0, \ldots, T-1\}$$

It suggests the optimal fraction allocated in risky asset is w_t^* given by:

$$w_t^* = \frac{1}{2\lambda}\frac{\mu_t}{\sigma_t^2}$$

D Volatility Forecasting of Time Series Momentum

This appendix explains the estimated ex-ante volatility needed in the dynamic weighting system explained in Section 4.1.

As explained in Section 4.1, the GARCH-GJR model (Glosten, Jagannathan, and Runkle, 1993) is firstly used to fit the monthly return series of the basic time series momentum strategy (BTSMOM) as shown below:

$$R_{BTSMOM,t} = \mu + \varepsilon_t\ , \qquad \varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + [\alpha + \gamma I(\varepsilon_{t-1} < 0)]\varepsilon_{t-1}^2$$

To combine the predicted information from GARCH process and existing information, then, we conduct a predicting regression of realised monthly volatility at month t on estimated monthly volatility of month t from GARCH-GJR model ($\hat{\sigma}_{GARCH,t}$) and
R^2
realised volatility at $t-1$ calculated from the past 6 month daily return ($\hat{\sigma}_{126,t-1}$). The results are displayed below:

	Intercept	$\hat{\sigma}_{GARCH,t}$	$\hat{\sigma}_{126,t-1}$	R^2
Coefficient	0.013	0.310	0.191	16%
(T-stat)	(6.34)	(5.09)	(2.40)	

Finally, the fitted estimate $\widehat{\sigma_t}$, together with predicted return, is used as an input into the dynamic weighting approach as shown by equation (8).