# Volatility of Volatility – The Uncertainties of Risks

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Stockholm School of Economics: Master Thesis in Finance

Spring 2015

## Abstract

This paper is an attempt to explore the characteristics of volatility of volatility on the aggregate level and investigate its role in the pricing of equity assets. Several measures of volatility of volatility for the S&P 500 index are elaborated and investigated in this study; realized, parametrized and implied. We explain differences between the measures and the uncertainties that are tied to the volatility of volatility. The measures are used in time series regression analyses on portfolios from which cross-sectional studies are undertaken. Here, the interplay with common risk factors for equities receives special attention. It is shown that volatility of volatility has similar effects in pricing as short-term reversal and momentum. Also, there is a lag between market downturns and spikes in volatility of volatility as the latter awaits volatility to revert. Finally, the numerical and qualitative findings are exploited for stocks in a long-short trading strategy, which in its simplified form, beats a hedge fund index and the S&P 500 thanks to its good performance during market turmoil. The main findings of this paper are the nature of VVOL as a source of hedging and the special characteristics of the VVIX index, probably due its inherent variance risk premium.

**Keywords: Empirical Asset Pricing, Volatility of Volatility, Volatility, Risk factors in Asset Pricing, Long-Short Trading Strategy**

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Date: 05/18/2015

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# Acknowledgements

We would like to thank Michael Halling, our beloved supervisor, for always having been ready to discuss the course of our thesis and for having been an inexhaustible spring of ideas and constructive suggestions.

*To Hsu, for devoted support.*

*To Ana, for having waited one last spring.*

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# 1 Introduction

Volatility plays a vital role in the investing activities of market participants. In essence, it is generated by these very activities and it is well known that trading volume has a positive relation with the market volatility. From the laws of asset pricing, higher risk is compensated by higher return. Investors are always faced with the dilemma of seeking the best risk-return tradeoff that fits their risk tolerance level. On certain occasions, investors might want to find ways to hedge for volatility in order to protect themselves against risks. Understanding market risk is one of the most essential things for investors if they are to achieve their desired results.

The recent price developments in the crude oil market and the favorable environment this volatility has created for some of the quantitative global macro funds is also part of the inspiration to investigate the mechanisms behind volatility. Some of the most volatile financial markets are commodity markets. The reason, especially in the crude oil market, can be partly attributed to its derivatives market, which, for the needs of hedging and speculation, has grown to become 14 times the size of the physical market (Ewing and Malik, 2009).

Volatility can be quantified in many ways, the two most common ones being realized past volatility and implied volatility. The different paths that can be taken make up a manifold of starting points and directions when creating a measure for the volatility of volatility. In the option markets, the CBOE has launched an implied volatility index, the VIX, commonly referred to as the "fear index" by mainstream media. The procedure of creating a volatility index through a series of puts and calls (Neuberger, 1994) has become standard in the financial industry. The market data provider SIX group quotes a Swedish volatility index, which was replicated in a master thesis at the Stockholm School of Economics by Dahlman and Wallmark (2007).

As volatility takes on a more prominent role in the sober, risk-conscious post-2008 financial community, the inadequacies caused by the uncertainties of volatility, we believe, will receive more attention. For example, regulation on capital requirements calls for accurate Value-at-Risk estimation models. For these models, volatility is key. This volatility, however, can only be based on historical data, even if bootstrapped and simulated. To get an adequate variance risk premium for modeled volatilities that are allowed to follow stochastic processes, volatility of volatility is fundamental. Thus volatility of volatility is crucial for precise quantification of tail risk.

In the financial markets, the volatility of volatility has been directly tradable ever since the Chicago Board Options Exchange (CBOE) first launched options on the VIX in 2006. Despite worries about the liquidity for these options, trading volume increased exponentially, reaching a turnover of 1 million contracts per day in 2011 (CBOE). The size of this market motivated the creation of a new index of the implied volatility of the VIX. This index is the VVIX, which has quotes dating back to 2006. VVIX has become a straightforward measure for investors to track and visualize volatility of volatility. Even though volatility of volatility is a rare animal in stochastic models, it is also the key variable in a multi-billion dollar market.

There is a discrepancy between realized and implied volatility which has been investigated by, among others, Christensen and Prabhala (1998). They show that implied volatility outperforms past realized volatility in forecasting future volatility and that it contains incremental information beyond that of realized volatility. We have to some extent tried to make the same distinction for volatility of volatility. Our results show that the VVIX, as the purely implied measure for volatility of volatility, stands out from other measures.

This paper takes a comprehensive approach in testing volatility of volatility as a pricing factor together with other stock-market factors and volatility for various equity portfolios. To the best of our knowledge, it is the first time that as many as eight different measures of volatility of volatility, realized, parametric and purely implied, are derived and tested in a portfolio pricing setting. Also, it is the first time a trading strategy is explored for individual stocks based on their loadings on volatility of volatility for five selected measures in parallel. We find that the volatility of volatility – return relationship is negative in general, that the option implied volatility of volatility (VVIX) has partly different characteristics, that volatility of volatility as a pricing and explanatory factor is complemented by volatility and that volatility of volatility has stronger power for individual stocks compared to portfolios. We also find that a trading strategy based on volatility of volatility acts as a good hedge to the S&P 500 index, especially during times of stock market decline.

A few stylistic conventions are worth mentioning. The volatility of volatility will, for simplicity, and its character of a source of risk, usually be abbreviated as VVOL. In qualitative discussions, the term volatility of volatility might still be used. Portfolios will usually be denoted by the name of the stock market characteristic they are sorted on, e.g. *Size* for portfolios sorted on market capitalization. Common risk factors, on the other hand, will usually be referred to by their abbreviation, i.e. *HML* for high-minus-low. Linear regression proportionality constants will interchangeably be called *betas*, *slopes*, *sensitivities* or *loadings*.

This paper is organized as follows. Chapter 2 reviews selected literature covering the issues with uncertainties behind volatility and caused by volatility. We move from the volatile commodities market through global macro and volatility transmission to equity markets and the impact from volatility of volatility on these. Next we address the problem of defining and calculating volatility of volatility. We conclude the literature review with insights from papers that use volatility of volatility in the pricing of equities and define and evaluate methods of trading on volatility of volatility. Chapter 3 describes the methods employed and chapter 4 provides a detailed description of the data used. In Chapter 5, we present the most important empirical results of our studies. These results are then further discussed and elaborated on in the final part of the same chapter. In Chapter 6 we conclude the paper by pointing out our most important findings and discussing opportunities for future research.

# 2 Literature Review

The starting point of a discussion on volatility of volatility is volatility. The recent developments in the crude oil market have shown signs of high volatility and dramatic price erosions and reversals that have led to a large impact on prices of other assets. One account of the influence from volatility on asset prices, especially commodities, and the macroeconomic impact this has had on the global economy was made by Ebrahim et al (2014). They coin the acronym OPV (oil price volatility) and state that OPV has been "advancing at a faster rate" compared to the volatility of other commodities in the past decade. They thereby touch upon the topic of VVOL that apparently is relatively high for crude oil. Ebrahim et al (2014) point at three factors that make up the VVOL of the oil market. Oil derivatives markets are showing a strengthened relation with seemingly unrelated financial markets, such as the EuroStoxx 600 and volatility spillovers from the oil market to other financial markets have become a reality. This transmission of volatility from commodities to equity markets is further investigated by Ewing and Malik (2009) and Hammoudeh and Malik (2007). The latter investigated the spillover effects from oil market volatility to equity markets in the Gulf States and the US. They identified two underlying explanations behind the connection between US capital markets and oil prices, the first being the impact on companies' cash flow projections and the second being the large amount of petrodollars from the Gulf States that are invested in US equities. The paper models volatility by a multivariate GARCH model. Similarly, Ewing and Malik (2009) use bivariate GARCH models to estimate mean and variance equations for US sector indices and oil prices in order to illustrate the volatility transmission mechanism over time, especially when price shocks occur.

The time-varying nature of volatility has received considerable attention in the financial economics literature. In Ewing and Malik (2009), the authors apply financial time series and model time-varying volatilities to examine the volatility transmission mechanism between the oil market and five major sectors of the US equity market. Now that financial literature recognizes the importance of time variations of volatility, the journey on the waves of volatility leads us to the next question. It is about how to measure the uncertainties in the volatility series, the volatility of volatility. The practical definition of several measures of VVOL has been performed by Clark et al (2013). In that paper, six measures of monthly VVOL in both nonparametric settings and GARCH models have been constructed and tested, together with the VIX index, to check their influence on the equity risk premium. Two of the parametric models, one of which is a two stage model based on returns, by Clark et al called the nested model, make up the foundations for the GARCH models constructed in this paper. Clark et al also have a mix of implied and realized (and realized of implied) measures of VVOL. The authors also perform some empirical tests where they regress the market equity risk premium on the VVOL measures and successively add factors such as implied volatility, the variance risk premium and others. They find that together with implied volatility, the VVOL reaches $R^2$ levels of 20%. Standalone, the VVOL accounts for an $R^2$ of 5-11%. An attempt to test the predictive power of VVOL for equity returns was also made with low power in the results.

The discussion on variance risk premium (VRP) in Clark et al (2013) has been an inspiration to the separation of our VVOL measures into implied and realized. The variance risk premium is implied variance minus realized variance. Usually, the implied variance is higher, which makes the risk premium positive and reflects the risks that make up the distribution of the forward-looking measure. According to Clark et al, when the VRP is negative, the market surprises investors and asset prices fall sharply. This paper also tries to investigate whether realized and implied volatility of volatility exhibit different behaviors.

The VVOL, in order to be calculated, requires its input. Some of the VVOL models in this paper (GARCH-3) use returns in a nested GARCH estimation, but for most models and realized measures, a realized volatility is needed. There are many ways to obtain realized volatility and a thorough review for Value-at-Risk estimation purposes was given by Brownlees and Gallo (2009). The investigated measures in their paper are realized volatility, realized kernel, bipower realized volatility and others.

There is a vast literature that has examined the time-series relation between the volatility of the market and the expected return on the market. Campbell and Hentschel (1992) find that the volatility discount in stock prices varied from 1% to 25% on monthly data in the early 1930's, and it increased from 0.5% to 13% over a few days when looking at daily data around the period of the market crash of October 1987. Market volatility has been shown to be a significant cross-sectional asset pricing factor by Ang et al. (2006). In Adrian and Rosenberg (2008), it is found that the prices of risk from market volatility are negative and significant for both short- and long-term components. Inspired by this literature, we want to push forward the studies of VVOL and to investigate the performance of VVOL as a determinant in the pricing of assets.

In the first half of the paper, the influence on asset prices from VVOL loadings is inspired by multifactor models. The multifactor models were first introduced with Arbitrage Pricing Theory (APT) due to Ross (1976). It is in this context that the main inspiration for the empirical investigations of portfolio prices versus VVOL has been the work once undertaken by Fama and French (1992, 1993 and 1996). The authors there show that the prices of stocks can be decomposed by a multifactor model into three main stock market drivers, small-minus-big (SMB), high-minus-low (HML) and the market risk premium (MRP). It is the methodology of cross-sectional analysis by constructing portfolios that mimic the same characteristics as the risk factors and finding patterns in the development of loadings as a function of portfolio deciles that has been an inspiration to partly investigate whether VVOL is a proxy for some of these factors. It is also the idea of convergence towards completeness in the linear space by the successive addition of factors, inspired by the APT, which has nurtured the hopes of VVOL completing the space of asset returns by increasing the goodness of fit and minimizing intercepts in linear regressions. The Fama and French (1992, 1993 and 1996) methodology also comes nicely at hand with the availability of portfolios in the Kenneth French Data Library. Furthermore, the Fama and French methodology includes a discussion on how the correlation between different risk factors is manifested in the multivariate regressions. Finally, Fama and French (1992) perform an analysis of the residuals of the previous regressions to test for market anomalies and see if the three-factor model leaves unexplained patterns. The anomalies are further investigated in Fama and French (1996), with the conclusion that the three-factor model captures stock market anomalies well. An assumption in that paper is central to one of the main assumptions in both the portfolio regression and the cross-sectional analysis part of this paper, namely that slopes through time on the factors are roughly constant and that the variations in returns arise due to variations in the factors themselves. They find that the three-factor model captures returns formed on earnings-to-price, cashflow-to-price and long-term-reversal. It is also stated that one necessary condition for the APT multifactor models is the existence of multiple undiversifiable sources of variance in returns. It is a tempting question for this paper to ask whether VVOL is such a linearly independent source of variance. The use of VVOL as a risk factor in a trading strategy is also explored by Huang and Shaliastovich (2014), this time for options. The idea is to make up a portfolio of delta hedged options and trade on the Volga exposure. They show that VVOL affects time series of VIX option returns. They also find that VVOL predicts returns of delta-hedged options with a negative sign. The main drawback of this kind of delta-hedged strategy is that the exposure, which is assumed to be mainly to Volga, might actually load on other risk factors. For this and other reasons, the investigation in this paper has not focused on options.

A paper that uses VVOL to formulate a strategy for equity trading is the one by Baltussen et al (2014). They claim that not only do uncertainties about the stock price in terms of volatility matter, but it also matters if the expected value of a distribution itself follows a distribution with some expected mean.

Investors look differently upon assets with fixed distributions and expected distributions. This was first touched upon by Segal (1987) as part of utility theory. The best proxy for this second distribution is the VVOL. Baltussen et al (2014) perform a study of the effects of VVOL on equity prices directly, an approach that resembles the last part of the empirical analysis in this paper. They also find that VVOL is negatively related to stock performance and that it is distinctive from more than 20 other pricing factors, including those of the Carhart (1997) four-factor model. The tested stocks are from the European and US equity markets. Baltussen et al (2014) find that VVOL is a factor that affects prices as a stock-level characteristic rather than as a traditional pricing factor based on some index like the VVIX, which is how the VVOL is represented in this paper.

Based on utility theory, their explanation to the VVOL effect is that investors have a preference for stocks with uncertainties about the risk, which drives up the prices of these stocks. Their alternative theory is that when the uncertainty preferences or expectations are heterogeneous, only the most optimistic investors will participate in the pricing of the high VVOL stocks (and thus drive prices up).

The VVOL measure that they use is the realized measure of implied volatilities backed out from stock options, similar to our implied rolling window measure of the VIX index, but as mentioned earlier with the difference that their VVOL is calculated individually for each stock. However, whereas Baltussen et al (2014) perform a normalization of the VVOL by dividing with the average implied volatility, our paper uses a different approach to control for volatility. Another difference is that Baltussen et al (2014) use a monthly resolution in their VVOL data as opposed to the daily data that appears in the subsequent chapters of this paper.

The main stylized fact takeaway from this paper is that VVOL affects stock prices negatively and thus a long-short portfolio would be long low VVOL stocks and short high VVOL stocks to achieve positive returns. This paper has been a benchmark for the trading strategy of our paper and has given useful insights about the fact that VVOL performs better in the pricing of stocks rather than portfolios.

A paper that supports the view of aggregate volatility, as opposed to individual volatility on the security level is the work carried out Cremers, Halling and Weinbaum (2014). In this paper, jump risk and aggregate volatility are studied as orthogonal risk factors. They find that jump and volatility risk are separately priced and that the sensitivities on each factor are practically uncorrelated. Extrapolating from their findings, we feel comfortable with the use of aggregate VVOL of the S&P 500 index.

# 3 Methodology

This study aims to investigate whether volatility of volatility works as a risk factor for equity market prices. The VVOL we focus on in our study will be equity market VVOL. We concentrate our study to the US equity market, since it has the most sufficient data for volatility and VVOL. No matter if investigating the impact of VVOL on US equity portfolios or US stocks, we use volatility of volatility of the S&P 500 index. The whole study applies multiple methods to help demonstrate the features of VVOL, such as time series regressions, in which we regress on VVOL and Fama-French factors, equity portfolios formed by five factors from the Kenneth French Data Library, and industry portfolios on our VVOL measures and other factors. We also apply cross-sectional analysis between VVOL regression coefficients and portfolio returns to see whether higher exposure to VVOL might generate higher returns. In the last part of our study, we try to design a trading strategy using the characteristics of VVOL we find and check whether this strategy can generate positive returns compared to a benchmark. VVOL is something that has not been observable in financial markets until the VVIX was introduced by CBOE. However VVIX is an implied measure of VVOL backed out from VIX option prices. We intend to have other VVOL measures, both realized and implied, to fully investigate the connection between VVOL and equity market, and difference between realized and implied measures. The first step is to find ways to model the measure of volatility of volatility.

## 3.1 Measures for Volatility of Volatility

To get the measures for VVOL, we start with two of volatility measures of US equity markets, one being realized daily S&P 500 volatility derived from intra-daily (5 minute intervals with no subsamples) returns retrieved from the Oxford-MAN Institute and the other one being the implied volatility (VIX) index from CBOE, which indicates the daily volatility level of the S&P 500 index. Both realized volatility and VIX datasets range from January 2000 to February 2015. From both the realized and implied volatility measures we apply models, some based on the previous work by Clark, Kirby and Wang (2013) and Baltussen et al (2014), to get the VVOL series. As a result we get four realized VVOL measures from the realized daily S&P 500 volatility and four implied VVOL measures from the VIX. Hereafter follow the models we use in generating the VVOL measures from the volatility. The following table illustrates the eight measures of VVOL that will be used.

| Realized VVOL Measures | Implied VVOL Measures |
|---:|:---|
| Rolling Window from Realized Volatility | Rolling Window from VIX |
| EWMA from Realized Volatility | EWMA from VIX |
| GARCH-2 from Realized Volatility | GARCH-2 from VIX |
| GARCH-3 from Realized Volatility | VVIX |

*Table 3.1. Overview of elaborated VVOL measures.*

### *3.1.1 Rolling Window*

Rolling Window is our nonparametric and realized approach for the VVOL series, similar to the VVOL measure in Baltussen et al (2014), who use a standard deviation of implied volatilities over a one-month time window. What we do here is to take a rolling window of the past 22 trading days including the current day and then calculate the standard deviation of the 22 volatility values in this window. One reason why we choose to use the window size of 22 trading days is that VVIX, which will be used as an implied measure for VVOL, is derived from the prices of options on VIX expiring in one month looking forward. Thus, the selection of a 22-day window is made to have consistency across our VVOL measures. Another reason is that we believe that it is more intuitive for investors to judge the volatility in a with a one month window. In other words, we believe investors will look back one month at a time at the realized measure to get a feeling of how volatile volatility is. We apply the rolling window approach to both the realized S&P 500 volatility and the VIX index and we denote our obtained measures realized ROLWIN and implied ROLWIN.

$$VVOL_n^{RW} = \sqrt{\frac{1}{22}\sum_{k=n-21}^{n}(VOL_k - \overline{VOL_k})^2}$$

### *3.1.2 EWMA*

As one of models to estimate volatility, the Exponentially Weighted Moving Average is used in our study to model the daily VVOL series from daily volatilities, both realized and implied. A warning flag should be raised here; the input in the normal EWMA model for volatility is returns, but what we have is a volatility series and there are potential pitfalls in using returns of volatilities. That is why we demean the volatility series first. The way we think of demeaned series is that if the volatility has a long term mean, the residuals left are actually the innovations in the volatility updating process. And as expected, the squared demeaned volatility series turns out to be a stationary series, based on the autocorrelation and partial autocorrelation (ACF and PACF) plots, Ljung-Box, and Augmented Dickey–Fuller tests. We thus choose to use the demeaned volatility series in the model to estimate the volatility of the original volatility series. We estimate the EWMA lambda parameter by a maximum likelihood function. Then we run the EWMA model based on the estimated lambda coefficient and compute the estimated VVOL series. We apply this EWMA approach to both realized the realized S&P 500 volatility series and the VIX index.

$$VVOL^{EWMA}{}_{n}^{2} = \lambda * VVOL^{EWMA}{}_{n-1}^{2} + (1-\lambda) VOL_{n-1}^{demeaned}{}^{2}$$

### *3.1.3 GARCH 2 – the Volatility of Realized/Implied Volatility*

Based on the second GARCH model applied in Clark, Kirby and Wang (2013), we start with the realized or implied volatility and apply a GARCH model on this series to get the VVOL series. According to Corsi et al. (2006), we can define realized volatility as

$$RV_t^2 \equiv \sum_{i=1}^{n} \left[ p_{t+\frac{1}{n}\Delta} - p_{t+\frac{i-1}{n}\Delta} \right]^2.$$

Then the logarithm of realized volatility follows a normal distribution with a time-varying variance.

$$\frac{RV_t - \sqrt{\int_{t-1}^{t} \sigma^2(s) ds}}{\sqrt{\frac{Q_t^*}{2M\ RV_t^2}}} \xrightarrow{d} N(0,1)$$

In the equation, $\sqrt{\frac{Q_t^*}{2M\ RV_t^2}}$ is an approximation of the standard deviation of the realized volatility. As a result, we can apply this and generally assume that the logarithm of realized volatility actually follows a GARCH (*p*,*q*) process.

In this approach we start by demeaning the volatility series and saving the residuals, which we consider as innovations in the volatility updating process. Before running the residuals in the GARCH model, we run several tests to check the autocorrelations and stationarity of the squared demeaned volatility series, such ACF, PACF, Ljung-Box and Augmented Dickey–Fuller tests. Graphs and tables for the tests are included in the Appendix. Having finished the tests and checked the series, we run the residual series into a GARCH estimation algorithm and estimate the optimal lags by the Akaike information criterion (AIC), which gives GARCH (*p*,*q*) model (the lags, p and q are shown in Table 3.1for our volatility series).

$$\begin{cases} RV_t = \mu + \sqrt{h_t}\epsilon_t \\ h_t = \omega + \alpha_1 h_{t-1} + \beta_1 u_{t-1}^2 \end{cases}$$

where $\{\epsilon_t\}$ is a white noise sequence and the second equation is a GARCH(1,1). The model is thus a standard geometrical Brownian motion for volatility, with VVOL as a parameter. Since here we have daily volatility series, we can apply this method to both implied and realized volatility and get the daily VVOL.

### *3.1.4 GARCH 3 – the Nested GARCH Model*

According to the third extended GARCH model from Wang, Kirby, and Clark (2013), to get the VVOL sequence, we can start by running the demeaned return series into a GARCH (*p*,*q*) model and then work on the residuals from the GARCH model. This algorithm can, however, only be applied to the realized measures as the implied measures are based on the volatility in form of the VIX, but there is no underlying "implied return series". Firstly we assume that the return series follow

$$r_t = \mu_r + \sigma_t \epsilon_{r,t}$$

where $\{\epsilon_{r,t}\}$ is a white noise process and $\mu_r$ theoretically could be an ARMA process, but since we are focusing on the second-level volatility, for simplicity, we just demean the series (assuming a constant $\mu_r$) and treat the demeaned series as innovations in the return process and elaborate on those. For the volatility series of the returns, the paper assumes part of it is a deterministic GARCH process, but the other part, which is the residual series from the first GARCH model, is stochastic.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 u_{t-1}^2 + q_t \epsilon_{\sigma,t}$$

In this GARCH model, $\{\epsilon_{\sigma,t}\}$ is a white noise process that is independent from $\{\epsilon_{r,t}\}$, $\{q_t\}$ is the sequence of volatility of volatility, $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$. We assume that $\{q_t\}$ also follows a GARCH process according to:

$$q_t^2 = \alpha_q + p_q q_{t-1}^2 + \phi_q \eta_{t-1}^2$$

where $\eta_{t-1}^2 = q_{t-1}\epsilon_{\sigma,t-1}$, and $\alpha_q > 0, p_q \geq 0, \phi_q \geq 0$, and $p_q + \phi_q < 1$.

Following this theoretical framework, we start by demeaning the original returns of the S&P 500 index and check the autocorrelation and stationarity on the squared residuals by several tests. We then estimate the optimal lags for the first GARCH model, which gives us a GARCH ($p$,$q$) process. From the GARCH ($p$,$q$) model, we estimate the first-level volatility, but more importantly we save the residual series in the model to proceed the next step. Before running the second GARCH model, we run another stationary test on the residual series that is treated as a series of second moments. Then, based on the Akaike information criterion, we estimate the lags of the second GARCH model to be GARCH ($p$,$q$) and that gives us the VVOL series. Finally we run the GARCH ($p$,$q$) model on the residual series from the first GARCH ($p$,$q$) and the GARCH-3 series is obtained as the output. Actual lags for the estimated models are shown in Table 3.2.

Among the implied VVOLs, this measure is replaced with a modified VVIX as the fourth implied measure. One thing important to note at this stage is that VVIX is the only "real" implied VVOL measure, in the sense *implied volatility of implied volatility*, since the three other measures that we label as implied are simply *realized or modeled volatility of implied volatility*. The modification made to the VVIX is that since the original VVIX series is annualized, we divide the VVIX sequence by the square-root of 252 to convert it from an annualized to daily series and thus comparable to our other series which are based on daily returns. The same modification is made to the VIX index which caters to the three remaining implied measures.

SUMMARY STATISTICS FOR THE ESTIMATED LAGS IN GARCH models - REALIZED AND IMPLIED

| | | REALIZED GARCH-2 | REALIZED GARCH-3 | IMPLIED GARCH-2 |
|---|---|---|---|---|
| **STAGE 1 GARCH(p,q)** | *p* | 1 | 2 | 2 |
| | *q* | 1 | 1 | 2 |
| **STAGE 2 GARCH(p,q)** | *p* | - | 2 | - |
| | *q* | - | 1 | - |

*Table 3.2. Summary Statistics for the Estimated Lags in GARCH Models.*

## 3.2 A Framework to Study VVOL as a Factor Based on Factors and Equity Portfolios

After the eight VVOL measures have been defined and modeled, the next step for is to find ways to investigate how volatility of volatility is connected with or is having an impact on the equity markets, or more explicitly, on equity returns. In this part of our research, we have tried to apply the methods and approaches from Fama and French (1992) and the paper has served us as a guidebook. Instead of using log changes of the VVOL series, we regress on the levels of VVOL and this approach has been chosen for two reasons. Firstly, we believe that volatility by nature *is* the very changes of the process. It captures the fluctuations in the returns and represents them. Every point in the VVOL series is a separate piece of information that shows the innovation at that point of time. Secondly, since the

VVOL sequence has extremely high jumps, we could get extreme and discontinuous values in the log changes. Thus the log change series, we suppose, would be very different from a normal distribution process. This could become hard to handle due to the limited amount of studies on the characteristics of VVOL.

### *3.2.1 Time-Series Regression*

According to the study by Fama and French (1992), we can first apply time-series regressions to investigate the linear relation between equity portfolio returns and our VVOL measures. There are four types of dependent variables to be explained in this regression analysis. These are Fama-French risk factors, returns of the S&P 500 index, and returns of two types of portfolios. The first type consists of portfolios formed from common stock risk factors, such as size, book-to-market, Momentum, short-term reversal, and long-term reversal, and the second type are industry sector portfolios. All of them are downloaded from the Kenneth French Data Library. Every group of factor-based portfolios has ten deciles, from the highest exposure to the lowest exposure with respect to the related factor. By using returns of factor portfolios as dependent variables, we want to investigate whether the coefficients or *loadings* on VVOL in the regressions increase or decrease monotonically with portfolio decile to identify trends, and in that way find a relation between the VVOL and existing common equity risk factors. In other words, we search for patterns to see whether VVOL shares some characteristics with the common risk factors and whether VVOL could be a proxy partial proxy any of them. The reason why we use 10-industry and 49-industry portfolios, is mostly that we want to save the VVOL loadings of these regressions and use them in the next stage which is the cross-sectional analysis of VVOL betas and returns.

To regress these portfolio returns, we apply five different (multivariate) regressions. (We will call them the five *families* of regressions in the rest of the paper.) The first family is simply the univariate regression on VVOL only. In the second family, we control for the original volatility series of the VVOL measure. Baltussen et al (2014) do this differently by dividing the “realized-of-implied” VVOL with the mean of the implied volatility. The third family regresses on VVOL while controlling for the original factor that sorts the portfolio. The reason we control for the original factor is to show whether VVOL as any marginal explanatory power beyond the main determinant. The fourth regression family is a multivariate regression on VVOL, the related factor, and the market risk premium (MRP). The fifth family is made up from regressions of returns on volatility series, VVOL, and the related factor. The five families of regressions are as follows,

$$
\begin{cases}
R_t^{pf} = \alpha_1 + \beta_1^{vvol} \cdot VVOL_t + \varepsilon_{1,t} \\
R_t^{pf} = \alpha_2 + \beta_2^{vol} \cdot VOL_t + \beta_2^{vvol} \cdot VVOL_t + \varepsilon_{2,t} \\
R_t^{pf} = \alpha_3 + \beta_3^{vvol} \cdot VVOL_t + \beta_3^{F} \cdot F_t + \varepsilon_{3,t} \\
R_t^{pf} = \alpha_4 + \beta_4^{vvol} \cdot VVOL_t + \beta_4^{F} \cdot F_t + \beta_4^{MRP} \cdot MRP_t + \varepsilon_{4,t} \\
R_t^{pf} = \alpha_5 + \beta_5^{VOL} \cdot VOL_t + \beta_5^{vvol} \cdot VVOL_t + \beta_5^{F} \cdot F_t + \varepsilon_{5,t}
\end{cases}
$$

We focus in the regressions on the VVOL betas, alphas, and $R^2$. What we care about the most are the VVOL betas, their sign, and their significance level. We would expect that if, for factor portfolio regressions, the VVOL betas follow an ascending/descending order, it means that the VVOL factor is positively/negatively related to the common risk factor in question. Another thing we want to investigate is whether, when controlling for other factors such as volatility, the common risk factor and the MRP, the betas on VVOL remain statistically significant and whether we still have a reasonably clear ascending/descending pattern in the VVOL beta series. If not, then the takeaway from the regression is that the VVOL as a source of risk is embedded in the other risk factors that are controlled for in the regression. The completeness of the linear space is also diagnosed by checking the $R^2$ and intercepts of the regressions.

### *3.2.2 Cross-Sectional Analysis*

Following the time-series regression analysis, we apply the cross-sectional approach to investigate whether VVOL could be a determinant for equity prices. Enlightened by the assumption in Fama and French (1996) that the slopes on factors in the factor model are roughly constant over time and that the variations in returns are due to the variations in the factor themselves, we also assume in our studies that the VVOL beta for a portfolio (or an asset in general) is constant across time. We calculate a single beta for average returns over all years. This is done for all tested portfolios, including factor-based portfolios, 10-industry portfolios, and 49-industry portfolios. We use betas from the all the regression families to estimate cross-sectional slopes on betas, but will focus our results on the second family regressions which are controlled for volatility. This is because in these regressions, we find the highest significance levels for VVOL betas and based on our findings, VVOL as factor in pricing seems to work better when controlling for its original volatility, something that will be discussed later. The cross sectional analysis is carried out by regressing portfolio returns on VVOL betas for each portfolio type and each regression family. We plot a graph of a least-squares line that fits the average annual return – $\beta_{VVOL}$ scatter plot. The cross -sectional regression coefficients and their significance levels are printed on the top right corner on the graph, see the Empirical Results section.

## 3.3 Long-Short Trading Strategy to study the Impact from VVOL on Individual Stocks

Research on the subject of pricing factors in recent years has shifted focus from portfolios to stocks in order to study the idiosyncratic noise and capture particular characteristics of individual stocks. In Baltussen et al (2014), the authors even go as far as modeling the VVOL individually on the security level, which they claim gives a better performing trading strategy. In an attempt to further investigate the influence of VVOL in the pricing of assets, a different approach is now tried with more focus on assets with idiosyncratic characteristics rather than portfolios. This time the assets under test are stocks provided by CRSP. The stocks are all NYSE, AMEX and Nasdaq listed common stocks from the period starting in June 2006 and ending in December 2014. This is the time window of available data for the VVIX index. Special attention is given to the VVIX measure in this section as it is the measure that has the most clearly negative loadings when used as a regressor for the S&P 500 index in section **5.2.2**. The data consists of daily closing prices for 6693 stocks. The way the investigation is undertaken in this part is complementary to the previous regressions in that it follows as a logical next step of formulating a trading strategy based on VVOL and the results from the cross-sectional analysis. The data is treated in the way described in 3.3.1.

### *3.3.1 Construction of the Trading Strategy*

The time window of the investigation is 103 months long. Inside each of the months, the returns of the stocks are then regressed on the corresponding volatility and VVOL measure and one $\beta_{VVOL}$ for every month is saved. When regressing on VVIX, we leave a one-month lag between dependent and independent variables, since VVIX is a purely implied measure of VVOL backed up by the prices of options expiring in the next 30 days, which we believe, makes it a natural measure of future volatility of volatility in a month. At the start of every month, the stocks are sorted on the VVOL betas of last month. The trading strategy is now to short the top $\beta_{VVOL}$ stocks and long the lowest $\beta_{VVOL}$ stocks, because as the results of the cross-sectional analysis show[1], low $\beta_{VVOL}$ stocks tend to outperform high $\beta_{VVOL}$ stocks. The size of the portfolios is chosen to be 1.5% of all available stocks, which in a perfect month would be 100 stocks. Some months have a subset of the 6693 stocks available for investing due to unavailability of intersecting data between the stock price and the VVOL measure or too few available days to solve the OLS equation. The portfolio is rebalanced at the start of every month and kept up-to-date based on the latest $\beta_{VVOL}$ values. The long-short portfolio is then aggregated to monthly returns and the returns of the strategy on VVIX are plotted over time in Figure 5.6.

[1] This will be discussed further in the cross-sectional part of the Empirical Results.

There are mainly two reasons why we want to base $\beta_{VVOL}$ on bivariate regression on volatility and VVOL to form our long-short trading strategy. Firstly, the results from portfolio regressions show on increased power on VVOL once volatility is included in the regressions. Secondly, as a sanity check, the above described procedure was tried using univariate regressions of stock returns only on previous VVOL time series. An inspection (Figure A.28 in the Appendix) shows that the trading strategy has a clearly worse performance compared to the bivariate regression setting that controls for volatility.

This trading strategy is carried out using the VVIX and the four realized measures, with a lagged return-VVOL regression for VVIX. The VVIX is especially interesting as it is practically useful, i.e. it is the measure of implied VVOL a hypothetical fund manager would use for its betting. The realized VVOL measures are added to test the strategy for alternative, empirical methods of obtaining the VVOL, as we assume that it is more intuitive for investors to rely on realized volatility, and for continuity with the studies in previous chapters.

In the design for our trading strategy, we have two ways to reinvest our profits over time. For each month, we enter into our long-short zero-cost position on the first day and clear the position on the last day, which leaves us with a profit or a loss. We need to reinvest our profits when there are any in order to put all our capital at work. The first alternative is to reinvest all profits at the beginning of the month into the long-only leg of the newly-rebalanced portfolio, i.e. the 1.5% of stocks with the lowest $\beta_{VVOL}$ of that month. The second alternative is to reinvest profits at the risk-free rate every month. In the results section, only the trading strategy using the first alternative of reinvesting is included and analyzed, since our calculation shows that the first alternative outperforms the second one for all VVOL measures. Later in the result section 5, a time-plot, Figure 5.7, of profits of the trading strategies using five different VVOL measures will be shown.

### *3.3.2 Evaluation of the Trading Strategy*

The paper applies two methods to evaluate the performance of the VVOL trading strategy to investigate the pricing power of volatility of volatility. In the first method, we regress the monthly returns of our trading strategy in the Carhart four-factor model (Carhart, 1997) to investigate intercepts and the loadings of the risk factors. In the second evaluation method, one of the four realized VVOL measures is chosen to represent the trading strategy together with the VVIX in comparison with three benchmarks; the S&P 500, the HFRI Equity Hedge Index, a widely used benchmark for long-short hedge fund strategies, and the Carhart long-only portfolio. This Carhart long-only strategy follows the same procedure of data treatment as the VVOL long-short portfolio until the monthly regressions on

VVOL, which instead are made on the four factors, namely MRP, SMB, HML and Momentum. The portfolio of stocks is now the equally weighted top quantiles of stocks sorted on loadings on each factor. The returns are then also aggregated monthly. There will be a figure comparing the performance or more explicitly profits from investing into these five “assets” from August 2006 to February 2015. To make all five ways of investing consistent and comparable, we normalized the starting value of each index to 100. This means the long and short positions in our VVOL trading strategy are both 100 for each month. For a closer comparison between our VVIX strategy and the HFRI index, we plot the active returns and cumulative active gains between across the whole sample period.

### *3.3.3 A Cross-Sectional Study of Stocks*

To further investigate the impact of VVOL on assets with idiosyncrasies, another cross-sectional study is added in the result interpretation part on the 6693 individual stocks. Using the previous architecture from the trading strategy, the monthly returns are sorted in a matrix structure over months and stocks. Each month, the daily excess return data is regressed on volatility and VVOL. A matrix of monthly betas is obtained. The average monthly excess returns for each stock are plotted against the average monthly beta. Extreme outlier betas are neglected as they have an unproportioned influence on the regression. This is done for the four realized VVOL measures and the VVIX.

The daily risk-free rate, used to achieve excess returns of the stocks, is obtained from the Kenneth French Data Library. This cross sectional analysis has been made to resemble the methods of Fama and MacBeth (1973) and Halling et al (2014). There is thus a distinction between this analysis and the cross sectional analysis of portfolios performed earlier. The betas are now an average over months, whereas in the portfolio analysis, they were assumed constant.

# 4 Data Description

As was described in the Methodology, the starting point is four daily time series which generate eight different VVOL series. The four original series are the S&P 500 returns, realized volatility of the S&P 500, and the daily compounded VIX and VVIX indices. The first two data series range from January 2000 to the start of February 2015, the VIX data ranges from March 1995 to the start of February 2015, and the VVIX series ranges from June 2006 to February 2015. The S&P 500 return and realized volatility (5 minute intervals with no subsamples, denoted $R_{vol}$) data is retrieved from the database of the Oxford-MAN Institute of Quantitative Finance. The CBOE VIX and VVIX indices are extracted from Datastream. Below is a table of summary statistics, Table 4.1, and a time-plot, Figure 4.1, of the two volatility series that some of the VVOL models are based on.

Data Summary Statistics

| | S&P return | Rvol | VIX | VVIX |
|---|---|---|---|---|
| Mean | 0.00009 | 0.00937 | 0.01317 | 0.05416 |
| Min | -0.09351 | 0.00127 | 0.00623 | 0.02277 |
| Median | 0.00064 | 0.00780 | 0.01191 | 0.05322 |
| Max | 0.10220 | 0.08802 | 0.05094 | 0.09142 |
| Variance | 0.00015 | 0.00004 | 0.00003 | 0.00007 |
| Stdev | 0.01227 | 0.00626 | 0.00564 | 0.00817 |

*Table 4.1. Table of summary statistics of the four starting-point series. In the table, six standard statistics numbers are calculated for the four time series used to generate the eight series of volatility of volatility in the studies Daily compounded data.*

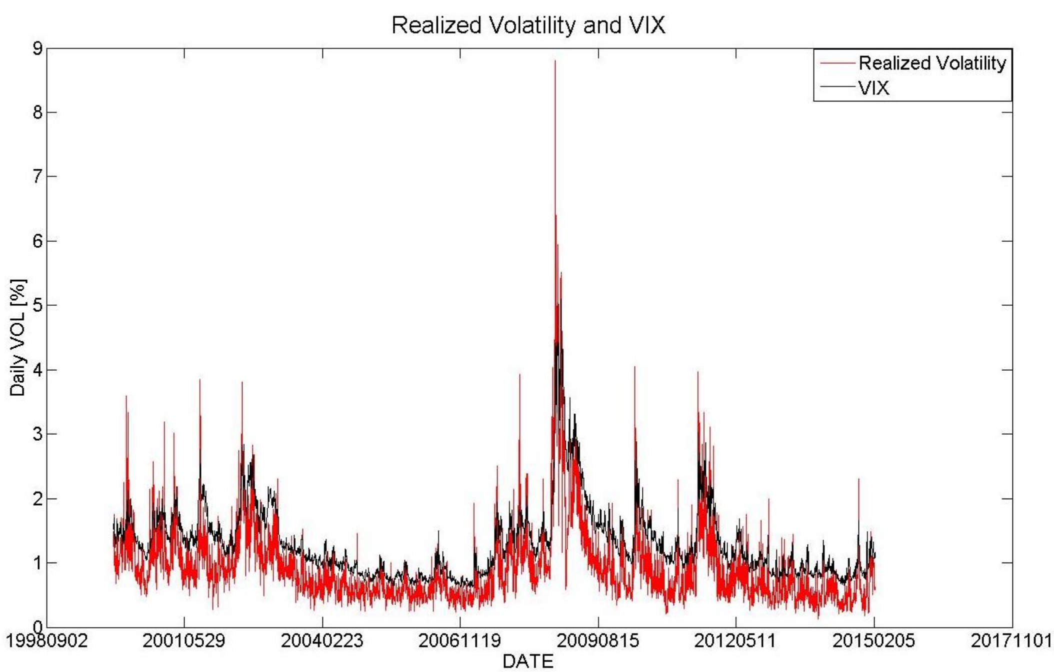


*Figure 4.1. Time series plot of the realized volatility and VIX series.*

As can be seen from the table, both realized volatility and VIX are shown to have a long-term mean above 0, with the mean of VIX higher than that of realized volatility. This evidence from the data backs up the approach we use to deal with the volatility series, which is to demean the volatility series and work on the residuals as the innovations in the process. However, the variance and standard deviation of the realized volatility series are higher than those of the VIX series, something that is easily observable from the time-plot as well. In Figure 4.1, the realized volatility and the VIX share a similar pattern in terms of spikes and trends, but the realized volatility has certainly more fluctuations in the process. It could be the case that VIX, being an implied measure from the market, reflects the ability of the market to discount, which makes the series smoother than the realized volatilities. The VVIX shows a mean that is quite higher than the standard deviation of the two volatility series, something that will be attribute to the variance risk premium of volatility below and is visible in the VVOL plots of chapter 5.1. The VVIX seems to be a different animal compared to other VVOL measures generated from $R_{vol}$ and VIX.

SUMMARY STATISTICS FOR THE UNIT ROOT TEST - THREE VOL MEASURES

| | | Realized VOL | GARCH VOL from Returns | VIX |
|---|---|---|---|---|
| | **p_Dickey-Fuller1** | 0 | 0 | 0 |
| | **p_Dickey-Fuller2** | 0 | 0,0621 | 0 |
| **p_Ljung-Box** | LAG:1 | 0 | 0 | 0 |
| | LAG:2 | 0 | 0 | 0 |
| | LAG:3 | 0 | 0 | 0 |
| | LAG:4 | 0 | 0 | 0 |
| | LAG:5 | 0 | 0 | 0 |
| | LAG:6 | 0 | 0 | 0 |

*Table 4.2. Summary Statistics for the Unit Root Test. Augmented Dickey-Fuller and Ljung-Box tests are applied to test the Stationarity of the series. P-values from the tests show that all three volatility series that have been used in generating VVOL are stationary, mostly at 99% level except for GARCH volatility in the second Augmented Dickey-Fuller test.*

For the regression and cross-sectional analysis part, we mainly use data and portfolios extracted from the Kenneth French Data Library, except for the index returns that are from the Oxford-MAN Institute database in the regression of S&P 500 returns. We use six market factors; MRP, SMB, HML, MOM, STR, and LTR to for the regressions. On the portfolio side, we use portfolios sorted on the five last factor characteristics and industry sector portfolios, also due to Kenneth French. All the data mentioned is matched to have the same time range as the volatilities, from the start of 2000 to the end of 2014. In the last section, the part about the trading strategy, we study the impact of VVOL on individual stocks. We use the US stock price data that contains 6693 stocks from CRSP, through the Wharton Research Data Services, in a time period from June 2006 until the end of January 2015.

# 5 Empirical Results

## 5.1 Eight Series of realized VVOL and implied VVOL

### *5.1.1 Plots in the Time Domain*

Below the time-series plots of the eight VVOL series generated according to the description in the Methodology are shown.

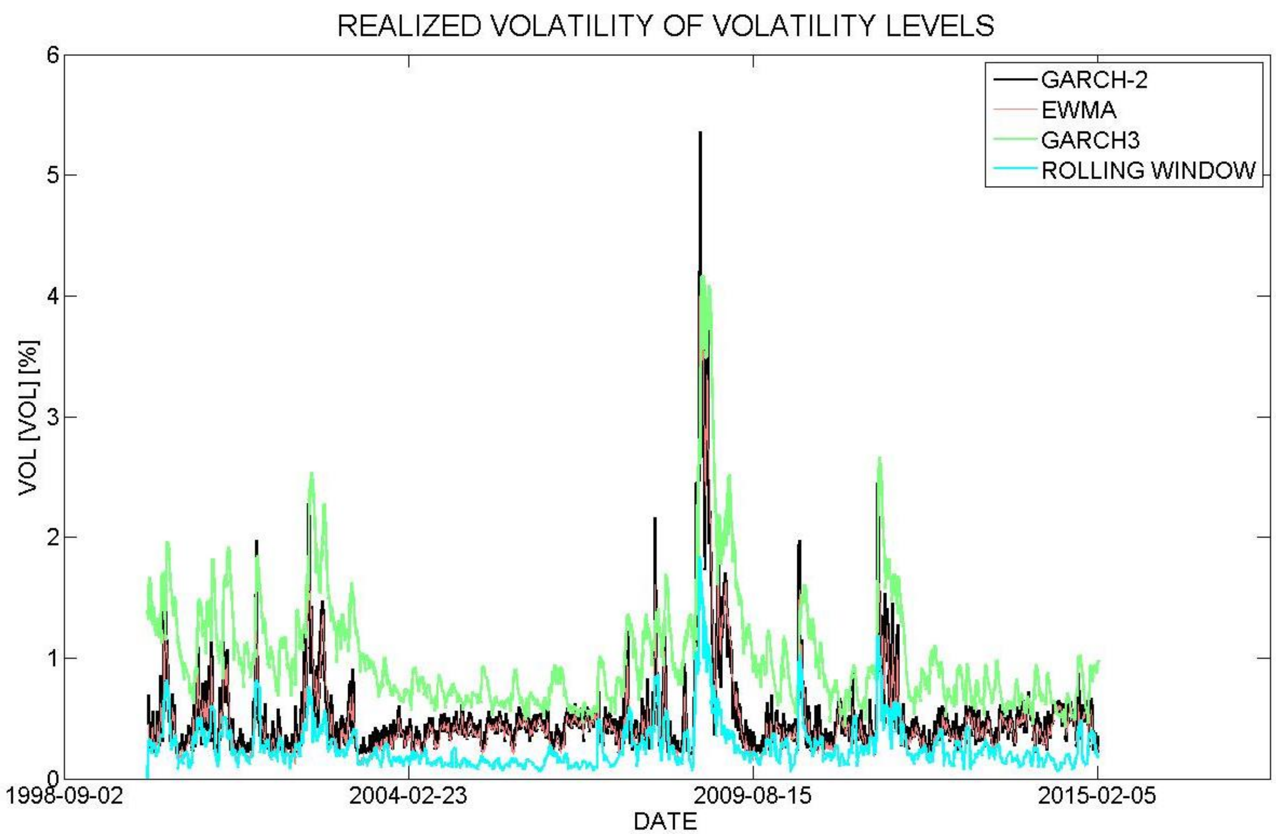


*Figure 5.1. Plot of the four realized VVOL Measures. The four series shown in the graph are derived by, respectively, the rolling window method, the EWMA model, the GARCH-2 model, and the GARCH-3 model described in the Methodology. All four series follow similar patterns in the graph with spikes that reflect the financial market events in the years after 2000, but usually with a lag of one or two months. Comparing between the four series, it can be seen that the GARCH-3 measure is on the largest scale and is the most volatile one. Rolling window, on the contrary, has the lowest scale, and the remaining two VVOL measures, EWMA and GARCH-2 are similar to each other being in the middle in terms of both magnitudes and volatilities.*

Relying on the graph above, it can be stated that the four realized VVOLs across the whole time period show similar patterns. In particular we see how the recent two market crises caused the spikes in realized volatility of volatility in all four series during the financial crisis in 2008 and the Eurozone crisis starting in the summer of 2011. The realized VVOL plot also show a series of spikes at the start of the series, indicating the burst of the dotcom bubble in the early 2000s and the 9/11 attacks. Also, the Flash Crash of May 2010, known for its sudden volatility spike, is also visible represented in the VVOL graph by the spike between financial crisis and Eurozone crisis. From the mapping of spikes in the graphs on events, we realize that VVOL has a delayed reaction to them. In other words, there is usually a lag between the financial market event and a spike in the VVOL series. It is our view that this is a characteristic of the VVOL, which peaks when the underlying volatility series experiences a downturn. As a result, the peaks in the VVOL series occur after the peaks in the volatility sequence, when volatility has been reversed. Comparing the four measures, we observe that the realized GARCH-3 VVOL is the most volatile measure among all realized measure. The realized rolling window is the least volatile one, and the two modeled realized VVOL series that are derived from the realized volatility sequence, EWMA and GARCH-2, have volatilities close to that of rolling window. The most probable reason why the GARCH-3 measure is the most volatile one lies in its theoretical framework. In GARCH-3, we are dealing with residuals from the first-stage GARCH model. And the in the residual series, the first-level noise is more prominent and so it distorts the second stage VVOL estimation.

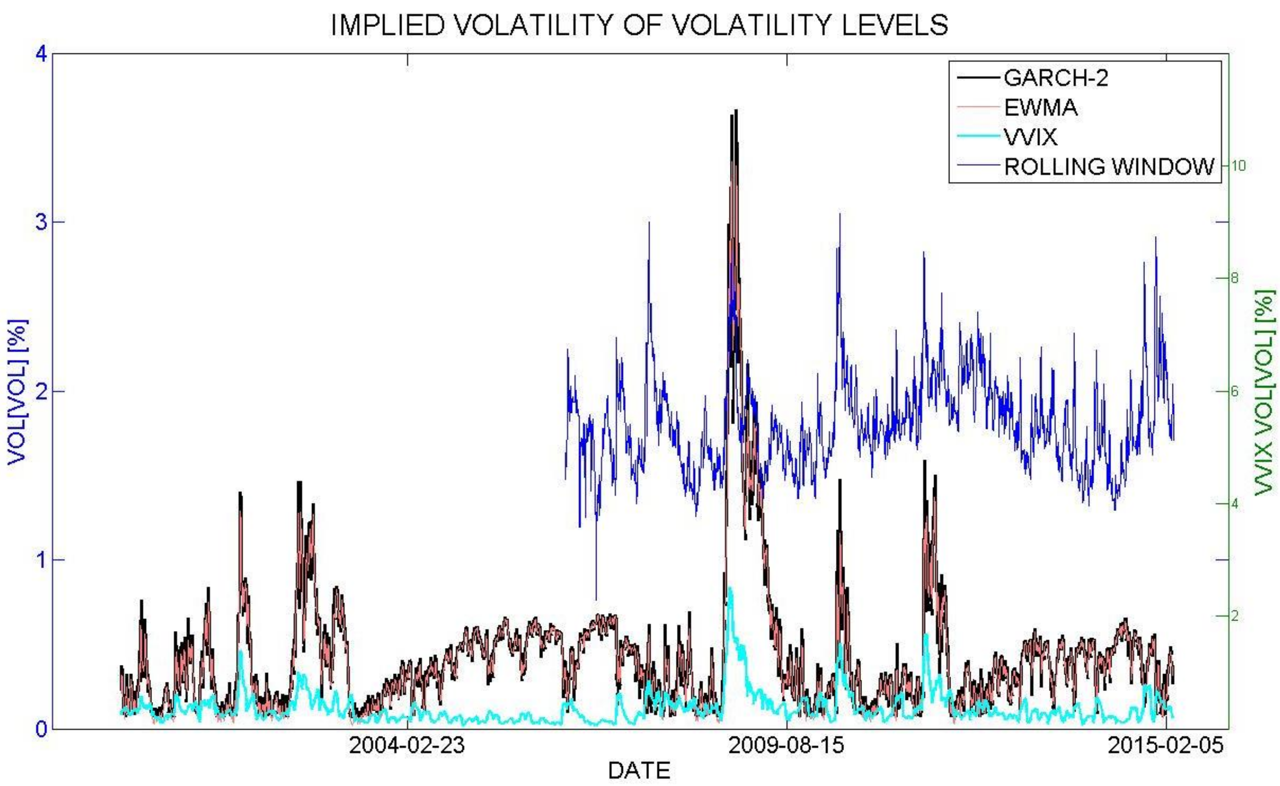


*Figure 5.2. Plot of the four implied VVOL series. The VVIX in the graph is just a daily compounded version of the CBOE VVIX data, and it is plotted with respect to the right-hand scale. The remaining three series are generated by the rolling window method, the EWMA model, and the GARCH-2 model from the Methodology part on the treatment of the implied volatility (VIX) series. VVIX shows very different features compared to the rest of the series, which in all fairness are "realized of implied" and "modeled of implied". It is on a much higher scale, has a partly different pattern of spikes and is more volatile than the rest of series. The remaining three series roughly share a similar pattern with coordinated peaks, reflecting the financial market crashes. Between them, the two most similar modeled series are EWMA and GARCH-2 which have a higher scale and larger fluctuations.*

In the implied VVOL graphs, it is visible that except for VVIX, all three other measures, rolling window, EWMA, and GARCH-2, share similar patterns in terms of the spikes that occur in the sequences. Several salient spikes in these three series reflect the same events or market crashes shown in the realized VVOL series, namely the dotcom bubble in the beginning of the 2000s, the 9/11 terrorist attacks, the 2008 financial crisis, the 2010 Flash Crash, and the Eurozone crisis. Interestingly we observe in the series that for the two modeled EWMA and GARCH-2 implied VVOL series, they both similarly follow some kind of random walk in two phases in the calm period between 2003 and 2007. And at the end of the sequence, after June 2012, we see the two modeled implied VVOL sequences starting to follow patterns reminiscent of random walks again. We guess this is due to a combination of the features of the models and the implied VIX, but since these observations are not the focus of this research, we would leave it for future studies. Between the three VIX-based series, the EWMA and GARCH-2 implied measures resemble each other the most and evolve on a higher scale than the rolling window implied VVOL sequence.

The daily compounded VVIX series shown in the graph is very different from the other three implied VVOL measures, and it deserves a separate analysis. First of all, it does not have its highest peak after the 2008 financial crisis. There are two higher peaks that occurred before and after 2008, respectively, on 8/16/2007 and 5/20/2010. This means that the VVIX, with a lag of one to two weeks, reacted more to the first signs of the 2008 Financial Crisis, namely the withdrawal of BNP Paribas from three hedge funds which occurred on 8/9/2007, and to the Flash Crash that happened on 5/6/2010. Secondly, only in the VVIX series is the recent spike from 12/14/2014, which we argue is likely due to the collapse of the oil price, which dropped to 60.50 dollars on the 11th of the same month. Thirdly, the scale of the VVIX series is a lot higher than the rest of the three implied VVOL measures derived from the VIX, and the series itself is really volatile but stays in a roughly constant range. So it seems to us that VVIX, as a measure of implied VVOL, is quite outstanding from the other VVOL measures. Once again, we underline the possible variance risk premium of volatility that is inherent in this measure.

### 5.1.2 Correlations Between the VVOL Measures

CORRELATION BETWEEN VVOL MEASURES on intersecting dates of VIX, VVIX and S&P500

| | | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| Realized Measure | ROLWIN | 1.0000 | 0.8168 | 0.7430 | 0.8381 | 0.8935 | 0.6230 | 0.6122 | 0.4413 |
| | EWMA | 0.8168 | 1.0000 | 0.9567 | 0.7572 | 0.7648 | 0.8829 | 0.8750 | 0.2960 |
| | GARCH-2 | 0.7430 | 0.9567 | 1.0000 | 0.6677 | 0.7050 | 0.8262 | 0.8301 | 0.2786 |
| | GARCH-3 | 0.8381 | 0.7572 | 0.6677 | 1.0000 | 0.8046 | 0.6852 | 0.6710 | 0.2817 |
| Implied Measure | ROLWIN | 0.8935 | 0.7648 | 0.7050 | 0.8046 | 1.0000 | 0.6381 | 0.6268 | 0.5046 |
| | EWMA | 0.6230 | 0.8829 | 0.8262 | 0.6852 | 0.6381 | 1.0000 | 0.9905 | 0.1944 |
| | GARCH-2 | 0.6122 | 0.8750 | 0.8301 | 0.6710 | 0.6268 | 0.9905 | 1.0000 | 0.1836 |
| | VVIX | 0.4413 | 0.2960 | 0.2786 | 0.2817 | 0.5046 | 0.1944 | 0.1836 | 1.0000 |

*Table 5.1. Correlations between the eight different measures of VVOL. For every pair of measures, an individual date matching time period is used to maximize the number of samples. According to this table, all VVOL measures excluding the VVIX show quite consistent correlations between each other.*

Table 5.1 above shows that all VVOL measures, both realized and implied (excluding the VVIX), have high correlations between each other. This might indicate that in terms of the original volatility measures, $R_{vol}$ and VIX, the realized and the implied measures are telling similar stories and are not so much different. However, we see that VVIX, as mentioned in previous sections, has a quite different feature than other measures, resulting in the correlation coefficients between it and other series to be below 50%, except for the correlation with implied rolling window, which is slightly above 50%. We also observe two interesting things from the correlations. First, the implied rolling window series has a really high correlation of over 70% with all four realized VVOL measures. This could mean that the realized feature of the rolling window method (realized of implied) makes it more similar to realized measures than to the other three implied measures. Second, the correlations between realized EWMA and realized GARCH-2, and between implied EWMA and GARCH-2 are extremely high, nearly 100%. This is consistent with the strong resemblance in of the two series in the time-plots in Figure 5.2.

### 5.1.3 Regressions of Volatility on VVOL

To investigate the influence and explanatory power of VVOL on volatility, a series of univariate regressions was run for the eight measures. The results are shown below in Table 5.2. Three things can be observed. Firstly, the betas are significant and positive. They are around 1 for five of the measures and above one for two. Only the VIX on VVIX has a coefficient far below 1 at 0.3 (but then again the VVIX is on roughly five times the level of the other measures). Secondly, the $R^2$ values are fairly high for a univariate regression with values between 40%-50% for most measures, except EWMA implied (33%) and VVIX (14%). This means that roughly, the VVOL explains half of the variations in volatility. The VVOL seems to be a large determinant underlying the volatility. The effect is least obvious for the regression of VIX on VVIX, which has a unique feature in that it is observable by the market. The third and final notable observations from this table are the intercepts. The alphas are positive and significant at the 99% level, implying that there are other elements besides VVOL that explain volatility. The magnitude of the alphas is half of the level of the daily VOL series at about 0.010 for the realized measures and roughly the same order of magnitude for the implied VOL series (with a mean of 0.13). Only for VVIX on VIX is the alpha significant and negative which means that VVIX as a regressor is underestimating the VIX.

Univariate Regressions of VOL measures on VVOL measures

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| alpha | 0.0039*** | 0.0060*** | 0.0040*** | 0.0002* | 0.0091*** | 0.0103*** | 0.0092*** | -0.0026* |
| beta | 1.1320*** | 1.4414*** | 1.0753*** | 0.8876*** | 0.9635*** | 2.6605*** | 0.9494*** | 0.2952** |
| R2 | 0.5329 | 0.4046 | 0.4462 | 0.5436 | 0.4237 | 0.3293 | 0.4080 | 0.1429 |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table 5.2. Regressions of volatility time series on VVOL - betas. The underlying is the S&P 500 equity index for subsets of dates after January 2000. The regressions on the eight VVOL measures are univariate linear regressions with a constant (alpha). The volatility measures are adapted to fit the VVOL measures by using realized measures, due to the Oxford-Man Institute for the realized VVOL measures and the VIX for the implied VVOL measures. Furthermore date matching is employed for every individual measure. This maximizes the length of each data pair. The $R^2$ values of these regressions are above 40% in 6 out of 8 cases.*

## 5.2 Results from Time-Series Regression Analyses with Equity Portfolios

As described in the Methodology part, five families of regressions were used to analyze the linear relation between our eight VVOL measures and the equity market. Staring with *Family 1*, we see that the univariate regression does not produce high significance level for VVOL betas. As soon as we include and control for the original volatility series in the regressions of *Family 2*, the VVOL coefficients shift statistically not significant to significant, usually at the 99% level. This effect appears consistently across all portfolio types. Significance levels are also high for *Family 5* regressions, where we conduct a multivariate regression controlling for volatility and the original factor. The *Family 5* regressions basically render all the VVOL betas significant. This indicates that VVOL still has a role to play and is significant beyond that of the factor, even though $R^2$ and intercepts are close to ideal. The rest of the regression families do not produce significant VVOL betas, so we will focus the analysis on *Family 2* and *Family 5* regressions. We will also discuss other regression statistics qualitatively but we will only include tables of VVOL betas in the main content and leave tables of intercepts and $R^2$ values in the Appendix. By including *Family 5* betas we enable an investigation of the explanatory power of

VVOL when we strip the factor effect out from the factor portfolios, and whether there is a pattern in this marginal VVOL effect.

In the *Family 4* regressions, when we control for MRP in the regression, the significance level of VVOL coefficients drops to the extent that not a single beta across all deciles of a portfolio is significant. What we see here is probably due to the outstanding explanatory power of the MRP in the factor portfolios. The construction of the portfolios makes them diversified and “similar” to the market. Another possible explanation of the weak VVOL explanatory power beyond MRP for portfolios could be that the VVOL we calculate is the VVOL of the market portfolio so a large part of this VVOL might be embedded in the market risk premium already.

### 5.2.1 Correlation Between Fama-French Factors and VVOL

Correlations Between Risk Factors and VVOL Measures

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| MRP | -0,0158 | 0,0005 | 0,0268 | -0,0055 | -0,0228 | -0,0209 | 0,0401 | -0,1616 |
| SMB | -0,0323 | -0,0419 | -0,0619 | -0,0062 | -0,026 | -0,0389 | -0,0467 | -0,0665 |
| HML | -0,0454 | -0,0439 | -0,0347 | -0,042 | -0,0585 | -0,0641 | -0,0563 | -0,107 |
| Momentum | -0,0182 | -0,0382 | -0,0278 | -0,0396 | -0,0051 | -0,0246 | -0,049 | 0,0872 |
| STR | 0,0839 | 0,0992 | 0,1353 | 0,0829 | 0,0694 | 0,0455 | 0,0756 | -0,021 |
| LTR | -0,0493 | -0,0586 | -0,0513 | -0,0211 | -0,0467 | -0,0607 | -0,068 | -0,1059 |

*Table 5.3. Correlations between VVOL measures and risk factor returns. The risk factors are taken from the Kenneth French Data Library. MRP is the market risk premium, the return of a value weighted portfolio of a broad market index minus the risk-free rate. SMB, Small minus Big, and HML, High minus Low, are the famous Fama-French factors formed by a 2x3 sorting on size and book-to-market of all NYSE, AMEX and Nasdaq stocks from 7/1963 to 1/2015. The Momentum factor as well as the STR, Short Term Reversal and LTR, Long Term Reversal factors are also due to K. French. The Momentum factor is the average of a portfolio of small company stocks with high past returns (in the past 2-12 months) and big company stocks with high past returns minus the average of a portfolio of small company stocks and big company stocks, both with low returns in the past 2-12 months. The STR factor is defined as the average of a portfolio of small stocks with low past returns (previous month) and big stocks with low past returns minus a portfolio of small and big stocks with high returns in the previous month. The percentiles used for high and low past returns are the 70$^{th}$ and 30$^{th}$ percentiles, respectively. The LTR factor is defined in the same way, with the evaluation period of past returns being set to 13-60 months prior to the evaluation period. All stocks belong to companies listed on NYSE, AMEX and Nasdaq at the time of the recording. More information is provided in the Kenneth French Data Library available online.*

From Table 5.3 shown above, we observe that in general VVOL has negative correlation with the common Fama-French risk factors, except for the short-term reversal factor. The results seem to imply that VVOL works more like a hedging factor that acts oppositely with the common risk factors. With STR, quite interestingly, all VVOL measures have positive correlation except for the VVIX which has negative correlation with STR. The correlation signs for the Momentum factor and STR with VVOL are opposite. By shedding light on the definition of the two factors, we think this pattern reflects the opposite nature of them.

Multivariate Regressions of Fama French factors on VOL and VVOL measures

VVOL betas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| MRP | 0.5489*** | 0.3878*** | 0.4689*** | 0.2789*** | 2.6515*** | 0.3570*** | 0.6922*** | -0.2178*** |
| SMB | -0.0960 | -0.0795** | -0.1306*** | 0.0290 | -0.0545 | -0.0554 | -0.0761** | -0.0478*** |
| HML | -0.0794 | -0.0349 | -0.0117 | -0.0216 | -0.3515** | -0.0929*** | -0.0700** | -0.0646*** |
| momentum | 0.1278 | -0.0455 | -0.0019 | -0.0409 | -0.2082 | -0.1257** | -0.2362*** | 0.1304*** |
| STR | 0.4905*** | 0.3271*** | 0.4668*** | 0.1902*** | 1.4207*** | 0.1394** | 0.2707*** | -0.0374 |
| LTR | -0.0179 | -0.0362 | -0.0229 | 0.0598** | -0.3923** | -0.1097*** | -0.1261*** | -0.0730*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table 5.4. Multivariate regressions of risk factor returns on volatility and VVOL – VVOL betas. The VVOL is the volatility of volatility of the S&P 500 equity index calculated in eight different ways, using returns, realized variance and implied volatility as input. The VVOL data covers various subsamples of the post 2000 period depending on measure, the most constrained subset being that of the VVIX data. The VOL measures (also of the S&P 500 index) are adapted to fit the VVOL measures in subsets of time, i.e. applying date matching and by using realized measures, due to the Oxford-Man Institute for the realized VVOL measures and the VIX for the implied VVOL measures.*

In this part we, regress the six Fama-French factors on our eight VVOL measures, by controlling for the corresponding volatility series. As far as we can tell, there are only two factors that have most of the VVOL beta coefficients significant, namely MRP and STR. With market risk premium, we see that all the VVOL measures but VVIX have a positive linear relation. On the contrary, the beta coefficient of VVIX has a negative sign, indicating the different features that VVIX, as a fully implied VVOL sequence, has compared with other measures. In the STR regression, we see that the first seven VVOL measures have positive significant beta coefficients, but VVIX has a negative insignificant beta. The close relation between VVOL and market risk premium is intuitive since simply volatility of volatility of the S&P 500 index is part of the total market risk. Nevertheless, the high significance level of VVOL coefficients in the STR regression might be suggesting that short-term reversal is tightly connected to VVOL. We dig deeper into this in the following parts of the paper.

### *5.2.2 Regressions of the S&P 500 Index on VVOL and Additional Factors*

The quantitative results from the S&P 500 regressions are shown below in Table 5.5. Coefficients on most of the VVOL measures are significant, but not for the realized rolling window when controlling for STR and implied EWMA, implied GARCH-2, VVIX are not when including MRP in the regressions. All measures but the VVIX have positive coefficients in the regressions as long as we do not control for MRP, but have negative coefficients when we do. The VVIX, as argued before, is a the most different measure and has the coefficient sign reversed. Since the explanatory power of MRP is so high that it could distort the regression, we will focus on the regressions without MRP. That means, with VVOL being always positive, the more volatile the volatility is, the more VVOL is going to drag down the S&P 500 returns in the case of VVIX. In this case VVIX, as a VVOL measure that has unique features from others, might seem to work as a hedging factor given the sign of the coefficients. Other VVOL measures seem to work more like risk factors. Additionally, for regressions using one additional factor beyond volatility and VVOL, only the HML factors has difficulties in having significant loadings; it is significant only when VVIX acts as VVOL.

In terms of alphas in the regressions, we see that when we include MRP as an explanatory variable in the regression, the value of alpha drops quite significantly to close to 0. Concerning the $R^2$ statistics in the regressions, we see that with MRP as an explanatory variable in the regression, it increases dramatically to a high figure close to 100%. Both of these findings are reasonable, we believe, since market risk premium we use here is simply the excess return of a larger set of US stocks than the S&P 500. For $R^2$ in the other regressions we observe that the Momentum factor and STR actually increase $R^2$ quite a lot to around 13% and 14%. A quick regression of the S&P 500 returns on volatility and STR, excluding VVOL, shows an $R^2$ of 13.83%. Thus the additional effect from VVOL in these regressions is minimal. The relation between VVOL and the Fama-French stock market factors will be discussed further in the sections that follow.

Multivariate Regressions of the S&P500 on VOL, VVOL and Fama-French Factors
factor loadings and intercepts

| | | Factor loading | | | | | | | | Intercept | Fit |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | VOL | VVOL | MRP | SMB | HML | MOM | STR | LTR | alpha | R2 |
| Realized Measure | ROLWIN | -0.2434*** | 0.4211*** | | | | | | | 0.0011*** | 0,0076 |
| | | 0.0267*** | -0.0910*** | 0.9380*** | | | | | | -0.0001* | 0,9769 |
| | | -0.2439*** | 0.4326*** | | 0.0915*** | | | | | 0.0011*** | 0,0098 |
| | | -0.2434*** | 0.4254*** | | | 0.0195 | | | | 0.0011*** | 0,0078 |
| | | -0.2854*** | 0.4780*** | | | | -0.4230*** | | | 0.0014*** | 0,1349 |
| | | -0.2277*** | 0.2032 | | | | | 0.4498*** | | 0.0014*** | 0,1388 |
| | | -0.2514*** | 0.4213*** | | | | | | -0.1443*** | 0.0012*** | 0,0119 |
| | | 0.0298*** | -0.1075*** | 0.9464*** | -0.1447*** | 0.0251*** | | | | -0.0000 | 0,9825 |
| | | 0.0276*** | -0.0977*** | 0.9471*** | -0.1453*** | 0.0094* | -0.0181*** | -0.0183*** | 0.0108* | -0.0000 | 0,9828 |
| | EWMA | -0.2962*** | 0.3240*** | | | | | | | 0.0013*** | 0,0107 |
| | | 0.0224*** | -0.0367*** | 0.9380*** | | | | | | -0.0001*** | 0,9768 |
| | | -0.2989*** | 0.3346*** | | 0.0948*** | | | | | 0.0012*** | 0,0130 |
| | | -0.2970*** | 0.3278*** | | | 0.0200 | | | | 0.0012*** | 0,0109 |
| | | -0.3171*** | 0.3079*** | | | | -0.4209*** | | | 0.0016*** | 0,1368 |
| | | -0.2645*** | 0.1808*** | | | | | 0.4472*** | | 0.0014*** | 0,1400 |
| | | -0.3031*** | 0.3219*** | | | | | | -0.1419*** | 0.0013*** | 0,0149 |
| | | 0.0282*** | -0.0507*** | 0.9467*** | -0.1450*** | 0.0251*** | | | | -0.0001** | 0,9824 |
| | | 0.0261*** | -0.0459*** | 0.9472*** | -0.1455*** | 0.0093* | -0.0184*** | -0.0182*** | 0.0107* | -0.0001** | 0,9828 |
| | GARCH-2 | -0.3035*** | 0.3851*** | | | | | | | 0.0010*** | 0,0136 |
| | | 0.0270*** | -0.0523*** | 0.9386*** | | | | | | -0.0001* | 0,9769 |
| | | -0.3076*** | 0.4010*** | | 0.1012*** | | | | | 0.0009*** | 0,0163 |
| | | -0.3037*** | 0.3880*** | | | 0.0188 | | | | 0.0010*** | 0,0138 |
| | | -0.3327*** | 0.3869*** | | | | -0.4219*** | | | 0.0013*** | 0,1403 |
| | | -0.2539*** | 0.1802*** | | | | | 0.4446*** | | 0.0013*** | 0,1401 |
| | | -0.3111*** | 0.3845*** | | | | | | -0.1423*** | 0.0011*** | 0,0179 |
| | | 0.0357*** | -0.0754*** | 0.9476*** | -0.1467*** | 0.0250*** | | | | -0.0000 | 0,9826 |
| | | 0.0326*** | -0.0677*** | 0.9479*** | -0.1472*** | 0.0096* | -0.0176*** | -0.0168*** | 0.0113** | -0.0000 | 0,9829 |
| | GARCH-3 | -0.2804*** | 0.2232*** | | | | | | | 0.0004 | 0,0095 |
| | | 0.0283*** | -0.0377*** | 0.9382*** | | | | | | 0.0000 | 0,9769 |
| | | -0.2772*** | 0.2213*** | | 0.0873*** | | | | | 0.0004 | 0,0114 |
| | | -0.2801*** | 0.2243*** | | | 0.0193 | | | | 0.0004 | 0,0096 |
| | | -0.2987*** | 0.2066*** | | | | -0.4209*** | | | 0.0008** | 0,1355 |
| | | -0.2644*** | 0.1386*** | | | | | 0.4485*** | | 0.0009** | 0,1399 |
| | | -0.2947*** | 0.2329*** | | | | | | -0.1504*** | 0.0004 | 0,0141 |
| | | 0.0260*** | -0.0353*** | 0.9464*** | -0.1438*** | 0.0254*** | | | | 0.0000 | 0,9824 |
| | | 0.0252*** | -0.0337*** | 0.9470*** | -0.1447*** | 0.0090* | -0.0187*** | -0.0185*** | 0.0118** | 0.0000 | 0,9828 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Implied Measure | ROLWIN | -0.5794*** | 2.3754*** | | | | | | | 0.0049*** | 0,0287 |
| | | 0.0011 | -0.0906* | 0.9371*** | | | | | | -0.0000 | 0,9768 |
| | | -0.5764*** | 2.3756*** | | 0.0869*** | | | | | 0.0048*** | 0,0304 |
| | | -0.5781*** | 2.3767*** | | | 0.0185 | | | | 0.0049*** | 0,0287 |
| | | -0.5661*** | 2.2924*** | | | | -0.4159*** | | | 0.0049*** | 0,1517 |
| | | -0.5167*** | 1.7482*** | | | | | 0.4416*** | | 0.0046*** | 0,1549 |
| | | -0.5758*** | 2.3219*** | | | | | | -0.1327*** | 0.0049*** | 0,0322 |
| | | 0.0033 | -0.1130** | 0.9456*** | -0.1445*** | 0.0245*** | | | | -0.0000 | 0,9824 |
| | | 0.0010 | -0.0932** | 0.9459*** | -0.1447*** | 0.0087* | -0.0190*** | -0.0182*** | 0.0102* | 0.0000 | 0,9827 |
| | EWMA | -0.4193*** | 0.3274*** | | | | | | | 0.0042*** | 0,0220 |
| | | -0.0097 | -0.0021 | 0.9367*** | | | | | | 0.0000 | 0,9768 |
| | | -0.4190*** | 0.3342*** | | 0.0899*** | | | | | 0.0041*** | 0,0240 |
| | | -0.4192*** | 0.3309*** | | | 0.0184 | | | | 0.0042*** | 0,0221 |
| | | -0.3951*** | 0.2792*** | | | | -0.4146*** | | | 0.0041*** | 0,1443 |
| | | -0.4094*** | 0.2644*** | | | | | 0.4468*** | | 0.0041*** | 0,1520 |
| | | -0.4167*** | 0.3144*** | | | | | | -0.1325*** | 0.0042*** | 0,0256 |
| | | -0.0065 | -0.0108 | 0.9452*** | -0.1445*** | 0.0246*** | | | | 0.0000 | 0,9823 |
| | | -0.0060 | -0.0110 | 0.9456*** | -0.1447*** | 0.0086* | -0.0192*** | -0.0184*** | 0.0102* | 0.0000 | 0,9827 |
| | GARCH-2 | -0.5557*** | 0.6480*** | | | | | | | 0.0046*** | 0,0399 |
| | | -0.0126* | 0.0044 | 0.9365*** | | | | | | 0.0000 | 0,9768 |
| | | -0.5565*** | 0.6581*** | | 0.0973*** | | | | | 0.0046*** | 0,0424 |
| | | -0.5557*** | 0.6521*** | | | 0.0217 | | | | 0.0046*** | 0,0402 |
| | | -0.5132*** | 0.5567*** | | | | -0.4069*** | | | 0.0045*** | 0,1575 |
| | | -0.5220*** | 0.5301*** | | | | | 0.4373*** | | 0.0045*** | 0,1640 |
| | | -0.5530*** | 0.6354*** | | | | | | -0.1195*** | 0.0047*** | 0,0430 |
| | | -0.0064 | -0.0111 | 0.9454*** | -0.1446*** | 0.0246*** | | | | 0.0000 | 0,9823 |
| | | -0.0063 | -0.0106 | 0.9457*** | -0.1448*** | 0.0086* | -0.0192*** | -0.0183*** | 0.0102* | 0.0000 | 0,9827 |
| | VVIX | -0.1705*** | -0.1932*** | | | | | | | 0.0130*** | 0,0311 |
| | | -0.0127** | 0.0061 | 0.9286*** | | | | | | -0.0002 | 0,9818 |
| | | -0.1680*** | -0.1843*** | | 0.2636*** | | | | | 0.0124*** | 0,0460 |
| | | -0.1277*** | -0.1402*** | | | 0.8744*** | | | | 0.0095*** | 0,1981 |
| | | -0.1969*** | -0.1330*** | | | | -0.4866*** | | | 0.0100*** | 0,1926 |
| | | -0.1873*** | -0.1801*** | | | | | 0.4661*** | | 0.0123*** | 0,1563 |
| | | -0.1677*** | -0.1831*** | | | | | | 0.1864*** | 0.0124*** | 0,0381 |
| | | -0.0115* | 0.0037 | 0.9325*** | -0.0856*** | 0.0119* | | | | -0.0001 | 0,9834 |
| | | -0.0124** | 0.0060 | 0.9361*** | -0.0986*** | -0.0165* | -0.0059 | -0.0029 | 0.0452*** | -0.0002 | 0,9836 |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table 5.5. Statistics for Regressions of S&P 500 Index Returns on VVOL and Additional Factors. For each of the VVOL measures, our regressions start by using only VVOL and volatility series as explanatory variables, We then proceed by including different single factors into the regression, and finally control for multiple Fama-French factors in a multivariable setting. In total we have 9 regressions for each of the VVOL measures. VVOL betas in the table are generally positive. Only in the case of implied EWMA, implied GARCH-2, and VVIX, do the betas become insignificant when we control for MRP. In the case of realized rolling window the beta turns insignificant when we control for a single factor of Short-Term Reversal. All VVOL measures except for VVIX have positive betas without controlling for MRP but they turn negative or insignificant with MRP. The VVIX coefficients are negative without controlling for MRP and insignificant with the control. The HML factor does not actually have a significant impact to the S&P 500 index in most of the cases except for VVIX.*

### *5.2.3 Regressions of Ten Portfolios Sorted on Size*

#### 5.2.3.1 *Family 2* Regressions

Multivariate Regressions of Size Portfolios on VOL and VVOL- Betas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| Low | 0.5408*** | 0.3058*** | 0.3534*** | 0.3932*** | 1.8034*** | 0.1465** | 0.3726*** | -0.2487*** |
| 2 | 0.4508** | 0.3037*** | 0.3308*** | 0.3218*** | 2.5340*** | 0.2669*** | 0.5963*** | -0.2818*** |
| 3 | 0.4401** | 0.2952*** | 0.3145*** | 0.2912*** | 2.6559*** | 0.2953*** | 0.6201*** | -0.2785*** |
| 4 | 0.4507*** | 0.3149*** | 0.3462*** | 0.2904*** | 2.5791*** | 0.3044*** | 0.6275*** | -0.2762*** |
| 5 | 0.5212*** | 0.3377*** | 0.3845*** | 0.2947*** | 2.7746*** | 0.3244*** | 0.6575*** | -0.2645*** |
| 6 | 0.6264*** | 0.3907*** | 0.4462*** | 0.3236*** | 2.6647*** | 0.3289*** | 0.6445*** | -0.2584*** |
| 7 | 0.6659*** | 0.4082*** | 0.4935*** | 0.3549*** | 2.6451*** | 0.3337*** | 0.6648*** | -0.2528*** |
| 8 | 0.6259*** | 0.3953*** | 0.4847*** | 0.3365*** | 2.6229*** | 0.3365*** | 0.6753*** | -0.2438*** |
| 9 | 0.5222*** | 0.3461*** | 0.4326*** | 0.2891*** | 2.4754*** | 0.3265*** | 0.6624*** | -0.2259*** |
| high | 0.5270*** | 0.3795*** | 0.4632*** | 0.2487*** | 2.5802*** | 0.3558*** | 0.6836*** | -0.1943*** |

*** Significant at 1%
** Significant at 5%
*Significant at 10%

*Table 5.6. Regression of size portfolios on volatility and VVOL – VVOL betas. The risk factors are taken from the Kenneth French Data Library. SMB, Small minus Big, is the Fama-French factor formed by a sorting on size in two portfolios of equally weighted small and big companies, respectively and by subtracting the portfolio of big companies from the portfolio of small companies. All stocks belong to companies listed on NYSE, AMEX and Nasdaq at the time of the recording. More information is provided in the Kenneth French Data Library. The regressions on the eight volatility and VVOL measures are multivariate linear regressions with a constant (alpha). The volatility measures are adapted to fit the VVOL measures in subsets of time, i.e.by applying date matching and by using realized measures, due to the Oxford-MAN Institute for the realized VVOL measures, and the VIX for the implied VVOL measures.*

Table 5.6 shows the VVOL betas and the significance levels of the coefficients in the *Family 2* multivariate regression of ten portfolios sorted on size. All the VVOL coefficients are significant at the 99% level, except for rolling window realized that has 2 out of 10 coefficients significant at the 95% level. Similar to what was seen in the S&P 500 regressions, VVIX is the only measure that exhibits negative loadings on VVOL. The values of VVOL betas increase with size deciles for the four implied VVOL measures, with the absolute loading on VVIX in a mild decline and a 5% difference between the biggest and the smallest deciles. Realized EWMA and realized GARCH-2 experience ascending VVOL loadings with size deciles, while the VVOL betas of realized GARCH-3 are in a decreasing trend. Realized rolling window does not have a clear difference between the smallest and biggest deciles. The behaviors of $R^2$ vary quite a bit. The $R^2$ values of implied rolling window and implied EWMA swing in a certain level across all portfolio deciles. For the rest of the measures, the $R^2$ statistics decrease as the portfolio decile increases. The alphas of this regression also decrease slightly with size deciles but are all significant. The tables of $R^2$ and intercepts are shown in the Appendix.

#### 5.2.3.2 *Family 5* Regressions

Multivariate Regressions of Size Portfolios on VOL, VVOL and SMB Factor- Betas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH2 | GARCH3 | ROLWIN | EWMA | GARCH2 | VVIX |
| Low | 0.6345*** | 0.3836*** | 0.4822*** | 0.3651*** | 1.8562*** | 0.2003*** | 0.4468*** | -0.1942*** |
| 2 | 0.5765*** | 0.4081*** | 0.5032*** | 0.2840*** | 2.6048*** | 0.3391*** | 0.6960*** | -0.2083*** |
| 3 | 0.5586*** | 0.3936*** | 0.4771*** | 0.2556*** | 2.7228*** | 0.3634*** | 0.7142*** | -0.2111*** |
| 4 | 0.5565*** | 0.4028*** | 0.4915*** | 0.2586*** | 2.6387*** | 0.3652*** | 0.7115*** | -0.2167*** |
| 5 | 0.6150*** | 0.4156*** | 0.5135*** | 0.2666*** | 2.8275*** | 0.3783*** | 0.7322*** | -0.2132*** |
| 6 | 0.6942*** | 0.4471*** | 0.5400*** | 0.3033*** | 2.7028*** | 0.3678*** | 0.6984*** | -0.2190*** |
| 7 | 0.7227*** | 0.4555*** | 0.5725*** | 0.3380*** | 2.6769*** | 0.3662*** | 0.7101*** | -0.2214*** |
| 8 | 0.6737*** | 0.4352*** | 0.5514*** | 0.3222*** | 2.6496*** | 0.3638*** | 0.7134*** | -0.2174*** |
| 9 | 0.5502*** | 0.3696*** | 0.4722*** | 0.2808*** | 2.4909*** | 0.3424*** | 0.6848*** | -0.2066*** |
| high | 0.5291*** | 0.3815*** | 0.4675*** | 0.2482*** | 2.5811*** | 0.3569*** | 0.6856*** | -0.1867*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table 5.7. Regression of size portfolios on volatility, VVOL and SMB – VVOL betas. The risk factors are taken from the Kenneth French Data Library. SMB, Small minus Big, is the Fama-French factor formed by a sorting on size in two portfolios of equally weighted small and big companies, respectively and by subtracting the portfolio of big companies from the portfolio of small companies. All stocks belong to companies listed on NYSE, AMEX and Nasdaq at the time of the recording. More information is provided in the Kenneth French Data Library available online. The regressions on volatility, VVOL and SMB are multivariate linear regressions with a constant (alpha). The SMB and VOL returns are adapted to fit the VVOL measures in subsets of time, i.e. by applying date matching and by using realized measures, due to the Oxford-Man Institute for the realized VVOL measures, and the VIX for the implied VVOL measures.*

By adding the SMB factor as an explanatory variable in the regression, a multivariate regression on volatility, VVOL and SMB is obtained. The VVOL loadings are shown in Table 5.7 above. We can clearly see that all the $\beta_{VVOL}$ coefficients are significant in this group of regressions. VVOL loadings of the three implied measures apart from VVIX rise with the portfolio decile and $\beta_{VVIX}$ follows a U-shaped pattern. Realized rolling window and realized GARCH-3 have decreasing $\beta_{VVOL}$ and the remaining two of the realized measures do not have a clear pattern for $\beta_{VVOL}$ with size deciles. The $R^2$statistics for all eight VVOL measures follow a clearly decreasing trend as the portfolio decile increases. However, that does not provide any new information since the decreasing $R^2$ statistics are due to the decreasing exposure of the portfolios to SMB factor. Alphas are all significant in these regressions.

To conclude the findings from the tables, it should be said that there is not a clear pattern that indicates a strong relationship between the SMB factor and VVOL. This set of tables shows that VVIX exhibits different features than other measures in the regressions. This could be because VVIX is a much more volatile measure and operates on a higher absolute level of VVOL, as mentioned earlier in this chapter.

Throughout our regression studies on factor portfolios, we observe that neither HML nor LTR show a clear relationship to VVOL, because just like the coefficients in the size portfolio regressions, their

VVOL betas do not have a clear monotonic pattern with the increase of portfolio decile. So for the sake of simplicity, we only analyze size portfolio regressions as representative for the HML and LTR portfolios and will not discuss regression results for these as they are similar to the regressions results of size portfolios. We will, however discuss the regressions of STR and Momentum portfolios in the next section.

### 5.2.4 Regressions of Ten Portfolios Sorted by Short-Term Reversal Factor and Momentum Factor

#### 5.2.4.1 *Family 2* Regressions

Multivariate Regressions of Portfolios Formed by S-T Reversal Factor on VOL and VVOL- Betas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| Low | 1.3584*** | 1.0110*** | 1.2289*** | 0.6666*** | 5.0958*** | 0.7019*** | 1.2328*** | -0.3244*** |
| 2 | 0.7815*** | 0.5977*** | 0.7810*** | 0.3964*** | 3.7113*** | 0.4680*** | 0.9165*** | -0.2719*** |
| 3 | 0.6934*** | 0.5285*** | 0.6819*** | 0.3568*** | 3.1604*** | 0.4298*** | 0.8366*** | -0.2564*** |
| 4 | 0.6390*** | 0.4958*** | 0.6054*** | 0.3786*** | 2.7040*** | 0.4151*** | 0.7803*** | -0.2278*** |
| 5 | 0.5218*** | 0.3615*** | 0.4450*** | 0.2559*** | 2.5632*** | 0.3325*** | 0.6737*** | -0.2213*** |
| 6 | 0.4746*** | 0.3526*** | 0.4281*** | 0.2551*** | 2.3180*** | 0.3275*** | 0.6538*** | -0.2203*** |
| 7 | 0.4708*** | 0.3431*** | 0.4076*** | 0.2318*** | 2.2656*** | 0.3423*** | 0.6554*** | -0.2031*** |
| 8 | 0.5050*** | 0.3780*** | 0.4512*** | 0.2556*** | 2.3739*** | 0.3615*** | 0.6536*** | -0.2174*** |
| 9 | 0.4854*** | 0.3282*** | 0.3442*** | 0.2461*** | 2.4515*** | 0.3565*** | 0.6591*** | -0.2314*** |
| high | 0.3527* | 0.2966*** | 0.3026*** | 0.2315*** | 2.5299*** | 0.3914*** | 0.7188*** | -0.2508*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table 5.8. Regression of short-term reversal portfolios on volatility and VVOL– VVOL betas. The risk factors are taken from the Kenneth French Data Library. The STR factor is defined as the average of a portfolio of small stocks with low past returns (previous month) and big (high market capitalization) stocks with low past returns minus a portfolio of small and big stocks with high returns in the previous month. The percentiles used for high and low past returns are the 70$^{th}$ and 30$^{th}$ percentiles, respectively. All stocks belong to companies listed on NYSE, AMEX and Nasdaq at the time of the recording. More information is provided in the Kenneth French Data Library. The regressions on volatility and VVOL are multivariate linear regressions with a constant (alpha). The volatility measures are adapted to fit the VVOL measures in subsets of time, i.e. by using date matching and realized measures, due to the Oxford-MAN Institute for the realized VVOL measures, and the VIX for the implied VVOL measures.*

The VVOL betas in the *Family 2* multivariate regressions of the ten STR portfolios on volatility and VVOL are summarized and presented in Table 5.8 above. Most of the $\beta_{VVOL}$ coefficients in these multivariate regressions are significant at the 99% level; there is only one $\beta_{VVOL}$ significant at the 90% level, namely that of realized rolling window in the highest portfolio decile. Just as was seen in previous regressions, VVOL betas are positive for all the VVOL measures but VVIX. VVIX has negative VVOL loadings for all the portfolio deciles. Interestingly, there is a consistent monotonic pattern for VVOL betas with the portfolio decile. For all VVOL measures, the absolute value of VVOL loadings decreases as the portfolio decile increases. Most VVOL measures have increasing $R^2$ with the increasing portfolio decile. The $R^2$ of realized EWMA does not vary much while the $R^2$ statistics of

GARCH-2 realized decrease. Detailed tables of these patterns are available in the Appendix. Most of the regression intercepts remain significant in the regressions. Realized GARCH-3 and the three implied VVOL measures except VVIX have their alphas roughly unchanged with portfolio decile, while the alphas of VVIX decrease slightly. The first three realized VVOL measures have increasing alphas with portfolio decile.

#### 5.2.4.2 *Family 5* Regressions

Multivariate Regressions of Portfolios by S-T Reversal Factor on VOL, VVOL and STR- Betas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| Low | 0.5436*** | 0.4760*** | 0.4672*** | 0.3559*** | 2.7402*** | 0.4767*** | 0.7934*** | -0.2665*** |
| 2 | 0.2111 | 0.2228*** | 0.2470*** | 0.1789*** | 2.0688*** | 0.3109*** | 0.6102*** | -0.2300*** |
| 3 | 0.2570* | 0.2421*** | 0.2746*** | 0.1905*** | 1.9081*** | 0.3099*** | 0.6035*** | -0.2235*** |
| 4 | 0.3188** | 0.2861*** | 0.3079*** | 0.2569*** | 1.7861*** | 0.3270*** | 0.6101*** | -0.2040*** |
| 5 | 0.2704* | 0.1965*** | 0.2108*** | 0.1601*** | 1.8473*** | 0.2636*** | 0.5407*** | -0.2020*** |
| 6 | 0.2839** | 0.2278*** | 0.2517*** | 0.1826*** | 1.7786*** | 0.2754*** | 0.5540*** | -0.2050*** |
| 7 | 0.3304** | 0.2516*** | 0.2790*** | 0.1784*** | 1.8717*** | 0.3040*** | 0.5829*** | -0.1918*** |
| 8 | 0.4367*** | 0.3344*** | 0.3922*** | 0.2298*** | 2.1887*** | 0.3429*** | 0.6199*** | -0.2112*** |
| 9 | 0.4736*** | 0.3216*** | 0.3372*** | 0.2418*** | 2.4320*** | 0.3536*** | 0.6564*** | -0.2298*** |
| high | 0.3863** | 0.3202*** | 0.3388*** | 0.2448*** | 2.6473*** | 0.4015*** | 0.7419*** | -0.2527*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table 5.9. Regression of portfolios sorted on short-term reversal on volatility, VVOL and the STR Factor– VVOL betas. The risk factors are taken from the Kenneth French Data Library. The STR factor is defined as the average of a portfolio of small stocks with low past returns (previous month) and big (high market capitalization) stocks with low past returns minus a portfolio of small and big stocks with high returns in the previous month. The percentiles used for high and low past returns are the 70th and 30th percentiles, respectively. All stocks belong to companies listed on NYSE, AMEX and Nasdaq at the time of the recording. More information is provided in the Kenneth French Data Library available online. The regressions on volatility, VVOL and STR are multivariate linear regressions with a constant (alpha). The volatility measures are adapted to fit the VVOL measures in subsets of time, i.e. by applying date matching and by using realized measures, due to the Oxford-MAN Institute for the realized VVOL measures, and the VIX for the implied VVOL measures.*

In the *Family 5* regressions for short-term reversal portfolios, we add the short-term reversal factor as an explanatory variable into the regressions. The VVOL betas are shown in Table 5.9 above. Controlling for volatility, almost all VVOL betas are significant at the 99% level. Only VVIX has a negative sign for the loadings while the remaining seven measures have positive loadings. As for the pattern of betas along portfolio deciles, the loadings of the realized measures decrease slightly with portfolio deciles, but the effect is weaker compared to the *Family 2* regressions. For the implied measures, the pattern is not visible. $R^2$ statistics decrease with portfolio deciles for all measures but alphas do not vary.

This set of results reveals a special connection between the STR factor and VVOL. The monotonic pattern in the regressions on volatility and VVOL is visible because VVOL might be the very *cause* behind short-term reversal. Once controlled for the STR factor in the regression, we observe a faded version of this effect, which means that the STR takes away most of the effect. VVOL and STR seem to overlap in this sense.

#### 5.2.4.3 The Mirror Image of Momentum

Not surprisingly, the *Family 2* and *Family 5* regressions of the Momentum factor sorted portfolios show the same patterns and similar results as the ones for portfolios sorted on STR. The reason is most probably that the construction of these two types of portfolios is similar. Simply put, the Momentum factor is the opposite of short-term reversal, as defined in the Kenneth French Data Library. It is thus intuitive that high exposure to one factor will transfer to the high exposure to the other one. Since the Momentum portfolios are basically telling the same story as the STR portfolios, we will neither analyze the regressions nor include the tables here. Instead, regression tables are available in the Appendix.

## 5.3 Cross-Sectional Analysis of Returns over VVOL Sensitivities for Equity Portfolios

It has been hard to draw any conclusions on whether VVOL is a proxy for any of the Fama-French factors by studying trends across deciles within portfolios. One exception are the *Family 2* regressions of the STR portfolios which show a monotonically decreasing exposure to VVOL with increasing STR deciles, compared with the *Family 5* regressions of the same portfolios where this pattern becomes a lot less clear. Instead, cross sectional studies have been performed in our thesis to see if VVOL exposure affects returns. As mentioned in chapter 3, Methodology, cross sectional analyses are performed for all Fama-French portfolios and equity portfolios sorted on sectors. Furthermore, the analysis has been done with regressions of all families. Here, for simplicity, three of the analyses have been selected for a closer presentation.

### *5.3.1 Cross-Sectional Analysis of STR portfolios*

The cross-sectional study of the STR portfolios illustrates the strong and positive relationship between VVOL and STR factor by a strong positive relationship between VVOL loadings and returns. From these results, it is tempting to state that VVOL shares a lot of characteristics with short-term reversal. A factor that is similarly but oppositely defined as STR is Momentum, which as far as VVOL is concerned, in much has the similarly close relation to VVOL as STR but with an inverted sign. The cross sectional plot in Figure 5.3 below is thus used here as representative of both STR and Momentum portfolios in the cross-sectional regression on VVOL. As can be seen in the portfolio regression tables, the inclusion of volatility in the regressions make the $\beta_{VVOL}$ significant and, as will be discussed later, the volatility seems to have a complementary effect on VVOL and vice versa. The regression is of *Family 2*, i.e. controlling for volatility and not taking into consideration neither the MRP nor the factor itself. The coefficients of the linear regression in the plots are printed directly on the plot area.

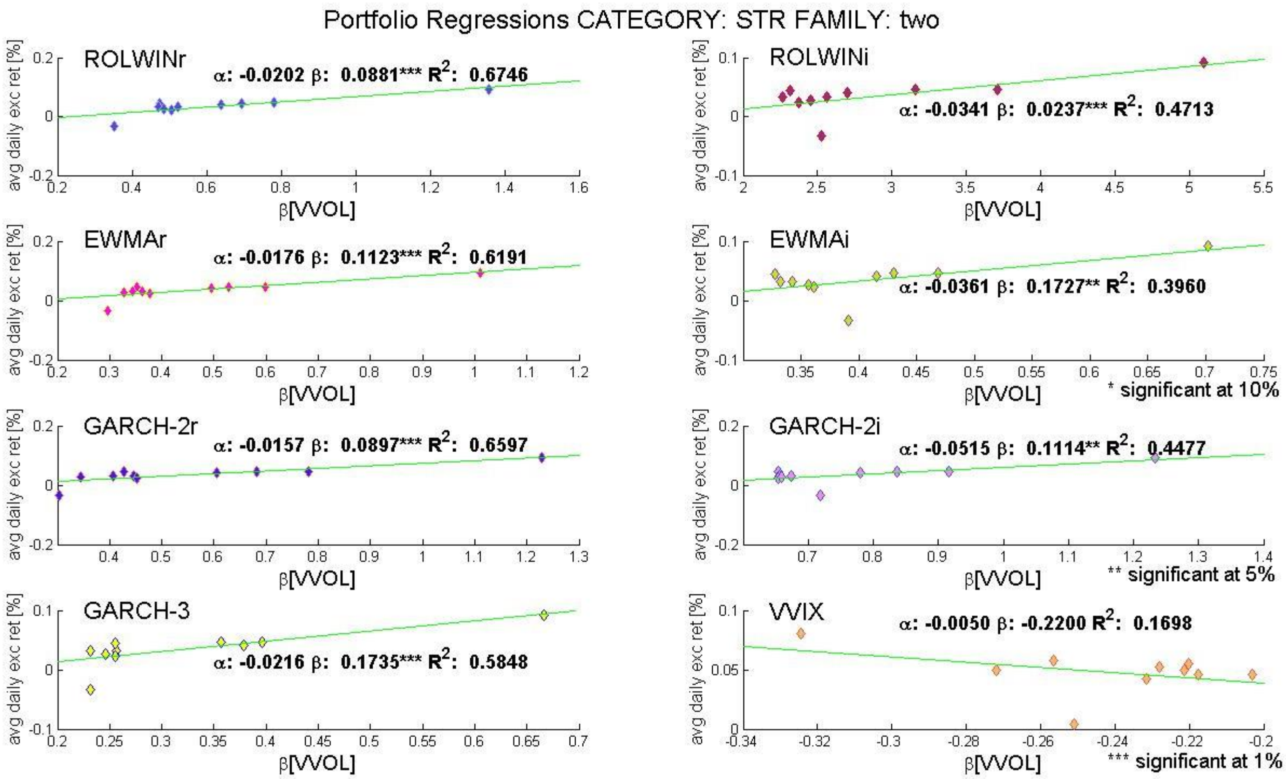


*Figure 5.3. Cross-sectional analysis of the STR portfolios. Scatter plots of mean returns versus loadings on VVOL. The regressions have been controlled for volatility. The regressions assume a constant beta over time. The plots show a positive relation between returns and VVOL, just as is the case if they were regressed on the STR factor. The exception is the case when VVIX acts as a measure for VVOL. As the only implied volatility of implied volatility, it stands out. The corresponding Momentum portfolios are qualitatively identical, but with inverted slopes for all measures.*

As can be seen in Figure 5.3, there is a positive linear relationship between increasing returns of STR portfolios and exposure to VVOL. The only VVOL measure that stands out in this figure is the VVIX. This could be explained by the variance risk premium of the VVIX, i.e. that the implied VVOL from option trading on VIX is on higher levels than are the realized VVOL measures obtained from past data. A quick glance at the VVOL plots in section 5.1.1 witnesses of a VVIX on significantly higher levels than the other VVOL measures. What is found out from the regression analyses of factor portfolios and S&P 500 also backs up the existence of this variance risk premium inside VVIX sequence. That is, mostly we see the VVOL loadings are negative for VVIX and positive for all other seven VVOL measures. The volatility variance risk premium, i.e. the VRP of the VIX is positive in this case, which means that realized volatility is lower than anticipated and so returns turn out to be higher, i.e. the high VRP pushes prices down which enables higher returns.

### 5.3.2 Cross-Sectional Analysis of 10 Industry Sectors

A more interesting case is the analysis of equity portfolios categorized based on industry sectors. Two different sets were used, a 10-portfolio analysis and a 49-portfolio analysis. The results of the 10-portfolio cross-sectional study are shown in Figure 5.4.

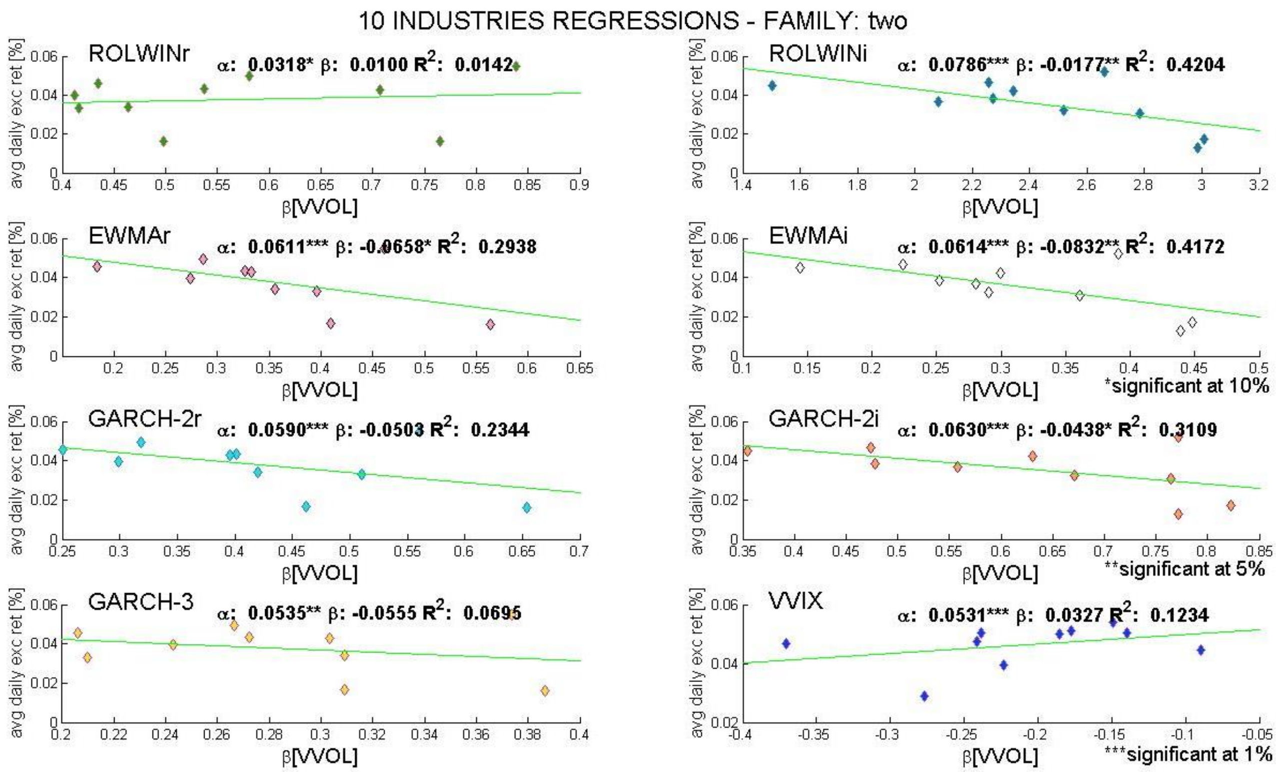


*Figure 5.4. Cross-sectional analysis of the 10 industry portfolios. Scatter plots of mean returns versus loadings on VVOL. The regressions have been controlled for volatility. The regressions assume a constant beta over time. The plots show a negative relation between returns and VVOL, with statistically significant slopes for four of the VVOL measures. The exception is the case when VVIX acts as a measure for VVOL, where the slope shows an upward going, but statistically insignificant, trend. The negative trend is exploited for in the trading strategy for stocks in chapter 5.4*

Four of the 10-industry cross sectional slopes (of regression *Family 2*) have significant and negative slopes on $\beta_{VVOL}$. The scatterplots in Figure 5.4 above have $R^2$ reaching 42% which shows that the points are not wildly dispersed. The VVOL measures that have significant slopes on $\beta_{VVOL}$ are EWMA realized, and EWMA, rolling window and GARCH-2 implied. The slope for VVIX is positive, but insignificant.

### 5.3.3 Cross-Sectional Analysis of 49 Industry Sectors

To test this approach for less diversified portfolios, the same cross-sectional analysis is made for 49 industry portfolios which this time are expected to harbor idiosyncrasies. The cross-sectional slopes of the regressions that control for volatility (*Family 2*) are shown in Figure 5.5. This time, the $\beta_{VVOL}$-return relationship is more dispersed, with lower $R^2$ coefficients in the fits. Also only one of the scatterplots (EWMA implied) has a significant slope on $\beta_{VVOL}$. Again the VVIX, although insignificant, stands out with an upward-sloping pattern. This testifies that either the idiosyncrasies need to come from even smaller portfolios, perhaps on the security level, or that the assumption of $\beta_{VVOL}$ being constant over time is inadequate.

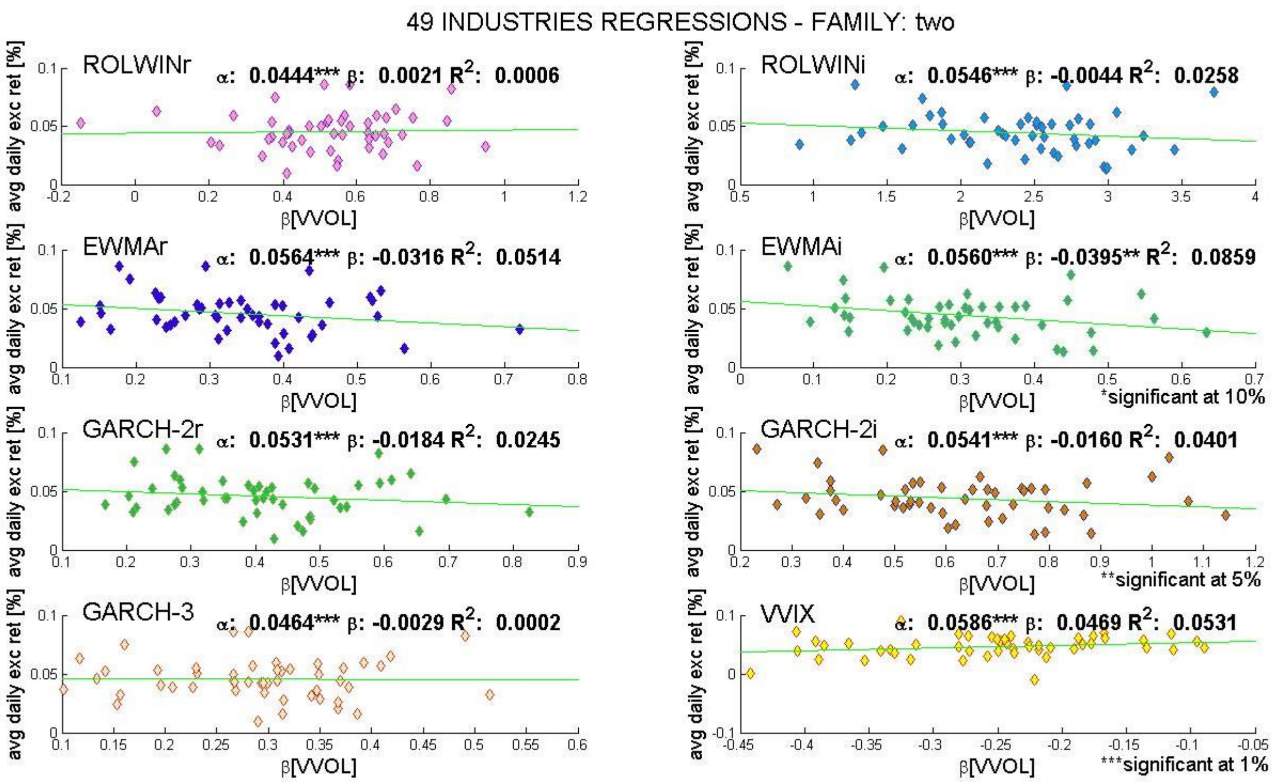


*Figure 5.5. Cross-sectional analysis of the 10 industry portfolios. Scatter plots of mean returns versus loadings on VVOL. The regressions have been controlled for volatility. The regressions assume a constant beta over time. The plots show a negative relation between returns and VVOL, with a statistically significant slope for one of the VVOL measures. The exception is the case when VVIX acts as a measure for VVOL, where the slope shows an upward going, but statistically insignificant, trend. The addition of more portfolios complicates the relation between $\beta_{VVOL}$ and average returns. The higher degree of idiosyncrasies seems to add more noise, without (strongly) capturing the VVOL influence on the returns. The negative trend is exploited for individual stocks in chapter 5.4.*

The same cross-sectional analyses were done for all portfolios studies in the regressions. Among the Fama-French portfolios, the STR and Momentum portfolios exhibit the clearest slopes as shown above. As for the rest, the slopes on $\beta_{VVOL}$ are insignificant and the $\beta_{VVOL}$ points themselves are more concentrated around the mean across portfolio deciles.

### 5.3.4 *Cross-Sectional Analysis of VVOL Loadings Controlled for Additional Risk Factor*

As mentioned earlier the STR and Momentum portfolios exhibit strong results once controlled for volatility. The cross-sectional slopes are also significant for most $\beta_{VVOL}$ from the regressions where the factor in question *and* the volatility are included (*Family 5*). From the rest of the regressions that do not control for volatility, $\beta_{VVOL}$ do not have significant slopes in the cross-sectional regression analyses.

Interestingly, the Size, Book-to-Market and LTR portfolios exhibit significant cross-sectional loadings on $\beta_{VVOL}$ *only* when no other factor, including volatility, is included in the regressions (*Family1*). The same goes for the industry portfolios, which do have some significant slopes for $\beta_{VVOL}$ *of Family 1* and especially for $\beta_{VVOL}$ *of Family 2*, but not the ones from the remaining multivariate regression families. This raises a question if VVOL is a partial replacement for volatility for these portfolios.

## 5.4 Studies of Long-Short Trading Strategy Based on VVOL Using Individual US Stocks

### 5.4.1 *Performance of the VVOL Trading Strategy*

The returns of the VVIX long-short portfolio based on the strategy described in Methodology is shown in Figure 5.6. The returns are monthly aggregated returns of the monthly-rebalanced long-short portfolio of stocks sorted on $\beta_{VVOL}$. This figure shows our zero-cost trading strategy has two big spikes over the whole time period from June 2006 to February 2015. Both spikes appear after the financial market crashes, namely the 2008 Financial Crisis and the 2010 Eurozone Crisis. Consistent with our previous findings on VVOL, the spike in our return series occurs with a lag behind the crash in the market. To conclude from this figure, our trading strategy favors volatility. That is, the strategy makes profits from the market crashes.

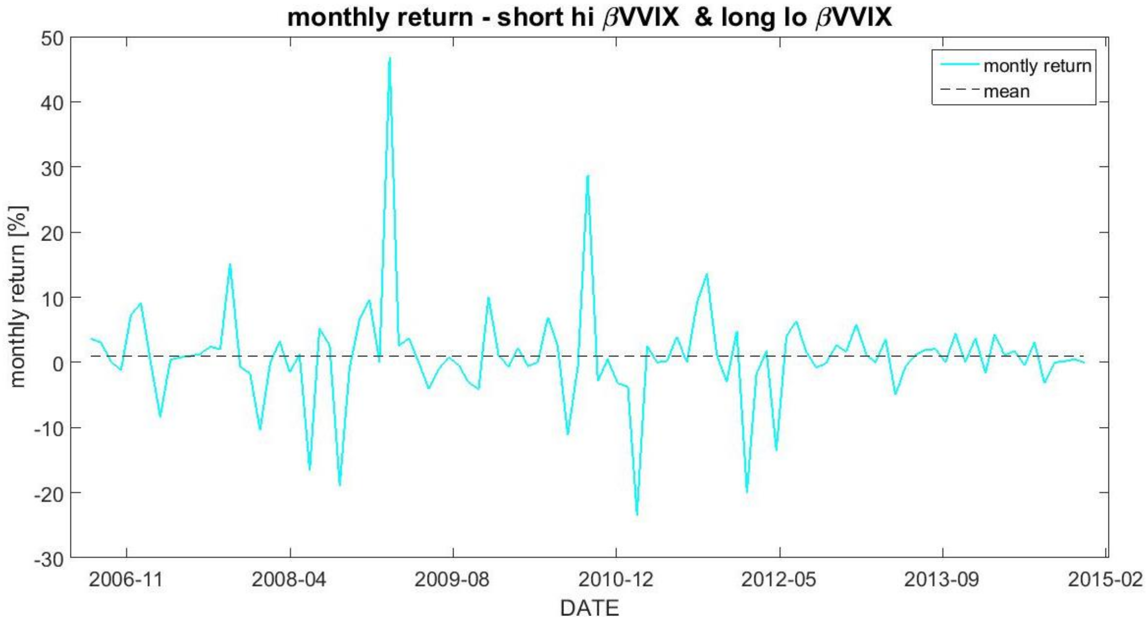


*Figure 5.6. Monthly returns of the long-short portfolio of stocks that have low loadings and high loadings on the S&P 500 VVOL, respectively. The return series consists of aggregated monthly returns for a portfolio that is rebalanced monthly. The size of either leg of the portfolio is maximally 100 stocks, but varies across months as a function of availability of intersecting data for VVOL and returns. From the monthly return series, the cumulative profits or the value gain of the portfolio are calculated. As was stated earlier, the cumulative profits are obtained from the long-short profits for every month with profits reinvested in the long-only leg. These cumulative profits across the whole time period are then compared to those of alternative reinvestment strategies of reinvesting profits at the risk-free rate and making no reinvestments. The comparison is not shown here, but the graphs clearly show that reinvesting in the long-only leg beats the alternative reinvestment strategies by more than 10% after the full test period. Another comparison is made between the trading strategy based on regressions of Family 1, univariate on VVOL, and that based on Family 2 regressions that control for volatility. The outcome shows clearly better performance of the latter trading strategy. The cumulative value gains shown here, however, illustrate the more interesting comparison of cumulative returns (reinvested in long leg) for the four realized VVOL measures and the VVIX. They are shown in Figure 5.7.*

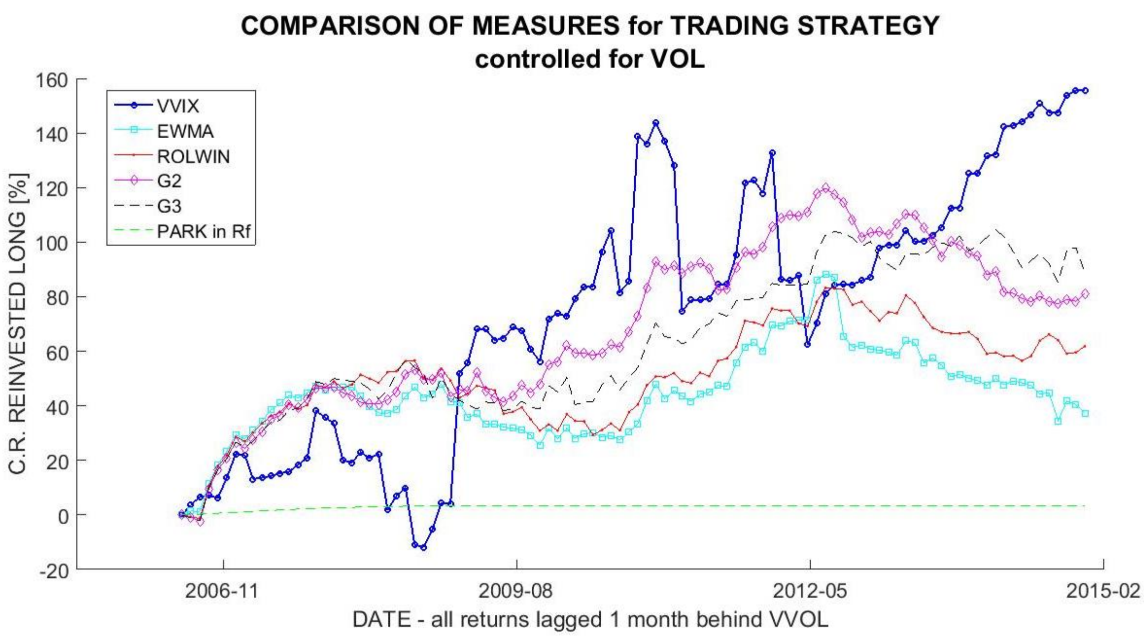


*Figure 5.7. Cumulative return for the long-short VVOL trading strategy with profits reinvested in the long leg. The strategy is controlled for VOL, in accordance with the findings from the regressions of the second family in the section on portfolio regressions on VVOL, chapter5.2. The two VVOL measures that perform the best for the longest time period are VVIX and GARCH-2 realized. The VVIX strategy exhibits more fluctuations and instability compared than the realized measures. The VVOL long-short strategy beats the alternative four-factor trading strategy and the S&P500 index, no matter what VVOL definition is used, see the next figure below.*

As can be seen in Figure 5.7, the VVOL trading strategy can yield very different cumulative returns depending on which VVOL measure is used. The realized measures are relatively close to each other in terms of the pattern and scale of fluctuations, but the real outlier is the VVIX, which exhibits more discontinuities and a more volatile behavior than the other measures. In the first 30 months of the period, the trading strategy based on the VVIX measure for VVOL actually strongly underperforms compared to the realized VVOL measures. This might per se be an indication of its real-life character and the uncertainties inherent in a forward-looking measure. We will analyze more in details about the performance of the strategy and the respective market environment in the next performance evaluation part.

### *5.4.2 Performance Evaluation on the VVOL Trading Strategy*

Regression of VVOL Long-Short Strategy Returns on FF3F+MOM

| Coefficients | ROLWINr | EWMAr | GARCH-2r | GARCH-3r | VVIX |
|---|---|---|---|---|---|
| $\alpha$ | 0,0050* | 0,0025 | 0,0060** | 0,0066* | 0,0069 |
| $\beta_{MRP}$ | -0,0003 | -0,0014 | -0,0015** | -0,0017* | 0,0018 |
| $\beta_{SMB}$ | -0,0010 | 0,0020 | 0,0005 | 0,0012 | 0,0020 |
| $\beta_{HML}$ | 0,0000 | 0,0000 | 0,0011 | 0,0000 | -0,0119*** |
| $\beta_{MOM}$ | 0,0010 | 0,0009 | 0,0007 | 0,0007 | -0,0027* |
| R2 | 0,0490 | 0,0480 | 0,0610 | 0,0610 | 0,1140 |

* significant at 10%

** significant at 5%

*** significant at 1%

*Table 5.10 The monthly returns are regressed on the four factors MRP, SMB, HML and MOM which make up one of the benchmarks used to test it. The table indicates that the strategy can generate positive alphas of roughly 0.5% per month for some of the realized measures. The VVIX alpha is, however, insignificant. The loadings on the Fama-French (1992) stock market factors and the Carhart (1997) Momentum factor are mostly insignificant for the realized measures, but the VVIX based strategy has statistically significant negative loadings on the HML and MOM factors, which is in accordance with the section of factor correlation with VVOL in chapter 5.2.1.*

To evaluate the performance and shed light on the potential driving factors of our trading strategy, these returns are regressed on the four factors MRP, SMB, HML and MOM. The regression coefficients are shown above in Table 5.10. As described in the previous sections, five VVOL measures, realized rolling window, realized EWMA, realized GARCH-2, realized GARCH-3, and VVIX, are applied to form five long-short and time-varying portfolios with the same strategy setup. It is observed that three measures out of five have significant alphas above the 10% level. MRP has significant explanatory power in the case of GARCH-2 and GARCH-3, and, for the VVIX-based portfolio, HML and Momentum factors show significant explanatory power. It is intuitive for the market risk premium exposure to be significant and negative because as shown in Figure 5.6 and later in Figure 5.8, these VVOL-based portfolios tend to perform well when the market crashes and vice versa. In the case of the portfolio formed on VVIX, the significant and negative loadings on HML might indicate that our stock selection process is actually negatively exposed to "value premium"; the significance level we see in the Momentum factor is consistent with what we find from the regression analysis and cross-sectional analysis, namely that VVOL shows strong relation with short-term reversal factor.

Next, the comparison between the two best-performing VVOL long-short strategies and three possible benchmarks or alternative trading strategies is shown. The three benchmarks are the S&P 500 index, which is the underlying asset of our VVOL measures, the Hedge Fund Research Equity Hedge Index (HFRI), and the Carhart (1997) four factor long-only trading strategy. The results are in Figure 5.8.

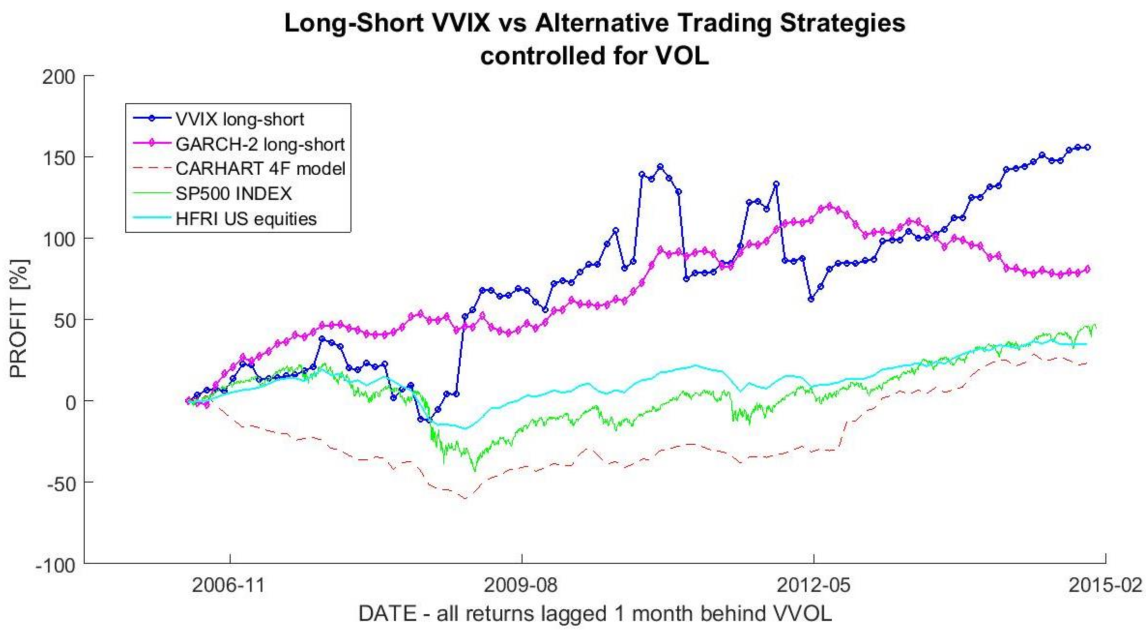


*Figure 5.8 Profits from the VVOL trading strategy compared to the Carhart four-factor model and the S&P 500 equity index. The Carhart four-factor model (Carhart, 1997), the way it is implemented here, basically tracks the index. The VVIX trading strategy is consistently above index, although in an index tracking pattern until the financial crisis of 2008. During that and other crises, the VVIX trading strategy acts as a hedging alternative. Thanks to the upsides during market turmoil, the VVOL trading strategy outperforms the index. The VVIX strategy is more volatile than the GARCH-2 realized strategy but it also offered higher profits in the last years with low market volatility.*

As shown in Figure 5.8, the VVOL trading strategy outperforms alternative trading strategies. The four-factor model based long-only trading strategy that has been designed here is made in a similar fashion as the VVOL trading strategy, with regressions on the four factors instead of VVOL. The VVOL strategy seems to be a good hedge at times of high volatility, and by especially looking at VVIX, it is clear that it is at those times that the invested capital is saved from market downturns. Interestingly, we find something consistent with what we observe from the time-plot of the VVOL series. The sharp increase of profits happens with a lag after the drop in the S&P 500 index. We attribute this to the lagging effect of the spike realization in the volatility of volatility. This enables the strategy to beat the index. The downturns or drops in portfolio profits, as shown in the graph, usually happen when the S&P 500 index recovers back from a dip.

Next, an investigation of the difference in returns and profits between our VVOL trading strategy and the HFRI Equity Hedge Index as a widely used benchmark for long-short equity strategy is illustrated. In the following Figure 5.9, the Active Return, defined as the difference in returns between the two is plotted on the right-hand scale. Also the Cumulative Active Gain is plotted on the left-hand scale, as a measure of relative profit of VVIX trading strategy compared to the profit in investing in the hedge fund index. In the active return series, there are spikes for the period of crises in 2008 and 2010. This once again supports our arguments that our VVOL trading strategy works as a good hedge of volatility and market crash, and that VVOL seems to be a hedging factor for equities. As shown in the graph, even though the active returns are negative in the period after 2011, the value of the portfolio still grows faster than the hedge index because the reinvestments into the long-only leg, which is the portfolio of low $\beta_{VVIX}$ stocks, gives positive returns for the period of low volatility in the market.

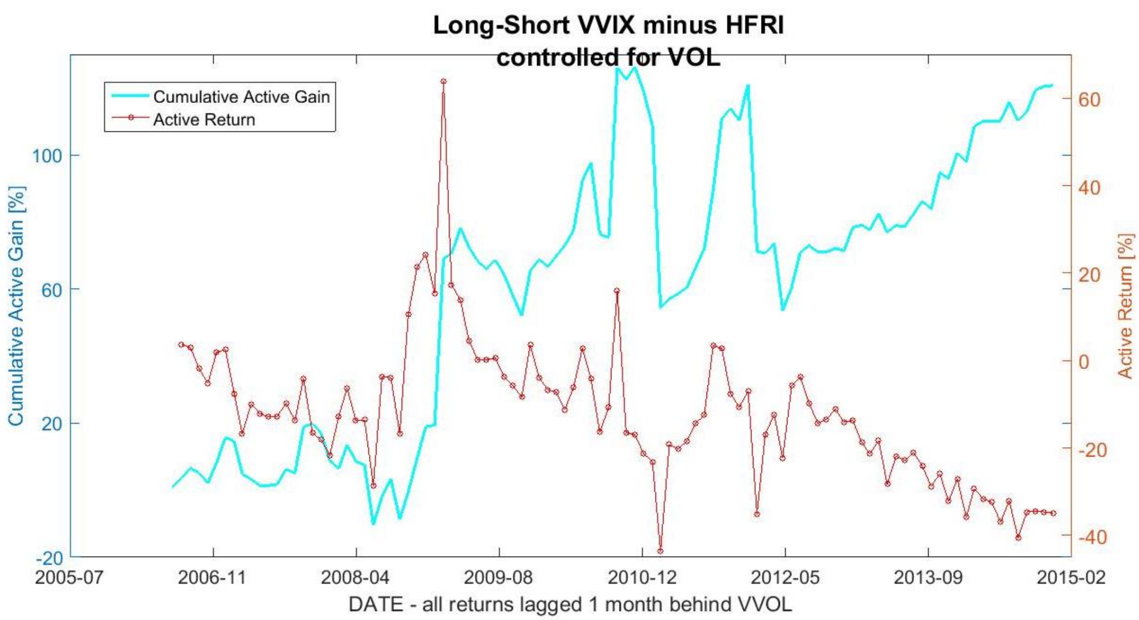


*Figure 5.9 The Active Gain (right-hand scale) and the Cumulative Active Gain (left-hand scale) illustrate the increasing relative profits of choosing the VVOL trading strategy over the four-factor model-based trading strategy. The active gain can be thought of as an active alpha relative to some specific benchmark performance.*

Due to time constraints, Sharpe ratios of the trading strategy were not looked into. Risk adjustment is most essential when comparing returns. However, the regressions control for volatility, which means that the VVOL trading strategy theoretically has little exposure to volatility. The VVOL of the individual stocks might have a larger impact in explaining returns and is certainly an interesting alternative way that we would like our trading strategy to be tried out on the same data set.

Finally, the trading strategy has neglected from practical issues such as trading costs arising from monthly rebalancing of the long and short legs and the investment of profits into the long leg. As mentioned earlier, the ideal month has 100 stocks in each leg. Surely, these costs are not negligible and should be taken into account for a fair comparison to other performance indices.

## 5.5 Interpretation of the Results

### *5.5.1 The Uniqueness of VVIX*

The VVIX is the measure that stands out from the others in that it is on higher levels (the long-term mean of the VVIX index in annual volatility terms is 85% while the long-term mean of the VIX is around 15%). It stands out in the *Family 2* and *Family 5* regressions, with negative loadings on VVOL and in the trading strategy as the measure that yields the most volatile profits. The explanation, we believe, lies in the definition of this measure. VVIX is the only implied volatility of implied volatility. The three remaining measures that we have denoted as implied are rather “realized of implied” and “modeled of implied”. The truly implied nature that makes the VVIX higher than the rest is the variance risk premium of volatility, i.e. the risk premium that buyers of VVIX call options are ready to pay to obtain certainties about the distribution of volatility, or as Baltussen et al (2014) would put it, to freeze the unknowns of the unknowns. This is also, in part, the reason why the $\beta_{VVIX}$ coefficients stand out in the regressions, but it is not the whole truth, since the regressions focus on co-changes of the data.

A further investigation of the data shows that the correlations between VVOL and the portfolios of returns exhibit a jump of one order of magnitude for the VVIX. For the first seven VVOL measures, the correlation is less than 10% below zero. For VVIX, the correlations are -15% or more. This pattern is recurrent for all Fama-French factor based and industry portfolios and the S&P 500 index. The only remark to this statement is that the STR portfolios have slightly positive correlations with all VVOL measures except with the VVIX. An illustration that might go to the bottom with this effect is shown in Figure 5.10, which shows the VVIX with two of the modeled VVOL measures in a separate-scale plot. The plot shows that the VVIX is more volatile and has a higher frequency of changes compared to the other VVOL measures. The modeled VVOL curves are smoother and only the envelope of the VVIX can be claimed to track their pattern. A proof of this would require the application of a smoothening filter of the VVIX graph for comparison of its envelope with the other measures. We leave that for future studies. The correlations between the portfolios and the VVOL measures are shown in Table 5.11. The correlations are weakly negative for the EWMA and rolling window measures and slightly positive for the parametric GARCH measures. The regression coefficients from the tables in section 5.2 are usually insignificant for the GARCH measures and significant for the other measures. The shift in sign between VVIX and the other VVOL measures is attributable to the correlation overbalance of the VVIX. We now claim that this has more to do with the (relatively) high volatility of the VVIX than a fundamentally flawed pattern vis-à-vis the other VVOL measures. A summarizing table of the correlations with VVOL of the 6693 stocks used in the trading strategy is shown in Table 5.12. This table shows percentiles and standard deviations of correlations over all stocks. Again the VVIX stands out with lower (more negative) correlations. All measures show a compact correlational structure over securities in that the standard deviation of the correlations to every measure is low, VVIX having the lowest.

The main question, however is not about the sign of the regression beta of assets, but rather the trend over securities and the $\beta_{VVOL}$-return relationship. This has been shown to follow a downward sloping pattern in the cross-sectional and trading strategy analyses.

The inference is that VVIX is a different animal compared to the rest of the VVOL measures with higher volatility and uncertainty. The best explanation to this is that it is the implied property of VVIX that manifests itself with a VVOL variance risk premium that is transmitted from investor sentiments and trading in VIX options and that is not observable in the three realized and parametrized volatility of implied volatility measures.

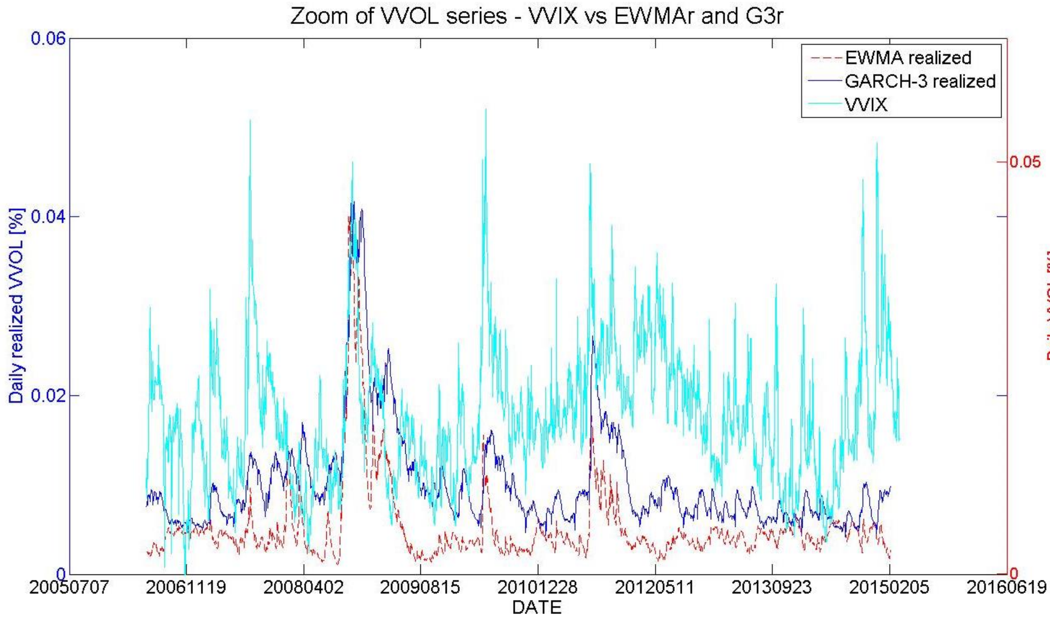


*Figure 5.10 A closer look at the VVIX vis-à-vis two other VVOL measures shows that the VVIX has a more complicated and volatile pattern, yet with an envelope that roughly resembles that of the other two measures. A filtering of the VVIX graph could confirm or reject this theory.*

Correlation between VVOL measures and portfolios (mean over deciles)

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH- | VVIX |
| S&P 500 | -2,09% | -0,37% | 1,86% | -1,09% | -2,90% | -2,49% | 3,66% | -15,73% |
| SMB | -2,62% | -1,57% | 0,56% | -0,97% | -3,35% | -3,58% | 1,83% | -16,74% |
| HML | -2,97% | -2,94% | 2,83% | -1,94% | -0,58% | 1,90% | -0,84% | -15,80% |
| momentum | -1,87% | -1,54% | 4,09% | -1,08% | 0,58% | 2,88% | 0,00% | -15,00% |
| STR | -1,57% | -1,16% | 4,51% | -0,95% | 0,88% | 3,42% | 0,18% | -14,67% |
| LTR | -2,66% | -2,87% | 2,91% | -1,75% | -0,62% | 1,94% | -0,63% | -15,63% |
| 10 industries | -2,07% | -2,20% | 3,00% | -1,23% | -0,38% | 1,77% | -0,28% | -14,35% |
| 49 industries | -2,48% | -2,51% | 2,07% | -1,68% | -0,86% | 0,97% | -0,60% | -14,28% |

*Table 5.11 The table shows mean correlations between the regressed portfolios (means over deciles) and the VVOL measures. The VVIX is outstanding in that it has correlations that are at least one order of magnitude higher than the other measures, which is seen in the $\beta_{VVOL}$ coefficients in the chapter on portfolio regressions, chapter 5.2.*

Correlation between VVOL measures and 6693 stocks from CRSP

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH- | VVIX |
| 25th pctile | -3,27% | -2,36% | -1,87% | -2,20% | -3,31% | -2,85% | -1,39% | -9,66% |
| 50th pctile | -1,35% | -0,38% | 0,24% | -0,47% | -1,47% | -0,88% | 1,00% | -6,63% |
| 75th pctile | 0,82% | 1,92% | 2,52% | 1,34% | 0,59% | 1,23% | 3,36% | -0,03% |
| std. | 6,14% | 6,06% | 6,12% | 6,21% | 6,08% | 6,08% | 6,25% | 5,88% |

*Table 5.12 The table shows the $25^{th}$, $50^{th}$ and $75^{th}$ percentiles of correlations over the 6693 stocks obtained from CRSP that are used in the trading strategy, together with the standard deviation of correlations over all stocks. Also on the security level, the VVIX stands out with lower correlations compared to other VVOL measures.*

### 5.5.2 The Volatility Interdependence and VVOL as a Process of Reversion

One of the most recurrent themes across the results from the regressions of both portfolios and stocks (for the purpose of the trading strategy) has been the interplay between volatility and VVOL. The VVOL as a factor in asset and security pricing seems to work better when controlling for the volatility. It is as if the volatility and VVOL share similar properties and are disjunct at the same time. A look at Table 5.2 shows that VVOL alone accounts for up to 53% for volatility in univariate regressions. As a standalone regressor, VVOL has difficulties capturing the volatility characteristics while volatility is complemented by and adds significance to VVOL when they are both included. Also taken together with other risk factors, as long as volatility is in the regression, the VVOL has a significant explanatory effect, if ever so marginal for the Fama-French portfolios. The conclusion is that VVOL, like the first order derivative, captures the changes of the volatility but, just like the derivative, it cannot say anything about the levels of volatility. This paper shows that VVOL is only relevant as a pricing factor once those levels are accounted for. In other words, when pricing an asset, both the changes and the changes of the changes are important. To this end, it is appropriate to mention the strong relationship that VVOL has with STR and Momentum. The definition of STR, given in the caption of Table 5.3, captures reversal, which intuitively is tied to second derivatives and local vertices. From the same caption, it can be seen that the definition of momentum, continuation, qualitatively is STR with a shifted sign. It is thus not surprising the regressions of STR on VVOL in Table 5.3 show significant coefficients and $R^2$ values that are one order of magnitude higher compared to those of other factors. It is also STR and Momentum portfolios that exhibit the clearest patterns with high slopes on $\beta_{VVOL}$ in the cross-sectional studies.

### 5.5.3 The Negative VVOL-Return Relationship

In the regressions of the S&P 500 returns on volatility and VVOL, we find positive loadings on VVOL (except VVIX) and negative loadings on volatility. The same effect is observed by Clark et al (2013). However these results differ from the results of Baltussen et al (2014) who find negative loadings on stock-individual VVOL with a measure similar to our rolling window implied. Nevertheless, apart from absolute values of βVVOL, the cross sectional trends that we observe (VVOL affects stock returns negatively) are the same as those in Baltussen et al (2014). The VVOL-return relationship has two competing explanations. The first one, formulated by Clark et al, assumes that investors are unaware of the volatility of volatility risk. Thus, even with increasing VVOL, returns will adjust only to the risks from implied volatility. The uncovered risks from VVOL will drive returns higher. The competing theory, due to Baltussen et al, states that investors are aware of volatility of volatility as a parameter in the distribution of uncertainties themselves (i.e. as a standard deviation of volatility with a mean that equals the implied volatility), and that they actually prefer stocks with these uncertainties in expected volatility, which thus drives prices up and leaves less room for returns. Only our regressions of the S&P 500 index on volatility, VVOL and single additional factors do not concur with these findings, probably because MRP explains too much.

### *5.5.4 VVOL as a Way of Hedging*

From the analysis of correlations, it has been seen that the correlations between VVOL and Fama-French factors are negative. Once controlled for volatility, the portfolio regressions also show a highly significant and negative relationship between VVOL and returns. This was used for the betting in the trading strategy for stocks with positive returns.

The VIX index is a good hedge for equity. The results in this paper show that the VVIX and other VVOL measures are a good hedge for equity as well. This is perhaps even truer for single stocks (based on the strong results in the trading strategy). Once controlled for volatility, the VVOL behaves like an "anti-risk factor". The assumption in multifactor asset pricing models is that the risk factors increase in expectation and so positive loadings yield positive returns. With an anti-risk factor, or a hedging factor, the assumption of growth of risk factors makes the anti-risk factor go down and so the portfolios with the most negative loadings on VVOL have the highest returns. So the cross sectional line is downward sloping for a hedge factor, which has been confirmed in all cross sectional studies, albeit with low power for the Fama-French portfolios. The Fama-French STR factor also seems to behave like an "anti-risk factor" with positive correlation with VVOL. In the cross sectional analysis of STR portfolios, the loadings on VVOL, controlled for volatility, are positive. For stocks, the cross sectional analysis has a slightly different approach that accounts for time-varying loadings on VVOL. The betas in this analysis are also negative and significant, although it would be of interest to see how the cross sectional analysis of stocks would look like if a security-specific VVOL measure was used instead, as in Baltussen et al (2014).

VVOL has properties that are typical of alternative asset classes, like trend-following CTAs and quantitative hedge funds, which yield positive returns in times of market declines and high volatility, something that is visible in the profits in Figure 5.8.

### 5.5.5 Is There a Free Lunch in VVOL? - The Delay between Returns and VVOL

There is, for all measures of VVOL a lagging feature of VVOL peaks relative to market downturns. As was seen in the plots of Figure 5.1, the VVOL lags the returns. This means that VVOL is awaiting a rebound in volatility, from high to low, before the effect is visible in the VVOL graph. Another explanation, specific for VVIX, is that during a market crash, investors drive up the VVIX index through options on VIX. The implied volatility of the VIX thus adjusts, as a reaction, when returns move downward. Theoretically, one could trade on the lag between current and future VVOL levels as soon as one observes a market downturn. The problem, however, is to know when the volatility will rebound, i.e. when prices will stabilize at some lower level. A trading strategy based on this would require the use of VVIX and synthetic replication of realized VVOL. An attempt to trade on VVOL by the use of delta-hedged option portfolios was made by Shaliastovich et al (2014). In our trading strategy, we exploit the movements of VVOL by forming long-short portfolios of stocks and capturing their VVOL exposure. As shown in the Figure 5.6 and Figure 5.8, there is also a delay between the spikes of the long-short portfolio returns and crashes in the market. In the simulation for the performance of the trading strategy, it can be seen that entering into the position from the time point of the crash would capture the spike of returns and generate profits, if not taking into account transaction costs. Accordingly, this is a strategy that does not need to time the volatility rebound as it is relative.

### 5.5.6 Capturing the Pricing of VVOL in Idiosyncrasies

Experience is the path to wisdom. Having investigated the trading strategy on stocks, we have seen that the impact of VVOL on stock prices, even after controlling for volatility, is strong and can yield profits. The portfolio regressions told us that when controlling for volatility, the $\beta_{VVOL}$ becomes negative and significant at the 99% level. Perhaps it is the idiosyncratic and undiversified nature of the asset that is needed for VVOL to play out. At this stage, one wonders what the cross sectional analysis performed on stocks would give. It is expected that the cross sectional analysis will give more power to VVOL in the cross section.

That is why we could not help but performing a cross sectional analysis of the 6693 stocks based on *Family-2* regressions on volatility and VVOL. The results are shown in Figure 5.11. The betas are now an average over months, whereas in earlier cross-sectional studies, they were assumed to be constant. Just as before, the returns are excess returns. For simplicity, only one realized measure and the VVIX are shown in the cross sectional graph below. The cross sectional regression coefficients are printed directly on the graph area.

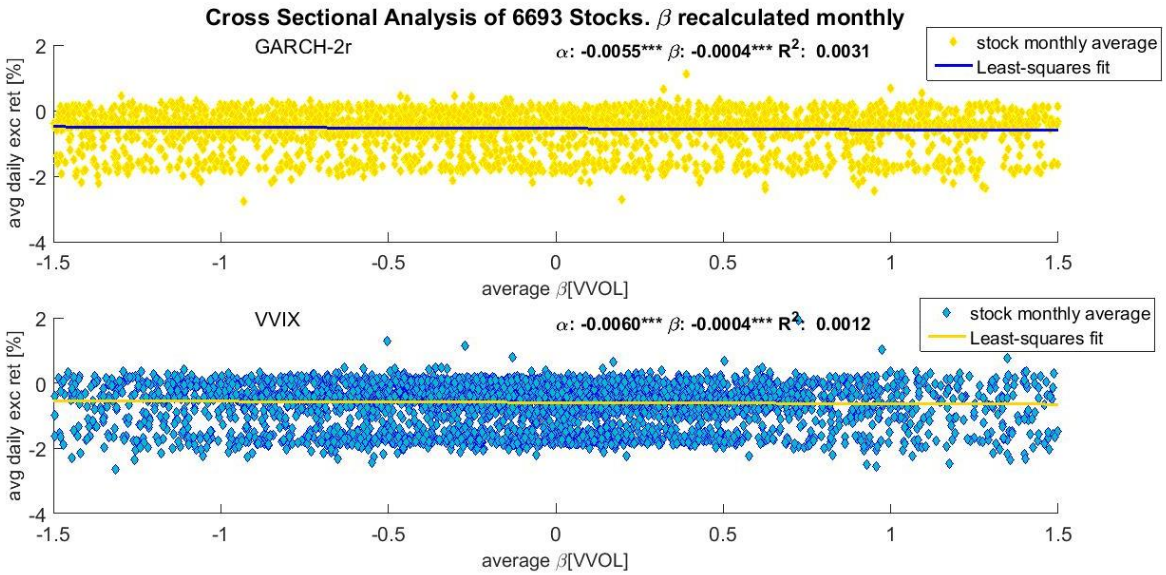


*Figure 5.11 Cross-sectional analysis of 6693 stocks returns over average annual VVOL sensitivity. The regressions are multivariate and control for volatility according to the regressions of Family 2 in **chapter 3.2.1**. Extreme outlier betas have been neglected in the least-squares linear fit due to the heavy clustering around 0. Despite the clustering, significant negative betas are obtained.*

The scatter plots in Figure 5.11 show that the stock data is rather dispersed around the fitted lines. Nevertheless, both the illustrated VVOL measures, GARCH-2 realized and VVIX, have significant negative slopes on $\beta_{VVOL}$. The value of the slope shows on a rather weak relationship. We saw earlier in section 5.3 that 49 industry portfolios rendered more dispersed data than the 10 industry portfolios. We still believe that using more idiosyncratic data in the analysis gives better cross-sectional relationships, relying on Figure 5.11 and the success of the trading strategy.

# 6 Conclusion

This study has tried to explore the lands of uncertainty. Volatility of volatility, as a parameter that defines the distribution of the (uncertain) volatility has been investigated, both as a standalone phenomenon and as a factor in the pricing of portfolios and stocks. Eight measures of VVOL have been elaborated; they are parametric, realized and purely implied. Each measure has had its own complications and, due to the lack of conventions for VVOL calculation, it has been a matter of judgment and theoretical analysis when manipulating time series of volatilities and returns. The VVOL time series have been plotted and analyzed quantitatively, with parallels to historical returns, volatilities and financial market events.

It has been seen that VVOL reflects market downturns with a lag of a couple of months, as a mechanism that awaits the stabilizing of volatility as negative returns level out. We have shown empirically that VVOL has a negative relation with returns of assets, both equity portfolios and individual stocks. Different types of equity portfolios have been tried in the search of risk factors that can be proxied by VVOL. One such candidate is short-term reversal (STR) and another is Momentum. The STR findings that VVOL is related to reversal are further supported when VVOL is considered as a second-order derivative of the returns. The cross-sectional analyses have shown that increased loadings on VVOL yield lower returns. These results are in line with previous studies, together with which they form the logic behind the design of our trading strategy. The trading strategy has been applied to individual stocks as a bet on relative exposure to a universal VVOL of the S&P 500 equity index. It has been shown that such a trading strategy performs well for all tested VVOL measures and especially for the VVIX. The trading strategy, in its simple and theoretical form, outperforms alternative indices and an alternative strategy based on multifactor models.

Along the road, several new questions have been raised and alternative ideas have been born. The weak results from several equity portfolio regressions, led us in the direction of individual stocks in the trading strategy and we suggest a thorough analysis of the VVOL on the security level as a pricing factor for stocks. Further pricing investigations on specifically-formed portfolios, again with particular VVOL measures are also suggested for future work. For the cross-sectional analysis, we would like to see whether models with time varying loadings on VVOL would yield the same results. In the cross sectional analysis of stocks from the trading strategy, we adopt such an approach of Fama-MacBeth (1973) regressions. We also welcome a study of the variance risk premium for volatility that would clarify the exclusive nature of the VVIX. Other things that would be interesting to investigate are the correlations between a smoothened version of the VVIX and other VVOL measures and the complementary effect of volatility on VVOL. Finally, in the modeling of VVOL, there are issues in the treatment of volatilities for the GARCH models, which are based on returns as inputs. Despite our best efforts to demean the series and treat residuals, we believe an elaborate analysis of the distribution and characteristics of volatility is needed to set up a systematic theoretical framework.

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# Appendix

Multivariate Regressions of Size Portfolios on VOL and VVOL - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0027*** | 0.0029*** | 0.0026*** | 0.0012*** | 0.0055*** | 0.0048*** | 0.0052*** | 0.0164*** |
| 2 | 0.0019*** | 0.0021*** | 0.0018*** | 0.0007 | 0.0060*** | 0.0051*** | 0.0056*** | 0.0179*** |
| 3 | 0.0020*** | 0.0021*** | 0.0019*** | 0.0009* | 0.0063*** | 0.0054*** | 0.0058*** | 0.0180*** |
| 4 | 0.0017*** | 0.0019*** | 0.0016*** | 0.0007 | 0.0059*** | 0.0051*** | 0.0055*** | 0.0176*** |
| 5 | 0.0017*** | 0.0019*** | 0.0016*** | 0.0007 | 0.0060*** | 0.0051*** | 0.0056*** | 0.0172*** |
| 6 | 0.0018*** | 0.0020*** | 0.0017*** | 0.0007 | 0.0058*** | 0.0050*** | 0.0054*** | 0.0167*** |
| 7 | 0.0020*** | 0.0022*** | 0.0018*** | 0.0007 | 0.0059*** | 0.0051*** | 0.0056*** | 0.0165*** |
| 8 | 0.0019*** | 0.0021*** | 0.0017*** | 0.0007 | 0.0058*** | 0.0050*** | 0.0055*** | 0.0160*** |
| 9 | 0.0017*** | 0.0019*** | 0.0016*** | 0.0008* | 0.0056*** | 0.0048*** | 0.0053*** | 0.0151*** |
| 10 | 0.0011*** | 0.0013*** | 0.0009*** | 0.0003 | 0.0049*** | 0.0042*** | 0.0046*** | 0.0130*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.1. Intercepts of Size portfolios Family 2.*

Multivariate Regressions of Size Portfolios on VOL and VVOL - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0251 | 0.0262 | 0.0286 | 0.0348 | 0.0312 | 0.0247 | 0.0319 | 0.0431 |
| 2 | 0.0073 | 0.0085 | 0.0094 | 0.0108 | 0.0224 | 0.0161 | 0.0260 | 0.0271 |
| 3 | 0.0072 | 0.0084 | 0.0091 | 0.0100 | 0.0251 | 0.0183 | 0.0290 | 0.0303 |
| 4 | 0.0070 | 0.0086 | 0.0097 | 0.0099 | 0.0246 | 0.0181 | 0.0297 | 0.0316 |
| 5 | 0.0078 | 0.0094 | 0.0110 | 0.0103 | 0.0266 | 0.0189 | 0.0316 | 0.0325 |
| 6 | 0.0115 | 0.0137 | 0.0165 | 0.0144 | 0.0294 | 0.0212 | 0.0358 | 0.0361 |
| 7 | 0.0138 | 0.0160 | 0.0202 | 0.0176 | 0.0301 | 0.0224 | 0.0379 | 0.0360 |
| 8 | 0.0119 | 0.0142 | 0.0183 | 0.0153 | 0.0286 | 0.0213 | 0.0370 | 0.0340 |
| 9 | 0.0104 | 0.0126 | 0.0164 | 0.0133 | 0.0284 | 0.0217 | 0.0382 | 0.0328 |
| 10 | 0.0079 | 0.0116 | 0.0161 | 0.0094 | 0.0282 | 0.0207 | 0.0395 | 0.0290 |

*Table A.2. $R^2$ of Size portfolios Family 2.*

Multivariate Regressions of Size Portfolios on VOL, VVOL and SMB - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0023*** | 0.0025*** | 0.0021*** | 0.0010*** | 0.0050*** | 0.0044*** | 0.0047*** | 0.0134*** |
| 2 | 0.0014*** | 0.0015*** | 0.0012*** | 0.0004 | 0.0053*** | 0.0045*** | 0.0050*** | 0.0139*** |
| 3 | 0.0014*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0056*** | 0.0048*** | 0.0053*** | 0.0143*** |
| 4 | 0.0013*** | 0.0015*** | 0.0011*** | 0.0004 | 0.0053*** | 0.0046*** | 0.0051*** | 0.0144*** |
| 5 | 0.0013*** | 0.0015*** | 0.0012*** | 0.0005 | 0.0055*** | 0.0047*** | 0.0052*** | 0.0144*** |
| 6 | 0.0015*** | 0.0017*** | 0.0013*** | 0.0005 | 0.0054*** | 0.0046*** | 0.0051*** | 0.0146*** |
| 7 | 0.0017*** | 0.0020*** | 0.0015*** | 0.0006 | 0.0056*** | 0.0048*** | 0.0053*** | 0.0148*** |
| 8 | 0.0016*** | 0.0019*** | 0.0015*** | 0.0006 | 0.0056*** | 0.0048*** | 0.0053*** | 0.0146*** |
| 9 | 0.0016*** | 0.0018*** | 0.0015*** | 0.0007 | 0.0054*** | 0.0047*** | 0.0052*** | 0.0140*** |
| 10 | 0.0011*** | 0.0013*** | 0.0009*** | 0.0003 | 0.0049*** | 0.0042*** | 0.0046*** | 0.0126*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.3. Intercepts of Size portfolios Family 5.*

Multivariate Regressions of Size Portfolios on VOL, VVOL and SMB - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.2786 | 0.2811 | 0.2870 | 0.2849 | 0.2820 | 0.2761 | 0.2859 | 0.2868 |
| 2 | 0.2638 | 0.2663 | 0.2697 | 0.2652 | 0.2756 | 0.2703 | 0.2831 | 0.2808 |
| 3 | 0.2436 | 0.2460 | 0.2490 | 0.2444 | 0.2587 | 0.2528 | 0.2664 | 0.2654 |
| 4 | 0.2087 | 0.2115 | 0.2150 | 0.2097 | 0.2237 | 0.2182 | 0.2326 | 0.2335 |
| 5 | 0.1685 | 0.1712 | 0.1752 | 0.1692 | 0.1851 | 0.1784 | 0.1937 | 0.1917 |
| 6 | 0.1140 | 0.1173 | 0.1223 | 0.1151 | 0.1294 | 0.1220 | 0.1388 | 0.1493 |
| 7 | 0.0844 | 0.0875 | 0.0937 | 0.0865 | 0.0990 | 0.0920 | 0.1093 | 0.1062 |
| 8 | 0.0607 | 0.0637 | 0.0695 | 0.0628 | 0.0758 | 0.0691 | 0.0863 | 0.0836 |
| 9 | 0.0287 | 0.0314 | 0.0361 | 0.0310 | 0.0458 | 0.0394 | 0.0569 | 0.0610 |
| 10 | 0.0081 | 0.0117 | 0.0163 | 0.0095 | 0.0283 | 0.0208 | 0.0397 | 0.0342 |

*Table A.4. $R^2$ of Size portfolios Family 5.*

Multivariate Regressions of Momentum Portfolios on VOL and VVOL - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0011 | 0.0012* | 0.0007 | -0.0004 | 0.0069*** | 0.0060*** | 0.0067*** | 0.0257*** |
| 2 | 0.0008 | 0.0009* | 0.0005 | -0.0001 | 0.0057*** | 0.0049*** | 0.0056*** | 0.0204*** |
| 3 | 0.0012** | 0.0013*** | 0.0010** | 0.0001 | 0.0055*** | 0.0047*** | 0.0053*** | 0.0177*** |
| 4 | 0.0011*** | 0.0012*** | 0.0009** | 0.0005 | 0.0052*** | 0.0045*** | 0.0050*** | 0.0163*** |
| 5 | 0.0011*** | 0.0012*** | 0.0009** | 0.0002 | 0.0047*** | 0.0039*** | 0.0044*** | 0.0150*** |
| 6 | 0.0011*** | 0.0012*** | 0.0009** | 0.0002 | 0.0044*** | 0.0038*** | 0.0042*** | 0.0139*** |
| 7 | 0.0015*** | 0.0017*** | 0.0013*** | 0.0006 | 0.0050*** | 0.0042*** | 0.0046*** | 0.0138*** |
| 8 | 0.0015*** | 0.0017*** | 0.0014*** | 0.0006 | 0.0049*** | 0.0042*** | 0.0046*** | 0.0131*** |
| 9 | 0.0019*** | 0.0022*** | 0.0018*** | 0.0010** | 0.0056*** | 0.0048*** | 0.0052*** | 0.0138*** |
| 10 | 0.0023*** | 0.0026*** | 0.0022*** | 0.0012** | 0.0068*** | 0.0057*** | 0.0062*** | 0.0156*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.5. Intercepts of Momentum portfolios Family 2.*

Multivariate Regressions of Momentum Portfolios on VOL and VVOL - BETAS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.6229** | 0.6607*** | 0.7081*** | 0.4347*** | 3.7658*** | 0.6647*** | 1.1860*** | -0.4374*** |
| 2 | 0.4717** | 0.4860*** | 0.5926*** | 0.2837*** | 3.0629*** | 0.4982*** | 0.9729*** | -0.3394*** |
| 3 | 0.4555** | 0.3830*** | 0.4502*** | 0.2879*** | 2.7402*** | 0.3931*** | 0.7847*** | -0.2811*** |
| 4 | 0.3652** | 0.3396*** | 0.4304*** | 0.1980*** | 2.4914*** | 0.3374*** | 0.7125*** | -0.2552*** |
| 5 | 0.5040*** | 0.3501*** | 0.4249*** | 0.2716*** | 2.4680*** | 0.3278*** | 0.6823*** | -0.2372*** |
| 6 | 0.4624*** | 0.3387*** | 0.4066*** | 0.2601*** | 2.1744*** | 0.3252*** | 0.6558*** | -0.2196*** |
| 7 | 0.5703*** | 0.3783*** | 0.4436*** | 0.2738*** | 2.5161*** | 0.3288*** | 0.6288*** | -0.2066*** |
| 8 | 0.5550*** | 0.3735*** | 0.4545*** | 0.2868*** | 2.3544*** | 0.3190*** | 0.6150*** | -0.1986*** |
| 9 | 0.6504*** | 0.3912*** | 0.4829*** | 0.2854*** | 2.5607*** | 0.3103*** | 0.5976*** | -0.1983*** |
| 10 | 0.7842*** | 0.4766*** | 0.5891*** | 0.3579*** | 3.2030*** | 0.3659*** | 0.6868*** | -0.2220*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.6. Betas of Momentum portfolios Family 2.*

Multivariate Regressions of Momentum Portfolios on VOL and VVOL - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0027 | 0.0070 | 0.0085 | 0.0053 | 0.0152 | 0.0134 | 0.0273 | 0.0217 |
| 2 | 0.0018 | 0.0055 | 0.0087 | 0.0033 | 0.0157 | 0.0128 | 0.0291 | 0.0217 |
| 3 | 0.0038 | 0.0066 | 0.0089 | 0.0062 | 0.0203 | 0.0155 | 0.0311 | 0.0254 |
| 4 | 0.0032 | 0.0062 | 0.0094 | 0.0043 | 0.0209 | 0.0155 | 0.0316 | 0.0261 |
| 5 | 0.0051 | 0.0075 | 0.0106 | 0.0074 | 0.0202 | 0.0141 | 0.0306 | 0.0237 |
| 6 | 0.0052 | 0.0081 | 0.0113 | 0.0079 | 0.0196 | 0.0152 | 0.0326 | 0.0246 |
| 7 | 0.0113 | 0.0145 | 0.0183 | 0.0134 | 0.0291 | 0.0207 | 0.0376 | 0.0299 |
| 8 | 0.0104 | 0.0136 | 0.0181 | 0.0133 | 0.0262 | 0.0194 | 0.0355 | 0.0266 |
| 9 | 0.0163 | 0.0184 | 0.0233 | 0.0174 | 0.0319 | 0.0233 | 0.0371 | 0.0327 |
| 10 | 0.0139 | 0.0159 | 0.0203 | 0.0154 | 0.0281 | 0.0196 | 0.0303 | 0.0303 |

*Table A.7. $R^2$ of Size portfolios Family 2.*

Multivariate Regressions of Momentum Portfolios on VOL, VVOL and MOM - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0022*** | 0.0025*** | 0.0020*** | 0.0010* | 0.0068*** | 0.0058*** | 0.0063*** | 0.0143*** |
| 2 | 0.0016*** | 0.0018*** | 0.0014*** | 0.0010* | 0.0056*** | 0.0047*** | 0.0052*** | 0.0115*** |
| 3 | 0.0018*** | 0.0020*** | 0.0017*** | 0.0010** | 0.0055*** | 0.0046*** | 0.0050*** | 0.0107*** |
| 4 | 0.0016*** | 0.0018*** | 0.0015*** | 0.0011*** | 0.0052*** | 0.0044*** | 0.0048*** | 0.0108*** |
| 5 | 0.0015*** | 0.0017*** | 0.0014*** | 0.0007* | 0.0046*** | 0.0038*** | 0.0043*** | 0.0102*** |
| 6 | 0.0014*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0043*** | 0.0037*** | 0.0041*** | 0.0104*** |
| 7 | 0.0017*** | 0.0019*** | 0.0016*** | 0.0009** | 0.0050*** | 0.0042*** | 0.0046*** | 0.0112*** |
| 8 | 0.0016*** | 0.0018*** | 0.0015*** | 0.0007* | 0.0049*** | 0.0042*** | 0.0046*** | 0.0114*** |
| 9 | 0.0020*** | 0.0022*** | 0.0019*** | 0.0011** | 0.0056*** | 0.0048*** | 0.0052*** | 0.0126*** |
| 10 | 0.0022*** | 0.0025*** | 0.0021*** | 0.0010* | 0.0068*** | 0.0058*** | 0.0062*** | 0.0157*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.8. Intercepts of Momentum portfolios Family 5.*

Multivariate Regressions of Momentum Portfolios on VOL, VVOL and MOM - BETAS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.8200*** | 0.5873*** | 0.7059*** | 0.3710*** | 3.4068*** | 0.4670*** | 0.8143*** | -0.1987*** |
| 2 | 0.6237*** | 0.4294*** | 0.5909*** | 0.2346*** | 2.7861*** | 0.3457*** | 0.6866*** | -0.1542*** |
| 3 | 0.5713*** | 0.3399*** | 0.4489*** | 0.2505*** | 2.5305*** | 0.2776*** | 0.5680*** | -0.1355*** |
| 4 | 0.4529*** | 0.3069*** | 0.4294*** | 0.1697*** | 2.3331*** | 0.2502*** | 0.5493*** | -0.1396*** |
| 5 | 0.5780*** | 0.3226*** | 0.4241*** | 0.2477*** | 2.3352*** | 0.2547*** | 0.5457*** | -0.1379*** |
| 6 | 0.5168*** | 0.3185*** | 0.4060*** | 0.2426*** | 2.0762*** | 0.2713*** | 0.5554*** | -0.1449*** |
| 7 | 0.6087*** | 0.3641*** | 0.4431*** | 0.2614*** | 2.4475*** | 0.2911*** | 0.5593*** | -0.1529*** |
| 8 | 0.5763*** | 0.3657*** | 0.4542*** | 0.2800*** | 2.3171*** | 0.2986*** | 0.5782*** | -0.1632*** |
| 9 | 0.6605*** | 0.3875*** | 0.4828*** | 0.2822*** | 2.5442*** | 0.3013*** | 0.5825*** | -0.1744*** |
| 10 | 0.7642*** | 0.4843*** | 0.5893*** | 0.3645*** | 3.2415*** | 0.3873*** | 0.7297*** | -0.2240*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.9. Betas of Momentum portfolios Family 5.*

Multivariate Regressions of Momentum Portfolios on VOL, VVOL and MOM - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.4645 | 0.4668 | 0.4694 | 0.4652 | 0.4764 | 0.4729 | 0.4793 | 0.5456 |
| 2 | 0.4471 | 0.4489 | 0.4531 | 0.4469 | 0.4605 | 0.4562 | 0.4644 | 0.5214 |
| 3 | 0.3769 | 0.3779 | 0.3811 | 0.3775 | 0.3899 | 0.3839 | 0.3922 | 0.4733 |
| 4 | 0.2612 | 0.2627 | 0.2666 | 0.2610 | 0.2753 | 0.2692 | 0.2789 | 0.3656 |
| 5 | 0.2118 | 0.2126 | 0.2165 | 0.2125 | 0.2224 | 0.2155 | 0.2262 | 0.3025 |
| 6 | 0.1363 | 0.1380 | 0.1418 | 0.1377 | 0.1486 | 0.1433 | 0.1559 | 0.2104 |
| 7 | 0.0822 | 0.0842 | 0.0885 | 0.0831 | 0.0976 | 0.0887 | 0.1020 | 0.1392 |
| 8 | 0.0321 | 0.0346 | 0.0395 | 0.0343 | 0.0465 | 0.0393 | 0.0535 | 0.0752 |
| 9 | 0.0208 | 0.0226 | 0.0276 | 0.0216 | 0.0355 | 0.0268 | 0.0399 | 0.0533 |
| 10 | 0.0244 | 0.0268 | 0.0310 | 0.0263 | 0.0399 | 0.0315 | 0.0436 | 0.0304 |

*Table A.10. $R^2$ of Momentum portfolios Family 5.*

Multivariate Regressions of BookToMarket Portfolios on VOL and VVOL - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0012*** | 0.0013*** | 0.0010*** | 0.0003 | 0.0048*** | 0.0041*** | 0.0045*** | 0.0130*** |
| 2 | 0.0013*** | 0.0015*** | 0.0012*** | 0.0004 | 0.0048*** | 0.0041*** | 0.0045*** | 0.0130*** |
| 3 | 0.0012*** | 0.0014*** | 0.0011*** | 0.0004 | 0.0047*** | 0.0040*** | 0.0044*** | 0.0124*** |
| 4 | 0.0016*** | 0.0018*** | 0.0015*** | 0.0008* | 0.0055*** | 0.0047*** | 0.0052*** | 0.0150*** |
| 5 | 0.0014*** | 0.0016*** | 0.0012*** | 0.0005 | 0.0054*** | 0.0046*** | 0.0051*** | 0.0144*** |
| 6 | 0.0017*** | 0.0019*** | 0.0015*** | 0.0007 | 0.0054*** | 0.0047*** | 0.0052*** | 0.0155*** |
| 7 | 0.0014*** | 0.0016*** | 0.0012*** | 0.0006 | 0.0051*** | 0.0044*** | 0.0048*** | 0.0134*** |
| 8 | 0.0018*** | 0.0019*** | 0.0016*** | 0.0010** | 0.0059*** | 0.0050*** | 0.0055*** | 0.0168*** |
| 9 | 0.0021*** | 0.0023*** | 0.0019*** | 0.0010** | 0.0061*** | 0.0053*** | 0.0058*** | 0.0180*** |
| 10 | 0.0026*** | 0.0027*** | 0.0023*** | 0.0012** | 0.0065*** | 0.0058*** | 0.0063*** | 0.0221*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.11. Intercepts of Book-to-Market portfolios Family 2.*

Multivariate Regressions of BookToMarket Portfolios on VOL and VVOL - BETAS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.5042*** | 0.3641*** | 0.4435*** | 0.2633*** | 2.5659*** | 0.3571*** | 0.6701** | -0.1943** |
| 2 | 0.5407*** | 0.3318*** | 0.4013*** | 0.2731*** | 2.3443*** | 0.3063*** | 0.6153** | -0.1970** |
| 3 | 0.5368*** | 0.3474*** | 0.4186*** | 0.2593*** | 2.3659*** | 0.3199*** | 0.6249** | -0.1892** |
| 4 | 0.4377*** | 0.3362*** | 0.4027*** | 0.2414*** | 2.4028*** | 0.2999*** | 0.6295** | -0.2250** |
| 5 | 0.6007*** | 0.3971*** | 0.4854*** | 0.2868*** | 2.6512*** | 0.3334*** | 0.6744** | -0.2189** |
| 6 | 0.5309*** | 0.3632*** | 0.4387*** | 0.2851*** | 2.3435*** | 0.3089*** | 0.6270** | -0.2349** |
| 7 | 0.5171*** | 0.3523*** | 0.4334*** | 0.2502*** | 2.4552*** | 0.3077*** | 0.6333** | -0.2005** |
| 8 | 0.4823*** | 0.3117*** | 0.4062*** | 0.2305*** | 2.4173*** | 0.2335*** | 0.5850** | -0.2494** |
| 9 | 0.5819*** | 0.3864*** | 0.4659*** | 0.3172*** | 2.4510*** | 0.3119*** | 0.6643** | -0.2754** |
| 10 | 0.5886*** | 0.4617*** | 0.5523*** | 0.3910*** | 2.4102*** | 0.3213*** | 0.6698** | -0.3508** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.12. Betas of Book-to-Market portfolios Family 2.*

Multivariate Regressions of BookToMarket Portfolios on VOL and VVOL - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0073 | 0.0104 | 0.0142 | 0.0095 | 0.0250 | 0.0182 | 0.0345 | 0.0293 |
| 2 | 0.0088 | 0.0106 | 0.0139 | 0.0111 | 0.0253 | 0.0184 | 0.0344 | 0.0301 |
| 3 | 0.0082 | 0.0108 | 0.0146 | 0.0102 | 0.0260 | 0.0188 | 0.0361 | 0.0263 |
| 4 | 0.0080 | 0.0109 | 0.0137 | 0.0100 | 0.0271 | 0.0203 | 0.0354 | 0.0308 |
| 5 | 0.0084 | 0.0111 | 0.0154 | 0.0101 | 0.0266 | 0.0187 | 0.0350 | 0.0250 |
| 6 | 0.0108 | 0.0135 | 0.0171 | 0.0135 | 0.0278 | 0.0216 | 0.0367 | 0.0332 |
| 7 | 0.0081 | 0.0107 | 0.0146 | 0.0097 | 0.0265 | 0.0189 | 0.0351 | 0.0259 |
| 8 | 0.0080 | 0.0094 | 0.0125 | 0.0090 | 0.0254 | 0.0182 | 0.0300 | 0.0287 |
| 9 | 0.0118 | 0.0142 | 0.0175 | 0.0147 | 0.0276 | 0.0217 | 0.0363 | 0.0346 |
| 10 | 0.0133 | 0.0171 | 0.0207 | 0.0181 | 0.0250 | 0.0207 | 0.0324 | 0.0374 |

*Table A.13. $R^2$ of Book-to-Market portfolios Family 2.*

Multivariate Regressions of BookToMarket Portfolios on VOL, VVOL and HML - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0015*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0052*** | 0.0044*** | 0.0049*** | 0.0111*** |
| 2 | 0.0013*** | 0.0015*** | 0.0012*** | 0.0004 | 0.0049*** | 0.0042*** | 0.0046*** | 0.0105*** |
| 3 | 0.0012*** | 0.0014*** | 0.0011*** | 0.0004 | 0.0047*** | 0.0040*** | 0.0045*** | 0.0099*** |
| 4 | 0.0014*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0053*** | 0.0045*** | 0.0050*** | 0.0107*** |
| 5 | 0.0011*** | 0.0013*** | 0.0010*** | 0.0002 | 0.0051*** | 0.0043*** | 0.0047*** | 0.0097*** |
| 6 | 0.0014*** | 0.0016*** | 0.0013*** | 0.0005 | 0.0052*** | 0.0045*** | 0.0049*** | 0.0112*** |
| 7 | 0.0011*** | 0.0013*** | 0.0009*** | 0.0002 | 0.0048*** | 0.0040*** | 0.0045*** | 0.0087*** |
| 8 | 0.0012*** | 0.0014*** | 0.0011*** | 0.0004 | 0.0053*** | 0.0044*** | 0.0048*** | 0.0094*** |
| 9 | 0.0016*** | 0.0018*** | 0.0014*** | 0.0004 | 0.0055*** | 0.0048*** | 0.0052*** | 0.0116*** |
| 10 | 0.0020*** | 0.0022*** | 0.0018*** | 0.0005 | 0.0059*** | 0.0052*** | 0.0056*** | 0.0146*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.14. Intercepts of Book-to-Market portfolios Family 5.*

Multivariate Regressions of BookToMarket Portfolios on VOL, VVOL and HML - BETAS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.4691*** | 0.3487*** | 0.4383*** | 0.2538*** | 2.4067*** | 0.3149*** | 0.6386** | -0.1649** |
| 2 | 0.5333*** | 0.3285*** | 0.4002*** | 0.2711*** | 2.3086*** | 0.2967*** | 0.6084** | -0.1580** |
| 3 | 0.5355*** | 0.3468*** | 0.4184*** | 0.2590*** | 2.3569*** | 0.3174*** | 0.6232** | -0.1492** |
| 4 | 0.4611*** | 0.3465*** | 0.4062*** | 0.2477*** | 2.5031*** | 0.3263*** | 0.6496** | -0.1566** |
| 5 | 0.6312*** | 0.4106*** | 0.4899*** | 0.2951*** | 2.7824*** | 0.3680*** | 0.7007** | -0.1442** |
| 6 | 0.5573*** | 0.3748*** | 0.4426*** | 0.2923*** | 2.4580*** | 0.3391*** | 0.6500** | -0.1663** |
| 7 | 0.5522*** | 0.3677*** | 0.4386*** | 0.2597*** | 2.6074*** | 0.3479*** | 0.6638** | -0.1258** |
| 8 | 0.5478*** | 0.3405*** | 0.4159*** | 0.2483*** | 2.7035*** | 0.3090*** | 0.6422** | -0.1334** |
| 9 | 0.6389*** | 0.4115*** | 0.4743*** | 0.3327*** | 2.7003*** | 0.3777*** | 0.7142** | -0.1746** |
| 10 | 0.6553*** | 0.4911*** | 0.5621*** | 0.4092*** | 2.7029*** | 0.3987*** | 0.7283** | -0.2313** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.15.Betas of Book-to-Market portfolios Family 5.*

Multivariate Regressions of BookToMarket Portfolios on VOL, VVOL and HML - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0542 | 0.0571 | 0.0612 | 0.0564 | 0.0744 | 0.0678 | 0.0834 | 0.0748 |
| 2 | 0.0111 | 0.0129 | 0.0163 | 0.0135 | 0.0281 | 0.0212 | 0.0370 | 0.1140 |
| 3 | 0.0083 | 0.0109 | 0.0147 | 0.0102 | 0.0262 | 0.0190 | 0.0363 | 0.1117 |
| 4 | 0.0281 | 0.0310 | 0.0337 | 0.0301 | 0.0461 | 0.0392 | 0.0547 | 0.2195 |
| 5 | 0.0412 | 0.0440 | 0.0480 | 0.0429 | 0.0580 | 0.0499 | 0.0668 | 0.2285 |
| 6 | 0.0371 | 0.0399 | 0.0432 | 0.0398 | 0.0534 | 0.0470 | 0.0627 | 0.2241 |
| 7 | 0.0566 | 0.0593 | 0.0629 | 0.0581 | 0.0735 | 0.0657 | 0.0826 | 0.2570 |
| 8 | 0.1396 | 0.1411 | 0.1438 | 0.1405 | 0.1552 | 0.1472 | 0.1603 | 0.4187 |
| 9 | 0.1147 | 0.1171 | 0.1201 | 0.1175 | 0.1291 | 0.1229 | 0.1385 | 0.3707 |
| 10 | 0.1257 | 0.1296 | 0.1328 | 0.1306 | 0.1365 | 0.1321 | 0.1447 | 0.3927 |

*Table A.16. $R^2$ of Book-to-Market portfolios Family 5.*

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0007 | 0.0011* | 0.0002 | -0.0014* | 0.0066*** | 0.0051*** | 0.0058*** | 0.0193*** |
| 2 | 0.0011** | 0.0013*** | 0.0007 | -0.0001 | 0.0062*** | 0.0050*** | 0.0056*** | 0.0175*** |
| 3 | 0.0009** | 0.0011*** | 0.0006 | -0.0002 | 0.0053*** | 0.0044*** | 0.0049*** | 0.0161*** |
| 4 | 0.0012*** | 0.0014*** | 0.0010** | -0.0000 | 0.0049*** | 0.0042*** | 0.0047*** | 0.0145*** |
| 5 | 0.0014*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0053*** | 0.0045*** | 0.0050*** | 0.0147*** |
| 6 | 0.0015*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0050*** | 0.0044*** | 0.0048*** | 0.0143*** |
| 7 | 0.0016*** | 0.0017*** | 0.0014*** | 0.0008* | 0.0052*** | 0.0046*** | 0.0050*** | 0.0138*** |
| 8 | 0.0015*** | 0.0016*** | 0.0013*** | 0.0006 | 0.0050*** | 0.0043*** | 0.0047*** | 0.0143*** |
| 9 | 0.0015*** | 0.0016*** | 0.0014*** | 0.0007 | 0.0052*** | 0.0045*** | 0.0050*** | 0.0148*** |
| 10 | 0.0020*** | 0.0021*** | 0.0019*** | 0.0012** | 0.0062*** | 0.0055*** | 0.0060*** | 0.0168*** |

Multivariate Regressions of Short Term Reversal Portfolios on VOL and VVOL - INTERCEPTS

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.17. Intercepts of Short-term-Reversal portfolios Family 2.*

Multivariate Regressions of Short Term Reversal Portfolios on VOL and VVOL - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0067 | 0.0144 | 0.0233 | 0.0102 | 0.0188 | 0.0105 | 0.0265 | 0.0133 |
| 2 | 0.0055 | 0.0103 | 0.0177 | 0.0078 | 0.0228 | 0.0141 | 0.0308 | 0.0218 |
| 3 | 0.0055 | 0.0106 | 0.0180 | 0.0082 | 0.0223 | 0.0147 | 0.0333 | 0.0224 |
| 4 | 0.0076 | 0.0131 | 0.0193 | 0.0126 | 0.0221 | 0.0169 | 0.0361 | 0.0235 |
| 5 | 0.0085 | 0.0112 | 0.0151 | 0.0102 | 0.0275 | 0.0201 | 0.0371 | 0.0299 |
| 6 | 0.0078 | 0.0109 | 0.0145 | 0.0101 | 0.0242 | 0.0186 | 0.0354 | 0.0279 |
| 7 | 0.0095 | 0.0124 | 0.0155 | 0.0110 | 0.0268 | 0.0221 | 0.0389 | 0.0303 |
| 8 | 0.0097 | 0.0133 | 0.0171 | 0.0116 | 0.0254 | 0.0205 | 0.0360 | 0.0286 |
| 9 | 0.0079 | 0.0098 | 0.0107 | 0.0095 | 0.0240 | 0.0190 | 0.0328 | 0.0258 |
| 10 | 0.0121 | 0.0137 | 0.0141 | 0.0137 | 0.0275 | 0.0242 | 0.0360 | 0.0315 |

*Table A.18. $R^2$ of Short-term-Reversal portfolios Family 2.*

Multivariate Regressions of Short Term Reversal Portfolios on VOL, VVOL and STR - INTERCEPTS

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.0016*** | 0.0017*** | 0.0014*** | 0.0003 | 0.0054*** | 0.0047*** | 0.0051*** | 0.0165*** |
| 2 | 0.0017*** | 0.0018*** | 0.0016*** | 0.0011** | 0.0054*** | 0.0048*** | 0.0052*** | 0.0154*** |
| 3 | 0.0014*** | 0.0015*** | 0.0013*** | 0.0007* | 0.0047*** | 0.0042*** | 0.0046*** | 0.0145*** |
| 4 | 0.0016*** | 0.0017*** | 0.0015*** | 0.0006 | 0.0045*** | 0.0041*** | 0.0045*** | 0.0133*** |
| 5 | 0.0017*** | 0.0018*** | 0.0017*** | 0.0012*** | 0.0050*** | 0.0044*** | 0.0048*** | 0.0138*** |
| 6 | 0.0017*** | 0.0018*** | 0.0016*** | 0.0011** | 0.0048*** | 0.0043*** | 0.0047*** | 0.0136*** |
| 7 | 0.0017*** | 0.0018*** | 0.0016*** | 0.0011*** | 0.0050*** | 0.0045*** | 0.0049*** | 0.0133*** |
| 8 | 0.0016*** | 0.0017*** | 0.0014*** | 0.0008* | 0.0049*** | 0.0043*** | 0.0047*** | 0.0140*** |
| 9 | 0.0015*** | 0.0017*** | 0.0014*** | 0.0007 | 0.0052*** | 0.0045*** | 0.0049*** | 0.0147*** |
| 10 | 0.0020*** | 0.0021*** | 0.0018*** | 0.0011* | 0.0063*** | 0.0055*** | 0.0060*** | 0.0169*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.19. Intercepts of Short-term-Reversal portfolios Family 5.*

Multivariate Regressions of Short Term Reversal Portfolios on VOL, VVOL and STR - R2

| decile | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| 1 | 0.4885 | 0.4906 | 0.4908 | 0.4903 | 0.4943 | 0.4932 | 0.4994 | 0.4778 |
| 2 | 0.4108 | 0.4117 | 0.4121 | 0.4117 | 0.4202 | 0.4183 | 0.4256 | 0.4230 |
| 3 | 0.3263 | 0.3277 | 0.3284 | 0.3277 | 0.3349 | 0.3332 | 0.3428 | 0.3461 |
| 4 | 0.2168 | 0.2190 | 0.2198 | 0.2200 | 0.2256 | 0.2247 | 0.2363 | 0.2352 |
| 5 | 0.1543 | 0.1551 | 0.1556 | 0.1553 | 0.1675 | 0.1643 | 0.1753 | 0.1861 |
| 6 | 0.0963 | 0.0978 | 0.0985 | 0.0979 | 0.1080 | 0.1054 | 0.1174 | 0.1288 |
| 7 | 0.0586 | 0.0603 | 0.0613 | 0.0597 | 0.0726 | 0.0702 | 0.0834 | 0.0880 |
| 8 | 0.0210 | 0.0239 | 0.0264 | 0.0227 | 0.0352 | 0.0315 | 0.0452 | 0.0450 |
| 9 | 0.0082 | 0.0100 | 0.0108 | 0.0097 | 0.0241 | 0.0192 | 0.0328 | 0.0267 |
| 10 | 0.0138 | 0.0156 | 0.0163 | 0.0155 | 0.0299 | 0.0262 | 0.0387 | 0.0325 |

*Table A.20. $R^2$ of Short-term-Reversal portfolios Family 5.*

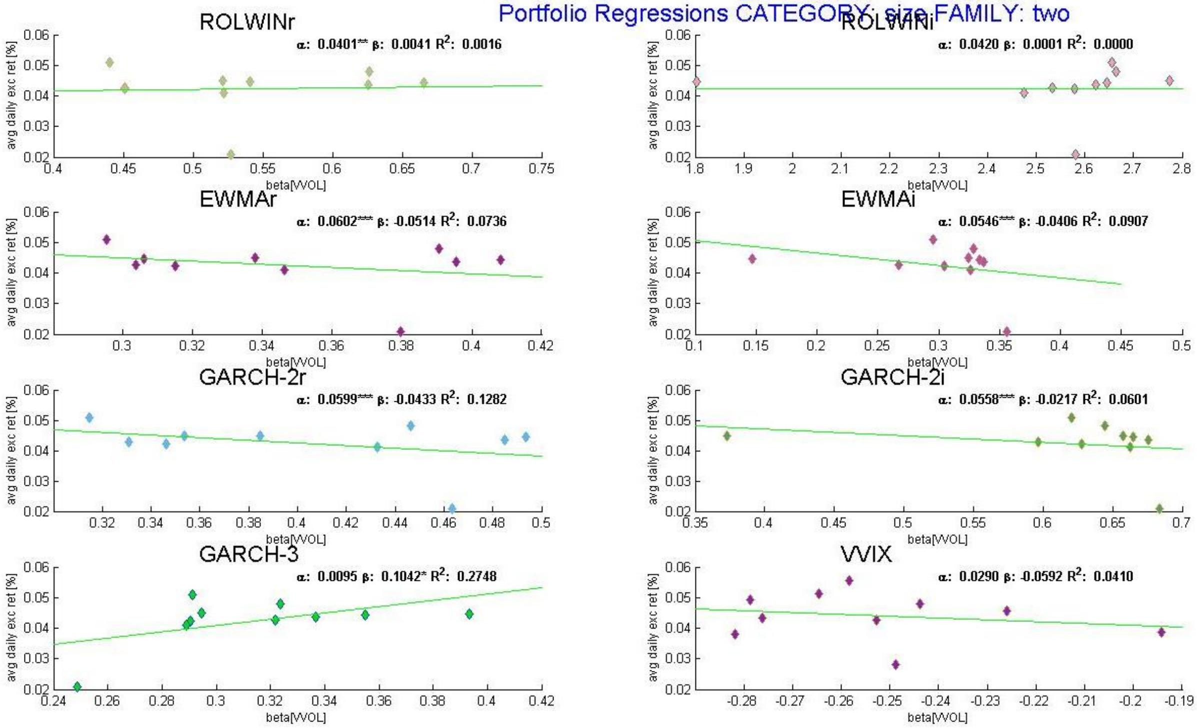


*Figure A.1. Cross-sectional analysis Size portfolios Family 2.*

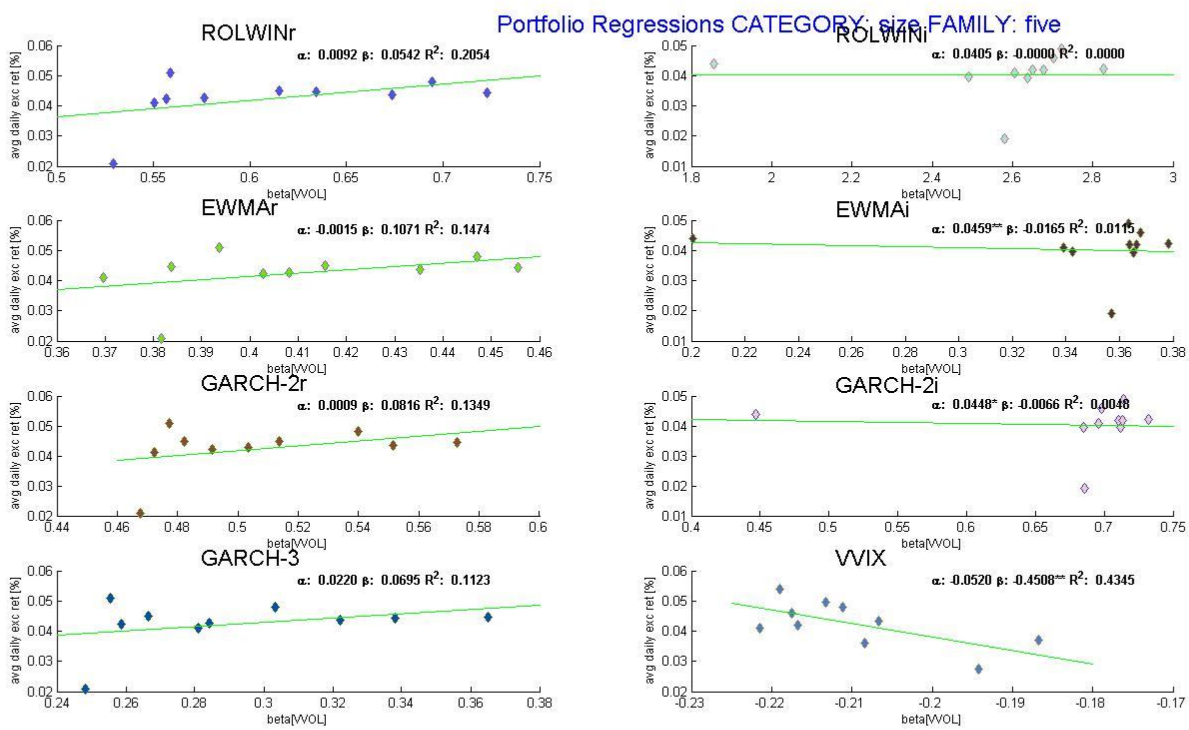


*Figure A.2. Cross-sectional analysis Size portfolios Family 5.*

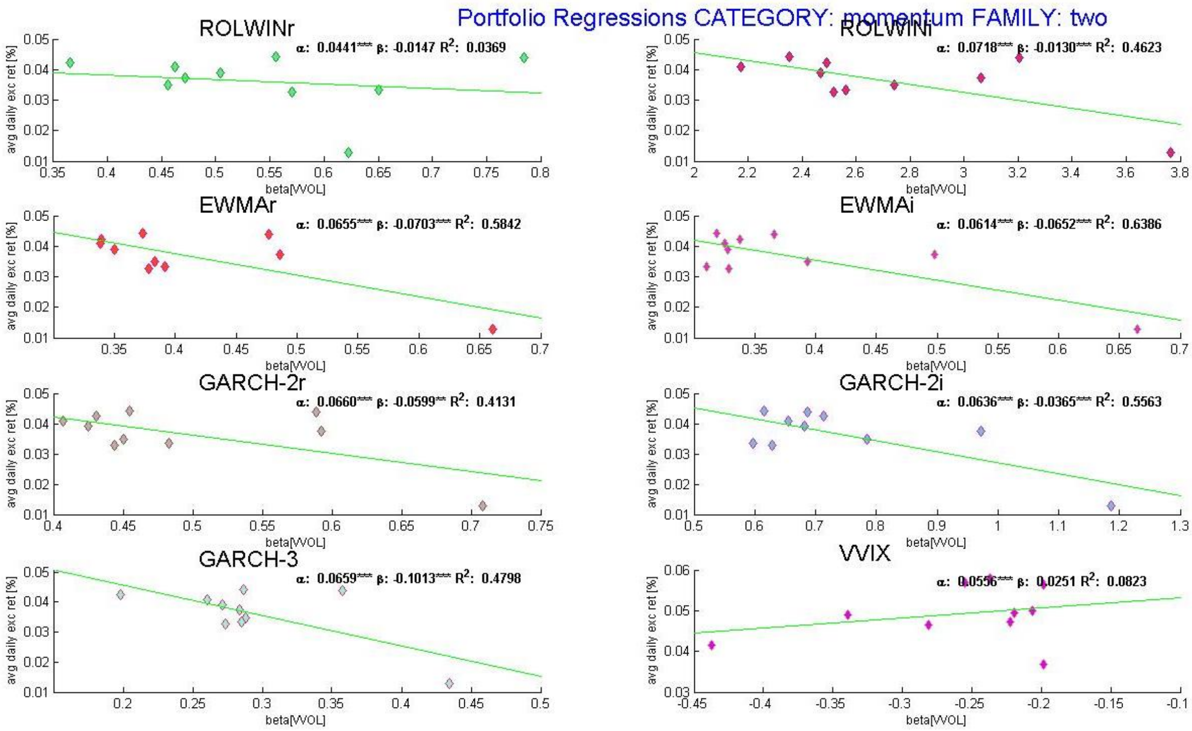


*Figure A.3. Cross-sectional analysis Momentum portfolios Family 2.*

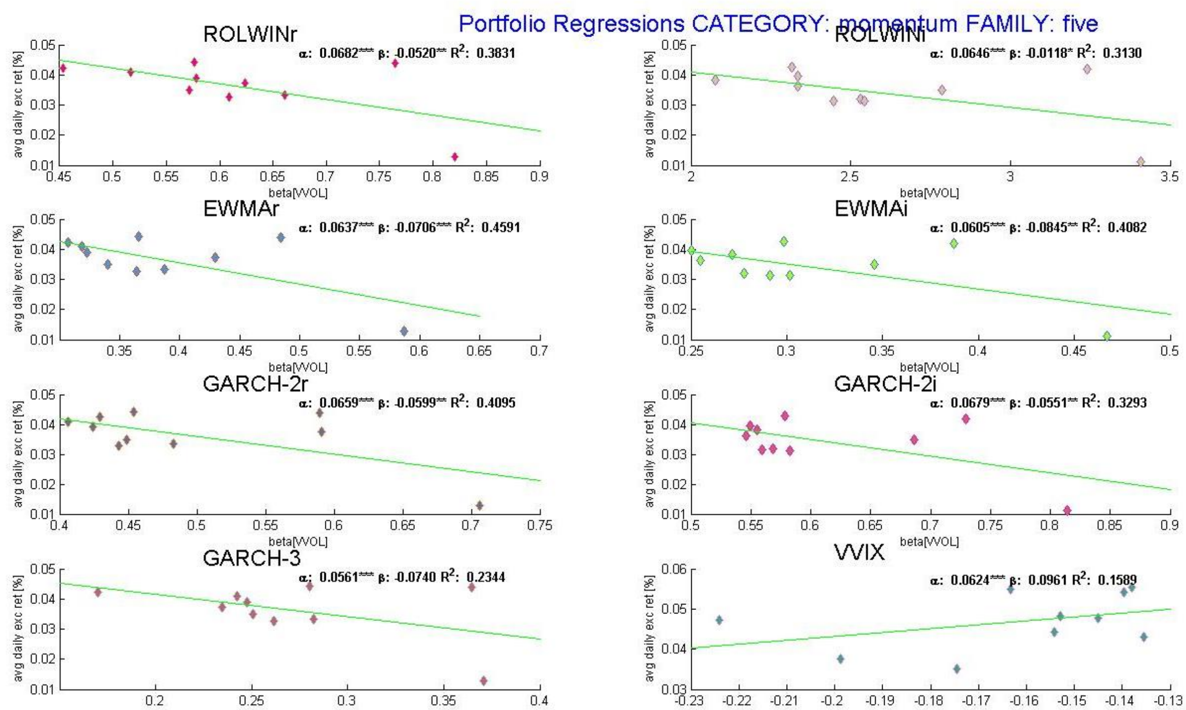


*Figure A.4. Cross-sectional analysis Momentum portfolios Family 5.*

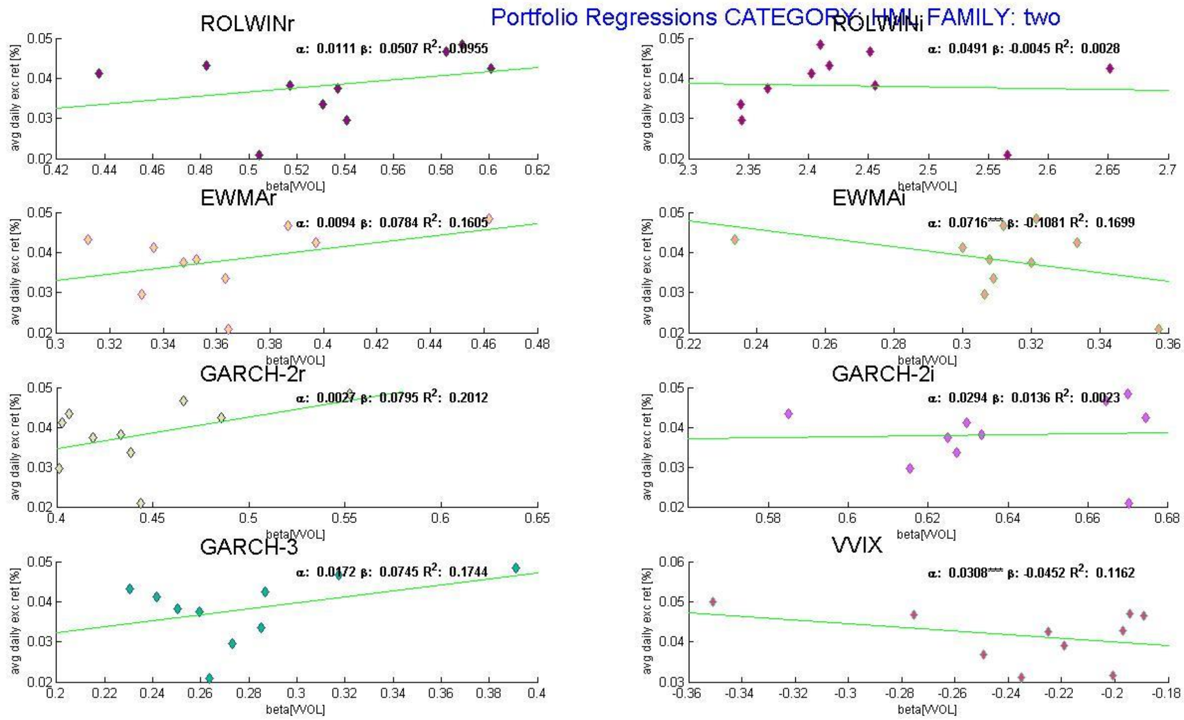


*Figure A.5. Cross-sectional analysis Book-to-Market portfolios Family 2.*

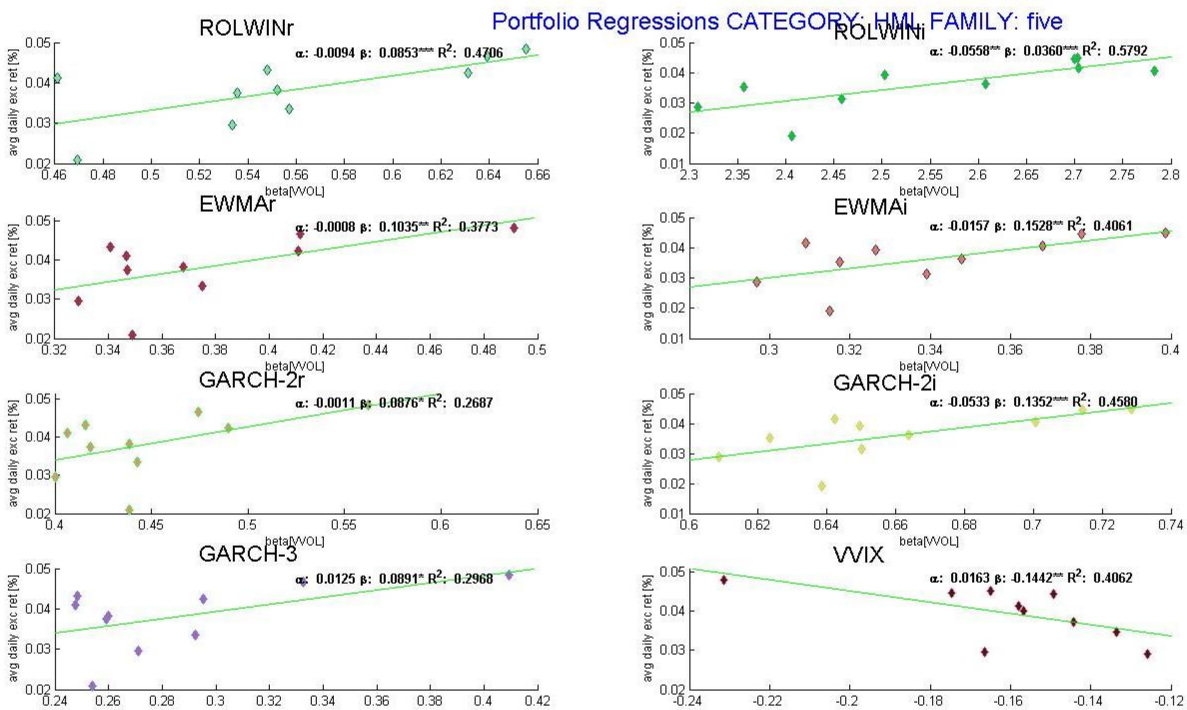


*Figure A.6. Cross-sectional analysis Book-to-Market portfolios Family 5.*

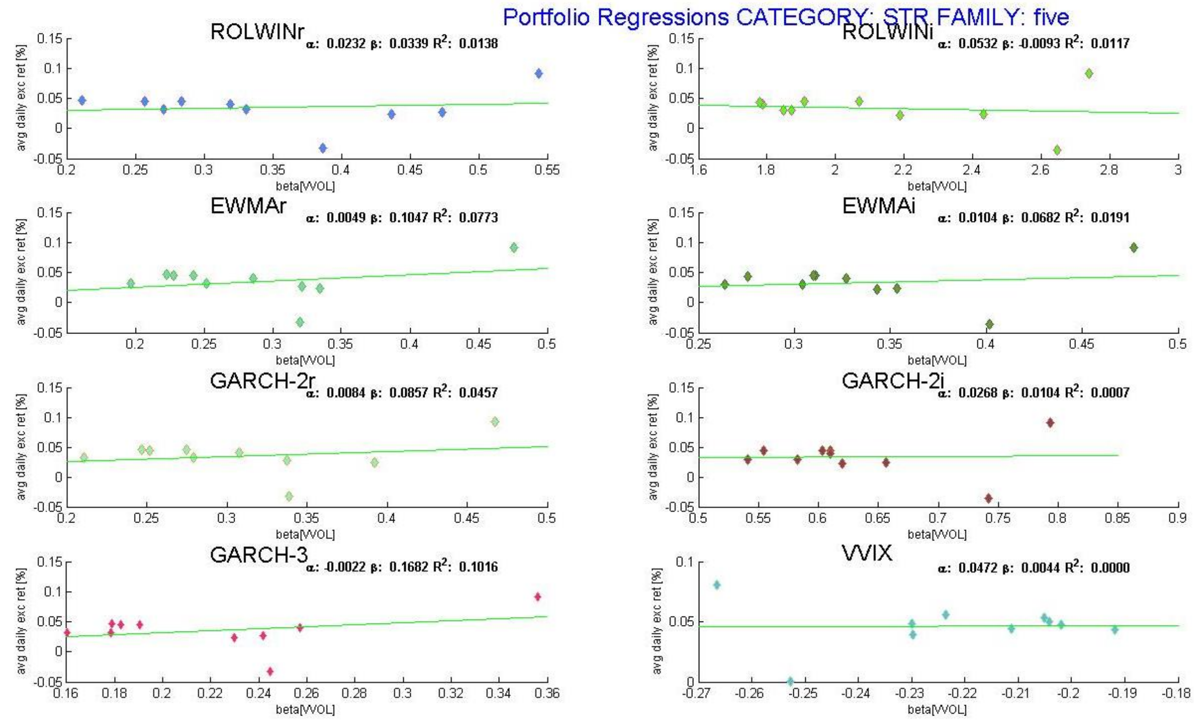


*Figure A.7. Cross-sectional analysis Short-term-Reversal portfolios Family 5.*

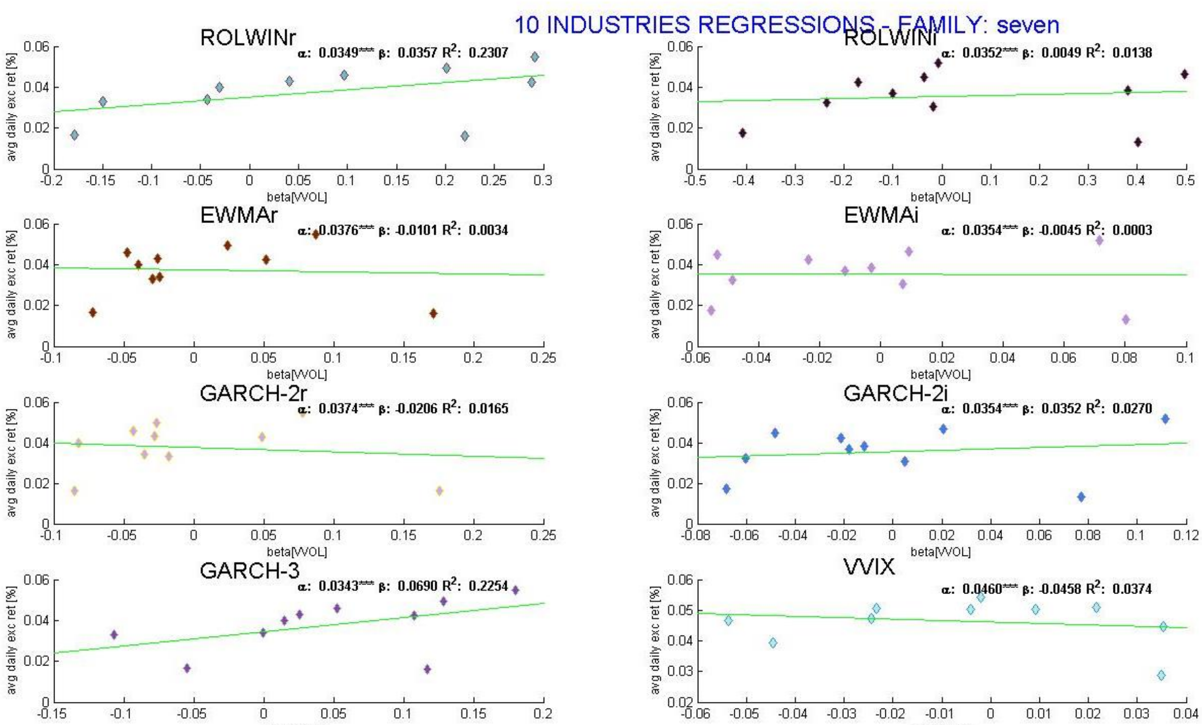


*Figure A.8. Cross-sectional analysis 10 Industries on VOL, VVOL, MRP, SMB, HML, MOM, STR and LTR.*

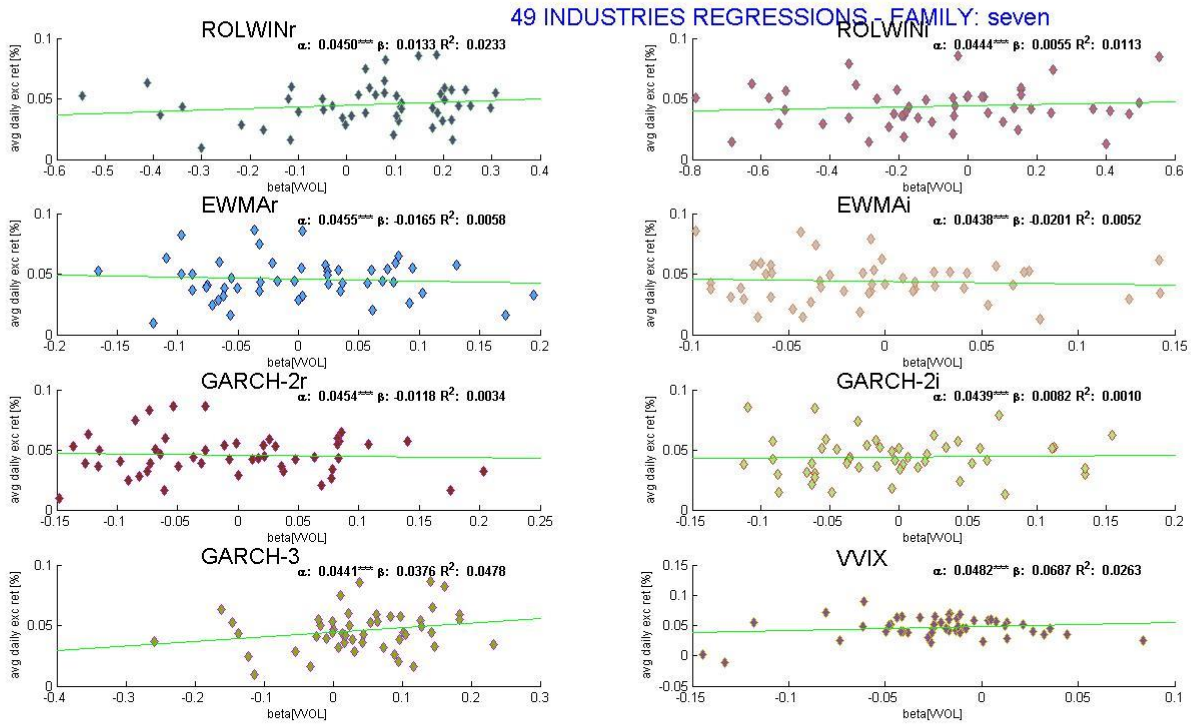


*Figure A.9. Cross-sectional analysis 49 Industries on VOL, VVOL, MRP, SMB, HML, MOM, STR and LTR.*

10 Industry Portfolios on VOL and VVOL

alphas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| **NoDur** | 0.0013*** | 0.0015*** | 0.0013*** | 0.0006** | 0.0035*** | 0.0030*** | 0.0033*** | 0.0097*** |
| **Durbl** | 0.0021*** | 0.0022*** | 0.0019*** | 0.0009 | 0.0064*** | 0.0056*** | 0.0061*** | 0.0230*** |
| **Manuf** | 0.0018*** | 0.0020*** | 0.0017*** | 0.0009* | 0.0056*** | 0.0048*** | 0.0053*** | 0.0159*** |
| **Enrgy** | 0.0017*** | 0.0021*** | 0.0017*** | 0.0006 | 0.0056*** | 0.0049*** | 0.0054*** | 0.0147*** |
| **HiTec** | 0.0013** | 0.0014*** | 0.0011** | 0.0002 | 0.0057*** | 0.0049*** | 0.0054*** | 0.0154*** |
| **Telcm** | 0.0014*** | 0.0016*** | 0.0011*** | 0.0001 | 0.0052*** | 0.0044*** | 0.0048*** | 0.0122*** |
| **Shops** | 0.0012*** | 0.0013*** | 0.0011*** | 0.0003 | 0.0043*** | 0.0036*** | 0.0040*** | 0.0120*** |
| **Hlth** | 0.0012*** | 0.0015*** | 0.0012*** | 0.0003 | 0.0041*** | 0.0034*** | 0.0037*** | 0.0105*** |
| **Utils** | 0.0014*** | 0.0016*** | 0.0014*** | 0.0005 | 0.0044*** | 0.0036*** | 0.0040*** | 0.0073*** |
| **Other** | 0.0014*** | 0.0015*** | 0.0011** | 0.0007 | 0.0061*** | 0.0052*** | 0.0058*** | 0.0180*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.21. Regression of 10 Industries on volatility and VVOL – Intercepts.*

10 Industry Portfolios on VOL and VVOL

Beta[VVOL]

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| **NoDur** | 0.4345*** | 0.1836*** | 0.2507*** | 0.2061*** | 1.5038*** | 0.1444*** | 0.3548*** | -0.1399*** |
| **Durbl** | 0.4640** | 0.3549*** | 0.4201*** | 0.3092*** | 2.5190*** | 0.2902*** | 0.6706*** | -0.3706*** |
| **Manuf** | 0.5375*** | 0.3260*** | 0.4013*** | 0.2720*** | 2.3423*** | 0.2995*** | 0.6304*** | -0.2414*** |
| **Enrgy** | 0.8379*** | 0.4608*** | 0.5610*** | 0.3735*** | 2.6586*** | 0.3907*** | 0.7720*** | -0.2229*** |
| **HiTec** | 0.4977** | 0.4096*** | 0.4616*** | 0.3091*** | 3.0060*** | 0.4477*** | 0.8223*** | -0.2385*** |
| **Telcm** | 0.7647*** | 0.5639*** | 0.6539*** | 0.3867*** | 2.9846*** | 0.4390*** | 0.7717*** | -0.1775*** |
| **Shops** | 0.4115*** | 0.2737*** | 0.2985*** | 0.2429*** | 2.0819*** | 0.2806*** | 0.5579*** | -0.1852*** |
| **Hlth** | 0.7066*** | 0.3330*** | 0.3957*** | 0.3033*** | 2.2702*** | 0.2524*** | 0.4785*** | -0.1494*** |
| **Utils** | 0.5803*** | 0.2860*** | 0.3182*** | 0.2663*** | 2.2563*** | 0.2242*** | 0.4736*** | -0.0896** |
| **Other** | 0.4163** | 0.3956*** | 0.5102*** | 0.2098*** | 2.7851*** | 0.3609*** | 0.7648*** | -0.2771*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.22. Regression of 10 Industries on volatility and VVOL – Betas.*

10 Industry Portfolios on VOL and VVOL

R2

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| **NoDur** | 0.0104 | 0.0092 | 0.0122 | 0.0121 | 0.0197 | 0.0132 | 0.0232 | 0.0260 |
| **Durbl** | 0.0075 | 0.0093 | 0.0110 | 0.0100 | 0.0210 | 0.0163 | 0.0266 | 0.0361 |
| **Manuf** | 0.0105 | 0.0119 | 0.0149 | 0.0125 | 0.0265 | 0.0204 | 0.0352 | 0.0329 |
| **Enrgy** | 0.0090 | 0.0099 | 0.0133 | 0.0102 | 0.0167 | 0.0131 | 0.0259 | 0.0170 |
| **HiTec** | 0.0043 | 0.0068 | 0.0084 | 0.0065 | 0.0185 | 0.0143 | 0.0269 | 0.0303 |
| **Telcm** | 0.0114 | 0.0176 | 0.0232 | 0.0149 | 0.0254 | 0.0188 | 0.0354 | 0.0216 |
| **Shops** | 0.0052 | 0.0067 | 0.0078 | 0.0077 | 0.0180 | 0.0131 | 0.0254 | 0.0239 |
| **Hlth** | 0.0122 | 0.0113 | 0.0148 | 0.0135 | 0.0221 | 0.0132 | 0.0238 | 0.0270 |
| **Utils** | 0.0092 | 0.0090 | 0.0106 | 0.0108 | 0.0215 | 0.0128 | 0.0229 | 0.0138 |
| **Other** | 0.0046 | 0.0079 | 0.0118 | 0.0054 | 0.0239 | 0.0180 | 0.0332 | 0.0279 |

*Table A.23. Regression of 10 Industries on volatility and VVOL – $R^2$.*

10 Industry Portfolios on VOL, VVOL, MRP, SMB, HML and MOM

alphas

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| **NoDur** | 0.0003** | 0.0004** | 0.0004*** | 0.0001 | 0.0002 | 0.0001 | 0.0001 | 0.0007 |
| **Durbl** | 0.0002 | 0.0001 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0032*** |
| **Manuf** | 0.0002** | 0.0003** | 0.0003** | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0011** |
| **Enrgy** | -0.0001 | -0.0000 | -0.0001 | -0.0005 | -0.0002 | -0.0001 | -0.0000 | -0.0009 |
| **HiTec** | 0.0004** | 0.0003* | 0.0004** | 0.0005** | 0.0002 | 0.0003 | 0.0003 | 0.0016*** |
| **Telcm** | 0.0002 | 0.0002 | 0.0001 | -0.0002 | 0.0002 | 0.0001 | 0.0001 | -0.0010 |
| **Shops** | 0.0000 | 0.0000 | 0.0001 | -0.0000 | -0.0001 | -0.0001 | -0.0001 | 0.0000 |
| **Hlth** | 0.0002 | 0.0004* | 0.0003* | -0.0000 | 0.0004 | 0.0002 | 0.0001 | 0.0011 |
| **Utils** | 0.0000 | 0.0001 | 0.0002 | -0.0002 | 0.0004 | 0.0002 | 0.0002 | -0.0026*** |
| **Other** | -0.0003*** | -0.0004*** | -0.0004*** | -0.0000 | -0.0002 | -0.0002 | -0.0002 | -0.0013*** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.24. Regression of 10 Industries on volatility, VVOL, MRP, SMB, HML and MOM – Intercepts.*

10 Industry Portfolios on VOL, VVOL, MRP, SMB, HML and MOM

Beta[VVOL]

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| **NoDur** | 0.0752 | -0.0605* | -0.0639** | 0.0470* | -0.0819 | -0.0547* | -0.0521* | -0.0069 |
| **Durbl** | -0.0503 | -0.0302 | -0.0434 | 0.0005 | -0.2626 | -0.0507 | -0.0640 | -0.0590*** |
| **Manuf** | 0.0158 | -0.0386 | -0.0499* | 0.0079 | -0.1729 | -0.0175 | -0.0158 | -0.0171* |
| **Enrgy** | 0.2500* | 0.0734 | 0.0476 | 0.1165** | 0.1318 | 0.1040 | 0.1489** | 0.0021 |
| **HiTec** | -0.1560** | -0.0590 | -0.0630* | -0.0422 | -0.3898** | -0.0587 | -0.0698* | -0.0295** |
| **Telcm** | 0.2231*** | 0.1721*** | 0.1768*** | 0.1229*** | 0.3855* | 0.0769* | 0.0729* | 0.0204 |
| **Shops** | -0.0440 | -0.0487 | -0.0957** | 0.0130 | -0.1372 | -0.0136 | -0.0219 | -0.0017 |
| **Hlth** | 0.2695*** | 0.0388 | 0.0284 | 0.1083*** | 0.3160 | -0.0079 | -0.0203 | -0.0123 |
| **Utils** | 0.1726* | 0.0123 | -0.0494 | 0.0948** | 0.5463** | 0.0247 | 0.0372 | 0.0527*** |
| **Other** | -0.1315*** | -0.0197 | -0.0015 | -0.0924*** | -0.0194 | 0.0018 | 0.0002 | 0.0256** |

*** Significant at 1%

** Significant at 5%

*Significant at 10%

*Table A.25. Regression of 10 Industries on volatility, VVOL, MRP, SMB, HML and MOM – Betas.*

10 Industry Portfolios on VOL, VVOL, MRP, SMB, HML and MOM
R2

| | Realized Measure | | | | Implied Measure | | | |
|---|---|---|---|---|---|---|---|---|
| | ROLWIN | EWMA | GARCH-2 | GARCH-3 | ROLWIN | EWMA | GARCH-2 | VVIX |
| **NoDur** | 0.6619 | 0.6620 | 0.6621 | 0.6620 | 0.6612 | 0.6614 | 0.6614 | 0.8315 |
| **Durbl** | 0.7864 | 0.7864 | 0.7864 | 0.7864 | 0.7856 | 0.7856 | 0.7856 | 0.8732 |
| **Manuf** | 0.8810 | 0.8810 | 0.8811 | 0.8810 | 0.8809 | 0.8808 | 0.8808 | 0.9435 |
| **Enrgy** | 0.5733 | 0.5730 | 0.5729 | 0.5734 | 0.5704 | 0.5707 | 0.5710 | 0.7408 |
| **HiTec** | 0.8725 | 0.8725 | 0.8725 | 0.8725 | 0.8721 | 0.8720 | 0.8721 | 0.9147 |
| **Telcm** | 0.7691 | 0.7697 | 0.7699 | 0.7695 | 0.7679 | 0.7679 | 0.7679 | 0.8568 |
| **Shops** | 0.7145 | 0.7146 | 0.7150 | 0.7145 | 0.7153 | 0.7153 | 0.7153 | 0.8453 |
| **Hlth** | 0.6189 | 0.6179 | 0.6179 | 0.6189 | 0.6147 | 0.6144 | 0.6144 | 0.8003 |
| **Utils** | 0.5626 | 0.5622 | 0.5624 | 0.5630 | 0.5612 | 0.5606 | 0.5606 | 0.7156 |
| **Other** | 0.9302 | 0.9300 | 0.9300 | 0.9304 | 0.9294 | 0.9294 | 0.9294 | 0.9613 |

*Table A.26. Regression of 10 Industries on volatility, VVOL, MRP, SMB, HML and MOM – $R^2$.*

SUMMARY STATISTICS FOR THE ESTIMATED GARCH models - REALIZED AND IMPLIED

| | | REALIZED GARCH-2 | REALIZED GARCH-3 | IMPLIED GARCH-2 |
|---|---|---|---|---|
| **STAGE 1 GARCH(p,q)** | *p* | 1 | 2 | 2 |
| | *q* | 1 | 1 | 2 |
| **STAGE 2 GARCH(p,q)** | *p* | - | 2 | - |
| | *q* | - | 1 | - |
| | **p_Dickey-Fuller1** | 0 | 0 | 0 |
| | **p_Dickey-Fuller2** | 0 | 0,0621 | 0 |
| **p_Ljung-Box** | LAG:1 | 0 | 0 | 0 |
| | LAG:2 | 0 | 0 | 0 |
| | LAG:3 | 0 | 0 | 0 |
| | LAG:4 | 0 | 0 | 0 |
| | LAG:5 | 0 | 0 | 0 |
| | LAG:6 | 0 | 0 | 0 |

*Table A.27. Parameters from the GARCH model estimations. See the descriptive text below.*

For the GARCH models, correlograms for three types of series are plotted. The first one is denoted *res1* and it shows the ACF/PACF plots of the demeaned original series. The second series is eps (eps3 for GARCH-3) and shows the ACF/PACF plots of the (second) residual series after the (two stage) GARCH2 (3) has been applied. The third series is denoted *sigmasq* and shows the ACF/PACF plots of our modeled VVOL used in the study.

The Dickey Fuller test statistic p-values are denoted p_Dickey-Fuller1 and p_Dickey-Fuller2. Dickey-Fuller 1 refers to the augmented Dickey Fuller test of the original squared and demeaned series.

Dickey-Fuller 2 refers to the augmented Dickey Fuller test of the residual series after the GARCH model has been estimated. The Ljung-Box test p-values refer to the autocorrelation test of the squared and demeaned original series (volatility for GARCH-2 and returns for GARCH-3).

*Textbox A.1. Descriptive text for the ACF and PACF plots below and the GARCH estimation statistics in Table A.35 above.*

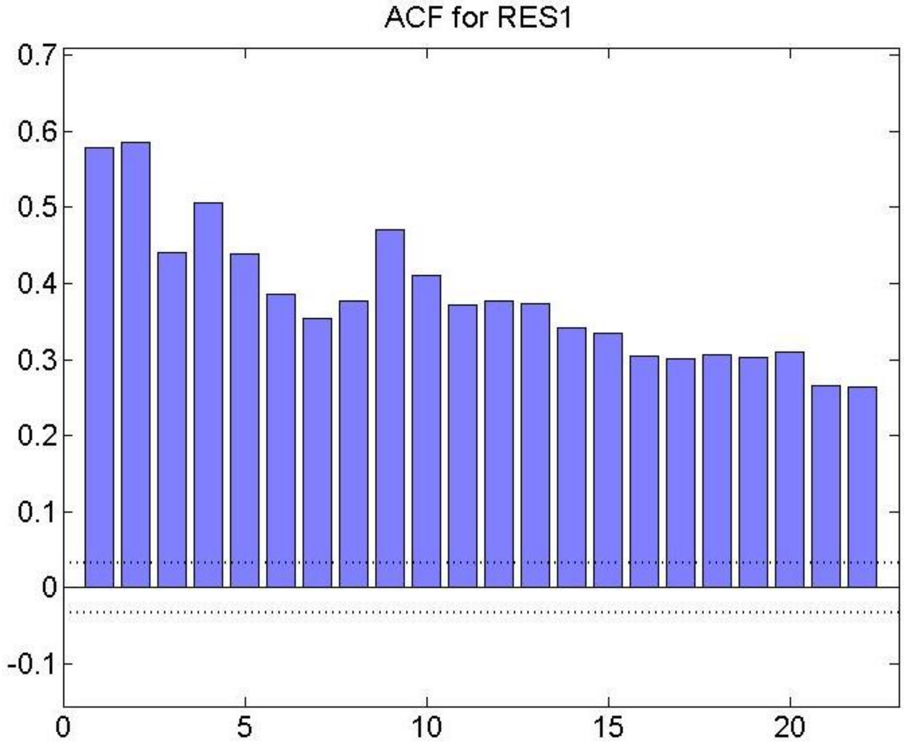

*Figure A.10. ACF plot of the demeaned volatility series for GARCH-2 realized.*

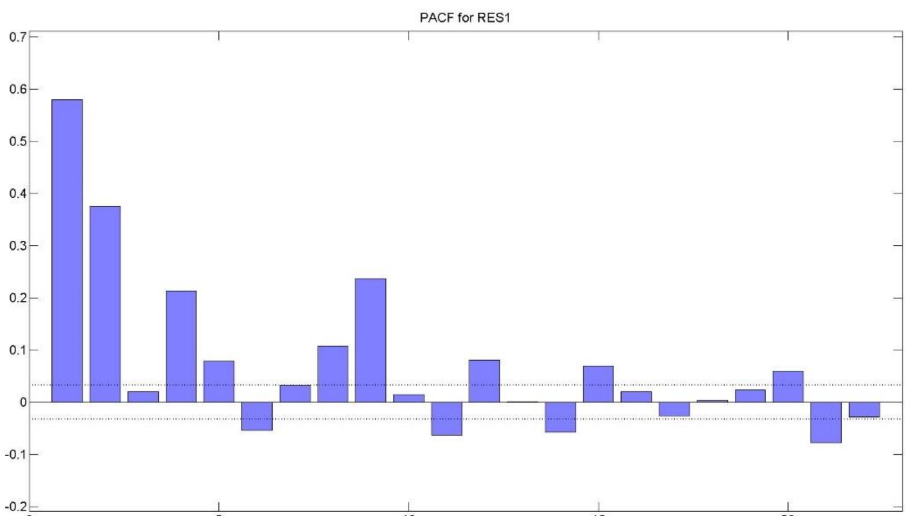

*Figure A.11. PACF plot of the demeaned volatility series for GARCH-2 realized.*

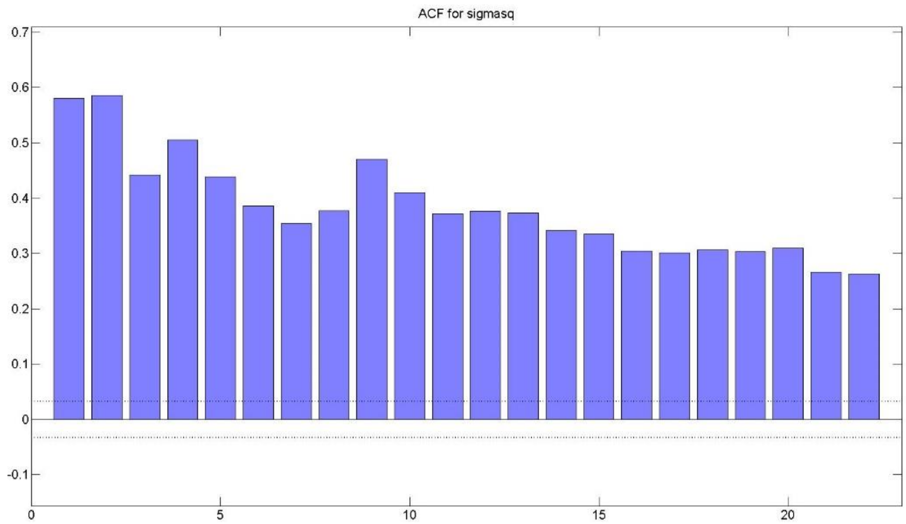

*Figure A.12. ACF plot of the modeled VVOL series for GARCH-2 realized.*

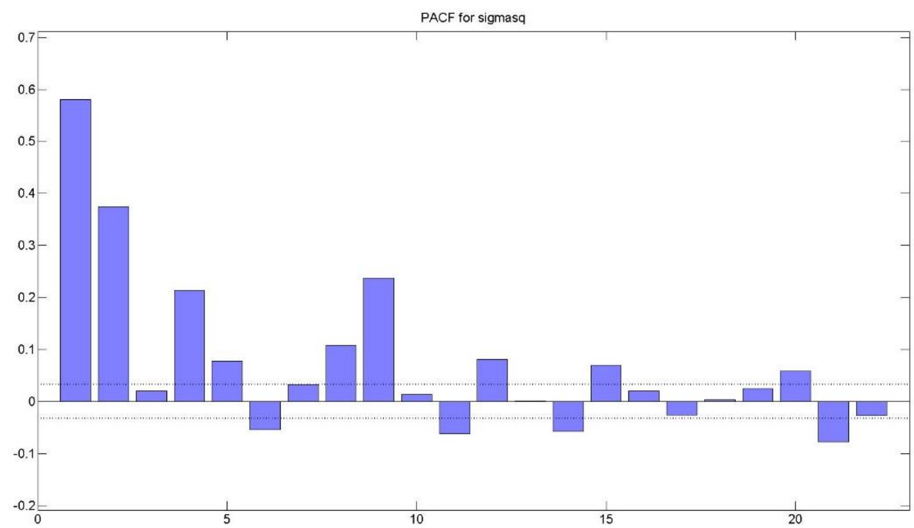

*Figure A.13. PACF plot of the modeled VVOL series for GARCH-2 realized.*

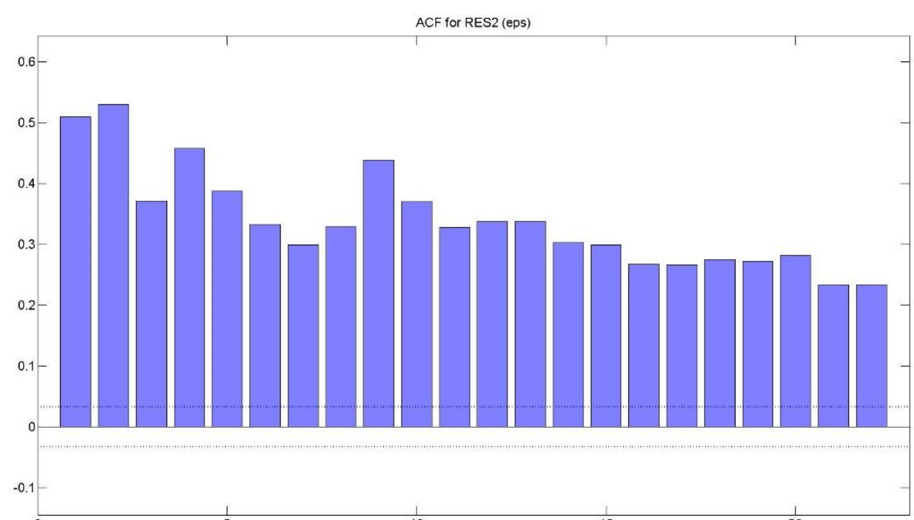

*Figure A.14. ACF plot of the residual series remaining after GARCH-2 realized is modeled.*

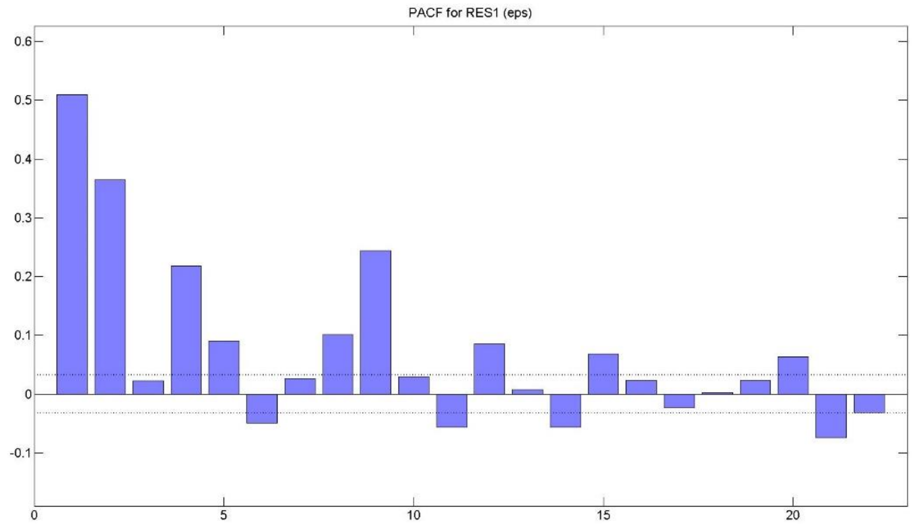


*Figure A.15. PACF plot of the residual series remaining after GARCH-2 realized is modeled.*

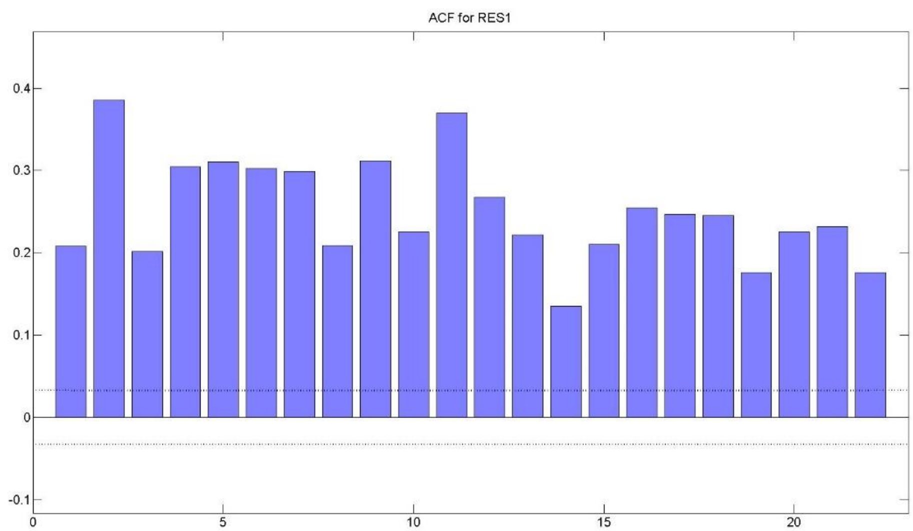


*Figure A.16. ACF plot of the demeaned volatility series for GARCH-3 realized.*

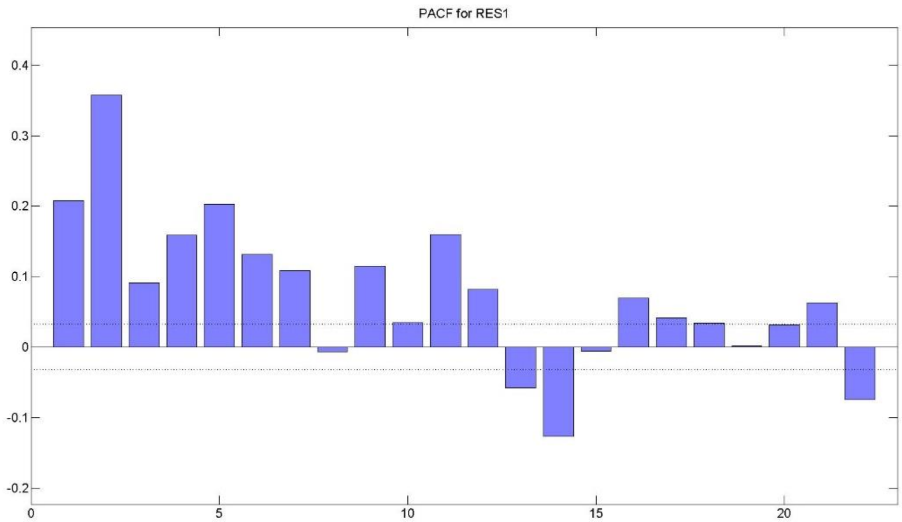

*Figure A.17. PACF plot of the demeaned volatility series for GARCH-3 realized.*

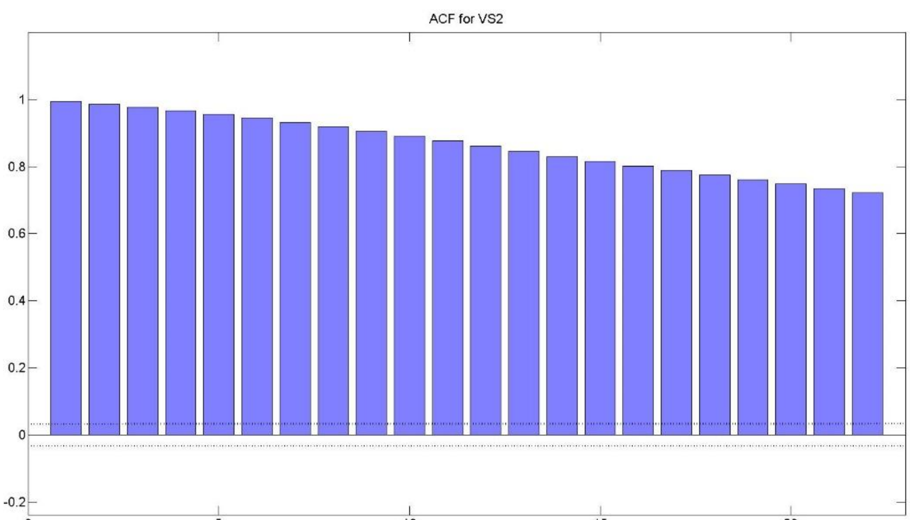

*Figure A.18. ACF plot of the modeled VVOL series for GARCH-3 realized.*

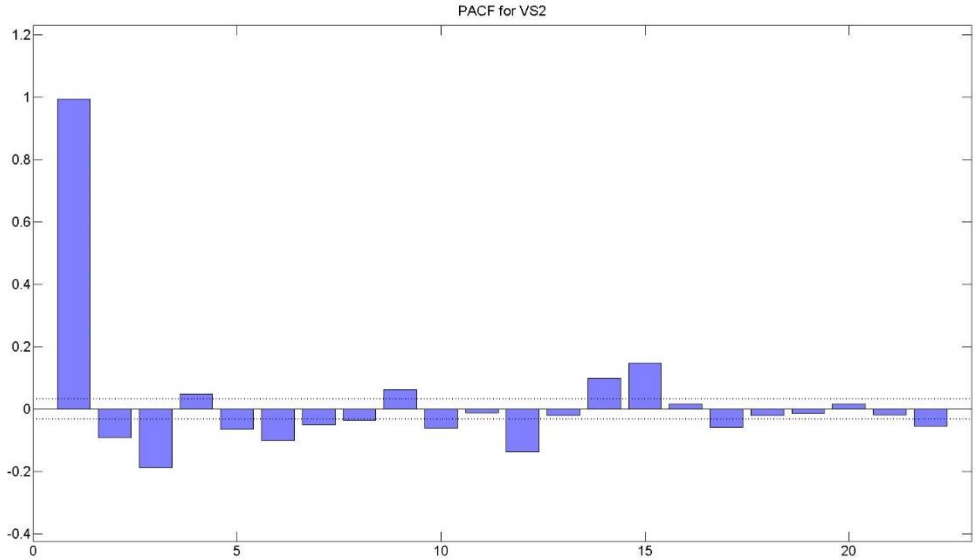

*Figure A.19. PACF plot of the modeled volatility series for GARCH-3 realized.*

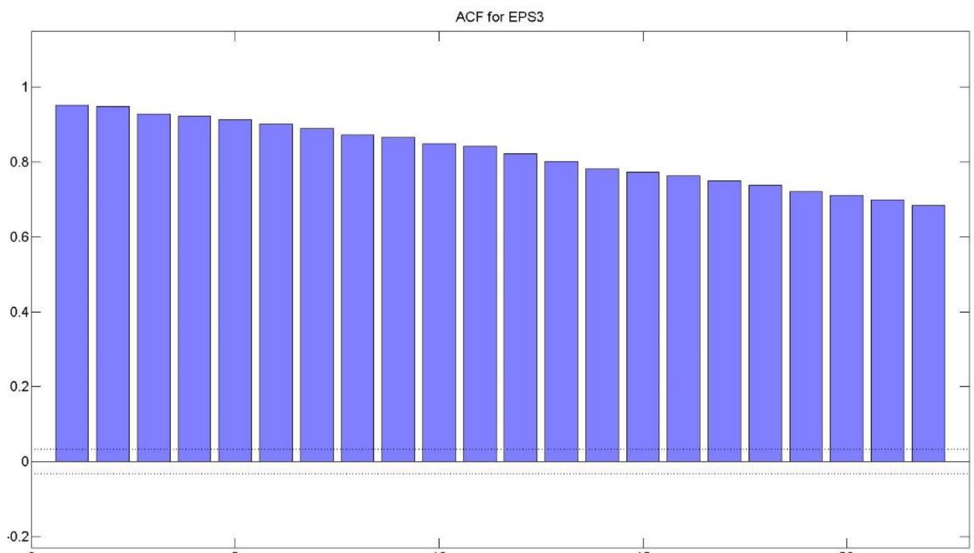

*Figure A.20. ACF plot of the residual series remaining after GARCH-3 realized is modeled.*

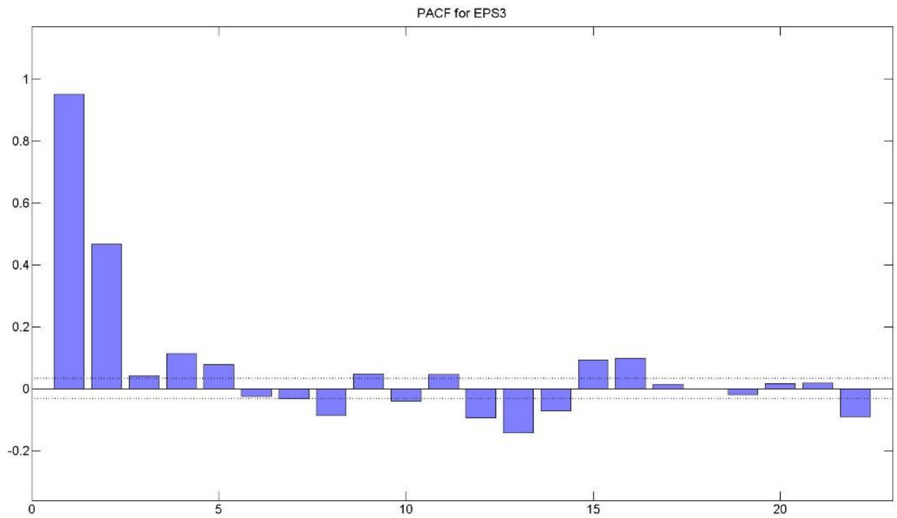

*Figure A.21. PACF plot of the residual series remaining after GARCH-3 realized is modeled.*

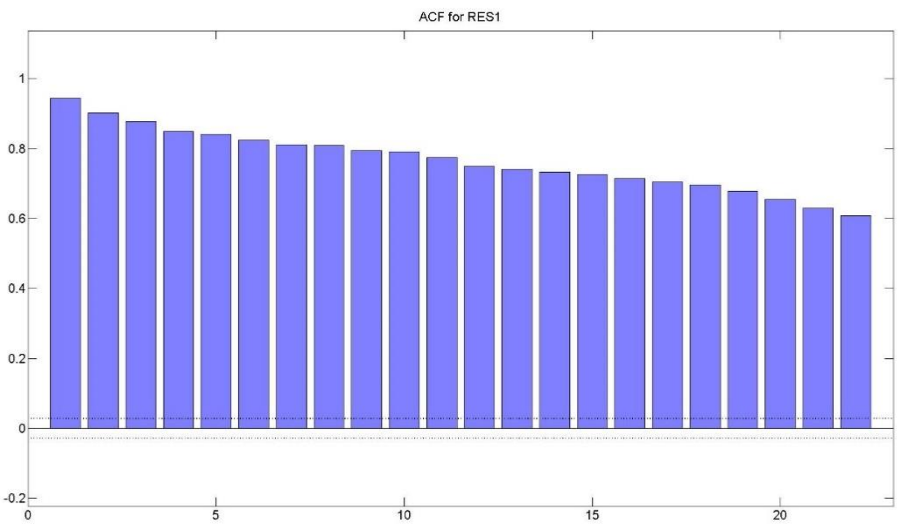

*Figure A.22. ACF plot of the demeaned volatility series for GARCH-2 implied.*

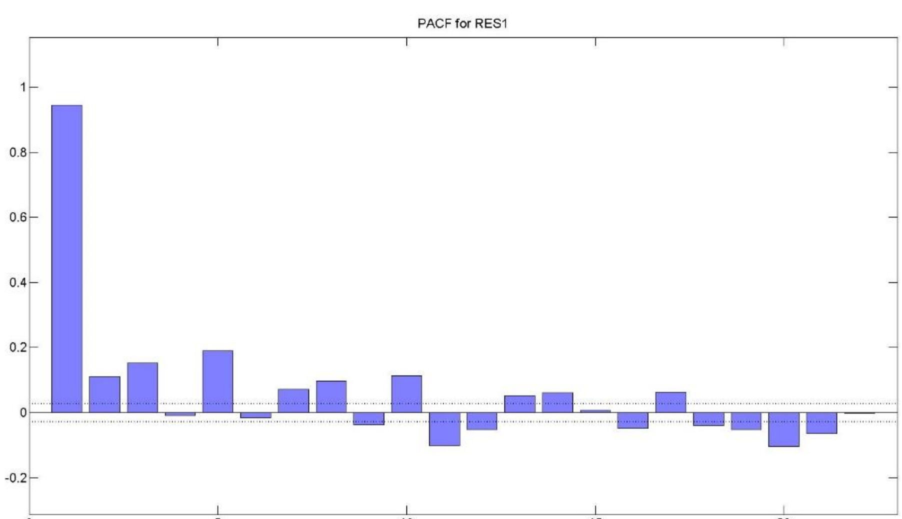

*Figure A.23. PACF plot of the demeaned volatility series for GARCH-2 implied.*

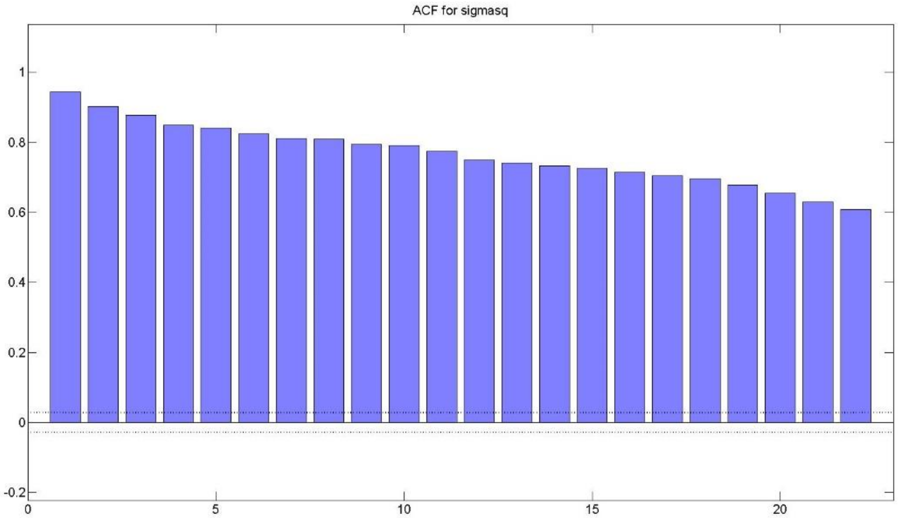


*Figure A.24. PACF plot of the demeaned volatility series for GARCH-2 implied.*

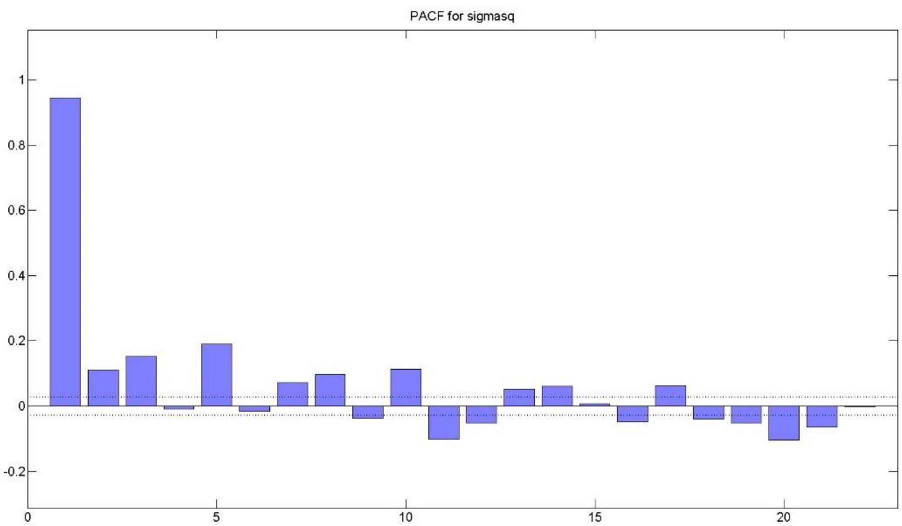


*Figure A.25. ACF plot of the modeled VVOL series for GARCH-2 implied.*

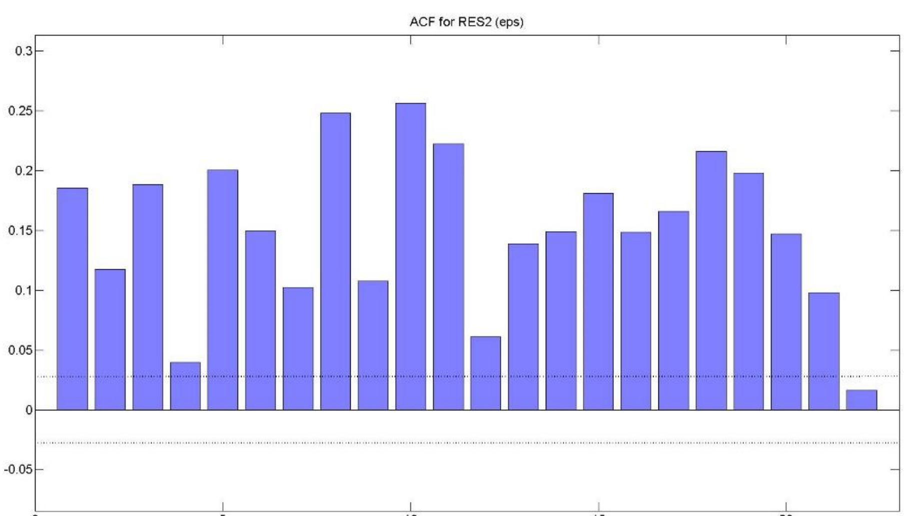


*Figure A.26. ACF plot of the residual series remaining after GARCH-2 implied is modeled.*

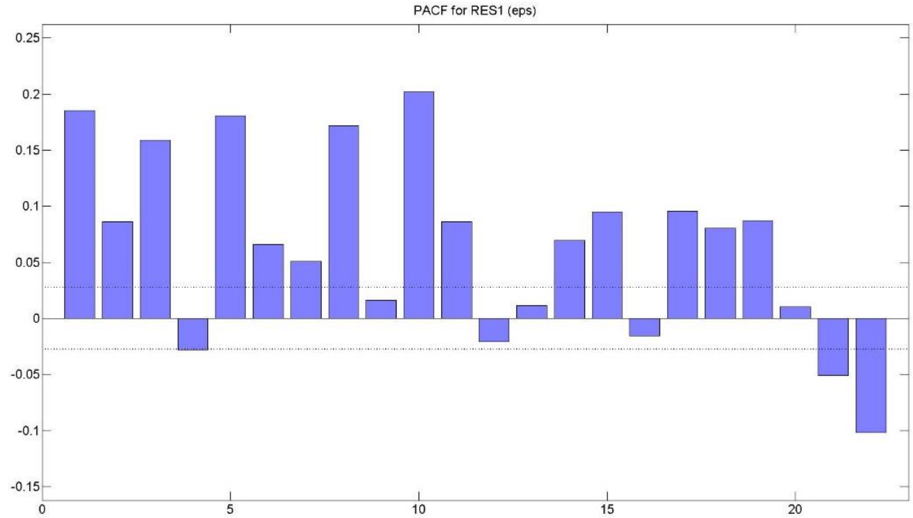

*Figure A.27. PACF plot of the residual series remaining after GARCH-2 implied is modeled.*

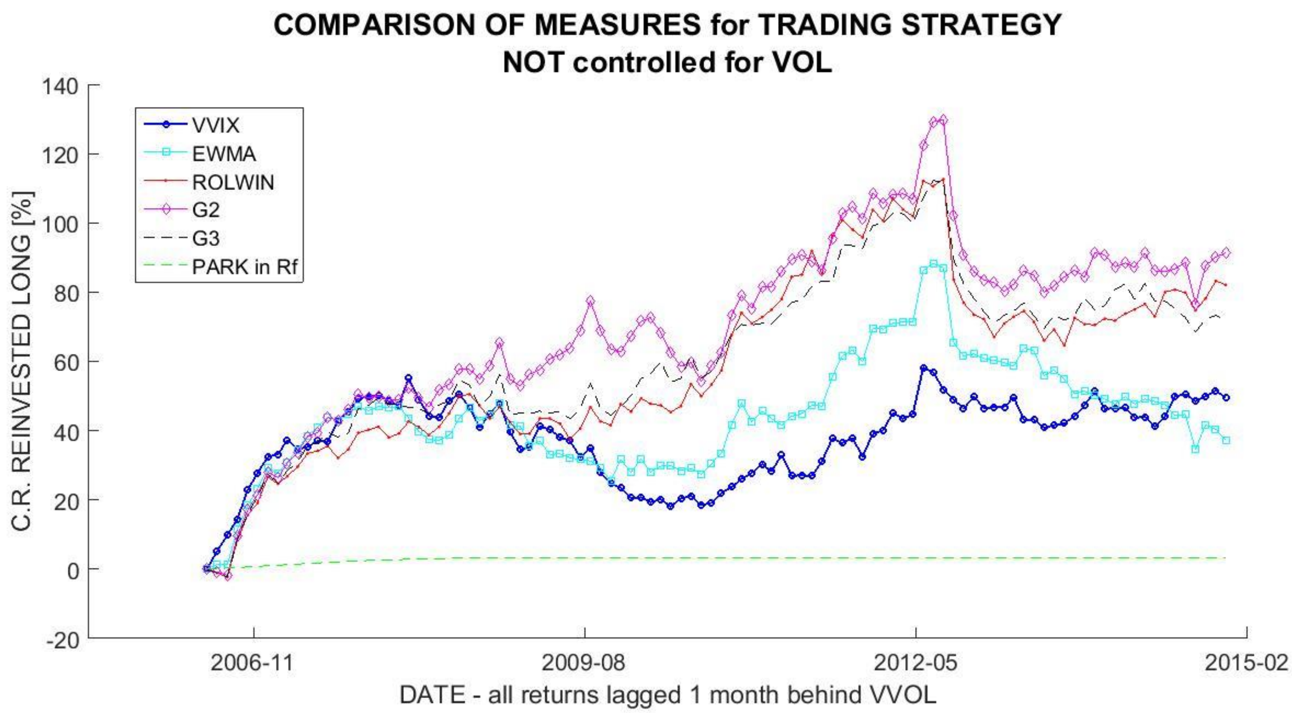


*Figure A.28. The long-short trading strategy profits for five VVOL measure when the loadings of stocks on VVOL have not been controlled for volatility.*