STOCKHOLM SCHOOL OF ECONOMICS
Department of Economics
5350 Master's thesis in economics
Academic year 2015–2016

GDP-linked Bonds: The Case for Greece

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Abstract: This thesis examines the benefits and drawbacks of GDP-linked government bonds as a financing tool for the Greek government related to the Greek debt crisis from 2009–2014. Such bonds are characterised by coupon payments, which vary in proportion to the GDP growth of the issuing country. Analysing the effect of GDP-linked bonds on the sovereign debt dynamics with a model of endogenous sovereign default, I find evidence that such bonds would have reduced Greece's debt servicing costs, stabilized the Greek debt to GDP ratio as well as lowered the sovereign default probability by up to 19%. The overall magnitude of the benefit of introducing GDP-linked bonds for Greece crucially depends on the GDP risk premium. Although GDP-indexed government bonds alone could not have averted the Greek debt crisis, such debt instruments would have contributed to macroeconomic stabilization by allowing the Greek government to pursue a more countercyclical fiscal policy with interest payments being deferred until the recovery materializes.

Keywords: GDP-linked bonds, sovereign debt, Greek debt crisis
JEL: F34, H63, H81

Supervisor: Federica Romei
Date submitted: 16 May 2016
Date examined: 25 May 2016
Discussant: Jenny Dreier
Examiner: Kelly Ragan

Acknowledgements

I am grateful to my supervisor Federica Romei for her support and insightful feedback during the thesis writing process.

Table of contents

1 Introduction

The Greek debt crisis has been one of the dominating economic issues of the last couple of years. Greece's struggles with an exploding debt stock and a collapsing economy as well as the ensuing official sector bailout programmes have received substantial attention among economists and press alike. Many different solutions have been suggested to stabilize Greece's government debt burden and revive the economy, however, a viable solution to the crisis has still not materialized.[1] This thesis therefore wants to analyse an alternative approach to bring Greece's fiscal position back to a sustainable path by investigating the impact of introducing GDP-linked government bonds for Greece.

A GDP-linked bond is a security whose coupon payments depend on nominal or real GDP outturns. The return on such a bond varies in proportion to GDP growth. That means, if growth declines, the debt servicing cost of the bond automatically decreases in line with GDP growth and when growth is strong, the interest rate on the bond increases. In this way, GDP-linked securities provide a sort of recession insurance for the issuing sovereign by transferring some of the GDP growth risk to the bondholder and thereby facilitating a higher degree international risk-sharing. By eliminating the impact of negative growth shocks on the sovereign debt dynamics, such bonds could therefore be an interesting financing tool for the Greek government to manage its huge debt burden more efficiently and stabilize the debt to GDP ratio.

Although the effect of GDP-linked bonds on the sovereign debt dynamics has been analysed in the literature before, this was primarily with models calibrated to emerging economies. Especially the case of Argentina has been analysed widely, as it has been one of the first countries that introduced GDP-linked warrants as part of its sovereign debt restructuring. In this thesis, I want to contribute to the previous literature on GDP-linked bonds by analysing the effects of introducing such bonds for Greece, a country, which has struggled with sovereign debt issues since the global financial crisis in 2009.

Therefore, my research question for this thesis is:

> *Would GDP-linked bonds be a beneficial financing tool for the Greek government and could GDP-linked bonds have helped Greece avoid its debt and austerity crisis?*

I want to investigate if some of the benefits of GDP-indexed debt instruments apply to the case of Greece and whether such merits would have outweighed the drawbacks of such indexed bonds and helped Greece during the debt crisis years of 2009–2014. To answer my research question, I intend to specifically look at the effect of GDP-linked bonds on four different measures in the case of Greece: the interest payments on government debt, fiscal policy, the debt to GDP ratio and the probability of sovereign default.

This type of exercise is valuable because this topic is of high current importance as the Greek debt and austerity crisis is far from over. Legacy issues from the Eurozone crisis such as high debt to GDP ratios in many European economies will continue to be a drag on future economic growth. The stringent austerity measures imposed on Greece by its European creditor nations have often been severely criticised in the past years. Therefore, it is important to analyse alternative approaches to bring Greece's fiscal conditions to a sustainable path and to stabilize sovereign debt for the future. GDP-linked bonds could be one such alternative.

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[1] For a short summary of the Greek debt crisis see Appendix section 1.

Greece is a particularly interesting candidate for studying the effects of GDP-linked bonds, because it is not only one of the few developed countries, which has struggled with unsustainably high public debt and as a consequence experienced a sovereign debt crisis in recent years, but also because it is part of the Eurozone and therefore exhibits many characteristics that would facilitate an easier implementation of GDP-indexed bonds. Furthermore, this analysis would be interesting because the country is part of the Eurozone monetary union and it has been suggested that the benefits of introducing GDP-linked bonds would be particularly high for countries where monetary policy is constrained.

2 Theoretical background

2.1 Advantages of GDP-linked bonds

Through the indexation feature of GDP-linked bonds, the government faces higher debt servicing costs when the economy is expanding and benefits from lower borrowing costs when economic output is contracting. By reducing or eliminating – depending on the degree and type of indexation – the impact of GDP growth fluctuations on the sovereign debt dynamics, GDP-linked bonds reduce the volatility of the debt to GDP ratio compared to conventional bonds (Borensztein and Mauro, 2004).

By breaking the vicious cycle between declining growth rates and exploding debt burdens, GDP-linked bonds reduce the likelihood and frequency of sovereign debt crises. If the economy enters a recession, the smaller increases in the debt to GDP ratio resulting from GDP-indexation of the bonds facilitates a lower probability of default and of debt crises. Such debt crises usually involve large costs in terms of financial sector turmoil, capital flight, job cuts, currency depreciations, output contractions and sovereign debt restructurings and therefore should be prevented by all means (Borensztein and Mauro, 2004). There is ample empirical evidence that the sustainability of a country's debt position depends crucially on domestic economic growth and adverse growth shocks have triggered many of the previous sovereign debt crises for example in Latin America in the 1980s (see Easterly (2001); Detragiache and Spilimbergo (2001)).

A second advantage of GDP-linked debt securities is that they reduce the need of the government to engage in procyclical fiscal policies. In times of economic weakness, the government faces lower borrowing costs and thereby has less pressure to reduce public spending to stabilize its debt to GDP ratio and to ensure access to capital market financing. Because of this characteristic, GDP-linked bonds have been suggested as a particularly suitable debt instrument for emerging markets (Borensztein and Mauro, 2004). These countries are often forced to react to growth slowdowns with contractionary fiscal measures to limit deteriorations in their debt to GDP ratios and to maintain credibility and access to international financial markets. Due to their ability to reduce the need for procyclical fiscal measures, GDP-linked bonds can also be an interesting financing tool for advanced countries, which seek to stabilize their debt to GDP ratios due to legal constraints (Stability and Growth Pact in the EU) or whose capacity for independent monetary policy is constrained (currency union as the Eurozone). GDP-indexed debt could also lead to smoother paths for taxes and private consumption, as governments will have less need to vary public expenditures with variations in economic growth (Barro, 1995). The lower debt servicing costs the sovereign has with GDP-linked securities during recession episodes, also means that the government faces higher interest rates on debt during years of strong GDP growth. This in turn would make it more difficult for irresponsible politicians to boost frivolous public spending during economic booms and facilitates a more countercyclical and growth-stabilizing fiscal policy with GDP-linked bonds (Borensztein and Mauro, 2004).

As this study wants to investigate the merits of GDP-indexed debt for the issuing debtor country, the aforementioned advantages have been derived from the sovereign's point-of-view. However, one can also conceive of benefits of GDP-linked bonds from the investor or creditor point-of-view. As GDP-linked securities facilitate international risk-sharing and limit the incidence of sovereign default, Borensztein and Mauro (2004) also discuss a range of benefits for international investors.

Domestic investors would benefit from the opportunity to invest in assets whose return is correlated with another country's GDP in order to diversify their portfolios.[2] Since growth rates across countries tend to be fairly uncorrelated, such instruments would create a diversification opportunity, which potentially lowers the volatility of returns for international investors. As mentioned above, GDP-indexed debt instruments lower the occurrence of sovereign debt crises and thereby avoid the costly consequences of a formal default. Not only debtors, but also creditors would profit from a lower likelihood and incidence of sovereign default and financial crises, as this often involves high costs for investors in terms of litigation, renegotiation and haircuts on debt. Finally, market participants might be interested in the opportunity to take a position in countries' future economic growth prospects. To some degree, this is already possible through stock markets, but – especially in emerging market economies – equity markets are often not very liquid and representative of the economy as a whole (Borensztein and Mauro, 2004).

2.2 Drawbacks and other issues related to GDP-linked bonds

The question remains why these debt instruments do not yet exist on a large scale if they have such attractive characteristics for debt issuing countries. This is mostly due to the fact, that GDP-linked bonds also have a range of disadvantages, which reduce their appeal as funding source for governments and make their implementation challenging in practice.

GDP-linked bonds can be seen as a form of financial innovation. However, in reality, market failures often prevent such advantageous financial innovation from taking place (Borensztein and Mauro, 2004). The first challenge for the implementation of GDP-linked bonds would be that a concerted effort from both issuing countries and investors would be needed. New and complex debt instruments often involve high expenses for investors in terms of pricing such novel instruments correctly. Market liquidity is also an issue, as a range of such debt instruments would need to be issued by different countries to ensure liquidity in the market for GDP-linked bonds and give investors the aforementioned portfolio diversification benefits. As with all innovative financial instruments, GDP-linked bonds would have to convince investors of their merits and overcome initial investor scepticism. Financial market participants would most likely demand a GDP risk premium for such a novel debt instrument with variable coupon payments, which may deter borrowing countries from issuing such bonds in the first place as the costs are just too high and outweigh the indexation benefits.

GDP-linked bonds also face difficulties in their implementation due to externalities and coordination issues. To price GDP-indexed securities correctly and diversify risk, investors would ideally need a large number of countries to issue them. However, an individual country planning to issue a GDP-linked bond will not take into account that by enabling investors to price its own instrument, it also supports the pricing of those bonds issued by other sovereigns. Furthermore, an individual GDP-linked bond issuing country does usually not take into account the social benefits arising from the international risk-sharing characteristics of the bond. Due to this externality, sovereigns might underestimate the merits of GDP-linked debt and continue to use conventional debt instruments instead. GDP-indexed bonds also suffer from another kind of externality. As holders of GDP-linked bonds help reducing the likelihood of default of the debtor country, they effectively subsidize holders of conventional bonds, which also profit from the lower incidence of sovereign default but do not pay in terms of variability of their returns (Borensztein and Mauro, 2004).

[2] The magnitude of the additional diversification benefits resulting from the ability to invest in claims to the GDP of other countries would of course depend on investors' existing portfolio and the range of countries that issue indexed bonds (Borensztein and Mauro, 2004).

To reduce the GDP risk premium that investors would demand for novel GDP-linked securities, it would be important that such debt instruments have common features and standards for all countries, which issue them, for example in terms of growth measures used (real or nominal GDP) and the indexation system (linear or non-linear). This would not only simplify pricing such securities by financial market participants but also support the creation of a liquid secondary market for GDP-linked government debt (Borensztein and Mauro, 2004). This is a real challenge in practice, as countries so far have designed their individual GDP-linked securities to maximize their own benefits and neglected the wider implications for financial market participants. Therefore, to increase acceptance of this debt instrument, the issuing country has to find the right balance of designing a bond, which provides the desired recession insurance but also remains an attractive investment tool for investors.

Moreover, as coupon payments of GDP-linked bonds are conditional on GDP growth outturns, it is crucial to have unambiguous and verifiable standards in place to verify whether those conditions are satisfied. Moral hazard problems have been highlighted as one of the major impediments for the acceptance of GDP-linked bonds. The idea behind this is, that governments – especially in emerging countries – could report false GDP figures to keep their coupon payments low. Although this is a valid objection in the case of developing countries, moral hazard considerations seem to be less of an issue for advanced countries as in the EU. Although Greece previously had issues related to misreporting of debt and deficit figures[3], which might impede the successful issuance of GDP-linked bonds due to credibility concerns, it would be difficult for the government to underreport GDP growth for many years in a row, as its political reputation also hinges on a successful economic policy. Furthermore, although the GDP growth figures published by Eurostat are provided by the member states, other EU institutions such as the ECB would most likely raise concerns about dubious growth figures. Therefore, GDP-linked debt seems to be particularly appealing for EU countries, which can credibly commit to coherent statistical standards and where a concerted action to issue such bonds in several countries would be easier to achieve.

This section has sought to provide some theoretical background of GDP-linked bonds and tried to compare some of the major benefits and drawbacks of these novel debt instruments. An extensive discussion of the concept and implementation challenges can be found in Borensztein and Mauro (2004), Griffith-Jones and Sharma (2006) and Brooke et al. (2013). Overall, there are quite a few impediments that a country needs to overcome to issue GDP-linked bonds. Nevertheless, such securities already exist on a small scale in practice and often were issued as part of a sovereign debt restructuring to increase investor participation as for example in Argentina or Greece.[4] The aim of this study, however, is not to investigate the circumstances under which GDP-linked bonds could be issued in Greece nor the optimal contract design, indexed debt share and maturity profile to maximize financial market acceptance, as this would go beyond the scope of this thesis. I rather want to investigate the benefits of GDP-linked bonds for Greece related to the debt and austerity crisis, assuming the country could issue such bonds on a sufficiently large scale.

[3] See Appendix section 1.

[4] For a full list of all countries, which issued a form of a GDP-linked bond see Borensztein and Mauro (2004), Griffith-Jones and Sharma (2006) and Miyajima (2006). For details on the existing Greek GDP-linked warrants see Appendix section 2.

3 Literature review

3.1 Previous literature

Although GDP-indexed debt instruments are a rather recent phenomenon, there is already a rich literature on the subject. Most of the existing literature has been published during the last two decades and is of a policy-oriented nature, as full scale GDP-indexed bonds are still very rare in practice. This policy orientation is also reflected in the high share of working papers compared to academic journal publications on the topic of GDP-indexed debt.

The first literature on indexing debt to GDP, exports or commodity prices was inspired by the academic interest in the debt crises of the 1980s in Latin America. Krugman (1988) and Froot, Scharfstein and Stein (1989) investigate ways to reduce the incidence of sovereign debt crises in developing countries. The authors compare the benefits of debt indexation to variables out of the debtor country's control (commodity prices) to variables that are at least partially under the sovereign's control (export revenues or output). In these early works, the authors favoured indexing debt to variables that are out of governments' control rather than indexing to GDP to limit moral hazard concerns.

The first economist to promote the benefits of GDP-indexed securities in terms of their risk diversification and insurance properties, was Shiller (1993; 2003). He proposed so called macro markets for GDP-linked securities. These were of the form of perpetual claims on a fraction of a country's GDP. Similar to shares issued by companies, which pay a fraction of corporate earnings as dividends, such a security would pay a fraction of the earnings of the country. Consequently, these GDP-linked securities would allow households or companies to take an equity stake in a country's economic development and thereby advance international risk-sharing and hedging.

Kamstra and Shiller (2009) further develop the idea of Shiller (1993; 2003) in their discussion paper and outline the benefits of so-called Trills – securities, which pay a coupon of one-trillionth of the GDP of a country. They argue that such a debt instrument would not only be of interest for the issuing government due its budget stabilizing properties, but also to international institutional and private investors due to portfolio diversification and hedging opportunities. Using the Capital Asset Pricing Model (CAPM), they seek to estimate the risk premium of such GDP-linked securities for the US government. The authors find that US Trills would have a risk premium of around 150 basis points over short-term US government debt and conclude that due to the low cost of Trills, they would be an attractive instrument for the US government.

Some studies have also looked at the benefits of GDP-indexed debt instrument for European countries specifically. In their study on the macroeconomic shock resilience of the European Economic and Monetary Union (EMU), Obstfeld and Peri (1998) find a high persistence of shocks in Europe and conclude that intercountry stabilization transfers will hence play a substantial role in the EMU. As an alternative to an extended European transfer union and the accompanying political obstacles, they suggest that EU governments issue perpetual Euro-denominated liabilities indexed to domestic nominal per-capita GDP growth to reduce their GDP risk. Inspired by Shiller (1993), Drèze (2000) advocates the issuance of perpetual bonds indexed to a country's nominal GDP by European countries as a way of improving international risk-sharing in the EMU. He suggests, that the EMU member countries would exchange these bonds among themselves to further diversify their portfolios and maximize the benefits of risk-sharing. However, the author also acknowledges that moral hazard considerations might impede the implementation of such a scheme in practice.

One of the most influential academic studies on GDP-linked bonds, Borensztein and Mauro (2004) seek to revive the case for countries to issue GDP-linked bonds. They illustrate how GDP-linked bonds decrease the volatility in the debt to GDP ratio and hence could reduce the incidence of sovereign debt crises. Apart from the debt stabilizing properties, the authors also find evidence that GDP-indexed debt would enable the government more space for countercyclical fiscal measures. Furthermore, Borensztein and Mauro stress that GDP-linked debt would promote greater risk sharing between sovereigns and international investors. Emerging market economies where growth slowdowns may be amplified by worsening investor sentiment and countries where fiscal and monetary policy is constrained (such as the EMU member states) would likely benefit most from the introduction of GDP-linked bonds. Potential obstacles to the implementation of such bonds include the verifiability of GDP data, the trade-off between insurance and moral hazard concerns as well as the need for liquidity in the market. The authors also estimate the premium required to compensate investors for taking on GDP volatility risk using a CAPM approach. They conclude that the risk premium would likely be small, because of the low degree of cross-country co-movement of GDP growth rates, which means that an investor with a diversified portfolio of GDP-indexed bonds could largely eliminate the GDP growth risk.

As it has been argued that the issuance and evaluation of GDP-linked bonds in practice would be challenging, a separate branch of the literature on GDP-indexed government debt has focused on the pricing and acceptance of such instruments among financial investors. Influenced by the growing international interest in GDP-indexed financial instruments and Argentina's recent issue of a GDP-linked warrant as part of its sovereign debt restructuring, Kruse, Meitner and Schröder (2005) develop a pricing approach for GDP-linked bonds analogous to the Black-Scholes model for stock options. Building on this earlier work, Schröder et al. (2007) examine the applicability of GDP-linked bonds as a financing tool of developing countries. As part of their study they conduct a survey among financial market participants and conclude that under certain conditions GDP-linked bonds can be successfully issued and traded on international capital markets due to their diversification opportunities. The authors argue that such bonds might be an appropriate financing instrument for countries without current capital markets access and typical bondholders would be different kinds of institutional investors such as hedge funds, insurance companies and pension funds. Additionally, they stress that investors would be willing to invest in GDP-linked bonds if there are diversification benefits, i.e. if the GDP of the issuing country has a negative or low positive correlation with the GDP of the reference region of the investor.

Chamon and Mauro (2006) seek to develop a simple way of pricing growth-indexed bonds. Their pricing approach relies on extracting as much information as possible from observed prices of plain-vanilla bonds and applying this information then to the case of growth-indexed bonds through Monte Carlo simulation. The authors study the effects of introducing GDP-indexed bonds using a debt sustainability framework, where default occurs as soon as the ratio of total debt to GDP exceeds a predetermined debt limit. They calibrate their model economy to emerging markets and compare three different debt compositions. Chamon and Mauro find that large issuances of GDP-linked bonds would result in a lower incidence of sovereign default: moving from no indexation to indexation of all debt reduces the default probability by one third. The authors also note that by reducing the sovereign default probability, GDP-indexed bonds would also diminish the required coupon on standard bonds as well as reduce the sensitivity of bond payoffs to surprises in growth outturns.

As a response to Chamon and Mauro's publication in 2006 and also influenced by Argentina's issue of a GDP-linked warrant in 2005, the academic community has sought to develop richer models, which consider the effects of GDP-linked bonds on the sovereign debt dynamics and default probability.

However, a dominant framework for analysing the effects of GDP-linked bonds has still not materialized, highlighted by the variety of working papers on the subject of modelling GDP-indexed debt.

Using a general equilibrium model with endogenous default calibrated to a typical emerging market economy, Faria (2007) evaluates the effect of GDP-indexed bonds on spreads, debt to GDP ratios and the likelihood of default in the presence of output shocks in his working paper. He finds that perfectly-indexed bonds lead to a higher debt to GDP ratio in equilibrium and reduce the spread and probability of default compared to standard bonds. The impact of perfectly-indexed bonds on the debt to output ratio, spreads and likelihood of default is larger when shocks to output are very persistent and volatile. However, if the sovereign issues bonds that are only partly indexed to GDP growth, changes in debt ratios, spreads and default probabilities are negligible. The author concludes that for his parametrization, the model suggested welfare benefits from GDP-indexed bonds are only small.

Relaxing the assumption of risk neutrality of Chamon and Mauro (2006), Ruban, Poon and Vonatsos (2008) develop a model of endogenous sovereign default in their working paper that depends on the sovereign's capacity to pay driven by the real exchange rate and potential output. In contrast to Chamon and Mauro (2006), who provide only a limited range of simulations on a particular set of macroeconomic variables using a hypothetical contract design, Ruban, Poon and Vonatsos test the model's empirical performance by pricing Argentina's GDP warrants and investigate several different GDP indexation schemes to find the optimal contract design of such indexed bonds. Calibrating their model to emerging markets (Brazil, Mexico and Argentina), they find that bonds linked to a country's nominal GDP growth measured in US dollar provide the greatest risk-sharing benefits and have considerably lower default rates than conventional bonds or bonds linked to real growth.

Prompted by the growing interest in indexed sovereign debt among policymakers during the financial and Eurozone crisis, Sandleris, Sapriza and Taddei (2011) analyse the effects of GDP-linked bonds on the probability of default, government borrowing and welfare in a dynamic stochastic general equilibrium (DSGE) model with endogenous sovereign default risk. They calibrate their model to the Argentine economy and show that the introduction of sovereign debt contracts indexed to real GDP substantially reduce the frequency of default and increase the amount of debt the sovereign can borrow and sustain in equilibrium. As a result of this improved risk-sharing and lower default incidence, the authors find that GDP-indexed debt instruments could improve welfare by 0.5% of aggregate average consumption.

In their working paper, Hatchondo and Martinez (2012) seek to extend earlier work on the effects of GDP-linked bonds by Chamon and Mauro (2006) and Sandleris, Sapriza and Taddei (2011). The authors develop an endogenous sovereign default DSGE model in which the government faces countercyclical borrowing costs and use this model to study the effects of introducing GDP-indexed bonds. Using again a parametrization for the Argentine economy, the authors find positive welfare gains from the introduction of GDP-indexed bonds as they allow the government to avoid costly default episodes, increase the amount of debt the government can borrow and improve consumption smoothing.

Based on earlier work by Brooke et al. (2013) and Ghosh et al. (2013), Barr, Bush and Pienkowski (2014) develop a model of endogenous sovereign default in their working paper, which incorporates the concept of fiscal fatigue and explore the impact of GDP-linked bonds on the maximum sustainable debt level of a country and the incidence of default. They extend their model by adding risk-averse investors and feedback effects from the primary balance to GDP. Comparing the cases of GDP-linked debt with standard government debt instruments, the authors conclude that significant benefits can be achieved by indexing government debt to GDP as such bonds can reduce the frequency of sovereign debt crises and increase the amount of debt the sovereign can sustain.

So far, the only study which has considered a form of GDP-linked debt instrument for Greece is a policy discussion paper by Fratzscher, Steffen and Rieth (2014). They investigate the effects of converting Greek Loan Facility (GLF) loans, which were granted by Euro area member states as part of the first Greek bailout programme, into GDP-linked loans. Using the same simulation model as Chamon and Mauro (2006), the authors find that GDP-linked loans would reduce the likelihood of an additional Greek debt restructuring by decreasing the probability of default by 59% and reduce the volatility of the GDP growth rate by around 20%. Furthermore, they argue that such GDP-linked loans would stabilize the Greek debt ratio, increase the sustainability of Greece's debt burden and provide stronger incentives for Greece to take responsibility for its own reforms.

3.2 Contribution to literature

By analysing the impact of GDP-linked bonds on the Greek debt dynamics, this thesis will contribute to the current literature on GDP-linked bonds and the Greek debt crisis in a number of ways. So far there have been few academic studies on the effects of GDP-linked bonds on the debt dynamics of an advanced economy in a currency union. Furthermore, only one previous study has specifically investigated GDP-indexed loans as a possible solution for the unsustainable Greek debt burden. However, my approach of investigating GDP-indexed debt will differ from Fratzscher, Steffen and Rieth (2014) in a number of ways.

First of all, whereas the authors consider the impact of indexing GLF loans following the Greek debt crisis from an ex post point of view, I intend to study the effect of GDP-linked government bonds for Greece before and during the crisis years and analyse if such bonds could have helped avoid the Greek debt and austerity crisis. Furthermore, the authors use a model by Chamon and Mauro (2006) to derive the sovereign default probability. Instead, to answer my research question and estimate the probability of sovereign default, I will use the simpler and more recent framework by Barr, Bush and Pienkowski (2014), which is also less sensitive to the chosen parameterization as complex DSGE models as in Sandleris, Sapriza and Taddei (2011) and Hatchondo and Martinez (2012). Whereas in the original paper Barr, Bush and Pienkowski calibrate their model to an average advanced economy, I will calibrate this model specifically to the Greek economy and use a number of model extensions and characteristics of Eurozone monetary policy to make it more relevant to the case of Greece.

4 Analysis

To analyse whether GDP-linked bonds would be a beneficial financing tool for the Greek government, I will investigate the effects of GDP-indexed debt on four different measures: the interest payments on government debt, fiscal policy, the debt to GDP ratio and the probability of sovereign default. The purpose of this section is to study whether the suggested benefits or drawbacks of GDP-linked bonds from the previous literature also apply to the case of Greece and whether they could have helped the country avoid its debt and austerity crisis.

4.1 Data and calibration

I obtained Greek real GDP growth rate and real GDP data as well as data on the Greek debt to GDP ratio and the amount of outstanding government debt securities (both short-term and long-term) from Eurostat (Eurostat, 2015a; 2015b; 2016a; 2016b). The data for the Greek primary balance is taken from the IMF World Economic Outlook (WEO) (IMF, 2015b).

For my investigation of the benefits of introducing GDP-linked bonds for Greece, I limit my analysis to the time period 1996–2014. Although this decision is mainly data driven, I also argue that the advantages and disadvantages of GDP-linked bonds can be sufficiently studied over a period of 19 years as it provides a snapshot of the country's economic cycle both in terms of expansionary (1999–2003) as well as recession years (2009–2013).

For the calculation of the interest rate on normal government bonds, I use the standard debt evolution equation

$$\frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} = \Delta \frac{D_t}{Y_t} = (r_t - g_t)\frac{D_{t-1}}{Y_{t-1}} - s_t$$

and defining $\frac{D_t}{Y_t} = d_t$

$$d_t - d_{t-1} = (r_t - g_t)d_{t-1} - s_t$$

$$d_t = (1 + r_t - g_t)d_{t-1} - s_t \tag{1.1}$$

where d_t is the debt to GDP ratio in period t, g_t is the growth rate in period t, r_t is the interest rate on government debt in period t and s_t is the primary balance in period t as percent of GDP. Government debt in the next period depends on the difference between the interest rate on debt and GDP growth rate, the debt in the last period as well as the primary surplus or deficit in the current period. Given the data on the actual debt to GDP ratio, real GDP growth rate and primary balance, I derive the corresponding interest rate r_t endogenously. The resulting interest rate can be found in the next to last column of Table 1. There is a clear increase in the interest rate from 2009 onwards. In 2012 Greece had a debt restructuring, which lead to a decline in the debt to GDP ratio from 172.0% to 159.4%. Looking at the table, we see that such a reduction in the debt ratio could according to the debt evolution equation only have been possible with a negative interest rate of -14.6 as the model equation cannot take account of a debt restructuring. As this negative interest rate would, however, bias the results, I assume that there was no debt restructuring in 2012 and hence use as approximation the same interest rate as in 2011, 8.5%, for 2012 in my calculations to overcome this complication.

For the indexation formula for the GDP-linked bonds, I follow Borensztein and Mauro (2004). Although many different indexation formulas have been proposed in the literature, each one with different advantages and drawbacks (see Ruban, Poon and Vonatsos (2008)), I use the following indexation formula as it is both relatively simple, linear and enables the setting of a maximum ($r^{\max}$) and minimum interest rate ($r^{\min}$).

$$r_t^{GDP} = \begin{cases} r^{\max} & \text{for } r_t^{GDP} \geq r^{\max} \\ \overline{r} + \Omega + \alpha(g_t - \overline{g}) & \text{for } r^{\max} > r_t^{GDP} > r^{\min} \\ r^{\min} & \text{for } r_t^{GDP} \leq r^{\min} \end{cases} \tag{1.2}$$

The interest rate on GDP-linked bonds r_t^{GDP} is constrained by a maximum and minimum rate and equal to an average interest rate $\overline{r}$, an indexation premium Ω and the difference between actual GDP growth from trend growth $(g_t - \overline{g})$ with α measuring the degree of growth indexation. I set the minimum interest rate to 0.5% as it has been argued that a positive minimum rate would increase investor demand for GDP-linked bonds and many big pension funds for example have to hold assets, which pay a strictly positive coupon. The higher one sets the minimum rate, the more interesting GDP-linked bonds would become for investors as it would guarantee them a positive, fixed coupon no matter how much the economy is in recession. At the same time, this would also limit the benefit of the indexation for the issuing country as a sufficiently severe decline in the growth rate is not translated into the interest rate paid on debt anymore. The opposite is the case for the maximum interest rate. The higher $r^{\max}$, the more investors would profit from positive growth outcomes and the more the government would have to pay in boom years. I set the maximum rate at 12%, to ensure that investors can sufficiently profit from the indexation in boom years, but at the same time also limit the debt servicing costs for the government to a threshold above which it would arguably become difficult to remain solvent. I set the average interest rate $\overline{r}$ equal to the average of the actual interest rate r_t from 1996–2008, which is equal to 4.4%. The trend growth rate $\overline{g}$ is set to equal the average real GDP growth rate from 1980–2008, which is 2.1% (IMF, 2015c).

It has been argued in the previous literature that investors would demand a premium for accepting and holding GDP-linked bonds compared to normal bonds due to their novelty and the uncertainty in the coupon payments. The argument behind a positive indexation premium is, that investors would demand compensation for holding bonds, which have an uncertain return due to GDP growth volatility. However, the variability in payoffs is not necessarily a drawback. The size of the GDP risk premium will ultimately depend on the correlation of the coupon payments of the GDP-linked bond with the investors own consumption. An international bondholder whose own consumption is currently low for example because her country of residence undergoes an recessionary episode, might in turn benefit from higher interest payments from a GDP-indexed bond from another county, which is currently in a boom phase. Even a negative indexation premium could be justified, if such bonds facilitate greater income and consumption smoothing due to the negative correlation between payoffs and consumption. Previous studies on the size of the indexation premium of GDP-linked bonds have also stressed that such a premium would be negligible due to portfolio diversification possibilities and the low degree of cross-country co-movement of GDP growth rates (see Miyajima (2006); Borensztein and Mauro (2004); Kamstra and Shiller (2009); Bacchiocchi and Missale (2015)). Potential holders of Greek GDP-linked

Table 1

Data: Greece

Unit	Year	Real GDP growth rate	Real GDP	Government debt securities	Debt to GDP	Primary balance	Interest rate on conventional government bonds r_t	Interest rate on GDP-linked bonds r_t^{GDP}
		%	Million €	Million €	% of GDP	% of GDP	%	%
	1995				98.9			
	1996	2.9	163,320.9	81,138.7	101.2	4.1	5.3	5.6
	1997	4.5	170,644.5	89,703.9	99.3	3.6	2.7	7.2
	1998	3.9	177,291.0	95,179.6	97.2	4.2	1.8	6.6
	1999	3.1	182,738.4	103,534.8	98.6	4.2	4.6	5.8
	2000	3.9	189,901.3	109,068.9	104.4	3.5	9.8	6.6
	2001	4.1	197,747.3	115,137.0	106.8	1.9	6.4	6.8
	2002	3.9	205,504.7	125,282.0	104.6	0.7	1.8	6.6
	2003	5.8	217,412.7	136,866.0	101.2	-0.7	2.5	8.5
	2004	5.1	228,416.0	155,706.0	102.7	-2.5	6.6	7.8
	2005	0.6	229,784.5	172,451.0	107.3	-0.9	5.1	3.3
	2006	5.7	242,772.9	184,156.0	103.5	-1.6	2.1	8.4
	2007	3.3	250,720.7	201,108.0	103.1	-2.2	2.9	6.0
	2008	-0.3	249,880.4	225,449.0	109.4	-5.0	5.8	2.4
	2009	-4.3	239,133.7	262,773.0	126.7	-10.3	11.4	0.5
	2010	-5.5	226,031.4	262,555.0	146.2	-5.3	9.8	0.5
	2011	-9.1	205,389.1	251,676.0	172.0	-3.0	8.5	0.5
*	**2012**	-7.3	190,394.7	93,443.0	159.4	-1.4	-14.6	0.5
	2013	-3.2	184,305.4	79,205.0	177.0	1.0	7.8	0.5
	2014	0.7	185,510.8	70,847.0	178.6	0.0	1.6	3.4

Source: Eurostat (2015a; 2015b; 2016a; 2016b), IMF (2015b) and own calculations

Note: *For the calculation of the interest rate on conventional government bonds, I use the standard debt evolution equation and given data on the actual debt to GDP ratio, real GDP growth rate and primary balance, I derive the corresponding interest rate endogenously.

Greece had a debt restructuring in 2012. A similar reduction in the debt to GDP ratio would only have been achievable with a negative interest rate of -14.6. As this negative interest rate would however bias the results, I assume for comparison that there was no debt restructuring in 2012 and hence use the same interest rate as in 2011 for my calculations (8.5).

For the calculation of the interest rate on GDP-linked bonds, I use the following baseline calibration: $\alpha = 1$, $r^{min} = 0.5$, $\bar{g} = 2.1$, $r^{max} = 12$, $\bar{r} = 4.4$, $\Omega = 0.4$

bonds would most likely be international institutional investors with ample diversification opportunities.[5] Therefore, the GDP risk premium for such bonds should be relatively small. As estimating the exact indexation premium for Greece goes beyond the scope of this thesis, I instead use a small, but positive value for Ω of 0.4% as approximation adopted from Borensztein and Mauro (2004).

Furthermore, α, the degree of growth indexation, measures the extent to which coupon payments are adjusted to changes in the growth rate. Setting α equal to 1 would lead to a complete translation of growth changes into coupon payments. On the other hand, setting α equal to 0.7 means that if growth falls short of trend growth by 1%, the interest rate on the GDP-linked bonds will only be reduced by 0.7% thereby limiting the effect of indexation. Generally, $\alpha < 1$ will decrease the degree of growth indexation whereas $\alpha > 1$ will magnify the degree of growth indexation. Setting the optimal α for the issuing country can be difficult, as an $\alpha < 1$ on the one hand limits the higher coupon payments in high growth years for the country, but on the other hand also bounds the indexation benefits during a recession in terms of lower coupon payments. As this study does not seek to find the optimal indexation formula or indexation degree for Greece, I leave this open to future research and instead use the standard indexation of $\alpha = 1$. In years when the economy grows exactly at the baseline rate, the interest rate on GDP-linked bonds will be equal to $\bar{r} + \Omega = 4.8\%$. If the economy grows above or below trend growth, the coupon rate will increase or decrease one for one with the actual GDP growth rate, respectively.

Parameter values		
$r^{\min}$	0.5	Minimum interest rate on GDP-linked bonds
$r^{\max}$	12	Maximum interest rate on GDP-linked bonds
$\bar{r}$	4.4	Average interest rate on government debt
$\bar{g}$	2.1	Trend GDP growth rate
Ω	0.4	GDP risk premium
α	1	Degree of indexation

Plugging in these parameters into (1.2), I use the following equation to calculate the interest rate on GDP-linked bonds.

$$r_t^{GDP} = \begin{cases} 12 & \text{for } r_t^{GDP} \geq 12 \\ 4.4 + 0.4 + (g_t - 2.1) & \text{for } 12 > r_t^{GDP} > 0.5 \\ 0.5 & \text{for } r_t^{GDP} \leq 0.5 \end{cases}$$

Using data on the real GDP growth rate from Eurostat for g_t, I obtain the interest rates in the last column of Table 1. The maximum rate constraint is never binding during the time period 1996–2014, however from 2009–2013, the fall in growth in Greece is sufficiently severe to make the minimum rate a binding constraint. Comparing the interest rates on normal and GDP-linked bonds over time, one clearly sees the effects of indexation. In the high growth years up to 2007, GDP-linked bonds tend to have a higher interest rate than normal bonds, but with the onset of the recession in 2008, interest rates on normal bonds exceed the rates on GDP-linked bonds considerably.

[5] Previous studies have stressed that GDP-linked bonds would be particularly interesting for institutional investors (Schröder et al., 2007). These would have better resources to price novel GDP-indexed debt securities and often also have a higher risk appetite compared to private investors, which makes them more likely to invest in innovative financial instruments.

4.2 Interest payments on government debt

For my first application, I am interested in the potential interest rate savings over the economic cycle when introducing GDP-linked bonds for Greece. Before the debt crisis in 2009, the majority of Greek government debt consisted of government bonds.[6] I assume that in 1996 half of these government debt securities were indexed to GDP and half of them were normal government bonds. I make the strong assumption that the composition of debt had no impact on the development of any other variables in the economy: the GDP growth rate and primary balance would behave exactly as in the actual data from 1996–2014. Furthermore, I use Eurostat data on the amount of outstanding government securities from 1996–2014 and multiply it with the interest rate on GDP-linked bonds and normal bonds, respectively. Finally, I compare the resulting interest payments over time with the case that all debt securities were instead conventional bonds and compute the potential interest savings as the difference between the two cases for each year.[7]

The result is shown in Figure 1. The interest rate on GDP-linked bonds mirrors the economic growth rate over time and declines to the minimum rate of 0.5% during the severe recession in 2009–2013. The interest rate on normal bonds, on the other hand, seems to be countercyclical to the economic growth rate and shoots up particularly when economic growth is very weak from 2009 onwards. Up to the year 2007, the interest bill savings from indexation are mostly negative, meaning that Greece would have had higher debt servicing costs. This is the case because economic growth during this time period – except for 2005 – was considerably above trend growth, leading to higher-than-average coupon payments on the indexed portion of the debt securities. However, with the onset of the recession in 2009, when output contracted by up to 9.1%, the coupon rate would have fallen to its minimum of 0.5% and interest bill savings from GDP-linked debt increase considerably, amounting to up to 6% of GDP. Expressed in Euro, converting half of government debt to GDP-linked bonds in 1996 would have resulted in cumulative interest bill savings of €42.74 billion from 2009–2014 for Greece. Of course Greece would have faced higher debt costs in the boom years, however cumulative interest bill savings from indexing half of debt securities to GDP from 1996–2014 would still have amounted to €27.5 billion.

However, indexing 50% of the outstanding government debt securities to GDP growth is still a sizable amount and might not be easily realizable in practice. Therefore, I also consider a more conservative scenario, where only 25% of government debt securities are GDP-linked.[8] The potential interest bill savings are smaller now as less debt benefits from growth indexation. However, converting only a quarter of outstanding government debt securities to GDP-indexed bonds would still have resulted in cumulative interest bill savings of €21.4 billion from 2009–2014 and €13.8 billion from 1996–2014. It seems that even with a relatively small amount of indexed government debt, Greece would have obtained a reduction in the interest bill over the studied time period, leaving more room for the government to avoid procyclical fiscal and austerity measures, especially during the recession and crisis years of 2009–2014.

To test the robustness of the results, I consider a variety of different parameter values for $\alpha, \Omega, r^{\min}$ and $r^{\max}$. Overall, the results are not very sensitive to changes in the parameters above. As expected, increasing Ω and $r^{\min}$ leads to weaker benefits of indexation, as servicing GDP-linked bonds becomes

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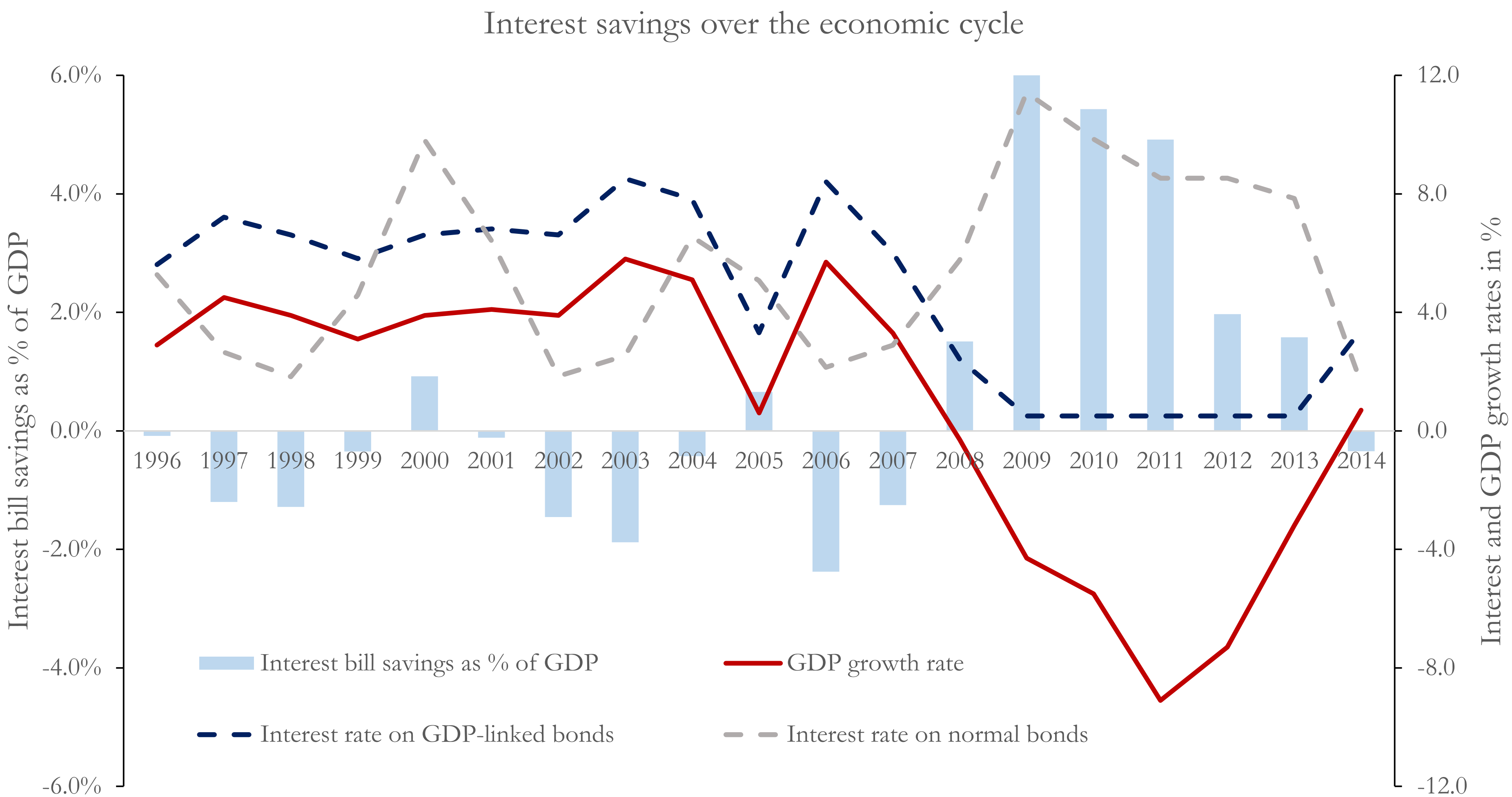
[6] For details on the Greek government debt composition see Appendix section 3.

[7] Interest payments$_t^{GDP}$ = 0.5 · government debt securities · r_t + 0.5 · government debt securities · r_t^{GDP}
Interest payments$_t^{Normal}$ = government debt securities · r_t
Interest savings$_t$ = Interest payments$_t^{Normal}$ - Interest payments$_t^{GDP}$
For details see Appendix section 4 Table A2.

[8] For details see Appendix section 4 Table A3.

Figure 1



Source: Eurostat (2015a; 2015b; 2016a; 2016b) and own calculations
Note: 50% of debt is conventional bonds, 50% GDP-linked bonds

relatively more expensive for the issuing country. Setting a lower threshold for $r^{\max}$ is beneficial for the issuing country as it has to pay less interest when the economy is performing strongly. Reducing the degree of indexation α to 0.7 – and holding all other parameters constant – initially increases the benefits of debt indexation with cumulative interest bill savings of €15.3 billion from 1996–2014 if 25% of government debt securities are GDP-linked. However, choosing $\alpha = 0.3$ reduces the degree of indexation to a point where the benefit of indexation in low growth years does not outweigh the cost of indexation in high growth years anymore, leading to cumulative interest bill savings of €13.2 billion from 1996–2014 if 25% of government debt securities are GDP-linked. Nevertheless, considering the extremely adverse scenario of $\alpha = 0.3$, $r^{\min} = 2$, $r^{\max} = 12$ and $\Omega = 2$ still results in cumulative interest bill savings of €1.9 billion from 1996–2014 if 25% of government debt securities are indexed to GDP.[9] Therefore, it seems that even a relatively small amount of GDP-linked bonds would have enabled Greece to reduce its interest bill during the crisis years between 2 to 6 percent of GDP depending on the chosen parameters and indexed debt amount.

These results show how GDP-linked bonds could have reduced Greece's debt servicing costs and provided a form of recession insurance by transferring some of the GDP growth risk from the sovereign to the bondholder through lower coupon payments. However, so far this analysis has focused exclusively on the debt servicing costs without considering the impact of lower interest payments on government spending and fiscal policy. The question remains whether the lower coupon payments on debt would also have reduced the need of Greece to engage in procyclical fiscal measures during the recession. In the next section, I therefore turn to the question of whether GDP-linked bonds would have created additional room for countercyclical fiscal policy in Greece.

4.3 Fiscal policy and primary balance

To study the relationship between GDP-indexed debt and procyclical fiscal policy more in depth, I look at the country's primary balance. In theory, a country with GDP growth below trend would be able to have a lower primary surplus with debt indexation than without it. Consequently, the country would have more room to increase government spending in crisis years and support the economy. On the other hand, in years of strong economic growth the government would need to have a higher primary balance to service its debt costs and hence would need to reduce its fiscal spending. The result would be a countercyclical fiscal policy, which supports growth in weak environments and reduces public spending during boom years. This would not only potentially lead to smoother paths of primary surplus and taxes over the economic cycle, but also avoid the necessity to adjust the primary deficit in times of economic and financial distress through costly measures such as tax rises, job cuts in the public sector and reduction in social welfare spending.

To investigate the effects of GDP-linked bonds on Greek fiscal policy, I assume that all government debt had been indexed to GDP in 1996. Furthermore, I assume that the debt to GDP ratio, GDP growth rate and interest rates on government debt would have evolved exactly as observed in reality and summarized in Table 1. Using again the standard debt evolution equation (1.1) from above and rearranging, I can solve for the primary balance with and without GDP indexation.

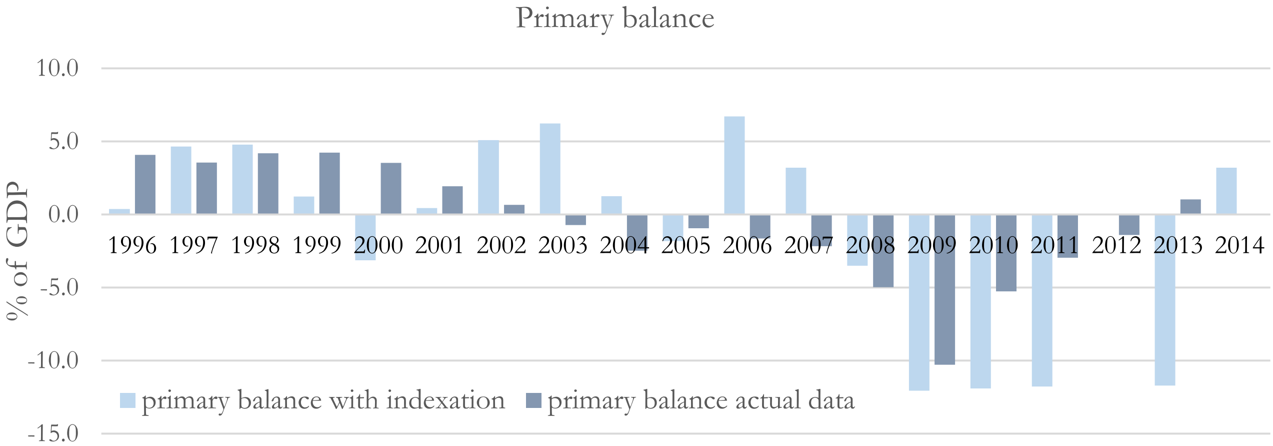
$$\begin{aligned} s_t &= (1 + r_t - g_t)d_{t-1} - d_t \\ s_t^{GDP} &= (1 + r_t^{GDP} - g_t)d_{t-1} - d_t \end{aligned} \tag{1.3}$$

[9] For figure see Appendix section 4 Figure A5.

Once I have solved for the primary balance with indexation, I compute the correlation of the primary balance with the real GDP growth rate. This correlation can be seen as a summary measure of the government's ability to conduct countercyclical fiscal policy. Finally I compare it to the same measure based on actual primary balance data for Greece.

Figure 2 compares the primary balances with and without GDP-indexed debt Greece would have needed to achieve to ensure that the debt to GDP ratio would have evolved as in reality.[10] The actual primary balance of Greece over the time period seems to constantly deteriorate up to the year 2009 after which it finally begins to improve again. As GDP growth rates and debt to GDP ratios are the same in the two scenarios, the differences in the primary balances with and without GDP indexation are driven by the different interest rates r_t^{GDP} and r_t. We see that due to the higher growth rates up to 2007, Greece would have to run a higher primary balance in these years if debt was indexed to GDP. This, however, changes with the crisis from 2009 onwards. In this case, the Greek government would have had to run an even higher primary deficit with indexation than without it, to achieve the same increase in the debt to GDP ratio as in reality. Due to the Greek debt restructuring in 2012, I do not calculate a primary balance with indexation for the year 2012 and also exclude this year in my calculations of the correlation.

Figure 2



Source: Eurostat (2015b; 2016a), IMF (2015b) and own calculations
Note: No primary balance with indexation calculated for the year 2012.

From the graph, I would already expect a higher correlation of the primary balance with GDP-indexed debt with GDP growth as the actual primary balance. This is confirmed in Table 2, which indicates that the correlation with indexation is considerably higher (0.90) than without GDP-linked bonds (0.55).

Table 2

Correlation of primary balance and GDP growth rate	
without indexation	0.55
with indexation	0.90

Source: Eurostat (2015b; 2016a), IMF (2015b) and own calculations
Note: time period 1996-2014. I exclude the year 2012 due to the Greek debt structuring.

[10] For details see Appendix section 4 Table A4.

To check the robustness of the results, I consider again a range of parameter values for $\alpha, \Omega, r^{\min}$ and $r^{\max}$ and compute the correlation of the primary balance and GDP growth rate as above.[11] All in all, the correlation between the primary balance with indexation and the GDP growth rate is fairly resilient to parameter changes and still in the range of 0.80 to 0.91. Unsurprisingly, the largest impact on the correlation seems to come from the degree of indexation α . Nevertheless, even with a fairly low degree of indexation of $\alpha = 0.2$, the correlation with indexation would have been substantially higher (0.80) than it was in the actual data (0.55).

Hence, there is some evidence that the introduction of GDP-linked bonds would have created additional room for Greece for countercyclical fiscal policy. Especially during the economic crisis years 2009–2014, debt indexation to GDP would have enabled the country to run a much smaller primary balance and hence restrain from painful austerity measures such as tax rises, pension cuts and public sector job losses to control fiscal spending and thereby aggravating the negative impact on economic growth further.

I have discussed how GDP-linked bonds would have lowered debt servicing costs and created additional room for countercyclical fiscal policy in Greece. However, the question remains whether the lower interest payments on GDP-indexed debt would also have led to a stabilization in the Greek debt GDP ratio and helped the Greek government manage its huge debt stock more efficiently. Therefore, I will now look at the ability of GDP-linked bonds to stabilize the debt to GDP ratio if a country faces a recession in more detail in the case of Greece.

4.4 Debt to GDP ratio

To simulate the different debt to GDP ratio profiles, I use actual GDP growth rate and primary balance data from Table 1 as well as the respective interest rates with and without GDP indexation of government debt. I then turn to the question, how the debt to GDP ratio of Greece would have evolved if all debt was converted to GDP-linked bonds in 2008. I compare the debt path under indexation with actual debt ratio data and a case without debt indexation and no debt restructuring in 2012. The result is shown in Figure 3.

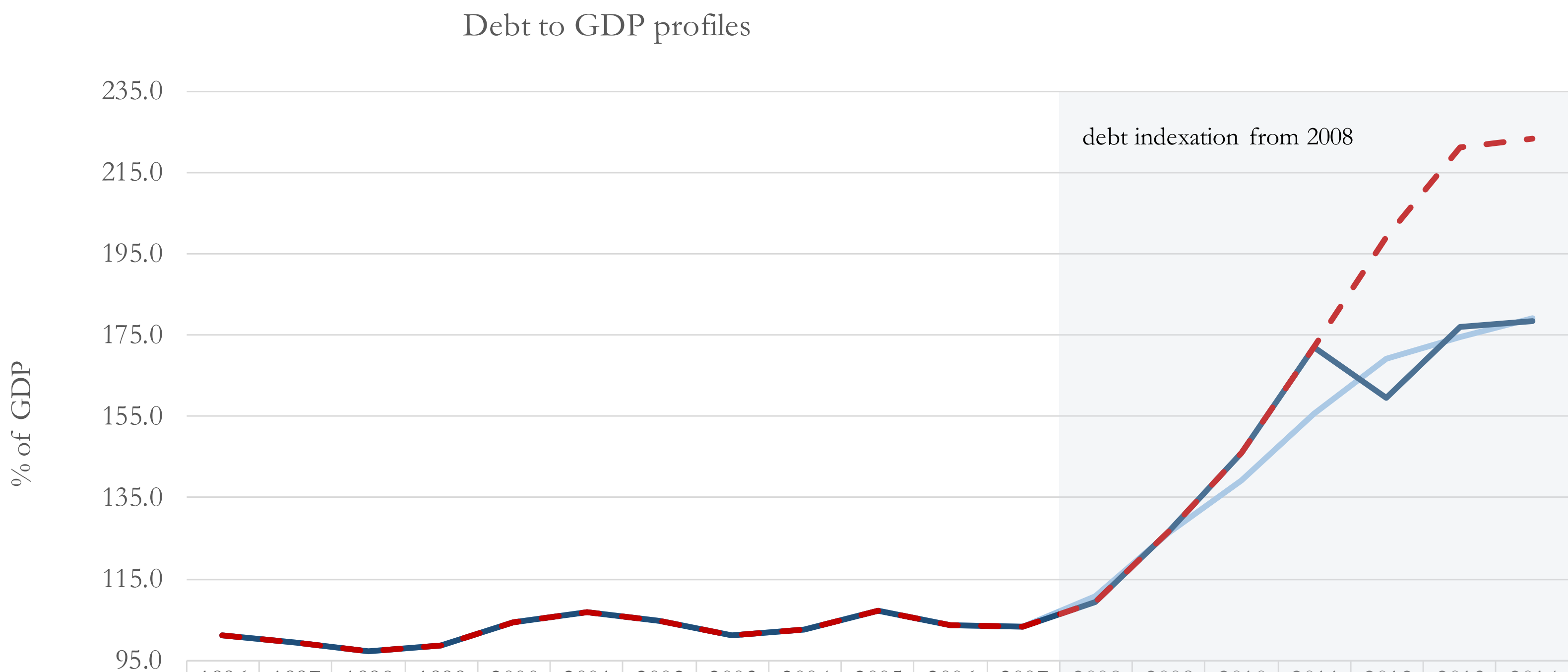
$$\begin{aligned} d_t &= (1 + r_t - g_t) d_{t-1} - s_t \\ d_t^{GDP} &= (1 + r_t^{GDP} - g_t) d_{t-1}^{GDP} - s_t \end{aligned} \tag{1.4}$$

We see that up to the year 2007, the debt profiles coincide. But from 2008 onwards, the debt to GDP ratios without indexation and without indexation and restructuring rise much faster than with GDP indexation. This is the case as the interest rate on GDP-linked bonds is much lower than on conventional bonds due to the severe recession Greece is undergoing during these years. The debt restructuring in 2012 leads to a temporary decline of the actual debt ratio to 159.4%, which is lower than the debt ratio with GDP indexation (169.2%) in 2012. However, the effect is only short-lived as the actual debt ratio increases again to 178.6% in 2014, which is nearly identical with the debt ratio with GDP-linked bonds (179.1%) in 2014. Without the debt restructuring the Greek debt to GDP ratio would have increased to 223.2%, which is clearly unsustainable for the country.

Without GDP-indexed bonds, the debt restructuring for Greece was a clear necessity to bring the debt to GDP ratio back to a manageable path. However, if Greece would have instead indexed all its debt to GDP growth in the onset of the crisis, its debt ratio would have stabilized automatically

[11] For details see Appendix section 4 Table A5.

Figure 3



	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
with indexation	101.2	99.3	97.2	98.6	104.4	106.8	104.6	101.2	102.7	107.3	103.5	103.1	110.9	126.5	139.3	155.6	169.2	174.4	179.1
actual data	101.2	99.3	97.2	98.6	104.4	106.8	104.6	101.2	102.7	107.3	103.5	103.1	109.4	126.7	146.2	172.0	159.4	177.0	178.6
without indexation and debt restructuring	101.2	99.3	97.2	98.6	104.4	106.8	104.6	101.2	102.7	107.3	103.5	103.1	109.4	126.7	146.2	172.0	199.2	221.2	223.2

Source: Eurostat (2015b, 2016a), IMF (2015b) and own calculations around 180%. The outcome would have been very similar to the actual data, but without Greece having to go through a painful debt restructuring and negotiations with creditors that only further reduced the country's credibility and standing among fellow Eurozone member states as well as financial market participants.

With conventional debt securities, Greece's public finances would never have remained on a sustainable path without the debt restructuring in 2012, which reduced the debt ratio by 12.6%. Converting all debt into GDP-linked bonds in 2008 – although difficult to implement in practice – would have enabled the country to stabilize its debt to GDP ratio below 180% while still running the same primary deficits as in reality. Nevertheless, without the austerity measures that actually facilitated the improvement in the primary balance after 2009, even complete GDP indexation of debt would not have enabled Greece to stabilize its debt burden. As can be seen from (1.4), the debt ratio is dependent on the relationship between r_t^{GDP} and g_t and although interest payments on GDP-indexed debt are already low at 0.5% during the crisis years, this is still not low enough in the face of negative growth rates of up to -9.1%. Therefore, it seems likely that Greece would still have had to achieve some adjustments in its public spending even if part of government debt consisted of GDP-linked bonds, as the decline in economic growth was just too severe for GDP-linked bonds to outweigh the negative effects of the recession on the debt to GDP ratio.

4.5 Sovereign default probability

Although the modelling framework used above has allowed me to gain new insights on the benefits of GDP-linked bonds for Greece, it is also rather simplistic and so far has neglected considerations of sovereign default and debt restructurings. Therefore, I will now introduce a more sophisticated model of endogenous sovereign default in my final application, to study the effects of GDP-linked bonds on the sovereign default probability in the case of Greece.

I use a model of the sovereign debt dynamics with endogenous default by Barr, Bush and Pienkowski (2014), which is built on Ghosh et al. (2013). This model combines the concept of fiscal fatigue[12] with a standard debt dynamics equation to estimate a country's debt limit, which in turn is used to model sovereign default for the case with and without GDP-linked bonds. I then compare the results of the standard model to two model extensions with risk-averse investors and varying inflation targets.

4.5.1 Model

The model from Barr, Bush and Pienkowski (2014) consists of four main parts, which govern the sovereign debt dynamics. The evolution of the government debt follows

$$d_{t+1} - d_t = \frac{(r_t - g_{t+1})d_t}{(1 + g_{t+1})} - s_{t+1} + \varepsilon_{t+1} \tag{1.5}$$

where d_t is the debt level as proportion of GDP, r_t is the real interest rate on debt in period t and due in period t+1, which applies to the entire debt stock, g_{t+1} is the real GDP growth rate from period t to period t+1, s_{t+1} is the primary balance as proportion of GDP and ε_{t+1} is a shock to the debt to GDP ratio. The ε_t term represents debt shocks, which are not caused by the primary balance or interest

[12] The concept of fiscal fatigue is introduced in Ghosh et al. (2013). The idea is that if the debt burden reaches a certain threshold, the primary balance is no longer large enough to counteract the negative influence of the interest rate-growth-differential. Government debt becomes explosive.

payments. An example for this could be unexpected contingent liabilities that the government faces, such as banking sector bailouts.

Furthermore, with risk-neutral investors, the expected returns from investing in risk-free and risky bonds must be equal.[13] This means that the endogenous interest rate on government debt r_t satisfies the following no arbitrage equilibrium condition

$$(1-p_{t+1})(1+r_t)+p_{t+1}\theta(1+r^*)=(1+r^*)$$

where p_{t+1} is the default probability and $\theta(1+r^*)$ is the recovery value on a bond if a default occurs.[14] Rearranging yields the following equation for the risky interest rate on government debt

$$(1+r_t)=(1+r^*)\frac{(1-p_{t+1}\theta)}{(1-p_{t+1})} \tag{1.6}$$

The risky interest rate r_t consists of an exogenously given constant risk-free rate r^* and a credit spread that compensates investors for potential losses due to sovereign default and which depends on the default probability as well as recovery rate θ.

Additionally to the debt shock ε_t, the model also incorporates a GDP growth shock, which is embedded in the GDP growth equation

$$g_t = g^* + u_t \tag{1.7}$$

where g^* is the steady state or trend real GDP growth rate and u_t is a stochastic shock to GDP growth. This shock is independent from the debt shock ε_t.

The final component of the model is a fiscal reaction function, which governs the evolution of the primary balance. The government reacts to an increasing debt to GDP ratio by raising the primary balance in order to stabilize debt. However, beyond a certain debt threshold, fiscal fatigue sets in – for example due to political resistance to additional austerity measures – and the primary balance does not increase in line with debt anymore. The primary balance s_{t+1} is given by

$$s_{t+1}=\min(\alpha+\beta(r_t-g^*)d_t,\gamma) \tag{1.8}$$

where γ is the maximum primary balance the government can sustain before fiscal fatigue prevents further consolidation. The constant α intercept captures other factors that influence the primary balance such as political stability. The β coefficient determines how strong the government reacts to shocks pushing the debt level away from its steady state value.

The primary balance reacts to all non-stochastic factors affecting debt dynamics – the debt to GDP ratio d_t, the interest rate r_t as well as the trend growth rate g^*. There are no exogenous shocks to the primary balance and no automatic stabilizers in the sense that the primary balance directly reacts to growth shocks.

[13] Risk-neutral investors derive the same utility from holding risk-free or risky bonds, as long as their expected return is the same.

[14] Given a default probability of zero ($p_{t+1}=0$), the interest rate on debt would be equal to the risk-free rate r^*.

Combining (1.5)-(1.8) yields the following equation which describes the sovereign debt dynamics[15]

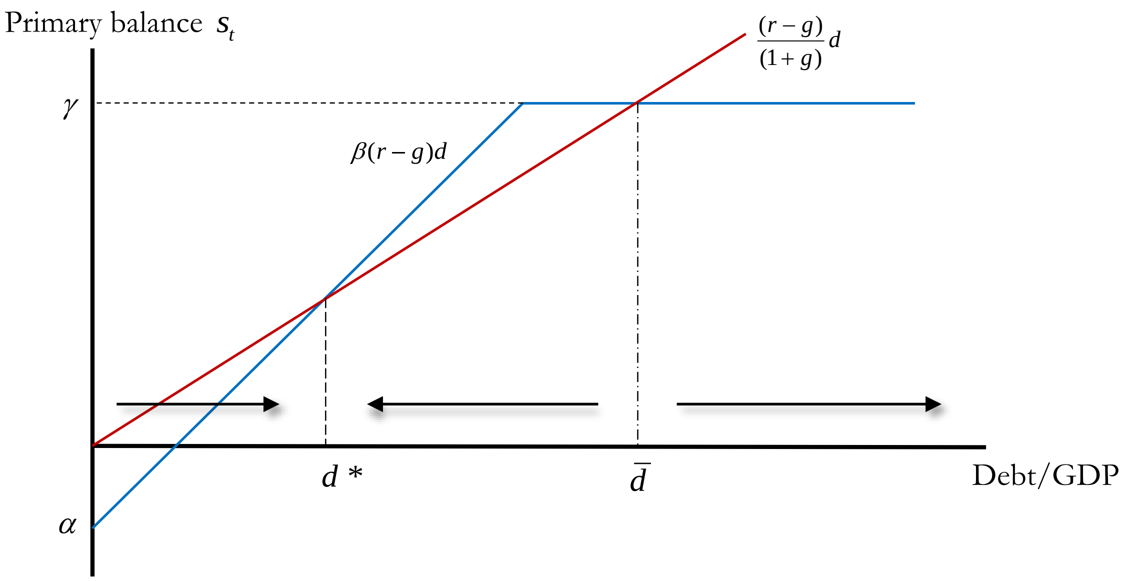
$$d_{t+1} - d_t = \frac{(r^* - g^* - u_{t+1})}{(1 + g^* + u_{t+1})} d_t + (1-\theta)(1+r^*)d_t \frac{1}{(1 + g^* + u_{t+1})} \frac{p_{t+1}}{(1 - p_{t+1})} - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1} \qquad (1.9)$$

The evolution of government debt is governed by two stochastic elements (GDP growth shock u_t and debt shock ε_t), a behavioural component (fiscal reaction function s_{t+1}) and the endogenous default probability p_{t+1}. All other parameters are exogenously given and constant.

Here a simple example to illustrate the basic structure of the model: assume that there are no stochastic shocks ($\varepsilon_t = 0, u_t = 0$). Consequently, without any shocks the probability of default must be zero $(p_{t+1} = 0)$. In this case, the debt dynamics are driven by two factors – the effect of the interest rate-growth-differential $\frac{(r^* - g^*)}{(1+g^*)} d_t$ and the primary balance $\alpha + \beta(r_t - g^*)d_t$. These two components are also illustrated in the debt dynamic phase diagram in Figure 4. There are two steady state debt values where the lines intersect. For $0 < d_t < \bar{d}$, debt will revert to the steady state value of d^* as the government responds to an increasing debt stock with a higher primary balance, which in turn leads to a decrease in the debt level back to its steady state value. However, once the debt burden becomes sufficiently high and fiscal fatigue sets in, the primary balance will not suffice anymore to stabilize the debt to GDP ratio. The government's debt becomes explosive and as markets will not finance such an unsustainable debt burden, the government is forced to default. Therefore, $\bar{d}$ constitutes the country's debt limit.

Figure 4

Debt dynamic phase diagram



Source: own calculations based on Barr, Bush and Pienkowski (2014)

[15] For full derivation see Appendix section 5.1.

The stochastic elements of (1.9) create the risk that negative shocks force the sovereign to breach its debt limit. Hence there is a positive, non-zero probability of default, which means that creditors demand a positive credit spread as compensation for facing this risk in holding government bonds.

The probability of default is defined as the probability that debt in the next period will breach the debt limit.

$$p_{t+1} = \Pr(d_{t+1} > \bar{d}) \tag{1.10}$$

This occurs when the two exogenous shocks u_t and ε_t are large enough to counter other factors influencing debt – such as the primary balance – and push the sovereign beyond its debt limit. Hence, the debt limit and probability of default are jointly determined. To solve for the debt limit $\bar{d}$ numerically, I search for the highest d_t with a default probability below one, where $\bar{d}$ and p_{t+1} satisfy

$$p_{t+1} = \Pr(d_{t+1} - \bar{d} > 0)$$

And plugging in (1.9), I obtain

$$p_{t+1} = \Pr\left[\frac{(r^* - g^* - u_{t+1})}{(1+g^* + u_{t+1})}\bar{d} + (1-\theta)(1+r^*)\bar{d}\,\frac{1}{(1+g^*+u_{t+1})}\frac{p_{t+1}}{(1-p_{t+1})} - \min(\alpha + \beta(r_t - g^*)\bar{d}, \gamma) + \varepsilon_{t+1} > 0\right] \tag{1.11}$$

Using (1.11) and proceeding as described above, I can solve for the debt limit $\bar{d}$, probability of default p_{t+1} and risky interest rate r_t simultaneously. For the computation of the model results I use MATLAB.

As the debt dynamics below the debt limit are also of interest, I also solve for the steady state debt level d^*. In steady state $d_{t+1} - d_t = 0$, and using (1.5), I get the following expression

$$0 = d_{t+1} - d_t = \frac{(r_t - g^* - u_{t+1})d_t}{1+g^*+u_{t+1}} - s_{t+1} + \varepsilon_{t+1}$$

Rearranging and solving for d^* yields[16]

$$d^* = \frac{(\alpha - \varepsilon)(1+g^*+u)}{[(r-g^*-u) - \beta(r-g^*)(1+g^*+u)]} \tag{1.12}$$

where r is the risky interest rate on government debt in steady state.

GDP-linked bonds in the model are of the following form

$$b_{t+1}^{GDP} = b_t^{GDP}(1+g_{t+1}) = b_t^{GDP}(1+g^*+u_{t+1}) \tag{1.13}$$

The redemption value of the GDP-linked bond b_t^{GDP} is linked to GDP growth, which means that if all sovereign debt consists of such bonds, the entire debt stock will adjust in proportion to GDP. The interest rate on GDP-linked bonds is defined as a fixed percent of the principal and so adjusts to GDP, too. A negative growth shock would originally have pushed the debt to GDP ratio upwards, however, the redemption value on the bond and the interest rate will now decrease in line with the shock.

[16] For full derivation see Appendix section 5.2.

Consequently, shocks to GDP growth u_t now no longer affect the debt to GDP ratio and therefore do not enter the sovereign debt dynamics equation anymore.

$$d_{t+1}^{GDP} - d_t^{GDP} = \frac{(r^* - g^*)}{(1+g^*)} d_t^{GDP} + (1-\theta)(1+r^*) d_t^{GDP} \frac{1}{(1+g^*)} \frac{p_{t+1}^{GDP}}{(1-p_{t+1}^{GDP})} - \min(\alpha + \beta(r_t^{GDP} - g^*) d_t^{GDP}, \gamma) + \varepsilon_{t+1} \quad (1.14)$$

This means that GDP-linked bonds reduce the volatility of a country's debt to GDP ratio. Equation (1.14) shows that the difference between the risk-free interest rate and GDP trend growth $r^* - g^*$ is important for the sovereign debt dynamics. Although GDP-linked bonds protect the government from unexpected variations in the growth rate, they still provide no protection against a decline in the trend growth rate g^*.

$$p_{t+1}^{GDP} = \Pr\left[\frac{(r^* - g^*)}{(1+g^*)} \bar{d}^{GDP} + (1-\theta)(1+r^*) \bar{d}^{GDP} \frac{1}{(1+g^*)} \frac{p_{t+1}^{GDP}}{(1-p_{t+1}^{GDP})} - \min(\alpha + \beta(r_t^{GDP} - g^*) \bar{d}^{GDP}, \gamma) + \varepsilon_{t+1} > 0 \right] \quad (1.15)$$

The absence of growth shocks from (1.15) reduces the probability of default at any given debt level and consequently also lowers the credit spread demanded by investors according to (1.6). As a result, in the absence of risk aversion, the risky interest rate on government debt r_t^{GDP} is lower with GDP-linked bonds than with conventional bonds, which in turn implies an increase in the debt limit $\bar{d}^{GDP}$.

Proceeding in a similar fashion as above, I also solve for the steady state debt level d^{*GDP} with GDP-linked bonds.[17]

$$d^{*GDP} = \frac{(\alpha - \varepsilon)(1+g^*)}{[(r^{GDP} - g^*) - \beta(r^{GDP} - g^*)(1+g^*)]} \quad (1.16)$$

4.5.2 Calibration

As this analysis is interested in the effects of GDP-linked bonds for Greece, I calibrate the model parameters to match the Greek economy. Table 3 summarizes the key parameter values of the model.

Table 3: model parameters		
$\sigma(u_t)$	0.024	Standard deviation of GDP growth shocks
$\sigma(\varepsilon_t)$	0.052	Standard deviation of debt shocks
α	-0.03	Intercept on fiscal reaction function
β	8.89	Responsiveness of the primary balance to changes in $(r_t - g^*)d_t$
γ	0.1	Maximum primary balance as a proportion of GDP
g^*	0.021	Trend GDP growth rate
r^*	0.03	Risk-free interest rate
θ	0.7	Recovery rate

Stochastic shocks

I assume that the two exogenous shocks u_t and ε_t follow a normal distribution with mean zero and calibrate the standard deviation of the shocks from actual data. The standard deviation of the GDP

[17] For full derivation see Appendix section 5.3.

growth shock u_t is calculated as the standard deviation of the Greek real GDP growth rate from 1980–2008 from IMF WEO data (IMF, 2015c). The standard deviation of the debt shock ε_t is calculated by finding the change in the Greek debt to GDP ratio each year and then subtracting the fiscal balance to obtain the unexplained part of the evolution of sovereign debt (Barr, Bush and Pienkowski, 2014). To calculate this, I use data on the Greek debt to GDP ratio and government net borrowing (lending) from the IMF Historical Public Debt Database and the IMF WEO from 1980–2008 (IMF, 2015a; 2015d).[18]

Fiscal reaction function

The intercept of the fiscal reaction function α is set to equal -0.03 to ensure a positive steady state debt level as in Figure 4.[19] Plödt and Reicher (2014) estimate a fiscal reaction function for Euro area countries and find a primary balance response coefficient to changes in the debt level of about 0.08. As Greece is part of the Eurozone and there have been no academic studies that estimate a fiscal reaction function for Greece specifically so far, I use the empirical findings of Plödt and Reicher (2014) as an approximation and calibrate β to match a responsiveness of the primary balance to changes in the debt level of 0.08. The value for the maximum primary balance γ is set equal to 0.1 or 10% of GDP. This seems a relatively high value, but similar primary surpluses have been observed for advanced economies in the past. Escolano et al. (2014) for example find a maximum primary balance achieved by advanced countries in the time period from 1945–2012 of 10.6% of GDP.

Growth rate, risk-free rate and recovery rate

The trend GDP growth rate g^* is set to equal the Greek average real GDP growth rate from 1980–2008, which is again 2.1% (IMF, 2015c). To get a good approximation for the risk-free interest rate faced by Greece, I obtain daily yield curve spot rate data from the ECB for Euro area AAA-rated government bonds with maturities from 3 months to 15 years (European Central Bank, 2016). I take the average from this data ranging from 2004–2008 and obtain a risk-free rate of 3%. The parameter for the recovery rate θ is set equal to 0.7, which is based on the empirical findings of Cruces and Trebesch (2013). In their study of 180 sovereign debt restructuring cases in 68 countries from 1970–2010, the authors find an average haircut size on debt of 30% when excluding highly indebted poor countries.

The sensitivity of the model results to the parameters will be tested in the next section.

4.5.3 Baseline model results

To implement the model shocks, I take one hundred thousand random draws from the distribution. Figure 5 plots the two shock distributions for the GDP growth and debt shocks and both follow the shape of a normal distribution. However, the debt shocks are wider dispersed and have a higher volatility because their standard deviation is more than twice as large as the standard deviation of the growth shocks. Hence, I would also expect the debt shocks to drive the model results much more than the growth shocks.

Solving (1.11) and (1.15) with the parameter values in Table 3 generates the baseline model results summarized in Table 4. The main interest here is to compare the cases with conventional government debt securities and GDP-linked bonds.

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[18] I limit myself to the time period up to 2008, as I assume throughout the study that GDP-linked bonds would have been introduced before the debt crisis 2009–2014.

[19] This means that without any debt, the government could run a primary deficit of 3% according to the fiscal reaction function.

Figure 5

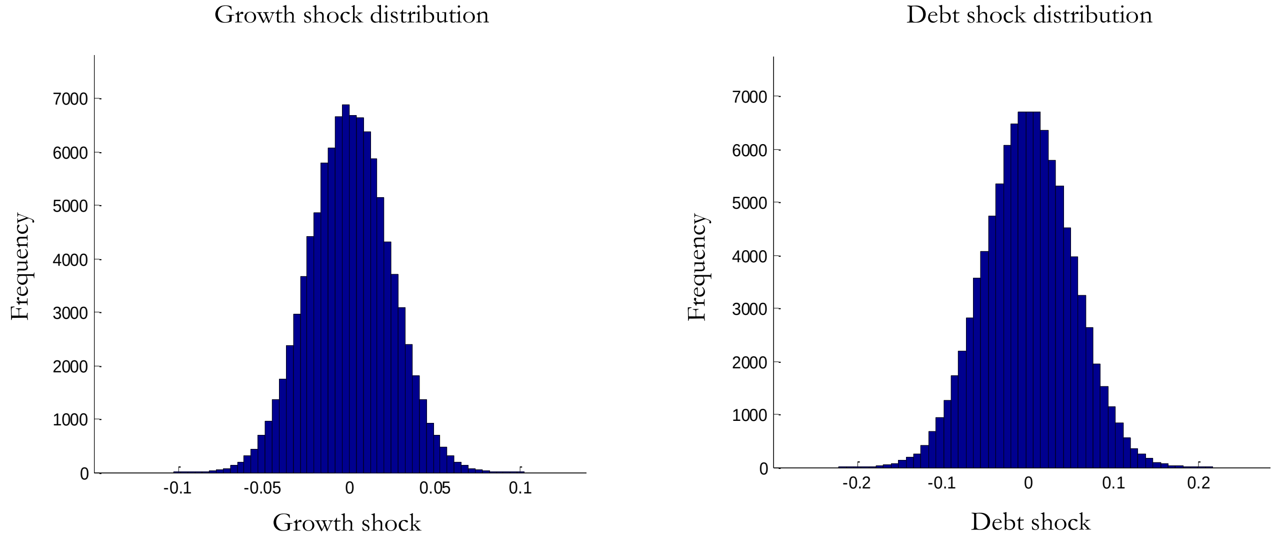


Table 4: baseline model results

	Conventional bonds	GDP-linked bonds
Debt limit $\bar{d}$	0.6900	0.7800
Steady state debt d^*	0.3057	0.3309
Debt space $\bar{d} - d^*$	0.3843	0.4491
Risky interest rate r_t	0.0628	0.0594
Credit spread $r_t - r^*$	0.0328	0.0294
Probability of default p_{t+1}	0.0960	0.0870

In this analysis, I am particularly interested in the effects of GDP-linked bonds on the debt limit and probability of default. We can see that with conventional bonds, the model generates a debt limit of 69% of GDP. Beyond this point, fiscal fatigue prevents the government from increasing the primary balance sufficiently to counteract the effect of the interest rate-growth-differential. Consequently, government debt will become explosive and the sovereign is unable to fund its debt at any finite interest rate as market investors will not finance such an unsustainable debt burden. As expected, the debt limit rises by 9 percentage points to 78% of GDP if the sovereign's debt consists only of GDP-linked bonds instead. The steady state debt value, on the other hand, is fairly constant between the scenarios and rises from 30.6% to 33.1% with GDP-linked bonds. The increase in the space between the steady state debt value d^* and the debt limit $\bar{d}$ from 38.4% to 44.9%, is what facilitates the lower incident of sovereign default in the case with GDP-linked bonds.

The estimated debt thresholds seem rather low compared to the capacity of a modern advanced economy to bear debt. However, this can be attributed to the simplifying assumptions of the model, which abstracts from many real world characteristics. Given the simplicity of the model, the debt limits derived should be seen as an indicative rather than a perfect estimation. The attention should therefore be focused on the ability of GDP-linked bonds to increase the amount of debt the sovereign can sustain in equilibrium and raise in the debt limit, rather than pinpoint the exact values as such.

As expected, the risky interest rate on government debt is lower with GDP-linked bonds than with conventional bonds without risk aversion. The absence of growth shocks with GDP-indexed debt reduces the probability that the sovereign will be pushed beyond its debt limit by an adverse shock. Consequently, the probability of default is lowered and government bond investors demand a smaller credit spread as compensation for the risk of sovereign default.

Sensitivity analysis

To evaluate the robustness of the model results, I investigate how the chosen parameter values affect the estimated debt limit and steady state debt level. Following Barr, Bush and Pienkowski (2014), Table 5 shows the change in each parameter value necessary to increase the debt limit by 10 percentage points *ceteris paribus*.

Table 5: sensitivity analysis of the model parameters

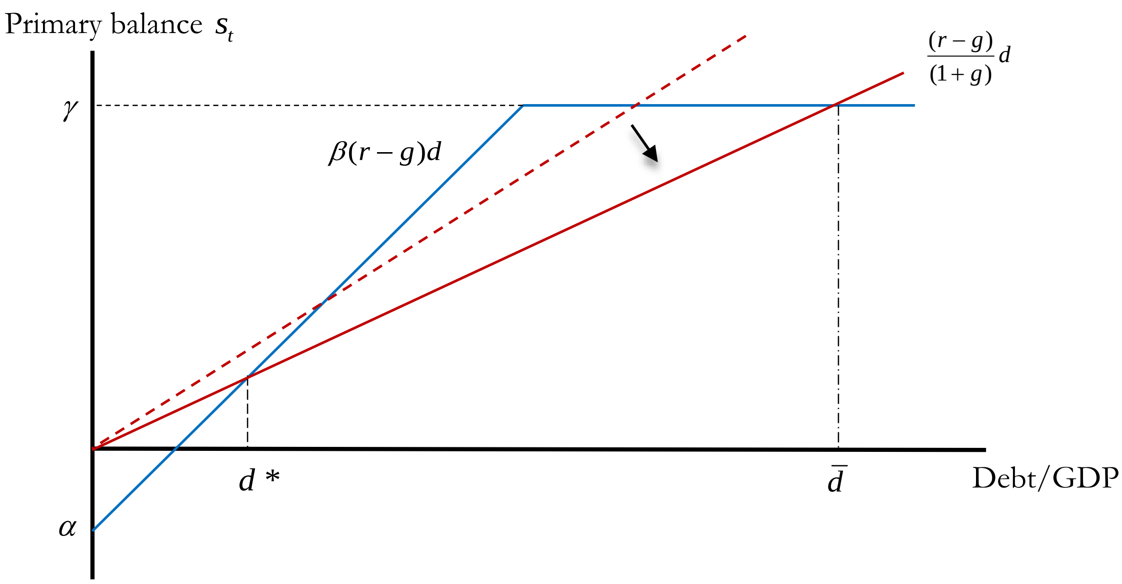
	Baseline parameter	Change in parameter	Change in debt limit $\bar{d}$	Change in steady state debt d^*
Conventional bonds				
r^*	0.03	-0.007	0.1	-0.2196
θ	0.7	0.05	0.1	0.0271
α	-0.03	0.02	No change	-0.0608
β	8.89	-1	No change	0.0445
γ	0.1	0.005	0.1	0.0214
$\sigma(u_t)$	0.024	-0.017	0.1	-0.0218
$\sigma(\varepsilon_t)$	0.052	-0.004	0.1	-0.0133
GDP-linked bonds				
r^*	0.03	-0.004	0.1	-0.1188
θ	0.7	0.04	0.1	0.0375
α	-0.03	0.02	No change	-0.0657
β	8.89	-1	No change	0.0484
γ	0.1	0.004	0.1	0.0589
$\sigma(u_t)$	-	-	-	-
$\sigma(\varepsilon_t)$	0.052	-0.006	0.1	0.0944

The model results are driven by the interest rate-growth-differential as can also be seen from Figure 6. Even a small reduction in the risk-free rate r^* by 0.007 (or 0.004 for GDP-linked bonds), leads to flatter red line in Figure 6, which in turn increases the debt limit while decreasing the steady state debt level.

One can also see from Table 5 that a higher recovery rate θ has a positive effect on the debt limit and steady state debt level. A higher recovery rate decreases the credit spread in (1.6) that investors demand and in turn leads to a lower interest rate on government debt and hence higher debt limit and lower probability of default.

Figure 6

Debt dynamic phase diagram



Source: own calculations

Changing the parameters of the fiscal reaction function α and β has no effect on the debt limit. Although this seems puzzling at first glance, it is due to the set-up of the model, as the debt limit lies beyond the point at which the primary balance has reached its maximum value γ and fiscal fatigue takes over. This means that changes to the intercept α and slope β of the fiscal reaction function have no effect on the debt limit, but they still affect the steady state debt level. Decreasing β leads to a flattening of the blue line in Figure 6 and consequently to a higher $d*$, whereas a higher intercept α shifts the blue line upwards, resulting in a lower $d*$. Unsurprisingly, an increase in the maximum fiscal balance γ has a positive influence on the debt limit. Now, it takes longer until fiscal fatigue sets in and the government breaches its debt limit and defaults.

Apart from the interest rate-growth-differential, the model results are also highly sensitive to the volatility of the debt shock $\sigma(\varepsilon_t)$. For both conventional and GDP-linked bonds, a reduction of the standard deviation of the debt shock by less the 0.01 will increase the debt limit by 10 percentage points. The influence of the variance in the growth shock $\sigma(u_t)$, on the other hand, seems to be of much less importance than the debt shock for conventional bonds. As presumed earlier, the debt shock determines the model results much more than the growth shock due to its higher magnitude. This also explains why the difference in the debt limits with conventional and GDP-linked bonds is only 9 percentage points in the baseline model. With values for the standard deviation of the growth and debt shocks which lie closer to each other, one would observe a much more marked increase in the debt limit with GDP-linked bonds.

4.5.4 Model extension 1: risk-averse investors

So far GDP-linked bonds seem to be an attractive instrument for the Greek government. However, one of its major implementation challenges in reality would be that investors demand a risk premium for holding such a novel instrument with varying coupon payments. To investigate whether it would still be beneficial for the Greek government to issue GDP-linked bonds under such circumstances, this section will introduce risk-averse investors to the model following Barr, Bush and Pienkowski (2014).

The introduction of risk aversion has two effects in the model. Firstly, government bond investors now demand a risk premium for holding bonds with default risk. This default risk premium increases the credit spread and leads to a higher risky interest rate on both conventional (r_t) and GDP-linked bonds (r_t^{GDP}). Additionally to the default risk premium, risk-averse investors also demand a GDP risk premium for holding bonds, which have an uncertain return due to GDP growth volatility. Therefore, GDP-linked bonds now face an extra GDP risk premium that only affects the risky interest rate on such government bonds (r_t^{GDP}).

To model the default risk premium with risk-averse investors, I incorporate a utility function into the model. Following Barr, Bush and Pienkowski (2014), I use a constant relative risk aversion (CRRA) utility function of the following form

$$U = \begin{cases} \dfrac{V^{1-\tau} - 1}{1-\tau} \end{cases} \qquad \text{for } \tau > 0, \tau \neq 1 \tag{1.17}$$

where τ is the coefficient of relative risk aversion and V is the asset value. Applying the utility function to the no arbitrage equilibrium condition (1.6) yields[20]

$$\begin{aligned} &\text{Conventional bonds} \qquad (1+r_t) = (1+r^*)\left[\frac{(1-p_{t+1}\theta^{1-\tau})}{(1-p_{t+1})}\right]^{\frac{1}{1-\tau}} \\ &\text{GDP-linked bonds} \qquad (1+r_t^{GDP}) = (1+r^{*GDP})\left[\frac{(1-p_{t+1}{}^{GDP}\theta^{1-\tau})}{(1-p_{t+1}{}^{GDP})}\right]^{\frac{1}{1-\tau}} \end{aligned} \tag{1.18}$$

These are the new equations that substitute (1.6) with risk averse investors. For any $\tau > 0$, the credit spread now will be larger and the risky interest rate on government debt higher compared to the case with risk-neutral investors.

GDP-linked bonds additionally face a GDP risk premium with risk-averse investors as they demand a compensation for the variance in growth outturns. Consequently, the risk-free rate for GDP-linked bonds now includes a GDP risk premium of the following form

$$r^{*GDP} = r^* + \Omega \tag{1.19}$$

where r^* is the risk-free rate for conventional bonds and Ω is the GDP risk premium, which is constant and exogenously given. From (1.18) it follows that investors in GDP-linked bonds now demand a higher interest rate compared to conventional bonds for any given level of risk aversion.

[20] For full derivation see Appendix section 5.4.

Model results

For the model with risk-averse investors, I use the same parameter values as in the baseline calibration. Additionally, I use a GDP risk premium Ω of 0.4% as above and a coefficient of relative risk aversion τ of 1.5, similarly to prior studies of the effects of GDP-linked bonds on the sovereign debt dynamics with risk-averse investors (see Sandleris, Sapriza and Taddei (2011); Hatchondo and Martinez (2012)).

Table 6: model parameters		
$r*$	0.03	Risk-free interest rate
$g*$	0.021	Trend GDP growth rate
$\sigma(u_t)$	0.024	Standard deviation of GDP growth shocks
$\sigma(\varepsilon_t)$	0.052	Standard deviation of debt shocks
θ	0.7	Recovery rate
α	-0.03	Intercept on fiscal reaction function
β	8.89	Responsiveness of the primary balance to changes in $(r_t - g^*)d_t$
γ	0.1	Maximum primary balance as a proportion of GDP
Ω	0.004	GDP risk premium
τ	1.5	Coefficient of relative risk aversion

I also use the same growth and debt shock distributions as in the baseline model. Table 7 summarizes the results of the model. The introduction of risk aversion has decreased the debt limit by 21 percentage points to 57% of GDP for GDP-linked bonds and reduced the debt limit for conventional bonds by 15 percentage points to 54% of GDP. As before, the debt limit with GDP-linked bonds is still higher than with conventional bonds. However, the difference in debt limits has shrunk between the two bond types. The debt space $\bar{d} - d*$ is also still higher for GDP-linked bonds compared to conventional bonds, but has shrunk somewhat compared to the results from the baseline model with risk-neutral investors.

Table 7: model results with risk-averse investors

	risk-averse investors		risk-neutral investors	
	Conventional bonds	GDP-linked bonds	Conventional bonds	GDP-linked bonds
Debt limit $\bar{d}$	0.5400	0.5700	0.6900	0.7800
Steady state debt $d*$	0.2997	0.2421	0.3057	0.3309
Debt space $\bar{d} - d*$	0.2403	0.3279	0.3843	0.4491
Risky interest rate r_t	0.0649	0.0736	0.0628	0.0594
Credit spread $r_t - r*$	0.0349	0.0396	0.0328	0.0294
Probability of default p_{t+1}	0.0780	0.0870	0.0960	0.0870

The incorporation of risk-averse investors and specifically the default risk premium has increased the risky interest rate for both types of bonds compared to the baseline model. However, whereas the interest rate for conventional bonds increased only by 0.21 percentage points from the baseline case to 6.49% , the interest rate for GDP-linked bonds showed a marked increase of 1.42 percentage points to 7.36%.

As expected, the interest rates increase due to the default risk premium in (1.18). Risk-averse investors demand a higher compensation for holding risky government bonds and hence require a higher interest rate. This in turn affects the sovereign debt dynamics negatively. The red line in Figure 4 from above becomes steeper and the debt limit is lowered.

I noted above that the difference between the risk-free interest rate and GDP trend growth $r^{*GDP} - g^*$ is important for the sovereign debt dynamics and default probability for GDP-linked bonds in (1.15). The additional GDP risk premium for indexed bonds now leads to a higher risk-free rate r^{*GDP} and risky interest rate on government debt r_t^{GDP}. This means that it also becomes more likely that a negative debt shock will push the government beyond its debt limit where risk-averse investors will not finance the debt burden at a finite interest rate anymore. Consequently, the additional GDP risk premium now leads to a higher probability of default with GDP-linked bonds than with conventional debt in this scenario.[21]

Overall, GDP-linked bonds seem to be somewhat more susceptible to risk aversion in the model. Especially the GDP risk premium combined with the default risk premium increases the interest rate on GDP indexed debt and thereby reduces the benefits of introducing GDP-linked bonds for the government in terms of the debt limit and probability of sovereign default.

Sensitivity analysis

To test the robustness of the results, I consider a range of different coefficients of relative risk aversion in Table 8 while holding all other parameters constant and investigate how they influence the debt limits with and without GDP-linked bonds.

Table 8: sensitivity analysis of the debt limit

Coefficient of relative risk aversion τ	**Conventional bonds**	**GDP-linked bonds** (with $\Omega = 0.004$)	**GDP-linked bonds** (with $\Omega = 0$)
0.5	0.63	0.66	0.72
1.5	0.54	0.57	0.60
2	0.51	0.51	0.57
3	0.42	0.42	0.45
4	0.33	0.33	0.36

The debt limit predicted by the model is very susceptible to the chosen degree of risk aversion and decreases with growing risk aversion. When comparing the debt limits between the two bond types in column two and three one can see, that for low degrees of risk aversion, the debt limit with GDP-linked bonds is still higher, but this changes with higher coefficients of relative risk aversion. With a higher level of risk aversion, the proportionate increase in the interest rate is much higher for GDP-linked bonds due to the extra GDP risk premium Ω and the debt limits become identical for the two bond types. Comparing column two and four also confirms that the debt limit of GDP-linked bonds is more sensitive to risk aversion in the model: the reduction in the debt limit is much stronger for GDP indexed bonds with rising risk aversion.

[21] To confirm that it is indeed the GDP risk premium, which drives the higher default probability, I consider the case $\Omega = 0$. That would have resulted in a probability of default of 0.0720 for GDP-linked bonds, which is lower than the default probability for conventional bonds (0.0780).

I conclude that whereas GDP-linked bonds would still benefit Greece in terms of a higher debt limit and lower probability of default if financial market participants exhibit only a relatively low degree of risk aversion, the advantage of growth indexed debt instruments vanishes completely if investors demand too much compensation for GDP growth volatility and the novelty of the instrument in the form of a higher GDP risk premium. Therefore, it seems as the benefits from introducing GDP-linked bonds for Greece would be maximized during an episode of sound macroeconomic fundamentals and strong growth when investors are eager to invest in such innovative debt instruments to diversify their portfolios and demand lower credit risk spreads.

4.5.5 Model extension 2: inflation targets

The second extension I introduce in the model, seeks to investigate the effect of varying central bank inflation targets on the sovereign default probability. As Greece is a member of the Eurozone and shares the Euro with the other member states, its central bank cannot engage in independent monetary policy. Instead monetary policy is set by the European Central Bank (ECB) in Frankfurt with the Bank of Greece implementing the ECB's decisions in practice.

One of the challenges for a country in a monetary union such as Greece is that it cannot appreciate or depreciate its currency nor determine its inflation rate independently. A country with a high debt burden, which sets its own monetary policy, on the other hand, could in principle devalue its currency and increase the domestic inflation rate and thereby inflate away the real value of its debt obligations.[22] Having a high inflation rate is certainly no universal remedy for countries struggling with debt, however, running a slightly higher inflation rate as normal during crisis years could give the government some fiscal breathing space to adjust public finances and achieve the needed correction in the primary balance for example. One question currently highly debated is whether the central bank might deviate from its inflation target if an economy faces a severe macroeconomic shock in order to support growth and fiscal stimulus during such a turbulent crisis episode.

I want to investigate this in more detail and therefore introduce an inflation variable in the debt equation (1.5).

$$d_{t+1} = \frac{(1+r_t)d_t}{(1+g_{t+1})(1+\pi)} - s_{t+1} + \varepsilon_{t+1} \tag{1.20}$$

where π is the inflation rate, which is constant and exogenously given since I do not model monetary policy explicitly. Rearranging yields the following new sovereign debt dynamics[23]

$$\begin{aligned} &\text{Conventional bonds} \qquad d_{t+1} - d_t = \frac{(r_t - g_{t+1} - \pi - g_{t+1}\pi)}{(1+g_{t+1})(1+\pi)} d_t - s_{t+1} + \varepsilon_{t+1} \\ &\text{GDP-linked bonds} \qquad d_{t+1}{}^{GDP} - d_t{}^{GDP} = \frac{(r_t^{GDP} - g^* - \pi - g^*\pi)}{(1+g^*)(1+\pi)} d_t{}^{GDP} - s_{t+1}{}^{GDP} + \varepsilon_{t+1} \end{aligned} \tag{1.21}$$

From (1.20) it follows that the debt burden next period will shrink with an increasing inflation rate π. The higher inflation in the country, the easier it will be for the government to manage and stabilize its debt burden by inflating some of the debt stock away.

[22] Assuming the debt burden is denominated in domestic currency.
[23] For full derivation see Appendix section 5.5.

The steady state debt levels for conventional and GDP-linked bonds are now given by[24]

$$
\begin{aligned}
&\text{Conventional bonds} \quad d^* = \frac{(\alpha-\varepsilon)(1+g^*+u)(1+\pi)}{[(r-g^*-u-\pi-(g^*+u)\pi)-\beta(r-g^*)(1+g^*+u)(1+\pi)]} \\
&\text{GDP-linked bonds} \quad d^{*GDP} = \frac{(\alpha-\varepsilon)(1+g^*)(1+\pi)}{[(r^{GDP}-g^*-\pi-g^*\pi)-\beta(r^{GDP}-g^*)(1+g^*)(1+\pi)]}
\end{aligned}
\tag{1.22}
$$

Substituting equation (1.5) with (1.21) and running the model with risk-averse investors otherwise as before with the same parameters, produces the results summarized in Table 9. I contrast the scenario with an inflation rate π of 2%, which is the official inflation target of the ECB, with the scenario of a higher inflation rate of 4%.

Table 9: model results with varying inflation targets

	$\pi = 0.04$		$\pi = 0.02$	
	Conventional bonds	GDP-linked bonds	Conventional bonds	GDP-linked bonds
Debt limit $\bar{d}$	1.1700	1.2900	0.8100	0.9600
Steady state debt d^*	1.0118	0.9980	0.4691	0.4783
Debt space $\bar{d}-d^*$	0.1582	0.2920	0.3409	0.4817
Risky interest rate r_t	0.0362	0.0364	0.0506	0.0506
Credit spread $r_t - r^*$	0.0062	0.0024	0.0206	0.0166
Default probability p_{t+1}	0.0150	0.0060	0.0480	0.0390

We see from the table that for all scenarios Greece would still benefit from GDP-linked bonds even in the presence of risk averse investors. With a maximum inflation target of 2%, the debt limit would rise by 15 percentage points to 96% of GDP and the probability of default would decline by 19% from 4.8% to 3.9%. I compare this to the scenario where the ECB would target instead a higher inflation rate of 4%, for example to facilitate additional stimulus for an economy hit by a sizable adverse growth shock such as was the case in Greece during 2009–2014. The higher inflation rate leads to a higher debt limit in the model as part of the debt stock can now be inflated away. Consequently, the debt limit for conventional bonds rises to 117% of GDP and 129% of GDP for GDP-linked bonds. The probability of default declines and hence the credit spread and risky interest rate on debt are lower. According to (1.22) the steady state debt level increases with a higher inflation rate and this is also what can be observed in the table.

Overall, it seems as if the merits from introducing GDP indexed debt securities decline with an increasing inflation rate. Whereas the debt limit increases by 15 percentage points with GDP-linked bonds in the scenario with an inflation rate of 2%, the debt limit rises merely by 12 percentage points with GDP-linked bonds and an inflation rate of 4%. The higher the inflation rate, the easier it is for a country with a high debt burden to inflate away part of the real value of its debt obligations and therefore the less it will benefit from the additional debt stabilizing features of GDP-linked bonds in turn.

[24] For full derivation see Appendix section 5.5.

This investigation of varying inflation targets and the effect of GDP-indexed debt has shown that Greece would achieve a higher increase in its debt limit in the model (from 81% to 117%) if the ECB could have been induced to pursue a more expansionary monetary policy and target a higher inflation rate of 4%, which would have helped Greece managing its huge debt burden more easily. However, given that the ECB's monetary policy is targeted towards the Eurozone as a whole and not to a specific country such as Greece, such an adjustment in the inflation target seems rather unlikely in practice. In this case, GDP-linked bonds could be a viable alternative that could help Greece extend its debt limit still by a sizable amount (from 81% to 96%) even if the inflation channel cannot be used for debt reduction. Hence, GDP-linked bonds seem to be an attractive instrument for Greece, especially as it is part of a monetary union and unable to alleviate its debt burden with its own tailored monetary and inflation policy.

4.6 Limitations

The results from this analysis are subject to a number of limitations. I have made many simplifying assumptions and in particular assumed that the paths of the variables from the data are not affected by the debt composition. Furthermore, indexing all government debt to GDP growth might be difficult to achieve in practice due to the novelty of the instrument for investors as well as the government's preference for traditional debt instruments. Therefore, the results should be interpreted as an upper bound for the benefits that can arise from GDP indexation of debt. The results are sensitive to the chosen indexation formula and parameterization and therefore the simulated values for debt to GDP ratios, primary balances and interest savings should be seen as approximations rather than perfect forecasts.

In my analysis, I also abstracted from the actual government debt composition in Greece and assumed that all of the government debt consisted either of traditional or GDP-linked government bonds. In practice, the official sector bailout programmes as well as the debt restructuring in 2012 have led to a shift in the composition of Greece's public debt, from private sector investors to mainly public sector creditors.[25] However, for my analysis I was more interested in studying the benefits of GDP-linked bonds from a macro perspective for the government, rather than looking at each debt category and instrument separately.

The model of endogenous sovereign default used in section 4.5 has no DSGE framework and hence the results of the model are also subject to a number of limitations. Due to the relatively simple model set up, the derived debt limits are rather low compared to observed debt to GDP ratios in many European developed countries. Therefore, the simulation results should be seen as an indicative and the reader should focus her attention on the ability of GDP-linked bonds to increase the amount of debt the sovereign can sustain in equilibrium, rather than the exact values as such. I have tried to overcome this weakness by adding risk aversion and inflation to the model and indeed I obtain higher debt limits when including these extensions. Unfortunately, the model does not allow for partial indexation of government debt, so that 50% would consist of conventional bonds and 50% of GDP-linked bonds for example. Therefore, the reported results should again be seen as an upper bound for the benefits of introducing GDP-linked bonds for Greece. Another drawback of the model is that it is in part susceptible to the chosen parametrization. Especially the parameters for the interest rate-growth-differential, the volatility of the debt and growth shocks as well as the degree of risk aversion determine the model results. The modelling of risk aversion is also subject to a number of simplifications. Utility here depends not on per-period consumption as is usually the case, but the value of the asset. The model also assumes that

[25] For details see Appendix section 3 Figure A2.

investors can only hold either conventional or GDP-linked government debt securities, not both at the same time as would be the case in reality (Kawamura, 2014).

In terms of external validity, the analysis was exclusively focused on Greece and therefore the results cannot necessarily provide conclusions about the benefits of introducing GDP-linked bonds for other economies.

5 Conclusion

This thesis sought to investigate the question of whether GDP-linked bonds would be a beneficial financing tool for the Greek government in terms of their impact on the sovereign debt dynamics, fiscal policy, debt servicing costs and sovereign default probability. In my analysis, I find evidence that many of the suggested benefits of GDP-indexed debt from previous literature also apply to the case of Greece.

Studying the potential interest bill savings from indexing a part of government debt to GDP growth, I find that Greece could have reduced its interest bill by up to 6% of GDP during the crisis years. These positive interest bill savings are also robust under varying parameterization and debt composition scenarios. Furthermore, I looked at the effect of GDP-linked bonds on the primary balance and fiscal policy. Studying the correlation of GDP growth rates with the primary balances with and without debt indexation, I find that GDP-linked bonds would have increased the correlation from 0.55 to 0.90 and hence created additional room for countercyclical fiscal policy in Greece. Analysing the paths of the debt to GDP ratios with and without GDP-linked bonds, I find that debt indexation would have helped stabilizing the Greek debt to GDP ratio below 180% without a debt restructuring if GDP-linked bonds would have been introduced at the onset of the crisis. Using a model of endogenous sovereign default by Barr, Bush and Pienkowski (2014), I also studied the effect of GDP-linked bonds on the debt limit and sovereign default probability. Both in the baseline model specification as well as in subsequent model extensions with risk-averse investors, I find evidence that GDP indexed debt would have increased the amount of debt Greece could sustain. By eliminating the impact of growth shocks such bonds would not only have reduced the volatility of Greece's debt to GDP ratio but also decreased the probability of a sovereign default by up to 19%.

Greece also exhibits several favourable characteristics that would simplify implementation and support the debt stabilizing properties of GDP-indexed government debt and therefore make the country a particularly suitable candidate for growth-indexed debt. Firstly, GDP-linked bonds would be particularly appealing to EU countries, which can credibly commit to good statistical standards and where a concerted action to issue such bonds in several countries would be easier to achieve. Secondly, Greece is part of the Eurozone monetary union and therefore cannot devalue its own currency and spur inflation to reduce part of its debt burden. Previous studies have also stressed that the benefits of GDP-linked bonds would be particularly large for countries in a currency union whose monetary policy is constrained (Borensztein and Mauro, 2004; Barr, Bush and Pienkowski, 2014). Investigating the effect of GDP-linked bonds on the sovereign debt dynamics under varying inflation targets, I find evidence that this is indeed the case for Greece as the positive impact of GDP-linked bonds on the debt limit and probability of default is much more pronounced if the inflation rate is constrained by the ECB's official target of 2%.

Nevertheless, the aforementioned merits of GDP-linked bonds have to be weighed against the drawbacks of GDP-indexed government debt for Greece. Especially, the GDP risk premium and the higher debt servicing costs during an economic boom phase are the major drawbacks of such bonds in the case of Greece. The results from the model with risk-averse investors have shown how susceptible the benefits of GDP-indexed debt are to the GDP risk premium. Investors would want to be compensated in terms of a higher interest rate for holding such novel debt instruments whose return varies with GDP growth. The risk premium leads to a higher interest rate paid on GDP-linked government bonds compared to conventional bonds and in turn significantly reduces the positive impact of such bonds on the debt dynamics. Therefore, the magnitude of the actual benefit of GDP-linked bonds for Greece crucially depends on the size of the GDP risk premium. If this premium is very large, the Greek government might well prefer to continue issuing normal debt securities instead of GDP-linked bonds as the positive impact of the variable interest rate does not outweigh the cost of the GDP risk premium anymore. In my calibration, I used a small, but positive GDP risk premium similar to previous studies and I still find positive gains for the Greek government from introducing GDP-linked bonds.

Another disadvantage of GDP-indexed debt securities is that that Greece would face higher debt servicing costs if the economy is expanding above its trend growth rate. The magnitude of the difference between the debt servicing costs with normal and GDP-indexed bonds depends on the type and direction of macroeconomic shocks in the economy. A country whose economic growth is very stable will profit much less from the recession insurance properties of GDP-linked bonds than a country with relatively frequent and sizable growth shocks. Up to 2008, Greece had a relatively low volatility in GDP growth rates but the country suffered a substantial negative growth shock with the onset of the debt crisis in 2009. The crisis and its huge debt burden let to Greece facing forbiddingly high interest rates in financial markets on its government bonds. Although Greece would have paid higher interest rates on debt during the high growth years before the crisis, the positive interest savings from GDP-indexed debt during the crisis years 2009–2014 would have outweighed this as discussed in section 4.2. Furthermore, higher interest rates on debt during years of strong GDP growth are not necessarily a bad thing, as it also makes it more difficult for irresponsible politicians to boost frivolous public spending during economic booms and hence induce the Greek government to pursue a more responsible and countercyclical fiscal policy with GDP-linked bonds.

Based on these observations, I conclude that GDP-linked bonds would have been a beneficial financing tool for the Greek government as the benefits of such bonds in terms of lower debt servicing costs, more room for countercyclical fiscal policy, a more stable debt to GDP ratio as well as a lower probability of default would have outweighed the drawbacks in the form of the GDP risk premium and interest rate variability in the case of Greece.

However, could GDP-linked bonds also have helped Greece avoid its debt and austerity crisis? Even converting all of government debt to GDP-indexed debt securities would not have rescued Greece from some form of a debt crisis as its fiscal spending and public debt burden was unsustainable. The Greek debt crisis was a result of various intertwined factors such as fiscal profligacy, unsustainably high debt servicing costs on government debt and structural weaknesses in the Greek economy. GDP-indexed debt instruments would only have addressed some of these issues and hence structural reforms as envisioned by the bailout programmes would still have been important to strengthen the future growth prospects of the Greek economy. Although GDP-linked bonds could not have averted the debt crisis altogether, they still could have alleviated the austerity crisis and recession in Greece that followed. By allowing the Greek government to pursue a more countercyclical fiscal policy with interest payments being deferred until the recovery materializes, such debt instruments could have contributed to macroeconomic stabilization. Especially during the economic crisis years 2009–2014, debt indexation to GDP growth would have enabled the country to run a much smaller primary balance and help avoid painful austerity measures. Although Greece would still have needed to reign in public spending and adjust its primary deficit to bring fiscal measures back to a sustainable path even with GDP-indexed government bonds, the procyclical debt servicing costs and debt stabilizing properties of such bonds would have created some breathing space for the government to avoid procyclical austerity measures and mitigated the negative repercussions of the debt crisis and recession. Given the high interest rates Greece was facing in financial markets on its debt at the time, the bailout programmes and debt restructuring in 2012 were a clear necessity to bring Greece's public finances back to a sustainable path. However, GDP-indexed debt could not only have helped Greece avoiding a costly debt restructuring, which damaged the country's credibility and standing among fellow Eurozone member states as well as financial market participants, but also shortened the recession as less austere fiscal measures could have prevented the complete collapse in GDP growth that followed.

This thesis has contributed to the current literature on GDP-linked bonds and the Greek debt crisis in a number of ways. It is one of the few studies, which have investigated the impact of GDP-linked bonds on the sovereign debt dynamics of an advanced economy in a monetary union. Furthermore, by analysing the benefits of such debt instruments specifically for Greece, this thesis has proposed and analysed an alternative approach to austerity measures to solve issues related to the Greek debt crisis.

Only one previous study has also specifically investigated GDP-indexed debt instruments as a possible solution for the unsustainable Greek debt burden. My results conform with and support the earlier findings of Fratzscher, Steffen and Rieth (2014) in many ways. I also find that GDP-linked bonds would reduce the probability of default by up to 19%. This is a lower value than the 59% Fratzscher, Steffen and Rieth quote in their study, however, it is due to different modelling frameworks used and a slightly different research focus (GDP-indexed GLF loans vs. GDP-linked government bonds). Similar to the authors I also find that GDP-linked bonds would reduce the volatility of the Greek debt to GDP ratio and thereby increase the sustainability of Greece's debt burden.

Although I used the same model of endogenous sovereign default as Barr, Bush and Pienkowski (2014), I calibrated the model specifically to the Greek economy. Therefore, my results also differ in some aspects from the original paper. The increase on the debt limit when introducing GDP-linked bonds I find in the case of Greece is considerably lower compared to the findings of Barr, Bush and Pienkowski. This difference is mainly driven by the volatility and magnitude of the growth and debt shocks which differ from the original study, which was calibrated to an average advanced economy. Furthermore, other than the authors, I also investigated an additional model extension with varying inflation targets to include further characteristics of Eurozone monetary policy and make it more relevant to the case of Greece. Whereas the authors did not present quantitative evidence for their claim that GDP-linked bonds would be particularly beneficial for countries in a monetary union, this study has presented first results that suggest that GDP-linked bonds might be particularly suitable for Greece because it cannot pursue its own tailored monetary and inflation policy and therefore not use the inflation channel for debt reduction.

Therefore, the findings of this thesis support earlier research on the benefits of GDP-linked bonds for a country struggling with high public debt and create a compelling case for GDP-linked bonds in Greece in order to manage the country's debt burden more effectively and reduce the impact of growth shocks on the government debt dynamics.

5.1 Policy implications

GDP-linked bonds would be a beneficial tool for the Greek government to manage its debt burden more efficiently, but the magnitude of the benefit would crucially depend on the contract design, indexed debt amount and GDP risk premium.

GDP-linked bonds with a relatively simple contract design as used in this thesis would both benefit the Greek government and make it easier for investors to price these instruments. Although debtor countries would want to include clauses that limit payments of such bonds in times of high economic growth, this would also significantly decrease demand for such instruments from investors who want to be compensated for taking on the country's growth risk. Therefore, it is important that if Greece should ever issue such GDP-indexed government bonds on a large scale, it should aim for a simple contract design with clear trigger conditions, that creates the right balance between the desired recession insurance for the Greek government and sufficient investor remuneration in the form of coupon payments to make it an attractive investment and diversification tool for investors and hence lowers the GDP risk premium. This trade-off should also be considered by the Greek government when determining a suitable maturity profile for GDP-linked bonds. Debt instruments with a longer maturity might be more interesting for financial investors as there is a higher probability that they will also profit from higher interest payments during the economic cycle.

Furthermore, to maximize the impact of GDP-indexed government debt and eliminate the negative impact of growth shocks on the sovereign debt dynamics, it would be desirable that GDP-linked bonds constitute a significant part of Greek government debt. However, even indexing half of Greece's debt to GDP growth might be difficult to implement in practice due to the novelty of the instrument and investor scepticism. Nevertheless, the Greek government has already issued a form of a GDP-linked warrant as part of its debt restructuring in 2012 and thereby facilitated a greater familiarity of financial market participants with this type of instrument. This might also make it easier for Greece to continue to issue such bonds and gradually increase its GDP-indexed debt share in the future.

I have highlighted above that the magnitude of the benefit from introducing GDP-linked bonds in the case of Greece would crucially depend on the size of the GDP risk premium. To maximize the merits of GDP-indexed government bonds, they should therefore be introduced during an episode of sound macroeconomic fundamentals and strong growth when investors exhibit a relatively low degree of risk aversion and high readiness to invest in novel government debt securities to diversify their portfolios and demand lower GDP risk premia. Previous literature on the topic also stressed that GDP-linked bonds should best be issued when investor risk appetite is high and the GDP risk premium lower (Griffith-Jones and Sharma, 2006). In the case of Greece, GDP-linked bonds ideally should have been issued before the onset of the crisis in 2009, to profit from the positive investor sentiment at the time. Due to the aforementioned implementation challenges as well as investor scepticism, Greece was unable to use GDP-linked bonds as a debt stabilization tool in practice during the debt crisis and instead had to rely on an official sector bailout from the EU and IMF. It also seems questionable whether the country would be able to issue GDP-linked debt on a sufficiently large scale under current market conditions when investor risk aversion is high and Greece's credibility and standing among financial market participants diminished.

The magnitude of GDP risk premium would, however, not only depend on the degree of investor risk aversion, but also on the type of bondholder. Domestic investors would generally be less interested in holding Greek GDP-linked bonds as coupon payments would in most instances exhibit a high correlation with their on consumption pattern and therefore only amplify the volatility in their income and consumption path. International investors, on the other hand, would appreciate the opportunity to hold Greek GDP-indexed bonds, which exhibit only a low or even negative correlation with their own income and consumption. Due to these consumption smoothing and diversification possibilities, such international investors would also demand a much lower (or even negative) GDP risk premium. Therefore, it would be desirable if Greek GDP-linked government bondholders consist to a great part of international institutional investors with ample diversification opportunities as this would limit the size of the GDP risk premium.

5.2 Future research

In this thesis, I investigated the impact of introducing GDP-linked bonds for Greece on the sovereign debt dynamics. To derive the probability of default, I have worked with a simpler model setup following Barr, Bush and Pienkowski (2014). Therefore, the next step for future research about the benefits of GDP-indexed government debt for Greece would be to test whether the reported results also hold in a richer DSGE model framework such as in Hatchondo and Martinez (2012) and if only a part of government debt is indexed to GDP growth. Furthermore, due to time constraints, I used an approximation for the GDP risk premium for Greece in my analysis. A more accurate estimation of the Greek GDP risk premium for GDP-linked bonds would therefore be desirable for future work.

I also think it would be worthwhile to investigate the benefits of GDP-linked bonds for countries in a monetary union in more detail and in a richer model setting. So far, there has been no specific research focused on investigating the effects of GDP-linked bonds in a monetary union. Although it has been noted in the literature that countries whose monetary policy is constrained would likely benefit most from such debt instruments, no quantitative results have been presented yet.

Kawamura (2014) also outlines a range of model improvements for future research. These include a richer modelling of the fiscal fatigue concept, which endogenizes the maximum primary balance parameter γ . One could well imagine that the capacity of the government to reduce public spending and introduce tax hikes depends on individual social, economic and labour market characteristics of the economy. The model also assumes that for the GDP-linked bond scenario, the entire debt stock adjusts linearly to variations in GDP growth. The existing Greek GDP-linked warrants however, have a minimum threshold for the GDP growth rate below which the coupon is zero.[26] Therefore, it would be interesting to see if the model results also hold for such non-linear coupon rates.

[26] For details on the existing Greek GDP-linked warrants see Appendix section 2.

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Appendix

1 The Greek debt crisis

The Greek debt crisis originated from an announcement by the new government in October 2009 that the country had misreported its debt and deficit figures for years. The budget deficit, originally projected at 7%, ended up at 15.6%. In the following months, Greece was hit by further economic disappointments, rating downgrades by the major credit rating agencies and faced increasingly high debt servicing costs in the financial markets. Eventually, Greece had to turn to the Eurozone member states and the IMF for a rescue package, which was agreed on in May 2010 despite initial German resistance. The bailout package consisted of €80 billion EU loans as well as €30 billion IMF credit and should be paid out in tranches until 2012 conditional on fiscal adjustment measures on the side of Greece (Zettelmeyer, Trebesch and Gulati, 2013).

Although the initial market reaction was positive, euphoria vanished in the early months of 2011 and market participants became sceptic about the ability of the Greek government to bring its finances to a sustainable path in the face of continuing economic weaknesses and growing domestic opposition to the austerity programme. The Deauville announcement by Chancellor Merkel and President Sarkozy in October 2010, which emphasized the need for an adequate participation of private creditors, resulted in a particular marked increase in bond yields for all peripheral countries, and foremost for Greece. It became increasingly clear that Greece was not able to return to capital markets by early 2012 as envisioned by the first bailout plan (Ardagna and Caselli, 2014). A second Greek bailout programme was ratified in February 2012 and consisted of even harder austerity measures for Greece, additional €109 billion of funds from Eurozone member countries and a private sector involvement in the form of a debt restructuring on existing Greek government bonds (Ardagna and Caselli, 2014).

Just when the Greek economy seemed to recover, the Greek election of January 2015 produced a new government led by Syriza with a clear mandate to end the austerity measures. The confrontational course of the Syriza government, which halted the reforms and rejected all bailout conditions agreed on by the former government as well as the introduction of capital controls, sent the economy back into recession. Once again the Greek government was facing imminent default and only the ECB's Emergency Liquidity Assistance (ELA) funding prevented a complete collapse of the Greek banking system. After difficult and lengthy negotiations the parties reached a provisional agreement on a third bailout programme for Greece in July 2015.[27]

[27] For a detailed account of the Greek debt crisis and debt restructuring see Zettelmeyer, Trebesch and Gulati (2013) and Ardagna and Caselli (2014).

2 Existing Greek GDP-linked warrants

As part of its debt restructuring in 2012, Greece offered investors a detachable GDP-linked warrant issued along with each new government bond with interest payments determined by the development of the country's GDP. The security is constructed in the way that it will make payments on the 15th October every year, starting from 2015 with a final maturity of 2042. The condition is that the nominal GDP in the year preceding any payment equals or exceeds the reference nominal GDP (see Table A1).

$$\text{Payment amount} = \text{interest rate} \cdot \text{principal}$$

where $$\text{interest rate} = 1.5 \cdot [\text{real GDP growth rate} - \text{reference real GDP growth rate}]$$

and the principal is determined by the table below. The interest rate cannot exceed 1% and is equal to zero if real GDP growth is negative or below the reference real GDP growth rate. The principal declines by about 5% per year starting in 2024. The real GDP growth rate of any year is determined by data published by Eurostat. Investors are not entitled to the principal, which is only used to calculate the payments. The securities are callable after 1st January 2020 (Zettelmeyer, Trebesch and Gulati, 2013).

Table A1

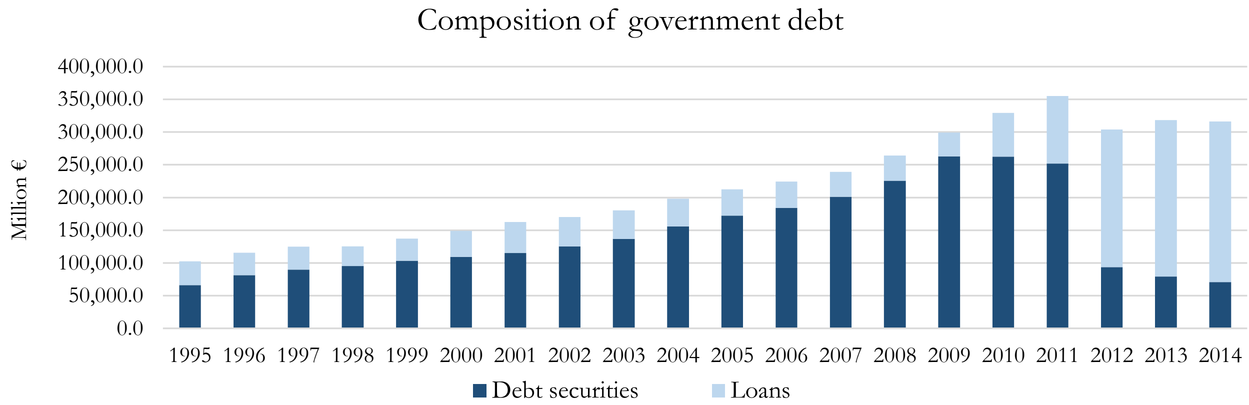
Year	Reference nominal GDP (€ billion)	Reference real GDP growth rate
2014	210.1014	2.345%
2015	217.9036	2.896%
2016	226.3532	2.845%
2017	235.7155	2.797%
2018	245.4696	2.597%
2019	255.8822	2.497%
2020	266.4703	2.247%
2021–2042	266.4703	2.000%

Date	Principal	Principal (% of original)
Up to 15-Oct-23	315	100.00%
15-Oct-24	300	95.24%
15-Oct-25	285	90.48%
15-Oct-26	270	85.71%
15-Oct-27	255	80.95%
15-Oct-28	240	76.19%
15-Oct-29	224	71.11%
15-Oct-30	208	66.03%
15-Oct-31	192	60.95%
15-Oct-32	176	55.87%
15-Oct-33	160	50.79%
15-Oct-34	144	45.71%
15-Oct-35	128	40.63%
15-Oct-36	112	35.56%
15-Oct-37	96	30.48%
15-Oct-38	80	25.40%
15-Oct-39	64	20.32%
15-Oct-40	48	15.24%
15-Oct-41	32	10.16%
15-Oct-42	16	5.08%

Source: Zettelmeyer, Trebesch and Gulati (2013)

3 Greek debt composition

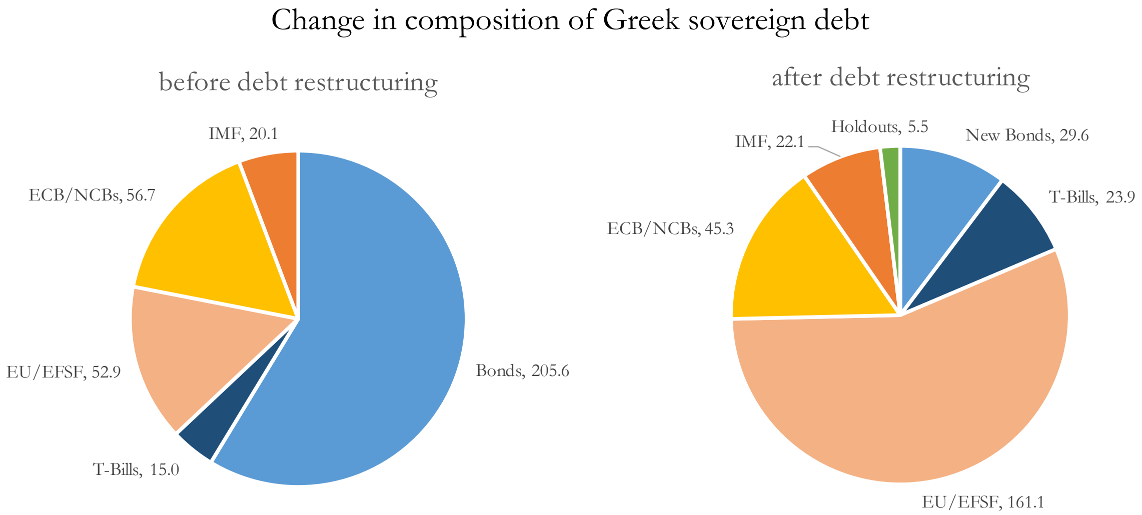
Figure A1



Source: Eurostat (2015a; 2015c) and own calculations

Figure A1 shows the evolution of Greek government debt over time. Before the onset of the crisis, the majority of government debt consisted of long-term government bonds. However, there is a clear change in the composition from mainly government debt securities to long-term loans over the time span. Whereas the amount of government debt securities declined significantly during the debt crisis years from 2009 onwards because Greece was losing access to capital market financing due to rising bond yields, the amount of government debt consisting of long-term loans increased due to the official sector bailout programmes for Greece. The debt restructuring in 2012 also led to a temporary decline in government debt and especially to a significant fall in the amount of outstanding government bonds. The changing composition of Greece's public debt, from private sector investors to mainly public sector creditors as a result of the official sector bailout programmes as well as the debt restructuring in 2012, can also be seen in Figure A2.

Figure A2

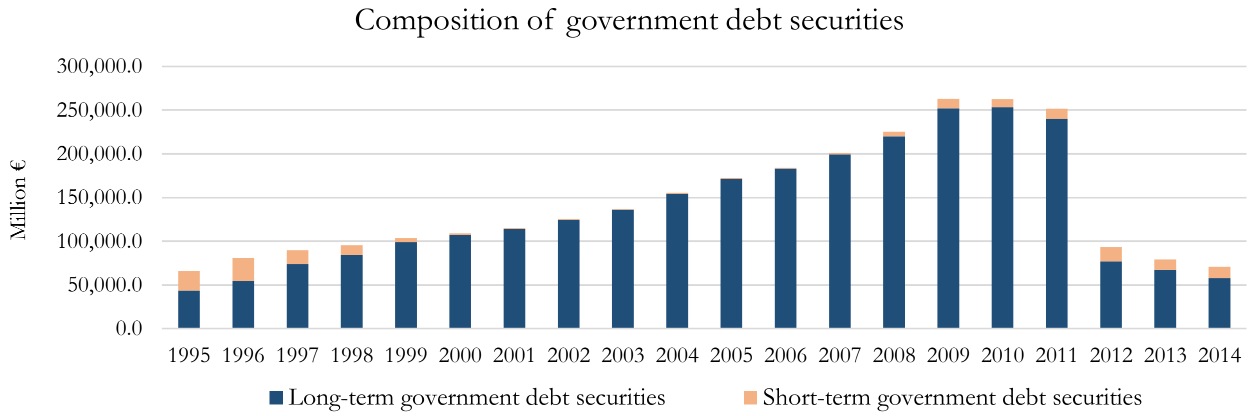


Source: Zettelmeyer, Trebesch and Gulati (2013)

Note: The Figure shows Greek government and government-guaranteed debt owed to private creditors (blue, bonds and T-bills only as well as guaranteed debt issued by banks) and official creditors (orange) in € billion as of end-December 2012. ECB/NCB debt refers to ECB SMP holdings as well as holdings by national central banks in the Eurozone. EU/EFSF loans include the bilateral GLF loans as well as the EFSF loans. T-bills are privately held short-term debt instruments.

Furthermore, there was also a shift in the maturity profile of Greek government debt securities as a result of the crisis from long-term to short-term debt. Whereas in 2008 only 2.4% of outstanding government bonds were of a short maturity, this increased to 18.6% in 2014. As Greece was facing unsustainably high interest rates on long-term debt securities in financial markets during the crisis, the country's debt issuance consequently shifted to a higher portion of short-term bonds.

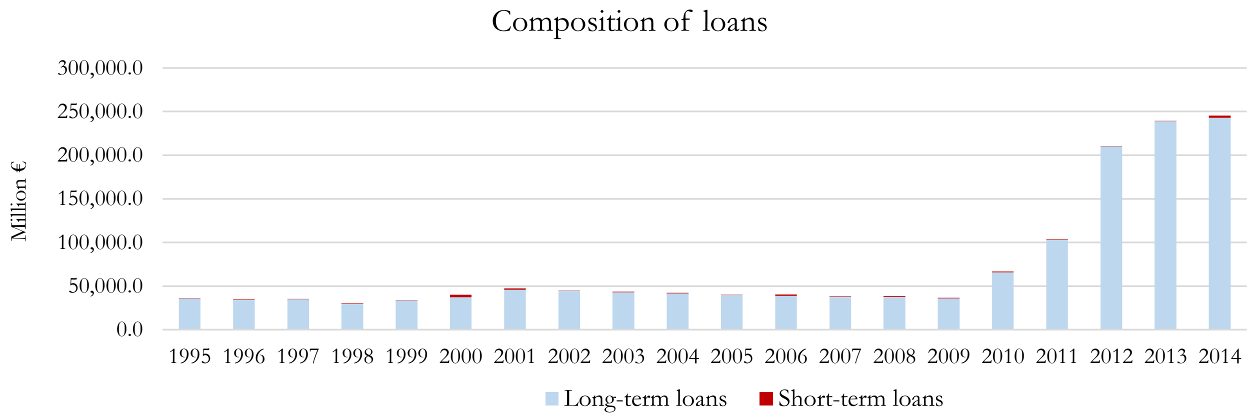
Figure A3



Source: Eurostat (2015a) and own calculations

Figure A4 shows that the majority of loans provided by the EU and IMF through the Greek bailout programmes are long-term loans.

Figure A4



Source: Eurostat (2015c) and own calculations

4 Tables and figures

Table A2

Unit / Year	Government interest payments (on GDP-linked bonds)	Government interest payments (on conventional bonds)	Total interest payments (GDP & conventional bonds)	Interest bill savings	Interest bill savings
	Million €	Million €	Million €	Million €	% of GDP
1996	2,277.7	2,136.7	4,414.5	-141.0	-0.1%
1997	3,235.8	1,192.0	4,427.8	-2,043.8	-1.2%
1998	3,147.8	869.7	4,017.5	-2,278.1	-1.3%
1999	3,009.9	2,372.9	5,382.9	-637.0	-0.3%
2000	3,607.1	5,354.4	8,961.5	1,747.3	0.9%
2001	3,922.9	3,694.3	7,617.3	-228.6	-0.1%
2002	4,143.3	1,156.6	5,299.9	-2,986.7	-1.5%
2003	5,826.6	1,740.1	7,566.7	-4,086.5	-1.9%
2004	6,083.7	5,105.4	11,189.1	-978.3	-0.4%
2005	2,857.8	4,371.5	7,229.3	1,513.7	0.7%
2006	7,747.8	1,973.5	9,721.3	-5,774.2	-2.4%
2007	6,047.7	2,908.7	8,956.4	-3,138.9	-1.3%
2008	2,721.6	6,495.5	9,217.1	3,773.9	1.5%
2009	656.9	15,003.9	15,660.8	14,346.9	6.0%
2010	656.4	12,929.8	13,586.2	12,273.4	5.4%
2011	629.2	10,730.0	11,359.1	10,100.8	4.9%
2012	233.6	3,983.8	4,217.5	3,750.2	2.0%
2013	198.0	3,108.0	3,306.0	2,910.0	1.6%
2014	1,209.5	568.2	1,777.6	-641.3	-0.3%

Source: Eurostat (2015a; 2015b; 2016a; 2016b) and own calculations

Note: I assume 50% of government securities are GDP indexed and 50% are normal bonds without any indexation. For comparison I assume that there was no debt restructuring in 2012 and hence use the same interest rate as in 2011 for my calculations.

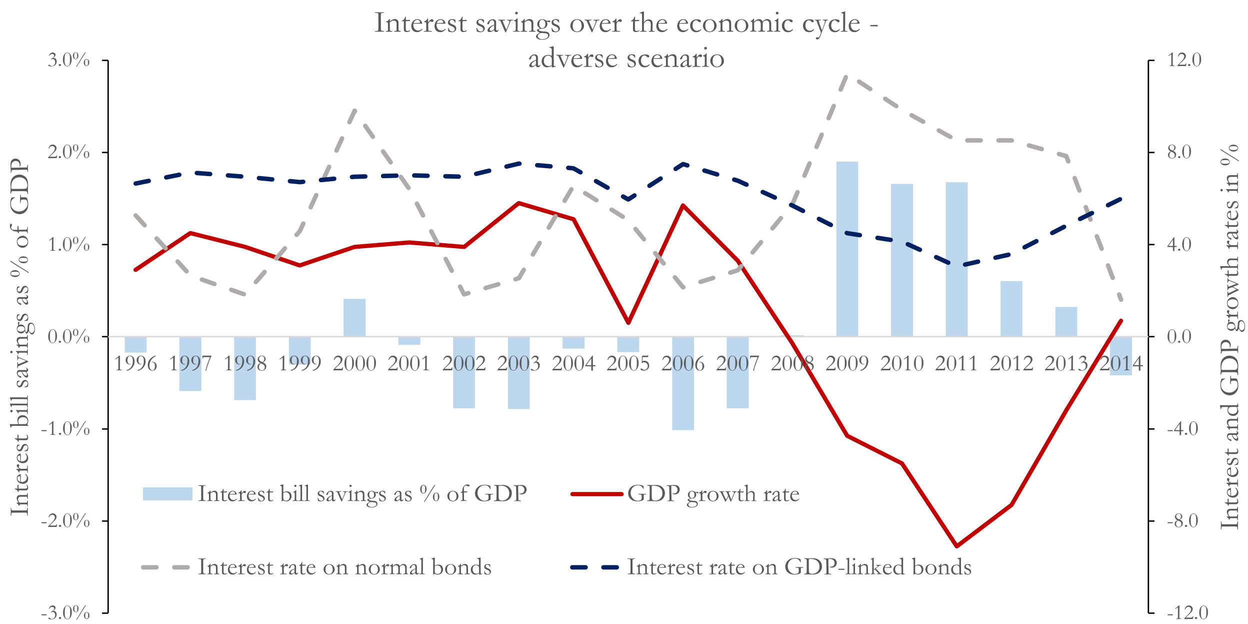
Table A3

Unit / Year	Government interest payments (on GDP-linked bonds)	Government interest payments (on conventional bonds)	Total interest payments (GDP & conventional bonds)	Interest bill savings	Interest bill savings
	Million €	Million €	Million €	Million €	% of GDP
1996	1,138.9	3,205.1	4,344.0	-70.5	0.0%
1997	1,617.9	1,788.0	3,405.9	-1,021.9	-0.6%
1998	1,573.9	1,304.6	2,878.4	-1,139.0	-0.6%
1999	1,505.0	3,559.4	5,064.4	-318.5	-0.2%
2000	1,803.5	8,031.5	9,835.1	873.6	0.5%
2001	1,961.5	5,541.5	7,503.0	-114.3	-0.1%
2002	2,071.6	1,734.8	3,806.5	-1,493.4	-0.7%
2003	2,913.3	2,610.1	5,523.4	-2,043.3	-0.9%
2004	3,041.9	7,658.1	10,700.0	-489.1	-0.2%
2005	1,428.9	6,557.3	7,986.2	756.9	0.3%
2006	3,873.9	2,960.3	6,834.2	-2,887.1	-1.2%
2007	3,023.8	4,363.1	7,386.9	-1,569.5	-0.6%
2008	1,360.8	9,743.3	11,104.0	1,887.0	0.8%
2009	328.5	22,505.8	22,834.3	7,173.5	3.0%
2010	328.2	19,394.7	19,722.9	6,136.7	2.7%
2011	314.6	16,094.9	16,409.5	5,050.4	2.5%
2012	116.8	5,975.8	6,092.6	1,875.1	1.0%
2013	99.0	4,662.0	4,761.0	1,455.0	0.8%
2014	604.7	852.2	1,457.0	-320.7	-0.2%

Source: Eurostat (2015a; 2015b; 2016a; 2016b) and own calculations

Note: I assume 25% of government securities are GDP indexed and 75% are normal bonds without any indexation. For comparison I assume that there was no debt restructuring in 2012 and hence use the same interest rate as in 2011 for my calculations.

Figure A5



Source: Eurostat (2015a; 2015b; 2016a; 2016b) and own calculations
Note: 75% of debt is normal bonds, 25% GDP-linked bonds

Table A4

Year	Primary balance % of GDP with indexation	Primary balance % of GDP actual data
1996	0.4	4.1
1997	4.6	3.6
1998	4.8	4.2
1999	1.2	4.2
2000	-3.1	3.5
2001	0.4	1.9
2002	5.1	0.7
2003	6.2	-0.7
2004	1.2	-2.5
2005	-1.8	-0.9
2006	6.7	-1.6
2007	3.2	-2.2
2008	-3.5	-5.0
2009	-12.0	-10.3
2010	-11.9	-5.3
2011	-11.8	-3.0
2012	-	-1.4
2013	-11.7	1.0
2014	3.2	0.0

Source: Eurostat (2015b; 2016a), IMF (2015b) and own calculations

Note: I exclude the year 2012 due to the Greek debt structuring.

Table A5

Correlation of primary balance and GDP growth rate

α	0.2	0.5	0.7	1
without indexation	0.55	0.55	0.55	0.55
with indexation	0.80	0.88	0.89	0.90

$\mathbf{r^{min}}$	0	0.5	1	1.5
without indexation	0.55	0.55	0.55	0.55
with indexation	0.91	0.90	0.90	0.89

$\mathbf{r^{max}}$	6	10	12	14
without indexation	0.55	0.55	0.55	0.55
with indexation	0.88	0.90	0.90	0.90

Ω	0	0.5	1	2.5
without indexation	0.55	0.55	0.55	0.55
with indexation	0.90	0.90	0.90	0.91

Source: Eurostat (2015b; 2016a), IMF (2015b) and own calculations
Note: time period 1996-2014. I exclude the year 2012 due to the Greek debt structuring.

5 Model derivation

5.1 Sovereign debt dynamics

$$d_{t+1} - d_t = \frac{(r_t - g_{t+1})d_t}{(1+g_{t+1})} - s_{t+1} + \varepsilon_{t+1} \tag{1.5}$$

$$(1+r_t) = (1+r^*)\frac{(1-p_{t+1}\theta)}{(1-p_{t+1})} \tag{1.6}$$

$$g_t = g^* + u_t \tag{1.7}$$

$$s_{t+1} = \min(\alpha + \beta(r_t - g^*)d_t, \gamma) \tag{1.8}$$

First plugging in (1.7) and (1.8) into (1.5)

$$d_{t+1} - d_t = \frac{(r_t - g^* - u_{t+1})d_t}{(1+g^* + u_{t+1})} - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1}$$

Then using (1.6)

$$d_{t+1} - d_t = \frac{\left(\left[(1+r^*)\frac{(1-p_{t+1}\theta)}{1-p_{t+1}} - 1\right] - g^* - u_{t+1}\right)d_t}{(1+g^* + u_{t+1})} - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1}$$

Rearranging

$$d_{t+1} - d_t = \frac{\left(\frac{(1+r^*)(1-p_{t+1}\theta) - (1-p_{t+1}) - g^*(1-p_{t+1}) - u_{t+1}(1-p_{t+1})}{1-p_{t+1}}\right)d_t}{(1+g^* + u_{t+1})} - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \frac{\left(\frac{1 + r^* - p_{t+1}\theta - r^* p_{t+1}\theta - 1 + p_{t+1} - g^* + g^* p_{t+1} - u_{t+1} + u_{t+1}p_{t+1}}{1-p_{t+1}}\right)d_t}{(1+g^* + u_{t+1})} - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \left(\frac{(r^* - g^* - u_{t+1}) - p_{t+1}(r^* - g^* - u_{t+1}) - p_{t+1}\theta + p_{t+1} - p_{t+1}r^*\theta + p_{t+1}r^*}{(1-p_{t+1})(1+g^* + u_{t+1})}\right)d_t - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \left(\frac{(1-p_{t+1})(r^* - g^* - u_{t+1}) - p_{t+1}(\theta - 1 + r^*\theta - r^*)}{(1-p_{t+1})(1+g^* + u_{t+1})}\right)d_t - \min(\alpha + \beta(r_t - g^*)d_t, \gamma) + \varepsilon_{t+1}$$

Simplifying and rearranging

$$d_{t+1} - d_t = \frac{(r^* - g^* - u_{t+1})}{(1 + g^* + u_{t+1})} d_t - \left(\frac{p_{t+1}(\theta - 1 + r^* \theta - r^*)}{(1 - p_{t+1})(1 + g^* + u_{t+1})} \right) d_t - \min(\alpha + \beta(r_t - g^*) d_t, \gamma) + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \frac{(r^* - g^* - u_{t+1})}{(1 + g^* + u_{t+1})} d_t - \left(\frac{p_{t+1}(1 + r^*)(\theta - 1)}{(1 - p_{t+1})(1 + g^* + u_{t+1})} \right) d_t - \min(\alpha + \beta(r_t - g^*) d_t, \gamma) + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \frac{(r^* - g^* - u_{t+1})}{(1 + g^* + u_{t+1})} d_t + \left(\frac{p_{t+1}(1 + r^*)(1 - \theta)}{(1 - p_{t+1})(1 + g^* + u_{t+1})} \right) d_t - \min(\alpha + \beta(r_t - g^*) d_t, \gamma) + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \frac{(r^* - g^* - u_{t+1})}{(1 + g^* + u_{t+1})} d_t + (1 - \theta)(1 + r^*) d_t \frac{1}{(1 + g^* + u_{t+1})} \frac{p_{t+1}}{(1 - p_{t+1})} - \min(\alpha + \beta(r_t - g^*) d_t, \gamma) + \varepsilon_{t+1}$$

5.2 Steady state debt level for conventional bonds

In steady state we know that $d_{t+1} - d_t = 0$, therefore

$$0 = d_{t+1} - d_t = \frac{(r_t - g^* - u_{t+1}) d_t}{1 + g^* + u_{t+1}} - s_{t+1} + \varepsilon_{t+1}$$

Rearranging and solving for d^*

$$\frac{(r - g^* - u) d^*}{1 + g^* + u} + \varepsilon = s$$

Using equation (1.8)

$$s_{t+1} = \min(\alpha + \beta(r_t - g^*) d_t, \gamma)$$

and plugging in

$$\frac{(r - g^* - u) d^*}{1 + g^* + u} + \varepsilon = \alpha + \beta(r - g^*) d^*$$

$$\frac{[(r - g^* - u) - \beta(r - g^*)(1 + g^* + u)] d^*}{1 + g^* + u} = \alpha - \varepsilon$$

$$[(r - g^* - u) - \beta(r - g^*)(1 + g^* + u)] d^* = (\alpha - \varepsilon)(1 + g^* + u)$$

$$d^* = \frac{(\alpha - \varepsilon)(1 + g^* + u)}{[(r - g^* - u) - \beta(r - g^*)(1 + g^* + u)]}$$

5.3 Steady state debt level for GDP-linked bonds

In steady state $d_{t+1}^{GDP} - d_t^{GDP} = 0$, therefore

$$0 = d_{t+1}^{GDP} - d_t^{GDP} = \frac{(r_t^{GDP} - g^*)d_t^{GDP}}{1+g^*} - s_{t+1} + \varepsilon_{t+1}$$

$$\frac{(r^{GDP} - g^*)d^{*GDP}}{1+g^*} + \varepsilon = s$$

$$\frac{(r^{GDP} - g^*)d^{*GDP}}{1+g^*} + \varepsilon = \alpha + \beta(r^{GDP} - g^*)d^{*GDP}$$

$$\frac{[(r^{GDP} - g^*) - \beta(r^{GDP} - g^*)(1+g^*)]d^{*GDP}}{1+g^*} = \alpha - \varepsilon$$

$$d^{*GDP} = \frac{(\alpha - \varepsilon)(1+g^*)}{[(r^{GDP} - g^*) - \beta(r^{GDP} - g^*)(1+g^*)]}$$

5.4 Risk-averse investors

Utility function is given by

$$U = \begin{cases} \dfrac{V^{1-\tau} - 1}{1-\tau} & \text{for } \tau > 0, \tau \neq 1 \end{cases}$$

and

$$U = \ln V \qquad \text{for } \tau = 1$$

Applying this to no arbitrage equilibrium condition (1.6)

$$(1 - p_{t+1})(1 + r_t) + p_{t+1}\theta(1 + r^*) = (1 + r^*)$$

yields

$$U[(1+r^*)] = (1-p_{t+1})U[(1+r_t)] + p_{t+1}U[\theta(1+r^*)]$$

$$\frac{(1+r^*)^{1-\tau} - 1}{1-\tau} = (1-p_{t+1})\left[\frac{(1+r_t)^{1-\tau} - 1}{1-\tau}\right] + p_{t+1}\left[\frac{(\theta(1+r^*))^{1-\tau} - 1}{1-\tau}\right]$$

$$(1+r^*)^{1-\tau} - 1 = (1-p_{t+1})\left[(1+r_t)^{1-\tau} - 1\right] + p_{t+1}\left[(\theta(1+r^*))^{1-\tau} - 1\right]$$

$$(1+r^*)^{1-\tau} = (1-p_{t+1})\left[(1+r_t)^{1-\tau} - 1\right] + p_{t+1}\left[(\theta(1+r^*))^{1-\tau} - 1\right] + 1$$

$$(1+r^*)^{1-\tau} = (1-p_{t+1})(1+r_t)^{1-\tau} - (1-p_{t+1}) + p_{t+1}(\theta(1+r^*))^{1-\tau} + 1 - p_{t+1}$$

$$(1+r^*)^{1-\tau} = (1-p_{t+1})(1+r_t)^{1-\tau} + p_{t+1}\theta^{1-\tau}(1+r^*)^{1-\tau}$$

Rearranging yields

$$(1+r_t) = (1+r^*)\left[\frac{(1-p_{t+1}\theta^{1-\tau})}{(1-p_{t+1})}\right]^{\frac{1}{1-\tau}}$$

5.5 Inflation targets

Introducing inflation into the sovereign debt dynamics

$$d_{t+1} = \frac{(1+r_t)d_t}{(1+g_{t+1})(1+\pi)} - s_{t+1} + \varepsilon_{t+1}$$

Rearranging

$$d_{t+1} - d_t = \frac{(1+r_t)d_t - (1+g_{t+1})(1+\pi)d_t}{(1+g_{t+1})(1+\pi)} - s_{t+1} + \varepsilon_{t+1}$$

$$d_{t+1} - d_t = \frac{(r_t - g_{t+1} - \pi - g_{t+1}\pi)}{(1+g_{t+1})(1+\pi)} d_t - s_{t+1} + \varepsilon_{t+1}$$

Deriving the steady state debt level

Steady state debt level d* for normal bonds

In steady state $d_{t+1} - d_t = 0$, therefore

$$d_{t+1} - d_t = 0 = \frac{(r_t - g_{t+1} - \pi - g_{t+1}\pi)}{(1+g_{t+1})(1+\pi)} d_t - s_{t+1} + \varepsilon_{t+1}$$

$$\frac{(r - g^* - u - \pi - (g^* + u)\pi)d^*}{(1+g^* + u)(1+\pi)} + \varepsilon = s$$

$$\frac{(r - g^* - u - \pi - (g^* + u)\pi)d^*}{(1+g^* + u)(1+\pi)} + \varepsilon = \alpha + \beta(r - g^*)d^*$$

$$\frac{[(r - g^* - u - \pi - (g^* + u)\pi) - \beta(r - g^*)(1+g^* + u)(1+\pi)]d^*}{(1+g^* + u)(1+\pi)} = \alpha - \varepsilon$$

$$[(r - g^* - u - \pi - (g^* + u)\pi) - \beta(r - g^*)(1+g^* + u)(1+\pi)]d^* = (\alpha - \varepsilon)(1+g^* + u)(1+\pi)$$

$$d^* = \frac{(\alpha - \varepsilon)(1+g^* + u)(1+\pi)}{[(r - g^* - u - \pi - (g^* + u)\pi) - \beta(r - g^*)(1+g^* + u)(1+\pi)]}$$

Steady state debt level d^{*GDP} for GDP-linked bonds

$$d_{t+1}^{GDP} - d_t^{GDP} = \frac{(r_t^{GDP} - g^* - \pi - g^*\pi)}{(1+g^*)(1+\pi)} d_t^{GDP} - s_{t+1} + \varepsilon_{t+1}$$

In steady state $d_{t+1}^{GDP} - d_t^{GDP} = 0$, therefore

$$\frac{(r^{GDP} - g^* - \pi - g^*\pi)d^{*GDP}}{(1+g^*)(1+\pi)} + \varepsilon = s$$

$$\frac{(r^{GDP} - g^* - \pi - g^*\pi)d^{*GDP}}{(1+g^*)(1+\pi)} + \varepsilon = \alpha + \beta(r^{GDP} - g^*)d^{*GDP}$$

$$\frac{[(r^{GDP} - g^* - \pi - g^* \pi) - \beta(r^{GDP} - g^*)(1+g^*)(1+\pi)]d^{*GDP}}{(1+g^*)(1+\pi)} = \alpha - \varepsilon$$

$$[(r^{GDP} - g^* - \pi - g^* \pi) - \beta(r^{GDP} - g^*)(1+g^*)(1+\pi)]d^{*GDP} = (\alpha - \varepsilon)(1+g^*)(1+\pi)$$

$$d^{*GDP} = \frac{(\alpha - \varepsilon)(1+g^*)(1+\pi)}{[(r^{GDP} - g^* - \pi - g^* \pi) - \beta(r^{GDP} - g^*)(1+g^*)(1+\pi)]}$$