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THE PROBLEM OF STABILITY AND THE BALANCE OF POWER – MARKETS AND DEMOCRACY WITHOUT EXTERNAL ENFORCEMENT FOR THREE PLAYERS

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Abstract. We analyze a non-cooperative game for three players intended to represent the stability of political systems. We compare a market and a democratic system to see how they modify an anarchic outcome in terms of stability. The market process is represented by a sequential bargaining game and the democratic process is represented by a voting game where players elect a leader that selects the allocation of resources. We find that under our assumptions markets and democracy have the same implications for stability when the two systems can be compared with each other.

Keywords: Anarchy, State of Nature, Markets, Democracy, Bargaining

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1 Introduction

1.1 The law—spontaneous or top-down

The democratic state is in many ways a natural way to think about the balance of power in society. The state regulates the population through laws and the population regulates the state through voting, which is part of some constitution that sets the basic rules of the game. However, scholars have studied some examples where law, in contrast to that of the top-down model through which we traditionally view it, have been left up to a more market-like order without intervention from the state.

For instance, medieval Iceland had for several hundred years a system where law resembled a commodity sold in a marketplace, and enforcement of the law was free from government intervention. The right to extract fines from criminals were given to the victim (or the survivors of the victim in the case of murder). People were free to move between different arbitrators of conflicts freely in contrast to having to use a monopolized court system for settling disputes (Friedman 1979a).

Several industries in the American West, including mining, created their own rules to define property rights, and used private enforcement to settle disputes. Only later did the rules developed become part of public law (Anderson and Hill 2004, Smith 2005, Umbeck 1977).

There are examples of stock markets that have evolved as what Stringham (2002) describes as "self-policing clubs." The London stock exchange developed in coffeehouses where rules were developed independently of the government legal system. The world's oldest stock exchange, in Amsterdam, similarly managed to develop financial instruments such as futures contracts and options even when they were not enforceable by government courts (Stringham 2003).[1]

Those few examples by no means show that law that develops in a more market-like process must be more efficient, equitable or feasible than some other alternative that could have been implemented instead. They are, however, interesting in the sense that they provide examples that, at least under some circumstances, show that the traditional notion of law as a top-down system need not be as obvious as initially thought.

[1]For further examples of similar developments, see, e.g., Smith 2005, Ostrom 1990.

1.2 The problem of enforcement

External enforcement, whether it comes from the state or some private actor, is often crucial for cooperation. If players do not face any punishment for defecting there are many circumstances where it is profitable, creating a destructive outcome where players benefit at the expense of one another, often creating a socially disadvantageous outcome. It is common to show how to change this outcome to a cooperative one simply by letting an outside enforcer add costs to defecting for the players large enough that they have an incentive to cooperate, from which we easily show that this improves the likelihood if achieving maximum social stability and utility (see, e.g., Shichijo 2012 for an analysis of a two-player prisoner's dilemma case).

As long as we assume that the enforcer has the incentive to enforce the peaceful outcome, we face few problems in this analysis. This changes drastically, however, if we consider the enforcer as a player with his own motives. If the enforcer is not benevolent, we must also have a feasible reason to believe that he will enforce the cooperation equilibrium, or we simply face a similar problem except for three players instead of two. If the enforcer can defect just like the two players initially involved in the game, we need to find a feasible reason for why he should not defect either.

1.3 The societal analogy

On a societal level, this problem has its analogy to the justice system. In most modern and many not so modern societies, the state works as the enforcer of many conflicts, both in the private sphere and those involving itself. But the state can also be viewed as just another player in the game of anarchy, except that it is a player that has different properties than the other players due to its special stance as an institution, or organization, with the most power, just like the enforcer in the two-player case. But if we ignore the mental models giving the state this special standing, the state can be viewed simply as another player with his own reasons to defect under certain circumstances.[2]

To get a complete picture, in other words, we must not only ask how the enforcer regulates the population but also how the population regulates the enforcer in order to find a reasonable model of cooperation and defection in society. Many economic models assume that markets are deterministic

[2]The social contract theory, stating that the state is an outgrowth of a need to provide e.g. public goods to solve the free rider problem, is frequently used in economic models (see, e.g., Buchanan 1975), which is a useful way of thinking about the state from an economic perspective, though social contract theory is defined very differently in the philosophical literature (see, e.g., Rawls 1971, Rousseau 2004, Huemer 2013 for various definitions). We restrict ourselves here, however, to the state simply viewed as an actor in the problem of enforcement.

while the government has "free will" to work in a variety of ways, with frictionless internal dynamics such that it can set the optimal policy. This is, however, a biased analysis as it makes stronger assumptions for one system that it does for the other.[3] In order to make a relevant analysis, we need to make no stronger assumptions for one system than the other.

1.4 Relevant comparisons

As we can easily show, markets are often dependent on enforcement from a higher force to not deteriorate into defective strategies, or even war. However, the same problem is not so commonly applied to the democratic process (see Caplan 2008 for an extension of this view). If we are not allowed to assume a higher enforcer for the market mechanism when we consider how feasible it is in regulating itself, then we should not reasonably be allowed to use a higher enforcer for a democratic state, or ignore the democratic process involved in creating policy, if we want to make a comparison of the systems. The reason why we consider democracy in particular is that the most relevant trade-off that modern societies face is that between markets and democratic rule, dictatorship is not a relevant alternative in modern society.[4]

This also applies to the problem of cooperation and defection when players want to benefit by using force against others. Therefore, we need to have a model that allows for the same generality in the assumptions, not using stronger assumptions for one than the other if we want to compare markets and democracy as general principles against each other in terms of creating stability. While markets do not always regulate themselves without an external enforcer, does democracy do so if we make equally strong assumptions?

It is not relevant to compare various systems by comparing their probability of arising from some other circumstances, but rather their ability to sustain themselves once established. States arise by definition from anarchy, but that does not mean that we have to analyze the stability of various systems of government based on their probability of arising out of anarchy. In order to make a relevant comparison, we need to assume that we start off with the given system already implemented, and then see whether the system will sustain peace or whether it will deteriorate into

[3] This is the standard public choice argument. See, e.g., Buchanan and Tullock 1962, p. 14, Downs 1957b. Here, the former argue that those assumptions the feasibility of government solutions might be overstated in comparison to their market alternative. However, the latter argues that treating government as this exogenous variable is a consequence of the premise that markets are self-regulating, opposed to the premise that governments are self-regulating, in many ways reaching the opposite conclusion.

[4] As Caplan 2008 (p.3) states: "In democracies the main alternative to majority rule is not dictatorship, but markets."

a defective outcome. Therefore we start with an analysis of anarchy and then allow for two different systems to operate before the game of anarchy starts. We model a democratic and a market system such that we can and consider the implications. Most importantly, we keep a number of variables free, and we assume that they vary equally with markets and democracy, not allowing one system to have a greater degree of freedom or stability between players than the other.

1.5 A comment on terminology

We choose to call anarchy the state of affairs in which there is no room for initial cooperation before the players can fight each other for resources. This is because it is common to call anarchy the state of affairs in which there are no institutions in place[5] building on the arguments in Hobbes 2012. However, Encyclopedia Britannica (2016) merely defines anarchism as a philosophy that argues that government is both "harmful and unnecessary," as opposed to the common notion that anarchy is synonymous to chaos or a state of nature.

Proponents in the political sense whether of the more collectivist types (see, e.g. Chomsky 2013, Graeber 2004) or individualist (e.g. Friedman 1979a, Spooner 1882) tend to have a definition of anarchism that allows for institutions to prevent this state of nature, (whether those are feasible in practice is another discussion however), so there is a double meaning to the term that we need to be aware of. The model in this paper might be called anarchic in the sense that we initially analyze a situation where fighting has no costs beyond those given by the natural state of things. But it might also be called anarchic in the sense that there is no state with a monopoly on enforcement or the use of force to settle disputes, which corresponds more closely to the normative definition.

For simplicity, however, we call anarchy the state of nature in the model and then move on to analyze markets and democracy. Even though they might be called "anarchic markets" or "anarchic democracy" as there is no state in the traditional sense in neither of the parts of our model, we stick to our chosen definition for simplicity, to allow the concept of anarchy to contrast with markets and democracy. Those definitions are, however, purely semantic.

1.6 Notes on simplification

Like any other economic model, we are forced to consider a simplified model of society in order to reach clear conclusions relevant to the question at hand. Most notably, we restrict ourselves to a

[5]For instance, the terminology of "The Price of Anarchy" which measures the cost of individual selfishness as opposed to cooperative behavior (see, e.g., Roughgarten 2005).

model of three players for the simple reason that a two-player society has no clear interpretation in the question we seek to address, and four players or above provide solutions that are too many and too abstract to be of much use. The three-player model can be viewed as an extension but where we allow the enforce to be an arbitrary player. We treat the "third part" between two arbitrary players like any other player, with his own interests and not having a special position in society. The other assumptions, while they should be somewhat uncontroversial in comparison to those made in other economic models, are discussed in more extensive detail after the results are presented so that we can make a more extensive analysis of the potential problems they bring with them.

1.7 Political philosophy and the comparison

We also need to be careful in pointing out that we do not seek to make a normative analysis of neither markets or democracy, even though we are inspired by a problem in political philosophy that is inherently normative, just like all selection of research topics are inspired by what humans consider to be important. The main question of the thesis has been inspired by some of the arguments in Caplan 2008. Although we do not necessarily share his positive or normative conclusions, the analogy posed with the extensive comparison between the market and democratic mechanism is the reason that we have considered the question at hand.[6]

1.8 Summary: Research Question

Those arguments sum up to the general research question: *Holding all other factors constant, which system out of markets and democracy is the most self-regulatory in terms of sustainability of the legal system?*

Of course, we can only get a partial insight into the question through our chosen method. We therefore specify the question more closely to: *In a model replicating a natural experiment, where markets and democracy are allowed to regulate an anarchic outcome, how will those systems change the stability of the subsequent anarchic game?* Obviously, we need to specify the model more carefully before this very specified version of the question can be answered. This will be clear once once the model is explained, and we then apply it to the broader research question.

[6]See also Tullock (2006, p. 48) for a similar argument that has been of some inspiration.

2 A Model of Anarchy

2.1 Preliminaries

We first model anarchy before we move on to the democratic and market cases. Consider a game with three players: Good, Bad and Ugly, where we use subscripts G, B and U, for them, respectively, and i, j and k, $i \neq j \neq k \neq i$ when we refer to arbitrary players. Each player i seeks to maximize the resources x_i^* that he has at the end of the game. The final resource distribution is given by the vector $\overline{x} = (x_G^*, x_B^*, x_U^*)$ and the initial resources are $\overline{x_0} = (x_G, x_B, x_U)$. We assume that resources can be split up in a continuous fashion and that all players start with resources $x_i > 0$. All players have all relevant information and behave rationally.

The game is sequential, where Good moves before Bad who moves before Ugly. A player, when it is his turn, can choose to fight, targeting one of the players of his choice, or pass, doing nothing. Define the fighting function of player i with the resource endowment x_i as $f_i(x_i) > 0$, where we call fighting power the value of f_i for a fixed value of x_i. We assume that the function is continuous and differentiable such that $\frac{\partial f_i}{\partial x_i} > 0$, which yields the intuition that power increases with the amount of resources that a player has, other things equal (though it might be trivially low in some cases). We do not assume, however, any particular properties of $\frac{\partial^2 f_i}{\partial x_i^2}$ as we can think of circumstances where it seems reasonable to assume both convexity and concavity, so our results are general with economies and diseconomies of scale.

All fights are won by the side with the highest fighting power. The winning player takes all of the loser's resources. A player that loses his resources is taken out of the game and do not get another chance to play. For convenience, we assume that if $f_i(x_i) = f_j(x_j)$, the fight ends in a draw and all resources are destroyed for both players.

So the strategy set for player i during any given turn where he remains in the game is

$S_i = \{\text{Pass, Attack } j\text{, Attack } k\} \backslash J$ where $J \subset \{\text{Attack i, Attack j}\}$, which is dependent on which of the players that remain in the game, as only reamining players can be attacked. The turns continue either until all remaining players have passed or there is only one player left in the game. When one of those conditions are fulfilled, the game stops.

Fighting is costly, however, reflected in the assumption that part of resources are destroyed from fighting, making the winning players always end up with less than the sum of his own resources and the loser's resources. After a fight where i beats j the resources that remain are

$g_i(x_i, x_j) = g_i^*(f_i(x_i), f_j(x_j)) \in]0, x_i + x_j[$ which therefore also depends on $f_i(x)$ and $f_j(x)$ such that a greater difference between them increases the retained resources. In other words, even if a player wins the fight, the destruction of resources might make him end up with fewer resources than before. It must hold that $g_i(x_i, x_j) > x_i$ or he has less resources after the fight than he has before it.

g_i is also continuous and differentiable when $g_i(x_i, x_j) > 0$. It has the property $\frac{\partial g_i}{\partial x_i} > 0$ yielding the intuition that a player with more resources, other things equal, will gain more from fighting an equivalent player, since he is stronger and wins the fight with a greater margin. We assume that $\frac{\partial g_i}{\partial x_j}$ is neither positive or negative definite, since we can think of circumstances where both orientations seem reasonable. On one hand, a player with more resources, if fighting power is constant or only increased slightly, is more profitable to attack, but the increase in fighting power as a consequence of the resource increase might make him less profitable to attack. However, we assume that $\frac{\partial g_i}{\partial x_j} < 1$ since it is unreasonable that it should change by more than the increased value of x_j.[7] We also assume that $\lim_{f_i(x_i) \to f_j(x_j)} g_i(x_i, x_j) = 0$.

We assume that Good has the highest fighting power at the start of the game. Bad has the second highest, and Ugly has the lowest, given the initial resource allocation, such that

$f_G(x_G) > f_B(x_B) > f_U(x_U)$.[8] Note that this does not necessarily imply that $x_G > x_B > x_U$ or that $f_G(x_i) > f_B(x_j) > f_U(x_k)$ for all x_i, x_j, x_k.

Our model also captures the intuition that an unequal distribution of power (which might be facilitated through an unequal distribution of wealth) might have a snowball effect where the inequality of resources and power is amplified if it is too dispersed initially.

[7] One example of g_i that we consider in the Appendix is, where i is the stronger or equally strong player to j,

$$g_i(x_i, x_j) = \begin{cases} \max\{x_j - \frac{1}{f_i(x_i) - f_j(x_j)}, 0\} \text{ if } f_i(x_i) > f_j(x_j) \\ 0 \text{ if } f_i(x_i) = f_j(x_j) \end{cases}$$

which satisfies both conditions of derivatives that we have specified above without losing generality, though it is perhaps rather conservative in predicting fights.

[8] Ideally, we would like to keep the move order arbitrary, but this proves to produce too many solutions to be useful. So we make the assumption that the player who is the strongest at the beginning of the game also moves first, which seems reasonable as we do not seek to artificially equalize the power between the players.

2.2 Bargaining before anarchy starts

We will later assume that before this game starts, we add the assumption that the players are allowed to bargain within some game that we have set up, which we seek to design so that it resembles a market and a democratic system. For our initial analysis of anarchy in the next chapter, however, this stage of the game is omitted.

2.3 Method for solving the game

Since players are rational, they know which strategy will be played by the player to come, and can predict what their outcome will be at the end of the game when the other players also behave rationally. We assume for simplicity that if a player will end up losing all his resources no matter which strategy he plays, he passes during his turn. Note that this implies that he always passes during fights that will end in a draw. Playing a given strategy is strictly dominated either if he loses the fight or if he wins the fight and is attacked by another player at a later turn.

3 Solving the Game Under Anarchy

We consider first the outcome under anarchy.

3.1 The intuitive solution to the game

The most obvious implication is that a player never chooses a fight where he will lose if he has the option of avoiding it. It is never beneficial for a player to have his fighting power reduced, as this by definition means that he has less resources than he did in the beginning, never increasing his chances of having more resources at the end of the game. Therefore, he will never choose to enter into such a fight voluntarily. So it must hold for any given attack to take place that $g(x_i, x_j) > x_i$ (though this is not a sufficient condition).

It is easiest to first consider Ugly's strategies. He only has the possibilities of fighting on his own initiative if he has not been defeated at a previous stage in the game. If no fighting has taken place at an earlier stage of the game, Ugly will pass, because then he remains the weakest player and will lose any fight he picks. So the only possible outcome where Ugly will attack is if Good and Bad have previously fought each other, reducing one of their fighting powers to such a low extent that Ugly can profit from attacking them. But, as we have seen above, such a situation never occurs in equilibrium since neither Good or Bad would choose to enter into such a fight. The equilibrium strategy is, hence, that Ugly never attacks anyone, but always passes if he gets the chance to play during his turn.

Bad only has the possibility of attacking Ugly if Good has not previously attacked Ugly and only gets a turn if he has not been attacked and lost the fight to Good. However, using the similar logic as in the case of Ugly, Good knows this and will not attack Ugly if he knows that his resources are reduced to such an extent that he will be attacked by Bad. (This also conflicts with the necessary condition that $g_i(x_i, x_j) > x_i$ for a fight to be profitable).

Note that if Good passes, Bad has the option of attacking Ugly as well (Bad will never attack Good during his first turn as he will lose). If he does so and becomes stronger than Good, Good will pass during his next turn as he will lose the fight. In other words, even if Good is the strongest player at the beginning of the game, Bad might in fact be the only player that has resources at the end of the game.[9]

Good always wins against an individual player during his first turn, so he determines who to attack based on the combination of resources he gets, and no strategy is always strictly dominated. Note also even if Good has such a position that he will attack both players, it matters in which order he does so. It does not necessarily hold that $g_G(g_G(x_G, x_B), x_U) = g_G(g_G(x_G, x_U), x_B)$.[10]

3.2 Formal solution

Call the set of equilibrium strategies at any given turn t $s_i^t \in S_i^t =$

(Strategy at first turn, Strategy at second turn, ..., Strategy at n:th turn). For simplicity of notation, assume that strategies are defined even if the player is out of the game but that it is unplayable (this does not make a difference to the results given our notation). Then, putting the reasoning above simplified calculations where we remove all strategies that are dominated in all sitautions, (where $|s_i^t| \in \{1, 2\}$ due to the limits we have specified, since the game at maximum runs during two turns for each player in equilibrium) we get:

$$s_G^t \in S_G^t : x_G^* = \max \begin{cases} g_G(g_G(x_G, x_B), x_U) \\ g_G(x_G, x_B) \\ g_G(g_G(x_G, x_U), x_B) \\ g_G(x_G, x_U) \\ x_G^* | s_B^t \end{cases}$$

$$s_B^t \in S_B^t : x_B^* = \max \begin{cases} g_B(g_B(x_B, x_U), x_G) \\ g_B(x_B, x_U) \\ x_B \end{cases}$$

$$s_U^t = (\text{Pass}, \text{Pass})$$

3.3 The possible equilibrium outcomes of anarchy

The following tree shows all possible outcomes under anarchy when we have eliminated all always dominated strategies. Which outcome that will actually prevail depends on our free variables. (We

[9]See Appendix 11.4.1 for an example of such an outcome.

[10]Obviously, if the equilibrium is that only Bad remains, he needs to attack Ugly first, so in this case it is not relevant to consider a similar inequality.

stop drawing the tree when there is only one set of strategies that dominate given any values of the free variables.) Note that the fact that there are several possibilities does not imply that they are all equally reasonable or realistic. We simply allow them to vary, and which interpretation is the most realistic depends on how we want to specify the functions. This allows for a large amount of generality.

We have removed the branch where we would have had (Good Passes, Bad attacks Ugly, Good attacks Bad...) because this is never an outcome of the game. If Bad knows that Good will attack him no matter what he plays, he will pass. Apart from this example, the solutions should be clear given the reasoning above.

$$
\text{Possible outcomes: }
\begin{cases}
\text{Good attacks Bad} \begin{cases}
\text{Good attacks Ugly. } \overline{x} = (g_G(g_G(x_G, x_B), x_U), 0, 0) \\
\text{Good passes. } \overline{x} = (g_G(x_G, x_B), 0, x_U)
\end{cases} \\
\text{Good attacks Ugly} \begin{cases}
\text{Good attacks Bad. } \overline{x} = (g_G(g_G(x_G, x_U), x_B), 0, 0) \\
\text{Good passes. } \overline{x} = (g_G(x_G, x_U), x_B, 0)
\end{cases} \\
\text{Good passes} \begin{cases}
\text{Bad attacks Ugly} \begin{cases}
\text{Bad attacks Good. } \overline{x} = (0, g_B(g_B(x_B, x_U), x_G), 0) \\
\text{Bad passes. } \overline{x} = (x_G, g_B(x_B, x_U), 0)
\end{cases} \\
\text{Bad passes. } \overline{x} = (x_G, x_B, x_U)
\end{cases}
\end{cases}
$$

3.4 Comments on solution

We see that there are seven possible solutions to the problem, when we allow our variables to vary freely within the bounds we have set up. In three of those solutions, only one player remains, in three solutions two players remain, and in one solution there is a peaceful outcome where all players remain. Which outcome will prevail depends on the resource allocation at the beginning of the game, the fighting functions of the individual players, and the cost of fighting function.

Note that this does not necessarily imply that it is the most likely outcome that players will fight: it depends on the free variables, and which values it is reasonable to assume depends on which empirical circumstances we find it the most reasonable to assume.

4 Transfer Systems

We now consider markets and democracy as possible modifications to the anarchic outcome. Those are in our model systems that are implemented before anarchy, and allows the players to redistribute resources according to a given mechanism before the anarchic outcome starts. As argued in the introduction, we must assume that a system is already implemented in order to make any kind of relevant comparison between systems. Which system that is the most likely to arise out of anarchy in practice is an empirical question, but the stability of them in themselves can not be compared based on this observation.

We first make some comments on the assumptions that are common for both systems.

4.1 Bargaining and force

The games of markets and democracy are games that serve as a mechanism by which players can redistribute resources between them, moving from the initial allocation if they so desire. Note that we assume that the initial allocation of resources has already been established, as we consider this a property of the state of nature that any of our systems have to operate within. In other words, we keep out assumptions of move order and an initial property allocation intact, so this is established not only before anarchy but also before the bargaining game starts.

Key to this comparison is that the assumptions for the market and democratic system must be equally strong, or at least as equal as possible. We therefore analyze some common characteristics, of the systems here, and the points here would hold for any system.

In neither of these systems can a player use force to achieve a given outcome. So in neither the market or democratic systems, a player can be forced to transfer resources. If there is a proposition in which he is affected, i.e. one where he gives away or receives resources, he can simply refuse and there is nothing the other players can do about it. The concerned players of a transfer, however, can internally do as they please given mutual consent (i.e. they do not require consent from the third player if the third player does not have any resources transferred to or from him).

The reason that we model markets and democracy in this way, as systems that can not use force, is that force is used at the stage of anarchy, and allowing players to use force at this stage of the game would essentially be the same in the analysis.

The fact that we do not allow players to fight at this stage of the game does not mean that we take the "stability" of the systems for granted. In contrast, it simply implies that we set it up initially and see whether negotiations under the system can cause stability. There is nothing inherent in the system that implies that it has to be able to solve the problem of anarchy: In contrast, we will see that both markets and democracy often fails to do so.

4.2 Incentives for transfers

The only reason that a player should agree to transfer resources away to another player is if it allows him to have more resources after anarchy has been released. If the anarchic equilibrium does not change as a consequence of the transfer of resources, the player is made worse off as he has his resources reduced. For instance, if a player is attacked under anarchy when no transfers have taken place he is willing to transfer almost all of his resources if it reduces the feasibility of attacking him to such an extent that he will not be attacked under the new equilibrium (this depends on the properties of the various functions). If there is such a point, the player will obviously hand away the minimum amount of resources he can to achieve this outcome in equilibrium.[11] If a player is indifferent between some proposition to transfer and not transferring, for convenience, we assume that he declines the offer.

For solving the problem, we consider a transfer for player i such that $t_i \in]-x_i, x_j + x_k[$, where $t_G + t_B + t_U = 0$, where a positive value implies that the net received amount is greater than zero and vice versa. In other words, a player can propose offers between himself or another player, or all three players. These player can choose to accept or decline the offer. If they accept, the proposed transfer goes through. If a player where $t \neq 0$ declines the offer, the proposed transfer does not go through and the game continues.

Note that this implies that if an offer proposes a transfer between all three players, it is dependent on all players accepting the offer in order for it to go through. If it is not accepted by any one player, the offer does not go through, even for the consenting players.

All players benefit from a transfer if the equilibrium solution strictly improves the outcomes for them over the pre-existing anarchy equilibrium solution. In other words, if the equilibrium does not change, the player which gives away the resources is made worse off and he will not agree to a

[11] A few additional assumptions have to be made in terms of decision rules when players are indifferent between the two or more different outcomes. We provide examples in the Appendix.

transfer, so it is a necessary condition for an equilibirum where transfer occurs.

As a transfer changes the resource allocation between the players, and hence the fighting functions, we can no longer assume that the initial strength relationship between the players hold after the transfer, i.e. it is not necessarily the case that $f_G(x_G + t_G) > f_B(x_B + t_B) > f_U(x_U + t_U)$. This means that under a transfer system we can not conclude that Ugly always passes like we could in the anarchy case, since it does not necessarily hold that he is the weakest player after a transfer. There might exist a solution where he has an incentive to attack another player.

One assumption that should be noted is that we restrict ourselves to allowing the players to transfer resources but not move orders. This is partially a limitation of the model as it could be kept arbitrary, but could also be considered a weak assumption as as we could consider this simply as resembling a commitment game, where a player could agree to pass during his turn in exchange resources from the other player(s). Since this would be pure cheap talk, however, this is unlikely to change the outcome of the game.

4.3 Transfers and equilibrium

The player that gains resources from the transfer must also gain in the new equilibrium. Since the equilibrium must change given that a transfer occurs, the gain in resources must move the player gaining resources to an equilibrium where he is not attacked either. (In a theoretical situation where the equilibrium solution remained the same, he would be better off due to the increase in resources, but as we have stated above, the equilibrium must change in order for a transfer to take place.) This is pointed out since he might become more feasible to attack if the resource effect dominates the fighting power effect of the change in resources.

This implies that if anarchy is peaceful, then a solution after a transfer is also necessarily peaceful. If the anarchy equilibrium is a peaceful one, no one will ever agree to hand away resources.

A transfer system will, therefore, never increase the amount of players that are attacked under anarchy.[12] However, this does not imply that transfer systems are inherently more peaceful under all conditions, in some circumstances it will merely change which player is attacked as opposed to reducing the number. However, as we will show, transfer systems can reduce the amount of players that are attacked under anarchy. This implies that if a transfer has occurred, there is never more than one player that is attacked under the following anarchic outcome, because at least two players

[12] This is partially a consequence that we do not make any assumptions about deeper possibilities of organization. See the discussion below for a discussion of this assumption.

are involved in the transfer and neither of them will agree to a transfer if they are attacked under the subsequent anarchy equilibrium.

Since we assume that players have the relevant information and are rational in their decisions, transfers must be beneficial also in the sense that the players must be protected from each other, not just for the player that stands outside of the exchange. All promises not to attack among trading parts are cheap talk, as there is never any credible commitment to such an agreement unless the resource allocation is such that no one has an incentive to fight.

4.4 Ultimatum offers and mutual optimality

As we will see, the systems that we compare at times end up in an equilibrium ending in an ultimatum game. This is described as a situation where one player gets to propose an allocation between himself and some concerned player(s). The concerned player(s) then gets to accept or decline the offer. If they accept, the proposed allocation of resources goes through. Else, the allocation of resources remain the same as before, even if only one of two concerned players.

The difference between this situation and an ultimatum game in the classical sense is that the distribution of resources has already been established in our model, so this is a transfer rather than a distribution of a cake in which neither player gets anything in the sense of the classical textbook model.[13]

In both the market and democratic cases the concept of ultimatum offers will be important. Since players can predict the response from the other players, they make the ultimatum offer that will be accepted at the optimal rate for themselves. If they can not make such an offer, they pass, by definition ending the game. We denote an equilibrium ultimatum offer for player i as u_i^*.

Note that this means that a player can make an offer, abusing the fact that he has the ultimatum offer, such that a disproportionate amount of the transfer comes from the other players. See Appendix 11.4.3 for an example of a trilateral accepted ultimatum offer where all three players accept the offer, but where the equilibrium allocation of resources is radically different depending on who makes the offer.

Note that not all offers have to be ultimatum offers in any transfer system. In contrast, we will see that there are situations where players benefit from declining offers of transfers in order to propose an offer themselves (This is by definition not an ultimatum offer.). If the game continues

[13] Andreoni and Blanchard (2006, p. 307), for instance, merely defines an ultimatum game as a take it or leave it game, not concerned about the specific form.

after an offer has been rejected, it is by definition not an ultimatum offer.

We call a transfer mutually optimal for a set of players if the equilibrium ultimatum offers have the properties that $t_i \neq 0$ for the other players in the set and no one outside of the set. For instance, if Good proposes an accepted offer between himself and Bad when he has the ultimatum offer, and Bad would propose an accepted offer between himself and Good if he had the ultimatum offer, then mutual optimality between Good and Bad holds.

4.5 Formal solution

As we have seen, given that a transfer has occurred, not all outcomes of anarchy are possible. However, in order for a transfer to take place it is necessary that the system has an equilibrium solution where it holds that some $t \neq 0$.

Given our reasoning above, the solution of anarchy is, given that a transfer has occurred, removing the strategies that are always strictly dominated (Note that we remove any $t = 0$, which is true for any player that is attacked in the equilibrium.).:

$$s_G^t \in S_G^t : x_G^* = \max \begin{cases} g_G(x_G + t_G, x_B) \\ g_G(x_G + t_G, x_U) \\ x_G^* | s_B^t \end{cases}$$

$$s_B^t \in S_B^t : x_B^* = \max \begin{cases} g_B(x_B + t_B, x_G) \\ g_B(x_B + t_B, x_U) \\ x_B^* | s_U^t \end{cases}$$

$$s_U^t \in S_U^t : x_U^* = \max \begin{cases} g_U(x_U + t_U, x_G) \\ g_U(x_U + t_U, x_B) \\ x_U + t_U \end{cases}$$

4.6 Possible equilibrium outcomes given accepted transfer

The following tree shows the possible outcomes given that a transfer has occurred. This is drawn using the same principles as that above.

$$
\text{Possible outcomes given a transfer:}
\begin{cases}
\text{Good attacks Bad: } \overline{x} = (g_G(x_G + t_G, x_B), 0, x_U + t_U) \\
\text{Good attacks Ugly. } \overline{x} = (g_G(x_G + t_G, x_U), x_B + t_B, 0) \\
\text{Good passes}
\begin{cases}
\text{Bad attacks Good. } \overline{x} = (0, g_B(x_B + t_B, x_G), x_U + t_U) \\
\text{Bad attacks Ugly. } \overline{x} = (x_G + t_G, g_B(x_B + t_B, x_U, 0)) \\
\text{Bad passes}
\begin{cases}
\text{Ugly attacks Good. } \overline{x} = (0, x_B + t_B, g_U(x_u + t_U, x_G)) \\
\text{Ugly attacks Bad. } \overline{x} = (x_G + t_G, 0, g_U(x_U + t_U, x_B)) \\
\text{Ugly passes. } \overline{x} = (x_G, x_B, x_U)
\end{cases}
\end{cases}
\end{cases}
$$

5 A Market Process

5.1 Sequential offers

A market can reasonably be described as an initially established property then governed by voluntary exchange. The first of those assumptions are already fulfilled in the assumptions of the model, though what we mean by property is merely that an individual has physical control over the given resource (i.e. there is no requirement for a mutual conception of what they legitimately are).[14]

To fulfill the second assumption, we also need to specify a concrete market bargaining system in which the players can operate. Using those assumptions as the cornerstone of our model, we assume that the market process works as follows: Good gets to propose a transfer. The concerned players can either accept or reject this offer. (By concerned we mean that they will hand away or receive some sum $t \neq 0$). Then Bad gets to propose and this is accepted or declined by the concerned players. Lastly, Ugly gets the same chance and then the process starts again from the beginning. This continues until neither of the players has proposed an accepted offer in their last rounds.[15]

The reason that we choose to model the market in this way is that a solution where players move at the same would lead to a bargaining game where we would have to make assumptions that do not seem to fit into our initial model, where we would need to consider a simultaneous game with possibly mixed strategy equilibria. The market in the real world moves in continuous time, so the assumption of a sequential process seems weak in this case, as we could consider the time between the turns arbitrarily small if we desire.

5.2 Intuitive solution

For simplicity, we assume that if a player is indifferent between two outcomes of the game, he will decline the offer from the other player if he can then make the same offer during his turn. We also assume that, as a player can predict the eventual outcome of the game given a certain transfer, taking this into account, that only one transfer takes part during the game. Neither of these assumptions are very strong given the rationality of the players.

[14] We discuss this assumption below.

[15] Like in the modelling of anarchy, the set move order is for simplicity as an arbitrary move order brings too many solutions.

5.2.1 Backward induction

Suppose that both Good and Bad have passed during their turns. Then Ugly has the ultimatum offer. If he does not make an offer, the game ends as one of the conditions for the game ending are fulfilled. Knowing this, if the solution is mutually optimal, Ugly will reject any offer and then offer his ultimatum offer, as this at least weakly dominates any other alternative.

Bad takes this into account when he makes his proposed offer. He is unable to make an accepted offer between himself and Ugly, since the latter will re-propose it at best. This also applies to trilateral trade. He can, however, make a proposition to Good which Good might accept if the solution is better for both of them than Ugly's ultimatum offer.

Knowing this, Good has few opportunities to make an accepted offer. If he offers trilateral trade or an offer between himself and Bad, Bad will re-propose them, so in that case Good passes.

However, there are conditions where Good can make an accepted offer between himself and Ugly. Suppose that (i) Good prefers this offer over Ugly's ultimatum offer and that Good prefers Bad's offer over Ugly's ultimatum offer. Ugly prefers to accept Good's offer over the offer Bad would make between himself and Good.

5.3 Formal solution

Define $\{j, k\} = M$. Call an offer from a player i to $m \subset M$: $\mathcal{O}_i^m$ with the rate that is the highest one that the other players will accept to the benefit of i and and a complete strategy O_i. Then the solution is

$$O_G = \begin{cases} \mathcal{O}_G^U : (x_G^*|\mathcal{O}_G^U) > (x_G^*|O_B) > (x_G^*|u_U^*) \text{ and } (x_U^*|\mathcal{O}_G^U) > (x_U^*|\mathcal{O}_B^G) \\ \text{Pass else} \end{cases}$$

$$O_B = \begin{cases} \mathcal{O}_B^G : (x_G^*|\mathcal{O}_B^G) > (x_G^*|u_U^*) \text{ and } (x_G^B|\mathcal{O}_B^G) > (x_B^*|u_U^*) \\ \text{Pass else} \end{cases}$$

$$O_U : \begin{cases} \mathcal{O}_U^m = u_U^* \text{ if accepted} \\ \text{Pass else} \end{cases}$$

5.4 The possible equilibrium outcomes of market bargaining

The tree below expresses all possibilities given our analysis above.

$$
\text{Possible outcomes under market:}
\begin{cases}
\text{Good makes accepted offer between himself an Ugly} \\
\text{Good passes}
\begin{cases}
\text{Bad makes accepted offer between himself and Good} \\
\text{Bad passes}
\begin{cases}
\text{Ugly makes accepted trilateral offer} \\
\text{Ugly makes accepted offer between himself and Good} \\
\text{Ugly makes accepted offer between himself and Bad} \\
\text{Ugly passes}
\end{cases}
\end{cases}
\end{cases}
$$

5.5 Brief comments on results

We consider the general implications more carefully in our comparison between the market and democratic processes. Generally, the outcome of the market process depends on very specific preferences of the players, where they are often conditional on the behavior of other players that are in turn dependent on the behavior of the third part.

We see that there are several possibilities where Ugly benefits from being the last one to offer, as it presents an ultimatum game to him and he can reject offers and then re-propose them at more feasible rates. However, this is partially a consequence of the assumption that we keep the move order in the game the same as that under anarchy.

As we have argued, we need to keep the assumptions as equal as possible between two systems in order to compare them. We will now consider a similar model, except using a democratic process rather than a market one. The comparison follows after the results of democracy have been presented.

6 A Democratic Process

We now consider a model of democracy. With democracy, we mean here a system of majority rule, where we first hold an election, and the player with the most votes, the democratically elected leader, gets to select the allocation of resources. This is intended to represent the representative democracy approach, as direct democracy is both harder to interpret and less relevant to consider given how modern states are structured.

Our model of democracy is, like the market, a sequential game. First Good gets to vote, and his vote is public information. Then Bad gets to vote, his vote is also public information, and finally Ugly votes. The player with the most votes is then elected the leader and gets to select the resource allocation through making an ultimatum offer. If no player has majority, the same outcome as anarchy ensues. The reason that we assume that democracy is a sequential game is to make the comparison to markets more equal. This means that the only difference between the games is this mechanism. It should make for a relevant comparison, but the problems with modelling democracy as a sequential game is discussed below.

It is reasonable to consider the leader as making an ultimatum offer as the democratic process in itself relies on the offer made by the leader. The main "bargaining" has already taken place through the voting system.

Like the market case, the democratic leader is unable to use force to enforce his preferences of resources, he can only propose an offer that reflects them. (Of course, the elected leader knows which offers will and will not be accepted, and so do the players before they vote, and they take this into account when they vote.)

It might seem curious that we assume complete information for the democratic process as it is generally anonymous and simultaneous in the real world. We choose to do this, however, for the same reason that we consider the market a sequential game, else it simply becomes a standard coordination problem in our model. This makes for the most relevant comparison.

6.1 Intuitive solution

We assume that if a player, by voting for himself, can achieve majority, he will always do so, because this is at least as good, i.e. it at least weakly dominates any other alternative. We also assume that if a player can not affect the outcome by his vote, he will vote for himself just like a market participant declines to accept transfers if he is indifferent between outcomes or can not affect it.

6.1.1 Ugly's strategy

Suppose that it is Ugly's turn to vote. Either his vote is decisive for the election, where no player has majority given the votes of Good and Bad, or it does not matter what his vote is (since either Good or Bad has majority already). In the latter case, he votes for himself.

In the former case, if his vote matters for who will win, that means Good and Bad have one vote each. This means Ugly can choose between the outcomes of Good's ultimatum offer, Bad's ultimatum offer, or the anarchic outcome where no transfers occur, and he will vote for the one which maximizes his resources, predicting what the ultimatum offers from Good and Bad will be.

6.1.2 Bad's strategy

Knowing what Ugly's strategy will be, Bad takes into consideration the vote that has been cast by Good at the previous turn. If Good has voted for Bad, Bad will always vote for himself. If Good has voted for Ugly, it does not matter for the outcome what Bad votes for since Ugly votes for himself, so Bad votes for himself.

If Good has voted for himself, however, there are multiple options for Bad that arise out of circumstance. A vote for Good means that Good will propose the ultimatum offer, a vote for Ugly means that Ugly will get to propose the ultimatum offer, and voting for himself means that Ugly decides on who makes the ultimatum offer as specified above.

6.1.3 Good's strategy

Knowing the optimal strategies of the other players, Good chooses his vote based on the principles of Bad and Ugly, using the same principles. The tree below should make all possibilities clear.

6.2 Formal solution

Call a player i voting for a player a $v_i = a$. Call the outcome under anarchy where no transfers have occurred, for player i, x_i^A Then the decision rule for the players are, given our reasoning above,

$$v_i = \begin{cases} j \text{ if } x_i^*|u_j^* > \max\{x_i^*|u_k^*, x_i^A\} \\ k \text{ if } x_i^*|u_k^* > \max\{x_i^*|u_j^*, x_i^A\} \\ i \text{ else} \end{cases}$$

This is then followed by the ultimatum offer of the given player with majority, or an outcome without any transfers if no player has achieved majority.

6.3 Possible equilibrium outcomes of democracy

Given our reasoning above, the possible solutions are:

Possible outcomes:

- Good votes for Good
  - Bad votes for Good (Good makes ultimatum offer)
  - Bad votes for Bad
    - Ugly votes for Good (Good makes ultimatum offer)
    - Ugly votes for Bad (Bad makes ultimatum offer)
    - Ugly votes for Ugly ($t_G = t_B = t_U = 0$)
  - Bad votes for Ugly (Ugly makes ultimatum offer)
- Good votes for Bad (Bad makes ultimatum offer)
- Good votes for Ugly (Ugly makes ultimatum offer)

6.4 Brief comments on results

We see that the democratic process leads to a rather simple pattern of solutions. Any player can, simply put, select another player to make the ultimatum offer, and if no player is willing to hand this responsibility over to someone else, we get an equilibrium where no transfers occur. If a solution is mutually optimal between two players, there is nothing the third player can do to avoid the outcome.

We also see that if a solution is mutually optimal, the last player to vote needs to vote for the first player in order for the transfer to take place. As he by definition prefers that the former player makes the ultimatum offer over anarchy, he must vote for the first player. This means that the first player gets to propose the transfer at an optimal rate for himself.

In the case of trilateral mutual optimality, the outcome depends on the rates at which a given player will make the ultimatum offer. Trilateral mutual optimality does not imply that the players are indifferent between who makes the offer. However, this directly mirrors the logic above about voting behavior, so the principles are exactly the same even under trilateral mutual optimality.

7 Comparison Between the Results

We see that in many cases, neither markets nor democracy succeeds in solving the problem of security, if fighting is not costly enough or the imbalance in initial fighting power is too high. This is in line with the seemingly reasonable conclusion that any system will fail to solve the problem of avoiding war under infeasible circumstances. We distinguish between two cases, that of mutual optimality and that in which we do not assume mutual optimality.

7.1 Mutual optimality

We see that if mutual optimality holds, both markets and democracy leads to the same outcome, except markets benefits the player who has his turn later and democracy benefits the player with the former turn, as this player gets to make the ultimatum offer (or, in the market case, an offer that is exactly the same as the ultimatum offer) and therefore can offer it at the best rate for himself.

The explanation is that mutual optimality holds, the players will find a way to transfer in both the market and democratic cases. In the market case, the player that moves last will simply decline the offer and then re-offer it. In the democratic case, the player that moves first has the advantage, as he votes for himself and the optimum response for the player that moves later is to vote for the first, as anarchy is the consequence else. This means that both the market and democratic mechanisms are sufficient for coordination in those instances.

7.2 General Cases

Assume now that mutual optimality does not necessarily hold. In the market the choice that players face is not between the various ultimatum offers but that between the various offers that are possible during bargaining. Players are forced to make offers within certain bounds that can be stricter than the ultimatum bounds such that they can not offer rates that are so feasible. For instance, in the case of Good making an accepted offer to Ugly, it is not certain that he can make the same offer that he could have made if he had the ultimatum offer, as this might cause Ugly to decline. In other words, since democracy only to consideration the ranking of ultimatum preferences and markets concern various preference ranking given conditions for offers that are not strictly those of ultimatum preferences. This, however, seems to be a reasonable analogy to reality as the ultimatum situation is close to that of democracy's elected leader, while the market has options to choose from where offers and counter-offers can be made in the market.

We consider now the, generally speaking, three different outcomes under the market mechanism, given specified preferences, and compare it to the outcome of the democratic process in terms of how the players would have voted if they operated in the democratic system instead.

7.2.1 Good's offer on the market

If Good makes an accepted offer to Ugly in the market, it must necessarily be specified within certain restrictive bounds, such that it is not the ultimatum offer, or another outcome would ensue due to Ugly's declining and re-proposition. However, this does not imply any certain limitations on the preferences of ultimatum offers on behalf of the players. All that can be said is that Good and Ugly prefer this bounded offer over Bad's bounded offer to Good, which Good in turn prefers to Ugly's ultimatum offer.

In the market, Good has no option of choosing to make this offer to Ugly, since there is no credible commitment for him to make this bounded offer even if they agree with one another beforehand (this is cheap talk). Instead, he will propose his equilibrium ultimatum offer if this is the case.

This means that we can not make a comparative analysis between markets and democracy in this case, as different preference rankings are relevant for the decision making. If we made an arbitrary ranking concerning all possibilities this would produce too many solutions to be useful, as we would have to specify a ranking for all players that span all of the different situations given all possible offers and counter-offers.

7.2.2 Bad's offer in the market

Suppose that in the market both Good and Bad prefer Bad's bounded offer over Ugly's ultimatum offer. Using these preferences for democracy, Bad might not be able to credibly commit to making this bounded offer as his ultimatum offer might be different. Good needs not prefer this over Ugly's ultimatum offer. In other words, in this case preferences are also not the same as those used to decide in the democratic process.

7.2.3 Ugly makes ultimatum offer in the market

If Ugly makes the ultimatum offer on the market, this means that it is preferred by both Good and Bad over the alternatives that would have been made given the possible bounded offers between them. In other words, this leads to the same situation as those mentioned above where we need to specify the preferences further in order to be able to compare the systems.

7.3 Summary

We see that the market and democratic processes, when the same preferences are relevant for the result, produces the same outcome, although they result in different allocations of resources if the ultimatum offers of the concerned players can operate at different rates. As a consequence, markets appear to protect the player that is weaker. However, this is mainly a consequence of our assumption of the processes as a sequential game and the fact that we have kept the move order fixed for simplicity, so this needs not be a result that should be taken with a grain of salt.

As the transfers under markets and democracy result in the same equilibrium outcomes, they also have the same amount of stability in those instances.

This result can be interpreted such that these systems enable cooperation between players that both strictly prefer to cooperate with one another. Both systems also allow for trilateral cooperation and therefore peace under several circumstances.

In the cases where different preferences are relevant for the optimal strategy, we can not say which of the systems that will produce which results without specifying further the ranking of the preferences, which can be made arbitrarily within our bounds.

8 Further Comments on Assumptions

We need to be careful in making analogies to the larger society when we have considered a simplified model of the world as the present one. However, like any other analysis, it is often reasonable to simplify the problem so that it becomes manageable. The assumptions that we have made are not overly strict in comparison to other economic models, and the shortcomings of those assumptions are well-known. One main reason that we have considered a model that is so theoretical is precisely because the problem of making proper empirical tests with available data. It is hard to find examples of natural experiments where unenforced markets are contrasted with unenforced democracy. In this sense, our model is one that is supposed to show at least some basic dynamics of organization under the two different systems, albeit on a simplified level.

Below follow some critical examinations of the assumptions we have made of the various assumptions of the model and the problem they might cause for the validity of our conclusions. Some are examples of similar theoretical models that have made a few assumptions differently, and some are empirical analogies. We restrict ourselves to giving a few examples that might be worth noting: any such analysis can go on forever in any model that simplifies reality.

Most importantly, we have assumed here that there are no particular market (or democracy) failures such as insufficient information, externalities or public goods problems. They are applicable to law and norms in both the market and democratic processes and hence the propensity of using violence. The fact that we have made those assumptions might be a contributing factor to our result, as market and democratic under idealized conditions are often similar in their outcomes also models that test other economic concepts (see, e.g., Frey 1970).

8.1 The larger analogy for markets and potential problems

It might be hard to imagine what a market system in its larger analogy to society would look like, as this is a more theoretical situation and the democratic state is the status quo in much of the world. We have provided a few historical examples, although they are imperfect by several means.

In its analogy to the larger society, a market in security might have to be represented by a form of competing agencies that are paid for protecting an individual. It might also be represented by some government that outsources (as the example of medieval Iceland presented above). This is, however, a more theoretical situation that has to be considered in itself, and we need to be careful when we extend the model to such a system. Our model should be viewed as a highly simplified version of this reality. The most famous works that have theorized further about this is Friedman 1979a and Rothbard 2006 (1970), where they have argued that market failures are at least overstated in comparison to the problems faced by governments, though they have a strong ideological bias.

Applied to the problem of security and the balance of power, this might affect the feasibility of stability in several ways. On one hand, a market operates in more continuous time than democracy in the real world, as elections are held only once ever few years while the market operates with continuous transactions.

It can also be argued that such services are natural monopolies where there is inevitable convergence towards a monopolized system (see, e.g. Nozick 1974).

8.2 Information problems

Information plays a crucial role in the process of conflict when agents decide to fight (see, e.g. Wärneryd 2003, Wärneryd 2012) and we have largely ignored those problems in the model for simplicity. In contrast, we have assumed complete information, which is a problem both for markets and democracy in their analogy to society.

However, this is probably a larger problem for democracy than it is for markets in the real world. When political information is costly to obtain, people are likely to obtain little of it (see, e.g., Downs 1957). Under the assumption of self-interest, this probably leads to larger problems for the democratic process than it does for markets, people have a very small chance of affecting the outcome of an election, virtually zero in larger societies.[16] In the market, however, people are directly affected by their own decisions, so they are more likely to obtain the relevant information that concerns their decisions.[17]

8.3 Individual and collective action

In our model, we have assumed that only individuals act in their respective turns and that there are no formal means to organize other than transferring resources before the game starts. This

[16] This argument relies on the notion that the "miracle of aggregation" does not hold. See Caplan 2000, Caplan 2001 for a critique of the assumption that voters have a mean error of zero in their selection of policy.

[17] DeCanio (2014) also argued that markets resemble the scientific method in its discovery of information and contrasts this to the democratic case.

seems largely reasonable to the extent that we consider individual self-interested. If we wanted to extend the game to take this into consideration, we would have to take into consideration further assumptions about the bargaining process in addition to the current assumption. This would not fit well with our initial model, as it would simply transform them into standard credible commitment games that are hard to interpret in the rest of our model.

However, there is an extensive literature on organization under anarchy that in other respects have some resemblances to our analysis but where anarchy is governed by forming formal coalitions. Herings et. al. 2007 build a similar model where players are allowed to organize and transfer resources between one another. Piccione and Razin 2009 makes a similar analogy. Jordan 2009 and adds a dimension of institutions to the problem.

One possibility that our model overlooks is that we can think of circumstances where it seems reasonable that the possibility of organization actually increases the propensity of violence. For instance, suppose that players in themselves are not strong enough to profit from fighting, but that they can do given that they organize and can solve any commitment problems. We can also think of circumstances, often implied in our model, where the possibilities of organization reduces the propensity of violence, even sticking to our assumption of methodological individualism.

8.4 Property rights

In our model, the initially established property rights are equivalent to formal control over a given resource, as opposed to a conception about what they legitimately are. Most similar models treat property rights as unestablished in the state of anarchy, (e.g. Hafer 2006) and this is probably a more realistic assumption when it comes to the comparison of what a more complex society of individuals looks like. However, our model merely looks at property as the initially established facts of nature: who has what at the beginning of the game in the sense that they have physical access to them, and the only assumption we have made is that other things equal, a player is a stronger fighter the more resources he starts off with. Who gets what property rights as a shared conception is often dependent on force (see, e.g., Umbeck 1981), but this analogy still holds for our model.

9 Conclusion

We need to be careful in the analogies to the larger society implied in our model due to the strong assumptions made, but in our model we find that when the same preferences are relevant, the market system and democracy have the same degree of stability. This is in line with the general result that systems tend to reach similar efficient results when no systematic failures, such as market failures, are present.

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11 Appendix: Examples of Solutions

We here provide examples of some solutions to the problem where we specify the fighting functions of the players more carefully. We are only considered here with solutions that might be counter-intuitive or not immediately obvious to the reader, as this might prove helpful for the understanding of some of the key points.

11.1 A few additional assumptions

As mentioned above, we need to make a few additional assumptions that concern the purely

mathematical sense of the model, as well as some arbitrary decision rules for how players should behave when they are indifferent between various outcomes. Those can be specified in a variety of ways.

First, here we assume that any transfer, and therefore any offer, can only be made in integers. This is such that we do not have to consider arbitrarily small variables, which would otherwise be beneficial for players to

We also need to make an assumption about what a player will offer when he is indifferent between some allocation between the remaining two players. If he can make an offer, he must sometimes decide how to split the resources between the other two players, given the constraints that they will accept. In this case, we assume that he simply declines to offer a trade. If he has to transfer some amount between the other two players in order to achieve his desired equilibrium, we assume that he transfers the minimal amount that he can given the constraints he faces.

11.2 A functional example

We consider the function

$$g_i(x_i, x_j) = \begin{cases} \max\{x_j - \frac{1}{f_i(x_i) - f_j(x_j)}, 0\} \text{ if } f_i(x_i) > f_j(x_j) \\ 0 \text{ if } f_i(x_i) = f_j(x_j) \end{cases}$$

as an example to analyze, and use it to get more concrete results than our previous calculations. We also assume that $f_i(x_i)$ and $f_j(x_j)$ are defined such that $g_i(x_i, x_j)$ is differentiable.

This is a relevant example to study because it fits our assumptions well. The only loss of generality is that we assume that the cost of fighting follows a given functional pattern rather than any functional pattern within our boundaries. Since

$$\frac{\partial g_i}{\partial x_i} = \begin{cases} \frac{f_i'(x_i)}{(f_i(x_i)-f_j(x_j))^2} \text{ if } f_i(x_i) > f_j(x_j) \text{ and } g_i(x_i, x_j) > 0 \\ 0 \text{ if } f_i(x_i) > f_j(x_j) \text{ and } g_i(x_i, x_j) = 0 \\ \text{n/a if } f_i(x_i) = f_j(x_j) \end{cases}$$

is positive definite where defined, our assumption of $\frac{\partial g_i}{\partial x_i} > 0$ when $g_i(x_i, x_j) > 0$ is satisfied.

We also have

$$\frac{\partial g_i}{\partial x_j} = \begin{cases} 1 - \frac{f_j'(x_j)}{(f_i(x_i)-f_j(x_j))^2} \text{ if } f_i(x_i) > f_j(x_j) \text{ and } g_i(x_i, x_j) > 0 \\ 0 \text{ if } f_i(x_i) > f_j(x_j) \text{ and } g_i(x_i, x_j) > 0 \\ \text{n/a if } f_i(x_i) = f_j(x_j) \end{cases}$$

which is in line with the assumption that $\frac{\partial g_i}{\partial x_j} < 1$ where defined (not restricting it) and $g_i(x_i, x_j) > 0$ as well as neither positive nor negative definite. It also fulfills the assumption that

$\lim_{f_i(x_i) \to f_j(x_j)} g_i(x_i, x_j) = 0.$

With simple calculations, a fight increases the winner's resources if

$$x_j - x_i > \frac{1}{f_i(x_i) - f_j(x_j)}$$

Consider matrices similar to that one below, where the first row specify the allocation of resources and the second represents the fighting functions. The columns represent Good, Bad and Ugly in order.

$$\begin{bmatrix} x_G & x_B & x_U \\ f_G(x) & f_B(x) & f_U(x) \end{bmatrix}$$

We now give examples of variables that yield outcomes in line with various assumptions.

11.3 Outcomes

11.3.1 Example where anarchy leads to an outcome where Bad is the only remaining player

Consider the following example. Assume that no trade has taken place for convenience (it is not relevant for the question that we seek to address here).

$$\begin{bmatrix} 100 & 9 & 80 \\ x & x^2 & x \end{bmatrix}$$

This starting point leads to the allocation (0;99.9998;0) under anarchy, and Bad is the only remaining player.

11.3.2 Example of mutual optimality between two players

Consider the following example:

$$\begin{bmatrix} 10 & 11 & 1 \\ x^{10} & x & x \end{bmatrix}$$

In this case, if Good gets the ultimatum offer, the allocation will be (20;1;1), transferring between himself and Bad. If Bad gets the ultimatum offer, the allocation will be (11;10;1), also transferring between himself and Good. So here mutual optimality holds strictly between the two player but not the third.

11.3.3 Example of trilateral mutual optimality and equilbrium

The following example is useful in the sense that it provides both an example of trilateral mutual optimality and an example where all players will agree to trade in all of those circumstances, illustrating two points at the same time.

$$\begin{bmatrix} 10 & 100 & 50 \\ x^{10} & x & x \end{bmatrix}$$

In the equilibrium anarchy solution, the only player that remains is Good, after attacking Bad followed by Ugly. If Good is given resources of 100 or abovve, he will accept any ultimatum transfer. Bad and Ugly also want to make any transfer that makes Good better off.

So if Good gets to make an ultimatum offer, the equilibrium outcome of resources is (158;1;1). If Bad gets to propose the allocation is (80;79;1) as he prefers that the maximum amount of resources are transferred from Bad. Finally, if Ugly proposes, the allocation is (54;52;54) since he transfers the resources from Bad as much as he can before offering his own resources. So in other words, who has the ultimatum offer is critical even though the outcome is a peaceful one.

This example also shows that transfers can reduce the amount of players that are attacked to obtain a peaceful outcome. Under anarchy, both Bad and Ugly are attack, but after any player has made an ultimatum offer where no player is attacked.