

STOCKHOLM SCHOOL OF ECONOMICS

Department of Economics

5350 Master's Thesis in Economics

Academic Year 2015–2016

Bank Capital in DSGE Models

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Abstract: This paper analyzes the implications of three different modeling approaches to bank capital for business cycle dynamics in a DSGE model with various real and financial frictions. Two approaches feature exogenous capital requirements but differ in their mathematical form. The third approach is novel to the DSGE literature and the capital requirement for banks emerges endogenously. Important features of the modeling approaches become visible only with higher-order Taylor approximations. We find that, relative to exogenous capital requirements, the endogenous capital requirement amplifies modestly business cycle dynamics and leads to opposite effects in some financial variables. In all models, short-run dynamics of bank capital are mainly driven by bank profits and in some cases of surprisingly strong magnitude. Surveying the literature we do not find any consensus on these effects and that most have not yet been tested empirically. We discuss major model results in light of basic empirical evidence and outline important aspects for future empirical work.

Keywords: banks, bank capital, financial intermediation, leverage

JEL Classification: E32, E44, G21, G28

Supervisor: Jesper Lindé

Date Submitted: May 16th, 2016

Date Examined: May 25th, 2016

Discussants: Kristina Boo and Malin Werin

Examiner: Federica Romei

Acknowledgements

We would like to thank our supervisor Jesper Lindé for helpful comments. Further we are grateful to Daria Finocchiaro for sharing the original code for the Gerali model with us and to Michael Kumhof for providing his mathematical appendix.

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"Economists have this outdated notion that economics advances through a progression of ever-better theories, with empirical testing serving to reject wrong models and confirm valid ones. In reality, we are really bad at formulating general models as social reality is malleable and contextual."

Dani Rodrik
Washington Post, March 2016

I Introduction

The effect of the Global Financial Crisis and ensuing Great Recession can be seen as having triggered a reassessment of the importance of financial intermediaries for macroeconomic analysis: financial shocks transmitted to the real economy through sharp contractions in bank credit and increased risk premia. Since then researchers have been busy improving the modeling of the financial sector in general equilibrium models. One important malaise was, and still is, that text books and workhorse models are scant of financial intermediaries that live up to requirements imposed by empirical findings and the corporate finance literature. At a more detailed level, the Global Financial Crisis can be thought of as a quasi-natural experiment. Large, unexpected losses from banks led to a sudden drop in capital levels that banks are still recovering from. While it is generally difficult to separate demand and supply effects, the fact that US firms largely shifted from bank to bond financing suggests that most of the following credit crunch might be due to a contraction in supply of bank credit. This in turn has put spotlight on the role of bank capital. Important questions are how it affects the steady state equilibrium of an economy but also monetary transmission and more generally business cycle dynamics.

Since then, a variety of approaches to model bank capital in macroeconomic general equilibrium models have been produced. Some of these modeling approaches share the same idea about the function of bank capital and they only differ in the exact mathematical form. Other modeling approaches distinguish themselves by emphasizing a specific function of bank capital. For instance, some researchers emphasize that bank capital reduces moral hazard between the banks and its depositors, allowing for an endogenous capital need, while others treat the banks' need to maintain a certain capital ratio as exogenous.

However, there is a lack of comparative studies trying to consolidate different streams and ideas in the literature. To provide reliable policy advice, these different approaches need to be compared and their results checked for consistency. Where model results are not reconcilable, data must tell which model might be more adequate in which situation. The

major purpose of this Master’s thesis is to provide a quantitative analysis of how different approaches to model bank capital affect the model equilibrium.

To do this comparison, we replicate the mid-scale DSGE model developed in Gerali et al. (2010), which includes the first of three approaches to model bank capital which we will consider. The second approach is taken from Jakab and Kumhof (2015). The third approach is developed by us and is based on the static model introduced by Disyatat (2011). In the first two approaches, bank capital is introduced through exogenous capital requirements. However, the mathematical forms of the latter differ between the two models. Therefore, these two models offer an opportunity to gauge the impact of seemingly small differences in the modeling approach on the quantitative results. In contrast to that, in the third approach to bank capital, the need of banks to hold capital emerges endogenously from the model. Comparing it to the first two models enables us to scrutinize the impact of fundamentally different approaches to bank capital on the model results. While there are other models with endogenous capital requirements readily available, for example Meh and Moran (2010), using our own approach facilitates comparison because, by construction, it fits well into the macro environment that we use. Besides that, we believe that it constitutes an important mechanism that deserves to be scrutinized in a DSGE model. Endogenous bank leverage determines the bank’s default probability because the higher leveraged the bank, the more vulnerable to credit risk shocks is the bank. Depositors know about the default risk of the bank and demand a premium on their deposit rates in excess of the risk-free rate. This risk premium varies with bank leverage and macroeconomic fluctuations.

Our model comparison doesn’t aim to reject any of the approaches or create a ranking among them. The aim is to tease out the implications of modeling decisions for the results. The quote by Dani Rodrik at the beginning tells more about the way economists should use models and the adequate degree of humbleness than about modeling decisions themselves. While there are some very fundamental issues, such as whether or not banks should play a role in determining business cycles, many model features and outcomes might be valid and relevant in one situation but not in another one. We provide some tentative empirics.

The rest of the thesis is organized as follows. Section II lays out a short review of the existing literature and of previous studies. A brief introduction into dynamic, stochastic general equilibrium models is done in Section III. In Section IV we describe the three models used in Section V in which the three models are compared and discussed. Section VI links the findings to the data. Section VII concludes.

II Literature review

Our Master's thesis is related to two broad areas of research in finance and macroeconomics. The first area is a response to the apparent lack of understanding of the behavior of financial agents. The research focuses on modeling the financial sector more realistically, both in a partial and general equilibrium context, and provides stronger micro-foundations of financial decision making. The models developed in this literature are often static but therefore involve complex financial sectors and decision making. The second area studies quantitatively the business cycle implications of financial frictions and caters the immediate need of policy makers to broaden their analysis to the financial factors which have garnered so much attention since the Great Recession. Researchers nest financial sectors in dynamic, stochastic general equilibrium models, in order to understand the quantitative importance of financial frictions and their interactions with real frictions which had previously dominated business cycle analysis. Borio and Zhu (2012) and Brunnermeier et al. (2012) provide excellent overviews.

In the following, we will discuss contributions to the aforementioned literature that are central to this Master's thesis. We start to discuss relevant papers which are mainly of static nature but provide important insight into the microeconomics of banking and the role of bank capital. Thereafter, we discuss important contributions in the DSGE literature.

Adrian et al. (2012) argue that the key driver of financial frictions are shocks to the supply of credit. They stage counter-cyclical risk premia and pro-cyclical bank leverage as important empirical findings which should be reflected in theoretical models. As determinants of risk premia, the evidence presented suggests that book leverage is more important than the level of net worth.¹ Moreover, motivated by the observation that bank lending changes one-to-one with changes in debt, Adrian et al. (2012) point out that bank lending is the consequence of the choice of leverage by banks. These insights are accommodated in their partial equilibrium model of bank credit supply, where banks maximize profits subject to a value-at-risk constraint.² The constraint means that the value-at-risk of the bank loans (expected losses) must not exceed the book value of the bank's equity.

The static model developed by Du and Miles (2014) extends Bernanke and Gertler (1990) and features banks which can finance their lending, exceeding their initial capital

¹In the literature "net worth" and "capital" are often used interchangeably and refer to funds provided by the owners of the undertaking, an important real-world example is shareholder equity in issue.

²First introduced in a 2011 working paper and finally published in Adrian and Shin (2014). In the paper discussed there is also a model part for bond financing, which is omitted from this discussion.

endowment, with either debt or equity provided by households.³ Absent any regulatory capital requirement, banks will choose to finance their lending solely with debt because equity finance is more costly than debt, the latter equalling the risk-free rate to rule out arbitrage opportunities. Under a regime with regulatory requirement, banks whose initial capital endowment is too small will seek equity injections from the household to meet (but not exceed) the regulatory minimum. For most banks, this leads to increased prudence as represented by an increase in the minimum probability of success a lending project must possess in order to receive a bank loan. Furthermore, a monetary contraction increases bank prudence by raising the opportunity cost of risky investments. The neutral monetary policy rate declines with increases in regulatory capital requirements.

Disyatat (2011) develops a static, general equilibrium framework in which banks finance themselves at a premium over the risk-free rate (henceforth Disyatat model).⁴ This is an important departure from the mainstream where the holding of bank debt is typically remunerated at the risk-free interest rate, either by assuming a deposit insurance or no-arbitrage conditions. Disyatat argues for risky deposits because, in reality, the marginal source of funding for a bank is always uninsured and therefore risk sensitive. In the model, the credit risk premium, compensating depositors for the possibility of bank default, is partly determined by the probability of bank default. The latter, in turn, depends positively on endogenous bank leverage and the riskiness of the real sector. This is an important departure from the conventional role of bank capital as e.g. in Meh and Moran (2010) where bank capital serves to mitigate the moral hazard problem of banks' incentive to shirk on their monitoring duty.⁵ The third model in our comparison builds on the Disyatat (2011).

Before the recent surge in interest in financial sectors in DSGE models, some very fundamental contributions have been made, from which each of the subsequent papers draws to some extent. Most importantly, Bernanke, Gilchrist and Gertler (1999) (henceforth BGG) formalize their views on the interaction of financial conditions and the real economy in a New Keynesian setting as they had expounded previously e.g. empirically in Bernanke (1983) or in a Neoclassical setting in Bernanke and Gertler (1989).⁶ In the BGG model, the cost of external financing of non-financial borrowers depends inversely on their net worth

³The coexistence of banks' limited liability and their private information on the probability of success of the lending projects induces a moral hazard problem which they can reduce by holding more capital.

⁴Another important feature of the Disyatat model is that deposits are driven by loans and not vice versa, the latter being commonly the case in the literature. While this is a very significant and welcome departure from the main stream, our analysis focuses on the role of bank capital and we therefore deliberately refrain from dealing with this feature in our analysis.

⁵This is the classic "double" moral hazard problem as in Diamond (1996).

⁶A very useful overview is provided in Bernanke (2007).

due to a moral hazard problem between the borrowers and the lender à la Townsend (1979). Borrower's net worth varies endogenously with the business cycle and hence their external finance premium does so too. This implies what BGG coin a "financial accelerator" where the evolution of financial variables amplifies real macroeconomic fluctuations. An important limitation is that financial frictions are placed only at the non-financial borrowers. The recent crisis experience however implies importance of considering shocks emanating from within the financial system. This concern is alleviated by Gertler and Kiyotaki (2010) who apply BGG's frictions to the banks in their model and therefore make it possible to analyze banks' influence on business cycle fluctuations.

The DSGE framework developed in Gerali et al. (2010) is our baseline model (henceforth Gerali model). It provides the macro environment in which we compare different approaches to model a banking sector. We use this model for two reasons. First, the main goal of Gerali et al. (2010) is to study the role of banking in business cycle fluctuations, which is well in line with the aim of our Master's thesis to compare different approaches to model bank capital in DSGE models. Gerali et al. (2010) focus specifically on monopolistic competition in the banking sector and bank capital. While our intend is not to study explicitly the role of real frictions such as monopolistic competition, it is nevertheless a desirable feature as other attempts to model bank capital have been nested into monopolistically competitive banking sectors as well and it therefore facilitates comparison. The second reason is that the Gerali model is used by policy makers, e.g. the Riksbank has conducted policy analysis with it, and therefore it is relevant to back-test its quantitative results with different modeling approaches. As in the Disyatat model, in the Gerali model, bank capital does not serve to mitigate a moral hazard problem. Instead, it matters because of an exogenously imposed capital requirement, which is modeled as a quadratic term in the bank's profit function, implying a pecuniary cost of deviating from the exogenous capital ratio requirement. However, the approach to bank capital is also different from the Disyatat model, as banks have unlimited access to a central bank facility and therefore fund themselves at the risk-free rate. Further details of the Gerali model are layed out in the section describing the model.

Jakab and Kumhof (2015), henceforth JK, provide us with an alternative way to model bank capital. However, the key contribution of the paper is that the authors develop a DSGE framework (henceforth "JK model"), in which loans drive deposits—one of the main features of the aforementioned Disyatat (2011). Besides that, their model features a banking sector which operates under an exogenously given capital requirement and where banking branches receive deposits and lend out loans in a monopolistically competitive way.

The capital requirement is imposed on banks in a slightly different way than it is done in the Gerali framework. Importantly, in the JK framework, banks are only punished if their capital ratio falls below the exogenous minimum whereas in the Gerali framework banks are punished both for downward and upward deviations. Another important difference is that the banks' pecuniary costs of deviating from the capital requirement are linear in the deviation for the JK model, while they are quadratic in Gerali et al (2010).

Dib (2010) provides another DSGE model including a banking sector and yet another approach to model the importance of bank capital. An exogenous capital requirement is imposed by letting the banks produce loans with deposits and capital using a Leontief technology. Banks can choose a capital level that is above the minimum requirement, in which case they are rewarded with quadratic gains. Another important feature is that banks can strategically default on their interbank borrowings, which induces a credit spread in the wholesale lending rate.

III Why using DSGEs and a non-technical introduction

The purpose of this Master's thesis is to tease out what different approaches to model bank capital in DSGE models imply for the quantitative results of these models. This clearly requires us to use DSGE models as our main method. However, spending so much time and effort to compare different DSGE models with each other implies that we grant importance to DSGE models as a method in economic research. As well, DSGE modeling is a method that may seem more technical than others. A non-technical introduction can be helpful, for readers that are not yet familiar with it. This holds particularly true given that the targeted audience of this Master's thesis includes other fellow students. Therefore, this section provides a rationale for using dynamic, stochastic general equilibrium modeling to answer macroeconomic questions and gives a non-technical overview of the topics involved. Readers familiar with the topic can easily skip this section.

I Why using DSGEs to answer macroeconomic questions

DSGE modeling is an abstract exercise. It involves setting up maximization problems and solving large systems of equations simultaneously. One main stream criticism often is that any kind of complex, mathematical models are not realistic enough, failing to capture all aspects of the world.⁷ However, it is a fallacy to draw the conclusion that they cannot be useful for solving analytical problems in macroeconomics. Indeed, DSGE models of economies are by definition not realistic. The very purpose of creating a model is to reduce complexity and to allow the analyst to focus on the important aspects of a certain analysis. Another decision is to choose a mathematical model. The use of mathematics is supposed to achieve consistency in arguments. It follows that the concern of economists should not be so much whether their models are realistic or not, they will never be. Instead, focus is placed on whether a model integrates all important aspects of the given analytical problem. Only then a model can yield relevant results. This is the judgemental part of mathematical modeling of economies. When considering which aspects to include in a model, e.g. what economic problems or type of agents, diverse factors can play a role, inter alia the (assumed) balance of cost and benefit of including a certain aspect or dogmatic believes in a certain strand of economic theory.

An alternative to DSGE models for studying macroeconomics would be the use of purely empirical methods such as time series analysis or cross-sectional data analysis. A

⁷For example, already in 1976, Robert Lucas wrote that the large-scale econometric models of that time were often deemed too complex and too simplistic at the same time (Lucas (1976), p. 20).

key advantage of DSGE modeling is that it withstands the so called *Lucas Critique*. The Lucas Critique, expounded in Lucas (1976), states that relationships between macroeconomic variables cannot be inferred solely from historic, aggregate data because they might not be time invariant. Lucas Robert made this point with respect to the assessment of macroeconomic policies. By the time of his writing, policy analysis commonly assumed certain functional forms of the relationship between macro variables. His concern was that the assumed functional forms might be affected themselves by economic policies, thereby undermining the validity of the assumption that they were time invariant. In DSGE models, the relationship between macroeconomic variables are functions of so called “deep parameters.” “Deep parameter” refer to parameters that specify primitive tastes of the respective agents. A famous deep parameter e.g. is the discount factor that households use in forming expectations. It is a reflection of households’ impatience. This primitive taste should not be affected by e.g. monetary policy. Thus, to the extent that the parameters used in DSGE models are indeed time invariant (because they are truly deep parameters), DSGE models offer a method that alleviates concerns expressed in the Lucas Critique.

II What is a DSGE and how does it work

Dynamic, stochastic general equilibrium models (DSGEs) are mathematical models used in modern economic research to study jointly the behavior of macroeconomic (aggregate) variables such as consumption, income and inflation etc. The name already indicates important features:

1. **Dynamic:** The model studies the behavior of the model economy over time.
2. **Stochastic:** The model includes shocks that affect the economy randomly, e.g. productivity shocks.
3. **General equilibrium:** All variables, that are deemed important for the analytical problem at hand, are endogenous, i.e. their behavior is determined from within the model.

A key feature of DSGEs, which distinguishes them from preceding theoretical macroeconomic models is that the behavior of macroeconomic variables is derived from microeconomic optimization problems representing the decision making of all key agents in an economy. Key players are e.g. households, firms, banks, governments and central banks. One example of a macroeconomic variable is the amount of goods and services consumed

by all households in the model economy in a given quarter of a year. A DSGE model would quantify this amount and its evolution over time as well as its relationship to other model variables. One important determinant of how consumption in the model behaves are, of course, the households who consume. They are usually modeled by a mathematical optimization problem that captures what basic decisions an archetypical household faces. For example by assuming that households maximize the utility (some abstract measure of happiness) they derive from consuming, taking into account that they have to give up some of their free time in order to work and finance their consumption. Given that these models are considering the developments in an economy over time, the household's optimization problem is dynamic, meaning that the household takes past and future into account. In summary, the behavior of most variables in a DSGE model is derived from maximization problems, reflecting the respective decision making of the associated agents. The different optimization problems yield a few equations that determine the variables of interest. The set of these equations represents jointly the most important part of a DSGE. In addition, there are some equations that ensure that all variables behave according to the basic principles of a market economy, e.g. that supply equals demand, and others that define composite macroeconomic variables, e.g. the total output of an economy as the sum of private and public consumption and investment. The system of all these equations together represents the DSGE model.

The equations are typically non-linear difference equations and it is therefore not trivial to solve this system. Often times, recourse to approximation techniques is required. The most common approximation technique is the so called log-linearization, which involves two principle steps:

- 1) Taking the natural logarithm of all variables.
- 2) Linearize the logged equations with a first-order Taylor approximations.

For illustration, we log-linearize the Coub-Douglas production function $y_t = A_t k_t^\alpha l_t^{1-\alpha}$. Taking natural logs:

$$\ln(y_t) = \ln(A_t) + \alpha \ln(k_t) + (1 - \alpha) \ln(l_t)$$

First order Taylor series expansion around steady states $(\bar{y}, \bar{k}, \bar{l}, \bar{A})$:

$$\begin{aligned} \ln(\bar{y}) + \frac{1}{\bar{y}}(y_t - \bar{y}) = \\ \ln(\bar{A}) + \frac{1}{\bar{A}}(A_t - \bar{A}) + \alpha \ln(\bar{k}) + \frac{\alpha}{\bar{k}}(k_t - \bar{k}) + (1 - \alpha) \ln(\bar{l}) + \frac{(1 - \alpha)}{\bar{l}}(l_t - \bar{l}) \end{aligned}$$

The steady state production function $\ln(\bar{y}) = \ln(\bar{A}) + \alpha \ln(\bar{k}) + (1 - \alpha) \ln(\bar{l})$ cancels out.

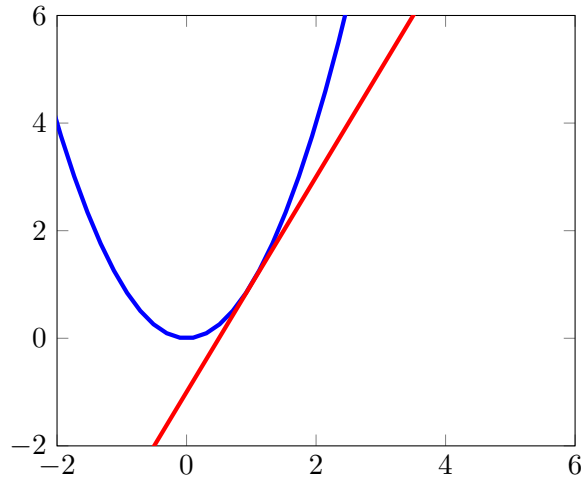
$$\frac{1}{\bar{y}}(y_t - \bar{y}) = \frac{1}{\bar{A}}(A_t - \bar{A}) + \frac{\alpha}{\bar{k}}(k_t - \bar{k}) + \frac{(1 - \alpha)}{\bar{l}}(l_t - \bar{l})$$

We can now define percentage deviations from the steady states, e.g. $\tilde{y}_t = \frac{1}{\bar{y}}(y_t - \bar{y})$. Finally we arrive at the log-linearized production function:

$$\tilde{y}_t = \tilde{A}_t + \alpha \tilde{k}_t + (1 - \alpha) \tilde{l}_t$$

As can be seen easily, the log-linearized production function is now linear in its arguments. This can be done with all equations representing the DSGE, converting a highly non-linear system into a linear one. One caveat of the standard first-order Taylor approximation technique is that the approximation gets less and less accurate the greater the distance from the steady state. To illustrate this, Figure 1 shows with the red line the first-order Taylor approximation of $y = x^2$ (blue line) around $(1,1)$, which is a straight line with a slope of 2. It is clear that the discrepancy between the blue and red line grows with the

Figure 1: Taylor approximation illustration



distance from $(1,1)$. This is exactly the reason why log-linearization is only an accurate approximation technique for the solution of DSGE models as long as one considers small deviations from the steady state of the economy under consideration. As we will show in our analysis, some of the features of the modeling approaches we consider will only become

visible with higher-order approximation techniques.

Dynare is a convenient tool to log-linearize the original system of equations following the steps outlined above.⁸ After Dynare has log-linearized the equations, it can solve for steady state values of the endogenous variables as well as for a dynamic representation of the system that allows to compute impulse response functions etc. Dynare is a free Matlab package that provides a large collection of useful functions that are specifically designed to solve DSGE models. There are many other very interesting details to learn. However, explaining all of them would exceed the scope of this short introduction.

⁸It is also possible to use second order Taylor approximations in Dynare. This is crucial if non-linearities implied by the original model should be taken into account.

IV The DSGE Models: Three approaches to model bank capital

This section lays out the models used in our comparative analysis. Given that we compare different approaches to model the banking sector, the models have exactly the same macro environment and differ only in their banking sectors. For the macro environment, we use the model economy introduced by Gerali et al. (2010). This section starts with an outline of the Gerali model and why we have chosen it to be our baseline. Thereafter, we describe the banking sector as modeled by Jakab and Kumhof (2015), which we nest into the Gerali model. Last but not least, we are presenting the third approach to bank capital in our comparison which is developed by us. This approach is novel to the DSGE literature and based on the static framework provided in Disyatat (2011). To simplify discussion throughout, we adopt the following abbreviations for each model

1. Gerali := The model as presented in Gerali et al. (2010).
2. Gerali-JK := The standard Gerali with a banking sector à la Jakab and Kumhof (2015).
3. Gerali-Disyatat := The standard Gerali with a banking sector à la Disyatat (2011).

I Baseline model including the first approach to bank capital

The Gerali model is a medium scale DSGE model. As such it is enriched with various features of the real economy such as consumption habit formation, capital utilisation as a choice of producers, sticky prices and wages. They typically increase the empirical fit of DSGE models. The main contribution by Gerali et al. was to complement this rich real economy with a banking sector. While details are explained later, an important factor that makes the Gerali model suitable as a baseline is that a bank consists of one perfectly competitive wholesale branch and two monopolistically competitive retail branches (deposit and lending branches). This structure is also featured by models who provide different modeling approaches of bank capital, most importantly also in the Jakab and Kumhof (2015). The common structure facilitates comparison and thus provides a good baseline model. Moreover, given its sophisticated real sector, paired with an interesting banking sector, the Gerali model is used by central banks for policy analysis. Back-testing its quantitative implications with different banking sectors is therefore also of highly practical relevance for policy maker.

i Macro environment

In the following, we describe the basic structure of the real economy in the Gerali model. We refrain from going through technical details because they are not essential to our analysis. However, we provide a thorough description with all equations in the appendix A. The economy consists of three main groups: Patient and impatient households, entrepreneurs, and banks. We will briefly discuss their activities in the given order. Besides that, there are capital goods producers, labour packers and unions. The model is closed by assuming a Taylor-type monetary policy rule that relates deviation from the target inflation rate, the potential real rate and output to the level of the nominal interest rate, allowing for some interest rate smoothing.

Both **households** consume, accumulate housing and supply differentiated labour. The two household groups are different in their degree of impatience: The *patient household* is relatively more patient and therefore prefers to smooth its consumption over time by saving in the form of deposits. To the contrary, the *impatient household* is considerably less patient (hence dubbed impatient) and borrows from the banks (in form of loans). The amount the borrower household can borrow is subject to a collateral constraint which depends on the future value of their stock of housing.

The **entrepreneurs** consume as well, which they finance through the production of intermediate goods which are sold to final goods producers. They also have the possibility to borrow from the bank, for which they have to pledge their physical capital. The production of the intermediate goods is subject to a standard Cobb-Douglas technology. The labour input is hired from a competitive labour packer which bundles the differentiated labour supplied through wage maximising unions by households. And the capital is bought from capital-goods producers. In each period these producers buy the not yet depreciated capital of the previous period from the entrepreneurs and i_t units of final goods, which they transform into capital.

ii Banking block following Gerali et al.

Given its importance in our analysis, we will here describe in detail the **banks**, which consist of three entities: 1. wholesale branches, 2. retail deposit branches and 3. retail lending branches.

The wholesale branch operates in a perfectly competitive market. It rents wholesale deposits (D_t) from the retail deposit branch at the interest rate R^d and lends wholesale loans (B_t) to the retail lending branch at the interest rate R^b . From the balance sheet

constraint follows that the amount lent cannot exceed the amount of wholesale deposits and bank capital (K_t^b). The bank capital is accumulated slowly over time and consists of retained profits from the retail branches. This is dictated by the bank capital law of motion:

$$\pi_t K_t^b = (1 - \delta^b) K_{t-1}^b + j_{t-1}^b \quad (1)$$

where j_{t-1}^b are the profits and δ^b are costs of managing bank capital, and π_t is the gross inflation rate.

The wholesale bank faces an exogenously given target to reach a certain capital ratio v^b . If the wholesale branch deviates from this target ratio it faces quadratic costs proportional to the outstanding capital.

The problem of the wholesale branch can be written as followed:

$$\begin{aligned} \max_{B_t, D_t} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P & \left[(1 + R_t^b) B_t - B_{t+1} \pi_{t+1} + D_{t+1} \pi_{t+1} - (1 + R_t^d) D_t \right. \\ & \left. + (K_{t+1}^b \pi_{t+1} - K_t^b) - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{B_t} - v^b \right)^2 K_t^b \right] \end{aligned}$$

subject to:

$$B_t = D_t + K_t^b$$

where $\Lambda_{0,t}^P = \beta^P \lambda_t^P$ and is the discount factor of the banks. κ_{Kb} is a parameter for the cost of capital ratio deviations.

After substituting the balance sheet constraint into the maximization problem and dividing by K_t^b the derivative with respect to B_t/K_t^b yields the following first order condition:

$$R_t^b = R_t^d - \kappa_{Kb} \left(\frac{K_t^b}{B_t} - v^b \right) \left(\frac{K_t^b}{B_t} \right)^2 \quad (2)$$

Gerali et al. allow the wholesale bank to have unlimited access to lending facility from the central bank at the risk free rate r_t . By the standard no-arbitrage condition, this implies that the wholesale deposit rate must equal the risk free rate, hence $R_t^d = r_t$.

The retail deposit branch is monopolistically competitive and raises funds from the patient households. These funds are then lent to the wholesale bank, remunerated at the risk-free interest rate r_t . Hence to maximise its profits, the retail deposit branch sets r_t^d with a mark-down from the risk-free rate. In order to incorporate interest rate stickiness the deposit branch is subject to quadratic rate adjustment costs. The maximisation problem is

the following:

$$\begin{aligned} \max_{r_t^d(j)} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P & \left[r_t D_t(j) - r_t^d(j) d_t^P(j) - \frac{\kappa_d}{2} \left(\frac{r_t^d(j)}{r_{t-1}^d(j)} - 1 \right)^2 r_t^d d_t \right] \\ \text{subject to: } & D_t(j) = d_t^P(j) \end{aligned}$$

where the constraint comes from the deposit demand curve.

After substituting the demand constraint, the first-order condition is derived and then symmetry imposed. This yields:

$$-1 + \epsilon_t^d - \epsilon_t^d \frac{r_t}{r_t^d} - \kappa_d \left(\frac{r_t^d}{r_{t-1}^d} - 1 \right) \frac{r_t^d}{r_{t-1}^d} + \beta_P E_t \left[\frac{\lambda_{t+1}^P}{\lambda_t^P} \kappa_d \left(\frac{r_{t+1}^d}{r_t^d} - 1 \right) \left(\frac{r_{t+1}^d}{r_t^d} \right)^2 \frac{d_{t+1}}{d_t} \right] = 0 \quad (3)$$

Similar to the retail deposit branch, the retail lending branch operates in a monopolistically competitive market. It receives funds at rate R_t^b from the wholesale branch and grants loans to impatient households and entrepreneurs. The rates offered to households and entrepreneurs may differ, and hence the lending branch also faces separate adjustment costs for each type of rate. Its maximization problem boils down to:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P & \left[r_t^{bH}(j) b_t^I(j) + r_t^{bE}(j) b_t^E(j) - R_t^b B_t(j) \right. \\ & \left. - \frac{\kappa_{bH}}{2} \left(\frac{r_t^{bH}(j)}{r_{t-1}^{bH}(j)} - 1 \right)^2 r_t^{bH} b_t^I - \frac{\kappa_{bE}}{2} \left(\frac{r_t^{bE}(j)}{r_{t-1}^{bE}(j)} - 1 \right)^2 r_t^{bE} b_t^E \right] \\ \text{subject to: } & B_t(j) = b_t^I(j) + b_t^E(j) \end{aligned}$$

where the constraint comes from the loan demand curves.

The first order conditions are then:

$$\begin{aligned} -1 + \epsilon_t^{bS} - \epsilon_t^{bS} \frac{R_t^b}{r_t^{bS}} - \kappa_{bS} \left(\frac{r_t^{bS}}{r_{t-1}^{bS}} - 1 \right) \frac{r_t^{bS}}{r_{t-1}^{bS}} \\ + \beta_P E_t \left[\frac{\lambda_{t+1}^P}{\lambda_t^P} \kappa_{bS} \left(\frac{r_{t+1}^{bS}}{r_t^{bS}} - 1 \right) \left(\frac{r_{t+1}^{bS}}{r_t^{bS}} \right)^2 \frac{b_{t+1}^s}{b_t^s} \right] = 0 \end{aligned} \quad (4)$$

where s is the index for households (H) and entrepreneurs (E).

II Second approach to bank capital, following Jakab and Kumhof (Gerali-JK)

In this section we describe the Gerali-JK model, in particular we focus on the banking sector of Jakab and Kumhof (2015) and explain how we implemented it into the Gerali model.

As mentioned before, to facilitate comparison, the Gerali-JK model has exactly the same macro environment as the original Gerali model, i.e. the same set of agents, optimization problems etc. Only the banking sector has been modified.

i Banking block following Jakab and Kumhof

Just as in the Gerali model the banking block consists of three separate entities: the wholesale branch, the retail deposit branch and the retail lending branch. Both retail branches are modeled in the same fashion, which results in a retail deposit rate with a markdown from the risk-free (central bank) rate, while the retail lending rates to borrower households and entrepreneurs respectively have a markup from the wholesale lending rate.

The difference between the two models is found in the wholesale branch. As stated earlier, in the Gerali model, the pecuniary costs a wholesale bank faces for deviations from the capital requirement take the form of the squared distance of the actual capital ratio to the given requirement. This means that the bank will face costs for both holding too little capital and holding more capital than necessary. Jakab and Kumhof (2015) choose a different way to incorporating such a capital ratio requirement. In their scenario, banks only face penalties for downward deviation. Their modeling approach is akin to the minimum capital ratio as required under Basel II and III. Jakab and Kumhof (2015) state that, in order to have realistic outcomes from the model, banks cannot be assumed to be homogeneous as this feature would result in all banks simultaneously either holding enough capital or not, and paying the penalty (Jakab and Kumhof (2015), p. 17). They resolve this problem by introducing the banks to an idiosyncratic shock. Through this idiosyncratic shock, banks now have to be treated heterogeneously, which, in equilibrium, results in a continuum of capital ratios across banks, and thus only a fraction of the banks face the penalties in each period.

Jakab and Kumhof (2015) introduce the shock to the bank's return on its loan book at the beginning of each period $t + 1$. The shock is modeled following BGG: it is defined by ω_t^b and is identically and independently distributed across time and across banks. It is unit mean log-normally distributed and has variance of $(\sigma_t^b)^2$. Incorporating such a shock in the Gerali setting results in an idiosyncratic shock ω_{t+1}^b to the repayment from the retail

lending branches to the wholesale bank $(1 + R_t^b)B_t$. Thus, the ex-post return for the bank is given by $\omega_{t+1}^b(1 + R_t^b)B_t$. As the bank chooses the amount it borrows and lends before it knows the size of the shock, there is a probability of facing a penalty ex-post. Given the size of the shock, we can write down the penalty condition as:

$$(1 + R_t^b)B_t\omega_{t+1}^b - (1 + R_t^d)D_t + j_{t+1}^B\pi_{t+1} < \gamma(1 + R_t^b)B_t\omega_{t+1}^b$$

where γ is the minimum capital ratio to be held by the banks. This means that, if the the ex-post net return of the wholesale branch and the profit transferred from the retail branches are lower than a certain fraction γ of the total amount of assets, the bank has to pay a penalty fee.⁹ This approach follows very closely the approach used by the Basel Committee, and the γ can be interpreted as the Basel minimum capital adequacy ratio (Jakab and Kumhof (2015), p. 18).

From the penalty condition we can derive the threshold value of ω_t^b which defines the smallest shock (i.e. biggest loss) a bank can suffer and still avoid the penalty. Conversely, if a bank has a shock below the threshold value, it doesn't hold enough capital and faces the penalty costs. The cutoff value of ω_t^b is found by setting the penalty condition equal and solving for $\bar{\omega}_t^b$, the threshold. This results in the following equation:

$$\bar{\omega}_{t+1}^b = \frac{(1 + R_t^d)D_t - j_{t+1}^B\pi_{t+1}}{(1 - \gamma)(1 + R_t^b)B_t} \quad (5)$$

The wholesale bank's optimization problem looks very similar to the Gerali's one. The two differences arise because of the new idiosyncratic shock. On the one hand, the bank faces the shock to its revenue and, on the other hand, it faces costs when falling short of the capital requirements. Given that the shock is idiosyncratic, we have to look at the behavior of a representative bank. Its maximization problem is given by:

$$\max_{B_t} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[(1 + R_t^b)B_t\omega_{t+1}^b - (1 + R_t^d)D_t - B_{t+1}\pi_{t+1}D_{t+1}\pi_{t+1} \right. \\ \left. + (K_{t+1}^b\pi_{t+1} - K_t^b) - \chi\pi_{t+1}B_tF(\bar{\omega}_{t+1}^b) \right]$$

subject to:

$$B_t = D_t + K_t$$

⁹The ex-post net return of the wholesale branch are ex-post payments received from the retail branch minus repayments to the retail deposit branch.

where $F(\bar{\omega}_t^b)$ is the cumulative density function of the idiosyncratic shock, and thus defines the probability of having a shock lower than the threshold value. χB_t are the costs of falling short of the capital requirements as share of the assets held.

After substituting the budget constraint for t and $t + 1$, dividing by K_t^b and taking the derivative with respect to $\frac{B_t}{K_t^b}$ this results in the following FoC:¹⁰

$$R_t^b - R_t^d - \chi \pi_{t+1} \left[F(\bar{\omega}_{t+1}^b) + f(\bar{\omega}_{t+1}^b) \left[\frac{(1 + R_t^d) + \frac{j_{t+1}^B \pi_{t+1}}{K_t^b}}{(1 - \gamma)(1 + R_t^b) \frac{B_t}{K_t^b}} \right] \right] = 0 \quad (6)$$

where $f(\bar{\omega}_t^b)$ is the probability density function of the idiosyncratic shock.

The FoC from the Gerali-JK model looks very similar to the FoC of the wholesale bank in the Gerali model. As in the original, the wholesale lending rate has a premium over the wholesale deposit rate, hence over the risk-free rate. And the margin is also defined by the cost of non-compliance with the capital ratio. The difference being, that the costs only occur if the bank falls short of the capital ratio (as opposed to up- and downward penalty in the Gerali model).

Before implementing the JK wholesale branch conditions into the model, j_t^b , the banks profit function, has to be adapted to the new penalty term. This results in:

$$j_t^b = r_t^{bH} + r_t^{bE} - r_t^d - Adj_t^b - \chi B_t F(\bar{\omega}_t^b) \quad (7)$$

where Adj_t^b are the different quadratic adjustment costs for changing rates over time faced by the two retail facilities.

The equations 5, 6, 7 and the balance sheet constraint make up the conditions needed to implement the JK's wholesale bank into the Gerali model. After implementation the Gerali-JK model can be calibrated and checked for its robustness.

ii Calibration of the parameters

We describe the calibration of the Gerali-JK model used in section V for comparison to the Gerali and the Gerali-Disyatat model. The calibration of the Gerali-JK model follows very closely the calibration used for the original Gerali model for the better part of the model. As the only changes made to the model happen in the financial sector the calibration of parameters regarding this sector differs from the Gerali model. For this calibration we look

¹⁰As $E_0[\omega_t^b] = 1$, $E_0[\omega_t^b R_t^d] = R_t^b$.

at the calibration of the original JK model, and calibrate the parameters accordingly.

Table 1: Calibrated parameters of the Gerali-JK model

| Parameter | Description | Value |
|--------------|---|---------|
| β_P | Patient households' discount factor | 0.9943 |
| β_I | Impatient households' discount factor | 0.975 |
| β_E | Entrepreneurs' discount factor | 0.975 |
| ϵ^h | Weight on housing | 0.2 |
| ϕ | Invers of the Frisch elasticity of labour supply | 1 |
| m_t^I | Impatient households' LTV ratio on mortgages | 0.7 |
| m_t^E | Entrepreneurs' LTV ration on loans | 0.35 |
| α | Capital share in the production function | 0.25 |
| δ | Depreciation rate of physical capital | 0.025 |
| γ | Minimum capital adequacy ratio | 0.09 |
| δ^b | Managing cost of bank capital | 0.0825 |
| χ | Penalty coefficient | 0.00325 |
| ξ_1 | Parameter of adjustment cost for capacity utilization | 0.0487 |
| ξ_2 | Parameter of adjustment cost for capacity utilization | 0.00478 |

Note: ξ_1 and ξ_2 taken as is from Gerali et al. (2010). Make up the adjustment costs for the utilization.

In essence this results in the same values for the discount factors of patient and impatient households and entrepreneurs which are set at 0.9943 for the former and 0.975 for the two latter following Gerali et al. (2010).¹¹ As in the Gerali model the weight on housing in the households utility is set at 0.2, while the weight on consumption is taken from the posterior estimates following a Bayesian estimation. The inverse of the Frisch elasticity (of labour supply) is kept at 1. The borrower households' LTV ratio for mortgages, m_t^I , is set at 0.7, the same goes for m_t^E , the LTV ratio for the entrepreneurs which is equal to 0.35. Regarding the entrepreneurs' production function we also follow Gerali and set α at 0.25. The natural depreciation rate of physical capital is set at 0.025. And the markups in the goods and in the labour market and for the three retail rates are kept the same. For comparison necessities, we set γ , the minimum capital adequacy ratio, equal to v^b from the Gerali model at 9%, which is one percentage point higher as in the JK model. The cost of managing capital, δ^b , is calibrated at 0.0825 such that, according to JK, in equilibrium

¹¹The discount factor for the saver household is calibrated in the original paper such that the r_t^d is equal to the rate on M2 deposits in their sample, while the discount factor for the borrower households and the entrepreneurs is taken from Iacoviello (2005).

banks hold a capital buffer of 2.5 percentage points, resulting in an effective capital ratio of 11.5% in our setting. Bank riskiness $(\sigma_t^b)^2$ is also calibrated following JK's description such that in equilibrium 2.5% of the banks violate the minimum capital requirement. In order to achieve these calibrations, χ the penalty coefficient is set at 0.00325.

For the exception of ρ_σ , the structural parameter for the exogenous process of the banks' riskiness given by $(\sigma_t^b)^2$, the remaining structural parameters for both endogenous equations, as well as the exogenous AR(1) processes are all taken from the posterior estimates from the Bayesian estimation done for the Gerali model. Bank riskiness is typically a fairly stable measure as we would expect the riskiness to stay low in a positive economic environment and high in a bad economic environment, ρ_σ has thus been set quite high at 0.920 which suggests a strong persistence of past values.

iii Robustness to changes in parameters

In this section we look at the robustness of the Gerali-JK model, and its behavior when we change one of the parameters at the time. Overall the model behaves as expected. Steady state values are robust and reasonable. Table 2 depicts steady state values for the main measures of interest for different values of the penalty coefficient χ . As we would expect, an increase of the penalty costs, increases the spread between the two wholesale interest rate given by *spread_w*. Looking at the Equation 6, an increase of χ means that the right term increases, and thus the difference between the two rates, all things equal, must increase. Furthermore, as the risk-free interest rate does not depend on χ , the increase in the spread is solely based on an increase in R_t^b . The consequence is that both retail lending rates increase, and therefore overall borrowings retract. Given the demand elasticities for loans from households and entrepreneurs, when increasing the penalty cost we see a slow shift of the loans held from entrepreneurs towards households. The capital ratio is increasing in the penalty coefficient. Due to the markup resulting from the monopolistically competitive setting of the retail branches, the impact of an increase in the penalty coefficient is higher for the retail lending rates than for bank capital. Hence bank capital falls by less than the amount of outstanding loans for a given increase. Similarly the overall output falls by less than the other macroeconomic variables, which results in a reduction of per output variables.

Table 3 shows the steady state values for different values of δ^b this time. Similar to Table 2, the center column are the steady state values of the baseline model with capital

Table 2: Calibration of Gerali-JK with respect to χ

| $\chi =$ | 0.000325 | 0.00325 (Baseline) | 0.0325 |
|---------------|-----------|--------------------|---------|
| C/Y | 0.89254 | 0.89049 | 0.88243 |
| I/Y | 0.14621 | 0.14358 | 0.1333 |
| K/Y | 5.8485 | 5.7434 | 5.332 |
| r^d | 2.2931 | 2.2931 | 2.2931 |
| r_{ib} | 3.8634 | 3.8634 | 3.8634 |
| $spread_w$ | 0.0049516 | 0.042996 | 0.2126 |
| r_{bh} | 6.0279 | 6.0872 | 6.3515 |
| r_{be} | 5.6929 | 5.7489 | 5.9985 |
| spread | 3.5673 | 3.625 | 3.8819 |
| bh/(bh+be)(%) | 30.493 | 30.75 | 31.7783 |
| be/(bh+be)(%) | 69.507 | 69.25 | 68.2217 |
| B/Y | 2.8311 | 2.7901 | 2.6277 |
| D/Y | 2.5085 | 2.4696 | 2.3143 |
| K^b/B | 0.11392 | 0.11487 | 0.11927 |
| K^b/Y | 0.32252 | 0.32049 | 0.31339 |

K^b is bank capital, B total loans, D total deposits, be loans to entrepreneurs, bh loans to HH, r_{ib} policy rate, $spread_w$ spread between wholesale lending and deposit rate, spread is the spread between the retail lending rate and the policy rate, r^d, r_{bh}, r_{be} are retail interest rates. All interest rates and spreads are annual, in percentages.

managing cost coefficient of 0.0825. Overall the model responds more strongly to a change in δ^b , than to the change in χ . While we change χ by a factor 10 the increase in the spread between the wholesale lending and deposit rate is of approximately a factor 5, compared to an 1.24 fold increase of δ^b resulting in a spread increase by a factor over 7. This stronger reaction can be observed through out the steady state values: while the sign of the change is the same as after a change in χ , the change in absolute as well as in relative terms is much more pronounced. With higher managing costs, wholesale banks need to increase their profits, thus they increase the retail lending rates. This results in a fall in loan demand, and thus production, and by consequence consumption falls—resulting in lower output, overall physical capital and investments. However, while in the case of a positive change in χ the capital ratio increases, when δ^b is increased, the capital ratio falls. The reason being that, in this scenario, the bank capital falls faster than the borrowings, as the bank tries to increase its profits by extending relatively more loans.

As in the case of χ , a shift of the loan demand from entrepreneurs to impatient households can be observed. The same reasoning can be applied here: the elasticity of substitution for entrepreneurs is higher and hence they respond more to a change in interest

rates.

Table 3: Calibration of Gerali-JK with respect to δ^b

| $\delta^{kb} =$ | 0.062 | 0.0825 (Baseline) | 0.1025 |
|-----------------|----------|-------------------|---------|
| C/Y | 0.89216 | 0.89049 | 0.87961 |
| I/Y | 0.14656 | 0.14358 | 0.12924 |
| K/Y | 5.8625 | 5.7434 | 5.1695 |
| r^d | 2.2931 | 2.2931 | 2.2931 |
| r_{ib} | 3.8634 | 3.8634 | 3.8634 |
| $spread_w$ | 2.06E-05 | 0.042996 | 0.29014 |
| r_{bh} | 6.0202 | 6.0872 | 6.4723 |
| r_{be} | 5.6856 | 5.7489 | 6.1126 |
| spread | 3.5599 | 3.625 | 3.9994 |
| bh/(bh+be)(%) | 30.4587 | 30.75 | 32.1933 |
| be/(bh+be)(%) | 69.5413 | 69.25 | 67.8067 |
| B/Y | 2.8365 | 2.7901 | 2.5625 |
| D/Y | 2.4 | 2.4696 | 2.3185 |
| K^b/B | 0.15389 | 0.11487 | 0.09523 |
| K^b/Y | 0.43652 | 0.32049 | 0.24403 |

K^b is bank capital, B total loans, D total deposits, be loans to entrepreneurs, bh loans to HH, r_{ib} policy rate, $spread_w$ spread between wholesale lending and deposit rate, spread is the spread between the retail lending rate and the policy rate, r^d, r_{bh}, r_{be} are retail interest rates. All interest rates and spreads are annual, in percentages.

As in the case depicted above, similar observations can be made when looking at the models reaction when comparing the changes in steady state values after changing the initial variance of the idiosyncratic shock. As we would expect, to counter this risk of falling below the capital adequacy ratio, banks increase their capital holdings and thus the capital buffer increases further. This is done by raising the interest rates on loans. The consequences of this are the same than in the first case after an increase in χ . The opposite is true if the variance is reduced. From this exercise we can conclude that the Gerali-JK model withstands different changes of parameters and yields robust results and steady state values.

iv Asymmetries with second-order approximations

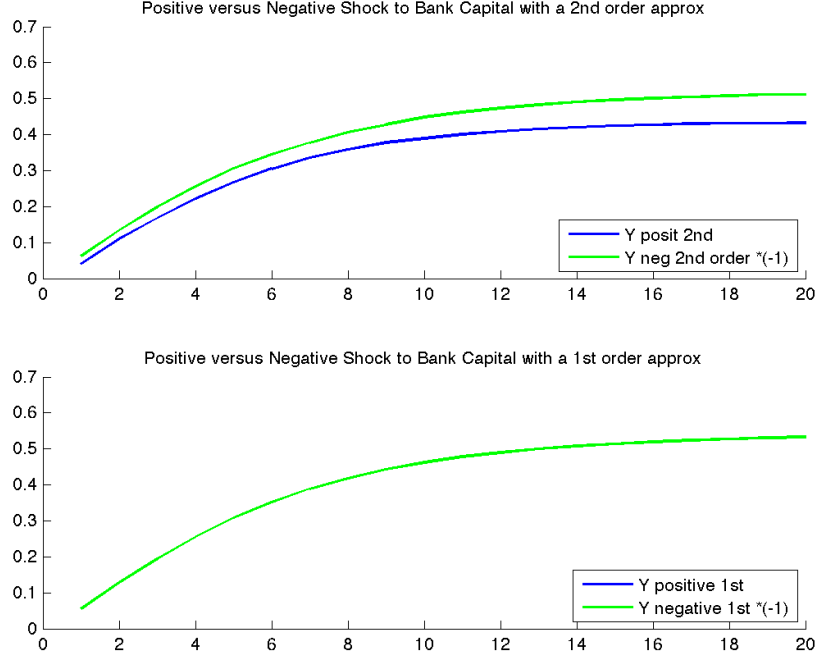
In the bank capital approach of JK only downward deviations are penalized. This should imply an asymmetric impact of a shock to bank capital: an unexpected *fall* of bank capital

will, without further action of the bank, trigger penalty payments because the capital ratio falls below the regulatory minimum. This is not the case for an unexpected *increase* in bank capital. Given the way the capital requirement is formulated, the sudden increase in capital would not entail any deviation costs.

However, this asymmetry is not going to show up in the Dynare simulations using default options. Dynare log-linearizes the equations before solving the model. Specifically, after applying the log-function to each variable, the variables are approximated by the first-order Taylor approximation. This implies that originally non-linear terms become linear. This technique strongly simplifies the solution of the system of equations—as it is now linear. In this respect, we conduct the following experiment to confirm our intuition and to quantify the difference. We solve the model again but this time with a second-order approximation and simulate both a positive and a negative shock to bank capital. While the solution with a second-order approximation is far more complex, the impulse responses will take into account non-linearities such as the aforementioned asymmetry. Therefore, we can verify whether the asymmetry exists by comparing the magnitude of the responses to a positive and to a negative shock to capital for a first order approximation with the magnitudes obtained with a second order approximation. This is shown in Figure 2. To facilitate the comparison between the magnitude of the response to a negative shock and the magnitude of the response to a positive shock, we have reversed the sign of the impulse response function of the negative shock, i.e. multiplied it by -1 . This is also reflected in the labels in Figure 2. The lower panel shows the results for the first-order approximation. The line for the impulse response to a positive shock matches perfectly the line for the reversed-sign impulse response to a negative shock—the responses are perfectly symmetric.

In the upper panel, showing the second-order approximations, this is not the case. The magnitude of response to a positive shock is smaller than the one of a negative shock, which is reflected in the line for a positive shock being below the line for the reversed-sign impulse response to a negative shock. In other words, under a second-order approximation, output falls more after an unexpected fall in bank capital than it increases after an unexpected increase in bank capital, which is in line with our expectations: given that banks only face a penalty fee if their capital ratio is lower than a target ratio, and the probability of this happening is higher with a negative shock, the economy’s trajectory should be more pronounced in case of a negative shock.

Figure 2: IRF: Response of output to bank capital shock, deviations in basis points

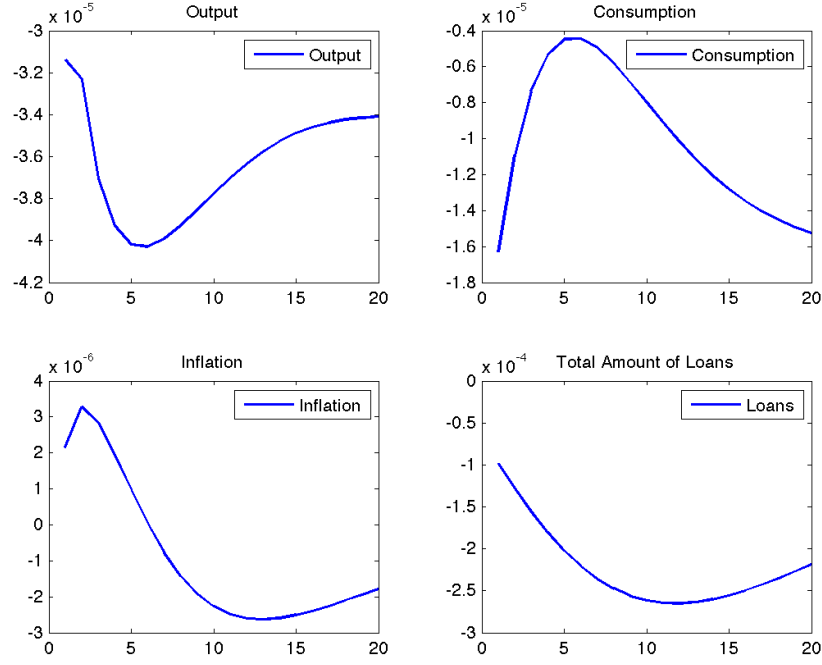


Note: Variables are shown in percentage deviations from the steady state.

v Impulse responses to an uncertainty shock

In this section we scrutinise the effect of a sudden increase in banks' riskiness. The variance of the idiosyncratic shock to the banks' loan books follows an AR(1) process. This set-up allows us to look at the impulse responses of the model economy to an unexpected shock to this bank riskiness measure. The reaction of four non-financial variables to an unanticipated increase of the banks' riskiness are shown in Table 3, while Table 4 shows the reaction of the financial sector. With an increase in the variance of the idiosyncratic shock to the loan book, the number of banks below the threshold $\bar{\omega}_t^b$ increases. As a consequence banks' capital holdings decrease in the first three quarter as a higher number of banks have to pay a penalty. Thereafter it starts to increase and reaches a positive deviation from the steady state. The increase starting in period four is caused by higher profits from the retail lending branches. The latter increases because the retail lending rates start to rise while the funding costs of the retail lending branch are already falling again. Faced with higher rates, borrowers from the household group as well as entrepreneurs reduce their demand for loans. After about 14 quarters the deviation of the bank capital decreases again as

Figure 3: Impulse responses of the non-financial sector to a shock to the variance $(\sigma^b)^2$

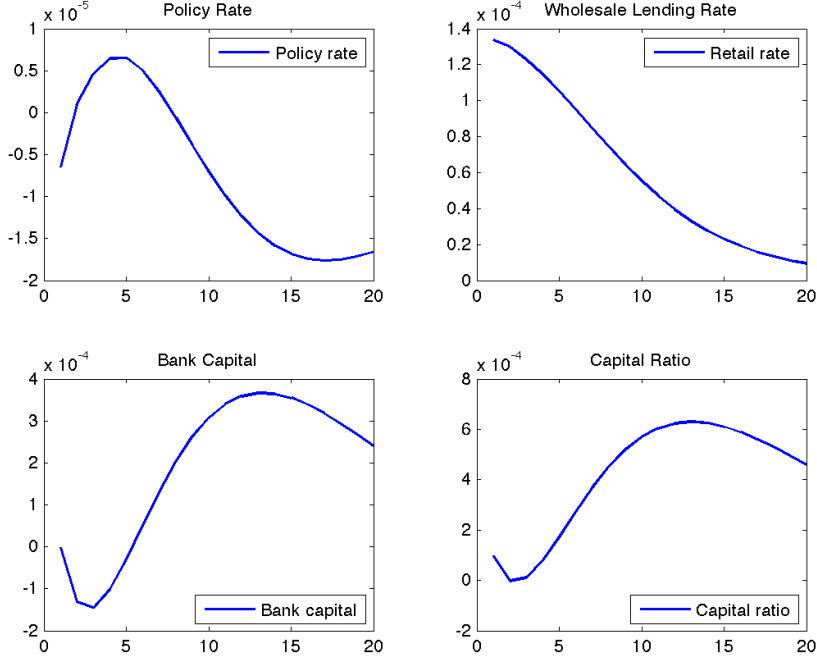


Note: All rates are given as percentage point deviations from the steady state while the other variables are percentage deviations from the steady state.

the shock on loans and deposits recovers less fast than the banks' margin falls back to its steady state. The reduction in loan demand reduces consumption and production. As the decrease in overall capital held by capital producers in the economy is lower, inflation initially increases. In response to this the central bank first increases the policy rate, but after about 5 quarters, in response to contractionary tendencies in the real economy, the central bank reduces its policy rate. The impulse response of the model economy's consumption reflects these developments: Consumption first falls drastically as, because of the suddenly higher rates, borrowers reduce their consumption. This reduction is then partly offset by the higher rates on deposits received by the patient households.¹² Once the central bank starts lowering the policy rate, patient households also cut back on their consumption, and thus overall consumption falls again.

¹²The deposit rate is at a fixed mark-down from the policy rate.

Figure 4: Impulse responses of the financial sector to a shock to the variance $(\sigma^b)^2$



Note: All rates are given as percentage point deviations from the steady state while the other variables are percentage deviations from the steady state.

III Third approach to bank capital, following Disyatat (Gerali-Disyatat)

This section describes our own approach to bank capital in a DSGE, which is based on the approach developed in a static context by Disyatat (2011). The macro environment of the Gerali-Disyatat model is, again, for comparison reasons derived from the original Gerali model. Except for the banking sector and the patient households, the model follows exactly Gerali et al. (2010).

This approach to bank capital is novel to the DSGE literature in that banks finance themselves at a credit risk premium on top of the risk-free rate. The credit risk premium is demanded by households as compensation for the default probability of banks. This default probability depends positively on the banks' leverage ratio or, conversely, negatively on the capital ratio. That is because the higher the banks' leverage ratio, the more vulnerable they are to shocks on their loan books. This incentivises banks to maintain a capital ratio that is above conventional levels of minimum capital ratios, absent of any exogenous regulatory requirement. A key financial frictions of the model are bankruptcy costs that make default

a wasteful event. In the event a bank defaults on its deposits, households get access to the bank's remaining assets and bank capital, but that is associated to value-destruction. Therefore households lose part of their claims in form of deposits.

Before the model details are developed, the next section briefly discusses the validity of Disyatat's argument for considering deposits on which banks can default and whose deposit rates need to compensate the depositors for this default risk.

Introducing risky deposits is a significant departure from the literature where deposits are almost in every case remunerated at the risk-free rate. Disyatat (2011) argues that the possibility of default on bank deposits is important as the marginal source of funding for banks is usually market funding and hence risk-sensitive. The sensitivity of funding costs to the debtor's credit risk should depend on the respective contract underlying the claim of the creditor. The assumption that banks can fund themselves at the risk-free rate is commonly justified with the existence of a deposit insurance scheme. Indeed, in the event of bank default, depositors are then guaranteed a payout from the deposit guarantee fund. The loss given default is then reduced by that amount which mitigates the credit risk inherent to the bank. Another type of claim that has reduced exposure to credit risk are repurchase agreements, which are collateralized. Bank liabilities that are sensitive to the credit risk of a bank are e.g. equity, subordinated and senior unsecured claims. Ferenius and Tietz (2016) document that the share of the latter instruments is around 22% on aggregate for the four major Swedish banks. The less granular data in BCBS (2010) indicates that this is also the case for a sample encompassing banks in 13 advanced economies between 1993 and 2007. These finding suggests that banks' funding costs are at least to some extent risk-sensitive. This is consistent with evidence reported by Gambacorta and Shin (2016) showing that banks' debt funding costs decrease as their capital ratio increases. Babihuga and Spaltro (2014) come to a similar conclusion. They find that a bank's unsecured funding costs react to changes in the bank's credit worthiness and more importantly on the bank's level and quality of capital.

i Banking block of the Gerali-Disyatat model

This section describes the approach taken to model the banking sector in the Disyatat approach, and the impact on the patient households maximization problem.

As in the Gerali-JK approach, a bank's loans are subject to an idiosyncratic shock ω_{t+1}^b that realises only after the lending decisions are made. ω_{t+1}^b is assumed to be log-normally distributed with a unit mean, implying that, in expectation, banks will always

receive back the full amount of their outstanding loans. However, a negative shock might realise and diminish the banks' resources to pay back their borrowings. We assume that bank equity holders have to absorb losses, before the bank defaults on its deposits. Hence, the default condition for a bank is given by

$$\omega_{t+1}^b(1 + R_t^b)B_t + K_t^b < (1 + R_t^d)D_t \quad (8)$$

The interpretation of Equation 8 is straight forward. It states that a bank defaults on its deposits when its resources, made up by repayments from loans outstanding and their own bank capital, are less than its repayment obligation for the deposits, that is the principal D_t and interest rate payments $R_t^d D_t$. Solving Equation 8 for ω_{t+1}^b with the balance sheet constraint $B_t = D_t + K_t^b$ plugged-in gives a threshold $\bar{\omega}_{t+1}^b$ below which the bank defaults:

$$\bar{\omega}_{t+1}^b = \frac{1 + R_t^d - (2 + R_t^d) \frac{K_t^b}{B_t}}{(1 + R_t^b)} \quad (9)$$

Note that, with $\frac{\partial \bar{\omega}_{t+1}^b}{\partial (\frac{B_t}{K_t^b})} = \frac{2 + R_t^d}{(1 + R_t^b)(\frac{B_t}{K_t^b})^2} > 0$, the threshold increases with the leverage ratio, meaning that the higher the leverage ratio, the smaller the minimum amount of losses that will push banks into default.

In case the bank defaults, it will lose the entirety of its loan book as well as its capital due to bankruptcy proceedings, formally expressed as $\int_0^{\bar{\omega}_{t+1}^b} (\omega(1 + R_t^b)B_t + K_t^b) f(\omega) d\omega$. The remaining value of a bank's loan book is given by $\omega(1 + R_t^b)B_t$. As all banks below the threshold $\bar{\omega}_{t+1}^b$ default, in order to have the overall value of the loan books of defaulting banks, we integrate over the values from 0 to the $\bar{\omega}_{t+1}^b$. To complete the equation the bank capital has to be taken into account as well.

The wholesale banks' optimization problem is

$$\begin{aligned} \max_{B_t} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P & \left[(1 + R_t^b)B_t \omega_{t+1}^b - (1 + R_t^d)D_t(1 - F(\bar{\omega}_{t+1}^b)) - B_{t+1}\pi_{t+1} + D_{t+1}\pi_{t+1} \right. \\ & \left. + (K_{t+1}^b\pi_{t+1} - K_t^b) - \int_0^{\bar{\omega}_{t+1}^b} (\omega(1 + R_t^b)B_t + K_t^b) f(\omega) d\omega \right] \end{aligned}$$

subject to:

$$B_t = D_t + K_t$$

where $f(\bar{\omega}_{t+1}^b)$ and $F(\bar{\omega}_{t+1}^b)$ are the probability and cumulative density functions for $\bar{\omega}_{t+1}^b$.

Substituting the budget constraint for t and $t + 1$ and dividing by K_t^b , before taking the derivative w.r.t. $\frac{B_t}{K_t^b}$ yields the first order condition:

$$\begin{aligned}
& (1 + R_t^b) - (1 - F(\bar{\omega}_{t+1}^b))(1 + R_t^d) + f(\bar{\omega}_{t+1}^b) \frac{2 + R_t^d}{(1 + R_t^b) \frac{B_t}{K_t^b}} (1 + R_t^d) \\
& - f(\bar{\omega}_{t+1}^b) \frac{2 + R_t^d}{(1 + R_t^b) \left(\frac{B_t}{K_t^b}\right)^2} (1 + R_t^d) - \bar{\omega}_{t+1}^b f(\bar{\omega}_{t+1}^b) (2 + R_t^d) \frac{K_t^b}{B_t} - G(\bar{\omega}_{t+1}^b) (1 + R_t^b) \\
& - f(\bar{\omega}_{t+1}^b) \frac{2 + R_t^d}{(1 + R_t^b) \left(\frac{B_t}{K_t^b}\right)^2} = 0
\end{aligned} \tag{10}$$

where $G(\bar{\omega}_{t+1}^b) = \int_0^{\bar{\omega}_{t+1}^b} \omega f(\omega) d\omega$ is the average ω_{t+1}^b , conditional on $\omega_{t+1}^b < \bar{\omega}_{t+1}^b$.

For the retail lending branch the same approach and equations are used as in the Gerali model. Retail lending branches get their funding from the wholesale branch at the wholesale lending rate R_t^b and set the retail lending rates r_t^{bE} and r_t^{bH} at a markup to maximize their profits. For simplicity the retail deposit branch operates in a perfectly competitive setting in the Gerali-Disyatat model. This allows us to set $r_t^d = R_t^d$ and to close the model.

Updating the banks' profit function gives:

$$\begin{aligned}
j_t^b = & r_t^{bH} b_t^H + r_t^{bE} b_t^E - (1 - F(\bar{\omega}_t^b)) r_t^d d_t - G(\bar{\omega}_t^b) (1 + R_t^b) B_t - F(\bar{\omega}_t^b) K_t^b \\
& - \frac{\kappa_{bH}}{2} \left(\frac{r_t^{bH}(j)}{r_{t-1}^{bH}(j)} - 1 \right)^2 r_t^{bH} b_t^H - \frac{\kappa_{bE}}{2} \left(\frac{r_t^{bE}(j)}{r_{t-1}^{bE}(j)} - 1 \right)^2 r_t^{bE} b_t^E
\end{aligned} \tag{11}$$

ii Patient households

Patient households deposit their savings at the banks, knowing about the risk of bank default and demanding a compensation for it. The opportunity cost of bank deposits is a risk-free deposit facility, an implicit investment alternative for the household. In case of bank default, the household will sue in court for its money and get access to the bank's remaining assets and equity, but only subject to a bankruptcy cost δ^c , expressed as a share of its original claims D_t . As explained above, the bankruptcy costs are an important financial friction as they imply value-destruction as result of default. This setup implies the following

no-arbitrage condition for the impatient households:

$$\begin{aligned} (1 + r_t^{ib})D_t &= [1 - F(\bar{\omega}_{t+1}^b)]D_t(1 + R_t^d) + \int_0^{\bar{\omega}_{t+1}^b} (\omega(1 + R_t^b)B_t + K_t^b - \delta^c D_t) f(\omega) d\omega \\ &= [1 - F(\bar{\omega}_{t+1}^b)]D_t(1 + R_t^d) + (1 + R_t^b)B_t G(\bar{\omega}_{t+1}^b) + (K_t^b - \delta^c D_t)F(\bar{\omega}_{t+1}^b) \end{aligned} \quad (12)$$

The no-arbitrage condition states that the return from holding deposits at the risky bank must be equal to the return from a risk-free deposit facility.

Given the new repayment schedule for the patient households, the budget constraint of a representative patient household is equal to:

$$\begin{aligned} c_t^P(i) + q_t^h \Delta h_t^P(i) + d_t^P(i) &\leq \\ w_t^P l_t^P(i) + (1 - F(\bar{\omega}_t^b)) \frac{1 + r_{t-1}^d}{\pi_t} d_{t-1} + G(\bar{\omega}_t^b) \frac{1 + R_{t-1}^b}{\pi_t} \hat{B}_{t-1} + F(\bar{\omega}_t^b) \frac{\hat{K}_{t-1}^b - \delta^c d_{t-1}}{\pi_t} + t_t^P(i) \end{aligned} \quad (13)$$

From this we get the FoCs:

$$\begin{aligned} \lambda_t^P &= \epsilon_t^z (1 - a^P) / (c_t^P - a^P c_{t-1}^P) \\ \lambda_t^P q_t^h &= \epsilon_t^h \frac{j}{h_t^P} + \beta^P E_t[\lambda_{t+1}^P q_{t+1}^h] \\ \lambda_t^P &= \beta^P E_t[\lambda_{t+1}^P \frac{1}{\pi_{t+1}} ((1 - F(\bar{\omega}_t^b))(1 + r_t^d) - F(\bar{\omega}_{t+1}^b)\delta^c)] \end{aligned} \quad (14)$$

where λ_t^P is the Lagrange multiplier of the budget constraint, and \hat{B}_t and \hat{K}_t^b are the average share of the loans and bank capital a representative household receives if the bank defaults. Equations 10, 11 and the borrowing constraint make up the system of equation implemented into Matlab for the banking sector, while 12, 13 and 14 make up the system of equations for the impatient households.

iii Calibration of the parameters

The calibration of the Gerali-Disyatat follows very closely the calibration of the Gerali model. Hence the discount factor for the patient household, impatient and entrepreneurs are kept equal to 0.9943, 0.975 and 0.975 respectively. Weights on housing and on consumption, as well as the inverse of the Frisch elasticity are kept the same. The two LTV ratios also have the same values as in the previous models. And the entrepreneurs parameters are all kept the same. For comparison necessities, δ^b is calibrated at 0.015. The bankruptcy costs δ^c for household is calibrated at 0.3.

Table 4: Calibrated parameters of the Gerali-Disyatat model

| Parameter | Description | Value |
|--------------|---|--------|
| β_P | Patient households' discount factor | 0.9943 |
| β_I | Impatient households' discount factor | 0.975 |
| β_E | Entrepreneurs' discount factor | 0.975 |
| ϵ^h | Weight on housing | 0.2 |
| ϕ | Invers of the Frisch elasticity of labour supply | 1 |
| m_t^I | Impatient households' LTV ratio on mortgages | 0.7 |
| m_t^E | Entrepreneurs' LTV ration on loans | 0.35 |
| α | Capital share in the production function | 0.25 |
| δ | Depreciation rate of physical capital | 0.025 |
| c | Bankruptcy costs for patient households | 0.3 |
| δ^b | Managing cost of bank capital | 0.015 |
| ξ_1 | Parameter of adjustment cost for capacity utilization | 0.045 |
| ξ_2 | Parameter of adjustment cost for capacity utilization | 0.0045 |

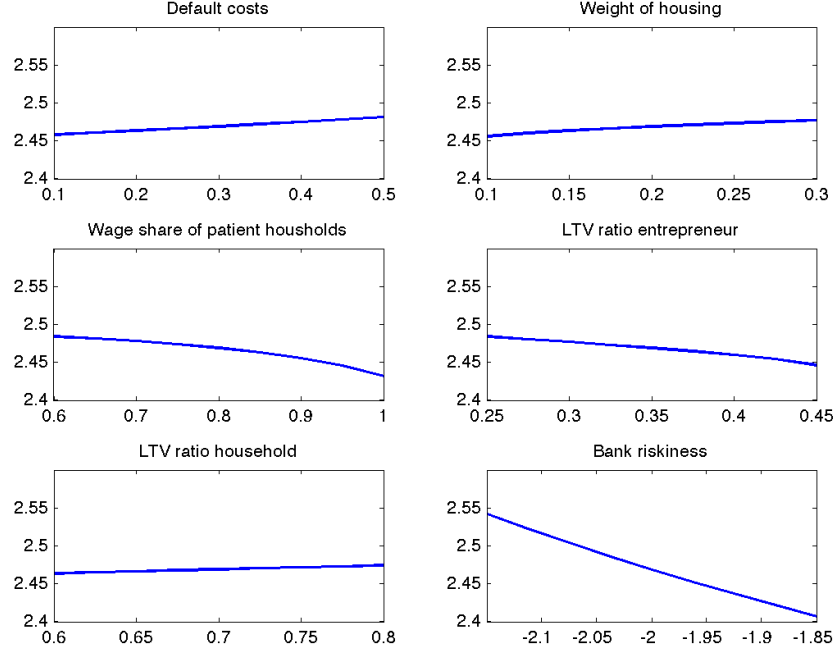
The structural parameters of the exogenous processes are all taken from the posterior from the Bayesian estimation done by Gerali et al. (2010). The structural parameter for the banks' riskiness is kept at 0.920 as in the Gerali-JK model.

iv Robustness to changes in parameters

In this section the behavior of the steady state values of the Gerali-Disyatat model is investigated. First off, the steady state values of the baseline model have reasonable and consistent values. However, in the Gerali-Disyatat model the relationship between the policy rate and the retail deposit rate differs from the relationship in the Gerali and Gerali-JK model. In the two previous models the bank financed itself at a markdown from the policy rate, while in the Gerali-Disyatat model the bank faces a premium on the policy rate. Therefore it is of interest to look at the steady state policy rate for different parametrizations.

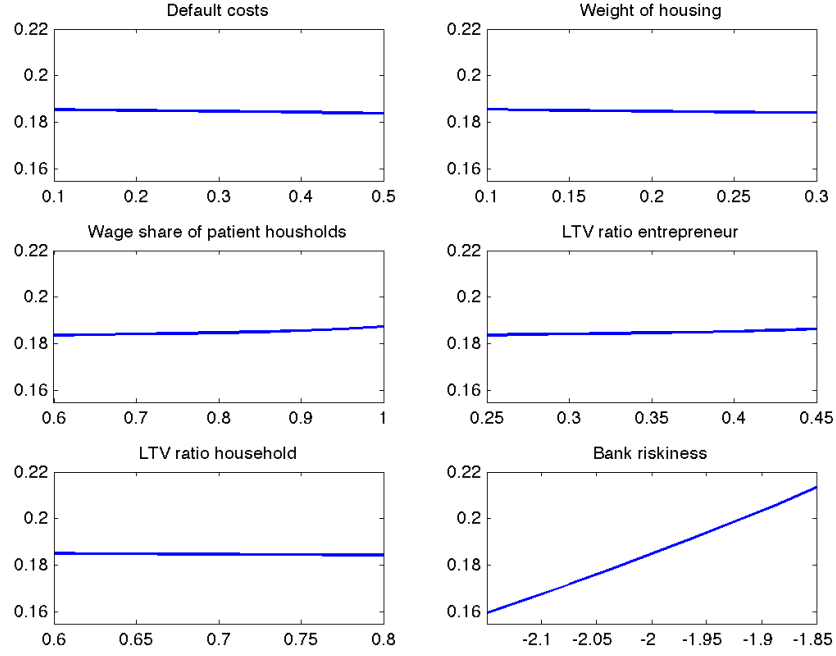
Figure 5 shows, for different parameters, the steady policy rate for a range of parameter values. The same applies for Figure 6 but steady state values of the bank leverage ratio are depicted. Each time holding the remaining parameters at their baseline values. The Figure 6 shows that, while an increase in the bankruptcy costs δ^c , the weight on housing j or the LTV-ratio of households m^I increase the natural policy rate, an increase in the wage share of patient households μ , in the entrepreneurs' LTV ratio or in the bank riskiness $(\sigma_t^b)^2$ lowers the steady state policy rate. The mechanism behind the positive relationship

Figure 5: Steady state values of the policy rate (horizontal axis) for different parameter values.



between the steady state policy rate and the weight on housing and the LTV ratio for households can be explained by the same reasoning. In the first case, the higher weight on housing leads to a higher steady state price for housing, which allows impatient households to borrow more. To mitigate this, the natural policy rate is set at a higher level. This has two effects: First, it increases the costs for borrowers, which *ceteris paribus* reduces the demand pressure for borrowings. Second, it also leads to higher deposit rates, which, all else equal, raises deposit demand and thus helps the banks to accommodate the increased loan demand by taking more deposits. In the second case the higher LTV ratio also allows households borrow more, and thus for the same reasons the policy rate is increased. Increasing the wage share of patient households results in a lower natural policy rate. The explanation is that by lowering the policy rate, the central bank wants to push the patient household to consume more and save relatively less. When the bank riskiness is increased, patient households demand a higher credit risk premium. This would lead to higher costs for both impatient households and entrepreneurs resulting in an overall lower economic activity. By lowering the natural interest rate, all else equal, retail interest rate decrease in lock-step which counteracts the increase in the level of interest rates due to the higher

Figure 6: Steady state values of the capital ratio (horizontal axis) for different parameter values.



risk premium. Overall, it must be noted, that except of the impact of changes in the bank riskiness, the other parameters only have a small impact on the steady state policy rate. Regarding Figure 6, Du and Miles (2014) point out that, as capital requirements increase, the natural policy rate of an economy decreases. Given that the lines in Figure 6 move in the opposite direction than the lines in Figure 5, our findings are consistent with theirs.

V Model comparison

In this section, we first compare the steady state values of the original Gerali, the Gerali-JK and the Gerali-Disyatat model with the respective baseline calibrations. In a second stage, we analyze quantitatively the dynamics of the model economies with their different banking sectors. We conduct different experiments and compare the trajectories of different economic variables in each model economy in response to exogenous shocks.

I Steady states

In Table 5, steady states values of relevant variables in the three models with their baseline calibration are shown. Consumption-to-Output is relatively constant across the three

Table 5: Steady state values of the three different models

| | Gerali baseline | Gerali-JK | Gerali-Disyatat |
|------------------------|-----------------|-----------|-----------------|
| C/Y | 0.86642 | 0.89049 | 0.88101 |
| I/Y | 0.11091 | 0.14359 | 0.12528 |
| K/Y | 4.4366 | 5.7435 | 5.0111 |
| r^d (%annual) | 2.2931 | 2.2931 | 2.5276 |
| r^{ib} (%annual) | 3.8634 | 3.8634 | 2.469 |
| spread_policy(%annual) | -1.5703 | -1.5703 | 0.0587 |
| r^{bh} (%annual) | 6.0385 | 6.0872 | 3.4236 |
| r^{be} (%annual) | 5.7029 | 5.7489 | 3.2333 |
| bh/(bh+be)(%) | 36.6162 | 30.7496 | 41.1501 |
| be/(bh+be)(%) | 63.3838 | 69.2504 | 58.8499 |
| B/Y | 2.355 | 2.7901 | 2.8825 |
| D/Y | 2.1438 | 2.4696 | 2.3499 |
| K^b/B | 9% | 11.5% | 18.5% |
| K^b/Y | 0.21117 | 0.3205 | 0.53252 |

models. Physical capital seems to differ between the three models. However this is only a consequence of the small differences in the consumption-to-output ratio, given that entrepreneurs need to produce more to satisfy the higher demand and hence need more physical capital as input factor. The share of household borrowing is the lowest in the Gerali-JK, with a gradual increase to the Gerali and then to the Gerali-Disyatat model. This difference is explained by the different interest rates and elasticities of loan demand. The steady state policy rate is much lower in the third model. However as opposed to the two

other models, in the Gerali-Disyatat, banks finance themselves at a risk premium, hence to avoid an extremely high deposit rate, the central bank sets the policy rate at a lower level. This is confirmed by the `spread_policy` variable: the spread between the deposit rate and the policy rate is negative for the two first models as banks finance themselves at a markdown from the risk-free rate, while in the Gerali-Disyatat model this spread is positive for the aforementioned reasons. Borrowing-to-output is the highest in the Gerali-Disyatat model which is a consequence of the relatively low retail borrowing rates resulting from the wholesale banks' maximization problem. These additional loans are mostly financed with the banks' own capital, and not with extra deposits, as seen in the capital ratio and in the capital-to-output.

Another interesting difference across the models is found in the capital ratio banks hold. In the Gerali model, banks hold capital amounting to 9% of total assets as requested by the regulator. In Gerali-JK, banks hold 11.5%, which is 2.5% above the minimum requirement of 9%. The 2.5% is supposed to reflect the Basel capital conservation buffer and follows from the parameterization chosen by Jakab and Kumhof (2015). Banks in the Gerali-Disyatat hold far more capital as a percentage of total assets than the other two models. This might seem surprising given that the banks do not face a (exogenous) requirement to hold bank capital. However, the fact that bank leverage is an important determinant of the probability of bank default implies an endogenous bank capital requirement.

II Impulse responses to different shocks

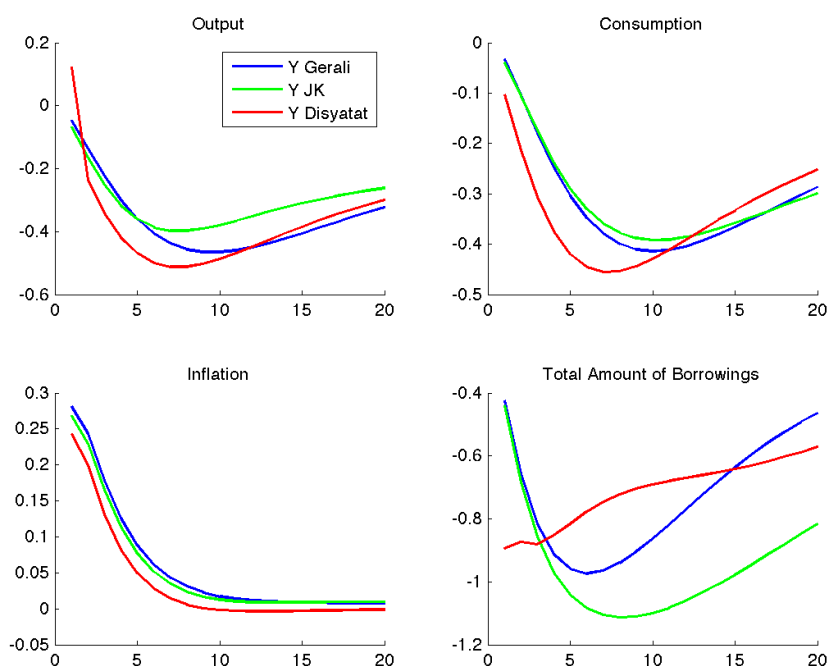
i Negative productivity shock

Figure 7 shows the responses of non-financial variables to a negative productivity shock. The shock size is one standard deviation and the same across models. The blue, green and red lines are the responses in the original Gerali model, Gerali-JK model, and Gerali-Disyatat respectively.

For output, consumption and inflation, the results are qualitatively very similar. Consumption falls because wage income of households decrease and the increase in the nominal interest rate leads to a temporary increase in the savings ratio. While output and consumption falls, inflation increases, reflecting the supply-side nature of the shock. Quantitatively, it's noteworthy that the Gerali-Disyatat produces slightly larger effects. The decline in output and consumption is slightly more pronounced. The differences in the response of total borrowings across models can be explained jointly by the different shares of entrepreneurial borrowings in total borrowings across the models, which is shown in Table 5, and the fact

that the entrepreneur is more directly affected by the productivity shock than the households. The productivity shock enters directly in the intermediate output, an important source of income in the entrepreneurial budget constraint. The entrepreneur knows that the lower output in the upcoming periods will constraint his repayment capacity and therefore reduces his borrowings by more than the households.

Figure 7: Impulse responses of non-financial variables to a negative productivity shock

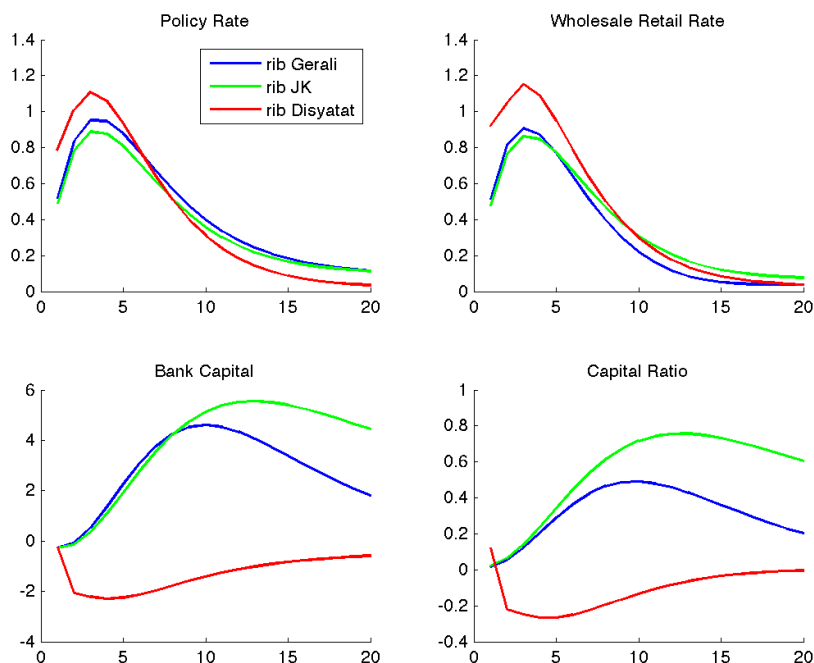


Note: All variables are percentage deviations from the steady state.

Figure 8 shows again the trajectories of the three model economies for the negative productivity shock but now for financial variables. Given that Figure 7 shows that the main real variables such as output and inflation behave relatively similar across the models, it is not surprising that the policy rate behaves similarly as well. This is simply the endogenous part of monetary policy, dictated by the same Taylor rule in each model. The relatively larger initial increase of the interest rates in the Gerali-Disyatat model is merely an artifact due to the fact that the steady state interest rates are lower than in the other models. A shock with the same absolute value will then have a relatively larger immediate impact

across the three models.

Figure 8: Impulse responses of financial variables to a negative productivity shock



Note: The interest rates and the capital ratio are given as absolute deviations from the steady state, expressed in percentage points. Bank capital is percentage deviations from the steady state.

There are some remarkable differences in how the financial variables react to the unexpected decrease in productivity. Importantly, bank capital and the capital ratio increase in the Gerali and Gerali-JK model while they decrease in the Gerali-Disyatat model. The decrease in bank capital is consistent with the results posted in Meh and Moran (2010) where bank capital has an endogenous purpose just as in the Gerali-Disyatat model. However, in Meh and Moran (2010), the capital ratio increases in contrast to what we observe in the Gerali-Disyatat model. This is not a qualitative but only a quantitative difference generated by a fact that, in Meh and Moran, credit outstanding falls faster than bank capital while the opposite holds for the Gerali-Disyatat. The qualitative difference between the reaction of bank capital between the Gerali-Disyatat and the Gerali and Gerali-JK model can be explained by differences in how bank profits react. Bank profits are affected through changes

in the interest rate margin (lending rate minus deposit rate) and changes in the volumes of lending. The Gerali-Disyatat model does not feature sticky deposit rates, therefore the funding costs increase faster than the banks' revenue, decreasing profits. Furthermore, the initial decrease in bank capital sets out second-round effects because lower capitalization will have a negative impact on the banks' funding costs.

Overall, the finding that the impulse responses of financial variables differ quite substantially across the models while the responses of real variables are more homogeneous is consistent with the results reported in Brzoza-Brzezina et al. (2013). Brzoza-Brzezina et al. (2013) compare two models that reflect the two fundamental approaches to financial frictions: collateral constraints as in Kiyotatki and Moore (1997) versus Bernanke et al. (1999).

ii Positive interest rate shock

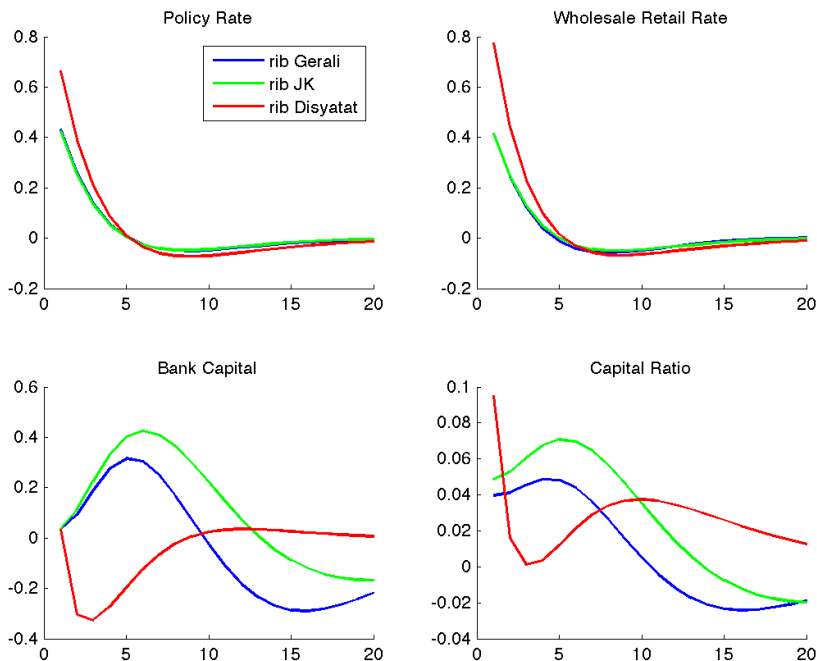
In Figure 9 the impulse response functions of a set of variables from the financial sector to a positive interest rate shock are shown, while Figure 10 depicts the impulse responses from the non-financial sector. The blue, green and red lines are the responses of the original Gerali model, Gerali-JK model, and Gerali-Disyatat respectively.

The shock is the same across the three models and corresponds to an unanticipated 50 basis points increase in the annualised policy rate. The difference observed in the upper left graph of Figure 9 showing the responses of the policy rate is simply due to the fact, that the natural policy rate in the Gerali-Disyatat model is lower than in the two other models, hence a shock of the same size increases the policy rate in the Gerali-Disyatat relatively more. This increase is transmitted to the wholesale lending rate.

The responses in Figure 10 of the non-financial variables are standard and very similar across the three models. After the interest rate increase demand for loans decreases from both households and entrepreneurs as shown in the bottom right graph of the Figure. However the amount borrowed in the Gerali-Disyatat decreases slightly more than in the two other models and takes longer to reach the steady state. In response to the increase in interest rates, the households reduce their demand for credit by more than the entrepreneurs, because of different elasticities. A higher share of households borrowings in total borrowings then implies a stronger decrease in total borrowings. The difference between the responses of borrowing can be jointly explained by this and the overall sharper increase in the lending rates in the Gerali-Disyatat model. The reduction of borrowing across all models results in a reduction of the supply of goods, thus output falls, while at

the same time demand is reduced. Hence overall consumption falls. And because demand-side effect dominate, inflation falls. The main difference between the three models is to be

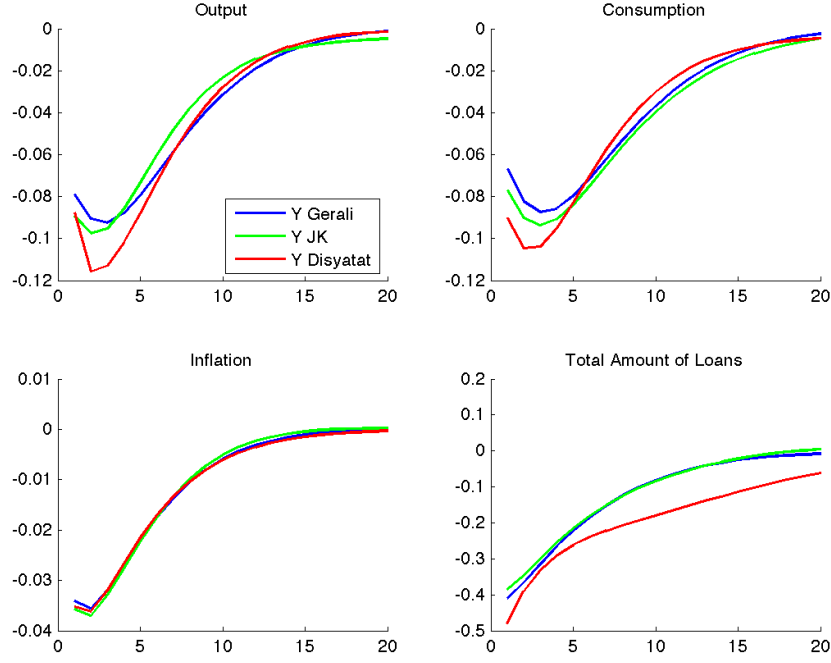
Figure 9: Impulse response of financial variables to a positive interest rate shock



Note: All rates are given as absolute deviations from the steady state, expressed in percentage points. Other variables are percentage deviations from the steady state.

found in the two graphs at the bottom of Figure 9 showing the impulse responses of bank capital and the capital ratio. While the movement in the Gerali and the Gerali-JK model are similar in their direction, the responses of the Gerali-Disyatat differ substantially. The initial increase in bank capital in the Gerali and the Gerali-JK model can be explained by the increase in interest rate which offsets the reduction in borrowing up to period 5, before the reduction in borrowings takes over and bank capital decreases again. The difference in magnitude between the two models is found in the set-up of the problem, and arises from higher costs from deviation faced by banks in the Gerali model. This reduces the banks profits, and thus the impulse responses of the Gerali are lower than for the Gerali-JK model.

Figure 10: Impulse responses of non-financial variables to a positive interest rate shock



Note: All variables are percentage deviations from the steady state.

The impulse response of bank capital in the Gerali-Disyatat follows a slightly different dynamic. In this model banks finance themselves in a perfectly competitive market which is devoid of any frictions. Hence the increase of the deposit rate is much faster and stronger than the increase of the retail lending rates. This results in a reduction in the banks margin channel, reducing overall banks profit and ultimately reducing bank capital. This trend is reversed from period 4 onwards, when the deposit interest rate is back in the long term equilibrium while the retail lending rates, because of the frictions, are still higher than in steady state. In addition the volume of outstanding loans is lower in the Gerali-Disyatat than in the two others, hence a volume effect is also at work.

The Gerali and the Gerali-JK model predict an increase in bank capital after both a positive interest rate shock, and a negative productivity shock, while the Gerali-Disyatat model predicts a decrease in bank capital after each of these shocks. The DSGE literature does not bring forward a clear consensus on the behavior of bank capital and the capital

ratio and reflects the findings of this thesis. In a model set up with risky borrowers and a stochastic risk of foreclosures faced by the banks, Takamura (2013) finds that bank capital has a negative deviation from the steady state after a negative productivity shock. Meh and Moran (2010) come to the same conclusion in their model characterised by a moral hazard problem between the banks and the borrowers, but unlike the trajectories of the capital ratio in the Gerali-Disyatat model, they find that the capital ratio increases after a negative productivity shock. While examining the role of bank lending and bank capital in the transmission of monetary policy, the model introduced by Van den Heuvel (2002) shows that the excess capital buffer held decreases after a positive interest rate shock. In a model that investigates the implications of various financial frictions and different capital requirements, the impulse response functions of Pariès et al. (2010) underline the mixed findings: while three out of their four set-ups show a decrease in bank capital and an increase in the bank's leverage ratio for a positive interest rate shock, following an unexpected decrease in productivity, bank capital first increases for three models. However the leverage ratio slightly increases too in the first periods after a negative productivity shock for all four approaches, hence the bank capital ratio falls after such a shock. It is apparent that the relationship between bank capital and the set of exogenous shocks is not well defined in the DSGE literature yet, and that future research in this directions is needed.

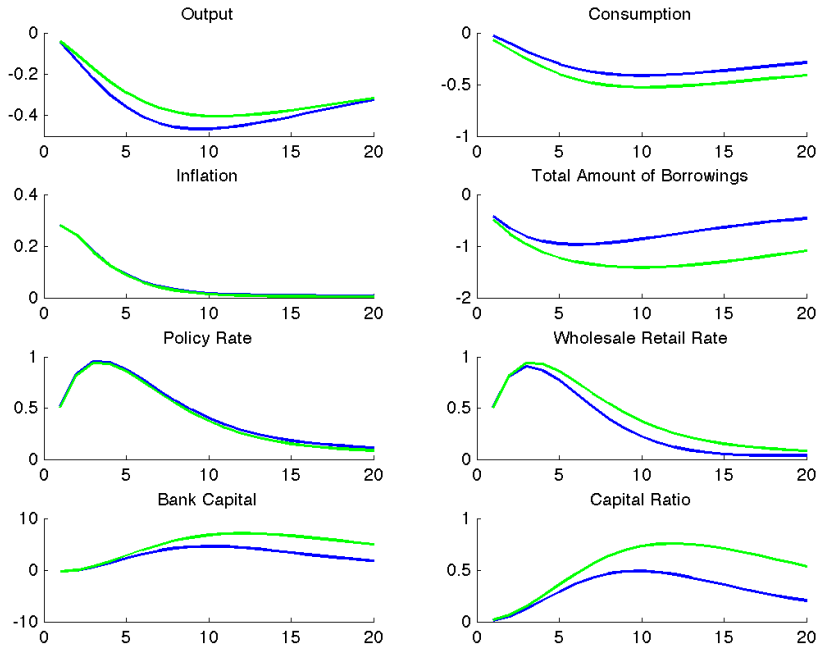
To sum up, our main observations from this comparison are the following

- The model with endogenous capital requirements (Gerali-Disyatat) amplifies slightly the responses of real variables to exogenous shocks, relative to the models with exogenous capital requirements.
- After both shocks considered, bank capital falls in the model with endogenous capital requirement while it increases in the models with exogenous capital requirements.
- The short-run dynamics of bank capital are mainly driven by the dynamics of bank profits.
- The magnitude of responses in bank capital is in some cases surprisingly strong.
- The dynamics of the capital ratio are more differentiated across models and shocks because it depends on how the numerator (bank capital) moves relative to the denominator.
- Surveying the literature, we conclude that there is no consensus on how bank capital and the capital ratio should respond to certain shocks.

iii Robustness of quantitative differences w.r.t. to parameterization

In the previous two sections, the impulse responses of the three model economies to two exogenous shocks have been investigated. As shown in the Figures 7 to 10 the trajectories of the three models are quantitatively different. However the models differ on two dimension: First, their parameterization is slightly different, and second as shown in Table 5 the steady state of the variables change considerably, and hence the three economies start off from different points when exposed to an exogenous shock. In order to attribute the differences in the impulse responses to the differences in modeling, this section investigates the behavior of the model economies when the parameters are calibrated such that the steady state values are the same across models. While this can be done for a reasonable parameterization for

Figure 11: Impulse responses to a negative productivity shock starting from the same steady state values

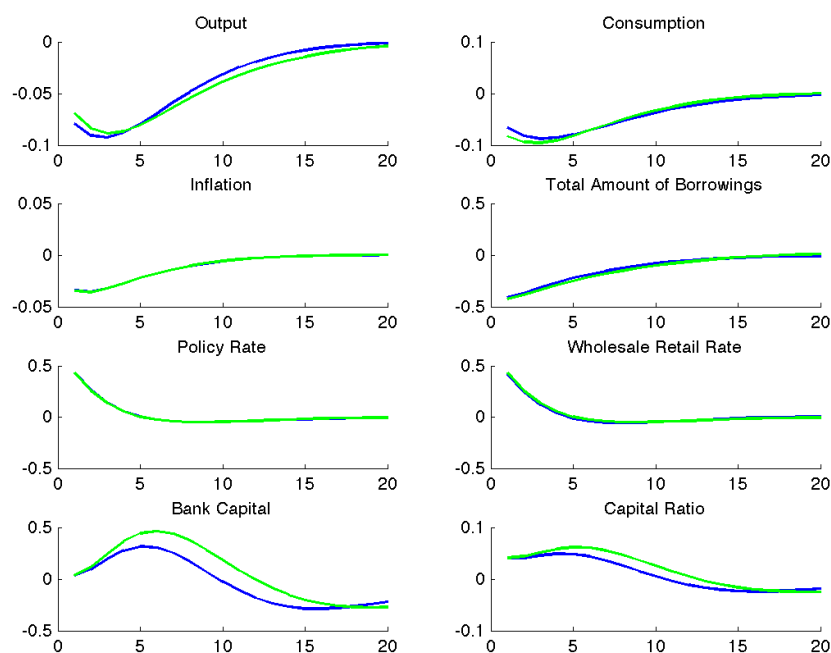


Note: Blue lines are the IRFs from the Gerali model, while the green lines represent the IRFs from the Gerali-JK model. All rates are given as absolute deviations from the steady state, expressed in percentage points. Other variables are percentage deviations from the steady state.

the Gerali-JK model, the model structure of the Gerali-Disyatat does not allow for such a

comparison. From the set up of the Gerali-Disyatat model, with regard to the No-Arbitrage condition given by Equation 12, the steady state value of the retail deposit rate will not be smaller than the natural policy rate for any reasonable parameterization. Hence, this robustness check is done for the two other models.

Figure 12: Impulse responses to a positive interest rate shock starting from the same steady state values



Note: Blue lines are the IRFs from the Gerali model, while the green lines represent the IRFs from the Gerali-JK model. All rates are given as absolute deviations from the steady state, expressed in percentage points. Other variables are percentage deviations from the steady state.

Figure 11 shows the impulse responses to a negative productivity shock of the same eight variables as previously investigated. The trajectories of the variables of the Gerali model are obviously the same, given that its parameterization has not been changed. While the trajectories of the variables of the Gerali-JK model differ slightly from the ones shown in Figures 7 and 8, they are still different from the trajectories of the variables in the Gerali model. Borrowing decreases further, resulting in a higher decrease in output and

consumption. The impulse responses of the financial variables do not seem to be affected by the new parameterization and are the same as in the Gerali-JK baseline parameterization. Figure 12 shows the impulse responses of models to a positive exogenous interest rate shock. Except for the impulse responses of output from the Gerali-JK model the trajectories are the same and starting from the same steady state does not alter the differences in the dynamics between the two models.

Hence, choosing a parameterization that yields the same steady state values does not alter our conclusion that the modeling approach of the banking sector plays a non-negligible role for the dynamics of the model economies and that the different approaches yield results which are not always consistent with each other.

VI Data section

The comparison of the dynamics resulting from the three DSGEs has shown that the three different approaches to model bank capital influence the trajectories of both real and financial variables. The latter being impacted more severely, we focus on the financial variables in this section. In particular, one of the three models yields impulse responses of bank capital that are not only quantitatively but also qualitatively different. This section provides some basic empirical insights into the relationships that were discussed in the model comparison and discusses interesting directions for future empirical research needed to verify the model predictions more thoroughly.

I Bringing implications for bank capital to the data

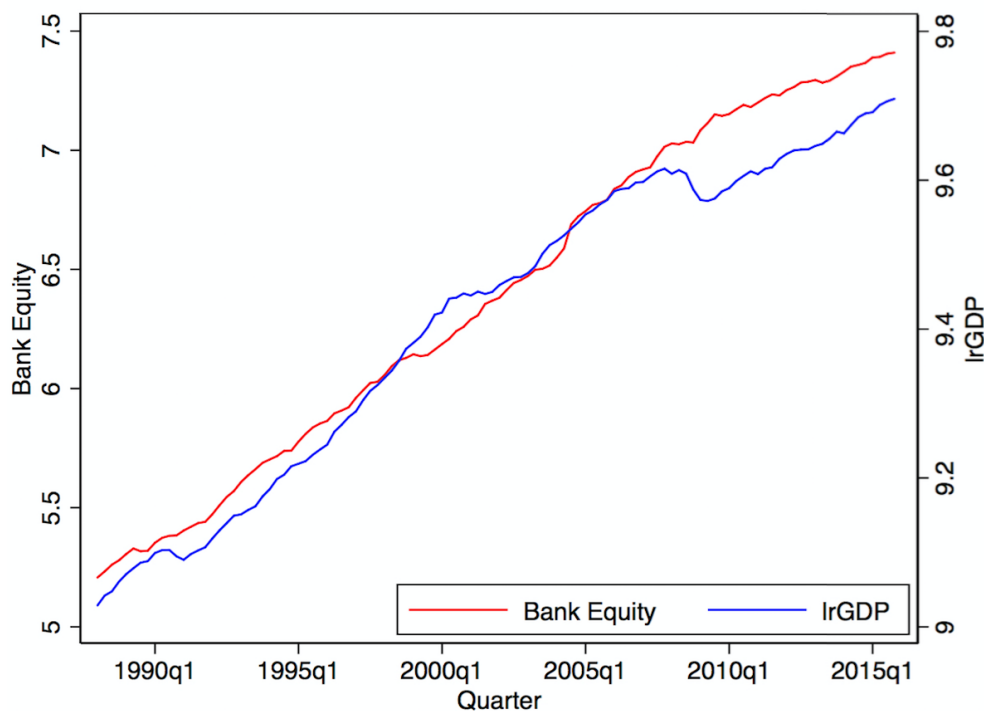
The impulse response functions discussed in previous sections implied some significant movement of the models' bank capital variable after the shocks considered. As we will argue, there is no formal empirical evidence available that could help to reconcile the implications of these models on short-term dynamics of bank capital with real-world data. This is likely due to various difficulties related to the empirical question at hand. To introduce the discussion of empirical evidence concerning these model implications, Figure 13 shows, in logarithmic scale, the evolution of aggregate bank capital and real GDP in the US. Figure 13 illustrates that aggregate bank capital is a slow moving variable, even if compared with GDP. E.g., the major downturn in 2009 led to a drastic drop in the level of US GDP, an event that is not mirrored in our measure of bank capital.

A first substantial challenge for macro-level empirical work on bank capital is the definition of bank capital. The measure we employ is plagued with far from trivial definitional issues. It is provided and aggregated by the Federal Reserve Bank St. Louis and based on the so called "Call Reports" under the reporting requirements for deposit-insured US commercial banks.¹³ As most other aggregate data on bank capital, the underlying definition is broad and includes, inter alia, unrealized losses or gains on securities held on the trading book. For properly testing the implications of the DSGE models, the definition of bank capital in these models and the one used in the data would have to be matched. In all three models, the dynamics of bank capital are effectively determined exclusively by retained earnings. Given that the models' banks business is limited to taking (retail) deposits

¹³The Fed's data dictionary, under <http://www.federalreserve.gov/apps/mdrm/data-dictionary>, gives a history of the exact definition. Total Bank Equity Capital can be found under item 3210. The MFI statistics of the ECB feature a similarly defined measure named "Bank Capital and Reserves," as set out in the Manual on MFI statistics <https://www.ecb.europa.eu/pub/pdf/other/manualmfbalancesheetstatistics201204en.pdf>

and granting loans to the non-financial private sector, the source of their earnings is far less diverse than those of real-world banks. The bank profit in the DSGE models is arguably closest to the Net Interest Income reported by real-world banks. In future work, we plan on scrutinizing this series as well. For the present analysis, the inclusion of unrealized trading losses or gains in the aggregate bank capital measure means that the model implications cannot straightforwardly be tested with this data.¹⁴ Besides the definitional issues, there is also considerable heterogeneity in the sample of our US data. It covers all deposit insured US commercial banks by adding up small and big banks. In Figure 17 in the Data Appendix D, we show how differently key balance sheet metrics behave within the sample.

Figure 13: Total equity of US commercial banks and real GDP, log-scale



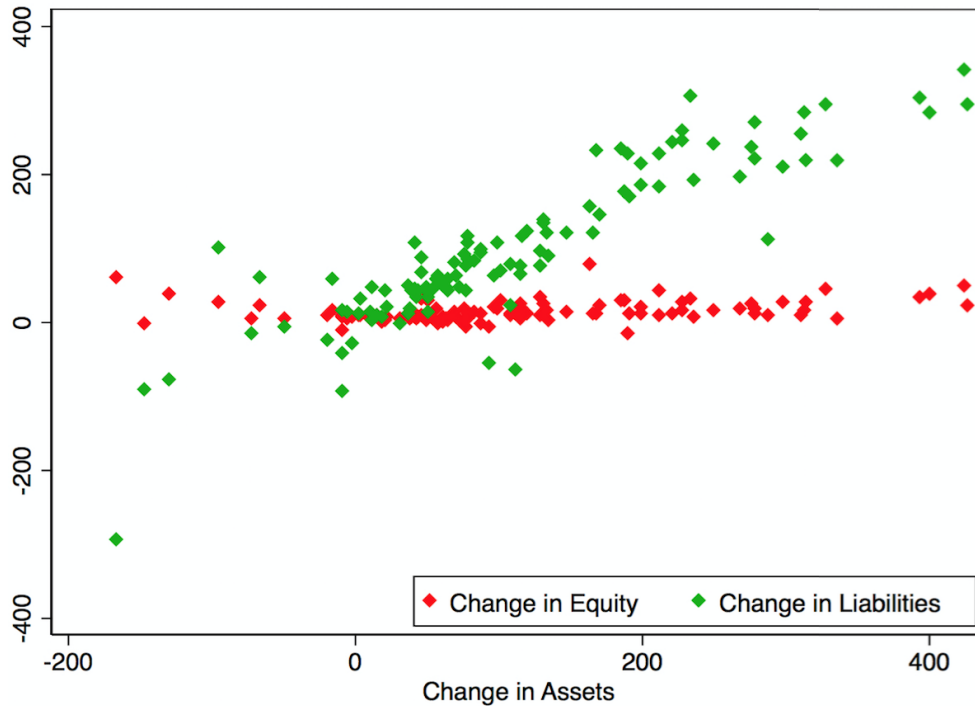
Note: Source: Federal Reserve Bank of St. Louis via Federal Financial Institutions Examination Council (US).

In addition to data issues, there are conceptual challenges concerning the short-run gyrations of bank equity. Figure 14 replicates a graph from Adrian et al. (2012). It shows, for US commercial banks for any quarter between first quarter of 1988 and fourth quarter

¹⁴This series is available separately, so that one could erase this mismatch. However, the new series starts only in 2009 and leaving us with a too short sample size to consider it.

2015, the absolute changes in aggregate total assets, liabilities and equity in billions US dollar. All data points represent a given quarter and given changes in total assets, depicted on the horizontal axis, in relation to either changes in total equity capital (red data points) or changes in total liabilities (green data points). The graph makes clear that quarterly

Figure 14: Changes in total assets, liabilities and equity in billions US dollars



Note: Author's own calculations, replicated from Adrian et al. (2012).
Source: Federal Reserve Bank of St. Louis via Federal Financial Institutions Examination Council (US).

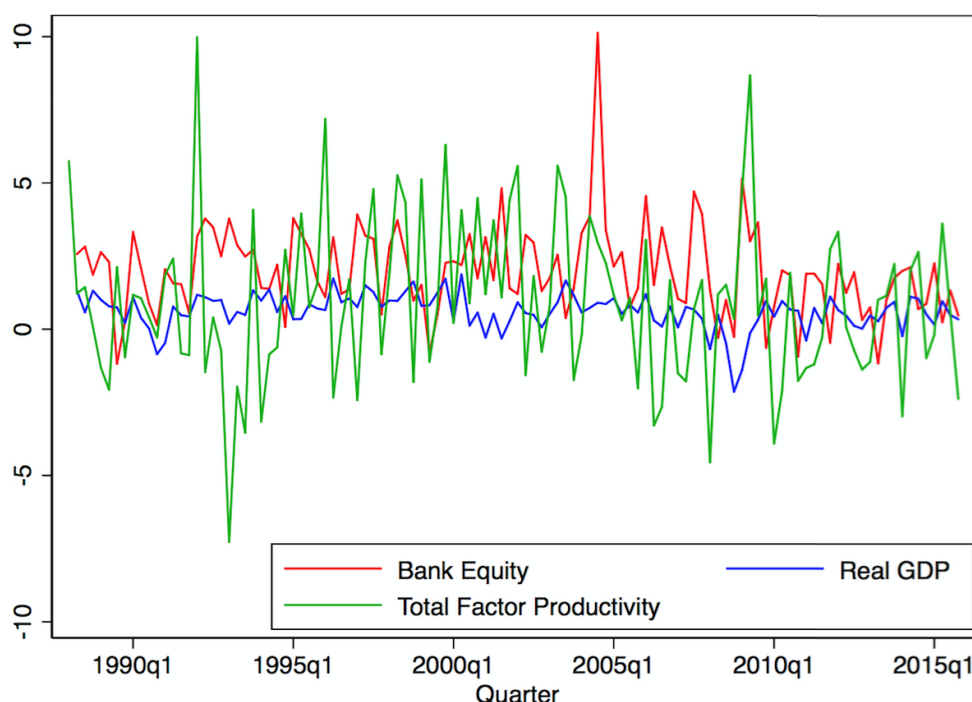
changes in assets are almost perfectly matched with quarterly changes in liabilities, while there is a near to zero correlation between changes in assets and changes in equity. Adrian et al. (2012) and, more formally, Adrian and Shin (2014) rationalize this with a model where bank lending is a consequence of a bank's leverage decision. Bank credit is clearly positively correlated with the business cycle, see e.g. in Biggs et al. (2009), Borio (2001) and Gross et al. (2016). The conjunction of the latter empirical findings and the stylized evidence from Figure 14 makes it seem unlikely that bank equity is significantly related to business cycle dynamics. This hypothesis is further supported by Kashyap et al. (2010) who differentiate between *stock costs* of equity and *flow costs* of equity, the latter being

substantial and making it therefore difficult for banks to react to unexpected capital needs by issuing equity swiftly.

i Response to a productivity shock

This section examines basic empirical relationships between total factor productivity (TFP) and aggregate bank capital and capital ratio respectively. We take as proxy for the exogenous productivity shocks in the DSGE model the quarterly change in total factor productivity in the US as measured by an estimate of Solow Residuals taken from Fernald (2012). A discussion of the validity and limitations of this exogeneity assumption is transferred to Appendix B.

Figure 15: Quarterly growth rates of bank equity, GDP and total factor productivity



Note: Bank equity and GDP is from Federal Reserve Bank of St. Louis. Total factor productivity is measured as "Solow Residuals," i.e. output growth minus growth in factor inputs, adjusted for utilization. Source: John G. Fernald, FRBSF Working Paper 2012-19.

Figure 15 shows the quarterly growth rates of bank equity, GDP and the TFP in the US. While GDP shows a very stable, slightly positive average growth rate, the growth rates of the two other variables are more volatile, although positive on average as well. From a

visual inspection no clear correlation between bank capital growth and either GDP growth or growth in TFP is observable and no special link seems to exist.

Table 6 shows the correlation between the TFP and a set of measures for bank capital and capital ratios respectively. As expected from the visual inspections of the Figure 15, the correlations are not significantly different from zero for all variables except for the correlation between the utilization rate adjusted total factor productivity and the first difference between the capital ratio. The correlation between these two variables is positive suggesting that an increase in one is followed by an increase in the other. Regarding the negative productivity shock examined in Section V this finding would support the trajectory of the capital ratio from the Gerali-Disyatat model. However this inference should be treated with caution, because first, all other correlations can clearly be rejected. Table 12

Table 6: Cross-correlations of aggregate bank equity and total factor productivity measures

| Variables | dIUSEQ | $dIUSEQ^{cyclical}$ | IUSEQ | $IUSEQ^{cyclical}$ | dEQTA | $EQTA^{cyclical}$ |
|---------------|----------------|---------------------|-----------------|--------------------|----------------|-------------------|
| dTFP | 0.07 (0.48) | 0.03 (0.76) | -0.03 (0.74) | -0.09 (0.37) | 0.12 (0.20) | 0.17 (0.07) |
| $dTFP^{Util}$ | 0.14 (0.15) | 0.11 (0.23) | -0.06 (0.54) | -0.10 (0.31) | 0.22 (0.02) | -0.14 (0.15) |

Note: Pairwise correlations with US data from 1988Q1 to 2015Q4. P-values in parentheses. dIUSEQ and IUSEQ are quarter-on-quarter log-change and log-level of aggregate bank equity respectively. "cyclical" indicates that the cyclical component of the respective variable was taken, obtained with a HP Filter with $\lambda = 1600$ for quarterly data. dTFP is quarterly change in total factor productivity, "util" indicates that the variable was adjusted for varying utilization rates. Sources: Bank equity data taken from FRED, total factor productivity data taken from Fernald (2012).

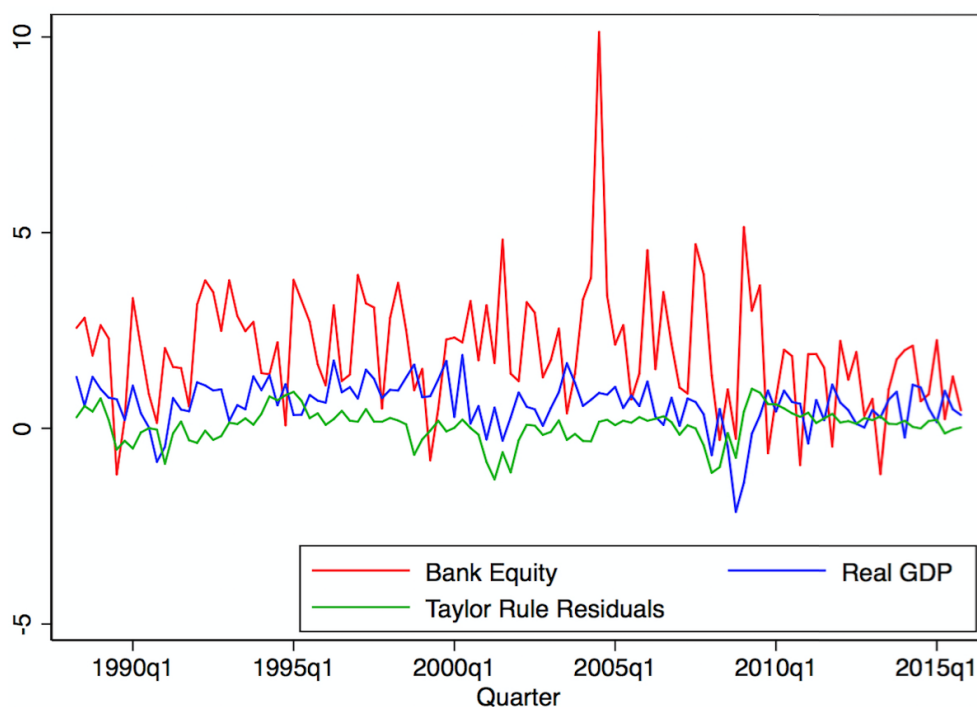
in the Data Appendix D lists further correlations between the banking variables and one to four lags of the TFP and utilization adjusted TFP. With the exception of the correlation between the fourth lag of the TFP and the cyclical component of the capital ratio all the others are not significantly different from zero underlining the fragile relationship between these variables. Second, a significant correlation does not mean that a causal relationship between the two variables can be established. In addition, to our knowledge, no formal empirical study has been done establishing such a relationship and to back this finding up. Hence the data does not clearly support one of the suggested impulse responses of the bank capital and the capital ratio of the three models. Further research in this area should be done to identify the relationship between the financial variables and the total factor productivity.

ii Response to a monetary policy shock

In this section the reaction of bank capital and the capital ratio to a monetary policy shock implied by the three models is examined. In order to bring these implications to the data, a measure of exogenous monetary policy shocks is required. Given the scope of this work, we settle with a simplistic measure of exogenous monetary policy shocks, namely the residuals from a Taylor rule regression. We discuss the validity and limitations of this measure in Appendix B.

Figure 16 plots the times series of the growth rates of bank equity, GDP and the Taylor rule residuals for the US over 1988Q1 to 2015Q4. Similar as in Figure 15 a visual inspection does not yield a clear relation between the Taylor rule residuals and the growth in bank equity.

Figure 16: Quarterly growth rates of bank equity, GDP and Taylor rule residuals



Note: Bank equity and GDP is from Federal Reserve Bank of St. Louis. Taylor rule residuals are percentage points deviations of the effective Fed Funds Rate from the prediction made with Taylor Rule regression, explained in detail Appendix C.

Table 7 lists the correlation between a set of financial sector variables and the Taylor rule residuals and the Fed Funds rate respectively. While the visual inspection did not yield

a clear result, the picture from the correlation table is more pronounced. The correlations between the Taylor rule residuals and the cyclical components of both bank capital and the capital ratio are all positive and significantly different from zero. This trend is also seen in Table 11 in the Data Appendix D. The one to three period lag of the Taylor rule residuals are positively correlated with the cyclical component of bank equity and of the capital ratio. The relationship between the Fed Funds rate and bank capital and the capital ratio is less striking, and does only yield one significant correlation, which however should be discarded. The negative correlations between the Fed Funds rate and the log of aggregate bank capital arises from the trend increase in bank capital as shown in Figure 13 and the steady decrease in the policy rate following the years of high rates in the seventies. Hence it is merely an artifact of the observed period.

The positive correlations between the Taylor rule residuals and the cyclical components of the log of the aggregate bank capital and its growth rate, and the cyclical components of the capital ratio suggests that a shock to the interest rate results in an increase in both the bank capital and the capital ratio. Regarding the implied impulse responses of the three models, this suggests that the Gerali and the Gerali-JK model better predict the reaction of the banking sector to an unexpected interest rate hike. However again, this results has to be treated with caution: a positive correlation doesn't suffice to clearly state a causal relationship. Once again future research is needed to identify the causality between these variables.

Table 7: Cross-correlations of aggregate bank equity and interest rate measures

| Variables | $dIUSEQ$ | $dIUSEQ^{cyclical}$ | $IUSEQ$ | $IUSEQ^{cyclical}$ | $dEQTA$ | $EQTA^{cyclical}$ |
|--------------|----------------|---------------------|-----------------|--------------------|-----------------|-------------------|
| Taylor-Resid | 0.12 (0.22) | 0.17 (0.08) | -0.02 (0.82) | 0.29 (0.00) | 0.10 (0.28) | 0.35 (0.00) |
| FFR | 0.09 (0.33) | -0.03 (0.72) | -0.81 (0.00) | 0.12 (0.20) | -0.04 (0.65) | -0.06 (0.55) |

Note: Pairwise correlations with US data from 1988Q1 to 2015Q4. P-values in parentheses. $dIUSEQ$ and $IUSEQ$ are quarter-on-quarter log-change and log-level of aggregate bank equity respectively. "cyclical" indicates that the cyclical component of the respective variable was taken, obtained with a HP Filter with $\lambda = 1600$ for quarterly data. FFR is the effective Fed Funds Rate. Taylor-Resid are percentage points deviations of the effective Fed Funds Rate from the prediction made with Taylor Rule regression. Sources: Bank equity data taken from FRED, Taylor-Resid are authors' own calculations, explained in detail Appendix C.

VII Concluding remarks

After the recent financial crisis increased attention has been devoted to the modeling of banks in DSGE models. This thesis compared three different approaches to model bank capital in a dynamic, stochastic general equilibrium model and teased out the implications of the different approaches for the model results.

Two of the three approaches compared feature an exogenous capital requirement faced by banks but they differ in the mathematical formulation of the problem. In the first, coming from Gerali et al. (2010), banks face pecuniary costs for both downward and upward deviations from the capital requirement. The second approach, coming from Jakab and Kumhof (2015), shares the rationale of an exogenous capital requirement, but banks only pay a penalty when they fall short of the target ratio. In the third approach, the capital requirement emerges endogenously. Banks finance themselves with unsecured funding whose interest rate depends on the capital ratio of the bank, hence inducing banks to hold a certain level of capital. The macroeconomic environment surrounding the three approaches comes from Gerali et al. (2010) and is kept the same, allowing for a sound comparison of the three ways to model bank capital.

As a first step, we analyse the three models separately both in terms of steady states as well as short run dynamics. A high importance has to be accorded to the parameterization of the models. While most parameters only affect the steady state marginally, some parameters have a strong impact on the steady states. In the two models that allow for bank riskiness, a sudden increase in that risk pushes banks to increase their capital holdings and their capital ratio. Further, we show that the asymmetry implied by penalizing only downward deviations from the target capital ratio only becomes relevant for the model results if the approximation technique employed allows for such non-linearities. This raises the question to what extent the currently conventional solution techniques in the DSGE literature must be updated, in order to model properly the financial sector which is prone to non-linearities.

In a second step, we compare the steady states and short-run dynamics across the three models. Specifically, we scrutinize the impulse responses to a contractionary monetary policy shock and to a negative shock to productivity. We show that the different modeling approaches yield sometimes qualitatively different results for short run dynamics of bank capital and the capital ratio. We find that there is no consensus in the literature on the directions of effects. Moreover even if sharing the same basic idea, different modeling approaches can generate noteworthy quantitative differences. However the qualitative inference is the same in non-financial variables for the three models analysed.

Overall, our findings suggest that the macro outcomes of the different approaches to bank capital considered are qualitatively reconcilable and yet quantitatively different. We show that the empirical relationship between the two analysed shocks and bank capital and the capital ratio respectively is not straightforward and no clear relationships are found in our basic empirical analyses.

In order to provide reliable and transparent policy advice, the model uncertainty should be researched more thoroughly. There are too few studies comparing different modeling approaches and thus shedding light on the consequences of them. In future, more work should be done on comparing different modeling approaches and try to identify the contexts in which they apply best. In addition, future empirical research is needed to identify how bank capital and the capital ratio relate to the business cycle. This is necessary in order to verify the modeling approaches that predict quite substantial variations in bank capital in response to business cycle dynamics.

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Appendices

A Gerali et al., original equations

I Households

Households are modeled as in the Gerali et al. (2010). They are divided into two groups. The first group consists of impatient households. They hold risk-free deposits at the retail deposit branch which pay an interest r_t^d . Their maximization problem is the following:

$$\max_{c_t^P, h_t^P, d_t^P} E_o \sum_{t=0}^{\infty} \beta_P^t \left[(1 - a^P) \epsilon_t^z \log(c_t^P(i) - a^P c_{t-1}^P) + \epsilon_j^h \log(h_t^b(i)) - \frac{l_t^b(i)^{1+\phi}}{1+\phi} \right] \quad (15)$$

subject to the impatient households budget constraint given by:

$$c_t^P(i) + q_t^h \Delta h_t^P(i) + d_t^P(i) \leq w_t^P l_t^P(i) + \frac{1 + r_{t-1}^d}{\pi_t} d_{t-1} + t_t^P(i) \quad (16)$$

The optimality conditions are given by the first order conditions of the maximization problem derived with respect to c_t , h_t and l_t respectively.

$$\begin{aligned} \lambda_t^P &= \epsilon_t^z (1 - a^P) / (c_t^P - a^P c_{t-1}^P) \\ \lambda_t^P q_t^h &= \epsilon_t^h \frac{j}{h_t^P} + \beta^P E_t [\lambda_{t+1}^P q_{t+1}^h] \\ \lambda_t^P &= \beta^P E_t [\lambda_{t+1}^P \frac{(1 + r_t^d)}{\pi_{t+1}}] \end{aligned} \quad (17)$$

where λ_t^P is the Lagrange multiplier of the budget constraint.

The three FoCs and equation 16 form the conditions implemented in Matlab.

The other group of households holds no deposits at the bank, but borrows money from it. The impatient household maximizes the following problem:

$$\max_{c_t^I, h_t^I, b_t^I} \sum_{t=0}^{\infty} \beta_I^t \left[(1 - a^I) \epsilon_t^z \log(c_t^I(i) - a^I c_{t-1}^I) + \epsilon_t^h \log(h_t^I(i)) - \frac{l_t^I(i)^{1+\phi}}{1+\phi} \right] \quad (18)$$

subject to its budget constraint given by:

$$c_t^I + q_t^h \Delta h_t^I(i) + \frac{1 + r_{t-1}^{bH}}{\pi_t} b_{t-1}^I \leq w_t^I l_t^I(i) + b_t^I(i) + t^I(I) \quad (19)$$

The impatient household has to pledge its accumulated housing stock as a collateral for its loans. Thus, additionally to the budget constraint, the household faces the following borrowing constraint:

$$(1 + r_t^{bH})b_t^I(i) \leq m_t^I E_t \left[q_{t+1}^h h_t^I(i) \pi_{t+1} \right] \quad (20)$$

where m_t^I is the loan-to-value ratio.

The impatient household's maximization problem boils down to the two constraint and the following first order conditions:

$$\lambda_t^I = \epsilon_t^z (1 - a^I) / (c_t^I - a^I c_{t-1}^I) \quad (21)$$

$$\lambda_t^I q_t^h = \epsilon_t^h \frac{j}{h_t^I} + E_t [\beta^I \lambda_{t+1}^I q_{t+1}^h + s_t^I m_t^I q_{t+1}^h \pi_{t+1}] \quad (22)$$

$$\lambda_t^I = s_t^I (1 + r_t^{bH}) + \beta^I E_t \left[\lambda_{t+1}^I \frac{(1 + r_t^{bH})}{\pi_{t+1}} \right] \quad (23)$$

These five equations form the set of equations for the impatient households, that will be implemented into Matlab.

II Capital goods producers

The capital goods producers face the following maximization problem:

$$\begin{aligned} & \max_{\bar{x}_t, i_t} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E (q_t^k \Delta \bar{x}_t - i_t) \\ & \text{subject to} \\ & \bar{x}_t = \bar{x}_{t-1} + \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1 \right)^2 \right] i_t \\ & \text{where } \Delta \bar{x}_t = k_t - (1 - \delta)k_{t-1} \text{ and } \Lambda_{0,t}^E = \beta_t^E \lambda_t^E \end{aligned} \quad (24)$$

This results in the two following equations to be implemented in Matlab:

$$\bar{x}_t = (1 - \delta)\bar{x}_{t-1} + \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2\right] i_t \quad (25)$$

$$1 = q_t^k \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2 - \kappa_i \left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right) \frac{i_t \epsilon_t^{qk}}{i_{t-1}}\right] + \beta_E E_t \left[\frac{\lambda_{t+1}^E}{\lambda_t^E} q_{t+1}^k \epsilon_{t+1}^{qk} \kappa_i \left(\frac{i_{t+1} \epsilon_{t+1}^{qk}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right)^2\right] \quad (26)$$

The first equation describes the amount of capital they can produce in every period, while the second equations sets the price of capital q_t^k .

III Entrepreneurs

Compared to the two household groups the entrepreneurs has a slightly simpler model. Only their consumption flows into their utility function, hence, they maximize:

$$E_0 \sum_{t=0}^{\infty} \beta_E^t \log(c_t^E(i) - a^E c_{t-1}^E) \quad (27)$$

subject to the budget constraint:

$$c_t^E(i) + w_t^P l_t^{E,P}(i) + w_t^I l_t^{E,I}(i) + \frac{1 + r_{t-1}^b E}{\pi_t} b_{t-1}^E(i) + q_t^k k_t^E(i) + \psi(u_t(i)) k_{t-1}^E(i) \leq \frac{y_t^E(i)}{x_t} + b_t^E(i) + q_t^k (1 - \delta) k_{t-1}^E(i) \quad (28)$$

where $y_t^E(i)$ is a standard Cobb-Douglas production function given by:

$$y_t^E(i) = a_t^E [k_{t-1}^E(i) u_t(i)]^\alpha \left[(l_t^{E,P})^\mu (l_t^{E,I})^{1-\mu} (i) \right]^{1-\alpha} \quad (29)$$

and $\psi(u_t)$ describe the real cost of setting a given level of capital utilization u_t and are equal to $\xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2$.

Like the impatient household, the entrepreneurs is also subject to a borrowing constraint. It cannot borrow more than a certain share of the capital held in period $t + 1$:

$$(1 + r_t^{bE}) b_t^E(i) \leq m_t^E E_t (q_{t+1}^k \pi_{t+1} (1 - \delta) k_t^E(i)) \quad (30)$$

The derivation of the Lagrangian with respect to the entrepreneur's choice variables, c_t^E , k_t^E , b_t^E and the two labour variables $l_t^{E,P}$ and $l_t^{E,I}$ yields the following conditions:

$$\lambda_t^E = \frac{1 - a^E}{c_t^E - a^E c_{t-1}^E} \quad (31)$$

$$\lambda_t^E q_t^k = s_t^E m_t^E q_{t+1}^k \pi_{t+1} (1 - \delta) + \beta_E \lambda_{t+1}^E [r_{t+1}^k u_{t+1} + (1 - \delta) q_{t+1}^k - \psi(u_{t+1})] \quad (32)$$

$$\lambda_t^E = [s_t^E (1 + r_t^{bE}) + \beta^E E_t [\lambda_{t+1}^E \frac{1 + r_t^{bE}}{\pi_{t+1}}]] \quad (33)$$

$$w_t^P = (1 - \alpha) \frac{y_t^E}{x_t} \frac{\mu}{l_t^{E,P}} \quad (34)$$

$$w_t^I = (1 - \alpha) \frac{y_t^E}{x_t} \frac{1 - \mu}{l_t^{E,I}} \quad (35)$$

$$r_t^k = \xi_1 + \xi_2 (u_t - 1) \quad (36)$$

$$\text{where } r_t^k = \frac{\alpha a_t^E [k_{t-1} u_t(i)]^{\alpha-1} l_t^E(i)^{1-\alpha}}{x_t} \quad (37)$$

IV Labour market

In the Gerali et al. (2010) households supply differentiated labour, which is sold by unions to perfectly competitive labour bundlers. They assemble the bought labour in a CES¹⁵ aggregator and sell the resulting homogenous labour to entrepreneurs who use it as input factor in the Cobb-Douglas Production function.

The economy consists of two different unions (one for each type of households) denoted by $s \in [P, I]$. The labour type is denoted by m , which belongs to the continuum of $[0, 1]$. Given this, the labour bundler maximises the over all labour supplied subject to overall wage costs faced by entrepreneurs. Hence his problem can be described by:

$$\begin{aligned} \max_{l_t^s(m)} l_t^s &= \left[\int_0^1 l_t^s(m)^{\frac{\epsilon_l - 1}{\epsilon_l}} dm \right]^{\frac{\epsilon_l}{\epsilon_l - 1}} \\ &\text{subject to} \\ \int_0^1 W_t^s(m) l_t^s(m) dm &\leq \bar{E}_t \end{aligned} \quad (38)$$

¹⁵Constant elasticity of substitution

From this problem we get the demand for differentiated labour.

$$\max_{U_{c_t^s(i,m)}} E_0 \sum_{t=0}^{\infty} \beta_s^t \left[U_{c_t^s(i,m)} \left[\frac{W_t^s(m)}{P_t} l_t^s(i,m) - \frac{\kappa_w}{2} \left(\frac{W_t^s(m)}{W_{t-1}^s(m)} - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \right) \frac{W_t^s}{P_t} \right] - \frac{l_t^s(i,m)^{1+\phi}}{1+\phi} \right]$$

subject to

$$l_t^s(i,m) = l_t^s(m) = \left[\frac{W_t^s(m)}{W_t^s} \right]^{-\epsilon^l} l_t^s \quad (39)$$

Assuming a symmetric equilibrium the wage-Philips curve is given by:

$$\begin{aligned} & \kappa_w (\pi_t^{w^s} - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w}) \pi_t^{w^s} \\ &= \beta_s E_t \left[\frac{\lambda_{t+1}^s}{\lambda_t^s} \kappa_w (\pi_{t+1}^{w^s} - \pi_t^{\iota_w} \pi^{1-\iota_w}) \left(\frac{\pi_{t+1}^{w^s}}{\pi_{t+1}} \right)^2 \right] + (1 - \epsilon^l) l_t^s + \frac{\epsilon^l l_t^{s1+\phi}}{w_t^s \lambda_t^s} \end{aligned} \quad (40)$$

$$\pi_t^{w^s} = \frac{w_t^s}{w_{t-1}^s} \pi_t \text{ for } s = P, I \quad (41)$$

V Retailers

The retailers buy the intermediate goods produced by the entrepreneurs and differentiate them and sell them in a monopolistically competitive market. Each retailer solves the following maximization problem:

$$\max_{P_t(j)} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[P_t(j) y_t(j) - P_t^W y_t(j) - \frac{\kappa_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^2 P_t y_t \right]$$

subject to the demand function:

$$y_t(j) = \frac{P_t(j)^{-\epsilon_t^y}}{P_t} y_t$$

The FoCs of this problem give the Phillips curve:

$$1 - \epsilon_t^y + \frac{\epsilon_t^y}{x} - \kappa_p (\pi_t - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}) \pi_t + \beta_P E_t \left[\frac{\lambda_{t+1}^P}{\lambda_t^P} \kappa_p (\pi_{t+1} - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] = 0 \quad (42)$$

While the profits of the sector are given by:

$$j_t^R = y_t \left[1 - \frac{1}{x_t} - \frac{\kappa_p}{2} (\pi_t - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}) \right] \quad (43)$$

which are transferred lump-sum to the patient households.

VI Loan and deposit demand

Before closing the model, and in order to be able to derive the three banking sectors equations, the demand for loans and deposits has to be established.

Loans and deposits are modeled as differentiated products offered by each bank and subject to a CES. Hence the bank sets its retail interest rate.

Thus, impatient households take this information as given and try to minimize the amount of repayment, hence:

$$\min_{b_t^I(j)} \int_0^1 r_t^{bH}(j) b_t^I(i, j) dj$$

subject to:

$$\left[\int_0^1 b_t^I(i, j)^{\frac{e_t^{bH}}{e_t^{bH}-1}} dj \right]^{\frac{e_t^{bH}}{e_t^{bH}-1}} \geq \bar{b}_t^I(i)$$

After solving the problem and aggregating over all impatient households, aggregate demand for loans of a given bank by households is given by:

$$b_t^I(j) = \left(\frac{r_t^{bH}(j)}{r_t^{bH}} \right)^{-\epsilon_t^{bI}} b_t^I \quad (44)$$

Demand by entrepreneurs can be derived in the same fashion, which yields:

$$b_t^E(j) = \left(\frac{r_t^{bE}(j)}{r_t^{bE}} \right)^{-\epsilon_t^{bE}} b_t^E \quad (45)$$

The setup for deposit demand follows the same concept. Except that households do not try to minimize their costs, but maximize their revenue from holding deposits. By symmetry this results in the aggregate demand for deposits at bank j of:

$$d_t^P(j) = \left(\frac{r_t^d(j)}{r_t^d} \right)^{-\epsilon_t^d} d_t \quad (46)$$

VII Monetary Policy and Rest

Aggregation, market clearing and the monetary policy equation close the model. The policy rate is given by a Taylor-type monetary policy rule that relates deviation from the target inflation rate and output to the level of the nominal interest rate, allowing for interest rate smoothing.

$$1 + r_t = (1 + r)^{1-\phi_R} (1 + r_{t-1})^{\phi_R} \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi(1-\phi_R)} \left(\frac{y_t}{y_{t-1}} \right)^{\phi_y(1-\phi_R)} \epsilon_t^r \quad (47)$$

and market clearing is defined by:

$$\begin{aligned} y_t &= c_t + q_t^k [k_t - (1 - \delta)k_{t-1}] + k_{t-1}\psi(u_t) + \delta^b \frac{K_{t-1}^b}{\pi_t} + Adj_t \\ c_t &= c_t^P + c_t^I + c_t^E \\ \bar{h} &= \gamma^P h_t^P(i) + \gamma^I h_t^I(i) \end{aligned} \quad (48)$$

B Discussion of exogeneity of measures for shocks

I Exogenous productivity shocks

As stated in the main text, we take as proxy for the exogenous productivity shocks in the DSGE model the quarterly change in total factor productivity in the US as measured by an estimate of Solow Residuals taken from Fernald (2012). The Solow Residuals, following Solow (1957), are essentially the share of output growth that cannot be accounted for by growth in factor inputs (assuming a certain production function and its parameterization), thus the term residual. The use of Solow Residuals is motivated by the tradition of Real Business Cycle models who treat variations in total factor productivity (also termed technology) as exogenous. It must be noted that this assumption is not unchallenged. Empirical work by Evans and Santos (2002) shows that, in their Advanced Economies sample, total factor productivity is significantly affected by monetary policy shocks. For our study more relevantly, Estevao and Severo (2010) identify a statistically and economically significant, negative effect of firms' funding costs on their productivity growth.¹⁶ However, considering these complications is not in the scope of this Master's thesis. Given that in the three DSGE models presented in this paper technology is an exogenous process, the use of a Solow Residual-type measure seems adequate while aforementioned caveats apply.

II Exogenous monetary policy shocks

We employ a simplistic measure for exogenous monetary policy shocks, namely the residuals from a Taylor rule regression. In macroeconometrics, a major branch of the literature identifies monetary policy shocks using aggregate data in Vector Autoregressions. A simple identification technique is a Choleski decomposition as proposed in Sims (1980), which imposes a recursive system on the variables. Others, e.g. Blanchard and Quah (1989) choose a more structural approach, imposing long-run restrictions guided by economic theory. Another method is to use high frequency movements in asset prices around monetary policy announcements, solving the severe endogeneity problems arising from low-frequency data. Gertler and Karadi (2015) show that using the latter measure produces results consistent with the large VAR literature on the effects of monetary policy such as Christiano, Evans and Eichenbaum (2005). For the scope of this Master's thesis, these techniques are too elaborate. We therefore recourse to deviations of the actual nominal short-term interest

¹⁶They rationalize this with a theoretical model in which financial shocks lead to inefficient factor allocation across firms.

rate targeted by the central bank from the same interest rate implied by a Taylor rule. Clarida, Gali and Gertler (2000) show that, under the Volcker and Greenspan regime, a forward-looking variant of the Taylor policy rule fits the US data well, suggesting that US monetary policy indeed followed relatively closely such a systematic rule. Deviations from such a rule could then be interpreted as exogenous to the normal conduct of monetary policy. In the DSGE models in this thesis, deviations from the Taylor rule clearly represent exogenous movements in monetary policy. However in reality, it seems unlikely that these deviations are truly exogenous shocks. For example, Kahn (2010) explores the linkages between financial stability and Taylor rule residuals, suggesting that central banks might not follow blindly the Taylor rule out of concerns that are not related to output and inflation. To truly test the implications of the DSGE models, we would need to better identify monetary policy shocks. Given the constraint for this work, we have to accept this limitation.

C Taylor rule regression

The time series presented in Figure 16 are the residuals originating from an estimated Taylor Rule Policy Function. The residuals are taken as a measure for exogenous monetary policy shocks because they reflect fluctuations in the short-term nominal interest rate targeted by the Federal Reserve that cannot be explained by a standard Taylor Rule of the following form

$$R_t = (1 - \phi_R)(R^* - \gamma_1(\pi - \pi^*) + \gamma_2 x_t) + \phi_R R_{t-1}$$

x_t is the difference between actual and potential real GDP, π_t is the annual inflation rate obtained from the GDP deflator, and R_t is the nominal short-term interest rate targeted by the central bank, in our case the effective Fed Funds rate. The equation follows the original rule proposed in Taylor (1993), allowing for interest rate smoothing as in Clarida et al. (1999). This policy rule says that the central bank has to raise interest rates if inflation is above target π^* or actual output above potential, i.e. if $x_t > 0$. If both inflation and output are on target, the nominal interest rate R_t will equal the equilibrium nominal interest rate R^* , here assumed to equal 4%. ϕ_R is the degree of interest rate smoothing and set to 0.9 in the estimation. The values assumed for R^* , π^* and ϕ_R follow closely Clarida et al. (1999).

Table 8: Taylor rule regression

| | Inflation | Output gap |
|-----|----------------------|----------------------|
| FFR | 1.425*** (0.2066) | 1.523*** (0.3100) |

Notes: Standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1.

The equation was estimated with Ordinary Least Squares on US data covering a sample from first quarter 1988 to third quarter 2008, Table 8 gives the results. The estimate on inflation is greater than 1, which is required to fulfill the Taylor principle: The nominal interest rate has to increase by more than one-to-one with inflation, in order to ensure determinacy of the system. The estimates are broadly consistent with Clarida et al. (1999), even though the coefficient on the output gap is quite large. As well, the residuals obtained are similar to those reported in Kahn (2010) and, given the scope of this thesis, we stick with this basic estimation result.

D Data appendix

Table 9: Data sources

| Abbr. | Description | Period | Source |
|-----------|------------------------------|-----------------|-------------------------------|
| nGDP | nominal GDP | 1988Q1 - 2015Q4 | FRED |
| GDP | real GDP | 1988Q1 - 2015Q4 | FRED |
| GDPPOT | real potential GDP | 1988Q1 - 2015Q4 | FRED |
| FFR | effective Fed Funds Rate | 1988Q1 - 2015Q4 | FRED |
| USEQTC | Total Equity Capital | 1988Q1 - 2015Q4 | FRED |
| EQTA | Total Equity to Total Assets | 1988Q1 - 2015Q4 | FRED |
| USTLI | Total Liabilities | 1988Q1 - 2015Q4 | FRED |
| dtfp | Business Sector TFP | 1947Q2 - 2015Q4 | Fernald, FRSBSF Working Paper |
| dtfp_util | Utilization-adjusted TFP | 1947Q2 - 2015Q4 | Fernald, FRSBSF Working Paper |

Table 10: Cross-correlations of aggregate bank equity and total factor productivity measures, pre-crisis sample

| Variables | dIUSEQ | $dIUSEQ^{cyclical}$ | IUSEQ | $IUSEQ^{cyclical}$ | dEQTA | $EQTA^{cyclical}$ |
|---------------|----------------|---------------------|-----------------|--------------------|----------------|-------------------|
| dTFP | 0.07 (0.54) | 0.03 (0.76) | 0.01 (0.91) | -0.22 (0.04) | 0.11 (0.32) | -0.02 (0.89) |
| $dTFP^{Util}$ | 0.08 (0.46) | 0.08 (0.48) | -0.00 (0.99) | -0.14 (0.22) | 0.14 (0.20) | -0.13 (0.25) |

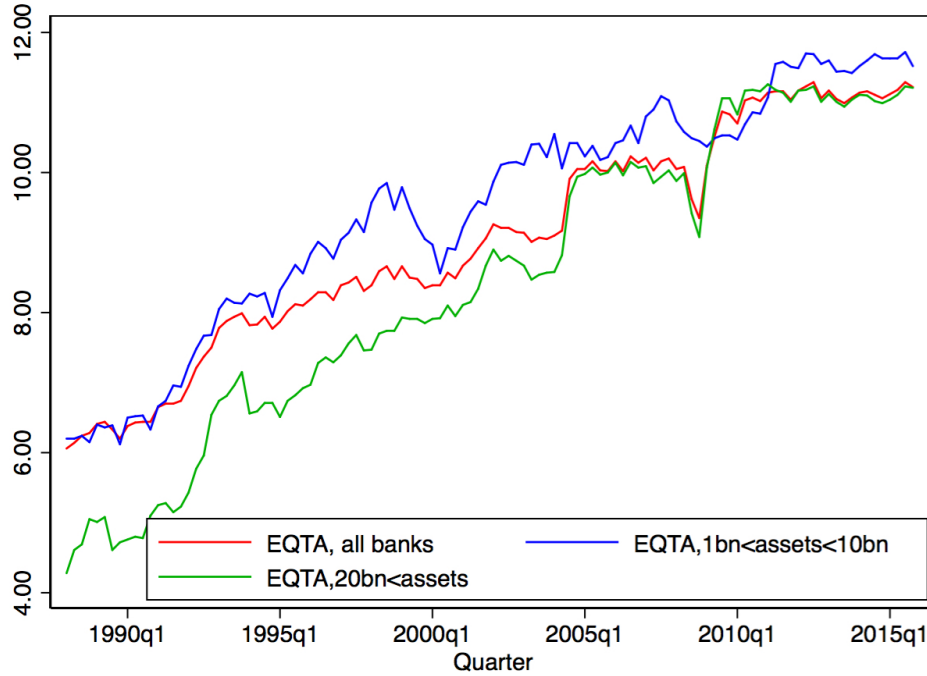
Note: Pairwise correlations with US data from 1988Q1 to 2008Q2. P-values in parentheses. dIUSEQ and IUSEQ are quarter-on-quarter log-change and log-level of aggregate bank equity respectively. "cyclical" indicates that the cyclical component of the respective variable was taken, obtained with a HP Filter with $\lambda = 1600$ for quarterly data. dTFP is quarterly change in total factor productivity, "util" indicates that the variable was adjusted for varying utilization rates. Sources: Bank equity data taken from FRED, total factor productivity data taken from Fernald (2012).

Table 11: Cross-correlations of aggregate bank equity and interest rate measures, pre-crisis sample

| Variables | $dIUSEQ$ | $dIUSEQ^{cyclical}$ | $IUSEQ$ | $IUSEQ^{cyclical}$ | $dEQTA$ | $EQTA^{cyclical}$ |
|--------------|-----------------|---------------------|-----------------|--------------------|-----------------|-------------------|
| Taylor-resid | 0.14 (0.21) | 0.14 (0.20) | -0.24 (0.03) | 0.24 (0.03) | -0.02 (0.84) | 0.29 (0.01) |
| FFR | -0.18 (0.11) | -0.04 (0.74) | -0.59 (0.00) | 0.17 (0.12) | -0.04 (0.74) | -0.07 (0.55) |

Note: Pairwise correlations with US data from 1988Q1 to 2008Q2. P-values in parentheses. $dIUSEQ$ and $IUSEQ$ are quarter-on-quarter log-change and log-level of aggregate bank equity respectively. "cyclical" indicates that the cyclical component of the respective variable was taken, obtained with a HP Filter with $\lambda = 1600$ for quarterly data. FFR is the effective Fed Funds Rate. Taylor-Resid are percentage points deviations of the effective Fed Funds Rate from the prediction made with Taylor Rule regression. Sources: Bank equity data taken from FRED, Taylor-Resid are authors' own calculations, explained in detail Appendix C.

Figure 17: Total equity to asset ratios of US commercial banks



Note: Total Equity to Asset Ratios are shown for different samples: All banks includes all deposit-insured US commercial banks, "1bn<assets<10bn" means all banks whose total assets range between 1 and 10 billion USD, "20bn<assets" means all banks whose total assets exceed 20 billion USD. Source: Federal Reserve Bank of St. Louis via Federal Financial Institutions Examination Council (US).

Table 12: Cross-correlations with different lags

| Variables | dIUSEQ | <i>dIUSEQ^{cyclical}</i> | IUSEQ | <i>IUSEQ^{cyclical}</i> | dEQTA | <i>EQTA^{cyclical}</i> |
|-------------------------------|-----------------|----------------------------------|-----------------|---------------------------------|-----------------|--------------------------------|
| L.Taylor-Resid | -0.05 (0.58) | -0.01 (0.93) | -0.02 (0.82) | 0.28 (0.00) | -0.05 (0.57) | 0.32 (0.00) |
| L2.Taylor-Resid | -0.13 (0.18) | -0.08 (0.40) | -0.02 (0.83) | 0.22 (0.02) | -0.05 (0.60) | 0.30 (0.00) |
| L3.Taylor-Resid | -0.10 (0.30) | -0.04 (0.69) | -0.02 (0.87) | 0.19 (0.05) | -0.14 (0.15) | 0.21 (0.03) |
| L4.Taylor-Resid | -0.13 (0.17) | -0.07 (0.47) | -0.02 (0.82) | 0.14 (0.15) | -0.18 (0.07) | 0.11 (0.27) |
| L.FFR | 0.07 (0.49) | -0.06 (0.52) | -0.81 (0.00) | 0.08 (0.40) | -0.05 (0.62) | -0.13 (0.19) |
| L2.FFR | 0.06 (0.53) | -0.07 (0.49) | -0.81 (0.00) | 0.03 (0.73) | -0.02 (0.81) | -0.18 (0.06) |
| L3.FFR | 0.06 (0.54) | -0.07 (0.48) | -0.80 (0.00) | -0.02 (0.87) | -0.02 (0.87) | -0.23 (0.02) |
| L4.FFR | 0.06 (0.52) | -0.07 (0.49) | -0.80 (0.00) | -0.06 (0.51) | 0.01 (0.94) | -0.27 (0.01) |
| L.dTFP | 0.05 (0.64) | 0.01 (0.92) | -0.01 (0.89) | -0.08 (0.39) | -0.11 (0.24) | 0.09 (0.36) |
| L2.dTFP | 0.01 (0.93) | -0.03 (0.73) | -0.01 (0.91) | -0.11 (0.27) | -0.00 (0.99) | 0.09 (0.36) |
| L3.dTFP | 0.19 (0.05) | 0.16 (0.09) | -0.01 (0.91) | 0.01 (0.90) | 0.10 (0.31) | 0.16 (0.09) |
| L4.dTFP | 0.10 (0.32) | 0.06 (0.54) | 0.00 (0.98) | 0.06 (0.57) | 0.09 (0.38) | 0.23 (0.02) |
| <i>L.dTFP^{Util}</i> | 0.11 (0.27) | 0.08 (0.38) | -0.04 (0.66) | -0.04 (0.71) | 0.08 (0.41) | -0.08 (0.42) |
| <i>L2.dTFP^{Util}</i> | 0.04 (0.67) | 0.01 (0.91) | -0.04 (0.68) | -0.03 (0.77) | 0.13 (0.17) | 0.02 (0.83) |
| <i>L3.dTFP^{Util}</i> | 0.06 (0.55) | 0.02 (0.84) | -0.05 (0.59) | -0.01 (0.90) | 0.02 (0.85) | 0.04 (0.72) |
| <i>L4.dTFP^{Util}</i> | 0.02 (0.84) | -0.02 (0.81) | -0.05 (0.63) | -0.03 (0.76) | 0.09 (0.33) | 0.10 (0.29) |

Note: Pairwise correlations with US data from 1988Q1 to 2015Q4. P-values in parentheses. "L." means variable is lagged by one quarter, "L2." means variable lagged by two quarters and so on. dIUSEQ and IUSEQ are quarter-on-quarter log-change and log-level of aggregate bank equity respectively. "cyclical" indicates that the cyclical component of the respective variable was taken, obtained with a HP Filter with $\lambda = 1600$ for quarterly data. FFR is the effective Fed Funds Rate. Taylor-Resid are percentage points deviations of the effective Fed Funds Rate from the prediction made with Taylor Rule regression. Sources: Bank equity data are taken from FRED. Taylor-Resid are authors' own calculations, explained in Appendix C.