Stockholm School of Economics
Department of Economics
Course 5210: Master's Thesis in Economics

Equilibrium Pricing in Decentralized Markets
The Case of the Equity Lending Market

Abstract

A central tenant to Efficient Market Theory is that arbitrage may be engaged in without cost. Research has shown that, in reality, equity markets are subject to numerous imperfections, one of which is short sale constraints. A primary constraint to arbitrage stems from the decentralized nature of the equity lending market. This paper models interaction in this decentralized marketplace. I conclude, based on the model, that regulators should specifically aim to decrease search costs associated with locating lendable shares of stocks with concentrated institutional ownership.

Author: Stefan Boettrich
Advisor: Peter Högfeldt
Discussants: Paul Elger and Gustav Lindqvist
Examiner: Mats Lundahl
Presentation: February 19, 2007. 10.15-12.00.

I. Introduction

Since the statement of Keynes that securities prices are effectively the result of changes in the "animal spirits" of investors a significant amount of financial economic research has been allocated to the study of equilibrium stock prices. Research on the topic culminated in a 1970 survey paper by Eugene Fama in which he proclaimed that efficient market theory (EMT) cannot be rejected and thus should be a fundamental pillar of empirical & theoretical financial research.

As is the case in nearly all financial research models, EMT relies heavily on assumptions made in justification of conclusions. One such assumption is that arbitrage is frictionless: in other words transactions costs equal zero for an agent wishing to take either a positive or negative position in the security. In a seminal 1977 paper Edward Miller theoretically and intuitively justified the contention that if investor beliefs are heterogeneous and short sale constraints exist that stock prices may become inflated above fundamental prices.

It is well understood, and extensively documented, that investor beliefs are heterogeneous and that without heterogeneity there would be no transaction of shares and thus no need for exchanges. Hence, this study focuses on constraints to short selling – namely those which arise due to the decentralized nature of the equity lending marketplace (ELM).

The ELM is a minimally standardized, decentralized network of borrowers and lenders of shares. In order to sell a stock that one does not own the shares must first be located, a lending fee negotiated, and finally the borrowing and selling of the security. At a later date the short seller must return the borrowed securities either by repurchasing shares on the open market or by borrowing shares from a third party.

Specifically, I develop a model of interaction where, in a decentralized market space, a short seller first locates a supplier of shares and is offered a fee for those shares by that lender. After considering the offer, the short seller has the option of either accepting the offer or searching for an alternate lender. If a secondary lender is found the firm's compete and the equilibrium lending fee is competitive, however if a competitor cannot be found the short seller pays the inflated initially offered fee.

In this paper I attempt to accomplish three objectives through the use of this model. Firstly, I illustrate how observed lending fees may be inflated due to the decentralized marketplace – thus posing a constraint to short selling. Secondly, I describe how the decentralized market structure adversely impacts the price efficiency of smaller stocks. Thirdly, I conclude that regulators aiming to increase market efficiency in general should focus on lowering the frictions associated with borrowing small cap stocks. Specifically, *I hypothesize that a reduction in marginal market search costs will yield asymmetrically positive efficiency gains to firms with fewer institutional lenders.*

The first part of this paper provides a background and framework to the reader. The second section describes the model – the fundamental interaction which occurs in a decentralized market. Thirdly, I model market interaction where a representative short seller interacts with only two supplying firms and then relax the assumption to allow for several suppliers of shares. The final section concludes with a discussion of regulatory initiatives which, if put in place, should increase overall market efficiency.

II. The Market for Securities Lending: Background and Framework

A. The ELM Transaction

The act of short selling occurs when an investor sells shares of a stock which are borrowed rather than owned. The borrowing of shares is facilitated by the equity loan marketplace (ELM). The ELM is a decentralized and unstandardized market where lenders and borrowers of shares intersect. When shares are lent from an incumbent owner to a short seller both the shares and voting rights of those shares are transferred. This is rational insofar as the future buyer of the shorted shares is, of course, purchasing voting rights. Thus, due to short selling, the float of shares in a stock may exceed the total number of shares outstanding although the total number of voting shares cannot change.

The incentive of an agent to engage in short selling is that he or she has a valuation of a stock which is lower than the prevailing market price. In other words they expect the price to fall - and profit from purchasing the shares at a lower price - known as *covering*.

Theoretically, short selling and the ELM are additive to overall market efficiency as described by Efficient Market Theory (EMT). Barring arbitrage frictions, the price of a stock is reflective of the average beliefs of those participating in the trading of that stock. More specifically, informed & rational market participants with information about the equity's true value counteract noise traders and other irrational investors who assign imperfect valuations to the stock.

Several types of imperfections exist in the ELM - known collectively as *short sale constraints*. Such constraints generally prevent negative information from becoming incorporated in the current price of the stock. The existence of short sale constraints have been documented to lead to overvaluation of share prices (see Lamont & Thaler, 2002). Moreover, these constraints are widely to believed to be the source of bubbles in equity markets (see Ofek & Richardson, 2003). The major sources of short sale constraints are discussed in the next section.

When engaging in a short sale the short seller must first *locate* shares to short. Duffie, et al (2002) note that "Occasionally, a significant amount of time may pass before the necessary stock can be located." Empirically, the aforementioned authors also note that factors related to the difficulty of location include liquidity, market capitalization, ownership concentration, and the size of the float – defined as the number of shares available for trading. D'Avolio (2002) furthers this contention by stating that "…the share search process is a real constraint to short selling."

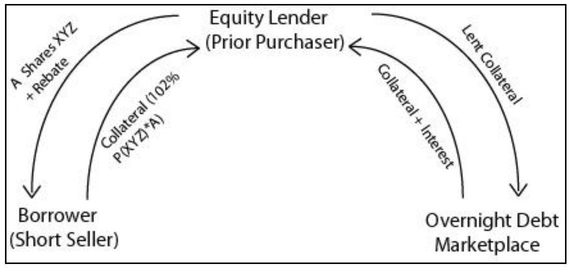

Figure 1: The dynamics of a standard ELM transaction. Note that this does not include any prior search process by the short seller.

Outside of the search process, the underlying dynamic of the equity loan marketplace is presented in Figure 1. As depicted, the short seller of a given stock XYZ utilizes the ELM to borrow shares and then sells them on the open market. Share suppliers are generally broker-dealers and large institutional investors. In order to collateralize the equity loan the short seller must make a deposit with the lender in for the duration of the loan period. The collateral is in the form of either cash or short-term government bonds. Moreover, these agreements are renewed and settled on a daily basis.

The collateral posted is generally invested by the equity lender. The standard fee returned to the short seller, known as the *rebate rate*, usually equals the overnight risk free rate, however the equilibrium price for the loanable assets is captured in the rebate rate as well. For all intensive purposes the ceiling on the rebate rate is the overnight rate, however the fee is not bound by a floor and thus negative rebate rates are possible and do occur. Stocks which are difficult to borrow due to a variety of factors typically are those bearing low, sometimes negative, rebate rates. Such equities are known by traders and academicians as being *on special*. Notable examples of stocks which have had negative rebate rates include Palm Inc., Delphi Automotive, and Lucent Technologies (Lamont/Thaler, 2002).

B. Constraints to Arbitrage: Imperfections in the ELM

Most securities markets are imperfect – either as a result of technical imperfections such a trading restrictions/barriers or due to regulatory/governance issues. The ELM is no exception, although all of the constraints listed below arise due to structural shortcomings of the market which may, and should be, dealt with. The major imperfections include the uptick rule, collateral requirements, recall risk and finally, non-centralization.

The "uptick rule" is an SEC regulation requiring that a short sales only be placed after the last trade was a purchase of shares – i.e. at the asking price (Duffie, et al. 2002). The rule was instated in order to counteract the so called "bear raids" in the markets during the 1920s in which a party would short sell a stock's price down significantly in order to induce panic selling. In reaction to this the short seller generally covered their short sales and repurchased stock for the inevitable upward correction. This rule poses a constraint to short sales as sale transactions by short sellers is limited – especially in times of rapid market corrections.

Under SEC Regulation T, short sellers must deposit 102 percent of the shorted stocks value with the equity lender. This requirement contrasts greatly to other markets such as that of stock options where writers of options contracts actually generate immediate income from the sale. Although, if market conditions are favorable, short sellers receive the risk-free interest rate on deposited collateral the collateral rule certainly is a deterrent to short selling.

As mentioned in the last section, share lending agreements are renewed on a daily basis. Moreover, the equity lender has the right to recall lent shares at any time. Obviously this represents a real risk to short sellers in that they are subject to the discretion of the lender. In the event of recall it is typically the case that a new lender is sought out and located. In the event that an alternate lender cannot be located a "short squeeze" may occur – meaning that the short seller is forced to cover their transaction with shares purchased on the open market.

Finally, the non-centralization of the ELM is a major constraint to short sales. As described above and modeled in this paper, the process of borrowing stock involves locating a suitable lender – a process which may be time consuming and expensive if arbitrage opportunities are fleeting. In the following sections I attempt to gain perspective on precisely how market decentralization leads to price imperfections.

C. Price Imperfections in a Decentralized Market: *The Tourist* Example

The concept reviewed in the latter sections of this paper is, perhaps, most effectively communicated with a simple example. Suppose that a tourist hungry for a hamburger is vacationing in a foreign town. This tourist doesn't know the exact location of restaurants in this town but knows that several exist and, after a search process, finds one of them. Being dressed as a tourist, the restaurateur knows the potential customer is, in fact, a tourist and unaware of the location of competing firms. The restaurateur offers the tourist a price for the hamburger, equal to P.

The tourist knows that the intrinsic value of the hamburger is less than P - actually it is equal to c. At this point the tourist has two choices: either pay the inflated offer price for the hamburger or search for one of the other restaurants in town. She knows that if a competing supplier is located that the firms will compete and she'll be able to buy the meal for c. The downside is that searching takes time and she is only getting hungrier. Besides, if she searches and doesn't find another restaurant then she'll have to pay the price P anyway.

Several interesting queries come to mind about this situation: What is the optimal action of the tourist? What is the optimal offer price of the local restaurant? Generally speaking do tourists in general pay a lot or a little to eat in this town? What if we change the total number of restaurants in the town or print maps for all the tourists in town - will that help?

The equity loan market shares many characteristics of this foreign town – namely it's decentralized nature. This research paper aims to answer these questions and also contemplate policy measures which aim to solve these competitive inefficiencies.

III. Lending Fees in a Decentralized Marketplace

The concept of agent interaction in a decentralized marketplace is most simply communicated via a comparison with a centralized marketplace. Fundamentally, centralized marketplaces exist to bring large numbers of buyers and sellers together. This promotes competition, facilitates standardization, and minimizes the costs of participation, price discovery, and information acquisition. In this market all agents are aware of equilibrium prices and can frictionlessly engage in transactions with any number of sellers.

Agent interaction in a decentralized market is relatively complex much less efficient. A consumer wishing to engage in a transaction must first search for a supplier, bargain with that supplier, and finally either engage in a transaction or search for a competing firm. Not only is price information difficult to obtain but, as I show, prices are generally inflated due to imperfect competition between lenders.

Figure 2 illustrates the relative spatial differences between these two contrasting market structures. The decentralized market is relatively disorganized, which is the primary source of the listed inefficiencies.

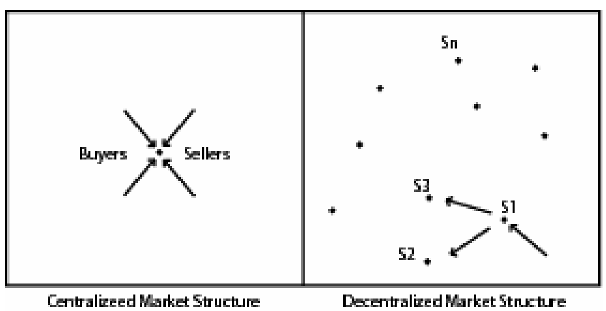

Figure 2: A centralized vs. decentralized market structure. In a centralized marketplace buyers agglomerate and as do competitive sellers. In a decentralized market, buyers a forced to search out sellers (Si) and the market power of sellers arise from their high degree of spatial separation from one another and the cost to buyers of exploring market space (Tirole, 1988).

This main section contains the basic model of interaction between lenders and borrowers in a decentralized marketplace. The key features of the model are: (i) Interaction occurs between one representative borrower[1], one lender with a known location, and one or more competing lenders which exist at unknown locations in the market space, (ii) The borrower demands a finite number of shares which are available to lenders at a marginal cost, c. (iii) Two equilibria are possible: If a competing lender is found then price competition forces the lending fee to marginal cost. In the event a competing lender is not found then the lending fee includes a profit margin captured by

[1] The representative agent is a hypothetical construct used in order to simplify aggregate interactions between a multitude of lenders and borrowers.

the local lender. (iv) Although two equilibrium states are possible in a one-shot interaction, in an environment where such an interaction is repeated then an average market price will be observed.

A. Description and Assumptions

Spatial markets have been described in a multitude of ways, each of which calls attention to specific unique attributes of the market. Hotelling (1929) modelled the classic linear market space in which he assumed that a finite number of consumers are equally distributed along a chord. Salop (1979) contributed to the field by considering spatial differentiation over a circular market space. More recent research has also considered multidimensional differentiation through the use of hybrid models. For instance such a model may incorporate intrinsic product diversity coupled with spatial differentiation.

The choice of a proper spatial environment is an important decision: the model must be a sufficient venue to describe the pertinent interaction, however be simplistic enough to effectively communicate the abstraction to the reader. I found that the optimal spatial structure to model ELM interaction is a linear chord – see $\overline{0B}$ below. Considering that, in reality, the short seller is unaware of the location of lending firms I assume that lenders are randomly distributed over the market space according to a uniform distribution. The short seller must search out institution(s) which are willing to lend shares.

Figure 3: The linear market space.

An alternative interpretation of the market space is a line with the total number of institutional lenders uniformly distributed over the interval – however only a fraction of those institutions are willing to lend the desired shares. It is sensible to assume that the representative short seller knows how many total institutions exist in the market space. The model captures the market size as the length B-A, which the short seller is aware.

The representative short seller is located at A, an end point of the market space. For the purposes of modelling interaction in this market we may assume that a single lender (the *local lender*)has been located by the short seller. Although a search of degree d_{INIT} was undertaken in locating the initial lender, the incurred cost of searching is sunk and thus inconsequential to further cost analysis.

Despite the suck nature of the initial search costs the degree of searching required to locate the initial lender yields important, albeit imperfect, information on the density of lending firms. If the size of the total market space is known as is the distance searched to find an initial lender then the short seller can make an ex ante estimation of how many lending firms exist a la the below equation.

$$n = \frac{(b-a)}{\mathrm{d}_{\mathrm{INIT}}} + e$$

Equation 1: The approximate number of lenders, n, in the market space plus a normally distributed error term with a mean of zero.

The short seller incurs a search cost t per unit of market searched, which captures the value of effort and opportunity lost during the time spent searching. Specifically, the model incorporates a search process with quadratic costs (Stahl, 1982). Thus, a short seller who searches a market segment of degree d incurs a cost of td^2.

The quadratic search cost assumption is realistic as it capture the fleeting nature of arbitrage and is more tractable mathematically. Literature on the topic of arbitrage highlights the swiftness in which such opportunities are corrected. Hence, an increasing marginal cost function captures the expediency of the situation. Moreover, the mathematical workup is simplified with a minimal loss of generality.

The short seller has memory the location of previously found equity lenders. Such an assumption has important competitive consequences: If a competitor is found during a search then the short seller does not incur additional search costs in re-locating the initial lender. On the same note, if a search is unsuccessful then the short seller is able to borrow from the initial lender without incurring additional search/transactions costs.

Competition between the lenders is in prices (Bertrand), as this is less presumptuous than assuming oligopolistic or Cournot interaction. Hence, if an alternate lender is located then the equilibrium lending fee equals c, the competitive price. If a competitor is not found then the equilibrium price is inflated and equal to P = (1+m)*c.

Researchers modelling the interaction of agents in spatial market models often encounter a dilemma in interpreting results. Specifically, intersections of supply and demand in such models reflect both the idiosyncratic demand function of consumers *and* the distortion caused by the spatial nature of the market. In order to disentangle this problem of causation, research by Hotelling (1929), Perloff and Salop (1985) and Salop (1979) assume that consumers have unit demands.

Rather than assuming unit demands by short sellers in the decentralized ELM my model corrects for the stated quandary by assuming that short seller demand is finite – equalling $\overline{q_d}$. An added benefit of this assumption is a simpler mathematical workup of the model and results which specifically illustrate the effect of market decentralization to the reader.

In summary, the stated model assumes that a short seller demanding a finite number of shares, $\overline{q_d}$, has located a single share lender located at point A on the chord $\overline{0B}$. The short seller knows the size of the remaining market and incurs a quadratic marginal cost in searching for a competitor. Share lenders compete in prices and, if a secondary lender is located, the equilibrium lending fee is the marginal cost of shares. If a competing lender is not found the lending fee is marked-up by the local lender.

B. Interaction between Borrowers and Lenders

The fundamental interactions between buyers and sellers is the source of equilibrium conditions in markets. In a monopolistic marketplace the single dominant firm is aware of how a consumer's demand for a product will change with prices – their elasticity of demand – and optimizes accordingly. Alternatively, in a purely competitive marketplace firms undercut one another and the result is equilibrium pricing at marginal cost. In short, rational agent behaviour in the context of the market's industrial organization is at the source of equilibrium pricing.

In spite of the fact that the IO of a decentralized market is considerably more complex than that of a purely monopolistic or competitive market, the underlying premise of the market's structure motivating the optimal conduct of agents and resulting in equilibrium performance (SCP) remains valid.

Based on the analytical framework described in the previous section I consider the optimal behaviour of borrowers and lenders in a general fashion. Firstly, I describe the optimal reaction of a short seller given their location in the market space. I then discuss how the profit-maximizing local firm will react to the short seller given their favourable market position. Finally, the interaction is reduced to a two-stage game between the short seller and local lender.

1. Behaviour of the Short Seller

As described in the prior section, the short seller in the model exists spatially at location A in Figure 3 or, in the more intuitive two-dimensional map in Figure 2, at the location S1. At this point the short seller is paired with a single local lender. The local lender offers to supply an unlimited number of shares to the short seller at the price P.

Given this market structure the short seller cannot frictionlessly request shares from an alternative lender – rather the short seller must first search for the secondary source. This spatial movement is denoted in Figure 2 as one of the two arrows representing movement to another lender.

As mentioned, all institutions are uniformly distributed over the interval $\overline{AB}$, however only a fraction of these institutions have or are willing to lend shares of the desired stock. The actual process of searching may be understood as a lottery in that, given the degree of searching, d, there is a probability of locating a competitor and a probability of not locating a competitor, and finite payoffs in each event.

The equation of the short seller's expected value function is simplified due to the assumption that a finite number of shares are demanded by the short seller. Since demand is finite then the utility generated to the short seller from those shares is also constant. Hence, we only must consider the relative expected cost of the shares to the short seller.

Consider that, given the search expenditure by the short seller, there are two levels of wealth: before the transaction and after the transaction. Prior to borrowing the shares the short seller has a wealth level W_1 while after the transaction it is W_2. The short seller's objective is to maximize the expected value of wealth in the second period, according to the equation for expected wealth below.

$$E[W_2] = W_1 - SearchCost - \Pr[FoundCompetitor] \cdot Cost[CompetitiveFee]$$
$$- \Pr[\neq FoundCompetitor] \cdot Cost[InflatedFee]$$

Equation 2: The expected wealth after the search process equals the first-period wealth minus the total search cost minus the relative probabilities of each state multiplied by the lending fee in each state.

The first obvious question is "under what conditions does the short seller search for shares versus accept the offer of the local firm?" Referring back to the hungry tourist analogy it is understandable that, being aware of the underlying prepared food cost to the restaurant that a tourist will *always* search for a competitor – even if the search only involves looking across the street. This is the basis of Proposition 1 which is below and proved in the appendix.

> **Proposition 1**: *It is a dominant strategy for a rational short seller to search for an competing lender if the price offered by the local lender is greater than the competitive lending fee.*

With the understanding that the short seller will search for shares if not offered the purely competitive price we may now delve deeper into the optimal behavior of the short seller and the local firm. The next question a skeptic may pose is "how much searching will the short seller engage in?" The answer to this question relates to the risk profile of the short seller and several characteristics of the environment, one of which is m - the degree to which the local firm inflates the offer price.

It's intuitive that the amount of searching done by the short seller is a direct function of how inflated the offer price is. Referring back to the example of the hungry tourist, it is quite understandable that even the hungriest of people would look far and wide for a competing firm if the offer price for a meal is sufficiently inflated.

Mathematically one may solve for the optimal degree of searching, denoted d*, by referring to the expected wealth equation earlier in this section. Given that lottery, the optimal degree of searching is the amount which maximizes the expected value of second period wealth. Formally, this is as follows:

$$d^*(m) \cong \{d \mid E[W_2] = Max[E[W_2]]\}$$

Equation 3: The optimal degree of searching is that amount of searching such that the expected second period wealth equals the maximum expected period wealth given market attributes and possible values of the variable d.

2. Behaviour of the Local Equity Lender

As is the case in all markets, rational firms optimize in such a way that profits are maximized. If the local equity lender was subject to a monopolistic market structure they would extract a significant percentage of the profit that the short seller expects to generate by selling an overvalued stock. Considering the other extreme, if the market was fully centralized, the local equity lender would be subject to the either the perceived or actual threat of alternative lenders – forcing the offered price to marginal cost.

The market under study, like most real-world markets, is neither of these extremes. The local firm doesn't price as a monopoly as it is subject to the perceived threat of outside competition given the likelihood that a competing supplier is located. However, due to the spatial differentiation of equity lenders the firm does not have an incentive to offer a competitive price.

Comparable to the perspective of a short seller, the local lender is subject to a lottery-style expected profit function. Given the search behaviour of the short seller a competitor is found with a probability *Pr[FoundCompetitor]*. In that event, price/Bertrand competition persists and lender profits are forced to zero. If a competitor is not located then the firm charges a marked-up price for the shares.

$$\begin{aligned} E[\pi_{LOCAL}] = {} & \Pr[FoundCompetitor] \cdot \Pr ofit[CompetitiveFee] \\ & + \Pr[\neq FoundCompetitor] \cdot \Pr ofit[InflatedFee] \end{aligned}$$

Equation 4: Expected profits of the local firm equals sum of the weighted amounts of profit in each state: when the competitor is found or not found.

Interestingly, and as noted previously, the probability of the short seller finding a competing firm in the market space is a direct function of the degree of searching undertaken. Furthermore, the optimal degree of searching is a function of how much the offer is inflated. The obvious implication is that the probability of the short seller finding a competitor is dependant on how much the local firm inflates the offer price.

Hence, from the perspective of the local lender a tension exists: If he offers a monopolistic[2] price then a competitor will certainly be located and firm profits will equal zero. Conversely, if he offers a purely competitive lending fee then the short seller will not search but profits will be zero.

[2] Bear in mind that if the local lender was a true monopolist that the optimal price charged is infinite due to the assumption that demand is finite and the short seller's price elasticity of demand is zero.

Thus, the local firm must take into account the reaction function of the short seller when deciding upon a fee to offer. Mathematically, he must maximize expected profits conditional upon the short seller's optimal distance searched in response. This is mathematically noted as equation 5.

$$m^* \cong \{m \mid \mathrm{E}[\pi_L] = MAX[\mathrm{E}[\pi_L \mid d^*(m)]]\}$$

Equation 5: The profit-maximizing markup of the local firm, m*, is the markup, m, such that the expected profits equal the maximum expected profits conditional upon the optimal degree of searching in reaction by the short seller, d*.

3. ELM Interaction as a Two-Stage Game

From a game-theoretic perspective this interaction may be understood as a two-stage game. The local firm has a first-mover advantage as he may choose the location on the short seller's reaction function which yields the greatest expected profits.

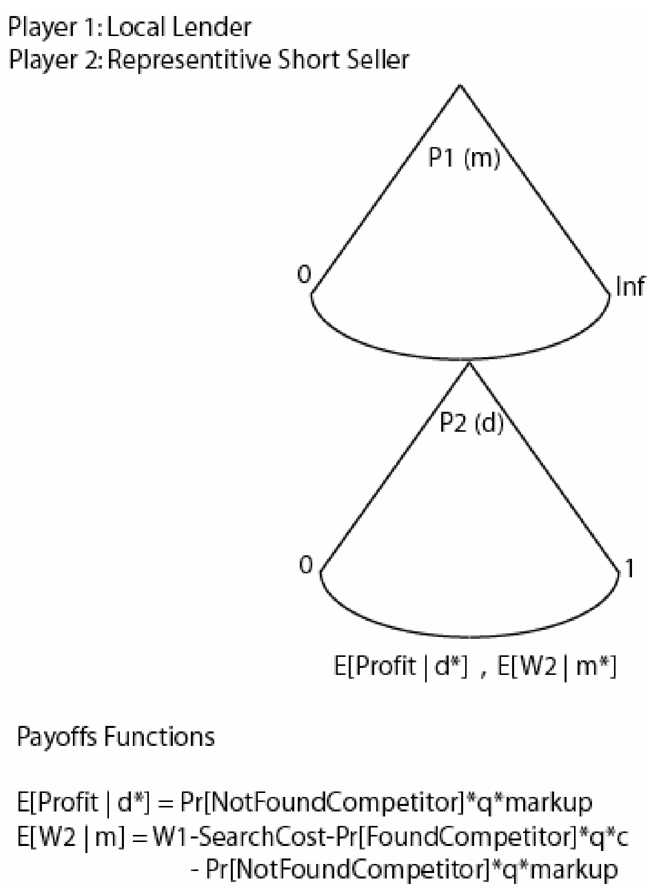

Figure 4: The extensive form game which illustrates the sequential nature of the game, the first mover advantage, and the payoff functions of the players.

The interaction is illustrated in the above extensive form game. The solution to the game may be solved using backward induction as dominated strategies are eliminated at each information set (Watson, 2002). Within this analytical framework it becomes clearer that the local firm has a first-mover advantage as the firm is able to maximize profits through selection of the ideal point on the short seller's reaction function.

C. Market Prices in the ELM

In a traditional exchange environment, such as the NYSE or NASDAQ, the market for securities is centralized. Buyers and sellers converge and a singular, market clearing price is determined. Buyers and sellers are aware of the single equilibrium price and face low transactions costs in information acquisition and market participation.

On the contrary, in a decentralized marketplace several intersections between buyers and sellers may exist – resulting in multiple price equilibria (Tirole, 1988). Consider that on this day people across Stockholm have paid a multitude of prices for apples, a commodity product. This is seemingly inefficient, however multiple equilibrium prices certainly have and will persist in the future.

The ELM is no exception to this paradigm[3]. Ofek, et al (2004) note that, empirically, although lender rebate rates may be correlated they generally are dispersed. In reality, the number of equilibria are limited only by agent heterogeneity and the degree of decentralization. We are working with an abstraction in which only two equilibria exist: either an alternative lender is found and competitive price is paid or a lender is not found and the short seller pays an inflated fee.

The prior analysis is done assuming a one-shot interaction between a representative short seller, a single local firm, and a many non-local firms. Rather, consider repeated interaction between several independent short sellers and the equity loan market space. In each case a lender is found, a price offered and a search undertaken. Sometimes a competitor is found; sometimes that's not the case.

Given the Law of Large Numbers, participation in the market by several short sellers yields a predictable trend in prices. In a two-lender model the probability of locating a competitor is simply 50%[4] and the average observed price is the average between the marked-up price and the marginal cost of shares. Simply put, *the mean observed price in a decentralized market is equal to the probabilistically weighted sum of each equilibrium price*.

---

[3] This is a severe drawback for empirical researchers studying securities lending. Most such studies rely on rebate rate data from single lenders. Hence, such data only reflects a small subset of equilibria in the marketplace.

[4] The equilibrium degree of searching, d*, can be shown to be exactly 50% regardless of market conditions. This implies that 50% of the time a competing firm is found!

Intuition may be gained by again considering to the market for apples in Stockholm. If with a 1/3 chance a consumer pays 6 SEK for an apple and 2/3 chance they pay 12 SEK then the mean market price is 10 SEK On the same note, in the N-lender case the mean observed price, given a sufficient number of iterations, equals the probability of locating a lender multiplied by the marginal cost of shares plus the probability of not locating a lender multiplied by the marked-up price:

$$\overline{P}_{MARKET}^{OBS} \approx \Pr[FoundCompetitor] \cdot MarginalShareCost$$
$$+ \Pr[\neq FoundCompetitor] \cdot InflatedFee$$

Equation 6: The average market price equals the weighted equilibrium prices in each state.

This observed market price provides a window into pricing in a decentralized marketplace and is of interest to social planners who regulate such markets. Given that price efficiency is of paramount concern to policy-makers then the goal should be to reduce the average market price as much as possible. The final section of this essay includes a discussion of policy initiatives which strive to accomplish just that.

IV. Equilibria and Trends: The Two-Lender Case

Before investigating the relatively complex case in which several competitors exist in the outside market space I discuss a simplified scenario in which the single representative short seller interacts with a local lender given the prospect of locating a single alternative lender.

Through this framework I provide insight into the process of solving for the optimal degree of searching and the optimal offer of the local firm – the next section utilizes a recursive method to solve for these equilibria. I follow with a decomposition of the equilibrium lending fee in order to illustrate the adverse effect of a decentralized market structure.

A. Preliminaries: The Mathematical Framework

In section III.A I described the applicable market space as a linear chord $\overline{AB}$. Institutional lenders are continuously distributed over that interval – in other words the expected density of institutions willing to lend the desired shares is equal over the space. Thus, we may mathematically represent the probability of locating a lender over the course of a search of d units of market space as follow:

$$P(Lender \in [A, A+d]) = \int_{A}^{A+d} \frac{dy}{b-a} = \frac{d-a}{b-a}$$

Equation 7: The probability of locating a lender over the interval [A,A+d] equals the value of the cumulative probability of locating a desired lender over that interval. Simply put, the probability of finding a lender over d units searched equals the ratio of d in the numerator and the size of the entire market in the denominator. (Weiss, 1999)

As mentioned, the local lender offers the short seller a fee of $P = (1+m) \cdot c$ to satisfy the short seller's demand for shares. Taking into account the above likelihood of finding an alternative lender, the short seller's *outside option* –the expected value of lottery representing the short seller's option to search the market space – may be mathematically expanded. The expected value of this lottery is the first period wealth net the total search cost, minus the weighted costs to the short seller if he finds a competitor or not.

$$E[W_2] = W_1 - d^2 t - \frac{d-a}{b-a}(c\bar{q}_d) - \frac{b-d}{b-a}(1+m)c\bar{q}_d$$

Equation 8: The post-search wealth of the short seller is the initial wealth minus search costs and the expected probabilities of success and failure multiplied by the respective costs.

Knowledge of the precise probability of the short seller locating a competitor over the course of a search may also be figured into the expected profits of the local lender. Specifically, if an alternative lender is located then the firms will compete in prices and profits will be zero. In the event that an alternate lender is not located then the profit equals the product of the percent markup, the marginal cost of shares and the quantity of transacted shares.

$$E[\pi_L] = \frac{d-a}{b-a} 0 + \frac{b-d}{b-a} cm\bar{q}_d = \underline{\underline{\frac{b-d}{b-a} cm\bar{q}_d}}$$

Equation 9: The expected profits to the local firm, given that they offer a price P=(1+m)c, equals the probability of the short seller locating a competitor multiplied by 0 (as price equals marginal cost) plus the probability that a competitor is not found multiplied by the associated profits.

In this section we have quantified the probability that the short seller finds a competing lender given a degree of searching. This derived probability was plugged into the objective functions of the short seller and local lender, respectively, thus solving for the expected second-period wealth of the agents. Given these valuations we may now compute the optimal response function of the short seller and the optimal first-mover reaction of the local lender.

B. Solving for d*

The first step in solving for equilibrium prices in this market is to solve for and understand the optimal search behaviour of short sellers. Considering the lottery faced by the short seller (Equation 7) one may solve for the optimal distance searched given a mark-up offered by the local firm. Given that the degree of searching, d, is the strategic variable one must maximize the expected second period wealth of the short seller subject to the distance searched in order to find the optimal response.

$$E[W_2] = W_1 - d^2 t - \frac{d-a}{b-a}(c\bar{q}_d) - \frac{b-d}{b-a}(1+m)c\bar{q}_d$$

$$\frac{\partial E[W_2]}{\partial d} = \frac{cmq + 2(a-b)dt}{b-a} = 0$$

$$\underline{\underline{d^* = \frac{cm\bar{q}_d}{2t(b-a)}}}$$

Equation 10: The expected post-search wealth is maximized with respect to the distance searched, which is the strategic variable of the short seller. The unconstrained optimization results in an intuitive optimal distance searched.

First consider the relationship between the exogenous search cost, t, and optimal search behaviour. One may understand the marginal search cost as the degree to which the market is decentralized. Centralized markets have search costs which are very low or zero. Thus, buyers and sellers in a centralized market are willing to engage in an exhaustive search process[5]. Accordingly, as a market becomes decentralized, higher search costs are a disincentive to search and thus the inverse mathematical relationship is understandable.

Like the marginal search cost, the size of the market space directly quantifies the degree of market decentralization. For similar reasons as those listed above the degree of market decentralization, given a finite number of desirable lenders and constant marginal search costs, is inversely related to the optimal amount of searching.

The numerator of the above equation equals the total expenditure by the short seller in the event that she must purchase shares locally. This value is known by the short seller prior to the search process. As the value of the expenditure increases this provides a higher incentive to search for a competitor. Intuition may be gained by, again, considering the hungry tourist: If the price offered by the local restaurant is highly inflated then this provides a greater incentive for the customer to search for an alternate eatery. The same incentive exists to a short seller searching for shares in the equity loan marketplace.

[5] In reality exchanges are structured such that searching is not even necessary as buyers and sellers have an incentive to agglomerate at the central location

C. Solving for m*

The local lender of shares has an incentive to maximize expected profits – defined as the probability of the short seller not finding a competitor multiplied by the total mark-up and the quantity of shares demanded. In optimizing, the local firm must take into account the effect that a higher mark-up has on profits from a higher fee in addition to how this changes the incentive for the short seller to search for a competitor vis-à-vis the response function, d*. This idea was conceptually defined in equation 5 and, given the mathematical preliminaries coupled with the response function d* we may solve for the precise mark-up.

$$\text{Given: } BR(ShortSeller) = d^* = \frac{cmq}{2(b-a)t}$$

Optimize:

$$E[\pi] = \frac{b - d^*(m)}{b-a} cm\bar{q}_d = \frac{b - \left[\dfrac{cmq}{2(b-a)t}\right]}{b-a} cm\bar{q}_d$$

$$\frac{\partial E[\pi]}{\partial m} = f(\mathrm{m}) = 0$$

$$\underline{\underline{m^* = \frac{(b-a)bt}{cq}}}$$

Equation 11: The optimization of the firm markup. Substitute the best response function of the short seller into the profit function of the local firm. Maximize this with respect to the strategic variable *m*. Note that the partial derivative of the expected profit with respect to the markup is f(m)[6]. This is substituted to save space as the FOC is a complex polynomial.

Consider the variables which compose the optimal markup. Those positively correlated with a markup in lending fees are the marginal search cost and the square of the size of the market space. Complete market centralization exists when either the size of the market is infinitely small or search costs are equal to zero - yielding a markup of zero. Moreover, the local lender is willing to offer a quantity discount to the short seller as the markup is a decreasing function of the quantity of shares demanded.

It is important that the reader understand that, although related, the offer price, equal to P = (1+m)*c, is not equivalent to the average market price. The offer price is the equilibrium price in the event that a competitor is not found – which only occurs some of the time. Estimated market prices are calculated in section III.c.

[6] The specific mathematical workup may be calculated with Matlab or Mathematica and is available on request.

D. Decomposition of Equilibrium Offered Fee

The stated goal of this thesis is to understand the impact that a decentralized market structure has on lending fees in the ELM and to make policy recommendations which aim to alleviate such inefficiencies. Given that a source of ELM inefficiencies is the market power of lending firms an important, and interesting, exercise would be to decompose the offer function into two components: the underlying share cost and the decentralized-market premium.

As mentioned, the equilibrium offer price is composed of the mark-up of the local firm and the underlying share cost. By plugging the optimal mark up, solved for in the last section, into the equilibrium offer function we may algebraically distinguish between the two effects.

$$P^* = (1 + m^*)c$$
$$P^* = \left(1 + \frac{(b-a)bt}{cq}\right)c$$
$$\underline{\underline{P^* = c + \frac{(b-a)bt}{\bar{q}_d}}}$$

Equation 12: The equilibrium offer price in the ELM depends on the cost of the share to the local firm plus a premium due to the decentralized nature of the market.

This simple decomposition highlights the two factors which affect the equilibrium price of shares and resulting arbitrage constraints.

1. The Marginal Share Cost

The first term in equation 12 captures the marginal cost of shares to the institutional lender. A logical question is "if there is an underlying cost to institutional lenders in holding shares, then why would a lender not simply sell those shares?". In most cases the marginal share cost to a lending firm is likely close to zero or even negative – generally institutional lenders have share valuations which exceed the current market price.

The cost of shares to the equity lender is not a literal cost but rather a shadow cost. A shadow cost represents the opportunity cost of the shares to the lender – the price that the lender could receive for the shares outside of the current transaction (Pepall, et al., 1999).

According to Duffie, et al (2002) the lending fee for shares of a stock may be positive in the event that equity lenders expect future demand for short selling to exceed current demand – hence waiting to lend shares in future periods is more profitable than lending now.

The most likely scenario in which this framework applies is during formation of a price bubble. Over the course of bubble's formation the price of a stock diverges materially from the underlying value of shares. Theoretically this may be due to imperfections in the equity loan market coupled with heterogeneity in beliefs (Miller, 1977). Over the course of a bubble's formation equity lenders may either sell or lend their shares, the choice of which depends on the relative share valuations of shorters and lenders[7].

The shadow cost, in this case, of lent shares is the discounted expected lending fee that equity lenders may receive in the future. During the formation of a price bubble the expected divergence in price from short seller valuations is definitively increasing – as is the lending fee which lenders expect to receive.

2. The Decentralized Market Premium

The next obvious question is "what implications does this have on share prices and which stocks are especially affected by the decentralized market premium?" I propose that the relative effects of this premium are most apparent in smaller stocks which trade in large markets.

Search costs are mostly affected by regulatory and institutional features – i.e. central exchange locations designated for share lending drive search costs lower (See Jones and Lamont, 2002). Additionally, electronic systems which facilitate agglomeration of lenders and borrowers such as Equilend or eSecLending also serve this purpose (D'Avolio, 2002).

Behaviorally, it is also possible that short sellers perceive the search process in such a way that marginal search costs perceived to be higher for stocks in which borrowers don't expect to easily locate shares. This relates to the psychological framing (see Kahneman & Tversky, 1999) of a situation whereby an agent will associate (or feel subject to) a loss of utility due solely to the context of the situation. Equity borrowers are most likely subject to such effects when trying to locate shares of a smaller stock which they believe has a lower supply of loanable shares.

As alluded to in a prior section, the quantity of shares demanded, q_d, is inversely related to the decentralized market premium. If the representative short seller demands a large block of loanable stock the decentralized market effect is lower. Conversely, it also holds that if short sellers demand fewer shares then the premium is higher. Assuming that the demand for shares by short sellers is correlated with the overall size of the stock – either measured by market capitalization or float – it's plausible that smaller stocks are more affected by imperfections arising from a decentralized market structure.

This contention is also supported empirically. Several studies highlight the negative correlation between firm size and the existence of arbitrage constraints due to ELM imperfections. As noted by Ofek and Richardson (2001), D'Avolio (2002), and Asquith et al (2005), and Lamont and Jones (2002), among others, institutions are the primary lenders of shares in the ELM. However, even after controlling for supply proxies in their analysis, D'Avolio, Geczy, et al (2002), and

[1] Consider the case where $P_{MARKET} > V_{INSTITUTION} > V_{SHORTER}$. If the valuation of the institutional lender is less than the current market price *and* the valuation of the short seller is less than the valuation of the institutional lender then the institutional lender finds it to be more profitable to lend the shares versus selling them.

Nagel (2005) find that firm size is significantly and positively correlated with subsequent abnormal returns. This implies that the market decentralization premium is a primary reason why firm size is a quality indicator of short sale costs.

V. Equilibria and Policy: The n-Lender Case

In this section I explore spatial competition and equilibrium pricing through relaxation of the assumption that only two-firms compete. Rather I consider that n-number of outside lenders exist in the spatial market space. Allowing for interaction of several firms is important as the addition of non-local competitors alters the incentive of short sellers to search. This, in turn, affects the lending fees offered by local firms and ELM market prices.

The key features of this section are comparable to that of the first model. In addition to the vector of market attributes listed previously, in this section equilibrium prices are also a function of n, the number of firms in the market space. Firstly, I review the mathematical framework for solving this model. I then solve for the model using dynamic programming. I then consider the impact of n-firms on mean ELM market prices. Most importantly, this section provides a framework in which policy actions to increase market efficiency may be considered.

A. Preliminaries: The Mathematical Framework

The objective functions of the short seller and the local firm are similar to that of the first section. These are listed below and include notable changes in the search parameter, d, and the probabilities used in calculating expected values.

As a mathematical simplification, a technical change is made in quantifying the search distance, d. The prior model assumed that the units of the variable were the same as that of the market space. As this model is solved in a different manner, d is assumed to be a percentage of the market space searched. Additionally, appropriate changes are made in calculating the total search cost. The results of the model are unaffected - this only is a technical change in the formulae.

Given the existence of several outside competitors, the expected profits of the local firm changes. The greater the number of firms in the marketplace the higher the probability of locating at least one firm over a give search distance, $\bar{d}$. Given a uniform random distribution of n lending institutions over a linear space the probability of not finding a single desired lender equals the amount of the market *not* searched to the power of the number of firms in the market space, n.

$$P(Lender \notin [0,d]) = (1-d)^n$$

Given that, the probability of *at least* one[8] lender existing within the searched domain equals:

[8] Obviously, searching agents don't consider the probability of finding either zero or exactly one competitor. Rather, they consider the sum of the probabilities of finding 1 → n lenders. Technically, this is the sum of the binomial search equations.

$$P(Lender \in [0,d]) = 1-(1-d)^n$$

Short sellers searching for shares in this marketplace incorporate such probabilistic changes into their expected value of searching. The relevant expected value function is below. The expected post-interaction wealth equals the current wealth net search costs and the expected cost of $\overline{q}_d$ shares.

$$E[W_2] = W_1 - t(db)^2 - \underbrace{(1-d)^n(1+m)c\overline{q}_d}_{\text{P(≠Find)*Pay(≠Find)}} - \underbrace{[1-(1-d)^n]c\overline{q}_d}_{\text{P(Find)*Pay(Find)}}$$

Equation 13: The expected second period wealth of a short seller equals the first period wealth net total search costs and the probabilistically weighted cost functions[9].

The profit maximizing local lender also understands the implications of the existence of several firms in the searchable market space. The expected value of offering a given markup, *m*, is:

$$E[\pi_L] = (1-d)^n mc\overline{q}_d$$

Equation 14: The expected profits of the local firm. Note that since the profits are zero in the event of competitive pricing that state is excluded.

As in the two-firm model, the local firm is at a clear first-mover advantage over the short seller, albeit feeling competitive pressure from additional share lenders in the unsearched market space. The methodology in solving for the equilibrium mark-up of the local firm is identical to that of the prior model – via backward induction of the two-player game established earlier and illustrated in Figure 4.

B. Recursively Solving for d*

The optimal degree of searching by the short seller, d*, is solved for via the standard first order condition. The equilibrium is expressed as an implicit function as the optima cannot be solved for with standard algebraic techniques. Thus, the optimal degree of searching is that which renders the following equation TRUE:

[9] Note, once again, that since the quantity of shares demanded is constant that the utility derived from them is also constant. Hence, the utility gained from q_d may be excluded from analysis.

$$\frac{\partial E[W_2]}{\partial d} = 0$$
$$\rightarrow \frac{(1-d^*)^{n-1}}{d^*} = \frac{2b^2 t}{cmnq}$$

Equation 15: The equilibrium relation between the vector of market attributes and the equilibrium distance searched.

Given that the above relation cannot be solved algebraically, I have devised a recursive method using dynamic programming in Visual Basic to efficiently and quickly find equilibrium solutions. This method is then paired with a script which computes the subgame perfect mark-up, m*, of the local firm. Through the application of this set of algorithms one is able to return statistics from the model and interpret relationships of interest between variables. These computations also aid understanding how different policy measures will theoretically affect price efficiency.

The stated objective is to figure out what value of d renders equation 15 to be TRUE. It can be shown through simple algebraic manipulation that the equation under study is equal to the following[10]:

$$\frac{(1-d^*)^{n-1}}{d^*} = \frac{2b^2 t}{cmnq} \leftrightarrow \frac{c(1-d)^n mnq + 2b^2(d-1)dt}{1-d} = 0$$

The objective is to find the point d*. In order to solve for this value recursively we must first show that the relationship is decreasing in *d*. When this is understood then the programming of the algorithm which solves for equilibria is much simpler. For a thorough understanding of this please refer to the source code in the appendix.

That the above relation is decreasing in d is somewhat intuitive in that one would expect, ceteris paribus, that the expected second period wealth is decreasing in the distance searched – due to the quadratic nature of search costs. It may be shown by considering the second derivative of the expected second period wealth with respect to *d*. If one considers the domain of each of the applicable variables it is clear that the second derivative is less than zero.

$$\frac{\partial^2 E[W_2]}{\partial^2 d} = -\frac{c(1-d)^n m(n-1)nq + 2b^2(d-1)^2 t}{(d-1)^2} < 0$$

Equation 16: Given the above equation, it is the case that the second derivative of the expected wealth function is negative – hence the function is always decreasing and crosses the x-axis only once.

[10] Reorgnizing the equation into this format simplifies the recursive process and the explanation thereof.

Now we must approximate the *single* point where distance is optimized. Consider that one may take *X* number of equidistant point samples[11], (B_L, $\check{S}_2$, … , $\check{S}_{x-1}$, B_U) between an upper and lower bound $\{B_L, B_U\}$ of equation 16. The optimal point lies between *two* of those point samples – specifically where the product of the $\check{S}_i$ and $\check{S}_{i+1}$ is negative[12]. Once the two critical points on either side of the optima are known one may consider these as a *new pair* of upper and lower bounds in the next iteration of the algorithm. This procedure may be iterated as many times as one likes with the accuracy of the estimate being proportional to the number of iterations. This concept is explained further in the figure below.

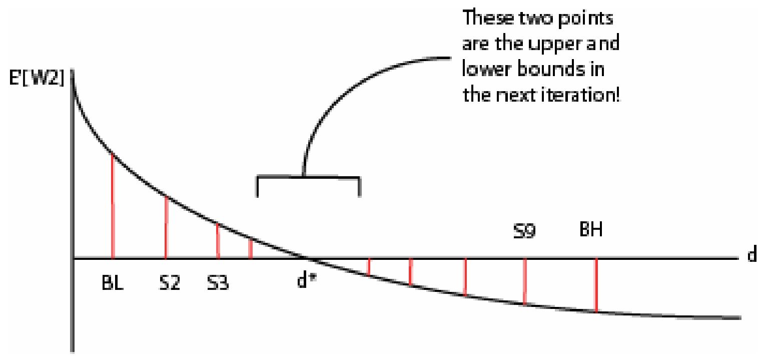

Figure 5: To find the solution one must consider X point samples within the bounds and determine the two points ($\check{S}_i$ and $\check{S}_{i+1}$)between which the inflection occurs. In order to estimate d* with a degree of accuracy these two points are considered as new boundary points in the dynamically programmed algorithm.

Given the nature of this method, an explicit equation-based optimization is not possible. Rather, the algorithm outputs a value, d*, which is a close approximation of equation 3. For the source code in Visual Basic 6 format please see the appendix.

C. Recursively Solving for m*

As stated in equation 5, the markup set by the local firm is that which maximizes the conditional expected profit of the firm. In the two-firm case the optimal markup was easily and explicitly found, however in the case where the optimal search behavior of the short seller cannot be explicitly found the expected profit must also be determined through non-algebraic means[13].

The process in which m* is calculated is similar to how d* was calculated in the last section. The conditional profit of the firm as a function of the markup has a maxima which must be

[11] Specifically, the array of samples are determined by the following equation:

$$\check{S}_i = U_L + \frac{U_H - U_L}{X} \cdot i$$

[12] Since the optimal point is where the equation equals zero then it must be the case that the two point samples on either side of the optimal point have different signs and thus multiply to a negative number.

[13] In other words, since we cannot explicitly solve for d* (as a function) we cannot solve for m*, which depends on d*, as an explicit function either.

recursively estimated. Bear-in-mind that the for each point *m* sampled the optimal reaction of the short seller (d*) is estimated (recursively) and plugged back into the profit maximization equation. The solution is a close approximation of equation 5.

As was the case in finding the optimal response of the short seller, an exact optimal markup cannot be found using this approach, however a close approximation can. The Visual Basic source code for this algorithm is in the appendix.

VI. Policy Recommendations

If the beliefs of investors are heterogeneous then competitive prices in the equity loan market are crucial for efficient pricing in the primary stock market (Boehme, 2002). As this model has shown, the decentralized nature of the equity loan market is theoretically a constraint to competitive pricing for lent shares. As a result, arbitrage is not costless and securities are at risk of price inflation. In order to increase efficiency of share prices significant reforms of the market for securities lending are needed.

In this section I justify two specific policy actions[14]. I first illustrate and discuss how, for a constant search expenditure by short sellers, average lending fees fall as additional lenders are added to the market space. Specifically, a higher density of lenders in the market space is a centralizing force in that the expected probability of finding a lender per unit search cost increases as more lenders enter the market. Hence, policy initiatives which encourage the entrance of lenders increase market efficiency.

My second, and primary, efficiency-enhancing policy recommendation is that regulatory efforts focus on decreasing the marginal cost of searching for shares of firms with fewer institutional owners. Specifically, I show that the efficiency gain per-unit reduction in the marginal search cost is decreasing as the number of institutional lenders increase. Thus, any concerted effort in ELM centralization should focus on smaller stocks which, generally, have fewer lenders.

A. Policy Recommendation I: Increase Lender Participation

> "Searching for information is a search that is concerned with…the density of choices." - Prof. R.W.J.

If the overall objective of policymakers is to decrease arbitrage costs via centralization of the ELM, one method that is tantamount would be to increase ELM participation by institutions who are willing to lend the desired shares. If there existed a higher density of favoured institutions then the expected search cost required for a short seller and second equity lender to intersect would be lower.

[14] Decreasing the size of the searchable market space would also increase efficiency. The implication with that policy is that the total number of institutions would be reduced which is not a reasonable alternative.

This can be simply proven by taking the first derivative of the probability of finding a second equity lender in the ELM with respect to n, the number of lenders in the market. If it is positive then this implies that as the density of desired lenders increases the result is a greater probability of success per unit of the market searched. The calculation in equation 17 confirms this.

$$\Pr[FindCompetitor] = 1-(1-d)^n$$

$$\frac{\partial 1-(1-d)^n}{\partial n} = -(1-d)^n Log(1-d) > 0$$

$$\frac{\partial^2 1-(1-d)^n}{\partial n} = -(1-d)^n Log(1-d)^2 < 0$$

Equation 17: The first and second derivatives of the probability of success in locating a second lender.

Now that we have determined that the probability of success is, in fact, increasing in the number of desirable lenders in the market space we may posit the following: at what point is the marginal benefit from the addition of a lender greatest – when there are already several lenders or when there are only a few?

The answer to this is captured in the second derivative of the probability function. If the second derivative is increasing (convex) this implies that the greatest efficiency gains occur when the market already has a high density of lenders. On the contrary, if the second derivative is decreasing (concave) this implies that the greatest efficiency gains stems from the addition of a competitor when the number of incumbent outside lenders is low.

Equation 17 shows that the second derivative is concave and, thus, the largest marginal efficiency gains arise from the addition of a competitive lender when the density of incumbent lenders is low. This idea is further, and perhaps more clearly, illustrated in the below figure.

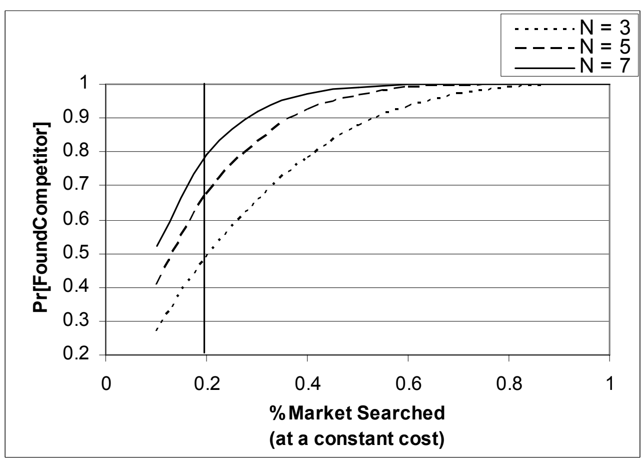

Figure 6: Given a constant expenditure on searching, the probability of finding a competing firm is increasing (at a decreasing rate) in the number of desired lenders in the market space.

The implication is that in order to maximize the gain in overall market price efficiency public policy should focus on encouraging institutions to diversity – especially into stocks with lower levels of institutional investment. This may be done by increasing the transparency of smaller firms which may otherwise not be viable candidates for institutional investment. Alternatively, higher transactions costs (i.e. liquidity costs) associated with smaller-cap stocks could be mitigated.

An additional policy measure which would encourage an excess supply of loanable shares in the relevant type of stock is the removal of the SEC requirement that no stock trading under $5 may be purchased on margin[15]. While shares loaned via margin accounts are not necessarily preferred by short sellers, this would increase the aggregate supply of a type of loanable share in high demand. The result would be more efficient pricing of smaller firms.

B. Policy Recommendation II: Focus Centralization Policy

A centralized marketplace is definitively a market in which either:

> 1. All buyers and sellers agglomerate at a single location to engage in transactions. In the model this is tantamount to reducing the linear chord $\overline{0B}$ to a point.
> 2. The transaction costs/frictions are eliminated. In the model this would equate to reducing the variable t to zero.
>
> Tirole, 1988

In this section I focus on the second attribute – and particularly, if reduction in search costs for *all* equities is not feasible, then what type of stocks should regulators focus on? I propose that the greatest efficiency gains arise when search costs are reduced in stocks with fewer institutional lenders – the vast majority of which are smaller firms.

I prove this proposition by first describing how the average market lending fee is a function of the degree of searching , the offer price of the local firm, and the competitive market price. This methodology is comparable to that in section III.c. I then illustrate, using output from the n-firm model, the effect that a reduction in marginal search costs has on the degree of searching and the offer price of the local firm. Finally, I illustrate how a reduction in marginal search costs is positively related to a reduction the average lending fee *and* that the marginal efficiency gain resulting from a reduction in t is decreasing in the number of outside lenders in the market space.

As described in Section III.c., the average lending fee in the ELM is the probability-weighted local offer price and the competitive fee. In the n-firm case the estimated market price is:

$$\overline{P}_{MARKET} \approx (1-(1-d^*)^n)\cdot c + (1-d^*)^n \cdot (1+m^*)c$$

Equation 18: The average market price equals the sum of the weighted-probabilities of each price equilibrium.

[15] SEC Regulation T, http://www.sec.gov

The objective of the socially optimizing regulator is to minimize P_{MARKET}. This may be accomplished by providing an incentive for the short seller to increase the degree of searching or by providing an incentive to the local lender to decrease the markup of the shares.

The most readily available tool to regulators/policy makers in altering agent incentives is to decrease search costs associated with transactions in this market. A lowering of the marginal search cost should encourage short sellers to search more and the local lender to reduce the offer price.

Analysis of the model provides specific insight into what the precise effect that changes in search costs has on the strategic variables of the relative agents. Firstly, a reduction in the marginal search cost in the ELM provides an incentive to short sellers to search a greater percentage of the marketplace for a competing firm. Resultantly, this increases the probability that a competitor is found, as illustrated in Figure 6. Moreover, the offer price is also reduced if search cost are decreased. Both of the described effects are pro-competitive in nature and beneficial to price efficiency in the primary stock market.

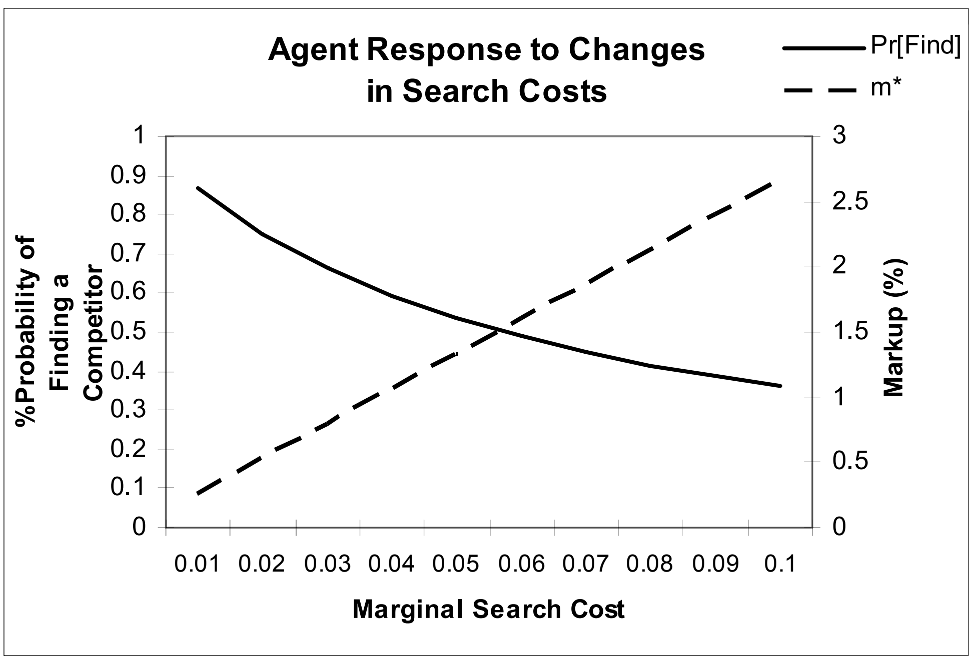

Figure 7[16]: Changes in search costs results in changes in the equilibrium markup of the local firm and increases the probability that the short seller will locate a competitor.

[16] **Note**: For the applicable relationships in this section the static variable values are as follows: c = 0.01; q = 5; b = 10; Other values have been tested in order to ensure the robustness of these findings. The output/calculations are available if requested.

Given the previously noted trends in the strategic variables, the next logical question is "what is the aggregate effect of search cost reductions on the average lending fee in the ELM? This may be determined by considering equation 17. The results are illustrated below.

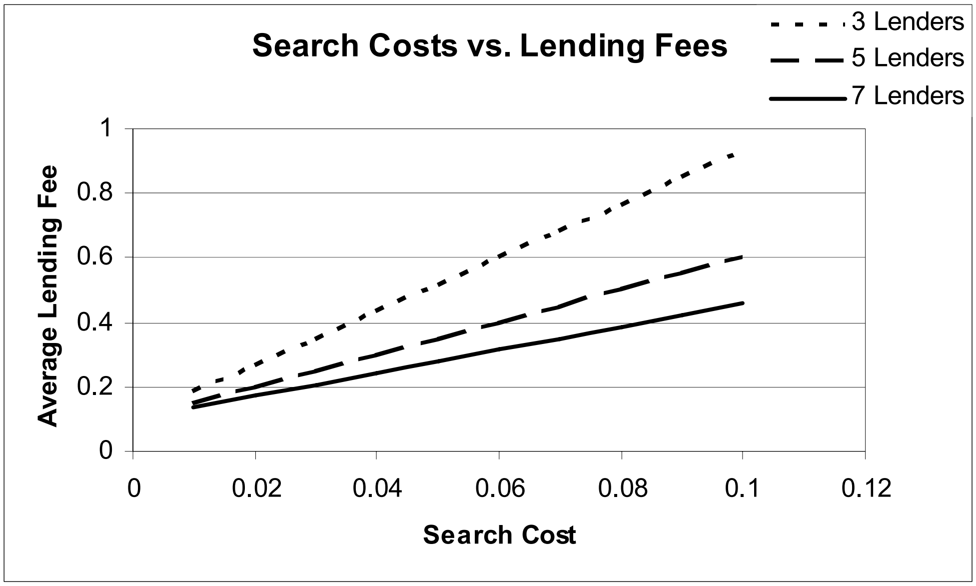

Figure 8: The relationship between the marginal search cost and the average lending fee. In all cases the relationship is positive, however lending fees of stocks with few institutional holders are most sensitive to changes in the marginal search cost.

Figure 7 shows that search costs and lending fees are positively correlated. The graphic also shows that lending fees are unambiguously higher if fewer institutional lenders exist in the market space.

An interesting and important trend in Figure 7, and one which alludes to the importance of targeted centralization, is the relative impact that a change in search costs has on the average lending fee. The slope of each line equals the change in the average lending fee over the change in the search cost. A higher slope implies that the marginal change in search costs has a larger impact on average lending fees. The relative slopes of each line implies that stocks with fewer lenders of desired shares gain more efficiency from a unit reduction in search costs.

This trend may be confirmed by specifically considering the gain in price-efficiency per unit reduction in search costs. For example, if ELM search costs are reduced by an equal amount in two types of stocks (i.e. large cap and small cap stocks) and the lending fee in type-A stocks falls by 40% while the lending fee of type-B stocks falls by only 20% then it is worthwhile concentrating efforts on reduction of the search costs for type-A stocks rather than the latter.

The implied trend is confirmed in the below diagrams. In Figure 8A the change in price per unit change in search cost is charted for several hypothetical levels of institutional ownership. Figure 8B charts the percentage change in price per percent change in search cost. The second chart was included in the analysis in order to indicate that the trend is not the result of a unique-level effect in the data[17].

The results indicate that, given the noted market characteristics, in a securities lending market with eight non-local equity lenders that a 10% reduction in search costs will result in approximately a 5.0% reduction in average share lending fees. However, in a market with only four non-local equity lenders a reduction of the same amount results in a 6.8% reduction in average lending fees. This research implies that by reducing search costs in the market with fewer institutional lenders yields a relative price efficiency gain of ~36%.

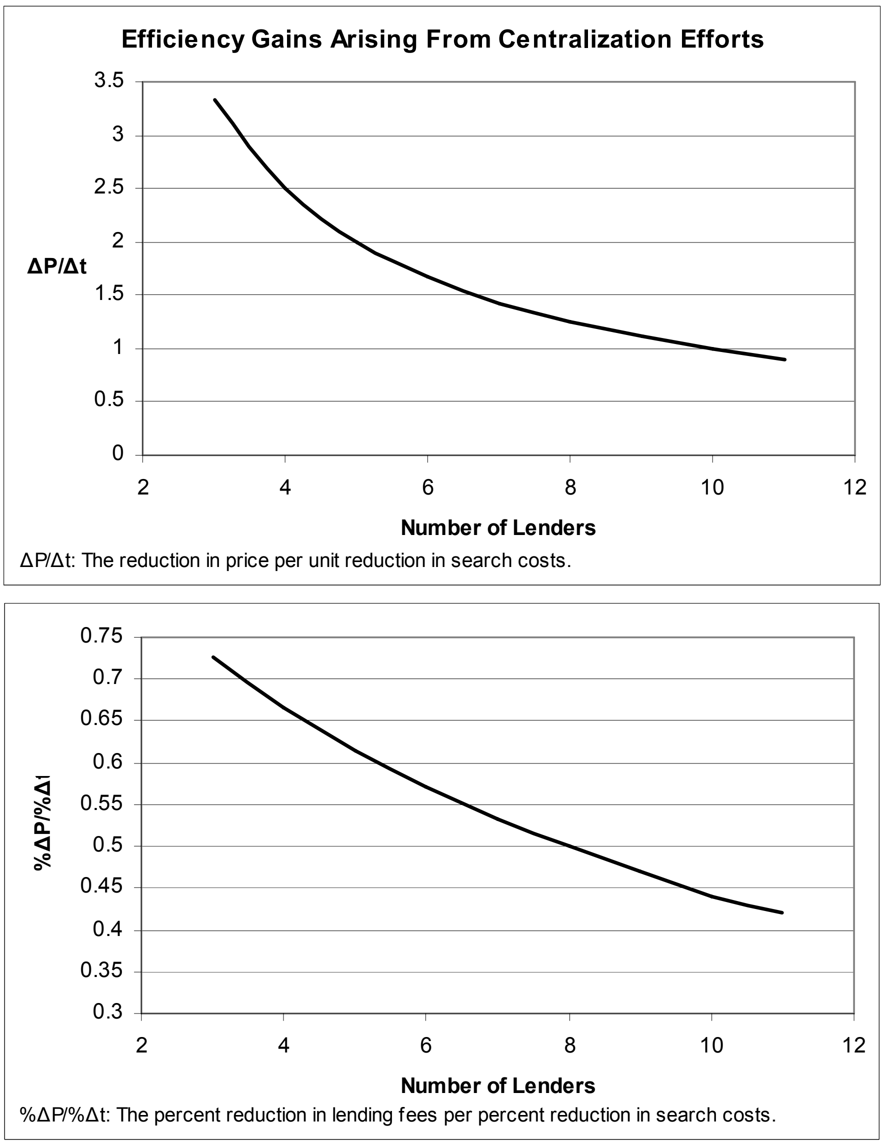

Figure 9: A (Above): The relationship between the number of lenders and the absolute change in average ELM prices per unit change in search costs.

[17] The robustness of this relationship was tested for several different levels of market size, quantity demanded, search costs, and underlying share cost. The specific data is available on request.

B (Below): The relationship between the number of lenders and the percent change in average ELM prices per percent change in search costs.

Based on the results of this second policy action tested, several conclusions may be cautiously drawn. Firstly, it is clear that a reduction in marginal search costs alters the incentives of both short sellers and institutional lenders. Moreover, these altered incentives are pro-competitive insofar as they result in a decrease in expected ELM lending fees.

It was also determined that lending fees for stocks with fewer institutional lenders decrease more than lending fees for stocks with a greater number of lenders – both on an absolute and percentage-basis. The final, and most important conclusion, is that regulatory policies which reduce marginal search costs result in greater efficiency gains to stocks which are most subject to price inflation as a result of short sale constraints – particularly smaller equities.

VII. Discussion and Conclusion

The US equity lending market must undergo significant reforms before equity markets may be deemed fully efficient, as defined by efficient market theory. The ELM has several constraints associated with it, not least of which is it's decentralized market structure. Given the results of this model/thesis, one should expect that the lending premium due to the market's decentralized structure is directly related to the size of the stock under consideration - smaller stocks are more apt to be overvalued due to this ELM imperfection.

The primary finding of the thesis is that policy measures aiming to reform the ELM should first and foremost attempt to reduce the search costs that short sellers wishing to short smaller-cap stocks are subject to. Such a policy will, theoretically, yield the greatest gain in price efficiency while boosting the confidence of small-cap investors.

Interestingly, a 2001 paper by Chen, Hong, and Stein researched the relative underperformance of stocks with concentrated institutional ownership. They interpreted the breadth of institutional ownership as a reflection of belief heterogeneity. If ownership was concentrated the implication was that beliefs were more heterogeneous - some institutions opted out - which signalled that future abnormal returns would be lower.

In the context of this model one may also interpret a low breadth in institutional ownership to be an indicator of lower future share returns. If institutional ownership is concentrated this implies that lenders are more difficult for short sellers to locate and, consequentially, arbitrage constraints are higher. If, all else equal, arbitrage costs are higher this points toward lower future returns.

Not since the early 1930s has there been any real attempt to centralize the equity lending marketplace (Jones and Lamont, 2002). Currently two private-sector attempts are being made at centralizing parts of the ELM. The first is Equilend, an electronic platform designed by several major broker-dealers to facilitate the lending of larger-cap stocks to their clients - primarily hedge funds. The second modern attempt is by eSecLending, a Boston-based firm which specializes in providing institutions with a venue to auction off lending rights to their entire portfolio of stocks.

The current efforts in ELM centralization are important first steps in the process of reducing frictions in the ELM, although they primarily focus on the lending of large and mid-cap equities. Present efforts must be expanded to not just include but emphasize the availability of a low-cost loanable supply of small-cap stocks.

References

Boehme, R., Danielsen, B., Sorescu, S. "Short sale constraints, differences of opinion, and overvaluation." Journal of Financial and Quantitative Analysis, 2004.

Chen, J., Hong, H., Stein, J., "Breadth of ownership and stock returns." 2002. Journal of Financial Economics 66, 171–205.

D'Avolio, G., "The market for borrowing stock." 2002. Journal of Financial Economics 66, 271–306.

Duffie, D., Garleanu, N., Pedersen, L. "Securities lending, shorting, and pricing." Journal of Financial Economics, 2002, vol. 66, issue 2-3, pages 307-339

Fama E. "Efficient Capital Markets: A Review of Theory and Empirical Work" The Journal of Finance, Vol. 25, No. 2, (May, 1970)

Geczy, C., Musto, D., Reed, A., "Stocks are special too: an analysis of the equity lending market." 2002. Journal of Financial Economics 66, 241–269.

Hotelling, H. "Stability in Competition" The Economic Journal, Vol. 39, No. 153 (Mar., 1929), pp. 41-57

Jones, C., Lamont, O., 2002. "Short-sale constraints and stock returns." Journal of Financial Economics 66, 207–239.

Kahneman, D, Tversky, A. Choices, Values, and Frames" in Choices, Values, and Frames, 2000. Cambridge University Press

Lamont, O., Thaler, R. "Can the Market Add and Subtract? Mispricing in Tech Stock Carve-outs" 2003 Journal of Political Economy, volume 111

Lucas, R. Stokey, N. & Prescott, E. Recursive methods in economic dynamics 1989. Harvard University Press

Miller, E., "Risk, uncertainty, and divergence ofopinion." Journal of Finance 32, 1977 1151–1168.

Nagel, S., "Short sales, institutional investors, and the cross-section of stock returns." Journal of Financial Economics, 2005

Ofek, E. Richardson, M "DotCom Mania: The Rise and Fall of Internet Stock Prices" 2003. The Journal of Finance 58 (3), 1113–1138.

Ofek, E., Richardson, M., Whitelaw, R. "Limited arbitrage and short sales restrictions: evidence from the options markets." 2004. Journal of Financial Economics 74, 305–342

Perloff, J., Salop, S "Equilibrium with Product Differentiation" The Review of Economic Studies, Vol. 52, No. 1 (Jan., 1985), pp. 107-120

Pepall, L., G. Norman, and D. Richards. Industrial Organization: Contemporary Theory and Practice, South-Western Publishing Company, 1999

Tirole, J. The Theory of Industrial Organization. 1988 MIT Press

Salop, S. "Monopolistic Competition with Outside Goods." 1979. Bell Journal of Economics 10: 141-156.

Simon, C. Blume, L. Mathematics for Economists, W.W. Norton & Company, 1994

Stahl, K. "Location and Spatial Pricing Theory with Nonconvex Transportation Cost Schedules." 1982. Bell Journal of Economics 13: 575-582.

Tijms, H. A first course in stochastic models. John Wiley & Sons Ltd, 2003

Watson, J. Strategy: An Introduction to Game Theory, New York: W. W. Norton and Company, 2002.

Appendix 1: Proof of Proposition 1

> **Proposition 1**: *It is a dominant strategy for a rational short seller to search for an competing lender if the price offered by the local lender is greater than the competitive lending fee.*

Step 1. If the agent does engage in a search the agent will search to the degree which maximizes the expected value of the search, d*.

$$E[W_2] = W_1 - t(db)^2 - dqc - (1-d)q(1+m)c$$

$$\frac{\partial E[W_2]}{\partial d} = c(1+m)q - 2b^2dt - cq = 0$$

$$\underline{\underline{d^* = \frac{cmq}{2b^2t}}}$$

Step 2. The expected value of the search, given the optimal degree of searching, is strictly higher than the value of purchasing from the local firm

$$W_1 - q(1+m)c < W_1 - t(d^*b)^2 - d^*qc - (1-d^*)q(1+m)c$$

$$-q(1+m)c < -t(d^*b)^2 - d^*qc - (1-d^*)q(1+m)c$$

$$q(1+m)c > t(d^*b)^2 + d^*qc + (1-d^*)q(1+m)c$$

$$q(1+m)c > t(d^*b)^2 + d^*qc + q(1+m)c - d^*q(1+m)c$$

$$0 > t(d^*b)^2 + d^*qc - d^*q(1+m)c$$

$$0 > t(d^*b)^2 + d^*qc - d^*qc - d^*qcm$$

$$0 > t(d^*b)^2 - d^*qcm$$

$$d^*qcm > t(d^*b)^2$$

$$qcm > t\frac{cmq}{2b^2t}b^2$$

$$qcm > \frac{qcm}{2} \Diamond$$

Appendix 2: Source Code to the Recursive Algorithms

```
Option Explicit
Dim recursive_dOPT_OUT As Single
Dim recursive_mOPT_OUT As Single

'compile a database of 1000 records
'cache 50 records at a time in an array which are then written
'to the file

Sub Main()
    Dim i, n, j, q As Integer
    Dim t, c As Single
    Dim arraycache(50, 4) As Single
    Dim outvector, outfile As Object
    Dim datacache As String

    Set outvector = CreateObject("Scripting.FileSystemObject")
    Set outfile = outvector.CreateTextFile("c:\searchdata.csv", True)

    outfile.WriteLine ("m,n,t,c,q")

    i = 1
    arraycache(0, 0) = 0

    While i <= 1500
        Randomize
        n = Fix(Rnd() * 5) + 3
        t = Round(Rnd() * 0.1, 2) + 0.01
        c = Round(Rnd() * 2.1 + 0.1, 1)
        q = Fix(Rnd() * 5) + 6
        arraycache(0, 0) = arraycache(0, 0) + 1
        Call recursive_mOPT(10, q, n, t, c, 0.01, 10, 1, 15)
        arraycache(arraycache(0, 0), 0) = recursive_mOPT_OUT
        arraycache(arraycache(0, 0), 1) = n
        arraycache(arraycache(0, 0), 2) = t
        arraycache(arraycache(0, 0), 3) = c
        arraycache(arraycache(0, 0), 4) = q
        If arraycache(0, 0) = 50 Then
            datacache = arraycache(1, 0) & "," & arraycache(1, 1) & "," & arraycache(1, 2) & "," _
            & arraycache(1, 3) & "," & arraycache(1, 4)
            j = 2
            While j <= 50
                datacache = datacache & Chr(13) & Chr(10) & arraycache(j, 0) & "," & arraycache(j, 1) & "," & arraycache(j, 2) _
                & "," & arraycache(j, 3) & "," & arraycache(j, 4)
                j = j + 1
            Wend
            outfile.WriteLine (datacache)
            arraycache(0, 0) = 0
        End If
        i = i + 1
    Wend
    outfile.Close
End Sub

'this dynamic progamming subroutine solves the implicit distance optimization function
```

```
'using a recursive method. The optimal value is returned to the recursive_dOPT_OUT variable
Sub recursive_dOPT(b, q, m, n, t, c, min, max, iterat, numiter)
    Dim strata(10, 1) As Single 'the vector of strata used in the analysis
    Dim strataat As Byte

    'populate the strata
    strata(0, 1) = -((c * ((1 - min) ^ n) * m * n * q + 2 * (b ^ 2) * (-1 + min) * min * t) / (-1 + min))
    strata(0, 0) = min
    strata(10, 1) = -((c * ((1 - max) ^ n) * m * n * q + 2 * (b ^ 2) * (-1 + max) * max * t) / (-1 + max))
    strata(10, 0) = max

    strataat = 1
    While strataat <= 9
        strata(strataat, 0) = ((max - min) * strataat / 10 + min)
        strata(strataat, 1) = -((c * ((1 - strata(strataat, 0)) ^ n) * m * n * q + 2 * (b ^ 2) * (-1 + strata(strataat, 0)) * _
        strata(strataat, 0) * t) / (-1 + strata(strataat, 0)))
        strataat = strataat + 1
    Wend

    'determine the inflection point. The bounds are strata(strataat) & strata(strataat+1)
    strataat = 0
    While strata(strataat, 1) * strata(strataat + 1, 1) > 0
        strataat = strataat + 1
    Wend

    'then call the same function again with newly determine bounds
    If iterat < numiter Then
        Call recursive_dOPT(b, q, m, n, t, c, strata(strataat, 0), strata(strataat + 1, 0), iterat + 1, numiter)
    Else
        recursive_dOPT_OUT = (strata(strataat, 0) + strata(strataat + 1, 0)) / 2
    End If
End Sub

Sub recursive_mOPT(b, q, n, t, c, min, max, iterat, numiter)
    Dim strata(10, 1) As Single 'the vector of strata used in the analysis
    Dim strataat As Byte
    Dim maxstrata(1) As Variant
    'maxstrata: (0) holds the vector location of the maximum strata
    '           (1) holds the value of the maximum strata

    'populate the strata
    strata(0, 0) = min
    Call recursive_dOPT(b, q, strata(0, 0), n, t, c, 0, 0.9999, 1, 10)
    strata(0, 1) = ((1 - recursive_dOPT_OUT) ^ n) * c * strata(0, 0) * q

    strata(10, 0) = max
    Call recursive_dOPT(b, q, strata(10, 0), n, t, c, 0, 0.9999, 1, 10)
    strata(10, 1) = ((1 - recursive_dOPT_OUT) ^ n) * c * strata(10, 0) * q

    strataat = 1
    While strataat <= 9
        strata(strataat, 0) = ((max - min) * strataat / 10 + min)
        Call recursive_dOPT(b, q, strata(strataat, 0), n, t, c, 0, 0.9999, 1, 10)
        strata(strataat, 1) = ((1 - recursive_dOPT_OUT) ^ n) * c * strata(strataat, 0) * q
```

```
        strataat = strataat + 1
    Wend

    'determine the maxima. the recursive bounds are the points to the left and right of the maxima
    strataat = 0
    maxstrata(1) = -1 * 10 ^ 20
    While strataat <= 9
        If strata(strataat, 1) > maxstrata(1) Then
            maxstrata(0) = strataat
            maxstrata(1) = strata(strataat, 1)
        End If
        strataat = strataat + 1
    Wend

    'then call the same function again with newly determine bounds
    If maxstrata(0) = 0 Then maxstrata(0) = 1
    If maxstrata(0) = 10 Then maxstrata(0) = 9
    If iterat < numiter Then
        Call recursive_mOPT(b, q, n, t, c, strata(maxstrata(0) - 1, 0), strata(maxstrata(0) + 1, 0), iterat + 1, numiter)
    Else
        recursive_mOPT_OUT = strata(maxstrata(0), 0)
    End If
End Sub
```