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Inflation Risk Premium in the Swedish Bond Market

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ABSTRACT

The purpose of this study is to investigate the dynamics of the inflation risk premium in the Swedish bond market using nominal and inflation-linked bond yields. We apply a no-arbitrage method extracting real yields from nominal and inflation-linked yields taking the three-month indexation lag inherent in Swedish bond yields into account. We estimate the inflation risk premium and impose a liquidity measure based on the dispersion of individual bond yields around the yield curve to account for potential illiquidity concerns in the inflation-linked bond market. We show that taking the indexation lag into account has an effect on breakeven inflation and the inflation risk premium. The estimated inflation risk premium is small or negative in recent years, on average -18 basis points for 5-year maturities in our sample of 10 years. We interpret the negative sign of the inflation risk premium as an indicator of periods during which markets were concerned about the risk of unexpected deflation. However, the inflation risk premium estimates could be negative due to an understated liquidity measure resulting from a limited number of inflation-linked bonds available when estimating the yield curve.

Keywords: Inflation risk premium, liquidity risk premium, expected inflation, inflation-linked bonds.

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1. Introduction

The interest in inflation-indexed government debt and more specifically inflation-linked bonds has increased lately. The market is fairly young and has experienced significant growth during the last decade where the global market for inflation-indexed debt has more than ten-folded. A nominal bond delivers a specified principal and interest without correcting for inflation. Markets and monetary policymakers generally assume that nominal yields include a risk premium for bearing that extra inflation risk, *the inflation risk premium*. An inflation-indexed bond is a bond that is indexed to inflation or more specifically to a Consumer Price Index (CPI) giving the holder protection against future inflation fluctuations and delivers a return adjusted for inflation, a *real return*. To visualize the inflation risk premium inherent in bond yields please consider the following formula for the yield of a nominal bond (Grishchenko and Huang (2013)):

$$y_t(\tau) = y_t^R(\tau) + E_t\pi_{t+\tau}(\tau) + IRP_t(\tau)$$

Where $y_t(\tau)$ is the nominal yield for period τ, $y_t^R(\tau)$ is the real yield for period τ, $E_t\pi_{t+\tau}(\tau)$ represent the expected inflation and finally $IRP_t(\tau)$ that is the inflation risk premium at time t for maturity τ. The yield spread between an inflation-linked bond and its nominal equivalent is often referred to as *breakeven inflation* or the *breakeven inflation rate* (BEIR) and is commonly seen as an approximate measure of markets inflation expectations. The breakeven inflation consist of two parts essentially, firstly the expected inflation $E_t\pi_{t+\tau}(\tau)$ and secondly the inflation risk premium $IRP_t(\tau)$.

The size of the inflation risk premium indicates the level of uncertainty in the perceived inflation expectations among bondholders, thus movements in the size of the premium can have a signalling effect regarding the effectiveness of policies introduced by monetary policymakers (Shen 1998). Knowing the size of the premium is thus a crucial tool for monetary policymakers when forming their policies or when issuing inflation-indexed debt. Previous empirical work on the estimation of the inflation risk premium is rather limited and inconclusive regarding both the sign and the magnitude of the premium.

Most previous empirical work estimating the inflation risk premium has been conducted using nominal bond yields due to the relative youth of the inflation-indexed bond market. There are studies using inflation-indexed bonds to estimate the inflation risk premium but they are limited and a majority of them focus on the US or the UK market. Markets for inflation-linked debt run a risk of being illiquid, particularly during the first years after issuance. To a large extent this liquidity factor has been disregarded in the past but during recent years it has gained notoriety in academia.

Campbell and Schiller (1996) were among the first ground-breaking studies that examined inflation-indexed government debt as an instrument and more specifically if the US Treasury should issue such an asset class or not. They argued in favour of issuance of inflation-indexed debt and when estimating the inflation risk premium they used US nominal bonds. They found a positive premium in the range of 50-100 b. p. depending on maturity. Evans (1998) introduced a new methodology where he used real zero-coupon yields extracted from nominal and inflation-linked bonds for the UK. He also developed a methodology to estimate the impact of the three-month indexation lag on the difference between real yields and inflation-linked bond yields using a no-arbitrage condition. He did not focus on the size of the inflation risk premium but according to his findings the premium could be both positive and negative.

Following the methodology introduced by Evans (1998), Grishchenko and Huang (2013) estimate the inflation risk premium using TIPS yields from the US market during the period 2000-2008. In their study they also impose a liquidity measure to be able to account for the potential liquidity risk premium inherent in inflation-linked bond yields. They find a time-varying inflation risk premium, negative in the first half of the sample period and then positive in the second half.

Previous empirical work does not give a general consensus neither regarding the size or the sign of the inflation risk premium which makes it an interesting area to examine further. We use an arbitrage free simple methodology following Evans (1998) and Grishchenko and Huang (2013) and we give it a new angle by examining the Swedish market that is still somewhat of an uncharted territory when it comes to studying the inflation risk premium.

Our study aims at estimating the inflation risk premium for the Swedish market using data on nominal zero-coupon bonds and inflation-linked bonds. To conduct our study we use data from The Swedish Central Bank, *Riksbanken.* We use Swedish monthly nominal zero-coupon bonds as well as inflation-linked bonds with 2-year and 5-year maturities. The novelty of our analysis lies in the estimation of real yields using nominal and inflation-linked bond yields for the Swedish market while correcting for the three-month indexation lag inherent in Swedish bond yields. We address the potential illiquidity in inflation-linked bond markets when we introduce a liquidity measure to estimate the liquidity risk premium. On average we find a 26 b. p. increase in inflation-linked bond yields due to illiquidity concerns. Then we proceed with the estimation of the inflation risk premium and before correcting for liquidity we find a mostly negative inflation risk premium with a downward sloping term structure. After controlling for liquidity the term structure shifts upwards but the inflation risk premium is still mostly negative, on average in the range from -15 b. p. to -18 b. p. for 2 and 5-year maturities.

Our main contribution is that we are able to estimate the real breakeven inflation rate, as opposed to the inflation-linked bond breakeven rate for Sweden using a no-arbitrage model; showing that the three-month indexation lag makes a difference of 11 b. p. on average for 5 year maturities in our sample. We are also able to estimate a potential proxy for the inflation-linked bond liquidity premium for Sweden. Finally we estimate the inflation risk premium for the Swedish market and briefly discuss the potential underlying causes for the negative sign of the estimate during Fall 2008 to 2009 and mid-2011 to 2013.

The reminder of this paper is structured as follows. In Section 2 we review previous literature covering the estimation of the inflation risk premium. The methodology and an outline of the estimation process are described in Section 3 while the data used in the estimation of our model is outlined in Section 4. The empirical results are presented and discussed in Section 5 and Section 6 concludes.

2. Literature review

When estimating the inflation risk premium the most common practice historically has been to use nominal bond yields which partly can be explained by the relative youth of the market for inflation-indexed bonds. Studies using inflation-indexed bonds have grown together with the growth of the market itself but are still limited. The study by Campbell and Schiller (1996) were an important starting point for studies covering the estimation of the inflation risk premium and the information embodied in bond yields. They studied the US market using nominal bond yields since inflation indexed government debt had not yet been introduced in the US. They estimated an inflation risk premium in the range of 50-100 b.p depending on the maturity of the bonds.

Firstly they evaluate indexed government debt as a debt instrument and secondly they aim at clarifying whether or not the US Treasury should issue inflation-indexed debt. They conclude their study by prompting the US Treasury to introduce indexed debt and two months after publication of their study the US Treasury announced the plans for issuance of inflation-indexed bonds known as Treasury Inflation Protected Securities, TIPS. A more recent study by Bekaert and Wang (2010) evaluates the role of inflation-indexed securities in the US since its inception in 1997. Overall they have a positive view on the issuance of inflation-indexed bonds but they also stresses the case that the benefits quickly vanish when liquidity in the markets surge.

Evans (1998) studies the term structure of real rates and the inflation risk premium using real zero-coupon yields extracted from nominal and inflation-linked bonds for the UK. The main focus of his study is not to estimate the magnitude of the inflation risk premium but according to his findings the inflation risk premium can be both positive and negative. The indexation lag effect on the difference between real yields and inflation-linked bond yields is mostly ignored in the previous literature; however, Evans (1998) takes it into consideration. This methodology pioneered the field and introduced a new way of estimating the inflation risk premium and since then many studies have followed the same concept (e.g. Joyce, Kaminska and Lildholt (2008), Garcia and Werner (2010)).

Grishchenko and Huang (2013) follow the framework set by Evans (1998) while estimating the inflation risk premium using TIPS yields, where they impose a new TIPS liquidity measure to their study (D'Amico, Kim and Wei (2014), Bergroth and Carlsson (2014)). They apply the Nelson-Siegel-Svensson model[1] with the specific individual TIPS yields and take the average fitting error between the Svensson (1994) yield curve and the respective TIPS yields and use as their liquidity measure. Their results indicate a slight lag-correction effect on short maturities that increases gradually with yield maturity. They study the period 2000-2008 and find a time-varying inflation risk premium, which is negative in the first half of the sample period, and then turns positive in the second half, even after controlling for liquidity.

2.1 Affine term structure models

Another common practise when estimating the inflation risk premium is the use of affine term structure models and below we present the findings linked to this practise. An important study examining the inflation risk premium using affine term structure models is D'Amico, Kim and Wei (2014) who introduce an inflation-linked bond liquidity premium to their model. They study nominal bond yields in the US during the period 1990-2013, but due to availability restrictions they only incorporate TIPS yields between 1999-2013.

When estimating the inflation risk premium without taking the liquidity factor into account they find a negative premium of around -50 b. p. that increases during the period studied. When the liquidity factor is introduced they improve the fit of their model and the estimated inflation risk premium is positive around 0 to 0.5%. Their main contribution to the literature lies in the finding of a liquidity premium, an important step in forming a better understanding of TIPS yields.

Bergroth and Carlsson (2014) follow the same concept as D'amico, Kim and Wei (2014) when estimating the liquidity premium for Swedish Inflation-indexed bonds during the period 2005-2014. They use a five-factor model where they introduce an inflation-linked liquidity premium. Their findings indicate negative inflation risk premiums for the two shortest maturities studied, 3-month and 6-month, while being positive for all other maturities.

[1] Nelson and Siegel (1987), Svensson (1994)

Garcia and Werner (2010) apply a no-arbitrage three-factor model on Eurozone data during the period 1995 to 2006. They focus on perceived inflation risks inherent in macroeconomic forecasts and its link to the inflation risk premium. Their estimations of the inflation risk premium are small but positive ranging from 3 to 23 b. p., increasing with yield maturity. The correlation between inflation risks and the inflation risk premium is found to be negative while they find a positive correlation between the inflation risk premium and inflation skewness.

Hördahl and Tristani (2012) include both macroeconomic and term structure dynamics into their model when examining both the US and the European market. Their results indicate similarities across the two major markets: the inflation risk premium is found to be small but positive and increasing with yield maturity, for both the US and the European market.

3. Methodology

We estimate the inflation risk premium for Sweden following the methodology proposed by Grishchenko and Huang (2013). Three relevant yield curves are identified for nominal yields, inflation-linked bond yields and real yields. Inflation-linked bond yields are viewed as different from real yields because of the indexation lag. This lag causes the holder of an index-linked bond to not be protected from inflation before the end of the bond's lifetime. Real yields are defined as yields of hypothetical bonds that fully protect investors from effects of inflation.

We compare real and nominal yields to calculate breakeven inflation. This is the inflation rate at which holding a real bond would generate a return equal to holding a nominal bond with the same maturity. Breakeven inflation rate is assumed to consist of three components: expected inflation, inflation risk premium and liquidity premium. In other words, the difference between nominal and real yields is determined by market inflation expectations, compensation for the risk of unexpected inflationary shocks and compensation for trading in the market for inflation indexed securities, which can be less liquid relative to the market for nominal government bonds.

To estimate inflation risk premium we first extract real yields from inflation-linked bond yields by adjusting for the indexation lag, which in Sweden is 3 months. Breakeven inflation can then be calculated by taking the difference between nominal and real yields. Second, survey data is used as the indicator for expected inflation. Finally, a liquidity measure for inflation-linked bonds is estimated based on the fit of individual bonds to the Svensson yield curve. This liquidity measure is based on the assumption that the Svensson yield curve represents fundamental bond prices and that differences between model-implied benchmark prices and observed market prices are due to mispricing, which is not immediately corrected due to illiquidity.

3.1 Real yields

For consistency we use the same notation as Grishchenko and Huang (2013) who apply the model for calculating real yields by Evans (1998).

$Q_t(h)$ is the nominal price at time t of a zero-coupon bond paying SEK 1 on period h. The continuously compounded yield for this bond is:

$$y_t(h) = -\frac{1}{h} lnQ_t(h)$$

Thus, we can use the following formula to transform zero-coupon yield data into prices:

$$Q_t(h) = \exp\big(-h \times y_t(h)\big)$$

Following Evans (1998) we define continuously compounded forward rates from bond prices using the following formula[2]:

$$F_t(h,k) = \left[\frac{Q_t(h)}{Q_t(h+k)}\right]^{1/k}$$

Here $F_t(h,k)$ is the rate between periods $(t+h)$ and $(t+h+k)$.

Prices, yields and forward rates of real bonds are calculated similarly. Let P_t be the known price level at time t. $Q_t^R(h)$ is the nominal price at time t of a zero-coupon bond that pays SEK (P_{t+h}/P_t) on period h.
Real yields and forward rates are calculated in a similar fashion:

$$y_t^R(h) = -\frac{1}{h} lnQ_t^R(h) \text{ and } F_t^R(h,k) = \left[\frac{Q_t^R(h)}{Q_t^R(h+k)}\right]^{1/k}$$

As mentioned previously, inflation-linked bonds do not provide complete indexation against changes in price levels h periods ahead. This is because inflation-linked bonds are indexed to price levels with a lag. Thus, $Q_t^{IL}(h)$ is the nominal price of an inflation-linked bond that pays SEK (P_{t+h-l}/P_t), where $l > 0$. Swedish inflation-linked bonds are indexed with a 3 month lag, i.e. $l = 3$.

[2] Grishchenko and Huang (2013) change the formula by subtracting 1. We keep this version for clarity in later steps where the log forward rate is used.

Yields and forward rates for inflation-linked bonds are defined as:

$$y_t^{IL}(h) = -\frac{1}{h} lnQ_t^{IL}(h) \text{ and } F_t^{IL}(h,k) = \left[\frac{Q_t^{IL}(h)}{Q_t^{IL}(h+k)}\right]^{1/k}$$

Under a no arbitrage condition Evans (1998) shows that the prices of real, nominal and inflation-linked bonds can be linked in the following way:

$$q_t^{IL}(h) = q_t^R(\tau) + [q_t(h) - q_t(\tau)] + \gamma_t(\tau), \qquad \tau = h - l$$

Here lower case characters represent the natural logarithms of their uppercase counterparts, e.g. $q_t(h) = \ln\big(Q_t(h)\big)$ while $\gamma_t(\tau)$ is a covariance term defined as:

$$\gamma_t(\tau) \equiv Cov_t(q_{t+\tau}(l), \Delta^\tau p_{t+\tau}), \text{ where } \Delta^\tau p_{t+\tau} \equiv \ln(\frac{P_{t+\tau}}{P_t})$$

$\gamma_t(\tau)$ represents the compensation for the risk of high inflation during the period when the inflation-linked bond is no longer protected against price level movements because of indexation lag.

The nominal payout of an inflation-linked bond is SEK $(P_{t+\tau}/P_t)$, which is the same as the nominal payout of a real bond with maturity τ. However, the inflation-linked bond will only provide this payout at time h. As a result, the price of the inflation-linked bond is affected by the nominal bond price $q_\tau(h)$.

If there is no arbitrage, at time τ the log price of the inflation-linked bond is equal to the price of the nominal bond $q_\tau(h)$. In case of high unexpected inflation between t and $t+\tau$, the nominal bond price $q_\tau(h)$ will typically drop, causing a negative $\gamma_t(\tau)$. Thus, inflation-linked bonds will require a discount compared to real bonds due to this risk.

To estimate the covariance term we use the $VAR(1)$ model in Grishchenko and Huang (2013):

$$z_{t+1} = Az_t + e_{t+1}, \text{ where}$$

$z_t \equiv [\Delta p_t, q_t(l), x_t]$, x_t is a (Tx1) vector of ones and e_{t+1} is the error term

After estimating the $VAR(1)$ model, $\gamma_t(\tau)$ is calculated from the coefficients A and the innovation variances $Var(e_{t+j}|\, z_t)$:

$$\gamma_t(\tau) = i_1' \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^{i} A^{i-j} Var(e_{t+j}|z_t) A^{i-j'} \right) \right] i_2$$

Here i_1 is the selection vector [1 0]' so that $\Delta p_t = i_1' z_t$, and i_2 is [0 1]' so that $q_t(l) = i_2' z_t$. Annual inflation rate is defined as $\Delta p_t = \ln(\frac{P_t}{P_{t-12}})$. $\gamma_t(\tau)$ is estimated using information available at time t. An example of deriving the formula for $\gamma_t(\tau)$ where $\tau = 2$ is provided in the Appendix.

Based on the link between prices of real, nominal and inflation-linked bonds, Evans (1998) and Grishchenko and Huang (2013) provide the following formula for calculating real yields from inflation-linked bond rates, log nominal forward rates and the covariance term:

$$y_t^R(\tau) = \frac{h}{\tau} y_t^{IL}(h) - \frac{l}{\tau} f_t(\tau, l) + \frac{1}{\tau} \gamma_t(\tau)$$

3.2 Expected inflation

Market expectations at time t of τ-period ahead inflation are defined as $E_t \pi_{t+\tau}(\tau)$. Grishchenko and Huang (2013) use three different estimates for expected inflation in their study: historical average of seasonally un-adjusted and core CPI, a vector autoregression including real activity variables based on the Phillips curve, and surveys on inflation forecasts. We use surveys as our measure of expected inflation in the estimation of our model. We know that such surveys run a risk of being biased, and the precision of survey inflation forecasts has been debated in the literature (see e.g. Kim (2009)). However, Mehra (2002) and the more recent study by Ang, Bekaert and Wei (2007) focus on the use of survey forecasts as a measure of expected inflation and they both find survey forecast to be the best approximation for expected inflation.

3.3 Inflation risk premium

Following Grishchenko and Huang (2013), we estimate the inflation risk premium using the formula:

$$y_t(\tau) = y_t^R(\tau) + E_t\pi_{t+\tau}(\tau) + IRP_t(\tau)$$

Here $y_t(\tau) - y_t^R(\tau)$ is the breakeven inflation from time t to $t+\tau$. $E_t\pi_{t+\tau}(\tau)$ is the τ period ahead expected inflation and $IRP_t(\tau)$ is the inflation risk premium at time t for maturity τ.

Grishchenko and Huang (2013) define the inflation risk premium using the stochastic discount factor. In an economy with no arbitrage, there exists a discount factor M_t, for which $E_t\left[M_{t+1}R_{i,t+1}\right] = 1$. Here $R_{i,t+1}$ is the gross return for any traded asset i. They derive the inflation risk premium as dependant on the covariance between log stochastic discount factor $m_t = lnM_t$ and inflation π_t:

$$IRP_t(\tau) = \frac{1}{\tau} Cov_t\left(-\sum_{i=1}^{\tau} m_{t+i}, \sum_{i=1}^{\tau} \pi_{t+i}\right)$$

This covariance can have positive or negative signs. Evans (2008) derives the inflation risk premium in a slightly different way and uses covariance with the real, instead of nominal, stochastic discount factor $m_{t+1}^R = m_{t+i} + \Delta p_{t+1}$. According to him, the inflation risk premium can be negative depending on the covariance between inflation risk and the marginal utility of consumption. This is because in economic models with a representative agent, m_t^R is the same as the marginal choice between consumption and saving.

More recently, Imakubo and Nakajima (2015) interpret m_t^R as investors’ marginal rate of substitution, based on the capital asset pricing model (CAPM). They then follow Campbell et al. (2009) and explain the covariance as the relationship between the intertemporal rate of marginal substitution and inflation expectations. In this case when the marginal utility is high, investors holding nominal bonds benefit from an unexpected drop in inflation, which causes a negative inflation risk premium.

3.4 Liquidity premium

Following Grishchenko and Huang (2013) and Hu, Pan, and Wang (2013), we attempt to measure the impact of liquidity on inflation-linked bond yields based on the difference between benchmark bond yields and observed bond yields. Hu et al. (2013) originally propose this measure for the US Treasuries market. This measure relies on two main assumptions.

First, Hu et al. (2013) assume that arbitrageurs smooth out the yield curve and reduce the mean dispersion of individual bond yields by doing relative value trades across different maturities. The second assumption is that when markets function normally, arbitrageurs rapidly correct large differences of bond prices from their fundamental values. However, during liquidity crisis periods in the markets institutional investors are no longer able or willing to quickly take advantage of mispriced assets. As a result, bond yields can move away from their fundamental values because of limits of arbitrage.

Based on these assumptions Hu et al. (2013) argue that the average distance of individual bond yields from the yield curve, which they call the level of "noise" in the market, can give an indication of the presence of arbitrage capital, which in turn acts as a proxy for overall market liquidity.

Grishchenko and Huang (2013) apply this measure to the US TIPS market, arguing that the fit of individual bonds to the yield curve indicates the level of available arbitrage capital and the market liquidity also for inflation linked bonds. They note that the Federal Reserve Board also calculates this level of "noise" to observe whether the TIPS market is functioning normally.

We use this measure to try to improve our estimates of real yields and inflation risk premium for Sweden. This is done under the assumption that the same arguments used for US nominal Treasuries and TIPS apply also for the Swedish inflation linked bond market: i.e. that the yield curve fitting errors represent asset mispricing, which is possible due to a lack of arbitrage capital in the market, which then signals a lack of liquidity in the market.

This liquidity measure is calculated following Grishchenko and Huang (2013) and Hu et al (2013):

$$y_t^L = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\left[y_t^{i,o} - y_t^{i,b}\right]^2},$$

Here y_t^L is the estimated impact of liquidity on the inflation-linked bond yields at time t, $y_t^{i,o}$ is an observed yield of an inflation-linked bond at time t, and $y_t^{i,b}$ is an benchmark yield at time t for an inflation-linked bond with the same maturity. N_t is the number of bonds at time t used to calculate this measure.

Following Grishchenko and Huang (2013) and Gürkaynak, Sack, and Wright (2010) we estimate the benchmark inflation-linked bond term structure using the Nelson and Siegel (1987) and Svensson (1994) model. Gürkaynak, Sack, and Wright (2010) explain that a parametric function such as the Svensson model smoothes individual movements in yields and can represent the shape of the fundamental term structure more accurately for macroeconomic analysis purposes. Hu, Pan, and Wang (2013) also use this functional form for estimating the liquidity measure for US government nominal bonds. The Svensson model postulates the following functional form for the instantaneous forward rate *f*:

$$f(h,b) = \beta_0 + \beta_1 \exp\left(-\frac{h}{\tau_1}\right) + \beta_2 \frac{h}{\tau_1} \exp\left(-\frac{h}{\tau_1}\right) + \beta_3 \frac{h}{\tau_2} \exp\left(-\frac{h}{\tau_2}\right),$$

where h is the time to maturity and b is a vector of model parameters $(\beta_0\ \beta_1\ \beta_2\ \beta_3\ \tau_1\ \tau_2)$ to be estimated. Looking at the limits of the function, $f = \beta_0$ as $h \to \infty$ and $f = \beta_0 + \beta_1$ as $h \to 0$. Thus, in this functional form, β_0 is the forward rate at an infinitely long maturity, and $\beta_0 + \beta_1$ is the zero maturity forward rate. The remaining parameters allow the function to match highly nonlinear term structures (Gürkaynak, Sack, and Wright, 2010).

The model parameters b_t are estimated by minimizing the duration-weighted sum of square differences between observed and model-implied prices:

$$b_t = \underset{\mathrm{b}}{\operatorname{argmin}} \sum_{i=1}^{N_t} \left[(Q^{IL,i}(b) - Q_t^{IL,i}) \times \frac{1}{D_i}\right]^2,$$

where $Q^{IL,i}(b)$ is the model-implied price given parameters b, and D is the MaCaulay's duration for bond i. Weighted by inverse duration the differences are transformed from pricing errors to yield fitting errors (Gürkaynak, Sack, and Wright, 2010).

An example of the calculation of the liquidity measure for one daily point can be found in the Appendix.

4. Data

4.1 Bonds

In our study we use data from The Swedish Central Bank, *Riksbanken*. In the estimation of our model we use data on yields of nominal zero-coupon bonds and inflation-linked bonds with the same maturities. To account for the three-month indexation lag inherent in Swedish bonds we use the 3-month T-bill benchmark rate. We used the 3-month T-bill benchmark rate instead of the 3-month nominal zero-coupon rate because when doing the estimation, the short end of the yield curve is noisier and somewhat altered in favour of a better fit at the long end (Hu, Pan and Wang (2013)). We use data on individual inflation-linked bonds to calculate the liquidity measure. Table 3:1 shows the summary statistics for these bonds.

For the nominal and inflation-linked bonds we have used 2 year and 5 year maturities. We did not include the 1-year maturities because of the noise inherent in the 1-year inflation-linked bonds, resulting from the 3-month indexation lag. Our data sample reaches from February 2005 until November 2015.

4.2 Inflation

To account for the inflations expectations we use data on inflations expectations from Prospera, an organisation assigned by The Swedish Central Bank to map the inflation expectations in Sweden. Prospera gather data by conducting monthly surveys with Money Market Players[3] present in the Swedish market. We estimate the inflation risk premium using Prospera data on "Expected annual increase in CPI the coming 2 and 5 years". Table 3:2 shows the summary statistics for these surveys.

In addition, our estimates for the impact of the three-month indexation lag also use the monthly consumer price index provided by Statistics Sweden.

[3] Swedish and International money market players active in the Swedish fixed income market.

5. Empirical Results

5.1 Real yields

To calculate the real yields we first estimate the covariance term $\gamma_t(\tau)$ with maturities of 1 to 7 years. We estimate the VAR(1) model for each observation in our sample using data available at time t. Since inflation data for a reference month is released the following month, we estimate the model using data up to t-1. The annualised covariance term is calculated as $hly\, \gamma(\tau) \times (-\frac{12}{\tau})$. It shows the impact of uncertainty in future inflation and nominal bond prices on annual yields of index-linked bonds.

The results for this estimation are summarized in Table 1. Similarly to Grishchenko and Huang (2013) and Evans (1998), the sample averages of the monthly covariance term are negative, indicating that in Sweden if there is high unexpected inflation between t and t+τ, the nominal bond price $q_\tau(h)$ will typically drop. Also similarly to the paper on TIPS, the covariance term is greater in absolute terms for longer maturities. Grishchenko and Huang (2013) explain that for bonds with longer maturities there is greater uncertainty about the relationship between inflation over an expanded time period and the nominal bond that determines the price of the linker at the end of the indexation period. Our estimates show that the risk measured by gamma increases the yields of Swedish inflation-linked bonds from 0.4 b. p. for bonds maturing in 12 months to 1.5 b. p. for 5 year bonds on average in our sample.

We use data on Swedish inflation-linked bond yields with maturity intervals of 6 months. However, the formula for real yields requires inflation-linked bond yields with a maturity greater by the indexation lag, which is 3 months in the case of Sweden. We use linear interpolation to estimate such yields. For example, to calculate real yields with 24 month maturity, we need yields of inflation-linked bonds with 27 month maturity, which we estimate from inflation-linked bonds with 24 and 30 month maturities.

Using the estimates of gamma and interpolated nominal and inflation-linked yields, we can then calculate real yields by correcting inflation-linked bond yields for indexation lag. Table 2 summarises the results and Figure 2 shows the dynamics of the adjustment. The adjustment is greater and more volatile for yields with 24 month maturity: real yields are on average lower by 21 b. p. compared with inflation-linked bond yields. 5-year and 7-year bonds have a smaller correction, at 11 b. p. and 9 b. p. respectively.

While this correction may not seem very large, it has a significant effect later on when we calculate the breakeven inflation rate and then estimate the inflation risk premium, especially for shorter maturities. We would argue that this measure of the indexation lag impact on yields illustrates the importance of distinguishing between inflation-linked bond breakeven inflation rate[4] and real breakeven inflation rate.

5.2 Breakeven inflation

Estimates of real rates allow us to calculate breakeven inflation, which is defined as the difference between nominal and real rates: $y_t^B(\tau) = y_t(\tau) - y_t^R(\tau)$. Figure 3 shows that survey inflation expectations have been mostly greater than breakeven inflation in recent years. If the effect of liquidity in the inflation-linked bond market is not taken into account, this would indicate a generally negative inflation risk premium.

5.3 Liquidity measure

Following Gürkaynak, Sack, and Wright (2010) we select inflation-linked bonds with at least 18 months remaining maturity to estimate the Svensson yield curve parameters. This is done because the indexation lag has a greater effect on the yields of inflation-linked bonds towards the end of their maturity (see the no arbitrage condition on the real yield estimation chapter in the methodology section). In our sample, the bid-ask spreads and yield volatility can rise dramatically during end of the life of an inflation-linked bond and create additional noise for the yield curve estimation.

We estimate the daily Svensson yield curve parameters from February 2005 to November 2015. If cases where the procedure is unable to find optimal values for the yield curve with starting values equal to estimated parameters of previous day, we use starting values based on the yield curve provided by Riksbank.

4 For the US, Grishchenko and Huang (2013) refer to it as the TIPS breakeven rate.

To calculate the liquidity measure we select bonds with at least 3 years remaining maturity. According to Hu, Pan, and Wang (2013), bonds that are close to maturity are necessary to fit the short end of the yield curve. However, this part of the yield curve is noisier, and arbitrage is less likely. Thus, for calculating the liquidity measure only longer maturity bonds are relevant. Hu, Pan, and Wang (2013) select bonds with maturity of at least 1 year; however, for TIPS Grishchenko and Huang (2013) suggest a minimum maturity of 3 years. This is because unlike nominal bonds, no short-term TIPS are issued. In Sweden most inflation-linked bonds are long-term; therefore, we apply a minimum maturity of 3 years as well. Due to the small number of Swedish bonds available for calculating this measure, we also include bonds with remaining maturity above 10 years.

Table 4 summarizes the results of this estimation. The number of bonds available for estimating the Svensson model parameters ranges from 5 to 7 in our sample period. This amount of data points for estimation is quite low. In comparison, Grishchenko and Huang (2013) estimate the liquidity measure using up to 16 outstanding TIPS. Individual bond price movements are therefore more likely to cause shifts in the yield curve. As a result, the effect of a mispriced bond on the RMSE of the fit might be diminished and cause the liquidity measure to be understated.

As shown in Figure 4, the liquidity measure varies substantially over time. Prior to Fall 2008 it is lower, although still above the mean of 5 basis points for TIPS over 2004-2008 (Grishchenko and Huang, 2013). This might indicate that the Swedish inflation-linked market was less liquid than the US TIPS market before the financial crisis. In later periods the estimates increase and reach a monthly sample maximum of 61 basis points. This is similar to Grishchenko and Huang (2013) estimates for US TIPS during the financial crisis. Overall, these estimates would suggest that illiquidity concerns increase Swedish inflation-linked bond yield by an average of 26 b. p. throughout our sample.

5.4 Inflation risk premium

Results of our estimates of the inflation risk premium for Sweden are shown before and after adjustment for the liquidity measure in Table 5 and Figure 5. As already indicated by comparing breakeven inflation rate with inflation expectations in Figure 3, the inflation risk premium is mostly negative and has a downward sloping term structure before adjusting it for liquidity.

Following Grishchenko and Huang (2013) we estimate the liquidity-adjusted inflation risk premium using the formula:

$$y_t(\tau) = y_t^R(\tau) - y_t^L + E_t\pi_{t+\tau}(\tau) + IRP_t(\tau),$$

where y_t^L is the average fitting error at time t. The inflation risk premium is adjusted upwards, but remains negative throughout most of our sample. On average, it ranges from -15 b. p. to -18 b. p. for 2 and 5 year maturities.

Imakubo and Nakajima (2015) provide an intuitive economic explanation of negative inflation risk premium. The inflation swap rate is equal to the difference between nominal and real rates, i.e. the breakeven rate. They express the payoff of this swap as a synthetic position of a call and a put option on inflation with expected inflation as the strike price for both options. The inflation risk premium becomes negative if the inflation put premium is greater than the inflation call premium, i.e. if markets are more concerned with unexpected deflation rather than unexpected inflation. Therefore, our estimates of negative interest rate premium imply that investors in Swedish government bonds believe that unexpected disinflation is more likely than an inflation shock.

Chen, Engstrom and Grishchenko (2016) explain that a negative inflation risk premium implies that it is more cost effective for the government to issue nominal securities rather than inflation-linked bonds. In other words, when the inflation risk premium is negative, investors are willing to pay a premium for nominal bonds relative to inflation-linked bonds. Based on this reasoning, estimates of the sign of the inflation risk premium could be taken into consideration when selecting the share of inflation-indexed bonds in government debt issues.

Our estimates of inflation risk premium decrease significantly during Fall 2008 – 2009 and mid-2011 – 2013, reaching minimums for 5 year liquidity-adjusted premiums of -1.7% and -0.8% respectively. The first drop coincides with the global financial crisis. In December 2008 the Swedish consumer price index decreased 1.3%. Figure 1 shows that nominal yields dropped substantially and Figure 3 shows a significant decrease in breakeven rates during this period. Expected inflation remained relatively stable and therefore, the inflation risk premium estimates decreased substantially, indicating market concerns about deflation.

An alternative explanation for this drop in breakeven inflation could be a sharp increase in the liquidity premium during Fall 2008 – 2009 not captured by our liquidity measure. Bergroth and Carlsson (2014) estimate the liquidity premium using a five factor latent variable model, and find the liquidity premium to be equal to around 2% during this period for 5 year inflation-linked bonds. Such a liquidity premium would cause our inflation risk premium estimate to become positive for this period.

The second significant drop in estimated inflation risk premium occurs in mid-2011 – 2013. As before, a drop in nominal rates pushed the breakeven rates downward, while expected inflation decreased gradually and to a lesser extent, resulting in our negative estimate of inflation risk premium. During this period, the Swedish inflation decreased to approximately zero, and the policy rate of the Swedish central bank was lowered from 2% to 0.75%. Such developments in the economy seem to be consistent with a higher likelihood of unexpected disinflationary shock indicated by the negative estimate of inflation risk premium.

5.5 Limitations and further research

In our opinion, the main limitation of our thesis is the suitability of the liquidity measure for Sweden during our sample period. We use a liquidity measure which assumes that the estimated yield curve reflects the fundamental value of fitted bonds. As a result, it is essential to be able to arrive at a reliable estimate of the yield curve. This might not be possible, as there are relatively few Swedish inflation-linked bonds and a mispricing of one bond is more likely to affect the parameters of the yield curve. As a result, our estimates may understate the liquidity premium and, as a consequence, the risk premium. Moreover, the small number of Swedish inflation linked bonds available for trading may reduce the opportunities for relative value trades for arbitrage capital, which is one of the key assumptions of using this measure as a proxy for liquidity.

Regarding data used, the expected inflation variable is not directly observed and therefore cannot be measured exactly. As discussed earlier several studies have found survey forecasts to be the best approximation for expected inflation (Kim (2009), Ang, Bekaert and Wei (2007)). However it is important to consider that expected inflation in our sample is usually close to or even greater than breakeven inflation. Because of this, even relatively small errors in our measure of expected inflation can have a significant impact on the inflation risk premium, possibly even changing its sign.

Our main finding of a negative interest rate premium can also be found in the current literature for several countries. We closely follow the methodology of Grishchenko and Huang (2013), who specify deflation fears as one of the potential explanations for their estimate of a negative inflation risk premium for US in 2000 – 2004 and at the end of their sample period in Fall 2008. Bergroth and Carlsson (2014) also estimate a negative inflation risk premium in recent periods in Sweden. Imakubo and Nakajima (2015) estimate a negative inflation risk premium for Japan from 2007 to 2012 using an affine term structure model and interpret it as an indication that market participants are worried about unexpected deflationary shocks. ECB Monthly Bulletin reports negative estimates of inflation risk premium for the euro area in the beginning of 2014 (European Central Bank, 2014) and explains them with market participant perceptions of the likelihood of a disinflationary shock.

Further research could involve using other proxies to estimate the liquidity premium for Swedish inflation-linked bonds. Such proxies should be valid even for a small sample of bonds with different maturities. For example, Hu Pan and Wang (2013) estimate the on-the-run premium for US Treasuries, which requires having an off-the-run bond with similar maturity for comparison, which may be difficult in a market with few traded bonds. Also, comparing the results from several proxies of expected inflation might help to find the most reasonable estimates of Swedish inflation risk premium. Inflation-linked swap rates observed in the market could be decomposed into expected inflation, liquidity premium and inflation risk premium. Finally, more complicated models such as models of term structure of interest rates with features from consumption-based asset pricing suggested by Chen, Engstrom and Grishchenko (2016) could be implemented.

6. Conclusion

In this paper we estimated the inflation risk premium in the Swedish bond market using nominal and inflation-linked bonds. We applied a no-arbitrage model following Evans (1998) and Grishchenko and Huang (2013) using historical bond yields and survey forecasts as a proxy for expected inflation. When doing the estimation we accounted for the three-month indexation lag inherent in Swedish inflation-linked bonds and added a liquidity measure to address the potential concern of illiquidity premium in the Swedish inflation-linked bond market. Our results indicated an average increase of 26 b. p. throughout our sample due to illiquidity concerns.

Our estimates of the inflation risk premium are mostly negative in the range of -15 b. p. to -18 b. p. for 2 and 5-year maturities, with a downward sloping term structure. Despite an upward shift in the term structure when correcting for liquidity, the inflation risk premium stays negative to a large extent. The negative inflation risk premium could be an effect of the understated liquidity measure resulting from the rather few number of inflation-linked bonds available when estimating the yield curve. A better fit of the yield curve could be achieved with another sample with more data points after applying the maturity filter, thus decreasing the risk of understatement of the liquidity measure.

Our main contribution is that we are able to estimate the real breakeven inflation rate, as opposed to the inflation-linked bond breakeven rate for Sweden using a no-arbitrage model; showing that taking the three-month indexation lag into account makes a difference. We are also able to estimate a proxy for the inflation-linked bond liquidity premium for Sweden and finally we estimate a negative inflation risk premium and discuss the economic intuition behind it.

In the future we encourage more work to be done to disentangle the liquidity premium from estimates of the inflation risk premium for Sweden. With a deeper knowledge regarding the size and the sign of the inflation risk premium monetary policy makers can improve their policies and ultimately decrease their cost of debt.

Table 1: Gamma estimation

The following equation shows the calculation of monthly gamma using outputs from a vector autoregressive model with 1 lag.

$$\gamma_t(\tau) = i_1' \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^{i} A^{i-j} Var\left(e_{t+j} \middle| z_t\right) A^{i-j'} \right) \right] i_2$$

The covariance is between $\Delta^\tau p_{t+\tau}$ and $q_{t+\tau}(l)$, conditional to z_t.

i_1 is the selection vector [1 0]' so that $\Delta p_t = i_1{}'z_t$, and i_2 is [0 1]' so that $q_t(l) = i_2{}'z_t$.

$VAR(1)$ model coefficients A and innovation variances $V\left(e_{t+j} \middle| z_t\right)$ are constant for all j > 0

Annual inflation rate is defined as $\Delta p_t = \ln(\frac{P_t}{P_{t-12}})$. $\Delta^\tau p_{t+\tau}$ is the log inflation from time t to time $t + \tau$.

$q_{t+\tau}(l)$ is the log price at time $t + \tau$ of a nominal bond maturing in l months.

$\gamma_t(\tau)$ is estimated using information available at time t. For each t, $\gamma_t(\tau)$ is estimated using a $VAR(1)$ model calculated using 10 years of data up to $t - 1$. Sample averages of the coefficient matrix A are reported below with standard errors in brackets:

0.958 (0.032)	0.036 (0.058)
-0.012 (0.004)	0.972 (0.014)

Annualised $\gamma_t(\tau)$ is calculated as $monthly\ \gamma_t(\tau) \times (-\frac{12}{\tau})$.

Results are reported in percentages as sample averages.

Maturity	1 year	2 years	3 years	4 years	5 years
$\gamma(\tau)$	-0.004	-0.016	-0.032	-0.052	-0.076
$\gamma(\tau) \times (-\frac{12}{\tau})$	0.004	0.008	0.011	0.013	0.015

Table 2: Estimated real yields

Summary statistics for estimated real yields are compared to inflation-linked bond yields over the sample period. We use the following formula to estimate the real yields based on inflation-linked bond rates, log nominal forward rates and a covariance term described in Table 1.

$$y_t^R(\tau) = \frac{h}{\tau} y_t^{IL}(h) - \frac{l}{\tau} f_t(\tau, l) + \frac{1}{\tau} \gamma_t(\tau)$$

$y_t^R(\tau)$ is the real yield of a zero coupon bond with τ month maturity.
$f_t(\tau,l)$ is the l = 3 month log nominal forward rate τ months ahead.
$\gamma_t(\tau)$ is the covariance between future inflation $\Delta^\tau p_{t+\tau}$ and nominal bond price $q_{t+\tau}(l)$.
Results are reported in percentages.

	Maturity	2 years	5 years	7 years
Inflation-linked bond yields	Mean	0.44	0.72	0.86
	St. dev	0.89	0.91	0.86
Real yield estimation	Mean	0.23	0.61	0.77
	St. dev	0.88	0.90	0.85

Table 3:1 Summary statistics of inflation-linked bonds

Sample statistics for February 2005 to November 2015. Source: Riksbank.

	Coupon (%)	Bid/Ask (%)	Maturity (Years)	Yield (%)
Mean	1.66	-0.03	6.50	0.60
Median	2.12	-0.03	5.35	0.61
Standard deviation	1.28	0.00	4.39	0.20

Sample includes 11 inflation-linked bonds traded during the sample period. Statistics are reported for bonds used for fitting the yield curve, i.e. bonds with remaining maturity of at least 18 months. Calculations are based on the average of time series of cross-sectional means, medians and standard deviations.

Table 3:2 Summary statistics of expected inflation surveys

Mean, minimum and maximum are calculated for February 2005 to November 2015. Standard deviation is calculated from September 2009, when the survey frequency became monthly. Results are reported in percentages. Source: Prospera.

Horizon (years)	Mean	St. dev.	Min	Max
1	1.62	0.55	0.49	3.25
2	1.94	0.38	1.05	2.93
5	2.12	0.19	1.65	2.61

Table 4: Liquidity risk premium

The measure of liquidity risk premium is based on the RMSE between actual yields and model-implied yields:

$$y_t^L = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\left[y_t^{i,o} - y_t^{i,b}\right]^2},$$

where $y_t^{i,o}$ is the observed yield of inflation-linked bond at time t, and $y_t^{i,b}$ is the benchmark yield at time t for an inflation-linked bond with the same maturity.

N_t is the number of nominal inflation-linked bonds with remaining maturity greater than 3 years at time t. The benchmark yields are calculated by fitting the Svensson functional form to daily market data in our sample. Fitting is performed if at least 3 applicable bonds are available. Monthly values are constructed by a simple average of daily liquidity premium correction measure. Results are reported for monthly values in basis points.

	Liquidity risk premium	# bonds fitted
Mean	25.81	5.41
Standard deviation	12.49	0.58
Minimum	2.32	5
Maximum	61.45	7

Table 5: **Inflation risk premium**

Average estimates for inflation risk premium. These estimates are based on survey forecasts of expected inflation.

Inflation risk premium not corrected for liquidity premium is calculated as follows:

$$y_t(\tau) = y_t^R(\tau) + E_t\pi_{t+\tau}(\tau) + IRP_t(\tau),$$

where γt(τ) is the nominal yield at time t with horizon τ, $y_t^R(\tau)$ is the estimated real yield at time t with horizon τ. $E_t\pi_{t+\tau}(\tau)$ is the τ period ahead expected inflation and $IRP_t(\tau)$ is the inflation risk premium at time t for maturity τ.

Inflation risk premium corrected for liquidity premium has one additional variable $y_t^L(\tau)$:

$$y_t(\tau) = y_t^R(\tau) - y_t^L(\tau) + E_t\pi_{t+\tau}(\tau) + IRP_t(\tau),$$

$y_t^L(\tau)$ is defined as the average fitting error at time t and is calculated as described in Table 3.

Results are shown in percentages.

Maturity	Non-adjusted for liquidity premium	Adjusted for liquidity premium
2 years	-0.407	-0.149
5 years	-0.441	-0.183

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Appendix

Figure 1: Real and nominal yields

In the graph below we have plotted the zero-coupon nominal and real yields for the 2 year and 5 year maturities. Real yields are calculated by adjusting yields of inflation-linked bonds to account for the 3 month indexation lag, as described in Table 2.

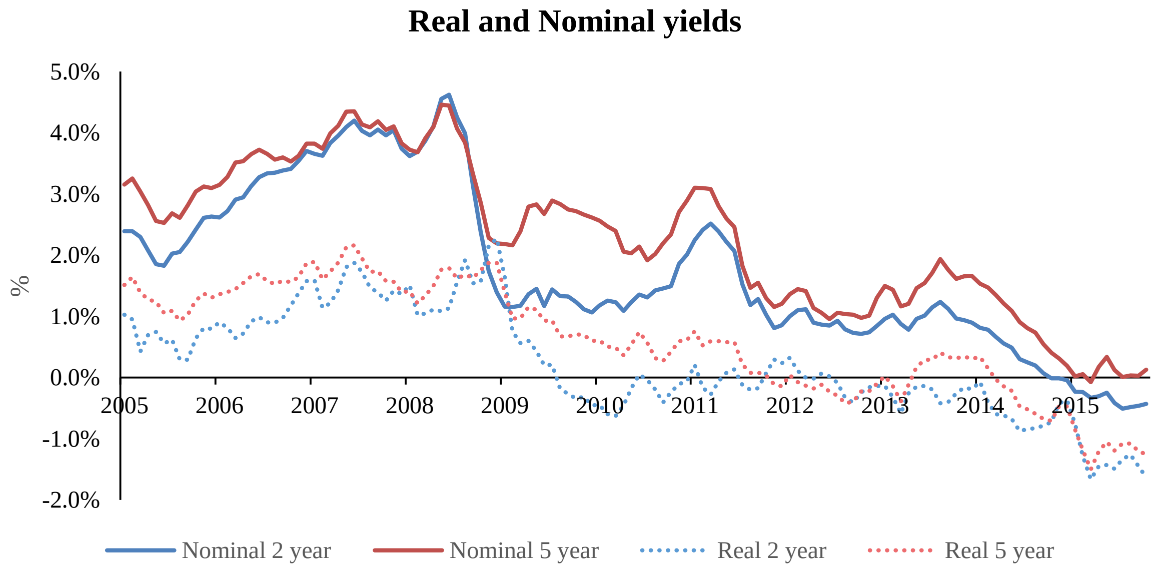


Figure 2: Real Yield Adjustment Dynamics

The chart below show the evolution of the estimated difference between inflation-linked and real yields $y_t^{IL}(\tau) - y_t^{R}(\tau)$ over our sample period. Monthly values of yields are obtained by calculating an unweighted average. Real yields are calculated by adjusting yields of inflation-linked bonds to account for the 3 month indexation lag, as described in Table 2. The adjustment is shown for yields with maturities of 2 and 5 years.

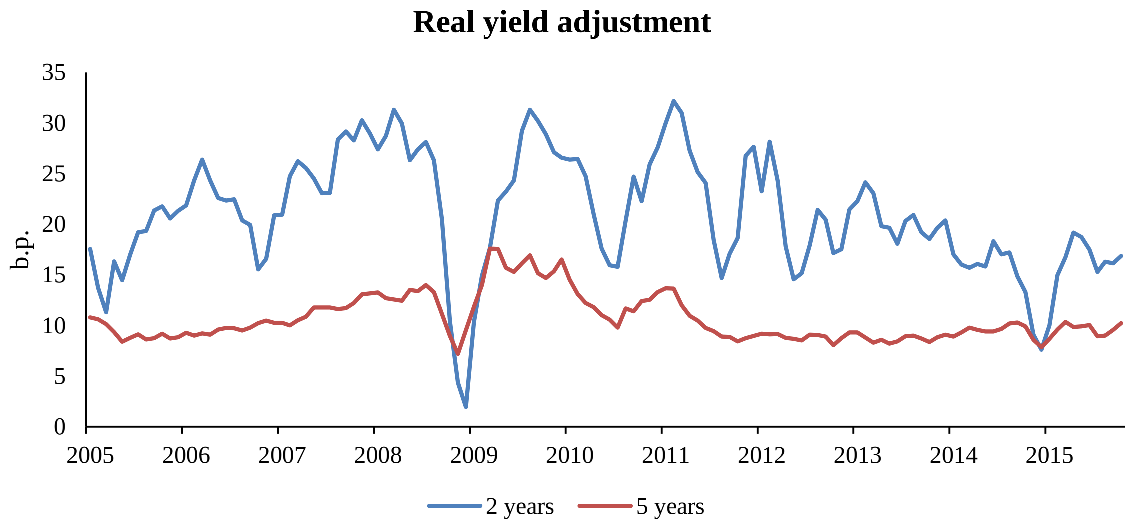
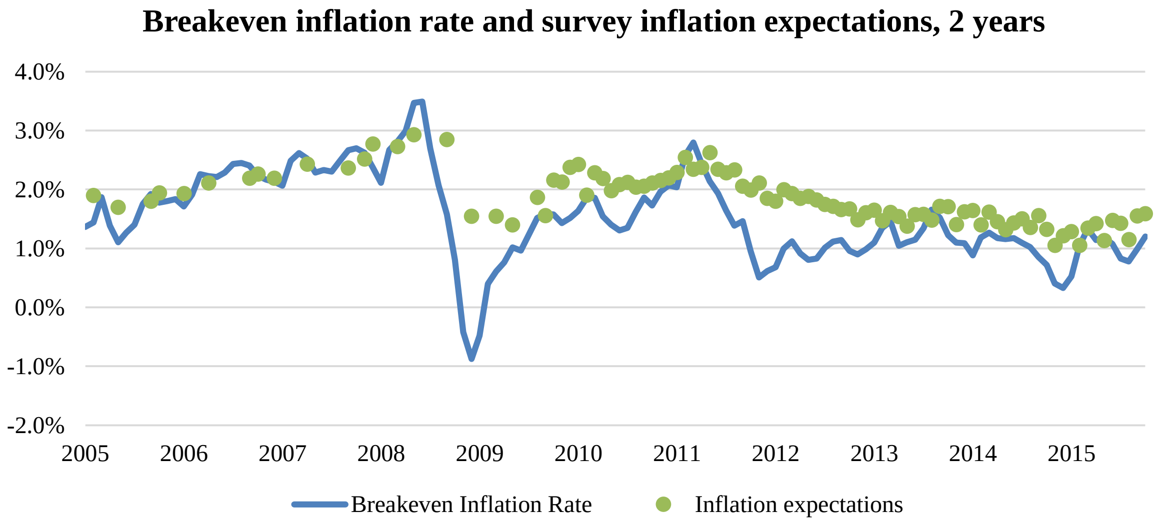


Figure 3: Breakeven inflation rate and survey inflation expectations

These charts show the evolution of the estimated breakeven inflation rate and survey inflation expectations over our sample period. Breakeven inflation is defined as the difference between nominal and real rates: $y_t^B(\tau) = y_t(\tau) - y_t^R(\tau)$. Inflation expectation data are taken from Prospera surveys conducted monthly with Money Market Players present in the Swedish market. Breakeven inflation and survey expectations are shown for 2 and 5 year maturities.



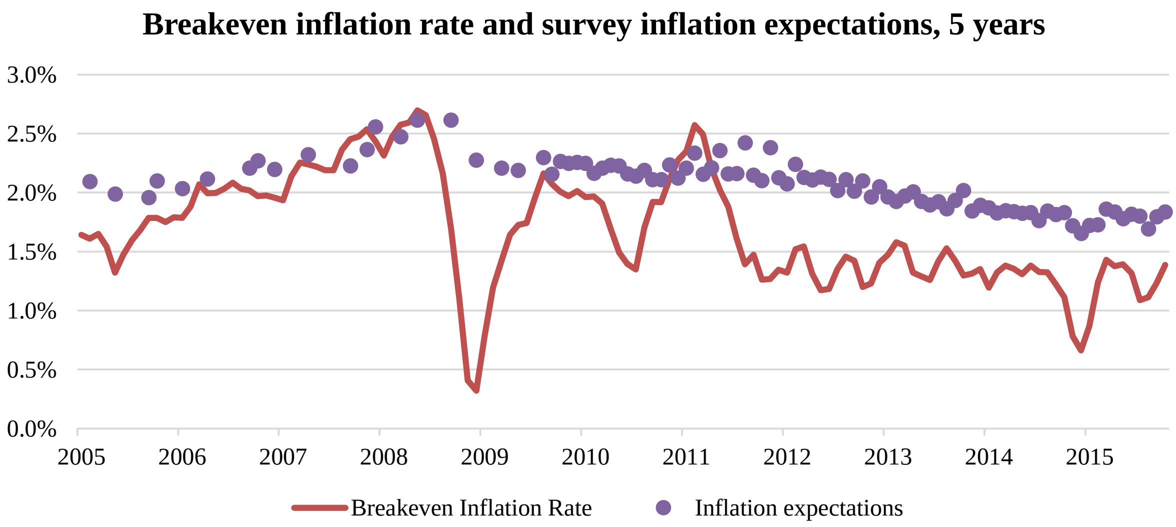
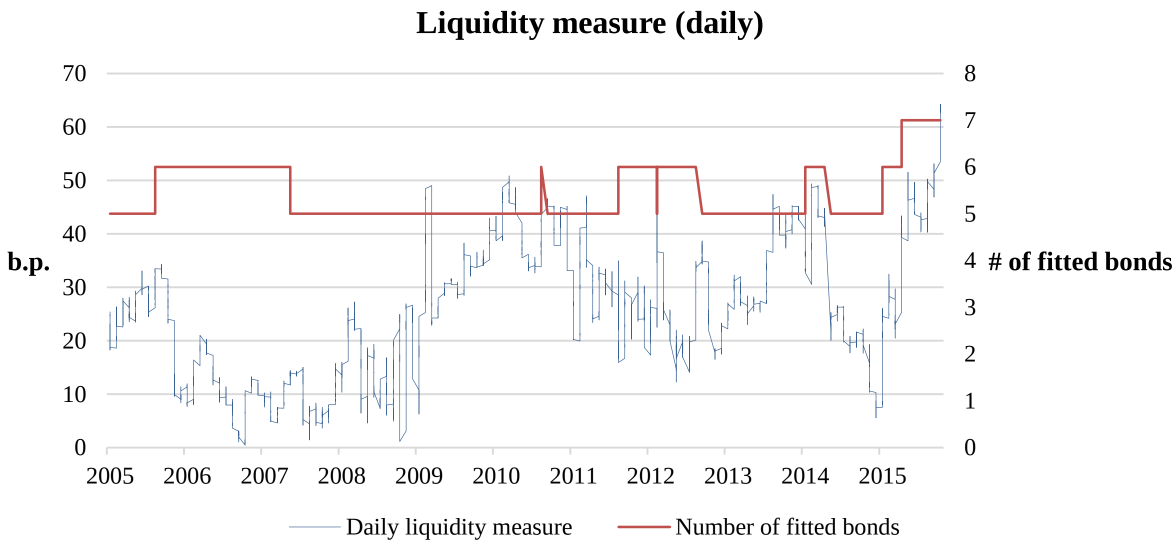


Figure 4: Liquidity Adjustment Dynamics

These charts show the evolution of the estimated liquidity risk premium over our sample period. Monthly values of the liquidity measure are obtained by calculating an unweighted average. All selected bonds have at least 3 years remaining maturity. The liquidity measure is calculated by fitting the selected bonds to a parametric term structure function and obtaining the root mean squared error of the fit, as described in Table 4.



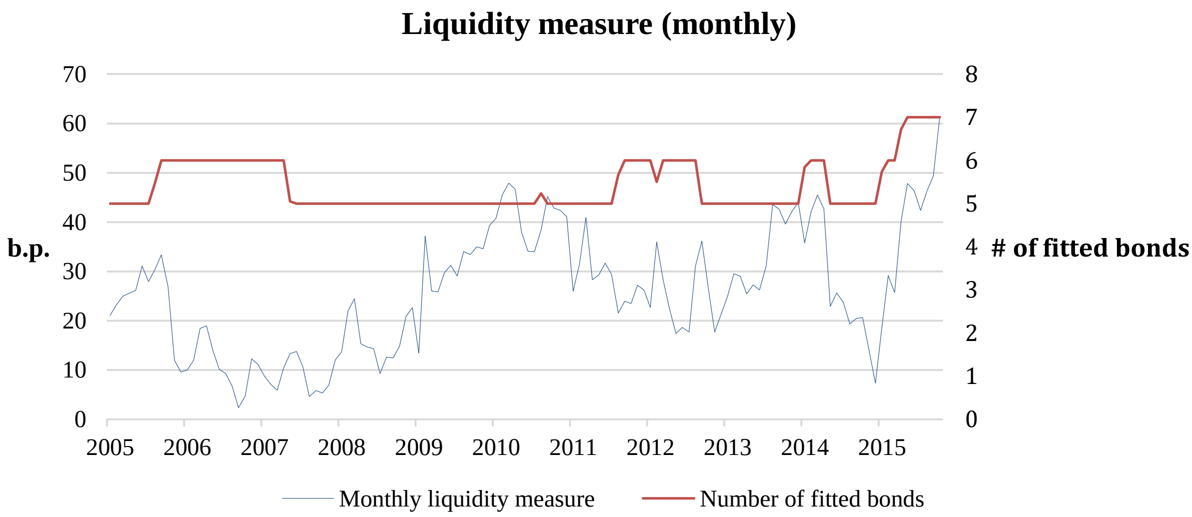
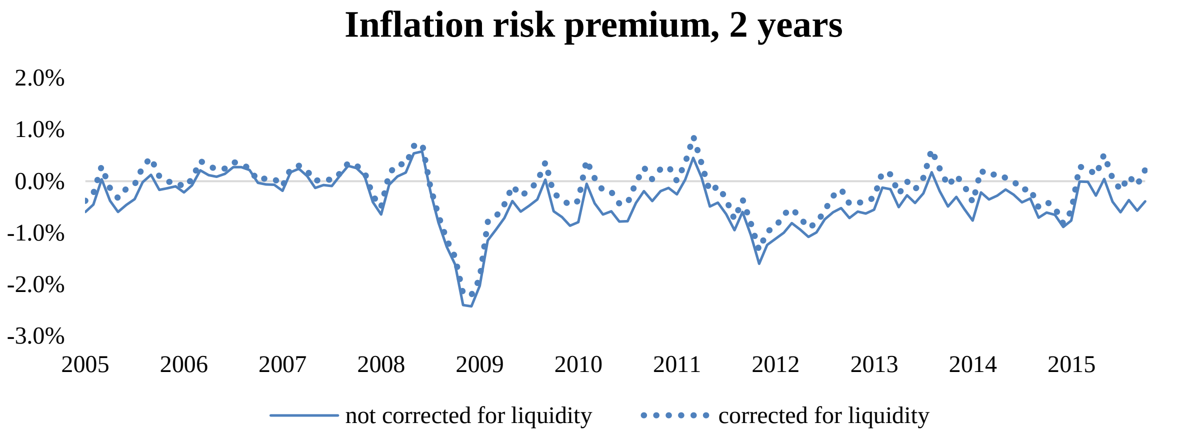
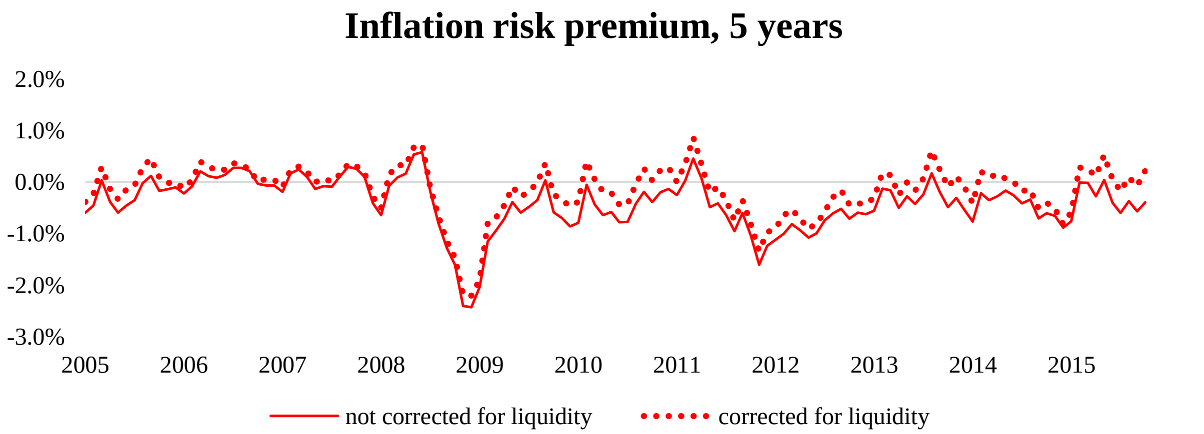


Figure 5: Inflation Risk Premium

The graphs below show the estimated Inflation risk premium for the 2-year and 5-year maturities, as calculated in Table 5.





Deriving gamma from VAR(1)

The covariance term gamma is defined as: $\gamma_t(\tau) \equiv Cov_t(q_{t+\tau}(l), \Delta^\tau p_{t+\tau})$

VAR(1) model used in estimating gamma: $z_{t+1} = Az_t + e_{t+1}$, where $z_t \equiv [\Delta p_t, q_t(l), x_t]$ and x_t is a (Tx1) vector of ones

Evans (1998) and Grishchenko and Huang (2013) estimate gamma based on the following VAR properties:

$Cov\left(z_{t+j}, z'_{t+i} \middle| z_t\right) = A^{j-i} Var(z_{t+i}|z_t), for\ j > i$

$Var\left(z_{t+j} \middle| z_t\right) = AVar\left(z_{t+j-1} \middle| z_t\right)A' + Var\left(e_{t+j} \middle| z_t\right), for\ j > 0$

As well as the following property of changes in log price index: $\Delta^\tau p_{t+\tau} \equiv \sum_{i=1}^{\tau} \Delta p_{t+i}$

For example, when $\tau = 2$:

$\gamma_t(2) \equiv Cov_t(q_{t+2}(l), \Delta^2 p_{t+2}) = Cov_t(q_{t+2}(l), \Delta p_{t+1} + \Delta p_{t+2}) = Cov_t(q_{t+2}(l), \Delta p_{t+1}) + Cov_t(q_{t+2}(l), \Delta p_{t+2})$

$Cov_t(q_{t+2}(l), \Delta p_{t+1})$ is shown by the off-diagonal element of $Cov(z_{t+2}, z'_{t+1}|z_t) = A^{2-1} Var(z_{t+1}|z_t) = A \times \left(AVar(z_{t+1-1}|z_t)A' + Var(e_{t+1}|z_t)\right) = AVar(e_{t+1}|z_t)$

$Cov_t(q_{t+2}(l), \Delta p_{t+2})$ is shown by the off-diagonal element of $Var(z_{t+2}|z_t) = AVar(z_{t+2-1}|z_t)A' + Var(e_{t+2}|z_t) = AVar(e_{t+1}|z_t)A' + Var(e_{t+2}|z_t)$

Then $\gamma_t(2) = i'_1\left(AVar(e_{t+1}|z_t) + AVar(e_{t+1}|z_t)A' + Var(e_{t+2}|z_t)\right)i_2$, where i_1 and i_2 are selection vectors for choosing the off-diagonal element.

Grishchenko and Huang (2013) provide a general formula for all τ:

$$\gamma_t(\tau) = i'_1 \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^{i} A^{i-j} Var\left(e_{t+j} \middle| z_t\right) A^{i-j'}\right)\right] i_2$$

Applying this formula to our example of $\tau = 2$, we get:

$\gamma_t(2) = i'_1\left[\sum_{i=1}^{2} A^{2-i}\left(\sum_{j=1}^{i} A^{i-j} Var\left(e_{t+j} \middle| z_t\right) A^{i-j'}\right)\right] i_2 = i'_1\left[A^{2-1}\left(A^{1-1} Var(e_{t+1}|z_t) A^{1-1'}\right) + A^{2-2}\left(A^{2-1} Var(e_{t+1}|z_t) A^{2-1'} + A^{2-2} Var(e_{t+2}|z_t) A^{2-2'}\right)\right] i_2 = i'_1[AVar(e_{t+1}|z_t) + AVar(e_{t+1}|z_t)A' + Var(e_{t+2}|z_t)] i_2$, which is the same result as above.

Example of the liquidity premium calculation

First, we estimate the parameters $b_t = (\beta_0\ \beta_1\ \beta_2\ \beta_3\ \tau_1\ \tau_2)$ for the daily Svensson yield curve:

$$f(h,b) = \beta_0 + \beta_1 \exp\left(-\frac{h}{\tau_1}\right) + \beta_2 \frac{h}{\tau_1} \exp\left(-\frac{h}{\tau_1}\right) + \beta_3 \frac{h}{\tau_2} \exp\left(-\frac{h}{\tau_2}\right)$$

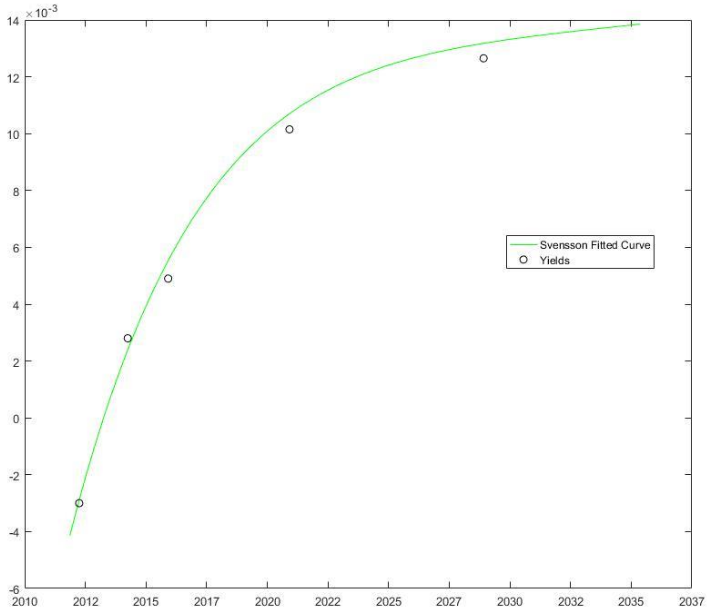
We use two sets of starting parameters: the first set is equal to the parameters of the previous estimation. The second set of starting parameters is based on the Riksbank yield curve estimates. We select the yield curve with lower average fitting errors.

We then calculate the average distance of the observed yields to benchmark yields. A benchmark yield corresponding to an observed yield is calculated by inserting the estimated parameters b_t and the remaining maturity of the observed bond h to the Svensson functional form for zero-coupon yields, which according to Gürkaynak, Sack, and Wright (2010) is:

$$y_t^{i,b}(h) = \beta_0 + \beta_1 \frac{1 - \exp\left(-\frac{h}{\tau_1}\right)}{\frac{h}{\tau_1}} + \beta_2 \left(\frac{1 - \exp\left(-\frac{h}{\tau_1}\right)}{\frac{h}{\tau_1}} - \exp\left(-\frac{h}{\tau_1}\right) \right) + \beta_3 \left(\frac{1 - \exp\left(-\frac{h}{\tau_2}\right)}{\frac{h}{\tau_2}} - \exp\left(-\frac{h}{\tau_2}\right) \right)$$

An example of the Svensson zero-coupon yield curve fitted to Swedish inflation-linked bond data is shown below. The green line represents benchmark yields, with the estimated parameters equal to: $b_t = (1.800, -1.801, -1.568, 0.006, 2.904, 0.007) \times 10^3$

Figure 6. Example of a fitted Svensson yield curve



Please note that one bond used in the fitting of the yield curve is excluded from this calculation, as its remaining maturity is below 3 years.

Remaining maturity (years)	3.94	5.63	10.71	18.82
Observed yield (%)	0.28	0.49	1.01	1.27
Benchmark yield (%)	0.24	0.56	1.07	1.32

$$y_t^L = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\left[y_t^{i,o} - y_t^{i,b}\right]^2}$$

$$= \sqrt{\frac{1}{4}((0.0028-0.0024)^2 + (0.0049-0.0056)^2 + (0.0101-0.0107)^2 + (0.0127-0.0132)^2)}$$

$$= 0.0048$$