Pricing of CO_2 Emission Allowance Derivatives

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Abstract

The aim of this paper is to analyse the pricing of carbon emission allowance futures and futures options to see how they can help us understand the intuition behind spot prices of the underlying emission allowance. We use data from the third time period within the European Union Emissions Trading Scheme. There have been studies within this area before but only a few have incorporated the possibility of jumps in the spot price movement. We estimated risk-neutral parameters for the futures options using the Merton (1976) jump-diffusion model and the Duan (1995) GARCH-(1,1) model built on locally risk-neutral valuation relationship where consequently the model parameters can be estimated from the spot returns using maximum likelihood. Our results show that even though the estimates of the risk-neutral parameters are highly variable for the Merton model, it performs better than the GARCH-model and our chosen benchmark, Black's model (1976). The GARCH-model is applied using Monte Carlo simulations and greatly underperforms all other models in our study, most likely due to errors in the estimation or model specification.

Keywords: CO2, EUA, GARCH, JUMP-DIFFUSION

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Contents

1. Introduction

Global warming is becoming an increasing threat to the environment, the economy and even human health in the future. Society has more or less started to embrace the fact of human activity affecting the global climate change through pollution, resulting in an increasing concentration of greenhouse gases[1] (GHG) in the atmosphere (Stern, 2007). An important step towards limiting the climate change was taken already in 1992 when the United Nations Framework Convention on Climate Change (UNFCCC) worked out the foundation for what would later be the Kyoto protocol. The core content of the Kyoto protocol was internationally binding emission targets and a framework for trading of emission allowances connected to man-made GHG emissions. All stated in effort to reach the goal of reducing emissions during the period of 2008-2012 with five percent, compared to emission level of year 1990. With the 2012 Doha amendment to the Kyoto protocol, higher goals were set for the 2013-2020 period with an 18 percent reduction of GHG emissions compared to 1990 levels. (UNFCCC 2012)

The main message of the Kyoto protocol is for countries to reduce their GHG emissions and in support of that three main market mechanisms were stated – international emissions trading (IET), clean development mechanism (CDM) and joint implementation (JI) (UNFCCC 2007). When stating an economic value to emission reduction targets, a new commodity was created. In order to make the commodity tradable and to follow the intuitions behind the IET mechanism, the European Commission launched the European Union Emissions Trading Scheme (EU ETS) in 2005. The main tradable asset within EU ETS is the European Union Allowance (EUA). One EUA gives the holder the right to emit one tonne of carbon dioxide (CO_2). (European Commission 2013)

The Scheme is designed as a cap-and-trade system; a "cap" is set on the amount of emission rights released in the market for certain high-emitting industries each year, which are then allocated between companies through trading. The market of trading allowances then promotes companies with high emissions to choose the most cost-efficient alternative, either invest in low-carbon technologies or buy more emission rights in the market. The EU ETS covers approximately 45 percent of the GHG emissions from

[1] Carbon dioxide (CO_2), Nitrous oxide (N_2O), Perfluorocarbons (PFCs)

the EU member states[2] and concerns heavy energy-using industries as power generation, manufacturing and aviation. (European Commission 2013)

Besides emissions trading with EUA there are the two other mechanisms of the Kyoto protocol, the projects under CDM and JI. The CDM is a program that offers industrialized countries with commitments under the Kyoto protocol to invest in emission-reduction projects in developing countries and through that earn certified emission reduction units (CER), which then can be traded and used to meet parts of emission targets. Finally, there is also the mechanism of JI that opens the possibility for countries, again with commitments under the protocol, to invest in emission-reduction projects in other industrialized countries and from that earn emission reduction units (ERU), which in turn can be used to meet emission targets. With the 2004 passing of a legislation package called the "linking directive" the EU ETS opened up the possibility to convert credits obtained from CDM- and JI-projects into EUAs that can be used or traded under the EU ETS. Just like the EUA, one unit of ERU or CER gives the right to emit one tonne of CO_2. (UNFCCC, 2007)

Since the launch in 2005 the EU ETS has been divided into 4 different trading periods, often denoted "phases", depending on the structure and goals of each time period. *Phase 1* started with the release of the system in 2005, spanned for two years, and was more or less a "learning by doing" period. When the EU ETS market was established it became, and still is, the biggest carbon market in the world. The number of allowances released in the market was based on an estimated need, which turned out to be too generous, resulting in prices of allowances in the end of the first phase to drop to zero. *Phase 2* spanned from 2008-2012 and important changes were the addition of new states (Iceland, Norway and Liechtenstein), a reduction of allowances and the inclusion of aviation (from 2012). Because of the economic crisis emissions went down and thus also the demand for emission allowances, creating again a surplus of allowances. The current period, *Phase 3*, covers 2013-2020 and added Croatia as a member but most significant for this period was the imposition of an EU-wide cap on emissions and a shift towards auctioning of allowances instead of distributing them for free as was previously done. Finally, *Phase 4* is scheduled over 2021-2028. (European Commission 2016)

[2] From 2008 (beginning of phase 2) EEA EFTA-states Iceland, Norway and Liechtenstein are included.

The main markets for trading emission allowance derivatives under the EU ETS are the Intercontinental Exchange (ICE), Nasdaq OMX Commodities and the European Energy Exchange (EEX). Both ICE, Nasdaq and EEX offer same day trading with daily futures contracts but for spot trading of EUA contracts EEX is the only current marketplace. Our paper studies the EUA spot prices from the third phase, 2013-2016.

There are several other countries that have, like the European Union, developed carbon emission allowance trading platforms. The EU ETS, however, is by far the biggest emissions trading systems in the world, covering over 75% of the international trading volume in 2013 (European Commission, 2013). Emission allowance certificates is therefore an important tool to deal with global warming and it is thus of great importance to understand the mechanics of pricing allowances and derivatives written on them. With focus on the third phase of trading we will examine which pricing model that best captures the characteristics of emission allowances prices when trying to price allowance derivatives. We take the possibility of jumps into account when modelling the price by using Merton's jump diffusion model and Duan's (1995) GARCH (1,1) model. We compare the models by measuring the in-sample and out-of-sample pricing error of the respective model. As a further benchmark we will compare the models results with Black's model from 1976 for pricing option futures.

We conclude that even though the Merton model's risk-neutral parameters are highly variable, it outperforms all other models in our study when compared to market data. The calibration procedure is quite time-consuming but the model provides a higher accuracy both in- and out-of-sample in almost all moneyness and maturity cases. The GARCH specification suffers greatly from approximation errors and or incorrect model specification and consequently performs very poorly compare to both the jump-diffusion model and Black's model.

The remainder of our paper is organized as follows; Section 2 presents a review of the current academic literature on the subject. Section 3 describes the data used in our study and outline the methodology for our empirical analysis. Further on in Section 4 we analyse the spot and futures prices for EUA and in Section 5 we specify the models used. In Section 6 we estimate our parameters and in Section 7 we present our results. Section 8 concludes.

2. Literature review

The academic literature on the EU ETS and trading of emission allowances has increased during recent years, from being a rather unexplored subject to gaining more attention as the societal importance of environmental issues have risen. On the basis of this trend, the major focus has been towards analysing whether the creation of emission allowance markets has led to actual reductions of GHG emissions (Vlachou, 2013) (Laing et al., 2013).

We direct our focus more towards pricing of emission allowance futures and futures options for the European market. There are a number of studies that have had similar focus, early in the field of examining the pricing of emission allowances was Daskalakis et al. (2009) finding evidence of market participants using no-arbitrage theory for pricing and indications of a non-normal distribution with heavy tails for the logarithmic returns. Their study focuses on the relationship between futures and spot prices, adding the factor of inter- and intra-period trade between EU ETS phases. They find that the prohibition of banking allowances between different phases had an important effect on derivatives prices, thus that separation based on maturity makes sense since prices differ. Their empirical study shows that emission allowance spot prices are characterised by jumps and non-stationarity, best approximated with a two-factor equilibrium model based on jump diffusion. More specifically that, both in and out of sample, Merton's (1976) jump-diffusion model delivers a better fit than Black's (1976) model when pricing EUA futures options. Further in the same area Borovkov et al. (2011) investigate pricing of emission allowance futures with help of continuous time and jump-diffusion models.

Seifert et al. (2008) investigate CO_2 spot price dynamics and develop a tractable equilibrium model incorporating stylized characteristics of the EU ETS market. They show that volatility increased when approaching the end of the trading period, recommending the use of models with conditional variance in order to capture the uncertain fact of the market experiencing high or low volatility. Concerning prices, they conclude that spot prices must always be positive and bound by the penalty cost plus the cost for having to deliver potentially lacking certificates. Further on, Benz and Trück (2009) analyse the short-term behaviour of spot prices for emission allowances and compare the performance of different pricing models and find GARCH or regime-switching models to provide the best model for modelling returns of emission allowances.

Wagner and Uhrig-Homburg (2009) evaluate the relationship between spots and futures prices in the carbon market. Their analysis indicate a cost-of-carry approach for the relationship between spot and futures prices, similar results were found by Daskalakis et al. (2009) and Gorenflo (2013). More specifically, Gorenflo (2013) analyse the pricing and lead-lag relationship between spot and futures prices of emission allowances under the EU ETS and conclude that the cost-of-carry hypothesis holds for phase 1 of trading but not phase 2. A similar result, with a non-significant cost-of-carry relationship for phase 2, was found by Chevallier et al. (2009).

Paolella and Taschini (2008) examine the heteroskedasdic dynamics and unconditional tail behaviour in the spot market returns of two emission permit markets, sulphur dioxide (SO_2) in the US and carbon dioxide (CO_2) in Europe. The results indicate that unconditional tails can fairly well be characterized by a Pareto distribution and for the conditional dynamics a GARCH-type structure provides a sound goodness-of-fit.

The latest contribution in the field, which is also close to the focus of our paper, is Yang et al. (2015). They focus on the pricing of emission allowances under phase 2 of EU ETS and incorporate the possibility of price jumps. The log-returns on emission allowance prices indicate a positive autocorrelation and the volatility indicate variations from both time and jump effect. To incorporate these effects when modelling a framework for pricing emission allowance futures they use an ARMA-GARCH model and find a superior fit compared to regular pricing models that does not incorporate jumps.

With the information and views gathered from previous research we believe that there is a possibility to develop the field by analysing data from the most recent time period. We have not found any study covering emission allowance spot, futures and option prices from phase 3. The development in the financial markets and the trading platforms since the start of the EU ETS makes the current phase an interesting subject to evaluate.

3. Dataset and Methodology

3.1 Dataset

Daily EUA spot prices from phase 3 during the period 02/01/2013 to 29/01/2016 were retrieved from EEX, which is the only secondary market for EUA spot at present. EUA futures prices for the same time-period were obtained from EEX and ICE and our analysis will focus on contracts maturing in December, as they are the most actively traded contracts, during 2013-2016. The futures options where only available from ICE, and we will limit our study to call options maturing in December 2015. The data is obtained from the third phase of the EU ETS meaning that we will only analyse intra-period[3] futures and futures options. The studied EEX and ICE futures contracts are both written on one thousand EUAs.

The Euro Overnight Index Average (EONIA) rates were obtained from Bloomberg. A total of 21 daily quoted rates, during 02/01/2013 to 29/01/2016, ranging from 1 day to 6 years were used to construct a simple swap curve.

3.2 Methodology

In this paper we will start of by doing an analysis of the EUA spot and try to estimate the process based on fairly simple models. We then turn to the futures contract and try to establish that they follow the same process as the spot in order to price futures options correctly.

We will price futures options using the jump-diffusion model by Merton (1976) and the GARCH(1,1) model from Duan (1995). As most other option models with multiple parameters, the jump-diffusion model is calibrated to option prices by minimizing the squared pricing error. The GARCH(1,1) model builds on the locally risk-neutral valuation relationship and can consequently be estimated using maximum likelihood on EUA spot data.

When pricing futures option, we restrict ourselves to options with the December 2015 futures contract as the underlying, trading in 2015. The reason for this is twofold. Firstly, we are using maximum likelihood estimation in one of the models using spot for

[3] When the future and the underlying emission allowance are traded during the entire duration of the contract.

phase 3. This requires as many observations as possible, which is why we focus only on 2015. Secondly, the futures options are not very liquid so we focus on the most liquid options with the most liquid underlying futures contract.

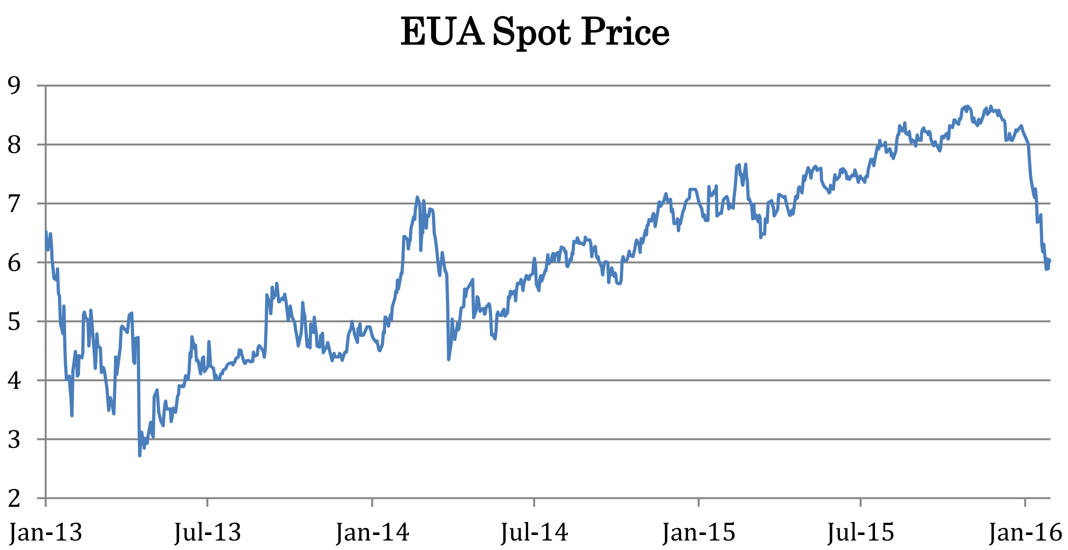
We compare the models by measuring the in-sample and out-of-sample pricing error of the respective model, as well as Black's model from 1976 for pricing option futures as our benchmark. Lastly, we conclude our paper by comparing the stability of the parameters across models and question the models' underlying assumptions.

4. Analysis of Spot and Futures

4.1 EUA Spot

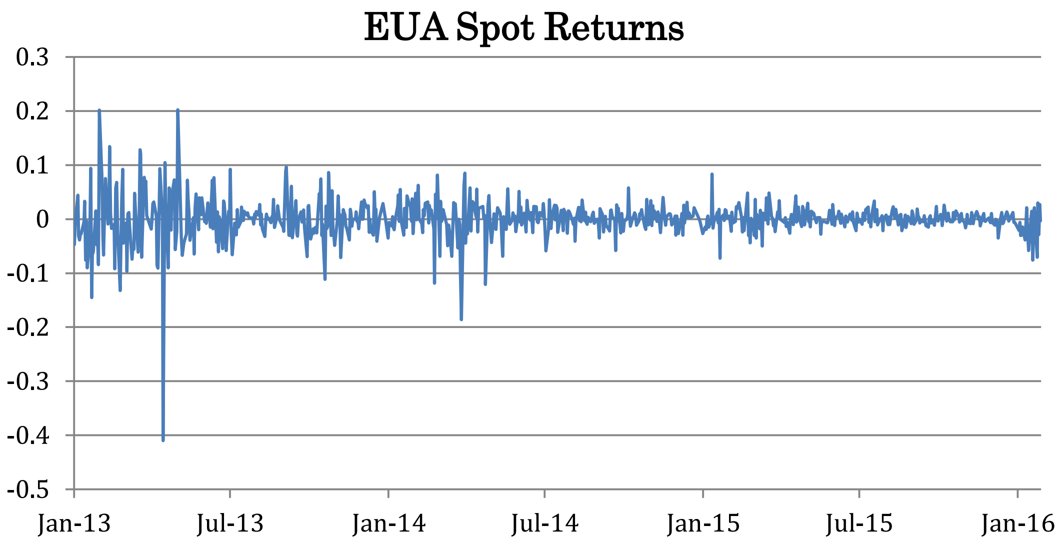
We use daily closing prices for the EUA spot prices, the movement in prices and log-returns are shown in Figure 4.1 and Figure 4.2 below and summarized descriptive statistics for prices and returns are presented in Table 4.1. The period shows a decrease in the first 4 months of trading and then a stable increase, with a spike in January 2014, until the end of our sample where the price falls back to the entry level price for the period. We observe high volatility (around 58% annualized) when analysing the daily returns of emission allowances, presented in Figure 4.2. Over the sample period daily returns fluctuate more than three standard deviations, indicating the presence of jump-risk in the emission allowance market. Moreover, the graphs suggests that there may be conditional heteroscedasticity

Figure 4.1



EUA Spot Prices from EEX during 02/01/2013 to 29/01/16.

Figure 4.2



EUA Spot log returns from EEX during 02/01/2013 to 29/01/16.

Table 4.1

Descriptive Statistics

EUA Spot Price (776 observations)							
Mean	Max	Median	Min	St. Dev.	Skewness	Kurtosis	Jarque-Bera
6.061	8.650	6.070	2.720	1.453	-0.021	-1.039	34.957**
Logarithmic returns (775 observations)							
Mean	Max	Median	Min	St. Dev.	Skewness	Kurtosis	Jarque-Bera
-0.010%	20.187%	0.000%	-40.970%	3.683%	-1.664	23.695	18487.319**

Sample from 02/01/2013 to 29/01/16.
*Significant at the 5% level. **Significant at the 1% level.

Similarly to findings for phase 1 (Daskalakis et al., 2009) and phase 2 (Chevallier, 2009) there is negative skew and a high kurtosis in our sample, which indicates that the returns have a leptokurtic distribution. Moreover, the Jarque-Bera test rejects the null hypothesis that the returns are normally distributed.

We continue our analysis of the spot by testing for stationarity. Contrary to Daskalakis et al. (2009), we reject the null hypothesis of a unit root using the Augmented Dickey-Fuller Test (Dickey and Fuller 1979).

Table 4.2
Augmented Dickey-Fuller Test

Model	Statistic	P-value
Autoregressive	-23.810	0.001
Autoregressive with drift	-23.794	0.001
Trend stationary	-23.779	0.001

Number of lags is 1 in all cases, which was selected based on the Bayesian Information Criterion (BIC).

Now that we have established some characteristics of the EUA spot returns we turn to how one could model these. The most well-known stochastic process for returns is undoubtedly the Geometric Brownian Motion (in continuous time):

$$dS_t = \mu S_t dt + \sigma S_t dz_t \qquad (4.1)$$

where S_t is the asset's price at time t, μ is the expected return, z_t is a standard Weiner process and σ is the time-invariant volatility.

Daskalakis et al. (2009) found that a Geometric Brownian motion augmented by jumps (Merton, 1976) had the best fit for the EUA spot price during the period 25/10/2005 to 28/12/2007. The process in the real world is as follows:

$$dS_t = (\mu - \lambda\mu_j)S_t dt + \sigma S_t dz_t + (y-1)S_t dq_t \qquad (4.2)$$

where λ is the jump intensity measured as the average number of jumps per annum and y, the jump size, is normally distributed as follows:

$$\mathrm{f(y)} = \frac{1}{\sqrt{2\pi\sigma_j}} \exp\left(\frac{(y-\mu_j)^2}{2\sigma_j^2}\right) \qquad (4.3)$$

where μ_j and σ_j^2 are the mean and variance. The jump component q_t is a Poisson process, independent of the Weiner process z_t. λ is the arrival intensity of the Poisson process so that $Pr\{dq_t = 1\} = \lambda dt$ and $Pr\{dq_t = 0\} = 1 - dt$.

The model helps to explain excess kurtosis and negative skew, which makes it a good fit in theory considering the characteristics of the EUA spot returns. Merton (1976) assumed that the jump-risk was diversifiable. However, looking at the sharp drops of the

EUA spot makes one question that assumption in this case. Moreover, as in the case of the Black-Scholes framework and the Geometric Brownian motion, the volatility is assumed to be constant over time. These assumptions are of course questionable but they allow us to use a fairly simple model with jumps when pricing options.

Models based on generalized autoregressive conditional heteroscedasticity (GARCH) are used in many areas of finance. In the EUA research, Benz and Trück (2009) had success with a GARCH framework. We will now present the model by Duan (1995) that we will use to price options. This model deals with heteroskedasticity of asset returns in a different way than most other models (such as the jump-diffusion model above): the price of an option depends on the risk premium of the underlying asset. Moreover, the model is non-Markovian. The lognormally distributed return of an asset under the real world probability measure P:

$$\ln\frac{S_t}{S_{t-1}} = r + \gamma\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t \tag{4.4}$$

where r is the constant one-period risk-free rate, γ is the risk premium[4] and ε_t follows a GARCH(1, 1) process of Bollerslev (1986),

$$\varepsilon_t|\phi_{t-1} \sim N(0, h_t) \text{ under measure P,}$$
$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{4.5}$$

where ϕ_t is the information set of all information up to and including t; $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$. Moreover, $\alpha_1 + \beta_1 < 1$ to ensure covariance stationarity.

We estimate the three abovementioned models by using log-likelihood[5] and present the results below in table 4.3. As indicated by the non-normality above, the Geometric Brownian Motion underperforms compared to the other models, while the winner in terms of log-likelihood is the GARCH(1,1) specification. Turning to the estimated real world parameters, we notice that volatility is high and fairly close to what Daskalakis et al. (2009) found in the Powernext market during phase 1 while the drift parameter, μ, is very off. By augmenting the Geometric Brownian Motion with jumps, we see that the diffusion coefficient more than halves, suggesting that a large part of the volatility can

[4] The unit risk premium is more commonly known as λ in the literature but we use γ to not confuse with the jump intensity parameter in the jump-diffusion model.
[5] Please see Brigo et al. (2008) for maximum likelihood estimation of GBM and JGBM.

be explained by jumps. The higher log-likelihood indicates that it better captures the leptokurtic features and negative skew found in the data. The jump intensity parameter, λ, is quite extreme as it suggests almost 91 jumps per year. Furthermore, we see that the mean of the jump is positive, albeit close to zero, whereas Daskalakis et al. (2009) estimated a negative mean. For the GARCH model we see fairly odd parameters as the magnitude of α_1 and β_1 is reversed as what is usually seen in the literature. The large β_1-coefficient indicates that the conditional variance is highly dependent on past conditional variance, while a relatively small contribution is attributed to past squared disturbances (ε_{t-1}^2). Assuming the estimation is accurate, this suggests that there is very persistent volatility clustering in the EUA spot returns.

Table 4.3
Maximum Likelihood Estimation (02/01/2013 - 29/01/2016)

Parameters	**GBM**	**JGBM**	**GARCH(1,1)**
μ	0.0254	0.0254	
σ	0.5840	0.2413	
λ		90.6291	
μ_j		0.0033	
σ_j		0.0522	
α_0			5.806×10^6
α_1			0.1053
β_1			0.8947
γ			0.0843
$\mathcal{L}$	1459.345	1650.773	1756.116

4.2 EUA Futures

Before we can price options on futures, we need to make sure that the cost-of-carry relationship holds and that they follow the same process as the spot price, as we intend to apply models based on this process.

The standard cost-of-carry relationship which says that a futures contract F at time t, maturing at T, can be calculated as follows:

$$F_{t,T} = S_t e^{(r+s-c)(T-t)} \qquad (4.6)$$

where S is the underlying asset, r is the risk-free rate, s is the storage cost and c is the convenience yield. As assumed by Daskalakis et al. (2009), the storage cost is zero, due to the non-physical nature of EUA contracts, and the convenience yield is zero as there is no benefit of having the spot contract compared to a futures contract for intra-phase contracts. Following Daskalakis et al. (2009), we test the accuracy of this relationship by comparing the actual closing prices of futures contract on EEX and ICE, maturing in December 2013, 2014, 2015 and 2016, with theoretical futures prices. We use daily prices from 02/01/2013 up to expiry for contracts maturing in 2013-2015 and to 29/01/2016 for the contract maturing in December 2016. As the secondary EUA spot is only traded on the EEX, it will also be used when calculating the theoretical price for futures on ICE.

We deviate from their methodology by using a daily quoted interest rate when pricing the futures. The 3 month (e.g. Yang et al., 2016) or 6 month (e.g. Daskalakis et al., 2009) EURIBOR rate is often times used as a proxy for the risk-free rate. However, Hull and White (2012) argues that overnight indexed swap rates should be used as the risk-free rate. When calculating the theoretical futures prices we got considerably more accurate prices when we used an interpolated swap curve using the Euro Overnight Index Average (EONIA), rather than using the 3 month or 6 month EURIBOR.

Daskalakis et al. (2009) calculate the mean squared pricing error, MSE (%), which we also present for comparability:

$$MSE\ (\%) = \frac{100}{N}\sum_{t=1}^{N}\left(\frac{F_t^T - F_t^A}{F_t^A}\right)^2$$

where F_t^T denotes a theoretical futures price and F_t^A is the actual futures closing price. We prefer the root mean squared error, RMSE (%), as it is easier to interpret. It is calculated as follows:

$$RMSE\ (\%) = 100\sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\frac{F_t^T - F_t^A}{F_t^A}\right)^2}$$

Moreover, the mean transaction cost (MTC) was calculated as the mean of fixed fees[6] per EUA as a fraction of the actual futures price. Our results are presented in table 4.2. Looking at MSE, it is quite comparable to what Daskalakis et al. (2009) found for intra-phase contracts in phase 1 and suggests that the cost-of-carry relationship holds. The

[6] The exchange and clearing fees for ICE is €0.0035 per contract for member's proprietary business. For EEX it was €0.0018 during 2013 and 2014, while it was €0.002 in 2015 and 2016.

RMSE is quite large at 6.1% for the ICE December 2016 futures. However, this can at least partially be explained by the discrepancy of using spot prices from another exchange and the fact that a larger weight is put on observations further away from maturity as it started trading in January 2013 which compounds any error of our proxy of the risk-free rate.

Table 4.4
Pricing Accuracy of EUA futures

	ICE				EEX			
	Dec-13	Dec-14	Dec-15	Dec-16	Dec-13	Dec-14	Dec-15	Dec-16
MSE (%)	0.0046	0.0386	0.1379	0.3781	0.0936	0.1550	0.0591	0.1377
RMSE (%)	0.6758	1.9642	3.7137	6.1490	3.0587	3.9372	2.4315	3.7110
MTC (%)	0.0741	0.0633	0.0457	0.0540	0.0401	0.0325	0.0312	0.0256

Futures are calculated using the cost-of-carry relationship in Eq. (4.6), where cost of carry and convenience yield have been assumed to be zero. MSE (%) is the mean squared pricing error in percentage while RMSE (%) is the squared root of the mean squared pricing error in percentage. MTC (%) refers to the transaction cost as a fraction of the actual futures price.

To further test if the cost-of-carry relationship holds, we also conduct an Engle-Granger test. This is done by OLS regression to find a stationary linear combination of two processes with minimum variance. As can be seen in table 4.3, we clearly reject the null of no co-integration between the series. This leads us to believe that the EUA spot and the futures contract follow the same trend and that the cost-of-carry relationship holds in the long run.

Table 4.5
Engle-Granger Test

	Statistic	P-value
Price levels	-3.993	0.001
Returns	-44.560	0.001

The Engle-Granger test is done on EUA spot and the 2015 December futures.

Although there is some conflicting evidence in previous research regarding the existence of a convenience yield, at least some of it could be subscribed to the choice of the risk-free rate. In the present paper, we simplify and assume it being non-stochastic. Hence it will have no real impact on our option pricing as we focus on European options expiring three business days before the expiry of the futures contract and there is no price difference between the spot and the futures contract at expiry of the option.

5. Theory and Model Specification

5.1. Risk-Neutral Valuation

An essential concept in derivatives pricing is risk-neutral valuation. In the risk-neutral world, investors do not care about risk. While in the real world, or so called physical world, investors demands a premium to compensate for systematic risks. As derivatives such as options depend on the price of the underlying asset in a relative sense, they themselves do not depend on risk aversion. Rather, the underlying asset's value may change due to shifts in investors risk appetite.

An asset can be valued by discounting with the risk-free rate, given that expected cash flows are calculated using risk-neutral probabilities. In this setting, all fairly priced assets earn the risk-free rate. If an asset wasn't fairly priced, an arbitrageur would exploit this by either selling the overpriced asset or buying the underpriced asset and hedging the exposure.

In a no arbitrage setting, a market is complete if and only if there is only one risk-neutral probability measure (Donaldson & Danthine, 2014). In the case of the jump-diffusion model, however, the underlying process is discontinuous which leads to an incomplete market as it's not possible to perfectly hedge ones exposure. This will cause some issues, not the least when estimating the risk-neutral parameters for this model as there is more than one probability measure and consequently different option prices (Cheang and Chiarella, 2011).

As it is costless to enter a futures contract, the risk-neutral expected return is zero. For a futures contract in the risk-neutral world, price F is often assumed to follow the below process (Hull, 2012):

$$dF = \sigma F dz \quad (5.1)$$

where the volatility σ is constant. The process behaves just like a stock with the dividend yield equal to the risk free rate.

5.3. Futures Options

A European call option gives the holder the right, but not the obligation, to buy the underlying asset at a predetermined price, the strike price. A European futures call option instead gives the holder the right to enter a long position in a futures contract. As in the case with many other commodity markets, the EUA spot is not as actively traded as the futures and hence it is more convenient to have futures contract as the underlying asset. At the expiry of the option, the payoff is as follows:

$$\max[F - X, 0]$$

where F is the futures price at expiry of the option and X is the strike price. If the option is in the money at expiry (i.e. F>X), the option holder will enter the underlying futures contract, with a price equal to the strike price, while otherwise the option expires worthless. In the EUA options market, there are futures trading with a quarterly expiry (March, June, September and December) up to 16 contract months. The underlying futures contract, however, is always the December contract for the particular year of expiry.

5.4. Black's Model

In 1976, Fischer Black showed how to value futures options in a simple formula:

$$C = e^{-rT}[F_0 N(d_1) - XN(d_2)] \qquad (5.2)$$

$$d_1 = \frac{\ln(F_0/X) + \sigma^2 T/2}{\sigma\sqrt{T}} \, d_2 = d_1 - \sigma\sqrt{T} \qquad (5.3)$$

It is based on the Black-Scholes framework and assumes that the futures price follows a log normal distribution and where volatility is constant. This is a very standard model when pricing futures option and consequently we will use it as our benchmark.

5. 5. Option Pricing Under Jump-Diffusion

Option models based on a simple diffusion process often undervalues short-term out of the money options. These models require very high volatilities to match the market's expectation of a large price movement in a short time-period. By allowing for discontinuous jumps in returns, the jump-diffusion model helps to explain these differences between market and model prices. The equivalent process for equation 4.2 under the risk neutral measure Q is given by:

$$dS_t = (r - q - \lambda^* \mu_j^*) S_t dt + \sigma S_t dz_t + (y^* - 1) S_t dq_t \qquad (5.4)$$

Where * indicates the equivalent risk adjusted parameters. When calibrating the model, we will however estimate all parameters from option's data to see if the option prices contain useful information and to see if the model is correctly specified. For convenience, the superscript * is dropped from here on.

The value of a European call option using the jump-diffusion model by Merton (1976) can be expressed as an infinite sum of weighted Black-Scholes prices. The weights are the Poisson probability that the underlying will jump n times during the time to maturity. Following Murphy and Ronn (2015), we present a version for European calls on futures, using the assumptions that futures price behaves like the underlying asset with a dividend yield q equal to r:

$$C = \sum_{n=0}^{\infty} \frac{e^{-\lambda' T}(\lambda' T)^n}{n!} C_n(F_T, X, T, r_n, q, \sigma_n) \qquad (5.5)$$

where $C_n(F_T, X, T, r_n, q, \sigma_n)$ is the Black-Scholes European call option value and

$\lambda' = \lambda(1 + \mu_j)$

$\sigma_n^2 = \sigma^2 + \dfrac{n\sigma_j{}^2}{T}$

$r_n = r - \lambda\mu_j + \dfrac{n\ln\left(1 + \mu_j\right)}{T}$

$q = r$

Although the sum is infinite, it can be well approximated by a truncated sum. Gilli and Schumann (2010) recommended including about 20 jumps. We still had some noticeable price differences up to around 50 jumps, but we include 150 jumps in the sum for good measure.

5.6. GARCH Option Pricing

Duan's GARCH option pricing model uses the locally risk-neutral valuation relationship which in practise means that one can estimate parameters, to be used when pricing options, using the underlying returns. Duan uses some assumptions with regards to the utility function and shows that the conditional variances under Q and P are equal. Under the pricing measure Q with normally distributed conditional returns, the model looks as follows:

$$\ln\frac{S_t}{S_{t-1}} = r - \frac{1}{2}h_t + \xi_t \qquad (5.6)$$

$$\xi_t | \phi_{t-1} \sim N(0, h_t) \qquad (5.7)$$

$$h_t = \alpha_0 + \alpha_1(\xi_{t-1} - \gamma\sqrt{h_{t-1}})^2 + \beta_1 h_{t-1} \qquad (5.8)$$

The European call option price at time t and maturing at time T is given by:

$$C = e^{-(T-t)r} E^{Q}[\max(S_T - X, 0) \mid \phi_t]$$

As no closed form expression really exists for this particular model, we will approximate the price using Monte Carlo simulations. A large number of paths of the process will be generated and the average payoff of the call option is then discounted back to time t using the risk-free rate. In our case, we will use 10,000 simulations per day. Monte Carlo simulations can of course be a bit time-consuming to use in practice, but it is not something we will consider notably in this paper as our primary goal is to compare models that are fairly easy to apply.

6. Parameter Estimation

6.1. Calibrating the Jump-Diffusion Model

In order to price options using the jump-diffusion model, we need to find four risk-neutral parameters: This is often done by calibrating the model to real option prices by using maximum likelihood estimation or by minimum least squares estimation. We use the latter and follow a methodology similar to Bakshi et al. (1997) and minimize the squared difference between market and model prices in order to find the parameter vector, θ.

$$\theta = \arg\min \sum_{n=1}^{N} [C_n^{obs}(t, X_n) - C_n^{model}(t, X_n)]^2$$

Instead of having the objective function as the sum of squared euro pricing errors as in the case above, one could also minimize the squared percentage pricing errors such as suggested by Cont & Tankov (2002). Naturally, this would put more weight to cheaper options (such as options out-of-the-money). However, to generate more comparable results to Daskalakis et al. (2009) and to follow what seems to be consensus, we will use the sum of squared euro pricing errors throughout when calibrating the jump-diffusion model.

In our case the observed market price will be the daily settlement price, which is the trade weighted average price during the settlement period (which is the last 10 minutes of a trading day). Should there be no executed price at this time it will be the last executed price within the closing bid and ask prices or if that is not possible; the mid-point of the closing quote. Due to liquidity reasons, we restrict our analysis to call options and five strike prices. These are options struck at the money as well as the four closest strike prices (two out-of-the-money and two in-the-money strike prices). Moreover, we focus on options expiring in December each year as these options, as well as the underlying futures contracts expiring the same month, are generally the most liquid.

The calibration was carried out using the abovementioned options over samples of 25 observations (i.e. 5 days with five option prices each) during 2015. The sample size was selected by considering the stability of the parameters and convergence of the optimization.

This was done in MATLAB using two local optimizers: lsqnonlin and fmincon. Lsqnonlin is especially suitable for nonlinear least-squares problems and was used with the default trust-region-reflective algorithm. Fmincon was used with the sequential quadratic programming (SQP) algorithm to look for any other smaller minima. These optimizers can find quite different points depending on the starting values of the parameters. Gilli and Schumann (2010) found that a best-of-N-restarts strategy with random starting points gave about the same or better results than picking "good starting values" (from economic or mathematic intuition) when calibrating a yield curve model. In the spirit of their findings, we tried to mitigate the problem with local minima by rerunning each minimization with random starting values 100 times using both optimizers. In order double-check whether there was a more optimal minimum which the other two methods did not find, we also used simulannealbnd which searches for a global minimum using simulated annealing.

The optimization routines took about three days to run on an Intel i5-4670K (five cores) at 3.40 GHz with 8 GB RAM. In practice, one might speed up the process by implementing the option valuation technique using fast Fourier transform by Carr and Madan (1999).

6.2. GARCH Maximum Likelihood Estimation

For the GARCH(1,1) model by Duan (1995), will use maximum likelihood when estimating the parameters for the GARCH model. Following Christoffersen and Jacobs (2004), we maximize the log-likelihood function conditional on the first observation:

$$\ln \mathrm{L} = -(T-1)\ln(2\pi)/2 - \sum_{t=2}^{T} \ln(h_t)/2 - \sum_{t=2}^{T} \left(R_t - r_f - \gamma\sqrt{h_t} + 0.5h_t\right)^2/(2h_t)$$

This is done on daily historical spot returns from 02/01/2013 up to and including the sample period of options we are pricing in-sample. In the out-of-sample case, we only include historical data preceding the sample period. Given the assumptions made on the model, we are able to estimate the parameters α_0, α_1, β_1 and λ that can be used to price options. As there is no analytical solution to this function, we optimize in Matlab using fmincon which allows us to optimize the negative log-likelihood with the correct bounds on the parameters. Since it's quite difficult to appropriately estimate GARCH models using returns (Christoffersen and Jacobs, 2004), we focus our efforts estimating the model in 2015 which ensures us at least two years of daily data from phase 3.

7. Option Pricing Results

We test our models accuracy in pricing EUA futures options by using a sample of 1125 European futures call option prices. The measure of accuracy in this case will be the mean absolute pricing error (MAPE) :

$$MAPE = \frac{1}{N}\sum_{n=1}^{N}\frac{\left|C_n^{obs} - C_n^{model}\right|}{C_n^{obs}}$$

This is calculated on a maturity and moneyness basis and we present the in-sample pricing errors below in table 7.1. We use 45 samples with 25 trading days and 125 option prices each. The GARCH model performs extremely poorly which can be cause by errors in the approximation, perhaps due to the short sample used in the maximum likelihood estimation, and or model specification errors. This can partially be caused by the previously noted reversed magnitude of α_1 and β_1 which causes rather low but very persistence volatility. Merton's model outperforms Black's model in all cases. As previously mentioned, the jump-diffusion model is often more accurate in pricing out-of-the-money options, especially when they the time to maturity is short. This is also the case in our data, where we see that Merton's model halves the short maturity OTM pricing error by Black's model.

Table 7.1
In-Sample Pricing Errors

	Strike	Black	Merton	GARCH
Long Maturity	Total	1.942%	1.315%	56.511%
	ATM	1.443%	1.285%	55.636%
	ITM	1.654%	0.999%	40.952%
	OTM	2.480%	1.645%	72.507%
Medium Maturity	Total	1.978%	1.139%	123.943%
	ATM	1.125%	1.048%	107.270%
	ITM	1.331%	0.821%	77.976%
	OTM	3.052%	1.504%	178.247%
Short Maturity	Total	3.137%	1.842%	124.059%
	ATM	2.070%	1.705%	108.297%
	ITM	1.203%	0.932%	48.643%
	OTM	5.605%	2.821%	207.356%

In-Sample Option Pricing using MAPE as the measure of error. The sample is 2 January to 13 November.
Long Maturity: 341 to 236 calender days.
Medium Maturity: 233 to 131 calender days.
Short Maturity: 128 to 26 calender days.

In table 7.2 we present the out-of-sample pricing errors, where the parameters from the previous sample have been applied to the next 25 trading days. As in the in-of-sample case, the Merton Jump-Diffusion model is generally the most accurate. As mentioned before, it is more accurate in pricing options out-of-the money. However, it falls short to Black's model in two cases: long maturity at-the-money calls and short maturity in-the-money calls.

Table 7.2

Out-of-Sample Pricing Errors

	Strike	Black	Merton	GARCH
Long Maturity	Total	3.348%	3.090%	55.009%
	ATM	2.986%	3.088%	54.758%
	ITM	2.702%	2.244%	39.873%
	OTM	4.175%	3.937%	70.269%
Medium Maturity	Total	2.856%	2.524%	121.938%
	ATM	2.258%	2.019%	104.715%
	ITM	2.394%	1.764%	74.354%
	OTM	3.616%	3.537%	178.135%
Short Maturity	Total	4.041%	3.860%	122.967%
	ATM	3.913%	3.130%	107.910%
	ITM	1.407%	1.616%	45.807%
	OTM	6.740%	6.468%	207.656%

Out-of-Sample Option Pricing using MAPE as the measure of error.
The sample is 9 January to 13 November.
Long Maturity: 334 to 229 calender days.
Medium Maturity: 226 to 117 calender days
Short Maturity: 114 to 26 calender days

Although the Merton model is much more tedious to work with due to the complex calibration procedure, there is still some gain in most cases compared to the standard futures option pricing model of Black. As can be seen in the Appendix in figures A.1 and A.2 the calibrated risk-neutral parameters are highly variable. Moreover, they are not really reflecting what we see in our maximum likelihood estimation of the parameters under the physical measures, where for example the jump intensity λ where over 90. Either the options do not contain very precise information or we need more options in our calibration to more accurately estimate the risk-neutral parameters. In the figure A.3 in the Appendix we also show the implied volatilities of Black's model, which shows a decreasing trend. This highlights the fact that a constant volatility is not present in the EUA market.

8. Conclusions

We first looked at the EUA spot returns characteristics and found that they follow a non-normal distribution with high kurtosis and negative skew. We applied maximum likelihood to estimate three models: the Geometric Brownian Motion, Merton's Jump-Diffusion model and the GARCH specification of Duan (1995). In the spot returns data, we noted that the GARCH model had the best fit in terms of log-likelihood, followed by the jump-diffusion model. After describing the spot return distribution we looked at the futures and found that they are co-integrated with the EUA spot. We then estimated parameters by calibrating the Merton model to options data and by using maximum likelihood on spot returns for the GARCH model. The pricing accuracy was measured using mean absolute percentage error in- and out-of-sample on futures options maturing in December 2015.

Our study of phase 3 confirms the findings of Daskalakis et al. (2009): that options are more accurately priced using jump-diffusion as they most likely better capture the high kurtosis and negative skew observed in the EUA spot returns. The GARCH model performed very poorly, most likely due to estimation errors or an incorrect model specification.

We acknowledge that our approach has limitations. Firstly, we assume that the convenience yield is zero while both our own futures pricing and prior research produce ambiguous evidence. Secondly we use non-constant risk-free rates in all our models. Moreover, we use Merton's jump-diffusion model which assumes constant risk-neutral parameters, which we estimate using 25 call settlement prices for each sample. As there is low liquidity in these options, many of the settlement prices are based on quotes rather than actual closes on the exchange which might bias the study. As for our GARCH specification, we apply Duan's (1995) locally risk-neutral relationship to the spot while pricing futures option. Unfortunately, we cannot really disentangle the effect of this misspecification from any potential estimation errors on the underperformance of the model.

As previously mentioned, there are still not many published papers on EUA derivatives. For future research on this topic, it would be highly interesting to apply models that use stochastic volatility as well as jumps, such as Bates (1996) to see if they price EUA option futures more accurately.

References

Bakshi, G., Cao, C. & Chen, Z. (1997). Empirical Performance of Alternative Option Pricing Models. Journal of Finance, 52, 5, 2003-2049.

Bates, D. "Jump and Stochastic Volatility: Exchange Rate Processes Implict in Deutche Mark in Options", Review of Financial Studies, vol. 9, 1996, pp. 69-107.

Benz, E., & Trück, S. (2009). Modeling the price dynamics of CO 2 emission allowances. Energy Economics, 31(1), 4-15.

Black, F. (1976). The Pricing of Commodity Contracts. Journal of Financial Economics, 3, 167-179.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics, 31, 307-327.

Brigo, D., Dalessandro, A., Neugebauer, M., & Triki, F. (2008). Available at SSRN: http://ssrn.com/abstract=1109160 or http://dx.doi.org/10.2139/ssrn.1109160

Carr, P., & Madan, D. (1999). Option valuation using the fast Fourier transform. Journal of Computational Finance, 2(4), 61–73.

Cont, R. & Tankov, P. (2004). Calibration of jump-diffusion option-pricing models: a robust non-parametric approach. Journal of Computational Finance, Vol. 7, No. 3, 1-49.

Cheang, G. H. L. & Chiarella C. (2011). A Modern View on Merton's Jump-Diffusion Model, Quantitative Finance Research Centre, Reseach Paper 287.

Chevallier, J., Ielpo, F., & Mercier, L. (2009). Risk aversion and institutional information disclosure on the European carbon market: a case-study of the 2006 compliance event. Energy Policy, 37(1), 15-28.

Christoffersen, P. & Jacobs, K. (2004). Which GARCH Model for Option Valuation?. Management Science, Vol. 50, No. 9, 1204-1221.

Danthine, J.-P. & Donaldson, J. B. (2014). Intermediate Financial Theory (Academic Press, Waltham, MA)

Daskalakis, G., Psychoyios, D., & Markellos, R. N. (2009). Modeling CO 2 emission allowance prices and derivatives: evidence from the European trading scheme. Journal of Banking & Finance, 33(7), 1230-1241.

Dickey, D.A., Fuller, W.A., 1979. Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association 74, 427–431.

Duan, J.-C. (1995). The GARCH Option Pricing Model. Mathematical Finance, Vol. 5, No. 1, 13-32.

European Comission (2013), The EU Emissions Trading System (EU ETS), European union publications office [online], Available at: http://ec.europa.eu/clima/publications/docs/factsheet_ets_en.pdf [Accessed 2016-05-01]

European Comission (2016), Climate Action, European Comission [online], Available at: http://ec.europa.eu/clima/policies/ets/index_en.htm, [Accessed 2016-05-01]

Gilli, M. & Schumann, E. (2010). A note on 'good starting values' in numerical optimization. COMISEF WORKING PAPERS SERIES, WPS-044 18/09/2010

Gilli, M. & Schumann, E. (2010). Calibrating Option Pricing Models with Heuristics. COMISEF WORKING PAPERS SERIES, WPS-030 08/03/2010

Gorenflo, M. (2013). Futures price dynamics of CO2 emission allowances. Empirical Economics, 45(3), 1025-1047.

Hull, J. C. (2012). Options, Futures, And Other Derivatives (Pearson Education Limited, Harlow).

Hull, J.C. & White, H. (2012). LIBOR vs. OIS: The Derivatives Discounting Dilemma, Journal of Investment Management, 11(3), 14-27.

Laing, T., Sato, M., Grubb, M., & Comberti, C. (2013). Assessing the effectiveness of the EU Emissions Trading System. Grantham Research Institute on Climate Change and the Environment Working Paper, (106).

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. Journal of financial economics, 3(1-2), 125-144.

Murphy, F. & Ronn, E. I. (2015). The valuation and information content of options on crude-oil futures contracts. Review of Derivatives Research, 18, 2, 95-106.

Paolella, M. S., & Taschini, L. (2008). An econometric analysis of emission allowance prices. Journal of Banking & Finance, 32(10), 2022-2032.

Seifert, J., Uhrig-Homburg, M., & Wagner, M. (2008). Dynamic behavior of CO 2 spot prices. Journal of Environmental Economics and Management, 56(2), 180-194.

Stern, N. (2007), The economics of climate change: the Stern review. Cambridge University press.

Tankov, P. & Voltchakova E. (2009). Jump-diffusion models: a practitioner's guide. Banque et Marchés, No. 99, March-April 2009

UNFCCC (2007), The Kyoto Protocol Mechanisms, UNFCCC [online], Available at: http://unfccc.int/resource/docs/publications/mechanisms.pdf, [Accessed 2016-05-01]

UNFCCC (2012), Kyoto protocol, UNFCCC [online], Available at: http://unfccc.int/kyoto_protocol/items/2830.php, [Accessed 2016-05-01]

Vlachou, A. (2013). The European union's emissions trading system. Cambridge Journal of Economics, bet028.

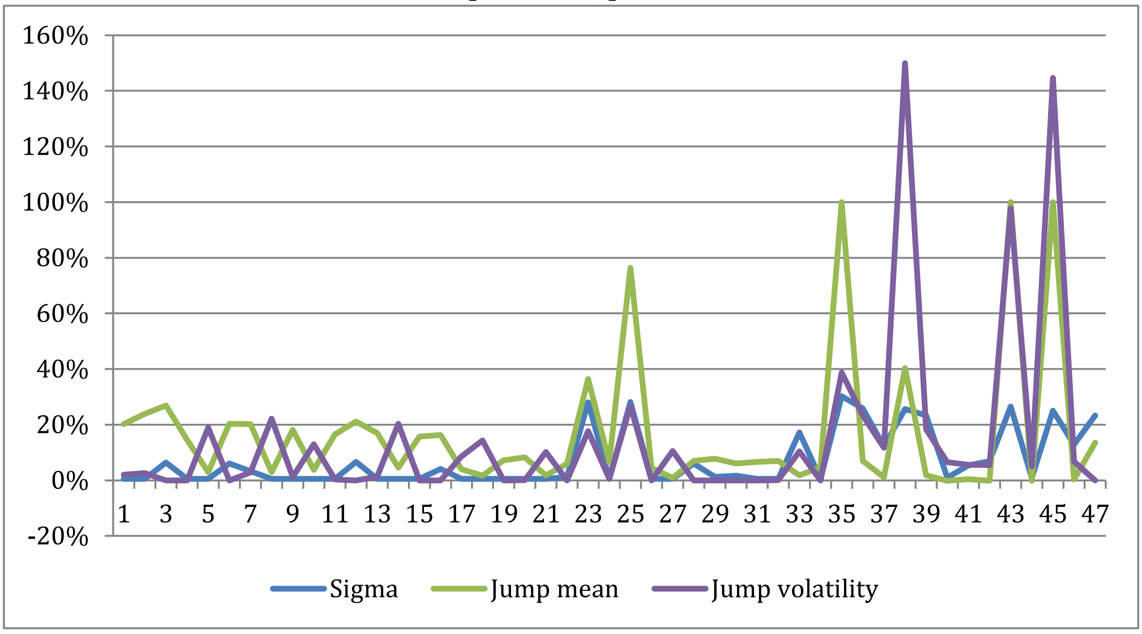
Wagner, M. W., & Uhrig-Homburg, M. (2009). Futures price dynamics of CO2 emission certificates-An empirical analysis. Journal of Derivatives, 17(2), 73-88.

Yang, S. S., Huang, J.-W. & Chang, C.-C. (2016). Detecting and modelling the jump risk of CO2 emission allowances and their impact on the valuation of option on futures contracts, Quantitative Finance, 16(5), 749-762.

Appendix

Figure A.1

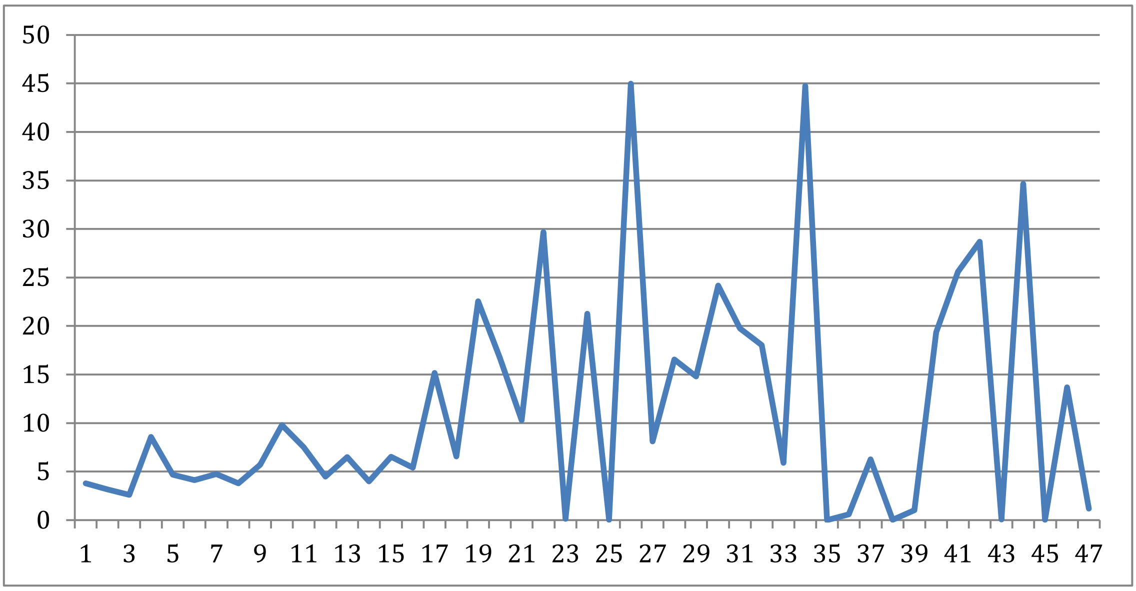
Calibrated Estimates over 47 Samples for σ, μ_j and σ_j



Each sample includes 25 trading days. Ranging from 02/01/2015 to 27/11/2015

Figure A.2

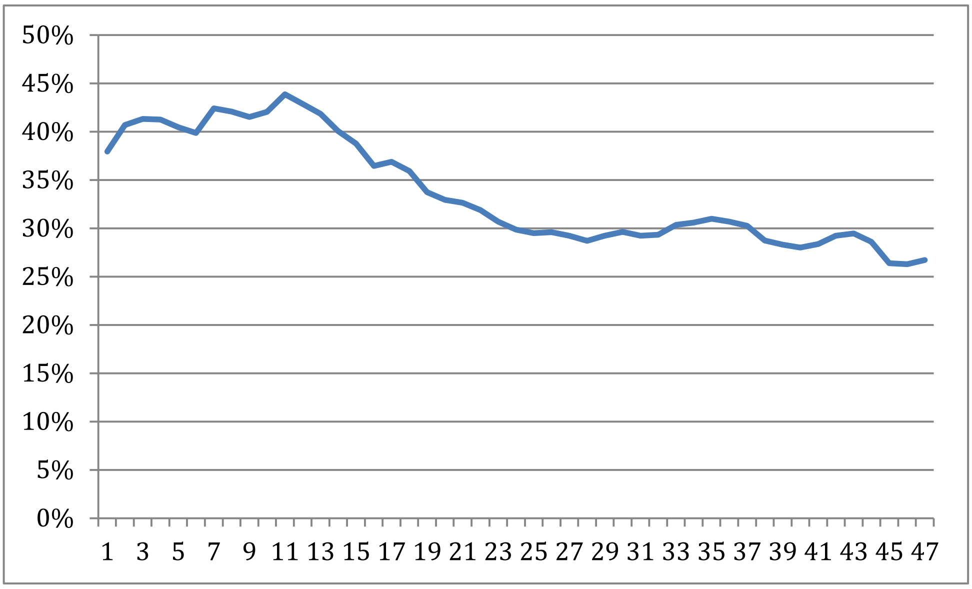
Calibrated Estimates over 47 Samples for λ



Each sample includes 25 trading days. Ranging from 02/01/2015 to 27/11/2015

Figure A.3

Implied volatilities in Black's model in 47 samples



Each sample includes 25 trading days. Ranging from 02/01/2015 to 27/11/2015