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Welfare effects in the Cournot model of oligopoly: An application on the Stockholm housing construction market

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Abstract

With high demand and rising prices, housing in the Stockholm region has become an important topic. Studies indicate that high concentration in the housing construction market may contribute to the record price levels. This paper is constructed as a thought experiment where we apply a Cournot model of competition on the Stockholm tenant-owned multi-dwelling housing construction market with the assumption of fixed cost, linear demand, homogenous tastes and goods, and heterogeneous firms. The model is applied to 30 scenarios where that fixed cost is lowered from its higher bound value to allow marginal entry in order to calculate the percentage change in welfare. The findings imply that the percentage change in welfare is small within the confines of the model.

Keywords: Welfare Economics, Cournot Model, Oligopoly Market Structure

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Definitions and abbreviations

BPI – Statistics Sweden’s building price index (*Byggnadsprisindex*)

CLS – Cost Leadership Structure

CPI – Consumer Price Index (*Konsumentprisindex*)

CS – Consumer Surplus

DTI – Debt to income

EMS – Even Market Structure

F – Fixed Cost

LTV – Loan To Value

MC – Marginal Cost

PS – Producer Surplus

PWC – Percentage Welfare Change

PWL – Percentage Welfare Loss

SCA – Swedish Competition Authority (*Konkurrensverket*)

SCB – Statistics Sweden (*Statistiska Centralbyrån*)

SKL – Swedish Association of Local Authorities and Regions (*Sveriges Kommuner och Landsting*)

SOU – Official Reports of the Swedish Government (*Statens Offentliga Utredningar*)

Tenant-owned housing – *Bostadsrätt* in Swedish. There are other translations in use but we choose to use the same as SCB. Sometimes referred to as co-op share.

The Big 4 – Refers to JM, Peab, Skanska and NCC

TOMD – Tenant-owned multi-dwelling

WL – Welfare Loss

“No one has the right, and few the ability, to lure economists into reading another article on oligopoly theory without some advance indication of its alleged contribution.”

(Stigler 1964)

1 Introduction

The housing construction market in Sweden has gone through major changes since the 1990s and has transformed from being a restricted and subsidized market, to a market exposed to competition. Construction of new housing has not matched the increased demand in the big cities. Production costs are said to have increased more than in other countries which, combined with a stagnant productivity, have resulted in hefty price increases. Several studies have been conducted based on this, with the aim of stressing the importance of competition in the housing construction market (SOU 2015:105).

Blame for the lack in supply is sometimes attributed to policymakers and the long and complex process of housing construction. Red tape and detailed development plans hinder and delay construction. Sweden has very limited competition from foreign companies and relatively long time to completion for projects.

The importance of competition in the construction market has been increasingly recognized by both the Official Reports of the Swedish Government (SOU) and the Swedish Competition Authority (SCA). Lack of competition in a market will push prices upwards and, in key markets such as housing, possibly stall economic growth. There is a concerning competition problem in the Swedish housing construction market that has contributed to the low level of construction in the country, despite the considerable demand increase. Four companies account for around 80% of all housing developing in Stockholm (SOU 2015:105). The dominating status of these four companies results in an oligopolistic market structure associated with subpar competition.

Research has also been conducted on oligopolies' effect on welfare and the possible societal welfare losses through Cournot competition. Whilst a lot of it has been of a more theoretical nature, there are also examples of applications on specific markets. Under certain conditions, the theoretical framework can lead to substantial welfare losses. It is often taken for a fact that high concentration in a market leads to output

below the social optimum. This line of thought seems prevalent in the housing debate in Sweden as well, although not always described in those terms. In this thesis, we wish to combine these and from an economic perspective examine how competition and oligopolistic market structures affect welfare in the Stockholm housing market. We will do this by applying a Cournot model on the tenant-owned multi-dwelling housing construction market in Stockholm to see if there is measurable welfare loss and consequently if this could be improved by increasing the number of operative firms in the market.

This paper is organized as follows. Section 2 provides the backdrop for the paper and why it is an important area of research. In section 3 we look at the current state of research on welfare effects through a Cournot model of competition as well as the research on housing construction with a focus on Sweden. Section 4 refines the purpose and direction of the paper and presents the hypothesis. We further study the market and develop the model. We also give a brief description of the data and how we will put it to use given the background. In section 5 we analyse the results of the calculations and the implications it has. This is followed by section 6 where we draw conclusions from the results and relate them to the earlier sections, primarily the current state of research. Section 7 provides a summary.

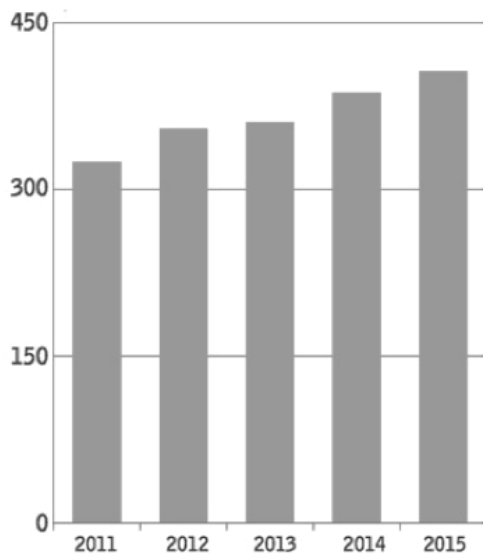
2 Background

2.1 Historical price development of housing in Sweden

The financial crisis led to falling housing prices in many countries and put an end to a continuous price growth. Sweden however, did not experience this notable price fall. The small decrease it faced following the crisis was quickly recovered and prices continued their upwards trajectory. Since then, Sweden has had an exceptional price growth of housing.

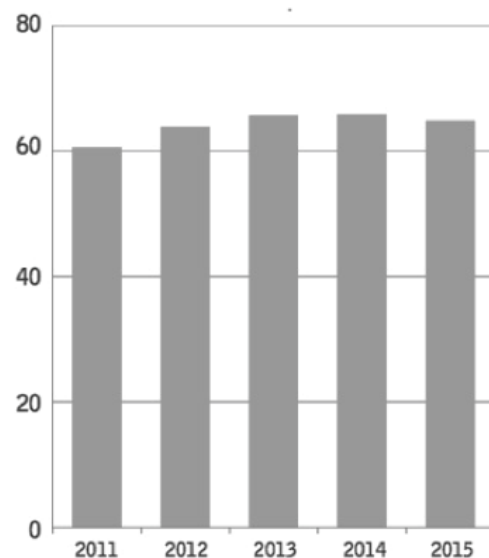
The dramatic price increase of housing has followed approximately the same growth rate as households' debt to income ratio. The lending to tenant-owned housing experiences an annual growth of 12% and the increase in mortgage debt has contributed to the rise of the debt to income ratio (Holmberg 2016). Two possible reasons for the price development of housing are the increase in people's disposable income as well as the decrease of actual mortgage rates. Between the years 1997 and 2009, the disposable income increased by approximately 75%. Mortgage rates in Sweden are among the lowest in the world. The low rates are an obvious advantage for loan takers since loaning becomes easier and cheaper. With lowered demands of down payments and amortization requirements, it is easier for people, especially those with low wealth, to enter the housing market. To get a complete understanding of the households' debt levels, it is important to look at both the debt to income ratio and the loan to value ratio.

Average debt to income ratio (percent)



Source: Finansinspektionen

Average loan to value ratio (percent)



Source: Finansinspektionen

The current housing situation in Sweden is to many alarming and widely considered to be quite extraordinary (Edwards 2016). Some even say it imposes a significant risk to the country's economy. With approximately 3%, Sweden has one of the fastest annual growth rates among developed countries (OECD 2015). Interest rates are below zero and inflation is close to zero, leading to cheap loans for homebuyers. The Swedish Riksbank is not raising the rates to calm the market, instead they are keeping them at a low level for fear of upsetting the economy (Edwards 2016). A combination of the housing supply shortage and the low interest rates has resulted in a rapid increase in housing prices. Tenant-owned housing prices in Stockholm rose by 18% in 2015 (Valueguard 2016).

Stockholm experiences a large annual inflow of people and an increase in demand for housing (SCB 2015a). The population growth in combination with the continuously low construction of new apartments results in an accelerating increase in price. Another reason for the high housing prices is the improved interest situation seen out of a buyer perspective, making it cheaper to take out mortgage loans. The yearly growth rate of mortgage debts is 8.4%, compared to 4.6% in 2012 (Holmberg 2016).

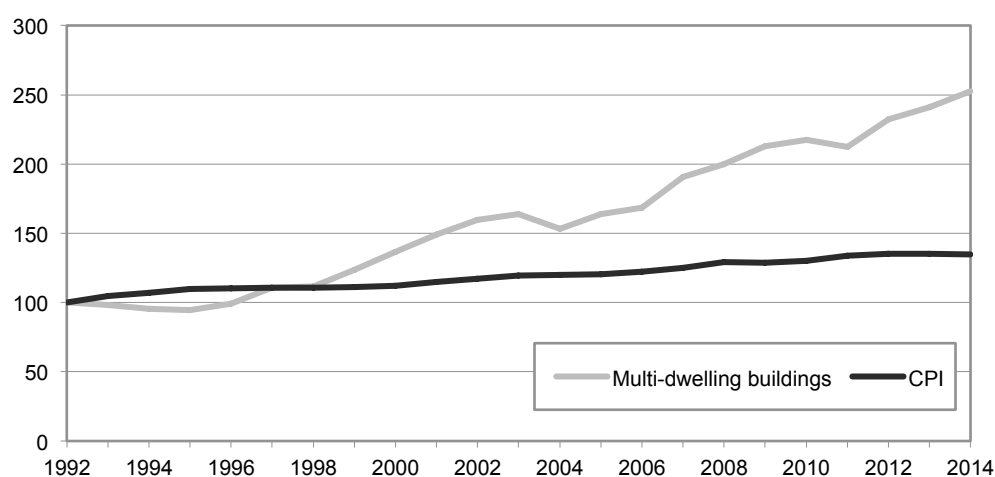
2.2 Housing situation in Stockholm

The supply of housing in Stockholm has been an issue for many years and it does not keep up with the population growth and consequently the increased need for housing. Several studies show that there is a big gap between supply and demand of housing in Sweden (SCA 2015a). From a socio-economic perspective, this housing-shortage could potentially stall the growth of the Swedish economy and put economic stability at risk.

While the demand for housing in Sweden's big cities has increased drastically the last decade, the construction of new apartments is continuously relatively low. Growing wages, lowered interest rates and amortization requirements, and an increasing inflow of people has been contributing to this rise (Hansson 2014, SOU 2015:105). Since there is little movement in the already existing housing stock, making it less sensitive to price signals, there is more pressure on the construction of new housing, thus making the price increase on housing larger than it would have been otherwise.

Statistics Sweden's (SCB) building price index (BPI) is a measure that works as an indication of the cost development of housing and it has increased substantially more than the consumer price index (CPI) during the last 15 years. The marginal cost of production for a building has doubled between the years 1992 and 2014. Increased costs might originate from low efficiency and higher demands from customers, while the increased price possibly derives from the low interest levels, abolished real estate tax and competition problems in the construction market, such as market concentration, entry costs and the fact that the four biggest construction companies own land in attractive areas without exploiting it (SCA 2015b).

BPI and CPI development in Sweden



Source: SCB

2.3 Competition in the housing construction market

Historically, the housing construction market in Sweden has been very restricted and closed for competition. Even though foreign competition is increasing, the competition in the Swedish construction market is still low compared to many other markets (SOU 2015:105). The conditions for competition in a market are often measured with the concentration rate, id est the collective market share of the three to five largest firms. Without sufficient competition in the market, the dominant firms can take advantage of their superior position. A high concentration rate does not per se imply that competition is restricted if there is foreign competition or if the entry barriers are low. The housing construction market is very capital demanding and banks do not always grant capable companies loans, hindering them from competing with the biggest firms (Hansson 2014). Construction accounts for around 10% of Sweden's GDP and thus plays an important

roll in the growth of the Swedish economy (Sveriges Byggindustrier 2015). It is of vital importance that the competition in this market is well functioning in order to push productivity, efficiency, and cost development further.

Construction costs have increased more than other production costs in the economy for a long period. The productivity has not evolved as well as in other parts of the industry. Increased construction costs together with a low productivity development can indicate weak competition (SOU 2015:105). Several studies in different countries have portrayed the presence of high housing prices, increased construction costs, low productivity development and a low production of housing as reasons for supply shortage and also possibly for the hampering of the economic growth. Further problems impacting housing construction are the limited supply of exploitable land as well as the protracted and unpredictable planning- and building process.

The Swedish housing construction market is indeed very complex and intertwined. The construction process goes through several levels and is really a part of a larger market structure. This paper aims to study the developers as they are gatekeepers in housing construction and whose total output will decide the number of apartments built. They are also the group primarily studied by the SCA. In a minimalistic form, a developer would decide to build, acquire property, hire a construction company and then sell the property, sometimes through an intermediary. However, the reality is much more complex. First off, the land is usually owned by either local government, who has their own development companies, or by private developers. For instance, The Big 4 had 50,000 construction rights. Secondly, in the case of the large private developers, they are often doubling as construction companies and retail their own projects afterwards (SCA 2015b). Additionally, although not covered in this paper, the supply of construction material is an important factor as they contribute to 45% of the costs in housing development and can sometimes exert influence due to high concentration. Cementa AB for example has 99% of the cement production in Sweden (SOU 2015:105). Research has shown that the efficiency in a given market may be compromised by intermediary markets, despite signs of an otherwise healthy competition (Ghosh, Morita 2007).

3 Literature review

The apparent lack of competition in the Swedish construction market has received much attention in recent years. As competition is an important component for a market to be able to thrive, and thus a deficiency could disturb the economic growth, it has brought many researchers to thoroughly evaluate this issue (SOU 2015:105, SCA 2015a). The effect of competition on welfare has been subject to several studies and there are numerous models for computing the effects of deficient competition on a market, one of which is the Cournot model.

3.1 The Cournot model of oligopolies

It is worth mentioning some of the applications of the Cournot model and research on welfare loss associated with it, although we can in no way provide an exhaustive coverage of such a widely used model. The classic Cournot model of competition dates from the 19th century but shows, despite its old age, continued relevance. In modern research it is often applied in multi-stage models, although that is beyond the scope of this paper (Daughety 2008).

Calculations of welfare loss in Cournot competition have seen a fair bit of attention over the years. The basic case of homogenous goods and firms and linear demand does not yield any substantial welfare losses. However, the introduction of heterogeneous goods and/or firms does increase the effect. The most substantial effects may be found when varying the demand function. From a theoretical point of view, the percentage welfare loss can, if parameters are chosen correctly, vary within a wide range between near zero and 100% (Corchón 2008).

Relaxing the assumption of homogenous products may possibly result in very large welfare loss. However, while in a sense perhaps more realistic, it is beyond the scope of this paper due to the immense complexity of the housing market in which the product ends up. Adding heterogeneity in goods would in any case bias the size of welfare loss upwards, implying that we will underestimate the effects (Corchón, Zudenkova 2009).

The same model has been tested loosening the demand constraint to see what effects a concave or convex demand curve might have on percentage welfare loss (PWL)

concluding that the division of surplus varies with the concavity (or convexity) of the demand curve (Andersen, Renault 2003). This has interesting implications for future research as it is beyond the scope of this paper; we will only look at the linear case.

A basic thought experiment implies that even when ignoring fixed costs (F), more heterogeneity among firms will cause the marginal increase in welfare (due to a marginal increase in the number of firms) to diminish. It is important to note that although free entry with fixed costs can yield insufficient entry, it can also lead to excessive entry. This is because in oligopolistic competition, a new entrant will reduce the output of incumbents so that the total marginal change in welfare is the entrant's added output less the incumbents' reduction in output. However, given an integer number of firms, the free entry number of firms will not differ with more than one from the socially optimal number (Mankiw, Whinston 1986).

3.2 State of the Swedish housing market

The general wisdom is that housing prices should rise in tandem with income over time, although there may certainly be discrepancies in the short-term. With a growing economy, low interest rates, and having avoided the worst of the financial crisis, Stockholm should see steadily rising prices. However, there has been a growing divergence over the past decades indicating that prices may in part be driven by other factors than higher income and cheap money, such as limited supply (BKN 2010).

Research conducted on Oslo's housing market indicated that impediments in the construction market contributed to the high price levels (Krakstad, Oust 2015). Conversely, research has also shown that housing investment in Sweden has become more demand driven over time (Berg, Berger 2006).

There has also been a considerable amount of research on the Swedish housing market, although it has generally taken a more macroeconomic perspective (Brownstone, Englund 1991).

SCA, as well as Boverket, describe the four largest construction firms as a group of dominant players, in part due to their size but also because they are the only nationwide construction companies. The annual revenue of the smallest is almost twice that of the following firm's and they are both horizontally and vertically integrated (SOU 2015:105).

The Swedish construction market has also had declining productivity relative to other industries, which may be caused by a lack of competition (SCA 2015b). This in turn would imply that the real welfare loss would be larger than calculated, as it is not taken into account in this paper.

While SOU ascertains that profits are high in housing construction (SOU 2015:105), oligopolistic competition and production below the socially efficient amount does not necessitate high profits (Gokhale, Tremblay 2012). In view of a lack of competition, managers may be incentivized to pursue other goals than cost minimization, leading to an inefficient market (Maudos, de Guevara 2007).

The dominant view is that the construction market is highly concentrated in certain segments and regions and that this is a cause of high prices. As previously mentioned, the driving force behind high prices is demand. Low interest rates, a relatively high economic growth, and a large influx of people all contribute to strong demand. However, in an efficient market one should expect not only price to rise, but also quantity. This has not really been the case, which is why the concentrated supply market has been named as a possible culprit (BKN 2010).

4 Research design

4.1 Research question

Previous studies seem to agree that there is a lack of competition in the Swedish housing construction market, leading to market inefficiency and associated welfare loss. They point to several possible causes for this, such as entry barriers caused by a lack of available property and firms' profit maximizing approach (as opposed to market share maximizing) that in the current environment leads to lower than socially optimal quantities. There is a flora of studies on welfare losses in various competition settings, though, as far as we know, none applied to the Stockholm multi-dwelling housing construction market. The complexity of the housing market does not readily lend itself to simplistic models. However, we believe there is some value in attempting to quantify the welfare losses associated with a specific market. Due to the sheer size of the Stockholm housing market, because of soaring prices and Stockholm's large share (30%) of GDP, quantifying losses may be helpful in policymaking. We will use the classic Cournot model of competition in order to attain this goal.

The basic though experiment is this: assuming free entry, firms enter a market as long as profits are positive. The function for profit depends on demand, what all other firms in the market are currently doing, and fixed costs. Fixed costs have several components. Some of these are technological, such as equipment, office space, etc. Others however are policy-driven due to things such as long approval times and detailed development planning. We consider the latter type as something that can be lowered through market reforms. It is the effect of such policymaking that may lead to the additional entry that we wish to calculate in this paper. Or, to put it in a different way, the welfare cost of having these policies in place.

Hence, this paper is based on the notion that:

The high concentration in the Stockholm tenant-owned multi-dwelling housing construction market causes a measurable welfare loss.

And as a direct consequence of the above, the hypothesis of this paper is that:

There are substantial welfare gains associated with marginal entry caused by a reduction in fixed costs.

We will further attempt to quantify a possible change in demand or cost of construction in order to account for different scenarios.

4.2 The choice of market and its characteristics

It is important to take into account that the housing construction market is highly localized. There are only four companies that operate nationwide but when it comes to housing they compete locally with each other as well as large local developers. Due to the high localization in construction, this paper will study a regional market. The choice of region befalls to Stockholm¹ as it has roughly 30% of GDP (SCB 2015b) and accounts for roughly half of all new housing construction (SCB 2016). It is thus undoubtedly the most important market. The market will refer to Stockholm unless otherwise stated.

SCB defines two main groups of housing, multi-dwelling units and one or two dwelling units, where the prior category is further divided into tenant-owned dwellings, rental dwellings and owner-occupied dwellings. The last group is very small and often categorized within tenant-owned dwellings (there were only 4 built in the past 5 years in Stockholm). This paper focuses solely on multi-dwelling housing, as the market is quite differentiated both in terms of customer group and in terms of supplier. This still leaves one division between rental dwellings and tenant-owned. Tenant-owned is arguably the more profitable, whereas rental regulation adds inconvenient complexity. With an effective cap on rents and a very low vacancy rate due to the high demand for housing, limiting quantities produced will not lead to higher prices and, by extension, profits. It therefore makes sense to limit the scope to tenant-owned housing in multi-dwelling buildings. This division finds support in previous research (Brownstone, Englund 1991).

¹ Stockholm in this paper refers to what SCB has defined as the Greater Stockholm area, which is the same as Stockholm county.

4.3 Method and model

In order to quantify welfare, we need a model that allows us to estimate welfare caused by a certain scenario. We turn our attention to the Cournot model. Based on the notion that housing developers choose a quantity to produce, rather than produce on demand for a price they set, it seems a prime candidate. The Bertrand model also has the drawback that it leads to perfect competition even with two firms in its basic form² (Baye, Kovenock 2008).

The basic model used in this paper is the Cournot model of competition, which assumes that players compete on quantity, taking demand as given. While the model dates from 1836 it is still a workhorse in use today. The quantity each firm procures is based on a best response function where each firm maximizes its profit given what all other firms are doing, leading to a Nash equilibrium. If there is a single firm, it will act as a monopolist causing substantial welfare loss as it procures far below the socially optimal quantity. However, in the basic Cournot model, welfare loss rapidly approaches zero as the number of firms increase.

Firms will enter as long there are positive profits and each time a firm enters they steal business from incumbents, whilst at the same time increasing total output. As price correlates negatively with quantity in the model, more entry also implies lower price, all to the benefit of consumers. Hence, the classic approach in economics dictates that free entry is good. On the contrary, research has shown that free entry may however not be optimal due to excessive entry if there are fixed costs involved (Suzumura, Kiyono 1987). This is not an issue in this paper as it studies the incremental change from a given market structure and not the optimum.

Two of the most common variations of the model are to remove the assumption of homogenous firms and to remove the assumption of homogenous goods. Because homogenous firms in the Cournot model would necessitate all firms to produce the same quantity, it would make it very unrealistic when testing a market for oligopolistic competition due to the dominance of a few large firms. Heterogeneous goods, however, add a complication, as it is very hard to measure the substitutability of goods.

² This is also known as the Bertrand paradox. The paradox is that with one firm, it will act as a monopolist and earn positive profits but with two firms there will be no profits at all. This assumes the basic model with only one homogenous good and identical firms (Baye, Kovenock 2008).

Additionally, as SKL notes, the product supplied by construction companies is quite similar and they compete with a ‘similar supply’ lending credibility to the approximation homogenous goods (Johansson, Westerberg et al. 2013). It is true that there are big differences between buildings but these differ on a project basis rather than between companies (SOU 2015:105).³ In short, we assume firms cannot charge a premium for having a differentiated product since their competitors could offer the same product.

Furthermore, there is the question of free entry and the related question of fixed costs. Free entry implies that any firm could enter the market as long as it is profitable. SCA states that there are barriers to entry in the form of a lack of available land and a protracted and complicated planning and construction process. We assume that the only reason a firm does not enter is profit based and so any actual barriers to entry appear in the form of fixed costs.

Firms take demand as given and enter if profits are positive. They produce according to the profit maximizing first order condition. I.e. they produce at the profit maximizing level given what all other firms are doing.

$$P = a - bX \quad \pi_i = \frac{(a + \sum c_j - Nc_i)^2}{b(N + 1)^2} - F \quad x_i = \frac{1}{b(N + 1)} \left(a + \sum_{i \neq j}^N c_j - Nc_i \right)$$

Where:

x_i =Quantity produced by firm i

b=Slope of the function

N=Number of firms

c_i =Marginal cost of firm i, where $c_1 \leq c_i$

a=Intercept

F=Fixed cost

The data will be sourced from estimates by the SCB and other government agencies. For a more detailed walkthrough of the algebra behind this please see Appendix C.

³ Projects differ primarily on type based on target group income and regulation, which depends partly on micro location such as environmental issues and macro location, what municipality it is in. However, this depends on each building and not the company that builds it.

4.4 The data

Ideally, to use the model we would have all the exogenous variables (a , b , c_i , and F) and then calculate the rest. However, these are generally not readily observable, especially a and b which can only be estimated. The observable variables (N , X , and P) do not provide enough information to calculate the welfare on their own.

Preferably, we would have X as m^2 of liveable housing. However, this has proven difficult to find and so we use the second best, number of apartments. A quick scan of annual reports and SCB's statistics show that the distribution of apartment sizes does not vary substantially year-to-year. Most buildings contain a mix of all sizes so while it decreases precision it ought not to invalidate the analysis. Consequently, P refers to $kr/\text{apartment}$ and not kr/m^2 . Additionally, there are no reliable measures for price so we must infer that from the model.

The data we have provides estimates for X , N , s_1 , and \bar{c} . This is not enough, as we would need to know the demand function and each firm's marginal costs in order to use to model. While F is exogenous, the way we use it in the model means we want to calculate it as an error term that makes the rest of the model work out. It is also not easily observable, the approach here aside.

While there are no estimates for the demand function, there has been a lot of research on the elasticity of demand for housing. This is somewhat superseded due to the increased usage of hedonic demand models, which mean that aggregate demand elasticity becomes less relevant. Furthermore, most estimates for demand elasticity have been on income as it was more valuable to researchers at the time. There are still estimates for the price elasticity of demand although they vary substantially. The majority however are in the range of -0.5 and -1. Much of this research is from the 1970s and primarily done in the United States although there are some more modern estimates in Sweden for the rental market which may serve as an indicator. It is worth reiterating an obvious weakness of the model, the assumption of linear demand. The elasticity changes on quantity if demand is linear so that we must assume it hold true for current quantities but the estimates made are for other markets so had they been linear, they must most certainly be wrong. Had they been isoelastic, not an entirely unrealistic assumption, the model used here is based on a demand curve that is incorrect.

In general, the data consist of estimates. As they come primarily from government agencies we deem them quite objective. Whilst we acknowledge the lack of precision, we do not consider it all too detrimental to our efforts, as we will model different scenarios. The data most lacking is a solid estimate for demand as it underpins the whole model. This is however also the biggest flaw of the model itself, as with housing, the assumption of a simple representative consumer utility function that does not differentiate between different units of housing as demand cannot be readily calculated, especially outside the current.

4.5 Application of the model

As most parameters are unknown we proceed based on the assumption that there are interior solutions such that we can infer the true value of the exogenous and endogenous parameters. An important point here is that N can only take on integer values. That is strictly speaking true for x_i as well but we choose to assume that the effect of rounding down is marginal.

We analyse percentage welfare change of lowering fixed cost just enough to let in one more firm. To account for ambiguity in estimates we test for three different elasticities in two different market structure and changes in two parameters, X and \bar{c} (rise, fall, and the baseline) for a total of 30 scenarios.

The market structures are based on three assumptions and consist of two rather stylized cases. The assumptions are that firm 1 is dominant with 40% of the whole market and the second is that the 4 biggest firms (please take note that these are *not* The Big 4) account for 80% of the market (SOU 2015:105). The last assumption is also underpinned by literature and is that these dominant firms make up a market segment with the remaining firms as a competitive fringe or competing in a different market (Johansson, Westerberg et al. 2013). As for the first structure it is what we will refer to as the cost leadership structure (CLS) where each firm has a clear advantage over the next firm in cost. The second is what we call the even market structure (EMS) where firm 1 still dominates and is followed by firm 2. However, firm 3 is only slightly less productive than firm 2 and the remaining firms have the same cost function.

Total quantity of multi-dwelling housing produced varies vastly over time and so we do the baseline case on the official data for 2013. We then run two additional scenarios based on a rise and fall of 1500 in X for all elasticities. This will aid in giving the paper more practical implications for public policy.

The elasticities we use are -1.2, -0.9, and -0.6. There are few sources citing elasticities much beyond unity and the model cannot be solved for elasticities as low as -0.5, leaving us with these three to provide a good range.

Lastly, the individual marginal cost is essentially just the independent variable in the function for x_i , which in turn is the product of total quantity and market share; the former, which we know and the latter, which we estimate in using the two abovementioned market structures. However, as costs have been rising faster than inflation, and faster than costs in other industries, we will try two scenarios with higher and lower marginal costs respectively. The case for trying a lower marginal cost is that much of the rise in costs in Stockholm has to do with rising cost of land, which may be affected by public policy. It is further noted that costs have been rising even though wages have not done so, disproportionately suggesting that such effects, as previously mentioned, of high concentration of building material suppliers and regulations may impact these.

Using this, the fixed cost associated with the last firm making zero profit is calculated. The fixed cost is then lowered just enough to let one more firm in, a firm that will be making zero profit. Welfare is calculated as the sum of producer surplus (PS) and consumer surplus (CS). Producer surplus is the sum of the difference between the cost and price of each unit. Consumer surplus is simply the area between the demand curve and the price.

Because the scenarios we test are not expected future values, but possible directions that the market may take, we perform a simple sign analysis. As we have best guesses, this provides a tool to see in what direction our welfare analysis might be headed.

5 Results

Reiterating the research design, we assume fixed costs contain a policy-driven part that may be reduced via market reforms. We wish to calculate the welfare effect of this by reducing fixed costs just enough to allow one more firm to enter. After inputting the estimated parameters for the market and inferring the highest fixed cost consistent with the model, we then lower fixed cost just enough to let one more firm in. Then we replicate this process, adjusting the parameters according to our scenarios.

5.2 Percentage welfare change

Welfare change is quite small in all cases, generally around 2.25% for the CLS and just over 1% for EMS. This suggests that perhaps the welfare loss caused by oligopolistic completion is not that large. It is of course a very stylized model and the values may have little real world relevance. The sign however ought to be correct, *ceteris paribus*, which supports the hypothesis.

It is also somewhat surprising that the percentage welfare change (PWC) is largely independent of elasticity. This is likely due to the behaviour of the dominant firm only being marginally altered by the slope of the demand curve, as its profits are largely unaffected. Remember that we adjust elasticities for a given quantity in the market.

The perhaps most noticeable difference is that between the market structures as CLS provides consistently larger PWC, except in the case of inelastic and declining demand. This seems counterintuitive, as more asymmetry should lead to larger welfare loss due to concentration, because firms have more market power. However, in EMS the marginal entrant is as efficient as the next firm up so it captures a larger share of the market, effectively stealing more from the more efficient firms than in CLS. An interesting implication of this is that not only does the market structure matter but also the firms not yet in the market.

Table 1: percentage welfare change

Elasticity	-1.2	-0.9	-0.6
Cost leadership structure			
No adjustment	2.25%	2.25%	2.25%
Q up	4.35%	0.86%	0.86%
Q down	4.25%	4.25%	4.25%
C up	3.49%	2.25%	2.25%
C down	2.25%	2.25%	2.25%
Even market structure			
No adjustment	1.07%	1.07%	1.07%
Q up	1.86%	1.86%	1.86%
Q down	0.20%	0.20%	0.20%
C up	1.07%	1.07%	1.07%
C down	1.07%	1.07%	1.07%

Table 1: percentage welfare change of one more firm entering market. Increase on baseline value shaded darker and decrease shaded lighter.

5.2 Fixed cost as a share of percentage welfare change

The only parameter that is directly changed in each scenario is F . However, fixed costs eat away at welfare because they reduce producer surplus. In the limit of $N \rightarrow 5$, PWC rises, but as the fifth firm enters it falls significantly as the fifth firm steals business from more efficient incumbents. Within the model, the change in F is actually larger than the change in welfare suggesting that, ceteris paribus, welfare actually falls. While the model implies that society could benefit from lower fixed costs (assuming these incorporate red tape and not only investments in capital), it also implies that this is not caused by a more efficient market. In fact, the opposite is true; the market grows less efficient because the business stealing effect is larger than benefit from reduced market power on the margin. All scenarios except diminished demand in CLS point at this conclusion.

Cost leadership structure

Elasticity	Baseline	Q up	Q down	C up	C down
-1.20	201%	127%	63%	130%	201%
-0.90	201%	638%	63%	201%	201%
-0.60	201%	638%	63%	201%	201%

Even market structure

Elasticity	Baseline	Q up	Q down	C up	C down
-1.20	173%	130%	561%	173%	173%
-0.90	173%	130%	561%	173%	173%
-0.60	173%	130%	561%	173%	173%

Table 2: fixed cost as a share of percentage welfare loss associated with marginal entry.

The diminished demand scenario of CLS is the only one where the fall in F is less than the total improvement in welfare of increased competition, prompting further scrutiny. The data shows that the required change in F is roughly $1/3$ of the baseline case with diminished demand in CLS, whereas with EMS it is roughly $1/2$. Meanwhile, the change in welfare is comparable in CLS but becomes quite paltry in EMS leading to the huge difference in F as share of PWC. This is probably due to two interrelated causes. Firstly, the depressed demand causes the dominant firm to receive a much larger share of the market, increasing overall efficiency. When F is subsequently lowered in CLS the dominant firm's cost advantage allows it to preserve much of its market share whereas in EMS, the symmetry among other firms means they are less at a disadvantage and so they steal more business from the leader. The second is that when demand is sufficiently depressed and firms are asymmetric enough, the fixed cost required to deter entry becomes so small that only a minor (relative to the other scenarios) adjustment is required to allow another entrant.

This is a bias in the model we have made, as more symmetry implicitly means a larger fixed cost is required to deter entry. In the Cournot framework, the only thing that differentiates firms is their marginal cost and so the entire difference in market share must be directly attributable to it. With one firm clearly dominant, its marginal cost must be substantially lower and so all new entrants' business stealing effect on welfare will be large in relation to the welfare effect of lower price and higher quantity.

5.3 The distribution of welfare

An important aspect to welfare is the distribution of it. Intuitively, increased competition ought to benefit consumers to the detriment of producers. However, looking at the data it is clear that this is not always the case. In PLS marginal entry actually increases producer surplus, except when demand is inelastic and increases. With EMS producers consistently lose out except when demand is inelastic and average marginal cost is higher.

Price leadership structure						
	Elasticity	Baseline	Q up	Q down	C up	C down
Elasticity	-1.2	-1.20	+	-	-	+
	-0.9	-0.90	+	-	-	+
	-0.6	-0.60	+	-	-	+
PWL	-1.2	2.25%	+	+	+	+
	-0.9	2.25%	-	+	+	+
	-0.6	2.25%	-	+	+	-
Change in PS	-1.2	0.88%	+	+	+	Same
	-0.9	0.88%	-	+	Same	Same
	-0.6	0.88%	-	+	Same	Same
Change in CS	-1.2	3.02%	+	-	-	Same
	-0.9	3.02%	+	-	Same	Same
	-0.6	3.02%	+	-	Same	Same
Even market structure						
	Elasticity	Baseline	Q up	Q down	C up	C down
Elasticity	-1.2	-1.20	+	-	-	+
	-0.9	-0.90	+	-	-	+
	-0.6	-0.60	+	-	-	+
PWL	-1.2	1.07%	+	-	-	-
	-0.9	1.07%	+	-	-	-
	-0.6	1.07%	+	-	Same	Same
Change in PS	-1.2	-8.30%	-	+	Same	Same
	-0.9	-8.30%	-	+	Same	Same
	-0.6	-8.30%	-	+	Same	Same
Change in CS	-1.2	5.40%	+	-	Same	Same
	-0.9	5.40%	+	-	Same	Same
	-0.6	5.40%	+	-	Same	Same

Table 3: changes in distribution of welfare associated with changes in market parameters when fixed costs are lowered to let one more firm in. A + implies that PWC is larger in the scenario than in the baseline and vice versa for -.

A lot of attention in the setting public policy focuses on the distribution of wealth. It is noteworthy, and quite intuitive, that there is a change not only in welfare but also in the distribution of it. As more firms enter, quantity rises and price falls. This in itself causes a reduction of PS and an increase in CS. This may seem quite interesting at first but is wholly attributable to the fall in fixed costs. As marginal cost for each firm remains the same and prices are unequivocally lower in all scenarios the only thing affecting profit, and therefore PS, is quantity and fixed cost; and the change in quantity is quite marginal.

From a public policy point of view this does give rise to interesting implications. First, there is a redistribution effect from producers to consumers, which may find political support and hence help policymakers wanting to increase competition. The second is of more interest as the increased competition may be completely outweighed by the fall in fixed costs. This is total PS, so there is also a redistribution effect among firms where incumbents are losing in favour of new entrants. However, it is important to remember that the model specifies that fixed costs are lowered just enough to let in one more firm making zero profit so the PS still befalls the incumbents as an aggregate.

As can be seen below, the increase in Q is marginal, as is the rise in price. Quite intuitively, price is affected more when demand is more inelastic.

Price leadership structure						
	Elasticity	Baseline	Q up	Q down	C up	C down
Change in P	-1.2	-1.11%	-1.82%	-0.33%	-0.91%	-1.28%
	-0.9	-1.48%	-2.40%	-0.44%	-1.28%	-1.65%
	-0.6	-2.22%	-3.49%	-0.69%	-2.04%	-2.37%
Change in Q	-1.2	1.33%	1.86%	0.48%	1.33%	1.33%
	-0.9	1.33%	1.86%	0.48%	1.33%	1.33%
	-0.6	1.33%	1.86%	0.48%	1.33%	1.33%

Even market structure						
	Elasticity	Baseline	Q up	Q down	C up	C down
Change in P	-1.2	-2.22%	-2.88%	-1.49%	-1.81%	-2.57%
	-0.9	-2.96%	-3.79%	-2.02%	-2.55%	-3.31%
	-0.6	-4.44%	-5.52%	-3.14%	-4.06%	-4.75%
Change in Q	-1.2	2.67%	2.95%	2.22%	2.67%	2.67%
	-0.9	2.67%	2.95%	2.22%	2.67%	2.67%
	-0.6	2.67%	2.95%	2.22%	2.67%	2.67%

Table 4: changes in price and quantity associated with marginal entry.

6 Conclusion

It should be noted that some scenarios have larger welfare implications than others, depending on the assumed values of the parameters. However, the parameters affecting this are exogenous. Strictly speaking so is F ; but we assume that it also incorporates barriers to entry such as red tape that could be affected by policy making. It is true that policymakers have attempted to influence other exogenous variables such as demand and so future research may want to study these effects in more detail.

For the results in the model to hold true in the real world, all the assumptions would have to be undoubtedly true. Some, while approximations, are not necessarily all that off. However, the assumption of perfectly linear demand and homogenous goods are very strong and very unlikely to be true.

It is noteworthy that the effect of the scenario has a substantial impact on PWC. While this paper's focus is on the PWC from lowering fixed costs these implications are still important. As brought up earlier, the cost of construction has grown much faster than inflation and faster than costs in other industries. Likewise, the total output has varied a lot decade-to-decade and even year-to-year. This means that analysing only a static "screenshot" of the market may be misleading. From a policy perspective it is therefore relevant to analyse different scenarios as the market changes. It should of course be noted that this is a very simplistic model ignoring many relevant real world factors.

On one hand, the model is biased towards increased competition. On the other, the assumption that the difference in market share is wholly caused by a difference in marginal cost will heavily bias the model towards less entry the more asymmetric the market. As the Stockholm tenant-owned multi-dwelling (TOMD) market is highly concentrated and dominated by one firm the latter apparently outweighs the former. This has both theoretical and practical implications. The theoretical is the one alluded to above, that you could easily construct a market with a dominant firm such that the rise in welfare from increased competition becomes insignificant. The practical is that *if* asymmetric market shares are at least associated with asymmetries in efficiency, increased competition might not be very beneficial.

We also had a result on the contrary. When firms are sufficiently asymmetric the fixed cost required to deter entry grows small enough that the welfare change from a marginal increase in the number of firms becomes strictly positive, even accounting for the reduction in fixed cost. The way these two combine is that having one dominant firm will reduce the welfare gain from additional entry due to the reduction in overall efficiency while symmetry among the remaining firms reduced the welfare gain as it implies higher fixed cost. In essence, having one dominant firm and the remaining rather symmetric is a move towards a mixed Stackelberg-Cournot competition with a market leader that acts more like a monopolist and the remaining engaging in something akin to perfect competition. In the real world of course, fixed cost is an exogenous variable and so testing different scenarios whilst holding the fixed cost fixed would lead to different and more realistic results.

Future research may want to look at more advanced demand functions as well as including the rental market. This would require a more complex model where there is a price ceiling calculated as the total value of future rents. Another issue would be to use m^2 instead of number of apartments. The perhaps most interesting opening would be to estimate fixed costs and market price and better estimate marginal costs and then calculate the welfare loss directly. Also of interest would be to include learning effects as it could be true that the higher efficiency of incumbents are due to experience and so over time the welfare loss cause by decreased efficiency due to new entrants may subside.

7 Summary

The Stockholm TOMD market is inarguably an important market with large and far-reaching effects on the economy as a whole. Housing is undoubtedly a very complex market and even though this paper attempts to isolate a smaller part of it, the supply of a specific type of dwelling in a limited region, it is still based on strong assumption and simplifications. There are furthermore strong arguments against looking at part of a market in isolation as it can only provide a partial equilibrium. All of this aside, the data used consists of estimates and is in part lacking. This has been worked around using a set of 30 scenarios to produce predictions for possible variations from the baseline that is based on 2013 data.

The purpose of this paper was to show that lowering barriers to entry, modelled as fixed costs, would have substantial effects on welfare. With a change in welfare of around 1.5% the effects are not large, although it does translate to a lot in real terms due to the sheer size of the market. The most significant result however was that the change in fixed costs actually is larger than change in welfare so net fixed costs, welfare would fall had fixed costs been lowered. This is a direct implication of the model as market share depends on marginal costs and so a very asymmetric market implies that incumbents are vastly more efficient than new entrants.

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Appendices

Appendix A Output summaries

Price leadership structure

	Elasticity	Baseline	Q up	Q down	C up	C down
Elasticity	-1.2	-1.20	-1.02	-1.48	-1.47	-1.04
	-0.9	-0.90	-0.78	-1.09	-1.05	-0.81
	-0.6	-0.60	-0.53	-0.70	-0.65	-0.56
PWL	-1.2	2.25%	4.35%	4.25%	3.49%	2.25%
	-0.9	2.25%	0.86%	4.25%	2.25%	2.25%
	-0.6	2.25%	0.86%	4.25%	2.25%	2.25%
Change in PS	-1.2	0.88%	5.65%	12.26%	4.94%	0.88%
	-0.9	0.88%	-10.65%	12.26%	0.88%	0.88%
	-0.6	0.88%	-10.65%	12.26%	0.88%	0.88%
Change in CS	-1.2	3.02%	3.76%	-1.72%	2.68%	3.02%
	-0.9	3.02%	6.04%	-1.72%	3.02%	3.02%
	-0.6	3.02%	6.04%	-1.72%	3.02%	3.02%

Even market structure

	Elasticity	Baseline	Q up	Q down	C up	C down
Elasticity	-1.2	-1.20	-1.02	-1.49	-1.47	-1.04
	-0.9	-0.90	-0.78	-1.09	-1.05	-0.81
	-0.6	-0.60	-0.53	-0.70	-0.66	-0.56
PWL	-1.2	1.07%	1.86%	0.20%	1.07%	1.07%
	-0.9	1.07%	1.86%	0.20%	1.07%	1.07%
	-0.6	1.07%	1.86%	0.20%	1.07%	1.07%
Change in PS	-1.2	-8.30%	-9.58%	-6.47%	8.30%	-8.30%
	-0.9	-8.30%	-9.58%	-6.47%	8.30%	-8.30%
	-0.6	-8.30%	-9.58%	-6.47%	8.30%	-8.30%
Change in CS	-1.2	5.40%	5.98%	4.48%	5.40%	5.40%
	-0.9	5.40%	5.98%	4.48%	5.40%	5.40%
	-0.6	5.40%	5.98%	4.48%	5.40%	5.40%

Appendix B Data output

Appendix B.1 Cost leadership structure with -1.2 baseline elasticity

Structure		Cost leadership structure			
Scenario	Baseline	Q up	Q down	C up	C down
Elasticity	-1.20	-1.02	-1.48	-1.47	-1.04
N	4	4	4	4	4
a	8 732 595	9 877 598	7 587 584	8 732 600	8 684 201
b	611	611	611	611	607
F	371 532 339	814 649 940	100 165 411	294 837 812	427 696 480
P	4 763 232	4 992 233	4 534 230	5 196 587	4 437 189
Q	6 500	8 000	5 000	5 790	6 993
PS	7 244 880 615	8 792 924 832	5 696 825 824	5 749 330 076	8 340 081 455
CS	12 900 428 446	19 541 461 417	7 633 387 168	10 237 414 560	14 850 572 346
Welfare	20 145 309 061	28 334 386 249	13 330 212 992	15 986 744 636	23 190 653 801
c average	3 420 000	3 485 792	3 314 733	4 000 000	3 000 000
Market share					
Firm 1	50%	45%	57%	50%	50%
Firm 2	22%	23%	21%	22%	22%
Firm 3	16%	18%	13%	16%	16%
Firm 4	12%	14%	8%	12%	12%
One more entry					
F	114 670 475	339 696 161	8 916 215	90 999 493	132 005 084
N	5	5	5	5	5
P	4 710 307	4 901 141	4 519 472	5 149 440	4 380 562
Q	6 587	8 149	5 024	5 868	7 087
PS	7 308 451 060	9 289 624 003	6 394 994 659	6 033 247 067	8 413 261 808
CS	13 290 343 896	20 276 989 005	7 502 279 478	10 512 232 652	15 299 430 883
Welfare	20 598 794 956	29 566 613 008	13 897 274 136	16 545 479 719	23 712 692 691
Change in PS	63 570 445	496 699 170	698 168 834	283 916 991	73 180 354
Change in CS	389 915 450	735 527 589	-131 107 690	274 818 091	448 858 536
Change in welfare	453 485 895	1 232 226 759	567 061 144	558 735 083	522 038 890
PWL	2.2511%	4.3489%	4.2540%	3.4950%	2.2511%
F as share of PWL	201.280%	126.610%	62.794%	129.642%	201.280%
Change in F total	-912 776 982	-1 560 118 953	-356 080 567	-724 353 781	-1 050 760 501
% change F total	-61.4198%	-47.8770%	-88.8731%	-61.4197%	-61.4198%
elasticity	-1.17105	-0.98487	-1.47305	-1.43712	-1.01787
c average	3 469 011	3 552 769	3 333 156	4 043 660	3 052 439
Change in PS	0.88%	5.65%	12.26%	4.94%	0.88%
Change in CS	3.02%	3.76%	-1.72%	2.68%	3.02%
Change in P	-1.11%	-1.82%	-0.33%	-0.91%	-1.28%
Change in Q	1.33%	1.86%	0.48%	1.33%	1.33%
Change in welfare	2.25%	4.35%	4.25%	3.49%	2.25%
Market share					
Firm 1	48%	43%	57%	48%	48%
Firm 2	20%	20%	21%	20%	20%
Firm 3	14%	16%	13%	14%	14%
Firm 4	11%	12%	8%	11%	11%
Firm 5	7%	9%	2%	7%	7%

Appendix B.2 Cost leadership structure with -0.9 baseline elasticity

Structure		Cost leadership structure				
Scenario	Baseline	Q up	Q down	C up	C down	
Elasticity	-0.90	-0.78	-1.09	-1.05	-0.81	
N	4	4	4	4	4	
a	11 570 519	13 327 193	9 813 871	11 570 521	11 570 512	
b	937	937	937	937	937	
F	570 000 403	1 249 829 118	153 672 698	491 763 256	630 258 003	
P	5 480 771	5 832 106	5 129 442	5 914 125	5 166 962	
Q	6 500	8 000	5 000	6 037	6 835	
PS	11 115 007 854	13 490 031 668	8 740 020 517	9 589 383 655	12 290 031 251	
CS	19 791 680 651	29 980 346 518	11 711 072 859	17 075 113 255	21 883 958 666	
Welfare	30 906 688 505	43 470 378 186	20 451 093 376	26 664 496 910	34 173 989 917	
c average	3 420 000	3 520 938	3 258 500	4 000 000	3 000 000	
Market share						
Firm 1	50%	45%	57%	50%	50%	
Firm 2	22%	23%	21%	22%	22%	
Firm 3	16%	18%	13%	16%	16%	
Firm 4	12%	14%	8%	12%	12%	
One more entry						
F	175 926 050	521 159 007	13 679 183	151 778 780	194 524 072	
N	5	5	5	5	5	
P	5 399 574	5 692 354	5 106 800	5 838 706	5 081 581	
Q	6 587	8 149	5 024	6 118	6 926	
PS	11 212 536 859	12 053 571 930	9 811 145 459	9 673 526 060	12 397 870 581	
CS	20 389 884 199	31 791 676 382	11 509 928 046	17 591 208 533	22 545 401 295	
Welfare	31 602 421 059	43 845 248 312	21 321 073 505	27 264 734 594	34 943 271 875	
Change in PS	97 529 006	-1 436 459 739	1 071 124 941	84 142 405	107 839 329	
Change in CS	598 203 548	1 811 329 864	-201 144 812	516 095 279	661 442 629	
Change in welfare	695 732 554	374 870 126	869 980 129	600 237 684	769 281 958	
PWL	2.2511%	0.8624%	4.2540%	2.2511%	2.2511%	
F as share of PWL	201.280%	638.494%	62.794%	201.280%	201.280%	
Change in F total	-1 400 371 361	-2 393 521 436	-546 294 878	-1 208 159 124	-1 548 411 650	
% change F total	-61.4198%	-47.8770%	-88.8731%	-61.4198%	-61.4198%	
elasticity	-0.87500	-0.74558	-1.08492	-1.01865	-0.78312	
c average	3 495 192	3 623 694	3 286 764	4 069 842	3 079 067	
Change in PS	0.88%	-10.65%	12.26%	0.88%	0.88%	
Change in CS	3.02%	6.04%	-1.72%	3.02%	3.02%	
Change in P	-1.48%	-2.40%	-0.44%	-1.28%	-1.65%	
Change in Q	1.33%	1.86%	0.48%	1.33%	1.33%	
Change in welfare	2.25%	0.86%	4.25%	2.25%	2.25%	
Market share						
Firm 1	48%	43%	57%	48%	48%	
Firm 2	20%	20%	21%	20%	20%	
Firm 3	14%	16%	13%	14%	14%	
Firm 4	11%	12%	8%	11%	11%	
Firm 5	7%	9%	2%	7%	7%	

Appendix B.3 Cost leadership structure with -0.6 baseline elasticity

Structure		Cost leadership structure				
Scenario	Baseline	Q up	Q down	C up	C down	
Elasticity	-0.60	-0.53	-0.70	-0.65	-0.56	
N	4	4	4	4	4	
a	20 917 451	24 688 688	17 146 315	20 917 457	20 917 451	
b	2 011	2 011	2 011	2 011	2 011	
F	1 223 671 086	2 683 123 896	329 903 410	1 143 891 922	1 283 120 903	
P	7 844 042	8 598 292	7 089 817	8 277 399	7 530 234	
Q	6 500	8 000	5 000	6 285	6 656	
PS	23 861 586 174	28 960 300 108	18 763 011 337	22 305 891 675	25 020 857 612	
CS	42 488 579 374	64 361 585 925	25 141 244 502	39 718 468 622	44 552 809 129	
Welfare	66 350 165 547	93 321 886 033	43 904 255 839	62 024 360 297	69 573 666 741	
c average	3 420 000	3 636 692	3 073 292	4 000 003	3 000 000	
Market share						
Firm 1	50%	45%	58%	50%	50%	
Firm 2	22%	23%	21%	22%	22%	
Firm 3	16%	18%	13%	16%	16%	
Firm 4	12%	14%	8%	12%	12%	
One more entry						
F	377 675 921	1 118 820 298	29 366 370	353 053 059	396 024 970	
N	5	5	5	5	5	
P	7 669 729	8 298 273	7 041 211	8 108 865	7 351 738	
Q	6 587	8 149	5 024	6 368	6 745	
PS	24 070 962 829	25 876 518 972	21 062 494 423	22 501 615 551	25 240 403 948	
CS	43 772 796 686	68 250 135 465	24 709 428 307	40 918 959 331	45 899 417 789	
Welfare	67 843 759 515	94 126 654 437	45 771 922 730	63 420 574 883	71 139 821 737	
Change in PS	209 376 656	-3 083 781 136	2 299 483 085	195 723 877	219 546 336	
Change in CS	1 284 217 312	3 888 549 540	-431 816 195	1 200 490 709	1 346 608 659	
Change in welfare	1 493 593 968	804 768 404	1 867 666 891	1 396 214 586	1 566 154 996	
PWL	2.2511%	0.8624%	4.2540%	2.2511%	2.2511%	
F as share of PWL	201.280%	638.494%	62.794%	201.280%	201.280%	
Change in F total	-3 006 304 739	-5 138 394 093	-1 172 781 791	-2 810 302 395	-3 152 358 761	
% change F total	-61.4198%	-47.8770%	-88.8731%	-61.4198%	-61.4198%	
elasticity	-0.57895	-0.50629	-0.69680	-0.63308	-0.54194	
c average	3 581 422	3 857 288	3 133 971	4 156 075	3 165 297	
Change in PS	0.88%	-10.65%	12.26%	0.88%	0.88%	
Change in CS	3.02%	6.04%	-1.72%	3.02%	3.02%	
Change in P	-2.22%	-3.49%	-0.69%	-2.04%	-2.37%	
Change in Q	1.33%	1.86%	0.48%	1.33%	1.33%	
Change in welfare	2.25%	0.86%	4.25%	2.25%	2.25%	
Market share						
Firm 1	48%	43%	57%	48%	48%	
Firm 2	20%	20%	21%	20%	20%	
Firm 3	14%	16%	13%	14%	14%	
Firm 4	11%	12%	8%	11%	11%	
Firm 5	7%	9%	2%	7%	7%	

Appendix B.4 Even market structure with -1.2 baseline elasticity

Structure		Even market structure				
Scenario	Baseline	Q up	Q down	C up	C down	
Elasticity	-1.20	-1.02	-1.49	-1.47	-1.04	
N	4	4	4	4	4	
a	8 684 211	9 822 875	7 545 546	8 684 208	8 684 201	
b	607	607	607	607	607	
F	656 842 105	1 215 928 145	268 557 663	520 075 477	765 833 291	
P	4 736 842	4 964 575	4 509 109	5 171 757	4 421 900	
Q	6 500	8 000	5 000	5 784	7 019	
PS	5 932 105 257	6 997 894 973	4 866 315 265	4 696 928 547	6 916 435 098	
CS	12 828 947 360	19 433 217 789	7 591 092 300	10 157 723 132	14 957 684 889	
Welfare	18 761 052 617	26 431 112 762	12 457 407 565	14 854 651 679	21 874 119 987	
c average	3 420 000	3 481 875	3 321 000	4 000 003	2 999 997	
Market share						
Firm 1	50%	45%	58%	50%	50%	
Firm 2	18%	19%	16%	18%	18%	
Firm 3	16%	18%	13%	16%	16%	
Firm 4	16%	18%	13%	16%	16%	
One more entry						
F	456 140 350	844 394 544	186 498 377	361 164 212	531 828 674	
N	5	5	5	5	5	
P	4 631 579	4 821 356	4 441 802	5 078 091	4 308 239	
Q	6 673	8 236	5 111	5 938	7 206	
PS	5 439 473 682	6 327 631 469	4 551 315 301	4 306 868 330	6 342 059 947	
CS	13 522 280 693	20 595 856 197	7 931 360 693	10 706 692 193	15 766 064 565	
Welfare	18 961 754 375	26 923 487 666	12 482 675 994	15 013 560 523	22 108 124 513	
Change in PS	-492 631 575	-670 263 504	-314 999 964	-390 060 217	-574 375 151	
Change in CS	693 333 333	1 162 638 408	340 268 393	548 969 061	808 379 676	
Change in welfare	200 701 758	492 374 904	25 268 428	158 908 844	234 004 526	
PWL	1.0698%	1.8629%	0.2028%	1.0698%	1.0698%	
F as share of PWL	172.727%	130.336%	560.932%	172.728%	172.727%	
Change in F total	-346 666 670	-641 739 858	-141 738 768	-274 480 849	-404 189 793	
% change F total	-13.1944%	-13.1944%	-13.1944%	-13.1943%	-13.1944%	
elasticity	-1.14286	-0.96398	-1.43111	-1.40819	-0.98452	
c average	3 474 710	3 540 418	3 368 824	4 048 685	3 059 071	
Change in PS	-8.30%	-9.58%	-6.47%	-8.30%	-8.30%	
Change in CS	5.40%	5.98%	4.48%	5.40%	5.40%	
Change in P	-2.22%	-2.88%	-1.49%	-1.81%	-2.57%	
Change in Q	2.67%	2.95%	2.22%	2.67%	2.67%	
Change in welfare	1.07%	1.86%	0.20%	1.07%	1.07%	
Market share						
Firm 1	46%	41%	54%	46%	46%	
Firm 2	15%	16%	13%	15%	15%	
Firm 3	13%	14%	11%	13%	13%	
Firm 4	13%	14%	11%	13%	13%	
Firm 5	13%	14%	11%	13%	13%	

Appendix B.5 Even market structure with -0.9 baseline elasticity

Structure		Even market structure				
Scenario	Baseline	Q up	Q down	C up	C down	
Elasticity	-0.90	-0.78	-1.09	-1.05	-0.81	
N	4	4	4	4	4	
a	11 472 462	13 214 249	9 730 709	11 472 467	11 472 456	
b	929	929	929	929	929	
F	1 004 746 362	1 859 958 890	410 803 641	865 219 764	1 112 289 450	
P	5 434 323	5 782 681	5 085 974	5 869 240	5 119 385	
Q	6 500	8 000	5 000	6 032	6 839	
PS	9 074 115 586	10 704 430 458	7 443 839 081	7 814 015 489	10 045 364 245	
CS	19 623 952 392	29 726 271 756	11 611 839 030	16 898 823 184	21 724 403 486	
Welfare	28 698 067 977	40 430 702 214	19 055 678 111	24 712 838 673	31 769 767 731	
c average	3 420 000	3 514 648	3 268 563	4 000 003	3 000 000	
Market share						
Firm 1	50%	45%	57%	50%	50%	
Firm 2	18%	19%	16%	18%	18%	
Firm 3	16%	18%	13%	16%	16%	
Firm 4	16%	18%	13%	16%	16%	
One more entry						
F	697 740 529	1 291 638 117	285 280 306	600 847 093	772 423 229	
N	5	5	5	5	5	
P	5 273 306	5 563 605	4 983 015	5 719 821	4 949 969	
Q	6 673	8 236	5 111	6 193	7 021	
PS	8 320 555 814	9 679 152 771	6 961 994 210	7 165 100 498	9 211 147 150	
CS	20 684 517 996	31 504 715 958	12 132 336 167	17 812 110 731	22 898 486 799	
Welfare	29 005 073 810	41 183 868 728	19 094 330 377	24 977 211 229	32 109 633 949	
Change in PS	-753 559 772	-1 025 277 687	-481 844 870	-648 914 990	-834 217 095	
Change in CS	1 060 565 605	1 778 444 201	520 497 137	913 287 547	1 174 083 313	
Change in welfare	307 005 833	753 166 514	38 652 266	264 372 556	339 866 218	
PWL	1.0698%	1.8629%	0.2028%	1.0698%	1.0698%	
F as share of PWL	172.727%	130.336%	560.932%	172.727%	172.727%	
Change in F total	-530 282 802	-981 644 974	-216 813 035	-456 643 591	-587 041 656	
% change F total	-13.1944%	-13.1944%	-13.1944%	-13.1944%	-13.1944%	
elasticity	-0.85065	-0.72721	-1.04957	-0.99429	-0.75891	
c average	3 503 687	3 604 199	3 341 718	4 077 663	3 088 052	
Change in PS	-8.30%	-9.58%	-6.47%	-8.30%	-8.30%	
Change in CS	5.40%	5.98%	4.48%	5.40%	5.40%	
Change in P	-2.96%	-3.79%	-2.02%	-2.55%	-3.31%	
Change in Q	2.67%	2.95%	2.22%	2.67%	2.67%	
Change in welfare	1.07%	1.86%	0.20%	1.07%	1.07%	
Market share						
Firm 1	46%	41%	54%	46%	46%	
Firm 2	15%	16%	13%	15%	15%	
Firm 3	13%	14%	11%	13%	13%	
Firm 4	13%	14%	11%	13%	13%	
Firm 5	13%	14%	11%	13%	13%	

Appendix B.6 Even market structure with -0.6 baseline elasticity

Structure		Even market structure				
Scenario	Baseline	Q up	Q down	C up	C down	
Elasticity	-0.60	-0.53	-0.70	-0.66	-0.56	
N	4	4	4	4	4	
a	20 540 566	24 243 799	16 837 320	20 540 565	20 540 561	
b	1 975	1 975	1 975	1 975	1 975	
F	2 136 219 381	3 954 505 289	873 417 486	1 993 931 120	2 242 315 470	
P	7 702 709	8 443 356	6 962 059	8 137 624	7 387 771	
Q	6 500	8 000	5 000	6 280	6 659	
PS	19 292 731 282	22 758 958 334	15 826 488 775	18 007 689 818	20 250 911 607	
CS	41 723 034 780	63 201 772 657	24 688 153 262	38 943 966 588	43 795 224 048	
Welfare	61 015 766 063	85 960 730 991	40 514 642 037	56 951 656 406	64 046 135 655	
c average	3 420 000	3 621 233	3 098 027	4 000 003	3 000 000	
Market share						
Firm 1	50%	45%	57%	50%	50%	
Firm 2	18%	19%	16%	18%	18%	
Firm 3	16%	18%	13%	16%	16%	
Firm 4	16%	18%	13%	16%	16%	
One more entry						
F	1 483 485 681	2 746 184 228	606 539 921	1 384 674 388	1 557 163 521	
N	5	5	5	5	5	
P	7 360 366	7 977 572	6 743 157	7 806 879	7 037 030	
Q	6 673	8 236	5 111	6 447	6 837	
PS	17 690 566 747	20 579 089 697	14 802 029 167	16 512 241 515	18 569 175 004	
CS	43 977 933 016	66 982 967 521	25 794 792 188	41 048 671 643	46 162 112 601	
Welfare	61 668 499 763	87 562 057 219	40 596 821 355	57 560 913 158	64 731 287 605	
Change in PS	-1 602 164 535	-2 179 868 636	-1 024 459 608	-1 495 448 304	-1 681 736 603	
Change in CS	2 254 898 235	3 781 194 864	1 106 638 927	2 104 705 055	2 366 888 553	
Change in welfare	652 733 700	1 601 326 228	82 179 318	609 256 752	685 151 950	
PWL	1.0698%	1.8629%	0.2028%	1.0698%	1.0698%	
F as share of PWL	172.727%	130.336%	560.932%	172.727%	172.727%	
Change in F total	-1 127 449 118	-2 087 100 017	-460 970 340	-1 052 352 540	-1 183 444 277	
% change F total	-13.1944%	-13.1944%	-13.1944%	-13.1944%	-13.1944%	
elasticity	-0.55844	-0.49044	-0.66803	-0.61309	-0.52113	
c average	3 597 929	3 811 629	3 253 564	4 171 905	3 182 294	
Change in PS	-8.30%	-9.58%	-6.47%	-8.30%	-8.30%	
Change in CS	5.40%	5.98%	4.48%	5.40%	5.40%	
Change in P	-4.44%	-5.52%	-3.14%	-4.06%	-4.75%	
Change in Q	2.67%	2.95%	2.22%	2.67%	2.67%	
Change in welfare	1.07%	1.86%	0.20%	1.07%	1.07%	
Market share						
Firm 1	46%	41%	54%	46%	46%	
Firm 2	15%	16%	13%	15%	15%	
Firm 3	13%	14%	11%	13%	13%	
Firm 4	13%	14%	11%	13%	13%	
Firm 5	13%	14%	11%	13%	13%	

Appendix C The Cournot model for heterogeneous firms

$$P = a - bQ$$

$$Q = \sum q_i$$

$$\pi_i = \text{Total revenue} - \text{Total cost} = p * q_i - c_i * q_i - F$$

P = price

Q = Total quantity

a = constant

b = constant

F = Entry cost

q_i = quantity produced by firm i

c_i = marginal cost of firm i

Calculating Q

$$\max \pi_i(q_i, q_{-i}) = p * q_i - c * q_i - F = (a - bQ)q_i - c_i q_i - F$$

$$= \left(a - b \sum_{i=1}^N q_i \right) q_i - c_i q_i - F = \left(a - b q_i - b \sum_{i \neq j} q_j \right) q_i - c_i q_i - F$$

$$\pi'_i = (a - b q_i - b \sum_{i \neq j} q_j + (-b) q_i - c_i) = a - 2b q_i - b \sum_{i \neq j} q_j - c_i$$

$$= a - b q_i - b \sum_{i=1}^N q_i - c_i = 0 \rightarrow \text{When profit is maximized}$$

$$c_i = a - b q_i - bQ$$

$$\text{Summing over all firms} \rightarrow \sum_{i=1}^N c_i = Na - b \sum_{i=1}^N q_i - bNQ = Na - bQ - bNQ$$

$$bQ + bNQ = Na - \sum_{i=1}^N c_i \rightarrow Q(b + bN) = Na - \sum_{i=1}^N c_i \rightarrow Q = \frac{Na - \sum_{i=1}^N c_i}{b + bN}$$

$$= \frac{Na}{b(N + 1)} - \frac{\sum c_i}{b(N + 1)}$$

Calculating Price

$$\begin{aligned} P = a - bQ &= a - b \left(\frac{Na}{b(N+1)} - \frac{\sum_{i=1}^N c_i}{b(N+1)} \right) = a - \frac{Na}{N+1} + \frac{\sum c_i}{N+1} \\ &= \frac{a(N+1)}{N+1} - \frac{Na}{N+1} + \frac{\sum c_i}{N+1} = \frac{Na + a - Na}{N+1} + \frac{\sum c_i}{N+1} \\ &= \left[\frac{a}{N+1} + \frac{\sum c_i}{N+1} \right] \end{aligned}$$

Calculating q_i

$$\pi_i = pq_i - c_i q_i = \left(a - b \sum_{i=1}^N q_i \right) q_i - c_i q_i = a q_i - b q_i \sum_{i \neq j}^N q_j - b q_i^2 - c_i q_i$$

$$\text{FOC: } \pi'_i = a - 2b q_i - b \sum_{i \neq j}^N q_j - c_i$$

$$\text{Firm best response, BR: } q_i = \frac{1}{2b} \left(a - c_i - b \sum_{i \neq j}^N q_j \right)$$

$$2b q_i = a - b \sum_{i \neq j}^N q_j - c_i$$

$$\text{Summing over all firms yields: } 2b \sum_{i=1}^N q_i = Na - \sum_{i=1}^N c_i - (N-1)b \sum_{i=1}^N q_i$$

$$= Na - \sum_{i=1}^N c_i - Nb \sum_{i=1}^N q_i + b \sum_{i=1}^N q_i$$

$$b \sum_{i=1}^N q_i = Na - \sum_{i=1}^N c_i - Nb \sum_{i=1}^N q_i \rightarrow b(N+1) \sum_{i=1}^N q_i = Na - \sum_{i=1}^N c_i \rightarrow b \sum_{i=1}^N q_i$$

$$= \frac{N}{N+1} a - \frac{1}{N+1} \sum c_i \rightarrow b \sum_{i \neq j}^N q_j + b q_i = \frac{N}{N+1} a - \frac{1}{N+1} \sum c_i$$

$$b \sum_{i \neq j}^N q_j = \frac{N}{N+1} a - \frac{1}{N+1} \sum c_i - b q_i$$

$$\text{FOC: } 2b q_i = a - c_i - b \sum_{i \neq j}^N q_j \text{ combined with the above } \rightarrow$$

$$\begin{aligned}
2bq_i &= a - c_i - \left(\frac{N}{N+1}a - \frac{1}{N+1} \sum c_i - bq_i \right) \\
&= \frac{(N+1)a}{N+1} - \frac{Na}{N+1} + \frac{1}{N+1}c_i - c_i + \frac{1}{N+1} \sum_{i \neq j}^N c_j + bq_i \\
bq_i &= \frac{1}{N+1}a + \frac{c_i - (N+1)c_i}{N+1} + \frac{1}{N+1} \sum_{i \neq j}^N c_j \\
bq_i &= \frac{1}{N+1}a - \frac{N}{N+1}c_i + \frac{1}{N+1} \sum_{i \neq j}^N c_j = \frac{1}{N+1} \left(a - Nc_i - \sum_{i \neq j}^N c_j \right) \\
&\rightarrow \left[q_i = \frac{1}{b(N+1)} \left(a + \sum_{i \neq j}^N c_j - Nc_i \right) \right]
\end{aligned}$$

$$\begin{aligned}
\pi_i &= pq_i - c_i q_i - F = (p_i - c_i)q_i - F = \left(\frac{a + \sum c_i}{N+1} - c_i \right) q_i - F \\
&= \left(\frac{a + \sum c_i - Nc_i - c_i}{N+1} \right) q_i - F = \left(\frac{a + \sum c_j + c_i - Nc_i - c_i}{N+1} \right) q_i - F \\
&= \left(\frac{a + \sum c_j - Nc_i}{N+1} \right) * \left(\frac{a + \sum c_j - Nc_i}{b(N+1)} \right) - F \\
&= \frac{(a + \sum c_j - Nc_i)^2}{b(N+1)^2} - F \\
&\rightarrow \pi_i = \frac{(a + \sum c_j - Nc_i)^2}{b(N+1)^2} - F
\end{aligned}$$

No one will enter the market if $\pi = 0$, meaning $F < \pi \rightarrow F$

$$\begin{aligned}
&= \frac{(a + \sum c_j - Nc_z)^2}{b(N+1)^2} \\
(N+1)^2 &= \frac{(a + \sum c_j - Nc_z)^2}{Fb} \rightarrow N = \frac{a + \sum c_j - Nc_z}{\sqrt{Fb}} - 1
\end{aligned}$$

where c_z is the marginal cost of the least productive firm

$$\begin{aligned}
q_z &= \frac{a + \sum c_j - Nc_z}{b \left(\frac{a + \sum c_j - Nc_z}{\sqrt{Fb}} \right)} = \frac{a + \sum c_j - Nc_z}{b(a + \sum c_j - Nc_z)} = \frac{\sqrt{Fb}(a + \sum c_j - Nc_z)}{b(a + \sum c_j - Nc_z)} = \frac{\sqrt{Fb}}{b} \\
q_z &= \frac{\sqrt{Fb}}{b} \quad \text{The quantity of the firm with zero profit}
\end{aligned}$$