STOCKHOLM SCHOOL OF ECONOMICS Department of Economics 5350 Master's thesis in economics Academic Year 2016-17

Robin who?

Using indivisible labor and systemic misapprehension to solve the Robin Hood paradox of the Meltzer and Richard model on government size.

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Abstract

To better understand why "history reveals a 'Robin Hood paradox,' in which redistribution from rich to poor is least present when and where it seems to be most needed." (Lindert, 2004, p.15), I make two modifications to the model of Meltzer and Richard (1981) on government size. Their model predicts that a lower productivity of the median voter, relative to the mean, leads to higher taxation, the opposite to what is observed in empirical studies. First, I assume indivisible labor, a binary choice of employment instead of the original assumption of a continuous choice, and find that for plausible parameterization, this solves the Robin Hood paradox. Then, I examine the potential effects on government size of voters that overestimate others productivity, using evidence from behavioral research (Karadja et al., 2014). Systemic overvaluation of others productivity increases the demand for redistribution, thus if the misapprehension is larger where the productivity distribution is more unequal, the Robin Hood paradox is solved.

Keywords: Political economy, Robin Hood paradox, Meltzer-Richard Model, income redistribution, voting equilibrium **JEL:** D72, F50, H2, H31.

Supervisor: Professor Jörgen Weibull Date submitted: December 11, 2016. Date examined: December 20, 2016. Discussants: Carl Larsson Examiner: Maria Perrotta Berlin

Acknowledgments

First and foremost I offer my sincerest gratitude to my supervisor, Professor Jörgen Weibull, who have supported me throughout the process of turning an abstract idea into this thesis. Without his guidance and persistent help, this would not have been possible.

I am indebted to my colleagues at the research institute Ratio for their persistent support and for enabling my participation at a conference of the European Public Choice Society last spring where I originally got this idea.

I would like to thank Chloé Le Coq and the participants at the midterm seminar for valuable input.

Finally, I want to acknowledge Maria Perrotta Berlin and Carl Larsson for valuable comments on the final seminar.

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1 Introduction

Building on earlier research by Romer (1975) and Roberts (1977), Meltzer and Richard (1981), from now on 'MR', develop a model on government size. Their seminal contribution became the 'workhorse' of political economy. Assuming rational individuals in a competitive market who maximize utility from own consumption and leisure and a government who balances its budget and use flat taxes to pay an equal cash transfer to everyone, they model the dynamics of taxes and redistribution on incentives to work and how this, in turn, changes the government budget. Their conclusion is that the further the median voter is from the mean productivity, the higher taxes there will be.

However, "history reveals a 'Robin Hood paradox,' in which redistribution from rich to poor is least present when and where it seems to be most needed." (Lindert, 2004, p.15). More specifically, among OECD-countries, more economically equal economies, such as the Nordics, redistribute more than to more unequal economies, such as the US. In an attempt to better understand this paradox I develop two thought experiments, making two, to my knowledge, new modifications of the model assumptions on the labor choice and the perception of others' productivity. More specifically, I hypothesize that small revisions of the original model can explain the Robin Hood paradox.

Research question: Can the Meltzer and Richard model more accurately explain relative cross-country variation in taxation by changing the underlying assumptions of a continuous choice of labor and/or by introducing a systemic misapprehension of others' productivity in the model?

In sections 2 I present a literature review on explanations for the Robin Hood paradox, and my contribution to the field.

In section 3 I present Meltzer and Richard's model (1981) on government size, from now on the 'MR-model'. To provide an intuitive illustration of the dynamics, I use the (logarithmic) Cobb-Douglas utility function and a mean-preserving spread of the log-normal distribution for productivity.

In section 4 I present my first model modification and show how a binary choice of employment can explain the Robin Hood paradox. This restriction has an effect on both the productivity limit of entering the labor market or not, and on average income and consequently on preferences for taxation.

In section 5 I instead assume a systemic overvaluation of others' productivity, using an observation from behavioral research (Karadja et al., 2014). Perceived redistribution will be higher, and if a more equal economy displays larger misapprehension, predicted taxes are *lower* than in the unequal economy, providing a second alteration of the original model which explains the Robin Hood paradox.

I discuss the results and their implications for the Robin Hood paradox in section 6. The interested reader will find more detailed analysis of a few of the model- and

distribution characteristics in the appendix, as well as some robustness checks.

2 Literature review

In this section, I first present the current state of knowledge about the two central questions in understanding the Robin Hood paradox: What are the current theories for explaining the failure of the MR-model to predict the actual levels of taxes, and is the relative income of the median to mean voter the relevant measure of inequality? Then I justify my modifications of the original model.

The influential paper by Meltzer and Richard (1981), who relied on earlier theoretical work by Romer (1975) and Roberts (1977), have become the 'workhorse' model of political economy and have been subject to extensive debate and empirical investigation. The conclusion of their model, that rising inequality increase support for redistributive policies, have become the standard prediction of median-voter theory.¹ Kenworthy and Pontusson (2005) and Milanovic (2000) show that patterns of within-country variation broadly conform to the core prediction of the model. Many others, however, have been unable to find empirical support for these predictions, rather, the opposite relationship is often confirmed. To my knowledge, this puzzle became known as "the Robin Hood paradox" after the following observation by Lindert (2004, p.15) "history reveals a 'Robin Hood paradox', in which redistribution from rich to poor is least present when and where it seems to be most needed." Recent literature in comparative political economy identifies and seeks to resolve this paradox. A recent report by the United Nations Development Programme (2013, p. 197)[footnote is my own] explains that

There are essentially two ways to understand the failure of the Meltzer-Richard model to accurately predict the actual levels of inequality reduction pursued by governments. The first set of explanations - which could be called 'demand-side' theories - points to a number of factors, including prospects of upward mobility (Benabou and Ok, 2001) and a belief in the fairness of current distributional outcomes (Bénabou and Tirole, 2006), which may reduce the demand for equalizing policies among those who would stand to gain from them. Demand-side explanations also note that those who would stand to gain from greater equality often are less able to engage in collective action to ensure that their demand for inequality reduction is fully reflected in the policy agenda (for instance, Cleaver, 2005).² Another body of work - consisting of what could be described as 'supply-side' theories - analyses specific features of the political system, such as clientelism or identity politics, that enable the anti-equalization minority, when in power, to ignore, circumvent or neutralize the demand for inequality reduction (see, for instance, Lizzeri and Persico (2001) and Robinson and Verdier (2002) on clientelism, Khemani (2013) on vote-buying and patterns of service provision, and Roemer (1998)

¹See Congleton (2004) for a comprehensive review of the properties, implications and problems of, and support for, the median voter model.

²As pointed out by Benabou (2000) and emphasized by Stiglitz (2012), political power is not necessarily more evenly distributed than economic power, if the rich have more political influence than the poor. See also Larcinese (2007) on voter turnout explanations, the rich are more likely to vote.

and Chandra (2007) on identity politics).

Other important extensions to the MR-model are multidimensional policy spaces, see Iversen (2006) for a comprehensive review of these models, and dynamic versions. Alesina and Rodrik (1991), Persson and Tabellini (1994) and Meltzer and Richard (2015) and many others have analyzed simple dynamic versions of the MR model, in which higher redistributive income taxation hurts the incentives to invest in human or physical capital and therefore economic growth. Recent work by Pontusson (2013) show that, across OECD countries, levels of earnings inequality and redistribution are associated with levels of unionization in the manner posited by power resources theory, and thus solve the Robin Hood paradox. One alternative to the power resource theory, arguing that partisanship and union power are endogenous to more fundamental differences in the organization of capitalist democracies, is developed by Iversen and Soskice (2009).

Regarding inequality and demand for redistribution, recent work by Scheve and Stasavage (2016) presents a historical review of the volatility in tax rates for the rich, using data from 20 countries over two centuries. They provide a summary of this hotly debated topic, and some explanations for the shifting popular demand for tax progressivity. The issue of inequality presents many problems, not least in measuring and comparing across time, groups and countries. The Robin Hood paradox concerns the conclusion that higher inequality, measured by mean to median income, should increase political equilibrium tax rate. There are many other income inequality metrics, the Gini index being one of the most popular, and evaluating their respective benefits and drawbacks is a field on its own, see ex. De Maio (2007) for an overview. Even the underlying price indices may be questioned, see for example Almås and Sorensen (2012) who find a systematic bias in the relative wealth of a country to the estimation of income. However, the ratio used by MR has the benefit of providing an intuitive method of relating the median voter theorem to the incentive for redistribution.

2.1 On my contribution

In the business cycle literature, models with discrete labor supply choice are better at matching the variation in aggregate hours (or employment) over the business cycle than a model with a continuous hours choice. Two key references are Hansen (1985) and Rogerson (1988). Worth noting is also that a binary choice of labor is sometimes assumed in theoretical research, see for example Lindbeck and Weibull (1987). And, as the title of Prescott's paper "Why do Americans work so much more than Europeans?" (2004) observes; the hours spent on labor vary a lot across OECD-countries, and we don't know why. While he suggests that this can be explained by tax rates, this doesn't explain why at a given moment in time workers vote for the observed tax increases. Also, before the 1970's, the decade he uses as a benchmark for comparing labor development across the G-7 countries, we saw a simultaneous increase of taxes and labor market participation, as women entered the workforce.³ Further, an increasing problem in the OECD countries is the growing difficulty to find low-productivity jobs, explained by

³See for example Hakim (2002) and Ferber (1982).

Blanchflower et al. (1993, p.1) by technological changes that increase the demand for workers able to learn at least cost (Berman et al., 1994; Krueger, 1993; Mincer, 1991), the transfer of jobs requiring relatively routinized tasks to low-wage countries (Reich, 1991), and the declining influence of unions (Freeman, 1993), the erosion of the real value of the minimum wage (DiNardo et al., 1995), and changes in pay-setting norms (Mitchell, 1989).

Regardless of the underlying causality in changes of labor market characteristic and changing demand for taxes, be it globalization, technological development, social norms, economic shocks, urbanization, unionization and labor market rigidities, any such development that can be seen as exogenous to the individual will have implications for her choice to enter or exit the workforce. Therefore, I develop a model, in section 4, where there is some exogenous hourly requirement for employment. To my knowledge, the implications of a binary choice of labor on the predictions of the MR-model has not been investigated before.

Building on new findings of national differences in individual perception of relative productivity, I develop a second model modification where voters misapprehend their relative position in the distribution of productivity in section 5. While Chambers et al. (2014) finds, using US data, that individuals tend to underestimate average income, and Roth et al. (2016) that Americans tend to underestimate their relative income, Karadja et al. (2014) finds that 86 percent of their sample of Swedes *overestimate* others' income. This may have important implications for the demand for taxes and redistribution in an economy, and to my knowledge, I am the first to examine the changes to the MR-model when there is an exogenous source of misapprehension.

3 The Meltzer and Richard model

In this section, I present the model developed by Meltzer and Richard (1981). I use their notation. To make the important characteristics of the model clear, I will exemplify their general framework by introducing a specific, rather than a general, utility function and specify a productivity distribution.

Key assumptions in the MR-model are: the budget is balanced.⁴ The size of the government is measured by the share of income redistributed by government, in cash and services, as is determined by the rational choices of utility maximizing individuals who are fully informed about the state of the economy and the consequences of taxation and

⁴All variables are real. There is no inflation. Budget balance means that redistribution uses real resources. Public goods are neglected.

income redistribution.⁵ Voters do not suffer from "fiscal illusion" ⁶ and are not myopic⁷. They know that the government must extract resources to pay for redistribution.

They show that the size of government depends on the relation of mean income to the income of the decisive voter. With universal suffrage and majority rule, the median voter is the decisive voter, as shown by Roberts (1977).⁸ I will assume this voting rule throughout the thesis. Studies of the distribution of income show that the distribution is skewed to the right, so the median income lies below the mean income. The median voter will consequently have an incentive for redistribution of income, financed by taxes on incomes that are (relatively) high. The conclusion of the MR-model is that the demand for redistribution is increasing with the distance between the productivity of the mean to median income. Once we take account of incentives, there is a limit to the size of government, where increasing the tax rate decrease the net utility from working and increase leisure in the economy, thus creating a Laffer curve of budget-balancing combinations of redistribution and tax rates. To bring together the effect of incentives, the desire for redistribution, and the absence of fiscal illusion or myopia, they develop a general equilibrium model.

After elections, taxes and redistribution are exogenous to any individual in the economy, and thus the labor-leisure choice can be represented by a static model. Individuals who differ in productivity, and therefore earned income, choose their preferred combination of consumption and leisure. Not all individuals work, but those who do pay a portion of their income in taxes. The choice between labor and leisure, and the amount of earned income and taxes, depend on the tax rate and the size of transfer payments.

The tax rate and the amount of income redistributed depend on the voting rule and the distribution of income or, as they show, the underlying productivity. MR develop a rule for the general case, but for an intuitive representation of the key features of the model, I restrict the representation by using a logarithmic Cobb-Douglas utility function,⁹ a log-normal distribution of productivity and a majority voting rule where

⁵Ideally the size of government would be measured by the net burden imposed (or removed) by government programs.

⁶Fiscal illusion is a public choice theory of government expenditure, revolving around the proposition that the true costs and benefits of government may be consistently misconstrued by voters, the costs of government are seen as less expensive and the benefits as more beneficial. The origins of this argument can be traced back at least as far as J.R. McCullock (1975) and J.S. Mill (1884). For a discussion of the origin and empirical analysis of the five main hypotheses subsumed under the generic term fiscal illusion, see Dollery and Worthington (1996)

⁷Myopic voters are short-sighted, for example, they condition their decision to vote on the current, short-term, state of the economy, which is largely due to chance. Fun study on curing this by Lenz (2011). Policy myopia explains why rational voters allow politicians to bias public investments towards short-term goals. To vote myopically is to ignore the effect of current political decisions on future political outcomes, such as taking on debt to finance current welfare.

⁸Who showed that Black's Theorem (1958) held also when preferences for a flat tax schedule were not single-peaked.

⁹Meltzer and Richard (1983) used a Stone-Geary utility function to approximate the median voters' preferred tax rate, there is a two-page summary of the article in the appendix, chapter A.2 on page 34. The application of the Stone-Geary function leads under certain parameter values to prohibited parameter values, such as working more than 100 percent of the time, why I choose to use the logarith-

the median voter is the decisive voter.

3.1 The Economic Environment

The economy considered has relatively standard features. There are a large number of individuals. Each treats prices, wages and tax rates as givens, determined in the markets for goods and labor and by the political process, respectively. Differences in the choice of labor, leisure, and consumption and differences in wages arise solely because of differences in endowments which reflect differences in productivity. Consider a log-



Figure 1: A productivity distribution

normal distribution of productivity, illustrated in figure 1. As we will see, this is the situation of interest, where the median voter has a lower productivity than the mean individual, but a higher productivity than the employment cutoff, which is the lowest level of productivity at which people will have sufficient incentives to work. The exact position of the median voter in relation to the mean is a measure of the inequality of the population, the further away from the mean the more unequal it is.

The utility function is assumed to be a strictly concave function, u(c, l), for consumption, c, and leisure, l. Consumption and leisure are normal goods, and the marginal utility of consumption or leisure is infinite when the level of consumption or leisure is zero, respectively. There is no capital and no uncertainty. I use a (logarithmic) Cobb-Douglas utility function, which is consistent with the mentioned criteria, to exemplify their general model.

$$u(c,l) = \ln(c) + a\ln(l) \tag{1}$$

mic Cobb-Douglas utility function, which is a specialization of the Stone-Geary function, and instead consequently have favorable characteristics such as marginal utility of leisure or consumption of infinity at zero, and consequently reasonable values for the labor-leisure choice. I explain this further in chapter A.3 on page 35 in the appendix.

The individual's endowment consists of ability to produce, or productivity, and a unit of time that she allocates to labor, n, or leisure, l = 1 - n. Individual incomes reflect the differences in individual productivity and the use of a common, constant-returnsto-scale technology to produce consumption goods. An individual with productivity xearns pretax income, y, measured in units of consumption:

$$y(x) = nx \tag{2}$$

Tax revenues finance lump-sum redistribution of r units of consumption per capita. Individual productivity cannot be observed directly, so taxes are levied against earned income. The tax rate, t, is a constant fraction of earned income but a declining fraction of disposable income, i.e. a flat tax schedule.¹⁰ The fraction of income paid in taxes net of transfers, however, rises with income. There is no saving; consumption equals disposable income:

$$c(x) = (1-t)nx + r.$$
 (3)

Each individual is a price taker in the labor market, takes taxes, t, and transfer payments, r, as given and chooses n to maximize her utility. The maximization problem is:

$$\max_{n \in [0,1]} \ln((1-t)nx + r) + a\ln(1-n)$$
(4)

The first-order condition¹¹

$$\frac{du(c,1-n)}{dn} = \frac{(1-t)x}{(1-t)nx+r} - \frac{a}{1-n} = 0$$
(5)

determines the optimal labor choice, $n^*(r, t, x)$, for those who choose to work. The choice depends only on the size of the welfare payment, r, and the after-tax wage, (1 - t)x.

Some people subsist on welfare payments. From (5) we know that the employment cutoff, the productivity level at which n = 0 is the optimal choice is

$$x_0 = \frac{ar}{1-t}.\tag{6}$$

Individuals with productivity below x_0 subsist on welfare payments and choose not to work: $n^* = 0$ for $x \leq x_0$. The solution to the labor choice is,

$$n^{*}(r,t,x) = \begin{cases} \frac{1}{1+a} \left(1 - \frac{x_{0}}{x}\right), & \text{if } x > x_{0} \\ 0 & \text{if } x \le x_{0} \end{cases}$$
(7)

¹⁰Reliance on a linear tax schedule follows a well-established tradition, see ex. Roberts (1977); Romer (1975) or Sheshinski (1972). The degree to which actual taxes differ from linear taxes has generated a large literature, see Pechman et al. (1974), King (1980) or Browning and Johnson (1979). Related work is on the welfare-maximizing degree of tax progressivity, see ex ongoing work by Conesa et al. (2009); Diamond and Saez (2011); Erosa and Koreshkova (2007); Heathcote et al. (2014).

¹¹We know that the (logarithmic) Cobb-Douglas utility function is strictly concave, so this defines a maximum.

and thus the amount of leisure any individual will enjoy is

$$l^{*}(r,t,x) = \begin{cases} \frac{1}{1+a} \left(a + \frac{x_{0}}{x}\right), & \text{if } x > x_{0} \\ 1 & \text{if } x \le x_{0} \end{cases}$$
(8)

The labor-leisure choice is a function of own productivity, given incentives from taxes



Figure 2: The relationship between labor, leisure, productivity and preferences.

and redistribution, represented in figure 2. We see that as productivity increases, both choices will approach the fraction $\frac{1}{1+a}$ which is determined by the utility-weight on leisure. An individual with a higher preference for leisure will work less, the limit, $\frac{1}{1+a}$, is decreasing in the utility-weight on leisure, a, and someone who doesn't value leisure at all, a = 0, will always work.

Given taxes and redistribution, individual gross income is

$$y^*(r,t,x) = n^*(r,t,x)x = \frac{(x-x_0)_+}{(1+a)}.$$
(9)

Under the assumption of (logarithmic) Cobb-Douglas utility, gross income is linear and increasing in productivity, x, for any $x > x_0$,¹² and zero for any non-working individual, $x \leq x_0$ ¹³ This has the important implication that productivity and gross income, x and y, are ordered pairs: the person with median productivity is the same person as she with median gross income. This facilitates the analysis of the political process, where the outcome is determined by the median voter.

Increases in redistribution increase consumption and taxes decrease net wages. For those who do not work, consumption equals redistribution. Those who do work must consider not only the direct effect on consumption but also the effect of redistribution on their labor-leisure choice. MR show that the assumption that consumption is a normal

 $^{{}^{12}\}frac{\partial y(r,t,x)}{\partial x} = \frac{1}{a+1} > 0, \forall a > -1.$ ${}^{13}MR$ (1981) show that gross income is increasing in productivity for any strictly concave utility function, even allowing for the supply of labor being backward bending, provided consumption and leisure being normal goods.

good is sufficient for consumption to be increasing in redistribution for both workers and non-workers. Consequently, it is possible to establish a unique equilibrium solution for any tax rate.

A key assumption in this model is that the government budget is balanced and all government spending is for redistributive income. This implies that, if taxes is t, redistribution is r and per capita income is \bar{y} , then $r = t\bar{y}(r,t)$ must always hold.

Let F denote the cumulative density function (CDF) of productivity and f as the probability density function (PDF).¹⁴ Average gross income is obtained by integrating:

$$\bar{y}(r,t) = \int_{x_0}^{\infty} \frac{(x-x_0)_+}{(1+a)} dF(x) = \frac{1}{1+a} \int_{\frac{ar}{1-t}}^{\infty} \left(x - \frac{ar}{(1-t)}\right) f(x) dx.$$
(10)

and is determined once we know x_0, t , and r. From (6), we know that x_0 depends only on t and r, and from the budget balancing requirement we know that,

$$r_{BC}(t,\bar{y}) = t\bar{y}(r,t) = \frac{t}{1+a} \int_{\frac{ar}{1-t}}^{\infty} \left(x - \frac{ar}{(1-t)}\right) f(x)dx.$$
 (11)

Since the left side is non-negative and increasing in r and the right side is a non-negative, continuous and decreasing function of r,¹⁵ there is a unique budget maximizing level of redistribution for any tax rate.¹⁶ Once r or t is chosen, the other is determined. The individual's choices of consumption and the distribution of his time between labor and leisure are determined also. The choice of redistribution, r, or the tax rate, t, uniquely determines each individual's welfare and sets the size of government.

3.2 The size of Government

Now I will show how the model predicts that relative placement of the median voter relative to the mean voter in terms of gross income determines the size of government in terms of tax rate. A more unequal distribution of productivity, where the median voter is further away from the mean, will result in a higher equilibrium tax rate.¹⁷

Any voter chooses the tax rate that maximizes her utility. In making her choice, she is aware that the tax rate affects everyone's decision to work and consume. Increases in the tax rate has two effects. Each dollar of earned income raises more revenue but earned income declines; everyone chooses more leisure and more people choose to subsist on redistribution. "High" and "low" tax rates have opposite effects on the choice of labor or leisure and, therefore, on earned income. Formally, the individual is constrained to find a tax rate that balances the government budget, $r = t\bar{y}(r,t)$, and maximizes her indirect utility function, denoted v(r,t,x). A Laffer curve is a representation of the

 $^{^{14}\}mathrm{For}$ discrete distributions of productivity; use integrals over F.

¹⁵Upper bound, at r = 0, is $\frac{tE(x)}{1+a}$, where E(x) is the average of x.

¹⁶This is true for any concave utility function under the assumption that leisure is a normal good, as shown by MR (1981).

¹⁷MR show this using a shortened presentation of their section on their corresponding chapter can be found in the appendix, chapter A.1 on page 33.

relationship between tax rates and the resulting levels of government revenue, here the rate of redistribution, see figure 4 on page 14. I will use Laffer curves to represent the condition of a budget-balancing government throughout this thesis. The indirect utility function, given the function $n^*(r, t, x)$, is

$$v(r,t,x) = \begin{cases} \ln\left(\frac{(1-t)x+r}{1+a}\right) + a\ln\left(\frac{a}{1+a}\left[1+\frac{r}{(1-t)x}\right]\right), & \text{if } x > x_0\\ \ln(r), & \text{if } x \le x_0 \end{cases}$$
(12)

Then, the optimal tax rate for any individual is the solution to

$$\hat{t} \in \underset{t \in (0,1)}{\operatorname{arg max}} \quad v(r,t,x) \\
\text{s.t.} \quad r = t\bar{y}(r,t)$$
(13)

The non-working individual has no personal disincentive of an increased tax rate but realizes that an increase of the tax rate will decrease average income. She maximizes redistribution by choosing the tax rate that corresponds to the top of the Laffer curve. If the working individual has an above-average productivity, she cannot benefit from a (flat) transfer system and will vote for a zero tax rate. The problem of finding an optimal tax rate for a working individual with below average productivity can be solved by using Lagrangian multipliers,

$$\mathcal{L}(r,t,x,\lambda) = v(r,t,x) - \lambda \left(t\bar{y}(r,t) - r \right) = 0, \tag{14}$$

using the following system of equations,¹⁸

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial r} = \frac{1+a}{x(1-t)+r} + \lambda \left(1 - t \frac{\partial \bar{y}(r,t)}{\partial r}\right) = 0\\ \frac{\partial \mathcal{L}}{\partial t} = \frac{a}{(1-t)} - \frac{(1+a)x}{(1-t)x+r} - \lambda \left(t \frac{\partial \bar{y}(r,t)}{\partial t} + \bar{y}\right) = 0\\ \frac{\partial \mathcal{L}}{\partial \lambda} = r - t\bar{y} = 0 \end{cases}$$

Due to lengthy calculations that do not provide much intuition, and that in the end must be solved numerically anyway, I instead choose to solve this problem graphically, by representing Laffer curves together with indifference curves of the median voter.

I represent the productivity distribution using a mean-preserving spread of the lognormal distribution, meaning I can adjust the relative productivity of median voter holding everything else constant. See figure 3 for an illustration.¹⁹ I normalize mean productivity to $\bar{x} = 1$. The PDF and CDF of the log normal distribution of productivity, x, are

$$f = PDF(x) = \frac{1}{x\sigma\sqrt{2\pi}} * exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right]$$
(15)

$$F = CDF(x) = \frac{1}{2} + \frac{1}{2}erf\left[\frac{\ln x - \mu}{\sqrt{2}\sigma}\right]$$
(16)

¹⁸A paraphrase of the indirect utility function for those who work that is easier for handling derivatives is: $v(r, t, x) = (1 + a) \ln ((1 - t)x + r) - a \ln ((1 - t)x) - (1 + a) \ln (1 + a) + a \ln (a)$

¹⁹For more variation of σ , see the appendix, chapter A.4 on page 37.



Figure 3: The relative position of the median voter with $\sigma \in \{0.3, 0.6, 0.9\}$.

Where ln () is the natural logarithm, $exp(w) = e^w$ and erf is the complementary error function²⁰. The distribution parameters are productivity, x, the scale parameter, $\mu \in \mathbb{R}$, and location parameter, $\sigma > 0$, such that the mean is $e^{\mu + \sigma^2/2}$ and the median is e^{μ} . The following relationship between the location parameter, μ , and the scale parameter, σ must hold in a mean-preserving spread of the distribution:²¹

$$\mu = -\frac{\sigma^2}{2} \tag{17}$$

Note that $\mu < 0 \ \forall \sigma \in \mathbb{R}$.



Figure 4: The effect of varying σ and a on the Laffer curve

 ${}^{20}erf(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-w^{2}} dw$ ${}^{21}\bar{x} = e^{\mu + \sigma^{2}/2} = 1, \text{ so if } \bar{x} = 1; \ \mu + \frac{\sigma^{2}}{2} = 0$ Interestingly, in figure 4 we see that the Laffer curve, the budget-balancing combinations of redistribution and taxes specified by equation (11), varies less with the underlying distribution, σ (holding a = 2 constant), than with the utility weight on leisure, a (holding $\sigma = 0.3$ constant). However, much of the effect of the utility weight on leisure on the Laffer curve is offset by the corresponding change in the voter's indifference curves, see figure 6 on page 16.

Now, the voters' preference for taxation is determined by the maximum of her indirect utility function, equation (12). Given a fixed productivity, x_i , I can rewrite her indirect utility function to represent all combinations of redistribution and taxes that will yield the same utility; her indifference curves for any level of utility, u, are those which fulfills the following equation,

$$r_{IC}(u,t,x_i) = \begin{cases} exp\left(\frac{u+a\ln\left((1-t)x_i\right)-a\ln\left(\frac{a}{1+a}\right)+\ln\left(1+a\right)}{1+a}\right) - (1-t)x_i, & \text{if } x_i > x_0 \\ u, & \text{if } x_i \le x_0 \end{cases}$$
(18)

which is non-linear in the level of utility, u, individual productivity, x_i , and utilityweight on leisure, a. Importantly, the inclination of the indifference curve depend on the individuals' productivity, x_i , as represented by figure 5, and her vote will be determined by the point at which the indifference curve is tangent to the Laffer curve. The final



Figure 5: Indifference curves for $x_i \in \{0.6, 0.8, 1\}$.

step in understanding the outcome of this single-issue election is understanding the implication of the relative productivity of the median voter to the mean. We already know that the voter with above average productivity, $x_i \ge \bar{x}$, will prefer a zero tax rate, that the voter who choose not to work, $x_i \le x_0$, will vote for the tax rate that corresponds to the top of the Laffer curve, and that as σ increases and the productivity distribution



Figure 6: Preffered tax rates, varying σ and a.

becomes more unequal, the relative productivity of the median voter decreases with respect to the mean. The left side of figure 4 represent a situation where the median voter is shifting in relative productivity and the right side where she is shifting in utility-weight on leisure. Including the corresponding indifference curves into figure 4 gives figure 6 and we see that the preferred tax rate of the median voter is increasing in both the productivity spread, σ , and the utility weight on leisure, a.

3.3 Illustrating the Robin Hood paradox

The purpose of this thesis is to understand the Robin Hood paradox, so I will use two hypothetical economies, Swedistan and Usistan, who are identical in all but the explicit parameter choices, in each of the models in this thesis to see whether the relative tax rates change with the model assumptions. Estimation of population productivity distribution and labor market characteristics are subject to extensive economic research, and in this thesis I make no attempt to contribute to that field. Instead, I parameterize my fictional economies by roughly approximating OECD-data on two economies who fit the description of the Robin Hood paradox, where the more unequal economy (the US) distribute less and have lower taxes than the more equal economy (Sweden). There are many other important differences, such as progressivity of their respective taxing systems, but this approximation is useful for the intuition of the characteristics of the models.

Using the mean and median disposable income in Sweden and the US, total population in 2013 (OECD.stat, 2016b), I calculate a rough estimate of the scale parameters for the mean-preserving spread of the log-normal distribution, using the relationship between the location and scale parameter described in equation (17) on page 14, and the mean-to-median ratio of disposable income in Swedistan and Usistan, see table 1. I use

Parameter	Economy	Value	\mathbf{unit}
Mean disp. income	Sweden	$276 \ 728$	SEK
	US	38 257	USD
Median disp. income	Sweden	$249\ 476$	SEK
	US	30 473	USD
Estimated σ	Swedistan	0.46	σ_S
	Usistan	0.68	σ_U

Table 1: OECD data and estimated productivity parameters.

an utility weight on leisure, a = 2, which corresponds to a work week of 56 hours for the most productive, as we will see in figure 2 on page 11. Figure 7 shows how the model predicts that the tax rate in the country with the more equal productivity distribution, Swedistan, should have lower taxes than in Usistan, 17 verses 31 percent.²² To see where the indifference curve is tangent to the Laffer curve, the right side of the figure is a zoomed version of the left side, a method of illustration which I use consistently throughout this thesis. This is consistent with the predictions by MR, but the opposite relationship to what we observe for Sweden and the US.



Figure 7: The Robin Hood paradox

²²For the curious reader: Given these predicted tax rates, the predicted unemployment cutoffs are $x_{0,S} = 0.10$ and $x_{0,S} = 0.21$ and the corresponding rates of unemployment are zero and 2.5 percent for Swedistan and Usistan, respectively.

4 A model with indivisible labor

Building on the business cycle literature on indivisible labor, discussed in section 2 on page 5, I examine the predictions of preferred tax rate if the labor-leisure choice is restricted to a non-continuous set. I illustrate this by a binary choice: Either you work or you don't. I assume that there is some exogenous constant $\kappa \in (0, 1)$ that is the hour requirement for employment. I find that under this assumption, the Robin Hood paradox is solved for plausible parameter values.

As in the original model, I want to find a productivity threshold, $x_{0,\kappa}$, such that people with productivity $x \leq x_{0,\kappa}$ will choose not to work and entirely subsist on the lump-sum redistribution transfer, r. I again assume that productivity follows a lognormal distribution and that preferences are defined by a (logarithmic) Cobb-Douglas utility function. In this problem, gross income and consumption are:

$$y(x) = x\kappa \tag{19}$$

$$c(x) = (1-t)\kappa x + r \tag{20}$$

And the new maximization problem for the labor-leisure choice is

$$\max_{n \in \{0,\kappa\}} \ln((1-t)nx + r) + a\ln(1-n)$$
(21)

such that

$$\begin{cases} n = \kappa & \text{if } x > x_{0,\kappa} \\ n = 0 & \text{if } x \le x_{0,\kappa}. \end{cases}$$
(22)

A person will choose to work if and only if she gets higher utility from working than not:

$$\ln\left((1-t)\kappa x + r\right) + a\ln\left(1-\kappa\right) > \ln\left(r\right) \tag{23}$$

so, the employment cutoff in this model is,

$$x_{0,\kappa}(r,t) = \frac{r}{\kappa(1-t)} \left[\frac{1}{(1-\kappa)^a} - 1 \right]$$
(24)

Which is non-monotonic in the hour condition for employment, and higher than in the original model.²³ Intuitively, when there is no option to dedicate a small portion of one's time to work, where working hours are increasing with productivity, one needs higher productivity to have sufficient incentive to work. To better understand the incentives to

²³The conditions under which $x_{0,\kappa} > x_0 = \frac{ar}{(1-t)}$ from the original model is:

$$x_{0,\kappa} = \frac{1}{1-t} \frac{r}{\kappa} [(1-\kappa)^{-a} - 1] > x_0 = \frac{ar}{1-t}$$
$$\frac{(1-\kappa)^{-a} - 1}{\kappa} > a$$

Which is true $\forall \kappa \in (0, 1), a > 0$

enter the workforce, I examine the sensitivity of the employment cutoff to κ in figure 8. Note that the employment cutoff is very sensitive to changes in the hour condition for employment, κ , if the redistribution transfers, r, are relatively high. I've marked mean productivity as a reference point because if the employment condition is that high, more than half of the population will be unemployed.



Figure 8: How $x_{0,\kappa}$ vary with redistribution, r, and taxes, t.

The choice to enter/exit the labor market depends only on the tax rate, t, the size of the welfare payment, r, and the hour requirement for working, κ and on the weight on leisure in the utility function, a. Gross income is,

$$y^*(r,t) = \begin{cases} \kappa x & \text{if } x > x_{0,\kappa}(r,t) \\ 0 & \text{if } x \le x_{0,\kappa}(r,t) \end{cases}$$
(25)

implying that,

$$\bar{y}(r,t) = \kappa \int_{x_{0,\kappa}(r,t)}^{\infty} x f(x) dx.$$
(26)

I use the same assumptions in the original MR-model, so consumption and leisure are normal goods, consumption is increasing in redistribution and income, and thus gross income is increasing in productivity. The government is still required to balance its budget, so the following relationship between taxes and redistribution must always hold,

$$r_{BC,\kappa}(t,\bar{y}) = t\kappa \int_{x_{0,\kappa}(r,t)}^{\infty} xf(x)dx.$$
(27)

Now, the optimal tax rate, t, for any voter will be determined by her productivity, x, in relation to the hourly requirement for employment, κ , and the redistribution,

 $r = t\bar{y}(r,t)$. As before, if her productivity is above average she will prefer a zero tax rate, and if she is unproductive, $x_i < x_{0,\kappa}$, she will not work and choose the tax rate that maximizes the distribution, i.e. the top of the Laffer curve. The productivity set of interest is thus the same as in the original model, where the voter is working but have a below-average productivity, $x_i \in (x_{0,\kappa}, \bar{x})$, where x_i is the productivity of the voter, and \bar{x} is the average productivity of the economy. It also follows from equation (24) that there will only be solutions in the interval,

$$0 \le t < 1 - \frac{r}{\kappa x_i} \left[\frac{1}{(1-\kappa)^a} - 1 \right] = 1 - (1-t) \frac{x_{0,\kappa}}{x_i}$$
(28)

Given the labor-leisure choice, specified by equations (22) and (24), I can specify an indirect utility function,

$$v_{\kappa}(r,t,x) = \begin{cases} \ln\left((1-t)\kappa x + r\right) + a\ln\left(1-\kappa\right), & \text{if } x > x_{0,\kappa} \\ \ln\left(r\right), & \text{if } x \le x_{0,\kappa} \end{cases}$$
(29)

Then, the optimal tax rate for any individual is the solution to

$$\hat{t}_{\kappa} \in \underset{t \in (0,1)}{\operatorname{arg max}} \quad v_{\kappa}(r,t,x) \\
\text{s.t.} \quad r = t\bar{y}(\kappa,r)$$
(30)

Which, for the working individual, $x_i \in (x_{0,\kappa}, \bar{x})$, can be solved using Lagrangian multipliers.

$$\mathcal{L}_{\kappa}(r,t,x,\lambda) = v_{\kappa}(r,t,x) - \lambda \left(t\bar{y}(r,t) - r \right) = 0 \tag{31}$$

using the following system of equations,

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial r} = \frac{1}{(1-t)\kappa x + r} - \lambda \left(t \frac{\partial \bar{y}}{\partial r} - 1 \right) = 0\\ \frac{\partial \mathcal{L}}{\partial t} = \frac{-\kappa x}{(1-t)\kappa x + r} - \lambda \left(t \frac{\partial \bar{y}}{\partial t} + \bar{y}(r, t) \right) = 0\\ \frac{\partial \mathcal{L}}{\partial \lambda} = r - t \bar{y}(r, t) = 0 \end{cases}$$
(32)

But as in the illustration of the original model, section 3.2 on page 12, I instead solve the problem graphically in order to provide better intuition.

First, from equation (27) we know that the Laffer curves will vary with the hour requirement for employment, κ , and the underlying distribution, σ . As we can see in figure 9, the latter has a smaller effect on the Laffer curve than does labor, determined by κ . Specially, for any given level productivity, an individual will prefer lower tax rates when more hours are required for employment, as we soon will see in figure 10.

When solving for the preferred tax rates for any individual with productivity x_i , we know that she will vote for the tax rate that corresponds to the point at which her indifference curve is tangent to the government budget constraint. From her indirect



Figure 9: Laffer curves with indivisible labor.

utility function, equation (29), we know that she is indifferent between all combinations of redistribution, r, and tax rates, t, that fulfill the following equation,

$$r_{IC}(u, t, x_i) = \begin{cases} \frac{e^u}{(1-\kappa)^a} - (1-t)\kappa x_i, & \text{if } x_i > x_{0,\kappa} \\ u, & \text{if } x_i \le x_{0,\kappa}. \end{cases}$$
(33)



Figure 10: Variation of preferred tax rate with indivisible labor.

In the bottom part of figure 10 we see that the preferred tax rate of the median voter, $x_i = exp\left(-\frac{0.3^2}{2}\right)$, is decreasing in the hour condition for employment, κ . Interestingly, holding κ constant and instead changing the underlying distribution, so that we have three different Laffer curves and three different median voters, $x_i = exp\left(-\frac{\sigma^2}{2}\right)$, the change in the preferred tax rate is non-monotonic. Instead, the preferred tax rate of the first and third median voter, $\sigma = 0.3$ and $\sigma = 0.9$, are very close, and the tax rate in the second distribution, $\sigma = 0.6$, is lower. However, these parametrizations of the scale parameter, σ , are far from realistic and their purpose is only to show the principles

of the underlying distribution. In the next section, I show the results using parameter values from approximating real countries.

4.1 Robin Hood and indivisible labor

Now I return to the situation in Swedistan and Usistan. I again use OECD data, on average usual weekly hours worked on the main job in 2014 (OECD.stat, 2016a), to approximate the hour condition for employment. First, I compare worked hours to total available hours in a week, defined as all hours in a week (168h). Second, as a check of robustness, I define 'available hours' as the time it is not unhealthy to be awake, thus subtract eight of every 24 hours (112h). Using the first specification of the hour

Parameter	Economy	Value	\mathbf{unit}
Week hours	Sweden	35.8	h
	US	38.6	h
Share of 168h	Swedistan	0.21	κ_S
	Usistan	0.23	κ_S
Share of 112h	Swedistan	0.32	κ_S
	Usistan	0.34	κ_S

Table 2: OECD data and estimation of hour condition.

condition, we see that the preferred tax rate of the median voter in Swedistan is *higher* than in Usistan, see figure 11, and thus solving the Robin Hood paradox.²⁴

Interestingly, this relationship holds even when the hour requirement is the same in both economies, see chapter A.5 on page 37 in the appendix for global $\kappa = 0.22$ and global $\kappa = 0.1$, nor from calculating the hour condition on more limited available time. I also control for a larger utility-weight on leisure, a.

In conclusion, when simulating the Laffer curves and the median voters indifference curve for two hypothetical countries, Swedistan and Usistan, with similar relative productivity distributions as Sweden and the US, the preferred tax rate of the median Swedistan voter is higher than that of the Usistan voter. This result, which holds for several robustness checks, implies that the assumption of a continuous choice of labor is a potential explanation for the Robin Hood paradox.

²⁴The predicted tax rates are $t_S = 48.5\%$ and $t_U = 46.2\%$, and the corresponding unemployment cutoffs are $x_{0,\kappa}^S = 0.5833$ and $x_{0,\kappa}^S = 0.5915$, and the fraction of the population outside the workforce are 17 and 33 percent.



Figure 11: Robin hood with indivisible labor, $\kappa_S = 0.21$ and $\kappa_U = 0.23$.

5 The misapprehension model

Now we go back to the original setting in section 3, with a continuous choice of labor, and introduce a new assumption. Building on the findings by Karadja et al. (2014), I assume that every individual in the economy overestimates everyone else's productivity by an equal fraction θ . I define $\theta \in \mathbb{R}$ so that if $\theta = 1.1$ then all individuals overestimate others' productivity by 10 percent. This will have important implications for the model, namely increased demand for redistribution and if the misapprehension is larger in the more equal economy, as empirical findings support, this modification of the original model can solve the Robin Hood paradox.

I assume every individual knows their own productivity and income, and observe the true tax rate. The labor-leisure choice, utility function and preferred relationship between taxes and redistribution thus remain the same as in the original model. The misapprehension of others' productivity only affect the perceived average income and thus redistribution, and consequently the employment condition, $x_{0,\theta}$, and the voting outcome. Assuming a large number of individuals in the economy, I disregard the effect of the individual voter's income on the aggregate average income, an underestimation of one's own productivity compared to *everyone else's* productivity (or income) is thus the same (in the limit) as an overestimation of *everyone's* productivity in forming expectations of redistribution from taxes.

Evidently, while misinformed voters will expect a higher redistribution after the elections, the actual redistribution will depend on the actual average income and thus be lower than expected. A possible reason for voters not discovering this are imperfect information of the per capita cash value of the redistribution, in cash and services, for example, insurance, defense, infrastructure and other public services. This is a step away

from the assumption that voters don't suffer from fiscal illusion, but given the evidence presented in section 2, the assumption of a systemic misapprehension still constitutes an interesting thought experiment.



Figure 12: The labor-leisure choice in a misinformed economy.

In short, while every voter's own decisions are the same as in the original MR model, every individual predicts that everyone else's labor choice is based on a higher productivity,

$$n_{\theta}^{*}(r_{\theta}, t, \theta x) = \begin{cases} \frac{1}{1+a} \left(1 - \frac{x_{0,\theta}}{\theta x}\right), & \text{if } \theta x > x_{0,\theta} \\ 0 & \text{if } \theta x \le x_{0,\theta} \end{cases}$$
(34)

and disregarding the effect of the increased redistributions' effect on the employment cutoff constant, we can make the comparison in figure 12 to the labor-leisure choice in the original model, see figure 2, with $\theta = 1.3$.

We see that when there is an overestimation of others' productivity, people will believe that others work more and enjoy less leisure than they actually do. Consequently, perceived average income is,

$$\bar{y}_{\theta}(r_{\theta},t) = \frac{1}{1+a} \int_{x_{0,\theta}}^{\infty} \left(\theta x - x_0\right) f(x) dx = \frac{1}{1+a} \int_{\frac{ar_{\theta}}{1-t}}^{\infty} \left(\theta x - \frac{ar_{\theta}}{1-t}\right) f(x) dx \tag{35}$$

and it follows that the new government budget constraint is,

$$r_{BC,\theta}(t,\bar{y}) = t\bar{y}_{\theta}(r_{\theta},t) = \frac{t}{1+a} \int_{\frac{ar_{\theta}}{1-t}}^{\infty} \left(\theta x - \frac{ar_{\theta}}{1-t}\right) f(x)dx,$$
(36)

(where the only difference to equation (11) is θx instead of x). Note that while $\theta x > x$, the lower limit of the integral is potentially higher, $x_{0,\theta} > x_0$ because when the perceived redistribution is larger, voters will need a larger incentive to work. However, it follows from the overestimation of *everyones* productivity that every voter also expects there to be fewer people with productivities below the employment cutoff. Figure 13 shows how



Figure 13: Laffer curves with $\theta \in \{1, 1.1, 1.3\}, \sigma = 0.3$.

the Laffer curves changes with θ (holding $\sigma = 0.3$ constant), where $\theta = 1$ is the Laffer curve of the original model.

The indifference curve, specified by equation 18 on page 15, of any voter varies only with her own productivity, and level of utility and consequently doesn't change with this new assumption, and I can compare the results of the misapprehension model to the original MR-model directly. We know from figure 4 on page 14 that while the productivity of the median voter does, the Laffer curves don't vary much with the underlying distribution. When we introduce misapprehension, however, the effects are larger. Now, the shape of the indifference curve of any voter is independent of the misapprehension parameter, θ , and it follows that there is a direct effect on the preferred tax rate. Figure 14 shows how the median voter's preference for taxation is increasing in the misapprehension parameter, θ and reaches a higher level of utility.

5.1 Robin Hood and misapprehension

To understand if systemic misapprehension of average productivity may explain the Robin Hood paradox I again present the median voters from Swedistan and Usistan. We know that, given a Laffer curve, voters prefer lower taxes the more productive they are, figure 5 on page 15, and that given a level of productivity, x_i , voters prefer higher taxes the larger the discrepancy between the belief of and true productivity of everyone else, figure 14.

The median voter in Usistan, with the more unequal productivity distribution, will have lower productivity than the median voter in Swedistan. Ceteris Paribus, Swedistan will have lower taxes than Usistan.²⁵ It follows that the answer to the Robin Hood

 $^{^{25}}$ An illustration can be found in figure 21 in the appendix, section 5.1 page 25.



Figure 14: Voter i's preference for taxes with $\theta \in \{1, 1.1, 1.3\}, \sigma = 0.3$.



Figure 15: Robin Hood in Swedistan and Usistan when $\theta_S = 1.2$ and $\theta_U = 1$.

paradox, assuming this model of misapprehension, lies in allowing the misapprehension parameter to be larger in the economy with the less equal distribution of productivity. Given the results discussed in section 2, I assume such national differences in misapprehension. If the misapprehension parameter in Swedistan is $\theta_S = 1.2$ and the citizens of Usistan are perfectly informed, $\theta_U = 1$, the Swedistan economy will have higher taxes than the USistan economy.²⁶

In conclusion, when simulating the Laffer curves and the median voter's indifference curve for two hypothetical countries, Swedistan and Usistan, with similar relative productivity distributions as Sweden and the US, the preferred tax rate of the median Swedistan voter is higher than that of the Usistan voter. This result is driven by a higher parameter of misapprehension in Swedistan and Usistan, an assumption that is supported by empirical findings, which implies that the voter's perception of her position in the income distribution is a potential explanation for the Robin Hood paradox.

6 Discussion and final remarks

I have presented two alterations of the original MR-model which both explains the Robin Hood paradox; predicted taxes in the more equal economy is actually higher than in the more unequal economy. While the relative change, reversing the originally predicted relationship between the productivity distribution and equilibrium tax rate, have potentially important implications for the literature on comparative political economy – Labor market restrictions and national differences in misapprehension of relative productivity need be considered when comparing demand for redistribution – the predicted difference in tax rates are very small. For example, the predicted difference between the equilibrium tax rates in Swedistan and Usistan in the misapprehension model, for a 20 percent difference in perception of productivity, is only three percentage points. Thus, the model modifications I've made seem to level out the difference in the underlying productivity distribution, rather than predicting actual levels of taxation.

In order not to mistakingly emphasize any numerical results from the models, I have put the predicted numerical values of tax rates, unemployment cutoffs and share of the population who choose not to work as footnotes when presenting the relative results of Swedistan and Usistan. An empirical investigation of the predictive power of these model modifications lies outside the scope of this paper; my sole intention has been to illustrate situations where the direction of the relative tax rates across populations with different mean to median income is the opposite of the prediction by Meltzer and Richard (1981). It is worth noting that the share of the population outside the workforce is larger in both alterations of the model, with the biggest increase in the model of indivisible labor. However, in the original model, the fraction was larger in Usistan than in Sweden, and this held true through the modifications. Consequently, this seems to not be the driving factor for these results.

The results of the model with indivisible labor, that the restriction of the laborleisure choice solved the Robin Hood paradox, also held when the hour requirement for employment was global. This indicates that it is the non-monotonic relationship (illustrated in figure 10 on page 21) between the scaling of the productivity distribution

²⁶Again, parameter values are a = 2, $\sigma_S = 0.46$, and $\sigma_U = 0.68$. Now, the predicted tax rates are $t_S = 32\%$ and $t_U = 29\%$, the corresponding employment cutoffs are $x_0^S = 0.2934$ and $x_0^U = 0.2196$, and fraction of population outside workforce are 0.74 and 2.94 percent respectively.

and the preferred tax rate that drive these results. I leave for future research to examine this further.

The results of the misapprehension model are entirely driven by inflating the Laffer curve by increasing the misapprehension parameter of the relatively equal economy enough to explain the higher preferred tax rate. However, given the empirical results discussed in the literature review, that US citizens *underestimate* others income while Swedish citizens *overestimate*, this model provides an analytical framework for the role of information in understanding demand for redistribution.

Some modifications, which may affect my conclusions, I leave for future research, in particular using other distributions of productivity and further comparisons using parameters from other countries. Furthermore, allowing for half-time employment and/or a continuous choice of overtime hours could have an effect on the political equilibrium.

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A Appendix

A.1 MR original Section III: The Size of Government

In this section they show that taxes are increasing in the relative distance between the mean to median productivity, using an additional assumption of constant partial elasticities of consumption and labor, how the political equilibrium tax rate is increasing in the relative distance between mean to median income. The first-order condition for the decisive voter, maximizing her utility subject to her own budget constraint, equation (4), is solved to find his preferred tax rate:

$$\bar{y} + t\frac{d\bar{y}}{dt} - y_d = 0, \tag{37}$$

where y_d is the gross income of the decisive voter.

Roberts (1977) showed that if the ordering of individual incomes is independent of the choice of r and t, individual choice of the tax rate is inversely ordered by income. This implies that with universal suffrage the voter with median income is decisive, and the higher one's income, the lower the preferred tax rate. By the assumption that consumption is a normal good, incomes are ordered by productivity for all r and t. Combining Roberts's lemma 1 (Roberts, 1977, p.334) with their results, they can order the choice of tax rate by the productivity of the decisive voter. We cannot deduce the effect of changes in productivity on t directly from equation (37). The reason is that \bar{y} depends on t, so finding the effect of changes in relative productivity requires the solution to a nonlinear equation in t. Instead, they rewrite equation (37) in a form which involves the partial elasticity of per capita income, \bar{y} , with respect to redistribution, r, and the wage rate, (1 - t)x.

Let $\tau = 1 - t$ be the fraction of earned income retained. From (10), \bar{y} depends on r and τ only. The total derivative

$$\frac{d\bar{y}}{dt} = \frac{\bar{y}_r \bar{y} - \bar{y}_\tau}{1 - t\bar{y}_r} \tag{38}$$

Where $\bar{y}_r, \bar{y}_{\tau}$ are partial derivatives. Substituting (38) into (37) and solve for t:

$$t = \frac{m - 1 + \eta(\bar{y}, r)}{m - 1 + \eta(\bar{y}, r) + m\eta(\bar{y}, \tau)}$$
(39)

where $m = \bar{y}/y_d$ is the ratio of mean income to the income of the decisive voter, and the η 's are partial elasticities. Using the common economic assumption that the elasticities are constant,²⁷ the tax rate rises as mean income rises relative to the income of the decisive voter, and taxes fall as m falls:

$$\frac{dt}{dm} = \frac{\eta(\bar{y},\tau)[1-\eta(\bar{y},r)]}{[m-1+\eta(\bar{y},r)+m\eta(\bar{y},\tau)]^2} > 0.$$
(40)

And this is the main result of their model.

 $^{^{27}}$ The assumption of constant elasticities is important, however they state that they expect that the sign remains positive, provided the change in elasticities is small.

A.2 The MR 1983 testing the model article

Meltzer and Richard approximate preferred tax rate of the median voter by specifying their original model with a Stone-Geary utility function,

$$u(c,l) = \ln(c+\gamma) + a\ln(l+\lambda), \lambda > -1$$
(41)

if $\gamma = \lambda = 0$, the utility function specializes to the Cobb-Douglas.

They find that the labor-leisure choice for those who choose to work and those who choose full-time leisure.

$$n(x) = \begin{cases} \frac{(1+\lambda)x(1-t)-a(r+\gamma)}{x(1-t)(1+a)}, & x > x_0\\ 0, & x \le x_0 \end{cases}$$
(42)

such that

$$x_0 = \frac{a(r+\gamma)}{(1-t)(1+\lambda)} \tag{43}$$

observe that $x_0 > 0$ if $\gamma > 0$. From (42) and (43) follows that as r increases, n falls and x_0 rises. Similarly, as the tax rate, t, rises, labor supply, n, falls and x_0 rises.

Average income is determined once we know the productivity of the last non-worker, x_0 ,

$$\bar{y}(x_0) = \frac{1+\lambda}{1+a} \int_{x_0}^{\infty} (x-x_0) dF(x).$$
(44)

A decisive voter with productivity below average but above x_0 will balance the utility gain from increased redistribution against the utility loss from higher taxes. Differentiate $\bar{y}(x_0)$ and use $x_0 = \text{and } t\bar{y} = r$:

$$\frac{d\bar{y}}{dt} = -\frac{a(1-F(x_0))(\bar{y}+\gamma)}{(1-t)[1+a-t(1+aF(x_0))]},\tag{45}$$

Substituting (45) into (37) and denoting $g = \frac{\lambda}{y_d}$ and $m = \frac{\bar{y}}{y_d}$,

$$0 = \frac{1 + aF(x_0)}{a(1 - F(x_0))}(m - 1)(1 - t)^2 + (2m + g - 1)(1 - t) - (m + g).$$
(46)

The solution to (46) is the optimal tax rate for a decisive voter who works and chooses t > 0. It is an equilibrium relation between t, m and F. For given productivity and tastes, the decisive voter's choice of t determines x_0, \bar{y} and m, and all other endogenous variables follow. When making his choice, the decisive voter is aware that he cannot treat (46) as a quadratic function in t. The reason is that m and x_0 depend on the choice of t.

To estimate this final equation they take a linear approximation, specify a decisive voter, and choose empirical counterparts for $m, F(x_0), g$ and t.

$$1 - t = \frac{-2m + 1 - g + (1 + g) \left[1 + \frac{4b}{1 + g} (m - 1) + \frac{4b}{(1 + g)^2} (m - 1)^2 \right]^{1/2}}{2(m - 1)(b - 1)},$$

such that $b = \frac{(1+a)}{a(1-F)}$. Expanding by means of a first-order approximation in m-1 and g gives an approximation ²⁸

$$t \approx \left(\frac{1+a}{a}\right) \frac{m-1}{1-F} \frac{1}{1+g}$$

Taking logs and letting g approximate $\ln(1+g)$ for small g, gives

$$\ln t + \ln \left(1 - F(x_0)\right) = \ln \frac{1+a}{a} + \ln \left(m - 1\right) - \frac{\gamma}{y_d}.$$
(47)

Remaining problem: Possibility of simultaneous equation bias (same process determines both t and $F(x_0)$, avoided by using $\ln t + \ln (1 - F)$ as the dependent variable.

A.3 Problems with the Stone-Geary utility function

In their paper from 1983, Meltzer and Richard Meltzer and Richard (1983) use a Stone-Geary utility function, which has the form:

$$u(c, l) = ln(c + \gamma) + aln(l + \lambda),$$

which is the Cobb Douglas utility function with two additional constants, γ and λ , on which the only restriction is that $\lambda > -1$. This has important implications for the marginal utilities of consumption and leisure, which the effect that if ex $\gamma = \lambda = 1$, zero leisure or zero consumption can be a reasonable outcome. As opposed to in Cobb Douglas where, if there is zero consumption (or leisure) the marginal benefit from very little consumption (or leisure) is infinite, and there will thus always be some consumption and leisure in equilibrium. Naturally, this has important implications for the choice of working hours, n(x). With this utility function, the continuous choice is the following:

$$n(x) = \begin{cases} \frac{(1+\lambda)x(1-t)-a(r+\gamma)}{x(1-t)(1+a)}, & x > x_0\\ 0, & x \le x_0 \end{cases}$$

such that

$$x_0 = \frac{a(r+\gamma)}{(1-t)(1+\lambda)}$$

Time is a limited resource, so the restriction $n(x) \in [0, 1]$ follows naturally. x_0 is the level at which individuals will choose n = 0. Is this a problem, i.e. when will they

$$(1 + px + qx^2)^{1/2} \approx 1 + \frac{p}{2}x + \frac{1}{2}\left(q - \frac{p^2}{4}\right)x^2 + o(x^3)$$

 $^{^{28}\}mathrm{They}$ use the following approximation for small x.

choose to work $n \ge 1$? Yes,²⁹ for

$$x \ge \frac{a(r+\gamma)}{(1-t)(\lambda-a)}$$

With the special case:

$$n(x) = 1 \Rightarrow x_1 = \frac{a}{\lambda - a} \frac{r + \gamma}{1 - t}$$

And, more importantly, if $x > x_1$ then it will be optimal to work more than 100 percent of the time, n > 1 if $x > x_1$.

For these equations to be reasonable, $x_1 > x_0$ must hold. A sufficient condition for this is that a > -1.³⁰ and that $\lambda > a$ (otherwise x_1 negative).

Note also that the function n(x), from the original form, can be written as:

$$n(x) = \begin{cases} \frac{(1+\lambda)}{(1+a)} - \frac{1}{x} \frac{a(r+\gamma)}{(1-t)(1+a)}, & x > x_0\\ 0, & x \le x_0 \end{cases}$$

and we can see that

$$\lim_{x \to \infty} n(x) = \frac{(1+\lambda)}{(1+a)}$$

and we can solve the issue of keeping $n \in [0, 1]$ in one in three ways:

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$$\begin{split} n(x) &\geq 1\\ \frac{(1+\lambda)x(1-t)-a(r+\gamma)}{x(1-t)(1+a)} &\geq 1\\ (1+\lambda)x(1-t)-a(r+\gamma) &\geq x(1-t)(1+a)\\ x\Big[(1-t)[(1+\lambda-1-a)]\Big] &\geq a(r+\gamma)\\ x &\geq \frac{a(r+\gamma)}{(1-t)(\lambda-a)} \end{split}$$

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$$\frac{a}{\lambda - a} \frac{r + \gamma}{1 - t} > \frac{a(r + \gamma)}{(1 - t)(1 + \lambda)}$$
$$\frac{1}{\lambda - a} > \frac{1}{(1 + \lambda)}$$
$$1 + \lambda > \lambda - a$$
$$a > -1$$

• Direct restriction in the n(x) function:

$$\begin{cases} 0, & x \le x_0\\ \frac{(1+\lambda)}{(1+a)} - \frac{1}{x} \frac{a}{1+a} \frac{r+\gamma}{1-t}, & x > x_0\\ 1, & x \ge x_1 \end{cases}$$

• Restrict limit of n(x) to be below 1 by choosing $a \ge \lambda$

$$\lim_{x \to \infty} n(x) = \frac{(1+\lambda)}{(1+a)} \le 1 \Rightarrow 1+\lambda \le 1+a$$

• Use the (logarithmic) Cobb-Douglas utility function: $\lambda = \gamma = 0$:

$$x_0 = \frac{a(r+\gamma)}{(1-t)(1+\lambda)} = \frac{ar}{(1-t)}$$
$$\lim_{x \to \infty} n(x) = \frac{(1+\lambda)}{(1+a)} = \frac{1}{1+a} < 1 \qquad \forall a \ge 0$$

A.4 The log-normal distribution



Figure 16: Mean preserving spread with $\bar{x} = 1$

A.5 Robustness of the model of indivisible labor

Using a global hour condition for employment does not change the conclusion in the model of indivisible labor. See figure 17 and 18.



Figure 17: Robin hood with indivisible labor, global $\kappa = 0.22$.



Figure 18: Robin hood with indivisible labor, global $\kappa = 0.1$.

As another check of robustness, I calculate a second set of employment conditions, where n = 1 corresponds to hundred percent of the time it is healthy to be awake, thus subtract eight hours per 24 hours, and $\kappa_S = 0.32$ and $\kappa_U = 0.34$. In that setting, taxes are lower but the relationship remains the same; predicted taxes are lower in Usistan than in Swedistan, see figure 19.

Figure 20 represents a final test of robustness, and how small the differences are between Swedistan and Usistan in this model, by varying the utility-weight on leisure.



Figure 19: Robin hood with indivisible labor, $\kappa_S = 0.32$ and $\kappa_U = 0.34$.

In the two previous illustrations in figure 11 and 19 I used a = 2. It turns out that the Robin Hood paradox is solved also for a = 3.



Figure 20: Swedistan and Usistan with a = 3 in the model of indivisible labor.

A.6 Robustness of the model of misapprehension

When the misapprehension parameter is the same for both countries, this version of the model doesn't change the original prediction of the model, that a more unequal economy will distribute more.



Figure 21: Robin Hood in Swedistan and Usistan when global $\theta = 1.2$.