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Stock Market Consolidation with Investor Heterogeneity

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Abstract: We investigate the impact on different types of investors of market consolidation and fragmentation, thought of as the number of objects listed on a stock exchange. We find that, at high levels of fragmentation, there are benefits to consolidation for buyers, as less information for a buyer's competitor enables him to make positive profits. At low levels of fragmentation, however, further consolidation impacts investors negatively. The optimal level of fragmentation depends on investor skill and the degree of correlation between stocks. Furthermore, our results constitute, to the best of our knowledge, new findings that are generally applicable to the theory of multiple correlated second-price sealed-bid auctions.

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1 Introduction

Consider the following hypothetical scenario: Different types of investors trade on a stock market which is constant in terms of total market capitalization but structurally changing in the sense that the number of firms listed is variable. Our question then is whether a particular class of investors gains or loses from consolidation. We will think of investors as being either private investors, mainly households saving for their pensions, or institutional investors such as banks and other financial institutions.

The main practical motivation for this problem is the heavy market consolidation in recent years. It has been speculated whether this has negative effects on investors, cf. Ljungqvist, Persson, and Tåg (2016). This has occurred partially due to an increased mergers and acquisitions activity, rendering the stock market a place with fewer, yet larger, firms, and partially due to stock market delistings by private equity investors.

Private investors face different constraints than institutional investors when purchasing stocks. They are on average less informed about the value of the underlying asset. There are various explanations for this, among them that institutional investors have more time to dedicate to research and, sometimes, may even obtain information through informal social channels. The informational advantage of institutional investors can thus arise due to more time dedicated to research or due to insider information. A change in the number of stocks listed could thereby affect private and institutional investors differently, because of the two differing information sets these actors face.

Market consolidation, then, impacts the choice set of private investors directly. The classical economic intuition is that less choice is unambiguously bad for investors. There are, however, other mechanisms in play. For instance, if there is a smaller number of different stocks traded, holding the number of trades executed fixed should yield a more correlated information structure. It is then conceivable that investors who are initially less informed partially benefit instead.

Thus we are interested in determining whether expected returns change when the number of stocks available for purchase decreases. In terms of the model we propose to study, this can be determined by investigating whether or not returns are invariant under changes of the parameter N, the number of stocks listed for auction (see Section 3 below for a detailed description of the framework).

1.1 Research Question

Are private and institutional investors affected differently by a shrinking stock market - and, if so, how?

1.2 Outline

The outline of this thesis is as follows. In Section 2, we offer an overview of the recent trend of market consolidation, as well as a brief description of how auctions work. We provide these concepts because we need auction theory to construct our model for studying market consolidation in Section 3, which is then solved in Section 4. Finally, a discussion on the implications of our findings and an agenda for further research both follow in Section 5.

2 Previous Research and Related Literature

Consider the following two markets. In the first market, there is a large number of firms being traded, where each firm has a low market value. In the second market, a smaller number of firms is traded - but each firm has a higher market value. The total market capitalization of these markets is the same, and we also assume that the firms are equivalent in risk profile and other relevant properties. Further, in this thought experiment, there are no economies of scale, oligopolies for price formation, or monopoly profits that make larger firms more profitable. Only the size of the average firm differs between our two markets.

Will these markets have different expected returns on stocks, so that investors prefer one market over the other? Our initial hypothesis is that they should not: if a portfolio contains many stocks yielding low returns it should be worth as much as a portfolio containing few stocks with higher returns, given that the total market capitalization for both asset classes is the same.

Why is this an interesting problem? First, we note in Figure 2.1 that the number of publicly traded firms in the United States has decreased by more than half between its peak of approximately 8 000 in 1996 to approximately 3 600 in 2015.

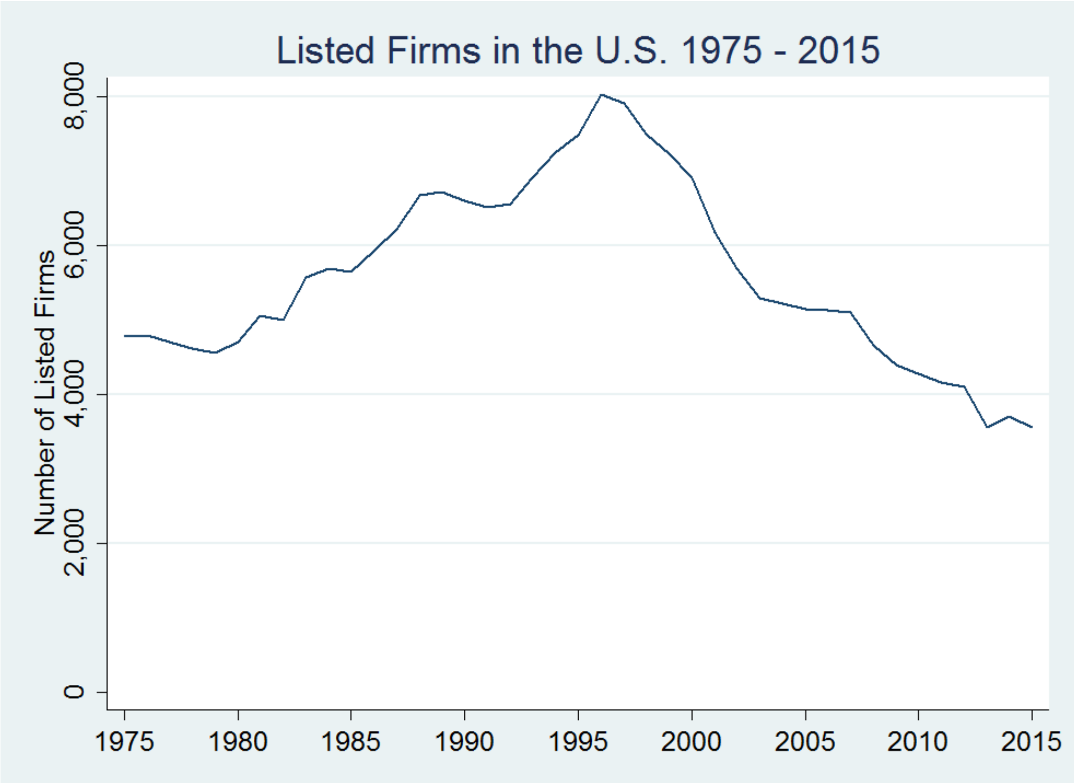

Figure 2.1. *Number of publicly listed firms in the United States 1975 - 2015. Based on data from Doidge, Karolyi, and Stulz (2015) between 1975 and 2012 and from Wilshire Associates (2013, 2014, 2015) between 2013 and 2015.*

Second, we note in Figure 2.2 that the total market capitalization in current USD of listed domestic companies in the United States during the period of decline in listings (from 1996 to 2015) has tripled. Inflation-adjusted, this works out to approximately double the 1996 value in real terms.

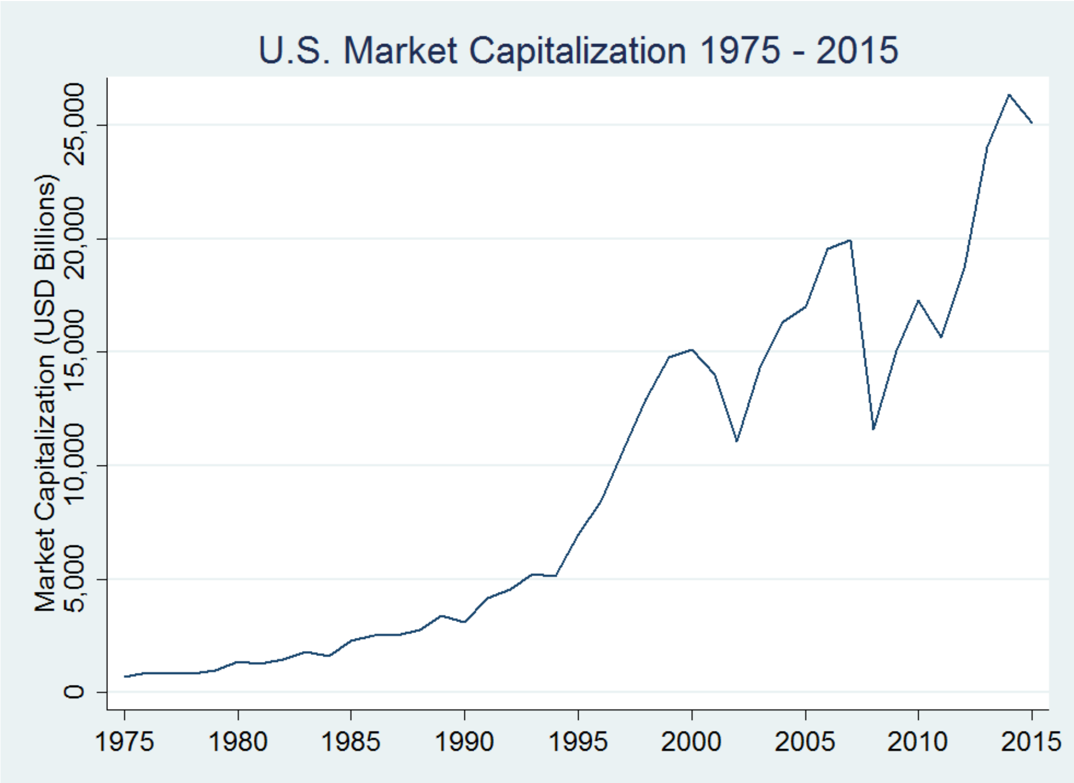

Figure 2.2. *Total market capitalization, in current USD, of publicly listed firms in the United States 1975 - 2015. Based on data from the World Bank (2016).*

Considering these two parallel developments, we can conclude that the decrease in listed firms is not due to the total market size (that is, its capitalization) shrinking. Doidge et al. (2015) point out that approximately half the decline in listed firms is due to delistings of existing firms, while the other half is due to fewer IPOs of new firms. The number of listed firms has thus decreased despite a growing market size, which has led to greater average market capitalization for firms.

The delisting phenomenon is not unique to the United States. Between 1995 and 2005, over 25% of listed firms in Europe went private, which removed 40% of the total asset value from these markets (Thomsen and Vinten, 2007). To study this aspect is therefore not only of interest to Americans; it could have wider implications.

Is this a welcome development for investors? This question is ambiguous and difficult to answer directly. We will proceed to investigate the property of number of listed firms and its effect on investors in our model in Section 3. While in the empirical problem, it is difficult to disentangle the effects of publicly listed firms and market size, we will hold the total market capitalization fixed and only look at changes in the number of listed firms to isolate one particular factor for analysis.

2.1 Delistings in the United States Market

Doidge et al. contrast the development in the United States with that of international stock markets. While the number of firms listed in the United States fell by 49% between 1996 and 2012, the number of firms listed in non-U.S. markets instead increased by 28% during this period, likely driven by new listings in markets outside both the U.S. and Europe. The authors claim that the decrease in the United States is not due to a lack of new firms entering the market, as the total number of companies (that is, the sum of listed and unlisted firms) was unchanged during the period and the number of startups increased. They also look into whether or not regulatory changes may have had an effect, but find that the decline in listings started long before major regulation was enacted. In a separate analysis, Gao, Ritter, and Zhu (2013), too, find that regulatory changes are unlikely to explain the decline.

Instead, Doidge et al. argue that the delistings during this period were caused by mergers and acquisitions, failures to meet listing requirements, and companies going private, often after buyouts by private equity firms. Mergers and acquisitions were the most important out of these three, and caused roughly half of all delistings. Still, existing research has not focused on mergers and acquisitions as a cause of delistings (Martinez, Serve, and Djama, 2015).

What could be the reason for this effect? Seven years before the decline started, Jensen (1989) predicted the eclipse of the public corporation, as it had "outlived its usefulness." He claimed that private ownership is better at resolving the inherent principal-agent conflicts between owners and managers of a company. Therefore, the method of raising capital to finance operations by publicly listing firms should become less common as private ownership by leveraged buyouts (LBOs) takes over.

Other researchers looking at financial markets have reached different conclusions about the significance of the public corporation. In the works of e.g. La Porta, Lopez-de Silanes, Shleifer, and Vishny (1997) and Djankov, La Porta, Lopez-de Silanes, and Shleifer (2008) the number of publicly traded firms per capita is viewed as an indication of how well-developed a country's financial markets are. This runs contrary to Jensen's ideas of evolution towards more private ownership.

Firms can be delisted either voluntarily or involuntarily. Martinez et al. argue that firms that involuntarily delist are forced to do so as they cannot properly manage their cost structures and thus spend too much.

Voluntary delisting can therefore be a way to mitigate the Free Cash Flow (FCF) problem. The FCF problem states that agency costs arise when there are cash flows remaining after a firm funds all projects with a positive net present value. Jensen (1986) describes this problem in detail, and argues managers will want to spend the excess cash on pet projects with negative net present value rather than distribute the money to shareholders. In Jensen's view, shareholders should want to delist firms to mitigate the FCF problem.

Vismara and Signori (2014) look at a sample of firms that went public in Europe between 1998 and 2003, and find that more innovative firms were more likely to be delisted. If Jensen's FCF hypothesis is correct, this suggests that innovative firms to a larger extent see managers using free cash flows in ways not in the interest of the firms' owners. Thus, the shareholders of innovative firms were more likely to voluntarily delist their companies to remedy this problem.

Firms could also be delisted due to poor performance. Balios, Eriotis, Missiakoulis, and Vasiliou (2015) find that companies with poor liquidity, high leverage, and a large decline in stock price are more likely to be delisted.

While the reasons for delistings continue to be debated by scholars, we intend to focus on the outcome of this and the effect it has on investors rather than attempt to establish which explanation has better support. There have been a number of attempts to investigate this effect to date.

2.2 Are Investors Made Better or Worse Off by Delistings?

Renneboog, Simons, and Wright (2007) look at data from the United Kingdom and argue the shareholders of the particular firm that is the target of a buyout receive a large premium for their shares and thus are made better off. The sources of the value appreciation are improved incentives for management, increased interest tax shields, and undervaluation before the buyout. Loughran and Vijh (1997) reach similar conclusions about the impact of delistings following cash offers for shares, and Marais, Schipper, and Smith (1989) find increases in the price of public convertible securities and nonconvertible preferred stock following buyout proposals. The positive effects of buyout proposals appear to be well-documented in previous research, with a multitude of other scholars finding large gains for shareholders (Booth, 1985; Torabzadeh and Bertin, 1987; Palepu, 1990; Travlos and Cornett, 1993).

The presence of institutional investors as owners of a company tends to increase the premium paid for shares in a takeover (Bajo, Barbi, Bigelli, and Hillier, 2013). However, consistent with normal rules of supply and demand, the higher price also decreases the chance of success of the takeover.

Delistings are not always a welcome development for investors. Angel, Harris, Panchapagesan, and Werner (2004) and Harris, Panchapagesan, and Werner (2008) look at a sample of firms that underwent involuntary delistings (due to not following the rules of the stock exchange they were listed on) accompanied by large declines in liquidity. The authors find that investors were made significantly worse off, with an average wealth-loss of 19 per cent. There can thus be adverse effects of delistings, when these are not initiated by a buyer but by failure to comply with the rules of stock exchanges.

2.3 The Firm-Level Impact of Private Equity

Previous research on private equity and LBOs has shown a number of beneficial effects. Using a dataset on income, profits, and value, Kaplan (1989) finds that the average firm doubles in value between buyout and subsequent sale. The firms in the sample experienced increase in operating income, decrease in capital expenditures, and increase in net cash flow. Performance particularly increases when a firm needs to undergo a complex restructuring process that would be difficult with quarterly reporting to public shareholders (Giovannini, Caselli, Capizzi, and Pesic, 2011). The benefits are especially large in cases where improvements in governance are difficult for regular management teams to implement.

Moreover, Bloom, Sadun, and Van Reenen (2009) look at the management practices of manufacturing companies and find that those owned by private equity firms have better systems in place for managing employees - and have much stronger operations management practices. Investigating these properties over time, Bloom et al. suggest data support the hypothesis that private equity firms buy poorly managed companies and successfully improve them.

There also seems to be positive effects of LBOs on productivity. Lichtenberg and Siegel (1990) find that manufacturing plants sold in LBOs experience significantly higher rates of total factor productivity growth than other plants. Labor costs for non-production labor decrease, while wages for production workers increase. This suggests that improvements in productivity are related to changing the management practices of these plants. Davis, Haltiwanger, Handley, Jarmin, Lerner, and Miranda (2014) also find evidence of increases in total factor productivity after an LBO using another sample from the United States. Consistent with the findings in the United States, Harris, Siegel, and Wright (2005) find productivity increases from management buyouts in the United Kingdom.

Private equity involvement may also benefit firm innovation. Lichtenberg and Siegel find such effects in addition to the increases in productivity described above. In another study, Lerner, Sorensen, and Strömberg (2011) note that firm patents are more widely cited (which is a proxy for how economically important they are) and companies become more concentrated in their core areas of research after an LBO. Thus, empirical findings indicate that private equity delistings have a positive impact on the firm level. But what about the effects on the market and on society at large?

2.4 The Effect of Delistings on the Market

In the view of Jensen (1989), the introduction of private equity investment should improve the efficiency of the market as firms have to compete in a tougher business environment after one of their competitors is bought out and made more efficient. In the neoclassical framework, this should either lead to corresponding increases in productivity among competing firms, or that they are driven out of the market by more productive companies.

Indeed, Hsu, Reed, and Rocholl (2010) find that the stock market puts pressure on competitors to increase productivity after a company is bought by private equity firms. Further, Bernstein, Lerner, Sorensen, and Strömberg (2016) find that industries where private equity firms invest grow more quickly both in productivity and employment. This suggests such investment may have positive externalities.

However, there could also be negative effects on the market. With the underlying view that private equity has contributed to a shrinking stock market in the United States, Ljungqvist et al. (2016) study the consequences of this with a political economy model. They show that private actors and the general public are impacted differently by delistings. If private equity firms are good at picking future winners, which they are likely to be as they otherwise go out of business, they could engage in "cherry-picking," at the expense of the profits of investors in the wider economy. This could, in turn, lead to weakened support for free-market policies, as voters believe the top returns of the market all go to private equity firms. Ljungqvist et al. describe this effect as a negative externality of private equity on the economy.

Another potential negative side effect for society, at least in the short term, is that some workers may lose their jobs as a result of restructuring and efficiency improvements (Shleifer and Summers, 1988; Lichtenberg and Siegel, 1990; Davis et al., 2014). Olsson and Tåg (2015) show that unemployment incidence doubles for workers that perform tasks easy to automate or offshore, when firms with low productivity are bought out. This implies private equity's performance improvements may come at a cost to low-skilled workers.

Despite these effects, Dong (2015) finds that private equity firms successfully prepare delisted firms for re-listing in a secondary equity offering after restructuring and other firm improvements. Dong states that "the market can expect that the stand-alone public firm will operate effectively after the change in ownership structure associated with the exit of private equity." It is therefore likely there are positive societal effects of private equity, as well.

Ott (2004, 2008) argues that, because investors get to take part of the profits from corporate activity, a historic consensus was formed in the United States where voters supported business-friendly policies in exchange for stock market returns. Ljungqvist et al. point out that if this perspective is correct, private equity firms may decrease popular support for free exchange and thereby lead to long-term reductions in aggregate investment, productivity, and employment.

However, there could be other effects of LBOs as well. As we will see, when firms are bought out the pool of publicly traded companies shrinks. What effect the market size property will have on investors is not yet fully established in current research. We will attempt to investigate this property in the following sections.

To do this, we will need to develop a model using auction theory in Section 3. We provide an overview of a few helpful concepts for the reader in the remainder of this section, as well as comments on how they relate to our framework. To understand the market institution itself, let us first look at how auctions work.

2.5 Types of Auctions

Formally, an auction is "a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants" (McAfee and McMillan, 1987). We will outline the properties of the most important types of auctions below. In practice, there are a number of different types of auctions: English auctions, Dutch auctions, first-price auctions, and second-price (Vickrey) auctions. The concept of the all-pay auction has also been used in economic theory to model and explain various contests. Additionally, auctions can be divided into private value auctions, common value auctions, and interdependent auctions depending on which beliefs bidders have. These three categories are not different types of auctions, but rather derived from the underlying information structure available to bidders at the time of the auction. After we have described each of these types, we will also explain how insider information affects auctions.

In an English auction, the price of an item ascends. One option is for bidders to place increasing bids in public, overbidding the previous leader, until only one bidder remains at a price level only he is prepared to meet. Another option is that an auctioneer announces ascending prices, until all bidders but one have dropped out of the auction. Regardless of method, the winner pays the full amount of his bid. English auctions are often used to sell antiquities or art.[1]

In contrast to an English auction, a Dutch auction consists of an auctioneer announcing a bid so high that he expects no bidder to be prepared to pay the full amount. Then, the auctioneer successively lowers the price in increments until one bidder accepts the price. The first bidder to do so is the winner, and pays the full amount of his bid. Dutch auctions are not as widely used as English auctions, but have been used to sell flowers in the Netherlands, fish in Israel, and tobacco in Canada (McAfee and McMillan, 1987).

In first-price sealed-bid auctions, bidders privately submit bids on an item. This means that what others have bid is not public information, so it is not possible to overbid as in an English auction. A participant must therefore attempt to anticipate what others will do before placing his bid. The bidder with the highest bid wins the auction and pays the full amount of his bid.

[1] Interestingly, the word "auction" itself is derived from the Latin word augeō, meaning "I increase" (Krishna, 2002). This indicates that the ascending-price English auction should have been more common than the Dutch auction throughout history.

First-price sealed-bid auctions are strategically equivalent to Dutch auctions (Myerson, 1981; Riley and Samuelson, 1981), although in practice experiments have shown higher revenues for the seller in a sealed-bid auction (Cox, Roberson, and Smith, 1982). These auctions are often used for government procurement contracts, in which firms bid on the right to deliver products or services to government. In such cases, however, sellers attempt to underbid competitors rather than buyers attempting to overbid each other.

Wang (1991) shows how a first-price sealed-bid auction with private information can yield an equilibrium in which all bidders randomize their bid over particular intervals. He studies how the properties of the model change when the number of bidders increases, but only sets up a game with one auction. Since we are interested in investigating what happens when the number of simultaneous auctions changes (i.e. when the number of stocks available for purchase changes), Wang's approach is insufficient for our purposes and we will need a more extensive model.

In second-price sealed-bid auctions, too, bidders privately submit bids for an item. Unlike in first-price sealed-bid auctions, however, the winning bidder does not pay his own bid but rather the amount that the second-highest bidder proposed (Vickrey, 1961). These auctions are sometimes called Vickrey auctions. Second-price sealed-bid auctions are under certain conditions strategically equivalent to English auctions (Myerson, 1981; Riley and Samuelson, 1981), a fact that turns out to be important later on when we define our model in Section 3. Still, just as with first-price sealed-bid auctions, experimental evidence contradicts theory and shows higher revenue for the seller in the sealed-bid auction (Kagel, Harstad, and Levin, 1987).

While interesting from a theoretical standpoint, second-price auctions are rarely used directly in practice (Ausubel and Milgrom, 2006). However, in our framework described below, we argue that stock markets are analogous to continuous second-price auctions.

In an all-pay auction, all bidders must pay their bids regardless of whether or not they win (Milgrom, 2004). The highest bidder gets the item. These auctions are uncommon in practice, although they do exist (see e.g. Cobb, 2013). Instead, they are primarily studied by academics as an analogue when modeling political lobbying, bribery, or other conflicts. Here, revenues for sellers are typically higher than in other auctions (Dechenaux, Kovenock, and Sheremeta, 2015). Winning bids are often within 10 per cent of the item's value, or sometimes above it (Gneezy and Smorodinsky, 2006). Intuitively, increasing the number of bidders also increases the seller's revenue.

2.6 Private, Common, and Interdependent Value Auctions

In addition to which mechanism of auction is used, buyers can also have different structures of beliefs, which affects how auctions should be modeled.

In a private value auction, the value of an item is different among bidders. Each bidder's valuation is also independent of other bidders (Athey and Segal, 2013). This means that, given equal and sufficiently high allocation of funds, the bidder with the highest value of an item should win the auction. Of course, bidders do not always possess the same monetary resources, which gives rise to distortionary effects.

It is not entirely trivial to find examples of pure private value auctions in the real world, as most auctions contain both private and common value elements (see below for a discussion on interdependent value auctions). A hypothetical example of this type would be the following. A young child creates a drawing, and the child's parents bid on it. The drawing would have no market value, but some emotional value to each parent which would not necessarily be equal.

By contrast, in a common value auction the true value of the item sold is the same among all bidders - but bidders' information about its value may differ (Capen, Clapp, and Campbell, 1971). For example, a seller may auction off oil drilling rights to a plot of land. The true value of winning the auction is derived from the amount of oil that can be drilled, which is the same for all bidders, but bidders may have different estimates of this value. It is conceivable some bidders have obtained better information than others by analyzing geological data, or that they simply came to different conclusions of the value by guessing.

The basic property of bidders with different estimates of value gives rise to an interesting effect, known as the winner's curse (Thaler, 1988). Because the winner of the auction will be the bidder with the highest bid, whoever wins will normally be the one with the highest estimate of the value of the object. If estimates of value are distributed around a true value, the winning bidder will be the one who has most overestimated the item's true value.

The winner's curse cannot occur if all bidders are rational, because a rational agent will anticipate the effect and adjust his bids accordingly (Cox and Isaac, 1984). Yet, adjusting for the winner's curse has proven difficult for subjects in experimental studies (Bazerman and Samuelson, 1983; Samuelson and Bazerman, 1985; Lind and Plott, 1991). Charness and Levin (2009) suggest that the effect persists because it arises due to bounded rationality; people find it difficult to perform Bayesian updating or seeing through a complex problem. Therefore, the winner's curse is likely common in practice.

In the model we present in Section 3, underlying assets have a common market value that is the same to all players. However, not all players are equally well-informed about the true state of this value, which gives rise to different strategies for bidding.

Banerjee (2005) attempts to model asymmetry between bidders, and studies a common value first-price sealed-bid auction with two bidders. In the model, one bidder is better informed than the other. Banerjee describes why players may engage in aggressive bidding and what effect changes in the asymmetry has on revenue.

In this framework, however, there is only one auction. That means we cannot use this case to determine the impact of changes in market size, other than to generalize the result to a situation with multiple uncorrelated auctions. If the performance of all firms is completely uncorrelated, stock markets could be modeled as a number of parallel auctions where, for each firm, bidders receive asymmetric signals about the true value of the company's stock. Since the performance of firms would be uncorrelated, players' optimal strategies would be the same in all auctions (if there is no budget restriction or other constraints imposed). The case of uncorrelated auctions therefore reduces to the same problem as solving for one auction.

As we will see, this is not a sufficiently accurate description of stock markets. Not only do boom and bust cycles occur, in which all stocks rise or fall in value, but the performance of firms in the same sector may be correlated as well. To understand the effect of market size we instead need the value of stocks to be correlated in some manner. Later, we shall also see that first-price auctions are not a good enough approximation of stock markets because they are, as previously stated, strategically equivalent to Dutch auctions.

Let us finally look at interdependent value auctions. While most auctions contain a common value element, some types of auctions can have both common and private value elements. Kagel and Levin (2002) provide a painting as an example: while bidders may purchase the painting for personal enjoyment, which would constitute a private value element, there is also a resale market value of the painting for investment. This latter portion is the common value element of the painting. Therefore, in an interdependent value auction bidders' valuations ($v_i = v + u_i$) consist of one invariant element (v) and one idiosyncratic element (u_i).

Goeree and Offerman (2003) test a model with both common and private value elements, attempting to maximize the efficiency of the auction. They find that efficiency increases the smaller the difference is between the quality of the signals for bidders, and that efficiency also increases with a larger quantity of bidders.

2.7 Auctions with Insider Information

It is possible to imagine situations in which one bidder has an advantage over others with respect to information about the true value of the asset. In the most extreme case, one bidder knows the value of the item with certainty, while other bidders only have estimates drawn from some distribution.

Wilson (1967) investigates such a case. In a model where bidders only have access to public information, introducing one bidder with private, "insider," information, causes the seller's revenue to fall. The insider bids below the true value of the asset and earns positive profits at the expense of the seller. Using a different framework, Hendricks, Porter, and Wilson (1994), too, find that informed bidders are more likely to submit low bids. One reason for this could be the elimination of the winner's curse for a bidder who knows the value of the item with certainty.

In a similar asymmetric model with differences in the quality of the signal for two bidders, Hausch (1987) calculates equilibrium bidding strategies for both first-price and second-price auctions. He counter-intuitively shows that, under some conditions, it can be better for the seller that one bidder is more informed than others. The same result, that is, increased profits for the seller after the introduction of a bidder with insider information, is verified in an experimental setting by Kagel and Levin (1999). This raises interesting questions about how to align the incentives of the involved actors to avoid situations with insider information.

To this end, Boone, Chen, Goeree, and Polydoro (2009) perform a series of experiments to test how auction design affects the seller when there is a buyer with insider information. They find that the English auction yields the highest revenues and best protects uninformed bidders from losses out of the auctions tested. Singh and Povel (2004) suggest that an optimal auction, from the seller's point of view, needs to be biased against the bidder with insider information to yield optimal revenue.

Still, rather than using a true "insider" with perfect information, we will analyze a framework in which one bidder is somewhat more informed than the other. We argue this models the stock market better than introducing a bidder with perfect information as bidders that actually possess perfect information are forbidden to trade on it (and even if they engage in insider trading, they have to conceal it somehow or they quickly get in trouble with law enforcement agencies).

2.8 Our Contribution

What is missing in previous research is an investigation of whether or not investors are made better or worse off with a smaller number of publicly traded firms on the market, given that market capitalization and all other relevant variables are kept constant. Our contribution to the current state of knowledge consists of such an investigation, using a model building on auction theory, in the following sections.

Moreover, the results from our model constitute new findings that are generally applicable to auction theory. To the best of our knowledge, no one has previously investigated the properties of multiple correlated second-price sealed-bid auctions in the manner that we do, with bidders whose information sets differ. Thus, our results may be useful for future research in this area.

3 The Model

To be able to answer our research question, we first present a framework with which we can study changes in market size.

3.1 How to Approximate Stock Markets Using Auctions

As we have discussed above, in a first-price sealed-bid auction players submit their bids privately. The winner pays the full amount of his bid, and he does not know the bids of other players - only that he won. This means the final price paid, conditional on winning, does not depend on other bidders' valuations.

We argue that this is not an accurate characterization of the stock market. Successful bids, in the form of latest prices paid for a particular stock, are public information. This means that any investor can, if his valuation of the stock is higher than the last price paid for it, raise the amount that was last paid by making an offer himself that is higher. He acquires the stock, and the price goes up so that the next investor is likely to have to pay more to place a "bid." This view of stock market transactions is similar to the English, ascending-bid auction.

As we have seen, the English auction and the second-price sealed-bid auction are strategically equivalent (Myerson, 1981; Riley and Samuelson, 1981).[2] To see why, consider an English auction with 10 bidders that have asymmetric valuations v_i with $i \in [1, 10]$. Bidders will place ascending bids, and those who have lower valuations will progressively drop out. When only two bidders remain, bidder 9, with the second highest valuation will place his maximum bid: his valuation v_9. Bidder 10, with the highest valuation v_{10} will increase the bid by ε, and win the item at the price $v_9 + \varepsilon$. If ε is arbitrarily small, this is equivalent to the second-price sealed-bid auction; in equilibrium, bidders will submit their valuations v_i and bidder 10 will win the item - but only pay the second-highest bid, v_9.

Because of the equivalence of English auctions and second-price sealed-bid auctions, it is sufficient to show what results follow from a second-price sealed-bid auction. This type of framework is more convenient to model and thus preferable to a direct characterization of the English auction. Therefore, we will in the following model the stock market as a second-price sealed-bid auction.

We do, however, note that experiments such as the one by Kagel et al. (1987) we referred to in Section 2.5 find higher revenues in practice for the seller in second-price sealed-bid auctions than in English auctions. We should therefore use some caution when generalizing from our theoretical results to any real-world implications.

In our model, we will allow the value of objects, V_n, to take on negative values. While it is true that stocks in the real world cannot have a negative value (in case of company default without recoverable assets the value of the stock is 0), the value to investors in our model is the earnings they can retrieve after paying to acquire the stock. Thus, any stock values below 0 can be interpreted as the stock yielding less return than its expected value, resulting in a loss for the investor.

We will think of stock returns as normally distributed around their means. This is

[2] At least when there are only two players, see Krishna (2002).

consistent with previous financial research, historical returns, and what is usually assumed for convenience (Smith, 1997), although some scholars also argue the distribution of returns have fatter tails and other properties than the normal distribution (see e.g. Officer, 1972 or Taleb, 1997). Still, to simplify matters somewhat we assume returns are normally distributed as they will then have a meaningful interpretation for investors.

3.2 Model Specification

We formulate our model as a game of buying stocks in an exchange modeled as an auction. There are 2 idealized bidders, labeled $i = 1, 2$. We will think of the first as a private investor who is relatively uninformed and the second as an institutional investor with relatively better information. There are N objects, labeled $n = 1, ..., N$, each being auctioned in a second-price sealed-bid auction, meaning that the bidder with the highest bid wins but pays the second highest bidder's bid. In case of equal bids, we employ a tie-break rule in which the value is shared. The value of an object, V_n is distributed normally with mean r_n and standard deviation σ_n. Let also v be the random vector consisting of all the V_n. We assume that the covariance matrix of V is Σ, which then completely determines the joint distribution of the objects with mean value $r = (r_1, ..., r_N)$.

In order to make precise our notion of varying firm size, we make the following assumption: $\sum_{n=1}^{N} \mathbb{E}[v_n] = R_0$. The total market capitalization is thus fixed *a priori*. This ensures that while the total number of stocks in the market may vary, the market capitalization remains fixed, so that any effect we detect indeed stems from structural changes in the market and is not exogenous as a byproduct of a stronger overall market.

To model the differential information quality, we assume that each bidder i receives a signal $S_{i,n}$ where $S_{i,n}|V_n \sim N(V_n, \gamma_i)$. That is, the value of the object is observed but normally distributed noise with standard deviation γ_i. Thus, $\frac{1}{\gamma_i}$ can be thought of as the skill of an investor, assessing information available such as annual reports and other financial news. We also assume that the $S_{i,n}$ are conditionally independent for different n. This means that the covariance matrix of $S_i|V$ is $\Gamma_i = \text{diag}(\gamma_i^2)$. Moreover, we assume that, given the state of an asset, the signals for that asset are all conditionally independent, player-wise. This can be written more succinctly that the joint distribution (V, S_i, S_j) is distributed

$$N\left(\begin{pmatrix} r \\ r \\ r \end{pmatrix}, \begin{pmatrix} \Sigma & \Sigma & \Sigma \\ \Sigma & \Sigma + \Gamma_i & \Sigma \\ \Sigma & \Sigma & \Sigma + \Gamma_j \end{pmatrix}\right) \tag{1}$$

which is what we assume. This also has the more clear interpretation that the signal is simply a noisy observation of the value.

Denote by $\mathcal{S}_i$[3] all the information obtained by observing the signal vector $S_i =$

[3]Formally, this is the sigma-field generated by S_i.

$(S_{i,1}, ..., S_{i,N})^t$. We define $X_{i,n} = \mathbb{E}[V_n|\mathcal{S}_i]$ to be the conditional expectation of the value, having observed the signals. The strategy space is denoted B, with strategies (bids) $b_{i,n}$, which we for simplicity assume to be functions of the conditional expectation so that $b_{i,n} = b(X_{i,n})$.

4 Solving the Model

In the case of complete information, the second-price sealed-bid auction has an easily accessible equilibrium. However, since we have incomplete information, the standard argument from Vickrey (1961) no longer holds as winner's curse effects come into play. Instead, we get the following results.

Proposition 4.1. *There exists a symmetric continuously differentiable and monotone increasing bid function*

$$b(x) = \mathbb{E}[V_n|X_{i,n} = x, X_{i,n} = X_{j,n}],$$

that is a Nash equilibrium of the game.

Proof. The expected utility from the point of view of player i is

$$\mathbb{E}[u_{i,n}], \ \forall n.$$

By iterated expectation we have that $\mathbb{E}[u_{i,n}] = \mathbb{E}[\mathbb{E}[u_{i,n}|X_{i,n}]]$ so that it suffices to consider $\mathbb{E}[u_{i,n}|X_{i,n}]$ for each n. Let $\chi_{b(x)\leq b_{i,n}}$ be the indicator of the event $b(x) \leq b_{i,n}$ and write $Y_n^i = \max_{j\neq i} X_{j,n}$ for the random variable assuming the value of the maximal conditional expectation of the value for object n, where the maximum is taken over all players. We will now look for a bid function $b(\cdot)$ which applies for all players. To do this, suppose that all players $j \neq i$ play an increasing continuously differentiable bid function $b(\cdot)$ and write

$$\begin{aligned}
\mathbb{E}[u_{i,n}|X_{i,n}] &= \int_{-\infty}^{\infty} \mathbb{E}[u_{i,n}|X_{i,n}, Y_n^i = y] f(y|X_{i,n}) dy \\
&= \int_{-\infty}^{\infty} \big(\mathbb{E}[V_n|X_{i,n}, Y_n^i = y] - b(y)\big)\, \chi_{b(x)\leq b_{i,n}} f(y|X_{i,n}) dy \\
&= \int_{-\infty}^{b^{-1}(b_{i,n})} \big(\mathbb{E}[V_n|x_{i,n}, Y_n^i = y] - b(y)\big)\, f(y|X_{i,n}) dy
\end{aligned}$$

Differentiating with respect to $b_{i,n}$, the first order condition is

$$\frac{1}{b'(b^{-1}(b_{i,n}))}\Big(\mathbb{E}[v_n|X_{i,n}, Y_n^i = b^{-1}(b_{i,n})] - b(b^{-1}(b_{i,n}))\Big) f(b^{-1}(b_{i,n})|X_{i,n}) = 0$$

which holds precisely when

$$b_{i,n} = b(X_{i,n}) = \mathbb{E}[v_n | X_{i,n}, Y_n^i = b^{-1}(b_{i,n})] = \mathbb{E}[v_n | X_{i,n}, Y_n^i = X_{i,n}].$$

Note also that the first order condition changes sign at b and that it is negative to the right and positive to the left of the stationary point, so that this indeed is a (local) maximum. □

Remark 1. *The standard proof integrates over the maximum of the signals, not the conditional expectations of the values. However, as the signals are multi-dimensional this is no longer possible since monotonicity with respect to a vector is not well defined.*

Remark 2. *This continues to hold for more than* 2 *players. However, for computational reasons we will restrict our attention to the case with* 2 *players as described in the setup.*

The conditioning $X_{i,n} = Y_n$ takes into account the fact that if player i wins, he knows that he must have had a higher expectation about the value of the object than all his opponents but one.

For the in-depth analysis of the game at large, we need some results about the signal, found in Appendix A. However, before we proceed, let us first note the central importance of the signal. Fix an object n and consider what happens when the total number of auctions increases from N to $N+1$. Define $\mathcal{S}_N$ to be the relevant information set defined for N auctions. The following result states, simply put, that more information makes the best guess of the value of an object increasingly close to the true value of that object.

Proposition 4.2. *More information improves the accuracy of $X_{i,n}$:*

$$\mathbb{E}[|V_n - \mathbb{E}[V_n|\mathcal{S}_N]|^2] \geq \mathbb{E}[|V_n - \mathbb{E}[V_n|\mathcal{S}_{N+1}]|^2]$$

Proof. Intuitively obvious. This simply states that more signals on average conveys more accurate information. Geometrically, note that $\mathcal{S}_N \subseteq \mathcal{S}_{N+1}$. Hence the orthogonal projection[4] into the larger space is by construction closer to V_n than that of the smaller. □

4.1 Normality

Thus far, our results hold for arbitrary distributions of value and signal (and in fact the first two propositions hold for arbitrarily many players). Now, we will turn our attention to the specific Gaussian case described in the model.

[4] L^2 is the space of square integrable functions, or in our context, the space of random variables such that $\mathbb{E}[X^2] < \infty$. The conditional expectation operator in L^2 given a σ-field is an orthogonal projection of a random variable onto the measure space with that σ-field. For a thorough presentation of measure-theoretic probability, see Billingsley (2008).

Proposition 4.3. *The conditional expectation*

$$\mathbb{E}[V_n|X_{i,n}, X_{j,n}]$$

is an affine function of $X_{i,n}, X_{j,n}$. In particular, the equilibrium bid

$$b(x) = \mathbb{E}[V_n|X_{i,n} = x, X_{j,n} = x].$$

is affine in x.

Proof. We know that $S_i|V \sim N(V, \Gamma_i)$ and that $V \sim N(r, \Sigma)$. By Lemma A.3 it follows that $V|S_i \sim N((\Sigma^{-1} + \Gamma_i^{-1})^{-1}\Gamma_i S_i, (\Sigma^{-1} + \Gamma_i^{-1})^{-1})$ and in particular, we have that

$$X_i = \mathbb{E}[V|S_i] = (\Sigma^{-1} + \Gamma_i^{-1})^{-1}\Gamma_i^{-1}S_i + (\Sigma^{-1} + \Gamma_i^{-1})^{-1}\Sigma^{-1}r$$

or

$$X_{i,n} = e_n^t(\Sigma^{-1} + \Gamma_i^{-1})^{-1}\Gamma_i^{-1}S_i + e_n^t(\Sigma^{-1} + \Gamma_j^{-1})^{-1}\Sigma^{-1}r$$

where $\{e_n\}$ is the standard basis and e_n^t its transpose. Now, given that (V, S_i, S_j) is jointly normal with covariance matrix C, it follows that we can obtain the joint distribution $(V_n, X_{i,n}, X_{j,n})$ via

$$\begin{pmatrix} V_n \\ X_{i,n} \\ X_{j,n} \end{pmatrix} = \underbrace{\begin{pmatrix} e_n^t & 0 & 0 \\ 0 & e_n^t(\Sigma^{-1} + \Gamma_i^{-1})^{-1}\Gamma_i^{-1} & 0 \\ 0 & 0 & e_n^t(\Sigma^{-1} + \Gamma_j^{-1})^{-1}\Gamma_j^{-1} \end{pmatrix}}_{:=D_n} \begin{pmatrix} V \\ S_i \\ S_j \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ e_n^t(\Sigma^{-1} + \Gamma_i^{-1})^{-1}\Sigma^{-1}r \\ e_n^t(\Sigma^{-1} + \Gamma_j^{-1})^{-1}\Sigma^{-1}r \end{pmatrix}$$

which then is normal with covariance $D_nCD_n^t$ and mean $D_n(r^t, r^t, r^t)^t$. Explicitly

$$
\begin{aligned}
D_nCD_n^t = &\begin{pmatrix} e_n^t & 0 & 0 \\ 0 & e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}\Gamma_i^{-1} & 0 \\ 0 & 0 & e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}\Gamma_j^{-1} \end{pmatrix} \\
&\begin{pmatrix} \Sigma & \Sigma & \Sigma \\ \Sigma & \Sigma+\Gamma_i & \Sigma \\ \Sigma & \Sigma & \Sigma+\Gamma_j \end{pmatrix} \\
&\begin{pmatrix} e_n & 0 & 0 \\ 0 & (\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & 0 \\ 0 & 0 & (\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \end{pmatrix} \\
= &\begin{pmatrix} e_n^t & 0 & 0 \\ 0 & e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}\Gamma_i^{-1} & 0 \\ 0 & 0 & e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}(\Gamma_j)^{-1} \end{pmatrix} \\
&\begin{pmatrix} \Sigma e_n & \Sigma(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & \Sigma(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \\ \Sigma e_n & (\Sigma+\Gamma_i)(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & \Sigma(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \\ \Sigma e_n & \Sigma(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & (\Sigma+\Gamma_j)(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \end{pmatrix} \\
= &\begin{pmatrix} e_n^t\Sigma e_n & e_n^t\Sigma(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & e_n^t\Sigma(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \\ e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}(\Gamma_i)^{-1}\Sigma e_n & e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}(\Gamma_i)^{-1}(\Sigma+\Gamma_i)(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}\Gamma_i\Sigma(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \\ e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}\Gamma_j\Sigma e_n & e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}(\Gamma_j)^{-1}\Sigma(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}(\Gamma_j)^{-1}(\Sigma+\Gamma_j)(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \end{pmatrix}
\end{aligned}
$$

We thus obtain by an application of Lemma A.2 that

$$
\begin{aligned}
\mathbb{E}[V_n|X_{i,n}, X_{j,n}] =& r_n + \begin{pmatrix} e_n^t\Sigma(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & e_n^t\Sigma(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \end{pmatrix} \\
&\begin{pmatrix} e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}(\Gamma_i)^{-1}(\Sigma+\Gamma_i)(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & e_n^t(\Sigma^{-1}+\Gamma_i^{-1})^{-1}(\Gamma_i)^{-1}\Sigma(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \\ e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}(\Gamma_j)^{-1}\Sigma(\Gamma_i^t)^{-1}((\Sigma^{-1}+\Gamma_i^{-1})^{-1})^t e_n & e_n^t(\Sigma^{-1}+\Gamma_j^{-1})^{-1}(\Gamma_j)^{-1}(\Sigma+\Gamma_j)(\Gamma_j^t)^{-1}((\Sigma^{-1}+\Gamma_j^{-1})^{-1})^t e_n \end{pmatrix}^{-1} \\
&\left(\begin{pmatrix} X_{i,n} \\ X_{j,n} \end{pmatrix} - \begin{pmatrix} r_n \\ r_n \end{pmatrix}\right)
\end{aligned}
$$

□

Using above, we are ready to state an inequality about the sum of utilities. The result indicates what follows: Increasing the number of objects to be auctioned will in general reduce the inequality in returns between players. Heuristically, the reason for this is that the marginal utility of more information weighs more heavily for the less informed player but also because total utility tends to decline with more objects listed.

Proposition 4.4. *The sum of both players' utilities is in equilibrium bounded by*

$$0 < \mathbb{E}[u_{1,n} + u_{2,n}] \leq \mathbb{E}[\max(|V_n - b(X_{1,n})|, |V_n - b(X_{2,n})|)].$$

Hence, the domain for positive profits is shifting downward to zero as N increases.

Proof. We now begin by investigating what happens with the sum of expected utilities as the number of firms increases. Note that the event $\bar{X}_{i,n} = \bar{X}_{j,n}$ almost surely never occurs (i.e. it occurs with probability zero). This allows us to ignore tie-break rules and this degenerate event in the analysis that follows since it has no bearing on the expected utility.

Note that for any ω in the sample space either

$$u_{1,n} + u_{2,n} \leq V_n - b(X_{1,n})$$

or

$$u_{1,n} + u_{2,n} \leq V_n - b(X_{2,n}).$$

Hence for all ω

$$u_{1,n} + u_{2,n} \leq \max(V_n - b(X_{1,n}), V_n - b(X_{2,n)}, 0) \leq \max(|V_n - b(X_{1,n})|, |V_n - b(X_{2,n})|)$$

and taking expectations yields

$$\mathbb{E}[u_{1,n} + u_{2,n}] \leq \mathbb{E}[\max(|V_n - b(X_{1,n})|, |V_n - b(X_{2,n})|].$$

Thus we have established the upper bound.

To see that for any fixed N expected utilities are indeed positive, note that they are non-negative since

$$\mathbb{E}[u_{i,n}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Big(\mathbb{E}[V_n|x_{i,n}, x_{j,n}] - b(x_{j,n})\Big)\chi_{b(x_{i,n})\geq b(x_{j,n})} f_{X_{i,n},X_{j,n}}(x_{i,n}, x_{j,n})dx_{i,n}dx_{j,n}$$

and if we just consider the inner integral

$$\int_{-\infty}^{\infty} \Big(\mathbb{E}[V_n|x_{i,n}, x_{j,n}] - b(x_{j,n})\Big)\chi_{b(x_{i,n})\geq b(x_{j,n})} f_{X_{i,n},X_{j,n}}(x_{i,n}, x_{j,n})dx_{i,n}$$

we note that the integrand must be non-negative for each $x_{i,n}$ for else bidding zero would result in a better outcome, contradicting the fact that b is a Nash equilibrium. Finally, the integrand is non-zero and thus positive since $\mathbb{E}[V_n|x_{i,n}, x_{j,n}]$ is almost nowhere equal to $b(x_{j,n})$ since they are both affine functions but with different coefficients. □

What this means is that as the number of objects N increases, the possibility to make positive profits decreases in equilibrium. This does not without qualification say that the profits actually decrease as N increases for every possible step. However, we can state the following:

Proposition 4.5. *Suppose that as $N \to \infty$, $|V_n - b(x_{i,n})| \to 0$.*[5] *Then for every N' there exists an $N'' > N'$ such that the (normalized) sum of utilities at N'' are smaller than at N'.*

[5] This occurs for instance if all stocks always are sufficiently correlated.

Proof. Just observe that the sum of profits are bounded by intervals lying strictly in the positive half-space for each N. However, as $N \to \infty$ this interval converges to the point 0, hence by monotonicity of the accuracy, there exists an N'' such that the lower bound for N' exceeds the upper bound for N''. We write normalized because $\mathbb{E}[V_n]$ is not fixed in general, due to the assumption of total market size being fixed, whereas individual objects are not. $\square$

We can think of this as a monotonicity property but with the caveat that the step-length is not unity. Given the non-differentiability of our main parameter of interest, N - the number of objects listed, the problem of comparative statistics is naturally more difficult, other than the inequalities derived above. To investigate the problem in closer detail, we rely on the assumption of normality and then proceed with numerical estimation of utilities.

Now, we know that in the equilibrium of Proposition 4.1, the expected utility is of the form

$$\mathbb{E}[u_{i,n}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Big(\mathbb{E}[V_n|x_{i,n}, x_{j,n}] - b(x_{j,n})\Big)\chi_{b(x_{i,n})\geq b(x_{j,n})} f_{X_{i,n},X_{j,n}}(x_{i,n}, x_{j,n}) dx_{i,n} dx_{j,n}.$$

With the results of Proposition 4.3, all functions in the above integral are now explicitly known, so we are ready to evaluate the equilibrium utilities numerically.

4.2 Numerical Results and Figures

To evaluate our analytical results in the context of some plausible parameters, we use a numerical approach modeled in Matlab. We include the code used for evaluation in Appendix B below.

We specify the average correlation between stocks as 0.2, which is a cautious estimate given historical trends. We set the average return of a given stock to $r_n = 1$, and the average standard deviation (the root of the diagonal elements of Σ) to 0.6. As before, we keep the total market capitalization fixed when we vary the number of stocks. This is done by only evaluating the utility derived from one stock, $u_{i,n}$. Investor skill (γ_i) is shown on the plots below as the x and y axes (but note that investor skill really is $\frac{1}{\gamma_i}$).

All figures are generated using the averages of expected utilities over several auctions with random covariance matrices.

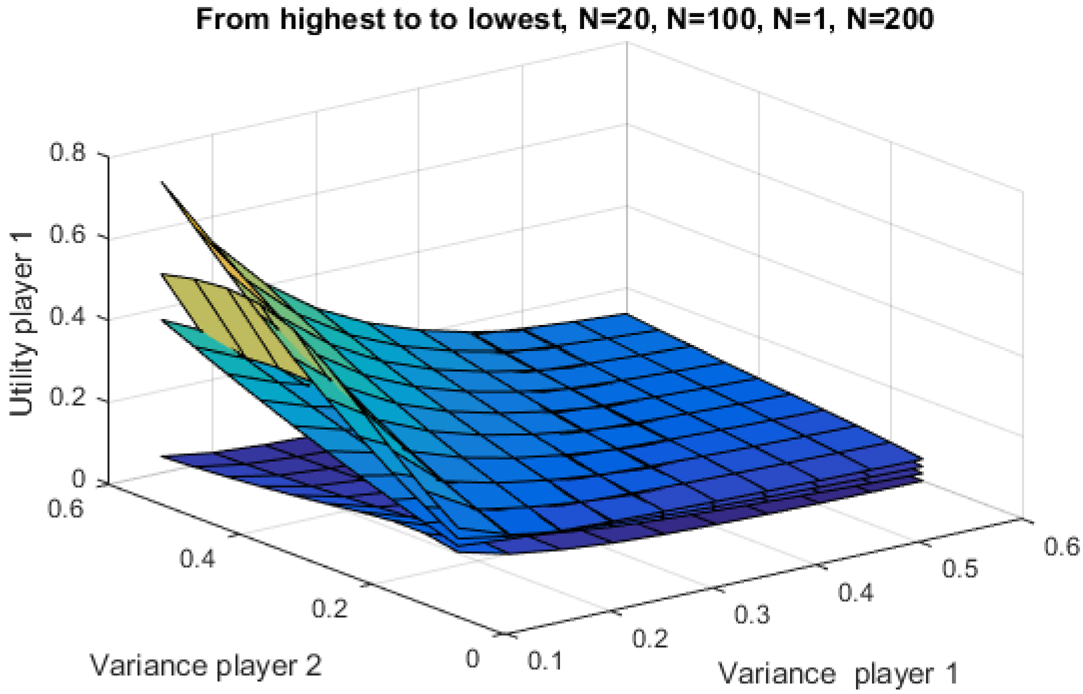

Figure 4.1. *Investor returns (utilities) for different market sizes, with N set to 1, 20, 100, and 200. Note how returns for player 1 vary with the investor skill of both players.*

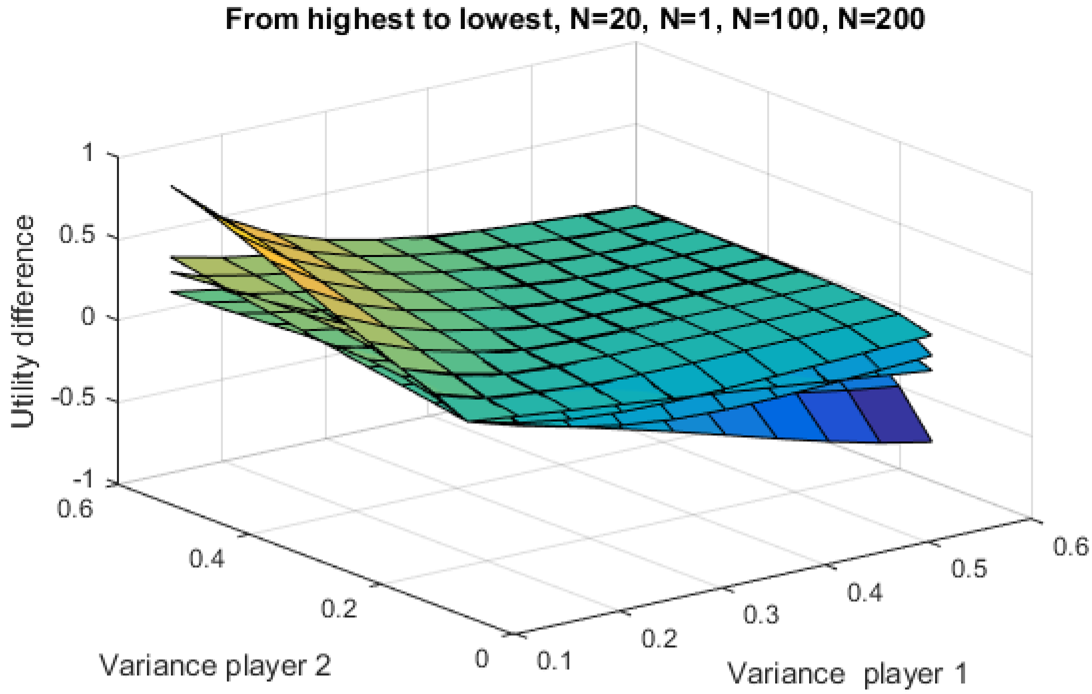

Figure 4.2. *The differences in investor returns (utilities) with N set to 1, 20, 100, and 200 from the perspective of player 1. Positive values indicate higher returns for player 1 relative to player 2.*

4.3 Interpretation

We note that the distribution of utility is not strictly monotone, but only approximately so. We know from Proposition 4.5 that for any N' we choose, it is possible to choose another N'' that yields a smaller sum of utilities. We can see from our graphs above that this N'' is not necessarily $N' + 1$. Instead, one may need to increase the number of stocks further to find such an N''.

When N becomes sufficiently large, the differences in utilities between the two players decrease. We can see this in the graphical representations above when we increase N. This means that the inequality, defined as the difference in returns between the players, decreases in N (although, as we saw above, sometimes this occurs non-linearly).

For small increases in N, going from, say, $N = 1$, increasing the number of stocks makes everyone better off. However, when N becomes large (at approximately the point of 100 or so firms), the inequality in returns falls. This is primarily driven by the fact that the information sets of the institutional investor and the private investor converge. With more accurate information, obtained through the correlations between stocks, the private investor is able to make a better estimate of the value of any given stock and the ability of the institutional investor to make positive profits at the expense of the private investor falls.

As N increases sufficiently much, the combined profits of both investors fall. When each investor approaches perfect information he will know with increasing certainty what to bid, which successively removes the ability of the other investor to profit at his expense. In the limit, there are no profits for either player - and thus no inequality in returns. Past the maximum, inequality is decreasing in N because overall profits decrease.

5 Discussion of Main Results

As we have shown, when the market becomes very fractioned (N is large and grows even larger) the ability of investors to make profits declines. As N grows large investors' information becomes asymptotically perfect. Essentially, this can be interpreted as our model being (asymptotically) consistent with contemporary financial models, which usually bar profits in equilibrium. Heuristically, $N \to \infty$ implies that neither player will be making a profit contrary to the economic intuition we described in the introductory section. Less choice can therefore be good for investors.

However, for small N, an increase in N yields larger profits for both players. We see that an increase in N at the low end is unambiguously good for both players, because it improves their available information as long as the correlation between stocks is not 0. This means that the effect of an increase in N depends on how large the current market is (in terms of number of stocks). Each player's utility is a second-order function of N in which utility first increases when N goes up initially, but then falls after a peak and goes asymptotically to 0 as $N \to \infty$. Where the peak is depends on the players' signal strengths, the probability of stocks to be of high value, and the correlation between stocks.

In general, the returns of an investor changes due to two factors. First, as N increases, his own information becomes better and so his returns improve. Second, simultaneously, the information of his opponent improves as well, which yields the opposite effect. The aggregates of these effects may have different signs for different investors, implying that there is a collection of N such that there is a strict trade-off between each player's utility.

Note that we can consider buying and selling as two sides of the same market. Every object bought is also an object sold, and with some abstraction we can think of selling a stock as buying a contract to sell a stock. This, then, could be thought of as an object in our auction model. Since every profit realized for the buyer is a symmetric loss realized for the seller, we essentially have a zero-sum game. Total utility, $\mathbb{E}[u_1 + u_2]$, is therefore not an interesting concept in our model but instead consider the return inequality between buyer and seller returns. From this point of view, we can focus solely on return inequality as a function of the number of objects listed, N. Ignoring any predictions about total utility, the model then says that large N is good for a "fair" stock market (if we think it is fair that buyers and sellers make similar returns). Stock market consolidation should therefore, in our framework, lead to increased inequality of returns given that the market starts in a sufficiently fragmented state. A caveat, however, is that prices are formed according to buyer expectations and not seller expectations. This makes the symmetry mentioned above a bit less straightforward.

We noted in Section 2 that Jensen (1989) argues the public corporation has outlived its usefulness, with the implicit assumption that more private ownership is a sign of more developed financial markets. If he is correct, we should see the trend with substantial decline in publicly traded firms continue. Our findings indicate that this might, at least initially, be good for investors. When the number of firms, N, decreases from a high level, investors are better off in our framework. However, if N drops sufficiently low, a further decrease could become bad for investors.

Regardless of its impact on the market, further private ownership may be good for firm owners as a way to mitigate the FCF problem as described by Jensen (1986). Less wasteful spending with better corporate control is unambiguously good for investors, regardless of the effect of delistings on the market.

Finally, a word of caution for interpreting our model. Note that we assumed firm size does not affect pricing power. This is inconsistent with the conventional literature in industrial organization, in which the the traditional paradigm of Bain (1951) states that market power is higher in more concentrated markets. Many studies have been made to verify this property (see e.g. Schwartzman, 1959; Miller, 1967; Levy, 1984; Bresnahan, 1989). In the cases where the mechanism driving delistings consists of mergers and acquisitions rather than buyouts, market concentration should increase. We would expect this to have an effect on the pricing power of firms, which should increase profits in oligopoly markets. To avoid overcomplication in our model we did not include this property, but it may be important to keep in mind when interpreting our results.

We also assumed the risk profile of firms is constant even when varying their size. This, too, may be an oversimplification. For example, as firms increase in size they may become less vulnerable to short-term fluctuations in the market due to better availability of spare capital, which would affect their risk profile. They could also conceivably become more bureaucratic and slower to adapt to new market trends, which would instead increase risk. When the average firm increases in size, the relative weight of the average stock in investors' portfolios also increases. This means higher idiosyncratic risk[6] because diversification decreases. Investors are thus more fragile to the misfortunes of individual firms. An empirical analysis of the impact of consolidation would do well to take this property into account. Could there be other areas in which future researchers may improve on our work?

5.1 An Agenda for Further Research

There are a number of potentially interesting changes or extensions to our model. One such extension would be to incorporate a mechanism for acquiring information at a cost. It is conceivable to imagine that the acquisition process for information is not free: institutional investors need to pay wages to analysts, and private investors must dedicate time to analyze companies. Building on our framework, one could design a mechanism where players must pay to receive a signal about a particular stock. Future research could then determine under which conditions it is rational for players to acquire the signal.

Another way to extend our model would be to increase the number of players. Clearly, in any real-world stock market there are more than two players. We chose to look at two players so that we could consider them representatives of their different types, with one more informed institutional investor and one less informed private investor. However, not all investors of one type are homogeneous. While Lehman Brothers may have gone bankrupt in 2008 during the subprime mortgage crisis, certainly not all financial institutions had made equally poor bets. Moreover, some private investors may have better information than some banks. For better fidelity to reality, it would therefore be interesting to increase the number of players and keep them heterogeneous with respect to signal strength.

Our intuitive prediction about a model with many players, however, is that returns should be decreasing in the number of players for two reasons: the winner's curse when

[6] The risk that one individual stock underperforms, as opposed to systemic or market risk of an event impacting the whole market.

the stock is of low value, and lower returns when the stock is of high value.

Consider the case where the stock is of low value. Increasing the number of players increases the probability that at least one player will have overestimated the value of any given stock, and since Proposition 4.1 still holds in such a model, there is an effect leaning towards that the player with the highest valuation wins the object. This creates the possibility for losses due to overbidding.

Even in cases where the stock is of high value returns will be lower, because a larger number of total players means more players will have received positive signals about this particular stock and are willing to place high bids. Thus, we expect the final price paid to go up compared to our model where only two players bid.

Finally, future research could investigate if our results hold under other types of auctions than second-price sealed-bid auctions. While we used this type because it is strategically equivalent to English auctions (in the case of only two players) and therefore a good approximation of stock markets, it could be interesting to look at what happens in a scenario with first-price auctions.

5.2 Summary and Conclusion

In this thesis, we have attempted to investigate what happens to private and institutional investors when the number of publicly listed firms decreases. Our results indicate that the effect is ambiguous and depends on the current number of listed stocks. When this number is high, a decrease in listed stocks increases the returns of both private and institutional investors. When this number is low, a decrease leads to lower returns for both types of investors. Where the cut-off point, and thus the peak in total utility, is depends on the strength of each player's information (i.e. his skill as an investor), as well as how correlated stocks are.

Our findings are also interesting from a purely auction-theoretic point of view. To the best of our knowledge, we are the first to investigate the properties of multiple second-price sealed-bid auctions with inter-auction correlations, where the information sets of bidders differ. Our results may therefore be useful for further research in this area; other scholars could build on the model we designed and introduce new elements to better fit it with how investors behave in the real world. Moreover, these types of auctions could have other applications outside the field of financial economics, in situations where heterogeneous actors place bids on multiple items simultaneously. In such cases, too, our findings will hopefully be of use.

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A Gaussian Probabilities

Lemma A.1. *Let* $X \sim N(\mu, \Sigma)$ *be multivariate normal where* $X = (X_1, X_2)'$, $\mu = (\mu_1, \mu_2)'$ *and with*

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma 22 \end{pmatrix} \tag{2}$$

Then if X_1, X_2 *are of the same dimension we have that* $X_1 - X_2 \sim N\Big(\mu_1 - \mu_2, \Sigma_{11} + \Sigma_{22} - \Sigma_{12} - \Sigma_{21}\Big)$.

Proof. We can write $X_1 - X_2 = BX$ where $B = (I, -I)$. Hence the covariance is $B\Sigma B' = \Sigma_{11} + \Sigma_{22} - \Sigma_{12} - \Sigma_{21}$. □

Lemma A.2. *Let* $X \sim N(\mu, \Sigma)$ *be multivariate normal where* $X = (X_1, X_2)'$, $\mu = (\mu_1, \mu_2)'$ *and with*

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \tag{3}$$

Then $X_1 | X_2 \sim N\Big(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\Big)$.

Proof. Let X_1 be the first partition and X_2 the second. Now define $Z = X_1 + AX_2$ where $A = -\Sigma_{12}\Sigma_{22}^{-1}$. Now we can write

$$\begin{aligned} \mathbb{CV}(Z, X_2) &= \mathbb{CV}(X_1, X_2) + \mathbb{CV}(AX_2, X_2) \\ &= \Sigma_{12} + A\,\mathbb{V}(X_2) \\ &= \Sigma_{12} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{22} \\ &= 0 \end{aligned}$$

Therefore z and x_2 are orthogonal and thus independent.[7] Next, $\mathbb{E}(Z) = \mu_1 + A\mu_2$. Thus, it follows that

$$\begin{aligned} \mathbb{E}(X_1 | X_2) &= \mathbb{E}(Z - AX_2 | X_2) \\ &= \mathbb{E}(Z | X_2) - \mathbb{E}(AX_2 | X_2) \\ &= \mathbb{E}(Z) - AX_2 \\ &= \mu_1 + A(\mu_2 - X_2) \\ &= \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_2) \end{aligned}$$

[7] This is a property of joint normality.

This establishes the first part. Next, for the covariance matrix

$$\begin{aligned}
\mathbb{V}(X_1|X_2) &= \mathbb{V}(Z - AX_2|X_2) \\
&= \mathbb{V}(Z|X_2) + \mathbb{V}(AX_2|X_2) - A\,\mathbb{CV}(Z, -X_2) - \mathbb{CV}(Z, -X_2)A^t \\
&= \mathbb{V}(Z)
\end{aligned}$$

Finally,

$$\begin{aligned}
\mathbb{V}(X_1|X_2) = \mathbb{V}(Z) &= \mathbb{V}(X_1 + AX_2) \\
&= \mathbb{V}(X_1) + A\,\mathbb{V}(X_2)A^t + A\,\mathbb{CV}(X_1, X_2) + \mathbb{CV}(X_2, X_1)A^t \\
&= \Sigma_{11} + \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{22}\Sigma_{22}^{-1}\Sigma_{21} - 2\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \\
&= \Sigma_{11} + \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - 2\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \\
&= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}
\end{aligned}$$

□

Lemma A.3. *Suppose that* $X_1 \sim N(\mu_1|\Sigma_1)$ *and* $X_2|X_1 \sim N(X_1|\Sigma_2)$. *Then* $X_2 \sim N(\mu_1, \Sigma_1 + \Sigma_2)$ *and* $X_1|X_2 \sim N\Big((\Sigma_1^{-1} + \Sigma_2^{-1})\Sigma_2 X_2 + (\Sigma_1^{-1} + \Sigma_2^{-1})\Sigma_1\mu_1, (\Sigma_1^{-1} + \Sigma_2^{-1})\Big)$

Proof. See Bishop (2006). □

B Matlab Code

The main code we use:

```
% runme4.m
clear all
close all
format short
global an bn signinv r n
nmax=50;
corr=0.2;
Sig=makeSigma2(nmax,corr);
n=1;  % object number
x=1.; y=1.;
imagelist=[1,3,10,30];
miter=10;
skillstep=0.05;
qq=zeros(miter,miter,length(imagelist));
tic
for k=1:length(imagelist)
    nn=imagelist(k);
    r=2*ones(1,nn);
    q=zeros(miter,miter);
    skill1=zeros(1,miter); skill2=skill1;
    for ii=1:miter
        skill1(ii)=0.1+ii*skillstep;
        for jj=1:miter
            skill2(jj)=0.1+jj*skillstep;
            Gami=makeGamma(nn,skill1(ii));
            Gamj=makeGamma(nn,skill2(jj));
            [result,an,bn,signinv]=calcC(Sig(1:nn,1:nn),Gami,Gamj,n,r,x,y);
            q(ii,jj)=integral2(@integrand,-15,15,-15,15);
        end
    end
    qq(:,:,k)=q;
    [XX,YY]=meshgrid(skill1,skill2);
    surf(XX,YY,q-q')
    xlabel('Variance  player 1')
    ylabel('Variance player 2')
    zlabel('Utility difference')
    title(['N=',num2str(imagelist(k)) ])
    pause(1)
    hold on
end
toc
save(datestr(now))

```

```
%% make plots separately below
k=8;
q=qq(:,:,k);
    [XX,YY]=meshgrid(skill1,skill2);
    surf(XX,YY,q) % or surfc(XX,YY,q)
    xlabel('Variance player 1')
    ylabel('Variance player 2')
    zlabel('Utility, u_1_n')
    title(['N=',num2str(imagelist(k)) ])
```

This generates the covariance matrix of objects:

```
% makeSigma.m
function Sig=makeSigma2(n,corr)
Sig=zeros(n);
s=0.1+0.5*rand(1,n);
s(1)=0.7;   % fix volatility of share to look at
for ii=1:n
    Sig(ii,ii)=s(ii)^2;
    for jj=ii+1:n
        Sig(ii,jj)=s(ii)*s(jj)*2*corr*(rand-0.5);
        Sig(jj,ii)=Sig(ii,jj);
    end
end
```

We generate the investor skill matrix, Γ:

```
function Gam=makeGamma(n,skill)
Gam=skill*eye(n);
```

Then, we generate bid and conditional expectation of the value used in the expected utility integral:

```
% calcC.m
function [out,an,bn,signinv]=calcC(Sig,Gami,Gamj,n,r,x,y)
nn=length(Sig);
qi=(inv(Sig)+inv(Gami));
qj=(inv(Sig)+inv(Gamj));
t1=Sig*(Gami')*qi';
```

```
t2=Sig*(Gamj')*qj';
m11=qi*(Gami)*(Sig+Gami)*(Gami')*qi';
m12=qi*(Gami)*Sig*(Gamj')*qj';
m21=qj*(Gamj)*Sig*(Gami')*qi';
m22=qj*(Gamj)*(Sig+Gamj)*(Gamj')*qj';
p1=(qi*inv(Gami))*r';
p2=(qj*inv(Gamj))*r';

signinv=inv([m11(n,n),m12(n,n);
      m21(n,n),m22(n,n)]);
out=r(n)+[t1(n,n),t2(n,n)]*signinv*[x-r(n);y-r(n)];

an=[t1(n,n),t2(n,n)]*inv([m11(n,n),m12(n,n);m21(n,n),m22(n,n)]);
bn=r(n)-an*[r(n) ; r(n)];
```

Finally, we generate the integrand in the expected utility integral:

```
function out=integrand(xx,yy)
global an bn signinv r n
x=xx-r(n);
y=yy-r(n);
psi=exp(-0.5*(signinv(1,1)*x.^2+2*signinv(1,2).*x.*y+signinv(2,2)*y.^2));
psi=psi*sqrt(det(signinv))/(2*pi);
out=(x>y).*psi.*an(2).*(x-y);
```