The Risk of Mini Flash Crashes

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ABSTRACT

This paper examines unique data on mini flash crashes in the American stock market in the time period ranging from 3 January 2006 to 3 February 2011. Data shows an autoregressive behaviour in the number of mini flash crashes (stock-day observations). However, the behaviour is complex and might differ a lot among individual stocks. Furthermore, results showed that the price change during a mini flash crash is bigger on Fridays and on days with abnormally negative daily stock returns. The price change also has a positive correlation with the number of sequential ticks during the crash.

In the extended version "Mini Flash Crashes in Logit Models" the findings about the autoregressive behaviour were supported. In addition, the probability of at least one mini flash crash was also higher on days with extremely negative returns.

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1. Introduction

"`We'd all sit there and stare at the screen and I'd have my finger over the Enter button. I'd count out loud to five…
`One…´
`Two… See, nothing's happened´
`Three… Offers are still there at forty-eight…´
`Four… Still no movement.´
`Five´ Then I'd hit the Enter button and –boom!- all hell would break loose. The offerings would all disappear, and the stock would pop higher."
At which point he turned to the guys standing behind him and said, "You see, I'm the event. I am the news."

This section is taken from the book "Flash Boys" (Lewis [2014]) and describes the moment when a trader starts to understand that he is being exploited by high frequency traders. In this very example the high frequency trader is using the time lag between when information arrives at different stock exchanges and the fact that he can send information faster.

Technological advancements have led to a lot of changes in the stock market over the years. Some for the better, some for the worse. As a consequence, many new trading strategies and phenomena have arisen. One of these phenomena is the mini flash crash. A mini flash crash is when the price of an individual security decreases or increases significantly and then returns to its original value within fractions of a second. Even though the price recovers and the event takes place within the blink of an eye, it can still be harmful to market participants. Imagine for example if a mini flash crash is triggered by a trader putting a market order to sell. The price before and after would be the same, but the average sell price for the trader would be somewhere between the price when he put the order and the lowest price during the crash. Furthermore, market participants using stop-losses might end up selling on local bottoms because of these temporary price changes. Some path dependant derivatives may also be very affected by the phenomena of mini flash crashes.

The topic of flash crashes is not widely researched and a lot of the previous work has been about finding out what triggered the big market wide flash crash of 2010. However, when it comes to the predictability of crashes, there are three papers that I want to highlight. Easley, López de Prado and O'Hara (2011) analysed the 2010 Flash Crash and found that it could have been predicted by measuring the increased levels of order toxicity. Aldridge (2015) shows that flash crashes are anticipated by an increase in the relative duration of negative runs compared to positive runs. Barany et al. (2012) observed patterns in the behaviour of the long term memory effect parameter connected to crashes. All of them suggest a certain predictability of crashes.

More knowledge about the behaviour of mini flash crashes is needed to make market participants able to protect themselves from the potential harm which the crashes can cause. In the attempt to shed some more light on the subject, my study will address the following research questions:

1. Under what conditions should one expect an increased occurrence of mini flash crashes?
2. What determines how big the price movement will be when a mini flash crash occurs?

In this paper I have studied a unique set of mini flash crashes in the American market between 3 January 2006 and 3 February 2011. From this data I have selected a sample of 23 stocks based on their number of flash crashes within the sample period (only stocks with more than 30 mini flash crashes). In this paper a mini flash crash needs to fulfil the following conditions:

1. The price movement has to be bigger than 0.8% up or down
2. The stock has to tick up/down at least 10 times before if recovers
3. The total time for both the crash and the recovery is maximum 1.5 seconds

The first part of the analysis was done with daily observations and a time series regression for each stock followed by a meta-analysis (Hunter-Schmidt method). The results showed that the number of flash crashes have an autoregressive behaviour for many individual stocks. This was particularly true for stocks with the highest number of flash crashes in the sample. Analysis of periods of mini flash crashes showed that after a certain stock has experienced mini flash crashes multiple days over a short time period, the risk of more mini flash crashes the coming days is even higher than after only one day of mini flash crashes. In addition, the risk of an extreme number of flash crashes in one day increases with the length of the flash crash period. This further supports the autoregressive behaviour, but also shows its complex nature and the need to look more than one day back to get enough information to understand the risk of flash crashes the next day. The last part of my study examines the absolute price change during a mini flash crash by looking at individual crashes in the selected stocks (1186 observations). The effect from selected factors were tested with an OLS regression. Results showed that the absolute price change is bigger on Fridays and on days with extremely negative stock returns. It also increases with the number of sequential ticks during the crash. Comparing my work to previous research there are three important differences which I think add value to the field of flash crashes:

1. Easley, López de Prado and O’Hara (2011) look at “The 2010 Flash Crash”, Barany et al. (2012) at crashes and Aldrige

(2015) at flash crashes (intraday basis). I look at mini flash crashes (fractions-of-a-second basis).

2. The intuitive risk assessment of mini flash crashes might be easier with less complicated variables like weekdays, daily stock return and the number of mini flash crashes yesterday.
3. My work does not only look at the occurrence of crashes, but also analyses the size of the price change during the crash.

2. Historical Background

2.1 The 2010 Flash Crash

May 6 2010, many people were shocked when the US market experienced one of its biggest intraday drops. Big equity indices were already down approximately 4% when they fell additionally 5% (CFTC-SEC [2010]). However, the additional drop was recovered within minutes. This event has become what most people think about when someone says "flash crash". Many traders, researchers and government employees have tried to understand what could have caused a trillion dollar market to crash and recover in a time as short as 36 minutes. Just after the crash happened some people thought it was triggered by a "fat-finger trade" in Procter & Gamble. That theory was rejected shortly after when the timeline of the events was mapped (Philips [2010]). Another possible factor that could have contributed to the flash crash is technical problems with reporting at NYSE and ARCA combined with delays in the consolidated tape (Flood [2010]). Furthermore, big short positions were taken just before the crash, such as the hedge fund Universa Investments buying put options (Patterson and Lauricella [2010]) and Wadell & Reed selling E-Mini contracts. That the latter was really contributing to the crash was disputed by the CME Group (2010). Some argued that changes in the currency market, more specifically the U.S. dollar/Japanese yen exchange rate, could be another underlying factor of the flash crash (Krasting [2010]). Many of the theories came shortly after the crash, but more recently fingers have been pointed to a single trader, Navinder Singh Sarao. He was charged with several crimes, among them the use of a spoofing algorithm (US Department of Justice [2015]). Spoofing is putting a buy or a sell order with the intention to cancel the order before it is filled. It is a way of trying to fool other market participants and manipulating the price of a security. Before the flash crash Sarao placed orders in E-mini contracts betting on that the market would fall. The orders had a total value of 200 million dollar and were replaced or amended 19000 times before they were cancelled. Even though Sarao’s spoofing algorithm probably was an important trigger, it is hard to give him full responsibility for the crash of a trillion dollar market. It is reasonable to believe that many factors combined must have caused the total harm on the market.

2.2 Mini Flash Crashes

The 2010 flash crash was a very unlikely event and could be classified as a black swan. However, flash crashes in general could not. Almost every day market participants experience several small flash crashes spread out over many individual securities. Prices fall and recover within fractions of a second and most market participants do not even notice when they do. During the worst days of the financial crisis 2008 the number of flash crashes that occurred in a single trading day was three digit. In August 24, 2015, the market faced another extreme flash crash event. This happened in times of high volatility and before the market opened one could observe high pressure from the sellers. Stocks opened at very low price levels and many stocks were not even open for trading in time. The selling pressure was amplified by a lot of market orders and stop-losses. During the first hour of trading many securities were halted by limit-up and limit-down rules because of the big movements in their prices. It happened in total more than 1200 times (Blackrock [2015]). Furthermore, the dysfunctionality in the stock market spread to other asset classes. For instance, it caused difficulties in pricing ETPs correctly. Comparing the two big flash crashes, the flash crash 2010 was a market wide flash crash while this event was rather the result of a huge number of mini flash crashes.

3. Previous Literature

3.1 Theory

3.1.1 Flash Crashes

A flash crash includes both a price movement up or down and a recovery phase. Without the recovery it is just a crash. The fact that the price recovers could suggest that someone made a mistake and that the price movement was not justified by new information. There are many potential mistakes that market participants can make that could trigger a mini flash crash. Fat-finger trade is probably a common explanation soon after an equity failure has occurred. The theory is based on a human error when typing the order size, accidently adding a few zeros more or putting billions instead of millions etc. This could result in an order that is taking more liquidity than the market can provide at the given time, causing rapid price movements. Investors putting very big orders is another possible source to a flash crash. In section 2.1 two such trades were mentioned, the hedge fund Universa Investments' purchase of put options and Wadell & Reed's sale in E-Mini contracts. The short term effect on the market liquidity is similar to the effect caused by a fat-finger trade.

The risk of big orders causing a flash crash is amplified by sudden order cancelations and withdrawal of liquidity. In today’s computerized trading environment liquidity can exit the market within the blink of an eye. One reason to draw back liquidity can be the lack of information. After the 2010 flash crash Nanex argued that quote stuffing forced market participants to reduce their activity in the market to catch up with the information flow (Nanex [2010]). Quote stuffing is the practice where the trader enters and withdraws many orders at a high speed. This creates latency and confusion in the market and can create opportunities for high frequency traders. Another reason for market makers to vanish from the market is that the risk of trading with a better informed counterpart is too high. Easley, López de Prado and O’hara [2010] developed a measure for the order toxicity in the market, the VPIN metric. Their theory is based on that insiders will trade on the same side of the market, buying or selling, thus creating an imbalance in the trading. VPIN metric measures the fraction of imbalanced trading, hence a high VPIN indicates high order toxicity. Furthermore, Easley, López de Prado and O’hara [2011] suggest three important factors in the current market structure which contributes to sudden illiquidity. These are concentration of liquidity provision, higher order toxicity because of reduced participation of retail investors and liquidity providers being more sensitive to intraday losses.

To conclude, on a very basic level there are two things which can cause a flash crash:

1. Aggressive market orders demanding too much liquidity
2. Sudden withdrawal of liquidity by liquidity suppliers

However, there are an endless amount of things in the market which could ultimately lead to one of these.

3.1.2 The Microstructure of Financial Markets

In the previous section it was mentioned that the recovery of a crash could indicate that the crash was caused by a mistake and not justified by new information. However, market microstructure theory offers several other explanations to temporary discrepancies from a security's "true value". An example is how the prices are affected by liquidity suppliers managing inventory risk. In a competitive dealer framework (De Jong and Rindi [2009]) every liquidity supplier is trying to maximise the expected utility of his terminal wealth $\widetilde{W}_j$ given his individual risk aversion A:

$$[1] \qquad \underset{q_j}{\text{Max}}\, E[\widetilde{W}_j] - \frac{A}{2}\text{Var}[\widetilde{W}_j]$$

Let us assume a batch market with M dealers supplying liquidity by submitting limit orders in the risky asset with the future value $\tilde{F} \sim N(\bar{F}, \sigma^2)$ and that I_j is the dealer's individual endowment. Then we get:

$$[2] \qquad E[\widetilde{W}_j] = (\bar{F} - p)q_j + I_j\bar{F}$$

$$[3] \qquad \text{Var}[\widetilde{W}_j] = \sigma^2[q_j^2 + I_j^2 + 2I_j q_j]$$

Solving the utility maximization problem we get that each dealer should submit the following quantity:

$$[4] \qquad q_j = \frac{\bar{F} - p}{A\sigma^2} - I_j = \frac{\bar{F} - I_j A\sigma^2 - p}{A\sigma^2} = \frac{\varphi_j - p}{A\sigma^2}$$

The dealer's marginal evaluation of the security is represented by $\varphi_j = \bar{F} - I_j A\sigma^2$ in the formula above. Furthermore, we assume Z traders submitting market orders and consequently taking liquidity from the market. With $\tilde{x} \sim (\bar{x}, \sigma_x^2)$ being the sum of all market orders we get the following market clearing condition:

$$[5] \qquad \sum_{j=1}^{M} q_j + \tilde{x} = 0$$

Using the market clearing condition together with the dealer's optimal quantity formula we can substitute q_j and get an expression for the price:

$$[6] \qquad \tilde{p} = \bar{F} - \bar{I}A\sigma^2 + \frac{A\sigma^2}{M}\tilde{x}$$

The interesting result achieved by using this model is that the price is not only dependant on its expected future value of the security, but also on the average inventory of the liquidity suppliers and the trade size of the market orders. The effect from these factors are in turn amplified by the level of volatility and risk aversion.

Furthermore, it is important to note that the price of a security does not usually jump to a new value instantly when news which changes the fundamental value of the security arrives. The information is gradually incorporated in the price through trading and sometimes the adjustment can be slow. However, since some trades are because of inventory management all trades do not necessarily help the price discovery. Looking at historical order flow and price development there are ways to isolate the permanent effect on the price, which is the information component, and the temporary effect, which is the inventory management component. Though, it is hard to make difference between trades as they occur in real time. Therefore, transactions to manage inventory could accidentally be interpreted to contain information about the security's value and consequently cause other participants to act on that.

3.1.3 High Frequency Trading

To understand how high frequency trading might affect the probability of flash crashes one needs to understand the fundamentals of the different trading strategies high frequency trading firms use. SEC divided the strategies into four groups (SEC Staff [2014]) – passive market making, arbitrage, structural and directional.

Passive market making provides liquidity by submitting non-marketable orders to the market and earns on the bid-ask spreads. This strategy does not involve any directional bet, but the prices need to be amended frequently as new information arrives in the market. Since high inventory turnover is essential for passive market making one can expect that presence from many high frequency firms with this strategy would decrease the bid-ask spread because of inexhaustible undercutting. However, it may also increase number of orders being amended or cancelled. A high frequency trader has an inventory turnover bigger than five times daily (Easley, López de Prado and O'Hara [2011]).

Arbitrage strategy is about finding pricing relations between assets and earn the difference when the relations are temporarily violated. It could be a security which is being traded on several markets to different prices. Another example is when there is a miss match between the price of an ETP and the sum of the underlying assets' prices.

Structural strategies earn from taking advantage of weaknesses in the market and other market participants. For example a trader with the lowest latency can profit by trading with market participants who offer prices which do not reflect changed market conditions. This strategy and the arbitrage strategy are both very sensitive to reporting errors and delayed information. That could cause illusions of opportunities in the market on which the algorithm would act.

Directional strategies, as the name suggests, involves anticipating the direction of price movements. One directional strategy is order anticipation in which one tries to identify big buyers or sellers to front run. Another strategy is momentum ignition which is about making several trades in one direction and hope for other market participants to follow. Hence, moving prices in the desired direction.

With a lot of high frequency trading activity and many of these strategies in play at the same time, at very high speed, it is reasonable to believe that the probability of mini flash crashes might increase.

3.2 Previous Empirical Work

3.2.1 Flash Crashes

An important paper examining the occurrence of flash crashes is Easley, López de Prado and O'hara (2011). Using their own theories about the VPIN measure (Easley, López de Prado and O'hara [2010]) they showed that before the big flash crash 2010 one could see abnormally much order toxicity in the market and that the timing of the crash could have been predicted minutes or even hours before it happened. High levels of VPIN could be observed already one week before the big crash. Aldridge (2014) showed that flash crashes can be predicted up to a day in advance by looking at the duration of runs[1]. A down crash is usually anticipated by an increase in the relative duration of negative

[1] When the price increases/decreases for n consecutive trade ticks the price is considered to be in a positive/negative run of duration n (where n is an arbitrary integer).

runs compared to the duration of positive runs. Another example of related research is Barany et al. (2012). By analysing oil stocks they found that before a crash one could see patterns in the behaviour of the long term memory effect parameter. This also suggests a certain predictability in the occurrence of crashes.

McInish, Upson and Wood (2014) studied the 2010 Flash Crash and came to the conclusion that intermarket sweep orders contributed to the crash. This is in line with Madhavan (2012) who argues that volume and quote fragmentation are important factors. Menkveld and Yueshen (2016) attribute the flash crash to cross-market arbitrage being malfunctioning right before the crash, causing lower liquidity in E-Mini. Studying the activity in E-Mini contracts during the 2010 Flash Crash Kirilenko et al. (2014) found that high frequency traders did not trigger the crash, but their activity made the market volatility worse. Cespa and Foucault (2014) showed that when liquidity goes down it leads to lower price informativeness, which in turn leads to even lower liquidity. This self-reinforcing relationship can cause liquidity crashes. Furthermore, they found that the liquidity status in one asset can affect and spread to other assets. A similar result was found by Cui and Gozluklu (2015) when they studied flash crashes and rally events during the SEC initiated single stock circuit breaker program. They showed that these events were not only detrimental for the halted stocks, but also for correlated stocks.

3.2.2 High Frequency Trading

Empirical studies suggests several attributes in the market and the trading associated with high frequency trading. Jovanovic and Menkveld (2011) analysed Dutch stocks and tested the effects from entry of middlemen in limit-order markets. They found that entry of a big high frequency firm was correlated with 15% lower effective spreads. Research by Bershova and Rakhlin (2013) came to a similar conclusion about high frequency firms being associated with lower spreads, but they could also observe an increased volatility in the short-term. Another interesting observation about spreads and high frequency trading is that the size of the spread seems to affect the aggressiveness of the high frequency firms. When the spread is smaller the high frequency firms tend to be more aggressive (Zhang and Riordan [2011] and Carrion [2013]). Furthermore, high frequency trading firms' reacted stronger to shocks during the financial crisis 2008 compared to before and after the crisis (Zhang [2013]).

By looking at OMXS trading data Breckenfelder (2013) found that competition among high frequency trading firms leads to lower liquidity and higher intraday volatility. However, interday volatility was not affected. Moreover, Breckenfelder discovered that when high frequency trading firms competed for volume their ratio of liquidity consuming trades increased substantially.

4. Data and Methodology

4.1 Data

The data set is provided by my supervisor Barbara Rindi and is a unique data set containing information about mini flash crashes in the American market. Included in the data one can find information about the size of the price change, the duration of the crash, how many sequential ticks it moved etc. In addition to the original mini flash crash data set I have added some complementary daily market data from the Reuter's database. The time period for the data set ranges from 3 January 2006 to 3 February 2011. The mini flash crash data set is based on Nanex's definition of mini flash crashes. According to the definition the following conditions need to be fulfilled:

1. The price movement has to be bigger than 0.8% up or down
2. The stock has to tick up/down at least 10 times before if recovers
3. The total time for both the crash and the recovery is maximum 1.5 seconds

Comparing the definition I am using to the definitions used in previously mentioned empirical work about flash crashes there are two important differences:

1. This definition includes both up crashes and down crashes while it has previously been more common to only look at down movements
2. This definition includes a restriction that the duration cannot exceed 1.5 seconds, which constitutes a significant difference between mini flash crashes and other flash crashes

From the total data set I have picked a sample of stocks to study based on the occurrence of mini flash crashes within the time period. Since I want to study how the number of flash crashes changes because of time varying factors rather than stock specific factors it is important to look at a sample of stocks with many flash crashes. Only stocks with more than a total of 30 mini flash crashes over the whole time period are included in this sample. Flash crashes in other securities than stocks are not regarded in this paper to avoid possible differences between asset classes. Furthermore, if a stock's underlying company has declared bankruptcy within the period I have excluded it as well. Table 1 gives a good overview of the selected stocks and how the mini flash crashes are distributed among them. To see how the mini flash crashes in the selected sample are distributed over time one can have a look at figure 1.

This can be compared to the distribution of flash crashes for the full data set, which is plotted in figure 2. Over the majority of the period the two graphs have roughly the same shape. The biggest deviation can be found May 6 2010 ("The 2010 Flash Crash"). As presented in table 2, 209 stocks had at least one mini flash crash that day, but only one is included in my chosen sample. This is particularly peculiar since the 23 stocks in my sample are the stocks with the most crashes in total over the whole time period. Another thing that makes this day stand out is the fact that even though it is one of the days with the highest number of mini flash crashes in total, no individual stock experiences more than three crashes that day. About 90% of the 209 stocks that crashed had only one crash.

Two sub-sets are created from the original data. The first one is used for analysing the number of mini flash crashes per day and stock, consequently it has 29,532 observations (1284 days x 23 stocks). A total of 671 of these observations have at least one mini flash crash. The second sub-set is used for analysing individual crash properties. Thus, it has as many observations as there are crashes in the sample period (1186 observations).

4.2 Variables

The variables are picked primarily based on theory and previous empirical work, but also on the easiness to use. In order to make the model to be useful for a trader or any other market participant it is important that the explanatory variables are easy to track in real time on a daily basis. In this section I will present the rationale behind looking at the selected variables and how they are defined. A Dickey-Fuller test is performed for each variable to ensure stationarity.

4.2.1 Number of Flash Crashes (($NFC_i(t)$)

One of the main variables I will analyse in this paper is the number of flash crashes that occur in an individual security during a day:

$NFC_i(t)$, where i is the security and t is the time (daily observations).

A possible problem with this variable is that if we assume that the probability of a flash crash is constant per volume traded, one will still see more flash crashes during a high trading activity day because of the increased trading volume. Consequently, adjustments for trading volumes are needed when analysing this variable.

4.2.2 Returns

When prices drop one can imagine that there are several reasons to why a mini flash crash might happen. One example is market makers drawing back the liquidity they are currently providing to the market. The effect could be amplified if they at the same time decide to liquidise their inventories to reduce the risk even further. Or simply the price reaches a level where several big stop-losses are triggered. Moreover, like discussed in the theory part it can be hard for market participants to distinguish between price movements which are justified by new information and the movements because of inventory management. This can be particularly true in today’s markets when responses on price movements to a larger extent are made in fractions of seconds by computerized trading and predefined algorithms. When news arrive, market participants need to re-evaluate the asset price and through the trading mechanism find the price. However, on the way to the new true value there might be unjustified price movements which could lead to mini flash crashes.

Returns are calculated according to the formula below:

$$[7] \qquad r_i(t) = \frac{P_i(t)}{P_i(t-1)} - 1$$

4.2.3 Volatility

There are many reasons why volatility is an interesting variable which could be correlated with the number of flash crashes. In the theory part one can see how the liquidity suppliers respond on traders' market orders by looking at the last term in the equation 6 ($\frac{A\sigma^2}{M}\tilde{x}$). It shows that traders' market orders have bigger impact on the liquidity supplier's valuation of the asset when the volatility is high. In addition, previous research has showed that high frequency firms react stronger to shocks in times of high volatility (Zhang [2013]).

In my research I have chosen to use a 3-day historical volatility:

$$[8] \qquad \sigma_i(t) = \sqrt{\frac{1}{3}\sum_{j=t-2}^{t}(r_i(j) - \bar{r})^2}$$

In today's fast-paced markets I believe it is more relevant to analyse volatility with a short window, in this case three days, than volatility based on a longer period.

Assuming the non-stationary behaviour of volatility I do not use the level of volatility itself, instead I look at the percentage of abnormal volatility. My abnormal volatility measure is constructed like returns, but instead of using yesterday's volatility as denominator I use the average level of volatility during the past two trading weeks:

$$[9] \qquad \sigma_{abn.,i}(t) = \frac{\sigma_i(t)}{\frac{1}{10}\sum_{j=t-10}^{t-1}\sigma_i(j)} - 1$$

This means that the variable can take both positive and negative values and because of its return like characteristics it can easily be compared among different stocks.

4.2.4 Lagged Number of Flash Crashes ($NFC_i(t-1)$)

If the probability of flash crashes for a stock is changing over time and is dependent on factors which are hard to identify or define, but are present over several days they might be captured by looking at the number of flash crashes the previous day, $NFC_i(t-1)$. For example the use of certain high frequency trading strategies might contribute to increased risk of mini flash crashes. This could be hard to define in a trackable variable, but if these strategies are present over several days it is possible that the lagged variable will be able to add their effect into the model. Another example of a variable which is hard to monitor and quantify is the risk aversion of market participants. Just like higher volatility increased liquidity suppliers' response to traders' market orders, increased risk aversion amplifies the effect even further. If we assume that risk aversion is non-stationary and has an effect on the likelihood of mini flash crashes, maybe the effect can be captured through the lagged variable. The possible reasons for autoregressive behaviour in NFC are countless.

4.2.5 Timing - Weekdays and Trading Hours

The timing of a trade can affect the risk of causing a mini flash crash. In the morning there might be news that the market needs to incorporate into the stock price. Mondays are extra sensitive since it is the first day after the weekend (when no trades are done). Furthermore, the last hour of trading and Fridays can be extra demanding for liquidity if many market participants reduce or exit their positions before the daily market closing or the weekends. The day with most mini flash crashes during the financial crisis 2008, October 10, was a Friday and the crashes of August 24, 2015, was a Monday.

Several time dummies are introduced:

d_{Monday} = 1 if it is a Monday

d_{Friday} = 1 if it is a Friday

$d_{Opening}$ = 1 if it is the first 30 minutes after market opening

$d_{Closing}$ = 1 if it is the last 30 minutes before market closing

$d_{Start\ of\ week} = d_{Monday} \times d_{Opening}$

$d_{End\ of\ Week} = d_{Friday} \times d_{Closing}$

4.2.6 Amplitude

Amplitude is simply the absolute price movement to the lowest/highest point (depending on if it is a down or up crash) during the flash crash divided by the price before the mini flash crash. In other words, the percentage price change. However, this measure is in absolute values and does not give any information about if the price movement is up or down.

$Amplitude_j$, where j is the individual mini flash crash ID. Each crash ID is connected to a single stock (i), day (t) and time of day.

4.2.7 Number of Ticks (NoT)

Some individual crash characteristics can be useful for understanding the amplitude of a mini flash crash. By looking at the number of sequential ticks one can learn about if crashes caused by withdrawal of liquidity or aggressive orders lead to bigger price changes.

NoT_j, where j is the individual mini flash crash

4.4 Method

There are three parts of the analysis. Frequency analysis and cluster analysis investigate the question "Under what conditions should one expect an increased occurrence of mini flash crashes?" and use the first data sub-set. Impact analysis investigates the question "What determines how big the price movement will be when a mini flash crash occurs?" and uses the second data sub-set.

4.4.1 Frequency Analysis

In this part of the analysis I look at the frequency of flash crashes in individual stocks and try to explain the distribution of them by looking at the correlation between the number of flash crashes in a stock and commonly monitored market variables as well as patterns connected to weekdays. The main part of this analysis is the following regression:

$$[10]\quad NFC_i(\mathrm{t}) = \alpha + \beta_1 \times NFC_i(\mathrm{t}-1) + \beta_2 \times r_i(\mathrm{t}) + \beta_3 \times \sigma_{abnormal,i}(t) + \beta_4 \times d_{Monday}(\mathrm{t}) + \beta_5 \times d_{Friday}(\mathrm{t}) + \varepsilon(t)$$

In this regression I am using Newey West errors. Newey West is a procedure to adjust for both autocorrelation and heteroscedasticity which is needed in my model because of the lagged NFC-variable. In addition, I am adjusting for trading volume.

Instead of doing a pooled analysis and only one panel data regression for all the stocks I am performing 23 different regressions, one for each stock. My conclusions are then based on a meta-analysis of all the results. The statistical method chosen for the meta-analysis is Hunter-Schmidt and is described in figure 3 in the appendix. An advantage with this method is that in the intermediate step when all the time series regressions are presented, individual differences are more transparent.

4.4.2 Cluster Analysis

Cluster analysis is about studying time periods when individual stocks have experienced mini flash crashes over several days. By looking at clusters in the individual stocks I hope to find patterns which can give deeper knowledge about how the probability of a flash crash changes over time. A flash crash cluster is defined by me as:

1. The longest possible period of at least two consecutive trading days
2. Which starts and ends with a day with at least one flash crash
3. And does not include two or more consecutive trading days with no flash crashes

This definition allows a cluster to include days with no flash crashes in a cluster as long as they are not in a row. The rationale behind this is that anomalies in one day should not be enough to give a false hope that the period of flash crashes is over. The clusters are identified and extracted from each stock in the data set used for the frequency analysis. In total

83 clusters are identified. All 23 stocks in the sample have at least one cluster during the sample period.

I categorise the clusters based on their duration (2-day-clusters, 3-day-clusters, 4-day clusters etc.). By doing so the analysis becomes more structured and I can more easily identify typical shapes and analyse the characteristics connected to when a security is affected by several flash crashes over a short period. The duration in days will from now on be called "cluster size". Days, with at least one mini flash crash, which are not part of a cluster are referred to as "non-clusters" or sometimes "1-clusters" even though 1-cluster is by my definition not possible.

In the analysis of the clusters I am focusing on understanding the attributes associated with different sizes of clusters. Besides descriptive statistics, data are analysed with the following OLS regression:

$$[11] \qquad ClusterSize_c = \alpha + \beta_1 \times NFC_{c,max} + \beta_2 \times \overline{NFC}_{c,max.adj} + \beta_3 \times DbC_c + \varepsilon$$

Where:

- C is the cluster ID
- $ClusterSize_c$ is the duration of the cluster period in days.
- $NFC_{c,max}$ is the highest value of NFC in the cluster.
- $\overline{NFC}_{c,max.adj}$ is the maximum adjusted average value of NFC in that cluster. Maximum adjusted means that the maximum value is excluded in the average calculation. This is done because the maximum value itself is an explanatory variable in the regression.
- DbC_c is the number of days between clusters (compared to the previous cluster).

The regression is adjusted for trading volume. Furthermore, I also run an alternative version of the regression where I adjust for the number of days with no crashes as a percentage of the cluster size ($\frac{days\ with\ zero\ crashes}{cluster\ size}$). This could be important since the most common cluster size, 2-cluster, is the smallest cluster size and cannot have days with NFC=0 by construction. If one does not adjust for zero days, there could be a bias towards smaller clusters having a higher maximum adjusted average NFC.

4.4.3 Impact Analysis

In the section impact analysis I study individual flash crashes. Given that a flash crash occurs, what factors affects how big the price change will be? The main part of the analysis is the following OLS regression:

$$
\begin{aligned}
[12]\quad Amplitude_j
&= \alpha + \beta_1 \times NoT_j + \beta_2 \times NFC_i(\mathrm{t}) + \beta_3 \times r_i(\mathrm{t}) + \beta_4 \times \sigma_{abn.,i}(t) \\
&+ \beta_5 \times d_{Monday}(\mathrm{t}) + \beta_6 \times d_{Friday}(\mathrm{t}) + \beta_7 \times d_{Opening,j} \\
&+ \beta_8 \times d_{Closing,j} + \beta_9 \times d_{Start\ of\ week,j} + \beta_{10} \times d_{End\ of\ week,j} + \varepsilon
\end{aligned}
$$

The regression is adjusted for fixed firm effects. This is particularly important since the observations are very unevenly distributed among the 23 stocks. Furthermore, I also run this regression on up and down crashes separately to see if they are differently affected by the factors.

5. Results

5.1 Frequency Results

5.1.1 Descriptive Results

Examining the interday correlation of the number of mini flash crashes, the data offers an interesting finding. The likelihood of a day having at least one flash crash seems to be conditioned on the outcome the day before. Figure 4 in the appendix shows that the probability of having at least one mini flash crash in time t is a lot higher if it was at least one mini flash crash in time t-1 compared to if there were no crashes (27.8% compared to 1.7%). However, one should remember that despite the big difference in conditional probability the majority of the days with at least one mini flash crash comes after days with no crashes. This is because a day having no mini flash crashes is still the most likely outcome when looking at individual stocks. Out of 671 days with at least one mini flash crash a total of 478 days had no crash the day before.

Analysing how daily price movements in the stock affects the number of mini flash crashes offers more ambiguous results. Descriptive statistics for daily stock returns and abnormal volatility are provided in table 3. The results are presented in two sub-groups, days with at least one mini flash crash and days with no crashes. For abnormal volatility the average and all the presented quartiles shift upwards for days with at least one crash, suggesting that days with mini flash crashes are correlated with higher abnormal volatility. However, the standard deviation is high for both the groups and values might vary a lot within the quartiles. The interpretation of descriptive statistics in returns is also somewhat dubious. On the one hand the first quartile, the median and the mean are lower for days with at least one mini flash crash, which indicates that crash days are negatively correlated with returns. On the other hand the third quartile shifts a lot in the opposite direction and it is possible that the occurrence of mini flash crashes are correlated with extreme returns in any direction and not only negative ones.

The weekday effect seems to have very little impact on the likelihood of mini flash crashes. Looking at figure 5 one can see that the number of flash crashes are approximately evenly spread among the weekdays. Fridays have slightly more crashes than the average day and Mondays have slightly less crashes than the average day. However, the deviations are within the range of what could be expected under random distribution with equal probabilities.

5.1.2 Statistical Analysis

Table 4 in the appendix presents the results from the time series regression for each stock in the sample separately. Out of 23 studied stocks 14 show statistically significant results on a 5% level that the number of flash crashes today (NFC(t)) are correlated with the number of crashes yesterday (NFC(t-1)). This is in line with what we could see in the descriptive results where days with at least one mini flash crash more likely occurred after another day with at least one mini flash crash. An interesting observation is that the correlation between NFC and lagged NFC seems to be stronger for the stocks which have a higher number of flash crashes over the whole sample period. Not only that more of them have significant results, the coefficients are in average a lot higher as well. This is very effectively visualised by the confidence intervals in figure 6. a). In this figure we can also see that for many of the stocks with no significant effect from lagged NFC, at least the confidence interval shifts in the same direction.

The possible correlation between high abnormal volatility and the occurrence of flash crashes suggested by the descriptive statistics is not supported by the regression results. A significant effect from abnormal volatility on a 5% significance level is only found in one of the studied stocks, and this result indicates an effect with the opposite sign to the shift seen in the descriptive statistics. Even though almost no individual stock show a statistically significant effect from returns on a 5% level, a general shift towards negative coefficients can be seen when looking at the confidence intervals in figure 6. b).

Few regression results show any significant effect from Mondays and Fridays on a 5% significance level. Neither do the confidence intervals have any obvious shift towards any direction.

In the meta-analysis (Hunter-Schmidt method) no significant results are found when looking at all the studies (see table 5). However, limiting the results to only looking at stocks with more than 50 mini flash crashes over the sample period (only 6 stocks included), then the effect from lagged NFC is strongly significant with a t-value of 4.67 and a coefficient of 0.521.

5.2 Cluster Analysis

5.2.1 Descriptive Results

Table 6 provides an overview of how the clusters are ditributed among the stocks in the sample. What is interesting is that all the 23 stocks have several clusters (4-7 clusters for the majority). The exceptions are the stocks IFN and ETP, which still have 1 cluster each. In addition, the cluster quota (days with crashes within clusters as a percentage of the total number of days with crashes) shows high values even for several stocks with no significant effect from lagged NFC in the frequency analysis (section 5.1.2). An extreme example is the stock POT (cluster quota 60%) which managed to form 4 clusters with an average cluster size of 6 days within the sample period, despite its lack of statistical significance in the lagged NFC variable. Noteworthy, 17% of all the days included in clusters are days with no mini flash crashes at all.

If one instead looks at how the NFC variable is distributed among different cluster sizes one can see how the average number of NFC increases with the cluster size (see table 7). The first quartile is approxemately the same for different cluster sizes. However, the median increase slightly with the cluster size (NFC=1 for the smaller half and NFC=2 for the bigger half) and the third quartile increase a lot (NFC is in the range 1-2 for the smaller half and 3-6 for the bigger half). Comparing the clusters to non-clusters one can see that in general the NFC values are relatively high in clusters. About 90% of the non-clusters have only one crash, and 8% have a NFC value of two (see table 8).

With a probable correlation between the cluster sizes and the frequency of mini flash crashes it is reasonable, when entering a cluster, to worry about the cluster being even longer. Figure 7 shows the sample probability of a cluster being larger than the size n, given the information that the cluster size is at least n. The biggest increase in probability happens between non-cluster (1-cluster) and 2-cluster. The probability increases from just over 20% to above 70%. For bigger cluster sizes the probabilities are more or less stable at 70%-80%. One thing that is not a lot higher in 2-clusters compared to non-clusters is the expected $NFC_i(t)$. Table 9 presents the descriptive statistics for the NFC variable in 2-clusters. What strikes the eye is that the first quartile, the median and the third quartile they all have the value 1 for both day 1 and day 2. This means that 2-

clusters are to a big extent standardised with one single crash the first day and another crash the second day. Even more interesting is that the 3-clusters are in general very similar to 2-clusters. Looking at 3-clusters, the first quartile, the median and the third quartile are all 1 for the first day, 0 for the second day and 1 for the third day (see table 10). In many cases one can take a 2-cluster and put a day with no mini flash crashes between the two days and the results is a typical 3-cluster. For bigger clusters the shapes are less standardised. Descriptive statistics for 4-clusters can be found in table 11 and used as an example.

Another interesting observation can be made looking at the number of days between clusters (see figure 8 for descriptive statistics). The median is 27.5 days, the first quartile is 7.5 days and the shortest duration between the clusters is 3 days. A stock has in average 3.61 clusters over the whole sample period, which means less than one per year. With that in mind, a median of 27.5 days is very low. In addition, 3 days is the lowest possible value according to the cluster definition. A lower value would have meant that the two periods are actually the same cluster.

5.2.2 Statistical Analysis

The results from the cluster analysis regression, which examine the attributes of cluster sizes, are presented in table 12 in the appendix. In the descriptive statistics one could see that the highest values of NFC could be found among the bigger clusters. The indicated correlation between a cluster's maximum NFC value and the cluster size is supported by the regression results. The maximum NFC variable has a coefficient of 0.577 with a t-value of 3.33. However, the maximum adjusted average NFC is not statistically significant in the basic version of the regression (column a), t-value of 1.53). When adjusting for the percentage of days with no crashes the t-value increases from 1.53 to 2.56, taking some of the explanatory power from Max NFC (t-value dropped from 3.33 to 2.10). Furthermore, the results do not support any significant effect from the number of days between clusters on the cluster size. By excluding the number of days between clusters variable, consequently adding another 23 observations to the regression, the statistical significance increases for the other two tested variables.

5.3 Impact Analysis

5.3.1 Descriptive Results

Looking at the descriptive statistics in figure 9 one can see that crash amplitudes seem to be higher for crashes with a higher number of sequential ticks. For crashes with 10-12 sequential ticks and crashes with 13-20 the statistics are approxemately the same. However, for the group with more than 20 sequential ticks all the descriptive statistics (mean, q1, meadian and q3) are shifted to 0.001-0.002 higher values. Looking at the correlation between the number of flash crashes during a day ($NFC_i(t)$) and the amplitude one can see that the descriptive statistics (figure 10) are relatively stable for NFC values within the range 1-6 crashes. However, when NFC exceeds that range amplitudes seem to be higher. Noteworthy, NFC is equal to a value between 1 and 6 in 929 of 1186 observations. Furthermore, for each group where NFC has a higher value than 11 crashes, descriptive statistics for amplitudes are based solely on observations from one stock at one date[2]. Hence, there is a big risk that amplitide values in these groups are heavily affected by other factors than NFC.

When it comes to timing effects the descriptive statistics do not indicate any differences in amplitude depending on in which

[2] Example: The group NFC=12 has only 12 observations. In other words all the crashes happened in the same stock at the same date.

trading hour the mini flash crash happens (see figure 11). However, weekdays might be of interest. Most of the days have descriptive statistics with similar values, but Fridays stand out (see figure 12). Amplitudes for the other four days have a median value of 0.011 and Fridays have a median value of 0.013. Furthermore, the third quartile for Fridays is 0.003 higher than for the other days. Combined with Fridays having a distinctly higher average amplitude, descriptive statistics suggest that mini flash crashes on Fridays are correlated with bigger price changes.

Figure 13 provides the distribution of amplitudes by daily stock returns. When sorting the mini flash crashes by groups of daily stock returns one can see that the amplitudes are similarly distributed among the majority. The exception is when returns drop below -20%, then the amplitudes are higher. The median for this group is as high as 0.016, compared to 0.011 for the total data set. For abnormal volatilites on the other hand, no obvious effect is observed (see figure 14).

5.3.2 Statistical Analysis

The regression results from testing how selected factors affect amplitudes are presented in table 13 in the appendix. These are presented for the total sample and for up/down crashes respectively. The results support the previously indicated correlation between amplitudes and the number of sequential ticks and are statistically significant on a 5% level for both up crashes and down crashes separately. However, the coefficient for down crashes is almost 50% bigger than for up crashes (0.00019 for down crashes and 0.00013 for up crashes). Any effect from the NFC variable is not supported by the regression results (t-value -0.91).

When it comes to trading hours and weekdays, regression results show that the previously indicated positive effect from Fridays on amplitudes is statistically significant (t-value 4.48). The coefficient suggests that Fridays have amplitude values which are 0.0049 higher. That number should be considered big since the median amplitude in the whole sample is 0.011. However, for up crashes this effect is offset in the last trading hour, indicated by the end of the week variable which has a coefficient of -0.0057. No other time dummy variable show any statistically significant effect.

Another variable which is statistically significant is return. The regression suggest a negative correlation between amplitude and daily stock returns with a coefficient of -0.0099 (t-value of -3.8). The coefficient for down crashes is more negative than for up crashes (-0.0114 compared to -0.0090). However, the economic significance is low. If a stock price drops 1% one day, the expected amplitude would be around 0.0001 higher. Consequently, exceptionally negative returns are needed in order to have an economically significant impact.

5.4 Limitations and Robustness

The stocks in the sample are chosen based on their high number of mini flash crashes, thus there might be a potential selection bias in the analyses. The results may not be applicable on stocks with a lower frequency of mini flash crashes. However, the biggest opportunity or risk arising from the phenomena of mini flash crashes can be found among the stocks with the highest probability of crashing. Hence, I think the knowledge about how these stocks behave in the landscape of mini flash crashes is the most important.

Moreover, a majority of the mini flash crashes are from 2008, the year of the financial crisis. It is possible that there might be specific underlying factors or conditions attributable to the crisis which are impacting the outcome of the number of mini flash crashes. Therefore it could be of interest to confirm my findings in a data set from a time period with no major financial crisis.

6. Discussion

In the frequency analysis one could see that lagged NFC had a positive correlation with NFC for the majority of the tested stocks. However, these results were hard to generalise, which was seen in the meta-analysis with the Hunter-Schmidt method. A significant result could only be observed when limiting the meta-analysis to the results from the stocks with the highest number of mini flash crashes over the sample period (more than 50 crashes). This could indicate that autoregressive behaviour in NFC is needed to get a high number of mini flash crashes over a longer time period. However, it is possible that using only one lag is a too simplified approach. The high percentage of days with NFC=0 in clusters (17%) suggests that introducing at least a second lag would probably describe the occurrence of mini flash crashes better. This could be particularly true in stocks with many 3-clusters. In addition, the likelihood of experiencing more mini flash crashes in the coming days increases substantially after two consecutive trading days of crashes. In other words, one might need to look at least two days back to get sufficient information about the risk of experiencing mini flash crashes in the near future. However, it is possible that the autoregressive behaviour in NFC is not necessarily an intrinsic stock specific property. What if stocks can have that property on and off over time? The finding that several stocks form more and bigger clusters than what would be expected without NFC having some sort of autoregressive behaviour makes this question relevant. A deeper study of clusters could be the key to understanding the underlying factors driving the occurrence of mini flash crashes. In the attempt of describing what affects the occurrence of mini flash crashes many other tested factors failed to present statistically significant results.

Descriptive statistics showed that the highest values of NFC are found within clusters of larger size. Furthermore, maximum NFC and maximum adjusted average NFC had significant positive correlations with the cluster size (the adjustment for the percentage of days with no crashes was needed in order to reach significance for the latter). Hence, data indicate that the risk of extreme values of NFC increases with the cluster size.

My results are in line with previous research on the point that there is a certain degree of predictability in the occurrence of flash crashes. I do not claim that my model predicts the flash crashes per se, but rather shows that past values of NFC can be useful for understanding the risk of mini flash crashes in the future. If one could use the lagged values of NFC to predict future values of NFC accurately, it would still have the disadvantage of not explaining the underlying factor to why mini flash crashes behave like they do. However, it offers several other advantages. Firstly, it is very intuitive and easy to use. There is no need to calculate complicated variables like VPIN (Easley, López de Prado and O'hara [2011]), relative duration of negative runs (Aldridge [2014]) and long term memory effect parameter (Barany et al. [2012]). Furthermore, since volatility is commonly modelled with GARCH models it might come more naturally to use an autoregressive model for the occurrence of mini flash crashes as well. After all, mini flash crashes are a kind of price volatility on a fractions-of-a-second basis.

When it comes to explaining the size of amplitudes several factors were of particular interest. If big price movements were mainly caused by sudden withdrawal of liquidity one would expect to see big amplitudes together with a small number of sequential ticks, hence a negative correlation. Since the results showed a positive correlation, my interpretation is that big amplitudes are rather driven by aggressive trading patterns and extensive use of market orders. Furthermore, results indicated that mini flash crashes have bigger amplitudes on Fridays, which could be an effect from market participants closing their positions before the weekend. The weekend could cause a sense of urgency, which makes people to prioritise minimising the risk of their trades not being executed at the cost of price. Hence, leading to an excessive use of market orders. These explanations are in line with the previous findings by Easley, López de Prado and O'hara (2011) since excessive use of market orders easily can lead to imbalances in the limit order book.

Another factor which is likely to have an effect on amplitudes is the daily stock return. The regression showed a strong statistical significance, but the coefficient has a very low value. Furthermore, descriptive statistics for amplitudes are similar for daily returns in the range -20% to -5% and the range -5% to 0%. However, the most extreme group (returns < -20%) have considerably higher values. This could indicate that the regression results are driven by the effect from extremely negative returns. Maybe a linear model is not the best way to describe the relationship between returns and amplitudes. Nevertheless, all the results suggest that extremely negative stock returns have an effect on amplitudes. It is reasonable to believe that extremely negative returns are driven by relatively long negative runs. Hence, this result is in line with the findings of Aldridge (2014).

7. Implications and Conclusions

For many of the tested stocks, data suggest a correlation between the number of flash crashes one day and the number of flash crashes the day before. This is particularly true for the stocks with the highest number of mini flash crashes in the sample. After a certain stock has experienced mini flash crashes multiple days over a short time period, the risk of more mini flash crashes the coming days is even higher than after only one day of mini flash crashes. In addition, the risk of an extreme number of flash crashes in one day increases with the length of the flash crash period. Hence, one should be particularly careful when a day with at least one mini flash crash is followed by another one.

The absolute price change during a mini flash crash increases with the number of sequential ticks. It is also bigger on Fridays which could be explained by more inventory management to adjust the positions before weekends. Furthermore, negative daily stock return have a positive effect on the absolute price change. However, extreme values are needed to have an economically significant impact.

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9. Appendix

Table 1. Stocks included in the sample

The table shows the stocks that were selected to be included in the examination of mini flash crashes. It is also presented how many crashes they had over the sample time period (3 January 2006 to 3 February 2011) and how many days the crashes were distributed on.

Company Name	Ticker	Mini Flash Crashes	Days with at Least One Crash
Morgan Stanley	MS	125	39
Goldman Sachs	GS	104	50
Wells Fargo	WFC	98	44
JP Morgan	JPM	90	49
MetLife	MET	81	31
Bank of America	BAC	66	39
American International Group	AIG	47	22
Citigroup	C	47	32
Bank of New York Mellon	BK	43	26
Prudential Financial	PRU	41	27
Potash Corp of Saskatchewan	POT	40	30
American Express	AXP	37	20
Freeport-McMoRan	FCX	37	27
General Electric	GE	37	29
Mosaic	MOS	37	19
State Street	STT	35	22
Cliffs Natural Resources	CLF	33	30
India Fund	IFN	32	26
US Bancorp	USB	32	23
Chesapeake Energy	CHK	31	22
Chevron	CVX	31	17
Energy Transfer Partners	ETP	31	20
SL Green Realty	SLG	31	27
	Total	1186	671

Figure 1. Number of Flash Crashes in the Selected Sample

The figure shows how the total number of mini flash crashes in the selected sample are distributed over the time period ranging from 3 January 2006 to 3 February 2011. Daily observations.

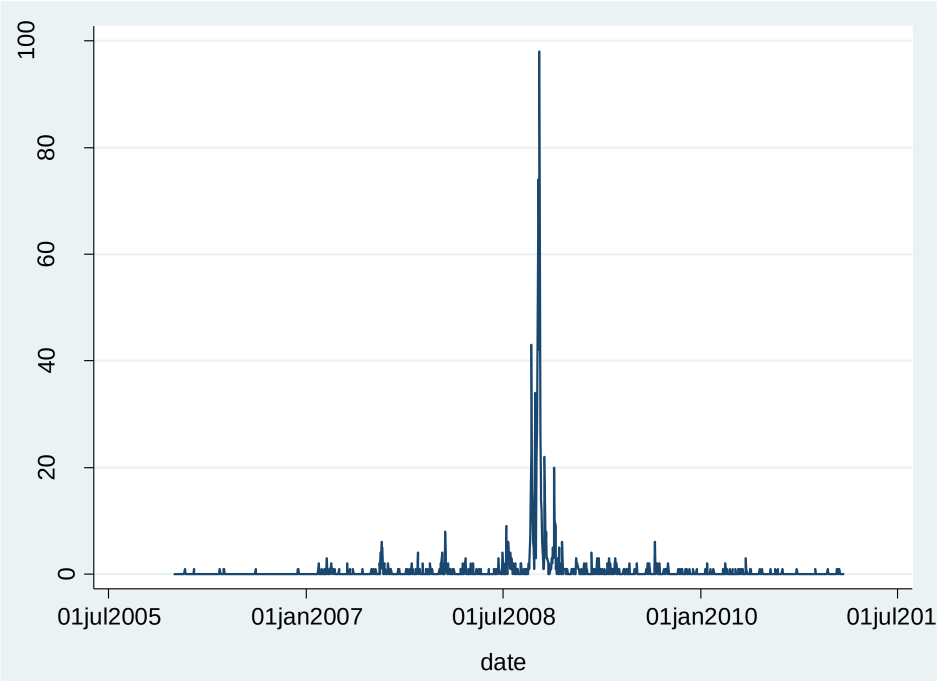


Figure 2. Number of Flash Crashes in the American Market

The figure shows how the total number of mini flash crashes in the American market are distributed over the time period ranging from 3 January 2006 to 3 February 2011. Daily observations.

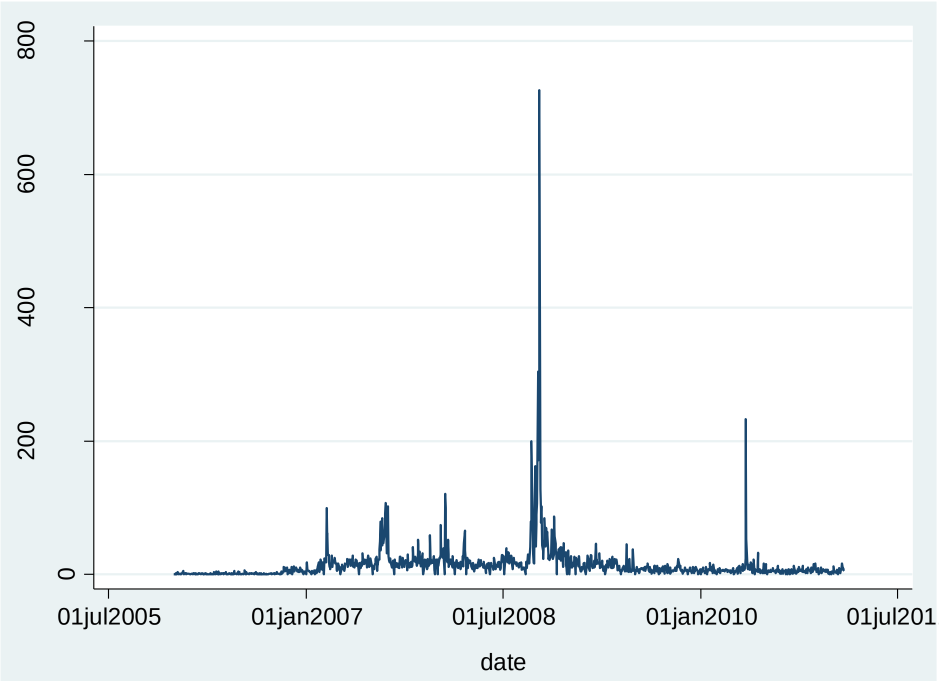


Table 2. An Overview of "The 2010 Flash Crash"

The table shows the number of stocks which had at least one mini flash crash during 6 May 2010. The result is presented by the number of times a stock crashed during that day. No individual stock had more than three crashes.

	Total American Market	Sample
1 Crash	187	0
2 Crashes	20	1
3 Crashes	2	0
Total	209	1

Figure 3. Hunter-Schmidt Method

The figure describes the way that the weighted mean effect size and the population variance is calculated using the Hunter-Schmidt method in a set of k studies.

With r being the sample effect size and n being the number of observations in the study the weighted mean effect size can be calculated for the k studies:

$$\bar{r} = \frac{\sum_{i=1}^{k} n_i r_i}{\sum_{i=1}^{k} n_i}$$

The variance of sample effect sizes is calculated by the formula:

$$\sigma_r^2 = \frac{\sum_{i=1}^{k} n_i (r_i - \bar{r})^2}{\sum_{i=1}^{k} n_i}$$

The sampling error variance is calculated with the formula:

$$\sigma_e^2 = \frac{(1 - \bar{r}^2)^2}{\bar{N} - 1}$$

The population variance is calculated by subtracting the sampling error variance from the variance of sample effect sizes:

$$\sigma_p^2 = \sigma_r^2 - \sigma_e^2$$

Figure 4. Conditional Probabilities of a Mini Flash Crash

The figure describes the sample probability of a stock having or not having at least one mini flash crash in time t (today) given the information if it had or did not have a least one mini flash crash in time t-1 (yesterday). It also shows how many observations (n) took the different paths in my sample.

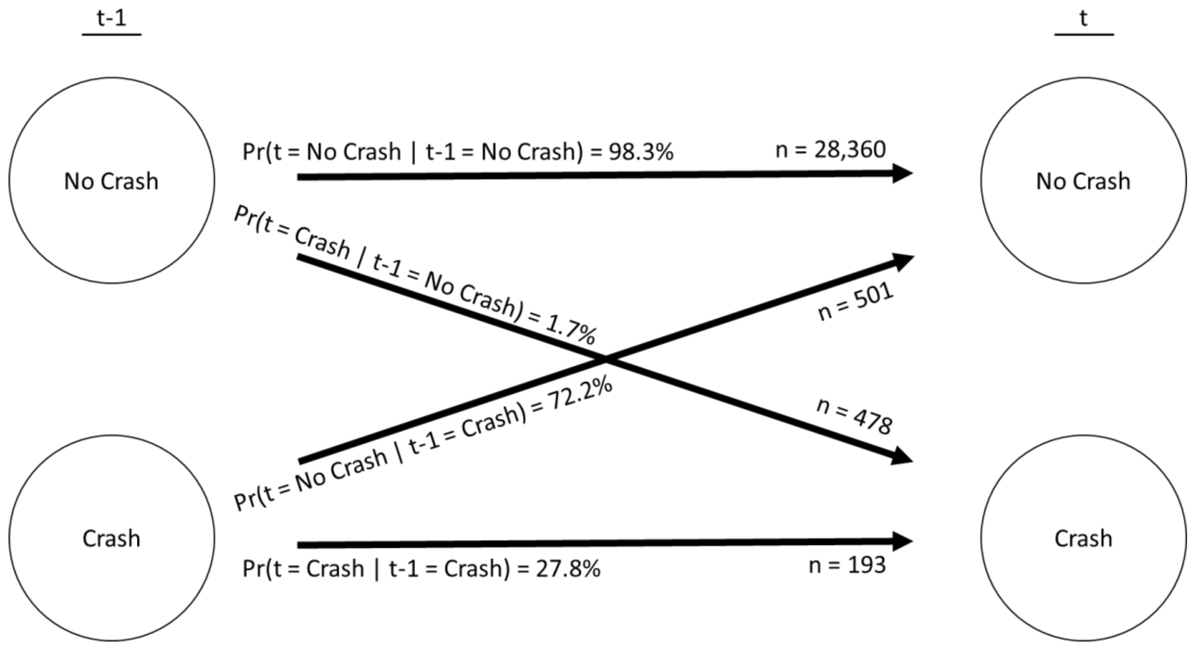


Table 3. Descriptive Statistics – Return and Abnormal Volatility

The table shows descriptive statistics for daily stock returns and abnormal volatility over the sample period (3 January 2006 to 3 February 2011). The results are presented for days with no mini flash crashes and days with at least one mini flash crash respectively.

	Daily Stock Return		Abnormal Volatility	
	No Crashes	At Least One Crash	No Crashes	At Least One Crash
Average	0.0009	-0.0057	0.0673	0.3313
StDev	0.0366	0.1021	0.6659	0.9504
Q1	-0.0120	-0.0546	-0.4005	-0.3085
Median	0.0003	-0.0091	-0.0482	0.1310
Q3	0.0132	0.0369	0.3921	0.6982
Observations	28861	671	28861	671

Figure 5. Distribution of Mini Flash Crashes among Weekdays

The figure shows the number of mini flash crashes which have happened in the sample on a certain weekday. The time period is ranging from 3 January 2006 to 3 February 2011.

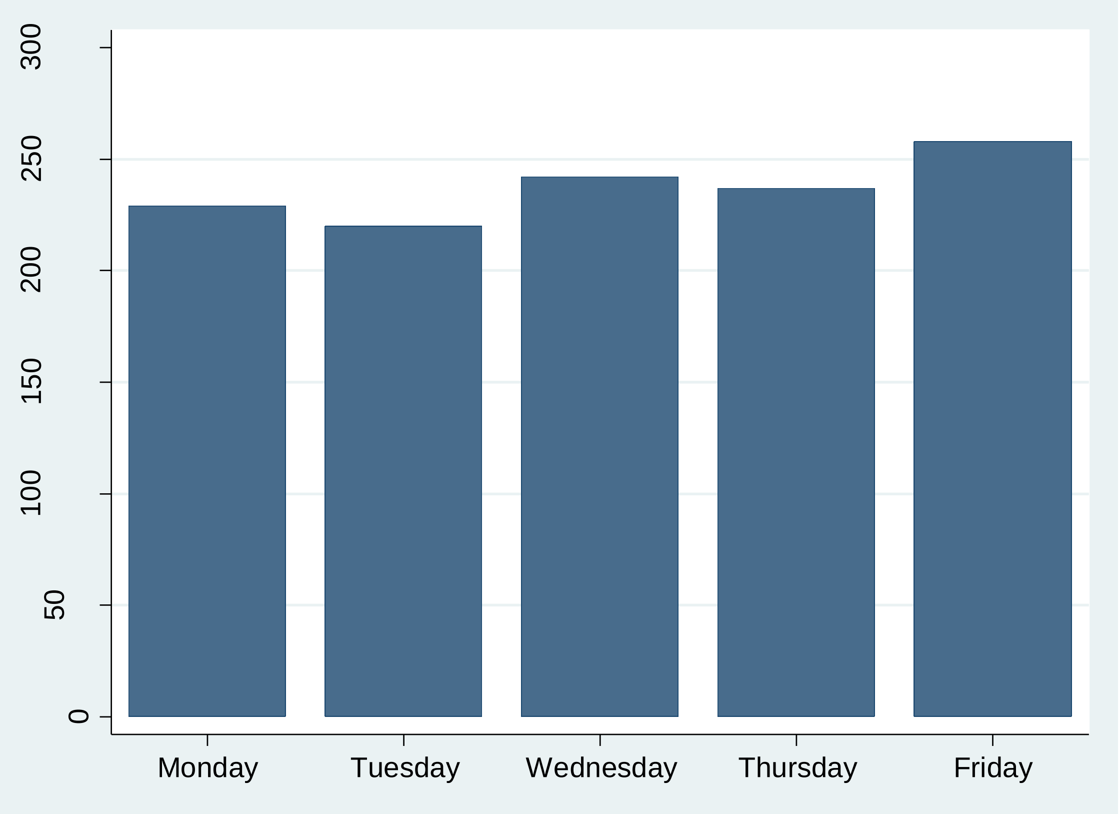


Table 4. Time Series Regressions – Effect on NFC

The table shows the coefficients and t-values from the time series regressions on the number of mini flash crashes in the 23 selected stocks. The regressions are using Newey-West standard errors and the following specification:

$$NFC(\mathrm{t}) = \alpha + \beta_1 \times NFC(\mathrm{t}-1) + \beta_2 \times \mathrm{r}(\mathrm{t}) + \beta_3 \times \sigma_{abnormal}(t) + \beta_4 \times d_{Monday}(\mathrm{t}) + \beta_5 \times d_{Friday}(\mathrm{t}) + \varepsilon(t)$$

The time period is ranging from 3 January 2006 to 3 February 2011 and each regression is based on 1216 observations. Results highlighted in dark grey are statistically significant on a 5% level. Light Grey means that they are statistically significant only on a 10% level. Stocks are sorted by their total number of mini flash crashes in the data set (the stock with the highest number at the top and the lowest at the bottom).

	Lag NFC		Returns		Abnormal Volatility		Monday		Friday	
Ticker	Coefficient	*t*	Coefficient	*t*	Coefficient	*t*	Coefficient	*t*	Coefficient	*t*
ms	0.43	*3.14*	-4.21	*-2.68*	-0.01	*-0.26*	0.01	*0.30*	-0.04	*-1.16*
gs	0.36	*2.53*	-3.62	*-2.60*	-0.05	*-1.64*	-0.01	*-0.47*	0.03	*0.59*
wfc	0.69	*4.09*	-0.60	*-0.90*	-0.02	*-0.96*	0.02	*0.56*	0.03	*0.71*
jpm	0.58	*4.18*	-0.54	*-0.66*	-0.02	*-1.32*	-0.02	*-0.84*	0.03	*0.88*
met	0.45	*2.18*	-3.39	*-1.76*	0.02	*1.14*	0.00	*0.01*	-0.01	*-0.23*
bac	0.60	*4.84*	0.12	*0.37*	0.00	*0.05*	0.02	*0.50*	0.01	*0.48*
c	0.41	*6.09*	-0.27	*-0.55*	0.00	*0.27*	-0.01	*-0.49*	0.05	*1.15*
aig	-0.13	*-1.40*	-0.64	*-1.43*	0.00	*0.15*	0.01	*0.60*	0.00	*-0.15*
bk	0.53	*4.67*	0.14	*0.34*	-0.01	*-1.74*	-0.02	*-1.98*	0.01	*0.62*
pru	0.22	*4.76*	-1.37	*-1.48*	0.00	*0.31*	0.02	*0.67*	-0.02	*-1.45*
pot	0.08	*1.06*	-0.28	*-0.55*	-0.02	*-1.79*	0.00	*0.25*	0.02	*1.17*
ge	0.08	*1.46*	-1.94	*-1.34*	0.01	*0.46*	0.05	*1.02*	-0.02	*-1.24*
fcx	0.35	*2.45*	0.35	*0.69*	0.01	*0.52*	0.02	*1.07*	0.02	*1.36*
mos	0.31	*2.36*	-0.53	*-1.70*	-0.02	*-2.32*	0.02	*0.88*	0.00	*-0.43*
axp	0.24	*1.47*	-0.68	*-1.54*	-0.02	*-1.37*	0.01	*0.24*	-0.02	*-0.86*
stt	-0.06	*-1.60*	-0.18	*-0.31*	-0.03	*-0.74*	0.02	*1.48*	0.00	*-0.05*
ifn	0.03	*0.58*	-0.31	*-1.43*	-0.01	*-0.60*	0.00	*0.29*	0.01	*0.47*
clf	0.26	*2.26*	-0.29	*-1.94*	-0.01	*-0.87*	-0.01	*-0.42*	0.00	*0.04*
usb	0.17	*2.07*	0.08	*0.26*	0.00	*0.18*	-0.02	*-2.03*	0.03	*1.26*
etp	0.03	*0.60*	-0.33	*-0.33*	0.03	*1.89*	0.03	*0.97*	-0.01	*-1.02*
chk	-0.01	*-0.17*	-0.24	*-0.63*	-0.02	*-1.00*	0.02	*0.98*	0.04	*0.93*
cvx	0.10	*3.68*	-2.00	*-1.64*	-0.01	*-0.58*	0.02	*1.07*	0.03	*0.83*
slg	0.05	*0.86*	-0.04	*-0.19*	0.02	*1.08*	-0.01	*-0.53*	-0.01	*-0.83*

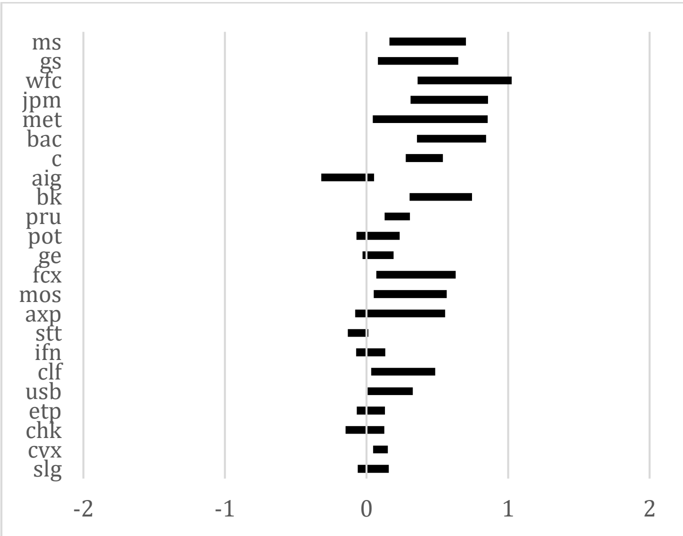
Figure 6. Confidence Intervals – Effect on NFC

The figures show the 95% confidence interval for the explanatory variables in the regressions on the number of mini flash crashes in the 23 selected stocks. The regressions are using Newey-West standard errors and the following specification:

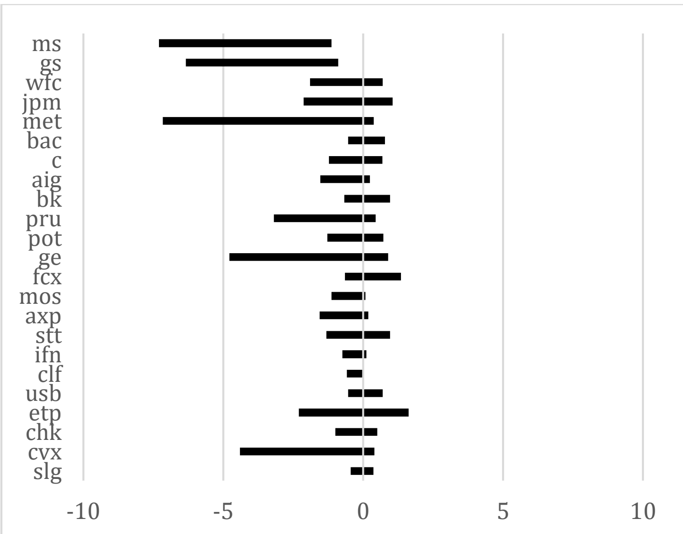
$$NFC(\mathrm{t}) = \alpha + \beta_1 \times NFC(\mathrm{t}-1) + \beta_2 \times \mathrm{r(t)} + \beta_3 \times \sigma_{abnormal}(t) + \beta_4 \times d_{Monday}(\mathrm{t}) + \beta_5 \times d_{Friday}(\mathrm{t}) + \varepsilon(t)$$

The time period is ranging from 3 January 2006 to 3 February 2011. Each regression is based on 1216 observations. Stocks are sorted by their total number of mini flash crashes in the data set (the stock with the highest number at the top and the lowest at the bottom).

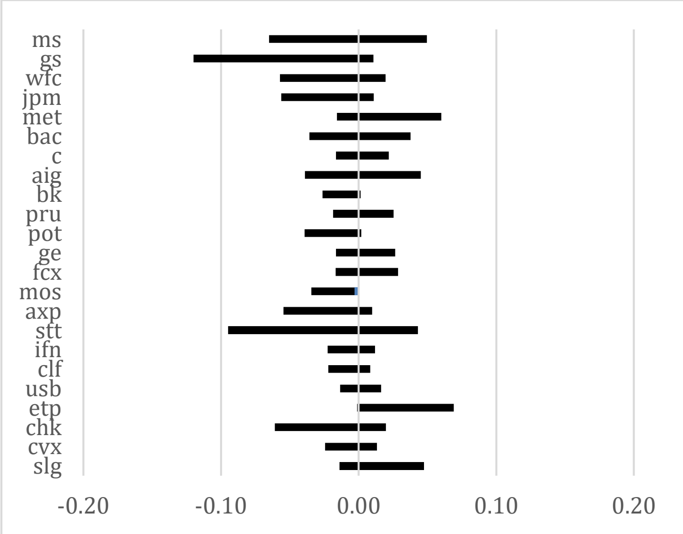
6. a) Lag NFC – Confidence Interval (95%)



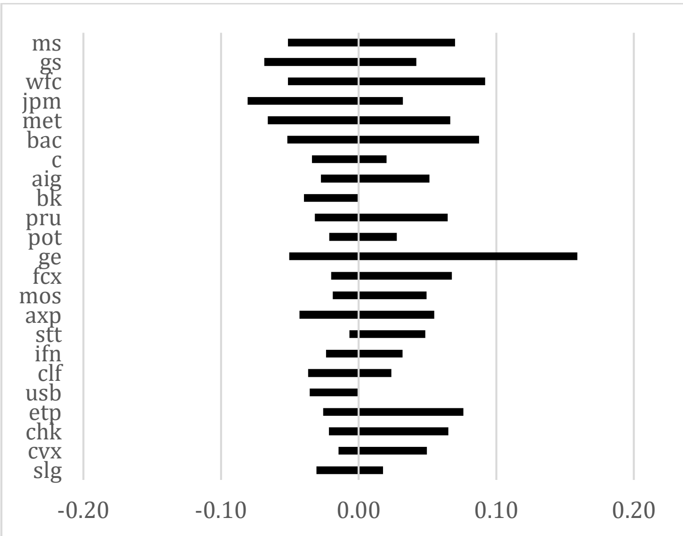
6. b) Return – Confidence Interval (95%)



6. c) Abnormal Volatility – Confidence Interval (95%)



6. d) Monday – Confidence Interval (95%)



6. e) Friday – Confidence Interval (95%)

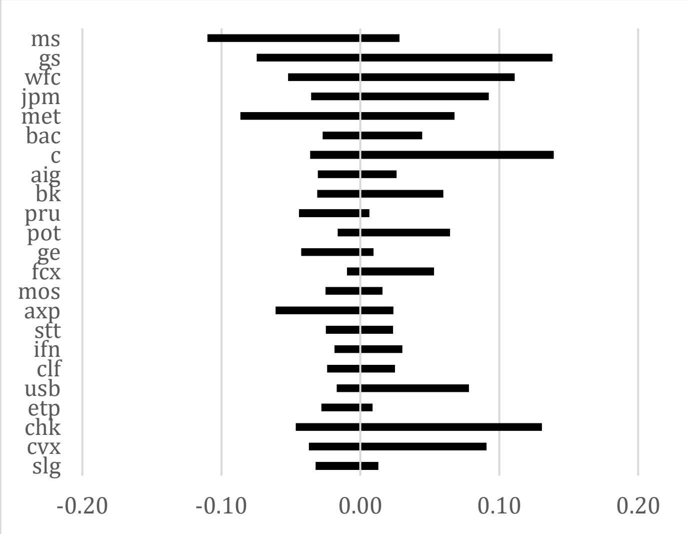


Table 5. The Number of Flash Crashes – Meta Analysis

The table shows the result from the meta-analysis of all the 23 time series studies described in table 4. The chosen method for performing the meta-analysis is Hunter and Schmidt. In the first column is the result from using all 23 studies and in the second is the result from limiting the meta-analysis to only looking at the stocks with a total of >50 mini flash crashes during the period ranging from 3 January 2006 to 3 February 2011 (6 stocks included).

Number of Stocks Included	23	6
Lag NFC	0.251	0.521
t-value	*1.12*	*4.67*
Returns	-0.903	-2.039
t-value	*-0.72*	*-1.18*
Abnormal Volatility	-0.006	-0.013
t-value	*-0.35*	*-0.57*
Monday	0.008	0.002
t-value	*0.43*	*0.11*
Friday	0.008	0.008
t-value	*0.36*	*0.31*

Table 6. Overview – Clusters by Stocks

The table shows how the number of clusters are distributed among the stocks in the sample. In addition, it provides the total number of days (NFC observations) within clusters. Cluster quota is the number of days in clusters with at least one flash crash divided by all the days with at least one flash crash.

	Number of Clusters	Number of Days in Clusters	Number of NFC=0 Days in Clusters	Zero Days Percentage	Cluster Quota
ms	4	28	3	11%	64%
gs	6	35	4	11%	62%
wfc	6	29	3	10%	59%
jpm	9	36	5	14%	63%
met	4	27	5	19%	71%
bac	4	23	2	9%	54%
c	3	21	4	19%	77%
aig	4	14	3	21%	34%
bk	2	17	2	12%	58%
pru	4	26	6	23%	74%
pot	4	24	6	25%	60%
ge	3	17	2	12%	75%
fcx	3	12	0	0%	44%
mos	5	20	5	25%	52%
axp	2	13	4	31%	47%
stt	3	12	4	33%	36%
ifn	1	4	1	25%	10%
clf	3	14	3	21%	42%
usb	5	17	3	18%	61%
etp	1	10	4	40%	27%
chk	2	9	2	22%	41%
cvx	2	8	1	13%	35%
slg	3	6	0	0%	22%
Total	83	422	72	17%	52%

Table 7. Descriptive Statistics – Number of Flash Crashes by Cluster Size

The table provides descriptive statistics for the NFC variable for days included in clusters. The result is presented by cluster sizes. Noteworthy, N is the number of days per group and not the number of clusters. The number of clusters in every group is presented separately and calculated by dividing N by the cluster size.

	Average	StDev	p25	Median	p75	Max	N	Clusters
2-Cluster	1.22	0.55	1	1	1	3	46	23
3-Cluster	0.94	0.86	0	1	1	4	51	17
4-Cluster	1.56	2.09	1	1	2	12	48	12
5-Cluster	1.16	1.31	1	1	1	6	25	5
6-Cluster	1.57	1.30	1	1	2	5	30	5
7-Cluster	1.71	2.09	1	1	2	11	28	4
8-Cluster	1.78	2.03	1	1	2	9	32	4
9-Cluster	3.11	2.89	1	2	3	9	9	1
10-Cluster	1.50	1.61	0	1	2	5	30	3
11-Cluster	4.23	3.68	2	2	6	11	22	2
12-Cluster	2.11	2.79	1	1	3	13	36	3
13-Cluster	1.85	1.21	1	2	3	4	13	1
15-Cluster	2.97	3.39	1	2	4	15	30	2
22-Cluster	4.59	4.94	1	3	6	19	22	1
Total	1.93	2.47	1	1	2	19	422	83

Table 8. Distribution of NFC for Non-Clusters

The table shows how the number of non-clusters are distributed over the different outcomes of NFC in the sample. The table is sorted so the stock with the highest NFC value is at the top and lowest at the bottom.

	Number of Non-Clusters with NFC=n									
	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
stt	12	1								1
axp	9				1					
aig	18	2		1						
slg	20			1						
wfc	16	1	1							
pot	11		1							
bac	15	3								
met	6	3								
jpm	16	2								
etp	14	2								
fcx	13	2								
bk	9	2								
chk	8	2								
ifn	26	1								
clf	14	1								
ms	13	1								
cvx	12	1								
usb	8	1								
c	4	1								
gs	19									
mos	14									
pru	7									
ge	5									
total sample	289	26	2	2	1	0	0	0	0	1
percent	90.03%	8.10%	0.62%	0.62%	0.31%	0.00%	0.00%	0.00%	0.00%	0.31%
cumulative	90.03%	98.13%	98.75%	99.38%	99.69%	99.69%	99.69%	99.69%	99.69%	100.00%

Figure 7. Conditional Probability of a Bigger Cluster

The figure shows the sample probability of a cluster which is at least an n-cluster to be a cluster of the size >n. There are bigger clusters than 8-clusters in the data sample, but these are not plotted because of uncertainty of estimate due to low number of observations.

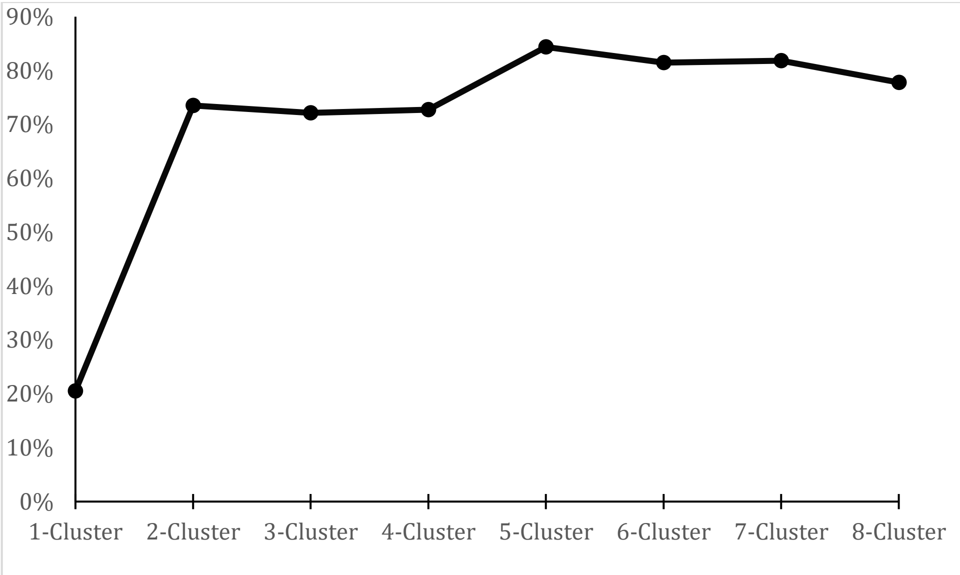


Table 9. Descriptive statistics – 2-Clusters by Cluster Day

The table includes descriptive statistics for the NFC variable in 2-clusters. The statistics are presented for each cluster day respectively.

2-Cluster							
Day	Average	StDev	p25	Median	p75	Max	N
1st	1.18	0.50	1	1	1	3	22
2nd	1.27	0.63	1	1	1	3	22
Total	1.23	0.57	1	1	1	3	44

Table 10. Descriptive statistics – 3-Clusters by Cluster Day

The table includes descriptive statistics for the NFC variable in 3-clusters. The statistics are presented for each cluster day respectively.

3-Cluster

Day	Average	StDev	p25	Median	p75	Max	N
1st	1.41	0.87	1	1	1	4	17
2nd	0.18	0.53	0	0	0	2	17
3rd	1.24	0.56	1	1	1	3	17
Total	0.94	0.86	0	1	1	4	51

Table 11. Descriptive statistics – 4-Clusters by Cluster Day

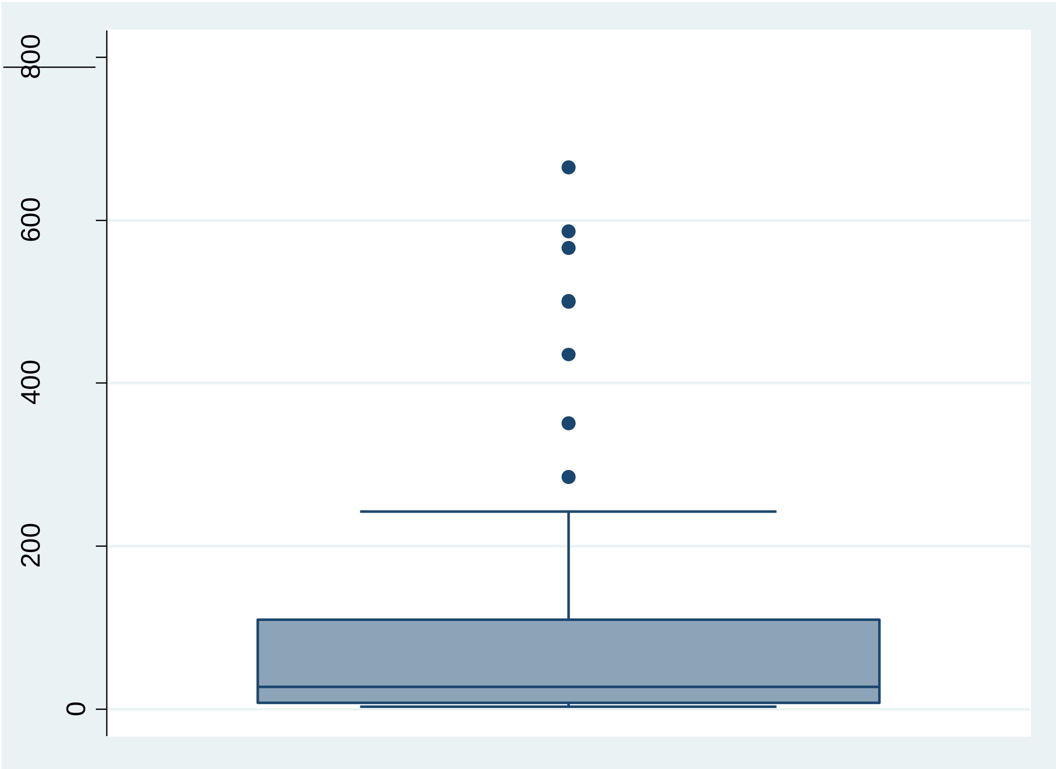
The table includes descriptive statistics for the NFC variable in 4-clusters. The statistics are presented for each cluster day respectively.

4-Cluster

Day	Average	StDev	p25	Median	p75	Max	N
1st	2.92	3.70	1	1	3	12	12
2nd	1.17	0.94	0.5	1	2	3	12
3rd	0.83	0.94	0	1	1	3	12
4th	1.33	0.65	1	1	1.5	3	12
Total	1.56	2.09	1	1	2	12	48

Figure 8. Descriptive Statistics – Number of Days between Clusters

The figure shows the distribution of the number of days between clusters. In other words, how long time did it take from a cluster until the same stock entered another cluster. The first cluster in each stock has, by definition, no previous cluster as reference. Consequently, 23 values are missing.



Average	StDev	Min	p25	Median	p75	Max	N
104.7	166.0	3	7.5	27.5	109.5	665	60

Table 12. Regression Results - Clusters

The table provides results from the cluster analysis, more specifically from the regression with the following specification:

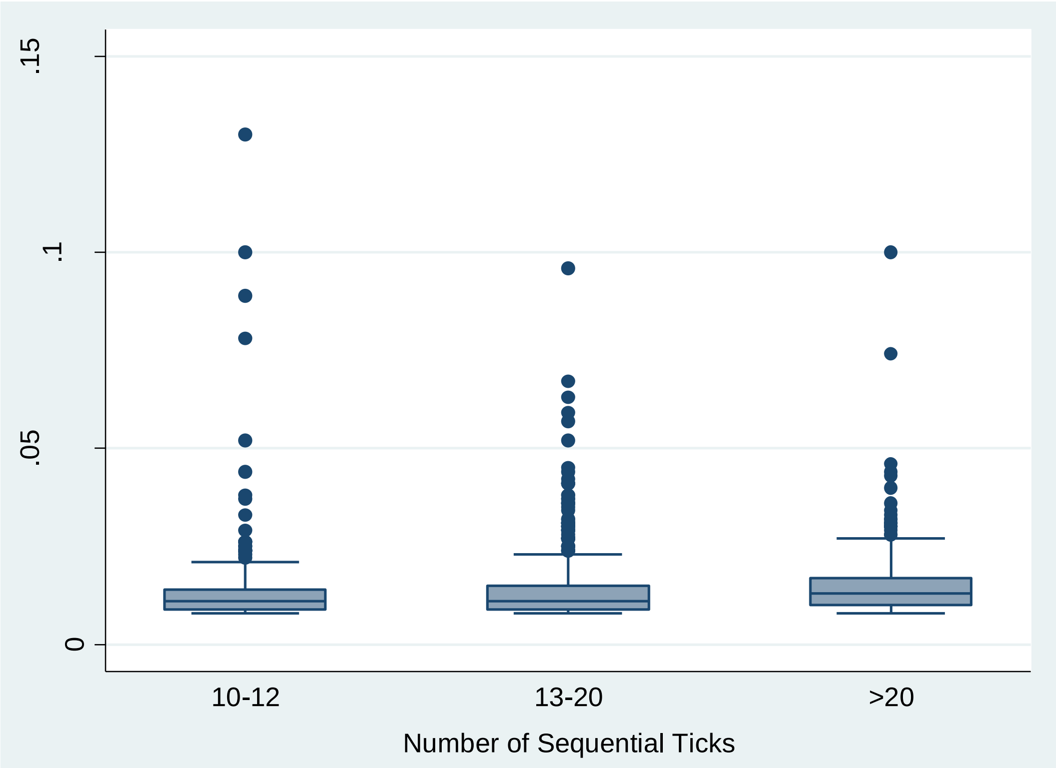
$$ClusterSize_c = \alpha + \beta_1 \times NFC_{c,max} + \beta_2 \times \overline{NFC}_{c,max.adj} + \beta_3 \times DbC_c + \varepsilon$$

The regression analysis is performed in four versions presented in columns a)-d). The differences between the versions are provided at the bottom of the table. The period is ranging from 3 January 2006 to 3 February 2011.

	a)	b)	c)	d)
Max NFC	0.577	0.365	0.591	0.345
t-value	*3.33*	*2.10*	*3.60*	*2.07*
Average NFC	1.33	2.85	1.39	3.12
t-value	*1.53*	*2.56*	*1.68*	*2.68*
Days between Cluster	-0.00238	-0.00179		
t-value	*-1.45*	*-1.29*		
N	60	60	83	83
Adjusted for percentage of NFC=0 days	no	yes	no	yes
Adjusted for Trading Volume	yes	yes	yes	yes
Days between Cluster Excluded	no	no	yes	yes

Figure 9. Descriptive Statistics - Amplitudes by the Number of Sequential Ticks

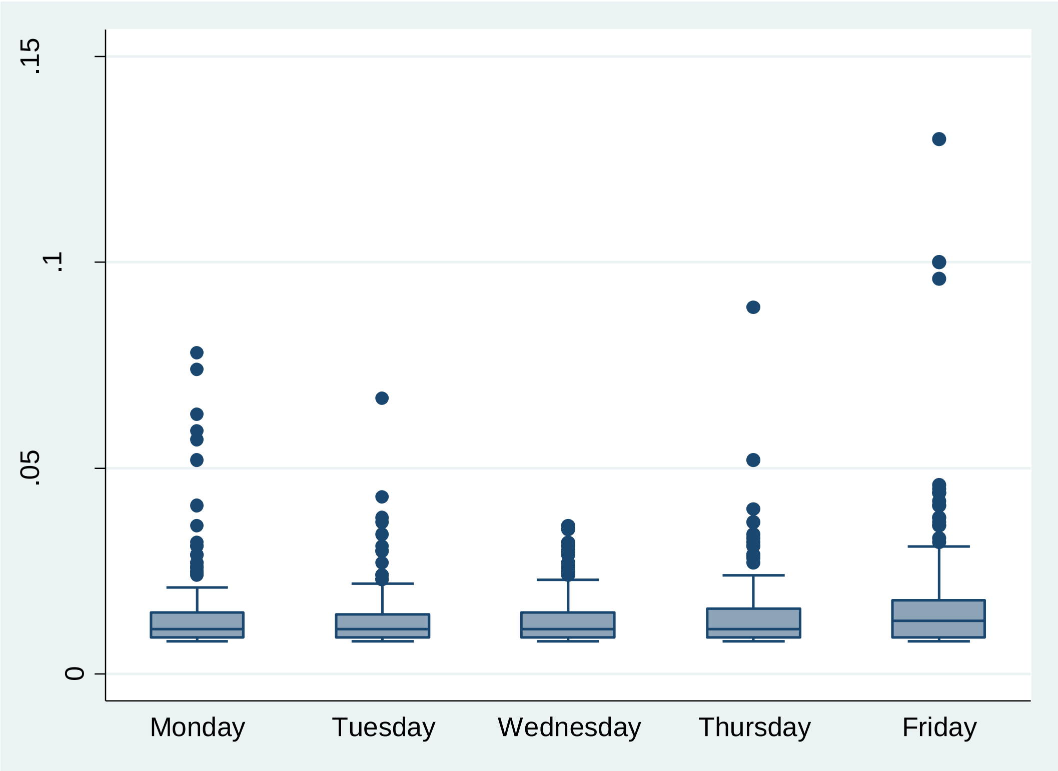
The figure provides descriptive statistics for amplitudes by the number of sequential ticks. The observations are categorised in three groups. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011.



Number of Ticks	Average	StDev	p25	Median	p75	N
10 to 12	0.0136	0.0116	0.009	0.011	0.014	301
13 to 20	0.0138	0.0085	0.009	0.011	0.015	585
more than 20	0.0152	0.0090	0.01	0.013	0.017	300
Total	0.0141	0.0095	0.009	0.011	0.015	1186

Figure 10. Descriptive Statistics - Amplitudes by Weekdays

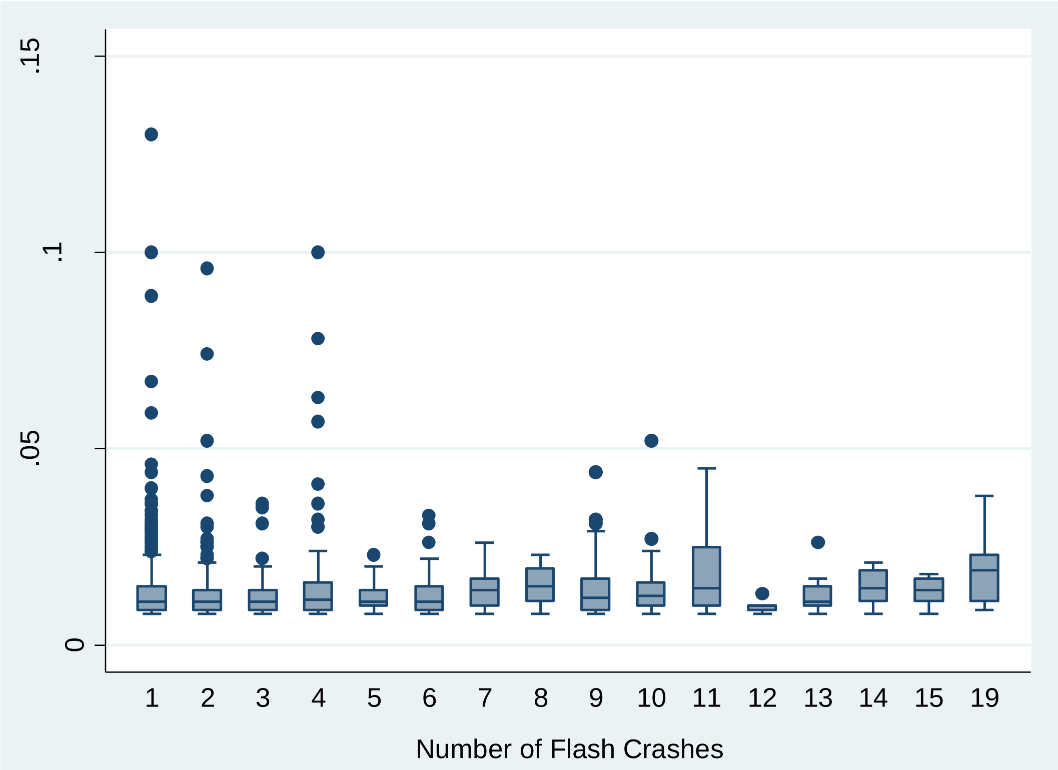
The figure presents the descriptive statistics for amplitudes for each weekday respectively. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011.



Weekday	Average	StDev	p25	Median	p75	N
Monday	0.0141	0.0099	0.009	0.011	0.015	229
Tuesday	0.0130	0.0068	0.009	0.011	0.0145	220
Wednesday	0.0128	0.0053	0.009	0.011	0.015	242
Thursday	0.0137	0.0079	0.009	0.011	0.016	237
Friday	0.0165	0.0140	0.009	0.013	0.018	258
Total	0.0141	0.0095	0.009	0.011	0.015	1186

Figure 11. Descriptive Statistics - Amplitudes by the Number of Mini Flash Crashes (NFC)

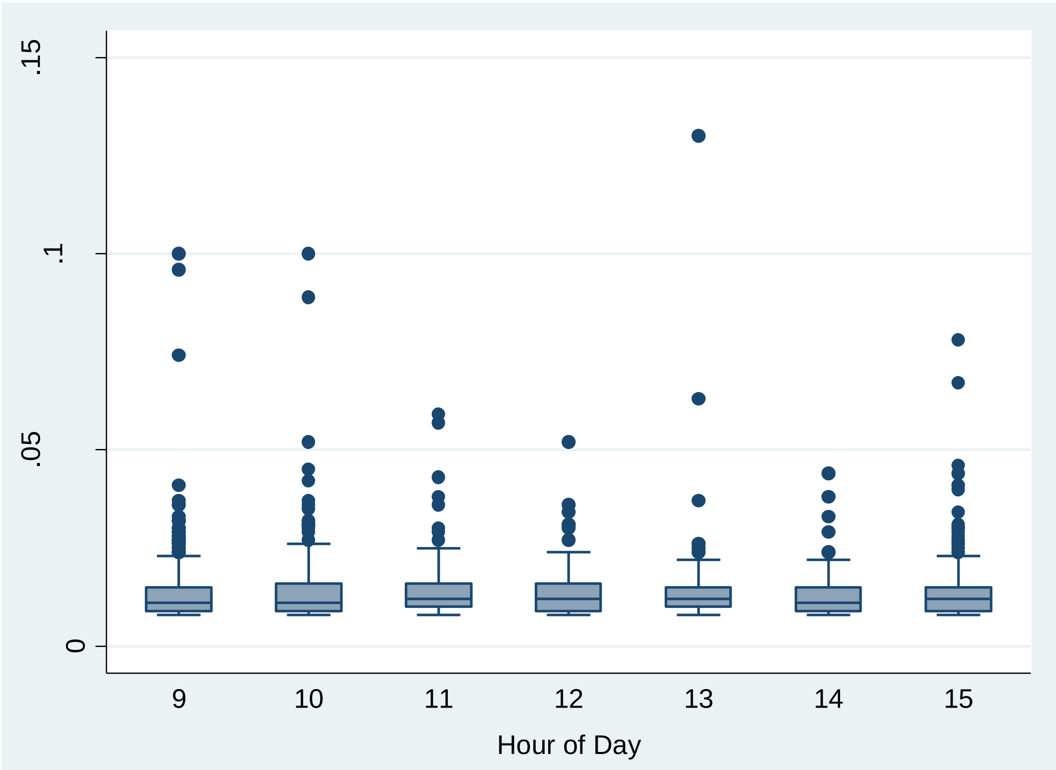
The figure presents the descriptive statistics for amplitudes for each value of NFC respectively. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011.



NFC	Average	StDev	p25	Median	p75	N
1	0.0139	0.0101	0.009	0.011	0.015	486
2	0.0134	0.0097	0.009	0.011	0.014	186
3	0.0125	0.0053	0.009	0.011	0.014	90
4	0.0159	0.0141	0.009	0.0115	0.016	96
5	0.0126	0.0033	0.01	0.011	0.014	35
6	0.0132	0.0062	0.009	0.011	0.015	36
7	0.0140	0.0050	0.01	0.014	0.017	21
8	0.0149	0.0048	0.011	0.015	0.0195	16
9	0.0145	0.0069	0.009	0.012	0.017	63
10	0.0143	0.0076	0.01	0.0125	0.016	40
11	0.0189	0.0115	0.01	0.0145	0.025	44
12	0.0097	0.0013	0.009	0.01	0.01	12
13	0.0128	0.0051	0.01	0.011	0.015	13
14	0.0144	0.0043	0.011	0.0145	0.019	14
15	0.0137	0.0032	0.011	0.014	0.017	15
19	0.0187	0.0085	0.011	0.019	0.023	19
Total	0.0141	0.0095	0.009	0.011	0.015	1186

Figure 12. Descriptive Statistics - Amplitudes by Hour of the Day

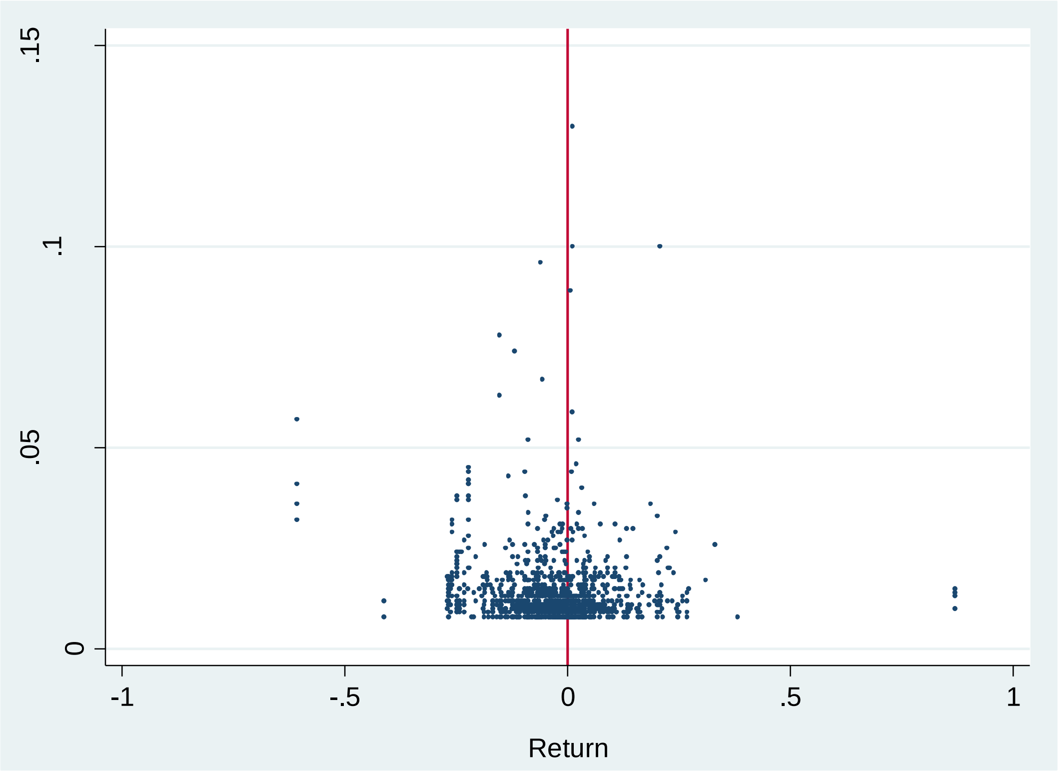
The figure presents the descriptive statistics for amplitudes for each hour of the day respectively (restricted to the hours when the stock market is open). The first "hour" is only 30 minutes because the stock market opens 09:30 am. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011.



Hour	Average	StDev	p25	Median	p75	N
9	0.0136	0.0089	0.009	0.011	0.015	374
10	0.0146	0.0116	0.009	0.011	0.016	173
11	0.0150	0.0095	0.01	0.012	0.016	91
12	0.0148	0.0085	0.009	0.012	0.016	59
13	0.0154	0.0148	0.01	0.012	0.015	81
14	0.0131	0.0062	0.009	0.011	0.015	119
15	0.0139	0.0081	0.009	0.012	0.015	289
Total	0.0141	0.0095	0.009	0.011	0.015	1186

Figure 13. Descriptive Statistics - Amplitudes by Daily Stock Returns

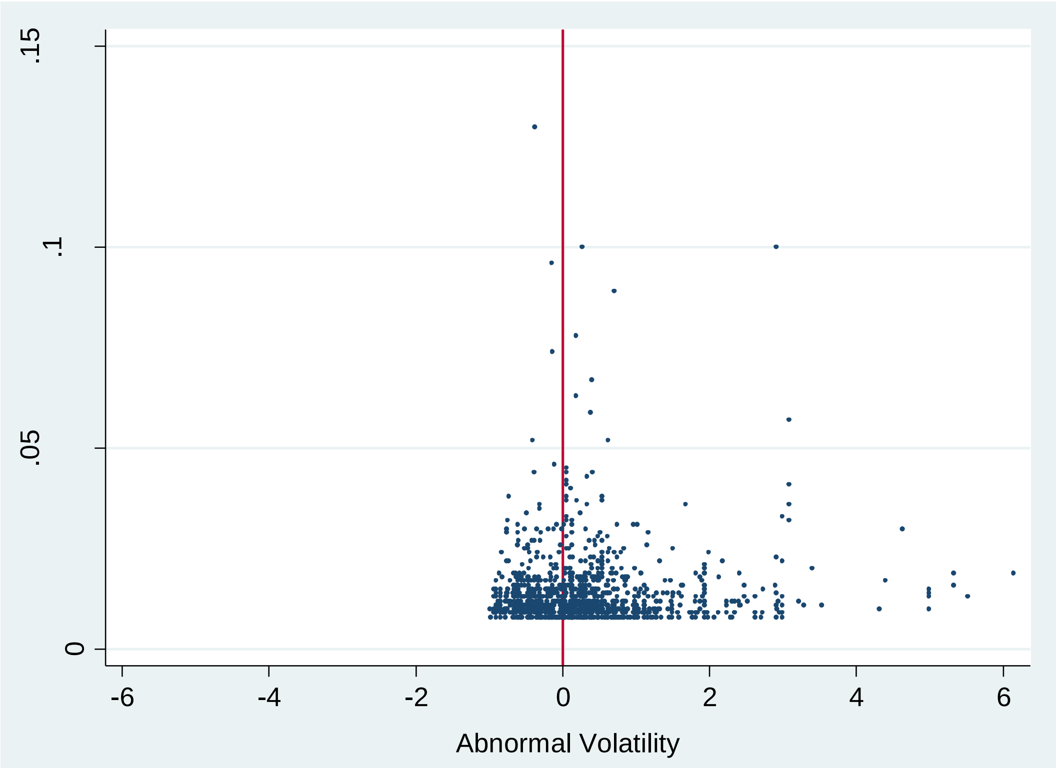
The figure presents the descriptive statistics for amplitudes by different daily stock returns. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011.



Return	Average	StDev	p25	Median	p75	N
smaller than -20%	0.0190	0.0104	0.011	0.016	0.023	98
-20% to -5%	0.0139	0.0094	0.009	0.011	0.015	362
-5% to 0%	0.0128	0.0057	0.009	0.011	0.014	282
0% to 5%	0.0142	0.0131	0.009	0.011	0.014	228
5% to 20%	0.0131	0.0054	0.009	0.011	0.016	167
bigger than 20%	0.0157	0.0136	0.01	0.012	0.016	49
Total	0.0141	0.0095	0.009	0.011	0.015	1186

Figure 14. Descriptive Statistics - Amplitudes by Abnormal Volatility

The figure presents the descriptive statistics for amplitudes by different values of abnormal volatility. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011. Please note that the lowest theoretically possible value for abnormal volatility is -1 (-100%).



Abnormal Volatility	Average	StDev	p25	Median	p75	N
smaller than -50%	0.0133	0.0054	0.0095	0.012	0.015	172
-50% to -15%	0.0137	0.0101	0.009	0.011	0.015	218
-15% to 15%	0.0153	0.0109	0.009	0.012	0.017	199
15% to 50%	0.0143	0.0104	0.009	0.011	0.015	251
50% to 100%	0.0138	0.0088	0.009	0.011	0.016	171
bigger than 100%	0.0139	0.0095	0.009	0.012	0.015	175
Total	0.0141	0.0095	0.009	0.011	0.015	1186

Table 13. Regression Results - Effect on Amplitudes

The figure presents the results from the main regression in the impact analysis which examines different factors' effect on amplitudes. The sample is based on 23 stocks during the period ranging from 3 January 2006 to 3 February 2011. The regression is adjusted for fixed stock effects and has the following specification:

$$Amplitude_j = \alpha + \beta_1 \times NoT_j + \beta_2 \times NFC_i(\mathrm{t}) + \beta_3 \times r_i(\mathrm{t}) + \beta_4 \times \sigma_{abn.,i}(t) \quad + \beta_5 \times d_{Monday}(\mathrm{t}) + \beta_6 \times d_{Friday}(\mathrm{t}) + \beta_7 \times d_{Opening,j} + \beta_8 \times d_{Closing,j} + \beta_9 \times d_{Start\ of\ week,j} + \beta_{10} \times d_{End\ of\ week,j} + \varepsilon$$

	All Crashes	Only Up	Only Down
	N=1186	N=764	N=422
Number of Ticks	0.000153	0.000130	0.000190
	3.73	*2.02*	*3.72*
Number of Flash Crashes	-0.000068	-0.000146	0.000100
	-0.91	*-1.49*	*0.71*
Return	-0.009875	-0.008989	-0.011432
	-3.8	*-2.58*	*-3.51*
Abnormal Volatility	0.000296	0.000671	0.000024
	0.92	*1.29*	*0.06*
Monday	0.001168	0.000921	0.001394
	1.36	*0.75*	*1.36*
Friday	0.004871	0.004739	0.005307
	4.48	*3.19*	*3.09*
Opening	-0.000833	-0.001629	0.000397
	-1.21	*-1.64*	*0.43*
Closing	0.000644	0.000439	0.001005
	0.88	*0.46*	*0.71*
Start of Week	0.001064	0.002044	0.000085
	0.77	*0.95*	*0.06*
End of Week	-0.004111	-0.005651	-0.000340
	-2.49	*-3.13*	*-0.1*

10. Mini Flash Crashes in Logit Models

10.1 Introduction

This part is an extension to my original paper "Mapping the Landscape of Mini Flash Crashes" (hereinafter referred to as "my previous paper"). Consequently, this part is based on theory, previous empirical work, variables, references and results presented there. In my previous paper a linear model was used to describe the occurrence of mini flash crashes, while a logit model is used in this part. The logit model complements the previous model in several ways, not least through its possibility to express the outcome as probabilities which simplifies the interpretation of the results.

10.2 Method

In my previous paper the occurrence of mini flash crashes was examined by looking at the NFC variable. In this part the exact number of flash crashes is not of interest, but instead a binomial case is examined. In other words the focus is on understanding the risk of having at least on mini flash crash. Hence, the explained variable is the dummy variable:

$d_{NFC>0}(t) = 1$ if the NFC variable has a value of one or higher

In my previous paper there were several results showing an autoregressive behaviour in the NFC variable. Consequently, this model will as well include a lagged variable. However, the single crash case (NFC(t-1)=1) and the multiple crash case (NFC(t-1)>1) will be separated and their effects will be captured by looking at the following dummy variables:

$d_{NFC(t-1)=1}(t) = 1$ if there was one mini flash crash in the previous time period

$d_{NFC(t-1)>1}(t) = 1$ if there were more than one mini flash crashes in the previous time period

Another factor which is interesting to include in the model is the effect from returns. In my previous the results did not support any significant effect from returns on a 5% level. However, for almost all of the 23 stocks examined the confidence intervals for the coefficients were shifting towards the negative side. In addition, negative returns had a significant effect on the percentage price change during the crash. Since the price change is a part of the definition of mini flash crashes, how much the price changes could be the difference between a movement being classified as a mini flash crash or not. Consequently, it would be wise to include the effect from returns in the model. The effect from returns is measured with two dummy variables:

$d_{-5\% < r(t) < 0\%}(t) = 1$ if the daily stock return is between -5% and 0%

$d_{r(t) < -5\%}(t) = 1$ if the daily stock return is lower than -5%

The main regression in this part is based on a logit model with the following specification:

$$[13] \quad P_{d_{NFC>0}(t)}(t) = \frac{1}{1+e^{-(\alpha + \beta_1 \times d_{NFC(t-1)=1}(t) + \beta_2 \times d_{NFC(t-1)>1}(t) + \beta_3 \times d_{-5\% < r(t) < 0\%}(t) + \beta_4 \times d_{r(t) < -5\%}(t) + \varepsilon)}}$$

$P_{d_{NFC>0}(t)}(t)$ is the probability of $d_{NFC>0}(t)$ having the value 1. The regression is volume adjusted and performed on the 23 stocks separately. After that a meta-analysis is done using the Hunter-Schmidt method.

The scenario when all the explanatory factors have the value of zero and only the effect from the constant is present is referred to as the base case.

10.3 Results

Looking at the descriptive results in table 14 (10.5 Appendix) one can see that the dummy variables $d_{NFC(t-1)=1}(t)$ and $d_{NFC(t-1)>1}(t)$ are more frequently taking the value of one on days with at least one mini flash crash compared to days with no crashes. This could suggest that there is a correlation between having at least one mini flash crash one day and having at least one mini flash crash the next day. The shift in frequency is even clearer in the multiple crash case where the $d_{NFC(t-1)>1}(t) = 1$ rarely exists on no crash days but occurs in 16% of the times on days with at least one crash. A similar behaviour is observed when looking at the variable $d_{r(t)<-5\%}(t)$ which represents days with extremely negative daily stock return. However, the opposite shift is seen in the variable $d_{-5\%<r(t)<0\%}(t)$. Hence, the total effect from returns is somewhat more dubious.

The results from the main regression can be found in table 15 (10.5 Appendix). What strikes the eye is that many of the tested stocks seem to be affected by the lagged value of NFC. This is particularly true for the multiple crash case ($d_{NFC(t-1)>1} = 1$) where 17 of 23 stocks shows an effect on a 5% significance level. In addition, the effect is in most cases big and increases the probability of having at least one mini flash crash with two digit percentage points. Negative returns do not seem to have any effect in general. However, when returns are sufficiently negative the probability of at least one mini flash crash increases for many of the tested stocks. This is in line with what could be seen in the descriptive statistics. In table 16

(10.5 Appendix) the results are presented as risk ratios.[3] Because of the low probability in the base case we can see that the significant factors multiply the risk of having at least one mini flash crash many times over.

Since the factors can be combined to a certain extent (extremely negative returns and the occurrence of mini flash crashes the previous day) there might be a very big difference between the probability of a mini flash crash in one scenario and another. Figure 15 (10.5 Appendix) shows the whole range from the lowest probability and the highest probability suggested by the logit model.[4] In average probabilities are almost 30 times higher in the worst case compared to the base case. However, despite the big multiple, the intervals only include probabilities above 50% in four of the stocks.

Table 17 (10.5 Appendix) shows the results from the meta-analysis with the Hunter-Schmidt method. No factor is significant on a 5% level.

10.4 Discussion

In my previous paper results suggested an autoregressive behaviour in the NFC variable. The logit model in this part supports that behaviour. In addition, the logit model shows that more stocks are affected by the lagged NFC than we could see in my previous paper. However, the finding that more stocks are affected by $d_{NFC(t-1)>1}(t)$ than $d_{NFC(t-1)=1}(t)$ could suggest that this relationship is not adequately described by a linear model.

Another interesting finding in the logit model is that returns can increase the probability of having at least one mini flash crash if the return is negative enough. The relationship between

[3] Risk ratio for a certain factor is the probability of a mini flash crash given that the factor is present divided by the probability when the factor is not present (base case).
[4] Only factors which are significant on a 5% level are included in the probability calculation for each stock and scenario

the occurrence of mini flash crashes and returns was not well described by the linear model and could not be supported in my previous paper. The lack of effect from $d_{-5\% < r(t) < 0\%}(t)$ could indicate that market participants can handle a certain amount of declining prices, but when the return goes below what could be considered the normal range of negative daily stock returns their responses to price changes are more aggressive. Hence, increasing the risk of mini flash crashes. Another explanation is that very big price drops are more likely to have broken important price support levels or triggered big stop-losses than small price drops.

Just like in my previous paper the results were hard to generalize through the meta-analysis. The information that several mini flash crashes occurred yesterday could mean everything from 2.6% (pot) to 69.7% (bac) increased probability of having at least one mini flash crash today, depending on what stock one is looking at.

One should remember that in the base case probabilities are very low (0.1%-2.1%) and explanatory variables being present is needed for mini flash crashes to really pose a threat to the markets.

10.4 Conclusions

The probability of mini flash crashes is higher when mini flash crashes occurred the previous day. If several crashes occurred the risk is substantially higher compared to if only one crash happened. The risk is also higher on days with extremely negative daily stock returns. However, the size of the effect from these factors vary a lot among individual stocks. In average the probability of having at least one mini flash crash is about 30 times bigger in the worst case compared to the base case (increasing the risk from less than 1% to about 20%).

10.5 Appendix

Table 14. Percentage of Values Being Equal to One

The table presents the percentage of values being equal to one for the explanatory dummy variables. The result is showed for days with at least one crash and days without crashes separately. The result is based on 1216 observations in the time period ranging from 3 January 2006 to 3 February 2011.

	No Crash	At Least One Crash	Total
Lagged NFC = 1	1.5%	12.9%	1.7%
Lagged NFC > 1	0.0%	16.0%	0.7%
- 5% < Return < 0%	44.1%	28.6%	43.7%
Return < -5%	4.8%	27.2%	5.3%

Table 15. Probability of at Least One Mini Flash Crash

The table presents the results from the 23 time series regressions and shows the contribution from every factor to the probability of having at least one mini flash crash. The regressions are logit models which are using the following specification:

$$P_{d_{NFC>0}(t)}(t) = \frac{1}{1 + e^{-(\alpha + \beta_1 \times d_{NFC(t-1)=1}(t) + \beta_2 \times d_{NFC(t-1)>1}(t) + \beta_3 \times d_{-5\% < r(t) < 0\%}(t) + \beta_4 \times d_{r(t) < -5\%}(t) + \varepsilon)}}$$

The time period is ranging from 3 January 2006 to 3 February 2011 and each regression is based on 1216 observations. Results highlighted in dark grey are statistically significant on a 5% level. Light Grey means that they are statistically significant only on a 10% level. Stocks are sorted by their total number of mini flash crashes in the data set (the stock with the highest number at the top and the lowest at the bottom).

	Constant (Base Case)		Lagged NFC = 1		Lagged NFC > 1		- 5% < Return < 0%		Return < -5%	
Ticker	Probability	z	Probability	z	Probability	z	Probability	z	Probability	z
ms	0.6%	-11.92	1.4%	1.58	27.1%	5.83	-0.2%	-0.67	1.0%	1.89
gs	0.9%	-12.57	3.6%	3.24	17.2%	4.62	-0.3%	-0.97	4.8%	4.04
wfc	1.4%	-13.03	6.6%	3.14	52.0%	6.83	-0.1%	-0.19	4.1%	2.92
jpm	0.9%	-12.79	1.5%	1.84	23.7%	6.13	-0.2%	-0.89	0.3%	0.64
met	0.6%	-12.18	3.9%	3.01	10.8%	4.55	-0.1%	-0.23	1.3%	2.17
bac	2.1%	-12.23	18.1%	4.66	69.7%	6.67	-1.0%	-1.49	2.1%	1.33
c	0.9%	-8.93	34.1%	5.18	56.1%	6.35	-0.1%	-0.24	9.3%	3.84
aig	0.8%	-11.95	-0.5%	-0.83	-0.4%	-0.64	0.1%	0.13	3.9%	3.71
bk	0.5%	-11.23	2.5%	2.25	33.1%	6.11	-0.1%	-0.49	0.6%	1.24
pru	0.4%	-11.06	3.7%	4.22	99.6%	0	0.1%	0.24	1.4%	3.02
pot	0.4%	-12.64	0.9%	1.68	2.6%	2.1	-0.2%	-0.99	0.1%	0.53
ge	0.3%	-9.01	44.8%	7.4	20.5%	3.72	0.0%	0.03	6.2%	3.54
fcx	0.5%	-10.33	6.3%	4.1	8.1%	3.15	-0.3%	-1.71	0.5%	1.28
mos	0.6%	-12.6	0.3%	0.35	3.4%	1.85	0.0%	-0.11	0.3%	0.64
axp	0.3%	-10.72	0.5%	0.88	5.2%	3.19	0.2%	0.89	0.3%	1.05
stt	0.7%	-11.38	-0.4%	-0.59	-0.7%	-0.01	0.1%	0.38	1.2%	1.5
ifn	1.4%	-12.69	0.5%	0.3	-1.4%	-0.01	0.5%	0.75	2.3%	1.54
clf	0.7%	-11.79	4.7%	2.97	11.1%	2.95	0.3%	0.66	0.4%	0.75
usb	0.5%	-11.7	3.0%	2.61	20.9%	3.64	-0.2%	-0.89	-0.1%	-0.37
etp	1.1%	-12.98	-1.1%	-0.02	10.9%	2.15	-0.1%	-0.13	8.0%	3.21
chk	0.2%	-10.19	0.9%	1.63	0.2%	0.49	0.2%	1.03	-0.1%	-0.35
cvx	0.1%	-9.32	0.1%	0.9	3.2%	2.88	0.0%	0.88	0.1%	1.08
slg	0.5%	-11.81	1.3%	1.71	-0.5%	-0.01	0.5%	1.45	0.7%	1.51

Table 16. Risk Ratios

The table presents the results from the 23 time series regressions and shows the contribution from every factor to the probability of having at least one mini flash crash. The probabilities are expressed as risk ratios. The regressions are logit models which are using the following specification:

$$P_{d_{NFC>0}(t)}(t) = \frac{1}{1 + e^{-(\alpha + \beta_1 \times d_{NFC(t-1)=1}(t) + \beta_2 \times d_{NFC(t-1)>1}(t) + \beta_3 \times d_{-5\% < r(t) < 0\%}(t) + \beta_4 \times d_{r(t) < -5\%}(t) + \varepsilon)}}$$

The time period is ranging from 3 January 2006 to 3 February 2011 and each regression is based on 1216 observations. Results highlighted in dark grey are statistically significant on a 5% level. Light Grey means that they are statistically significant only on a 10% level. Stocks are sorted by their total number of mini flash crashes in the data set (the stock with the highest number at the top and the lowest at the bottom).

	Constant (Base Case)		Lagged NFC = 1		Lagged NFC > 1		- 5% < Return < 0%		Return < -5%	
Ticker	Probability	z	Risk Ratio	z	Risk Ratio	z	Risk Ratio	z	Risk Ratio	z
ms	0.6%	-11.92	3.37	1.58	47.75	5.83	0.71	-0.67	2.73	1.89
gs	0.9%	-12.57	5.03	3.24	20.27	4.62	0.67	-0.97	6.34	4.04
wfc	1.4%	-13.03	5.76	3.14	38.54	6.83	0.93	-0.19	3.96	2.92
jpm	0.9%	-12.79	2.78	1.84	28.71	6.13	0.71	-0.89	1.36	0.64
met	0.6%	-12.18	7.78	3.01	19.56	4.55	0.89	-0.23	3.27	2.17
bac	2.1%	-12.23	9.61	4.66	34.16	6.67	0.52	-1.49	1.98	1.33
c	0.9%	-8.93	37.02	5.18	60.23	6.35	0.87	-0.24	10.86	3.84
aig	0.8%	-11.95	0.36	-0.83	0.49	-0.64	1.08	0.13	5.79	3.71
bk	0.5%	-11.23	6.29	2.25	71.07	6.11	0.77	-0.49	2.26	1.24
pru	0.4%	-11.06	11.48	4.22	284.40	0	1.16	0.24	5.10	3.02
pot	0.4%	-12.64	3.05	1.68	7.05	2.1	0.60	-0.99	1.35	0.53
ge	0.3%	-9.01	129.96	7.4	60.00	3.72	1.02	0.03	18.93	3.54
fcx	0.5%	-10.33	13.85	4.1	17.38	3.15	0.35	-1.71	1.97	1.28
mos	0.6%	-12.6	1.45	0.35	6.60	1.85	0.95	-0.11	1.47	0.64
axp	0.3%	-10.72	2.70	0.88	19.40	3.19	1.63	0.89	2.12	1.05
stt	0.7%	-11.38	0.40	-0.59	0.00	-0.01	1.22	0.38	2.80	1.5
ifn	1.4%	-12.69	1.37	0.3	0.00	-0.01	1.34	0.75	2.66	1.54
clf	0.7%	-11.79	7.30	2.97	15.83	2.95	1.36	0.66	1.56	0.75
usb	0.5%	-11.7	6.81	2.61	40.93	3.64	0.62	-0.89	0.76	-0.37
etp	1.1%	-12.98	0.00	-0.02	10.56	2.15	0.94	-0.13	8.03	3.21
chk	0.2%	-10.19	5.29	1.63	2.09	0.49	1.83	1.03	0.70	-0.35
cvx	0.1%	-9.32	2.23	0.9	51.16	2.88	1.61	0.88	2.93	1.08
slg	0.5%	-11.81	3.29	1.71	0.00	-0.01	1.97	1.45	2.33	1.51

Figure 15. Probability of Mini Flash Crashes – Risk Interval

The figure shows the risk interval from the lowest probability of having at least one mini flash crash to the highest probability suggested by the model:

$$P_{d_{NFC>0}(t)}(t) = \frac{1}{1 + e^{-(\alpha + \beta_1 \times d_{NFC(t-1)=1}(t) + \beta_2 \times d_{NFC(t-1)>1}(t) + \beta_3 \times d_{-5\% < r(t) < 0\%}(t) + \beta_4 \times d_{r(t) < -5\%}(t) + \varepsilon)}}$$

A multiple is presented to the right of the bars, showing how many times bigger the risk of having at least one mini flash crash is in the worst case (all risk increasing variables being present) compared to the base case (only the effect from the constant in the model). Only factors which are significant on a 5% level are included in the probability calculation for each stock and scenario.

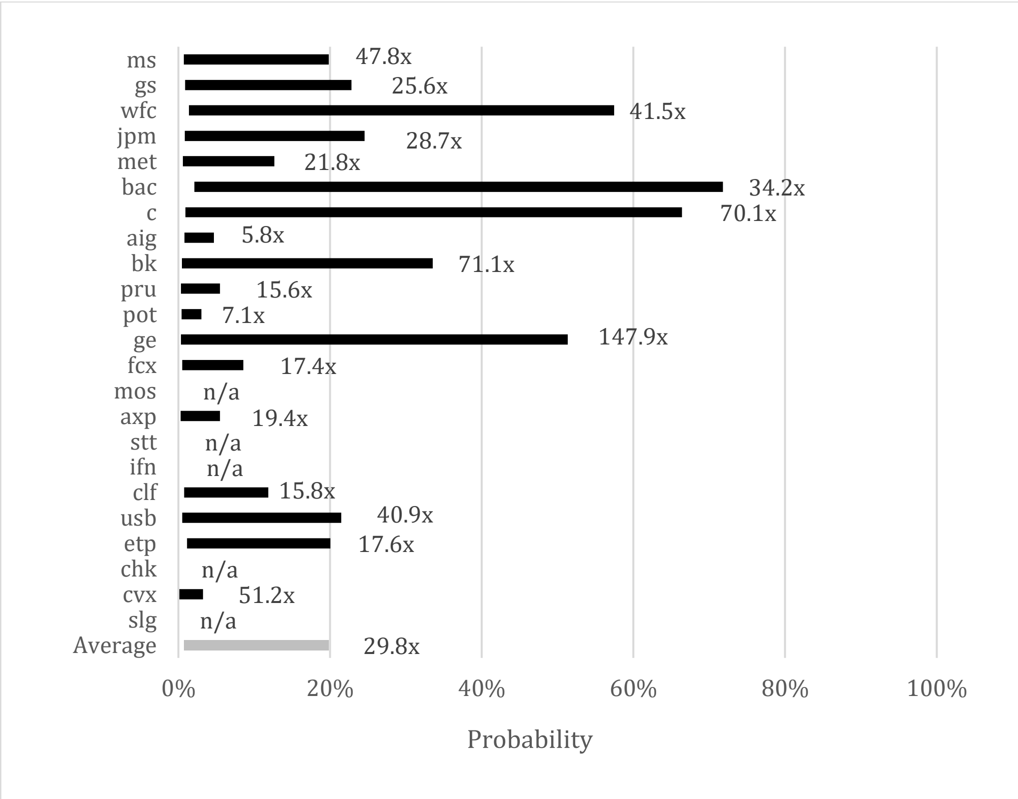


Table 17. Probability of Mini Flash Crashes - Meta Analysis

The table shows the result from the meta-analysis of all the 23 time series studies described in table 15. The chosen method for performing the meta-analysis is Hunter and Schmidt.

	Constant (Base Case)	Lagged NFC = 1	Lagged NFC > 1	- 5% < Return < 0%	Return < -5%
Probability	0.71%	5.94%	20.54%	-0.04%	2.12%
z-value	*1.60*	*0.55*	*0.81*	*-0.14*	*0.81*