Conditional Value-at-Risk targeted portfolio optimisation

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Abstract

New financial regulations have constantly forced market participants to adapt to changing rules. Recent regulatory iterations require them to focus on tail risk in portfolios of financial assets. One metric to quantify tail risk in portfolios is the Conditional Value-at-Risk (cVaR). While academic research has recently enhanced the importance of constructing optimal portfolios from a risk management perspective these results have not been incorporated into business models of firms in the asset management industry. Therefore, this thesis focuses on the practical aspect of implementing a process to optimise portfolios from a risk perspective. It gives step by step instructions to the optimal risk controlling construction of a portfolio from different asset universes including equity, bonds and commodity indices. Backtest results show that risk focussing strategies deliver superior risk-adjusted returns compared to traditional strategies like buy and hold and equal-weight. In particular, the cVaR Deviation and Minimum Variance portfolios achieve the highest Sharpe Ratio. Additionally, a replication consisting of exchange traded funds validates the importance of the results as it shows that retail investors are able to follow the developed investment approaches.

Keywords: Conditional Value-at-Risk, Value-at-Risk, Coherent Risk Measures, Portfolio Optimisation, Monte Carlo Simulation

Supervisor: Michael Halling[1], Associate Professor Department of Finance

[1] We are thankful for the valuable input and guidance throughout the writing of this thesis.

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1 Introduction

The asset management industry has gone through significant structural changes over last decades. Exchange traded funds (ETFs) and funds managed by quantitative asset managers generated massive inflows during recent years. Consequently, the share of funds managed by traditional money managers diminished. This process has also lead to changes in how capital is invested by professionals. A larger set of investment strategies and portfolio analyses is in place as a consequence of the 'quantification' of the industry. Most noticeably, investment strategies that shift their focus away from a pure return perspective towards the inclusion of embedded risks analysis have grown attention among investors.

However, the industry standard for measuring risk has long been the standard deviation. Common portfolio metrics like the Sharpe Ratio or Tracking Error apply the simple standard deviation as a proxy for risk. Despite well-known downsides like the penalization of returns equally to losses and the limited application to normally distributed returns has the standard deviation remained the undisputed standard (Rockafellar et al. (2002)).

Nowadays, new risk measures rapidly gain popularity among investment professionals and investors mainly due to two reasons. First, financial regulation, mostly represented by the Basel accords, has introduced rules that require financial market participants to adapt to new rules, including new risk metrics that focus on tail events. Furthermore, the Financial Crisis has reminded investors to not only invest based on a pure return perspective but also to embrace the potential loss in exceptional market environments.

As a consequence, the Value-at-Risk (VaR) and related risk metrics like the Conditional Value-at-Risk (cVaR) have gained tremendous importance in the industry as they are able to capture the portfolio risk in extreme market scenarios. The portfolio management industry, however, has yet to fully recognize the potential of pure risk focussing strategies. In active money management, risk objectives are, if at all, only constraints in the optimisation process. For passive products, minimum volatility strategies for downside protection are common. The lack of efficient and empirical approaches to optimise large sets of assets might be an explanation for the limited applicability of academic results. Thereby, there is significant potential for enhancements to the process.

The aim of this work is to provide a guide to the implementation of a portfolio optimisation process that minimizes the cVaR. The focus hereby is on presenting an empirical step by step execution in Matlab.

The methodology of the thesis is mainly based on current research results and an optimisation approach by Rockafellar and Uryasev (2000). However, the implementation itself follows a less academic but rather practical approach.

Therefore, the paper is structured as follows: Section 2 gives an overview of relevant literature on cVaR portfolio optimisation and important research results for the derivation of required inputs into the process. Section 3 describes the data used in this thesis. Section 4 introduces the methodology applied throughout this thesis including the relevant risk metrics, variance and covariance estimation using sample, exponentially weighted moving average and generalized autoregressive conditional heteroscedasticity approaches, simulation and price forecast process, calculation of portfolio risk measures and portfolio construction and optimisation. Section 5 reports the results of the proposed strategies including a comparison of the impact of different (co)variance estimation methods on portfolio characteristics, followed by a robustness review in section 6. Section 7 shows an ETF strategy to replicate the approach. Section 8 gives implications of the results. An outlook for further research is provided in section 9. Section 10 concludes.

2 Related Literature

The importance of the concept of VaR has not only got attention within academic research but also the financial industry has increasingly focussed on analysing the tails of distributions and effects of extreme market scenarios on portfolios of assets. Of particular interest for the present thesis are research results on portfolio optimisation problems based on VaR and cVaR.

Portfolio optimisation first gained traction with the introduction of the risk-return framework by Markowitz (1952) in which capital was allocated according to the mean-variance approach. Simultaneously, Roy (1952) pioneered the portfolio selection under shortfall constraints. In his “safety-first theory”, the construction of a portfolio is conducted such that only with a specific disaster probability, the value of a portfolio falls below a certain disaster level.

In more recent years, focus has shifted towards portfolio optimisation applying certain risk measures. As the pioneers in this field, Rockafellar and Uryasev (2000) introduced a formal technique for portfolio construction and later extended their approach to a linear model. A simultaneous optimisation of a portfolio VaR and cVaR is their main finding. In addition, Rockafellar and Uryasev (2002) discuss the application of the cVaR to continuous as well as discrete loss distributions. Furthermore, Rockafellar and Uryasev (2006) establish a cVaR deviation risk measure which will be further outlined in section 4.1.

One major step in the research on risk measures including the VaR has been the introduction of a formal definition for coherent risk measures. Artzner et al. (1999) introduced four axioms a risk metric has to fulfil in order to be a coherent risk measures. The properties, which are outlined in detail in section 4.1 and desired for portfolio optimisation, guarantee that the respective risk metric behaves according to expectations. Artzner et al. (1999) demonstrate how the VaR fails to satisfy all axioms, a finding that is often referred to in other papers dealing with VaR and akin risk measures.

Yamai and Yoshiba (2002) found instable results in cVaR estimations for distributions that are characterized by fat tails, typically a feature present in asset returns. A stable estimation of a portfolio cVaR is desired to ensure empirical applicability. Therefore, results are tested for stability in the robustness section 6.

In order to provide the reader with more background of the process of deriving input data needed for the optimisation, the following papers are of particular relevance.

A main aspect of the concept of VaR and cVaR are methods for volatility estimation. Especially relevant for this thesis are the exponentially weighted moving averages (EWMA) and generalized autoregressive conditional heteroscedasticity (GARCH) models further described in section 4.2. For EWMA models, research results by RiskMetrics (1996) are of importance. The authors are the leading developer of the methodology and are famous for their decay factor estimation utilizing multiple asset classes. The results are of high significance as many practitioners apply the given values for their volatility estimation models without an attempt to derive more precise results. This master thesis offers an empirical approach to optimise results for the EWMA models for each specific asset individually and, thereby, extends the approach taken by RiskMetrics.

The leading researcher on GARCH models is Bollerslev (1986), who generalized autoregressive conditional heteroskedastic models (ARCH). Bollerslev (1986) extended results by Engle to allow the conditional variance to dynamically vary over time as a function of past errors terms. GARCH (p,q) processes are of substantial importance in the risk management industry and are utilized in its simplest form, the GARCH (1,1) in this thesis. Adding the sample (co)variance, the three estimation approaches are an integral part of this thesis and will be compared in the result section.

3 Data description

In this thesis three different datasets are considered to cover a broad range of asset classes and allow comparisons of results. The first dataset consists entirely of equity indices of different countries. The second asset universe represents a multi asset opportunity set consisting of equity, bond and commodity indices. Lastly, a pure US stock universe is constructed. As this thesis aims to evaluate the cVaR portfolio optimisation concept, a thorough look on its performance for different asset classes is essential. To evaluate investment approaches based on financial data, a long history of data for a backtest is crucial. For all three asset universes, the longest available daily data history was gathered. The trade-off between a long history for each individual asset and a broad coverage within the universes led to the dataset compositions. All prices are total return prices to incorporate dividend reinvestments. Furthermore, all assets are converted into USD to make international assets comparable in terms of price changes and losses. This excludes the additional impact of foreign exchange fluctuations as a separate asset class within the optimisation.

Table 1 shows the composition of the first dataset.

Table 1 Equity Indices Universe

Table 1 shows the composition of the equity indices universe.

Name	Bloomberg Ticker
DAX Performance Index	DAX Index
Nikkei 250 Index	NKY Index
S&P 500 Index	SPX Index
FTSE 100 Index	UKX Index
CAC 40 Index	CAC Index
Hang Seng Index	HSI Index
IBEX 35 Index	IBEX Index
KOSPI Index	KOSPI Index
MSCI Emerging Market Index	MXEF Index
MSCI Australia Index	MXAU Index
OMX Stockholm 30 Index	OMX Index

Data source: Bloomberg

This dataset consists of prices between December 31st, 1987 and December 30th, 2016. Table 2 shows the correlation structure between all equity indices over this period. The lowest correlation between two indices is 0.12 and the highest is 0.82 which will be further discussed in section 8.

Table 2 Correlation Matrix Equity Indices Universe

Table 2 shows the correlation structure within the equity indices universe

	DAX	NKY	SPX	UKX	CAC	HSI	IBEX	KOSPI	MXEF	MXAU	OMX
DAX	1.00	0.27	0.51	0.73	0.82	0.35	0.73	0.22	0.51	0.31	0.71
NKY	0.27	1.00	0.12	0.29	0.28	0.44	0.27	0.35	0.46	0.48	0.28
SPX	0.51	0.12	1.00	0.51	0.50	0.17	0.46	0.13	0.40	0.12	0.44
UKX	0.73	0.29	0.51	1.00	0.81	0.35	0.71	0.23	0.53	0.32	0.70
CAC	0.82	0.28	0.50	0.81	1.00	0.33	0.80	0.22	0.52	0.31	0.74
HSI	0.35	0.44	0.17	0.35	0.33	1.00	0.32	0.38	0.61	0.51	0.33
IBEX	0.73	0.27	0.46	0.71	0.80	0.32	1.00	0.21	0.52	0.31	0.68
KOSPI	0.22	0.35	0.13	0.23	0.22	0.38	0.21	1.00	0.53	0.35	0.23
MXEF	0.51	0.46	0.40	0.53	0.52	0.61	0.52	0.53	1.00	0.52	0.50
MXAU	0.31	0.48	0.12	0.32	0.31	0.51	0.31	0.35	0.52	1.00	0.32
OMX	0.71	0.28	0.44	0.70	0.74	0.33	0.68	0.23	0.50	0.32	1.00

The second dataset, shown in table 3 with further information provided in appendix II, consists of the following twenty indices with correlations reported in table 4.

Table 3 Multi Asset Universe

Table 3 shows the composition of the multi asset universe.

Name	**Bloomberg Ticker/ Datastream**
Barclays Long U.S. Corporate - Investment Grade	LHCCORP(IN)+100
Barclays U.S. Corporate High Yield - Speculative Grade	LHYIELD(IN)+100
S&P GSCI Commodity Total Return - RETURN IND. (OFCL)	GSCITOT
US-DS Real Estate - TOT RETURN IND	RLESTUS
DAX Performance Index	DAX Index
Nikkei 250 Index	NKY Index
S&P 500 Index	SPX Index
FTSE 100 Index	UKX Index
CAC 40 Index	CAC Index
Hang Seng Index	HSI Index
IBEX 35 Index	IBEX Index
KOSPI Index	KOSPI Index
FTSE MIB Index	FTSEMIB Index
MSCI Emerging Market Index	MXEF Index
MSCI Australia Index	MXAU Index
OMX Stockholm 30 Index	OMX Index
US BENCHMARK 10 YEAR DS GOVT. INDEX	BMUS10Y
UK BENCHMARK 10 YEAR DS GOVT. INDEX	BMUK10Y
BD BENCHMARK 10 YEAR DS GOVT. INDEX	BMBD10Y
JP BENCHMARK 10 YEAR DS GOVT. INDEX	BMJP10Y

Data source: Bloomberg and Datastream

This dataset consists of prices between December 31st, 1998 and December 30th, 2016. All indices possess daily prices starting December 31st, 1998. Especially for high yield and Emerging Market indices, availability of daily prices is limited, resulting in a history of less than 20 years.

Table 4 Correlation Matrix Multi Asset Universe

Table 4 shows the correlation structure within the multi asset universe.

	US IG	US HY	GSCITOT	RLESTUS	DAX	NKY	SPX	UKX	CAC	HSI	IBEX	KOSPI	FTSEMIB	MXEF	MXAU	OMX	BMUS10Y	BMUK10Y	BMBD10Y	BMJP10Y
US IG	1.00	0.18	-0.13	-0.13	-0.20	0.05	-0.23	-0.16	-0.18	0.03	-0.17	0.01	-0.17	-0.04	0.08	-0.16	0.89	0.30	0.29	0.18
US HY	0.18	1.00	0.20	0.13	0.34	0.33	0.22	0.40	0.37	0.34	0.37	0.27	0.38	0.47	0.36	0.33	-0.08	0.08	0.03	-0.15
GSCITOT	-0.13	0.20	1.00	0.16	0.23	0.12	0.24	0.29	0.27	0.16	0.25	0.13	0.28	0.32	0.13	0.23	-0.18	0.13	0.10	-0.06
RLESTUS	-0.13	0.13	0.16	1.00	0.35	0.02	0.69	0.33	0.34	0.11	0.32	0.09	0.32	0.29	0.06	0.32	-0.18	0.03	0.00	-0.12
DAX	-0.20	0.34	0.23	0.35	1.00	0.27	0.61	0.80	0.89	0.36	0.79	0.29	0.82	0.56	0.28	0.78	-0.34	-0.09	-0.16	-0.27
NKY	0.05	0.33	0.12	0.02	0.27	1.00	0.13	0.30	0.30	0.55	0.27	0.52	0.26	0.55	0.56	0.28	-0.09	0.02	-0.03	-0.27
SPX	-0.23	0.22	0.24	0.69	0.61	0.13	1.00	0.55	0.57	0.20	0.52	0.16	0.54	0.44	0.13	0.51	-0.33	-0.02	-0.07	-0.18
UKX	-0.16	0.40	0.29	0.33	0.80	0.30	0.55	1.00	0.88	0.40	0.78	0.30	0.78	0.61	0.34	0.78	-0.30	-0.09	-0.11	-0.28
CAC	-0.18	0.37	0.27	0.34	0.89	0.30	0.57	0.88	1.00	0.38	0.87	0.30	0.88	0.59	0.33	0.82	-0.33	-0.08	-0.16	-0.30
HSI	0.03	0.34	0.16	0.11	0.36	0.55	0.20	0.40	0.38	1.00	0.35	0.57	0.34	0.75	0.58	0.36	-0.11	0.06	-0.01	-0.17
IBEX	-0.17	0.37	0.25	0.32	0.79	0.27	0.52	0.78	0.87	0.35	1.00	0.28	0.87	0.56	0.31	0.74	-0.31	-0.04	-0.11	-0.27
KOSPI	0.01	0.27	0.13	0.09	0.29	0.52	0.16	0.30	0.30	0.57	0.28	1.00	0.27	0.69	0.50	0.30	-0.10	0.00	-0.03	-0.16
FTSEMIB	-0.17	0.38	0.28	0.32	0.82	0.26	0.54	0.78	0.88	0.34	0.87	0.27	1.00	0.55	0.29	0.74	-0.32	-0.05	-0.12	-0.28
MXEF	-0.04	0.47	0.32	0.29	0.56	0.55	0.44	0.61	0.59	0.75	0.56	0.69	0.55	1.00	0.60	0.57	-0.22	0.11	0.07	-0.20
MXAU	0.08	0.36	0.13	0.06	0.28	0.56	0.13	0.34	0.33	0.58	0.31	0.50	0.29	0.60	1.00	0.31	-0.07	0.06	0.03	-0.17
OMX	-0.16	0.33	0.23	0.32	0.78	0.28	0.51	0.78	0.82	0.36	0.74	0.30	0.74	0.57	0.31	1.00	-0.29	-0.08	-0.13	-0.25
BMUS10Y	0.89	-0.08	-0.18	-0.18	-0.34	-0.09	-0.33	-0.30	-0.33	-0.11	-0.31	-0.10	-0.32	-0.22	-0.07	-0.29	1.00	0.29	0.30	0.26
BMUK10Y	0.30	0.08	0.13	0.03	-0.09	0.02	-0.02	-0.09	-0.08	0.06	-0.04	0.00	-0.05	0.11	0.06	-0.08	0.29	1.00	0.69	0.29
BMBD10Y	0.29	0.03	0.10	0.00	-0.16	-0.03	-0.07	-0.11	-0.16	-0.01	-0.11	-0.03	-0.12	0.07	0.03	-0.13	0.30	0.69	1.00	0.39
BMJP10Y	0.18	-0.15	-0.06	-0.12	-0.27	-0.27	-0.18	-0.28	-0.30	-0.17	-0.27	-0.16	-0.28	-0.20	-0.17	-0.25	0.26	0.29	0.39	1.00

The third dataset consists entirely of individual stocks. It is constructed by applying the following selection rules to US-stocks. Each stock must have a stock price history of at least three years on a selection day to become eligible. Its country of risk[2] must be the United States of America and it also must be listed on an exchange in the US. Furthermore, only one share class per company is considered for the selection process. On each selection day, the 50 largest companies according to their market capitalization are chosen from the eligible stock universe. The selection day is the last trading day in December of each year. Stocks are selected between 1995 and 2016. The entire dataset consists of 121 stocks which fulfilled the selection criteria in at least one of the 21 selections. For all 121 companies, stock prices and market capitalizations in USD are compiled. The prices are total return prices to account for reinvestments of dividends into the respective stock and adjusted for stock splits.

Following this selection methodology helps to overcome a survivorship bias as stocks can enter and exit the eligible universe depending on their market capitalization at the time of the selection day. This includes companies that fulfil the criteria at that time but might be delisted over the analysed period.

4 Methodology

4.1 Motivation for conditional Value-at-Risk approach

Academic research has long focussed on the choice of the superior risk metric considering the VaR and cVaR as relevant measures. The argumentation is based on different mathematical properties, robustness of statistical estimation, simplicity in the process of optimising with regard to the relevant metric and the regulatory environment (Sarykalin (2008)).

Following the argumentation of Quaranta and Zaffaroni (2008), a quantile based measure of risk such as the VaR and cVaR allow for capturing the difference between positive and

[2] The country of risk is a country classification set by Bloomberg. Bloomberg defines this country assignment in the following way: Returns the International Organization for Standardization (ISO) country code of the issuer's country of risk. Methodology consists of four factors listed in order of importance: management location, country of primary listing, country of revenue and reporting currency of the issuer. Management location is defined by country of domicile unless location of such key players as Chief Executive Officer (CEO), Chief Financial Officer (CFO), Chief Operating Officer (COO), and/or General Counsel is proven to be otherwise.

negative deviations of returns from their means. As both market participants as well as regulators are mainly concerned about deviations to the downside, the attention towards quantile based measures has increased recently.

A motivation for the application of the cVaR rather than the VaR requires an exact definition of the two concepts in order to point out the shortcomings of the VaR in relation to the cVaR.

Formally, let X be a random variable with cumulative distribution function $F_X(z) = P\{X \leq z\}$, the VaR is defined as follows

$$VaR_\alpha(X) = min\{z|F_X(z) \geq \alpha\} \quad (1)$$

with $\alpha \in (0,1)$ representing the confidence level. In this case, X is representing asset losses, causing the relation in the formula to be 'greater or equal to' (Sarykalin et al. (2008)).

According to Inui and Kijima (2005), the VaR is one of the most popular risk measures used by financial institutions. However, it has been criticized by several researchers for not being a coherent risk measure as it entails severe undesirable mathematical characteristics (Inui and Kijima (2005), Quaranta and Zaffaroni (2008) and Rockafellar and Uryasev (2002)). Referring to Artzner et al. (1999), a risk measure is coherent if it satisfies the following four axioms where $\rho(X)$ stands for a certain risk measure ρ of X, a random variable denoting the final net worth of a position or a portfolio:

Axiom 1: Translation invariance

Let X be defined as above, G the set of all risks and r the total return on an asset, then for all $X \in G$ and all real numbers α, the following holds:

$$\rho(X + \alpha * r) = \rho(X) - \alpha \quad (2)$$

The term α can be interpreted as an initial amount of capital α that has a certain expected return. Axiom 1 implies that adding the safe asset (e.g. cash) to the existing position decreases the risk measure by the same amount. The opposite applies for decreasing the position by α. Axiom 1 is also often referred to as the 'Risk Free Condition'.

Axiom 2: Subadditivity

Let X and G be defined as in Axiom 1. Then for all X_1 and $X_2 \in G$, the following holds:

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2) \tag{3}$$

Subadditivity implies that the risk measure for portfolios consisting of multiple positions is bounded from above by the sum of the same risk measure for all positions individually. In other words, combining positions in a portfolio allows for diversification.

Axiom 3: Positive homogeneity

For all $X \in G$ and all $\lambda \geq 0$,

$$\rho(\lambda X) = \lambda \rho(X) \tag{4}$$

Axiom 3 eliminates any non-linear effects of position sizes. Positive homogeneity guarantees that the relationship between the amount invested in a position and the associated risk is linear.

Axiom 4: Monotonicity

For all X and $Y \in G$ with $X \leq Y$, the following holds:

$$\rho(Y) \leq \rho(X) \tag{5}$$

If $X \leq Y$ and X *and* $Y \in G$, then in all instances the outcome of Y is more positive than (or equal to) X, i.e. the expected return of Y is superior to the one of X. Consequently, the risk associated with the more positive outcome (Y) is smaller than (or equal to) the risk of the less positive outcome.

In the context of this thesis it is of particular interest to point out the shortcoming of the VaR. For this risk measure, the most cited critique is the lack of subadditivity, which implies that a higher degree of diversification of a portfolio is not necessarily associated with a reduced VaR. Only in the case of normally distributed losses for a portfolio where the VaR is simply the standard deviation of the loss distribution scaled up by the z-score of the analysed percentile, the VaR conforms to the axioms and is, consequently, a coherent risk measure. However, as widely accepted in financial research, empirically observed return distributions are rarely normal, leaving the VaR breaching the Subadditivity axiom as shown in Rockafellar and Uryasev (2000)). An illustration is given by the following example adapted from Kisiala (2015):

Given the following two assets A and B with three different scenarios s_1, s_2 and s_3 and their probabilities p(s_i), the table below shows the losses for each asset in the respective state of the scenarios:

	p(s_1)= 0.06	p(s_2)= 0.06	p(s_3)= 0.88
A	500	0	0
B	0	500	0
A+B	500	500	0

The $VaR_{0.90}(A)$ is equal to the $VaR_{0.90}(B)$ and both have a value of 0 as the probability of a loss greater than zero for A and B is lower than the confidence level of the VaR (with a probability of 0.94 there will be no losses for both assets). However, the portfolio consisting of A and B has a $VaR_{0.90}(A + B) = 500$ and thereby violates axiom 2 – the subadditivity criteria as $VaR_{0.90}(A + B) \geq VaR_{0.90}(A) + VaR_{0.90}(B)$.

Furthermore, Krokhmal et al. (2001) point out that the VaR is difficult to optimise for discrete distributions which are generated using a simulation as multiple extrema impede the construction of a risk optimal portfolio. This thesis tries to construct a portfolio using a risk measure from simulated non-normal returns. As a simulation of future asset returns results in a discrete distribution of portfolio losses, the VaR does not serve as an appropriate measure for the purpose of this thesis.

In contrast to the VaR, the cVaR is defined by Artzner et al. (1999) as a coherent measure of risk. As the cVaR is the weighted average of the VaR and losses strictly exceeding the VaR, this methodology combines the benefits of being a coherent risk measure with the quantile based approach. Following the approach by Sarykalin et al. (2008), the cVaR is determined as the following:

$$cVaR_{\alpha}(X) = E[X \mid X \geq VaR_{\alpha}(X)] \tag{6}$$

In addition, constructing the cVaR as a weighted average of the mass points in the tail, it includes the entire tail of the loss distribution by design. Not only does this approach result in a more conservative risk measure compared to the VaR (Sarykalin et al. (2008)), but also incorporates insight into the distribution within the tail as show in figure 1. Especially in extreme scenario testing, the cVaR controls for losses exceeding the VaR and, therefore, gives a more adequate picture of the risk embedded in the portfolio positioning.

Following the introduction of the cVaR, it is worth looking at the aforementioned example in which the VaR lacked to fulfil the second axiom. The cVaR, however, is able to overcome this drawback of the VaR as it is a coherent risk measure. For each individual asset the $cVaR_{0.90}(A) = cVaR_{0.90}(B) = \frac{0.06}{0.1} * 500 = 300$ and the portfolio consisting of A and B has a $cVaR_{0.9}(A+B) = VaR_{0.9}(A+B) = 500$ since there are no losses strictly exceeding the $VaR_{0.9}(A+B)$. Consequently, the cVaR fulfils the subadditivity axiom as $cVaR_{0.9}(A+B) \leq cVaR_{0.90}(A) + cVaR_{0.90}(B)$.

Given the importance of overall quantile based concepts and the lack of coherence of the VaR, the cVaR was chosen as the risk measure for optimising and constructing portfolios of assets described in section 3.

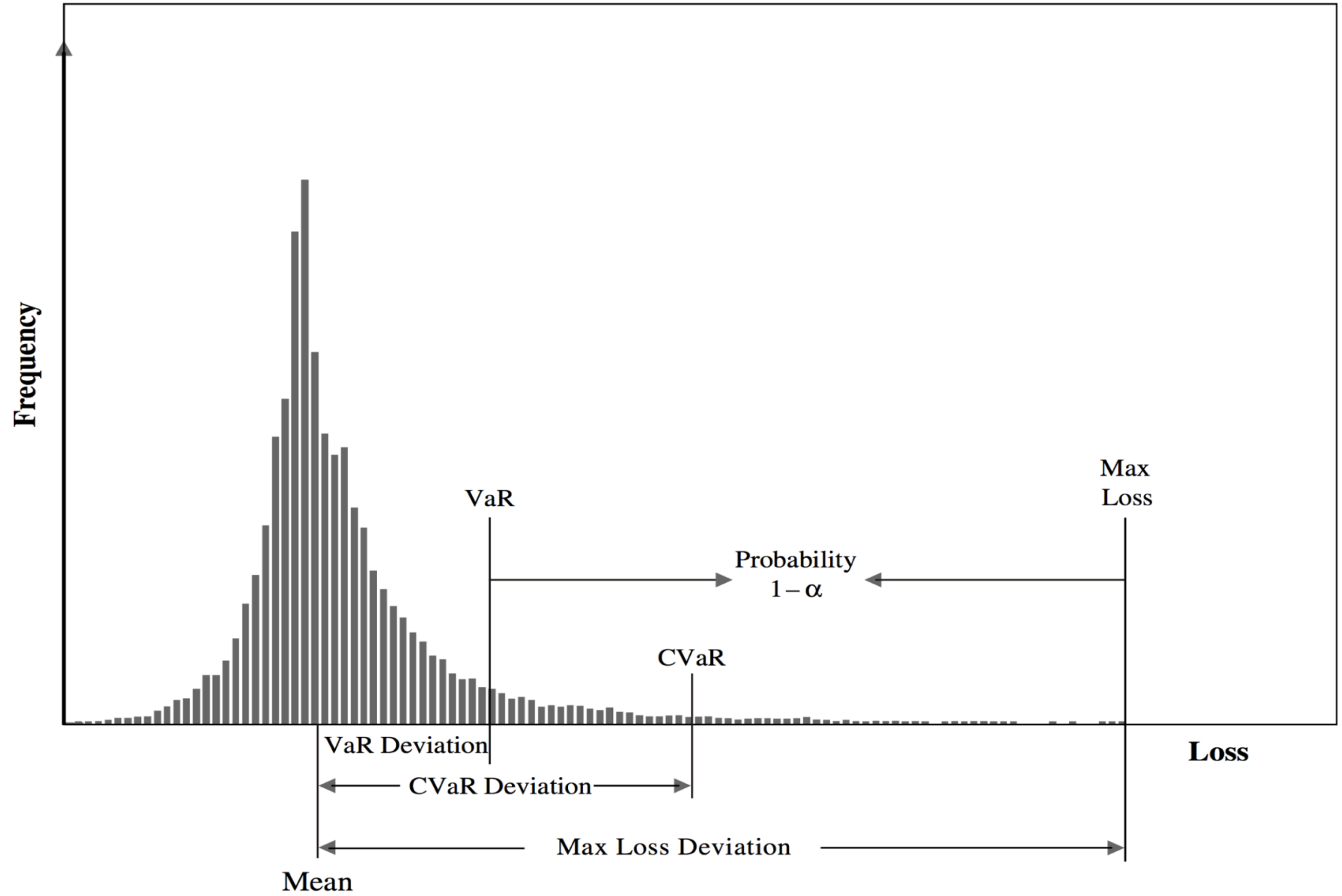
More recently, Rockafellar et al. (2006) have brought an additional risk metric to the researcher 's and practitioner 's attention. Of the multiple deviation metrics in risk analysis raised by them, the cVaR deviation is of major importance for this thesis.
Formally, the cVaR deviation for all $\alpha \in (0,1)$ is defined as follows:

$$cVaR_{deviation} = cVaR_{\alpha}(X - EX) \tag{7}$$

with $cVaR_{\alpha}(X)$ as the cVaR calculated following formula 6, X denoting the simulated asset losses and EX representation the expected simulated asset loss, i.e. the mean asset loss. As the calculation requires the same inputs as the usual cVaR and only adds the mean of the forecasted losses, no further computational effort is required in order to determine the cVaR deviation for a specific asset.
Following the superior properties of the cVaR over the VaR, Rockafellar et al. (2006) and Sarykalin et al. (2008) show that the cVaR deviation does as well fulfil all four axioms introduced by Artzner et al. (1999) in order to be classified as a coherent risk metric.
This fairly new measure is also included in figure 1 which shows the differences between the aforementioned risk metrics graphically.
A particularly appealing feature of the cVaR deviation is that it not only has the embedded information about the shape of the tail but also incorporates additional insight into the distribution of all losses larger than the mean. In other words, the cVaR deviation measure gives a more comprehensive picture about the loss distribution and allows for further inferences in terms of the risks involved.
Furthermore, common portfolio metrics including a deviation metric, e.g. the Sharpe Ratio with the standard deviation in its denominator, should be calculated with the cVaR deviation metric rather than the cVaR (Sarykalin et al. (2008)). It is worth noting that the above-mentioned risk metrics cannot be compared in absolute terms since the deviation measures are calculated by subtracting the mean.

Figure 1 VaR and cVaR

Figure 1 shows the determination of VaR, cVaR and cVaR deviation.



Source: Sarykalin et al. (2008)

4.2 Variance and covariance estimation

In this master thesis three different volatility and covariance estimation procedures are considered and compared in terms of robustness and accuracy of the results. First, simple sample variances and covariances from the above described datasets are calculated. Second, the exponentially weighted moving average (EWMA) following the approach by RiskMetrics is constructed. Lastly, a generalized autoregressive conditional heteroscedasticity (GARCH) model, a model which is widely used in risk management, is estimated. The following sub-chapters give an overview of the motivation and the calculation methodology of each method.

4.2.1 Sample variance and covariance

A simple sample variance and covariance estimation is by far the easiest method to determine a measure for volatility and correlations. As it requires no further optimisation and can simply be derived from the observed data, it offers the least computational power demanding method to base further risk analyses on. The sample variance and covariance are calculated in the following way:

$$S^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2 \qquad (8)$$

$$Cov(r_{1,i}, r_{2,i}) = \frac{1}{T-1}\sum_{t=1}^{T}(r_{1,t} - \bar{r}_1)(r_{2,t} - \bar{r}_2) \qquad (9)$$

where $\bar{r}$ is calculated as the average return

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t \qquad (10)$$

Calculating the variance and covariance as described above, it is of importance to bear in mind that every observation is weighted equally and has the same impact on the estimate. This feature might result in less appropriate results if one assumes general shifts in absolute volatility over time.

4.2.2 Exponentially weighted moving average variance and covariance

In contrast to the sample variance, the EWMA variance is calculated using a different weighting scheme. It weights every observation exponentially using a decay factor λ. This provides, in general, two advantages compared to the equally weighted estimation. By using λ to weight observations, the EWMA variance reacts faster to current market shocks as recent data has a higher weight within the variance estimation. Furthermore, more distant observations that potentially incorporate a deviating volatility regime or lack significance on the current estimate have less impact on the result. The same logic applies to the covariance estimation (J.P. Morgan, 1996), allowing to incorporate higher weight on recent co-movements of assets' returns.

The variance and the covariance are calculated in the following way:

$$\sigma_{t+1|t}^2 = \lambda\sigma_{t|t-1}^2 + (1-\lambda)r_t^2 \tag{11}$$

$$\sigma_{12,t+1|t} = \lambda\sigma_{12,t|t-1}^2 + (1-\lambda)r_{1,t}r_{2,t} \tag{12}$$

where $\sigma_{t+1|t}^2$ and $\sigma_{12,t+1|t}$ are the one day forecasts based on information in time t (J.P. Morgan, 1996). r_t^2 and $r_{1,t}r_{2,t}$ are the lagged squared returns and lagged co-returns, respectively.

J.P. Morgan proposes to use a decay factor $\lambda = 0.94$ for a daily data set and $\lambda = 0.97$ for a monthly data. J.P. Morgan performs the same optimisation described below to determine a decay factor for each asset individually. Consequently, lambda is calculated as a weighted average of the individual optimal decay factors. These decay factors are weighted according to their forecast accuracy measured by the root mean squared error. Given the history of analysed asset returns, J.P. Morgan derives the optimal decay factors of 0.94 and 0.97, respectively.

However, this approach might not reflect the optimal lambda for each individual data series and potentially does not capture differences between decay factors for variances and covariances. As J.P. Morgan determines the values using a wider universe of asset classes including foreign exchanges, 5-year swaps, 10-year zero prices, 1-year money market rates and equity indices, the values might substantially deviate from the optimal results for a pure equity or multi asset universe. In order to find the optimal lambda for each asset within the analysed universe, a root mean squared error (RMSE) minimization is performed to determine the optimal lambda for each asset (variance) or each pair of two assets (covariance) individually. It tries to minimize the difference between the squared return in time $t+1$ and the variance forecast for $t+1$ by changing the decay factor lambda (J.P. Morgan (1996)).

$$RMSE_V = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{t+1}^2 - \sigma_{t+1|t}^2(\lambda))^2} \tag{13}$$

$$RMSE_C = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{1,t+1}r_{2,t+1} - \sigma_{12,t+1|t}(\lambda))^2} \tag{14}$$

4.2.3 GARCH (1,1) variance and covariance

The GARCH approach is an extension of the autoregressive conditional heteroscedasticity (ARCH) process introduced by Engle (1982). The ARCH model allows for different modelling of unconditional and conditional variance where the conditional variance is thereby allowed to dynamically change over time.

Bollerslev (1986) extended the ARCH and developed the GARCH model which additionally allowed the implementation of long-term variance terms and multiple lag terms to modify the impact of past variance realizations on current forward estimates. Formally, Bollerslev (1986) defined a GARCH (p,q) process as

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{15}$$

where α_0 represents a long-term variance, ε_{t-i}^2 denotes the a real-valued stochastic process in discrete time (for this application the squared return) and h_{t-i} the observed variance in $t-i$. While p stands for the order of GARCH terms, q is the length of ARCH lags. α_0, α_i, β_i, q and p are all non-negative.

In order to illustrate the connections between an EWMA, an ARCH and a GARCH process, it is worth noting that in case of $p = 0$, the GARCH process collapses to an ARCH process, i.e. a GARCH (0,q) process equals an ARCH (q) process. Additionally, assuming α_0 to be equal to zero, the GARCH (1,1) and EWMA approaches are identical.

In an initial step, the GARCH (p,q) model requires the determination of p, the length of the period of past variance realizations that influence the current forward estimate and q, the length of the period of past squared return with impact on the current forward estimate. The simplest and especially least computational power demanding specification is the GARCH (1,1) model which has been used for this thesis. The final model for variance forecasting was specified as follows

$$\sigma_{t+1|t}^2 = V_L + \alpha \, \sigma_{t|t-1}^2 + \beta \, r_t^2 \tag{16}$$

with the long-term variance V_L, lagged variance $\sigma_{t|t-1}^2$ and lagged, squared return r_t^2. The same approach was chosen for modelling the covariances over time. Worth noting in that context is the inclusion of a long-term covariance variable. The formula can be expressed in the following way

$$\sigma_{12,t+1|t} = Cov_L + \alpha \, \sigma_{12,t|t-1} + \beta \, r_{1,t} r_{2,t} \tag{17}$$

with the long-term covariance Cov_L, lagged covariance $\sigma_{12,t|t-1}$ and lagged co-return $r_{1,t}r_{2,t}$.

The idea for optimising the GARCH (1,1) parameters is identical to the approach chosen for the EWMA model where the objective function is to minimize the root of the mean squared errors by changing the GARCH parameters $\alpha_0, \alpha_i, \beta_i$.

$$RMSE_V = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{t+1}^2 - \sigma_{t+1|t}^2(\alpha_0, \alpha_i, \beta_i))^2} \quad (18)$$

$$RMSE_C = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{1,t+1} * r_{2,t+1} - \sigma_{12,t+1|t}^2(\alpha_0, \alpha_i, \beta_i))^2} \quad (19)$$

4.3 Cholesky Decomposition

For the asset price forecasts resulting from a Monte Carlo simulation further described in section 4.5, random variables are required.

Imposing a correlation structure in order to account for possible co-movement in the prices of the analysed assets proved to be necessary. A widely accepted method to generate correlated random variables both in the industry as well in academic research is the so-called Cholesky decomposition (Higham (1990)).

Pointed out in the first step in Schmidt‘s (2007) algorithm, based on the Variance-Covariance matrix obtained from the variance and covariance estimation in section 4.2, the correlation between asset i and j is calculated according to formula 20:

$$p_{i,j} = \frac{\sigma_{i,j}}{\sigma_i * \sigma_j} \quad (20)$$

with the covariance between asset i and j equal to $\sigma_{i,j}$ and σ_i representing the standard deviation of asset i. The correlation matrix forms the basis of the Cholesky approach and is manipulated to incorporate the desired correlation structure on the randomly drawn variables. Therefore, the correlation matrix $Corr$ is decomposed into an upper triangular matrix R with positive diagonal elements in the following way (Higham (1990)):

$$Corr = R^T R \tag{21}$$

As outlined in the second step of the approach by Schmidt (2007), the resulting matrix R is consequently multiplied with a vector of standard-normally distributed random variables to obtain correlated random variables.

A severe limitation of the Cholesky decomposition is the applicability on only non-negative-definite correlation matrices (Brissette et al. (2007)). Rebonato and Jäckel (2011) argue that outliers and noise might lead to truly observed correlation matrices not being positive-definite. Brissette et al. (2007) find that computational problems in programs used to estimate correlations can also lead to results that do not allow to apply the Cholesky decomposition.
As the data and the program used indeed resulted in non-positive-definite correlation results, the correction method by Rebonato and Jäckel (2011) was applied. Their approach transforms a formerly ill-defined correlation matrix into a positive-definite one and, therefore, allows to apply the usual Cholesky manipulation. In order to do so, the matrix is diagonalized and the negative Eigenvalues that cause the matrix to be non-positive-definite are replaced with epsilons, a small positive value close to zero. The new correlation matrix is then calculated in two steps. First, the adjusted positive-definite matrix is calculated as follows:

$$Corr_{adj} = E * D * E^{-1} \tag{22}$$

where E is the matrix of column eigenvectors and D represents the diagonal matrix of eigenvalues of the unadjusted matrix. As the diagonal elements are different from 1, the matrix $Corr_{adj}$ needs to be normalized as shown in formula 23:

$$Corr_{final} = \frac{Corr_{adj}}{\sqrt{diag(Corr_{adj}) * diag(Corr_{adj})^T}} \tag{23}$$

The resulting matrix fulfils the requirement of being positive-definite and can, therefore, be manipulated following the steps in the standard Cholesky decomposition (Brissette et al. (2007)).

4.4 Student t-copula

Two copula approaches were considered in this thesis. First, the Gaussian copula which constructs a multivariate normal distribution and second, the t-copula which builds a multivariate student t-distribution. However, only the t-copula was used as it incorporates beneficial properties compared to the Gaussian copula. As stated by Schmidt (2007), the t-copula allows for much more probability mass in the extreme cases. Consequently, it is more likely that all random variables are either strongly negative or positive which is oftentimes a suitable feature for financial assets. Thereby, the t-copula is able to map the true relationship between the used financial assets more closely. This is especially relevant if the examined financial data does not follow a normal distribution but rather possesses excess kurtosis and hence, experiences a higher probability of events in the tails of the distribution. All asset universes in this thesis have a sample kurtosis of greater than 3 which is defined as excess kurtosis (Madan and Seneta (1990)) as can be seen in appendix I and II. Consequently, the t-copula is chosen and a multivariate t-distribution is constructed using the correlated random variables ($X = X_1, \dots, X_d$) resulting from the Cholesky decomposition. This is done by following the approach by Schmidt (2007). The multivariate t-distribution possesses v degrees of freedom and ξ follows a χ_v^2-distribution.

$$(\eta_1, \dots, \eta_d) = (\frac{X_1}{\sqrt{\frac{\xi}{v}}}, \dots, \frac{X_d}{\sqrt{\frac{\xi}{v}}}) \tag{24}$$

The degrees of freedom v are estimated using the sample kurtosis of the respective financial asset at time t. Time t represents a yearly rebalancing of the portfolios. By calculating the sample kurtosis at each rebalancing date, an inclusion of new information regarding the kurtosis of the financial assets can be taken into consideration when constructing the multivariate student t-distribution. The approach follows the idea of a conditional kurtosis by Brooks et al. (2005):

$$v = \frac{2(2k_t - 3)}{k_t - 3} = \frac{6}{k_t - 3} + 4 \tag{25}$$

The conditional kurtosis is calculated at each rebalancing for each financial asset in the specified universe. Furthermore, the degrees of freedom of the chi-square distribution used in the t-copula is determined as the median of the conditional kurtosis' across all financial assets used in the portfolio construction to control for outliers.

Finally, these correlated student t-distributed random variables are then used in the Monte Carlo simulation to forecast asset prices.

4.5 Monte Carlo simulation and Geometric Brownian Motion

There is a wide range of models designated for forecasting asset prices with many different methods having not only found acceptance in research but also in the financial industry.

Following the approach of Rockafellar and Uryasev (2000), a Monte Carlo simulation in combination of a Geometric Brownian Motion is used to forecast future asset prices. The Geometric Brownian Motion is defined as:

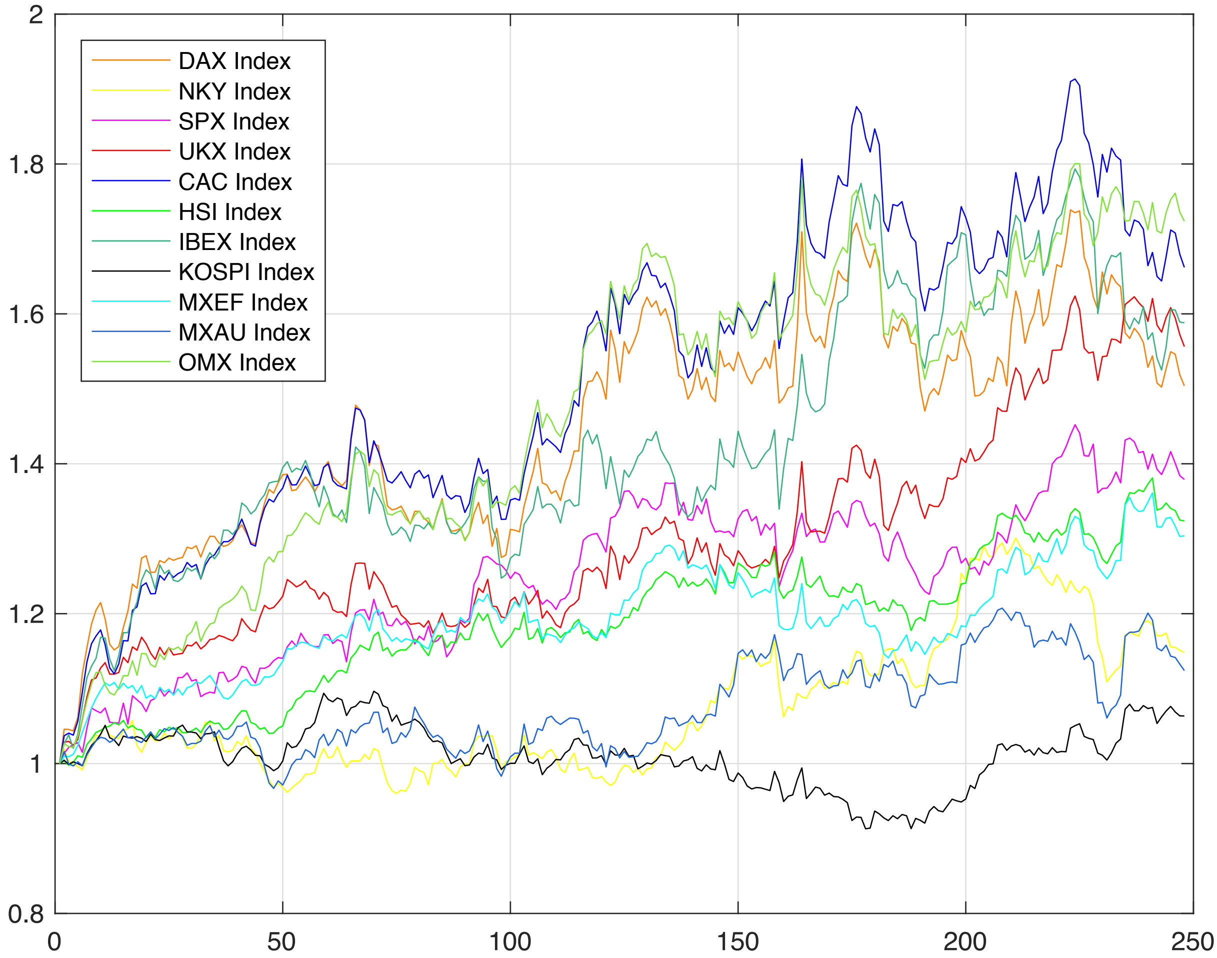
$$S_1 = S_0 * e^{\left[\left(\bar{r} - \frac{\sigma^2}{2}\right) + \sigma * \varepsilon\right]} \tag{26}$$

where $\bar{r}$ and σ are the respective daily mean and standard deviation of the different assets and ε is one of the correlated random variables which was constructed in the copula approach.

The computation is conducted using one of the three volatilities methods defined in chapter 4.2 at a time. The daily price forecast is repeated for all assets between the current and the next rebalancing. A sample price forecast is reported below in figure 2 and shows the forecast for the equity index universe. This process is run 50000 times for each rebalancing of the portfolio and future asset losses for each asset are calculated. These losses are weighted to calculate both the portfolio's VaR and cVaR.

Figure 2 Monte Carlo simulation

Figure 2 shows one run out of the 50000 Monte Carlo simulations for the equity indices universe.



4.6 Conditional Value-at-Risk on a portfolio level

As outlined in chapter 4.1, the VaR is defined as the possible maximum loss of a portfolio over a given risk horizon within a fixed confidence level. The cVaR is calculated as the average of all losses larger than or equal to the VaR. In addition, a cVaR deviation is calculated following the methodology outlined in chapter 4.1. Besides having the full benefits of the cVaR compared to the VaR, the cVaR deviation incorporates more information about the distribution of losses into the optimisation process. In order to calculate a cVaR and cVaR deviation on a portfolio level, it is therefore necessary to both determine the VaR and identify all the losses above this critical threshold. First, the forecasted asset losses derived as described in chapter 4.5 are weighted with an asset weight vector, resulting in a single column vector with 50000 portfolio losses on an aggregated portfolio level.

Further, in order to define the threshold of losses to calculate the desired risk metrics, a confidence level must be defined. The current Basel Committee on Banking Supervision (BCBS) recommends to estimate the VaR with 99% confidence level (Campbell et al. (2001)).

For the threshold definition in this thesis, the 99^{th} percentile was set as the confidence level to base further calculations on. Other values for confidence levels have also been tested to evaluate how the composition of the portfolio changes depending on the chosen threshold for the loss distribution. As the resulting weights in the optimisation and, consequently, performance of the strategies in the backtest described in chapter 5 showed negligible differences to the 99% confidence level, results are only reported for the 99^{th} percentile.

Depending on the chosen confidence level, the calculation of the cVaR risk measure is adjusted accordingly.

4.7 Portfolio construction and optimisation

The following chapter covers the methodology of portfolio optimisations and gives theoretical background on the goal function determining the composition of an ideal portfolio depending on the chosen risk measure. As stated in the introduction, the objective of this thesis is a comparison between optimised portfolios using volatility as the risk measure and portfolios using the cVaR measure as a risk proxy.

4.7.1 Markowitz mean-variance portfolio optimisation

As a comparison for the cVaR strategies, the traditional Markowitz Mean-Variance portfolio optimisation is applied to the respective asset universes. Under Markowitz investors face a trade-off between the desirable expected return and the undesirable variance of the portfolio (Markowitz (1952)). This trade-off becomes:

$$\max_{\{w\}} w' * (E(R) - R_f) - \frac{\gamma}{2} w' \Sigma w \tag{27}$$

where w denotes the weight vector, R the expected returns, R_f the risk-free return, Σ the covariance matrix and γ the risk aversion coefficient of the respective investor (Campbell and Viceira (2002)).

Furthermore, the risk-free rate is assumed to be zero and the portfolio is long-only which means that short sales of assets are prohibited. Thereby, the optimisation problem reduces to the following form:

$$\max_{\{w\}} w' * E(R) - \frac{\gamma}{2} w' \Sigma w \tag{28}$$

such that

$$w_i \geq 0 \tag{29}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{30}$$

By varying the risk aversion coefficient, the frontier of efficient portfolios can be constructed. However, this thesis wants to examine specifically two portfolios. First the Minimum Variance portfolio which is an extreme case of the above mentioned equation. If γ approaches infinity, the investor only cares about the risk of the portfolio and hence, tries to minimize it. Consequently, she will choose the Minimum Variance portfolio.

It is constructed by varying the weights of the assets in order to minimize the portfolio variance:

$$\min_{\{w\}} w'\Sigma w \tag{31}$$

Second, the maximum Sharpe Ratio portfolio which is the portfolio with the highest trade-off between return and risk of the portfolio is determined. The Sharpe Ratio of a portfolio is generally defined as (Campbell and Viceira (2002)):

$$SR = \frac{w' * E(R) - R_f}{\sqrt{w'\Sigma w}} \tag{32}$$

The maximum Sharpe Ratio portfolio is as well constructed by changing the weights but it tries to maximize the trade-off between the expected returns and standard deviation. Under the assumption that the risk free rate is equal to zero, the optimisation problem becomes:

$$\max_{\{w\}} \frac{w'E(R)}{\sqrt{w'\Sigma w}} \tag{33}$$

4.7.2 Conditional Value-at-Risk portfolio optimisation

Following the methodology of the traditional mean-variance portfolio construction by Markowitz, similar portfolios are constructed for the cVaR optimisations. The main difference is the substitution of the volatility by the cVaR as the risk measure. More precisely, the portfolio cVaR is calculated following a stepwise approach. First, future asset prices are simulated using a Geometric Brownian Motion with correlated random variables. Second, individual asset losses are calculated. Third, asset losses are weighted to construct a portfolio loss. Fourth, this simulation is run 50000 times and, hence, 50000 portfolios are generated in order to calculated the VaR with a given confidence level. Fifth, as described in chapter 4.1, the cVaR is calculated as the average of all losses greater than or equal to the VaR. Lastly, the portfolio is optimised by changing the individual asset weights to create a Minimum cVaR portfolio. This allows a comparison with the Minimum Variance portfolio.

$$\min_{\{w\}} cVaR_{\propto}(w) \tag{34}$$

A second portfolio which maximizes the mean-return over cVaR is created. This portfolio is compared to the maximum Sharpe Ratio portfolio originating from the Mean-Variance portfolio optimisation.

$$\max_{\{w\}} \frac{w'E(R)}{cVaR_{\propto}(w)} \tag{35}$$

In total four different cVaR portfolios are constructed. The above-mentioned cVaR portfolios are also replicated for the cVaR deviation risk measure. The methodology for the portfolio construction using the cVaR deviation as the risk proxy equals the approach chosen for the cVaR optimisation.

4.7.3 Additional Benchmarks

In addition to constructing cVaR and Mean-Variance efficient portfolios, a buy and hold portfolio and a yearly equal weighted portfolio are calculated for all asset universes.

For the buy and hold portfolio one invests equally weighted into all eligible assets at the first rebalancing and does not rebalance over the entire investment horizon.

The yearly equal weighted portfolio invests in the same manner as the buy and hold portfolio, however it rebalances once a year to equal weights. In addition, a portfolio weighted according to the company's market capitalization is constructed for the third asset universe, the US stock dataset. This market capitalization weighted portfolio is as well constructed once a year using the 50 largest companies determined by the selection process outlined in chapter 3.

4.8 Backtest procedure

For all asset universes, a backtest is calculated to evaluate the historical performance of the above-mentioned strategies. On each selection day, weights for all assets are calculated following the optimisation methodology outlined in chapter 4.7. A maximum single asset weight of 20% for the first two universes and 10% for the third universe is applied in order to avoid too concentrated portfolios. Index shares are calculated using the optimal weight and closing price of the respective asset i on the selection day and implemented as opening shares on the next trading day.

$$Shares_i = \frac{weight_i * index\ level}{close\ price_i} \tag{36}$$

In the event of a potential delisting of a stock, a cash dividend amounting to the stock price is assumed to be paid out and invested across the portfolio on a pro rata basis. This is only relevant for the US stock universe as delistings are not applicable to the analysed indices. All backtests are calculated using the longest available daily price history of the respective universe. The backtests start three years after the first available data point to calibrate the input parameters for the optimisation process.

5 Results

For all strategies returns, volatilities, Sharpe Ratio, sample kurtosis and maximum drawdown are calculated. The returns and volatilities are annualized and the Sharpe Ratio is calculated using these two parameters without a risk-free asset as defined in chapter 4.7.1. The maximum drawdown is defined as the maximum loss from the peak of the index over the whole investment period:

$$MD(T) = min_{\tau\epsilon(0,T)}\,(\frac{Index(\tau)}{max_{t\epsilon(0,\tau)}(index(t))} - 1) \tag{37}$$

It can be interpreted as the maximum loss for an investor who has invested at the highest index level and stayed invested until the minimum index level is reached. In the following three paragraphs, strategies are compared to each other for all three universes.

5.1 Equity Indices Universe

As described in the previous chapter, all strategies are calculated using either an exponentially weighted variance, a variance calculated by the GARCH (1,1) model or the sample variance. Performance and maximum drawdown for each strategy are shown in appendices XIII and XIV.

For the equity indices universe the two best performing variance methods in terms of Sharpe Ratio across all strategies are the GARCH (1,1) model and the sample variance. However, the differences in performance to the exponential weighted variance are relatively small. All portfolios have a maximum drawdown of between 50% to 55% and the Sharpe Ratio varies between 0.57 and 0.69 across all strategies and universes. Using the exponential weighted variance, the portfolios which try to maximize the Sharpe Ratio have not only the lowest maximum drawdown but also one of the highest annualized returns among the strategies. This is also persistent using the GARCH (1,1) model. However, the yearly equal weighted portfolio has a slightly lower maximum drawdown in this case. Nevertheless, it experiences a higher annualized volatility and lower annualized return which results in an overall lower Sharpe Ratio. Calculating all strategies with the sample variance, the results are similar to results when the GARCH (1,1) is applied. All strategies with the objective to minimize their specific risk factor experience a lower annualized volatility compared to the results using GARCH (1,1).

Yearly allocations to the eleven equity indices for the optimised portfolios are reported in appendices V-VIII. The results indicate that the applied optimisation approach clearly prefers some assets. Depending on the goal function of the optimisation, these assets typically show a low volatility or a high risk-adjusted return. Nevertheless, all portfolios are invested into the entire opportunity set, resulting in diversified portfolios.

Table 5 EWMA Equity Indices Universe - Performance Statistics

Table 5 shows the performance of all strategies using the EWMA variance calculation.

Name	**Return p.a.**	**Annualized Volatility**	**Sharpe Ratio**	**Sample Kurtosis**	**Maximum Drawdown**
Min Variance	7.88%	13.54%	0.58	11.63	-53.17%
Sharpe Ratio	9.25%	14.46%	0.64	10.77	-50.93%
Min cVaR	8.01%	13.92%	0.58	12.38	-54.02%
Min cVaR Deviation	7.72%	13.31%	0.58	11.38	-53.59%
cVaR Sharpe Ratio	9.48%	14.68%	0.65	10.61	-51.55%
cVaR Deviation Sharpe Ratio	9.42%	14.70%	0.64	10.61	-51.47%
Buy and Hold	9.08%	15.82%	0.57	9.72	-54.92%
Yearly Equal Weighted	9.21%	15.04%	0.61	10.19	-53.24%

Table 6 GARCH Equity Indices Universe - Performance Statistics

Table 6 shows the performance of all strategies using the GARCH variance calculation.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	8.84%	13.59%	0.65	11.60	-53.91%
Sharpe Ratio	9.83%	14.23%	0.69	11.21	-53.48%
Min cVaR	9.47%	13.90%	0.68	12.42	-55.10%
Min cVaR Deviation	8.68%	13.44%	0.65	12.06	-53.85%
cVaR Sharpe Ratio	9.85%	14.39%	0.68	11.08	-53.99%
cVaR Deviation Sharpe Ratio	9.90%	14.41%	0.69	11.06	-53.85%
Buy and Hold	9.08%	15.82%	0.57	9.72	-54.92%
Yearly Equal Weighted	9.21%	15.04%	0.61	10.19	-53.24%

Table 7 Sample Variance Equity Indices Universe - Performance Statistics

Table 7 shows the performance of all strategies using the sample variance calculation.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	9.03%	13.47%	0.67	10.93	-52.63%
Sharpe Ratio	9.74%	14.39%	0.68	10.27	-53.74%
Min cVaR	9.49%	13.69%	0.69	11.26	-53.44%
Min cVaR Deviation	8.58%	13.34%	0.64	10.86	-52.79%
cVaR Sharpe Ratio	9.95%	14.77%	0.67	10.51	-54.95%
cVaR Deviation Sharpe Ratio	10.00%	14.78%	0.68	10.53	-54.98%
Buy and Hold	9.08%	15.82%	0.57	9.72	-54.92%
Yearly Equal Weighted	9.21%	15.04%	0.61	10.19	-53.24%

5.2 Multi Asset Universe

In this section, the methodology outlined in chapter 4 is applied to the multi asset universe. Performance and maximum drawdown for each strategy are shown in appendices XV and XVI. In table 8 the results for the exponentially weighted variance are shown. The return of the Yearly Equal Weighted Portfolio exceeds all other strategies but the portfolio has the highest volatility and maximum drawdown. Because of the elevated volatility, it has one of the lowest Sharpe Ratios of all strategies. The Minimum cVaR Deviation Portfolio has the highest Sharpe Ratio and also the lowest volatility. Over the whole backtest horizon it loses at most 22.93% compared to the yearly equal weighted portfolio which lost almost 44%.

When using the GARCH (1,1) model, all portfolios besides the Minimum Variance portfolio have a higher Sharpe Ratio. This stems foremost from the increased return rather than the decreased volatility. The Minimum cVaR Deviation portfolio still achieves the highest Sharpe Ratio. Interestingly, using the GARCH (1,1) model leads to a smaller maximum drawdown for every portfolio besides the Minimum Variance portfolio. The Minimum cVaR Deviation portfolio has the highest risk adjusted return and the lowest maximum drawdown of –22.67%.

The Sharpe Ratios of all portfolios are even higher for all portfolios when a sample (co)variance is calculated. However, the return per annum across all portfolios is lower compared to the variance calculation using the GARCH (1,1) model. Conversely, the annualized volatility overcompensates the loss in performance, resulting in an increased Sharpe Ratio. The picture is less clear for the maximum drawdown. On the one hand, it is even lower for the Minimum cVaR Deviation portfolio and the Minimum Variance portfolio, on the other hand all other portfolios experience a higher maximum drawdown.
The optimised weights for the Minimum Variance, Maximum Sharpe Ratio, Minimum cVaR Deviation and cVaR Deviation Sharpe Ratio portfolios can be found in appendix IX-XII. When comparing the asset allocation of the Minimum Variance with the Minimum cVaR Deviation portfolio, a noticeable difference is the more concentrated

asset allocation of the latter. While the Minimum Variance portfolio invests approximately at least 1% into every single asset, the Minimum cVaR Deviation portfolio allocates almost no weight at all to several assets.

Table 8 EWMA Multi Asset Universe - Performance Statistics

Table 8 shows the performance of all strategies using the EWMA variance calculation.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	6.21%	5.18%	1.20	7.72	-21.98%
Sharpe Ratio	5.30%	5.52%	0.96	9.48	-28.16%
Min cVaR	6.41%	5.40%	1.19	8.54	-25.45%
Min cVaR Deviation	6.06%	4.95%	1.23	10.24	-22.93%
cVaR Sharpe Ratio	6.07%	5.97%	1.02	8.55	-29.35%
cVaR Deviation Sharpe Ratio	6.12%	6.04%	1.01	8.75	-29.23%
Buy and Hold	6.20%	10.20%	0.61	10.53	-43.88%
Yearly Equal Weighted	6.80%	10.21%	0.67	8.66	-41.25%

Table 9 GARCH Multi Asset Universe - Performance Statistics

Table 9 shows the performance of all strategies using the GARCH variance calculation.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	6.30%	6.02%	1.05	10.87	-26.15%
Sharpe Ratio	6.26%	5.66%	1.11	9.52	-24.20%
Min cVaR	6.78%	5.54%	1.23	8.18	-24.19%
Min cVaR Deviation	6.72%	5.29%	1.27	8.92	-22.67%
cVaR Sharpe Ratio	6.91%	5.72%	1.21	7.89	-22.88%
cVaR Deviation Sharpe Ratio	6.81%	5.76%	1.18	7.90	-24.16%
Buy and Hold	6.20%	10.20%	0.61	10.53	-43.88%
Yearly Equal Weighted	6.80%	10.21%	0.67	8.66	-41.25%

Table 10 Sample Variance Multi Asset Universe - Performance Statistics

Table 10 shows the performance of all strategies using the sample variance calculation.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	6.56%	4.68%	1.40	7.36	-21.52%
Sharpe Ratio	6.12%	5.08%	1.21	7.19	-27.57%
Min cVaR	6.90%	4.80%	1.44	7.06	-24.65%
Min cVaR Deviation	6.96%	4.08%	1.70	7.24	-16.77%
cVaR Sharpe Ratio	6.42%	5.76%	1.12	8.08	-32.06%
cVaR Deviation Sharpe Ratio	6.43%	5.77%	1.11	7.98	-32.07%
Buy and Hold	6.20%	10.20%	0.61	10.53	-43.88%
Yearly Equal Weighted	6.80%	10.21%	0.67	8.66	-41.25%

5.3 US Stocks Universe

When looking at the results for the US stock universe, no model significantly outperforms any other strategy. Performance and maximum drawdown for each strategy are shown in appendices XVII and XVIII. However, the Minimum cVaR Deviation portfolio has the highest Sharpe Ratio in case of the variance being calculated either as an exponential weighted average or as a sample variance and just a slightly lower Sharpe Ratio compared to the Minimum Variance portfolio when the GARCH model is used. In addition, the Minimum cVaR Deviation portfolio has the lowest maximum drawdown and volatility across strategies for all variance calculation methods. The yearly equal weighted portfolio has the highest annualized return but also a comparatively high annualized volatility and a maximum drawdown of almost 50%. Interestingly, the performance and volatility is worse for the yearly equal weighted portfolio which means that the portfolio does not benefit from being rebalanced over the whole horizon. One explanation for this observation could be the applied selection process. As only the 50 largest companies in the US are selected and then equally weighted, there is some overlap between the buy and hold portfolio and the yearly equally weighted portfolio. Thirteen stocks from the first selection stay in the yearly equal weighted portfolio over the entire investment horizon. The market capitalization weighted portfolio has the highest annualized volatility and hence one of the lowest Sharpe Ratios.

The maximum drawdown for all Sharpe Ratio maximizing portfolios increases for the sample variance compared to both the exponentially weighted and GARCH (1,1) variances.

Similar to the asset allocation patterns of the other two universes, optimisations for the US stock dataset result in more concentrated portfolios except for the Minimum Variance portfolio. Due to the dimension of the universe, weight figures are not reported in this thesis.

Table 11 EWMA US Stocks Universe - Performance Statistics

Table 11 shows the performance of all strategies using the EWMA variance calculation.

Name	**Return p.a.**	**Annualized Volatility**	**Sharpe Ratio**	**Sample Kurtosis**	**Maximum Drawdown**
Min Variance	7.91%	15.25%	0.52	10.20	-42.11%
Sharpe Ratio	6.45%	16.88%	0.38	9.37	-51.82%
Min cVaR	6.22%	15.81%	0.39	9.81	-53.79%
Min cVaR Deviation	8.06%	14.87%	0.54	9.90	-38.11%
cVaR Sharpe Ratio	7.04%	17.76%	0.40	8.95	-57.93%
cVaR Deviation Sharpe Ratio	7.12%	17.29%	0.41	9.36	-54.74%
Buy and Hold	9.50%	18.17%	0.52	10.20	-49.98%
Yearly Equal Weighted	8.09%	18.89%	0.43	10.01	-50.88%
Market Capitalization Weighted	7.58%	19.04%	0.40	9.88	-54.77%

Table 12 GARCH US Stocks Universe - Performance Statistics

Table 12 shows the performance of all strategies using the GARCH variance calculation.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	7.46%	15.62%	0.48	10.23	-47.34%
Sharpe Ratio	6.95%	16.72%	0.42	9.82	-51.57%
Min cVaR	6.14%	16.42%	0.37	10.26	-53.35%
Min cVaR Deviation	6.98%	15.24%	0.46	10.59	-44.73%
cVaR Sharpe Ratio	7.22%	16.95%	0.43	9.84	-53.03%
cVaR Deviation Sharpe Ratio	6.57%	16.80%	0.39	10.06	-53.16%
Buy and Hold	9.50%	18.17%	0.52	10.20	-49.98%
Yearly Equal Weighted	8.09%	18.89%	0.43	10.01	-50.88%
Market Capitalization Weighted	7.58%	19.04%	0.40	9.88	-54.77%

Table 13 Sample Variance US Stocks Universe - Performance Statistics

Table 13 shows the performance of all strategies using the sample variance calculation.

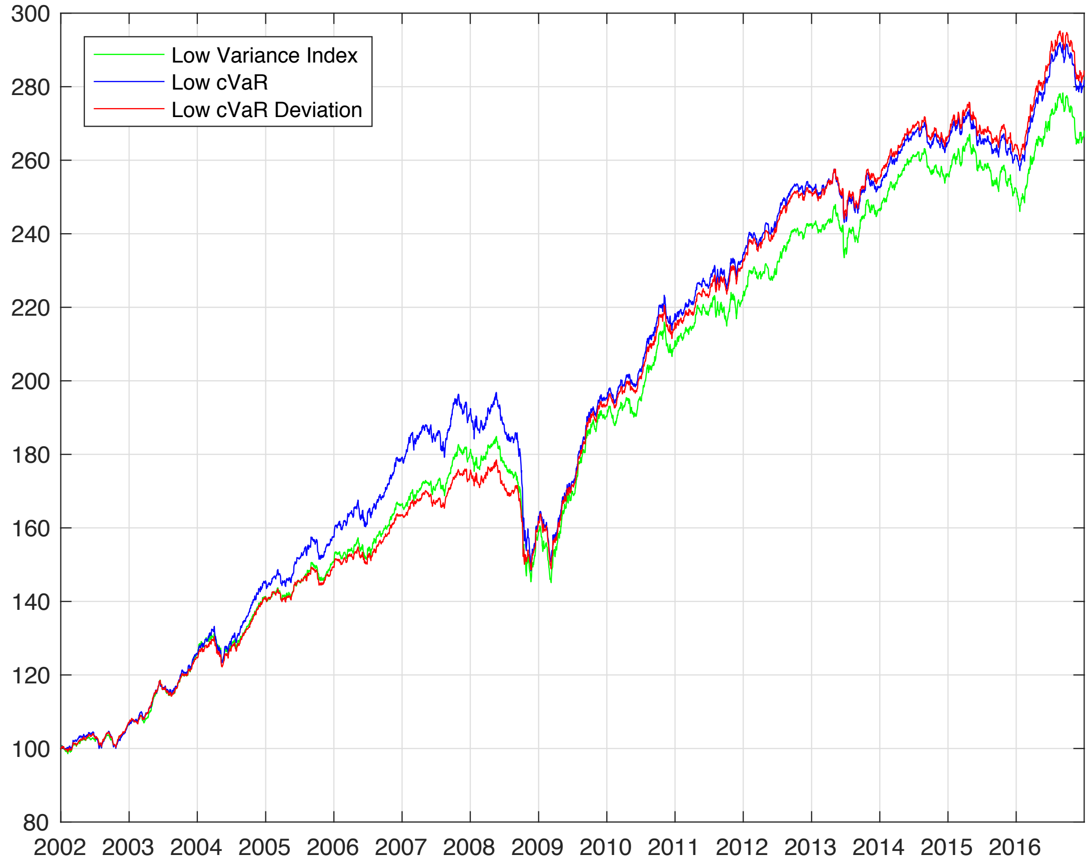
Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	8.78%	15.07%	0.58	10.95	-40.73%
Sharpe Ratio	7.13%	17.96%	0.40	8.93	-64.77%
Min cVaR	7.96%	15.54%	0.51	10.86	-47.52%
Min cVaR Deviation	8.89%	14.83%	0.60	10.44	-39.86%
cVaR Sharpe Ratio	6.74%	18.56%	0.36	8.97	-67.23%
cVaR Deviation Sharpe Ratio	7.47%	18.27%	0.41	9.10	-62.48%
Buy and Hold	9.50%	18.17%	0.52	10.20	-49.98%
Yearly Equal Weighted	8.09%	18.89%	0.43	10.01	-50.88%
Market Capitalization Weighted	7.58%	19.04%	0.40	9.88	-54.77%

5.4 Minimum Variance and Minimum conditional Value-at-Risk Deviation

Besides comparing different variance calculation methods and strategies in general, a more detailed comparison between the Minimum Variance and Minimum cVaR Deviation strategy is conducted. For both the US equity and the equity indices universe the differences in performance, volatility or maximum drawdown are marginal. However, within the multi asset universe with either a GARCH (1,1) or sample variance the cVaR has some appealing properties. As already mentioned 5.2, the cVaR deviation portfolio has a higher Sharpe Ratio with both annualized return and annualized volatility being superior to the Minimum Variance portfolios. The maximum drawdown of the cVaR deviation portfolios is lower in both scenarios as well. When looking at the figure below, the performance over the whole investment horizon for the three strategies can be seen. Before the financial crisis in 2008/2009, the Minimum cVaR portfolio was the best performing strategy followed by the Minimum Variance and the cVaR Deviation. However, during the crisis the benefit of the Minimum cVaR Deviation strategy becomes evident. It has the lowest drawdown and loses only 16.77% compared to 21.52% for the Minimum Variance portfolio and even 24.65% for the Minimum cVaR portfolio. One explanation for this outperformance to the Minimum cVaR could be the superior properties of the cVaR deviation measure outlined in chapter 4.1. As the cVaR deviation takes the distribution of asset losses into account by subtracting the average loss from the calculated cVaR, it might help to incorporate the excessive risk taking in the loom of the financial crisis. If the loss distribution becomes less favourable, i.e. is shifted towards higher losses, but the average loss that exceed the 99% VaR stays relatively constant, the Minimum cVaR is not able to grasp this change in riskiness whereas the Minimum cVaR Deviation can account for the higher risk. The two cVaR strategies have an advantage over the Minimum Variance strategy as the variance does not differentiate between fluctuations resulting from positive and negative returns. In contrast, the cVaR only accounts for the right tail of the loss distribution and thereby clearly distinguishes between positive and negative losses. This might be an explanation for the higher return following the financial crisis.

Figure 3 Minimum Variance and Minimum cVaR strategies

Figure 3 shows a comparison between the Minimum Variance and Minimum cVaR portfolios using sample variance in the multi asset universe.



6 Robustness of empirical results

In order to examine whether the results are sensitive to model specifications, several iterations were run. This was implemented for all three universes and all three variance and covariance calculation methods except for the US stock universe where a quarterly rebalancing was not conducted as the universe itself is constructed on a yearly basis due to the selection process. However, only the results for the multi asset universe with sample variance and covariances are discussed in length in the thesis. In table 14, results for quarterly rebalancings instead of yearly rebalancings are shown.

Table 14 Sample Variance Multi Asset Universe - Performance Statistics – Robustness I

Table 14 shows the performance of all strategies using quarterly rebalancing with 99% quantile in the multi asset universe applying the sample variance.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	6.44%	4.74%	1.36	8.17	-22.10%
Sharpe Ratio	6.33%	4.92%	1.29	6.90	-25.83%
Min cVaR	6.70%	4.40%	1.52	7.55	-21.38%
Min cVaR Deviation	6.64%	4.18%	1.59	8.06	-19.74%
cVaR Sharpe Ratio	6.57%	5.52%	1.19	8.32	-30.26%
cVaR Deviation Sharpe Ratio	6.57%	5.53%	1.19	8.32	-30.35%
Buy and Hold	6.20%	10.20%	0.61	10.53	-43.88%
Quarterly Equal Weighted	6.69%	10.51%	0.64	9.86	-41.89%

In case of a quarterly rebalancing, all strategies which try to maximize a Sharpe Ratio have a slightly higher risk adjusted return compared to the yearly rebalancing. This positive impact is also present in lower maximum drawdowns for all Sharpe Ratio strategies. On the contrary, the low risk strategies have a lower Sharpe Ratio and a higher maximum drawdown. However, all results are similar to the yearly rebalancing. The average Sharpe Ratio across all strategies increases by only 0.02 and the average absolute difference is 0.06. Both measures represent only a small influence of applying a quarterly rebalancing.

Table 15 Sample Variance Multi Asset Universe - Performance Statistics – Robustness II

Table 15 shows the performance of all strategies using yearly rebalancing with 90% quantile in the multi asset universe applying the sample variance.

Name	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Min Variance	6.56%	4.68%	1.40	7.36	-21.52%
Sharpe Ratio	6.12%	5.08%	1.21	7.19	-27.57%
Min cVaR	6.59%	5.21%	1.26	7.94	-29.14%
Min cVaR Deviation	6.91%	4.07%	1.70	7.26	-16.95%
cVaR Sharpe Ratio	6.32%	5.58%	1.13	8.03	-31.17%
cVaR Deviation Sharpe Ratio	6.42%	5.59%	1.15	7.77	-30.92%
Buy and Hold	6.20%	10.20%	0.61	10.53	-43.88%
Yearly Equal Weighted	6.80%	10.21%	0.67	8.66	-41.25%

In the second robustness test the strategies are rebalanced yearly but the quantile for the cVaR was changed to 90% to assess the dependency of the results on the chosen quantile. By desing, this variation only affects the strategies using the cVaR. The average return of all cVaR strategies decreases by 0.12% and the average Sharpe Ratio decreases by 0.03. This is mainly due to the Minimum cVaR portfolio which loses the most in this variation. The Minimum cVaR Deviation portfolios are almost not affected at all.

Table 16 Sample Variance Multi Asset Universe - Performance Statistics – Robustness III

Table 16 shows the performance of all strategies using yearly rebalancing with 99% quantile in the multi asset universe applying the sample variance with an upper bound of 50% per asset within the optimisation.

Name	**Return p.a.**	**Annualized Volatility**	**Sharpe Ratio**	**Sample Kurtosis**	**Maximum Drawdown**
Min Variance	6.51%	4.41%	1.48	8.14	-22.16%
Sharpe Ratio	5.64%	4.32%	1.31	11.54	-29.43%
Min cVaR	7.07%	4.08%	1.73	13.82	-28.41%
Min cVaR Deviation	7.36%	3.70%	1.99	16.20	-24.57%
cVaR Sharpe Ratio	6.24%	4.73%	1.32	11.04	-31.23%
cVaR Deviation Sharpe Ratio	6.33%	4.73%	1.34	11.08	-31.19%
Buy and Hold	6.20%	10.20%	0.61	10.53	-43.88%
Yearly Equal Weighted	6.80%	10.21%	0.67	8.66	-41.25%

In the third variation, the upper bounds within the portfolio optimisation are set to 50% instead of 20% to allow for a more extreme investment approach. All strategies with flexible weights have a lower annualized standard deviation. It decreases on average by 0.7%. Furthermore, both the Minimum cVaR and the Minimum cVaR Deviation portfolios have a higher return whereas all other portfolios have lower annual return. Overall all portfolios benefit from an increased upper bound as the average Sharpe Ratio of all affected portfolios increases by 0.2 and is positive for all strategies. The average maximum drawdown decreases on average as well. The Minimum cVaR Deviation portfolio profits the most by a decrease of almost 8%.

These analyses show that the differences for the multi asset universe across the above-mentioned variations are small. This yields a first insight in the robustness of the conducted optimisation. Similar changes are also observed for the other two universes with one exception. In the case of the equity universe with sample variances and covariances and the 50% upper bound, the cVaR and cVaR deviation Sharpe Ratio portfolios experience a sharp drop in their Sharpe Ratio and their maximum drawdown increases to almost 85%. By increasing the upper bound within the optimisation, it allows for extreme results which is especially relevant for the US stock universe as one invest not in country indices but instead in individual stocks. This bears a much higher risk and allowing for such a concentrated allocation in this setting might not be optimal for an investor.

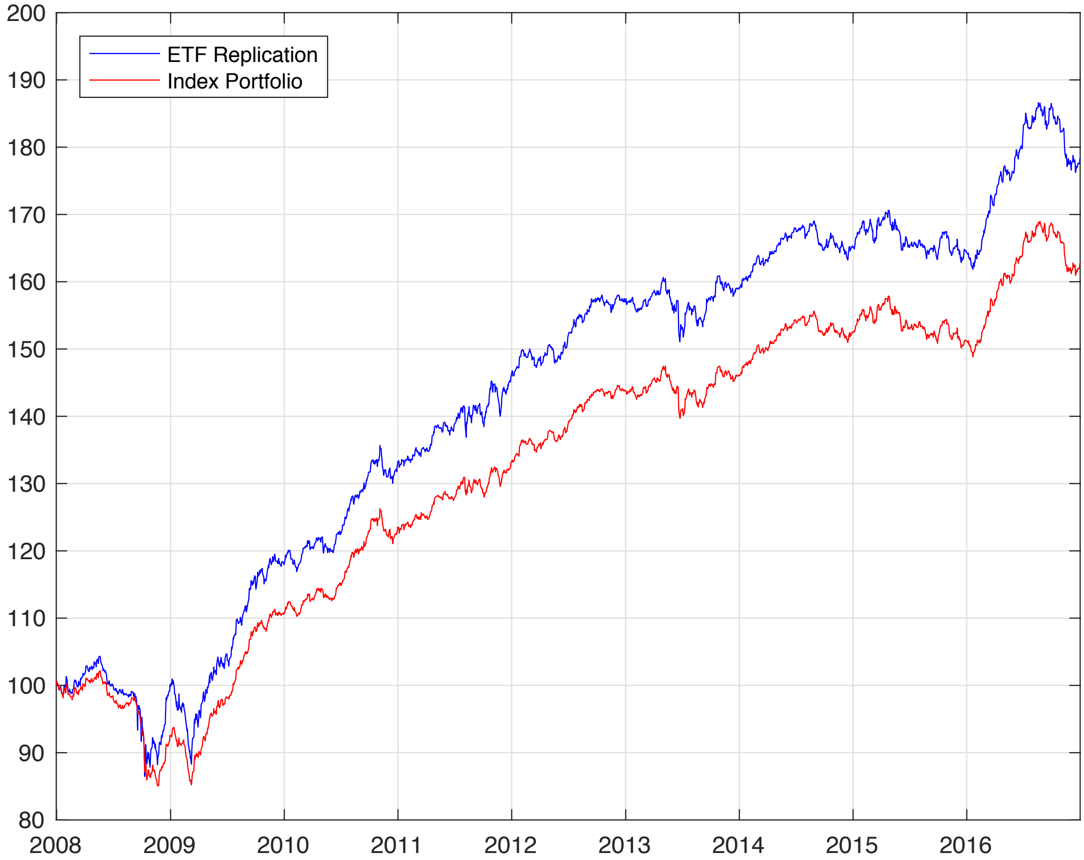
7 Exchange traded funds replication strategy

Following the discussion of the results in the previous chapters and the potential benefits of a Minimum cVaR Deviation strategy, a practical implementation of the aforementioned strategies is shown in this chapter. As one cannot invest into indices, it is necessary to find products linked to these indices. However, as these products, in general, provide less historical price data than the indices themselves, it might not be optimal to perform the optimisation procedure outlined in chapter 4. Hence, the weights from the optimisation in chapter 5.2 are applied. The evaluation of a practical implementation is conducted from the perspective of a retail investor. Only one exception has to be made in order to invest into a product which at least to some extend follows the Japanese Government Bond index. As it was not possible to find any ETF that uses either the 10-year Japanese Government Bond index or a similar benchmark for Japanese Government bonds as their benchmark, a passive index tracking fund was utilized. However, this fund possesses a minimum investment of USD 100,000. Another issue arises as not every ETF replicating one of the desired benchmarks provides at least a few years of history to allow a meaningful comparison. Consequently, ETFs on similar indices were used to accommodate for the lack of investment vehicles which track the used indices closely. However, a second ETF replication portfolio with a shorter history is constructed using the ETFs in table 20 which follows the optimised indices more closely. Both ETF portfolios already incorporate all management fees (shown in appendix III and IV) that have to be paid by the investor and, therefore, serve as a realistic proxy of the true performance when investing in the strategy. However, transaction costs arise due to the yearly rebalancing which must be deducted from the performance shown in tables 17 and 18.

In figures 4 and 5, the performance of both ETF replications and the index portfolio is shown. The weights of the Minimum cVaR Deviation optimisation are implemented in this comparison. However, one could have taken weights of any of the aforementioned strategies.

Figure 4 Comparison between ETF and index portfolio – long history

Figure 4 shows the comparison between the ETF portfolio and the underlying index portfolio starting in December 31st, 2007.



Using the ETFs of table 19, a comparison starting December 31st, 2007 can be conducted. As several ETFs are distinct to the indices used for the optimisation and implementation of the index portfolio, a divergence of this portfolio was expected. This divergence can be seen in the performance data and is especially captured by a higher Tracking Error in table 17 below. The Tracking Error is the annualized standard deviation of the difference in returns between the ETF portfolio and the index portfolio and is a measure of how closely the ETF portfolio follows the index portfolio.

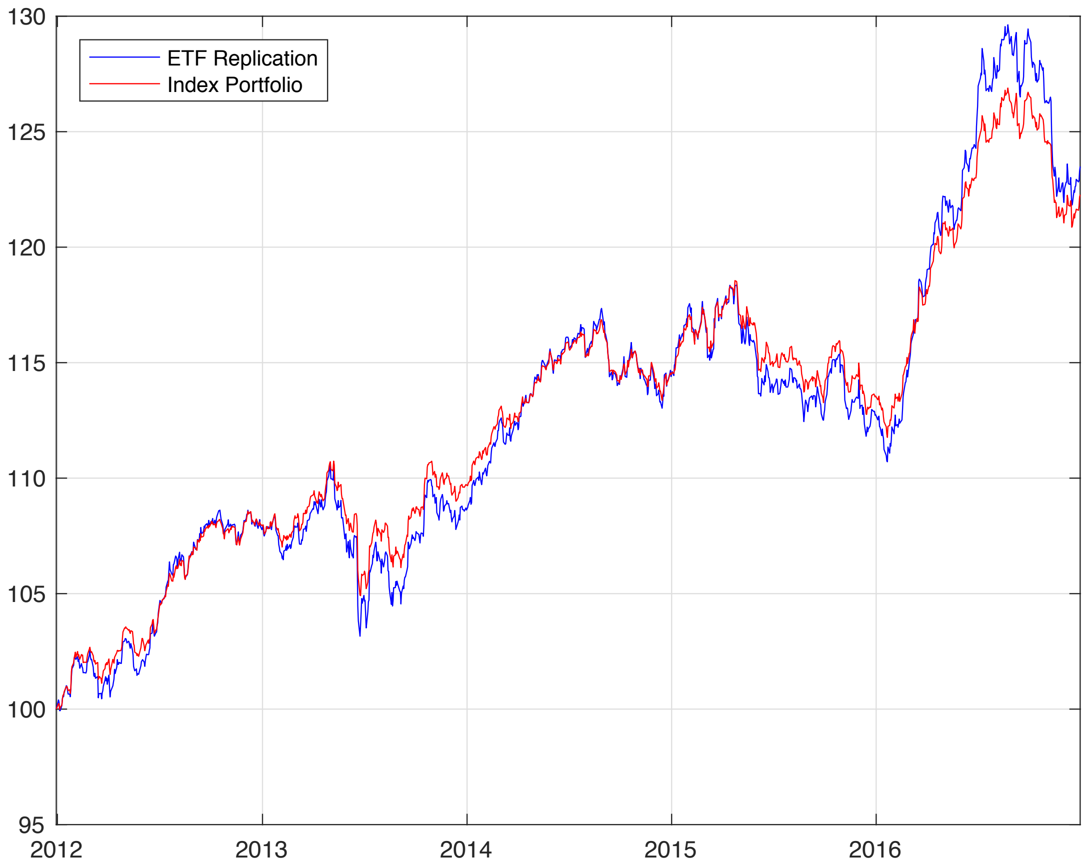
Table 17 Comparison between ETF and index portfolio – long history

Table 17 shows the comparison between the ETF portfolio and the underlying index portfolio starting in December 31st, 2007.

	ETF Portfolio	Index Portfolio
Return p.a.	6.64%	5.57%
Volatility p.a.	6.50%	4.41%
Tracking Error p.a.	4.30%	

Figure 5 Comparison between ETF and index portfolio – short history

Figure 5 shows the comparison between the ETF portfolio and the underlying index portfolio starting in December 30th, 2011.



A noticeable reduction in divergence of the two portfolios can be observed if the second ETF universe is utilized. This dataset is tracking the underlying indices more closely but only allows a comparatively short backtest. This can be inferred by looking at portfolio levels figure 5 but also from the performance data in table 18 below.

Table 18 Comparison between ETF and index portfolio – short history

Table 18 shows the comparison between the ETF portfolio and the underlying index portfolio starting in December 30th, 2011.

	ETF Portfolio	Index Portfolio
Return p.a.	4.31%	4.11%
Volatility p.a.	4.43%	3.45%
Tracking Error p.a.	2.00%	

Comparing the results for both ETF universes, the second universe clearly manages to follow the index portfolio more closely than the first universe. This is also observable in the lower Tracking Error. The first universe experience a Tracking Error of 4.3% p.a. whereas the Tracking Error for the second universe only amounts to 2%.

8 Implications of results

The results for the equity indices universe are in contrast to how the respective strategies perform when applied to the multi asset and the US equity universe. For both universes, the portfolio which tries to maximize the Sharpe Ratio are among the worst performing assets. However, they belong to the best performing strategies in the equity indices universe. In general, these strategies invest into riskier assets and pay less attention to lower the overall risk in the portfolio, whereas the minimum variance or minimum cVaR strategies focus on minimizing the risk in the portfolio. This proves to be superior to maximizing a Sharpe Ratio within the multi asset and US equity universe. One explanation could be the high correlation among all equity indices. As shown in table 2, all equity indices have a rather high and always positive correlation to each other. This impedes the construction of a well-diversified portfolio and might explain why a risk-return measure is more suitable for an investment objective. However, it is noteworthy that the construction of a portfolio using either of the above-mentioned strategies reduces the overall risk of investing compared to an investment into a single equity index significantly. As shown in appendix I, the two equity indices with the highest individual Sharpe Ratio (S&P 500 and OMX Stockholm 30 Index) have annualized volatilities of 17.84% and 22.88% and maximum drawdowns of 55.22% and even 71.18%, respectively. Using the Minimum cVaR strategy with the sample variance calculation, the annualized volatility can be reduced to as low as 13.35%.

This structure of highly correlated assets might also contribute to the results of the US equity universe. Despite having 50 assets in the eligible universe at each rebalancing date and hence a greater number of investable assets compared to the equity indices or multi asset universe, all stocks in the universe are listed in the US, their country of risk is in the US and all are large caps by design of the selection process (50 largest companies). Consequently, they all have a positive correlation with each other and might face similar risk factors. This again limits the ability to construct a well-diversified portfolio but not to the same extent as in the equity indices universe as all indices are already constructed using several individual stocks. Hence, it might still be beneficial to construct low risk portfolios rather than focusing on the maximization of a risk-return objective.

The multi asset universe clearly benefits the most from applying different optimisation strategies compared to the other universes. The two assets with the highest risk-return payoff (US Corporate Bonds IG and US Corporate Bonds HY) have a Sharpe Ratio of 1.05 and 1.29 and a maximum drawdown of 16.03% and 35.34%. The Minimum cVaR Deviation portfolio which is optimised using the sample variance has a higher annualized return (6.96%) and a lower annualized volatility (4.08%) than either of the above-mentioned indices and, hence, achieves a higher Sharpe Ratio. Besides having such a high risk-adjusted return, its maximum drawdown (16.77%) is almost as low as the minimum in the whole multi asset universe. Another benefit of the Minimum cVaR Deviation is the reduced kurtosis compared to the US High Yield Bond Index. However, this effect is also present across all strategies and all variance calculation methods. Following the discussion of the correlation structure in the previous paragraphs, it is worth looking at the correlations within the multi asset universe in table 4. The correlations vary substantially across assets, even showing negative relationships between several assets. This constitutes a remarkable difference compared to the two other universes and might be an explanation for the better results of the proposed optimisation methods. Assets with a negative correlation allow the construction of diversified portfolios in a more efficient manner.

9 Outlook for further research

Following the promising results for the multi asset universe, it would be interesting to see how the proposed strategy of a cVaR deviation optimisation performs when applied to a multi asset universe of individual stocks, bonds, commodities and across different currencies. One could also take the idea behind the US stock universe one step further and use a global stock universe which includes large, mid and small caps. This may help to overcome the high correlations and allow a construction of a well-diversified portfolio. Following the extensive discussion of different volatility calculation methods, it is obvious that further estimation procedures could be applied to approximate the volatility more precise. For instance, this could be done by either using a more advanced GARCH (p,q) model or a downside volatility measure. Following the academic discussion of excess kurtosis of stock market returns, a t-distribution with adjusted degrees of freedom by the sample kurtosis of the respective asset was applied when forecasting asset returns and imposing a correlation structures. This could be further improved by using a maximum likelihood estimation for calibrating the degrees of freedom. Furthermore, it would also be possible to take the skewness of the respective asset returns into account. This would allow a more precise estimation process and might improve the performance of the cVaR strategies even further.

One can also think of a combination of the minimum risk cVaR strategy with other types of strategies. This could be for example, following the typical minimum volatility and high dividend trend, a minimum cVaR and high dividend approach. This could be an alternative to the minimum volatility and high dividend portfolios currently used by investors.

10 Conclusion

In general, all above-mentioned results shed new light on the importance of constructing a diversified portfolio. In any possible setting used in this thesis, a superior result in terms of risk adjusted return to an investment into a single asset could be achieved. Even in the case of the equity indices where none of the proposed strategies yield superior returns a higher risk adjusted return is possible by combining different country indices together. The simple buy and hold portfolio already provides clear benefits by having a lower annualized volatility than all individual country indices. When looking at the US stock universe, a construction of a diversified portfolio rather than investing into an individual stock and consequently facing idiosyncratic risk is clearly beneficial for an investor. In contrast to the equity indices universe, however, two strategies prove to be slightly outperforming the other construction methodologies. Both the Minimum Variance and the Minimum cVaR Deviation portfolios have the highest Sharpe Ratios for the exponentially weighted and sample variance calculation. Furthermore, by constructing these two portfolios lower annualized volatilities and lower maximum drawdowns can be achieved across all variance methodologies. However, the equity indices as well as the US stock universe contain only assets of the same asset class. In addition, all assets from the US stock universe are from one country as mentioned in chapter 3. Hence, these assets show a positive correlation structure. The capability of constructing a diversified portfolio increases with both quantity and diversity of asset classes that are eligible for investing. This is observable in the results for the multi asset universe in chapter 5.2. Within the multi asset universe there is a clearer contrast between investing into simple strategies like the buy and hold or yearly equal weighted portfolio and optimisations which utilize this diversity in asset classes. As mentioned in chapter 3, the multi asset universe incorporates assets with negative correlations to each other. Within this universe, the Minimum cVaR Deviation strategy can outperform all other strategies including the Minimum Variance strategy. One explanation for the outperformance against the Minimum Variance strategy might be the superior properties of the cVaR deviation. In contrast to the standard deviation, the cVaR differentiates between positive and negative returns and can capture risk stemming from fat tails. Additionally, the construction of a portfolio using the cVaR deviation and

thereby including a more detailed picture of the distribution of losses into the optimisation leads to more efficient portfolios.
Besides finding a suitable optimisation strategy for constructing a portfolio using different assets, an analysis is provided in chapter 7 to assess the possibility of investing into these strategies from the perspective of a retail investor. All strategies are implementable for a retail investor using ETFs except for the Japanese government bond index where a passive index tracking fund is used. Using these replication scheme, it is possible to replicate the index strategy with a Tracking Error of 2.0%.

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Appendix I Equity Indices Universe

Name	Bloomberg Ticker	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
DAX Performance Index	DAX Index	8.85%	22.80%	0.39	8.77	-72.68%
Nikkei 250 Index	NKY Index	0.47%	23.36%	0.02	9.35	-79.38%
S&P 500 Index	SPX Index	10.26%	17.84%	0.58	11.83	-55.22%
FTSE 100 Index	UKX Index	7.33%	17.39%	0.42	9.17	-48.26%
CAC 40 Index	CAC Index	7.79%	21.99%	0.35	7.84	-64.09%
Hang Seng Index	HSI Index	11.06%	25.64%	0.43	18.39	-64.10%
IBEX 35 Index	IBEX Index	8.11%	22.04%	0.37	8.85	-56.49%
KOSPI Index	KOSPI Index	6.03%	27.06%	0.22	8.85	-74.98%
MSCI Emerging Market Index	MXEF Index	9.61%	18.46%	0.52	11.49	-65.16%
MSCI Australia Index	MXAU Index	8.46%	15.96%	0.53	9.22	-49.20%
OMX Stockholm 30 Index	OMX Index	13.19%	22.88%	0.58	7.36	-71.18%

Appendix II Multi Asset Universe

Name	Bloomberg Ticker/ Datastream	Return p.a.	Annualized Volatility	Sharpe Ratio	Sample Kurtosis	Maximum Drawdown
Barclays Long U.S. Corporate - Investment Grade	LHCCORP(IN)+100	5.40%	5.16%	1.05	5.09	-16.03%
Barclays U.S. Corporate High Yield - Speculative Grade	LHYIELD(IN)+100	6.21%	4.82%	1.29	53.23	-35.34%
S&P GSCI Commodity Total Return - RETURN IND.	GSCITOT	0.57%	23.11%	0.02	5.67	-80.51%
US-DS Real Estate - TOT RETURN IND	RLESTUS	9.83%	28.53%	0.34	23.63	-73.69%
DAX Performance Index	DAX Index	4.44%	24.18%	0.18	7.94	-72.68%
Nikkei 250 Index	NKY Index	3.17%	23.51%	0.13	9.90	-62.90%
S&P 500 Index	SPX Index	4.81%	19.62%	0.25	11.26	-55.22%
FTSE 100 Index	UKX Index	3.67%	18.98%	0.19	9.32	-48.26%
CAC 40 Index	CAC Index	3.73%	23.35%	0.16	8.23	-64.09%
Hang Seng Index	HSI Index	7.89%	24.02%	0.33	12.43	-64.10%
IBEX 35 Index	IBEX Index	3.13%	23.59%	0.13	8.64	-56.49%
KOSPI Index	KOSPI Index	9.15%	25.92%	0.35	9.15	-55.62%
FTSE MIB Index	FTSEMIB Index	-0.28%	24.01%	-0.01	8.42	-68.93%
MSCI Emerging Market Index	MXEF Index	8.27%	19.53%	0.42	12.14	-65.16%
MSCI Australia Index	MXAU Index	7.88%	16.45%	0.48	10.33	-49.20%
OMX Stockholm 30 Index	OMX Index	7.15%	23.82%	0.30	6.66	-71.18%
US BENCHMARK 10 YEAR DS GOVT. INDEX	BMUS10Y	4.76%	7.62%	0.62	5.62	-12.97%
UK BENCHMARK 10 YEAR DS GOVT. INDEX	BMUK10Y	4.67%	10.38%	0.45	6.05	-26.85%
BD BENCHMARK 10 YEAR DS GOVT. INDEX	BMBD10Y	4.92%	11.35%	0.43	6.40	-32.16%
JP BENCHMARK 10 YEAR DS GOVT. INDEX	BMJP10Y	2.44%	11.14%	0.22	6.54	-34.04%

Appendix III Replication starting December 31st 2007

Index Universe		ETF Replication		
Index Name	**Bloomberg Ticker/ Datastream**	**ETF Name**	**Bloomberg Ticker**	**Management Fee**
Barclays Long U.S. Corporate - Investment Grade	LHCCORP(IN)+100	ISHARES IBOXX INVESTMENT GRADE	LQD US Equity	0.15%
Barclays U.S. Corporate High Yield - Speculative Grade	LHYIELD(IN)+100	ISHARES IBOXX USD HIGH YIELD	HYG US Equity	0.50%
S&P GSCI Commodity Total Return - RETURN IND. (OFCL)	GSCITOT	PowerShares DB Commodity Index Tracking Fund	DBC US Equity	0.89%
US-DS Real Estate - TOT RETURN IND	RLESTUS	ISHARES US REAL ESTATE ETF	IYR US Equity	0.43%
DAX Performance Index	DAX Index	db x-trackers DAX UCITS ETF	XDAX GY Equity	0.09%
Nikkei 250 Index	NKY Index	Daiwa ETF - Nikkei 225	1320 JT Equity	0.16%
S&P 500 Index	SPX Index	ISHARES CORE S&P 500 ETF	IVV US Equity	0.04%
FTSE 100 Index	UKX Index	ISHARES CORE FTSE 100	ISF LN Equity	0.07%
CAC 40 Index	CAC Index	BNP Paribas Easy CAC 40 UCITS ETF	E40 FP Equity	0.25%
Hang Seng Index	HSI Index	HANG SENG H-SHARE IND ETF-HK	2828 HK Equity	0.64%
IBEX 35 Index	IBEX Index	LYXOR IBEX35 (DR) UCITS ETF	LYXIB SM Equity	0.30%
KOSPI Index	KOSPI Index	ISHARES MSCI KOREA UCITS ETF	IKOR LN Equity	0.74%
FTSE MIB Index	FTSEMIB Index	LYXOR FTSE MIB UCITS ETF	ETFMIB IM Equity	0.35%
MSCI Emerging Market Index	MXEF Index	iShares MSCI Emerging Markets ETF	EEM US Equity	0.72%
MSCI Australia Index	MXAU Index	ISHARES MSCI AUSTRALIA ETF	EWA US Equity	0.48%
OMX Stockholm 30 Index	OMX Index	iShares MSCI Sweden Capped ETF	EWD US Equity	0.48%
US BENCHMARK 10 YEAR DS GOVT. INDEX	BMUS10Y	iShares 7-10 Year Treasury Bond ETF	IEF US Equity	0.15%
UK BENCHMARK 10 YEAR DS GOVT. INDEX	BMUK10Y	iShares Core UK Gilts UCITS ETF	IGLT LN Equity	0.20%
BD BENCHMARK 10 YEAR DS GOVT. INDEX	BMBD10Y	iShares eb.rexx Government Germany 10.5+yr UCITS ETF	RXPXEX GY Equity	0.16%
JP BENCHMARK 10 YEAR DS GOVT. INDEX	BMJP10Y	VANGUARD JAP GV BD IDX- INV	VANJGBY ID Equity	0.25%

Appendix IV Replication starting December 30th 2011

Index Universe		ETF Replication		
Index Name	**Bloomberg Ticker/ Datastream**	**ETF Name**	**Bloomberg Ticker**	**Management Fee**
Barclays Long U.S. Corporate - Investment Grade	LHCCORP(IN)+100	SPDR Bloomberg Barclays Long Term Corporate Bond ETF	LWC US Equity	0.12%
Barclays U.S. Corporate High Yield - Speculative Grade	LHYIELD(IN)+100	SPDR Bloomberg Barclays High Yield Bond ETF	JNK US Equity	0.40%
S&P GSCI Commodity Total Return - RETURN IND. (OFCL)	GSCITOT	iShares S&P GSCI Commodity-Indexed Trust	GSG US Equity	0.89%
US-DS Real Estate - TOT RETURN IND	RLESTUS	ISHARES US REAL ESTATE ETF	IYR US Equity	0.43%
DAX Performance Index	DAX Index	db x-trackers DAX UCITS ETF	XDAX GY Equity	0.09%
Nikkei 250 Index	NKY Index	Daiwa ETF - Nikkei 225	1320 JT Equity	0.16%
S&P 500 Index	SPX Index	ISHARES CORE S&P 500 ETF	IVV US Equity	0.04%
FTSE 100 Index	UKX Index	ISHARES CORE FTSE 100	ISF LN Equity	0.07%
CAC 40 Index	CAC Index	BNP Paribas Easy CAC 40 UCITS ETF	E40 FP Equity	0.25%
Hang Seng Index	HSI Index	HANG SENG H-SHARE IND ETF-HK	2828 HK Equity	0.64%
IBEX 35 Index	IBEX Index	LYXOR IBEX35 (DR) UCITS ETF	LYXIB SM Equity	0.30%
KOSPI Index	KOSPI Index	MIRAE AS HORI KOSPI 200 ETF	2835 HK Equity	0.74%
FTSE MIB Index	FTSEMIB Index	LYXOR FTSE MIB UCITS ETF	ETFMIB IM Equity	0.35%
MSCI Emerging Market Index	MXEF Index	iShares MSCI Emerging Markets ETF	EEM US Equity	0.72%
MSCI Australia Index	MXAU Index	ISHARES MSCI AUSTRALIA ETF	EWA US Equity	0.48%
OMX Stockholm 30 Index	OMX Index	iShares MSCI Sweden Capped ETF	EWD US Equity	0.48%
US BENCHMARK 10 YEAR DS GOVT. INDEX	BMUS10Y	iShares 10-20 Year Treasury Bond ETF	TLH US Equity	0.15%
UK BENCHMARK 10 YEAR DS GOVT. INDEX	BMUK10Y	iShares Core UK Gilts UCITS ETF	IGLT LN Equity	0.20%
BD BENCHMARK 10 YEAR DS GOVT. INDEX	BMBD10Y	iShares € Govt Bond 10-15yr UCITS ETF	EUN8 GR Equity	0.20%
JP BENCHMARK 10 YEAR DS GOVT. INDEX	BMJP10Y	VANGUARD JAP GV BD IDX- INV	VANJGBY ID Equity	0.25%

Appendix V Weights (in %) for the equity indices universe resulting from the Minimum cVaR Deviation optimisation

	DAX Index	NKY Index	SPX Index	UKX Index	CAC Index	HSI Index	IBEX Index	KOSPI Index	MXEF Index	MXAU Index	OMX Index
31/12/1990	0.06	12.43	19.98	8.34	0.20	3.43	19.98	11.05	3.62	19.98	0.92
31/12/1991	0.30	15.56	19.99	5.88	0.15	3.05	19.95	11.87	1.27	19.96	2.02
31/12/1992	1.55	9.50	19.98	13.25	0.02	3.86	14.74	10.71	6.35	19.97	0.06
31/12/1993	3.61	9.14	19.99	19.22	0.03	1.36	10.49	9.44	6.74	19.95	0.04
30/12/1994	3.72	11.31	19.99	16.76	0.02	1.77	9.78	9.64	7.46	19.50	0.03
29/12/1995	5.73	9.28	19.99	17.07	0.03	0.67	7.63	11.10	8.68	19.80	0.03
31/12/1996	0.58	9.92	19.99	19.11	0.02	0.04	6.37	11.67	12.47	19.81	0.03
31/12/1997	1.91	11.28	20.00	17.25	0.01	0.01	3.15	12.16	14.24	19.99	0.00
31/12/1998	0.47	15.76	20.00	18.71	0.01	0.00	0.06	9.44	15.55	20.00	0.00
31/12/1999	0.47	16.52	20.00	17.04	0.01	0.01	2.62	6.99	16.35	20.00	0.01
29/12/2000	0.57	19.10	20.00	13.81	0.00	0.00	4.71	6.79	15.02	20.00	0.00
31/12/2001	0.65	17.86	20.00	15.39	0.01	0.00	1.81	5.52	18.76	20.00	0.00
31/12/2002	0.01	18.51	20.00	17.90	0.00	0.01	0.02	3.60	19.97	20.00	0.00
31/12/2003	0.00	18.44	20.00	18.45	0.00	0.01	0.00	3.10	20.00	20.00	0.00
31/12/2004	0.00	17.62	20.00	19.99	0.00	0.00	0.01	2.39	19.98	20.00	0.00
30/12/2005	0.00	16.81	20.00	19.96	0.00	0.75	0.03	2.68	19.76	20.00	0.00
29/12/2006	0.01	16.39	20.00	19.99	0.00	3.14	0.12	4.18	16.16	20.00	0.00
31/12/2007	0.02	19.93	20.00	19.99	0.01	0.09	1.40	4.19	14.37	20.00	0.00
31/12/2008	0.01	19.99	20.00	20.00	0.00	0.00	3.15	3.19	13.65	20.00	0.00
31/12/2009	0.40	19.99	20.00	20.00	0.02	0.01	4.17	5.46	9.95	20.00	0.00
31/12/2010	0.01	19.99	20.00	20.00	0.01	1.35	5.37	5.85	7.42	20.00	0.00
30/12/2011	0.01	19.99	20.00	20.00	0.01	0.30	3.41	6.39	9.91	20.00	0.00
31/12/2012	0.01	19.99	20.00	20.00	0.01	0.01	3.17	5.66	11.15	20.00	0.00
31/12/2013	0.01	19.31	20.00	20.00	0.01	1.62	2.90	6.98	9.17	20.00	0.00
31/12/2014	0.10	19.87	19.99	19.98	0.07	0.35	3.65	6.11	9.86	19.99	0.02
31/12/2015	0.04	19.42	20.00	20.00	0.01	0.73	3.38	7.23	9.21	20.00	0.00

Appendix VI Weights (in %) for the equity indices universe resulting from the Minimum Variance optimisation

	DAX Index	NKY Index	SPX Index	UKX Index	CAC Index	HSI Index	IBEX Index	KOSPI Index	MXEF Index	MXAU Index	OMX Index
31/12/1990	1.87	5.27	17.67	16.96	4.79	2.95	7.54	12.78	11.18	15.30	3.69
31/12/1991	2.25	4.90	18.15	17.49	3.70	3.84	6.20	12.04	11.14	16.19	4.11
31/12/1992	2.98	3.67	18.40	16.54	3.97	5.10	6.30	10.98	12.83	16.61	2.62
31/12/1993	3.24	4.21	18.31	16.35	3.99	4.76	6.10	11.05	13.15	16.03	2.82
30/12/1994	3.66	5.28	18.40	16.04	3.86	3.61	5.22	11.83	13.21	15.90	3.00
29/12/1995	3.83	5.26	18.39	15.91	3.77	3.67	5.31	11.88	12.78	16.01	3.21
31/12/1996	3.80	5.67	18.24	15.81	3.90	3.62	5.56	11.71	13.10	15.40	3.21
31/12/1997	3.47	6.50	18.44	16.55	3.90	2.55	5.56	9.79	13.66	16.61	2.96
31/12/1998	3.09	7.92	18.67	17.11	4.17	1.91	4.78	8.67	12.68	18.15	2.84
31/12/1999	2.95	8.66	18.70	16.97	4.10	1.95	4.85	7.63	12.75	18.29	3.15
29/12/2000	3.04	9.81	18.76	17.15	4.05	1.90	4.98	6.03	12.99	18.56	2.73
31/12/2001	2.67	10.30	18.81	17.56	3.73	1.97	4.76	5.31	13.71	18.80	2.38
31/12/2002	1.88	10.85	18.79	17.22	2.63	2.45	4.37	5.47	15.32	19.01	2.01
31/12/2003	1.65	11.06	18.82	16.99	2.28	2.76	4.37	5.06	15.88	19.06	2.06
31/12/2004	1.69	10.98	18.82	17.13	2.37	2.95	4.56	4.81	15.49	19.08	2.12
30/12/2005	1.76	10.84	18.80	17.12	2.42	3.15	4.71	4.79	15.16	19.04	2.20
29/12/2006	1.82	10.72	18.85	17.19	2.50	3.43	5.05	4.92	14.34	19.02	2.16
31/12/2007	1.95	11.19	18.90	17.06	2.62	3.25	5.63	4.98	13.27	19.00	2.15
31/12/2008	2.26	11.26	19.11	17.29	2.52	2.81	6.16	6.37	10.57	19.16	2.50
31/12/2009	2.17	11.87	19.10	17.46	2.59	2.72	6.20	7.18	8.87	19.20	2.63
31/12/2010	2.37	12.08	19.10	17.50	2.49	2.98	5.17	7.62	8.58	19.20	2.93
30/12/2011	2.20	12.68	19.10	17.60	2.31	3.12	5.10	7.73	8.12	19.19	2.85
31/12/2012	2.20	12.84	19.10	17.64	2.31	3.23	4.78	7.84	8.00	19.19	2.88
31/12/2013	2.27	12.33	19.09	17.61	2.35	3.33	4.72	8.09	8.07	19.17	2.97
31/12/2014	2.28	12.12	19.07	17.56	2.38	3.37	4.58	8.27	8.16	19.15	3.05
31/12/2015	2.26	12.04	19.07	17.48	2.30	3.45	4.41	8.67	8.11	19.13	3.07

Appendix VII Weights (in %) for the equity indices universe resulting from the maximum cVaR Sharpe Ratio optimisation

	DAX Index	NKY Index	SPX Index	UKX Index	CAC Index	HSI Index	IBEX Index	KOSPI Index	MXEF Index	MXAU Index	OMX Index
31/12/1990	1.54	0.00	20.00	0.22	19.99	4.79	0.00	13.49	20.00	0.00	19.97
31/12/1991	0.01	0.00	20.00	0.58	19.99	19.99	0.00	1.88	20.00	12.02	5.53
31/12/1992	0.00	0.00	20.00	18.20	9.97	20.00	0.00	4.83	20.00	2.07	4.93
31/12/1993	3.17	0.00	20.00	14.96	0.92	20.00	0.00	4.07	20.00	7.97	8.91
30/12/1994	5.12	0.00	20.00	8.70	0.00	19.99	0.00	9.42	20.00	0.02	16.73
29/12/1995	0.01	0.00	20.00	12.56	0.00	19.99	0.00	5.27	20.00	2.60	19.57
31/12/1996	2.84	0.00	20.00	12.15	0.00	20.00	2.06	0.01	20.00	2.95	20.00
31/12/1997	9.43	0.00	20.00	13.70	0.01	10.98	3.62	0.00	19.99	2.28	19.99
31/12/1998	7.30	0.00	20.00	13.90	0.00	7.01	10.65	0.00	11.44	9.71	20.00
31/12/1999	8.65	0.00	20.00	6.49	0.08	11.79	6.88	0.83	19.90	5.40	19.99
29/12/2000	11.60	0.00	20.00	2.24	4.36	13.58	0.14	0.00	10.70	17.39	20.00
31/12/2001	6.70	0.00	20.00	0.47	2.64	11.50	2.98	0.00	15.73	20.00	20.00
31/12/2002	0.00	0.00	20.00	4.96	0.01	15.04	0.01	0.00	19.99	20.00	20.00
31/12/2003	0.00	0.00	20.00	0.00	0.00	19.99	0.01	0.00	20.00	20.00	20.00
31/12/2004	0.00	0.00	20.00	0.01	0.00	18.40	1.59	0.00	20.00	20.00	20.00
30/12/2005	0.00	0.00	20.00	3.41	0.00	16.50	0.09	0.00	20.00	20.00	20.00
29/12/2006	0.00	0.00	20.00	0.01	0.00	17.10	2.89	0.00	20.00	20.00	20.00
31/12/2007	0.00	0.00	20.00	0.01	0.00	19.97	2.93	0.00	20.00	20.00	17.08
31/12/2008	0.00	0.00	20.00	0.00	0.00	16.44	3.56	0.00	20.00	20.00	20.00
31/12/2009	0.00	0.00	20.00	0.00	0.00	19.54	0.46	0.00	20.00	20.00	20.00
31/12/2010	0.00	0.00	20.00	0.01	0.00	19.99	0.00	0.00	20.00	20.00	20.00
30/12/2011	0.00	0.00	20.00	0.03	0.00	19.97	0.00	0.00	20.00	20.00	20.00
31/12/2012	0.00	0.00	20.00	0.11	0.00	19.89	0.00	0.00	20.00	20.00	20.00
31/12/2013	0.00	0.00	20.00	0.03	0.00	19.97	0.00	0.00	20.00	20.00	20.00
31/12/2014	0.00	0.00	20.00	2.20	0.00	17.80	0.00	0.00	20.00	20.00	20.00
31/12/2015	0.00	0.00	20.00	1.44	0.00	18.57	0.00	0.00	19.99	20.00	20.00

Appendix VIII Weights (in %) for the equity indices universe resulting from the maximum Sharpe Ratio optimisation

	DAX Index	NKY Index	SPX Index	UKX Index	CAC Index	HSI Index	IBEX Index	KOSPI Index	MXEF Index	MXAU Index	OMX Index
31/12/1990	0.00	0.00	20.00	13.94	19.20	1.95	0.00	14.12	20.00	0.00	10.79
31/12/1991	0.00	0.00	20.00	19.99	12.81	10.45	0.00	3.47	20.00	10.34	2.94
31/12/1992	0.00	0.00	20.00	20.00	11.08	17.36	0.00	5.58	20.00	4.06	1.91
31/12/1993	0.01	0.00	20.00	19.91	2.34	19.93	0.00	6.04	20.00	5.89	5.88
30/12/1994	1.05	0.00	20.00	18.07	0.01	13.23	0.00	11.37	20.00	4.14	12.14
29/12/1995	0.00	0.00	20.00	19.98	0.00	12.64	0.00	6.64	20.00	6.24	14.49
31/12/1996	0.01	0.00	20.00	19.99	0.00	15.14	0.01	1.07	20.00	3.80	19.99
31/12/1997	2.95	0.00	20.00	20.00	0.00	6.47	4.08	0.00	20.00	6.52	20.00
31/12/1998	0.85	0.00	20.00	20.00	0.01	2.77	8.44	0.00	11.68	16.27	20.00
31/12/1999	0.30	0.00	20.00	19.93	1.74	5.06	1.53	0.02	19.97	11.46	20.00
29/12/2000	0.11	0.00	20.00	18.26	6.82	6.58	0.01	0.00	8.25	19.97	20.00
31/12/2001	0.01	0.00	20.00	18.88	2.14	5.68	0.03	0.00	13.27	20.00	20.00
31/12/2002	0.00	0.00	20.00	10.94	0.01	9.07	0.01	0.00	19.98	20.00	19.99
31/12/2003	0.00	0.00	20.00	8.96	0.00	11.03	0.02	0.00	20.00	20.00	19.99
31/12/2004	0.00	0.00	20.00	9.06	0.00	10.71	0.25	0.00	20.00	20.00	19.98
30/12/2005	0.00	0.00	20.00	11.11	0.00	8.77	0.12	0.01	20.00	20.00	19.98
29/12/2006	0.00	0.00	20.00	7.32	0.00	10.03	3.92	0.00	20.00	20.00	18.72
31/12/2007	0.00	0.00	20.00	6.41	0.00	12.37	6.39	0.01	20.00	20.00	14.82
31/12/2008	0.00	0.00	20.00	0.10	0.00	12.51	7.40	0.00	19.99	20.00	19.99
31/12/2009	0.00	0.00	20.00	0.04	0.00	13.10	6.87	0.01	20.00	20.00	19.99
31/12/2010	0.00	0.00	20.00	6.11	0.00	13.84	0.02	0.04	20.00	20.00	20.00
30/12/2011	0.00	0.00	20.00	6.84	0.00	13.14	0.01	0.02	20.00	20.00	19.99
31/12/2012	0.00	0.00	20.00	5.62	0.00	14.37	0.01	0.01	20.00	20.00	19.99
31/12/2013	0.01	0.00	20.00	7.57	0.00	12.43	0.01	0.01	19.99	20.00	19.99
31/12/2014	0.01	0.00	20.00	7.23	0.00	12.76	0.01	0.01	19.99	20.00	20.00
31/12/2015	0.01	0.00	20.00	7.16	0.00	12.85	0.01	0.01	19.97	20.00	20.00

Appendix IX Weights (in %) for the multi asset universe resulting from the Minimum cVaR Deviation optimisation

	US CB IG	**US CB HY**	**GSCITOT**	**RLESTUS**	**DAX**	**NKY**	**SPX**	**UKX**	**CAC**	**HSI**	**IBEX**	**KOSPI**	**FTSEMIB**	**MXEF**	**MXAU**	**OMX**	**BMUS10Y**	**BMUK10Y**	**BMBD10Y**	**BMJP10Y**
31/12/2001	19.87	19.99	3.75	10.51	0.09	1.09	1.47	0.02	0.02	0.01	0.04	0.01	2.21	0.04	9.09	0.02	9.36	2.96	3.65	15.82
31/12/2002	10.67	19.99	3.21	9.25	0.49	0.87	2.06	1.16	0.02	0.02	0.04	0.01	3.18	0.04	10.78	0.02	16.18	4.68	2.44	14.87
31/12/2003	19.92	19.99	2.11	10.06	0.68	0.31	2.07	1.36	0.02	0.02	0.04	0.01	2.29	0.04	12.05	0.02	8.35	4.36	0.62	15.70
31/12/2004	19.96	19.99	2.13	6.24	0.62	0.09	3.40	1.38	0.02	0.01	0.03	0.01	2.55	0.02	14.44	0.02	12.70	2.30	0.05	14.03
30/12/2005	19.97	19.99	1.86	4.21	0.13	0.66	4.26	3.88	0.02	0.02	0.03	0.01	1.70	0.02	12.60	0.02	13.76	1.61	0.09	15.17
29/12/2006	19.99	20.00	1.45	2.95	0.06	1.00	5.52	2.70	0.00	0.01	0.01	0.00	1.84	0.00	11.54	0.00	17.95	0.32	0.18	14.48
31/12/2007	19.97	19.99	1.89	2.31	0.40	0.58	5.72	3.45	0.03	0.01	0.03	0.01	1.56	0.01	9.77	0.02	18.57	0.26	0.39	15.02
31/12/2008	19.98	19.99	0.98	1.92	0.19	0.79	6.07	2.84	0.03	0.01	0.03	0.02	2.36	0.01	5.12	0.09	19.53	3.99	0.07	15.97
31/12/2009	19.98	19.99	0.81	1.10	0.15	1.74	6.20	3.77	0.06	0.02	0.04	0.06	0.60	0.01	3.37	0.05	19.96	6.78	0.08	15.22
31/12/2010	19.98	19.99	1.32	1.25	0.89	1.53	5.74	3.98	0.04	0.01	0.06	0.04	0.18	0.01	2.96	0.02	19.93	5.02	0.20	16.86
30/12/2011	19.98	19.99	0.63	0.25	0.09	1.80	6.56	4.33	0.04	0.02	0.12	0.03	1.08	0.01	3.06	0.04	19.94	5.23	0.10	16.70
31/12/2012	19.98	19.99	0.91	0.73	0.77	1.51	6.37	4.78	0.05	0.02	0.10	0.08	0.06	0.01	3.08	0.03	19.94	5.37	0.47	15.76
31/12/2013	19.98	19.99	1.06	0.65	0.10	1.87	6.01	4.93	0.07	0.02	0.38	0.03	0.20	0.01	2.54	0.02	19.93	5.12	0.10	16.98
31/12/2014	19.97	19.99	1.53	0.60	0.05	2.05	6.62	3.48	0.03	0.02	0.13	0.05	1.19	0.01	2.44	0.02	19.89	4.84	0.12	16.96
31/12/2015	19.98	19.99	1.74	0.14	0.05	1.85	6.88	3.66	0.05	0.03	0.71	0.12	0.38	0.01	2.66	0.02	19.95	3.50	1.33	16.93

Appendix X Weights (in %) for the multi asset universe resulting from the Minimum Variance optimisation

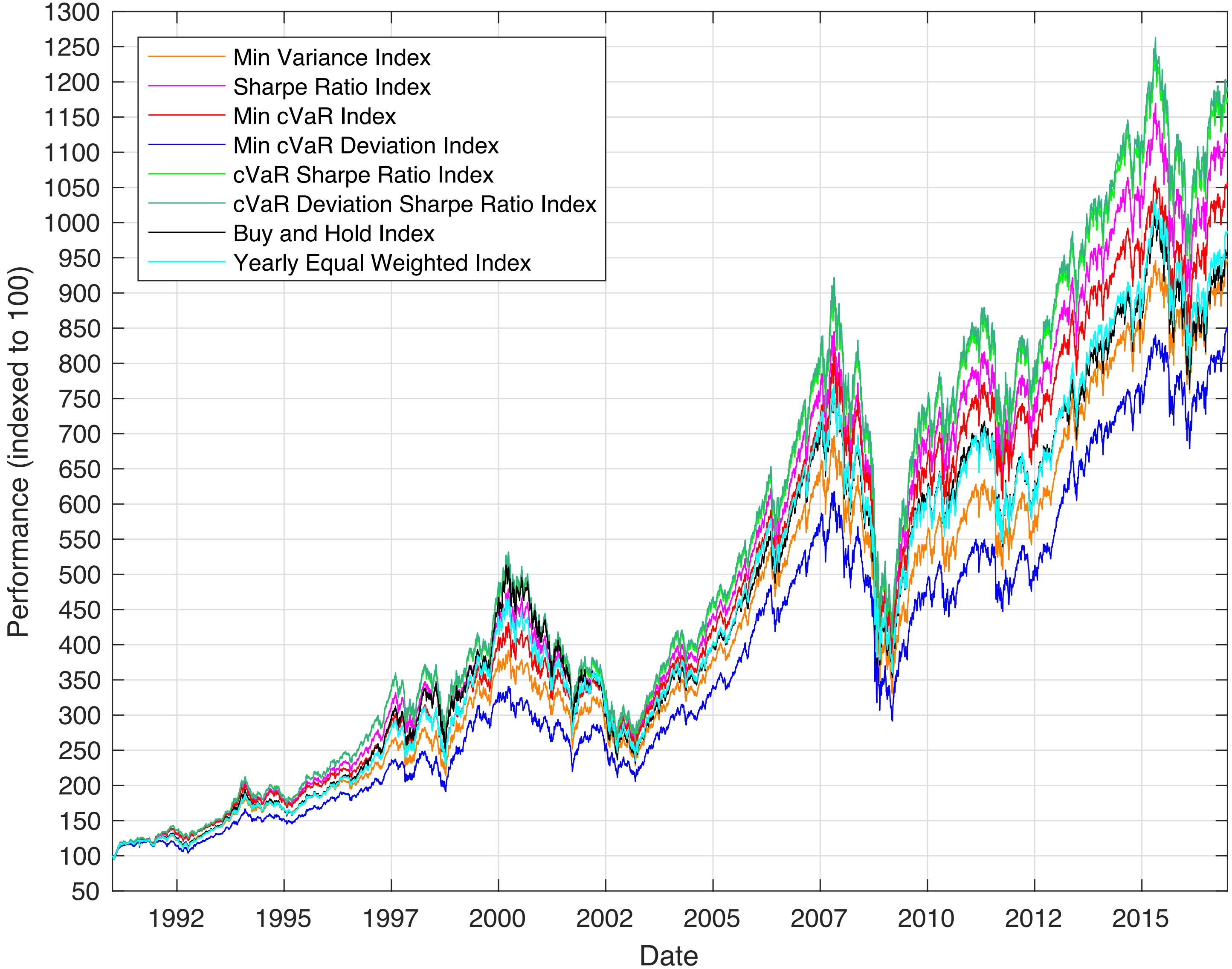
	US CB IG	US CB HY	GSCITOT	RLESTUS	DAX	NKY	SPX	UKX	CAC	HSI	IBEX	KOSPI	FTSEMIB	MXEF	MXAU	OMX	BMUS10Y	BMUK10Y	BMBD10Y	BMJP10Y
31/12/2001	13.89	15.47	4.56	5.25	1.28	2.02	2.49	1.83	1.27	1.25	1.28	0.95	1.34	1.54	3.62	1.33	12.53	9.41	6.93	11.76
31/12/2002	13.86	15.09	4.66	4.91	1.24	1.90	2.49	1.72	1.21	1.41	1.33	1.03	1.37	1.60	3.75	1.22	13.62	9.74	7.09	10.76
31/12/2003	13.74	15.08	4.29	5.34	1.31	1.96	2.64	1.82	1.29	1.55	1.43	1.09	1.45	1.69	4.03	1.33	13.30	9.47	6.76	10.45
31/12/2004	14.19	15.50	4.23	4.57	1.37	2.05	2.91	2.00	1.38	1.58	1.50	1.12	1.59	1.65	4.50	1.42	13.50	8.45	6.42	10.06
30/12/2005	14.24	15.42	4.03	4.26	1.43	2.20	3.02	2.10	1.45	1.68	1.57	1.18	1.66	1.69	4.62	1.52	13.51	8.18	6.42	9.84
29/12/2006	14.43	15.54	3.92	4.08	1.43	2.18	3.20	2.10	1.50	1.86	1.57	1.23	1.69	1.56	4.49	1.55	13.77	7.92	6.40	9.57
31/12/2007	14.39	15.38	3.82	3.72	1.51	2.26	3.30	2.13	1.51	1.68	1.65	1.28	1.80	1.51	4.05	1.53	13.72	7.74	6.33	10.71
31/12/2008	15.85	15.52	2.78	2.58	1.45	1.70	2.73	1.79	1.39	1.30	1.50	1.31	1.53	1.17	2.98	1.40	15.40	7.06	6.13	14.43
31/12/2009	16.22	15.92	2.65	1.99	1.44	1.81	2.62	1.78	1.38	1.23	1.49	1.39	1.45	1.15	2.79	1.39	15.77	6.65	6.10	14.78
31/12/2010	16.23	15.95	2.61	1.99	1.45	1.84	2.66	1.79	1.35	1.26	1.41	1.43	1.42	1.16	2.77	1.42	15.82	6.56	6.00	14.89
30/12/2011	16.40	15.86	2.52	1.94	1.40	1.89	2.65	1.78	1.32	1.26	1.41	1.43	1.37	1.16	2.67	1.39	15.94	6.78	5.91	14.90
31/12/2012	16.32	15.74	2.54	1.99	1.41	1.94	2.67	1.80	1.33	1.30	1.40	1.48	1.34	1.17	2.69	1.40	15.90	6.87	5.90	14.81
31/12/2013	16.21	15.77	2.60	1.98	1.44	2.06	2.68	1.87	1.34	1.30	1.44	1.53	1.38	1.19	2.79	1.40	15.87	6.69	5.94	14.52
31/12/2014	16.13	15.54	2.56	2.00	1.46	2.09	2.74	1.90	1.38	1.36	1.43	1.52	1.39	1.22	2.92	1.45	15.77	6.66	6.00	14.46
31/12/2015	16.03	15.52	2.57	2.00	1.47	2.13	2.77	1.90	1.40	1.39	1.44	1.57	1.40	1.24	2.87	1.47	15.66	6.56	6.12	14.49

Appendix XI Weights (in %) for the multi asset universe resulting from the cVaR deviation Sharpe Ratio optimisation

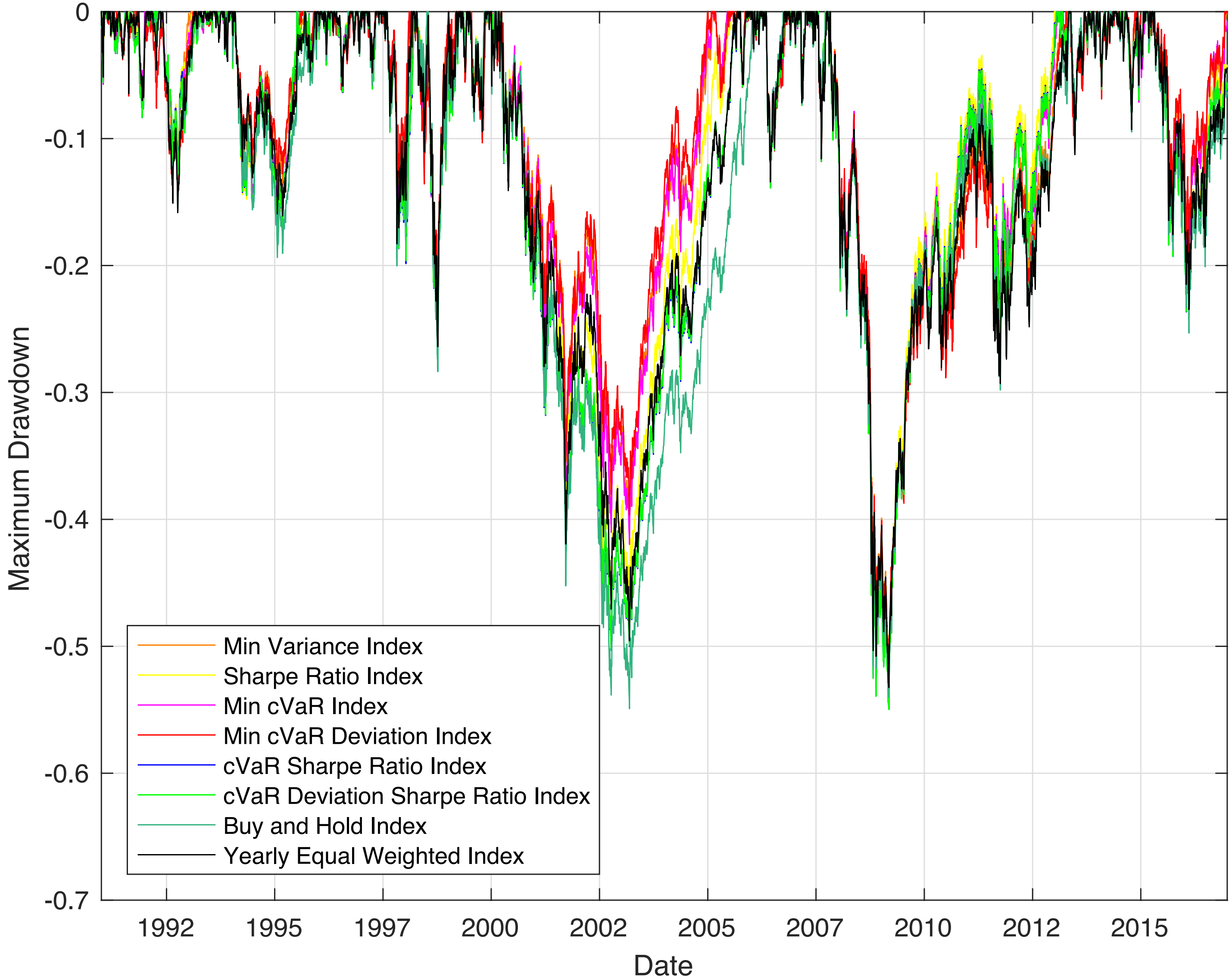
	US CB IG	US CB HY	GSCITOT	RLESTUS	DAX	NKY	SPX	UKX	CAC	HSI	IBEX	KOSPI	FTSEMIB	MXEF	MXAU	OMX	BMUS10Y	BMUK10Y	BMBD10Y	BMJP10Y
31/12/2001	20.00	19.99	13.56	20.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	20.00	0.00	4.83	0.00	0.00	1.61
31/12/2002	19.99	0.48	19.75	19.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	11.03	0.00	16.46	2.75	0.01	9.55
31/12/2003	19.98	19.99	8.76	19.97	0.01	0.00	0.01	0.01	0.01	0.02	0.01	0.01	0.01	5.97	8.21	0.01	3.73	3.64	0.43	9.24
31/12/2004	19.97	19.99	6.56	19.93	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	4.28	15.65	0.01	3.50	2.82	0.28	6.95
30/12/2005	19.99	20.00	6.13	17.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.79	17.98	0.00	3.73	1.91	0.34	3.41
29/12/2006	19.99	20.00	3.74	18.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.49	19.87	0.00	4.28	2.24	0.10	3.61
31/12/2007	19.95	19.98	5.05	10.92	0.02	0.01	0.02	0.02	0.02	0.02	0.03	0.01	0.01	11.08	16.12	0.01	7.95	1.53	1.35	5.90
31/12/2008	19.74	19.75	2.68	5.54	0.03	0.01	0.02	0.03	0.02	0.04	0.04	0.22	0.01	5.49	7.07	0.05	19.89	0.04	4.59	14.75
31/12/2009	19.99	20.00	1.24	3.86	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.00	11.81	5.65	0.01	19.92	0.03	3.81	13.64
31/12/2010	19.99	20.00	1.14	4.44	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.16	0.00	11.54	4.27	0.34	19.87	0.02	0.58	17.60
30/12/2011	19.97	19.99	1.13	4.53	0.02	0.01	0.04	0.09	0.02	0.03	0.03	0.61	0.01	7.60	3.73	0.75	19.93	1.95	1.66	17.91
31/12/2012	19.97	19.99	1.16	4.82	0.38	0.01	0.09	0.14	0.03	0.04	0.02	0.74	0.01	7.54	4.20	0.74	19.93	2.73	3.11	14.36
31/12/2013	19.99	20.00	1.28	5.16	0.05	0.00	1.54	0.05	0.01	0.01	0.01	0.27	0.00	4.34	8.56	1.25	19.53	1.15	5.89	10.89
31/12/2014	19.99	20.00	0.35	5.82	0.01	0.00	2.55	0.02	0.00	0.01	0.01	1.12	0.00	1.56	8.76	1.97	19.97	3.55	6.34	7.96
31/12/2015	19.99	20.00	0.01	5.94	0.03	0.00	3.31	0.03	0.01	0.05	0.01	1.94	0.00	0.03	10.67	1.74	19.98	1.48	5.05	9.74

Appendix XII Weights (in %) for the multi asset universe resulting from the cVaR deviation Sharpe Ratio optimisation

	US CB IG	US CB HY	GSCITOT	RLESTUS	DAX	NKY	SPX	UKX	CAC	HSI	IBEX	KOSPI	FTSEMIB	MXEF	MXAU	OMX	BMUS10Y	BMUK10Y	BMBD10Y	BMJP10Y
31/12/2001	20.00	14.29	11.69	19.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	15.15	0.46	18.26	0.00	0.00	0.15
31/12/2002	20.00	9.65	12.22	15.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.02	0.00	20.00	6.43	0.00	9.35
31/12/2003	20.00	20.00	7.96	17.56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.55	5.27	0.00	10.80	7.89	0.27	7.69
31/12/2004	20.00	20.00	7.58	16.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.03	10.27	0.00	7.79	7.19	1.71	6.56
30/12/2005	20.00	20.00	7.98	15.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.36	12.76	0.00	9.29	7.40	0.02	2.98
29/12/2006	20.00	20.00	5.83	16.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.16	13.84	0.00	7.66	10.35	0.01	1.52
31/12/2007	20.00	20.00	6.32	9.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.58	11.16	0.00	12.47	8.32	0.04	5.23
31/12/2008	20.00	19.99	3.55	4.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92	0.00	2.63	6.18	0.02	20.00	0.00	6.25	15.81
31/12/2009	20.00	20.00	2.89	3.14	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.12	0.00	6.90	5.51	0.61	20.00	0.00	5.20	15.61
31/12/2010	20.00	20.00	2.84	3.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.88	0.00	6.27	3.98	1.52	20.00	0.02	2.36	18.65
30/12/2011	20.00	20.00	2.83	3.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.71	0.00	2.98	3.28	1.85	20.00	3.86	1.77	17.93
31/12/2012	20.00	20.00	2.47	4.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	1.45	0.00	2.54	4.46	2.11	20.00	4.44	3.18	15.33
31/12/2013	20.00	20.00	2.49	3.03	0.00	0.00	2.47	0.00	0.00	0.00	0.00	1.72	0.00	0.01	6.92	2.73	20.00	1.99	7.15	11.48
31/12/2014	20.00	20.00	0.59	3.34	0.00	0.00	3.04	0.00	0.00	0.00	0.00	1.38	0.00	0.00	7.19	3.23	20.00	4.85	6.53	9.85
31/12/2015	20.00	20.00	0.00	3.48	0.00	0.00	3.19	0.00	0.00	0.00	0.00	1.79	0.00	0.00	7.15	3.45	20.00	5.09	4.66	11.18



Min Variance Index
Sharpe Ratio Index
Min cVaR Index
Min cVaR Deviation Index
cVaR Sharpe Ratio Index
cVaR Deviation Sharpe Ratio Index
Buy and Hold Index
Yearly Equal Weighted Index
Performance (indexed to 100)
1300
1250
1200
1150
1100
1050
1000
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
1992
1995
1997
2000
2002
2005
2007
2010
2012
2015
Date



Maximum Drawdown
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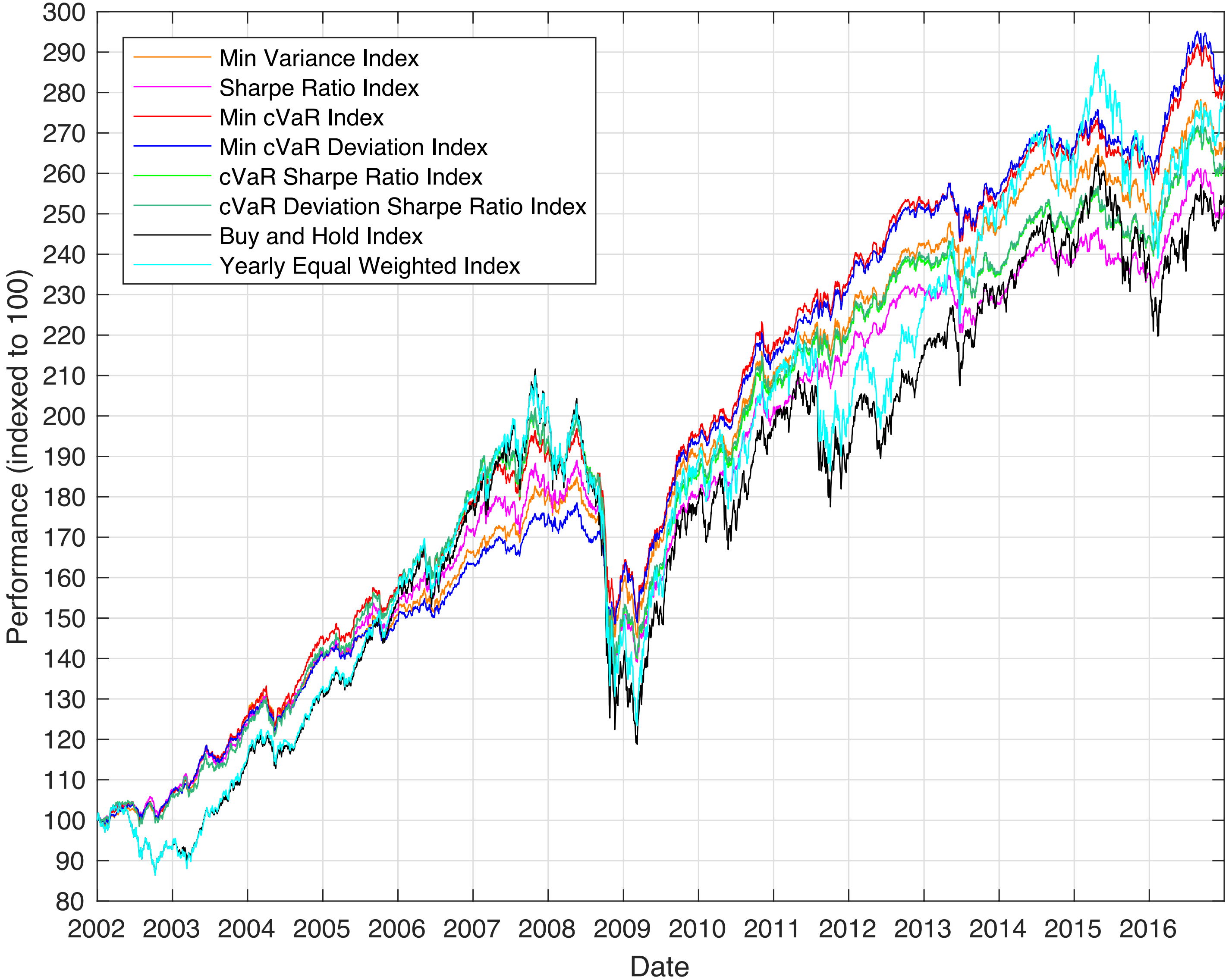
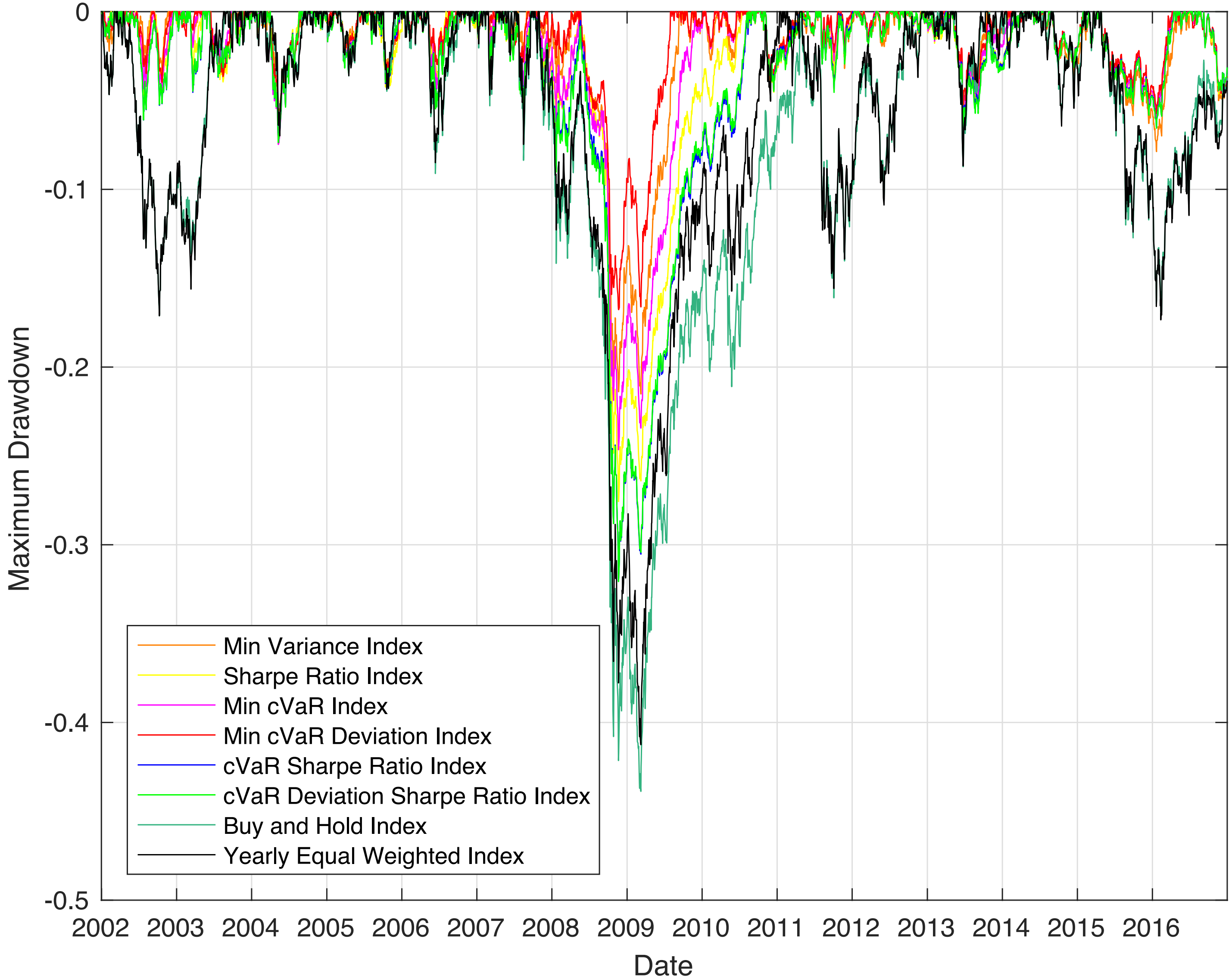
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
1992
1995
1997
2000
2002
2005
2007
2010
2012
2015
Date
Min Variance Index
Sharpe Ratio Index
Min cVaR Index
Min cVaR Deviation Index
cVaR Sharpe Ratio Index
cVaR Deviation Sharpe Ratio Index
Buy and Hold Index
Yearly Equal Weighted Index


Min Variance Index
Sharpe Ratio Index
Min cVaR Index
Min cVaR Deviation Index
cVaR Sharpe Ratio Index
cVaR Deviation Sharpe Ratio Index
Buy and Hold Index
Yearly Equal Weighted Index
Performance (indexed to 100)
300
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016
Date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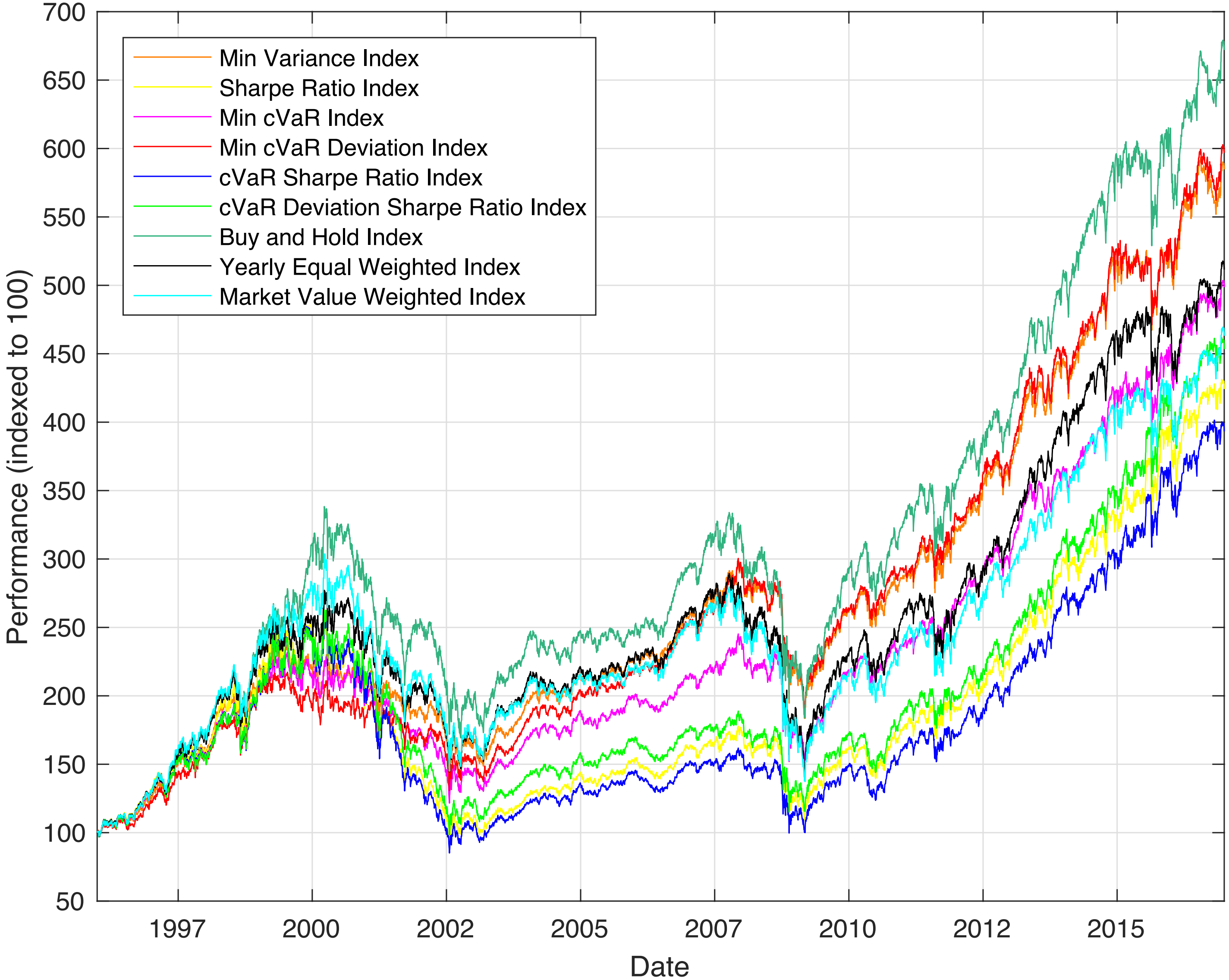
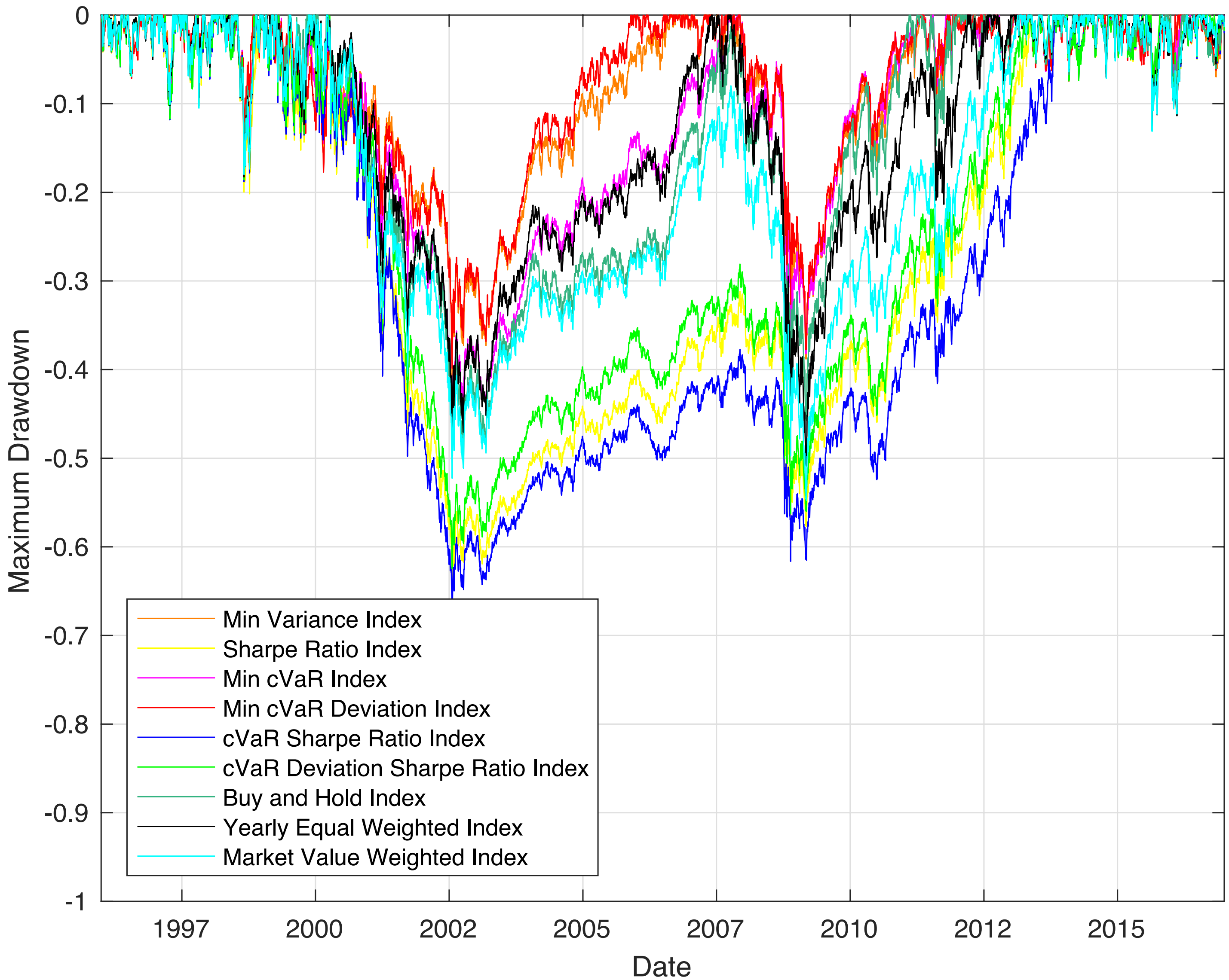
-0.1
-0.2
-0.3
-0.4
-0.5
Maximum Drawdown
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
Date
Min Variance Index
Sharpe Ratio Index
Min cVaR Index
Min cVaR Deviation Index
cVaR Sharpe Ratio Index
cVaR Deviation Sharpe Ratio Index
Buy and Hold Index
Yearly Equal Weighted Index


Min Variance Index
Sharpe Ratio Index
Min cVaR Index
Min cVaR Deviation Index
cVaR Sharpe Ratio Index
cVaR Deviation Sharpe Ratio Index
Buy and Hold Index
Yearly Equal Weighted Index
Market Value Weighted Index
Performance (indexed to 100)
700
650
600
550
500
450
400
350
300
250
200
150
100
50
1997
2000
2002
2005
2007
2010
2012
2015
Date


0
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-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
Maximum Drawdown
1997
2000
2002
2005
2007
2010
2012
2015
Date
Min Variance Index
Sharpe Ratio Index
Min cVaR Index
Min cVaR Deviation Index
cVaR Sharpe Ratio Index
cVaR Deviation Sharpe Ratio Index
Buy and Hold Index
Yearly Equal Weighted Index
Market Value Weighted Index