Factor-Based Hedge Fund Replication Using Exchange-Traded Funds

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Abstract

This paper studies the performance of factor-based hedge fund replication. We use monthly data of nine exchange-traded funds to estimate clone portfolios over the sample period 2008-2016 for eleven different hedge fund indices. We find that clones are capable of capturing a large part of the return characteristics of certain hedge fund strategies, although the clones generally underperform the indices. We further find that the use of factor selection and shrinkage methodologies improves replication results and leads to lower underperformance and portfolio turnover. The stepwise regression model shows the best out-of-sample performance, although a backtest shows that clones underperform the hedge fund indices more severely over a longer time span.

Keywords: Hedge Funds, Replication, Factor Model, Exchange-Traded Funds
JEL classification: G11, G12, G23

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1 Introduction

The hedge fund industry has seen an incredible growth in the years leading up to the 2007-2008 financial crisis. Assets under management have grown from about $500 billion in 2000 to over $2 trillion in 2007. Growth became negative during the crisis, but reached new highs by the end of 2016 at approximately $3 trillion in assets under management.[1] The growth in the hedge fund industry has also led to demand for passive hedge fund replication products, which brings up the question to what extent hedge fund returns can be replicated.

Hedge funds are alternative investment vehicles that employ certain strategies to generate returns for their investors. They are open only to accredited investors and benefit from low levels of regulation. This allows them to more heavily use leverage, derivatives, and short positions compared to better regulated vehicles such as mutual funds. Hedge fund strategies can therefore differ significantly from a long only strategy, which may enable them to exploit market inefficiencies.

These characteristics led to a claim that hedge funds could consistently outperform the market, and that managers with extensive investment experience would be able to provide steep returns to investors. Some studies indeed confirm that hedge funds provide superior results (e.g., see Brown et al., 1999; Edwards & Caglayan, 2001). It is difficult, however, to separate managerial skill from luck. Furthermore, findings indicate that outperformance is not persistent, or at best only present in the short-term (e.g., see Ackermann et al., 1999; Agarwal & Naik, 2000).

Perhaps the main advantage of hedge funds arrives from diversification benefits. As part of a portfolio, hedge funds can potentially provide a better risk-return trade off due to historical low correlation with traditional asset classes such as stocks and bonds. The low systematic risk can make it a good addition to an investment portfolio (e.g., see Fung & Hsieh, 1997; Amin & Kat, 2003). Diversification benefits are, however, reduced if the investment concerns a fund of funds, which consists of a portfolio of hedge funds. The high costs inherent to fund of funds do not justify including them into a portfolio, according to Ennis & Sebastian (2003).

[1] Source: Hedge Fund Research, Inc.

Other disadvantages of hedge funds include high fees and significant lockup periods. Fees traditionally follow a 2-20 structure, which translates into a 2% annual management fee charged against assets under management, and a 20% performance fee deducted from the fund's return. Lockup periods prevent investors from accessing their capital for a certain amount of time, and these periods can be substantial. Agarwal et al. (2009) find a median lockup period of one year using a broad sample of lockup imposing hedge funds.

These downsides have made researchers and practitioners search for methods to replicate hedge fund returns. By using liquid securities a replication product can potentially provide exposure to hedge fund like returns, while avoiding the high fees and lockup periods. Together with higher transparency this might provide investors with a serious alternative to hedge funds.

The fast growing exchange-traded funds (ETFs) market has been attracted by this concept as well. ETFs are traded securities that passively track an index. A recent innovation in this industry are ETFs that track a specific hedge fund index. The largest hedge fund ETF measured by assets under management is currently the IQ Hedge Multi Strategy Tracker (QAI). Incepted in early 2009 it is the first ETF that focused on replicating hedge fund returns. By using a portfolio composed of mainly other ETFs it attempts to follow its underlying index.

Our study will investigate to what extent hedge fund returns can be replicated using ETFs. We will use a linear factor model as a method for replication, which is common in hedge fund literature. Previous hedge fund replication studies mainly use financial indices or futures contracts as factors. Little focus has been put on the use of ETFs, however. A reason for this might be data availability, as ETFs are still a fairly recent development compared to, for example, futures contracts. In the last couple of years the ETF market has been growing rapidly. Assets under management in the global ETF industry have grown from about $200 billion in 2003 to over $3.5 trillion in 2016.[2] This has resulted in significantly lower costs and increased liquidity. The exposure that ETFs offer to a broad range of asset classes

[2] Source: Deutsche Bank Markets Research, ETF Annual Review & Outlook, 2017.

make them an interesting tool for replication strategies. Our study will therefore examine hedge fund replication using only ETFs as factors.

The remainder of the paper is organized as follows. Section 2 gives a brief overview of the existing literature on hedge fund replication. Section 3 provides a description and summary statistics of the data on hedge fund index and ETF returns. Section 4 lays out the factor model and the different replication methods. Section 5 will discuss the results using various performance measures including a backtest. Section 6 concludes.

2 Literature Review

There are two main approaches to replicate hedge fund returns. The first approach is based on replicating the distributional properties of hedge funds. This is done by using complex mathematical techniques such as those derived from Merton (1973) and Dybvig (1988). Using these techniques an algorithm is created to mimic the distribution of a hedge fund. Kat & Palaro (2005) show that this approach, using futures contracts, is capable of producing returns similar to those of hedge funds. Replicating individual hedge funds may be difficult, however, as data of individual hedge fund performance is scarcely available. Amenc et al. (2008) apply the distribution based approach on different types of hedge fund strategies and find that they are able to match the distribution of the convertible arbitrage strategy most accurately. However, a 96-month out-of-sample period is needed to successfully replicate the distribution, and out-of-sample performance is disappointing. The authors find that they are not able to perfectly match the average returns of the hedge funds, which leads to underperformance of the clones. They further argue that the distributional approach is unsuitable for hedge fund replication as time-series properties of returns are not captured. Other downsides of this approach are the reliance on a long time period to derive the distribution, and the difficulty in capturing dynamic trading strategies. Implementing a distribution based strategy may further prove difficult as it is highly complex and not sufficiently transparent (Wallerstein et al., 2010).

The alternative approach to replicate hedge funds is the factor-based approach, which provides a more intuitive way of cloning hedge fund returns compared to the distributional approach. It relies on the factor model pioneered by Sharpe (1992), who decomposes mutual fund returns in asset class factors and an uncorrelated residual. Sharpe (1992) argues that the mutual fund's exposure to different asset classes represents the fund's style, and the respective residuals show its selection. This approach of capturing a fund's performance with specific factors was first applied to hedge funds by Fung & Hsieh (1997). Using a principal component analysis on a group of 407 hedge funds, they construct five main investment styles of hedge funds. The authors then look how different asset classes explain the returns of each investment style. Although the asset classes explain part of the variation in hedge fund returns, a large part still remains unexplained.

Ennis & Sebastian (2003) apply the factor model of Sharpe (1992) to the Hedge Fund Research fund of funds index and find that it shows large exposure to six market factors. This is surprising given that the first hedge funds were created to provide security against market movements, and implies that hedge fund returns can be partially captured by market instruments. This is confirmed by Jaeger & Wagner (2005) who estimate that about 80% of hedge fund returns can be captured by systematic risk factors. Hasanhodzic & Lo (2007) build upon these findings by testing the out-of-sample performance of a replication strategy that uses a mix of six asset classes. The authors use a rolling window estimation period to allow the factor exposures to change over time, and thus mitigate part of the issues related to dynamic trading strategies. Findings indicate that for several investment styles a large fraction of the hedge fund returns can be captured by market factors. However, the clones are generating returns that are mostly inferior to those of the hedge funds. Underperformance seems to be the biggest issue with linear factor models, and made researchers look for alternative approaches to the factor-based model.

Amenc et al. (2010) analyse whether non-linear models are better able to replicate hedge fund returns compared to a linear factor model. They apply an option based factor model as suggested in the working paper of Diez De Los Rios & Garcia (2011), which includes a call option on an equity index combined with a standard factor model. Due to the non-linear or option like nature of hedge fund returns such models should be better able to replicate their returns. They find that the option model results in a better in-sample fit compared to the six factor model suggested by Hasanhodzic & Lo (2007). The authors recognize, however, that option and conditional models are more prone to estimation risk compared to the standard linear factor model. This is confirmed in the out-of-sample performance, where there appears to be little to no difference in the estimation errors between the different methodologies. As going beyond the linear case does not necessarily improve replication performance, Tancar et al. (2012) try using an asymmetric replication strategy that aims at minimizing only the negative sum of squared errors. This should allow the clone to freely outperform the hedge fund as long as its returns are above that of the fund, and thus eventually lead to better clone performance. Applying the asymmetric model on data between 1990 and 2008 the authors indeed find a higher Sharpe ratio, although the improvement is rather marginal.

To see what drives the underperformance of clones Dor et al. (2012) analyse the performance of several hedge fund replication indices against a weighted average of their target benchmarks. The authors find that market wide liquidity and reporting biases arising from attrition among hedge funds are the main drivers of tracking errors of clones. Missing factors or misspecified factors may further lead to poor clone performance (Bacmann et al., 2008).

A variety of different factors have been used to replicate hedge fund returns, but some may be impractical to implement as they are not directly investable. Bollen & Fisher (2013) adopt a similar methodology to Hasanhodzic & Lo (2007), but use five liquid futures contracts as factors. Their replication shows mixed results with high correlations between the clones and the hedge fund indices, but Sharpe ratios that are significantly lower for the clones. The authors argue that there is value in the underperforming clones as they allow investors to identify hedge fund alpha by eliminating the systematic risk, as defined by Kung & Pohlman (2004).

Using only five factors like Bollen & Fisher (2013) may be limiting in explaining hedge fund returns, given the wide variety in hedge fund styles and strategies. Chen & Tindall (2014) analyse a variety of different regression and variable selection techniques to improve the replication performance of hedge fund clones. Using a total of 27 different independent variables, they find that using ridge and LASSO shrinkage regression methods produce superior out-of-sample performance compared to other regression techniques. The use of econometric variable selection may thus help in generating better out-of-sample model fit. More recently, O'Doherty et al. (2016) introduced an approach to replicate hedge fund returns that combines four separate factor models, each consisting of three factors that represent different investments within the same larger asset class. These models are then pooled based on their log score, which essentially allows for variable selection by only including the factor models that have a positive log score. The authors find that the pooled model produces lower tracking errors and higher correlations with their hedge fund indices compared to the factor models suggested by Bollen & Fisher (2013) and Hasanhodzic & Lo (2007), who do not use econometric variable selection. Improvements are marginal, however, and the model combination approach still suffers from underperfoming clones. This seems to be a persistent problem among factor-based hedge fund products. Clones appear to provide systematically inferior returns compared to their hedge fund benchmark. Nevertheless, O'Doherty et al. (2016) show that inclusion of a hedge fund clone to an institutional investment portfolio can provide economic benefits, especially for investors with high levels of risk-aversion.

Our study will assess the performance of hedge fund clones that are constructed using a linear factor model of ETFs. The proposed strategy will be implementable for investors, which differs from most other studies. We will further contribute to the area of hedge fund replication by using more recent hedge fund data, performing a backtest to check robustness, and by including various regression techniques in addition to a standard OLS regression to see if replication can be improved.

3 Data

This study aims to replicate hedge fund returns using ETFs. Data on hedge fund returns is obtained from the Hedge Fund Research Index, while returns of ETFs are retrieved from Datastream. A detailed description of the data is given below.

3.1 Hedge Fund Index Returns

Monthly returns of eleven different hedge fund indices are collected from Hedge Fund Research for the period May 2006 to December 2016. These indices are used as a benchmark for hedge fund performance, and will be used as the target for replication. The eleven indices include a broad composite index that consists of over 2000 individual hedge funds, and ten sub-indices that fall into different strategy categories. We include the ten primary strategies: Equity Hedge, Event Driven, Macro, Relative Value, Emerging Markets, Equity Market Neutral, Short Bias, Convertible Arbitrage, Multi Strategy, and Fund of Funds, as described by Hedge Fund Research. Descriptions of the different strategies can be found in Appendix A. All indices are constructed from the Hedge Fund Research database, which consists of over 7300 individual hedge funds and fund of funds globally. To be included in the Hedge Fund Research database the fund must have a minimum of $50 million assets under management, or a track record that exceeds twelve months. All indices are equal-weighted and returns are reported net of fees. For most of our empirical research we use excess returns, calculated by deducting the risk-free rate from the index return. As a benchmark for the risk-free rate we use returns of the 3-month U.S. Treasury bill, obtained from Datastream.

Hedge fund data comes with several biases (e.g., see Brown et al., 1992; Fung & Hsieh, 2000; Baquero et al., 2005). Closed funds are generally not represented in the data, which creates a survivorship bias. A commonly known problem among mutual fund data as well. By only including active funds the overall returns are likely biased upwards. Secondly, a selection bias arises because hedge funds are not required to publicly report their returns. Funds seeking new investors are more likely to disclose returns, but will only do so if they have a good track record. This further creates a backfill bias as data providers generally include past returns of a new fund into their database. Aiken et al. (2013) find that hedge funds that report their returns to a commercial database show alpha that is more than twice as large compared to hedge funds that do not report their performance. To overcome these biases Fung & Hsieh (2002) argue that fund of funds returns are a better benchmark for hedge fund performance, because negative results of retired funds will still show up in the performance of fund of funds. Hence we also include the fund of funds index. Hedge Fund Research tries to mitigate the aforementioned biases by keeping returns of inactive funds in their database, and by not backfilling historical returns of newly included funds.

Summary statistics of total returns of all eleven hedge fund indices can be found in Table 1. The average annual return of the hedge funds index over the specified period is 3.59% compared to 1.55% for the fund of funds index. Relative value shows the highest mean annual return at 5.49%. Standard deviations are in the range of 2.70% and 11.66% annually. The hedge fund returns on average show relatively high levels of excess kurtosis combined with a negative skewness. This means that there is an increased risk of observing extreme negative returns, a common characteristic of hedge fund returns (e.g., see Brooks & Kat, 2002; Cremers et al., 2005). Macro and short bias strategies show different patterns with positive skewness and lower excess kurtosis. Highest tail risk is shown by the relative value, convertible arbitrage, and multi strategy indices. Correlations between the hedge fund returns and the S&P 500 are high at levels exceeding 65%, except for the macro, short bias and market neutral strategies. This is in contrast with the belief that hedge funds provide returns that are uncorrelated with the market. Although this may still be the case for individual hedge funds, it clearly does not hold for most hedge fund indices.

Table 1: Summary statistics of hedge fund index returns

	Mean (%)	**Std. Dev. (%)**	**Sharpe Ratio**	**Excess Kurtosis**	**Skewness**	**Correlation S&P 500**
Hedge Funds	3.59	6.13	0.43	2.56	-0.92	0.83
Equity Hedge	3.02	8.54	0.24	2.22	-0.90	0.87
Event Driven	4.38	6.49	0.52	3.38	-1.22	0.81
Macro	2.86	4.72	0.40	-0.14	0.38	0.19
Relative Value	5.49	4.93	0.91	11.93	-2.42	0.70
Emerging Markets	2.71	11.66	0.15	2.87	-0.87	0.75
Market Neutral	2.13	2.70	0.43	3.92	-1.46	0.52
Short Bias	-5.70	10.85	-0.61	0.61	0.24	-0.87
Convertible Arb.	4.96	8.92	0.45	17.26	-2.38	0.65
Multi Strategy	4.01	5.04	0.60	12.71	-2.43	0.67
Fund of Funds	1.55	5.31	0.11	3.88	-1.42	0.71

This table shows summary statistics of monthly total returns for all eleven hedge fund indices over the period May 2006 to December 2016. Means are calculated using a geometric average. Mean, standard deviation, and Sharpe ratio are annualized.

3.2 Exchange-Traded Fund Returns

We obtain data for nine different ETFs for the period May 2006 to December 2016, which are used as factors for replicating the hedge fund index returns. The following ETFs are included in our study: SPDR S&P 500 (**SPY**), iShares 7-10 Year Treasury Bond (**IEF**), SPDR Gold Shares (**GLD**), United States Oil Fund (**USO**), Guggenheim CurrencyShares Euro Trust (**FXE**), iShares Core U.S. Aggregate Bond (**AGG**), iShares U.S. Real Estate (**IYR**), iShares MSCI Emerging Markets (**EEM**), and iShares MSCI EAFE (**EFA**). These ETFs are selected to include a broad range of asset classes.

The first five securities are in line with Bollen & Fisher (2013) who use futures contracts covering the S&P 500, 10-year Treasury note, gold, crude oil, and the U.S. dollar index. Instead of using an ETF that tracks the U.S. dollar index we use one that follows the euro due to limited data availability. FXE was the first ETF tracking a currency and began trading late 2005. The first ETF tracking the U.S. dollar index was incepted in 2007, and would substantially shorten our sample period. We add an additional four ETFs to be used as factors since an advantage of ETFs is that they provide exposure to a wide range of different assets. For example, corporate bonds are not traded in the futures market, but can be invested in through ETFs. By including additional factors we hope to capture more of the variation in hedge fund returns. Covered asset classes are: domestic equity (SPY), foreign equity (EFA), emerging markets equity (EEM), government bonds (IEF), domestic bonds (AGG), real estate (IYR), currencies (FXE), gold (GLD), and crude oil (USO). Correlations among all ETFs can be found in Table 2. We observe that international equity markets are highly correlated with correlations of 0.81 and 0.90 between the SPY ETF and the other two equity market ETFs (EEM and EFA). The remainder of the ETFs show less correlation with SPY, with treasury bonds and gold displaying the lowest at -0.29 and 0.03 respectively. The bond ETFs IEF and AGG also show a high correlation with each other at 0.83. To adjust for this multicollinearity among factors we will perform different regression techniques, explained in more detail under methodology.

Table 2: Correlation matrix of ETF returns

	SPY	IEF	GLD	USO	FXE	AGG	IYR	EEM	EFA
SPY	1.00	-0.29	0.03	0.44	0.47	0.05	0.76	0.81	0.90
IEF	-0.29	1.00	0.34	-0.35	0.05	0.83	-0.04	-0.17	-0.19
GLD	0.03	0.34	1.00	0.20	0.35	0.39	0.10	0.26	0.13
USO	0.44	-0.35	0.20	1.00	0.45	-0.19	0.20	0.52	0.48
FXE	0.47	0.05	0.35	0.45	1.00	0.29	0.35	0.62	0.66
AGG	0.05	0.83	0.39	-0.19	0.29	1.00	0.31	0.18	0.19
IYR	0.76	-0.04	0.10	0.20	0.35	0.31	1.00	0.65	0.72
EEM	0.81	-0.17	0.26	0.52	0.62	0.18	0.65	1.00	0.89
EFA	0.90	-0.19	0.13	0.48	0.66	0.19	0.72	0.89	1.00

This table shows correlations of monthly total returns between all nine ETFs over the period May 2006 to December 2016.

Data availability is an obstacle as ETFs are still a relatively new development. The very first ETFs were developed in the beginning of the 1990s, but the real growth only began after 2003. We select the first ETF created in every asset class mentioned above to maximize the timespan for our analysis. The limiting factor for the timespan is the USO ETF that was incepted on April 10, 2006. To overcome the shortage of data we run a backtest by extending the returns of the selected ETFs back to January 1992 using their underlying benchmarks. This allows us to perform a robustness test to check how the replication would have performed further back in time. The approach of extending ETFs back in time is explained in the following section. Other criteria that the ETFs fulfil is that they are still active today, are denominated in U.S. dollar, and are sufficiently liquid. See Appendix B for more details regarding all selected ETFs.

Summary statistics of returns for all nine ETFs can be found in Table 3. The average annual return of the S&P 500 ETF is the highest at 7.32%. This is well above the average returns of the hedge funds over the same period. Volatility of SPY is, however, significantly higher at 14.86%. The lowest mean return of -15.39% is obtained by USO. Standard deviations are in a range of 3.87% for AGG to 32.53% for USO. Except for IEF, AGG, and GLD, the ETFs show negative skewness with some excess kurtosis. This again implies somewhat negative tails, although less than for the hedge fund indices. The gold ETF seems to follow a normal distribution.

Table 3: Summary statistics of ETF returns

	Mean (%)	**Std. Dev. (%)**	**Sharpe Ratio**	**Excess Kurtosis**	**Skewness**	**Correlation S&P 500**
SPY	7.32	14.86	0.43	1.72	-0.71	1.00
IEF	5.47	6.59	0.68	1.36	0.34	-0.29
GLD	5.01	19.02	0.21	0.01	-0.02	0.03
USO	-15.39	32.53	-0.50	0.88	-0.26	0.44
FXE	-1.18	10.68	-0.20	1.33	-0.19	0.47
AGG	4.37	3.87	0.88	7.30	1.08	0.05
IYR	5.17	24.00	0.17	5.78	-0.68	0.76
EEM	1.91	23.67	0.04	1.32	-0.22	0.81
EFA	1.45	18.61	0.03	1.56	-0.55	0.90

This table shows summary statistics of monthly total returns for all nine ETFs over the period May 2006 to December 2016. Means are calculated using a geometric average. Mean, standard deviation, and Sharpe ratio are annualized.

3.3 Backtest Data

We perform a backtest until January 1992 to check the robustness of hedge fund replication. ETF data only allows us to include one bull and one bear market, because the USO ETF was created in 2006. By extending the ETFs further back in time we can look at several other interesting periods in history, to see how hedge fund replication would have performed under different market conditions. We obtain returns of all eleven hedge fund indices from January 1992 to December 2016, and returns of all nine ETFs from their respective inception date until December 2016. Benchmarks of the ETFs are used to calculate data for the missing periods. Inception dates and benchmarks for all ETFs can be found in Appendix B.

Six of the nine ETFs use a specified index as their benchmark. For these ETFs we deduct fees (i.e. the expense ratio) from the total return of the index to obtain the missing data. The remaining ETFs (GLD, USO, and FXE) do not track a particular index. GLD tracks the gold bullion spot price by holding physical gold. We replicate this ETF by deducting fees from the total return of the gold spot price. Similarly, FXE follows the Euro/Dollar spot rate and missing data is calculated by deducting fees from returns of the exchange rate. Finally, USO tracks the near month crude oil futures traded on the NYMEX. When the near month futures contract is within two weeks of expiration the benchmark will change to the next month contract to expire. We obtain data of all crude oil futures contracts from 1992 onwards, and then apply above methodology by rolling over to the next months contract when it is within two weeks of expiration. We deduct monthly fees from this benchmark to get the missing data.

The approach outlined above allows us to run a backtest that includes all ETFs, although some estimation errors have to be accounted for. ETFs are unable to track their benchmark at 100% accuracy, and this tracking error is not captured in our backtested data. The fact that tracking errors change over time make them difficult to incorporate, and including a fixed tracking error introduces other estimation errors. Results will not be significantly affected if we exclude tracking errors from consideration, because these are generally very small between an ETF and the index. A second estimation error may arise from the premium or discount that can exist between a funds net asset value and its market price. This discrepancy can arise for a variety of reasons, but is generally only short lived in nature. The exclusion of this estimation error will also not significantly affect results.

Summary statistics for the hedge fund indices and the ETFs over the entire backtest period can be found in Table 4. The most notable difference between the period starting in 1992 and the shorter period starting in 2006 is that returns of hedge funds are significantly higher during the longer sample period as seen in Panel A. The average return of the hedge funds index is 9.32% over the period 1992-2016 compared to 3.59% in the shorter period 2006-2016, while having similar levels of standard deviation. Hedge funds thus delivered a significantly higher Sharpe ratio historically, which indicates they performed better during the 1990s and the early 2000s compared to more recent years. This corresponds with Joenväärä et al. (2016) who find that hedge fund outperformance has become significantly less during the period 2004-2012 compared to preceding years. They argue that the increase in assets under management in the industry has led to diminishing returns to scale. Correlations between hedge funds and the S&P 500 seem to have also increased over time. It moves from 0.73 for the hedge funds index over the entire backtest period to 0.83 during the more recent sample. This supports the case of performing a backtest to check the robustness of hedge fund replication when facing different market conditions. Tail risk as measured by excess kurtosis and skewness do not provide remarkably different results between the two periods.

Returns of the ETFs differ to a lesser extent when comparing the two sample periods. Average returns and standard deviations stay relatively similar for most ETFs although USO, IYR, EEM, and EFA show better performance over the longer period. The average returns of these ETFs have thus shown a decline in more recent years. Correlation among equity markets have further increased over the years as seen by the higher correlation of EEM and EFA with the S&P 500 during the more recent sample period. The other moments of the distribution of the ETF returns (i.e. standard deviation, excess kurtosis, and skewness) do not differ considerably between the two periods.

Table 4: Summary statistics of returns during the backtest period

	Mean (%)	**Std. Dev. (%)**	**Sharpe Ratio**	**Excess Kurtosis**	**Skewness**	**Correlation S&P 500**
Panel A: Hedge Fund Research Index						
Hedge Funds	9.32	6.63	0.99	2.85	-0.63	0.73
Equity Hedge	10.34	8.72	0.87	2.24	-0.26	0.75
Event Driven	10.39	6.44	1.18	4.18	-1.20	0.71
Macro	9.14	6.80	0.94	1.45	0.57	0.28
Relative Value	9.05	4.27	1.48	13.58	-2.06	0.54
Emerging Markets	10.16	13.29	0.56	4.14	-0.83	0.61
Market Neutral	5.65	3.04	0.98	2.08	-0.22	0.30
Short Bias	-2.02	17.02	-0.27	3.33	0.40	-0.69
Convertible Arb.	7.92	6.42	0.81	29.83	-3.04	0.48
Multi Strategy	7.21	4.06	1.11	15.62	-2.53	0.52
Fund of Funds	5.94	5.59	0.58	4.17	-0.66	0.58
Panel B: ETF Factors						
SPY	8.99	14.12	0.44	1.35	-0.65	1.00
IEF	6.03	6.31	0.53	0.88	0.01	-0.17
GLD	4.42	16.02	0.11	1.11	0.32	0.00
USO	-0.30	31.62	-0.09	0.86	-0.02	0.19
FXE	-1.15	10.00	-0.37	0.73	-0.09	0.17
AGG	5.49	3.86	0.73	3.22	0.20	0.05
IYR	9.41	18.68	0.36	7.73	-0.80	0.59
EEM	6.23	23.11	0.15	1.62	-0.50	0.72
EFA	4.85	16.53	0.13	1.24	-0.49	0.79

This table shows summary statistics of monthly total returns for all eleven hedge fund indices and all nine ETFs over the backtest period January 1992 to December 2016. ETF returns are extended back in time, as explained in the text. Means are calculated using a geometric average. Mean, standard deviation, and Sharpe ratio are annualized.

4 Methodology

Our goal is to develop a method that replicates hedge fund returns using ETFs. We follow existing literature by using a linear factor model to estimate holdings of the clone portfolio. Excess returns of each hedge fund index are regressed against excess returns of the nine previously specified ETFs, while suppressing the alpha. Estimated betas are then assigned as weights to each ETF. This is similar to the linear replication model specified by Hasanhodzic & Lo (2007), although they use five not directly investable indices as factors. We perform several regression techniques to estimate portfolio weights, explained in more detail in the sections below.

The general regression model can be specified as:

$$\begin{aligned} r_{i,t} = \beta_{i,1}SPY_t + \beta_{i,2}IEF_t + \beta_{i,3}GLD_t \\ +\beta_{i,4}USO_t + \beta_{i,5}FXE_t + \beta_{i,6}AGG_t \\ +\beta_{i,7}IYR_t + \beta_{i,8}EEM_t + \beta_{i,9}EFA_t \\ +\epsilon_{i,t} \end{aligned} \tag{1}$$

where $r_{i,t}$ is the excess return of hedge fund index i at time t. We will use $f_{n,t}$ as notation for excess returns of ETF factor n at time t from now on, such that the model becomes:

$$r_{i,t} = \sum_{n=1}^{N} \beta_{i,n} f_{n,t} + \epsilon_{i,t} \tag{2}$$

Intercepts in the model are set to zero such that the regression procedure is forced to fit any alpha to the beta estimates. This is common for these type of studies as the goal is to mirror hedge fund returns as close as possible. Setting alpha to zero means that betas will play a larger role in fitting the hedge fund's returns. In a factor model the alpha is the average unexplained return, and these are not possible to replicate as noted by both Bollen & Fisher (2013) and O'Doherty et al. (2016). This raises the concern that betas may be somewhat biased. We therefore look at the in-sample fit of our factor model to check the presence of alpha. Corresponding to the model developed by Sharpe (1992), in-sample fit is tested as follows:

$$r_{i,t} = \alpha_i + \sum_{n=1}^{N} \beta_{i,n} f_{n,t} + \epsilon_{i,t} \tag{3}$$

4.1 Rolling Window Linear Factor Model

A rolling window is used to capture changing exposures to factors over time and to prevent look-ahead bias. Similar to Hasanhodzic & Lo (2007) we use a 24-month estimation window. New positions in the clone portfolio are established every month using the regression estimates calculated over the previous 24 months. We also incorporate a one-month lag into our estimation period such that the strategy is implementable. Hedge fund returns are reported by the Hedge Fund Research Index in the middle of the month following the one where the return is achieved. The one-month lag thus makes sure that data is available for the entire estimation period. For example, returns of December are reported in mid January, starting of which weights can be estimated. Positions can then be entered by the start of February. The model can be specified as:

$$r_{i,t-k} = \sum_{n=1}^{N} \beta_{i,n,t} f_{n,t-k} + \epsilon_{i,t-k}, \qquad k = 2, ..., 25 \tag{4}$$

where k is the 24-month estimation period including the one-month lag.

The estimated coefficients $\hat{\beta}_{i,n,t}$ can be interpreted as weights in each ETF. Negative values correspond to short positions and positive ones to going long. Following Bollen & Fisher (2013) we assume positions in every ETF, both long and short, to be fully collateralized. This means that the amount of any short position is put aside into a margin account earning the risk-free rate. Total allocation to the ETFs is then given by:

$$\gamma_{i,t} = \sum_{n=1}^{N} |\hat{\beta}_{i,n,t}| \tag{5}$$

When $\gamma_{i,t}$ exceeds 100%, we assume that the excess capital needs to be borrowed at the risk-free rate plus 100 basis points. If $\gamma_{i,t}$ is lower than 100% the remainder is invested at the risk-free rate. We use the 3-month U.S. Treasury bill return as a proxy for the risk-free rate. Monthly returns of the clones are then given by:

$$\hat{R}_{i,t} = \begin{cases} R_{f,t} + \sum_{n=1}^{N} \hat{\beta}_{i,n,t} f_{n,t} & \text{if } \gamma_{i,t} \leq 1 \\ \\ R_{f,t} + \sum_{n=1}^{N} \hat{\beta}_{i,n,t} f_{n,t} - \frac{0.01}{12}(\gamma_{i,t} - 1) & \text{if } \gamma_{i,t} > 1 \end{cases} \tag{6}$$

where $R_{f,t}$ is the risk-free rate at time t.

Finally, we impose a limit to the amount of leverage that can be used as institutional or individual investors who might seek to replicate hedge fund returns will not have access to infinite amounts of leverage, which would also impose high costs. Analogous to O'Doherty et al. (2016) we use a maximum exposure of 400%. If $\gamma_{i,t}$ exceeds this level we reduce the weights by $\frac{4}{\gamma_{i,t}}$, such that gross leverage equals 4.

Downsides of using a rolling window is that turnover might be substantial and that estimation errors may increase due to the relatively short estimation period. An alternative would be to use a fixed estimation period as examined by Hasanhodzic & Lo (2007). However, a fixed window suffers from clear look-ahead bias and may not work out-of-sample as it does not take time variation into account. Our paper will therefore only focus on a rolling window estimation.

4.2 Regression Techniques

We use several regression techniques to estimate coefficients and corresponding positions of the clone portfolio. Next to an ordinary least squares (OLS) regression we employ a stepwise regression, a ridge regression, and a LASSO regression. These are all explained further in the sections below.

4.2.1 OLS Regression

OLS regressions are widely used in hedge fund replication studies, because of its simplicity. Coefficients in an OLS regression are estimated by minimizing the sum of squared residuals:

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'Y \tag{7}$$

where Y is the dependent variable and X the explanatory variable. We perform two variations of the OLS regression based on the factors included. The OLS where all nine ETF factors are included will be simply referred to as 'OLS'. Following Bollen & Fisher (2013) we will also look at a model where only a selection of five factors are included. The selected factors are SPY, IEF, GLD, USO, and FXE. We will refer to this method as 'OLS Select'.

4.2.2 Stepwise Regression

Besides an OLS regression we also perform a stepwise regression. A stepwise regression automatically fits the model by adding and removing factors based on their significance. It compares the explanatory power of the model using F-tests, and selects the one that provides the highest value. The technique gives a local optimum, which can deviate from the global maximum. It is an interesting approach as it allows the model to remove insignificant factors, potentially improving replication results. An issue with stepwise regression is that it might cause overfitting of the data since it evaluates a broad set of models. Results generally work significantly better in-sample than out-of-sample, while hedge fund replication depends upon out-of-sample performance. Furthermore, the short estimation period of 24 months may force the model to include only a relatively small number of factors each time. Nevertheless, stepwise regressions have been used in studying hedge fund returns (e.g., see Agarwal & Narayan, 2004; O'Doherty et al., 2016).

Similar to O'Doherty et al. (2016) we use an entry significance level of 10% and a retaining significance level of 25% as parameters for the stepwise regression. The stepwise regression is performed every month, and the number of included factors may vary over time. Coefficient estimates are determined as in (7).

4.2.3 Ridge Regression

Several ETFs in our sample show high correlation with each other, which raises the concern of multicollinearity. We use a ridge regression like Chen & Tindall (2014) to correct for collinearity among factors. A ridge regression introduces a penalty term λ that acts as a shrinkage parameter to the beta coefficients. By shrinking coefficients the variance of the model is reduced and this lowers collinearity among factors. The original ridge regression model of Hoerl & Kennard (1970) estimates coefficients as follows:

$$\hat{\beta}_{ridge} = (X'X + \lambda I)^{-1}X'Y, \qquad \lambda \geq 0 \tag{8}$$

where λ is the shrinkage parameter that gives a different solution to the model for each value it takes on. The model converges towards an OLS regression when lambda moves closer to zero, and coefficients become zero when lambda reaches infinity. We select the λ that minimizes the cross-validated mean squared error of the model. Cross-validation is used to reduce the problem of overfitting data and to improve prediction power of the model. Cross-validation means that the dataset is split up into several groups, which are all being tested in separate rounds and validated against the other groups. Final results are then determined as an average across the different rounds.

4.2.4 LASSO Regression

Similar to the ridge regression, a LASSO regression incorporates a shrinkage parameter λ that reduces the coefficients. The two techniques differ because coefficients in a ridge regression have to be all zero or all non-zero, while the LASSO regression allows individual coefficients to become zero. LASSO can therefore perform variable selection (like stepwise regression), while also correcting for multicollinearity problems. This results in a non-linear optimization problem as originally proposed by Tibshirani (1996). LASSO coefficients are given by the following minimization problem:

$$\hat{\beta}_{lasso} = argmin \left\{ \frac{1}{2} \sum_{i=1}^{n} (y_i - \sum_{j=1}^{p} x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}, \qquad \lambda \geq 0 \tag{9}$$

The problem is written in Langrangian form (Hastie et al., 2008) and without intercept. We again select the λ that minimizes the cross-validated mean squared error of the model.

4.3 Performance Measures

We use several measures to assess the performance of the clones compared to the hedge fund indices. First, we look at tracking error to see how large the deviations between returns of the clone and the target are. Tracking error is measured by the root mean squared error of the clone returns and the hedge fund index returns, which is defined as:

$$TE_i = \sqrt{\frac{1}{(T-25)} \sum_{t=26}^{T} (\hat{R}_{i,t} - R_{i,t})^2} \tag{10}$$

where T is the total number of months, $\hat{R}_{i,t}$ the return of the clone, and $R_{i,t}$ the return of the hedge fund index.

To check whether the clone is under- or outperforming the hedge fund index, we compare average returns and look at the geometric average excess return (AER), which is measured as:

$$AER_i = \left[\prod_{t=26}^{T} (1 + \hat{R}_{i,t} - R_{i,t}) \right]^{\frac{1}{(T-25)}} - 1 \tag{11}$$

where a negative AER indicates that the clone underperforms the respective hedge fund index.

Finally, we will look at the out-of-sample correlation and model fit of the clones with respect to the hedge fund indices. Out-of-sample fit is measured using the hedge fund index return as the dependent variable and the clone return as an explanatory variable, which can be written as:

$$R_{i,t} = \alpha_i + \beta_i \hat{R}_{i,t} + \epsilon_{i,t} \tag{12}$$

where an α of zero and a β of one would indicate that the variation in hedge fund index returns can be fully explained by the variation in clone returns, which would clearly be the ideal case for a replication strategy.

5 Results

5.1 In-Sample Model Fit

We examine the in-sample fit of the replication model to see how well the returns of the hedge fund indices can be captured by the ETF factors over the entire sample period. The in-sample period extends from May 2006 to December 2016. Results can be found in Table 5. Focusing on the OLS regression in Panel A, we find a high adjusted R-squared of 0.86 for the broad hedge funds index. A large part of the variation in hedge fund returns is thus explained by the variation in the nine ETF factors. The strategy indices equity hedge, event driven, emerging markets, and short bias also show relatively high adjusted R-squared values exceeding 0.70. The hedge fund indices least explained by the factors in terms of adjusted R-squared are the macro index at 0.22, and the market neutral index at 0.41. Panels B to E show comparable levels of adjusted R-squared for the other regression techniques although they are, on average, slightly lower than those of the OLS regressions. For example, the OLS select model for the hedge funds index has an adjusted R-squared that is 0.08 lower than that of the OLS model. This is not surprising given that the OLS select model only incorporates five factors instead of nine. Furthermore, the stepwise, ridge, and LASSO regressions show slightly lower adjusted R-squared values as a result of the variable selection and shrinkage of coefficients.

The OLS regression of the hedge funds index and the equity hedge index (Panel A) has significant coefficients for all factors at a 5% significance level. The remaining indices show different results with respect to significance of factors. For example, the macro and market neutral indices show significance to only three factors. There might be missing factors that explain returns of these strategies, which can also be concluded from the low adjusted R-squared. The emerging markets index shows the highest significance for EEM, which makes economic sense as EEM tracks an emerging markets equity index. Also the negative coefficient of the short bias index with respect to SPY is as expected. SPY follows the S&P 500, while short bias funds try to be on the other side of the market and hold a net short position. The convertible arbitrage index shows significant coefficients for the bond ETFs (i.e. IEF and AGG). This is not surprising as convertible arbitrage funds try to profit from the spread between convertible fixed income instruments and related securities. On average, the coefficients are the highest for the equity and bond ETFs (i.e. SPY, IEF, AGG, EEM, and EFA), and lowest for GLD, USO, and IYR. Betas are mostly positive across the factors except for IEF, FXE, and IYR.

Examining the OLS select regression model in Panel B we find that removing four factors significantly affects coefficients. For example, all betas with respect to FXE have become insignificant, and coefficients of SPY are considerably higher than before. This can be explained by the fact that two of the removed factors concern equity ETFs (i.e. EEM and EFA), that have a high correlation with SPY.

Looking at the stepwise, ridge, and LASSO regressions in Panels C, D, and E we observe similar results regarding coefficients compared to the OLS regression. However, it becomes clear that the stepwise and LASSO regressions perform variable selection. For example, the stepwise regression drops six out of the nine factors for the emerging markets index while LASSO drops five of them. We also see the effect of shrinkage of coefficients in the ridge and LASSO regressions as betas are on average smaller than in the OLS case. Panel D and E show no levels of significance for the coefficients as these are not well defined for ridge and LASSO regressions. No well accepted method for testing significance has been presented yet and regular p-values are not available (Lockhart et al., 2014).

Alpha shows the part of the hedge fund returns that is not explained by the factors. We see that the alphas of the OLS model are insignificant at a 5% level except for the event driven, relative value, and short bias indices. The average alpha of all the indices is 0.09%, with 0.22% being the highest for the event driven index, and -0.28% being the lowest for the short bias index. As noted earlier these alphas cannot be replicated. We find similar alphas for the other four regression techniques.

The high explanatory power measured by adjusted R-squared and the modest levels of alpha, provide a good basis for replicating returns. Macro and market neutral funds seem harder to replicate given their low levels of adjusted R-squared. In-sample fit does not warrant out-of-sample fit, however. This will be the focus of the next sections.

Table 5: In-sample model fit

	α (%)	β_{SPY}	β_{IEF}	β_{GLD}	β_{USO}	β_{FXE}	β_{AGG}	β_{IYR}	β_{EEM}	β_{EFA}	R^2
Panel A: OLS Regression											
Hedge Funds	0.11	0.15***	−0.30***	0.06***	0.02*	−0.13***	0.36**	−0.05***	0.07***	0.13**	0.86
Equity Hedge	0.05	0.25***	−0.40***	0.06***	0.02*	−0.12**	0.39*	−0.07***	0.11***	0.13**	0.90
Event Driven	0.22*	0.17**	−0.36***	0.04*	0.04**	−0.12**	0.34	−0.03	0.03	0.12*	0.78
Macro	0.09	−0.01	−0.02	0.10***	−0.01	−0.11	0.02	−0.06*	0.03	0.14*	0.22
Relative Value	0.29***	0.06	−0.39***	0.03	0.04***	−0.15***	0.67***	−0.02	0.04	0.09	0.66
Emerging Markets	0.10	0.04	−0.32*	0.05	0.04*	−0.14*	0.43	−0.09**	0.33***	0.17*	0.84
Market Neutral	0.10	0.08*	−0.08	0.01	0.00	−0.03	−0.29*	−0.03*	0.01	0.05	0.41
Short Bias	−0.28*	−0.40***	0.26	0.01	0.02	0.05	0.03	−0.07*	−0.06	−0.07	0.79
Convertible Arb.	0.15	0.13	−0.89***	0.06	0.05*	−0.18*	1.58***	−0.06	0.08	0.10	0.58
Multi Strategy	0.16	0.08	−0.40***	0.04	0.03*	−0.13**	0.67**	−0.02	0.01	0.11	0.57
Fund of Funds	−0.02	0.12*	−0.23*	0.05**	0.02	−0.17***	0.27	−0.08***	0.03	0.17**	0.68
Panel B: OLS Select Regression											
Hedge Funds	0.10	0.30***	−0.13**	0.07***	0.03**	−0.01					0.78
Equity Hedge	0.03	0.43***	−0.23***	0.08***	0.04**	0.02					0.84
Event Driven	0.20*	0.30***	−0.19***	0.04*	0.04***	−0.03					0.75
Macro	0.07	0.07*	0.02	0.10***	0.00	−0.02					0.18
Relative Value	0.29**	0.21***	−0.06	0.04*	0.04**	−0.05					0.57
Emerging Markets	0.02	0.46***	−0.14	0.11**	0.08**	0.13					0.68
Market Neutral	0.09	0.08***	−0.08*	0.01	0.01	−0.01					0.33
Short Bias	−0.20	−0.63***	0.23**	−0.01	0.03	−0.01					0.77
Convertible Arb.	0.19	0.34***	−0.12	0.09*	0.04	−0.02					0.47
Multi Strategy	0.16	0.21***	−0.07	0.05*	0.02	−0.03					0.49
Fund of Funds	−0.03	0.23***	−0.12*	0.05**	0.03*	−0.05					0.59
Panel C: Stepwise Regression											
Hedge Funds	0.11	0.15***	−0.30***	0.06***	0.02*	−0.13***	0.36**	−0.05***	0.07***	0.13**	0.86
Equity Hedge	0.05	0.25***	−0.40***	0.06***	0.02*	−0.12**	0.39*	−0.07***	0.11***	0.13**	0.90
Event Driven	0.24**	0.14**	−0.34***	0.04*	0.04***	−0.12**	0.27			0.15**	0.78
Macro	0.10			0.11***		−0.10*		−0.05*		0.15***	0.25
Relative Value	0.35***				0.04***	−0.14***			0.06*	0.15***	0.62
Emerging Markets	0.17		−0.12		0.05**				0.40***		0.83
Market Neutral	0.09	0.11***					−0.16**	−0.04**	0.03*		0.41
Short Bias	−0.26	−0.49***	0.25***					−0.09**			0.79
Convertible Arb.	0.25	0.17**			0.04				0.14**		0.50
Multi Strategy	0.21*			0.04*	0.03**	−0.15***				0.22***	0.53
Fund of Funds	0.00	0.11*	−0.11*	0.06***	0.02	−0.16***		−0.07***		0.22***	0.67
Panel D: Ridge Regression											
Hedge Funds	0.14	0.13	−0.18	0.05	0.02	−0.08	0.14	−0.02	0.08	0.10	0.83
Equity Hedge	0.10	0.20	−0.26	0.05	0.03	−0.07	0.11	−0.03	0.11	0.12	0.87
Event Driven	0.25	0.14	−0.23	0.03	0.03	−0.08	0.10	0.00	0.05	0.10	0.73
Macro	0.09	0.02	0.02	0.08	0.00	−0.04	0.05	−0.03	0.03	0.05	0.14
Relative Value	0.31	0.06	−0.21	0.02	0.03	−0.09	0.32	0.00	0.04	0.07	0.55
Emerging Markets	0.11	0.09	−0.21	0.05	0.05	−0.06	0.21	−0.05	0.25	0.15	0.80
Market Neutral	0.11	0.06	0.00	0.00	0.01	−0.02	−0.17	−0.02	0.02	0.03	0.29
Short Bias	−0.35	−0.28	0.22	0.02	0.02	0.04	0.10	−0.09	−0.07	−0.11	0.77
Convertible Arb.	0.21	0.11	−0.50	0.05	0.04	−0.13	0.84	−0.01	0.08	0.10	0.42
Multi Strategy	0.19	0.07	−0.18	0.03	0.02	−0.07	0.26	0.00	0.03	0.07	0.45
Fund of Funds	0.00	0.11	−0.16	0.04	0.02	−0.11	0.11	−0.05	0.05	0.11	0.61
Panel E: LASSO Regression											
Hedge Funds	0.11	0.15	−0.29	0.06	0.02	−0.12	0.34	−0.05	0.07	0.13	0.82
Equity Hedge	0.05	0.25	−0.38	0.06	0.02	−0.12	0.36	−0.07	0.11	0.13	0.88
Event Driven	0.25	0.16	−0.17	0.02	0.03	−0.04			0.04	0.08	0.73
Macro	0.09		0.01	0.10	0.00	−0.07	0.02	−0.04	0.02	0.10	0.13
Relative Value	0.31	0.04	−0.29	0.02	0.04	−0.12	0.47		0.04	0.08	0.50
Emerging Markets	0.14		−0.04		0.04				0.34	0.06	0.81
Market Neutral	0.10	0.06			0.00		−0.13		0.01	0.01	0.23
Short Bias	−0.28	−0.41	0.26	0.01	0.02	0.03		−0.07	−0.05	−0.05	0.77
Convertible Arb.	0.16	0.13	−0.85	0.06	0.05	−0.17	1.50	−0.05	0.08	0.09	0.38
Multi Strategy	0.20	0.08			0.01				0.03	0.06	0.39
Fund of Funds	0.01	0.10	−0.10	0.04	0.02	−0.12		−0.04	0.04	0.14	0.59

This table shows the in-sample fit of all regression models over the period May 2006 to December 2016. The ETF factors are the explanatory variables and the hedge fund index is the dependent variable. R^2 shows the adjusted R-squared of the model. Stars show the level of significance, with (*); (**); (***) being significant at a 5%, 1%, and 0.1% level respectively. For the shrinkage regressions ridge and LASSO the level of significance is not shown as this is not well defined, see text for further explanation.

5.2 Clone Performance

5.2.1 Tracking Accuracy

To assess the accuracy of which clones track the hedge fund indices we analyse monthly out-of-sample tracking errors and correlations, as shown in Table 6. The out-of-sample period extends from July 2008 to December 2016. When we look at the broad hedge funds index we observe that the correlation between the clones and the index ranges from 0.85 for the LASSO regression to 0.89 for the ridge regression. The various replication techniques thus produce clones that closely follow the movements of the hedge funds index. This strong linear relationship is also observed for the equity hedge, event driven, emerging markets, short bias, and fund of funds indices, which all show a correlation with clones that exceed 0.70. As expected from looking at the in-sample model fit, the macro and market neutral indices are more difficult to replicate with an out-of-sample correlation of just 0.32 and 0.56 to the OLS clone respectively. Other indices that show rather low correlation with the clones are relative value, convertible arbitrage, and multi strategy. However, correlations of these strategies are still above 0.40, which means that part of their movements are captured by the clones.

Correlations do not provide a full picture of performance as they only show to what extent returns of the clones and the indices move in line with each other. We therefore look at tracking errors to see how clones actually deviate from the target. Minimum tracking error for the broad hedge funds index is given by the ridge clone at 0.83%. Clones based on the other regression techniques only show slightly higher tracking errors. Since tracking error is defined as the standard deviation of the difference in the clone return and the index return, we can say that the return of the clone will fall within an interval of 0.83% above or below the hedge funds index return about 68% of the time. This is quite a substantial deviation given that we are looking at monthly tracking errors.

The strategy indices show tracking errors exceeding 1% except for the market neutral index. The highest tracking error is observed for the convertible arbitrage strategy. This is not surprising given the low in-sample fit and low out-of-sample correlation of this index. More remarkable are the relatively high tracking errors of the emerging markets and short bias indices, as these indices show a high in-sample fit and a high out-of-sample correlation with the clone.

Tracking error does not capture whether the clone under- or outperforms the index, but only to what extent its return deviates in absolute terms. Further analysis on actual returns will be provided in the next section.

Table 6: Monthly out-of-sample tracking error and correlation

	OLS		**OLS Select**		**Stepwise**		**Ridge**		**LASSO**	
	TE (%)	**Corr.**	**TE (%)**	**Corr.**	**TE (%)**	**Corr.**	**TE (%)**	**Corr.**	**TE (%)**	**Corr.**
Hedge Funds	0.92	0.87	0.91	0.88	0.94	0.86	0.83	0.89	0.95	0.85
Equity Hedge	1.13	0.90	1.19	0.90	1.23	0.88	1.07	0.92	1.18	0.89
Event Driven	1.30	0.76	1.14	0.82	1.39	0.74	1.18	0.81	1.29	0.75
Macro	1.66	0.32	1.52	0.31	1.41	0.33	1.37	0.29	1.44	0.22
Relative Value	1.33	0.56	1.26	0.62	1.38	0.52	1.20	0.61	1.21	0.60
Emerging Markets	1.85	0.85	2.15	0.81	2.02	0.82	1.62	0.89	1.72	0.87
Market Neutral	0.72	0.56	0.70	0.54	0.73	0.48	0.65	0.58	0.71	0.50
Short Bias	1.91	0.83	1.69	0.86	1.78	0.85	1.70	0.85	1.68	0.86
Convertible Arb.	2.71	0.41	2.72	0.45	2.85	0.42	2.47	0.49	2.51	0.48
Multi Strategy	1.51	0.48	1.41	0.53	1.45	0.48	1.31	0.54	1.33	0.53
Fund of Funds	1.15	0.73	1.04	0.78	1.10	0.72	0.96	0.77	0.99	0.76

This table shows monthly out-of-sample tracking error and correlation between the clone returns and the hedge fund index returns. The clones are constructed using a 24-month rolling window and five different regression techniques over the period July 2008 to December 2016.

5.2.2 Average Returns

Investors are obviously interested in the return and volatility that the clone portfolios produce compared to the hedge fund benchmarks. Average monthly returns and standard deviations of the clones and their target can be found in Table 7. We observe an average monthly return of 0.23% for the hedge funds index compared to a return of just 0.11% for the OLS clones. Stepwise does a better job of capturing the hedge funds index return with an average of 0.22%. Ridge underperforms the broad hedge funds index most severely with an average monthly return of only 0.05%.

The other indices show comparable results with clones that are generally underperforming. For example, clones of the event driven index show a significant lower average return. The event driven index has an average monthly return of 0.33% while the clones show average returns ranging from 0.07% for OLS to 0.18% for OLS select. This is surprising given that this index shows a relatively good in-sample fit and a high out-of-sample correlation with the clones. We find that clones of the five strategy indices that showed poor replication performance earlier, are also underperforming. The macro, relative value, market neutral, convertible arbitrage, and multi strategy indices show considerably higher average returns than the clones. For example, the macro index shows an average return of 0.12% while the clones have an average return ranging from -0.04% to 0.05%. Stepwise clones generally show the highest average returns among the different regression techniques, and are closest to the average return of the target. They also provide superior returns compared to the equity hedge, emerging markets, short bias, and fund of funds indices.

Monthly standard deviations of the clones are close to those of the hedge fund indices. The hedge funds index has a volatility of 1.82% compared to 1.80% for the OLS clone. On average the clones show lower standard deviations than the indices. There also seems to be variation in the level of volatility among the regression techniques. OLS and OLS select, on average, show the highest standard deviations while ridge and LASSO show the lowest. For example, for the broad hedge funds index we find a monthly standard deviation of 1.80% for the OLS clone compared to 1.46% for ridge. Given that ridge and LASSO regressions shrink coefficients they inherently put less weight in the risky assets, which leads to lower volatility.

Table 7: Monthly out-of-sample returns and standard deviations

	Index		**OLS**		**OLS Select**		**Stepwise**		**Ridge**		**LASSO**	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Hedge Funds	0.23	1.82	0.11	1.80	0.10	1.91	0.22	1.69	0.05	1.46	0.14	1.51
Equity Hedge	0.20	2.57	0.11	2.47	0.13	2.63	0.26	2.34	0.06	2.00	0.15	2.09
Event Driven	0.33	1.97	0.07	1.80	0.18	1.82	0.14	1.88	0.08	1.34	0.11	1.43
Macro	0.12	1.30	0.01	1.53	-0.04	1.28	0.05	1.11	-0.01	0.95	-0.04	0.98
Relative Value	0.42	1.51	0.18	1.30	0.19	1.36	0.15	1.26	0.11	1.03	0.07	1.04
Emerging Markets	0.02	3.46	0.03	3.30	0.00	3.45	0.13	3.26	0.06	2.79	0.05	2.81
Market Neutral	0.13	0.80	0.04	0.72	0.08	0.62	0.10	0.57	0.03	0.47	0.07	0.54
Short Bias	-0.68	3.25	-0.62	3.34	-0.51	3.10	-0.49	3.34	-0.14	2.57	-0.27	2.60
Convertible Arb.	0.43	2.80	0.14	2.06	0.30	2.34	0.32	2.46	0.21	1.70	0.20	1.86
Multi Strategy	0.36	1.55	0.20	1.40	0.20	1.33	0.10	1.26	0.14	0.98	0.12	1.01
Fund of Funds	0.05	1.51	0.04	1.62	0.05	1.59	0.20	1.44	0.01	1.22	0.02	1.30

This table shows monthly out-of-sample average returns and standard deviations of the clone returns and the hedge fund index returns. The clones are constructed using a 24-month rolling window and five different regression techniques over the period July 2008 to December 2016. Means are calculated using a geometric average. All values are shown in %.

To visualize returns of the clones and the indices we plot cumulative returns of the broad hedge funds index and the fund of funds index in Figure 1. We specifically focus on cumulative returns of these two indices as the hedge funds index provides the broadest picture of hedge fund performance, and the fund of funds index mitigates some of the data biases as discussed before. We also exclude the OLS clones from the plots as they show inferior out-of-sample performance and suffer from substantial multicollinearity problems. Plots for all other indices can be found in Appendix D.

The cumulative return plots show the value of one dollar invested over the out-of-sample period July 2008 to December 2016. It clearly shows the underperformance of the clones to the hedge funds index, although stepwise mimics the index closely. Deviations of the other regression techniques are noticeably larger. If we focus on the fund of funds index we see that the OLS select, ridge, and LASSO clones again underperform the index, although by a smaller margin. The stepwise clone shows a different pattern in which it outperforms the fund of funds index over the entire period. The cumulative return plots for all other indices (Appendix D), confirm the tendency of clones to underperform the underlying index, except for the emerging markets and short bias strategies.

Underperformance of the hedge fund clones is similar to findings of other literature (e.g., see Hasanhodzic & Lo, 2007; Amenc et al., 2008, 2010; Bollen & Fisher, 2013; O'Doherty et al., 2016). There are several reasons why clones may underperform the index. First of all, any alpha that is present in the model cannot be replicated as pointed out by Bollen & Fisher (2013). This means that the model is missing certain risk factors that can explain part of the hedge fund returns, or that the fund displays managerial skill. Skill is hard to distinguish from luck, however. Furthermore, the indices of Hedge Fund Research are non-investable, which creates an upward bias due to backfill and survivorship biases (e.g., see Jaeger & Wagner, 2005; Jurek & Stafford, 2015). This puts the clones at a disadvantage, because they are investable and do not suffer from these biases. Finally, hedge funds typically load up on illiquid assets to earn the liquidity premium. Clones, however, consist of highly liquid ETFs that do not earn the same liquidity premium. We take a closer look at liquidity in a later section.

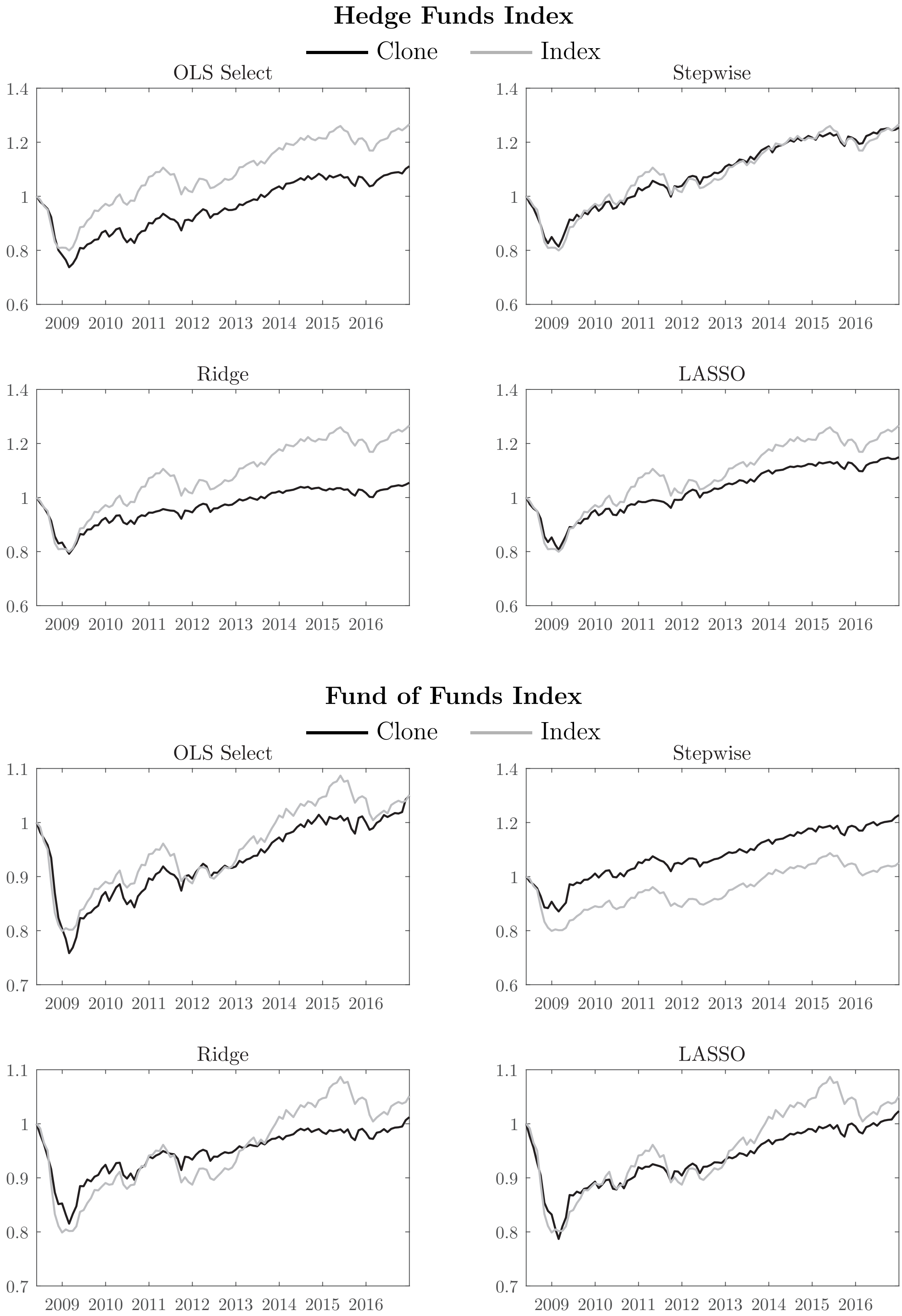

Figure 1: Cumulative returns of hedge funds index and fund of funds index

This figure shows cumulative returns of the clones and the hedge fund indices over the period July 2008 to December 2016. It shows the value of one dollar invested over time for the hedge funds index and the fund of funds index. The OLS regression is excluded.

To see how loss patterns of the clones compare to those of the hedge fund indices we look at maximum drawdown. Drawdown is the observed loss of a portfolio from a peak to a trough, and can be an indicator of risk. Maximum drawdown of the indices and the clones closely follow each other for most strategies, as can be seen in Table C1 in Appendix C. The maximum drawdown of the hedge funds index is about 20% compared to 22% for the OLS clone. The different regression techniques do not show large variations in maximum drawdown. Looking at the strategy indices we observe the highest drawdown for the short bias index at 64%, which also shows the largest deviation of drawdown by the clones. The lowest maximum drawdown is observed for the macro index at 8%. Except for the short bias index these maximum drawdowns are a result of the financial crisis of 2007-2008.

5.2.3 Out-of-Sample Model Fit

We test the out-of-sample model fit of the different replication methods by running an OLS regression of the hedge fund indices on the respective clones. Contrary to the replication regressions, we include an intercept to assess whether the hedge fund indices show any alpha that is not explained by the clone returns. We correct standard errors for heteroscedasticity and autocorrelation using methodology of Newey & West (1987). Results can be found in Table 8.

Looking at the adjusted R-squared of the different models we observe that the explanatory power of the clones is especially strong for the broad hedge funds, equity hedge, and the emerging markets indices. Levels of R-squared are exceeding 0.70 for these models. This is consistent with the relatively good out-of-sample performance and in-sample fit of these indices. The macro, market neutral, convertible arbitrage, and multi strategy indices show the lowest levels of adjusted R-squared, at 0.30 and below. No large differences are observed when comparing the various regression techniques, although the ridge model on average seems to provide a slightly higher explanatory power.

Betas are lower than one for all indices using the OLS, OLS select, and stepwise regression techniques. If clones would perfectly replicate the hedge fund indices, betas would equal one such that the systematic risk of the clone would be exactly equal to that of the respective hedge fund index. The clones tend to be less volatile than the hedge fund indices for the OLS, OLS select and stepwise models, however. The ridge and LASSO regressions show betas higher than one for certain strategies. The strategies that show the lowest levels of adjusted R-squared also show the highest deviation from a beta of one. Overall, the betas are highly significant across the different hedge fund indices and regressions. The clones are thus able to capture a significant part of the returns of the hedge fund indices.

Finally, we observe that alphas are mainly positive across the regression models. This implies that there is an unexplained part of the hedge fund index returns that cannot be captured by the clones. Given that alphas are mostly positive it indicates that clones generally underperform the indices. This conclusion is similar as the one drawn from the monthly out-of-sample average returns. Negative alphas are observed for the short bias index. There is some variation in alphas when looking at the different regression techniques. The stepwise model shows noticeably lower alphas for certain strategies. For example, it shows negative alphas for the equity hedge, emerging markets, and fund of funds indices. This is in line with earlier findings of outperformance by the stepwise clones for these indices. Furthermore, alphas are higher for the ridge and LASSO clones, which indicates more severe underperformance for these methods. Alphas are not significant except for one.

Table 8: Out-of-sample model fit

	OLS			**OLS Select**			**Stepwise**			**Ridge**			**LASSO**		
	α (%)	β	R^2	α (%)	β	R^2	α (%)	β	R^2	α (%)	β	R^2	α (%)	β	R^2
Hedge Funds	0.13	0.88***	0.76	0.15	0.84***	0.77	0.03	0.93***	0.73	0.18	1.11***	0.79	0.10	1.03***	0.73
Equity Hedge	0.10	0.94***	0.81	0.09	0.88***	0.80	−0.05	0.97***	0.77	0.13	1.18***	0.85	0.04	1.10***	0.79
Event Driven	0.29	0.83***	0.58	0.18	0.89***	0.67	0.23	0.77***	0.54	0.24	1.19***	0.65	0.23	1.04***	0.57
Macro	0.12	0.27*	0.09	0.14	0.31	0.08	0.11	0.38*	0.10	0.13	0.40*	0.08	0.14	0.30	0.04
Relative Value	0.31	0.65***	0.31	0.29	0.69***	0.38	0.33	0.62**	0.26	0.33	0.90**	0.37	0.36*	0.88**	0.36
Emerging Markets	0.00	0.89***	0.72	0.03	0.81***	0.65	−0.07	0.87***	0.67	−0.03	1.10***	0.78	−0.01	1.07***	0.75
Market Neutral	0.10	0.61***	0.30	0.08	0.70***	0.29	0.06	0.67***	0.22	0.10	0.99***	0.33	0.08	0.74**	0.24
Short Bias	−0.16	0.81***	0.69	−0.21	0.90***	0.74	−0.26	0.83***	0.73	−0.51	1.08***	0.73	−0.36	1.08***	0.74
Convertible Arb.	0.39	0.56	0.16	0.30	0.54*	0.20	0.31	0.48	0.17	0.30	0.80*	0.23	0.32	0.73*	0.23
Multi Strategy	0.25	0.53*	0.23	0.24	0.62**	0.27	0.30	0.59*	0.22	0.24	0.85**	0.29	0.27	0.81**	0.27
Fund of Funds	0.02	0.68***	0.53	0.02	0.74***	0.60	−0.10	0.76***	0.52	0.04	0.96***	0.59	0.03	0.88***	0.57

This table shows the out-of-sample fit of all regression models. The monthly clone return is the explanatory variable and the hedge fund index return is the dependent variable. R^2 shows the adjusted R-squared. The clones are constructed using a 24-month rolling window and five different regression techniques over the period July 2008 to December 2016. Standard errors are adjusted for heteroscedasticity and autocorrelation using Newey & West (1987) methodology with a lag of 4. Stars show the level of significance, with (*); (**); (***) being significant at a 5%, 1%, and 0.1% level respectively.

5.2.4 Portfolio Turnover

Implementing the hedge fund replication strategy involves adjusting positions on a monthly basis, which might lead to high transaction costs. We therefore examine differences in turnover between the regression techniques. Average annual portfolio turnover can be found in Figure 2. We clearly see that the OLS clones have the highest annual turnover across all indices. At its peak it shows an average annual turnover of over 400% for the emerging markets index. This is not surprising as the OLS model consists of all nine ETF factors, and does not perform any kind of econometric variable selection or shrinkage. The other regression techniques consistently show lower levels of turnover. OLS select has the lowest average turnover as this model only includes five factors. The other three regression techniques (i.e. stepwise, ridge, and LASSO) show more similar results, although on average the ridge clones shows the lowest turnover of the three.

When comparing the different strategies we observe the highest turnover for the emerging markets index, and the lowest for the market neutral index. This is somewhat remarkable as the market neutral strategy attempts to be indifferent to overall market moves, which generally requires frequent rebalancing. The broad hedge funds index shows an average annual turnover of about 100% across the different regression methods. This corresponds to an average holding period of one year, and means that the entire portfolio will on average change once during a year.

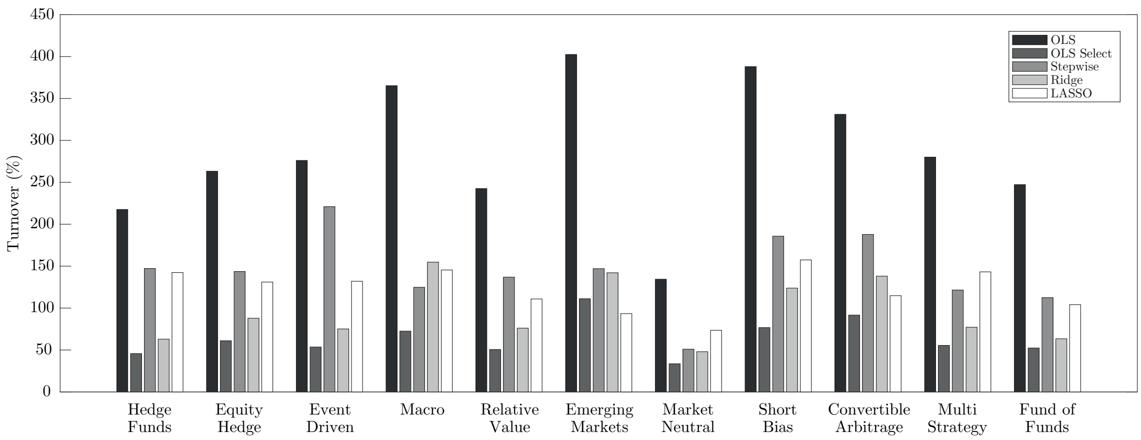

Figure 2: Average annual portfolio turnover

This figure shows the average annual turnover of the clone portfolio over the period July 2008 to December 2016 for all regression methods and indices. All values are shown in %.

5.2.5 Leverage

Another aspect that might lead to high costs of the clone portfolio is the leverage used in the replication process. A replication strategy might require the use of excessive leverage, which would make it no longer viable to implement. We attempt to mitigate these problems by limiting the amount of gross leverage to a maximum of 400%, take costs of leverage into account, and fully collateralize short positions as discussed under methodology. Table C2 in Appendix C displays the mean, minimum, and maximum amount of gross leverage used in the clone portfolios. The OLS clones employ leverage for all indices, with the exception of the market neutral strategy. This is in contrast with the other four regression models that on average invest a large part of the portfolio into the risk-free asset.

Average portfolio weights for the ETF factors are shown in Table C3, and clearly show why the OLS regression generally employs leverage compared to the other regression techniques. OLS has a relatively large factor loading with respect to AGG, which is a factor that is not included in the OLS select model and does not receive equally high weights in the other regression models. The high weights of AGG in the OLS portfolio are combined with high negative weights for IEF. This might be the result of multicollinearity among factors, because the two bond ETFs show a high correlation of 0.83 with each other. It highlights the weakness of the OLS clones, and shows why the other regression techniques are more preferable as they address the issue of multicollinearity.

Overall, the clone portfolios is heavily weighted in SPY, which makes sense given the relatively high correlation of the hedge fund indices with the S&P 500. Furthermore, it is worth noting that the econometric factor selection regressions (i.e. stepwise and LASSO) show average weights for all factors, even though they might not invest in every one of them each month. The average factor weights for these two regression techniques and the ridge regression are therefore lower in comparison to the OLS and OLS select regressions, which do not exclude or shrink coefficients. Average portfolio weights do not tell us anything about time variation in the composition of the portfolio, since weights may differ significantly from month to month.

5.2.6 Liquidity

Hedge funds generally have lockup periods that prevent investors from accessing their capital for a certain period of time. This makes hedge funds illiquid as an asset class, and is generally the result of investing in illiquid assets. Hedge fund strategies often involve buying illiquid assets and shorting liquid ones to capture the liquidity premium. For example, a specific relative value strategy focuses on buying off-the-run treasuries and shorting on-the-run treasuries, both having equal maturities. These securities should have an equal payoff, but differences occur due to liquidity. The illiquidity risk that hedge funds are exposed to is not shared by the hedge fund clones as these are composed of highly liquid securities (i.e. ETFs). Part of the difference between the hedge fund returns and the clone returns may thus be explained by differences in liquidity.

Lo (2001) and Getmansky et al. (2004) find that positive values of first-order autocorrelation can serve as a proxy for illiquidity risk. First-order autocorrelation is the correlation between the current month's return and the previous month's return. The authors argue that predictability in returns would normally be arbitraged away under the Efficient Markets Hypothesis (Samuelson, 1965), but that market frictions of illiquid assets, such as high transaction costs, may lead to significant autocorrelation. A second reason provided is that hedge fund managers may try to smooth returns of the fund by investing in illiquid asets. Managers have more discretion to mark the portfolio's value when assets are illiquid, as market prices are then not well established. Table 9 shows the first-order autocorrelation for the hedge fund indices and the clones. It also shows statistical significance of the autocorrelation tested with the Ljung & Box (1978) Q-statistic.

Autocorrelation is systematically higher for the indices compared to the clones. This confirms that the hedge fund indices are exposed to higher illiquidity risk than the clones, which is in line with findings of Hasanhodzic & Lo (2007). The hedge funds index has a first-order autocorrelation of 0.38 compared to 0.14 for the OLS clone. OLS select shows the highest average first-order autocorrelation among the different regression techniques, while stepwise shows the lowest. Autocorrelations are highly significant for the hedge fund indices except for the macro and short bias strategies. The clones show lower levels of significance, especially for the stepwise, ridge and LASSO regressions.

The highest autocorrelation is given by the relative value and convertible arbitrage indices at a level of 0.59 for both. This is not surprising as these strategies generally involve buying illiquid securities and shorting liquid ones to earn the existing spread. For example, the spread between off-the-run and on-the-run treasuries as mentioned before. The macro index shows the lowest autocorrelation at -0.12.

The hedge fund indices that show the highest autocorrelation also have the strongest underperforming clones (i.e. the event driven, relative value, convertible arbitrage, and multi strategy indices). The liquidity premium thus seems to explain part of the deviation in returns between the clone and the hedge fund index. This is similar to the conclusion of Dor et al. (2012), who find that liquidity is an important driver of the underperformance of clones.

Table 9: First-order autocorrelation of clones and hedge fund indices

	Index	OLS	OLS Select	Stepwise	Ridge	LASSO
Hedge Funds	0.38***	0.14*	0.29*	0.10	0.26	0.19
Equity Hedge	0.33***	0.17	0.29	0.05	0.21	0.20
Event Driven	0.48***	0.21	0.29	0.09	0.18	0.14
Macro	−0.12	−0.10**	−0.05*	−0.11	−0.10	−0.15
Relative Value	0.59***	0.18***	0.28**	0.14	0.15*	0.23***
Emerging Markets	0.41***	0.10	0.32***	0.04	0.16	0.13
Market Neutral	0.16***	−0.11***	0.14*	−0.21**	−0.08***	−0.26*
Short Bias	0.13	0.20	0.18	0.20	0.16	0.22*
Convertible Arb.	0.59***	0.12**	0.19*	−0.13**	−0.05***	−0.03*
Multi Strategy	0.58***	0.18***	0.25***	0.00	0.05	0.12**
Fund of Funds	0.45***	0.11*	0.36***	0.06	0.26	0.32

This table shows the first-order autocorrelation for all clones and indices over the period July 2008 to December 2016. Stars show the level of significance, with (*); (**); (***) being significant at a 5%, 1%, and 0.1% level respectively. Significance is tested using the Ljung & Box (1978) Q-statistic with a lag of 6.

5.3 Robustness

Hedge fund indices have shown lower returns and higher correlations with the overall equity market in more recent years as discussed under data description. Moreover, Bollen & Fisher (2013) find significant differences between hedge fund replication before and after the financial crisis. Changing market conditions thus seem to impact hedge fund replication, which underlines the importance of a backtest. Our out-of-sample backtest period extends from March 1994 to December 2016. This allows us to look at part of the 1990s during which hedge funds performed especially well, at the impact of the dotcom bubble that bursted in 2000, and at the period leading up to the financial crisis of 2007-2008.

5.3.1 Backtest Performance

We start by looking at the same performance measures as used for the shorter sample period. The tables can be found in Appendix C. Tracking errors over the longer sample period are noticeably higher (Table C7). For example, the OLS clone shows a monthly tracking error of 1.38% for the hedge funds index, compared to 0.92% during the shorter period. The other regression techniques show lower tracking errors compared to OLS, although also being considerably higher than during the shorter time period. Differences in tracking errors between the OLS and the other regression methods are more pronounced during the backtest period, which indicates that a standard OLS is not the optimal way to replicate hedge fund returns.

Strategy indices also show higher tracking errors on average, with the highest for the OLS clone of the short bias index at 4.12%. This is more than double compared to the shorter time frame. Clones thus seem to track their respective indices less accurately over the longer period.

This is further confirmed when looking at the out-of-sample correlation between the hedge fund indices and the clones, which are lower over the backtest period. The hedge funds index dropped from a correlation of 0.87 with the OLS clone over the shorter time period to 0.74 over the backtest period. Correlations do not differ widely across the various regression techniques although the alternative regression methods show higher correlation coefficients on average. For example, the stepwise and ridge clones show a correlation of 0.79 and 0.82 with the hedge funds index respectively. This is slightly below the 0.86 and 0.89 correlation of stepwise and ridge clones during the shorter time frame.

Other indices also show lower correlation over the longer sample period on average. The macro, relative value, market neutral, convertible arbitrage, and multi strategy indices still have the lowest correlation with the clones. Thus over a longer period these strategies still seem the most difficult to replicate. Interesting, however, is that the correlation for the macro index went up from about 0.29 for the ridge clones over the shorter time period to 0.54 during the backtest period.

The higher tracking errors of the hedge fund clones during the backtest period become more apparent when we look at actual returns (Table C8). Average out-of-sample returns of the clones are well below the hedge fund indices over the longer period. The stepwise clone of the hedge funds index shows an average monthly return of 0.26% compared to 0.63% for the hedge funds index. This is a considerable difference given that the stepwise clone is most accurate in replicating returns over the shorter time period. We take a closer look at returns across strategies and regression techniques in the next section, where we compare average excess returns.

Standard deviations of the clone and hedge fund index returns are again similar during the backtest period. The lowest standard deviations are shown by the ridge and LASSO regressions, while stepwise is positioned in the middle. This is equal to the observations drawn from the shorter time frame.

Finally, out-of-sample model fit of the backtest also confirms that clones are performing less during the backtest period compared to the more recent sample (Table C9). Alphas have increased significantly across all strategies and regression methods. For example, stepwise clones show an alpha of 0.03% with the hedge funds index during the short time period, while this has increased to 0.42% over the backtest period. Betas, on average, also deviate further from one over the backtest period. A larger part of the hedge fund returns thus seem to be unexplained by the clones. This is also seen by the lower levels of adjusted R-squared during the backtest, and the fact that nearly all alphas have turned significant at a 5% level, while being insignificant during the shorter time period.

The weaker performance of clones during the backtest period is in line with results of Bollen & Fisher (2013), who finds that out-of-sample correlation between the clone and the Dow Jones Credit Suisse hedge funds index changes from 0.40 during the pre-crisis period to 0.81 post-crisis.

5.3.2 Average Excess Returns

Figure 3 shows the average excess returns of the clones for both sample periods. A negative value means that the clone is underperforming the underlying hedge fund index. In Panel A we see the short sample period that extends from 2008 to 2016. We observe relatively small monthly excess returns, and is essentially a different representation of the results discussed before. As noted earlier the clones generally underperform the index, except for the short bias strategy and three of the stepwise clones. Underperformance is most significant for the event driven, macro, relative value, convertible arbitrage, and multi strategy indices. Variation in excess returns among the different regression techniques is relatively small, although stepwise clones perform slightly better on average.

Panel B shows average excess returns over the backtest period extending from 1994 to 2016. All clones are underperforming the indices over the backtest period, and it becomes clear that excess returns are substantially more negative compared to the shorter sample period. Especially the OLS clones show bad out-of-sample performance during the backtest, with negative average excess returns that far exceed those of the other replication techniques. This confirms the previous finding that the standard OLS regression that includes all factors is not the preferred way of replicating hedge fund returns. The other regression models show similar levels of excess returns, and on average underperform the hedge funds index by approximately 0.40% every month. This is considerably more than the excess return of 0.01% for the stepwise clone during the shorter sample period. Average excess returns do not differ widely across the hedge fund strategies during the backtest, although the short bias index shows a noticeably larger underperformance by the clones. Ridge and LASSO clones show lower deviations for this strategy.

There can be a variety of reasons why clones show greater underperformance against the hedge fund indices over the backtest period compared to the more recent sample period. When looking at Table C6 in Appendix C, we see that the hedge fund indices show significantly larger alphas during the backtest period compared to the shorter time period (see Table 5). Given that clones are unable to capture these alphas it explains part of the larger historical underperformance. For example, the alpha for the hedge funds index is 0.34% over the backtest period compared to 0.11% over the shorter sample period. Additionally, almost all alphas are significant for the backtest period while the opposite is true for the shorter period. The model is either missing factors that can explain part of the historical hedge fund returns, or funds have historically shown increased managerial skill that cannot be captured by common risk factors. Moreover, hedge fund index returns have shown a stronger correlation with the equity market during more recent years, which might explain why the ETF factors are now better able to explain the hedge fund returns.

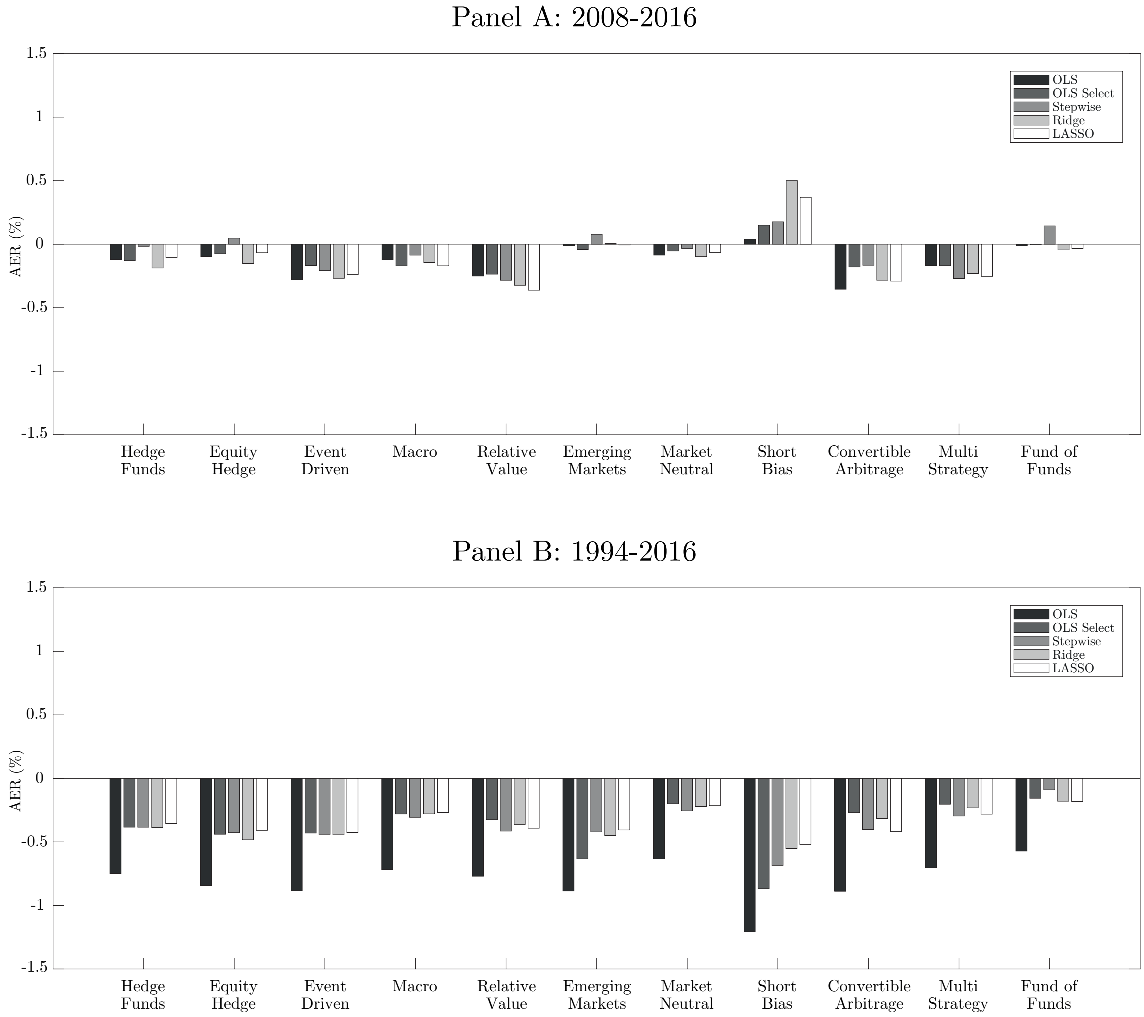

Figure 3: Average monthly excess returns

This figure shows monthly out-of-sample average excess returns (AER) of the clone over the underlying hedge fund index for both the standard period (July 2008-December 2016) and the backtest period (March 1994-December 2016). Means are calculated using a geometric average. All values are shown in %.

5.3.3 Time Variation of Model Fit

To see how the performance of hedge fund replication has evolved over time we look at the time variation of adjusted R-squared. We will focus on the stepwise regression model as it shows the most promising results during the recent sample period. Moreover, we will only look at the time variation of the hedge funds index and fund of funds index as these provide the broadest measures of hedge fund performance.

Figure 4 shows the time variation of adjusted R-squared for the stepwise regression model over the entire backtest period. The adjusted R-squared is the product of the 24-month rolling window regression model that is estimated every month from March 1994 to December 2016. The hedge funds index shows relatively high values of adjusted R-squared over the entire period, with an average of about 0.80. There seems to be a time difference in explanatory power of the model, however. During the 1990s the adjusted R-squared seems to fluctuate rather strongly. It moves from a level of 0.70 to below 0.60, before reaching 0.80 by the start of 2000. The factor model clearly explains less of the variation in hedge fund returns during the 1990s compared to the rest of the sample period. Another noticeable difference in adjusted R-squared is observed during the period directly after the year 2000 when the dot-com bubble came to a burst. Explanatory power of the model drops considerably to a low of about 0.50 during late 2000. It then remains low for a brief period of time before recovering to the old level of 0.80 during 2002. It thus seems that replication of hedge fund returns using an ETF factor model is more difficult in times of crisis. During the financial crisis of 2007-2008 we also observe a drop in adjusted R-squared although less pronounced. It declines from values that exceed 0.90 to about 0.80. The rest of the sample period shows more stable levels of adjusted R-squared.

The fund of funds index on average shows a lower adjusted R-squared compared to the broad hedge funds index. Adjusted R-squared levels fluctuate more strongly for the fund of funds index, but averages around 0.70 over the entire sample period. Similar to the hedge funds index we observe variation in explanatory power of the model during the 1990s, the 2000-2002 dotcom crash, and the 2007-2008 financial crisis. The drop in adjusted R-squared during the dotcom crash and the financial crisis is more evident for the fund of funds index. For example, adjusted R-squared lowers from 0.80 to about 0.60 during the financial crisis.

There seems to be a noticeable time variation in the performance of hedge fund replication as shown by the fluctuating levels of adjusted R-squared for the hedge funds and fund of funds indices. Adjusted R-squared, however, only reveals what part of the variation in hedge fund returns can be explained by the variation in ETF factors. In the next section we therefore look at the performance of clones over different subsamples.

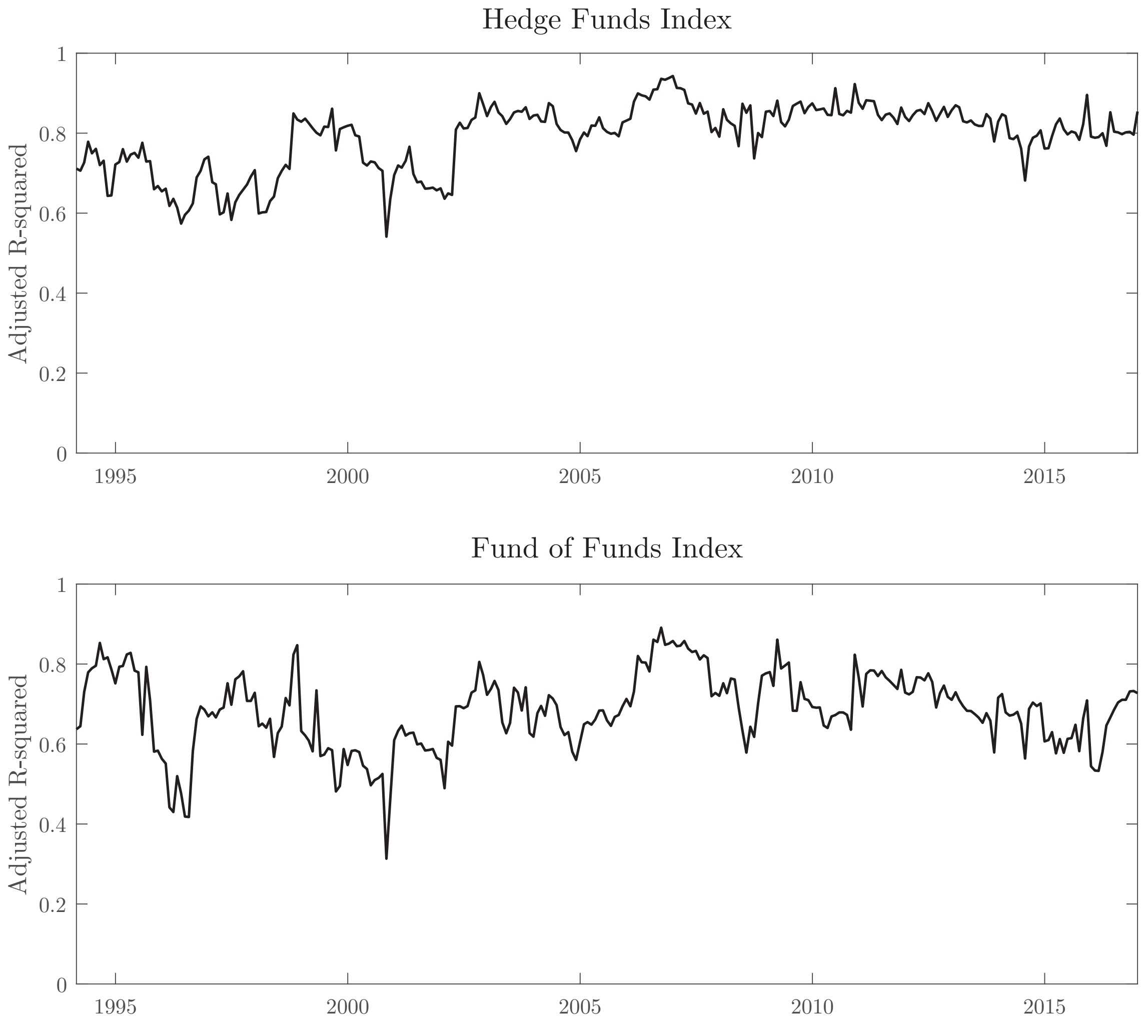

Figure 4: Adjusted R-squared over time for stepwise regression model

This figure shows how the adjusted R-squared evolves over the entire backtest period March 1994 to December 2016 for the stepwise regression model. The adjusted R-squared is the result of the rolling window regressions that are performed each month based on returns of the past 24 months including one-month lag. The explanatory variables are the nine ETF factors and the dependent variable is the hedge fund index. Plots are shown for stepwise clones of both the hedge funds index and the fund of funds index.

5.3.4 Subsamples

We divide our backtest period into five subsamples to check how the clones perform during different time periods. Subsamples are chosen to include the various bull and bear markets in the backtest period of 1994 to 2016. We again focus on the stepwise regression model, and measure performance using average excess returns, tracking error, and correlation. The results can be found in Table 10.

When looking at the average excess returns across the different sample periods, we see that these change significantly over the years. Monthly excess returns of the clones are on average -0.95% for the hedge funds index during the period 1994-1999, while being just -0.01% during the period 2010-2016. This confirms the previous finding that underperformance of the clone to the hedge fund target is smaller during more recent times. Tracking error between the clone and the hedge funds index also decreases over time, as it goes down from 1.36% during 1994-1999 to 0.76% during 2010-2016. Focusing on the other strategy indices we observe similar results when comparing time periods, where average excess returns become less negative or turn positive, and tracking errors go down on average. Especially the clones of the hedge funds, equity hedge, emerging markets, short bias, and fund of funds indices show improved performance in more recent years. The remaining six indices show less attractive results both historically and recently, even though performance of these clones also becomes better over the years. Excess returns for all clones improve from an average of -0.85% during 1994-1999 to -0.09% during 2010-2016. Furthermore, tracking error across all indices decreases from an average of 1.90% to 1.00%.

Out-of-sample correlations between the clones and the hedge fund indices do not differ widely across time, but go up on average with exception for the macro index. The macro index shows a decrease of out-of-sample correlation with its respective clone, as it drops from 0.62 during 1994-1999 to 0.33 during 2010-2016. The relative value, market neutral, and convertible arbitrage indices show the largest increase in correlation, but perform poorly when focusing on average excess returns. Out-of-sample correlation with the clones across all indices goes up from an average of 0.56 during 1994-1999 to 0.71 during 2010-2016.

Interesting is the performance of clones during the two bear markets of 2000-2002 and 2007-2009. On average we observe an improvement in average excess returns during these times. For example, the clone outperforms the fund of funds index by 0.33% during the financial crisis compared to an underperformance of -0.15% during the preceding period of 2003-2006. This pattern is found for most other clones as well, and is also observed during the period 2000-2002. Certain clones, however, exhibit higher tracking errors when markets are down. For example, the hedge funds index has a tracking error of 1.41% during the financial crisis compared to 0.75% during the period 2003-2006. Returns of the clones thus seem to show greater deviation from those of the target in times of crisis. This might be due to the increased volatility of markets during these periods. Tracking error of the clone with the hedge funds index again rebounds to 0.76% in the period following the crisis. Not all strategies show an increase in tracking error during bear markets, however, and the effect is almost non-existent during the period 2000-2002.

Table 10: Clone performance during subsamples for stepwise regression

	Full Sample			**1994-1999**			**2000-2002**		
	AER (%)	**TE (%)**	**Corr.**	**AER (%)**	**TE (%)**	**Corr.**	**AER (%)**	**TE (%)**	**Corr.**
Hedge Funds	-0.38	1.21	0.79	-0.95	1.36	0.80	-0.50	1.58	0.72
Equity Hedge	-0.43	1.69	0.76	-1.15	1.97	0.70	-0.76	2.54	0.62
Event Driven	-0.44	1.42	0.71	-0.85	1.74	0.65	-0.54	1.16	0.81
Macro	-0.31	1.67	0.53	-0.70	2.12	0.62	-0.19	1.94	0.25
Relative Value	-0.41	1.15	0.47	-0.58	1.32	0.35	-0.67	0.82	0.20
Emerging Markets	-0.42	2.19	0.82	-0.70	2.68	0.85	-1.02	2.20	0.84
Market Neutral	-0.26	0.84	0.41	-0.59	1.03	0.31	-0.35	1.01	0.33
Short Bias	-0.68	3.85	0.67	-2.15	4.49	0.65	-0.32	7.45	0.59
Convertible Arb.	-0.40	1.99	0.39	-0.79	1.36	0.29	-0.79	0.76	0.48
Multi Strategy	-0.30	1.17	0.45	-0.43	1.10	0.34	-0.43	0.93	0.49
Fund of Funds	-0.09	1.30	0.65	-0.40	1.70	0.59	-0.08	1.40	0.55
	2003-2006			**2007-2009**			**2010-2016**		
	AER (%)	**TE (%)**	**Corr.**	**AER (%)**	**TE (%)**	**Corr.**	**AER (%)**	**TE (%)**	**Corr.**
Hedge Funds	-0.27	0.75	0.84	-0.17	1.41	0.83	-0.01	0.76	0.85
Equity Hedge	-0.22	0.86	0.86	0.04	1.68	0.86	0.01	0.98	0.88
Event Driven	-0.37	1.10	0.68	-0.21	1.92	0.73	-0.19	0.97	0.78
Macro	-0.20	1.23	0.60	-0.48	1.44	0.52	-0.01	1.33	0.33
Relative Value	-0.29	0.45	0.67	-0.37	2.05	0.51	-0.25	0.78	0.61
Emerging Markets	-0.59	1.54	0.82	0.16	3.07	0.77	-0.08	1.30	0.88
Market Neutral	-0.18	0.48	0.45	0.08	1.01	0.35	-0.12	0.51	0.67
Short Bias	-0.66	1.13	0.89	0.24	1.85	0.90	0.00	1.70	0.79
Convertible Arb.	-0.19	0.99	0.24	-0.20	4.56	0.37	-0.11	1.16	0.63
Multi Strategy	-0.21	0.67	0.52	-0.06	2.25	0.42	-0.27	0.80	0.58
Fund of Funds	-0.15	0.71	0.76	0.33	1.67	0.70	0.03	0.75	0.77

This table shows the monthly out-of-sample performance of the stepwise regression clones during different subsamples. Performance is measured using average excess return, tracking error, and correlation. The clones are constructed using a 24-month rolling window and the stepwise regression over the period March 1994 to December 2016.

5.4 Comparison To Investable Hedge Fund ETF

The investable nature of the clones puts them at a disadvantage compared to the hedge fund indices as discussed previously (e.g., see Jaeger & Wagner, 2005; Jurek & Stafford, 2015). Comparing the clones to an investable benchmark might therefore provide a better view of their performance. Investable hedge fund replication products are offered in various forms, but one of the most popular ones is the IQ Hedge Multi Strategy Tracker ETF (QAI), incepted March 25, 2009. With over $1 billion in assets under management, the fund follows the underlying IQ Hedge Multi Strategy index. The index attempts to replicate hedge fund returns by optimally combining five different investment styles that are constructed using mainly other ETFs. The provider of QAI compares the returns of their ETF against those of the Hedge Fund Research fund of funds index. While not specifically tracking this index it is used as a benchmark for hedge fund performance, which allows for a comparison between our fund of funds clone and the QAI ETF. Since we are looking at an investable fund we have to adjust for fees to make a fair comparison. QAI has an annual expense ratio of 0.96%, which consists of a 0.75% management fee, a 0.01% fee for other direct expenses, and a residual of 0.20% for indirect costs (i.e. expense ratios) of investing in other ETFs. We add back the direct fees of 0.76% to the returns of QAI to obtain gross performance. Table 11 shows gross performance measures for the stepwise fund of funds clone and QAI. Performance is relatively similar, although we observe a small difference in average returns and Sharpe ratios. The clone has an average annual return of 4.27% compared to 3.65% for QAI. Standard deviations are also at similar levels, but slightly lower for the clone.

Table 11: Performance comparison of FOF clone and QAI ETF

	Mean (%)	**Std. Dev. (%)**	**Sharpe Ratio**	**Beta S&P 500**	**Turnover 2016 (%)**	**AER (%) FOF Index**	**TE (%) FOF Index**	**Corr. FOF Index**
FOF Clone	4.27	4.42	0.94	0.27	56	0.05	0.86	0.75
QAI ETF	3.65	4.67	0.76	0.27	312	0.01	0.94	0.73

This table shows a comparison of performance measures between the stepwise fund of funds clone and the QAI ETF over the period April 2009 to December 2016. Beta is a measure of systematic risk calculated using the S&P 500 as a benchmark for the market. Performance is shown gross of fees. Mean, standard deviation, Sharpe ratio, and turnover are annualized. The last three columns provide a comparison to the fund of funds index.

The two hedge fund replicators show relatively similar tracking performance compared to the fund of funds index, although the clone follows it somewhat better. This is not surprising given that the clone is constructed using the index as a target while QAI is not. Systematic risk is measured with beta and is identical for the clone and QAI at a level of 0.27. This is relatively low and is possibly one of the selling points of the ETF. The two replication products differ most noticeably in terms of annual portfolio turnover. The fund of funds clone has a portfolio turnover of 56% in 2016 compared to 312% for QAI.[3] This may result in significantly lower transaction costs for the clone. Another difference between the clone and QAI is the costs of managing the fund. The direct expenses of 0.76% are not incurred by the clone, which makes them more attractive when looking at net returns. However, the clone does not take transaction costs into account, while these are included in the ETF expenses. Figure 5 shows cumulative returns of the clone and QAI, both gross and net of fees. It is clear that cumulative performance of the clone and QAI are similar over the entire period when looking at gross returns. Focusing on net returns we see the negative impact of the relatively high expense ratio of QAI. The fund of funds clone thus seems to provide slightly better results compared to this specific hedge fund replication product.

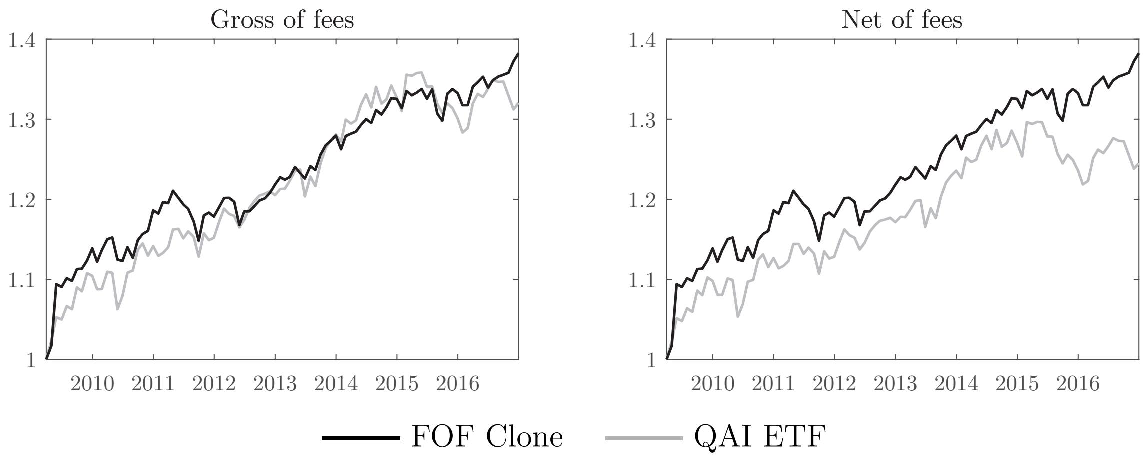

Figure 5: Cumulative returns of FOF clone and QAI ETF

This figure shows cumulative returns of the stepwise fund of funds clone and the QAI ETF over the period April 2009 to December 2016. It shows the value of one dollar invested over time. Returns of QAI are shown gross and net of the 0.76% direct fees.

[3] Source: IndexIQ ETF Trust, Summary Prospectus QAI, 2016.

6 Conclusion

Hedge fund replication has received increased attention over the years as ETF providers started offering hedge fund like products. Lower costs, higher transparency, and improved liquidity make it an interesting alternative for institutional investors compared to traditional hedge funds. Average returns of hedge fund have decreased over time, but their low volatility and exposure to tail risk may still provide benefits to an investment portfolio. This study investigates to what extent hedge funds can be replicated using a linear factor model of ETFs. The benefit of our replication strategy compared to most other studies is that it is actually implementable, because the factors are actively traded. We employ different regression techniques to correct for multicollinearity among factors.

Results show that clones are able to capture a large part of the return characteristics of certain hedge fund indices. Out-of-sample correlation and adjusted R-squared between the clone and several hedge fund indices are high over the sample period 2008-2016. The clones, however, underperform their respective index on average, which is in line with other literature on hedge fund replication. We further find that underperformance of the clones is higher historically when looking at the backtest period 1994-2016. This can possibly be explained by the larger alpha that hedge funds have shown further back in time. These alphas cannot be replicated by the model. Furthermore, tracking error of clones to the target are found to be higher during the financial crisis.

Replication works better for strategies that show less extreme levels of tail risk. The relative value, convertible arbitrage and multi strategy indices show high levels of excess kurtosis and skewness, which results in more severe underperformance of the clones. The presence of illiquidity risk further drives a wedge between returns of the hedge fund indices and the clones. Clones are significantly more liquid than the hedge fund indices, which leads to higher underperformance because of the liquidity premium. Finally, the clones are at a disadvantage because they are directly investable while the hedge fund indices are not. Comparison of the fund of funds clone against an investable hedge fund ETF shows that the clone is able to provide similar results, while having lower portfolio turnover and no management fee. Overall our replication model shows the best out-of-sample results for the hedge funds, equity hedge, emerging markets, short bias, and fund of funds indices.

The use of different regression techniques improves replication performance compared to a standard OLS. Using economic or econometric variable selection leads to lower underperformance and lower turnover of the clone portfolio. Shrinkage methodologies also perform better than the plain OLS on average. Overall, the stepwise regression methodology shows the most favourable results over the sample period 2008-2016, with lowest underperformance to the hedge fund indices.

While clones seem to be able to replicate hedge fund index returns with a certain precision, some limitations have to be discussed. Hedge fund data show survivorship, selection, and backfill biases, which may lead to an upward bias of hedge fund returns. We also look at fund of funds performance, which may be more appropriate in this case. Furthermore, differences between individual hedge funds can be substantial, which are not captured when using an index of hedge fund performance. Hedge fund managers have a far greater number of securities to invest in than the nine factors included in our model. Adding more risk factors might improve the model, but may also cause overfitting. Finally, transaction costs of constructing and rebalancing the clone portfolio are not taken into account. This would obviously decrease performance of the clones, where clones with lower turnover will be less expensive to implement.

To conclude, clones are able to closely replicate certain hedge fund indices, and consequently provide investors with a passive alternative to hedge funds. However, hedge fund indices have shown lower average returns and higher correlation with the equity market in recent years. This brings up the question if hedge funds still provide clear benefits to a portfolio, which an alternative investment would supposedly bring. Abnormal returns of hedge funds may become even lower in the future due to an increasing amount of assets under management and rising competition in the industry. The extent to which hedge funds and their clones are beneficial to an investment portfolio is subject for further research, and will eventually determine the popularity of hedge fund replication products.

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Appendix A: Hedge Fund Strategy Descriptions

These descriptions are obtained directly from Hedge Fund Research, Inc.

Equity Hedge

Investment Managers who maintain positions both long and short in primarily equity and equity derivative securities. A wide variety of investment processes can be employed to arrive at an investment decision, including both quantitative and fundamental techniques; strategies can be broadly diversified or narrowly focused on specific sectors and can range broadly in terms of levels of net exposure, leverage employed, holding period, concentrations of market capitalizations and valuation ranges of typical portfolios. EH managers would typically maintain at least 50% exposure to, and may in some cases be entirely invested in, equities, both long and short.

Event Driven

Investment Managers who maintain positions in companies currently or prospectively involved in corporate transactions of a wide variety including but not limited to mergers, restructurings, financial distress, tender offers, shareholder buybacks, debt exchanges, security issuance or other capital structure adjustments. Security types can range from most senior in the capital structure to most junior or subordinated, and frequently involve additional derivative securities. Event Driven exposure includes a combination of sensitivities to equity markets, credit markets and idiosyncratic, company specific developments. Investment theses are typically predicated on fundamental characteristics (as opposed to quantitative), with the realization of the thesis predicated on a specific development exogenous to the existing capital structure.

Macro

Investment Managers which trade a broad range of strategies in which the investment process is predicated on movements in underlying economic variables and the impact these have on equity, fixed income, hard currency and commodity markets. Managers employ a variety of techniques, both discretionary and systematic analysis, combinations of top down and bottom up theses, quantitative and fundamental approaches and long and short term holding periods. Although some strategies employ RV techniques, Macro strategies are distinct from RV strategies in that the primary investment thesis is predicated on predicted or future movements in the underlying instruments, rather than realization of a valuation discrepancy between securities. In a similar way, while both Macro and equity hedge managers may hold equity securities, the overriding investment thesis is predicated on the impact movements in underlying macroeconomic variables may have on security prices, as opposes to EH, in which the fundamental characteristics on the company are the most significant are integral to investment thesis.

Relative Value
Investment Managers who maintain positions in which the investment thesis is predicated on realization of a valuation discrepancy in the relationship between multiple securities. Managers employ a variety of fundamental and quantitative techniques to establish investment theses, and security types range broadly across equity, fixed income, derivative or other security types. Fixed income strategies are typically quantitatively driven to measure the existing relationship between instruments and, in some cases, identify attractive positions in which the risk adjusted spread between these instruments represents an attractive opportunity for the investment manager. RV position may be involved in corporate transactions also, but as opposed to ED exposures, the investment thesis is predicated on realization of a pricing discrepancy between related securities, as opposed to the outcome of the corporate transaction.

Emerging Markets
Emerging Markets funds invest, primarily long, in securities of companies or the sovereign debt of developing or 'emerging' countries. Emerging Markets regions include Africa, Asia ex-Japan, Latin America, the Middle East and Russia/Eastern Europe. Emerging Markets - Global funds will shift their weightings among these regions according to market conditions and manager perspectives.

Equity Market Neutral
Equity Market Neutral strategies employ sophisticated quantitative techniques of analyzing price data to ascertain information about future price movement and relationships between securities, select securities for purchase and sale. These can include both Factor-based and Statistical Arbitrage/Trading strategies. Factor-based investment strategies include strategies in which the investment thesis is predicated on the systematic analysis of common relationships between securities. In many but not all cases, portfolios are constructed to be neutral to one or multiple variables, such as broader equity markets in dollar or beta terms, and leverage is frequently employed to enhance the return profile of the positions identified. Statistical Arbitrage/Trading strategies consist of strategies in which the investment thesis is predicated on exploiting pricing anomalies which may occur as a function of expected mean reversion inherent in security prices; high frequency techniques may be employed and trading strategies may also be employed on the basis on technical analysis or opportunistically to exploit new information the investment manager believes has not been fully, completely or accurately discounted into current security prices. Equity Market Neutral Strategies typically maintain characteristic net equity market exposure no greater than 10% long or short.

Short Bias

Short-Biased strategies employ analytical techniques in which the investment thesis is predicated on assessment of the valuation characteristics on the underlying companies with the goal of identifying overvalued companies. Short Biased strategies may vary the investment level or the level of short exposure over market cycles, but the primary distinguishing characteristic is that the manager maintains consistent short exposure and expects to outperform traditional equity managers in declining equity markets. Investment theses may be fundamental or technical and nature and manager has a particular focus, above that of a market generalist, on identification of overvalued companies and would expect to maintain a net short equity position over various market cycles.

Convertible Arbitrage

Convertible Arbitrage includes strategies in which the investment thesis is predicated on realization of a spread between related instruments in which one or multiple components of the spread is a convertible fixed income instrument. Strategies employ an investment process designed to isolate attractive opportunities between the price of a convertible security and the price of a non-convertible security, typically of the same issuer. Convertible arbitrage positions maintain characteristic sensitivities to credit quality the issuer, implied and realized volatility of the underlying instruments, levels of interest rates and the valuation of the issuer's equity, among other more general market and idiosyncratic sensitivities.

Multi Strategy

Multi Strategies employ an investment thesis is predicated on realization of a spread between related yield instruments in which one or multiple components of the spread contains a fixed income, derivative, equity, real estate, MLP or combination of these or other instruments. Strategies are typically quantitatively driven to measure the existing relationship between instruments and, in some cases, identify attractive positions in which the risk adjusted spread between these instruments represents an attractive opportunity for the investment manager. In many cases these strategies may exist as distinct strategies across which a vehicle which allocates directly, or may exist as related strategies over which a single individual or decision making process manages. Multi strategy is not intended to provide broadest-based mass market investors appeal, but are most frequently distinguished from others arbitrage strategies in that they expect to maintain >30% of portfolio exposure in 2 or more strategies meaningfully distinct from each other that are expected to respond to diverse market influences.

Fund of Funds

Fund of Funds invest with multiple managers through funds or managed accounts. The strategy designs a diversified portfolio of managers with the objective of significantly lowering the risk (volatility) of investing with an individual manager. The Fund of Funds manager has discretion in choosing which strategies to invest in for the portfolio. A manager may allocate funds to numerous managers within a single strategy, or with numerous managers in multiple strategies. The minimum investment in a Fund of Funds may be lower than an investment in an individual hedge fund or managed account. The investor has the advantage of diversification among managers and styles with significantly less capital than investing with separate managers.

Appendix B: ETF Details

ETF Name	Ticker	Benchmark	Asset Class	Inception Date	Expense Ratio	ADV 3-Month	AUM $
SPDR S&P 500	SPY	S&P 500 Index	Domestic Equity	22-01-1993	0.09%	78.39m	231.98b
iShares 7-10 Year Treasury Bond	IEF	ICE US Treasury 7-10 Year Bond Index	Government Bonds	22-07-2002	0.15%	2.14m	7.78b
SPDR Gold Shares	GLD	Gold Bullion Spot Price	Metal Commodities	18-11-2004	0.40%	8.20m	33.69b
United States Oil Fund	USO	WTI Crude Oil Futures NYMEX	Energy Commodities	10-04-2006	0.72%	25.39m	2.78b
Guggenheim CurrencyShares Euro Trust	FXE	Euro/Dollar Spot Rate	Currencies	12-12-2005	0.40%	0.67m	0.32b
iShares Core U.S. Aggregate Bond	AGG	Barclays US Aggregate Bond Index	Domestic Bonds	22-09-2003	0.05%	2.54m	44.18b
iShares U.S. Real Estate	IYR	Dow Jones U.S. Real Estate Index	Real Estate Equity	12-06-2000	0.44%	7.38m	4.90b
iShares MSCI Emerging Markets	EEM	MSCI Emerging Markets Index	Emerging Markets Equity	07-04-2003	0.72%	54.21m	30.31b
iShares MSCI EAFE	EFA	MSCI EAFE Index	Foreign Equity	14-08-2001	0.33%	18.68m	71.31b

ADV is short for average daily volume, and AUM stands for assets under management. Source: ETF Database, ETFdb.com, 2017.

Appendix C: Additional Tables

Table C1: Maximum Drawdown

	Index	OLS	OLS Select	Stepwise	Ridge	LASSO
Hedge Funds	20.1	22.3	26.3	18.6	20.8	19.4
Equity Hedge	27.9	30.0	34.0	23.5	26.3	27.0
Event Driven	21.8	24.5	23.7	21.9	17.7	19.1
Macro	8.0	11.6	15.1	9.7	9.7	11.0
Relative Value	17.4	15.2	17.5	15.8	12.3	15.6
Emerging Markets	37.2	32.4	43.2	23.8	30.1	28.2
Market Neutral	9.2	7.0	6.9	2.2	3.7	2.4
Short Bias	64.2	104.0	85.6	96.2	58.4	73.3
Convertible Arb.	31.5	19.6	23.0	15.8	13.5	18.5
Multi Strategy	19.0	16.8	15.5	13.2	8.5	12.6
Fund of Funds	20.1	20.4	24.2	12.9	18.5	21.3

This table shows maximum drawdown for all clones and hedge fund indices for the period July 2008 to December 2016. All values are shown in %.

Table C2: Gross portfolio leverage

	OLS			OLS Select			Stepwise			Ridge			LASSO		
	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
Hedge Funds	144	64	236	60	44	85	46	24	199	52	18	116	40	9	190
Equity Hedge	171	86	255	84	59	116	61	43	179	71	24	131	55	12	202
Event Driven	186	74	400	63	39	88	57	25	400	49	11	142	43	0	400
Macro	174	55	400	50	14	97	44	0	127	42	0	279	44	0	400
Relative Value	182	46	391	53	32	98	38	14	270	42	8	169	36	0	288
Emerging Markets	229	94	400	107	65	164	61	38	279	90	42	182	51	17	175
Market Neutral	71	29	144	25	12	60	23	4	75	23	6	67	19	0	93
Short Bias	255	136	400	105	48	165	101	40	210	78	31	170	65	11	262
Convertible Arb.	237	56	400	74	25	168	58	13	400	65	8	308	37	0	288
Multi Strategy	168	51	374	52	28	106	32	13	253	38	9	171	30	4	242
Fund of Funds	139	59	258	50	33	81	36	18	204	43	14	116	37	10	206

This table shows the mean, minimum, and maximum amount of gross leverage used in the clone portfolio for the period July 2008 to December 2016. A weight above 100% implies that the excess money needs to be borrowed at the risk-free rate plus 100 basis points, while a weight lower than 100% means that the remaining money is invested at the risk-free rate. All values are shown in %.

Table C3: Average portfolio weights

	w_{SPY}	w_{IEF}	w_{GLD}	w_{USO}	w_{FXE}	w_{AGG}	w_{IYR}	w_{EEM}	w_{EFA}
Panel A: OLS Regression									
Hedge Funds	18.4	-25.6	5.7	1.8	-7.4	38.6	-4.3	5.3	10.2
Equity Hedge	28.2	-22.9	5.6	2.2	-7.8	15.0	-4.3	9.2	11.8
Event Driven	22.4	-38.5	3.6	1.8	-6.6	58.0	-3.6	1.3	9.6
Macro	-3.5	-12.2	9.1	0.9	-4.9	36.1	-4.6	2.2	10.0
Relative Value	17.5	-37.9	2.4	1.7	-9.4	85.3	-4.1	1.6	4.7
Emerging Markets	8.7	-22.7	5.0	3.2	-5.5	39.4	-7.6	28.6	16.5
Market Neutral	8.9	-4.3	0.2	0.7	-2.2	2.0	-2.1	-0.9	5.8
Short Bias	-55.8	-1.4	0.9	2.9	3.7	24.4	-1.4	-5.8	-2.4
Convertible Arb.	20.0	-47.2	5.6	1.2	-12.6	87.4	-5.5	3.7	8.5
Multi Strategy	17.2	-28.3	3.6	0.7	-10.6	60.0	-5.4	-1.2	10.2
Fund of Funds	14.6	-22.7	5.3	1.9	-8.1	27.2	-5.6	-0.9	14.1
Panel B: OLS Select Regression									
Hedge Funds	30.1	-10.7	6.3	2.2	2.1				
Equity Hedge	43.0	-20.3	6.9	2.9	4.9				
Event Driven	32.5	-13.1	4.1	2.4	-0.5				
Macro	5.7	0.7	8.5	0.9	1.6				
Relative Value	23.5	1.9	3.3	1.8	-2.8				
Emerging Markets	42.8	-9.0	8.5	5.5	19.4				
Market Neutral	9.9	-1.9	-0.5	1.5	-0.6				
Short Bias	-65.1	12.4	-0.5	3.1	-0.6				
Convertible Arb.	33.2	-10.0	8.7	1.5	-3.0				
Multi Strategy	22.1	-1.2	3.8	0.7	-2.2				
Fund of Funds	21.5	-11.5	4.6	2.6	-1.4				
Panel C: Stepwise Regression									
Hedge Funds	14.6	-5.1	2.4	0.8	-2.5	1.7	-0.5	8.8	7.3
Equity Hedge	24.4	-10.6	1.1	1.2	-0.6	1.5	-0.6	14.4	4.6
Event Driven	18.4	-9.8	1.1	1.4	-0.9	6.6	1.1	2.9	8.8
Macro	2.8	3.3	7.6	0.2	-3.8	6.8	-0.4	-0.1	4.7
Relative Value	8.0	-4.6	0.4	0.4	-2.3	9.9	1.6	5.2	5.3
Emerging Markets	4.5	-4.8	0.8	3.2	-0.2	1.6	-2.0	36.9	4.9
Market Neutral	10.3	-0.9	-0.4	0.9	-0.3	-2.1	-1.2	2.0	0.0
Short Bias	-57.0	8.2	0.8	2.5	3.4	7.0	-1.9	-4.3	-3.4
Convertible Arb.	8.7	-7.1	1.9	0.1	-3.3	14.9	1.6	9.1	9.2
Multi Strategy	5.7	-4.6	0.9	0.0	-1.5	5.6	1.7	4.6	6.6
Fund of Funds	10.2	-4.9	1.2	1.6	-3.0	-0.7	-0.8	4.1	7.9
Panel D: Ridge Regression									
Hedge Funds	9.4	-10.0	3.2	2.0	-1.0	5.4	1.0	5.0	6.7
Equity Hedge	14.0	-15.2	3.2	2.6	-0.2	0.1	2.2	7.5	9.3
Event Driven	9.4	-11.1	1.8	2.1	-0.7	1.9	2.1	3.9	6.4
Macro	2.0	-0.4	4.7	0.1	-1.0	8.4	-0.5	0.7	3.1
Relative Value	6.0	-6.9	1.8	1.6	-2.5	12.1	1.0	2.9	3.9
Emerging Markets	11.3	-13.0	4.5	3.4	5.0	9.8	-0.6	16.7	12.6
Market Neutral	3.4	-2.6	-0.1	0.8	0.1	-4.3	0.6	1.1	2.2
Short Bias	-19.5	12.5	0.7	-0.3	1.9	2.2	-8.1	-5.8	-9.6
Convertible Arb.	8.4	-12.8	3.8	1.9	-3.2	17.0	1.5	5.2	6.1
Multi Strategy	5.8	-6.3	1.9	1.1	-1.8	9.0	0.8	2.6	4.0
Fund of Funds	7.7	-8.3	2.6	1.8	-1.8	-1.0	-0.2	2.9	6.3
Panel E: LASSO Regression									
Hedge Funds	11.7	-4.9	1.7	1.1	-2.0	2.4	-0.2	6.5	5.8
Equity Hedge	19.8	-7.3	1.7	1.3	-2.2	-2.2	-0.3	9.7	6.7
Event Driven	12.8	-8.3	0.9	1.3	-0.9	5.0	1.3	2.6	6.5
Macro	1.0	-3.8	4.8	0.3	-2.7	12.6	-0.3	0.6	2.9
Relative Value	6.5	-6.4	1.1	1.2	-2.5	10.0	0.6	2.6	4.1
Emerging Markets	4.1	-1.6	1.4	2.5	-0.1	-0.5	-1.2	28.5	8.7
Market Neutral	4.6	-2.6	0.1	0.8	-0.1	-3.0	0.3	0.8	1.5
Short Bias	-34.7	5.3	0.8	0.6	3.3	3.7	-5.4	-2.6	-5.0
Convertible Arb.	7.1	-3.9	2.3	0.5	-2.3	4.3	1.5	6.9	6.4
Multi Strategy	6.2	-5.1	1.6	0.7	-2.1	5.3	0.4	2.5	4.7
Fund of Funds	8.3	-5.3	1.9	1.5	-3.0	-0.1	-0.8	2.5	7.7

This table shows the average weights invested into each ETF of the clone portfolio for the period May 2006 to December 2016. All values are shown in %.

Table C4: Minimum portfolio weights

	w_{SPY}	w_{IEF}	w_{GLD}	w_{USO}	w_{FXE}	w_{AGG}	w_{IYR}	w_{EEM}	w_{EFA}
Panel A: OLS Regression									
Hedge Funds	-4.9	-75.6	-2.1	-2.5	-30.0	-55.6	-15.9	-5.1	-13.2
Equity Hedge	0.2	-92.7	-1.2	-4.3	-36.6	-114.9	-19.6	-10.0	-11.3
Event Driven	-14.2	-138.4	-7.1	-4.7	-43.5	-75.6	-12.9	-14.0	-22.1
Macro	-66.6	-120.2	-6.4	-5.9	-40.2	-100.0	-22.6	-17.2	-40.2
Relative Value	-2.8	-103.0	-7.8	-4.8	-43.0	-4.5	-13.9	-11.8	-28.8
Emerging Markets	-31.2	-99.7	-7.2	-11.8	-53.5	-145.2	-32.7	-0.6	-31.4
Market Neutral	-7.7	-25.4	-6.4	-2.9	-18.0	-64.7	-14.6	-14.8	-8.0
Short Bias	-113.6	-114.6	-20.6	-5.5	-23.4	-131.3	-41.9	-42.9	-70.3
Convertible Arb.	-11.8	-118.6	-10.3	-5.2	-61.1	-93.4	-24.3	-31.9	-35.7
Multi Strategy	-10.9	-99.3	-10.2	-6.5	-64.4	-69.3	-16.7	-25.6	-27.0
Fund of Funds	-15.0	-82.5	-4.0	-2.1	-38.8	-75.1	-21.3	-10.3	-10.7
Panel B: OLS Select Regression									
Hedge Funds	24.8	-35.6	-0.1	-2.9	-15.9				
Equity Hedge	36.7	-48.3	-0.5	-3.1	-14.1				
Event Driven	25.5	-43.7	-3.6	-3.9	-15.7				
Macro	-15.6	-45.3	-1.8	-6.1	-25.3				
Relative Value	15.9	-38.0	-6.5	-3.5	-18.1				
Emerging Markets	25.7	-64.5	-10.1	-14.2	-17.9				
Market Neutral	-6.4	-39.2	-5.0	-2.1	-9.0				
Short Bias	-99.5	-18.3	-10.4	-8.0	-29.4				
Convertible Arb.	14.1	-73.6	-10.0	-4.1	-42.3				
Multi Strategy	12.6	-37.7	-8.4	-6.0	-19.4				
Fund of Funds	11.4	-36.5	-2.3	-1.8	-21.8				
Panel C: Stepwise Regression									
Hedge Funds	0.0	-56.3	0.0	0.0	-29.8	-50.1	-11.2	0.0	0.0
Equity Hedge	0.0	-57.8	0.0	0.0	-29.8	-39.8	-11.5	0.0	0.0
Event Driven	0.0	-134.2	-5.2	0.0	-40.1	-59.0	-10.5	0.0	0.0
Macro	-10.9	0.0	0.0	0.0	-29.5	-53.8	-16.8	-16.8	0.0
Relative Value	0.0	-78.5	-3.9	-4.9	-29.9	0.0	-14.3	0.0	0.0
Emerging Markets	-25.2	-75.9	0.0	0.0	-34.7	0.0	-33.4	0.0	0.0
Market Neutral	0.0	-14.2	-4.9	-2.0	-6.8	-66.9	-11.7	-5.3	0.0
Short Bias	-121.5	-33.9	-7.5	0.0	-24.9	-40.4	-36.2	-41.3	-39.2
Convertible Arb.	0.0	-125.5	-5.9	0.0	-57.5	0.0	-17.0	-12.4	0.0
Multi Strategy	0.0	-73.8	0.0	-5.5	-25.1	0.0	-12.1	0.0	0.0
Fund of Funds	0.0	-60.2	0.0	0.0	-18.7	-60.5	-19.4	0.0	0.0
Panel D: Ridge Regression									
Hedge Funds	2.9	-30.4	0.1	0.0	-11.2	-29.8	-5.8	1.6	2.0
Equity Hedge	4.4	-34.8	0.1	-0.5	-17.0	-31.6	-10.1	2.4	2.9
Event Driven	2.3	-31.0	-1.8	-2.4	-22.5	-25.9	-2.8	-0.4	1.5
Macro	-11.3	-72.8	0.0	-2.6	-21.3	-36.9	-9.8	-8.1	0.0
Relative Value	1.3	-40.2	-1.8	-0.3	-22.0	-5.3	-4.7	0.1	0.9
Emerging Markets	-4.9	-38.9	-4.5	-4.5	-14.8	-21.3	-23.6	4.1	4.2
Market Neutral	-1.8	-10.9	-2.8	-1.1	-8.8	-41.0	-9.9	-2.5	0.3
Short Bias	-41.1	-1.7	-8.0	-5.9	-11.3	-44.0	-23.4	-19.7	-31.6
Convertible Arb.	2.4	-72.8	-3.2	-1.8	-39.7	-5.1	-9.2	0.1	1.6
Multi Strategy	1.7	-37.4	-1.9	-3.7	-26.1	-4.6	-4.0	-0.4	1.2
Fund of Funds	2.3	-35.2	-1.7	-1.0	-16.5	-37.6	-9.7	-1.4	1.6
Panel E: LASSO Regression									
Hedge Funds	0.0	-59.7	0.0	0.0	-25.0	-50.0	-8.4	0.0	0.0
Equity Hedge	0.0	-47.7	0.0	0.0	-32.4	-56.9	-11.1	-4.8	0.0
Event Driven	0.0	-133.8	0.0	-2.3	-35.8	-42.0	-7.3	0.0	0.0
Macro	-26.0	-121.3	0.0	-2.6	-23.5	-51.7	-14.0	-5.3	0.0
Relative Value	0.0	-82.4	-0.5	-1.0	-33.6	0.0	-6.9	-0.5	0.0
Emerging Markets	0.0	-26.7	-1.5	-0.7	-26.6	-22.8	-30.2	10.0	0.0
Market Neutral	-0.2	-20.0	-3.5	-1.4	-11.1	-58.4	-13.7	-11.4	0.0
Short Bias	-63.5	0.0	-18.0	-3.9	-7.0	-36.0	-36.4	-29.1	-56.2
Convertible Arb.	-8.7	-84.3	-3.4	-1.7	-52.2	0.0	-11.1	-8.9	0.0
Multi Strategy	0.0	-65.2	-0.1	-2.8	-43.1	0.0	-10.7	-9.9	0.0
Fund of Funds	-2.7	-55.3	0.0	-0.3	-36.0	-63.9	-16.9	-3.7	0.0

This table shows the minimum weights invested into each ETF of the clone portfolio for the period May 2006 to December 2016. All values are shown in %.

Table C5: Maximum portfolio weights

	w_{SPY}	w_{IEF}	w_{GLD}	w_{USO}	w_{FXE}	w_{AGG}	w_{IYR}	w_{EEM}	w_{EFA}
Panel A: OLS Regression									
Hedge Funds	34.0	42.5	19.0	7.4	10.9	131.2	5.6	16.5	44.9
Equity Hedge	49.8	69.3	19.7	8.6	16.3	109.8	10.5	21.8	54.2
Event Driven	56.7	58.2	17.3	11.6	15.7	230.6	19.3	11.9	63.8
Macro	24.1	101.5	30.1	9.8	19.1	243.9	13.6	23.5	44.2
Relative Value	56.1	11.3	14.3	11.6	9.4	220.8	9.9	16.0	33.2
Emerging Markets	42.9	92.9	29.7	13.0	39.5	203.9	22.2	50.0	82.5
Market Neutral	22.2	44.1	7.5	5.0	12.1	90.9	10.1	8.3	22.9
Short Bias	22.5	125.3	18.0	13.1	52.0	240.5	27.0	37.3	62.6
Convertible Arb.	83.1	60.4	21.9	13.0	18.9	250.0	17.9	33.2	69.0
Multi Strategy	64.4	33.9	20.0	13.6	10.7	204.0	11.6	21.1	45.4
Fund of Funds	33.9	54.9	23.2	7.6	11.8	134.2	5.5	10.6	66.1
Panel B: OLS Select Regression									
Hedge Funds	38.9	11.6	14.2	8.8	15.4				
Equity Hedge	51.5	11.3	14.1	11.8	23.3				
Event Driven	42.3	18.7	13.0	8.8	14.9				
Macro	25.6	45.4	23.9	7.8	29.2				
Relative Value	34.1	37.3	16.0	10.7	10.2				
Emerging Markets	66.7	51.1	25.4	18.1	58.7				
Market Neutral	19.2	15.2	2.4	6.8	16.6				
Short Bias	-39.5	46.6	20.2	15.1	38.9				
Convertible Arb.	67.3	43.4	38.1	16.7	13.8				
Multi Strategy	36.7	36.3	18.7	9.9	14.6				
Fund of Funds	36.7	12.5	12.1	9.3	11.9				
Panel C: Stepwise Regression									
Hedge Funds	36.1	0.0	18.7	9.5	0.0	95.2	0.0	28.1	44.1
Equity Hedge	48.8	0.0	11.2	10.4	18.8	79.3	7.9	36.7	49.0
Event Driven	47.7	0.0	13.4	12.7	0.0	238.5	22.8	23.4	39.6
Macro	27.2	71.6	23.5	7.6	13.6	73.3	12.6	16.0	30.2
Relative Value	49.1	0.0	12.4	7.1	0.0	180.4	17.4	26.7	29.3
Emerging Markets	54.7	0.0	30.6	13.5	26.5	168.4	10.6	53.3	102.9
Market Neutral	20.2	12.2	5.8	4.7	0.0	23.2	8.6	10.8	0.0
Short Bias	0.0	61.8	20.0	16.0	67.8	141.8	24.6	0.0	40.5
Convertible Arb.	35.8	0.0	32.0	7.3	0.0	272.2	31.2	40.9	59.8
Multi Strategy	42.8	13.9	15.8	4.7	0.0	164.4	19.5	20.9	35.1
Fund of Funds	45.8	0.0	12.4	8.7	0.0	63.0	0.0	26.0	41.6
Panel D: Ridge Regression									
Hedge Funds	19.6	1.6	11.9	5.0	5.5	36.1	7.7	10.9	19.3
Equity Hedge	26.8	0.1	11.4	5.8	9.2	37.1	11.4	15.0	26.3
Event Driven	17.8	-0.5	9.1	7.8	5.7	47.7	11.9	8.5	26.8
Macro	17.1	20.7	21.4	6.0	18.8	160.9	5.0	8.8	15.4
Relative Value	20.1	3.4	9.2	7.1	3.7	71.7	6.6	9.3	12.8
Emerging Markets	24.4	23.4	18.3	9.3	22.9	74.9	12.7	27.8	36.1
Market Neutral	9.6	2.6	3.7	3.3	6.2	14.5	7.0	5.9	8.9
Short Bias	-6.9	50.6	12.9	6.2	33.1	39.8	0.3	12.2	3.4
Convertible Arb.	23.5	3.5	20.5	8.8	11.2	120.8	10.9	23.0	22.7
Multi Strategy	13.6	3.5	10.2	7.2	5.8	70.2	6.9	10.7	10.0
Fund of Funds	19.3	1.8	13.6	5.8	3.8	16.0	4.3	9.1	28.1
Panel E: LASSO Regression									
Hedge Funds	27.8	9.5	18.0	6.9	1.8	101.5	8.8	18.5	42.6
Equity Hedge	43.9	8.3	19.2	6.1	10.6	38.3	11.3	25.6	52.2
Event Driven	29.0	0.0	9.7	10.8	0.5	227.6	17.6	12.5	35.1
Macro	20.4	42.2	20.5	7.0	17.5	239.6	8.4	11.2	17.6
Relative Value	26.2	0.0	12.9	9.9	0.0	172.9	11.7	14.3	27.3
Emerging Markets	20.0	5.6	24.1	9.7	18.1	0.0	7.1	38.5	67.1
Market Neutral	14.1	5.9	5.3	4.3	6.9	14.9	8.2	7.7	18.2
Short Bias	0.0	57.7	15.4	9.5	36.3	126.6	8.1	29.4	2.3
Convertible Arb.	38.4	0.0	23.3	4.7	0.0	162.7	19.0	32.9	56.0
Multi Strategy	36.5	0.0	14.9	10.9	0.0	117.5	11.8	18.3	32.5
Fund of Funds	30.6	20.0	20.1	7.4	0.0	91.3	5.7	15.6	49.9

This table shows the maximum weights invested into each ETF of the clone portfolio for the period May 2006 to December 2016. All values are shown in %.

Table C6: In-sample model fit backtest

	α (%)	β_{SPY}	β_{IEF}	β_{GLD}	β_{USO}	β_{FXE}	β_{AGG}	β_{IYR}	β_{EEM}	β_{EFA}	R^2
Panel A: OLS Regression											
Hedge Funds	0.34***	0.12***	−0.20*	0.04**	0.02***	−0.12***	0.30*	−0.02	0.13***	0.07**	0.74
Equity Hedge	0.39***	0.23***	−0.26*	0.04	0.04***	−0.10**	0.32	−0.02	0.10***	0.10**	0.69
Event Driven	0.45***	0.12***	−0.40***	0.02	0.03***	−0.08**	0.52**	0.02	0.08***	0.05	0.65
Macro	0.32***	−0.05	0.25	0.07**	0.00	−0.21***	0.08	−0.04	0.11***	0.13***	0.31
Relative Value	0.39***	0.02	−0.44***	0.02	0.02***	−0.06*	0.70***	0.03*	0.03**	0.03	0.47
Emerging Markets	0.36***	−0.09	−0.20	0.00	0.02	−0.22***	0.37	−0.05	0.50***	0.12*	0.79
Market Neutral	0.22***	0.04*	0.08	0.00	0.01*	−0.01	−0.13	0.01	−0.02	0.04	0.11
Short Bias	0.17	−0.65***	−0.14	−0.02	0.01	0.14	0.44	0.04	−0.16**	−0.08	0.50
Convertible Arb.	0.26**	0.05	−0.98***	0.03	0.03**	−0.02	1.63***	0.01	0.03	0.02	0.45
Multi Strategy	0.26***	0.00	−0.50***	0.01	0.02***	−0.05*	0.84***	0.01	0.04**	0.04	0.49
Fund of Funds	0.12	0.02	−0.19*	0.02	0.02***	−0.15***	0.36*	−0.03	0.10***	0.10***	0.56
Panel B: OLS Select Regression											
Hedge Funds	0.34***	0.33***	−0.10*	0.09***	0.03***	−0.06*					0.62
Equity Hedge	0.39***	0.43***	−0.15**	0.08***	0.05***	−0.03					0.63
Event Driven	0.47***	0.30***	−0.16***	0.06***	0.03***	−0.03					0.57
Macro	0.31**	0.16***	0.22***	0.12***	0.02	−0.14***					0.20
Relative Value	0.43***	0.15***	−0.06	0.04**	0.02***	−0.01					0.35
Emerging Markets	0.33	0.54***	−0.24*	0.18***	0.06**	−0.11					0.44
Market Neutral	0.21***	0.06***	0.00	0.00	0.01*	0.01					0.10
Short Bias	0.22	−0.83***	0.20	−0.08	−0.01	0.11					0.49
Convertible Arb.	0.34***	0.20***	−0.08	0.06**	0.03**	0.04					0.30
Multi Strategy	0.30***	0.13***	−0.06	0.03*	0.02***	0.00					0.33
Fund of Funds	0.12	0.22***	−0.06	0.07***	0.03***	−0.08**					0.41
Panel C: Stepwise Regression											
Hedge Funds	0.34***	0.12***		0.03*	0.03***	−0.12***			0.13***	0.08**	0.73
Equity Hedge	0.39***	0.21***			0.05***	−0.09**			0.12***	0.11**	0.69
Event Driven	0.45***	0.12***	−0.41***		0.03***	−0.08**	0.57***		0.09***	0.05	0.65
Macro	0.32***		0.30***	0.08***		−0.19***		−0.04*	0.11***	0.11**	0.32
Relative Value	0.42***	0.05**			0.02***			0.04***	0.06***		0.41
Emerging Markets	0.36***				0.02	−0.16***			0.51***		0.78
Market Neutral	0.21***	0.06***			0.01**						0.11
Short Bias	0.24	−0.65***				0.13			−0.17***		0.51
Convertible Arb.	0.29***		−1.00***		0.03***		1.69***		0.05**	0.04	0.44
Multi Strategy	0.29***				0.02***	−0.04*		0.04**	0.05***	0.06***	0.42
Fund of Funds	0.15*				0.02***	−0.14***			0.11***	0.11***	0.56
Panel D: Ridge Regression											
Hedge Funds	0.36	0.12	−0.10	0.03	0.02	−0.10	0.13	0.00	0.12	0.08	0.73
Equity Hedge	0.41	0.20	−0.15	0.03	0.04	−0.08	0.12	0.00	0.10	0.11	0.67
Event Driven	0.47	0.12	−0.21	0.02	0.03	−0.07	0.20	0.03	0.08	0.06	0.63
Macro	0.33	−0.02	0.18	0.07	0.00	−0.18	0.16	−0.03	0.10	0.10	0.28
Relative Value	0.41	0.03	−0.23	0.02	0.02	−0.05	0.34	0.03	0.04	0.04	0.40
Emerging Markets	0.37	0.00	−0.12	0.03	0.02	−0.19	0.17	−0.02	0.40	0.13	0.76
Market Neutral	0.23	0.02	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.01	0.07
Short Bias	0.15	−0.53	0.01	−0.01	0.00	0.15	0.21	0.02	−0.15	−0.12	0.48
Convertible Arb.	0.29	0.05	−0.52	0.03	0.03	−0.01	0.86	0.03	0.04	0.04	0.32
Multi Strategy	0.27	0.01	−0.25	0.01	0.02	−0.04	0.42	0.02	0.04	0.05	0.42
Fund of Funds	0.13	0.04	−0.09	0.02	0.02	−0.13	0.18	−0.02	0.10	0.09	0.54
Panel E: LASSO Regression											
Hedge Funds	0.34	0.12	−0.18	0.04	0.02	−0.11	0.27	−0.01	0.13	0.07	0.72
Equity Hedge	0.39	0.22	−0.24	0.03	0.04	−0.09	0.28	−0.01	0.10	0.10	0.68
Event Driven	0.45	0.13	−0.35	0.02	0.03	−0.08	0.44	0.02	0.08	0.05	0.61
Macro	0.33	−0.03	0.25	0.07	0.00	−0.19	0.07	−0.04	0.11	0.11	0.27
Relative Value	0.40	0.03	−0.37	0.02	0.02	−0.05	0.59	0.03	0.04	0.03	0.38
Emerging Markets	0.38				0.01	−0.13		−0.01	0.49	0.04	0.77
Market Neutral	0.22	0.03			0.01					0.02	0.08
Short Bias	0.17	−0.63				0.05	0.13		−0.15		0.50
Convertible Arb.	0.28	0.05	−0.77	0.03	0.03		1.27	0.02	0.04	0.02	0.35
Multi Strategy	0.27		−0.40	0.00	0.02	−0.04	0.66	0.02	0.04	0.04	0.40
Fund of Funds	0.15	0.01		0.01	0.02	−0.11	0.01		0.11	0.09	0.52

This table shows the in-sample fit of all regression models over the backtest period January 1992 to December 2016. The ETF factors are the explanatory variables and the hedge fund index is the dependent variable. R^2 shows the adjusted R-squared of the model. Stars show the level of significance, with (*); (**); (***) being significant at a 5%, 1%, and 0.1% level respectively. For the shrinkage regressions ridge and LASSO the level of significance is not shown as this is not well defined, see text for further explanation.

Table C7: Monthly out-of-sample tracking error and correlation backtest

	OLS		OLS Select		Stepwise		Ridge		LASSO	
	TE (%)	Corr.	TE (%)	Corr.	TE (%)	Corr.	TE (%)	Corr.	TE (%)	Corr.
Hedge Funds	1.38	0.74	1.27	0.79	1.21	0.79	1.12	0.82	1.19	0.79
Equity Hedge	1.86	0.72	1.76	0.76	1.70	0.76	1.56	0.80	1.59	0.79
Event Driven	1.57	0.65	1.37	0.72	1.42	0.71	1.32	0.72	1.37	0.69
Macro	1.94	0.43	1.76	0.51	1.68	0.53	1.54	0.54	1.61	0.51
Relative Value	1.28	0.43	1.06	0.56	1.16	0.47	0.98	0.56	1.02	0.52
Emerging Markets	2.53	0.75	2.91	0.69	2.19	0.82	2.14	0.83	2.18	0.82
Market Neutral	1.13	0.17	0.88	0.35	0.84	0.41	0.77	0.48	0.79	0.43
Short Bias	4.12	0.58	3.98	0.65	3.86	0.67	3.84	0.64	3.75	0.66
Convertible Arb.	1.97	0.36	1.88	0.44	1.99	0.39	1.71	0.47	1.80	0.42
Multi Strategy	1.29	0.43	1.11	0.49	1.18	0.45	1.02	0.52	1.05	0.49
Fund of Funds	1.37	0.65	1.32	0.65	1.30	0.65	1.18	0.69	1.27	0.63

This table shows monthly out-of-sample tracking error and correlation between the clone returns and the hedge fund index returns for the entire backtest period. The clones are constructed using a 24-month rolling window and five different regression techniques over the period March 1994 to December 2016.

Table C8: Monthly out-of-sample returns backtest

	Index		OLS		OLS Select		Stepwise		Ridge		LASSO	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Hedge Funds	0.63	1.93	-0.10	1.92	0.26	1.95	0.26	1.82	0.26	1.49	0.29	1.49
Equity Hedge	0.73	2.56	-0.09	2.41	0.31	2.50	0.32	2.34	0.27	1.91	0.35	1.91
Event Driven	0.73	1.88	-0.14	1.89	0.32	1.74	0.30	1.88	0.31	1.24	0.33	1.26
Macro	0.55	1.80	-0.15	1.83	0.28	1.73	0.26	1.62	0.29	1.25	0.30	1.31
Relative Value	0.62	1.18	-0.14	1.21	0.30	1.07	0.21	1.06	0.27	0.73	0.24	0.76
Emerging Markets	0.57	3.81	-0.26	3.14	-0.01	3.55	0.19	3.52	0.17	2.97	0.22	2.81
Market Neutral	0.43	0.87	-0.20	0.88	0.23	0.64	0.18	0.63	0.21	0.42	0.22	0.46
Short Bias	-0.17	4.95	-1.24	3.60	-0.94	4.57	-0.76	4.54	-0.59	3.62	-0.57	3.62
Convertible Arb.	0.59	1.92	-0.28	1.52	0.34	1.56	0.21	1.65	0.30	1.10	0.20	1.25
Multi Strategy	0.51	1.19	-0.19	1.23	0.31	0.98	0.22	1.03	0.29	0.70	0.24	0.76
Fund of Funds	0.39	1.63	-0.17	1.61	0.25	1.52	0.31	1.47	0.23	1.18	0.23	1.22

This table shows monthly out-of-sample average returns and monthly standard deviations of the clone returns and the hedge fund index returns for the entire backtest period. The clones are constructed using a 24-month rolling window and five different regression techniques over the period March 1994 to December 2016. Means are calculated using a geometric average. All values are shown in %.

Table C9: Out-of-sample model fit backtest

	OLS			OLS Select			Stepwise			Ridge			LASSO		
	α (%)	β	R^2	α (%)	β	R^2	α (%)	β	R^2	α (%)	β	R^2	α (%)	β	R^2
Hedge Funds	0.72***	0.75***	0.55	0.44***	0.78***	0.62	0.42***	0.84***	0.63	0.36***	1.06***	0.66	0.34***	1.02***	0.62
Equity Hedge	0.81***	0.77***	0.52	0.50***	0.78***	0.57	0.47***	0.83***	0.58	0.45***	1.07***	0.63	0.38**	1.05***	0.62
Event Driven	0.83***	0.65***	0.42	0.49***	0.78***	0.51	0.52***	0.72***	0.51	0.41**	1.09***	0.51	0.41**	1.03***	0.47
Macro	0.62***	0.42***	0.18	0.41***	0.53***	0.25	0.40***	0.58***	0.27	0.33***	0.78***	0.29	0.35***	0.70***	0.25
Relative Value	0.68***	0.42***	0.18	0.44***	0.61***	0.31	0.51***	0.52***	0.22	0.38**	0.91***	0.31	0.44***	0.80***	0.26
Emerging Markets	0.84***	0.91***	0.56	0.61**	0.74***	0.47	0.42*	0.89***	0.68	0.41*	1.06***	0.69	0.35	1.12***	0.68
Market Neutral	0.47***	0.17*	0.03	0.32***	0.48***	0.12	0.33***	0.56***	0.16	0.22***	0.99***	0.22	0.25***	0.81***	0.18
Short Bias	0.88**	0.79***	0.33	0.54*	0.71***	0.42	0.43	0.73***	0.45	0.41	0.87***	0.41	0.40	0.90***	0.43
Convertible Arb.	0.73***	0.46**	0.13	0.41*	0.54**	0.19	0.50**	0.45*	0.15	0.36	0.82**	0.22	0.47*	0.65**	0.17
Multi Strategy	0.59***	0.42***	0.19	0.33**	0.59***	0.23	0.40***	0.51***	0.20	0.26*	0.88***	0.26	0.33**	0.77***	0.24
Fund of Funds	0.51***	0.65***	0.41	0.23*	0.70***	0.42	0.17	0.72***	0.42	0.18*	0.96***	0.48	0.21*	0.84***	0.40

This table shows the out-of-sample fit of all regression models. The monthly clone return is the explanatory variable and the hedge fund index return is the dependent variable. R^2 shows the adjusted R-squared. The clones are constructed using a 24-month rolling window and five different regression techniques over the entire backtest period period March 1994 to December 2016. Standard errors are adjusted for heteroscedasticity and autocorrelation using Newey & West (1987) methodology with a lag of 4. Stars show the level of significance, with (*); (**); (***) being significant at a 5%, 1%, and 0.1% level respectively.

Appendix D: Additional Figures

The figures below show cumulative returns of the clone and the hedge fund indices over the period July 2008 to December 2016. It shows the value of one dollar invested over time. The OLS regression is excluded.

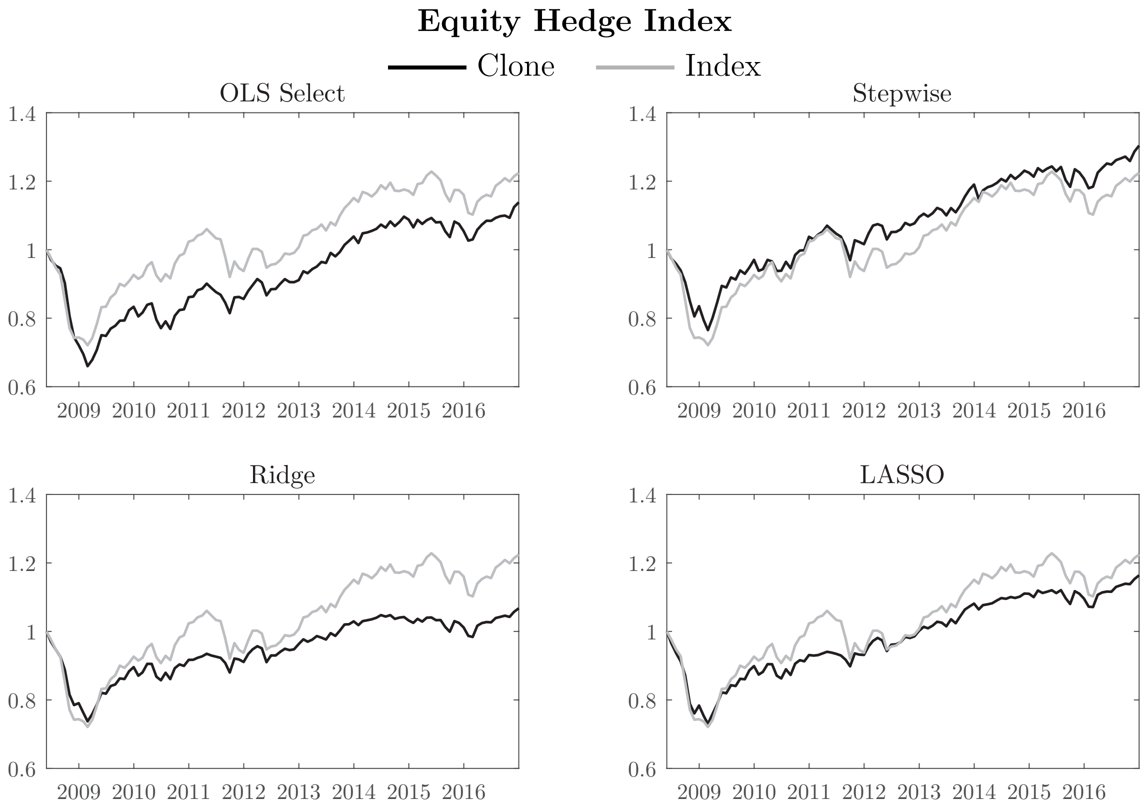

Figure D1: Cumulative returns of equity hedge index

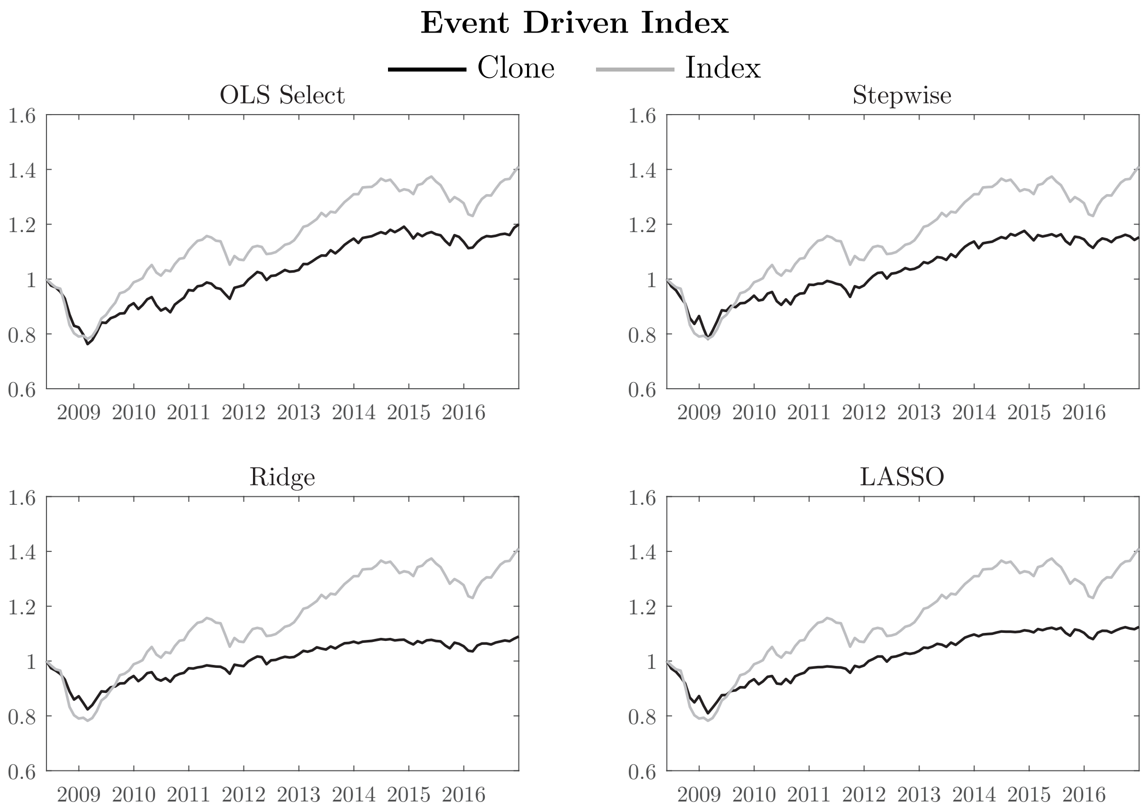

Figure D2: Cumulative returns of event driven index

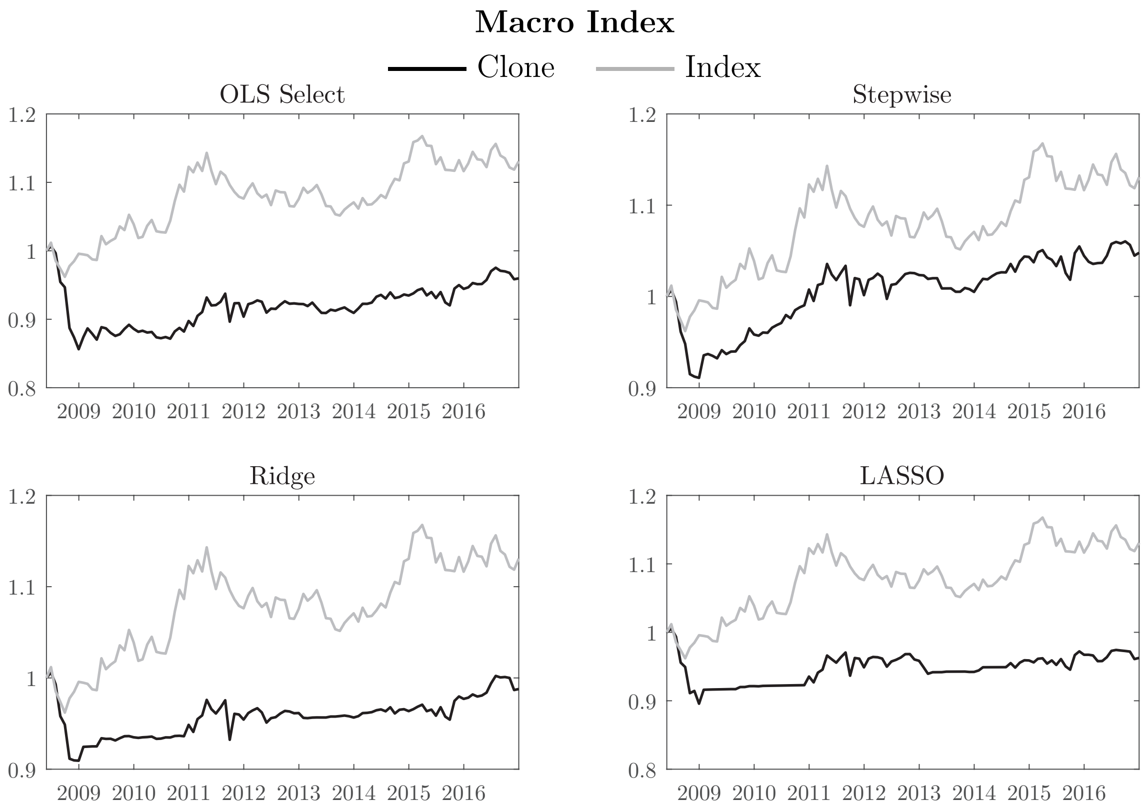

Figure D3: Cumulative returns of macro index

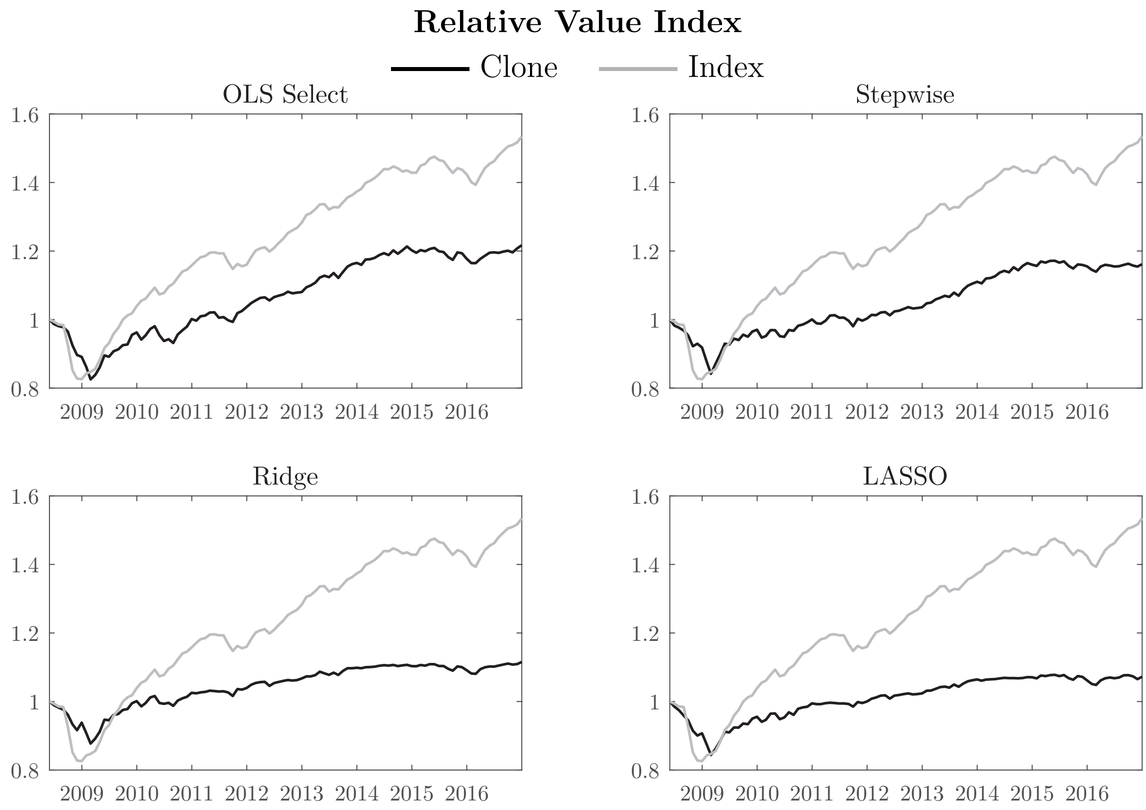

Figure D4: Cumulative returns of relative value index

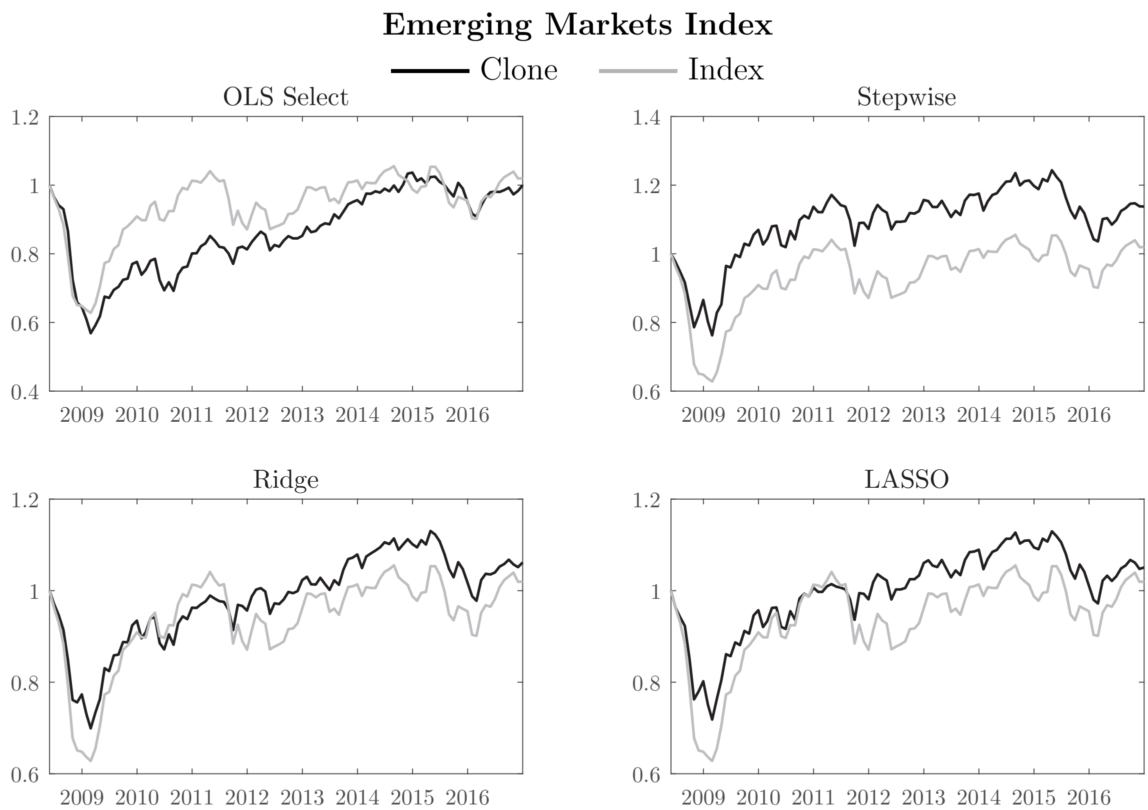

Figure D5: Cumulative returns of emerging markets index

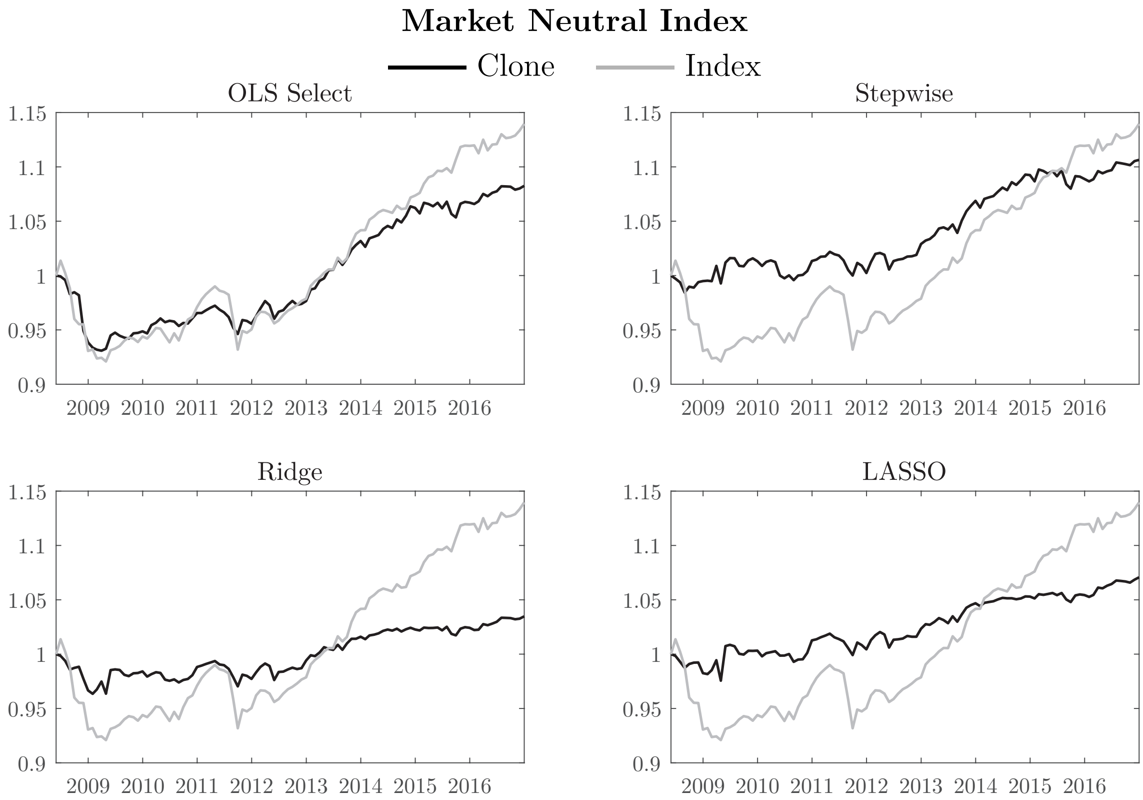

Figure D6: Cumulative returns of market neutral index

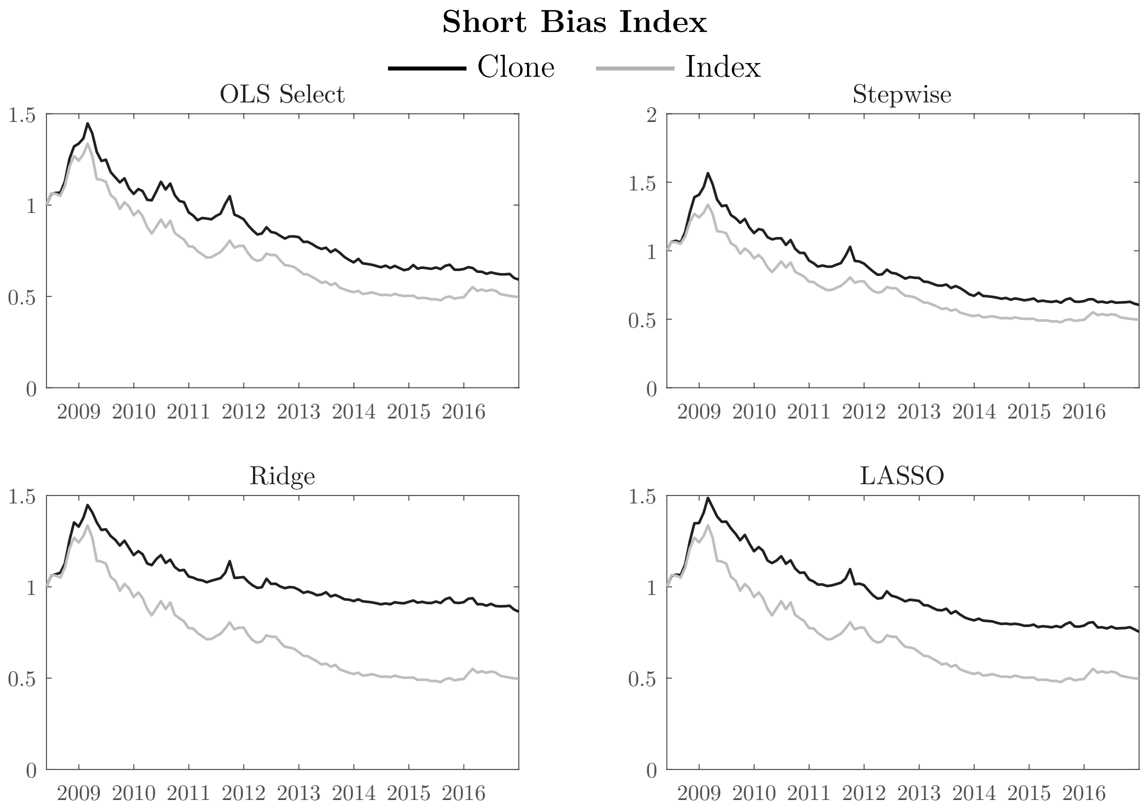

Figure D7: Cumulative returns of short bias index

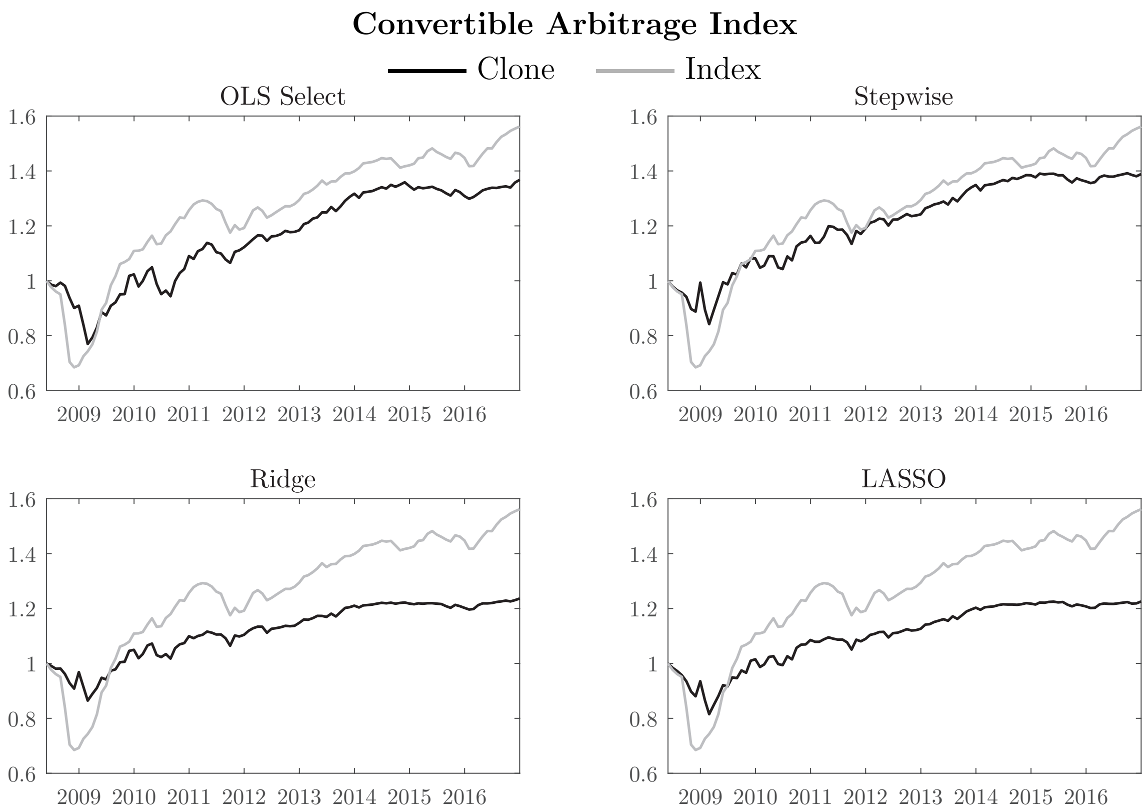

Figure D8: Cumulative returns of convertible arbitrage index

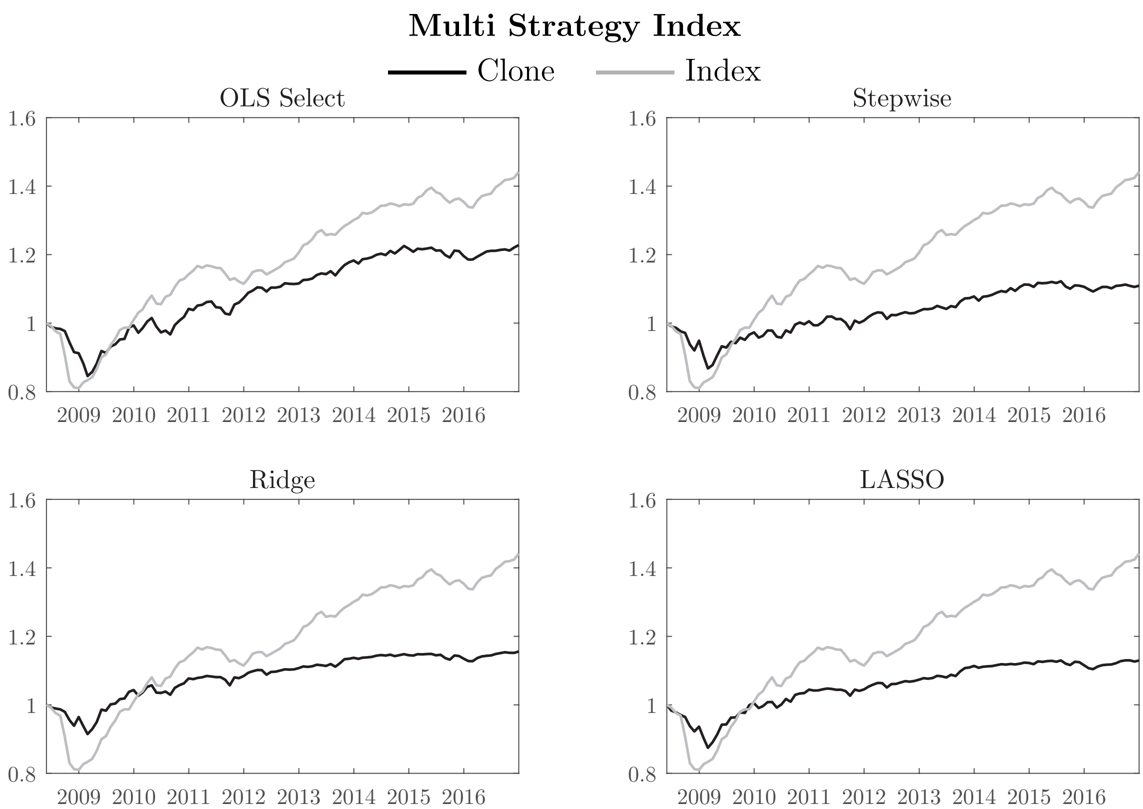

Figure D9: Cumulative returns of multi strategy index