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## Subscription loan facilities in private equity

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#### Abstract

Driven by low interest rates and an aim for private equity funds to reach higher performance rankings, the usage of subscription loan facilities has surged in recent years. In one of the pioneering in-depth analyses of these facilities, this paper aims to meet the private equity industry demand for a report that sheds light on the impact on fund performance and the incentives behind these facilities. In an academically backedup model, we verify and prove previous claims that subscription loans boost fund IRR in positive fund performance scenarios and magnify the negative fund return in negative fund performance scenarios. We elaborate on earlier claims by showing impacts on funds performing at different return levels. We show that fund managers (GPs) only gain money in certain IRR brackets when there is a catch-up in place and we show that having a catch-up is incentive aligning in regard to the loans. Looking at incentives we argue that the GPs care more about the IRR than actual money made since this will enable them to rank higher than other funds. This improves their chance of attracting capital from investors ahead of other PE funds as well as other asset classes. We also show that LPs are always affected negatively in terms of actual money made, suggesting that subscription loan facilities amplify the agency and control problems within the contract theories of private equity, and can be classified as return manipulation according to earlier literature definitions. $\alpha_{22327 @ \text { student.hhs.se }}$ $\beta_{22271 @ \text { student.hhs.se }}$


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## 1. Background and introduction

### 1.1 Introduction to subscription loan facilities in private equity

The theme of subscription loans is currently a buzzing subject within the private equity industry. The phenomenon of private equity funds using subscription loans when acquiring companies instead of making capital drawdowns from investors is new and has poor academic coverage. Initially used by funds as a mean to swiftly take advantage of acquisition opportunities, the phenomenon has recently been deemed as more of an IRR-boosting or return manipulation tool. Previously these facilities were held for short duration, such as one month, during which time they had limited effects on IRR. Now these facilities are systematically held for longer periods, of up to one year, under which condition they can have extensive impact on IRR.

Industry players show a high demand to see the impact, mainly on fund returns, of the facilities. Having talked to various private equity funds as well as investors, there is an apparent request for a research that sheds light on this new phenomenon ${ }^{2}$. The high demand from industry players together with the fact that subscription loan usage in private equity funds have increased rapidly in recent years ${ }^{3}$, motivated us to perform this research.

The aim of our research is to understand the effects of these loans for each party involved, in terms of IRR and actual money made, and to discuss the incentives to the parties connected to these facilities. The private equity covenant structure has been, in many regards, unchanged since its inception in the early eighties. This is specifically true for performance and compensation measures such as IRR, hurdle rates and carried interest (Phalippou and Gottschalg (2009)). Our paper will add much value to the knowledge about how the private equity business work by describing this new feature used by PE firms to boost return and shift compensation.

This research paper is based on qualitative interviews with different stakeholders in the private equity industry, including LPs, GPs and banks. We build a model for private equity returns, solidly backed up since it extends on a previously used private equity returns model in academic research (Metrick and Yasuda (2010)). We further complement it with a subscription loan feature with input from interviews with industry players and a bank involved with subscription loans. We then simulate the usage of the

[^1]bridge-loans in a private equity context to observe the effect of the loans to the fund and the different parties. In our model, we show the theoretical impacts of subscription loans.

We find that the subscription loans have an extensive impact on the reported fund return while reducing the value to the LPs. A fund that returns a net IRR of c. 18-19\% would be able to become a top quartile fund (Preqin (2016)) by increasing its net IRR up to $22 \%$. For funds with a catch-up ${ }^{4}$ feature we find that the compensation is increased for the GPs only if they return a mediocre amount of return between gross IRR of 8 and $12 \%$ while above that they lose money in terms of compensation. Regarding the subscription loans, we find that having a catch-up structure aligns the interest in compensation at higher return levels, above $12 \%$ gross return, while not having a catch-up increases the GP compensation in all cases where the GPs earn carry. This is true in a low interest environment and under the conditions we work with in this paper.

### 1.2 Purpose of our study and research questions

As outlined in the literature review chapter below, there exists very little earlier research on subscription loan facilities because it is such a new phenomenon. While previous research states that the subscription loan facilities have certain impact on fund return, our analysis provides a more thorough approach with detailed description of the elements of compensation. As one of the pioneering analyses of these facilities, the primary purpose of our paper is to shed light on the effects and incentives behind the usage of subscription loans. As outlined in the introduction section above, we aim to produce a report to meet the demand of further knowledge on the phenomenon among private equity industry players. Our research focus is within two main areas:
(1) We aim to study the theoretical implications of the subscription loan facilities. We discuss what the impact on fund performance (IRR) is, and how the loans affect actual money made to investors (LPs) and fund managers (GPs). As a part of this we aim to look at how the loan facilities impact the different sources of compensation earned in a private equity deal (pre-hurdle, hurdle, catch-up and profit sharing). Based on our model findings, we relate this to previous reports to verify their claims, but also to elaborate on under what conditions the loans have different effects on IRR and actual money made and thus become more optimal to have in place for the different parties. This we do by performing sensitivity analyses as well as IRR bracket analyses. We set up hypotheses that have their foundation in earlier research and applicable theories, outline these in section 3 of this paper and provide answers in section 5 .

[^2](2) Based on our findings in our first research area, as well as on correspondence with private equity industry players in interviews, we aim to describe the incentives behind the subscription loans. We try to discuss why the different parties involved in a private equity deal pursue this structure.

To continue, we aim to describe in our conclusion if the subscription loans drive the agency conflicts and control theory problem of financial contracting theory in private equity. We also aim to discuss whether our findings justify these facilities or classify the phenomenon as return manipulation according to definitions provided in previous research (Ingersoll et al. (2007)). There is also a game theory approach that could be used to describe the recent surge in these loans, in line with previous reports on the increased competition between private equity funds and asset classes. If certain funds start to use the bridge loans instead of committed capital, this magnifies the effect of their returns. Funds that are not using these loans are operating without that magnifying effect which might affect their IRR and therefore make them seem inferior to other funds effectively creating a need to use these facilities to be on the same terms in perceived performance as other funds.

### 1.3 Outline of this paper

The following section (2) outlines earlier research on subscription loan facilities in the private equity industry, including relevant economic theories that a discussion of the topic can be based on. After that, in section (3) we describe what we specifically want to look at in this paper based on the earlier research and interviews, and outline our specific hypotheses. In the following section (4) we describe our methodology and how we set up our model to shed light on the theoretical implications of these facilities. Then in section (5) we present and analyze our findings, and section (6) contains our conclusions and suggestions for further research.

## 2. Earlier research and literature review

Our paper is written within a new phenomenon of the private equity industry, which means the empirical reports on the subject are limited. The phenomenon of subscription loan facilities has only existed to a large extent for a few years ${ }^{5}$, especially in Europe and the Nordics.

This part of the paper (section 2.1) starts with describing the private equity deal structure and how subscription loans are different from common private equity deals. In the second part (2.2) we aim to depict what earlier reports say on incentives to parties involved and describe what has been found in terms of the theoretical implications, mainly the IRR impact. In the next section (2.3) we depict earlier reports on the relationship between contract design and fund performance. Finally, we include a section (2.4) discussing the relevant broader economic theories that we aim to relate our results to.

### 2.1. Description of private equity deals and subscription loans

As described by Kaplan and Strömberg (2009), a private equity deal, or leveraged buyout (LBO) is when a target company is acquired by a specialized investment firm (private equity, or "PE" firm) using a relatively large portion of debt (normally $60-90 \%$ ) in relation to equity to pay for the target. The debt is provided by for instance banks as a loan, and the PE firm raises equity capital through a private equity fund. The PE firms are organized as limited partnerships where the general partners (GPs) are managers, making the investment decisions, and normally provides approximately one percent of the equity capital. The equity is raised from investors such as pension funds, also known as limited partners (LPs), and most funds are "closed-end". The LPs commit to give a certain amount of capital to pay for investments and management fees to the PE firm, and closed-end means the LPs are not allowed to withdraw their funds until the fund is terminated. (Kaplan and Strömberg (2009)).

The equity portion is received from the LPs in a "capital draw-down", thus using up the capital that the LPs have committed to the life of the fund. It is common for funds to have a fixed life, normally 10 years (that can be extended by up to three years), before the fund is terminated. Before this, the fund has an investment horizon, of up to five years normally, to invest the committed capital and then five to eight years to give the LPs back the capital by exiting investments with the created return. As long as the covenants of the funds are followed, e.g. type of asset class investment, the LPs do not have much to say regarding the investment decisions of the GPs. (Kaplan and Strömberg (2009)).

The GP compensation is done in three ways. To start with, an annual management fee is charged as a percentage of the committed capital, and as investments are made the GPs charge a percentage of the

[^3]capital that is put into action. Secondly, GPs can charge a deal and monitoring fees to the target companies they invest in. Finally, the GPs make a share of the profits of the fund, known as "carried interest" or simply "carry", which is almost always $20 \%$ (Kaplan and Strömberg (2009)). However, normally private equity firms have a "hurdle rate", a minimum rate of return for the fund they must achieve before they can collect their carry. The hurdle rate is normally $8 \%$ internal rate of return (IRR), i.e. annual return on the invested equity. ${ }^{6,7}$ Except for this $20 \%$ profit sharing region, it is common for funds to have a catch-up phase where GPs gain more than $20 \%$ of the profits, e.g. $60 \%$ between fund returns of $8-12 \%{ }^{8}$

Appelbaum (2016) describes a PE deal involving a subscription loan facility as similar to a home equity line of credit. When people buy homes, they use the equity in the homes as collateral for the line of bank credit to be used as needed. In the same way, PE firms use the LP committed capital as collateral to get a line of credit on fund level that they can use as needed. In other words, instead of calling committed capital from investors when acquiring a company, the PE firms use a subscription loan to finance the acquisition, thus deferring the capital call, normally for one year. ${ }^{9}$ Previous studies of the effects of this replacement of the equity drawdown with a subscription loan facility are depicted in section 2.2 below.

### 2.2. Earlier reports on incentives of subscription loans and theoretical implications

The usage of subscription loans as described by Flood (2016), where a main incentive for GPs is underlined as being able to show a higher IRR and charge higher performance fees. Flood (2016) describes the usage of subscription loans as fairly little-known and how for instance Willis Towers Watson (one of the world's largest adviser to pension schemes) claims that it is a financial engineering technique used by private equity managers to boost the fees that private equity firms can charge the LPs. Flood (2016) depicts that since the LPs cash are put into work later, the IRRs calculations are made over fewer years, boosting the IRR. This phenomenon is expected to continue: "I suspect that all private equity fund managers are looking into this as they realize that without using subscription line financing, they are being left behind when it comes to their internal performance calculations." A main reason behind the increased usage of these loans is claimed to be the current low interest-rate environment, which allows private equity funds larger debt funding from banks (Flood (2016)). ${ }^{10}$

[^4]Flood (2016) also refers to professor Ludovic Phalippou at the University of Oxford, who says that LPs to some extent do not understand how bridge loan facilities work to the advantage of the GPs. Phalippou describes how the facilities enable the IRRs that are presented to the LPs to appear "stratospheric" where the actual money made to investors is modest, and again boost performance fees. In terms of incentives for GPs, Flood (2016) emphasizes that in the light of increased competition and current low interest rate environment, GPs are pushed towards using the bridge loan financing to reach the returns required to trigger their performance fees and quotes Christian Kvorning at the Danish pension fund PKA: "The standard 8 percent return hurdle that has to be met to trigger performance payments has become a higher bar to hit in the current low-rate environment". Kvorning states that it sometimes is necessary and understandable that the GPs use this structure, to swiftly make an attractive deal go through, but that they as investors must "pay the price" in terms of a higher performance fee, which thus does not boost the actual money (MoM) made to investors and can even decrease it. Flood (2016) does however also state that some LPs support bridge loan financing and even encourage GPs to use the facilities since some LP managers run funds that are paid performance bonuses based on the private equity investment IRRs. ${ }^{11}$

Appelbaum (2016) also looks into why the subscription loans exist. Pension fund investors, the largest group of investors in PE funds, have been rewarded large returns by their GPs in the past ${ }^{12}$. However, over the last decade (2006-2016), financial economists claim that the PE firms have failed to beat the stock market ${ }^{13}$. Based on past returns (prior to the last decade), pension funds continue to expect high returns on their investments in the PE funds. To maintain the pension fund money invested in the PE funds, and not switch to some other asset class, one incentive for the PE funds to use subscription loan facilities is to boost their IRRs. LPs are tricked by looking at the IRR reported to them by the GPs, and the IRR as a performance measure has been discredited by finance professors. Some pension funds and other LPs in the look at another, "truer", measure called the Public Market Equivalent (PME) ${ }^{13}$, but the IRR is still the most widely used measure of performance. The IRR has been manipulated by private equity funds in the past, one example being loading up portfolio companies with debt to pay dividends to investors in the first years of the fund's life. ${ }^{12}$ Kaplan et al (2016) also report that underperforming private equity funds boost their reported returns (looking at Net Asset Value manipulation) during fundraising times. They do however argue that these funds are unlikely to raise a next fund pointing towards the conclusion that investors will eventually see through the return manipulation. Due to the lower private equity fund

[^5]performance over the past decade, these manipulations are not enough to show the returns the LPs expect in order to maintain their money in the PE funds ahead of other asset classes such as public equities.

Appelbaum (2016) argues that the subscription line facilities, just as previous return manipulation tools, boost the IRR with no relation to what the investors ultimately receive in terms of money multiple. Appelbaum (2016) describes how the subscription line loans were originally used to move swiftly to benefit from an acquisition opportunity. However, Appelbaum (2016) argues that this used to be on relatively rare occasions and PE funds would use bridge loans when they needed to make a move before committed capital could be called, and when there was an imminent risk of missing out on an acquisition opportunity. Rapidly after the acquisition, the capital would be called and the subscription line loan would be repaid. ${ }^{14}$

The subscription lines are not anymore used as a bridge to finance an acquisition opportunity where committed capital could not be called, but instead as an alternative to calling commitments from LPs in the first couple of years of the fund life. Appelbaum (2016) depicts that if committed capital is drawn early on during a fund's investment horizon, IRR is negative during these years because capital flows out from LPs before there is any return on investment. Using a subscription facility to make acquisitions reduces the time under which LPs capital is held which increases the IRR of the fund, without increasing the actual returns to LP investors; the first capital draw-down from the LPs comes closer in time to the last repayment to the LPs. Furthermore, Appelbaum (2016) depicts that another incentive for the GPs of the PE funds is to attract capital ahead of not only other asset classes as described earlier, but also ahead of other PE funds. When certain funds use the subscription line strategy, and some do not, it becomes difficult for LPs to compare the funds. LPs make investment decisions to a large extent based on the reported IRR, making the subscription line strategy attract more capital to the certain PE funds that use it. ${ }^{15}$

TorreyCove Capital Partners (2016) report that there is a recent surge in the usage of these facilities. An incentive that is mentioned for the GPs is to smooth fund cash management and to close deals quickly. However, TorreyCove underlines what Flood (2016) and Appelbaum (2016) describe, that a likely reason is to take advantage of the current low interest rates and use fund-level debt to boost fund IRR. In terms of theoretical impact of the subscription loan phenomenon, TorreyCove (2016) provides examples of an equity only investment (normal capital draw-down from the LPs) versus subscription line scenarios for both profit and loss cases, see appendix (section 8.1). The low-cost debt subscription credit line defers

[^6]capital calls, lowering the investment holding period. IRR is boosted by this fact in the gain scenario, and the negative impact is magnified in the loss scenario, where the leakage from debt service becomes an additional cost that is not weighted up by positive returns. In line with the previous reports outlined earlier in this literature review, the actual money investors make, put across as the effect on the net multiple (MOIC) ${ }^{16}$ is negative for the LPs, due to the leakage from debt service. TorreyCove's (2016) specific scenario shows that absolute IRR is boosted, enabling GPs to grab hold of carried interest, but they also underline Appelbaum's (2016) conclusion that another important incentive for the GPs is that the credit lines boost relative performance to attract capital ahead of others. TorreyCove (2016) outlines that the credit line usage can push funds into a better "quartile ranking" and improve reputation and attract more capital.

To sum up, the previous reports on subscription loans outlined above do not provide any strong evidence to back up their conclusions on the impact of these loans. The TorreyCove (2016) example is for one fund setting with only one gain and one loss scenario, with equity only investments. We therefore believe that building an extensive model on fund returns will add a lot of depth and shed more light on the theoretical implications of the subscription loan facilities, to see under what conditions these loans have the most effect and when it benefits each party the most. As outlined in more detail later in the methodology section (4) and results section (5), we build a model for private equity returns based on an approach used by Metrick and Yasuda (2010), and then we add a subscription loan feature.

### 2.3. Earlier reports on contract design and private equity performance

Metrick and Yasuda (2010) build a model in which they examine the relationship between investor contract characteristics and expected revenue to the GPs, and how this varies across contract characteristics of funds. The input for their model is based on a sample, they use 238 funds (of which 144 are buyout funds), and find that many common differences in contracts leads to large differences in expected revenue to the GPs. In line with this, a subscription loan feature is another contract difference that we will look at, which makes our research add to theirs. In addition, it is not a common contract difference, such as having a variable payment structure to the GPs (profit sharing, or carry), but rather a new phenomenon that has not been looked at before. Even though Metrick and Yasuda (2010) do not specifically draw conclusions on the contract structure impact on fund performance (IRR), they include fund performance in their model, as a function of fund terms, since some GP revenue such as the carried interest is driven by the performance of the fund. In our methodology section 4, we describe how we use

[^7]the model of Metrick and Yasuda (2010) as a foundation, and then add a subscription loan feature to look at IRR effects and other theoretical implications. We would, in line with Metrick and Yasuda (2010) hypothesize that such a difference would in general lead to large differences in IRR and thus revenue to the GPs. The results we find are displayed in our analysis section (5).

In terms of further research, Caselli et al. (2013) study the relationship between contract design and PE returns, based on sample fund data in Italy. They perform a broader study of private equity contracts that have a lot of covenants and how that impacts the IRR and observe that covenant-heavy contracts are associated with higher returns. Caselli et al. (2013) look at the impact of how adding or removing several covenants impacts the IRR. They find for instance that adding an incentive covenant for the GP is associated with a higher return for the fund. In line with Drucker and Puri (2009), Caselli et al. (2013) look at the return impact of covenants that to a varying degree are common in private equity contracts, and have been so for a longer time, e.g. lockups, permitted transfers, first refusal and tag-along and drag-along rights. Our research adds to theirs, since it is focused on a new specific element of a private equity deal, namely the potential usage of a subscription loan facility.

### 2.4 Relevant economic literature and theories we relate our results to

Based on the alleged problems with the subscription loan facilities outlined above, we aim to discuss the phenomenon on the foundation of contracting theories in private equity. According to the outline by Kaplan and Strömberg (2003), this includes agency theory, control theory and debt theory. We argue that the first two are applicable for subscription loans. The agency theory, pioneered by Holmström (1979) describes the problems in agency relationships due to unaligned goals or levels of risk aversion of principals and agents. This situation may occur because the principal is not aware of the actions of the agent ${ }^{17}$. A principal could be a shareholder in a company and the agent, that is supposed to be acting in the principal's best interest, could be the company executive. In private equity, the principals are the LPs investing money in the fund, and the agents are the GPs that make the investment decisions. Axelson et al (2009) depict an agency conflict in that the GPs find the investments but then rely on external capital from the LPs to finance the investments. Since GPs have a limited liability and take less downside in the deals, they have an incentive to overstate potential values of investments and thus reported IRR when attracting financing from uninformed LPs, which is why this opens for potential agency problems between the LPs and GPs. Since previous reports claim that subscription loans boost IRR, we would say that the phenomenon could amplify this agency problem.

[^8]The agency problems are also emphasized by Robinson and Sensoy (2013), who describe that private equity contracts often do little to discipline GPs or to make GPs maximize LP returns. Subscription loan facilities might therefore enhance this statement, since many LPs are not measured on IRR, but rather on actual money made, which has been claimed to become lower with the facilities. For LPs that are so-called "sources of money" that are not measured on IRR, such as Swedish pension fund AP6, the allowance of subscription facilities is potentially a covenant that harms the LPs rather than benefits them. This since it has been argued by previous research that the subscription loan facilities lower the MoM, i.e. actual money made to LP investors. Also, as described by Kaplan and Strömberg (2009) as well as Gompers et al (2016), one value creation tool in private equity is to align incentives between LPs and GPs, this tool is called governance engineering. A subscription loan might therefore cause problems for these alignments of incentives. We aim to verify if these claims are true for various fund performances in our model.

The control theory in financial contract as outlined by Grossman and Hart (1986) and Hart and Moore (1990), elaborate on the classic agency theory and state that some actions are observable but not verifiable. Aghion and Bolton (1992) describe how some projects include private actions or benefits that are non-verifiable and only go to the entrepreneur (in VCs) and equivalently to the GPs (in BOs). Relating to this, Conner (2005) describes how LPs often end up with partnership terms and conditions without clearly understanding the relative value of the individual covenants put into the contracts with the private equity funds. We mean that if LPs do not understand what subscription loans mean for them in terms of money made, which has been speculated by previous research (e.g. Appelbaum (2016)) and mentioned in some of our interviews, the subscription loans can be viewed as benefits that are non-verifiable. If we in our analysis are also able to verify that the benefits do not go to the LPs, but rather to the GPs (in terms of actual money made), this loan phenomenon would be in line with and enhance the control theory problem.

Relating to this, as described earlier, Appelbaum (2016), Flood (2016) and Kaplan et al (2016) claim that using subscription loans is an example of fund return manipulation. Ingersoll et al. (2007) study return manipulation and state that there is a potential for moral hazard, in line with the limited liability put across by Axelson et al (2009) described above, when investors quantitatively evaluate and rank GPs of funds. Return is manipulated to attract capital ahead of other PE funds as well as other asset classes, in a competitive situation. In the evaluation process, GPs have an incentive to manipulate whatever performance measure they are ranked on. Ingersoll et al. (2007) define manipulation as "the action to increase fund return in a way that does not actually add value for the fund's investor". This is in line with
what earlier reports state is true for subscription loan facilities ${ }^{18}$. We aim to see if we with our extensive model, by looking at the NPV to LPs and/or MoM, can verify if this definition of return manipulation applies to the facilities.

In our conclusion, section 6, we add a brief comment to motive on the surge of these loans from a game theory approach; "making logical decisions in competitive environments" ${ }^{19}$. In line with e.g. Harsanyi and Selten (1988), we discuss rational solutions in non-cooperative games, and we use this part of game theory to discuss why funds might use these loans in a competitive situation.

To sum up, in our model we will test how subscription loans impact IRR, but also actual money made. This to verify whether actual money made goes down for LPs, and thus if the loans amplify the agency and control problems, and qualify as potential manipulation features.

[^9]
## 3. Specification of areas of research

This section outlines what we want to investigate connected to previous papers and theories and what answers in our interviews we want to put to a test.

### 3.1 IRR implications

We want to investigate the effect on IRR from implementing these loans, specifically the magnitude of the effect and what factors drive it. The factors we will look at include the loan spread and the so-called eligible investor ratio, described in the methodology section (4). Also, looking at our data-set, we want to investigate how many funds actually achieve a higher quartile ranking if they were to implement subscription loans.

The following four hypotheses address the earlier reports on IRR effects and the fact that the interest rate has been pointed out as a driver of the increase in the usage of the loans.

Based on previous research the loans should increase IRR in positive fund scenarios and decrease IRR in negative fund scenarios ${ }^{20}$. Our first hypothesis therefore becomes:

Hypothesis 1: The IRR effect of the loans on fund performance is positive for funds with positive return and negative for funds with negative return.

The magnitude of the effect has been estimated to be $6 \%$ on gross IRR and $4 \%$ on net IRR ${ }^{21}$. However, the magnitude is dependent on how the fund is performing which leads us to our second and third hypothesis:

Hypothesis 2: The magnitude of the positive effect on IRR imposed by the loans is dependent on how well the fund is performing.

Hypothesis 3: The magnitude of the positive effect on IRR is large enough to increase funds' quartile rankings in the sense that a fund can increase its IRR by several percentage points.

Hypotheses 1, 2 and 3 are addressed in section 5.2.1 in our IRR bracket analysis.
From our interviews, we have learned that an important factor for these loans to exist is the low interest rate charged by the banks ${ }^{22}$. We therefore formulate a simple hypothesis around this statement.

[^10]Hypothesis 4: As the cost of the loans increase (loan-spread) the ability of the loans to increase net IRR decreases.

Hypothesis 4 is addressed in section 5.1.1 in our sensitivity analysis.

In the way the loans are constructed we should expect that funds will prefer certain LPs over others to increase their eligible investor ratio which will allow them to take on more loans. This factor was emphasized by our bank contact as a major factor in the subscription loan allowance. Investigating this further we formulate our fifth hypothesis.

Hypothesis 5: A higher eligible investor ratio improves the funds ability to increase its IRR through loans.

Hypothesis 5 is addressed in section 5.1.2.

### 3.2 Value and incentives to parties involved

Here we investigate if GPs earn more carry and/or if they earn money faster in terms of earning carry at a lower return, since this has been stated in previous research and interviews ${ }^{23}$. We then examine the extent to which LPs and GPs pay for these facilities and how their compensations are connected to the loan facilities.

The following hypotheses address earlier research claims about value creation, but also to relate our results to financial contracting theory (agency, control) and return manipulation.

Hypothesis 6: GPs on average earn more money by using bridge-loan facilities.

Hypothesis 7: GPs earn money at lower IRRs when using the loans compared to when not using the loans.

Hypothesis 8: GPs start to collect more money earlier in the fund's life with loans.

Hypotheses 6, 7 and 8 are addressed in section 5.2.2.

Hypothesis 9: LPs lose money when these loans are used

Hypothesis 9 is addressed in section 5.2.3.

[^11]The data will be analyzed as a comparison between how a certain case looks like without a loan and then how that case would look like if loan was implemented. The change in distribution of capital between the GPs and LPs will be analyzed through looking at the carry distributed to GPs and LPs and we will elaborate if there is a change and if this change is large enough to warrant that there are agency costs associated to these loans.

## 4. Methodology

We build a theoretical model based on Metrick and Yasuda's approach (2010), where we simulate fund investment value developments with and without a subscription loan facility and look at the theoretical implications; IRR effects, actual money made to GPs (PV) and LPs (MoM and NPV). In running the simulations, we apply a risk neutral development of the price paths on our investments. The set-up of the model is described throughout this section, and the results are analyzed in the next section 5 . We use qualitative interviews to be able to get input to the structure of the subscription loans and to understand the incentives behind them, i.e. who benefits from them and how. We use interview correspondence with one of the major Nordic banks that offers these types of loan facilities, LPs, GPs and a lawyer experienced in the structure of these loans.

### 4.1 A model for theoretical implications of the subscription loan facilities

There are some limitations to our topic. First, the subscription loan phenomenon is so new that it is not possible to fully observe any effects on private equity fund returns in empirical data yet ${ }^{24}$. Second, it is generally very hard to get hold of data from the private equity industry due to a low requirement of public reporting. To be able to estimate the IRR impact of the subscription loans we therefore start with creating a theoretical model, based on a set-up provided by Metrick and Yasuda (2010) and based on the information we have gathered in our interviews.

Metrick and Yasuda (2010) use a fund sample for inputs to their model. Their primary fund dataset is anonymously provided by "one of the largest LPs in the world" and contains 238 funds raised between 1993 and 2006. They make a distinction between venture capital (VC) funds and buyout (BO) funds, where we focus on the 144 BO funds in the sample. This sample is complemented by other private equity databases, that they have access to such as Galante's venture capital and private equity directory. Even though they have access to this private equity return data, they still build a theoretical model to examine the revenue (fee, carried interest, etc.) to GPs under different fund development scenarios. They argue that instead of using the empirical data to estimate the actual revenue, or in our case return earned by the funds, a model approach makes more sense. They argue that this is partly because the number of funds for which fund term information is available is too small. Also, many sample funds in their dataset had been raised recently, i.e. with no information on finalized returns, which is in line with our problem of a new phenomenon with no return impact data available (Metrick and Yasuda (2010)).

[^12]In the rest of this section we describe our model. Section 4.1.1 contains our model input assumptions based on Metrick and Yasuda (2010) and on our interviews. We use a benchmark case with fixed inputs to describe our model, and in the findings and analysis part (section 5) we perform sensitivity analyses where we change these inputs. Section 4.1.2 contains a walk-through of the no-subscription loan scenario using an example of one of the simulation runs and section 4.1.3 depicts how we model the subscription loan scenario. The same inputs and investment value development simulations apply for both fund scenarios, and then we look at how the subscription loan feature affects the IRR and other outputs in our simulation runs, as outlined in section 4.1 .4 below.

### 4.1.1 Model inputs for the benchmark model

For input on the structure of the subscription loans, we use interview answers from a meeting with one of the major Nordic banks, which provides these loans, and phone interviews with LPs and GPs. Our benchmark case investment inputs are summarized in the appendix, exhibit 8.2.1. In our model, we use a committed capital amount of SEK 1 billion. According to AP6, a standard for private equity funds is to have a bridge loan of 20 percent of committed capital up to one year ${ }^{25}$, which is also verified by our bank contact. ${ }^{26}$ Of the committed capital, the fund uses 80 percent for investments, based on Metrick and Yasuda's model (2010). The rest of the capital will be kept as a buffer for fees. In line with Metrick and Yasuda (2010), we assume that funds are fully invested at the end of the investment period ${ }^{27}$, and assume a five-year investment period and a following five-year divestment and repayment period, which is also a common set-up according to Kaplan and Strömberg (2009). We assume that each fund make 10 investments ${ }^{28}$, divided over two investments each year. The management fee level is set to 2 percent, in line with Metrick and Yasuda (2010). The calculation of yearly management fee is depicted in more detail below relating to exhibit 4.1. To make our model simpler we have chosen to ignore the minor fees: transaction fees and monitoring fees of the fund. As for the holding period of each investment these are simplified and set to a certain time, in reality investments have unknown and varying exit dates. In Metrick and Yasuda's (2010) sample data, they show that a median first-round VC investment have an annual probability of exit of around 20 percent, which they also assume for BO funds. They then randomize exits each year based on this probability. However, a problem with Metrick and Yasuda's (2010) approach is that IRRs can be abnormally high if for example in one simulation-scenario an exit

[^13]would happen the first year at a very high value. This gives us an unstable model with big swings in simulated IRRs. Kaplan and Strömberg (2009) depict that the median holding period for a large sample of private equity investments from 1970 to 2007 is roughly six years. They do however argue that this has varied over time and more recently median holding periods have been less than five years, affected by the hot IPO market in the late 1990s and they argue that private equity firms have become more short-term oriented as of lately (Kaplan and Strömberg (2009)). In a more recent study, Gompers et al (2016) also find that most PE firms (almost 100 percent, see appendix, exhibit 8.2.5 for table) expect to hold their investments for five years. Based on this we assume a five-year holding period for all investments in our benchmark model. Furthermore, in line with Metrick and Yasuda (2010), we assume that exit dates are uncorrelated with the returns the investments make. They assume that if an investment is randomly drawn to be divested in one year, it will be regardless of the value of that investment. Similarly, we assume that the investments will be sold after five years, in all cases. While Metrick and Yasuda (2010) conclude that this assumption is false, they also state that it is computationally very hard to incorporate this type of correlation in a model.

As depicted in the literature review, most private equity funds use leverage when acquiring companies. Metrick and Yasuda (2010) assumes a leverage ratio of $2: 1$ in their benchmark case, which means that debt is $200 \%$ of equity in each acquisition on the company level. How we model the leverage component is depicted in more detail below relating to exhibit 4.3. We assume a hurdle rate of 8 percent, which is in line with industry standards (Metrick and Yasuda (2010)). We assume that the carried interest is 20 percent, in line with the indications of Kaplan and Strömberg (2009) and Metrick and Yasuda (2010). This means that the part of a simulated fund return that exceeds $12 \%$ IRR is shared with 20 percent to the GPs and 80 percent to the LPs. In between these two compensation parts we assume a catch-up phase with a 60 percent share to the GPs and 40 percent share to the LPs. The catch-up rates are set between 8 and 12 percent, which is common in a private equity fund ${ }^{29}$; Phalippou and Gottschalg (2009) present a sample of funds in which $83 \%$ had a hurdle rate of which $74 \%$ set it at 8 percent and $92 \%$ had a catch-up phase. Our catch-up means that the money returned from the fund that creates IRR between 8 and 12 percent is divided in the 60/40 fashion. How these parameters are modelled is outlined below relating to exhibit 4.5.

Exhibit 8.2.2 in the appendix contains a summary of the inputs in relation to the subscription loan facilities. These inputs have mainly been provided from our bank contact. In terms of interest rate on the subscription loans, our bank contact indicates that the interest rate is decided with a fundamental Interbank Offer Rate (IBOR) that depends on the currency of the fund plus an additional margin, today at around 2

[^14]percent. We assume our model funds are invested in Swedish Krona (SEK), why our interest rate used is the current Stockholm Interbank Offer Rate (STIBOR) plus 2 percent. The current STIBOR is negative for all maturities (one month - 6 months) ${ }^{30}$, but the IBOR-base for these loans cannot start below 0 percent. If the relevant IBOR is below 0 percent the base is set to 0 percent. This means the interest rate we use in the benchmark case for the bridge loans is 0 plus 2 percent, i.e. 2 percent. ${ }^{31}$

In terms of the time outstanding for the loans before they need to be repaid, both our bank contact and LP AP6 indicate that the funds hold the loans for a maximum of 365 days (one year), which is the timing setup we use for the bridge loans in our model. After one year, the bridge loan is replaced with another facility throughout the five-year investment period.

As mentioned above, it is common to have a subscription loan facility that cannot exceed 20 percent of the total committed capital. ${ }^{32}$ Additional drivers of the maximum allowed bridge loan facility are the eligible investor ratio and drawn debt ratio. Eligible LP investors are investors that banks assess to have lower risk of default, e.g. large pension funds, and less eligible LP investors are investors that are assessed to have higher risk of default, e.g. wealthy individuals. These ratios drive the maximum allowed bridge loan facility in the way that a fund with more eligible investors can borrow more. In the benchmark case, we assume that the fund has 100 percent eligible investors. This is done to allow for the funds to be able to take maximum amounts of subscription loans to fully assess the impact of the loans. How these inputs are modelled to drive the amount of bridge loans a fund can have is depicted below relating to exhibit 4.9. ${ }^{33}$

Below we outline our benchmark fund and investment schedule and respective implications for the scenario without subscription loans (section 4.1.2) and with subscription loans (section 4.1.3).

### 4.1.2 The no-subscription loan scenario

An input to our model we use from the Metrick and Yasuda (2010) sample data is the investment pace of the BO funds, where the pace from their sample is $26 \%, 23 \%, 25 \%, 18 \%$ and $8 \%$ of the investment capital in years 1 through 5, respectively. The investment pace drives the amount of invested capital over the investment period. In line with Metrick and Yasuda (2010), the management fee for the investment period is calculated as 2 percent of the total committed capital. After year 5, during the divestment period, the management fee is calculated as 2 percent of the net invested capital in the fund at the start of each year.

[^15]Net invested capital (the "Less realization (at cost)" row in exhibit 4.1) is calculated as total invested capital less the total realization at cost, i.e. less the initial investment values of the exited investments. For example, in year 1, the fund invests SEK 208 million. In year 6, the exit value at cost is SEK 208 million in total. For example for year 7, they start the year with SEK 800 m less SEK 208m, i.e. SEK 592 m of which 2 percent is SEK 11.840 m . Exit values at cost are outlined in exhibit 4.2. In the no-subscription loan case, the invested capital and the management fees make up the drawn capital amount from the LPs.

Exhibit 4.1 - Investment schedule without subscription loan

| Investments | Investment period |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\Gamma$ |
| Year |  | 1 | 2 | 3 | 4 | 51 |
| hrsestoneytava |  | $2 \%$ | $22 \%$ | 23\% | \%\% | $8 \% 1$ |
| Invested capital |  | 208000000 | 184000000 | 200000000 | 144000000 | 640000001 |
| Total invested capital |  | 208000000 | 392000000 | 592000000 | 736000000 | 8000000000 |
| Less realization (at cost) |  | 208000000 | 392000000 | 592000000 | 736000000 | 800000000 |
| Management fees | r | 20000000 | 20000000 | 20000000 | 20000000 | 20000000 |
| Total management fees |  | 20000000 | 40000000 | 60000000 | 80000000 | $100000000 \mid$ |
| Loan payments |  |  |  |  |  |  |
| Drawn capital |  | 228000000 | 204000000 | 220000000 | 164000000 | 84000000 |
| Cumulative total drawn capital |  | 228000000 | 432000000 | 652000000 | 816000000 | 900000000 |


| Investments | Divestment period |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | 6 | 7 | 8 | 9 | 10 |
| /7\%esthnerforace |  |  |  |  |  |  |
| Invested capital |  | 0 | 0 | 0 | 0 | 0 |
| Total invested capital |  | 800000000 | 800000000 | 800000000 | 800000000 | 800000000 |
| Less realization (at cost) |  | 592000000 | 408000000 | 208000000 | 64000000 | 0 |
| Management fees |  | 16000000 | 11840000 | 8160000 | 4160000 | 1280000 |
| Total management fees |  | 116000000 | 127840000 | 136000000 | 140160000 | 141440000 |
| Loan payments |  |  |  |  |  |  |
| Drawn capital |  | 16000000 | 11840000 | 8160000 | 4160000 | 1280000 |
| Cumulative total drawn capital | I | 916000000 | 927840000 | 936000000 | 940160000 | 941440000 |

Exhibit 4.2 - Investment exit schedule at cost for calculation of management fees beyond year 6*

| Yexir |  | 6 | 7 | 9 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Exitvalue at cost |  |  |  |  |  |  |
|  | 1 | 104,000,000 | 0 | 0 | 0 | 0 |
|  | 2 | 104,000,000 | 0 | 0 | 0 | 0 |
|  | 3 | 0 | 92,000,000 | 0 | 0 | 0 |
|  | 4 | 0 | $92,000,000$ | 0 | 0 | 0 |
|  | 5 | 0 | 0 | 100,000,000 | 0 | 0 |
|  | 6 | 0 | 0 | 100,000,000 | 0 | 0 |
|  | 7 | 0 | 0 | 0 | 72,000,000 | 0 |
|  | 9 | 0 | 0 | 0 | 72,000,000 | 0 |
|  | 9 | 0 | 0 | 0 | 0 | 32,000,000 |
|  | 10 | 0 | 0 | 0 | 0 | 32,000,000 |

*) Two investments per year, make up 10 investments in total. The SEK 104 m in year 6 is the SEK 208m invested in year 1 divided into two investments that are then realized in year 6 .

The leverage ratio of $200 \%$ debt to equity gives us a debt level for each investment the fund makes. For instance, in year 1 the fund makes an equity investment of SEK 208m, i.e. SEK 104m for two investments. This implies a debt level of SEK 208m for each investment. The debt is added to the initial equity values according to exhibit 4.3 below.

## Exhibit 4.3 - Initial equity and debt values



The numbers in exhibit 4.3 are the starting value points for looking at the value development of each investment. Exhibit 4.4 below outlines an example of simulated value developments and exit values. For example, looking at the first two investments made in year 1, they are exited in year 6, and the two investments made in year 2 are exited in year 7. Every simulation of exit values is used for both scenarios (with and without loan). To simulate the development of the value of the investments, we use a Geometric

Brownian Motion (GBM) approach ${ }^{34}$, in line with Metrick and Yasuda (2010).

## Exhibit 4.4-Investment value development

| Year | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Investment values and exits |  |  |  |  |  |
| 1 | 312000000 | 303871141 | 414269734 | 515742909 | 297074830 |
| 2 | 312000000 | 566255539 | 668935299 | 633743126 | 605248899 |
| 3 |  | 276000000 | 165281922 | 384061064 | 378683399 |
| 4 |  | 276000000 | 381276684 | 414536461 | 305324634 |
| 5 |  |  | 300000000 | 303875978 | 174140582 |
| 6 |  |  | 300000000 | 176062896 | 207660874 |
| 7 |  |  |  | 216000000 | 256320623 |
| 8 |  |  |  | 216000000 | 227029868 |
| 9 |  |  |  |  | 96000000 |
| 10 |  |  |  |  | 96000000 |
| Year | 6 | 7 | 8 | 9 | 10 |
| Investment values and exits |  |  |  |  |  |
| 1 | 488897460 |  |  |  |  |
| 2 | 1163918720 |  |  |  |  |
| 3 | 443400578 | 578658672 |  |  |  |
| 4 | 85740356 | 46287621 |  |  |  |
| 5 | 121395230 | 180632063 | 77204614 |  |  |
| 6 | 141253238 | 109779710 | 112921846 |  |  |
| 7 | 493112253 | 535268373 | 529996914 | 199715480 |  |
| 8 | 418965120 | 501299778 | 335811784 | 214026645 |  |
| 9 | 198898900 | 135974572 | 143590914 | 156172114 | 96924890 |
| 10 | 47621201 | 67389234 | 59781350 | 30900048 | 124001948 |

To simulate the value paths using GBM, we need to estimate the volatility and correlation of the investments. The volatility is set to $60 \%$ and the correlation to $20 \%$ in line with Metrick and Yasuda (2010), Campbell et al (2001) and Woodward (2010). Details regarding this are outlined in the appendix section 8.2. There we also describe our other GBM assumptions in closer detail.

With a five-year investment holding period, it is not until year 6 that distributions can be made. The distributions in year 6 and forward depends on both the simulated exit value and the debt the investment took on at a company level that must be paid back at exit. For example, looking at Exhibit 4.5 below in year 6, the distribution to LPs and GPs become c. SEK 1237m which is the total exit value in year 6 from

[^16]exhibit 4.4, i.e. SEK 489m and SEK 1164 m from the first two investments made in year 1 , less the debt that was initially used for those investments, two times SEK 208m in total from exhibit 4.3. ${ }^{35}$

Exhibit 4.5 further depicts how the LPs get back the capital that they have put into the fund, on the row "Returned pre-hurdle \& carry" that should eventually become the SEK 941m that the LPs initially put in. After that money has been returned to the LPs, the LPs get potential hurdle (the returned money creating between 0 and 8 percent IRR), while both LPs and GPs get catch-up (between 8 and 12 percent IRR) and profit sharing (above 12 percent IRR). In the example in Exhibit 4.5 it is at year 6 that some money is made available for hurdle and carry. The "Draw-down balance" for the period is the cumulative total capital drawn (see exhibit 4.1) less the cumulative distributions from exhibit 4.5, until the cumulative distributions become higher than the cumulative total capital drawn. When this happens, money becomes available for hurdle and carry.

## Exhibit 4.5 - Payoff to LPs and GPs

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Distributions | 0 | 0 | 0 | 0 | 0 |
| Cumulative distributions | 0 | 0 | 0 | 0 | 0 |
| Draw down balance | 228000000 | 432000000 | 652000000 | 816000000 | 900000000 |
| To be returned pre hurdle \& carry | 941440000 | 941440000 | 941440000 | 941440000 | 941440000 |
| Returned pre hurdle \& carry | 0 | 0 | 0 | 0 | 0 |
| Available for hurdle \& carry | 0 | 0 | 0 | 0 | 0 |
| Cumulative for hurdle \& carry | 0 | 0 | 0 | 0 | 0 |
|  | 6 | 7 | 8 | 9 | 10 |
| Distributions | 1236816180 | 394658672 | 0 | 125742126 | 92926838 |
| Cumulative distributions | 1236816180 | 1631474853 | 1631474853 | 1757216978 | 1850143817 |
| Draw down balance | 0 | 0 | 0 | 0 | 0 |
| To be returned pre hurdle \& carry | 941440000 | 941440000 | 941440000 | 941440000 | 941440000 |
| Returned pre hurdle \& carry | 941440000 | 941440000 | 941440000 | 941440000 | 941440000 |
| Available for hurdle \& carry | 295376180 | 394658672 | 0 | 125742126 | 92926838 |
| Curnulative for hurdle \& carry | 295376180 | 690034853 | 690034853 | 815776978 | 908703817 |

Exhibit 4.6 below continues from Exhibit 4.5 and depicts how the LPs get their hurdle. "Hurdle relating to period" is 8 percent of the draw-down balance for the period, found in Exhibit 4.5 above. When the drawdown balance has become zero, year 6 in exhibit 4.5, it means the LPs have received their initial put in capital, and any potential remaining distributions is paid out as hurdle. If the amount "Available for hurdle

[^17]\& carry" from exhibit 4.5 exceeds the hurdle paid to LPs, there is money left for catch-up and potential profit sharing.

Exhibit 4.6-Hurdle paid to LPs

| Hurdle | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Hurdle relating to period | 18240000 | 34560000 | 52160000 | 65280000 | 72000000 |
| Total hurdle outstanding | 18240000 | 52800000 | 104960000 | 170240000 | 242240000 |
| Balance | 18240000 | 52800000 | 104960000 | 170240000 | 242240000 |
| Hurdle paid to LP | 0 | 0 | 0 | 0 | 0 |
| Cumulative paid hurdle | 0 | 0 | 0 | 0 | 0 |
| Returns available to carry [GP \& LP] | 0 | 0 | 0 | 0 |  |
| Cumulative available carry | 0 | 0 | 0 | 0 | 0 |

## Hurdle

|  | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | ---: | ---: | ---: | ---: |
| Hurdle relating to period | 0 | 0 | 0 | 0 |
| Total hurdle outstanding | 242240000 | 0 | 0 | 0 |
| Balance | 0 | 0 | 0 | 0 |
| Hurdle paid to LP | 242240000 | 0 | 0 | 0 |
| Cumulative paid hurdle | 242240000 | 242240000 | 242240000 | 242240000 |
| Returns available to carry [GP \& LP] | 53136180 | 394658672 | 242240000 |  |
| Cumulative available carry | 53136180 | 447794853 | 447794853 | 573536978 |

Once the LPs have received a) their initially put in capital back and b) their hurdle for the returned capital creating up to 8 percent IRR, we include the catch-up phase. For the returned capital creating between 8 and 12 percent, the money is shared so that 60 percent go to the GPs and 40 percent go to the LPs ${ }^{36}$. We call this part "Carry tier 1" in the model. Once returned capital create between 8 and 12 percent IRR, it becomes available as "Returns available to carry (GP \& LP)", as seen in exhibit 4.6 above for year 6. Catch-up can then be paid out, as seen in exhibit 4.7 below for year 6 as "Paid carry (LP and GP)".

[^18]
## Exhibit 4.7-Catch-up paid to LPs and GPs

| Carry tier Lcatch-up] | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Net catch-up related to period | 9120000 | 17280000 | 26080000 | 32640000 | 36000000 |
| Total carry outstanding | 9120000 | 26400000 | 52480000 | 85120000 | 121120000 |
| Catch-up balance | 9120000 | 26400000 | 52480000 | 85120000 | 121120000 |
| Paid carry [LP \& GP] | 0 | 0 | 0 | 0 | 0 |
| Cumulative paid carry (LP \& GP] | 0 | 0 | 0 | 0 | 0 |
| Paid to GP |  | 0 | 0 | 0 | 0 |
| Paid to LP | 0 | 0 | 0 | 0 | 0 |

Carry tier U(catch-up)

| Net catch-up related to period | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | ---: | ---: | ---: | ---: |
| Total carry outstanding | 0 | 0 | 0 | 0 |
| Catch-up balance | 121120000 | 67983820 | 0 | 0 |
| Paid carry [LP \& GP] | 67983820 | 0 | 0 | 0 |
| Cumulative paid carry (LP \& GP) | 53136180 | 67983820 | 0 | 0 |
| Paid to GP | 53136180 | 121120000 | 121120000 | 121120000 |
| Paid to LP | 31881708 | 40790292 | 0 | 0 |

After the catch-up phase comes the profit sharing phase, which we call "Carry tier 2". For returned capital that creates above 12 percent IRR, the money is divided so that 80 percent go to the LPs and 20 percent to the GPs. Observed in exhibit 4.8 below, first in year 7, this is calculated on the "Total" row as the difference between the "Returns available to carry (GP \& LP)" from exhibit 4.6 less the "Paid carry (LP \& GP)" from exhibit 4.7.

## Exhibit 4.8 - Profit sharing 80/20

## Carrytier ll [80120]

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Total | 0 | 0 | 0 | 0 | 0 |
| Paid to GP | 0 | 0 | 0 | 0 | 0 |
| Paid to LP | 0 | 0 | 0 | 0 | 0 |


| Carry tier Ш(80120) | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :--- | :--- | :--- | :--- | :--- | ---: | :--- |
| Total | 0 | 326674853 | 0 | 125742126 | 92926838 |
| Paid to GP | 0 | 65334971 | 0 | 25148425 | 18585368 |
| Paid to LP | 0 | 261339882 | 0 | 100593700 | 74341471 |

Exhibit 8.2 .3 in the appendix depicts the IRR implication, as well as multiple-of-money (MoM), net present value to LPs and present value to GPs for the example investment value path development outlined in the exhibits above. In this simulation of the investment value paths, the no-subscription loan gives us a gross IRR of $23 \%$, net IRR of $16 \%$, MoM of 1.77 x , NPV to LPs of c. 727 m and PV to GPs of c .

SEK 323m. The gross IRR of the fund is the annual return of the fund of the divestment values from year 6 forward and the invested amounts done during the first five years. The net IRR considers the total cash flows to the LPs, including fees and potential interest ${ }^{37}$ on the cash outflow side and adjusting for catch-up and profit sharing on the cash inflow side. The MoM is presented on a net basis, and is the total positive cash flows for the LPs divided by the total negative cash flows, without taking the time perspective into account, assessing the money made in absolute amount expressed as a multiple. The NPV to LPs and PV to GPs are discounted back using the risk free rate (STIBOR) that is set to zero in the base case. The NPV to LPs looks at the same thing as the MoM, but expressed in actual money (here SEK), and the PV to GP looks at the actual money made to GPs.

### 4.1.3 The subscription loan scenario

For the subscription loan scenario, we use the same investment value paths as for the no-subscription loan scenario from exhibit 4.4. The investment schedule and management fee calculations are the same. The addition is the loan feature which is depicted in exhibit 4.9 below.

Exhibit 4.9- Investment schedule with subscription loan


[^19]| Investments | Divestment period |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 6 | 7 | 8 | 9 | 10 |
| h7wsinnerfarce |  |  |  |  |  |
| Invested capital | 0 | 0 | 0 | 0 | 0 |
| Total invested capital | 800000000 | 800000000 | 800000000 | 800000000 | 800000000 |
| Less realization (at cost) | 592000000 | 408000000 | 208000000 | 64000000 | 0 |
| Management fees | 16000000 | 11840000 | 8160000 | 4160000 | 1280000 |
| Total management fees | 116000000 | 127840000 | 136000000 | 140160000 | 141440000 |
| Loan payments |  |  |  |  |  |
| Amount allowed start |  |  |  |  |  |
| Loan | 84000000 | 0 | 0 | 0 | 0 |
| Interest rate | 1680000 | 0 | 0 | 0 | 0 |
| Total interest | 16960000 | 16960000 | 16960000 | 16960000 | 16960000 |
| Amount allowed | 0 | 0 | 0 | 0 | 0 |
| Amount taken | 0 | 0 | 0 | 0 | 0 |
| Residual |  |  |  |  |  |
| Loan for period |  |  |  |  |  |
| Dutflow for investment | 64000000 |  |  |  |  |
| Drawn capital | 101680000 | 11840000 | 8160000 | 4160000 | 1280000 |
| Cumulative total drawn capital | 932960000 | 944800000 | 952960000 | 957120000 | 958400000 |

As for subscription loan usage, the PE funds most of the time use it for all their investments over the investment horizon, if they have access to the facilities. ${ }^{38,39}$ Therefore we assume this in our model. For technical input on the subscription loan calculations in our model we rely on our bank contact.

The formula for loan allowed is:

$$
\operatorname{MIN}\left[\frac{(\text { Committed capital }- \text { Drawn capital }) * \text { \%Eligible investors }}{\text { Drawn debt ratio }} ; 20 \% * \text { committed capital }\right]
$$

The drawn debt ratio is between 1.5 and 2 , depending on how much risk the bank want to take ${ }^{40}$.
For the capital that is borrowed, the IRR calculated on that capital starts to count the moment that capital is drawn from the LPs. The general effect of this is that the investment period for parts of the capital invested by the fund has a shorter period of investment, at least from the LP perspective. This increases the IRR of a certain investment ${ }^{40}$.

[^20]In exhibit 4.10 below, the payoff in the example simulation to LPs and GPs is displayed for year 6 and forward where the values in terms of money available for hurdle and carry become slightly different from the no-subscription loan scenario in exhibit 4.5. The interest that comes with the subscription loans increase the draw-downs in the later years, which means the amount that LPs should eventually get back increases (SEK 958.4m compared to SEK 941.4m in the no-subscription loan case). As an effect, we see that the money available for hurdle and carry in year 6 becomes lower than the no-subscription loan case.

Exhibit 4.10 - Payoffs to LPs and GPs with subscription loans

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Distributions | 0 | 0 | 0 | 0 | 0 |
| Cumulative distributions | 0 | 0 | 0 | 0 | 0 |
| Draw down balance | 28000000 | 236000000 | 460000000 | 664000000 | 831280000 |
| To be returned pre hurdle \& carry | 958400000 | 958400000 | 958400000 | 958400000 | 958400000 |
| Returned pre hurdle \& carry | 0 | 0 | 0 | 0 | 0 |
| Available for hurdle \& carry | 0 | 0 | 0 | 0 | 0 |
| Cumulative for hurdle \& carry | 0 | 0 | 0 | 0 | 0 |
|  | 6 | 7 | 8 | 9 | 10 |
| Distributions | 1236816180 | 394658672 | 0 | 125742126 | 92926838 |
| Cumulative distributions | 1236816180 | 1631474853 | 1631474853 | 1757216978 | 1850143817 |
| Draw down balance | 0 | 0 | 0 | 0 | 0 |
| To be returned pre hurdle \& carry | 958400000 | 958400000 | 958400000 | 958400000 | 958400000 |
| Returned pre hurdle \& carry | 958400000 | 958400000 | 958400000 | 958400000 | 958400000 |
| Available for hurdle \& carry | 278416180 | 394658672 | 0 | 125742126 | 92926838 |
| Cumulative for hurdle \& carry | 278416180 | 673074853 | 673074853 | 798816978 | 891743817 |

In terms of hurdle, it becomes lower initially since the amount drawn down is lower in the first years when using subscription loans. See exhibit 4.11 below compared to exhibit 4.6 above. The hurdle starts counting when capital starts getting drawn, which is pushed forward a year when using a subscription loan. In the example simulation here, the effect is that less is paid out to LPs as hurdle with subscription loans in place.

## Exhibit 4.11 - Hurdle paid to LPs with subscription loans

| Hurdle |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 |
| Hurdle relating to period | 2240000 | 18880000 | 36800000 | 53120000 | 66502400 |
| Total hurdle outstanding | 2240000 | 21120000 | 57920000 | 111040000 | 177542400 |
| Balance | 2240000 | 21120000 | 57920000 | 111040000 | 177542400 |
| Hurdle paid to LP | 0 | 0 | 0 | 0 | 0 |
| Cumulative paid hurdle | 0 | 0 | 0 | 0 | 0 |
| Returns available to carry [GP \& LP] | 0 | 0 | 0 | 0 | 0 |
| Cumulative available carry | 0 | 0 | 0 | 0 | 0 |

Hurdle

| Hurdle relating to period | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | ---: | ---: | ---: | ---: |
| Total hurdle outstanding | 0 | 0 | 0 | 0 |
| Balance | 177542400 | 0 | 0 | 0 |
| Hurdle paid to LP | 0 | 0 | 0 | 0 |
| Cumulative paid hurdle | 177542400 | 0 | 0 | 0 |
| Returns available to carry [GP \& LP] | 177542400 | 177542400 | 177542400 | 177542400 |
| Cumulative available carry | 100873780 | 394658672 | 177542400 |  |

Once the hurdle is paid out to the LPs, the amount left available for catch-up and carry is higher in the subscription loan case the year the hurdle is paid out (year 6 in the example simulation). Exhibit 4.12 below shows the effects on the tier 1 carry (the catch-up phase). Just like with the hurdle, the catch-up part of the payoff is lower initially when subscription loans push forward the drawn capital. As more LP capital is put into work over the years, the catch-up part approaches the no-subscription loan case, but does not reach all the way there in the simulation example provided, before the IRR increases over 12 percent and the 80/20 profit sharing phase starts. However, since the IRR reaches above 12 percent faster with the subscription loan in place, all the catch-up is paid in year 6 rather than spread out over year 6 and 7 as when subscription loan is not used.

## Exhibit 4.12 - Catch-up paid to LPs and GPs with subscription loans

| Carry tier \ (catch-up] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 |
| Net catch-up related to period | 1120000 | 9440000 | 18400000 | 26560000 | 33251200 |
| Total carry outstanding | 1120000 | 10560000 | 28960000 | 55520000 | 88771200 |
| Catch-up balance | 1120000 | 10560000 | 28960000 | 55520000 | 88771200 |
| Paid carry [LP \& GP] | 0 | 0 | 0 | 0 | 0 |
| Cumulative paid carry (LP \& GP) | 0 | 0 | 0 | 0 | 0 |
| Paid to GP | 0 | 0 | 0 | 0 | 0 |
| Paid to LP | 0 | 0 | 0 | 0 | 0 |

Carry tier (catch-up)

|  | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | ---: | ---: | ---: | ---: |
| Net catch-up related to period | 0 | 0 | 0 | 0 |
| Total carry outstanding | 88771200 | 0 | 0 | 0 |
| Catch-up balance | 0 | 0 | 0 | 0 |
| Paid carry [LP \& GP] | 88771200 | 0 | 0 | 0 |
| Cumulative paid carry (LP \& GP) | 88771200 | 88771200 | 88771200 | 88771200 |
| Paid to GP | 53262720 |  | 0 | 88771200 |
| Paid to LP | 35508480 | 0 | 0 | 0 |

The profit sharing phase for the capital exceeding 12 percent IRR becomes larger in the example provided with the subscription loans, as seen in exhibit 4.13 below, compared to exhibit 4.8 above.

## Exhibit 4.13-Profit sharing 80/20 with subscription loans

Carry tier (180120)

|  | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Total | 12102580 | 394658672 | 0 | 125742126 | 92926838 |
| Paid to GP | 2420516 | 78931734 | 0 | 25148425 | 18585368 |
| Paid to LP | 9682064 | 315726938 | 0 | 100593700 | 74341471 |

As can be seen in exhibit 8.2.4 in the appendix, the IRR (gross and net) is boosted in this simulation example with the subscription loan facilities ( $31 \%$ gross IRR and $21 \%$ net IRR). The effect on MoM is negative. (See exhibit 8.2 .3 in the appendix for comparison). This is a gain scenario where the fund makes a profit, so the results are in line with previous reports presented in the literature chapter. Further reporting and analysis on the effects follow in section 5.

### 4.1.4 Monte Carlo simulation set-up

Section 4.1.2 and 4.1.3 depicts one possible fund development situation for the case without and with subscription loans respectively. In line with Metrick and Yasuda (2010), we create a fund universe by doing a Monte Carlo simulation with 100,000 runs, which means the investment value paths and subsequent payoffs to LPs and GPs and IRRs will vary each time. Exhibit 4.14 below shows the first five runs in the Monte Carlo simulation. We then look at average gross and net IRRs and other descriptive statistic outputs for both the no-subscription loan case and the subscription loan case.

## Exhibit 4.14-Monte Carlo simulation

| No subscription loan |  |  |  |  |  | Subscription Loan |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Runs | Gross IRR | Net IRR | MoM | NPV LP | PV GP | Gross IRR | Net IRR | MoM | NPV LP | PV GP |
|  | 23\% | 16\% | 1,8x | 726963054 | 323180763 | $31 \%$ | 21\% | 1.7x | 713395054 | 319788763 |
| 1 | 28\% | 23\% | 3,21x | 2079666311 | 661356578 | 35\% | 27\% | 3,16x | 2066098311 | 657964578 |
| 2 | -3\% | -7\% | 0.72x | -267182790 | 141440000 | -4\% | -8\% | 0.70x | -284 142790 | 141440000 |
| 3 | 6\% | 3\% | 1.18x | 172655479 | 141440000 | 7\% | 3\% | 1,16x | 155695479 | 141440000 |
| 4 | 48\% | 39\% | 3.89x | 2716559111 | 820579778 | 64\% | 50\% | 3,82x | 2702991111 | 817187778 |
| 5 | 53\% | 42\% | 3,53x | 2385991414 | 737937854 | $74 \%$ | 57\% | 3,48x | 2372423414 | 734545854 |

### 4.2 Approach for analyzing our results

For analysis, we run our simulations and save data on each of the following variables: gross IRR, net IRR, paid pre-hurdle, hurdle, catch-up and profit sharing (both to GPs and LPs). These and other variables are saved for each simulation to create our data-set.

### 4.2.1 Sensitivity analysis set-up

The above scenarios (without and with subscription loans) put across one simulation and explains how the set-up is looked at in our simulation runs to create different investment value paths and IRR. This is done on the benchmark case with the inputs described above. We then perform a sensitivity analysis where we allow the inputs to vary and then discuss the outcome. We start with looking at the whole dataset, analyze how the descriptive statistics vary when the inputs change, adding the perspective of no-subscription loan versus subscription loan. We also discuss statistical significance of our model results.

### 4.2.2 Bracket analysis

In the bracket analysis we sort the funds into different gross IRR brackets that we find interesting based on where most funds currently are performing or aim to perform. We also base the brackets on at what fund return levels previous reports and our interview responses have indicated that the usage of the facilities would be most desirable, e.g. at the hurdle rate of 8 percent. ${ }^{41,42}$

### 4.3 Model critique

Even though our model is academically backed up to a large extent, it makes several simplifications of reality. One of these could be the fact that GPs won't use bridge loans in the last year of their investment period to avoid drawing down capital in the post investment period (year 5 onwards). We do not know whether this is common or not; in our model, they can use bridge loans the last year of the investment period. To get the answer to this we would need comments from PE funds that use subscription loans,

[^21]which we have not been able to get. Another shortcoming of the model is that we only allow for the value developments of the investments to vary. Even though we argue that our inputs are solidly backed up either academically by Metrick and Yasuda (2010) or from interviews with relevant parties, in reality many other factors will vary such as exit dates, interest rates and investment pace. We therefore include a section in the appendix relating to how our results are affected by varying some of our fixed inputs, such as holding period. See appendix section 8.3. We would argue that the investment value development paths are the most important input to vary in the model, since it allows us to look at the breadth of different performing funds in order to come to valuable conclusions in regard to how the subscription loans work.

## 5. Findings and Analysis

This section is set up as follows. Initially, we describe our findings for our whole created data-set; we discuss our findings on average across all our simulations with a focus on the IRR impact. This initial section also includes findings for actual money made to GPs on average across all our simulations, with an elaboration on the catch-up versus no catch-up case. Then follows a sensitivity analysis section (5.1) aiming to discuss model sensitivity to inputs and more in detail address some of our specific hypotheses outlined in section 3. After that we present a bracket analysis (section 5.2), discussing IRR effects as well as actual money made for the parties for different fund performances in our created universe. This again to address some of the specific hypotheses and create a foundation to discuss incentives in our conclusion section.

Looking at our whole created data-set, we start by performing an analysis based on 100,000 investment value simulation runs in line with Metrick and Yasuda (2010). Then we elaborate on these findings with a $1,000,000$-simulation run analysis. With our 100,000 simulations, we find statistically solid differences on IRR levels, both gross and net, that support what we have heard in the interviews and what previous research claims - that these loans do increase reported IRR on a gross and net level in a gain scenario, as well as magnify the negative effects in loss scenarios ${ }^{43}$. What we do not find in running those simulations are statistically solid evidence on difference in NPV to LPs and PV to GPs. This should be found and the logic behind that is that there are costs associated to the loans which should decrease the total flow to LPs and GPs since some of the returns should now go to pay the bank interest on their loan. When we run $1,000,000$ simulations we do find statistically solid differences in these results for LPs. ${ }^{44}$ An exert from our model showing our results with $1,000,000$ runs is included below:

[^22]
## Exhibit 5.1 - Descriptive statistics with $\mathbf{1 , 0 0 0 , 0 0 0}$ simulation runs

|  | Median | Average | min | max | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gross IRR | 5.16\% | 7.415\% | -45\% | 246\% | 22\% | 0.062\% | -8.8\% | 20.7\% |
| Net IRR | 1.97\% | 2.631\% | -54\% | 223\% | 21\% | 0.058\% | -12.4\% | 15.1\% |
| MoM | 0.87x | 1.36 x | 0.00x | 171.94x | 1.76 x | 0.0029\% | 0.31 x | 1.75 x |
| NPV to LP | -122,852,271 | 336,614,129 | -941,440,000 | 160,932,767,179 | 1,654,866,253 | 4,262,653 | -648,072,030 | 701,410,285 |
| PV to GP | 141,440,000 | 297,739,894 | 141,440,000 | 40,374,631,795 | 373,593,422 | 962,313 | 141,440,000 | 316,792,571 |
| Loss in IRR calc. | 147306 |  |  |  |  |  |  |  |
| Max if loss IRR | -529,149,670 | <-- max NPV to LP if IRR not possible to calculate |  |  |  |  |  |  |
| Min if loss IRR | -941,440,000 | $<-\min$ NPV to LP if IRR not possible to calculate |  |  |  |  |  |  |
| $\alpha=0.01$ |  |  |  |  |  |  |  |  |


|  | Median | Average | min | max | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gross IRR | 6.43\% | 10.072\% | -52\% | 369\% | 29\% | 0.051\% | -10.8\% | 26.4\% |
| Net IRR | 1.96\% | 3.324\% | -77\% | 311\% | 26\% | 0.047\% | -15.4\% | 18.5\% |
| MoM | 0.85x | 1.33 x | 0.00x | 168.90 x | 1.73 x | 0.0028 x | 0.31 x | 1.72 x |
| NPV to LP | -139,812,271 | 319,866,531 | -958,400,000 | 160,919,199,179 | 1,655,903,279 | 4,265,324 | -665,032,030 | 687,842,285 |
| PV to GP | 141,440,000 | 297,527,491 | 141,440,000 | 40,371,239,795 | 372,386,606 | 959,204 | 141,440,000 | 313,400,571 |
| Loss in IRR calc. | 147306 |  |  |  |  |  |  |  |
| Max if loss IRR | -546,109,670 | <- max NPV to LP if IRR not possible to calculate |  |  |  |  |  |  |
| Min if loss IRR | -958,400,000 | $<-\min$ NPV to LP if IRR not possible to calculate |  |  |  |  |  |  |
| $\alpha=0.01$ |  |  |  |  |  |  |  |  |

These statistics show that there is an IRR increase on both gross and net IRR level in the loan case and that this increase, in average, amounts to $2.7 \%$ on gross level and $0.7 \%$ on net level ${ }^{45}$. This is lower than what some previous research has indicated (e.g. TorreyCove (2016)), as well than what was indicated in our interview with Aberdeen - their investor memo indicated a difference of $6 \%$ on gross and $4 \%$ net level. However, the results above are for our whole dataset, while Aberdeen's memos are on investments that achieve more common fund IRRs. We observe differences closer to theirs in our IRR bracket analysis below in section 5.2.

Furthermore, there are costs associated with these loans that are not beneficial for the LPs. The NPV to LPs are statistically different in the two cases and there is no overlap in confidence interval which solidifies this difference. The difference amounts to SEK 16.7 million $^{46}$ when comparing the averages which is roughly the same as the total amount of interest spent to finance the loans in the loan case which amounts to SEK 16.97 million. When looking at the whole data-set, we cannot say with confidence that the GPs are earning more in the loan case. There are two reasons for why this difference is not apparent when looking across all simulations. The first is that there is a significant amount of simulations ending up in negative territory in terms of fund returns and in these cases the GPs only earn their fees which are

[^23]the same in both cases. Secondly, as returns are increased, the GP compensation converges to the profit sharing of 80/20 which decreases the difference in GP compensation between the two cases at higher returns meaning that the amount gained in catch-up becomes a smaller part of the total return to GPs. Elaborating on this we look at outcomes with a positive IRR and find many cases where the GPs earn less in the loan case. This happens since the hurdle build up is lower in the loan case because less equity capital from the LPs is put into work. Since the catch-up is dependent on the hurdle there are cases where returns that would be labeled as catch-up and divided $60 / 40$ [GP/LP] in the no-loan case instead are divided as profit sharing in 20/80 [GP/LP] to the effect of decreasing the PV to GPs. There are cases when GPs earn more; these are the specific cases where the total return of the fund ends up in the catch-up region ( $8-12 \%$ IRR). See bracket analysis in section 5.2.

Having a catch-up phase is very common ${ }^{47}$, but some of the previous research (e.g. Kaplan and Strömberg (2009) and Appelbaum (2016)) and interview responses have based their input on a no catch-up structure. Removing the catch-up, and running $1,000,000$ simulations, we observe a statistically solid difference in the two cases showing GP compensation increasing in the loan case as seen in exhibit 5.2.

## Exhibit 5.2 - PV to GPs with no catch-up

|  | PV to GP |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Median | Average | min | max | Volatility | Confidence int. * | 3rd Quartile | 1st Quartile |
| No loan | 141,440,000 | 277,225,634 | 141,440,000 | 28,922,755,303 | 357,801,699 | 921,636 | 141,440,000 | 253,391,842 |
| Loan | 141,440,000 | 280,570,241 | 141,440,000 | 28,932,302,823 | 360,080,588 | 927,506 | 141,440,000 | 262,504,378 |

Without the catch-up, the difference in PV to GP without and with loans is explained by the fact that the hurdle build up to LPs is lower since with the loans the GPs reduce the time under which they claim LPs capital and part of the return must be allocated to paying the bank its interest. Thus, less hurdle to LPs translates into giving the GP a larger share from the 20/80 [GP/LP] profit sharing. Running the 1,000,000 simulations we observe a maximum difference between the two cases to an increase in PV to GPs at SEK 9.5 million in the loan case. The difference in hurdle for the first five years, when the loans are in place, amounts to SEK 64.7 million, that is, the funds pay SEK 64.7 million less in hurdle when taking loans. This is true for all simulations in the base case. Removing from this the interest we get SEK 47.7 million of which the GPs take $20 \%$, or correspondingly, SEK 9.5 million. We see that the positive increase in GP compensation is because GPs claim LPs capital for a shorter period which translates into a lower amount of hurdle to LPs; returns are pushed into the profit sharing region where GPs take $20 \%$ of the return. At a loan spread of $8 \%$ (instead of $2 \%$ as in the base case) the total interest on the loans amount to SEK 67.7

[^24]million, which is higher than the hurdle the GPs can off-set. In such cases the GPs would always lose money in the loan case as compared to the no loan case.

An additional comment on our IRR calculations is warranted: certain negative price paths yield an IRR that cannot be calculated. If this happens, we remove all the IRRs in that simulation for both the loan and no-loan case. Statistics on IRRs are therefore skewed upwards in our data-set when looking at all simulations in aggregate. The differences between the two cases however persist and we can show, with statistical assurance, that the loan case yields a higher IRR than the no loan case. The total amount of losses when running $1,000,000$ simulations amount to $14.7 \%$. That means that in $14.7 \%$ of the cases the IRR is not possible to calculate. This happens when the NPV to LPs is below SEK -529 million. We lose no IRR calculations on positive NPV cases.

### 5.1 Sensitivity analysis

The sensitivity analysis is conducted from the base case and then we change one variable at a time to see how these affect the model. In addition to removing the catch-up, where the effects are discussed above in the introduction to this chapter, we deem the following parameters worthy of investigation: (1) Interest rate and cost of loan, (2) Holding period, (3) Leverage, (4) Loan constraints and (5) discount rate. The interest rate and loan constraints relating to eligible investor ratio are included in this section, since they relate the most to previous research and interview responses. The other categories are discussed in the appendix section 8.3. We then discuss the effects on the descriptive statistics outlined above which are gross and net IRR, MoM, NPV to LP and PV to GP and look at the loan case vs the no-loan case. MoM (calculated as net of all costs) and NPV to LP explain the same result, just that MoM is expressed as a multiple and NPV to LP in actual currency. They are therefore not included together for all descriptive statistics analyzed.

### 5.1.1 Interest and cost of loan

## Interest

The risk-free interest rate [STIBOR] affects our model in three ways. (1) It is used in the drift of the GBM (2) it is used to calculate the interest on the loans to the banks and (3) it is used to discount the cash flows to GPs and LPs. Increasing the STIBOR increases the average returns of our funds, even after discounting the cash flows. Although the loans become more expensive when increasing the STIBOR input we initially observe that the net IRR difference between taking on loans and not taking on loans increase, yielding a higher average IRR in the loan case which is counter intuitive, see below.

## Exhibit 5.3 - Net IRR effects with increasing STIBOR

| Average Net IRR |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: |
| Stibor base | [No loan] | Conf int $^{*}$ | [Loan] | Conf int | IRR diff |
| Benchmark | $2,635 \%$ | $0,117 \%$ | $3,324 \%$ | $0,147 \%$ | $0,69 \%$ |
| $2 \%$ | $5,028 \%$ | $0,118 \%$ | $5,875 \%$ | $0,148 \%$ | $0,85 \%$ |
| $4 \%$ | $7,076 \%$ | $0,118 \%$ | $7,979 \%$ | $0,148 \%$ | $0,90 \%$ |
| $6 \%$ | $9,484 \%$ | $0,119 \%$ | $10,540 \%$ | $0,149 \%$ | $1,06 \%$ |
| $8 \%$ | $12,041 \%$ | $0,120 \%$ | $13,285 \%$ | $0,150 \%$ | $1,24 \%$ |
| $10 \%$ | $15,294 \%$ | $0,120 \%$ | $16,906 \%$ | $0,150 \%$ | $1,61 \%$ |

$7.92 \%$ covifarne intarya/

This is explained by the fact that our value development paths are skewed upward which offset the impact of more expensive loans. In addition, the loans in our base case amount to a maximum of SEK 200 million, on which interest is charged, while the invested capital amounts to SEK 800 million and which's value development is driven by the GBM drift which increases with the risk-free rate. Because of the riskfree interest rate's extensive impact on our value paths we choose to investigate what happens in a case where only the spread on the cost of the loans increase.

## Cost of loan

The effect of increasing the cost of loans (spread) is intuitive both when looking at the no-loan case and loan case individually and when comparing the two cases. Gross IRRs are not affected since it does not consider any costs associated with loan-financing. The same is true for the net IRR for the no-loan case individually. In the loan case, we see statistically solid decreasing benefits of the loans when the spread is increased, see exhibit 5.4 below. The difference in net IRR compared to the no loan case decreases from a solid difference in net IRR to a convergence while the IRR in the loan case steadily decrease as the spread on the loan increase. While it is interesting to look at a $2 \%$ vs $6 \%$ spread, one take-away is that we see that the cost of loans (spread) will have to go up quite a lot from today's indicated $2 \% \operatorname{spread}^{48}$ for the loans to become undesirable from a net IRR perspective on average

[^25]
## Exhibit 5.4 - Net IRR effect when increasing the cost of the loans (spread)

| Net IRR |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Average | Confidence int."] | Average | Confidence int."] | IRR diff |
| [Noan] |  |  |  |  |  |
| Benchmark case | $2,748 \%$ | $0,118 \%$ | $3,467 \%$ | $0,148 \%$ | $0,72 \%$ |
| $3 \%$ | $2,574 \%$ | $0,117 \%$ | $3,035 \%$ | $0,146 \%$ | $0,46 \%$ |
| $4 \%$ | $2,776 \%$ | $0,117 \%$ | $3,068 \%$ | $0,147 \%$ | $0,29 \%$ |
| $5 \%$ | $2,755 \%$ | $0,117 \%$ | $2,830 \%$ | $0,146 \%$ | $0,07 \%$ |
| $6 \%$ | $2,660 \%$ | $0,117 \%$ | $2,504 \%$ | $0,146 \%$ | $-0,16 \%$ |

### 7.97. \% wow fanne intaryal

As seen in exhibit 5.4 above, we see that at a spread of around $6 \%$ the average net IRR is lower with loans. We further test this in exhibit 5.5 below by running our data through $1,000,000$ simulations while testing for a confidence interval at $99 \%$ and setting the spread at $6 \%$.

Exhibit 5.5 - Comparison no-loan vs loan with a $6 \%$ spread


At a spread of $6 \%$ the average net IRR in the loan case becomes lower than in the no loan case. We see a statistically solid difference of at least $-0.04 \%{ }^{49}$ implying a break-point in the benefit of the loans ability to increase IRR on average in our base case. For further analysis, we add an in-depth analysis of what happens in funds with different IRRs as the loan spread changes:

[^26]Exhibit 5.6-2\% vs 6\% loan spread IRR impact-analysis in different IRR brackets

| Loan spread 6\% |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gross IRR | Net IRR |  |  |  | Net IRR [Loan] |  | $\begin{gathered} \text { Average difference } \\ \text { (loan - no loan) } \\ \hline \end{gathered}$ |  |
| Range | min | max | Average | min | max | Average |  | Observations |
| -2\%-0\% | -7\% | -2\% | -4.3\% | -13\% | -3\% | -6.6\% | -2.3\% | 3194 |
| 0\% - $2 \%$ | -5\% | 0\% | -2.3\% | -9\% | -1\% | -4.2\% | -1.9\% | 3133 |
| 2\% - 4\% | -3\% | 2\% | -0.2\% | -7\% | 1\% | -1.6\% | -1.4\% | 3102 |
| 4\%-6\% | -1\% | 4\% | 1.8\% | -3\% | 3\% | 0.8\% | -1.0\% | 1489 |
| 6\% - 8\% | 1\% | 6\% | 3.8\% | -1\% | 6\% | 3.3\% | -0.5\% | 1466 |
| 8\%-10\% | 3\% | 7\% | 5.8\% | 2\% | 7\% | 5.7\% | -0.1\% | 2965 |
| 10\% - 12\% | 5\% | 8\% | 7.1\% | 5\% | 9\% | 7.3\% | 0.3\% | 2887 |
| 12\% - 14\% | 7\% | 10\% | 8.4\% | 7\% | 11\% | 9.1\% | 0.7\% | 2822 |
| 14\% - 16\% | 8\% | 12\% | 10.1\% | 9\% | 13\% | 11.2\% | 1.1\% | 2647 |
| 16\%-18\% | 10\% | 14\% | 11.8\% | 11\% | 15\% | 13.4\% | 1.6\% | 2508 |
| 18\% - 20\% | 11\% | 15\% | 13.6\% | 14\% | 17\% | 15.6\% | 2.0\% | 2374 |
| 20\% - 22\% | 13\% | 17\% | 15.4\% | 16\% | 19\% | 17.8\% | 2.4\% | 2159 |
| 22\% - $24 \%$ | 15\% | 19\% | 17.1\% | 18\% | 22\% | 20.1\% | 2.9\% | 1947 |

Loan spread 2\%

| Gross IRR |  | Net IRR |  |  | Net IRR [Loan] |  | (loan - no loan) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Range | min | max | Average | min | max | Average |  | Observations |
| -2\%-0\% | -7\% | -2\% | -4.3\% | -11\% | -3\% | -5.7\% | -1.4\% | 3115 |
| 0\%-2\% | -5\% | 0\% | -2.3\% | -8\% | -1\% | -3.3\% | -1.0\% | 3165 |
| 2\%-4\% | -3\% | 2\% | -0.2\% | -5\% | 2\% | -0.8\% | -0.5\% | 3180 |
| 4\%-6\% | -1\% | 4\% | 1.8\% | -2\% | 4\% | 1.7\% | -0.1\% | 1530 |
| 6\%-8\% | 1\% | 6\% | 3.8\% | 1\% | 6\% | 4.2\% | 0.4\% | 1474 |
| 8\%-10\% | 3\% | 7\% | 5.8\% | 4\% | 7\% | 6.5\% | 0.7\% | 3023 |
| 10\% - 12\% | 5\% | 8\% | 7.1\% | 7\% | 9\% | 7.9\% | 0.8\% | 2950 |
| 12\% - 14\% | 7\% | 10\% | 8.4\% | 8\% | 11\% | 9.9\% | 1.5\% | 2761 |
| 14\%-16\% | 8\% | 12\% | 10.1\% | 10\% | 14\% | 12.1\% | 2.0\% | 2609 |
| 16\% - 18\% | 10\% | 14\% | 11.8\% | 13\% | 16\% | 14.3\% | 2.5\% | 2528 |
| 18\%-20\% | 11\% | 16\% | 13.6\% | 15\% | 18\% | 16.5\% | 2.9\% | 2239 |
| 20\% - 22\% | 13\% | 17\% | 15.4\% | 17\% | 20\% | 18.8\% | 3.4\% | 2232 |
| 22\% - $24 \%$ | 15\% | 19\% | 17.2\% | 20\% | 23\% | 21.0\% | 3.9\% | 2072 |

We see that the gross IRR needed to break even on a net level with loans increases as the loan spread increases. We see this primarily by looking at which brackets on gross IRR level contain a zero net IRR with loan in their range (see grey shaded areas in exhibit 5.6 above). For the loan case with a $2 \%$ loan spread, zero net IRR is found at gross IRR ranges between 2 and $6 \%$ while when increasing the loan spread to $6 \%$, zero net IRR is found in the gross IRR ranges of 2 to $8 \%$, an increase by $2 \%$ of the range. Increases in the loan spread has the effect of reducing the benefit of the loans by pushing up the return needed to break even on a net level; it could be viewed, from an LP perspective, as imposing more fees.

The difference in return ("Average difference (loan - no loan)" in exhibit 5.6) increases with higher gross IRRs. If the IRRs are high enough to offset the negative impact that the interest poses, taking on loans still has a positive effect on IRR even though the cost of the loan increases.

The average effect of the loans on net IRR is higher in the loan case at returns above $6 \%$ gross IRR when the spread is $2 \%$. If the spread is $6 \%$, the average effect of loans on net IRR is higher at above $10 \%$ gross IRR. At spreads of above $6 \%$ our model fund would have to have more than SEK 1 billion in committed capital which effectively sets a cap on how large the spreads can be for the loans to be in place in our current set-up. A conclusion to the effect that the spread poses on net IRRs is that in the boundaries of a spread between $2 \%$ and $6 \%$ the IRR increasing effect is evident for funds that can return above 6 (at a $2 \%$ spread) to $10 \%$ (at a $6 \%$ spread) gross IRR.

As for money made to LPs, outlined in exhibit 5.7 below, we see that LPs get a lower NPV as the cost of the loan increases, which is intuitive because the interest that goes to the banks increase. This effect is stronger as we increase the loan spread one percentage at a time from the benchmark case, but not statistically significant for small increases in the spread. Below we show results, which are statistically solid, comparing the benchmark with $6 \%$ spread at $1,000,000$ runs. The difference between the averages are c. SEK 33 million corresponding to the difference in interest payments made between a spread at 2 vs $6 \%$ which is SEK 33.9 million.

## Exhibit 5.7 - Comparison of money made to GPs and LPs with different loan spreads ( $\mathbf{2 \%}$ vs $\mathbf{6 \%}$ )

| Comparison of the loan case at $2 \mathrm{vs} 6 \%$ spread in loan cost |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2\% |  | Median | Average | min | max | Volatility | Confidencr | 3rd Quartile | 1st Quartile |
|  | NPV to LP | -139,812,271 | 319,866,531 | -958,400,000 | 160,919,199,179 | 1,655,903,279 | 4,265,324 | -665,032,030 | 687,842,285 |
|  | PV to GP | 141,440,000 | 297,527,491 | 141,440,000 | 40,371,239,795 | 372,386,606 | 959,204 | 141,440,000 | 313,400,571 |
| 6\% | NPV to LP | -178,501,697 | 287,087,013 | -992,320,000 | 94,593,488,150 | 1,656,849,671 | 4,267,762 | -699,923,434 | 658,306,590 |
|  | PV to GP | 141,440,000 | 294,183,510 | 141,440,000 | 23,789,812,037 | 370,403,318 | 954,096 | 141,440,000 | 306,016,647 |

The PV to GP also decreases and comparing the averages we see a difference of SEK 3.3 million. The reason why these averages together do not add up equal to the whole interest of SEK 33.9 million is because we are looking at an average across all simulations. For funds that do not return the invested capital, the downside for LPs is increased by the whole amount of interest paid while in the upside some of the cost of the loans are shared with the GPs through reducing their carry in the catch-up region. For a fund that yields very high return the intuition is that the amount of carry shared in the profit sharing region is reduced by the amount of interest paid, since this amounts to SEK 33.9 million the effect from a 20/80 split results in a reduction in the carry for GPs by SEK 6.8 million and SEK 27.1 million for the LPs.

As the loan spread increases, the amount to be returned pre-hurdle also increases since the GPs must draw additional capital to pay for interest on the bridge loans. The increased amount of draw-downs at each \%spread is static in our model since it is not dependent on the investment value development paths but rather on how much loan the funds take on. Exhibit 5.8 below shows the increase in cumulative drawn capital which is the amount to be returned pre-hurdle.

## Exhibit 5.8 - LP return pre-hurdle at different spreads



In our benchmark case, when increasing the loan spread above $6 \%$ the amount of drawn capital needed to fund fees, interest and investments exceed the amount of committed capital.

Relating back to the relevant hypothesis ${ }^{50}$, we conclude that the magnitude of the loans ability to increase IRR do decrease with increased levels of the loan spread. However, for funds that achieve a high return, say $22-24 \%$ on a gross level, loans still have an increasing effect on net IRR when the spread is increased to $6 \%$. For funds in that range the average net IRR increasing effect is still positive at $2.9 \%$, down from $3.9 \%$ if the spread increases from $2 \%$ to $6 \%$. Our model prohibits us to increase the interest spread over $6 \%$ as that would make our model fund break its budget on committed capital. These findings on loan spread are much in line with input from interviews in terms of the interest rate as a driver of the loans. They are also in line with previous research that the loans have a good possibility to continue to be in place since the spread will have to go up quite a bit before they do not improve IRR anymore.

[^27]
### 5.1.2 Loan constraints

## Tightening loan constraints

In analyzing sensitivity to loan constraints, we change the ratio of eligible investors. This has the effect of decreasing the amount of loan the fund can take on at fund level. There are other constraints available also, such as increasing or decreasing the max percentage of committed capital. Exhibit 5.9 below displays how these constraints affect our benchmark fund in terms of the amount of loans they can take and how much of this they actually use:

Exhibit 5.9 - Debt schedule with changing loan constraint inputs

|  |  | Debt schedu |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 1 | 2 | 3 | 4 | 5 | 6 |
| Benchmark case (Eligible investor ratio $=100 \%$ ) |  |  |  |  |  |  |
| Amount allowed | 200,000,000 | 200,000,000 | 200,000,000 | 200,000,000 | 112,480,000 | 0 |
| Amount taken | 200,000,000 | 200,000,000 | 200,000,000 | 164,000,000 | 84,000,000 | 0 |
| Eligible investor rate 50\% |  |  |  |  |  |  |
| Amount allowed | 200,000,000 | 200,000,000 | 186,666,667 | 112,088,889 | 56,674,963 | 0 |
| Eligible investor rate 30\% |  |  |  |  |  |  |
| Amount allowed | 200,000,000 | 153,600,000 | 112,185,600 | 67,736,858 | 34,665,910 | 0 |
| Eligible investor rate 10\% |  |  |  |  |  |  |
| Amount allowed | 66,666,667 | 51,377,778 | 37,709,274 | 22,992,328 | 12,028,339 | 0 |
| *Amount taken is amount allowed for all eligible investor ratios below benchmark |  |  |  |  |  |  |

Max of committed capital 50\%

Amount taken $\quad 228,000,000204,000,000 \quad 220,000,000 \quad 164,000,000 \quad 84,000,000 \quad 0$

As seen in exhibit 5.10 below, when decreasing the eligible investor rate from $100 \%$ to $50 \%$, the gross and net IRR are still increasing with loans, seen by the difference in averages. The increase in IRR is expectedly lower in the case with a lower eligible investor ratio.

## Exhibit 5.10 - Eligible investor ratio IRR impact*

| Gross IRR |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | [No loan] |  | [Loan] |  |  |
| Eligible investor rate | Median | Average | Median | Average | Diff average |
| Benchmark case | 5.31\% | 7.62\% | 6.62\% | 10.331\% | 2.714\% |
| 50\% | 5.10\% | 7.33\% | 6.13\% | 9.613\% | 2.281\% |
|  | Net IRR |  |  |  |  |
|  | [No loan] |  | [Loan] |  |  |
| Eligible investor rate | Median | Average | Median | Average | Diff average |
| Benchmark case | 2.15\% | 2.82\% | 2.19\% | 3.560\% | 0.738\% |
| 50\% | 1.91\% | 2.52\% | 1.89\% | 3.25\% | 0.726\% |

* The no-loan case should in theory not give a difference in gross and net IRR average/median when changing the eligible investor ratio. Due to changed investment value development paths we observe slight differences when changing the eligible investor ratio and running our simulations again. In this case the number of simulations is 100,000; if increasing the number of simulations, the no-loan case differences become lower.

Exhibit 5.11 below contains the eligible investor ratio impact for various gross IRR brackets. It underlines that the higher the gross IRR of funds and the more the eligible investor ratio is lowered, the higher impact on net IRR.

## Exhibit 5.11 - Eligible investor ratio net IRR impact

|  | Net IRR with loan <br> Eligible investor ratio |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gross IRR [no loan] | $\mathbf{y y y y}$ | $\mathbf{5 0 \%}$ | $\mathbf{2 5} \%$ | $\mathbf{1 0} \%$ |
| $-2 \%-0 \%$ | $-5.7 \%$ | $-5.5 \%$ | $-5.1 \%$ | $-4.6 \%$ |
| $0 \%-2 \%$ | $-3.2 \%$ | $-3.1 \%$ | $-2.8 \%$ | $-2.4 \%$ |
| $2 \%-4 \%$ | $-0.8 \%$ | $-0.7 \%$ | $-0.5 \%$ | $-0.3 \%$ |
| $4 \%-6 \%$ | $1.7 \%$ | $1.7 \%$ | $1.8 \%$ | $1.8 \%$ |
| $6 \%-8 \%$ | $4.2 \%$ | $4.2 \%$ | $4.0 \%$ | $3.9 \%$ |
| $8 \%-10 \%$ | $6.4 \%$ | $6.4 \%$ | $6.1 \%$ | $5.9 \%$ |
| $10 \%-12 \%$ | $7.9 \%$ | $7.8 \%$ | $7.5 \%$ | $7.2 \%$ |
| $12 \%-14 \%$ | $9.9 \%$ | $9.7 \%$ | $9.2 \%$ | $8.7 \%$ |
| $14 \%-16 \%$ | $12.1 \%$ | $11.8 \%$ | $11.1 \%$ | $10.5 \%$ |
| $16 \%-18 \%$ | $14.3 \%$ | $14.0 \%$ | $13.1 \%$ | $12.3 \%$ |
| $18 \%-20 \%$ | $16.5 \%$ | $16.2 \%$ | $15.1 \%$ | $14.1 \%$ |
| $20 \%-22 \%$ | $18.8 \%$ | $18.4 \%$ | $17.1 \%$ | $16.0 \%$ |
| $22 \%-24 \%$ | $21.0 \%$ | $20.6 \%$ | $19.2 \%$ | $17.9 \%$ |

In terms of money made to LPs, the effect of decreasing the eligible investor rate directly affects the NPV for the LPs by decreasing the amount of loans that can be taken which affects the total interest paid on the loans. The reduction in interest is therefore distributed as hurdle and carry, thus increasing the LPs NPV. For the GPs, the effects of the loans discussed in the bracket analysis throughout section 5.2 below, are reduced.

From exhibit 5.11 above, we see what happens to the net IRR at different levels of gross IRR a fund achieves when its eligible investor ratio goes gradually from $100 \%$ down to $10 \%$. Relating to hypothesis $5^{51}$ we see a small difference between $100 \%$ ratio and $50 \%$ ratio, while a $10 \%$ ratio yields a significantly larger difference, for all brackets. The size of the difference is dependent on the gross IRR, at lower levels of gross IRR the difference is small. This is because the loans are inefficient in boosting the IRRs at these levels. Looking at increasing levels of gross IRR we observe an increased difference in net IRR. At a gross IRR of between $22-24 \%$, the difference in net IRR comparing $100 \%$ and $10 \%$ eligible investor ratio is $3.1 \%$, which is larger than at lower gross IRRs. In relation to our interviews and to sum up the analysis of loan constraints, our bank contact underlines that the eligible investor ratio, stemming from the investor base of the private equity fund, is a fundamental factor in determining which funds that uses subscription loans. Our model-results, that a high eligible investor ratio is connected to a higher benefit of the subscription loans, are much in line with our correspondence with our bank contact. These "eligible"

[^28]investors are mainly large established pension funds, so-called "sources of money". Similarly, smaller funds that mainly have wealthy individuals as an investor base do not take on these loans to the same extent since their investors are not eligible. A conclusion is that to be able to get these loans, funds will start preferring certain LP over others in future capital fund raisings. This will be especially desirable for smaller funds GPs that today mostly have investors considered as less eligible, to be able to get these loans and become more comparable with the larger funds.

### 5.2 Bracket analysis

Since the hurdle, catch-up and profit-sharing are dependent on the amount of actual return on each investment, they depend heavily on the simulated value development paths of the investments. To present interesting analysis, that is based on returns that can be linked to long term stock market indices and connected to returns on other asset classes, we have chosen to present analysis on an IRR bracket level. The most common metric used by funds to compare themselves to their peers and to value investments is by using gross IRR (Gompers et al (2016)). In choosing the brackets we want to look at we also consider that much of the dynamics in fee structure comes into effect within our chosen range, such as hurdle rate and catch-up cap. We also know that in using these brackets on a gross IRR level, we capture the results that many funds achieve or aim to achieve in reality (Gompers et al (2016)). For our analysis, considering the dynamics of our model, the interesting return levels are (1) a limited negative return range, (2) positive return below the hurdle rate, (3) returns at and around the hurdle rate, (4) returns within the catch-up cap, (5) returns above the catch-up cap. (6) Higher returns at and above $20 \%$ as is promised to the investors by many GPs (Gompers et al (2016)). The results in the bracket analysis come from our base case model running 100,000 simulations, and in exhibit 5.12 below we outline the brackets.

## Exhibit 5.12 - IRR bracket overview

| Brackets on Gross IRR |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{m i n}$ | max | Average | Count |
| $-2 \%$ | $0 \%$ | $-0.99 \%$ | 3064 |
| $0 \%$ | $2 \%$ | $0.99 \%$ | 3226 |
| $2 \%$ | $4 \%$ | $3.00 \%$ | 3170 |
| $4 \%$ | $6 \%$ | $5.00 \%$ | 3076 |
| $6 \%$ | $8 \%$ | $7.01 \%$ | 3036 |
| $8 \%$ | $10 \%$ | $9.01 \%$ | 2915 |
| $10 \%$ | $12 \%$ | $11.00 \%$ | 2912 |
| $12 \%$ | $14 \%$ | $12.99 \%$ | 2726 |
| $14 \%$ | $16 \%$ | $15.00 \%$ | 2698 |
| $16 \%$ | $18 \%$ | $17.00 \%$ | 2408 |
| $18 \%$ | $20 \%$ | $19.00 \%$ | 2353 |
| $20 \%$ | $22 \%$ | $20.98 \%$ | 2206 |
| $22 \%$ | $24 \%$ | $22.98 \%$ | 2026 |
| $24 \%$ | $26 \%$ | $25.00 \%$ | 1865 |
| $26 \%$ | $28 \%$ | $26.98 \%$ | 1691 |
| $28 \%$ | $30 \%$ | $29.00 \%$ | 1578 |
| *Count constitutes the number of funds found in these brackets when running our simulations |  |  |  |

The following analyses are done through sorting our dataset and looking at resulting values (net IRR, PVs and NPVs etc.) on funds that have a no-loan gross IRR within the brackets outlined above. These brackets are created from $-2 \%$ ramping up to $30 \%$ in steps of $2 \%$.

### 5.2.1 IRR analysis

An analysis on IRR levels follows below, in exhibit 5.13.

## Exhibit 5.13 - IRR overview, no-loan vs loan, for different brackets

| Gross IRR [no loan] |  | Net IRR [no loan] |  |  | Gross IRR [Loan] |  |  | Net IRR [Loan] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Brackets | Average | min | max | Average | min | max | Average | min | max | Average |
| -2\%-0\% | -1.0\% | -7.2\% | -2.3\% | -4.3\% | -2.8\% | 0.0\% | -1.3\% | -11.0\% | -2.9\% | -5.7\% |
| 0\% - $2 \%$ | 1.0\% | -5.0\% | -0.3\% | -2.2\% | 0.0\% | 2.7\% | 1.3\% | -7.8\% | -0.6\% | -3.2\% |
| 2\% - 4\% | 3.0\% | -3.0\% | 1.7\% | -0.2\% | 2.4\% | 5.6\% | 3.7\% | -4.9\% | 1.7\% | -0.8\% |
| 4\% - $6 \%$ | 5.0\% | -0.9\% | 3.8\% | 1.8\% | 4.6\% | 8.4\% | 6.2\% | -2.0\% | 4.0\% | 1.7\% |
| 6\%-8\% | 7.0\% | 1.2\% | 5.8\% | 3.8\% | 7.0\% | 11.2\% | 8.7\% | 0.9\% | 6.4\% | 4.2\% |
| 8\%-10\% | 9.0\% | 3.3\% | 7.0\% | 5.8\% | 9.4\% | 14.0\% | 11.3\% | 3.9\% | 7.5\% | 6.4\% |
| 10\% - $12 \%$ | 11.0\% | 5.5\% | 8.3\% | 7.1\% | 11.7\% | 16.8\% | 13.8\% | 6.7\% | 9.3\% | 7.9\% |
| 12\% - 14\% | 13.0\% | 7.1\% | 10.1\% | 8.4\% | 14.1\% | 19.6\% | 16.4\% | 8.2\% | 11.3\% | 9.9\% |
| 14\% - 16\% | 15.0\% | 8.2\% | 11.9\% | 10.1\% | 16.4\% | 22.4\% | 19.0\% | 10.2\% | 13.5\% | 12.1\% |
| 16\% - 18\% | 17.0\% | 9.6\% | 13.6\% | 11.9\% | 18.8\% | 25.2\% | 21.5\% | 12.6\% | 15.9\% | 14.3\% |
| 18\% - 20\% | 19.0\% | 11.4\% | 15.3\% | 13.6\% | 21.1\% | 27.9\% | 24.1\% | 15.0\% | 18.2\% | 16.5\% |
| 20\% - $22 \%$ | 21.0\% | 13.1\% | 17.4\% | 15.4\% | 23.6\% | 30.7\% | 26.7\% | 17.4\% | 20.5\% | 18.8\% |
| 22\% - $24 \%$ | 23.0\% | 14.8\% | 19.2\% | 17.2\% | 26.0\% | 33.5\% | 29.4\% | 19.6\% | 22.8\% | 21.0\% |
| 24\% - $26 \%$ | 25.0\% | 16.6\% | 21.1\% | 18.9\% | 28.0\% | 36.4\% | 32.0\% | 21.8\% | 25.0\% | 23.3\% |
| 26\% - $28 \%$ | 27.0\% | 18.3\% | 23.0\% | 20.7\% | 30.7\% | 39.2\% | 34.7\% | 24.0\% | 27.2\% | 25.6\% |
| 28\%-30\% | 29.0\% | 20.1\% | 24.9\% | 22.5\% | 32.9\% | 42.1\% | 37.4\% | 26.3\% | 29.6\% | 27.9\% |

Relating to hypothesis $1^{52}$ and starting by looking at the negative bracket of -2 to $0 \%$ gross IRR, we see that the average gross IRR with loan is $-1.3 \%$ compared to $-1.0 \%$ without loan showing us the magnifying effect at negative levels we have elaborated on earlier, and in line with previous research ${ }^{53}$. In addition, if we look at more negative levels and observe that this difference is increased. Relating to hypothesis $2^{54}$ exhibit 5.13 further tells us that the more positive return a fund yields in gross IRR without loan, the more positive impact on gross and net IRR the loan will yield. However, the impact on net IRR is negative until a no loan gross IRR of $6-8 \%$ is achieved; at and above this level the loans have a positive impact on net IRR on average. At gross IRRs below $6-8 \%$ the negative impact from the interest paid on the loans outweigh their positive effect of reducing the time the fund holds the LP capital. The difference is increasing as gross IRR is increasing. Funds target a gross return at c. 25-30\% (Gompers et al (2016)); connecting this to our brackets, looking at averages, we see that at gross returns between 24-26 \%, the impact is $+7 \%$ on gross IRR and $+4.4 \%$ on net IRR. At gross returns between $26-28 \%$, the impact is

[^29]$+7.7 \%$ on gross IRR and $+4.9 \%$ at a net level. At our highest bracket of $28-30 \%$ gross IRR the difference in gross is an increase of $8.4 \%$ and net IRR is increased by $5.4 \%$. This is in line with interview indications of the effects. ${ }^{55}$

Our analysis simulates different investment value development paths; an investment value development path that yields a gross IRR at a certain percentage can be achieved in various ways and this will affect the hurdle differently. Private equity carry is dependent on the hurdle rate, especially when considering catchup. Our bracket analysis is therefore a better reflection of what can happen in private equity funds than what can be deducted through looking at static investment value development paths, such as the TorreyCove example in the literature review chapter and appendix, that yield a certain yearly return on each investment.

Furthermore, according to Preqin's 2016 report, the top quartile for PE funds stands at a net IRR of $22 \%{ }^{56}$. According to our findings it would be possible for funds that achieve a net IRR of around 18-19\% to use these loans to achieve a "top quartile" rank with a net IRR of $22 \%$. Relating to hypothesis $3^{57}$, this means that what we find underlines the previous research statements about one of the most important incentives for GPs behind the loan facilities - to attract more capital in today's increased competition between PE funds and across asset classes.

[^30]To continue, the below analysis aims to describe how the loan facilities affect the different parts of the return created in a private equity fund (hurdle, catch-up and profit sharing) for the GPs and LPs.

### 5.2.2 Compensation analysis to GPs

## Exhibit 5.14 - GP catch-up at different IRRs



In observing the average catch-up earned by GPs in exhibit 5.14 above, we see that the GPs on average earn more catch-up at lower returns in the loan case than in the no-loan case until a gross return of around $12 \%$ has been achieved at which point the GPs on average get more catch-up in the no-loan case. This difference exists because the catch-up by construction is dependent on the hurdle; less hurdle translates into less catch-up. At higher return levels the GPs get less catch-up since more of the returns translate into profit sharing, as shown in the next section (exhibit 5.15).

It is not clear in exhibit 5.14 whether GPs start to earn money at lower IRRs and we therefore look at our data-set to see at which return carry is started to be earned. In the no loan case carry is observed to start at a gross IRR of $8.6 \%$ while in the loan case this starts at $8.2 \%$ which is a very small difference in percentage points. Relating to hypothesis $7^{58}$, the statement from previous reports and interviews that an incentive for the loans is to enter the carry phase faster, in terms of lower IRR, is true for our simulated

[^31]fund universe, but only for funds that earn a certain gross IRR; the carry phase is not entered at a lower IRR but rather more of the profit is given to the GPs in terms of carry in the gross IRR brackets between 8-12\%.

## Exhibit 5.15 - Profit sharing to GPs at different IRRs



As seen in exhibit 5.15 above, at a profit sharing level we now observe that the GPs on average start collecting more profit sharing at a lower gross IRR levels with the loans.

Exhibit 5.16 - Total GP compensation at different IRRs


Furthermore, in exhibit 5.16 above we look at total GP compensation. We observe that the GP compensation is higher in the loan case only in the gross IRR brackets of $8-10 \%$ and $10-12 \%$. Below $8 \%$ the GPs only earn their fees, in both the no-loan and the loan case. Above a gross IRR of $12 \%$ two effects come into play that reduces the GP compensation in the loan case as compared to the no loan case. First, as GPs earn carry on $20 \%$ on the profits made in the profit sharing region and the total amount of profit made is reduced by the amount of interest paid, this has a negative effect on GP carry. Second, in the loancase the amount of hurdle is lower due to capital call deferrals which translates into less catch-up in which the GPs earn $60 \%$. Some of the carry that would be counted as catch-up is now instead counted as profit sharing. As an effect of this the GPs come into the profit sharing region faster, meaning at lower IRRs, in the loan-case in which the capital available for distribution is reduced by the interest paid to the banks. Since our base case has a risk-free interest rate of $0 \%$ the PV to GPs represent actual nominal amount paid to the GPs.

Looking at the timing of the carry flows to GPs we compare the amount of carry paid in the no loan case to the loan case in terms of year and brackets. Exhibit 5.17 below shows that difference.

### 5.17 - Average Carry to GPs for different fund IRRs over time

| Average difference of carry in distribution years, Loan minus no-loan |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Brackets | Year 6 | Year 7 | Year 8 | Year 9 | Year 10 | Sum |
| $-2 \%-0 \%$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $0 \%-2 \%$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $2 \%-4 \%$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $4 \%-6 \%$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $6 \%-8 \%$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $8 \%-10 \%$ | 0 | 13,874 | 557,148 | $2,710,318$ | $7,104,931$ | $10,386,271$ |
| $10 \%-12 \%$ | 231,916 | $1,209,082$ | $4,826,015$ | $8,163,636$ | $2,702,754$ | $17,133,403$ |
| $12 \%-14 \%$ | 638,643 | $2,133,759$ | $3,326,617$ | $-990,639$ | $-6,031,667$ | $-923,287$ |
| $14 \%-16 \%$ | 658,847 | $1,279,784$ | 637,207 | $-2,785,012$ | $-3,117,171$ | $-3,326,346$ |
| $16 \%-18 \%$ | 665,328 | $1,089,825$ | $-838,406$ | $-2,858,280$ | $-1,450,468$ | $-3,392,000$ |
| $18 \%-20 \%$ | 682,079 | 331,594 | $-1,113,578$ | $-2,426,790$ | $-865,305$ | $-3,392,000$ |
| $20 \%-22 \%$ | 399,760 | 82,122 | $-1,418,370$ | $-2,079,232$ | $-376,279$ | $-3,392,000$ |
| $22 \%-24 \%$ | 410,931 | $-91,750$ | $-1,746,441$ | $-1,816,256$ | $-148,485$ | $-3,392,000$ |
| $24 \%-26 \%$ | 316,452 | $-455,155$ | $-1,798,126$ | $-1,263,340$ | $-191,831$ | $-3,392,000$ |
| $26 \%-28 \%$ | 249,199 | $-631,627$ | $-1,920,232$ | $-930,464$ | $-158,876$ | $-3,392,000$ |
| $28 \%-30 \%$ | 216,805 | $-1,133,239$ | $-1,715,637$ | $-730,224$ | $-29,705$ | $-3,392,000$ |

We find that GPs do not earn carry faster to an extent that it would have a material impact on the decision on whether to use the loans or not. This considers that in the first year the carry difference is less than

SEK 1 million. Put in perspective of the total fees of SEK 141 million, these are small amounts. At higher Gross IRRs, this difference translates into a loss in carry. It is only in the gross IRR brackets of $8-12 \%$ when most of the carry is distributed as catch-up that the GPs earn more carry earlier because of reasons discussed previously. This concludes hypothesis $8^{59}$ and we can say that for our brackets, the GPs do earn more money faster, in the first year of divestments. Whether the GPs earn more money faster after the first year depends on the gross IRR achieved by their fund. The total impact on GP compensation becomes negative above $12 \%$ gross IRR.

Furthermore, relating to hypothesis $6^{60}$, for funds that achieve a high IRR we see that GPs receive less money. The only instance when the GPs receive more carry in the loan case is when the fund returns most of its carry in the catch-up region. The average money made is higher for GPs only in the gross return brackets of 8 to $12 \%$. For funds that return a gross IRRs above $12 \%$ the GPs receive less carry because the interest on the loan decreases the amount available to be distributed as carry.

We also find in the sensitivity analysis (section 5.1) that, when the catch-up is removed, GPs always earn more money in the loan case, when the loan spread is $2 \%$. This is because less hurdle is built up in the loan case because of the reduced time that GPs claim LP capital. This effectively moves returns, that would be calculated as hurdle into profit sharing, and the cost of the loans at a loan spread of $2 \%$ is not high enough to offset the benefits GPs gain by reducing the hurdle amount. We find that the catch-up works to align the incentives between the GPs and LPs at higher returns when the loan spread is at $2 \%$ by making the GPs pay for the subscription loan through a reduction in their carry. This reduction corresponds to their share of the interest on the loans.

### 5.2.3 Compensation analysis to LPs

When presenting the LP flows over brackets we start with total returned pre-hurdle, then we show their flows from hurdle, catch-up and profit sharing carry. Exhibit 5.18 below shows what is returned prehurdle at different gross IRRs to LPs.

[^32]
## Exhibit 5.18 -LP repayment pre hurdle at different IRRs



We observe a rising amount returned pre-hurdle as gross IRR goes from negative to positive. The total amount drawn increases in the loan case by the amount of interest on the loans which amount to SEK 16.96 million. The amount returned pre-hurdle is maxed out in the brackets of $6-8 \%$ for both the loan and no loan case, since at that level all drawn capital has been returned and hurdle start to be paid out.

Exhibit 5.19 below shows the average hurdle paid to LPs at different gross IRRs with a span of -1 to $+1 \%$ from the gross IRR values on the x -axis.

## Exhibit 5.19 - Hurdle to LPs at different IRR levels



| Gross range | No loan | Loan |
| :---: | :---: | :---: |
| $-2 \%-0 \%$ | 0 | 0 |
| $0 \%-2 \%$ | 0 | 0 |
| $2 \%-4 \%$ | $12,603,340$ | $6,886,639$ |
| $4 \%-6 \%$ | $95,429,667$ | $79,113,172$ |
| $6 \%-8 \%$ | $204,070,564$ | $187,110,564$ |
| $8 \%-10 \%$ | $309,595,788$ | $275,325,336$ |
| $10 \%-12 \%$ | $340,780,610$ | $280,631,033$ |
| $12 \%-14 \%$ | $335,642,289$ | $273,706,627$ |
| $14 \%-16 \%$ | $328,563,295$ | $266,366,677$ |
| $16 \%-18 \%$ | $324,936,901$ | $262,591,737$ |
| $18 \%-20 \%$ | $317,352,628$ | $254,820,044$ |
| $20 \%-22 \%$ | $310,184,785$ | $247,453,374$ |
| $22 \%-24 \%$ | $304,591,607$ | $241,669,238$ |
| $24 \%-26 \%$ | $304,424,345$ | $241,486,445$ |
| $26 \%-28 \%$ | $298,531,038$ | $235,429,883$ |
| $28 \%-30 \%$ | $293,012,618$ | $229,750,558$ |

We see that the average hurdle paid goes down with loan and increase with gross IRR until a gross return of $10-12 \%$ is achieved after which the average starts to decrease. In this gross IRR bracket the average hurdle is the highest in the whole data-set. At higher gross IRRs, the returns are high enough to allow for investments to return enough money to investors so that the fund surpasses the hurdle requirement and at higher returns this limit is surpassed earlier which is why we see the graphs panning down at higher returns.

## Exhibit 5.20 - LP catch-up at different IRRs



In exhibit 5.20 above, the average catch-up paid to LPs follow the same trend as that paid to the GPs. LPs start earning catch-up at a lower gross IRR and comparing these two cases the level at which this happens is the same as in the case for the GPs - at $8.2 \%$ in the loan case and $8.6 \%$ in the no loan case.

## Exhibit 5.21 - Profit sharing to LPs at different IRRs



As seen in exhibit 5.21 above, the average profit sharing earned by LPs also follow the same tendencies between the loan and no loan case as it does for GPs with more profit sharing starting to be collected at lower IRR levels when the loans are in place.

## Exhibit 5.22 - Total money invested and returned to LPs (NPV) at different IRRs



What we see when looking at the difference in total LP return is that for low returns, the NPV difference for the LPs is the total of the interest charged by the banks for the loans. Between 8 and $12 \%$ gross IRR the NPV is significantly lower in the loan case because less hurdle is built up and more return is distributed as catch-up to the GPs. At around $16 \%$ gross IRR and above we see that the loss for LPs is $80 \%$ of the interest cost. At these return points the GPs and LPs split the loan cost in accordance to the
profit sharing schedule. In conclusion, we see that the total actual money in SEK to LPs is always lower with the loan case. This is what has been indicated in previous research and interviews ${ }^{61}$.

To sum up the LP compensation. We find that the composition of return to the LPs change with the loans. For funds that yield positive net IRR the LPs gain more money returned pre-hurdle; the amount is equal to the total interest paid on the loans. Further, they gain less return in the form of hurdle but more in profit sharing. For funds that end up paying most of the carry as catch-up, the LPs lose the most money as compared to the no loan case. For higher return funds, the LPs return is reduced by $80 \%$ of the interest on the loans. Actual money made is in total always lower in the loan case verifying hypothesis $9^{62}$.

[^33]
## 6. Conclusions

### 6.1 IRR implications

In a broader perspective, the subscription loans impact IRR in line with previous research. We add depth in creating a fund universe in an academically backed up model with different investment value development paths where we for interesting brackets show that negative scenarios yield a magnifying negative effect and positive scenarios a boosting effect, both on gross and net IRR. The magnifying effect comes from the fact that the GPs claim the LP capital for a shorter period when using these loans.

More specifically, the higher IRR the fund ends up with the larger the effect on IRRs will be. We find that some funds would be able to boost their return by several percentage points in such a way as to increase its quartile ranking to a first quartile ranking $(22 \%)^{63}$ which should have a considerable positive effect for the GPs when they seek investors for future funds.

From our sensitivity analysis in section 5.1, one can draw several conclusions under what conditions the loans have the most impact. One of the most interesting parameters to look at is the interest rate as it is one of the main drivers of the usage of these facilities. Expectedly, in line with our interview responses, we see that these loans on average become less attractive when the loan spread increases, why we should expect less usage of these loans if the spread increases. However, interest rates would have to increase quite a lot before a stop in the usage of the loans happens. Furthermore, having eligible investors is a requirement for the loans but it takes a large difference in the eligible investor ratio to impact the IRRboosting effect from the loans. Still, even without the loans the GPs should prefer eligible investors because of the reputation and credibility these give to the fund. ${ }^{64}$

### 6.2 Value created and incentives for the parties involved

Our findings further emphasize how the subscription loans impact the different return steps of a PE fund (hurdle, catch-up and profit sharing). While our model fund structure (with a catch-up) is a common structure for a PE fund, we find that it impacts the effect of actual added or lost value to the parties. This is in line with Metrick and Yasuda's (2010) findings on contract differences impacts on fund return and GP compensation, and in our case the effect is on GP/LP compensation.

We find some cases where GPs earn more by using subscription loan financing. This always happens if the catch-up is removed from our base case, except for when the interest spread on the loans exceeds $8 \%$.

[^34]If the GPs have a catch-up, as in our benchmark model, they earn more money through this financing only if their actual return ends up below the catch-up cap (12\%). GPs with catch-up earn less money through carry if their fund returns carry as profit sharing (above the catch-up cap), and this is the amount of profit sharing lost due to interest paid on the loans. Our conclusion is that a catch-up functions as an incentive alignment between the GPs and the LPs for funds that achieve a return that yields carry in the profitsharing region. Kaplan and Strömberg (2009) and Gompers et al (2016) argue that governance engineering is one value creation tool in private equity deals that aim to align incentives between LPs and GPs. Since the GPs get a share of the down-side in the cost of the subscription loans when using a catch-up, our model findings relating to a PE structure with and without a catch-up phase therefore suggests that the catch-up is an example of governance engineering value creation. However, we can say that the value of increasing the IRR outweighs the loss in actual money to the GPs. Showing a higher IRR will make the GPs able to attract more investors in the future (e.g. Appelbaum (2016)).

With our benchmark model, we observe that the PV to GPs get the highest positive amplification around gross returns of $8 \%$. This suggests that GPs expecting that their return will end up in that area should desire these loans the most, which is what some of our interviews and previous research have indicated ${ }^{65}$. Regarding earlier statements on that GPs use the loans to get carry faster ${ }^{66}$, we find limited effects on this. We also find limited evidence that the GPs earn money at lower IRRs when the loans are applied.

Furthermore, these loans are not solely used by funds that perform around the hurdle, they are also used by high performing funds and the usage is overall increasing ${ }^{67}$. The general incentive for the GPs can be connected to the importance of boosting the IRR to improve the ranking of the fund, it is especially attractive for funds that achieve a net IRR of around $20 \%$ because these funds would be able to achieve a higher quartile ranking. PE funds have showcased less superior performance to other asset classes in the last 10 years compared to previous decades (Appelbaum (2016)) and the competition between PE funds is fierce, leading to difficulties in raising the next fund. The subscription loan usage is in line with game theory approach in a competitive situation as underlined by our interview correspondence; funds are being forced towards using these facilities because everyone else is using them ${ }^{68}$.

There are some LPs that are measured on IRR who benefit through being able to show better IRRs while this conduct is looked down upon by LPs that are measuring return in actual money (NPV to LP or MoM).

[^35]First, underlining the agency problem of financial contracting as described by e.g. Axelson et al (2009) and Robinson and Sensoy (2013) we can verify that subscription loans do not motivate GPs to maximize LP returns. Second, looking at the control theory (Aghion and Bolton (1992) and Conner (2005)) we also conclude that since the benefits go primarily to the GPs the subscription loans enhance the control theory problem in private equity. Subscription loans can therefore be added as an example of financial contracting problems outlined in earlier research.

Since we verify that LPs lose actual money with the bridge loan facilities, one may ask what is in it for LPs that are not measured on IRR. One common thing mentioned in our interviews as a benefit of the loans is that the capital draw-downs become fewer and larger. This is claimed to improve administration and "quality of life" for these LPs, and would arguably mitigate the cash flow risk if well planned and communicated to the LPs. ${ }^{69}$ If this is done, the LPs can more optimally manage their capital. ${ }^{69}$ By investing the capital that is not called the LPs could potentially match the interest cost of the loans by earning the same return as the interest somewhere else. However, there are some caveats for LPs in using their money successfully elsewhere.

First, Robinson and Sensoy (2015) describe the typical cash flow risk in private equity, that there is a risk in that capital calls come unexpectedly due to GPs' decisions on timing and magnitudes of investments, and that this might be at a sub-optimal time for the LPs. This relates to the cost of uncalled capital as outlined by Demaria (2010); the LPs must be able to supply the GPs with capital on short notice and the lack of visibility of capital calls makes it hard to plan the portfolio composition due to liquidity issues. Since LPs cannot plan the capital calls, some cash must stay in liquid positions with relatively low risk, creating an opportunity cost (Demaria (2010)) ${ }^{70}$.

Second, in today's economic environment it is difficult to find highly liquid, risk free, fixed income securities that yield a return equal to the interest rate the bank charges on the loans, as indicated in various interviews. ${ }^{71}$ Investing the money elsewhere puts the LPs in a significant risky position should they not be able to divest their assets or if they take on losses on that capital since missing out on paying a draw-down can have draconic consequences where the LP lose its interest in the fund (Pindyck-Costantino \& Corelli (2010)).

Regarding the LPs, we would argue that the benefits are limited with subscription loans. Since these loans mean fund increase their reported returns, but make investors lose money as described above, one could

[^36]argue that these facilities can be classified as a return manipulation tool, in line with the definition outlined by Ingersoll et al. (2007). The return manipulation definition outlined by Ingersoll et al. (2007) is:
"The action to increase fund return in a way that does not actually add value for the fund's investor".
This is true in our investigation through increased reported IRR and decreased value to LPs. One might however argue that the value for LPs of having the capital calls compressed could outweigh the loss of money. If this value, however measured, is larger than the decreased value to LPs through interest payments to the banks then according to Ingersoll et al. (2007), this does not constitute return manipulation. On the other hand, as indicated in interviews by certain LPs, this benefit is not strong enough to justify the facilities ${ }^{72}$.

We find it hard to justify these loans from an LP perspective, and it is understandable why LPs that are not measured on IRR oppose these loans. Our analysis of subscription loans underlines the flaws of the IRR as a performance measure in private equity, as argued previously by for instance Appelbaum (2016) and Kaplan et al (2016).

### 6.3. Suggestions for further research

An interesting question for further research is to look at how the discussion continues between LPs, e.g. AP6, and the GPs, and how the outcome effects the rapidly increasing usage of these loans. A suggestion for further research would also be to look at how the subscription loans affect other performance measurements, such as PME (Public Market Equivalent) and net asset values, which have been discussed as better performance measurement tools (Appelbaum (2016), Kaplan et al (2016)). Another important factor we have not analyzed in depth in this paper is the opportunity cost for LPs when capital drawdowns are delayed.

In the upcoming years, private equity firms that are using the subscription loans will start exiting their investments which means data will become available on the actual performance of these funds contra funds that do not use these loans. It would be interesting to look at these funds and adjust their performance, to add depth to and back up the findings we present in this paper. One would then be able to compare actual funds, and adjust their rankings. Since relative fund comparison decides where LPs will place their money in upcoming fund raisings, it will become more important to be able to compare returns properly, as the usage of and demand for these loans is increasing rapidly ${ }^{73}$. Furthermore, once empirical data on private equity deals with subscription loans become available one could solidify our model

[^37]findings by for instance looking at regression analysis, between e.g. interest rates and the usage of the subscription loans, as well as correlation between usage of the subscription loans and IRR performance.

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### 7.3 Interview correspondence

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Phone interview with Morten Welo, FSN Capital, February 16, 2017

Interview in person with a representative of a Nordic bank, 21 February 2017

Phone interview with Toby Suen, Aberdeen asset management, 3 March 2017

Phone interview with Mattias de Beau, Stepstone global, 3 March 2017

Phone interview with Stephen Short, Simpson Thacher \& Bartlett LLP, 3 March 2017

## 8. Appendix

### 8.1 Earlier research (TorreyCove) example of IRR impacts of subscription loans

TorreyCove assumes $100 \%$ debt financing of fund assets for two years, i.e. capital call deferral of two years in the credit line scenario. They use a fund size and investment of $\$ 100$ million, interest on the debt of 4 percent, a 2 percent management fee and a holding period of six years. Realization values of the investment are $\$ 200$ million and $\$ 50$ million in the respective profit and loss scenarios. In a second example, they use the same fund set-up, but only include a gain scenario with a realization value of $\$ 162$, which underlines a situation in which the 8 percent hurdle rate is reached and collection of carry (20 percent of the fund's profits) is allowed (TorreyCove (2016)).

Exhibit 8.1.1 - IRR implications of subscription loans, TorreyCove Capital Partners (USDm)*

| Transaction Type | Gain |  | Loss |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Traditional | Credit line | Traditional | Credit line |
| [ Investment | (100) |  | (100) |  |
| Year 1 Management Fee | (2) |  | (2) |  |
| L Interest on Debt |  |  |  |  |
| Year 2 [ Management Fee | (2) |  | (2) |  |
| Investment |  | (100) |  | (100) |
| Year $3\{$ Management Fee | (2) | (6) | (2) | (6) |
| Interest on Debt |  | (8) |  | (8) |
| Years [ Investment |  |  |  |  |
| 4,5 \& $6\left\{\begin{array}{l}\text { Management Fee }\end{array}\right.$ | (6) | (6) | (6) | (6) |
| 4,5\& Interest on Debt |  |  |  |  |
| End of $\{$ Realization | 200 | 200 | 50 | 50 |
| IRR | 10.56\% | 13.93\% | -13.28\% | -20.23\% |
| MOIC | 1.79x | 1.67x | 0.45x | 0.42x |

[^38]Exhibit 8.1.2-8 percent IRR hurdle reached with the help of a subscription credit line (USDm)*


### 8.2 Exhibits relating to our methodology and model set-up

## Exhibit 8.2.1 - Benchmark case investment inputs

| Investment inputs |  |
| :--- | ---: |
|  |  |
| Committed capital | 1000000000 |
| Capital used for investments | $80 \%$ |
| Investments per year | 2 |
| Years of investing | 5 |
| Total number of investments | 10 |
| Management fees year 1-5 (of committed capital) | $2 \%$ |
| Managememt fees year 6-10 (of invested capital) | $2 \%$ |
| Holding period | 5 |
| Leverage (as \% of equity) | $200 \%$ |
|  |  |
| Carried interest | $8 \%$ |
|  |  |
| Hurdle rate | $60 \%$ |
| Catch up rate (carried interest) | $12 \%$ |
| Catch up cap | $20 \%$ |
| Profit sharing rate (carried interest) |  |

Exhibit 8.2.2 - Benchmark case bridge loan terms

Bridge loan terms

|  |  |
| :--- | ---: |
| Length [years] | 1 |
| Libor spread | $2 \%$ |
| Interest rate | $2 \%$ |

Dther inputs

Stibor 0,00\%
Bridge loan duration (years) 1
Eligible investor ratio 100\%
Drawn debt ratio 150\%
Max of committed capital $20 \%$

## Exhibit 8.2.3-IRR calculation without subscription loan

| IRR Calculation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 1 | 2 | 3 | 4 | 5 |
| No bridge loan |  |  |  |  |  |
| Divestments - investments | -208 000000 | -184000 000 | -200 000000 | -144000 000 | -64000 000 |
| Gross IRR | 23\% | 23\% |  |  |  |
| LP flows |  |  |  |  |  |
| Drawdowns | -228 000000 | -204000000 | -220000000 | -164 000000 | -84000 000 |
| Returned pre hurdle | 0 | 0 | 0 | 0 | 0 |
| Hurdle | 0 | 0 | 0 | 0 | 0 |
| Carry tier I | 0 | 0 | 0 | 0 | 0 |
| Carry tier II | 0 | 0 | 0 | 0 | 0 |
| Total LP CF | -228 000000 | -204 000000 | -220 000000 | -164 000000 | -84 000000 |
| Net IRR | 16\% |  |  |  |  |
| MoM | 1.77x |  |  |  |  |
| Present values for LPs |  |  |  |  |  |
| PV of drawdowns | -228000000 | -204000000 | -220000000 | -164000000 | -84000000 |
| PV of pre hurdle | 0 | 0 | 0 | 0 | 0 |
| PV of hurdle | 0 | 0 | 0 | 0 | 0 |
| PV of carry tier I | 0 | 0 | 0 | 0 | 0 |
| PV of carry tier II | 0 | 0 | 0 | 0 | 0 |
| NPV | 726963054 |  |  |  |  |
| Present value for GPs |  |  |  |  |  |
| Fees | 20000000 | 20000000 | 20000000 | 20000000 | 20000000 |
| Carry tier 1 | 0 | 0 | 0 | 0 | 0 |
| Carry tier 2 | 0 | 0 | 0 | 0 | 0 |
| PV | 323180763 |  |  |  |  |

IRR Calculation

| Year | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No bridge loan |  |  |  |  |  |
| Divestments - investments | 1236816180 | 394658672 | 0 | 125742126 | 92926838 |
| Gross IRR | 23\% |  |  |  |  |

LP flows

| Drawdowns | -16000 000 | -11840000 | -8160 000 | -4160 000 | -1280000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Returned pre hurdle | 941440000 | 0 | 0 | 0 | 0 |
| Hurdle | 242240000 | 0 | 0 | 0 | 0 |
| Carry tier I | 21254472,15 | 27193527,85 | 0 | 0 | 0 |
| Carry tier II | 0 | 261339882 | 0 | 100593700 | 74341471 |
| Total LP CF | 1188934472 | 276693410 | -8160000 | 96433700 | 73061471 |
| Net IRR | 16\% |  |  |  |  |
| MoM | 1.77x |  |  |  |  |


| Present values for LPs |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| PV of drawdowns | -16000000 | -11840000 | -8160000 | -4160000 | 0 |
| PV of pre hurdle | 941440000 | 0 | 0 | 0 | 0 |
| PV of hurdle | 242240000 | 0 | 0 | 0 | 0 |
| PV of carry tier I | 21254472 | 27193528 | 0 | 0 |  |
| PV of carry tier II | 0 | 261339882 | 0 | 100593700 | 74341471 |

NPV

| Present value for GPs |  |  | 8160000 | 4160000 | 1280000 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Fees | 16000000 | 11840000 | 0 | 0 | 0 |
| Carry tier 1 | 31881708,22 | 40790291,78 | 0 | 25148425 |  |
| Carry tier 2 | 0 | 65334971 |  | 18585368 |  |
| PV | 323180763 |  |  |  |  |

## Exhibit 8.2.4-IRR calculation with subscription loans

| IRR Calculation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 1 | 2 | 3 | 4 | 5 |
| Bridge loan |  |  |  |  |  |
| Investments | -8000000 | -200000 000 | -184000 000 | -2000000000 | -144000 000 |
| Divestments | 0 | 0 | 0 | 0 | 0 |
| Net Cash flow | -8000000 | -200000 000 | -184000 000 | -2000000 000 | -144000 000 |
| Gross IRR | 31\% | 31\% |  |  |  |
| LP flows |  |  |  |  |  |
| Drawdowns | -28000 000 | -208000 000 | -224000 000 | -204 000000 | -167280 000 |
| Returned pre hurdle | 0 | 0 | 0 | 0 | 0 |
| Hurdle | 0 | 0 | 0 | 0 | 0 |
| Carry tier I | 0 | 0 | 0 | 0 | 0 |
| Carry tier II | 0 | 0 | 0 | 0 | 0 |
| Total LP CF | -28 000000 | -208000000 | -224000000 | -204000000 | -167280000 |
| Net IRR | 21\% |  |  |  |  |
| Molv | 1.74x |  |  |  |  |
| Present values for LPs |  |  |  |  |  |
| PV of drawdowns | -28000000 | -208000000 | -224000000 | -204000000 | -167280000 |
| PV of pre hurdle | 0 | 0 | 0 | 0 | 0 |
| PV of hurdle | 0 | 0 | 0 | 0 | 0 |
| PV of carry tier I | 0 | 0 | 0 | 0 | 0 |
| PV of carry tier II | 0 | 0 | 0 | 0 | 0 |
| NPV | 713395054 |  |  |  |  |
| Present value for GPs |  |  |  |  |  |
| Fees | 20000000 | 20000000 | 20000000 | 20000000 | 20000000 |
| Carry tier 1 | 0 | 0 | 0 | 0 | 0 |
| Carry tier 2 | 0 | 0 | 0 | 0 | 0 |
| PV | 319788763 |  |  |  |  |


| IRR Calculation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 6 | 7 | 8 | 9 | 10 |
| Bridge loan |  |  |  |  |  |
| Investments | -64000000 | 0 | 0 | 0 | 0 |
| Divestments | 1236816180 | 394658672 | 0 | 125742126 | 92926838 |
| Net Cash flow | 1172816180 | 394658672 | 0 | 125742126 | 92926838 |
| Gross IRR | 31\% |  |  |  |  |
| LP flows |  |  |  |  |  |
| Drawdowns | -101680 000 | -11840000 | -8160000 | -4 160000 | -1280 000 |
| Returned pre hurdle | 958400000 | 0 | 0 | 0 | 0 |
| Hurdle | 177542400 | 0 | 0 | 0 | 0 |
| Carry tier I | 35508480 | 0 | 0 | 0 | 0 |
| Carry tier II | 9682064,293 | 315726938 | 0 | 100593700, 5 | 74341470,77 |
| Total LP CF | 1079452944 | 303886938 | -8160000 | 96433700 | 73061471 |
| Net IRR | 21\% |  |  |  |  |
| MolM | 1.74x |  |  |  |  |
| Present values for LPs |  |  |  |  |  |
| PV of drawdowns | -101680000 | -11840000 | -8160000 | -4160000 | -1280000 |
| PV of pre hurdle | 958400000 | 0 | 0 | 0 | 0 |
| PV of hurdle | 177542400 | 0 | 0 | 0 | 0 |
| PV of carry tier I | 35508480 | 0 | 0 | 0 | 0 |
| PV of carry tier II | 9682064 | 315726938 | 0 | 100593700 | 74341471 |
| NPV | 713395054 |  |  |  |  |
| Present value for GPs |  |  |  |  |  |
| Fees | 16000000 | 11840000 | 8160000 | 4160000 | 1280000 |
| Carry tier 1 | 53262720 | 0 | 0 | 0 | 0 |
| Carry tier 2 | 2420516,073 | 78931734,49 | 0 | 25148425,12 | 18585367,69 |
| PV | 319788763 |  |  |  |  |

## Exhibit 8.2.5 - Private equity investor target investment horizon

Below depicts a survey made by Gompers et al (2016), where we see that almost $100 \%$ of the PE firms interviewed in their large sample use five years.


### 8.2.1 Note on GBM methodology

## Volatility and Correlation of investments

In terms of volatility of BO fund investments, Metrick and Yasuda (2010) rely on Campbell et al. (2001) using a volatility of 60 percent. The foundation behind that figure is the fact that BO funds invests in public companies (and take them private) or in private companies that are similar in size to small public companies (Metrick and Yasuda (2010)). Since BO funds buy low-beta companies and then load them up with debt, Woodward (2004) states that the mean beta of all BO funds is around 1. Metrick and Yasuda (2010) elaborates on this, explaining that the increased leverage of the target companies would affect the idiosyncratic risk of these companies, and estimate the volatility of BO investments to be the same as a unit beta public stock of similar size. Metrick and Yasuda (2010) follow Campbell et. al (2001), in using 60 percent as a small-stock volatility using the BO fund size and target company approach. Also, due to various reasons, e.g. that funds at times make many investments in the same industry, as argued by Metrick and Yasuda (2010), fund investments can co-move with each other. The correlation between any pair of investments of BO funds is assumed to be 20 percent in line with Campbell et al. (2001). They argue that this is the correlation chosen to match the high end of the correlation between small-company investments in the same industry.

## Geometric Brownian Motion elaboration including Cholesky decomposition

The GBM formula for simulating a value development is as follows: $\boldsymbol{S} \mathbf{1}=\boldsymbol{S 0} * \boldsymbol{e}^{\boldsymbol{\mu}-\boldsymbol{\sigma}^{\mathbf{2}} / \mathbf{2}+\boldsymbol{\sigma} \boldsymbol{\varepsilon}}$. S0 is the initial value and S 1 the value next year, and $\mu$ is the drift, or the annual expected return. We use a risk-free rate of zero, so $\mu$ is set to zero. $\sigma$ is the volatility of the investment value development, and is set to 60 percent according to Metrick and Yasuda (2010). $\varepsilon$ is the error term of each investment value development, following a standard normal distribution. The error term considers potential correlation between several random variables, which is the case for us. Below we depict how we handle this pair-wise correlation between investments in a fund. In the value development of the investments, the random shock of a value path development is driven by potential correlation between investments. As learned in the course 4320 - Quantitative methods in corporate finance ${ }^{74}$ at the Stockholm School of Economics, Cholesky decomposition is useful when handling multiple correlated random variables. Exhibit 8.2.6 outlines the correlation between any pair of investments at 20 percent, as given by Metrick and Yasuda (2010). Exhibit 8.2.7 contains the Cholesky decomposition step of the correlations, and Exhibit 8.2.8 the matrix of uncorrelated standard normal distributed random variables. By multiplying exhibit 8.2 .8 with the

[^39]Cholesky matrix in exhibit 8.2 .7 we arrive at the correlated standard normal distributed RVs, outlined in exhibit 8.2.9. While exhibit 8.2.6 and 8.2.7 below are used for all cases, exhibit 8.2.8 is randomly drawn. Therefore, the final matrix seen in exhibit 8.2.9 that goes into the value path development of the investments will vary.

Exhibit 8.2.6-Correlation matrix between investments

| Investment |  |  |  |  |  |  |  |  | Matrix |  |  |  |  |  |  |  | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Investment | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{1}$ | $100 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{2}$ | $20 \%$ | $100 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{3}$ | $20 \%$ | $20 \%$ | $100 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{4}$ | $20 \%$ | $20 \%$ | $20 \%$ | $100 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{5}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $100 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{6}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $100 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{7}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $100 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{8}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $100 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{9}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{1 0}$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ | $20 \%$ |  |  |  |  |  |  |  |  |  |  |  |  |  |

Exhibit 8.2.7-Cholesky decomposition matrix

|  | Decomposition matrix |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 2 | 0,20 | 0,98 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 3 | 0,20 | 0,16 | 0,97 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 4 | 0,20 | 0,16 | 0,14 | 0,96 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 5 | 0,20 | 0,16 | 0,14 | 0,12 | 0,95 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 6 | 0,20 | 0,16 | 0,14 | 0,12 | 0,11 | 0,94 | 0,00 | 0,00 | 0,00 | 0,00 |
| 7 | 0,20 | 0,16 | 0,14 | 0,12 | 0,11 | 0,09 | 0,94 | 0,00 | 0,00 | 0,00 |
| 8 | 0,20 | 0,16 | 0,14 | 0,12 | 0,11 | 0,09 | 0,09 | 0,93 | 0,00 | 0,00 |
| 9 | 0,20 | 0,16 | 0,14 | 0,12 | 0,11 | 0,09 | 0,09 | 0,08 | 0,93 | 0,00 |
| 10 | 0,20 | 0,16 | 0,14 | 0,12 | 0,11 | 0,09 | 0,09 | 0,08 | 0,07 | 0,93 |

Exhibit 8.2.8 - Uncorrelated standard normal distributed random variables*

| Year--> |  |  |  |  |  |  |  |  | $\mathbf{1}$ | $\mathbf{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Investment | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |  |  |
| $\mathbf{1}$ | $-1,43$ | 0,09 | 0,92 | 0,35 | $-1,11$ | $-0,57$ | $-0,65$ | 0,04 | $-0,31$ | $-1,32$ |
| $\mathbf{2}$ | $-0,91$ | $-0,10$ | 1,19 | $-0,36$ | $-1,83$ | $-1,02$ | $-0,82$ | 1,46 | 1,50 | $-0,92$ |
| $\mathbf{3}$ | $-0,10$ | $-1,91$ | 0,04 | $-0,33$ | 0,41 | $-0,03$ | 0,35 | $-0,97$ | 2,20 | $-1,13$ |
| $\mathbf{4}$ | 0,04 | $-0,36$ | 1,90 | 0,01 | 0,33 | 0,80 | $-0,78$ | $-0,57$ | $-1,32$ | 0,51 |
| $\mathbf{5}$ | $-1,05$ | 0,13 | 1,15 | 2,56 | $-0,28$ | $-0,11$ | $-0,07$ | $-0,62$ | 1,21 | 1,69 |
| $\mathbf{6}$ | $-0,86$ | 0,78 | $-0,84$ | 0,33 | $-2,34$ | $-0,55$ | 3,33 | 0,14 | $-0,03$ | 0,72 |
| $\mathbf{7}$ | $-0,46$ | 1,37 | $-0,28$ | 0,37 | 0,20 | 0,68 | $-1,51$ | $-0,81$ | $-1,22$ | 1,87 |
| $\mathbf{8}$ | 1,43 | $-0,73$ | 0,04 | $-0,02$ | 0,27 | $-1,06$ | $-0,80$ | 0,06 | 1,39 | $-0,16$ |
| $\mathbf{9}$ | 0,57 | $-1,07$ | $-2,71$ | 0,81 | 0,21 | 1,38 | 0,70 | 0,39 | 0,98 | $-0,12$ |
| $\mathbf{1 0}$ | $-1,13$ | $-0,02$ | 1,19 | $-0,52$ | 0,12 | 0,69 | $-2,46$ | 0,99 | $-0,78$ | $-0,83$ |

* An example of one simulation

Exhibit 8.2.9-Correlated standard normal distributed random variables*

| Year--> |  |  | Corr. Std. Norm. RV. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Investment | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | -1,43 | 0,09 | 0,92 | 0,35 | -1,11 | -0,57 | -0,65 | 0,04 | -0,31 | -1,32 |
| 2 | -1,17 | -0,08 | 1,35 | -0,28 | -2,01 | -1,12 | -0,93 | 1,44 | 1,41 | -1,16 |
| 3 | -0,53 | -1,84 | 0,42 | -0,31 | -0,12 | -0,31 | 0,08 | -0,69 | 2,31 | -1,50 |
| 4 | -0,41 | -0,60 | 2,20 | -0,02 | -0,15 | 0,48 | -0,96 | -0,44 | -0,77 | -0,08 |
| 5 | -1,44 | -0,18 | 1,70 | 2,39 | -0,69 | -0,29 | -0,37 | -0,55 | 1,48 | 1,09 |
| 6 | -1,37 | 0,45 | -0,06 | 0,54 | -2,66 | -0,72 | 2,82 | 0,11 | 0,43 | 0,35 |
| 7 | -1,07 | 1,07 | 0,39 | 0,61 | -0,49 | 0,38 | -1,41 | -0,77 | -0,69 | 1,49 |
| 8 | 0,67 | -0,78 | 0,66 | 0,28 | -0,41 | -1,18 | -0,88 | -0,03 | 1,65 | -0,26 |
| 9 | -0,03 | -1,15 | -1,90 | 1,06 | -0,44 | 1,01 | 0,46 | 0,29 | 1,37 | -0,23 |
| 10 | -1,57 | -0,25 | 1,54 | -0,12 | -0,51 | 0,46 | -2,43 | 0,87 | -0,19 | -0,89 |

[^40]
### 8.3 Further sensitivity analysis

### 8.3.1 Holding period

The IRRs are very sensitive to holding periods. When we decrease the holding period the average IRR changes drastically as shown below:

## Exhibit 8.3.1.1 - Holding period analysis

|  |  | Gross IRR |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Benchmark case | 4 years | 3 years | 2 years | 1 year |
|  |  | Average | $7.41 \%$ | $10.41 \%$ | $14.95 \%$ | $28.22 \%$ |
| No Loan | Median | $5.18 \%$ | $7.21 \%$ | $10.27 \%$ | $19.05 \%$ | $41.12 \%$ |
|  | Volatility | $22 \%$ | $26 \%$ | $32 \%$ | $44 \%$ | $105 \%$ |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Loan | Average | $10.067 \%$ | $15.544 \%$ | $27.188 \%$ | $84.077 \%$ | $2147.613 \%$ |
|  | Median | $6.45 \%$ | $9.61 \%$ | $15.34 \%$ | $37.24 \%$ | $403.27 \%$ |
|  | Volatility | $29 \%$ | $37 \%$ | $55 \%$ | $147 \%$ | $3407 \%$ |


|  |  | Net IRR |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No Loan |  | Benchmark case | 4 years | 3 years | 2 years | 1 year |
|  | Average | $2.62 \%$ | $4.90 \%$ | $8.09 \%$ | $17.95 \%$ | $49.72 \%$ |
|  | Median | $2.00 \%$ | $3.63 \%$ | $5.84 \%$ | $10.51 \%$ | $23.73 \%$ |
|  | Volatility | $21 \%$ | $24 \%$ | $28 \%$ | $37 \%$ | $85 \%$ |
|  |  |  |  |  |  |  |
| Loan | Average | $3.313 \%$ | $6.843 \%$ | $13.818 \%$ | $46.660 \%$ | $571.153 \%$ |
|  | Median | $2.00 \%$ | $4.13 \%$ | $7.17 \%$ | $18.85 \%$ | $146.91 \%$ |
|  | Volatility | $26 \%$ | $32 \%$ | $45 \%$ | $94 \%$ | $887 \%$ |
|  |  |  |  |  |  |  |
|  | Obs. lost | 14729 | 14427 | 15028 | 22651 | 37340 |

The number of lost observations is due to that it is not possible to calculate an IRR on investment paths that are highly negative. We see that the average and median IRR is increasing with shorter holding period which is expected. As described in the methodology chapter in section (4), when the holding period for investments go down, there is a potential that a very high value from the GBM simulation path coincides with realizing the investment after say only two or three years. This gives us a potential for very high IRRs. The IRR in the loan case is also always higher than the no-loan case while the volatility increases drastically in the loan case. Drawing a conclusion from an IRR on a holding period of one year does not make sense in the loan case since capital is drawn and returned in the same moment.

In terms of money made to LPs, the median MoM at a holding period of one year is larger than for our benchmark case while a two-year holding period yields the highest median MoM. This depends on two things (1) In the case of a one year holding period, the amount of hurdle that is "worked up" is low. This "worked up" hurdle increases with holding periods. This accounts for the increase in median MoM from 1 to 2 years. (2) At longer holding periods, more paths in the GBM go "bust", meaning that the equity in the individual holdings are allowed to deteriorate so that no value is left at divestment, which is why the median MoM decreases from a holding period of two years and up.

While looking at the average MoM we see that the average increases with holding period reflecting the fact that there is a large upside to the investments and that a longer holding period allows for more value to be created in total for investments that are positive. See below for table.

Exhibit 8.3.1.2 - Holding period impact on MoM, no-loan vs loan

| MoM |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
|  | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| Benchmark case | 0.867x | 1.359x | 1.75 x | 0.009x | 0.31 x | 1.75 x | 0.852x | 1.335x | 1.72 x | 0.009x | 0.30x | 1.72 x |
| 4 years | 0.937x | 1.309x | 1.43 x | 0.007x | 0.38x | 1.72 x | 0.920x | 1.286x | 1.40x | 0.007x | 0.38x | 1.70 x |
| 3 years | 0.990x | 1.237x | 1.10x | 0.006x | 0.48 x | 1.65 x | 0.972x | 1.215x | 1.09 x | 0.006x | 0.47x | 1.62 x |
| 2 years | 1.020x | 1.153x | 0.80x | 0.004x | 0.57x | 1.53 x | 1.001x | 1.132x | 0.79x | 0.004x | 0.56x | 1.50 x |
| 1 year | 0.985x | 1.026x | 0.49x | 0.003x | 0.67x | 1.31 x | 0.966x | 1.008x | 0.49x | 0.003x | 0.65 x | 1.29x |

In terms of actual money earned for GP (PV to GP), the average naturally decreases with shorter holding periods, both for the no-loan and loan cases individually. When comparing the two cases, we see that the difference become more and more negative with the loan case as the holding period is shortened. This is because if the fund is closed after a few years, the GPs lose out on fees for the upcoming years. The results are however only statistically significant (looking at the overlaps in confidence intervals between the no loan and loan case) for holding periods of 1 year. See below for table.

## Exhibit 8.3.1.3 - Holding period impact on PV to GP, no-loan vs loan

|  | PV to GP |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
|  | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| Benchmark case | 141,440,000 | 298,379,344 | 372,333,496 | 1,936,687 | 141,440,000 | 317,886,609 | 141,440,000 | 298,151,812 | 371,124,383 | 1,930,397 | 141,440,000 | 314,494,609 |
| 4 years | 125,440,000 | 261,583,172 | 289,512,928 | 1,505,897 | 125,440,000 | 292,694,942 | 125,440,000 | 261,198,651 | 288,216,044 | 1,499,151 | 125,440,000 | 289,302,942 |
| 3 years | 113,600,000 | 224,055,635 | 212,566,545 | 1,105,661 | 113,600,000 | 261,168,616 | 113,600,000 | 223,475,941 | 211,212,730 | 1,098,620 | 113,600,000 | 257,776,616 |
| 2 years | 105,440,000 | 187,742,592 | 143,838,045 | 748,171 | 105,440,000 | 224,538,195 | 105,440,000 | 186,868,001 | 142,481,238 | 741,114 | 105,440,000 | 221,146,195 |
| 1 year | 101,280,000 | 146,856,613 | 76,098,356 | 395,824 | 101,280,000 | 170,371,713 | 101,280,000 | 145,698,253 | 74,810,342 | 389,125 | 101,280,000 | 166,979,713 |

### 8.3.2 Leverage

Leverage is affecting our model in terms of IRR, NPV to LP and PV to GP. When running 100,000 simulations we observe statistical significant differences in within each case. There is a statistical significant difference between the cases in terms of IRR but not in terms of PV to GP and NPV to LP.

## Exhibit 8.3.2.1 - Overview of effects of leverage

|  | Gross IRR |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
|  | Median | Average | Volatility | Confidence int. | 3rd Quartile |  |  | Average | Volatility |  | 3rd Quartile | 1st Quartile |
| No leverage | -5.22\% | -3.71\% | 14\% | 0.072\% | -13.5\% | 4.2\% | -6.46\% | -4.287\% | 17\% | 0.1\% | -16.5\% | 5.3\% |
| 1:1 Leverage | -0.15\% | 1.81\% | 19\% | 0.108\% | -12.3\% | 13.3\% | -0.19\% | 2.859\% | 25\% | 0.1\% | -15.0\% | 16.8\% |
| Benchmark case | 5.19\% | 7.47\% | 22\% | 0.125\% | -8.7\% | 20.8\% | 6.48\% | 10.150\% | 29\% | 0.2\% | -10.7\% | 26.4\% |
| 4:1 Leverage | 13.39\% | 15.93\% | 26\% | 0.146\% | -3.0\% | 31.8\% | 16.79\% | 21.131\% | $34 \%$ | 0.2\% | -3.8\% | 40.8\% |
|  | Net IRR |  |  |  |  |  |  |  |  |  |  |  |
|  | [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
|  | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| No leverage | -8.64\% | -7.64\% | 14\% | 0.071\% | -17.3\% | 1.0\% | -10.96\% | -9.514\% | 17\% | 0.1\% | -21.3\% | 0.8\% |
| 1:1 Leverage | -3.60\% | -2.68\% | 18\% | 0.103\% | -16.1\% | 8.8\% | -4.86\% | -3.314\% | 23\% | 0.1\% | -19.9\% | 10.5\% |
| Benchmark case | 1.97\% | 2.66\% | 21\% | 0.117\% | -12.4\% | 15.2\% | 1.96\% | 3.355\% | 26\% | 0.1\% | -15.4\% | 18.6\% |
| 4:1 Leverage | 8.55\% | 10.37\% | 24\% | 0.135\% | -6.5\% | 24.7\% | 10.15\% | 13.065\% | 31\% | 0.2\% | -8.3\% | 30.6\% |
|  | NPV to LP |  |  |  |  |  |  |  |  |  |  |  |
|  | [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
|  | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| No leverage | -334,660,949 | -170,339,466 | 631,810,376 | 3,286,351 | -562,910,931 | 38,725,507 | -351,620,949 | -187,686,932 | 631,971,681 | 3,287,191 | -579,870,931 | 21,765,507 |
| 1:1 Leverage | $-287,849,199$ | 26,785,482 | 1,139,837,448 | 5,928,846 | -669,726,701 | 364,160,841 | -304,809,199 | 9,776,409 | 1,140,611,698 | 5,932,873 | -686,686,701 | 335,914,421 |
| Benchmark case | -121,588,993 | 335,596,378 | 1,611,152,349 | 8,380,383 | -648,056,693 | 707,152,375 | -138,548,993 | 318,864,905 | 1,612,215,847 | 8,385,915 | -665,016,693 | 693,584,375 |
| 4:1 Leverage | 239,282,397 | 968,992,331 | 2,713,074,188 | 14,112,012 | -572,650,289 | 1,465,790,267 | 214,382,695 | 952,721,744 | 2,714,223,387 | 14,117,989 | -589,610,289 | 1,452,222,267 |
|  | PV to GP |  |  |  |  |  |  |  |  |  |  |  |
|  | [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
|  | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3 rd Quartile | 1st Quartile |
| No leverage | 141,440,000 | 174,770,230 | 118,672,298 | 617,272 | 141,440,000 | 141,440,000 | 141,440,000 | 175,157,697 | 117,929,059 | 613,406 | 141,440,000 | 141,440,000 |
| 1:1 Leverage | 141,440,000 | 229,369,022 | 241,454,883 | 1,255,924 | 141,440,000 | 194,036,945 | 141,440,000 | 229,418,096 | 240,379,027 | 1,250,328 | 141,440,000 | 212,769,395 |
| Benchmark case | 141,440,000 | 297,560,278 | 361,488,983 | 1,880,279 | 141,440,000 | 318,228,094 | 141,440,000 | 297,331,751 | 360,248,902 | 1,873,829 | 141,440,000 | 314,836,094 |
| 4:1 Leverage | 141,440,000 | 443,887,920 | 643,991,126 | 3,349,709 | 141,440,000 | 507,887,567 | 141,440,000 | 443,198,507 | 642,791,280 | 3,343,468 | 141,440,000 | 504,495,567 |

Increased leverage has a magnifying effect on average returns and IRR by increasing the number of cases that yield very high returns, since leverage adds to the investment value development paths. The volatility in NPV to LPs also increases drastically by increasing leverage to $4: 1$ suggesting that there is a significant amount of funds that go bankrupt on the low end while many funds that had mediocre but positive returns are now increased to yield good returns while the best cases are significantly better in terms of returns.

### 8.3.3 Loosening loan constraints

From our base case, loosening loan constraints can be done by increasing the \% of committed capital that can be borrowed.

Exhibit 8.3.3.1 - Bridge loan usage when varying the constraint on maximum allowed

## Amount drawn

| Year | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Benchmark | $200,000,000$ | $200,000,000$ | $200,000,000$ | $164,000,000$ | $84,000,000$ | 0 |
| No constraints | $228,000,000$ | $204,000,000$ | $220,000,000$ | $164,000,000$ | $84,000,000$ | 0 |
| Difference | $28,000,000$ | $4,000,000$ | $20,000,000$ | 0 | 0 | 0 |

The difference in the amount that is allowed to be borrowed under a no constraint scenario is not significantly enough large to have a greater impact on IRRs and PVs. Results follow the same intuition as tightening the constraints, to the effect of increasing IRRs. A loan constraint of $20 \%$ of committed capital allows the funds in our base case to utilize a high ratio of debt to cover the actual investments.

### 8.3.4 Discount rate

When testing the discount rate, we set a separate discount rate for GPs and LPs and look what happens on average on the whole data-set running 100,000 simulations. It is important to test the discount rate, especially for LPs since in the loan case it is their capital that is claimed for a shorter period. The discount rate could represent, for example, the LPs opportunity cost.

Exhibit 8.3.4.1 - Discount rate analysis

| NPV to LP |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
| Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| Benchmark case - 122,205,230 | 337,173,056 | 1,607,933,679 | 13,097,404 | -648,436,695 | 713,412,990 | -139,165,230 | 320,430,490 | 1,608,998,741 | 13,106,079 | -665,396,695 | 699,844,990 |
| 2\% -193,982,026 | 206,039,417 | 1,446,025,869 | 11,778,586 | -653,048,127 | 529,820,885 | -193,982,026 | 206,217,704 | 1,446,932,384 | 11,785,970 | -653,048,127 | 532,739,205 |
| 4\% -248,447,992 | 108,102,397 | 1,271,451,889 | 10,356,595 | -653,704,558 | 397,570,056 | -233,163,033 | 123,572,205 | 1,272,261,713 | 10,363,192 | -638,419,599 | 415,365,992 |
| 6\% -295,504,222 | 23,995,489 | 1,135,503,705 | 9,249,231 | -654,326,657 | 276,706,992 | $-266,413,205$ | 53,233,829 | 1,136,221,235 | 9,255,076 | -625,235,639 | 307,844,127 |
| 8\% -332,345,198 | -43,738,878 | 1,053,624,227 | 8,582,283 | -651,435,194 | 177,723,990 | -290,757,201 | -2,012,770 | 1,054,244,656 | 8,587,337 | -609,847,197 | 220,904,548 |
| 10\% -368,125,785 | -109,412,667 | 929,197,435 | 7,568,766 | -647,232,461 | 85,531,740 | -315,202,426 | -56,371,440 | 929,753,103 | 7,573,292 | -594,309,101 | 139,686,763 |
| PV to GP |  |  |  |  |  |  |  |  |  |  |  |
| [No loan] |  |  |  |  |  | [Loan] |  |  |  |  |  |
| Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile | Median | Average | Volatility | Confidence int. | 3rd Quartile | 1st Quartile |
| Benchmark case $141,440,000$ | 297,828,680 | 360,580,678 | 1,875,555 | 141,440,000 | 319,793,247 | 141,440,000 | 297,611,245 | 359,334,960 | 1,869,075 | 141,440,000 | 316,401,247 |
| 2\% 132,885,186 | 267,611,339 | 324,966,995 | 1,690,311 | 132,885,186 | 282,974,832 | 132,885,186 | 267,433,052 | 323,918,447 | 1,684,857 | 132,885,186 | 280,162,257 |
| $4 \% \quad 125,245,974$ | 244,092,383 | 284,340,887 | 1,478,995 | 125,245,974 | 257,300,026 | 125,245,974 | 243,907,535 | 283,416,622 | 1,474,187 | 125,245,974 | 254,862,407 |
| 6\% 118,399,507 | 223,397,910 | 253,356,586 | 1,317,830 | 118,399,507 | 233,094,900 | 118,399,507 | 223,250,588 | 252,539,077 | 1,313,578 | 118,399,507 | 231,175,721 |
| 8\% 112,242,195 | 206,155,662 | 236,519,151 | 1,230,251 | 112,242,195 | 212,969,823 | 112,242,195 | 206,017,552 | 235,825,101 | 1,226,640 | 112,242,195 | 211,189,944 |
| 10\% 106,686,307 | 189,063,459 | 207,493,706 | 1,079,275 | 106,686,307 | 192,923,667 | 106,686,307 | 188,945,592 | 206,873,805 | 1,076,051 | 106,686,307 | 191,429,566 |
| Alpha $=0.01$ |  |  |  |  |  |  |  |  |  |  |  |

With increasing discount rates the loan case becomes more attractive to the LPs when comparing that NPV to the NPV in the no loan case on average. This is because the time value of money of the outflows are less worth today than in the future. The change in NPV to LPs gained by increasing the discount rate is driven by the reduction in NPV of the negative cases. Looking only at the positive cases with a $10 \%$ increase in discount rate we still see a decrease in NPV to LPs in the loan case. The GPs only have positive flows and thus their PV is decreased as the discount rate is increased.


[^0]:    ${ }^{1}$ Specifically we would like to thank FSN Capital, Alder, the AP6 pension fund, Aberdeen Asset Management, StepStone, Simpson Thacher \& Bartlett LLP and one of the major Nordic banks that has chosen to be anonymous

[^1]:    ${ }^{2}$ FSN Capital, Alder private equity are example of private equity funds showing demand for a report on subscription loan facilities. LPs such as Swedish pension fund AP6, as well as finance professor Per Strömberg at the Stockholm School of Economics have also shown interest for the topic.
    ${ }^{3}$ Interview with a Nordic bank, February 212017

[^2]:    ${ }^{4}$ A catch-up is a division of the profits between GPs and LPs within a certain IRR level, in our model this consists of a profit sharing ratio of $60 \%$ to GPs and $40 \%$ to LPs at returns between 8 and $12 \%$

[^3]:    ${ }^{5}$ E.g. phone interview with Jonas Lidholm, Sjätte AP-fonden, January 312017

[^4]:    ${ }^{6}$ http://www.investopedia.com/articles/stocks/09/abcs-of-private-equity.asp, viewed 9 February 2017
    ${ }^{7}$ http://www.investopedia.com/terms/h/hurdlerate.asp, viewed 9 February 2017
    ${ }^{8}$ FSN Capital, Alder
    ${ }^{9}$ http://cepr.net/blogs/cepr-blog/private-equity-s-latest-con-subscription-line-loans-boost-returns-and-deceive-investors, viewed 25 January 2017
    ${ }^{10}$ https://www.ft.com/content/c5c24c58-953c-11e6-a80e-bcd69f323a8b, viewed 25 January 2017

[^5]:    ${ }^{11}$ https://www.ft.com/content/c5c24c58-953c-11e6-a80e-bcd69f323a8b, viewed 25 January 2017
    12 http://cepr.net/blogs/cepr-blog/private-equity-s-latest-con-subscription-line-loans-boost-returns-and-deceive-investors, viewed 25 January 2017
    ${ }^{13} \mathrm{http}: / / c e p r . n e t / p u b l i c a t i o n s / r e p o r t s / a r e-l o w e r-p r i v a t e-e q u i t y-r e t u r n s-t h e-n e w-n o r m a l, ~ l a s t ~ v i e w e d ~ 25 ~ J a n u a r y ~ 2017 ~$

[^6]:    ${ }^{14} \mathrm{http}: / /$ cepr.net/blogs/cepr-blog/private-equity-s-latest-con-subscription-line-loans-boost-returns-and-deceive-investors, viewed 25 January 2017
    ${ }^{15} \mathrm{http}: / /$ cepr.net/publications/reports/are-lower-private-equity-returns-the-new-normal, last viewed 25 January 2017

[^7]:    ${ }^{16}$ MOIC: Money on Invested Capital = Realization net of interest and management fees / initial equity investment, irrespective of time (unlike the IRR). Also known as Multiple of Money (MoM).

[^8]:    ${ }^{17}$ http://www.investopedia.com/terms/a/agencytheory.asp, viewed 17 March 2017

[^9]:    ${ }^{18}$ http://cepr.net/blogs/cepr-blog/private-equity-s-latest-con-subscription-line-loans-boost-returns-and-deceive-investors,
    https://www.ft.com/content/c5c24c58-953c-11e6-a80e-bcd69f323a8b
    ${ }^{19}$ http://www.investopedia.com/terms/g/gametheory.aspt, viewed 7 April 2017

[^10]:    ${ }^{20}$ E.g. TorreyCove (2016) and Appelbaum (2016)
    ${ }^{21}$ Phone interview with Toby Suen, Aberdeen asset management, March 3, 2017
    ${ }^{22}$ E.g. Phone interview with Jonas Lidholm, Sjätte AP-fonden, January 31, 2017 and interview with a Nordic bank, February 21, 2017

[^11]:    ${ }^{23}$ E.g. Appelbaum (2016)

[^12]:    ${ }^{24}$ Phone interview with Morten Welo, COO at FSN Capital, February 16, 2017

[^13]:    ${ }^{25}$ Phone interview with Jonas Lidholm, Sjätte AP-fonden, January 312017
    ${ }^{26}$ Interview with a Nordic bank, February 21, 2017
    27 PE Performance Monitor 2006
    ${ }^{28}$ Metrick and Yasuda (2010) use 11 investments per fund, but to simplify the set-up of our model we use 10 . The outputs of the model are not affected significantly.

[^14]:    ${ }^{29}$ Input from GPs FSN Capital and Alder

[^15]:    ${ }^{30} \mathrm{http}: / / \mathrm{www} . n a s d a q o m x . c o m / t r a n s a c t i o n s / t r a d i n g / f i x e d i n c o m e / f i x e d i n c o m e / s w e d e n / s t i b o r s w a p t r e a s u r y f i x i n g ~, ~ 20 ~ M a r c h ~ 2017 ~$
    ${ }^{31}$ Interview with a Nordic bank, February 21, 2017. Phone interview with Jonas Lidholm, Sjätte AP-fonden, January 312017
    ${ }^{32}$ Phone interview with Jonas Lidholm, Sjätte AP-fonden, January 312017
    ${ }^{33}$ Interview with a Nordic bank, February 21, 2017

[^16]:    ${ }^{34}$ The Geometric Brownian Motion approach is used to model a stock price development, or in our case a value development where the value follows a random walk. Formula: $\boldsymbol{S} \mathbf{1}=\boldsymbol{S} \mathbf{0} * \boldsymbol{e}^{\boldsymbol{\mu}-\boldsymbol{\sigma}^{2} / 2+\boldsymbol{\sigma} \boldsymbol{\varepsilon}}$

[^17]:    ${ }^{35}$ The difference between the exit value and the initial debt is set so that it cannot be below zero. If the debt would exceed the exit value for an investment at exit, that acquisition is considered bankrupt. In year 8 we see that no distributions are made. This is because in the simulation looked at here, the values of the investments exited in year 8 were lower than the debt that needed to be repaid in year 8 .

[^18]:    ${ }^{36}$ Input from FSN Capital

[^19]:    ${ }^{37}$ Interest is only a factor for the subscription loan scenario

[^20]:    ${ }^{38}$ Phone interview with Jonas Lidholm, Sjätte AP-fonden, January 312017
    ${ }^{39}$ Phone interview with Toby Suen, Aberdeen asset management, March 3, 2017
    ${ }^{40}$ Interview with a Nordic bank, February 21, 2017

[^21]:    ${ }^{41} \mathrm{http}: / /$ cepr.net/blogs/cepr-blog/private-equity-s-latest-con-subscription-line-loans-boost-returns-and-deceive-investors, viewed 25 January 2017

    42 Interview with Toby Suen, Aberdeen asset management, 3 March 2017

[^22]:    $43 \mathrm{http}: / /$ cepr.net/blogs/cepr-blog/private-equity-s-latest-con-subscription-line-loans-boost-returns-and-deceive-investors, viewed 25 January 2017. https://www.ft.com/content/c5c24c58-953c-11e6-a80e-bcd69f323a8b, viewed 25 January 2017
    ${ }^{44}$ Statistical significance is assessed by looking at the confidence interval; from the average value, you add and deduct the confidence interval value, and the drawn value will lie within that range with 99 percent confidence if using an alpha of 1 percent

[^23]:    ${ }^{45}$ From exhibit 5.1, difference in average gross IRR: $10.072-7.415=2.7 \%$, difference in average net IRR: $3.324-2.631=0.7 \%$
    ${ }^{46}$ From exhibit 5.1, average NPV to LP from loan case - average NPV to LP from no loan case: $319.87-336.61=$ SEK 16.7 m

[^24]:    ${ }^{47}$ FSN Capital, Alder, Phalippou and Gottschalg (2009)

[^25]:    ${ }^{48}$ Interview with a Nordic bank, 21 February 2017

[^26]:    ${ }^{49}$ From exhibit 5.5, $99 \%$ confidence interval with loan: $2.55 \%+/-0.046 \%$ and without loan $2.697 \%+/-0.058 \%$

[^27]:    ${ }^{50}$ Hypothesis 4: As the cost of the loans increase (loan-spread) the ability of the loans to increase net IRR decreases.

[^28]:    ${ }^{51}$ Hypothesis 5: A higher eligible investor ratio improves the funds ability to increase its IRR through loans.

[^29]:    ${ }^{52}$ Hypothesis 1: The IRR effect of the loans on fund performance is positive for funds with positive return and negative for funds with negative return.
    ${ }^{53}$ E.g. TorreyCove (2016)
    ${ }^{54}$ Hypothesis 2: The magnitude of the positive effect on IRR imposed by the loans is dependent on how well the fund is performing.

[^30]:    ${ }^{55}$ Aberdeen asset management investor memo observed effects of the loans: $+6 \%$ on gross IRR, $+4 \%$ on net IRR
    ${ }^{56}$ Preqin Global Private Equity \& Venture Capital report, 2016
    ${ }^{57}$ Hypothesis 3: The magnitude of the positive effect on IRR is large enough to increase funds' quartile rankings in the sense that a fund can increase its IRR by several percentage points.

[^31]:    ${ }^{58}$ Hypothesis 7: GPs earn money at lower IRRs when using the loans compared to when not using the loans

[^32]:    ${ }^{59}$ Hypothesis 8: GPs start to collect more money earlier in the fund's life with loans.
    ${ }^{60}$ Hypothesis 6: GPs on average earn more money by using bridge-loan facilities.

[^33]:    ${ }^{61}$ E.g. TorreyCove (2016)
    ${ }^{62}$ Hypothesis 9: LPs lose money when these loans are used

[^34]:    ${ }^{63}$ Preqin Global Private Equity \& Venture Capital report, 2016
    ${ }^{64}$ Interview with a Nordic bank, 21 February 2017

[^35]:    ${ }^{65}$ E.g. TorreyCove (2016)
    ${ }^{66}$ E.g. Appelbaum (2016), Flood (2016)
    ${ }^{67}$ Interview with a Nordic bank, 21 February 2017
    ${ }^{68}$ Interview with a Nordic bank 21 February 2017

[^36]:    ${ }^{69}$ Phone interview with Jonas Lidholm, Sjätte AP-fonden, 31 January 2017. Verified in interview with a Nordic bank, 21 February 2017
    ${ }^{70} \mathrm{https}: / / \mathrm{www} . i p e . c o m /$ private-equity-dont-overlook-the-hidden-costs/34779.fullarticle, viewed 3 April 2017
    ${ }^{71}$ Interview with a Nordic bank, 21 February 2017. Interview with AP6, January 312017

[^37]:    ${ }^{72}$ Phone interview with Jonas Lidholm, Sjätte AP-fonden, 31 January 2017
    ${ }^{73}$ Interview with a Nordic bank, February 212017

[^38]:    * Note from TorreyCove: For illustrative purposes. Also, they did not account for transaction fees associated with the debt, and they did not include carried interest. Interest is assumed to be paid at end of term and the management fee is deferred during debt holding period

[^39]:    ${ }^{74}$ Michael Halling, Spring 2016

[^40]:    * An example of the one simulation in exhibit 8.2.8 that multiplied with exhibit 8.2.7 gives exhibit 8.2.9

