Currency Exposure and Hedging Strategies for Limited Partners Investing in Private Equity

Caroline Holtsjö* & Carl Åkerlind†

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Department of Finance
Stockholm School of Economics

Abstract

This paper sets out to examine how limited partners investing in private equity should define currency exposure and whether they should hedge it or not. To our knowledge, no previous research has investigated this field. The subject is important since foreign investments in private equity are increasing. Using investment data from a specific limited partner and simulated foreign exchange rates, we find that limited partners should define currency exposure as the currency that denotes the portfolio company of the private equity fund, and hedge accordingly. This strategy yields similar mean returns but reduces the riskiness of those returns, and thus yields the highest risk-return trade-off, compared to other hedging strategies. An implication of this result is that many limited partners investing in private equity mishedge their currency exposure. Furthermore, we find that limited partners investing in private equity should diversify their assets under management by investing in private equity funds exposed to several currencies, through the portfolio companies.

Keywords: Currency Exposure, Hedging, Private Equity, Limited Partners, Foreign Exchange Rates, Sharpe Ratio, Cross-Currency Arbitrage, Sixth Swedish National Pension Fund

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*22902@student.hhs.se
†22707@student.hhs.se

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1 Introduction

In 1958, Modigliani and Miller developed the capital structure irrelevance theorem, that in the absence of taxes, bankruptcy costs, agency costs and asymmetric information, in other words, in an efficient market, the value of a firm is independent of its capital structure. Since then, researchers have been studying the subject in greater detail. Furthermore, Modigliani and Miller argue that since risk management strategies are purely financial transactions, they do not affect the value of a company's operating assets, and therefore hedging do not increase firm value. However, outside an idealised Modigliani and Miller world and assuming no currency risk premium, Glen and Jorion (1993) argue that investors holding international portfolios can significantly improve the risk-return trade-off by hedging currency exposure.

This paper sets out to examine if and how limited partners investing in private equity should define and hedge currency exposure. According to OECD's *Annual Survey of Large Pension Funds and Public Pension Reserve Funds*, investments in private equity have increased over the recent time period. Furthermore, for most limited partners, foreign investments in private equity constitute a substantial fraction of total investments in private equity. Consequently, currency hedging for limited partners investing in private equity might play a more important role now than ever before. To our knowledge, no previous research has investigated the field of currency hedging for limited partners investing in private equity.

Based on several interviews with Swedish limited partners and private equity firms, we develop three hedging strategies. The hedging strategies are (i) unhedged, (ii) private equity fund currency hedged and (iii) portfolio company currency hedged. Returns and money multiple of each hedging strategy are simulated using data from three co-investments made by the Sixth Swedish National Pension Fund, hereafter AP6, combined with historical foreign exchange rates and risk-free interest rates.

The main result of this paper is that limited partners investing in private equity should define their currency exposure as the currency that denotes the portfolio company of the private equity fund, and hedge accordingly. The economic interpretation of this result is that limited partners can obtain higher Sharpe ra-

tios, without reducing expected return, by hedging their currency exposure. An implication of our findings is that many limited partners mishedge their currency exposure. However, important to note is that our portfolio consist of only three investments and hence, part of the results could stem from reduced idiosyncratic risk. Furthermore, we have assumed no currency risk premium. In our robustness tests, we find arguments supporting the efficient portfolio theory, where all idiosyncratic risk is diversified away. Consequently, we conclude that limited partners should invest in private equity funds exposed to several currencies, through the portfolio companies, to diversify their assets under management, which indicates higher Sharpe ratios.

In the main scenario, the limited partner invests in several private equity funds, denoted in different currencies. For this scenario, Sharpe ratios for the (i) unhedged strategy, (ii) private equity fund currency hedged strategy, and (iii) private equity portfolio company currency hedged strategy are 2.52, 2.30 and 3.20, respectively. In the first robustness test scenario, the limited partner invests in one private equity fund, that invests in portfolio companies denoted in different currencies. Sharpe ratios for the hedging strategies are 2.52, 2.41 and 3.19, respectively. In the second robustness test scenario, the limited partner invests in several private equity funds denoted in different currencies, which invest in portfolio companies denoted in the same currency. Sharpe ratios for the hedging strategies are 2.51, 2.30 and 3.12, respectively.

The remaining part of this thesis is organised as follows. In section 2, previous literature related to this thesis is presented and applied. In section 3, the hypotheses and underlying rationales are presented and section 4 presents the data. Section 5 describes the methodology leading up to the results and discussion, presented in section 6. Finally, in section 7 main conclusions and implications of this thesis are presented. Also, suggestions for future research are proposed.

2 Previous Literature

In the following section, previous literature is presented and applied to the topic of this thesis.

2.1 Previous Research

Numerous research articles have explored the field of risk management. Stulz (2008) explains that risk management has the role to identify and evaluate risks that firms face, as well as to monitor and manage those risks according to the desired risk exposure of the firms. If accepting the view of Modigliani and Miller, risk management strategies are purely financial transactions that do not affect the value of a company's operating assets, and therefore hedging do not increase firm value. However, Stulz (1999) concludes that risk management is worthwhile for companies in some situations, because a company without risk management faces the risk of bearing more direct bankruptcy costs than it should, and consequently risks being unable to invest in valuable projects. The author argues that, in case of bankruptcy, homemade risk management cannot be considered a substitute for risk management within the firm. In line with Stulz (1999), Froot et al. (1994) find that risk management is important for companies in scenarios where they can find additional positive NPV investment opportunities. The authors find that to develop a consistent risk management strategy, it is crucial to understand the relation between a company's key economic variables and its investment opportunities. Finally, the authors mention that companies should pay close attention to the hedging strategies of peers and introduce guidelines for managers.

According to Glen and Jorion (1993), investors holding international portfolios can significantly improve the risk-return trade-off by hedging currency exposure, under the assumption of a conditional hedging strategy of stock and bond portfolios. They conclude that even if currency hedging reduces the volatility of returns, hedging will be beneficial if and only if mentioned risk reduction is not accompanied by an offsetting decrease in returns. Important to note is that the authors have assumed no currency risk premium. In an international asset pricing model framework, the authors find that currency hedging will improve portfolio performance only if forward contracts are not fairly priced. The authors also find that,

with predetermined positions in either stocks or bonds and using an unconditional hedging strategy, there is little evidence of improvement from adding currencies to the portfolio.

Pérold and Schulman (1988) draw similar conclusions as Glen and Jorion (1993), and find that currency hedging reduces risk without any loss of expected return, in other words, currency hedging is a "free lunch". They argue that, from a long-term perspective, investors should think of currency hedging as having zero expected return. As Glen and Jorion (1993), the authors have assumed no currency risk premium.

On the other hand, Black (1990) shows that, under the assumptions of a CAPM world with many currencies, Siegel's paradox[1] makes investors want a positive amount of foreign exchange risk. Therefore, investors should never fully hedge their currency exposure. Black argues that, when the average risk tolerance is the same across countries, each investor will hold the same mix of market risk and foreign exchange risk, assuming that investors hold the world market portfolio of all assets as well as a diversified basket of foreign currencies.

De Santis and Gérard (1998) find that the currency risk premium is a substantial fraction of the total risk premium. The conclusion is based on the assumption that the purchasing power parity is violated, hence the expected return on any asset must include a market risk premium as well as a currency risk premium. Morey and Simpson (2001) compare different hedging strategies over different time horizons and for different foreign exchange rates. They find that an unhedged strategy of the foreign exchange risk outperforms a hedged strategy, in all samples and time horizon periods. Dufey and Srinivasulu (1983) also find that the foreign exchange risk should not be hedged. The authors sum up the arguments that oppose hedging at the level of the firm as (i) foreign exchange risk does not exist, (ii) even if it exists, it need not to be hedged, and (iii) even if it is to be hedged, corporations need not to hedge it. They refer to consequences from the purchasing power parity, CAPM, Modigliani-Miller theorem, efficient markets and uncertainties of

[1]If a fixed fraction f of a given amount of money M is lost, and then the same fraction f of the remaining amount is gained, the result is less than the original and equal to the final amount if a fraction f is first gained, and then lost. Mathematically, Siegel's paradox is due to that $E[1/X]$ is not the same as $1/E[X]$, where E is the expectation operator and X is a random variable.

future forward rates and future spot rates.

2.2 Applying Previous Literature

To our knowledge, no previous research has investigated the field of currency hedging for limited partners investing in private equity. However, previous literature can be applied to this topic. As investments in the private equity sector have grown to be more international over recent years, currency hedging is an area of increasing interest. The investment chain of private equity is exposed to different layers of foreign exchange risk, thus hedging could be a complex matter in this setting.

Previous literature supporting both hedged and unhedged investment strategies exist. Applying the framework of Glen and Jorion (1993), limited partners could receive an improved risk-return trade-off by hedging currency exposure. As long as expected returns do not decrease with decreasing volatility, hedging should be beneficial. These arguments are in line with Pérold and Schulman (1988).

On the contrary, since De Santis and Gérard (1998) find that the currency risk premium is a substantial fraction of the total risk premium, an unhedged strategy for limited partners could capture this risk premium and hence outperform hedged strategies. Following Froot et al. (1994), limited partners and private equity firms should pay attention to peers' hedging strategies.

3 Hypothesis

This thesis sets out to investigate if and how limited partners investing in private equity should define and hedge currency exposure. To evaluate this, it is important to understand currency exposures and hedging strategies throughout the investment chain, presented in Figure 1. A first step is to understand whether private equity firms hedge their currency exposure or not, since the currency exposure of limited partners depends on this. We believe that private equity firms in general should not hedge currency exposure since hedging foreign exchange risk means that the private equity firm is taking a view on foreign exchange movements rather than focusing on its core business.

The next step is to understand the limited partners' currency exposure. In our belief, limited partners hedge currency exposure as they aim at stable returns

over time. Furthermore, we believe that there are two main definitions of currency exposure, (i) private equity fund currency exposure and (ii) portfolio company currency exposure. The logic behind defining the currency exposure in the first way is that limited partners invest and receive capital in the private equity fund currency. The rationale behind defining the currency exposure in the second way is that the portfolio company currency is the real operational currency exposure. The value of the private equity fund, denoted in private equity fund currency, is dependent on the underlying value of the portfolio companies, denoted in portfolio company currencies. We believe that the second definition of currency exposure for limited partners is more accurate. In line with De Santis and Gérard (1998) and Simpson (2001), the null hypothesis of this thesis is

H_0: *Limited partners investing in private equity should not, regardless of definition of currency exposure, hedge the currency exposure, to obtain the highest possible Sharpe ratio.*

As stated above, our belief is that the currency exposure should be defined as the portfolio company currency exposure. Furthermore, according to Glen and Jorion (1993) hedging yield a higher risk-return trade-off. This leads to the alternative hypothesis

H_1: *Limited partners investing in private equity should define their currency exposure as the portfolio company currency and hedge the exposure accordingly, to obtain the highest possible Sharpe ratio.*

4 Data

Historical data of foreign exchange rates and risk-free interest rates originate from Bloomberg. Foreign exchange rates are collected for the time period January 1, 2009 to December 31, 2016. Risk-free interest rates are collected for the time period January 1, 2013 to December 31, 2016. Both foreign exchange rates and risk-free interest rates are collected on a daily basis. Regarding a proxy for the risk-free interest rate the yields on 10-year government bonds are used. Interviews

with Swedish limited partners and private equity firms active in Sweden have been made to collect information regarding their view on currency exposure and hedging strategies. In total, five private equity firms and six limited partners have been interviewed.

The data of investments is received from AP6, presented in Tables 13-21. In more detail, three investments made by AP6 are analysed. These investments are co-investments in portfolio companies of private equity funds. Important to note is that, to calculate returns of the investments, we have assumed that the investments are exited on the last valuation date in the data set. Also, the portfolio company currency is assumed to the currency where the headquarter is located. In some scenarios of the analysis, the currencies of the investments will be changed from the original currencies.

Data from the first investment is shown in Tables 13-15. Presented in Table 13, the investment is made May 20, 2014 and is assumed exited December 31, 2016. The committed capital is GBP 20,000,000 and the net asset value, hereafter NAV, on the last valuation date is GBP 34,000,000. NAV adjustments are made quarterly. Table 14 presents quarterly NAV valuations in NOK, the original portfolio company currency, and in GBP, the original private equity fund currency. In Table 15, hedging transactions made by AP6 are presented.

Data from the second investment is shown in Tables 16-18. Presented in Table 16, the investment is made April 22, 2016 and is assumed exited September 30, 2016. The committed capital is USD 50,000,000 and the NAV on the last valuation date is USD 49,966,873. NAV adjustments are made quarterly. Table 17 presents quarterly NAV valuations in USD, the original portfolio company currency and the original private equity fund currency. In Table 18, hedging transactions made by AP6 are presented.

Data from the third investment is shown in Tables 19-21. Presented in Table 19, the investment is made June 26, 2013 and is assumed exited September 30, 2016. The committed capital is EUR 8,170,056, split on two occasions. The first commitment of EUR 7,000,000 is made June 26, 2013 and the second commitment of EUR 1,170,056 is made September 23, 2013. The NAV on the last valuation date is EUR 4,712,662. NAV adjustments are made quarterly. Table 20 presents

quarterly NAV valuations in DKK, the original portfolio company currency, and in EUR, the original private equity fund currency. In Table 21, hedging transactions made by AP6 are presented.

Using the standard deviation of historical foreign exchange rates during the period January 1, 2009 to December 31, 2012, foreign exchange rates are simulated for the period January 1, 2013 to December 31, 2016. The historical foreign exchange rates are collected from 2009 since we want to exclude major effects of the financial crisis. Furthermore, forward rates are calculated using the simulated foreign exchange rates and the historical risk-free interest rates.

5 Methodology

In the following section, the methodology used for conducting the study is outlined. First, key variables are described and currency exposure is defined. Second, the simulation of foreign exchange rates and forward rates is described. Third, hedging strategies are introduced. Finally, robustness tests are presented.

5.1 Defining Currency Exposure

A starting point in this study is to define what currency exposure is. Below, key variables are introduced, followed by a discussion of currency exposure.

5.1.1 Variable Description

(i) **Limited Partner Currency**

The limited partner currency is defined as the currency the limited partner is denominated in.

(ii) **Fund Currency**

The private equity fund currency, hereafter fund currency, is defined as the currency the private equity fund is denominated in.

(iii) **Portfolio Company Currency**

The portfolio company currency, hereafter PC currency, is defined as the currency the portfolio company of the private equity fund is denominated in. In our opinion, it is the net exposure, in terms of cash inflows and cash outflows,

of the portfolio company that is the true currency exposure. However, we do not have access to this data, and therefore assume the PC currency to be solely decided based on the country where the portfolio company headquarter is located. Note that the portfolio company could hedge its currency exposure. In such a case, the company reporting currency would be the correct currency exposure.

5.1.2 Private Equity Currency Exposure

As the limited partner currency exposure depends on the private equity currency exposure, because of the investment chain structure presented in Figure 1, a natural first step is to define the private equity currency exposure. For a private equity firm, currency exposure can be found both in the income statement and in the balance sheet. The first type of exposure stems from revenues and costs in different currencies. The second type of exposure stems from equity investments. Independent of mentioned definitions, the private equity firms could face the foreign exchange risk between signing and closing of a deal. We believe that this exposure could be substantial in some cases.

5.1.3 Limited Partner Currency Exposure

Limited partners are at the top of the investment chain and are exposed to currency fluctuations at all levels in the investment chain, in other words fluctuations at the fund level as well as at the portfolio company level, see Figure 1. Currency exposure can be defined either as the fund currency or the PC currency.

The rationale behind defining the currency exposure as the fund currency is that limited partners invest and receive capital in the fund currency. If hedging, one must remember that limited partners have multiple investments in funds denoted in different currencies. These currencies might offset each other resulting in a decreased aggregated currency exposure. As a consequence, it could be too costly and time consuming to hedge the exposure at portfolio company level compared to the additional value created. Furthermore, if the private equity fund hedges its currency exposure from portfolio companies, the limited partners' exposure would be the fund currency.

The rationale behind defining the currency exposure as the PC currency is that it is the real operational currency exposure. The value of the portfolio company denoted in the fund currency depends on the underlying value of the portfolio company, in other words the value of the portfolio company in the currency which denominates it.

There are several goals of hedging. First, the limited partner could aim at minimising the standard deviation of returns. A second goal could be to maximise the expected return. Third, the limited partner could aim at maximising the risk-return trade-off. Moreover, if the limited partner determines to hedge, a subsequent issue is whether to hedge called or committed capital. The time between commitment and call date can be relatively long and foreign exchange rates could fluctuate during this period. However, this will not be analysed further in this thesis.

5.2 Simulating Foreign Exchange Rates

After defining currency exposure, the next step is to simulate foreign exchange rates and forward rates. To do this, historical foreign exchange rates, risk-free interest rates, correlations and arbitrage conditions need to be considered. Our procedure to do this is as follows. First, foreign exchange rates are modeled as geometric Brownian motions, where standard deviations of historical data are considered. Through Cholesky decomposition, correlation of foreign exchange rates for the investment period is simulated, using the correlation of historical foreign exchange rates. Furthermore, cross-currency arbitrage is described and applied. Finally, forward rates are calculated using the simulated foreign exchange rates and historical risk-free interest rates.

5.2.1 Geometric Brownian Motion

Foreign exchange rates are simulated as geometric Brownian motions.[2] Let Y_t be the foreign exchange rate at time t, e.g. Y_t is the number of SEK that one USD will buy at time t. Then, Y_t behaves like a geometric Brownian motion, that is, it solves the stochastic differential equation of the form

[2]See Appendix *B Brownian Motion* for an explanation of a Brownian motion.

$$dY_t = \mu Y_t dt + \sigma Y_t dB_t, \quad Y_0 = Y(0) \tag{1}$$

where, the dt-term is the drift and the dB_t-term is the diffusion. μ and σ are constants with $\sigma > 0$. Solving the stochastic differential equation (1) gives the explicit formula

$$Y_t = Y_0 \exp\left((\mu - \frac{1}{2}\sigma^2)t + \sigma B_t\right) \tag{2}$$

When simulating foreign exchange rates, we assume the expected change to be zero, i.e. $\mu = 0$. The rationale behind this is that we do not want to take a view of the evolvement of foreign exchange rates.

5.2.2 Cholesky Decomposition

To make the simulations of foreign exchange rates more accurate, it is important to take the correlation between foreign exchange rates into account. Using historical foreign exchange rates between the period January 1, 2009 to December 31, 2012, a correlation matrix is determined. Subsequently, a Cholesky factorisation of the correlation matrix is made. The Cholesky factorisation works in the following way. Consider a square matrix $\mathbf{A}$, that is symmetric and positive definite. Then $\mathbf{A}$ has a special, more efficient, triangular decomposition. Cholesky decomposition constructs a lower triangular matrix $\mathbf{L}$ whose transpose $\mathbf{L}^T$ can serve as the upper triangular part itself. Hence, the matrix $\mathbf{A}$ can be rewritten as

$$\mathbf{A} = \mathbf{L}\mathbf{L}^T \tag{3}$$

In this paper, the matrix $\mathbf{A}$ denotes the correlation matrix of historical data, i.e. the correlation between foreign exchange rates.

5.2.3 Correlated Random Variables

Consider an n-dimensional column vector $\mathbf{v}$, with elements $v_1, v_2, ..., v_n$, where

$$v_i \overset{iid}{\sim} N(0,1), \quad \text{for } i = 1, 2, ..., n \tag{4}$$

Taking the matrix $\mathbf{L}$ in equation (3) times the vector $\mathbf{v}$ will create a new vector $\mathbf{v}'$

$$\mathbf{v}' = \mathbf{L}\mathbf{v} \tag{5}$$

$\mathbf{v}'$ includes elements that are correlated and follow a standardised normal distribution

$$v_i' \sim N(0,1), \quad \text{for } i = 1, 2, ..., n \tag{6}$$

From the vector $\mathbf{v}'$ a new vector $\mathbf{v}''$ including values between 0 and 1 can be created by taking the cumulative distribution function of the standard normal distribution of each element in $\mathbf{v}'$

$$v_i'' = \Phi(v_i'), \quad \text{for } i = 1, 2, ..., n \tag{7}$$

where, Φ is the cumulative distribution function of the standard normal distribution

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt, \quad \text{for } i = 1, 2, ..., n \tag{8}$$

As a last step in this procedure, a vector $\mathbf{u}$ can be created taking the inverse normal cumulative distribution function using the elements in $\mathbf{v}''$ as random variables and using the standard deviation of historical foreign exchange rates as well as a mean of zero. $\mathbf{u}$ is used for simulating foreign exchange rates as geometric Brownian motions described in equation (2). Again, a mean of zero is used because we do not want to take a view of the evolvement of foreign exchange rates.

5.2.4 Cross-Currency Arbitrage

Assuming perfect capital markets, foreign exchange rates must hold for a cross-currency arbitrage. Hence, foreign exchange rates have to be in line with exchange rates, e.g. quoted relative the USD. Thus, the formula for the implicit exchange rate from two given foreign exchange rates is calculated as

$$S_{a/\$} = S_{b/\$} S_{a/b} \tag{9}$$

where, $S_{a/\$}$ is the implicit exchange rate for USD in terms of currency a, $S_{b/\$}$ is the quoted exchange rate for USD in terms of currency b, and $S_{a/b}$ is the quoted exchange rate for currency b in terms of currency a.

For each investment, three foreign exchange rates are needed, (i) the limited partner currency, (ii) the fund currency, and (iii) the PC currency. In this paper, we simulate two of these foreign exchange rates and calculate the third using the simulated exchange rates and cross-currency arbitrage.

5.2.5 Forward Rate

In this study, forward contracts are used to hedge the currency exposure arising from foreign investments. Following the covered interest rate parity, which implies a relationship between interest rates and foreign exchanges rates that results in a no arbitrage condition, the forward rate is calculated as

$$F_{t,T} = S_t \left(\frac{1 + r^*_{t,T}}{1 + r_{t,T}} \right)^{T-t} \tag{10}$$

where, $F_{t,T}$ is the forward rate, S_t is the spot rate, $r^*_{t,T}$ is the historical foreign risk-free interest rate, $r_{t,T}$ is the historical domestic (country of the limited partner) risk-free interest rate, t is the trade date and T is the maturity date.

In equation (10) covered interest rate parity is assumed. An alternative way to calculate the forward rate is through the uncovered interest rate parity, where the same rate of returns in different currencies is expected, on average. If this relation holds the unbiasedness hypothesis yields

$$E_t[S_T] = F_{t,T} \tag{11}$$

Important to note is that historical risk-free interest rates are used, which could lead to biased forward rates. One could argue that risk-free interest rates should also be simulated, since they correlate with foreign exchange rates. In a scenario where the currency of a specific country is too strong or too weak, its central bank can use interest rates to normalise the foreign exchange rate. Hence, risk-free interest rates should correlate with the foreign exchange rates. Using the uncovered interest rate parity and/or simulating risk-free interest rates could affect the forward rate and hence the extent of the hedges.

5.2.6 Time Period of Simulation

The simulation starts at January 1, 2013, hence the date of S_0 in equation (10) is January 1, 2013. Since the investments are made between June, 2013 and April, 2016, the initial foreign exchange rate for each investment will be different for every simulation. For the same reason, forward rates will also be different for each investment and simulation.

5.3 Hedging Strategies

To find the optimal hedging strategy for limited partners investing in private equity, three hedging strategies will be evaluated

(i) Unhedged Strategy

(ii) Fund Currency Hedged Strategy

(iii) Portfolio Company Currency Hedged Strategy

To assess each of the strategies, investment data from three co-investments made by AP6 is used, presented in Tables 13-21. For each co-investment, fund currency and PC currency are presented. The limited partner currency is always set to SEK, since we want to evaluate the performance of a limited partner. In the main scenario, the co-investments are modified to different fund currencies, as a proxy for investing in different private equity funds. The fund currencies are set to EUR, GBP and USD and the three PC currencies are set to NOK, USD and GBP, see Figure 4. For the simulation, the following is calculated for each of the hedging strategies of the limited partner

(i) Total Return

(ii) Annualised Return

(iii) Money Multiple

To evaluate the hedging strategies, 5,000 Monte Carlo simulation runs are made, for each investment. For the hedges, we assume no transaction costs and that all currency exposure is hedged. Furthermore, management fees and carried interest to

private equity funds are not considered. Mean returns are calculated to evaluate if there is a superior hedging strategy in terms of mean. As a measure of the riskiness of returns, standard deviation is used. Finally, to evaluate the risk-return trade-off, the Sharpe ratio is calculated. The Sharpe ratio is defined as

$$\textit{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p} \tag{12}$$

where, R_p is the mean annual return, R_f is the risk-free interest rate of the currency denoting the limited partner at exit date of the investment, and σ_p is the standard deviation of annual returns.

We expect the standard deviations of the unhedged scenario and the fund currency hedged scenario to be similar to each other and higher than in the PC currency hedged scenario. The intuition behind this is that the fund currency is a transaction currency, presented Figure 2. Furthermore, according to our alternative hypothesis, the Sharpe ratio is expected to be the highest for the scenario hedging PC currency. This because we expect that hedging the PC currency will result in the lowest standard deviation since the operational currency exposure is hedged. We believe that expected returns will be similar for all strategies. Note that a potential risk premium for bearing foreign exchange risk is disregarded. This would result in a higher mean return for the unhedged scenario and thus a higher Sharpe ratio, if standard deviations would remain unchanged.

5.3.1 Unhedged Strategy

The first of the hedging strategies is simply to not hedge. That is, under this strategy the limited partner never purchases a forward contract to cover currency exposure. Hence, the limited partner is exposed to foreign exchange risk. This strategy will be referred to as the unhedged strategy.

5.3.2 Fund Currency Hedged Strategy

The second hedging strategy is to hedge the fund currency. That is, the limited partner takes a short position in the fund currency using a forward contract, with trade date being the call date[3] and maturity date being the exit date of the in-

[3]Call date is when the commitment is drawn from the limited partner by the private equity fund.

vestment. We are aware of that we do not know the exit date ex ante, but this assumption is made for simplification purposes. NAV of the portfolio company, denoted in fund currency, is updated and reported to the limited partners on a quarterly basis by the private equity fund. In a perfect world, the short position would reflect the portfolio company exposure continuously. However, in line AP6 treatment, the short position is renewed only when NAV updates are substantial. In other words, we hedge when AP6 hedges.

We expect fund currency hedged scenario to result in similar returns and standard deviations as in an unhedged scenario. The intuition behind this is that even though the limited partner hedges the fund currency, it is exposed to fluctuations originating from the foreign exchange rate between the fund currency and the PC currency, see Figure 3. Hence, the limited partner is still exposed to fluctuations stemming from the foreign exchange rate of the fund currency and the PC currency.

5.3.3 Portfolio Company Currency Hedged Strategy

The third hedging strategy is to hedge the PC currency. That is, the limited partner takes a short position in the PC currency using a forward contract, with trade date being the call date and maturity date being the exit date of the investment. Similar to the hedge of the fund currency, the short position is renewed when NAV updates are substantial, in PC currency.

In line with the alternative hypothesis, we expect the PC currency hedged scenario to result in lower standard deviation compared to the unhedged scenario and fund currency hedged scenarios. The rationale of hedging the PC currency is that all value is created in the PC currency, hence the limited partner should protect the NAV in PC currency. This scenario builds on the assumption of no arbitrage in foreign exchange rates, thus the fund currency is only a transaction currency, see Figure 2, neither adding nor destroying value for the limited partner.

5.4 Robustness Tests

To examine the robustness of the results in the main scenario and further analyse them, two additional scenarios are considered.

5.4.1 First Robustness Test Scenario

In the first robustness test scenario, the limited partner currency is still always set to SEK. On the contrary to the main scenario, where the fund currencies are set to different currencies, the fund currencies are now set to the same currency. This to simulate that a private equity fund invests in several portfolio companies denoted in different currencies. In this scenario the fund currency is EUR and the PC currencies are still set to NOK, USD and GBP, see Figure 5.

According to the null hypothesis this would result in a lower Sharpe ratio, because of a lost diversification effect since the currency exposure is no longer split between different fund currencies. However, if beliefs are in line with the alternative hypothesis, this scenario would yield similar Sharpe ratio as the main scenario when hedging the PC currency, since the fund currency is regarded as a transaction currency, see Figure 2.

5.4.2 Second Robustness Test Scenario

In the second robustness test scenario, the limited partner currency is still always set to SEK and the fund currencies are similar to the main scenario. On the contrary to the main scenario, where the PC currencies are different for all co-investments, the PC currencies are now set to the same currency. This to simulate that a limited partner invests in several private equity funds, investing in portfolio companies denoted in the same currency. The fund currencies are set to EUR, GBP and USD. The PC currencies are set to NOK, see Figure 6.

The reason for this scenario is to see the effects of non-diversified investments in terms of PC currency. Again, according to the null hypothesis this would result in a lower Sharpe ratio, because of a lost diversification effect since the currency exposure is no longer split between different PC currencies. Similarly, if beliefs are in line with the alternative hypothesis, this scenario would yield a lower Sharpe ratio compared to the main scenario when hedging the PC currency. This is due

to a lost diversification effect.

6 Results and Discussion

In the following section the results and interpretations of this study are described in detail. The section begins with results from the interviews with limited partners and private equity firms. Results regarding the simulation of foreign exchange rates are then presented. After that, results and interpretations from the hedging strategies are shown. Later follows the robustness tests where again results and interpretations of the hedging strategies are presented. The section is completed by a comparison of the different hedging strategies.

6.1 Currency Exposure

Below, results and interpretations of the interviews are presented. To be consistent, the private equity currency exposure is presented first followed by the limited partner currency exposure.

6.1.1 Private Equity Currency Exposure

Based on interviews with private equity firms, we draw the conclusion that private equity firms focus on the balance sheet exposure in hedging situations, even though some firms expressed that the real exposure is revenues and costs stemming from operating currencies. The private equity firms reduce currency exposure by natural hedges such as debt financing in the PC currency, hence the fund is left with currency exposure from its equity investment. Also, many private equity firms influence the hedging strategies of the portfolio companies.

However, most private equity firms consider currency exposure to be a zero sum game, which only has a minor or no effect on the total return. This is in line with Pérold and Schulman (1988), arguing that from a long-term perspective, investors should think of currency hedging as having zero expected return. Some private equity firms have the rationale that a hedge is only a short term insurance during a specific time period. The core business of private equity firms is risky, hence there might be more important factors than currency exposure to consider. Furthermore, private equity firms reason that it is beyond their expertise to take a

view of foreign exchange rate evolvements and that they rather should focus on their core business. There is one exception where several private equity firms sometimes hedge, which is the period between signing and closing of a deal. However, this hedge is often not about performance of the fund. Rather, it is a cash management issue taken care of, where the private equity fund makes sure to only draw capital from the limited partner once. However, in times of uncertainty, the effects of the foreign exchange risk between signing and closing of a deal could be substantial.

When raising capital for a new fund, the private equity firm can choose to present past performance of deals in either the PC currency or the fund currency, creating a possibility for IRR gaming. If currency movements are in favour of the private equity fund, the fund is often likely to regard it as a skill. In an opposite scenario, the effect might be regarded as bad luck. In those cases, the private equity fund might be more likely to focus on the performance in PC currency.

6.1.2 Limited Partner Currency Exposure

Numerous Swedish limited partners were interviewed. We find that most limited partners choose to define their currency exposure as the fund currency in hedging situations, while a few define the exposure as the PC currency in hedging situations. Disregarding definition of currency exposure, all limited partners hedge. We also find that most limited partners reason that an overlay hedging strategy, with the investment in fund currency representing the exposure, is enough to cover a sufficient amount the currency exposure. This is due to the fact that most limited partners invest in several asset classes and want an aggregated hedging method for its portfolio. As most limited partners only have a small fraction of its portfolio invested in private equity, they think it is too costly and time consuming to use the PC currency as exposure, even though it would be more correct.

One exception is the AP6, the only limited partner in our study solely investing in private equity. AP6 is restricted by law to keep currency exposure below ten per cent. This implies that it is more important to control the currency exposure for AP6 compared to its peers. In line with its peers, AP6 hedges the aggregated exposure, as hedging each portfolio company by itself would imply higher transaction costs.

Interesting is that, in most cases, limited partners and private equity firms do not, on a regular basis, discuss currency exposure and hedging strategies with each other. This implies an enhanced risk of overhedging and/or mishedging.

6.2 Simulation

The simulated foreign exchange rates, presented in Figures 9-11, seem reasonable when comparing them to historical foreign exchange rates, presented in Figures 7-8. Results from the simulations are presented and discussed in the next sections.

6.3 Hedging Strategies

This section presents the results and interpretations of the (i) unhedged strategy, (ii) fund currency hedged strategy, and (iii) PC currency hedged strategy. The performance metrics shown are total return, annualised return and money multiple as well as standard deviations of these. Also, Sharpe ratios are presented.

6.3.1 Unhedged Strategy

Tables 1-3 present total return, annualised return and money multiple for the unhedged strategy. These results will be compared to the hedged strategies. The unhedged strategy implies that the limited partner is exposed to all foreign exchange risks, stemming from both the fund currency and the PC currency. Using a Monte Carlo simulation of 5,000 runs, the unhedged strategy yields a mean total return of 33.18% with a standard deviation of 14.43%. Annualised return and money multiple are 18.08% and 1.33x, with standard deviations of 6.97% and 0.14x, respectively. The Sharpe ratio is 2.52, presented in Table 4.

There are several reasons for applying an unhedged strategy as a limited partner investing in private equity. First, if exposed to several currencies, it would enjoy a diversification effect. Second, transaction costs from hedging are avoided. Lastly, additional resources might be needed to handle the currency exposure and hedging.

6.3.2 Fund Currency Hedged Strategy

Tables 1-3 present total return, annualised return and money multiple for the fund currency hedged strategy. The fund currency hedged strategy implies that the

limited partner is exposed to the foreign exchange risk between the limited partner currency and the PC currency, see Figure 3. Using a Monte Carlo simulation of 5,000 runs, the strategy hedging fund currency yields a mean total return of 34.02% with a standard deviation of 18.42%. Annualised return and money multiple are 17.92% and 1.34x, with standard deviations of 7.55% and 0.18x, respectively. The Sharpe ratio is 2.30, presented in Table 4.

In a scenario where the fund hedges the PC currency, the limited partner is exposed to the foreign exchange risk between the limited partner currency and the fund currency. Hence, in such a scenario, the optimal hedging strategy for the limited partner is to hedge against the fund currency, in order not to overhedge and/or mishedge.

6.3.3 Portfolio Company Currency Hedged Strategy

Tables 1-3 present total return, annualised return and money multiple for the PC currency hedged strategy. The PC currency hedged strategy implies that the limited partner is not exposed to foreign exchange risk. Using a Monte Carlo simulation of 5,000 runs, the strategy hedging PC currency yields a mean total return of 34.05% with a standard deviation of 12.27%. Annualised return and money multiple are 18.50% and 1.34x, with standard deviations of 5.62% and 0.12x, respectively. The Sharpe ratio is 3.20, presented in Table 4.

In line with the intuition behind hedging the PC currency, the fund currency will simply become a transaction currency, see Figure 2. Hence, returns and standard deviations will not be affected by fluctuations in the foreign exchange rates between the fund currency and PC currency or limited partner currency. Consequently, the limited partner will only handle fluctuations in the foreign exchange rate between PC- and limited partner currency.

In a scenario where the limited partner hedges the PC currency, but the private equity fund also hedges the PC currency, a reversed currency exposure is constructed. As the limited partner invests in several private equity funds, it is not always informed about the hedging strategy of a specific private equity fund. Hence, there is a risk of overhedging and/or mishedging.

6.4 Robustness Tests

This section presents the results and interpretations of the robustness tests of the (i) unhedged strategy, (ii) fund currency hedged strategy, and (iii) PC currency hedged strategy. The performance metrics shown are total return, annualised return and money multiple as well as standard deviations of these. Also, Sharpe ratios are presented.

6.4.1 First Robustness Test Scenario

Tables 5-8 present total returns, annualised returns, money multiples and Sharpe ratios for the first robustness test scenario. Using a Monte Carlo simulation of 5,000 runs, (i) the unhedged strategy, (ii) fund currency hedged strategy, and (iii) the PC currency hedged strategy yield mean total returns of 33.27%, 34.80% and 33.94% with standard deviations of 14.41%, 17.86% and 12.24%, respectively. Furthermore, the strategies yield annualised returns of 18.19%, 18.45% and 18.51%, with standard deviations of 7.01%, 7.42% and 5.63%, as well as money multiples of 1.33x, 1.35x and 1.34x, with standard deviations of 0.14x, 0.18x and 0.12x, respectively. The Sharpe ratio is still significantly higher for the PC currency hedged strategy compared to the unhedged strategy and fund currency hedged strategy. Sharpe ratios are 2.52, 2.41 and 3.19, respectively.

Worth noting is that this scenario yields similar Sharpe ratios for the unhedged scenario and the PC hedged scenario as in the main scenario. This supports the rationale behind the alternative hypothesis, that the fund currency is only a transaction currency, as presented in Figure 2. The Sharpe ratio for the fund currency hedged scenario is slightly higher compared to the main scenario. This might be explained by the relation between the risk-free interest rates of the currencies. The effect arises from not including the correlations between foreign exchange rates and risk-free interest rates. This problem might be solved by including these correlations in the simulation or assume the uncovered interest rate parity to hold. A third way to solve the problem is to include views of foreign exchange rate evolvements, in other words changing the value of μ in equation (1). However, these suggestions are beyond the scope of this thesis.

6.4.2 Second Robustness Test Scenario

Tables 9-12 present total returns, annualised returns, money multiples and Sharpe ratios for the second robustness test scenario. Using a Monte Carlo simulation of 5,000 runs, (i) the unhedged strategy, (ii) fund currency hedged strategy, and (iii) the PC currency hedged strategy yield mean total returns of 33.38%, 33.63% and 34.03% with standard deviations of 14.31%, 18.02% and 12.17%, respectively. Furthermore, the strategies yield annualised returns of 18.25%, 17.80% and 18.59%, with standard deviations of 7.07%, 7.50% and 5.78%, as well as money multiples of 1.33x, 1.34x and 1.34x, with standard deviations of 0.14x, 0.18x and 0.12x, respectively. The Sharpe ratio is still significantly higher for the PC currency hedged strategy compared to the unhedged- and fund currency hedged strategies. Sharpe ratios are 2.51, 2.30 and 3.12, respectively.

This scenario yields similar Sharpe ratios for the unhedged and the fund currency hedged scenarios as the main scenario. Note that the Sharpe ratio for the PC currency hedged strategy is lower compared to the main scenario, this might be due to a diversification effect.

6.5 Comparison of Hedging Strategies

The results in Tables 1-2 show that mean returns are similar but standard deviations differ between the hedging strategies. Therefore, it is the standard deviation alone that will affect the Sharpe ratios. These results also apply to the money multiple, presented in Table 3, where standard deviation is significantly lower for the scenario where PC currency is hedged, compared to the other strategies. For the unhedged scenario and the scenario where fund currency is hedged, the standard deviations are close to each other. These results are in line with Glen and Jorion (1993), concluding that hedging is beneficial since the risk reduction is not accompanied by an offsetting decrease in return.

Comparing Sharpe ratios in the main scenario, presented in Table 4, we find that hedging against PC currency yield by far the highest Sharpe ratio, in accordance with the alternative hypothesis. Hence the alternative hypothesis of this thesis can be accepted, and the null hypothesis can be rejected. The results from the robustness test scenarios, presented Tables 5-12, support the findings in the

main scenario. Importantly, we have assumed no currency risk premium. According to De Santis and Gérard (1998) the currency risk premium is a substantial fraction of the total risk premium, suggesting that the expected return of the unhedged strategy might be understated. Assuming that standard deviation for this strategy would be unchanged, the Sharpe ratio for the unhedged strategy would increase as a result of increased mean return.

Most interviewed limited partners have had the rationale in line with the main scenario when defining and hedging their currency exposure as the fund currency. However, the results presented in Table 4 suggest that the limited parters would perform even better, in terms of Sharpe ratio, if defining the currency exposure as the PC currency and hedge accordingly.

Comparing the main scenario and the second robustness test scenario, one can see that when hedging the PC currency the Sharpe ratio is higher in the main scenario. This might be explained by the diversification effect stemming from a diversified currency exposure from the portfolio companies denoted in different currencies. This diversification effect is not visible in the unhedged strategy as it should be. This is due to the construction of the simulation model using predetermined NAV of the portfolio companies. To solve this problem, NAVs must be simulated. However, this is beyond the scope of this thesis.

For a limited partner, hedging fund currency is simple compared to hedging PC currency, since it is easier to determine the fund currency. In addition, determining the PC currency exposure might require a lot of resources. For a perfect hedge, it is not enough to assume the PC currency exposure to be the denoted currency where the headquarter is located. The currency exposure of a portfolio company is determined by its net exposure, in other words net cash flows in different currencies, dependent on if the portfolio company is a net exporter or net importer. Moreover, the limited partner needs to investigate if and how the portfolio company hedges its currency exposure. These aspects have to be considered when hedging the PC currency. Nevertheless, this might be too much required of the limited partner.

On the contrary, if a limited partner determines to hedge the fund currency, it might as well consider leaving the portfolio unhedged, since the unhedged and fund

currency hedging strategy yield similar results. By leaving the portfolio unhedged, the limited partner could save resources, as transaction costs are avoided as well as that time can be spent on other tasks.

Worth mentioning is that opposed to the finding of Froot et al. (1994), that companies should pay attention to the hedging strategies of peers, no interviewed limited partner considers that type of peer analysis to be prioritised.

7 Conclusion

This study sets out to investigate how limited partners investing in private equity funds should define their currency exposure and whether they should hedge it or not. Inspiration has been gathered from previous literature investigating risk management and currency hedging. The conclusions of Glen and Jorion (1993) as well as Pérold and Schulman (1988), in other words, that currency hedging improves the risk-return trade-off, are applied in another setting. First, simulations of foreign exchange rates are made. Second, different hedging strategies are examined and discussed. The results found are significant and robust.

The main result of this study is that limited partners investing in private equity should define their currency exposure as the portfolio company currency and hedge accordingly. The economic interpretation of this result is that limited partners can obtain higher Sharpe ratios by hedging currency exposure. Therefore, the result is consistent with previous literature and the null hypothesis can be rejected. Furthermore, the alternative hypothesis can be accepted. From interviews, it is found that most limited partners investing in private equity funds hedge the fund currency. Hence, an implication of this result is that many limited partners investing in private equity funds mishedge their currency exposure.

In addition, it is found that limited partners investing in private equity should diversify their assets under management by investing in private equity funds exposed to several currencies, through the portfolio companies. By doing so, the limited partners are awarded with higher Sharpe ratios.

This study contributes to existing literature in two ways. First, as the results of this thesis indicate a higher Sharpe ratio when hedging, the finding contributes to research investigating the relationship between hedging and risk-return trade-off.

Second, this study also contributes to the field of research concerning private equity and limited partners investing in private equity.

Furthermore, there are limitations to this study that are important to highlight. The first issue is data related. As the data only covers three co-investments made by AP6, there is a risk that the results presented in this study are isolated to this particular investment universe. A second issue could be the time period of which historical data is used. During the period, the market might have had certain characteristics which are not representative for other time periods. Third, Swedish limited partners and private equity firms have been interviewed, which might not illustrate a global view. Last, there are limitations related to perfect market assumptions, such as no transaction costs and cross-currency arbitrage. The limitations could generate biased results.

Finally, since this area of finance is quite unexamined, there are interesting settings for future research to examine. First, a possible starting point could be to confirm the findings of this study by extending the data set, to include a complete limited partner portfolio and all cash flows between a limited partner, the private equity fund and its portfolio companies. In such a setting, it would also be interesting to take the true currency exposure of the portfolio company into account and investigate if systematic risk is reduced by hedging. Second, future research could examine whether the results of this thesis are time period dependent, by conducting the study during different time periods. Third, extensions to the simulation could be made. In addition to foreign exchange rates, risk-free interest rates, private equity portfolios as well as portfolio companies could be simulated. Another approach could be a simulation method bootstrapping historical foreign exchange rates and corresponding risk-free interest rates. Moreover, it would be interesting to include views on currency appreciations and depreciations in the simulation. At last, it would be interesting to include a currency risk premium.

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Appendices

A Tables and Figures

Table 1: Main Scenario, Total Return

Descriptive statistics	Unhedged	Hedged - FC	Hedged - PCC
Mean	33.18%	34.02%	34.05%
Standard deviation	14.43%	18.42%	12.27%

Table 1 summarises the performance of the hedging strategies in the main scenario, in terms of total return and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 2: Main Scenario, Annualised Return

Descriptive statistics	Unhedged	Hedged - FC	Hedged - PCC
Mean	18.08%	17.92%	18.50%
Standard deviation	6.97%	7.55%	5.62%

Table 2 summarises the performance of the hedging strategies in the main scenario, in terms of annualised return and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 3: Main Scenario, Money Multiple

Descriptive statistics	Unhedged	Hedged - FC	Hedged - PCC
Mean	1.33x	1.34x	1.34x
Standard deviation	0.14x	0.18x	0.12x

Table 3 summarises the performance of the hedging strategies in the main scenario, in terms of money multiple and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 4: Main Scenario, Sharpe Ratio

	Unhedged	Hedged - FC	Hedged - PCC
Sharpe Ratio	2.52	2.30	3.20

Table 4 summarises the performance of the hedging strategies in the main scenario, in terms of Sharpe ratio for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 5: Robustness Test Scenario 1, Total Return

Descriptive statistics	**Unhedged**	**Hedged - FC**	**Hedged - PCC**
Mean	33.27%	34.80%	33.94%
Standard deviation	14.41%	17.86%	12.24%

Table 5 summarises the performance of the hedging strategies in the first robustness test scenario, in terms of total return and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 6: Robustness Test Scenario 1, Annualised Return

Descriptive statistics	**Unhedged**	**Hedged - FC**	**Hedged - PCC**
Mean	18.19%	18.45%	18.51%
Standard deviation	7.01%	7.42%	5.63%

Table 6 summarises the performance of the hedging strategies in the first robustness test scenario, in terms of annualised return and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 7: Robustness Test Scenario 1, Money Multiple

Descriptive statistics	**Unhedged**	**Hedged - FC**	**Hedged - PCC**
Mean	1.33x	1.35x	1.34x
Standard deviation	0.14x	0.18x	0.12x

Table 7 summarises the performance of the hedging strategies in the first robustness test scenario, in terms of money multiple and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 8: Robustness Test Scenario 1, Sharpe Ratio

	Unhedged	**Hedged - FC**	**Hedged - PCC**
Sharpe ratio	2.52	2.41	3.19

Table 8 summarises the performance of the hedging strategies in the first robustness test scenario, in terms of Sharpe ratio for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 9: Robustness Test Scenario 2, Total Return

Descriptive statistics	**Unhedged**	**Hedged - FC**	**Hedged - PCC**
Mean	33.38%	33.63%	34.03%
Standard deviation	14.31%	18.02%	12.17%

Table 9 summarises the performance of the hedging strategies in the second robustness test scenario, in terms of total return and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 10: Robustness Test Scenario 2, Annualised Return

Descriptive statistics	**Unhedged**	**Hedged - FC**	**Hedged - PCC**
Mean	18.25%	17.80%	18.59%
Standard deviation	7.07%	7.50%	5.78%

Table 10 summarises the performance of the hedging strategies in the second robustness test scenario, in terms of annualised return and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 11: Robustness Test Scenario 2, Money Multiple

Descriptive statistics	**Unhedged**	**Hedged - FC**	**Hedged - PCC**
Mean	1.33x	1.34x	1.34x
Standard deviation	0.14x	0.18x	0.12x

Table 11 summarises the performance of the hedging strategies in the second robustness test scenario, in terms of money multiple and standard deviation for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 12: Robustness Test Scenario 2, Sharpe Ratio

	Unhedged	**Hedged - FC**	**Hedged - PCC**
Sharpe ratio	2.51	2.30	3.12

Table 12 summarises the performance of the hedging strategies in the second robustness test scenario, in terms of Sharpe ratio for an (i) unhedged-, (ii) fund currency hedged-, and (iii) portfolio company currency hedged strategy. FC is fund currency and PCC is portfolio company currency.

Table 13: Fund 1, AP6 Co-Investment

Date	Type	Operation	Commitment	Net Cash	Total Called	NAV	NAV Adjustment
2016-12-31	Official NAV	Adjusted Valuation Q4 2016				34,000,000	1,083,936
2016-09-30	Official NAV	Adjusted Valuation Q3 2016				32,916,064	2,365,764
2016-08-31	Accounting Valuation	Accounting Valuation T2 2016					
2016-06-30	Official NAV	Adjusted Valuation Q2 2016				30,550,300	6,281,134
2016-04-30	Accounting Valuation	Accounting Valuation T1 2016					
2016-03-31	Official NAV	Adjusted Valuation Q1 2016				24,269,166	4,079,158
2015-12-31	Accounting Valuation	Accounting Valuation T3 2015					
2015-12-31	Official NAV	Adjusted Valuation				20,190,008	118,138
2015-11-19	External Expense			(5,822)			
2015-08-31	Accounting Valuation	Accounting Valuation T2 2015					
2015-08-31	Official NAV	Adjusted Valuation				20,071,870	2,318,233
2015-04-30	Accounting Valuation	Accounting Valuation T1 2015					
2015-04-30	Official NAV	Adjusted Valuation				17,753,637	(1,439,746)
2014-12-31	Accounting Valuation	Accounting Valuation EOY					
2014-12-31	Official NAV	Adjusted Valuation				19,193,383	(806,617)
2014-08-31	Official NAV	Adjusted Valuation				20,000,000	
2014-08-19	Call	Rebate Prel. Exp.		989	(989)		(400)
2014-07-28	Call			(20,022,691)	20,022,691		20,000,400
2014-06-18	External Expense			(9,104)			
2014-05-20	Commitment	Commitment	20,000,000				
			20,000,000	20,036,628	20,021,702		

Table 13 presents data from a co-investment made by AP6. We have simplified the data and present the relevant parts for this thesis. Dates, type of event, NAV updates and NAV adjustments for the portfolio company in fund currency for AP6 during the investment period is presented. The original fund currency is GBP.

Table 14: Fund 1, Investment in the Portfolio Company

Instrument Currency	Date	Type	Amount (NOK)	Amount (GBP)	Force Cost (GBP)	Valuation (NOK)	Valuation (GBP)
NOK	2016-12-31	Valuation				360,546,577	34,000,000
NOK	2016-09-30	Valuation				343,268,407	32,916,064
NOK	2016-06-30	Purchase			592,072		
NOK	2016-06-30	Valuation				346,726,592	30,550,300
NOK	2016-03-31	Purchase			(592,072)		
NOK	2016-03-31	Valuation				280,082,032	23,547,836
NOK	2015-12-31	Valuation				261,049,859	20,190,008
NOK	2015-08-31	Valuation				256,335,368	20,071,870
NOK	2015-04-30	Valuation				206,478,964	17,753,637
NOK	2014-12-31	Valuation				222,558,418	19,193,383
NOK	2014-08-31	Valuation				205,674,287	20,000,000
NOK	2014-08-19	Purchase	4,246	400			
NOK	2014-07-28	Purchase	(208,673,569)	(20,000,400)			
			(208,669,323)	(20,000,000)			

Table 14 presents data from a co-investment made by AP6. We have simplified the data and present the relevant parts for this thesis. Dates, type of event, NAV updates in fund currency and portfolio company currency for the private equity fund during the investment period is presented. The original fund currency is GBP and the original portfolio company currency is NOK.

Table 15: Fund 1, Hedging Transactions Made by AP6

Hedge Instrument Type	Trade Date	Maturity Date	Spot Rate	Forward Rate	Inverse Rate	Amount (Buy)	Currency (Buy)	Amount (Sell)	Currency (Sell)	Hedged Currency
Fx Spot	2014-07-24	2014-07-28	11.6380			20,022,691	GBP	233,024,078	SEK	GBP
Fx Forward	2014-07-24	2015-05-29	1.0000	0.9168	1.0907	233,024,078	SEK	213,646,354	NOK	NOK
Fx Spot	2015-05-28	2015-05-29	1.0890			213,646,354	NOK	232,660,879	SEK	NOK
Fx Forward	2015-05-28	2016-05-31	0.9127	0.9326	1.0723	229,092,985	SEK	213,646,354	NOK	NOK
Fx Forward	2015-12-28	2016-11-30	1.0340	1.0519	0.9507	47,535,000	SEK	50,000,000	NOK	NOK
Fx Spot	2016-05-30	2016-05-31	0.9985			213,646,354	NOK	213,325,884	SEK	NOK
Fx Forward	2016-05-30	2017-05-31	0.9998	1.0166	0.9836	210,148,963	SEK	213,646,354	NOK	NOK
Fx Forward	2016-09-21	2017-05-31	0.9663	0.9769	1.0236	87,006,000	SEK	85,000,000	NOK	NOK
Fx Spot	2016-11-29	2016-11-30	1.0750			50,000,000	NOK	53,750,000	SEK	NOK
Fx Forward	2016-11-29	2017-11-30	0.9287	0.9469	1.0561	52,805,000	SEK	50,000,000	NOK	NOK
Fx Forward	2016-12-30	2017-11-30	0.9496	0.9654	1.0358	51,790,000	SEK	50,000,000	NOK	NOK

Table 15 presents hedging transactions made by AP6 to minimise its currency exposure from investments in portfolio companies. We have simplified the data and present the relevant parts for this thesis. The table presents hedge instrument type, trade date and maturity date for the transactions. Furthermore, spot rates, forward rates and inverse forward rates are presented. At last, the currency amount bought and sold as well as the hedged currency are presented.

Table 16: Fund 2, AP6 Co-Investment

Date	Type	Operation	Commitment	Net Cash	Total Called	NAV	NAV Adjustment
2016-09-30	Official NAV	Adjusted Valuation Q3 2016				49,966,873	247,764
2016-09-01	Call						(254,264)
2016-08-31	Accounting Valuation	Accounting Valuation T2 2016					
2016-06-30	Official NAV	Adjusted Valuation Q2 2016				49,973,373	
2016-06-30	Call	Adjusted Valuation					254,264
2016-04-28	External Expense			(2,008)			
2016-04-22	Commitment	Commitment Other Investors					
2016-04-22	Call			(50,000,000)	50,000,000		49,719,109
2016-04-22	Commitment	Commitment	50,000,000				
			50,000,000	(50,003,283)	50,000,000		

Table 16 presents data from a co-investment made by AP6. We have simplified the data and present the relevant parts for this thesis. Dates, type of event, NAV updates and NAV adjustments for the portfolio company in fund currency for AP6 during the investment period is presented. The original fund currency is USD.

Table 17: Fund 2, Investment in the Portfolio Company

Instrument Currency	Date	Type	Amount (USD)	Amount (USD)	Force Cost (USD)	Valuation (USD)	Valuation (USD)
USD	2016-09-30	Valuation				49,719,109	49,719,109
USD	2016-06-30	Valuation				49,719,109	49,719,109
USD	2016-04-22	Purchase	(49,719,109)	(49,719,109)			
			(49,719,109)	(49,719,109)			

Table 17 presents data from a co-investment made by AP6. We have simplified the data and present the relevant parts for this thesis. Dates, type of event, NAV updates in fund currency and portfolio company currency for the private equity fund during the investment period is presented. The original fund currency is USD and the original portfolio company currency is USD.

Table 18: Fund 2, Hedging Transactions Made by AP6

Hedge Instrument Type	Trade Date	Maturity Date	Spot Rate	Forward Rate	Inverse Rate	Amount (Buy)	Currency (Buy)	Amount (Sell)	Currency (Sell)	Hedged Currency
Fx Spot	2016-04-25	2016-04-26	8.1326			50,000,000	USD	406,300,000	SEK	USD
Fx Forward	2016-04-25	2017-03-31	0.1231	0.1247	8.0195	400,975,000	SEK	50,000,000	USD	USD

Table 18 presents hedging transactions made by AP6 to minimise its exposure from investments in portfolio companies. We have simplified the data and present the relevant parts for this thesis. The table presents hedge instrument type, trade date and maturity date for the transactions. Furthermore, spot rates, forward rates and inverse forward rates are presented. At last, the currency amount bought and sold as well as the hedged currency are presented.

Table 19: Fund 3, AP6 Co-Investment

Date	Type	Operation	Commitment	Net Cash	Total Called	NAV	NAV Adjustment
2016-09-30	Official NAV	Adjusted Valuation Q3 2016				4,712,662	739,371
2016-08-31	Accounting Valuation	Accounting Valuation T2 2016					
2016-06-30	Official NAV	Adjusted Valuation Q2 2016				3,973,291	(888,030)
2016-04-30	Accounting Valuation	Accounting Valuation T1 2016					
2016-03-31	Official NAV	Adjusted Valuation Q1 2016				4,861,321	(1,849,679)
2015-12-31	Accounting Valuation	Accounting Valuation T3 2015					
2015-12-31	Official NAV	Adjusted Valuation				6,711,000	(1,509,277)
2015-08-31	Accounting Valuation	Accounting Valuation T2 2015					
2015-08-31	Official NAV	Adjusted Valuation				8,220,277	(1,314,100)
2015-04-30	Accounting Valuation	Accounting Valuation T1 2015					
2015-04-30	Official NAV	Adjusted Valuation				9,535,277	1,219,485
2014-12-31	Accounting Valuation	Accounting Valuation EOY					
2014-12-31	Official NAV	Adjusted Valuation				8,315,792	244,000
2014-08-31	Official NAV	Adjusted Valuation				8,071,792	1,714,100
2014-07-25	Distribution			1,528,731			(1,528,731)
2014-05-02	Call	Adjustment					(30,547)
2014-04-30	Official NAV	Adjusted Valuation				7,916,071	
2013-12-31	Official NAV	Adjusted Valuation				7,916,071	
2013-10-03	Call	Call		(1,170,056)	1,170,056		1,170,056
2013-09-23	Commitment	Commitment	1,170,056				
2013-09-11	External Expense			(339)			
2013-08-31	Official NAV	Adjusted Valuation				6,746,015	8,515
2013-07-05	External Expense	Co-Investment		(10,614)			
2013-06-26	Call	Call		(6,737,500)	6,737,500		6,737,500
2013-06-26	Commitment	Commitment	7,000,000				
			8,170,056	(5,389,778)	7,907,556		

Table 19 presents data from a co-investment made by AP6. We have simplified the data and present the relevant parts for this thesis. Dates, type of event, NAV updates and NAV adjustments for the portfolio company in fund currency for AP6 during the investment period is presented. The original fund currency is EUR.

Table 20: Fund 3, Investment in the Portfolio Company

Instrument Currency	Date	Type	Amount (DKK)	Amount (EUR)	Force cost(EUR)	Valuation (DKK)	Valuation (EUR)
DKK	2016-09-30	Valuation				35,056,820	4,703,558
DKK	2016-06-30	Valuation				29,464,879	3,961,423
DKK	2016-03-31	Valuation				36,096,013	4,845,126
DKK	2015-12-31	Valuation				49,756,905	6,667,048
DKK	2015-08-31	Valuation				61,347,265	8,220,277
DKK	2015-04-30	Valuation				71,153,911	9,535,277
DKK	2014-12-31	Valuation				61,589,868	8,271,840
DKK	2014-08-31	Valuation				59,700,416	8,014,840
DKK	2014-07-25	Sale			1,528,731		
DKK	2014-07-25	Sale	11,398,034	1,528,731			
DKK	2014-05-02	Purchase	653,126	87,500			
DKK	2014-04-30	Valuation				59,087,862	7,916,071
DKK	2013-12-31	Valuation				59,056,340	7,916,071
DKK	2013-10-03	Purchase					
DKK	2013-10-03	Purchase					
DKK	2013-10-03	Purchase	(8,725,610)	(1,170,056)			
DKK	2013-08-31	Valuation				50,319,100	6,746,015
DKK	2013-06-26	Purchase	(49,574,674)	(6,650,000)			
DKK	2013-06-26	Purchase	(652,298)	(87,500)			
			(46,902,250)	(6,291,325)	1,528,731		

Table 20 presents data from a co-investment made by AP6. We have simplified the data and present the relevant parts for this thesis. Dates, type of event, NAV updates in fund currency and portfolio company currency for the private equity fund during the investment period is presented. The original fund currency is EUR and the original portfolio company currency is DKK.

Table 21: Fund 3, Hedging Transactions Made by AP6

Hedge Instrument Type	Trade Date	Maturity Date	Spot Rate	Forward Rate	Inverse Rate	Amount (Buy)	Currency (Buy)	Amount (Sell)	Currency (Sell)	Hedged Currency
Fx Forward	2014-02-24	2015-02-27	0.7821	0.8396	1.2054	69,573,277	SEK	57,718,000	DKK	DKK
Fx Spot	2015-02-26	2015-02-27	1.2606			57,718,000	DKK	72,759,311	SEK	DKK
Fx Forward	2015-02-26	2016-02-29	0.7916	0.7897	1.2664	73,091,189	SEK	57,718,000	DKK	DKK
Fx Spot	2016-02-26	2016-02-29	1.2535			9,718,000	DKK	12,181,513	SEK	DKK
Fx Spot	2016-02-28	2016-02-29	1.2535			48,000,000	DKK	60,168,000	SEK	DKK
Fx Forward	2016-02-26	2017-02-28	0.7964	0.7988	1.2518	60,086,400	SEK	48,000,000	DKK	DKK
Fx Forward	2016-10-20	2017-02-28	1.3039	1.3062	0.7656	18,000,000	DKK	23,510,700	SEK	DKK
Fx Spot	2017-02-27	2017-02-28	1.2853			30,000,000	DKK	38,559,000	SEK	DKK
Fx Forward	2017-02-27	2018-02-28	0.7805	0.7774	1.2863	38,590,200	SEK	30,000,000	DKK	DKK

Table 21 presents hedging transactions made by AP6 to minimise its exposure from investments in portfolio companies. We have simplified the data and present the relevant parts for this thesis. The table presents hedge instrument type, trade date and maturity date for the transactions. Furthermore, spot rates, forward rates and inverse forward rates are presented. At last, the currency amount bought and sold as well as the hedged currency are presented.

Figure 1: Investment Chain of Private Equity

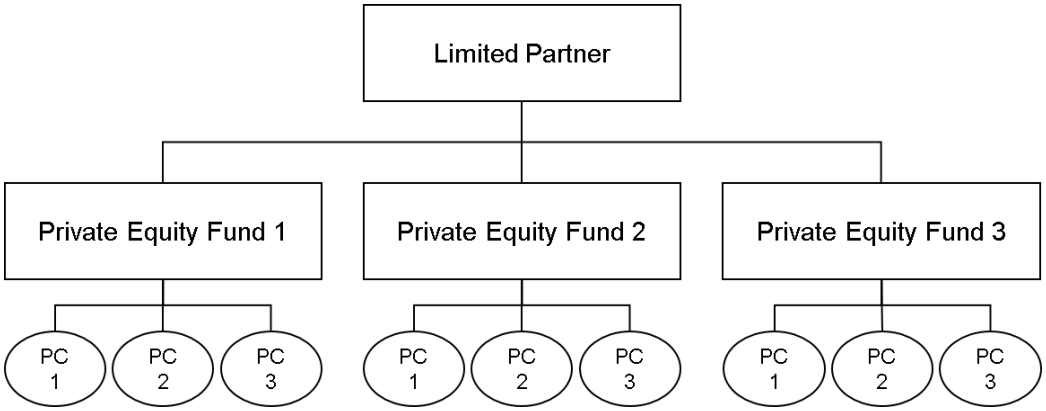

Figure 1 presents a simplified investment chain of private equity. First, the limited partner invests in private equity funds. Second, the private equity funds invest in portfolio companies. PC denoted portfolio company

Figure 2: Portfolio Company Currency Exposure

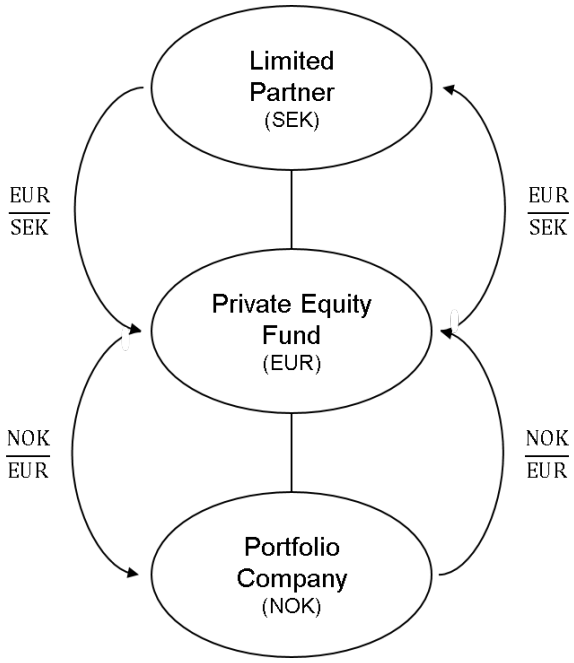

Figure 2 presents the currency exposure assuming the portfolio company exposure to be the true exposure. Hence, the fund currency is seen as a transaction currency according to the equation below

$$\frac{EUR}{SEK}\frac{NOK}{EUR} = \frac{NOK}{SEK}$$

Figure 3: Private Equity Fund Currency Exposure

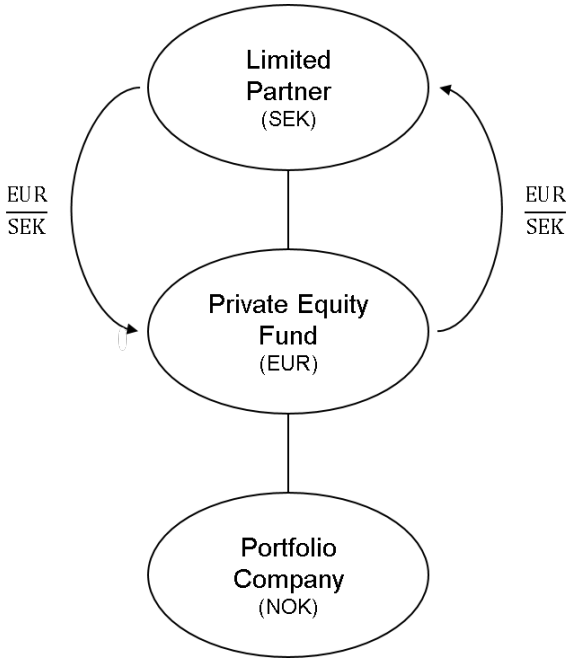

Figure 3 presents the currency exposure when assuming the private equity fund currency exposure to be the true exposure. Hence, the portfolio company currency is disregarded.

Figure 4: Investment Chain, Main Scenario

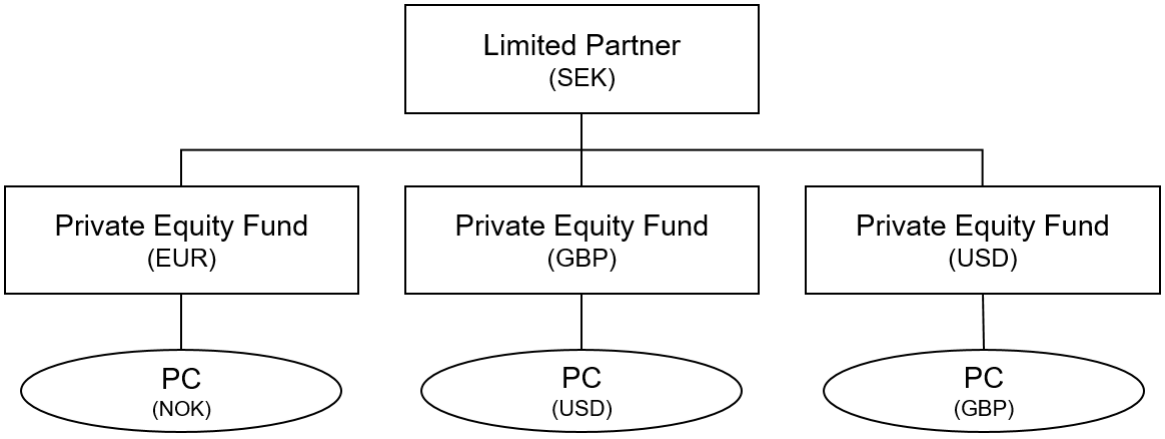

Figure 4 presents the investment chain for the main scenario. The limited partner currency is set to SEK, the private equity fund currencies are set to EUR, GBP and USD, to replicate multiple private equity funds. Portfolio company currencies are set to NOK, GBP and USD.

Figure 5: Investment Chain, Robustness Test Scenario 1

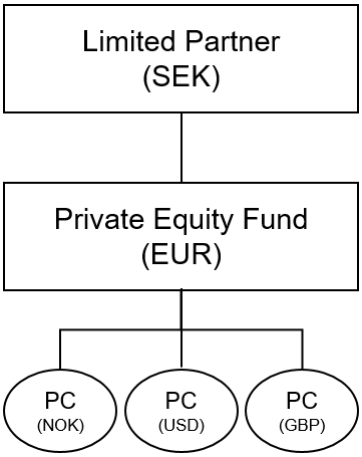

Figure 5 presents the investment chain for the first robustness test scenario. The limited partner currency is set to SEK. The private equity fund currency is set to EUR. This to replicate a private equity fund investing in several portfolio companies denoted in different currencies. Portfolio company currencies are set to NOK, USD and GBP.

Figure 6: Investment Chain, Robustness Test Scenario 2

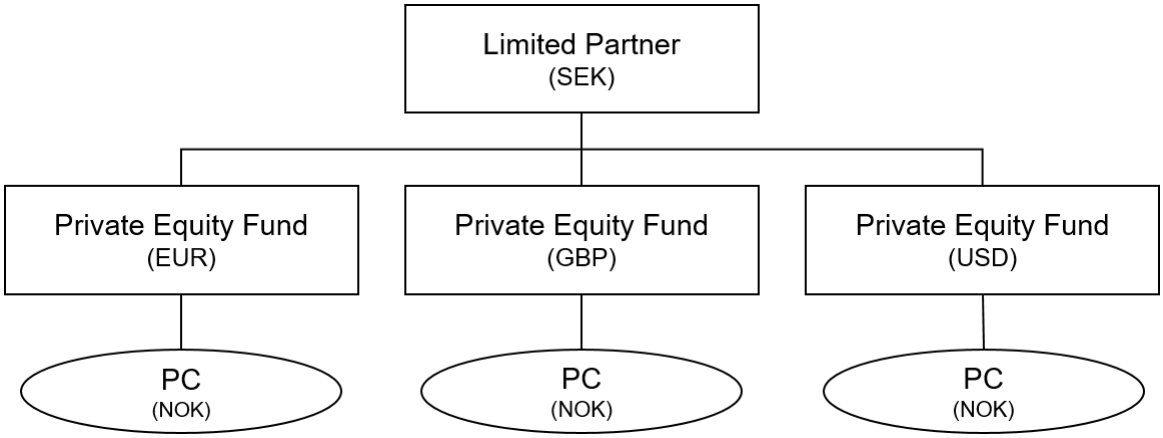

Figure 6 presents the investment chain for the second robustness test scenario. The limited partner currency is set to SEK. The private equity fund currencies are set to EUR, GBP and USD and all the PC currencies are set to NOK for all investments. This to replicate that the limited partner invests in several private equity funds denoted in different currencies, that all invest in portfolio companies denoted in the same currency.

Figure 7: Historical Foreign Exchange Rates, 2009-2012

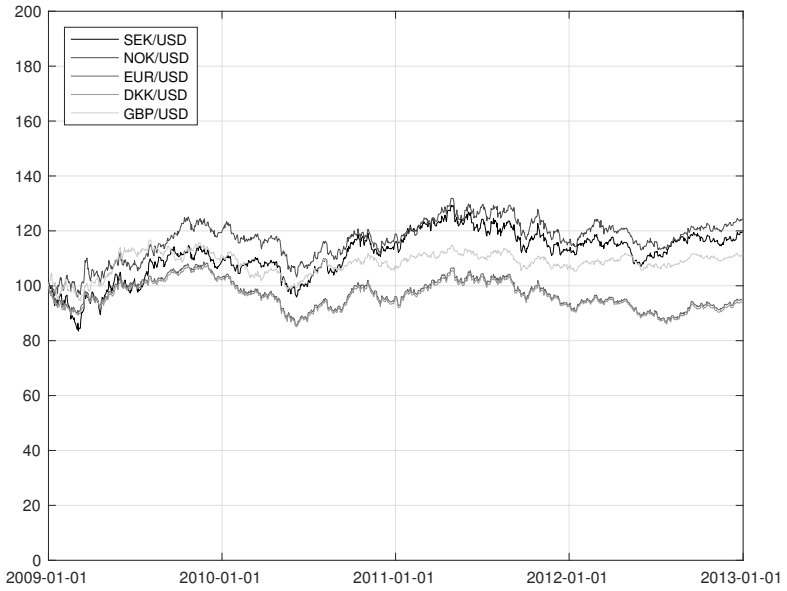

Figure 7 presents indexed historical foreign exchange rates relative to the USD. The foreign exchange rates are SEK/USD, NOK/USD, EUR/USD, DKK/USD and GBP/USD. January 1, 2009 = 100.

Figure 8: Historical Foreign Exchange Rates, 2013-2016

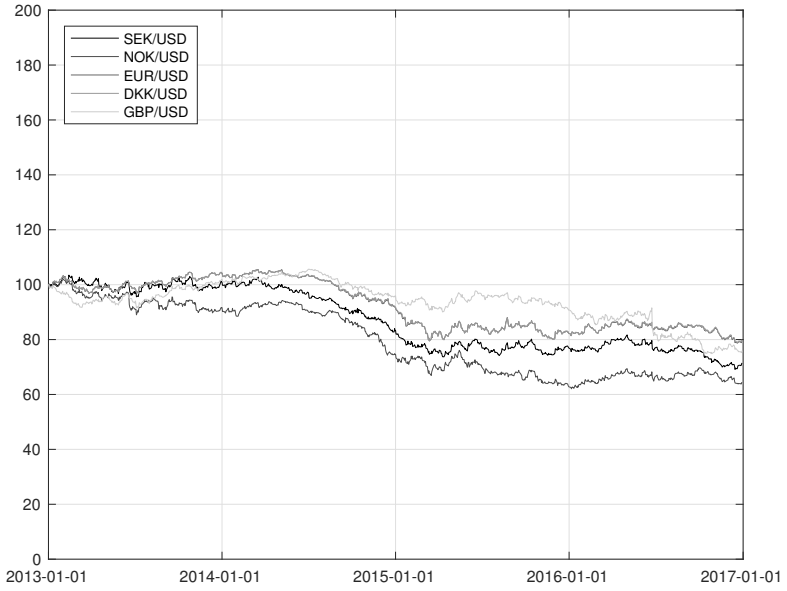

Figure 8 presents indexed historical foreign exchange rates relative to the USD. The foreign exchange rates are SEK/USD, NOK/USD, EUR/USD, DKK/USD and GBP/USD. January 1, 2013 = 100.

Figure 9: Simulated Foreign Exchange Rates (i), 2013-2016

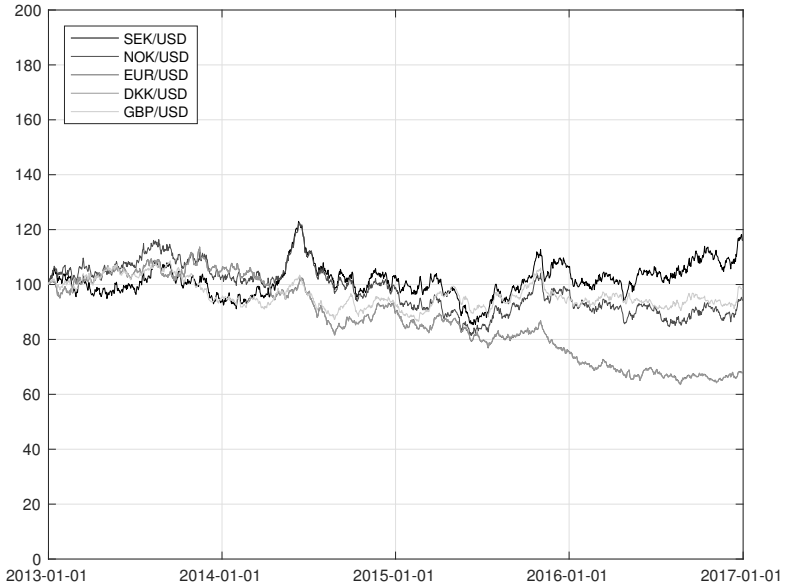

Figure 9 presents indexed simulated foreign exchange rates relative to the USD. The foreign exchange rates are SEK/USD, NOK/USD, EUR/USD, DKK/USD and GBP/USD. Simulation 1, January 1, 2013 = 100.

Figure 10: Simulated Foreign Exchange Rates (ii), 2013-2016

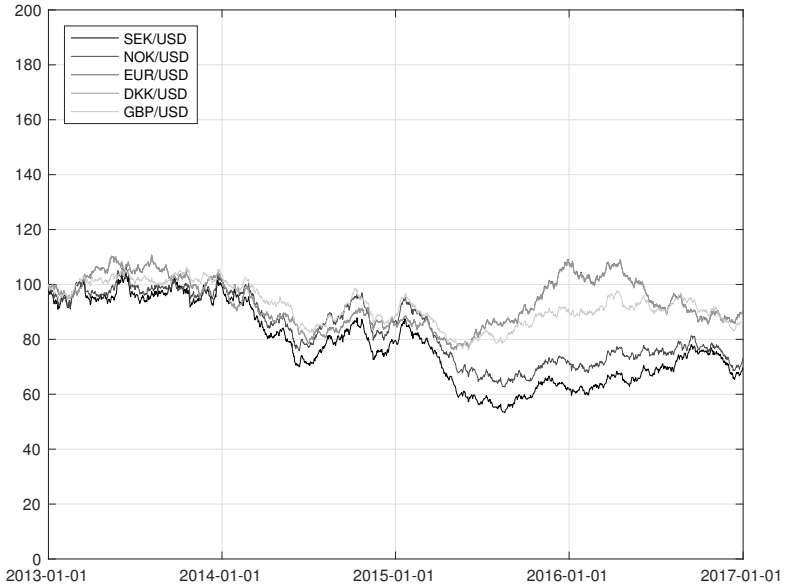

Figure 10 presents indexed simulated foreign exchange rates relative to the USD. The foreign exchange rates are SEK/USD, NOK/USD, EUR/USD, DKK/USD and GBP/USD. Simulation 2, January 1, 2013 = 100.

Figure 11: Simulated Foreign Exchange Rates (iii), 2013-2016

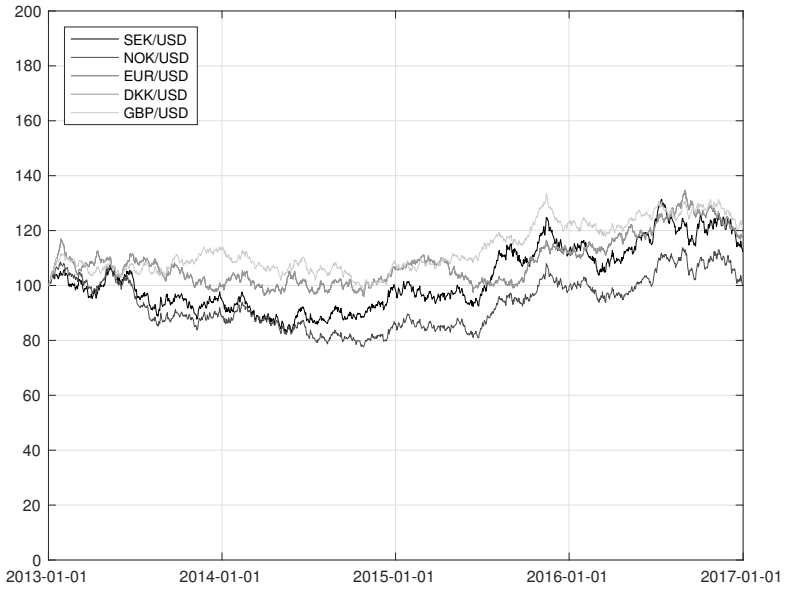

Figure 11 presents indexed simulated foreign exchange rates relative to the USD. The foreign exchange rates are SEK/USD, NOK/USD, EUR/USD, DKK/USD and GBP/USD. Simulation 3, January 1, 2013 = 100.

B Brownian Motion

A Brownian motion $(B_t, t \geq 0)$ defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$ is the real-valued continuous time extension of the discrete Brownian motion. Thus, it is required that

(i) $B_0 = 0$

(ii) for each $t \geq 1, B_t \sim N(0, t)$

(iii) (adaption) for every $t \geq 0$, B_t is $\mathcal{F}_t$-measurable

(iv) (continuity) $t \mapsto B_t$ is almost surely continuous

where, B is a Brownian motion, Ω is the sample space, $\mathcal{F}$ is a σ-algebra, $\mathbf{P}$ is a probability measure and t is the time. Hence, B_t is the Brownian motion at time t. $N(0, t)$ denotes the normal distribution with expected value 0 and variance t.