VIX ETPs as Portfolio Diversifiers

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Abstract

This paper studies whether the popularity of VIX ETPs can be explained by their suitability as portfolio diversifiers for retail investors having access to a typical set of ETFs. We first carry out an analysis from the perspective of investors with a quadratic utility function by employing the mean-variance spanning test and the mean-variance criterion. We then include skewness and kurtosis in the portfolio selection problem by applying the Lai (1991) polynomial goal programming model. We find that mean-variance investors seeking the global minimum-variance portfolio would have benefitted from adding volatility exposure to their portfolio, while the results are less promising for investors maximising the Sharpe Ratio. Investor preferences for higher moments, especially for skewness, are found to drive substantial allocations to volatility. The findings apply to different market conditions and therefore offer an alternative explanation for the undiminished investor interest in VIX ETPs. They are also robust for different investment intervals.

JEL classification: G11, G15, G23

Keywords: VIX, ETPs, Mean-Variance Spanning, Mean-Variance Criterion, Multi-Objective Portfolio Selection

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1 Introduction

During the global financial crisis (”GFC” hereafter), asset classes usually considered as portfolio diversifiers faced significant losses and correlations rose substantially. Thereafter, the increase in correlations was further fuelled by monetary policies (see for example Bahaji and Aberkane (2015)). This phenomenon intensified the quest for a robust diversifying asset class. During the course, volatility, long considered only a statistical metric for returns, emerged as a promising candidate. Potential diversification benefits of volatility-related assets stem from the well-documented fact that there exists a highly negative correlation with most other asset classes, especially equities. Further, this relationship is most pronounced during market downturns, and hence when needed the most (see for example Bekaert and Wu (2000)).

The introduction of the CBOE’s S&P 500 volatility index (”VIX” hereafter) in 1993 played an important role in the rise of volatility as an asset class. After changes to its calculation method in 2003, futures and options on the VIX were introduced and quickly embraced by investors. The development peaked with the introduction of exchange-traded products (”ETPs” hereafter) in the form of exchange-traded notes (”ETNs” hereafter) and exchange-traded funds (”ETFs” hereafter) based on VIX futures. What started out as a niche market became available to basically any investor, especially retail investors and institutional investors barred from trading in the derivatives market, as these products can be publicly accessed via the stock market. Eye-watering cumulative losses of 99.9% over less than eight years on some of these ETPs have not harmed investor interest, and new funds keep flowing into those products despite the massive amounts lost by investors.

Motivated by the enormous amounts lost by investors, this study aims to provide an explanation for the continued popularity of VIX ETPs by examining whether the statistical properties of volatility make them suitable portfolio diversifiers. This would add to existing arguments, namely that investors either lack sophistication (Whaley (2013)) or engage in highly speculative trades (Alexander and Korovilas (2012)). Previous studies on the diversification benefits of VIX related assets yield mixed results. While on one hand, VIX related assets are found to be valuable portfolio diversifiers during the GFC, the results are less promising for the years after the GFC. However, these studies are all conducted in the mean-variance space, and neglect the ”lottery-like” feature of volatility which appeases to investors with preferences for skewness (see for example Bahaji and Aberkane (2015)). Researchers have long argued that higher moments should be included in financial applications, and portfolio selection in particular. Nevertheless, academics only recently started to acknowledge the unique statistical properties of volatility by using different risk measures or considering skewness in their analysis. Yet, no ex-post portfolio selection problem has been solved to determine the suitability of VIX ETPs for investors with preferences for higher moments.

Expanding the literature on VIX ETPs, we conduct a comprehensive ex-post analysis on the suitability of long volatility exposure for retail investors, the main target group of the ETPs, that have access to a typical set of ETFs. We first revisit and update the existing findings in the mean-variance space with (a) a longer sample through backtesting VIX ETP returns and including more recent observations and (b) a broader set of benchmark assets. Building on that, we then want to fill the gap in the existing literature by employing the Lai (1991) polynomial goal programming model to solve the multi-objective portfolio selection problem for investors with preferences for higher moments.

This study is organised as follows: Section 2 introduces the VIX-related assets and their economics which motivates the research question at hand. Section 3 reviews the current literature on portfolio selection with VIX futures and VIX ETPs and the arguments in favour of considering higher moments. Section 4 provides an overview and analysis of the data and backtesting procedure used for the empirical approach, which is laid out in Section 5. Section 6 describes the results of the empirical analysis and Section 7 tests their robustness under different investment intervals. Section 8 concludes.

2 VIX, VIX Futures and VIX ETPs

To motivate our empirical analysis and shed light on the underlying economics of VIX-related assets, we provide an introduction to the VIX, VIX futures and VIX ETPs.

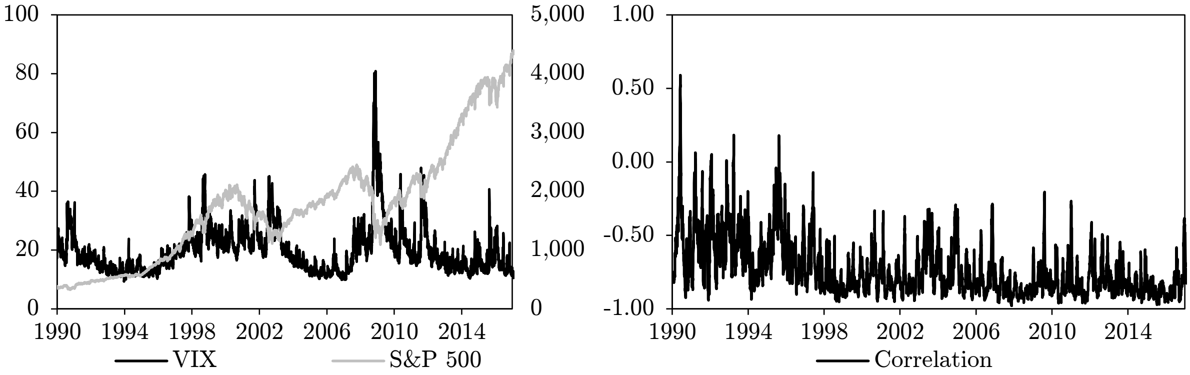
2.1 VIX

The VIX, often called "The Investor Fear Gauge" (Whaley 2000), was first introduced by the Chicago Board Options Exchange (CBOE) in 1993. At the time, it was a measure of the market's expectation of the 30-day annualised volatility of the S&P 100 quoted in percentage points and calculated based on the average of the Black-Scholes implied volatilities of eight near-the-money options on the S&P 100 with the two nearest maturities. Over time, the S&P 500 options market grew ever more popular and became the biggest and most actively traded options market overtaking the S&P 100. Moreover, while initially open interest for puts and calls was roughly equal, Whaley (2009) shows that puts in general, and out-of-the-money puts in particular, became increasingly popular due to portfolio insurance reasons. To account for these changes in the market and a bias introduced by the trading-day conversion used for the original VIX, the CBOE introduced a new method to calculate the VIX in 2003, while the original index continued as VXO. The new VIX is based on the market prices of options on the S&P 500, rather than implied volatilities of options on the S&P 100, and additionally includes out-of-the-money options using an appropriate day-counting convention. Carr and Wu (2006) show that the new VIX squared approximates the conditional risk-neutral expectation of the annualised return variance of the S&P 500 over the next 30 calendar days and can in theory be replicated through a static position in European options and a dynamic position in futures.

2.1.1 Statistical Properties of the VIX

Data on the VIX reaches back to 1990. Its level averages roughly 19 over that period with a strong mean-reversion characteristic. The VIX peaked during the heights of the GFC on 20 November 2008 with a closing level of more than 80. From Figure 1 we can see that the VIX has a strongly negative correlation with the S&P 500. The "leverage effect", as proposed by Black (1976), and time-varying risk premia (see for example Campbell and Hentschel (1992)) are two of the most prominent theories explaining this asymmetric relationship. Further, the negative correlation is more pronounced during times of large negative stock returns (see for example Schwert (1989) and Bekaert and Wu (2000)). Another distinct characteristic of implied volatility is the significantly positive skewness of its variations as noted for example by Bahaji and Aberkane (2015), Carr and Wu (2009) and Egloff et al. (2010). The correlation and skewness features are two of the main reasons for the attractiveness of the VIX and its derivatives as potential portfolio diversifiers.

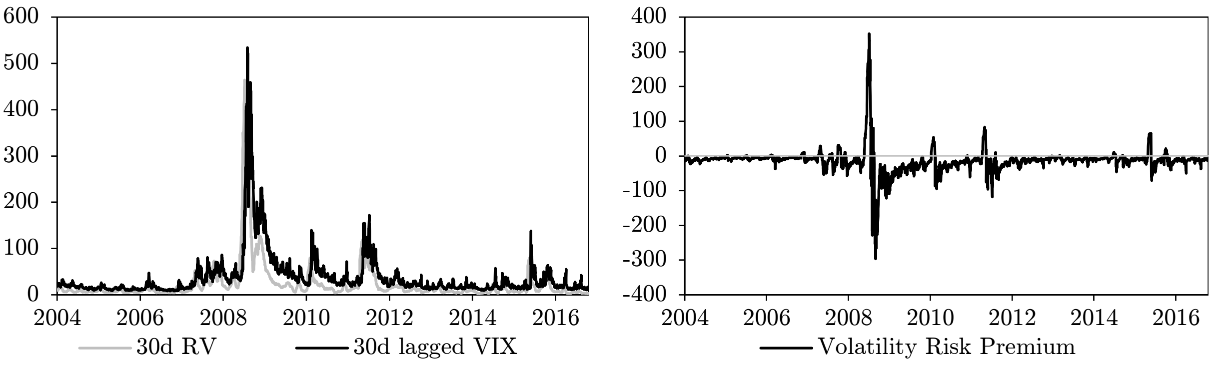
Figure 1. VIX and S&P 500 Performance & Correlation. The chart on the left shows the VIX level (left y-axis) and the S&P 500 Total Return Index level (right y-axis) from January 1990 to December 2016. The chart on the right shows the 22-trading-day rolling correlation between the returns of the VIX and the S&P 500. Data is sourced from Bloomberg.



2.1.2 Implied Volatility and Realised Volatility

Another important aspect of the VIX in the context of our research question is the well-established fact that the VIX typically overpredicts future realised volatility. This means investors pay a premium on options to insure against upward movements in volatility. The premium has been coined the "volatility risk premium" and is considered an independent risk factor as the returns cannot be explained by classical risk factors (see for example Bakshi and Kapadia (2003), Bollerslev et al. (2011), Carr and Wu (2009), Drechsler and Yaron (2011), and Eraker (2004)).[1]

Figure 2. Implied & realised Variance and Volatility Risk Premium. The chart on the left shows the realised variance of the S&P 500 over the past 30 calendar days multiplied by 10^4 and the 30 calendar days lagged squared VIX divided by 12 and quoted in percentage points as in Drechsler and Yaron (2011). The chart on the right shows the volatility risk premium as the difference between the two aforementioned. Realised variance is based on 5-minute subsamples obtained from the Oxford-Man Institute. The data ranges from April 2004 to December 2016.



From Figure 2 we can see that during times of low volatility, the volatility risk premium is large and negative. At the time of jumps in volatility (most notably at the heights of the GFC, the flash crash in 2010, the peak of the European debt crisis in 2011 and the Chinese stock market turbulences in 2015), the volatility risk premium

[1] This risk premium is subject to its own research area. Broadie et al. (2007), Eraker (2004) and Pan (2002) for example find that much of the risk premium stems from a jump risk premium rather than the volatility risk premium. For the sake of this paper, we make no further distinction and refer to the general risk premium as volatility risk premium.

turns positive and spikes briefly. Interestingly, the period leading up to the GFC displays a lower risk premium compared to the period following the GFC, indicating a change in the volatility regime.

2.2 VIX Futures

On 24 March 2004, not long after the new VIX methodology was introduced, the CBOE launched VIX futures on the CBOE Futures Exchange. VIX futures are contracts on forward 30-day implied volatilities as measured by the VIX. They are standard futures contracts settled in cash using a special opening quotation of the VIX based on opening prices of S&P 500 weekly options. It is important to note that, contrary to most other futures, there is no cost of carry relationship between the VIX and the VIX futures as the VIX itself is not an investable asset but a volatility forecast. Despite the changed calculation method, Asensio (2013) notes that it is costly and difficult to replicate the VIX due to the required volume of trades, the required trading frequency to rebalance and the high transaction costs for low-delta options. This results in VIX futures being the investors' principal way of achieving exposure to the VIX.

2.2.1 Descriptive Statistics of VIX Futures

Since introduction, open interest and daily volume of the VIX futures steadily increased until dropping during the heights of the GFC. Yet, Bahaji and Aberkane (2015) show that VIX futures proved to still be relatively liquid compared to for instance credit default swaps, at a time where investors arguable seek out hedging and diversification instruments the most. Shortly thereafter, the popularity of futures took off and open interest and volume rose significantly which is directly linked to the introduction of several VIX ETPs.

We can see from Figure 3 that VIX futures prices note on average higher than the VIX and increase with maturity. This displays the typical term structure of VIX futures called "contango", which means futures trade above the spot price and the futures curve is upward sloping. During times of high volatility, the GFC being a prime example, the term structure turns into "backwardation", meaning futures trade below the spot price and the futures curve is downward sloping.

Table 1 shows the VIX futures summary statistics. While the VIX lost an annualised 1.6% during the observed period, it is interesting to see that VIX futures lost dramatically more with the front-month future losing an annualised 57.6%. The extent of the losses decreases with the maturity of the futures. This also holds for the standard deviation of VIX futures owing mainly to the mean-reverting characteristic of volatility and the speed thereof. Similar to the VIX, VIX futures also exhibit the important feature of positively skewed returns, together with a high kurtosis.[2] However, both

[2] Note that with kurtosis we refer to excess-kurtosis in this paper.

skewness and kurtosis decrease with increasing maturity of the VIX futures. Moreover, the returns of VIX futures are highly correlated with the VIX as shown in Table 2, although the correlation is far from perfect. The correlation also decreases with increasing maturity of VIX futures.

Figure 3. VIX Futures Volume, Open Interest and Term Structure. The chart on the left shows the development of open interest and volume of the first seven VIX futures from the launch date until December 2016. The chart on the right shows the average term structure of the VIX futures and the spot VIX as well as examples for two different dates with the business days until settlement shown on the x-axis. The term structure over time is shown in Appendix A.I. We omit the eighth and ninth future due to low liquidity.

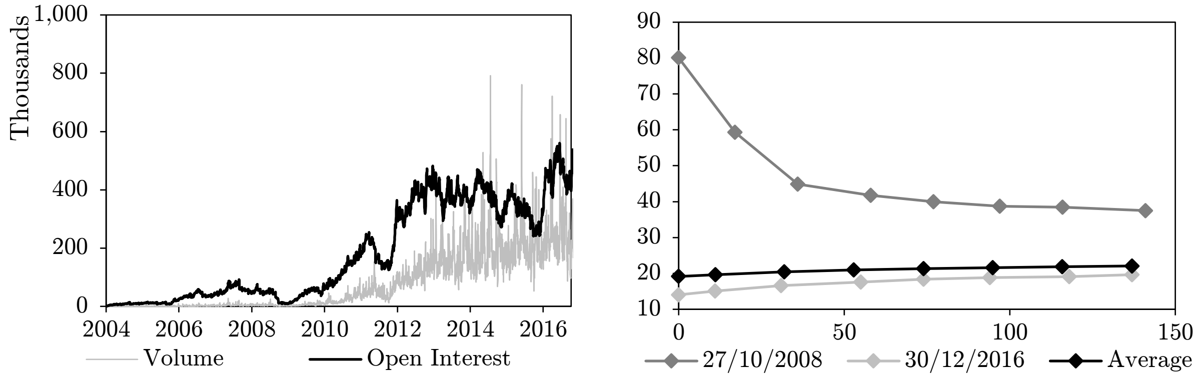


Table 1. VIX & VIX Futures Descriptive Statistics. The table shows the descriptive statistics for the daily returns of the VIX and the first seven VIX futures. We omit the eighth and ninth future due to low liquidity. We also report the annual geometric mean. Skewness and kurtosis are calculated based on standardised moments.

	VIX	VX1	VX2	VX3	VX4	VX5	VX6	VX7
Geo. Mean	-0.016	-0.576	-0.408	-0.273	-0.212	-0.178	-0.159	-0.208
Mean	0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001
Min	-0.296	-0.255	-0.159	-0.131	-0.108	-0.101	-0.093	-0.133
Max	0.642	0.358	0.255	0.195	0.159	0.139	0.121	0.114
StdDev	0.073	0.048	0.033	0.026	0.023	0.020	0.019	0.019
Skew	1.319	1.051	0.781	0.675	0.611	0.586	0.545	0.184
Kurt	6.582	5.397	4.801	4.323	4.079	4.146	4.094	5.254

Table 2. VIX & VIX Futures Correlation Matrix. The table shows the correlations of the daily returns of the VIX and the first seven VIX futures. We omit the eighth and ninth future due to low liquidity.

	VIX	VX1	VX2	VX3	VX4	VX5	VX6	VX7
VIX	1.00	0.89	0.85	0.83	0.81	0.79	0.76	0.71
VX1	0.89	1.00	0.94	0.91	0.89	0.86	0.83	0.78
VX2	0.85	0.94	1.00	0.97	0.95	0.92	0.89	0.84
VX3	0.83	0.91	0.97	1.00	0.98	0.96	0.93	0.87
VX4	0.81	0.89	0.95	0.98	1.00	0.98	0.95	0.89
VX5	0.79	0.86	0.92	0.96	0.98	1.00	0.97	0.91
VX6	0.76	0.83	0.89	0.93	0.95	0.97	1.00	0.92
VX7	0.71	0.78	0.84	0.87	0.89	0.91	0.92	1.00

2.2.2 Pricing of VIX Futures

The pricing of VIX futures is subject of ongoing academic research. According to the CBOE, the price of a VIX future can either be above, below or equal to the VIX depending on the market's expectation about the volatility in the 30-day forward period covered by the future compared to the 30-day spot period covered by the VIX. Nossman and Wilhelmsson (2009) tested the expectation hypothesis on VIX futures and found that when not adjusting for the volatility risk premium, the expectation hypothesis is rejected at a 5% significance level. When including the volatility risk premium, the expectation hypothesis cannot be rejected anymore. Johnson (2016) comes to the same conclusion and finds that a slope component describes nearly all information about the volatility risk premium in the VIX futures term structure. Further, Eraker and Wu (2016) show that the returns of VIX futures cannot be explained by classical risk factors. They use an equilibrium model that produces a sizeable volatility risk premium and an upward sloping futures curve. Others (Zhang and Zhu (2006), Lin (2007) and Zhu and Lian (2012)) study the model fit based on the class of Heston (1993) and Duffie et al. (2000) with the conclusion that models with more complex volatility characteristics (e.g. jumps) perform best.

Most importantly for this paper, Aıt-Sahalia et al. (2015), Dew-Becker et al. (2016), Eraker and Wu (2016) and Huskaj and Nossman (2013) find that volatility risk premia are downward sloping, meaning that the volatility risk premium is decreasing with the maturity of the volatility asset. Hence, volatility risk premia are largest for short-term VIX futures. To illustrate this further, we will look at the two most prominent VIX futures indices.

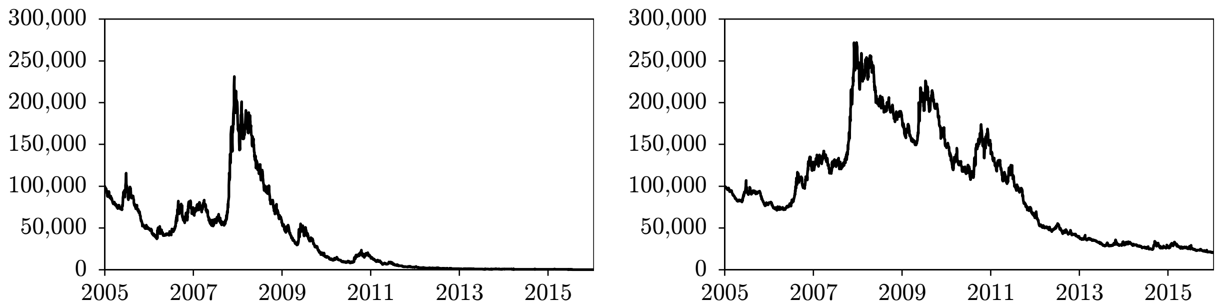
2.3 S&P 500 VIX Short-Term and Mid-Term Futures Indices

On 22 January 2009, Standard & Poor's introduced the two indices "S&P 500 VIX Short-Term Futures Index" ("Short-Term Index" hereafter) and "S&P 500 VIX Mid-Term Futures Index" ("Mid-Term Index" hereafter). The Short-Term Index measures the performance of continuously rolling a long position in the first month VIX future into the second month VIX future, maintaining a constant 30-day maturity. The Mid-Term Index measures the performance of continuously rolling a long position in the fourth month VIX future into the seventh month VIX future, while keeping constant positions in the fifth and sixth month futures, resulting in a constant 150-day maturity. The exact calculation for both indices are provided by Standard & Poor's and can be found in Appendix A.II.

2.3.1 Performance of Indices

The performance of the indices as displayed in Figure 4 is astonishing: The Short-Term Index has lost nearly all of its value with a cumulative loss of 99.9% since inception, which translates into a negative annualised return of 47.6%. While the Mid-Term Index cumulatively lost 81.5% of its value since inception, it performed better with a negative annualised return of 15.5% further lending support to the theory of downward sloping risk premia. Given the high correlation of VIX futures with the VIX, the spikes of the two indices are in line with the event-driven spikes in the VIX, most notably during the GFC when both funds posted significant gains. The Short-Term Index however quickly eroded gains from the GFC, while the losses have been less severe for the Mid-Term Index following the GFC. Both indices posted strong but short-lived gains following the flash crash in 2010 and the height of the European debt crisis in 2011. Afterwards, there have been periods of low volatility which resulted in significant deterioration in performance for both indices. On 24 June 2016, the day after the UK voted to leave the EU, both indices posted their biggest daily gain since inception with the Short-Term Index and the Medium-Term Index rising 32.7% and 13.4% respectively. Overall, both indices display a high volatility with an annualised daily standard deviation of 65.1% and 32.2% respectively.

Figure 4. Short-Term and Mid-Term Index Performance. The charts below show the performance of the Short-Term Index (left) and Mid-Term Index (right) from the base date 20 December 2005 until December 2016.



2.3.2 Return Decomposition

The extraordinary losses of the indices deserve a closer examination. Figure 5 shows the biasedness of the two indices as a predictor of the future VIX level, similar to the biasedness of the VIX as a predictor of future realised variance. The resulting spread for the Short-Term Index is on average lower than for the Mid-Term Index, owing to the fact that the term structure typically finds itself in contango. We can see that in more than two thirds of the time both constant maturity future prices overpredict the VIX. To get a grasp of the price impact of the volatility risk premium inherent in the futures, we follow Johnson (2016) and Whaley (2013) and derive the slope of the futures curve at the 30-day and 150-day constant maturity. We divide the price difference of the two futures straddling the respective constant maturity with the difference in business days to settlement of these futures. In Table 3, we can see that the 30-day constant maturity futures price is expected to drop by an average 0.038 (equivalent to 0.19%) per business day, while the 150-days constant maturity futures price is expected to drop by only 0.012 (equivalent to 0.06%) in comparison, explaining the significantly worse performance of the Short-Term Index. During market downturns, the slope turns negative as the term structure turns into backwardation as shown in Figure 6.

Figure 5. Constant-Maturity predicted VIX and realised VIX. The chart on the left (right) shows the realised VIX and 30 (150) calendar days lagged value of the 30 (150) day constant maturity VIX futures price ST (MT) from the base date 20 December 2005 until December 2016.

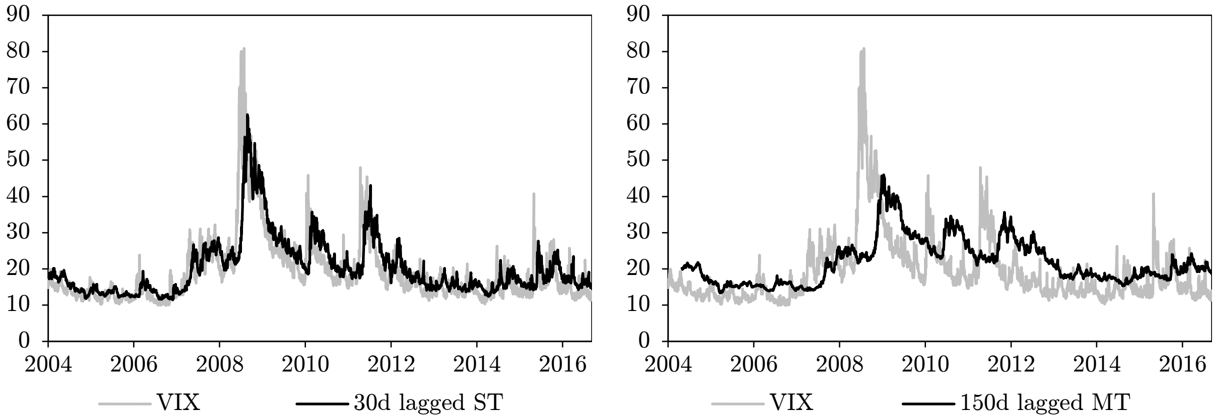


Figure 6. Short-Term and Mid-Term Index Slope. The chart below shows the slope of the futures curve at the point of the 30 (150) day constant maturity VIX future ST (MT) for the Short-Term (Mid-Term) Index calculated by dividing the price difference of the two futures straddling the respective constant maturity with the difference in business days to settlement of these two futures.

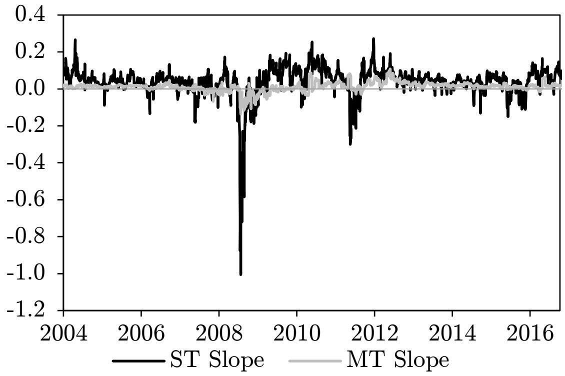


Table 3. Short-Term and Mid-Term Index Slope and Spread. The table shows the summary statistics for the slope as shown in Figure 6, and the spread calculated as the difference between the 30 (150) calendar days lagged value of the 30 (150) day constant-maturity VIX futures price and the realised VIX as shown in Figure 5.

	Slope		Spread	
	ST	MT	ST	MT
Mean	0.038	0.012	0.931	2.582
Min	-1.005	-0.136	-52.185	-57.451
Max	0.273	0.108	18.768	18.411
StdDev	0.084	0.024	5.723	8.841
% positive	84.1%	82.1%	70.6%	77.0%

We run OLS regressions to further shed light on the underlying factors of the performance of the two indices. Based on the CAPM and the Fama French 3-Factor Model, we find statistically significant negative alphas for the performance of the Short-Term Index while they are insignificant for the Mid-Term Index, which is consistent with the results in the literature. These findings can be explained by the theory of downward sloping volatility risk premia that finds comparably little evidence for risk premia in far-dated derivatives. Moreover, we observe that the negative correlation of VIX futures with equities translates into a substantial negative market beta for the two indices. The two indices also respond differently to a change in the VIX. When the VIX rises by one percent, the Short-Term Index rises by slightly less than half a percent, while the Mid-Term Index responds with about a fifth of a percent. This shows that the indices are unable to replicate the performance of the VIX.

Table 4. Short-Term and Mid-Term Index Regression Results. The table shows the OLS coefficients of regressing daily excess-returns of the Short-Term (ST) and Mid-Term (MT) Index on the market risk premium (CAPM) and the Fama-French size and value factors, as well as the results of regressing their daily total returns on the returns of the VIX and the slope as derived in Figure 6. Data has been sourced from the Kenneth French Library. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	ST	MT	ST	MT	ST	MT
Market	-2.449***	-1.204***	-2.501***	-1.226***		
Size			-0.215***	-0.057		
Value			0.413***	0.152***		
VIX					0.475***	0.211***
ST Slope					1.400***	
MT Slope						1.806***
Constant	-0.001**	-0.0002	-0.001**	-0.0002	0.0005	0.0002
Observations	3,231	3,231	3,231	3,231	3,231	3,231
Adj. R^2	0.577	0.560	0.582	0.563	0.788	0.626

2.4 VIX ETPs

On 29 January 2009, shortly after the introduction of the Short-Term Index and the Mid-Term Index, the first two VIX ETPs, namely the iPath S&P 500 VIX Short-Term Futures ETN (Ticker: VXX) and the iPath S&P 500 VIX Mid-Term Futures ETN (Ticker: VXZ), were launched by Barclays. They track the Short-Term Index and the Mid-Term Index respectively, and fill a gap that VIX futures themselves were not able to fill. Many institutional investors are restricted in their ability to buy futures and option contracts, while many retail investors are too small or lack the necessary sophistication to invest and trade in the derivatives market. The introduction of the ETPs finally allowed these investors to trade VIX related assets on the public stock market. Given the tremendous rise in popularity of the VXX and VXZ, additional VIX ETPs were launched within a short timeframe.

2.4.1 VIX ETP Universe

VIX ETPs usually come as either ETNs or ETFs. ETNs are notes that promise to pay the benchmark return over a certain maturity, which usually ranges between 10 and 40 years, without paying a coupon. There are some considerable differences between VIX ETNs and the underlying VIX futures themselves, that investors should be aware of. ETNs are callable at short notice at any time by the issuer and are not secured by the underlying assets themselves, bearing the credit risk of the issuer. ETFs on the other hand do not share the credit risk of the issuer as they represent claims on the underlying portfolio's assets. While the creation and redemption mechanism (a market maker delivers the shares of the underlying ETF basket and receives shares in the ETF from the issuer and vice versa) ensures efficient pricing for ETFs, it is a bit more complex for ETNs. The creation process requires an entirely new offering of notes. Further, notes can be redeemed by the holder of the ETN at the closing indicative value only one or more days after giving notice of the redemption. This could potentially give rise to mispricings. Moreover, ETN providers usually hedge their exposure to early redemption by trading VIX futures at the daily closing price. Alexander and Korovilas (2013) argue that this has led to large-scale front-running of issuers' hedging activity.

Table 5 shows an overview of the 10 largest VIX ETPs based on the Short-Term and Mid-Term indices, also including daily inverse levered ETPs. ETPs offering long exposure to the indices total to a combined market capitalisation of more than $2bn, while ETPs offering short exposure have a smaller combined market capitalisation of close to $1bn. The liquidity of some of the ETPs is noteworthy as the VXX and UVXY have for example been among the most actively traded securities in 2016.[3] The turnover time suggests that the ETPs based on the Short-Term Index are mainly used as highly speculative vehicles by traders, rather than by buy-and-hold investors. On the other hand, ETPs based on the Mid-Term Index show significantly longer turnover times, which lends support to the theory that investors might use them as portfolio diversifiers.

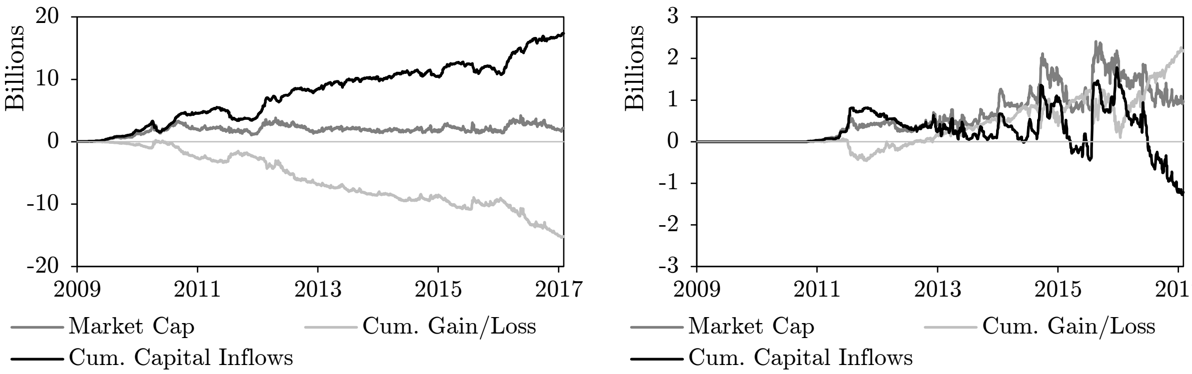
Table 5. VIX ETPs Overview. The table shows an overview of the ten largest ETPs on the Short-Term and Mid-Term Index. The sign of leverage shows the long or short-positioning on the underlying index of the ETP. Turnover time is the median of the daily market capitalisation divided by the daily dollar volume. Dynamic and strategy ETPs have been excluded from the scope of this paper.

Ticker	Inception Date	ETN/ETF	Index	Leverage	Expense Ratio	Market Cap ($m)	Average Volume (k)	Turnover Time (d)
VXX	29-Jan-09	ETN	ST	1x	0.89%	1,326.1	7,620.8	1.5
UVXY	04-Oct-11	ETF	ST	2x	0.95%	436.6	903.6	1.2
TVIX	29-Nov-10	ETN	ST	2x	1.65%	206.2	99.1	3.2
VIXY	04-Jan-11	ETF	ST	1x	0.85%	154.1	372.0	5.0
VIXM	04-Jan-11	ETF	MT	1x	0.85%	39.5	16.0	40.4
VXZ	29-Jan-09	ETN	MT	1x	0.89%	37.4	134.2	11.1
VIIX	29-Nov-10	ETN	ST	1x	0.89%	12.1	20.1	3.2
XIV	29-Nov-10	ETN	ST	-1x	1.35%	536.1	11,682.8	1.6
SVXY	04-Oct-11	ETF	ST	-1x	0.95%	336.9	1,753.8	1.7
ZIV	29-Nov-10	ETN	MT	-1x	1.35%	83.1	41.8	39.9

[3] According to Burger (2017), the VXX was the fifth and the UVXY was the tenth most traded security on the U.S. stock market in 2016 by daily average volume.

To further illustrate the popularity of VIX ETPs, we look at the development of market capitalisation, cumulative gains and losses, and cumulative capital inflows over time. Figure 7 shows the staggering amount of losses that the long-positioned VIX ETPs have accumulated totalling more than \$15bn since their inception in 2009. Nevertheless, this has not harmed the popularity of the ETPs in any way, as capital was continuously contributed keeping the market capitalisation at a relatively stable level of around \$2bn. Short-positioned VIX ETPs on the other hand show a significantly different picture. After initial losses, the funds started to rack up gains over time. Capital flows were much more volatile and jumped as soon as the ETPs suffered sudden and large losses.

Figure 7. VIX ETPs Market Capitalisation and Capital Flows. The charts below show the market capitalisation, cumulative gains or losses and cumulative capital inflows of the long-positioned (left chart) and short-positioned (right chart) VIX ETPs as in Table 5 since inception of the VXX and VXZ. Gains or losses are calculated based on the closing market capitalisation of the previous day and the returns of the current day. Capital inflows are calculated as the difference between the closing market capitalisation of the current day and the sum of the gain or loss on the current day and the closing market capitalisation of the previous day.



2.4.2 VXX and VXZ

In this paper, we want to focus on the VXX and VXZ for our empirical analysis as they offer the longest historical available data and are arguably the most popular long-positioned ETPs with respect to the Short-Term and Mid-Term Index.

As mentioned, the structure of the creation and redemption mechanism of ETNs might lead to tracking errors. Hence, before we proceed further in our analysis, we regress the returns of the two ETNs on their respective underlying index to see whether earlier findings on the underlying indices can also be assumed to hold for the ETNs. From Table 6 we can see that the VXX tracks the ST Index basically perfectly. This can be attributed to the earlier mentioned extreme liquidity that the ETN showcases. The VXZ on the other hand trails the VXX in tracking performance, which could be a result of the comparably lower liquidity, paired with the execution costs and timing issues of the creation and redemption arbitrage as noted by Whaley (2013). There is a considerable amount of noise when looking at the mean absolute return deviation. However, the mean return deviations suggest that both ETNs closely track the indices.

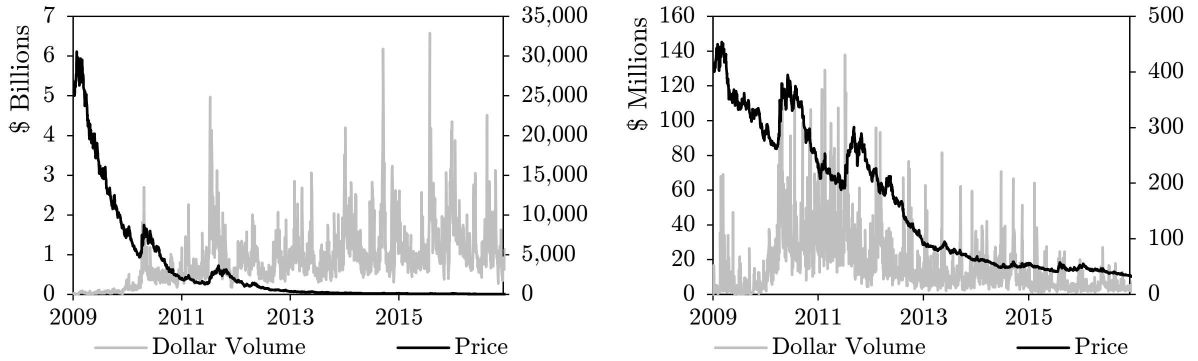
The negative intercept for both ETNs can be explained by the 0.89% annual management fee collected by the ETN issuer.

Table 6. VXX and VXZ Tracking Error Regression Results. The table shows the OLS coefficients of regressing the returns of the VXX and VXZ on the Short-Term and Mid-Term Index respectively following Whaley (2013). The mean deviation in returns as well as the mean of the absolute deviation in returns is also reported. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ
ST Return	1.000***	
MT Return		0.897***
Constant	-0.00004***	-0.0002
Observations	2,016	2,016
Adj. R^2	1.000	0.872
Mean Deviation	0.00016	0.00005
Mean Absolute Deviation	0.00776	0.00494

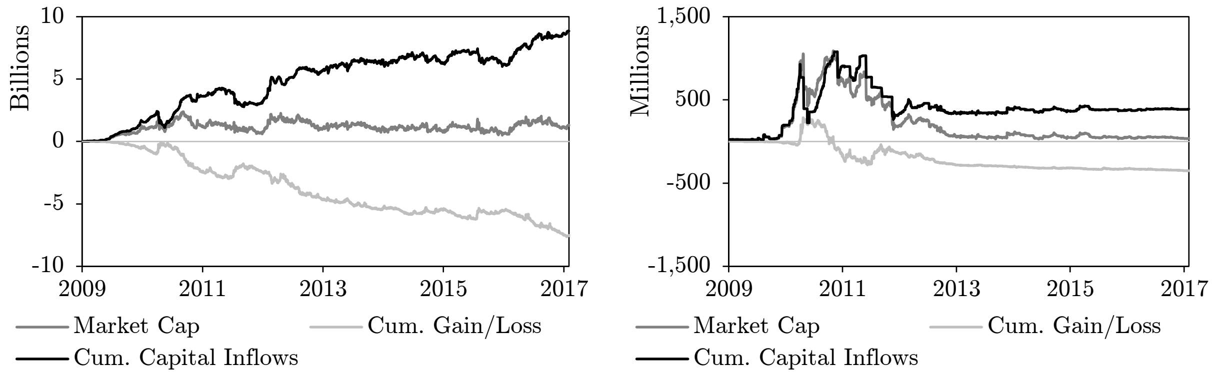
Figure 8 shows the performance of the two ETNs. In line with the earlier findings for the underlying indices, the VXX has rapidly lost enormous amounts after inception. The decline for the VXZ has been less pronounced. It is interesting to look at the development of dollar volume over time. While initially trading activity was rather subdued, it surged for both ETNs roughly a year after inception. Over time, the dollar volume traded steadily increased for the VXX with several significant spikes (e.g. on 25 August 2015, the peak of the Chinese stock market turbulences, nearly $7bn in the VXX was traded which compares to a total market capitalisation of roughly $3bn for all long-positioned VIX ETPs at the time). Clearly, the VXX is one of the main benefactors of the popularity of the VIX ETPs. The VXZ showed similar tendencies in the beginning, albeit on a much smaller scale. However, the daily dollar volume reversed course and declined steadily since 2011.

Figure 8. VXX and VXZ Performance and Volume. The charts below show the daily dollar volume (left y-axis) and price performance (right y-axis) since inception for the VXX on the left and the VXZ on the right. Dollar volume was chosen due to the large negative returns and hence limited information share volume conveys.



These points are further substantiated when looking at Figure 9. The VXX has enjoyed a steady inflow of large amounts of capital over the years keeping the market capitalisation consistently hovering above $1bn, despite racking up staggering losses of more than $7bn since inception. The VXZ on the contrary, after enjoying large capital inflows in 2010 and 2011, seems to have fallen out of favour with investors and traders in recent years. After significant gains in mid 2010, capital was withdrawn abruptly and re-contributed quickly only to see it being withdrawn stepwise from 2011 to 2012. Ever since, activity has been subdued with losses accumulating and the market capitalisation declining steadily with little movement in capital.

Figure 9. VXX and VXZ Market Capitalisation and Capital Flows. The charts below show the market capitalization, cumulative gains or losses and cumulative capital inflows of the VXX (left chart) and VXZ (right chart) since inception. Gains or losses are calculated based on the closing market capitalisation of the previous day and the returns of the current day. Capital inflows are calculated as the difference between the closing market capitalisation of the current day and the sum of the gain or loss on the current day and the closing market capitalisation of the previous day.



The popularity of the VXX in recent years has some important consequences. Bollen et al. (2016) for instance find that the VXX now leads VIX futures in intraday price discovery, which in turn also lead the VIX (and equivalently the S&P 500 options market). They conclude that VIX ETPs and VIX futures have now become the primary market for hedging volatility risk in a timely manner. Whaley (2013) suggests that the popularity of the VIX ETPs can be explained by the lack of sophistication of market participants. The VXX indeed has an institutional ownership of only around 40% while more than two thirds of the VXZ are owned by institutional investors.[4]

[4] Numbers are based on 13-F filings compiled by Nasdaq as of 29 March 2017.

3 Literature Review

In the following section, we summarise the findings in the literature with regards to the suitability of VIX futures and VIX ETPs as portfolio diversifiers. Researchers have almost exclusively restricted their analysis to the mean-variance space. We therefore also review the literature on the inclusion of higher moments in finance applications and more specifically portfolio selection problems.

3.1 Portfolio Selection with VIX Futures and VIX ETPs

One of the first to examine VIX futures are Moran and Dash (2007), who consider whether a portfolio of the S&P 500, spot VIX and VIX futures could have outperformed the S&P 500 on a risk-adjusted basis from 2004 to 2007. The paper finds that a small allocation to the spot VIX and VIX futures could have improved the risk-adjusted performance as measured by the Sharpe Ratio. In a more comprehensive study, Chen et al. (2011) examine whether the spot VIX, VIX-squared portfolios and VIX futures can enlarge the investment opportunity set of an investor. The authors conduct a mean-variance spanning test with VIX futures and several equity portfolios with data ranging from April 2004 to April 2008. They find that investors could have significantly expanded their efficient frontier. However, this stems mainly from the change in the global minimum-variance portfolio, while there are no significant improvements to the tangency portfolio.

Szado (2009) studies the diversification effects of adding long volatility exposure through VIX futures to static portfolios consisting of equities, bonds and alternative asset classes during the GFC. The examined period ranges from March 2006 to December 2008. The paper compares the performance of different portfolios with a 2.5% and a 10% allocation to near-month VIX futures, which is rolled over to the next month future whenever a future is about to expire. The author concludes that long volatility exposure might result in negative returns in the long term, but does provide considerable protection by increasing returns and decreasing standard deviation in times of significant market declines. In a similar setting, Warren (2012) concludes that monthly rolled third-month VIX futures would have improved the Sharpe Ratio of a typical pension fund portfolio consisting of equities, fixed income and real estate property over the period from May 2004 to March 2010.

Gantenbein and Rehrauer (2013) find that volatility exposure trough the VXX and VXZ would have improved the Sharpe Ratio of an S&P 500 portfolio from 2006 to 2011 using backtested returns. Post-crisis (2009-2011), they show that the positive effect only holds up for the VXZ. In a similar study, Bordonado et al. (2016) examine a portfolio of the S&P 500 as well as the VXX and the VXZ using backtested return series. During the sample period from June 2006 to April 2014, the authors find that a small allocation to the VXZ would have slightly increased the Sharpe Ratio of the portfolio. However, the paper argues that the performance improvement is only caused by the historically unique GFC, and consequently finds that during the period from February 2009 to April 2014 neither the VXX nor the VXZ have a positive effect on the risk-adjusted performance of the portfolio, due to the presence of a large volatility risk premium that outweighs the diversification benefits.

The first to depart from the mean-variance framework and to take into account the statistical properties of the VIX and its derivatives in a portfolio context are Briere et al. (2010), who employ a modified value-at-risk approach based on the Cornish-Fisher expansion. In the first ex-ante analysis with VIX futures, the study concludes that long VIX futures exposure in an S&P 500 portfolio significantly reduces the modified value-at-risk, both in-sample (1990-1999) and out-of-sample (1999-2008). They backtest VIX futures returns using a linear relationship with the VIX, an approach criticised by other researchers. Based on actual return data ranging from 2004 to 2011, Alexander and Korovilas (2012) try to incorporate skewness into their ex-ante analysis by using an approximation of the generalised Sharpe Ratio as an optimisation and performance criterion. The authors find larger allocations to VIX futures when considering skewness, but also find that these portfolios rarely outperform equity-only portfolios out-of-sample, even when measuring performance with the generalised Sharpe Ratio. In fact, outside of periods of high volatility, adding VIX futures is shown to be profitable only when investors have personal views based on precise VIX futures forecasts. This leads the authors to conclude that the success of volatility-diversified equity portfolios rests entirely on short-term speculation rather than long-term diversification. In contrast, Bahaji and Aberkane (2015) show that also uninformed investors with skewness preference can benefit from including VIX futures in a portfolio consisting of equities and bonds, although the optimal exposure entails majorly short-positions in VIX futures.

3.2 Portfolio Selection with Higher Moments

In the finance literature, there is a controversy regarding the role of higher moments in financial applications, and portfolio selection in particular. Many researchers (see for example Arditti (1967, 1971), Samuelson (1970), Rubinstein (1973) and Tobin (1958)) argue that higher moments can only be disregarded if asset returns are normally distributed, or if the utility function of investors is quadratic, which is equivalent to the notion that higher moments do not influence investors' preferences.

There is a large literature that provides strong evidence that individual asset and portfolio returns are not normally distributed (see for example Fama (1965), Ibbotson (1975), Arditti (1971), Simkowitz and Beedles (1978), and Singleton and Wingender (1968)). Further, the works of Arditti and Levy (1975), Kraus and Litzenberger (1976), and Jondeau and Rockinger (2003) have established that higher moments are an important factor in explaining security returns.

While Tsiang (1972, 1974) is supportive of using the quadratic approximation for the utility function under the assumption that the risk taken by the investors is small relative to their wealth, others (for example Bierwag (1974), Borch (1974) and Levy (1974)) remain sceptical. Hanoch and Levy (1970) note that the quadratic utility function implies increasing absolute risk aversion, which stands in conflict with the generally accepted assumption of decreasing absolute risk aversion. Levy and Sarnat (1972) also demonstrate that the quadratic utility function is only appropriate in case of relatively low returns. In a defence of quadratic utility, Levy and Markowitz (1979) and Markowitz (1991) suggest that at least for relatively small deviations in rates of return, the mean-variance approach approximately maximises expected utility even if distributions are not normal.

In the works of Samuelson (1970), Kraus and Litzenberger (1976), Stephens and Proffitt (1991), and Harvey and Siddique (1999, 2000) it is shown that higher moments are relevant to the investor's portfolio selection decision. Scott and Horvath (1980) show that investors exhibit a positive preference for odd moments and a negative preference for even moments, which is consistent with the notion of decreasing absolute risk aversion. Prakash et al. (2003) interpret the positive preference for skewness as a preference for decreasing the probability of large negative returns. Dittmar (2002) provides the intuition behind investors' negative preference for kurtosis, by noting that investors are averse to extreme outcomes. More specifically, Fang and Lai (1997) and Harvey and Siddique (1999, 2000) show that investors forego returns in pursuit of higher systematic skewness, while they receive higher returns in exchange for bearing systematic variance and kurtosis risk.

Even though it was demonstrated early on in the literature that portfolio selection should take into account higher moments, only few approaches were developed to construct portfolios accordingly. These attempts suffered from a number of defects as discussed in Chunhachinda et al. (1997), and an appropriate solution was not established until Lai (1991). He applied polynomial goal programming ("PGP" hereafter) as introduced by Tayi and Leonard (1988) to solve the multi-objective portfolio selection with skewness. Chunhachinda et al. (1997), Prakash et al. (2003), Sun and Yan (2003) and Davies et al. (2006) have subsequently applied and augmented the PGP approach, and show that incorporating investor preferences for skewness and kurtosis significantly changes the optimal portfolio allocation.

4 Data

In this section, we introduce the assets used in our empirical analyses. Further, the backtesting procedure and descriptive statistics are presented and discussed with a particular focus on the VIX ETNs.

4.1 Asset Overview and Sources

For our empirical work on the incorporation of VXX and VXZ in portfolios, we introduce a set of ETFs that represents a range of the most common and liquid assets, easily accessible to retail investors via the public stock market. Therefore, asset classes such as hedge funds and private equity are excluded from our analysis.

Based on the global multi-asset market portfolio as determined by Doeswijk et al. (2014), we introduce the asset classes of equities, bonds and real estate. Equities are represented by the iShares Core S&P 500 ETF ("S&P 500" hereafter), tracking the S&P 500 Index, and the Vanguard FTSE All-World ex-US ETF ("AW"), tracking the FTSE All-World ex US Index. Bonds are further split into treasuries, investment grade corporate bonds and high yield corporate bonds, represented by the iShares 1-3 Year Treasury Bond ETF ("UST") tracking the ICE U.S. Treasury 1-3 Year Bond Index, the iShares Global Corp Bond UCITS ETF ("IGCB") tracking the Bloomberg Barclays Global Aggregate Corporate Index and the S&P 500 High Yield Corporate Bond Total Return Index ("HYCB"), adjusted for an industry standard ETF expense fee. Real estate exposure is incorporated by the SPDR Dow Jones Global Real Estate ETF ("RE"), tracking the Dow Jones Global Select Real Estate Securities Index. Further, following the arguments made in the literature about diversification benefits of commodities (see for example Gorton (2006) and Jensen et al. (2000)), we incorporate commodity exposure through the iShares S&P GSCI Commodity-Indexed Trust ETF ("GSCI"), tracking the S&P GSCI Index. As precious metals constitute only around 2% of the GSCI, we include the SPDR Gold Shares ("GLD"), a physically-backed gold ETF, to capture the safe haven characteristics of gold as evidenced in the literature (see for example Baur and Dermott (2010)).

We source our data for VIX futures, which we need to backtest return series for the VXX and VXZ as explained in Section 4.2, from the CBOE. Data for the Gold Fixing Price is retrieved from the Federal Reserve Bank of St. Louis, while we use Bloomberg for all other indices and ETPs. To insure the quality and correctness of our data, we crosscheck it whenever possible with DataStream as well as the respective ETP provider. The observation period ranges from 1 April 2004, the first full month after the introduction of the VIX futures, to 31 December 2016. All returns are total returns in order to insure comparability among different asset classes. Furthermore, all return data is quoted in US Dollar.

4.2 VXX and VXZ Backtesting

The VXX and VXZ were launched in 2009 while the base date of the underlying Short-Term Index and Mid-Term Index goes back to 2005. VIX futures were already introduced in 2004. Therefore, in order to be able to carry out our empirical analysis over the full observation period, we have to backtest VXX and VXZ return series using VIX futures and indices data.

For the period from 1 April 2004 to 20 December 2005, we manually calculate the Short-Term and Mid-Term Index based on the index formulas provided by Standard & Poor's (see Appendix A.II). As some VIX futures did not exist or are not priced due to low liquidity during the first years, we interpolate missing VIX futures prices following the method described in Appendix A.III. Using the obtained indices data, we calculate the VXX and VXZ return series by subtracting the daily accrued annual investor fee using the method provided by Barclays (see Appendix A.IV).[5]

In order to separate actual return data from backtested return data in our empirical analysis, we introduce two different periods, namely the "main period" and the "backtested period". Consequently, the main period ranges from 29 January 2009 to 31 December 2016, and the backtested period ranges from 1 April 2004 to 28 January 2009. We also examine a third period, the "full period", which is a combination of the main period and backtested period and therefore spans the entire observation period.

4.3 Descriptive Statistics

For our main analyses, we use quarterly returns to reflect an investment interval that we deem appropriate for retail investors with a buy-and-hold ETF portfolio. Any larger investment interval would result in an insufficient number of observations to carry out a meaningful empirical analysis. As we test our results of the main analyses for robustness using different investment intervals, we follow Prakash et al. (2003) and scale the returns to a simple annual rate

$$R_t = \frac{price\ index_t - price\ index_{t-1}}{price\ index_{t-1}} \times \frac{365}{days\ in\ interval} \tag{1}$$

to ensure comparability.

In Table 7 we can see the descriptive statistics for the main period, which includes the last months of the GFC. Nevertheless, returns are overall shaped by a strong bull market. During the main period, the VXX and the VXZ lost on average 66.6% and 27.1% respectively. We also observe that not only do the VXX and the VXZ exhibit more extreme means, but they also have a substantially higher standard deviation than all other assets. In the equities space, the S&P 500 outperformed the AW with respect

[5] We employ the same investor fee calculation method to the set of ETFs representing non-volatility assets in case the respective ETF was launched after 1 April 2004.

to both mean and standard deviation. The rates of return of the bond ETFs, namely the UST, the IGCB and the HYCB are in line with their economic risk profile. The highest average return is achieved by the RE, which however also showcases a high level of standard deviation. The GSCI was the only non-volatility asset, which has a negative mean during the main period. In terms of risk-adjusted performance as measured by the Sharpe Ratio, the HYCB, the UST and the S&P 500 perform best, while the two volatility ETNs, due to the highly negative means and significantly higher standard deviation, perform worst.

The VXX and the VXZ together with the bond ETFs represent the assets with the highest values for skewness. The VXX has the highest skewness of all assets with a value of 1.445, while the VXZ trails the UST and IGCB with a skewness of 0.596. It is important to note that the volatility assets have a high skewness in particular when compared to equities. The S&P 500 has the lowest skewness value, while the AW has, at least for equities, a relatively high skewness during the main period.[6]

The VXX has one of the higher kurtoses, while the VXZ has a relatively low kurtosis, similar to the equities' kurtoses. The bond ETFs are among the assets with a higher kurtosis. In general, it is observable that most assets have positive or only slightly negative kurtosis, with the exception of the RE and the GLD.

We can further see that the VXX and the VXZ as well as the bond ETFs are not normally distributed according to the Anderson-Darling Test, while the null hypothesis of normal distribution cannot be rejected for the remaining assets at any significance level. However, we have to note that the statistical power of normality tests is generally low for small sample sizes, with the Anderson-Darling Test requiring at least 100 observations according to Razali and Wah (2011). Hence, we refer to the results for monthly investment intervals over the full period using 150 observations (see Appendix A.V), with the conclusion that with the exception of the GSCI and the GLD, all assets are not normally distributed.

Table 7. Descriptive Statistics Quarterly Return Main Period. The table shows descriptive statistics for quarterly asset returns scaled by a factor of four over the main period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test (see Stephens (1974) for a comparison of selected normality tests). ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	31	31	31	31	31	31	31	31	31	31
Mean	-0.666	-0.271	0.147	0.098	0.009	0.064	0.116	0.149	-0.052	0.050
StdDev	0.917	0.445	0.194	0.295	0.012	0.130	0.143	0.276	0.433	0.295
Skew	1.355	0.596	-0.486	0.446	1.224	1.186	0.593	0.043	-0.476	0.149
Kurt	1.445	0.041	-0.018	0.067	1.593	1.693	0.427	-0.766	-0.064	-0.784
SR	n.a.	n.a.	0.756	0.332	0.781	0.493	0.813	0.539	n.a.	0.171
AD	1.285***	0.702*	0.382	0.416	0.979**	0.924**	0.834**	0.159	0.335	0.245

[6] For more details on usually negatively skewed equity returns, we refer the reader to Christie (1982) and Campbell and Hentschel (1992).

Table 8 shows the descriptive statistics for the backtested period, which includes the majority of the GFC. As a consequence, we notice that most assets have a much lower mean, or in case of the S&P 500 even a negative one, when compared to the main period. On the other hand, the volatility ETNs have significantly higher rates of return, as volatility spikes during the crisis as shown in Section 2.1.1. However, the mean return of the VXX is still negative, while the VXZ produced the second largest average rate of return of all assets amounting to 9.4%. Moreover, the UST and the GLD have a considerably higher rate of return, as they are considered safe havens during crises. The GSCI, despite the large drop in oil prices in 2008, has a higher average return as well, given that the period captures much of the commodities super cycle of the 21st century. Overall, the UST displays by far the best risk-adjusted performance as measured by the Sharpe Ratio, with the GLD and VXZ trailing.

During the backtested period, we observe a very different skewness pattern. The returns of the VXZ are more positively skewed in the backtested period than in the main period, while the VXX's returns are less positively skewed. The returns of the remaining assets display large negative skewnesses, a direct consequence of the large negative returns during the crisis. Simultaneously, the returns of the UST remain positively skewed, given its safe haven nature. Interestingly, during the backtested period, no clear pattern of change for the kurtosis emerges when compared to the main period. Given the low number of observations, the normality tests mostly fail to reject the null-hypotheses, as they lack any statistical power as mentioned earlier.

Table 8. Descriptive Statistics Quarterly Return Backtested Period. The table shows descriptive statistics for quarterly asset returns scaled by a factor of four over the backtested period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	19	19	19	19	19	19	19	19	19	19
Mean	-0.159	0.094	-0.015	0.051	0.039	0.019	0.015	0.033	0.043	0.160
StdDev	1.112	0.650	0.248	0.373	0.043	0.136	0.149	0.487	0.729	0.297
Skew	1.232	0.974	-0.898	-0.831	0.276	-1.233	-0.939	-1.100	-0.716	-0.105
Kurt	1.190	0.308	-0.038	0.207	-1.193	2.525	0.306	0.964	-0.243	-0.666
SR	n.a.	0.144	n.a.	0.136	0.918	0.143	0.102	0.067	0.059	0.539
AD	0.734**	0.566	0.561	0.451	0.370	0.640*	0.733**	0.485	0.421	0.250

Over the full period, the mean and standard deviation of the assets not surprisingly resemble a combination of the main and the backtested period. It is interesting to note that the volatility ETNs show a significantly higher skewness, especially when compared to the other assets. The kurtosis is also showing more extreme values for the VXX and to a lesser extent for the VXZ. This behaviour results from the relatively constant negative returns during calm market periods, whereas outliers appear on the right tail of the distribution when volatility spikes. As a consequence of a higher number of observations, we also note a higher number of non-normally distributed assets.

Table 9. Descriptive Statistics Quarterly Return Full Period. The table shows descriptive statistics for quarterly asset returns scaled by a factor of four over the full period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	50	50	50	50	50	50	50	50	50	50
Mean	-0.387	-0.108	0.083	0.074	0.020	0.047	0.083	0.100	-0.022	0.095
Min	-0.493	-0.252	-0.174	-0.247	-0.007	-0.101	-0.082	-0.336	-0.399	-0.215
Max	1.859	0.553	0.168	0.173	0.029	0.128	0.166	0.245	0.280	0.190
StdDev	1.508	0.716	0.286	0.368	0.032	0.143	0.165	0.434	0.559	0.297
Skew	3.020	1.482	-0.681	-0.713	1.325	0.352	0.443	-0.834	-0.669	-0.494
Kurt	12.274	2.107	0.598	0.488	1.241	2.374	2.366	1.282	0.556	0.598
SR	n.a.	n.a.	0.289	0.200	0.638	0.331	0.505	0.231	n.a.	0.319
AD	3.196***	1.959***	0.906**	0.449	2.532***	0.925**	0.723*	0.741*	0.573	0.315

Table 10 shows the correlations during the main period. The strong negative correlation with most other assets is certainly one of the most notable features of the volatility ETNs. As expected, it is most pronounced with the equity ETFs, and surprisingly the GSCI, which has been classified as a portfolio diversifier itself in the literature. The positive correlation of volatility ETNs with the UST comes at no surprise, given the flight to treasuries during turbulent times. However, the volatility ETNs have nearly no correlation with the GLD, argued by many to be another safe haven. Given the lack of prolonged market turbulences, the lack of correlation can be attributed to the different return behaviour during quiet market times.

Table 10. Correlation Matrix Main Period. The table shows the correlation matrix for quarterly asset returns scaled by a factor of four over the main period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.838	-0.690	-0.620	0.180	-0.320	-0.467	-0.480	-0.588	-0.041
VXZ	0.838	1.000	-0.590	-0.522	0.453	-0.269	-0.254	-0.325	-0.507	0.081
S&P500	-0.690	-0.590	1.000	0.771	-0.358	0.413	0.563	0.702	0.706	0.223
AW	-0.620	-0.522	0.771	1.000	-0.196	0.685	0.762	0.670	0.696	0.339
UST	0.180	0.453	-0.358	-0.196	1.000	0.203	0.186	0.146	-0.306	0.344
IGCB	-0.320	-0.269	0.413	0.685	0.203	1.000	0.771	0.697	0.453	0.584
HYCB	-0.467	-0.254	0.563	0.762	0.186	0.771	1.000	0.703	0.503	0.427
RE	-0.480	-0.325	0.702	0.670	0.146	0.697	0.703	1.000	0.364	0.531
GSCI	-0.588	-0.507	0.706	0.696	-0.306	0.453	0.503	0.364	1.000	0.299
GLD	-0.041	0.081	0.223	0.339	0.344	0.584	0.427	0.531	0.299	1.000

Table A - 7 in Appendix A.VI shows the correlations during the backtested period. It is notable that the negative correlation of the volatility ETNs with the S&P 500, AW, HYCB and RE is stronger compared to the main period. This is in line with Bekaert and Wu (2000), who find that the negative correlation of volatility and equities is stronger in market declines. The positive correlation between the volatility ETNs and the UST is stronger as well. It is also noticeable that the correlation between the

S&P 500 and the AW, as well as between the S&P 500 and corporate bonds is higher during the backtested period. This portrays the phenomenon of increasing correlations, sometimes called "contagion", among assets during market downturns (see for example Bae et al. (2003) and Chiang et al. (2007)).

Table A - 8 shows the correlations between the different asset classes during the full period. Generally, the correlations are more similar to the backtested period than to the main period. Correlations between the volatility ETNs and equities interestingly are even more negative in the full period than in the backtested period. On the other hand, the correlations between the volatility ETNs and the non-equity assets are less pronounced than during the backtested period.

5 Methodology

In this section, we explain the empirical methods we use to determine whether adding volatility exposure can be beneficial for a retail investor. We introduce methods applicable to both the mean-variance and higher moments space.

5.1 Mean-Variance Analysis

5.1.1 Mean-Variance Spanning

In a first step, we want to empirically determine whether the VXX and the VXZ can significantly enlarge the investment opportunity set of a mean-variance investor holding a set of benchmark assets as introduced in Section 4. While Chen et al. (2011) found significant diversification benefits from adding VIX futures to equity portfolios during the period from April 2004 to April 2008, it remains to be seen whether we can reach a similar conclusion for the backtested period, given our broader set of benchmark assets. Further, we examine whether the benefits of adding the VXX and the VXZ to a portfolio hold up in the main period, as the beneficial negative correlations of the two ETNs are less pronounced than during the backtested period.

As we take a more practical approach applicable to retail investors, we assume for all our analyses that investors do not have access to the risk-free lending or borrowing rate. This implies that we only consider raw returns, and no excess-returns. No access to the risk-free rate also means that investors are not only interested in the tangency portfolio, but in the entire mean-variance efficient frontier. Thus, we analyse whether the frontier of the portfolio consisting of benchmark assets and the VXX and VXZ is identical with the frontier of the benchmark assets.

This will be examined via the mean-variance spanning test as first introduced by Huberman and Kandel (1987). The approach statistically tests the effects of adding a set of N new assets to an existing set of K benchmark assets. If the new frontier of the combined portfolio of N and K assets overlaps with the frontier of the portfolio of K assets, the outcome is known as "spanning", meaning that investors gain no benefit from adding the N assets to their portfolio. If the frontier of the combined portfolio is larger than the frontier of the portfolio of K assets, investors gain diversification benefits from adding the N assets. Further, to assess the economic magnitude of the diversification benefits, we will look at the change in Sharpe Ratio of the tangency portfolio and the change in standard deviation of the global minimum-variance portfolio as suggested by Bekaert and Urias (1996).

Let R_{2t} be the N-vector of raw returns on the N assets, whereas R_{1t} denotes the K-vector of raw returns on the K benchmark assets. We then estimate the following model

$$R_{2t} = \alpha + \beta R_{1t} + \epsilon_t, \quad t = 1,2, \dots, T. \tag{2}$$

As in Huberman and Kandel (1987), the null hypothesis of spanning is then

$$H_0: \alpha = 0_N, \; \delta = 1_N - \beta 1_K = 0_N. \tag{3}$$

We further denote the expected returns and the covariance matrix of the combined N and K assets as

$$\mu = E(R_t) = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \tag{4}$$

$$V = Var(R_t) = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}. \tag{5}$$

Following Kan and Zhou (2012), we then calculate the statistics of the Wald, Likelihood Ratio and Lagrange Multiplier tests as

$$W \; = T\,(\lambda_1 + \lambda_2) \sim \chi_2^2, \tag{6}$$

$$LR \; = T\,((\ln(1+\lambda_1) + \ln(1+\lambda_2)) \sim \chi_2^2, \tag{7}$$

$$LM = T\left(\frac{\lambda_1}{1+\lambda_1} + \frac{\lambda_2}{1+\lambda_2}\right) \sim \chi_2^2, \tag{8}$$

that follow an asymptotic chi-squared distribution with two degrees of freedom, and where λ_1 and λ_2 are the eigenvalues of the matrix $\hat{H}\hat{G}^{-1}$ with

$$\hat{H} = \begin{bmatrix} \hat{\alpha}'\,\hat{\Sigma}^{-1}\,\hat{\alpha} & \hat{\alpha}'\,\hat{\Sigma}^{-1}\,\hat{\delta} \\ \hat{\alpha}'\,\hat{\Sigma}^{-1}\,\hat{\delta} & \hat{\delta}'\,\hat{\Sigma}^{-1}\,\hat{\delta} \end{bmatrix}, \tag{9}$$

$$\hat{G} = \begin{bmatrix} 1 + \hat{\mu}_1{}'\hat{V}_{11}^{-1}\hat{\mu}_1 & \hat{\mu}_1{}'\hat{V}_{11}^{-1}1_K \\ \hat{\mu}_1{}'\hat{V}_{11}^{-1}1_K & 1_K'\hat{V}_{11}^{-1}1_K \end{bmatrix}, \tag{10}$$

where $\hat{\Sigma}$ stands for residual variance. For further details we refer to Kan and Zhou (2012).

To facilitate the discussion of the results, we also look at the tangency portfolio and global minimum-variance portfolio given by

$$w_T = \frac{V^{-1}\mu}{1'_{N+K}V^{-1}\mu}, \tag{11}$$

$$w_{GMV} = \frac{V^{-1}\mu}{1'_{N+K}V^{-1}1_{N+K}}. \tag{12}$$

5.1.2 Constrained Mean-Variance Efficient Frontier

The preceding mean-variance spanning analysis applies to unconstrained cases, and therefore allows short-selling by investors. However, as a big part of the investors in the VXX and VXZ are retail investors, who may lack sophistication or the required means, we impose a common short-selling constraint. As mentioned in the previous section, we assume that retail investors do not have access to the risk-free rate. Therefore, we construct the constrained mean-variance efficient frontier, determine whether the VXX and the VXZ are part of its composition, and derive the tangency portfolio as well as the global minimum-variance portfolio.

As summarised earlier, the empirical literature has so far not considered a portfolio optimisation that includes VIX related assets and a broader set of benchmark assets. Previous analyses were mostly restricted to equities represented by the S&P 500, with the conclusion that long positions in VIX futures provide increases in the risk-adjusted performance when looking at periods that include the GFC. However, the effect does not uphold since inception of the VXX and the VXZ in 2009. We expect to confirm the latter finding, and challenge the first by using a broader set of assets for the portfolio selection problem. Further, compared to the mean-variance spanning outcome, we would expect relatively lower utilisation of volatility overall given that short-selling could account for a large part of volatility exposure, as the highly negative return performance can outweigh diversification benefits.

To ensure portfolio diversity and avoid excessive exposure to a single asset, we further introduce an upper bound constraint on the individual portfolio weights of 35% for our analyses.[7] The constrained efficient frontier as in Markowitz (1952) can therefore be determined by

$$\min_w \quad \mathrm{w}'\, V\, \mathrm{w} - q\, \mu'\, w, \tag{13}$$

$$s.t. \quad \textstyle\sum_i w_i = 1, \tag{14}$$

$$w_i \geq 0, \tag{15}$$

$$w_i \leq 0.35, \tag{16}$$

$$q \geq 0, \tag{17}$$

[7] For further discussion on portfolio constraints, we refer the reader to Jagannathan and Ma (2003).

where w denotes the portfolio weights of the individual assets i, and q denotes a risk tolerance factor, which we vary to move along the efficient frontier, with $q = 0$ representing the global minimum-variance portfolio.

5.2 Polynomial Goal Programming

Given the non-normality of the underlying asset returns and the arguments made in the literature for including higher moments in portfolio selection decisions, we want to expand the portfolio optimisation approach to the higher moments space. As seen in Section 4.3, the VXZ and especially the VXX show highly positive skewness and in the case of the VXZ moderate kurtosis. Hence, we would expect that investors with a preference for (predominantly) higher moments embrace the VXX and VXZ to a greater extent than investors allocating assets under a quadratic utility function. As in our previous analyses, we assume that investors do not have access to the risk-free rate. To test whether higher moments can explain the popularity of the VIX ETNs and justify their inclusion in portfolios, we introduce the Lai (1991) multi-objective portfolio selection model, which has subsequently been employed and augmented by Chunhachinda et al. (1997), Prakash et al. (2003), Sun and Yan (2003) and Davies et al. (2006).

Keeping our previous notations and following Jondeau and Rockinger (2006), we denote the first four portfolio moments as

$$m_1 = \mathrm{w}'\,\mu, \tag{18}$$

$$m_2 = \mathrm{w}'\,V\,\mathrm{w}, \tag{19}$$

$$m_3 = \mathrm{w}'\,M_3\,(\mathrm{w} \otimes \mathrm{w}), \tag{20}$$

$$m_4 = \mathrm{w}'\,M_4\,(\mathrm{w} \otimes \mathrm{w} \otimes \mathrm{w}), \tag{21}$$

where M_3 is the (n, n^2) co-skewness matrix, M_4 is the (n, n^3) co-kurtosis matrix and $\otimes$ is the Kronecker product.

We then compute the portfolio skewness and kurtosis based on standardised moments as

$$S = \frac{m_3}{{m_2}^{3/2}} = \frac{\mathrm{w}'\,M_3\,(\mathrm{w} \otimes \mathrm{w})}{(\mathrm{w}'\,V\,\mathrm{w})^{3/2}}, \tag{22}$$

$$K = \frac{m_4}{{m_2}^{2}} - 3 = \frac{\mathrm{w}'\,M_4\,(\mathrm{w} \otimes \mathrm{w} \otimes \mathrm{w})}{(\mathrm{w}'\,V\,\mathrm{w})^{2}} - 3. \tag{23}$$

Recalling investors' positive preference for odd moments and negative preference for even moments, the constrained multi-objective problem can then be formulated as

$$\max_w \quad Z_1 = \mathrm{w}'\,\mu, \tag{24}$$

$$\min_w \quad Z_2 = \mathrm{w}'\,V\,\mathrm{w}, \tag{25}$$

$$\max_w \quad Z_3 = \frac{\mathrm{w}'\,M_3\,(\mathrm{w} \otimes \mathrm{w})}{(\mathrm{w}'\,V\,\mathrm{w})^{3/2}}, \tag{26}$$

$$\min_w \quad Z_4 = \frac{\mathrm{w}'\,M_4\,(\mathrm{w} \otimes \mathrm{w} \otimes \mathrm{w})}{(\mathrm{w}'\,V\,\mathrm{w})^{2}} - 3, \tag{27}$$

$$s.t. \quad \textstyle\sum_i w_i = 1, \tag{28}$$

$$w_i \geq 0, \tag{29}$$

$$w_i \leq 0.35. \tag{30}$$

As it is unlikely that any single solution of (24) to (27) satisfies all four objectives (Z_1, Z_2, Z_3 and Z_4) simultaneously, Lai (1991) proposes a two-step procedure. In a first step, a set of optimal solutions for each individual optimisation problem (24) to (27) is derived independent of investor preferences. As the portfolio selection only depends on the relative percentage invested in the assets, we can rescale and restrict the weights to the unit variance space. The first step then solves the following problem

$$\max_w \quad Z_1 = \mathrm{w}'\,\mu, \tag{31}$$

$$\max_w \quad Z_3 = \mathrm{w}'\,M_3\,(\mathrm{w} \otimes \mathrm{w}), \tag{32}$$

$$\min_w \quad Z_4 = \mathrm{w}'\,M_4\,(\mathrm{w} \otimes \mathrm{w} \otimes \mathrm{w}) - 3, \tag{33}$$

$$s.t. \quad \mathrm{w}'\,V\,\mathrm{w} = 1, \tag{34}$$

$$w_i \geq 0, \tag{35}$$

$$w_i \leq 0.35 \textstyle\sum_i w_i. \tag{36}$$

Based on the obtained set of results, investors then select the most suitable solution according to their own set of preferences for objectives. Lai (1991) incorporates these preferences in a PGP model, where the objective function does not contain choice variables, but rather minimises the sum of deviational variables.

The resulting PGP model is given by

$$\min_w \quad Z = (1+d_1)^{p_1} + (1+d_3)^{p_3} + (1+d_4)^{p_4}, \tag{37}$$

$$s.t. \quad \mathrm{w}'\,\mu + d_1 = Z_1^*, \tag{38}$$

$$\mathrm{w}'\,M_3\,(\mathrm{w} \otimes \mathrm{w}) + d_3 = Z_3^*, \tag{39}$$

$$\mathrm{w}'\,M_4\,(\mathrm{w} \otimes \mathrm{w} \otimes \mathrm{w}) - 3 - d_4 = Z_4^*, \tag{40}$$

$$\mathrm{w}'\,V\,\mathrm{w} = 1, \tag{41}$$

$$w_i \geq 0, \tag{42}$$

$$w_i \leq 0.35 \textstyle\sum_i w_i, \tag{43}$$

$$d_1, d_3, d_4 \geq 0, \tag{44}$$

where p_1, p_3 and p_4 denote the investors' preference values for mean, skewness and kurtosis. Z_1^* is the expected return of the mean-variance efficient portfolio obtained from solving (31), Z_3^* is the skewness value of the variance-skewness efficient portfolio obtained from solving (32) and Z_4^* is the kurtosis value of the variance-kurtosis efficient portfolio obtained from solving (33). Consequently, d_1, d_3 and d_4 are the values of deviation of the current solution from Z_1^*, Z_3^* and Z_4^* respectively. The specification of the objective function (37) follows Davies et al. (2006) and ensures that it is monotonically increasing in d_1, d_3 and d_4.

In order to derive meaningful results, the choice of preference values is crucial, especially with regards to the kurtosis preference as shown by Davies et al. (2006). Accordingly, we employ the values 0 (no preference), 1 (medium preference) and 2 (high preference) for both expected return preference p_1 and skewness preference p_3, while we use the values 0, 0.5 and 0.75 for kurtosis preference p_4. The ($p_1 = 1$, $p_3 = 0$, $p_4 = 0$) portfolio represents the mean-variance efficient portfolio, and should hence match the tangency portfolio from our mean-variance analysis. The (0, 1, 0) and (0, 0, 0.5) portfolios depict the variance-skewness and variance-kurtosis efficient portfolios respectively. Additional combinations of preference values will be looked at to determine the trade-offs between the different moments for investors with more balanced preferences.

We employ the evolutionary algorithm for non-linearly constrained global optimisation problems developed by Runarsson and Yao (2005). Although the algorithm has escape heuristics for local optima, no convergence proof exists yet. Therefore, we create one million random portfolios and sort them according to the respective objective function to be optimised. The best portfolios for each objective are then rescaled to the unit variance space, and fed to the algorithm alongside an equal-weighted portfolio. Finally, the obtained solution is rescaled back to a fully invested portfolio.

6 Results

This section lays out the results of our empirical approach as introduced in Section 5 and discusses them critically.

6.1 Mean-Variance Analysis

6.1.1 Mean-Variance Spanning

Table 11 reports the results of the main-variance spanning test for the three examined periods. The asymptotic tests reject the null hypothesis of spanning at a 1% significance level for the Likelihood Ratio and Wald test, and a 5% significance level for the Lagrange Multiplier test in the main period. In the backtested period, all three tests reject the null hypothesis at a 1% significance level. For the full period, the Wald test is statistically significant at a 1% level, while the Lagrange Multiplier and Likelihood Ratio tests are significant at a 5% level.[8] Hence, we can conclude that the VXX and VXZ expand the investors' mean-variance efficient frontier, and therefore their investment opportunity set. This confirms the results of Chen et al. (2011) on VIX futures, despite the addition of several more assets and the examination of periods of low volatility with less pronounced negative correlations.

Table 11. Mean-Variance Spanning Test Results. The table shows the results for the mean-variance spanning test of N-assets (VXX, VXZ) with K-assets (S&P500, AW, UST, IGCB, HYCB, RE, GSCI, GLD) for all three periods. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	Main Period		**Backtested Period**		**Full Period**	
	Value	P-Value	Value	P-Value	Value	P-Value
Lagrange Multiplier	12.778	0.012**	17.267	0.002***	12.279	0.015**
Likelihood Ratio	14.720	0.005***	25.404	0.000***	13.306	0.010**
Wald	17.136	0.002***	40.950	0.000***	14.468	0.006***

In Table 12, we can see that the VXX and the VXZ help to improve the Sharpe Ratio of the tangency portfolio and reduce the standard deviation of the global minimum-variance portfolio in the main period. The VXX and the VXZ are represented through a small long position and a comparably larger short position respectively. The results look similar for the full period. During the backtested period, the size of the positions grows given the performance of the VIX ETNs during the GFC, which results in significant Sharpe Ratio improvements. Moreover, the allocation to the VXZ is bigger than the allocation to the VXX for any given scenario. This could be explained by the magnitude of the latter's negative mean and standard deviation. A very interesting observation is that in most periods, for both the tangency portfolios as well as for the global minimum-variance portfolios, it would have been optimal to go long the VXX and to short the VXZ. This holds up for all periods and portfolios, except for

[8] For further details on the ranking of the three test statistics we refer the reader to Kan and Zhou (2012).

the tangency portfolio during the backtested period, where the positive returns of the VXZ result in a long position while simultaneously shorting the VXX. This dynamic suggests that the diversification benefits of the VXX outweigh the prospects of shorting the significantly more negative returns, when compared to the VXZ.

Briefly looking at the other assets, we can see that all portfolios show excessive loadings on the UST, emphasising the importance of the weight constraints introduced for all following analyses. Curiously, the Sharpe Ratio of the tangency portfolio in the full period is lower than the Sharpe Ratio in the main and backtested periods. A potential explanation could be that asset returns show a relatively monotonic behaviour during the main and backtested period, as the periods are shaped either by a bull or a bear market respectively, while they show a dispersed behaviour during the full period, which covers a full market cycle.

Table 12. Mean-Variance Spanning Portfolios. The table shows the tangency (T) and global minimum-variance (GMV) portfolios for the mean-variance spanning test of N-assets (VXX, VXZ) with K-benchmark-assets (S&P500, AW, UST, IGCB, HYCB, RE, GSCI, GLD) for all three periods. SR depicts the Sharpe Ratio. Δ denotes the change of the respective metric of the N+K portfolio with regards to the respective metric of the K benchmark portfolio.

	Main Period		Backtested Period		Full Period	
	T	GMV	T	GMV	T	GMV
VXX	0.6%	1.0%	-11.6%	0.7%	0.2%	1.2%
VXZ	-2.2%	-1.9%	15.4%	-9.9%	-3.3%	-3.7%
S&P500	12.0%	4.4%	-112.9%	13.6%	10.3%	5.5%
AW	-1.7%	1.0%	76.3%	0.0%	-5.5%	-1.4%
UST	92.1%	99.9%	180.1%	112.8%	92.8%	98.5%
IGCB	3.0%	-1.0%	-35.1%	12.2%	-6.9%	-2.9%
HYCB	4.1%	-1.3%	-3.9%	-14.9%	16.9%	7.7%
RE	-6.0%	-1.8%	10.7%	-9.9%	-5.6%	-3.2%
GSCI	-2.6%	0.2%	0.9%	0.1%	-1.6%	0.3%
GLD	0.6%	-0.6%	-19.8%	-4.7%	2.7%	-2.1%
Mean	0.026	0.010	0.124	0.021	0.034	0.023
StdDev	0.011	0.007	0.058	0.024	0.029	0.023
SR	2.388	1.467	2.131	0.880	1.195	0.970
Δ StdDev		-15.4%		-8.9%		-24.5%
Δ SR	9.0%		30.7%		7.8%	

6.1.2 Constrained Mean-Variance Efficient Frontier

Figure 10 shows the optimal portfolio weight allocations along the constrained efficient frontier for all three periods, starting with the global minimum-variance portfolio on the very left. We can clearly see that volatility is part of the global minimum-variance portfolio during all three periods. This confirms the economic impact of the volatility assets as seen in the mean-variance spanning test results, even after the introduction of constraints. Hence, we can note that fully invested highly risk-averse investors with a quadratic utility function can use volatility during all market times to decrease the standard deviation of their portfolio. However, volatility exposure quickly decreases and vanishes with the level of risk, as we move along the efficient frontier during the main and full period. For the backtested period, we note the opposite effect, as the exposure in the VXZ slightly increases with the level of risk. This is a direct consequence of the VXZ's mean, which is second only to the GLD during the period and outweighs the high standard deviation of the ETN.

Figure 10. Efficient Frontier Portfolio Allocation. The figure shows the portfolio weights along the constrained efficient frontier for the main period, backtested period and full period using quarterly asset returns scaled by a factor of four.

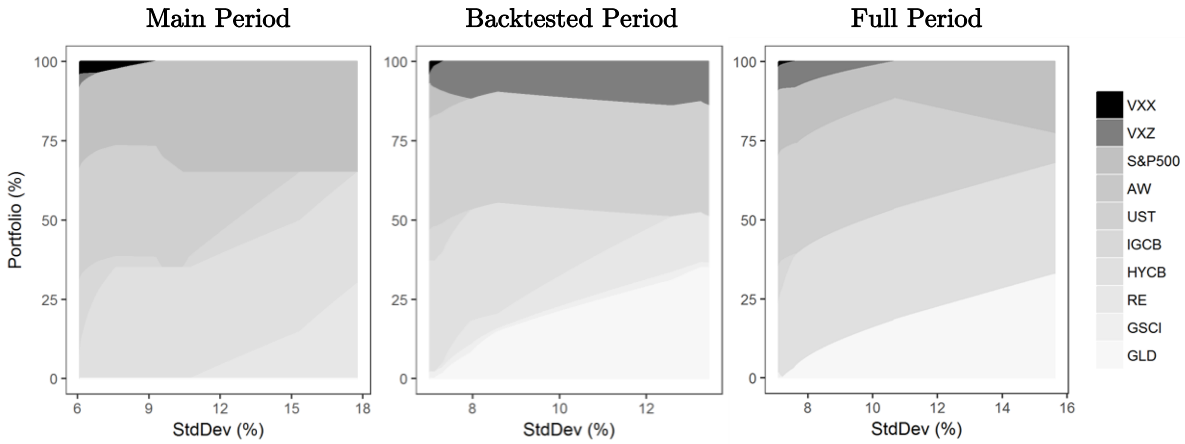


Table 13 shows the tangency and global minimum-variance portfolios of the three periods. As expected, the tangency portfolio of the main period does not include any volatility exposure given the abysmal performance and high standard deviation of the VXX and VXZ, which outweigh any diversification benefits. Hence, investors seeking the maximised risk-adjusted return would have not benefitted from adding volatility to their portfolio. This confirms the findings of Bordonado et al. (2016) in a broader portfolio context. As shown earlier, the global minimum-variance portfolio has relevant volatility exposure, with an allocation of 4.1% to the VXX and 4.4% to the VXZ. It is interesting to see that neither of the two VIX ETNs dominates the other, which can be attributed to the VXZ offering higher and less volatile returns, while not offering the same degree of diversification as the VXX. Looking at the other asset classes, we note that both portfolios exclusively utilise equities and bonds. Further, we observe that as a consequence of the bull market during the main period the Sharpe Ratio of the tangency portfolio is the highest among the three periods.

Contrary to the main period, the tangency portfolio of the backtested period includes volatility exposure by allocating a sizeable 14.0% to the VXZ, which is in line with the findings of Gantenbein and Rehrauer (2013). This allocation can be explained by the return performance of the VXZ during the backtested period. The global minimum-variance portfolio allocates 4.7% to the VXX and only 2.2% to the VXZ, emphasising the diversification characteristics of the VXX. Assets outside the equities and bond space now receive significant allocations, notably the GLD and the RE in the tangency portfolio. While allocations to the first are not surprising given the GLD's safe haven characteristics, the allocations to the RE are somewhat unexpected, given the housing crisis during the GFC and the therefore relatively low mean return.

Further, the Sharpe Ratio of the tangency portfolio is considerably lower than during the main period, given the relatively poor performance of most assets during the GFC.

The optimal tangency portfolio for the full period shows allocations that could be interpreted as a trade-off between the respective portfolios for the main period and the backtested period. In the tangency portfolio, we find a volatility exposure of 5.1% through the VXZ, but no allocation to the VXX. The global minimum-variance portfolio on the other hand exhibits higher exposure to both VIX ETNs, highlighting the diversification benefits that come at the cost of expected returns. As in the case of the mean-variance spanning results, the Sharpe Ratio of the tangency portfolio during the full period ranks lowest when compared to the other two periods.

Table 13. Tangency and Minimum-Variance Portfolios. The table shows the tangency (T) and global minimum-variance (GMV) portfolios for the constrained mean-variance optimisation using quarterly asset returns scaled by a factor of four.

	Main Period		Backtested Period		Full Period	
	T	GMV	T	GMV	T	GMV
VXX	0.0%	4.1%	0.0%	4.7%	0.0%	1.8%
VXZ	0.0%	4.4%	14.0%	2.2%	5.1%	7.3%
S&P500	30.0%	25.8%	0.0%	11.6%	15.7%	21.0%
AW	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.0%	25.6%	0.0%	9.3%	0.0%	12.8%
HYCB	35.0%	5.1%	0.4%	35.0%	35.0%	20.5%
RE	0.0%	0.0%	17.3%	0.0%	0.0%	0.0%
GSCI	0.0%	0.0%	2.4%	2.1%	0.0%	1.6%
GLD	0.0%	0.0%	30.8%	0.0%	9.3%	0.0%
Mean	0.088	0.024	0.083	0.015	0.052	0.032
StdDev	0.095	0.061	0.125	0.070	0.085	0.071
SR	0.922	0.398	0.663	0.207	0.621	0.455

Given the consistent exposure to the VXX and VXZ in all global minimum-variance portfolios, we have shown that there are indeed justifiable arguments that the two ETNs are beneficial to a certain set of constrained investors with high risk-aversion. However, the results look less promising when the investors' objective is to maximize the risk-adjusted performance as measured by the Sharpe Ratio.

6.2 Polynomial Goal Programming

Table 14 shows the results of applying the polynomial goal programming model to the multi-objective problem for ten different combinations of preference values during the main period. At first glance, we note that optimal portfolio allocations that take into account investors' preferences for higher moments differ significantly from the mean-variance portfolio allocations, in line with previous literature (see for example Prakash et al. (2003)).

Portfolio A is the mean-variance portfolio, matching the tangency portfolio from the results obtained for the constrained mean-variance optimisation in Section 6.1.2.[9] It is the portfolio that has by definition the highest Sharpe Ratio (0.922) of all portfolios. As observed earlier, investors with a quadratic utility function do not allocate any assets to the VXX and VXZ. We can also see that portfolio A has only a slightly positive skewness (0.092) and a moderately positive kurtosis (0.320), implying that the portfolio is not efficient in a higher moment space. This is in line with the results of Amin and Kat (2003), who find that mean-variance efficient portfolios tend to have a very low skewness.

Looking at the variance-skewness portfolio B, we find a much higher volatility exposure with a 30.0% weight in the VXX, but no allocation to the VXZ. When considering the skewness of the single assets, this result does not come as a surprise as the VXX has the highest skewness among all assets. The large positive portfolio skewness of 1.784 however comes at the cost of the other moments. The mean of the portfolio is with -15.3% highly negative, which is not unexpected given the high allocation to the VXX.[10] Further, the standard deviation is with 21.2% more than twice as high as it is in the optimal mean-variance portfolio, and the kurtosis is with 4.501 the highest of all portfolios. This indicates that maximising skewness is a relatively costly trade-off, in line with the literature.

The variance-kurtosis portfolio C has high volatility exposure as well, through an allocation of 29.8% to the VXZ and 1.1% to the VXX. Even though the two volatility ETNs have a relatively high kurtosis themselves, their desirable co-kurtosis features make them a valuable instrument to reduce the kurtosis on the portfolio level. The allocations to the RE and the GLD do not come as a surprise, as these are the two assets with the lowest kurtosis. The portfolio exhibits a negative mean of -2.7%, a comparably high standard deviation of 17.6% and a slightly positive skewness. It is interesting to see, that optimising kurtosis does not substantially decrease skewness compared to portfolio A, while it considerably worsens the mean return and standard deviation. Furthermore, the decrease in return and increase in standard deviation when optimising for kurtosis is less pronounced, than when optimising for skewness.

The mean-variance-skewness portfolio D has allocations of only 2.0% to the VXX and none to the VXZ. The low volatility exposure is a bit surprising, as volatility has very favourable skewness characteristics. Yet, the investor preference for returns is the limiting factor. Compared to the mean-variance portfolio, it obviously has a much higher skewness of 1.201, at the cost of a moderately lower Sharpe Ratio of 0.704, while

[9] Note that we terminate the algorithm after two million iterations which can result in minor deviations from the results in Section 6.1.2.

[10] Note that while skewness and kurtosis of the portfolios are comparable given that they are scale-invariant due to the standardisation, returns cannot be compared between portfolios directly but have to be compared using the Sharpe Ratio. Return and standard deviations will only be discussed in case of negative Sharpe Ratios.

at the same time exhibiting a much higher kurtosis. The results confirm again that skewness is generally traded for a lower risk-adjusted return and a higher kurtosis.

The mean-variance-kurtosis portfolio E has no volatility exposure at all, which can again be explained by the return preference. The portfolio is somewhat similar to the mean-variance portfolio, but it has a considerably more negative kurtosis of -0.645 and a somewhat lower Sharpe Ratio of 0.739. The skewness of the portfolio is surprisingly a bit higher than the skewness of the mean-variance portfolio, even though we did not optimise for skewness.

The variance-skewness-kurtosis portfolio F has the highest volatility exposure with allocations of 35.0% in the VXX and 12.2% in the VXZ. This also means that it is the portfolio with the lowest mean (-25.1%) and highest standard deviation (35.9%). The high allocation towards the VIX ETNs is not surprising, given the results for portfolios B and C that indicate that skewness and kurtosis seem to be the main drivers of volatility exposure. However, we can also see that kurtosis is relatively high, compared to the variance-kurtosis portfolio, while the skewness of portfolio F is close to the skewness of portfolio B. This shows that the preference for skewness dominates the preference for kurtosis to a certain extent.

The mean-variance-skewness-kurtosis portfolio G is probably the most balanced of all portfolios. It has a relatively high Sharpe Ratio, while its skewness is considerably higher and its kurtosis moderately lower. Interestingly, we do not find any volatility exposure, while we would have expected to find some due to the skewness and kurtosis properties of the VXX and VXZ. It seems that as long as the portfolio optimisation is sufficiently return-driven, volatility allocation is limited.

Portfolio H has an emphasised preference for returns and a relatively similar asset allocation compared to the mean-variance-skewness-kurtosis portfolio G. Strikingly, we find an allocation of 3.1% to the VXZ, despite an increased preference for returns, which has shown to be counterproductive to exposure to volatility. This follows as the portfolio does not include the GLD, an asset with a highly negative kurtosis. The VXZ potentially offers higher diversification benefits in the mean-variance space than the GLD, as we can see from the results in Section 6.1.2, where the GLD is not part of the global minimum-variance portfolios. Hence, with a stronger focus on the Sharpe Ratio, the GLD gets "traded" for the VXZ. When looking at the other statistical properties of the portfolio, we can see that the increased preference for returns has come at the cost of higher kurtosis, while skewness has remained relatively stable.

Portfolio I is a more skewness dominated version of portfolio G. Consequently, it comprises an allocation of 2.6% to the VXX, while not allocating any weight to the VXZ. The size of allocation to the VXX reflects the relation between the preferences for return and skewness as witnessed in portfolio D. Thus, we can see a lower, yet very positive Sharpe Ratio and a significantly higher kurtosis, a trade-off for the increase in skewness.

Portfolio J corresponds to a portfolio optimal for an investor with elevated kurtosis preferences. We do not find any volatility exposure in that setting, suggesting that kurtosis is not favouring volatility assets to the extent skewness does. We can observe a significantly lower, yet positive skewness and a Sharpe Ratio that is fairly close to portfolio I. This shows again that the trade-off between the higher moments is not asymmetric, in a sense that trading from one direction to the other demands similar return sacrifices.

Overall, we observe that all portfolios, which exhibit skewness preferences without a preference for mean, include a relatively high VXX exposure. Further, all portfolios which include kurtosis preferences without a preference for mean, include a relatively high VXZ exposure. Preferences for returns lower the exposure to the VXX and VXZ considerably, although investors with elevated preferences for skewness still find it optimal to invest in the VXX.

Table 14. Polynomial Goal Programming Results Main Period. The table shows the portfolios optimised with the polynomial goal programming model for the main period using quarterly asset returns scaled by a factor of four. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively. SR depicts the Sharpe Ratio (negative values omitted).

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	30.0%	1.1%	2.0%	0.0%	35.0%	0.0%	0.0%	2.6%	0.0%
VXZ	0.0%	0.0%	29.8%	0.0%	0.0%	12.2%	0.0%	3.1%	0.0%	0.0%
S&P500	30.0%	35.0%	0.0%	17.5%	26.1%	0.0%	19.4%	14.4%	15.8%	24.6%
AW	0.0%	0.0%	28.2%	0.0%	0.0%	0.0%	0.0%	0.0%	4.2%	0.0%
UST	35.0%	14.2%	0.6%	35.0%	35.0%	0.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.0%	0.0%	1.2%	35.0%	0.0%	6.0%	35.0%	12.5%	35.0%	19.2%
HYCB	35.0%	0.0%	0.0%	10.5%	20.0%	0.0%	0.6%	35.0%	7.3%	0.6%
RE	0.0%	0.0%	13.2%	0.0%	2.9%	35.0%	0.0%	0.0%	0.0%	0.0%
GSCI	0.0%	15.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GLD	0.0%	5.3%	25.9%	0.0%	16.1%	11.8%	10.0%	0.0%	0.0%	20.5%
Mean	0.088	-0.153	-0.027	0.050	0.077	-0.251	0.060	0.065	0.044	0.063
StdDev	0.095	0.212	0.176	0.071	0.104	0.359	0.090	0.078	0.073	0.104
Skew	0.092	1.784	0.059	1.201	0.364	1.382	0.797	0.731	1.266	0.553
Kurt	0.320	4.501	-1.624	1.951	-0.645	1.494	-0.029	0.557	2.184	-0.740
SR	0.922	n.a.	n.a.	0.704	0.739	n.a.	0.669	0.828	0.600	0.603

Table 15 reports the PGP results for the backtested period. We find considerably different results compared to the main period. As expected, volatility exposure is significantly higher. In the case of the VXX, return preferences are not as punishing as in the main period, given its less negative returns and the worse performance of the other assets. In the case of the VXZ, return preferences are actually beneficial as shown by the mean-variance portfolio A, due to the positive performance during the GFC. Moreover, most remaining assets show significantly lower or even negative skewness compared to the main period, caused by the large negative returns during the GFC. This benefits the volatility assets, as they exhibit similar or higher skewness values. Exposure to the VXX is almost exclusively driven by single preferences for either skewness or kurtosis, while volatility exposure is achieved via the VXZ in all other cases. The total exposure to volatility does not breach 20% only in portfolios A and E. Hence, we find that during the backtested period investors would have benefitted from adding volatility to their portfolio regardless of their preferences. Preferences for higher moments amplify volatility exposure, when compared to the mean-variance portfolio.

We refrain from describing the individual portfolios in more detail, as the direction and extent of the trade-offs between the different moments remain largely unchanged when compared to the main period.

Table 15. Polynomial Goal Programming Results Backtested Period. This table shows the portfolios optimised with the polynomial goal programming model for the backtested period using quarterly asset returns scaled by a factor of four. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively. SR depicts the Sharpe Ratio (negative values omitted).

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	25.6%	22.7%	0.0%	0.0%	1.2%	0.0%	0.0%	0.0%	0.0%
VXZ	13.8%	22.8%	0.4%	24.3%	7.0%	34.4%	21.9%	23.1%	25.2%	20.3%
S&P500	0.0%	0.0%	2.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AW	0.0%	0.0%	3.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.8%
UST	35.0%	0.0%	28.1%	35.0%	35.0%	0.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.0%	35.0%	6.5%	0.0%	0.0%	17.7%	0.0%	0.0%	0.0%	3.2%
HYCB	1.4%	1.0%	5.6%	0.0%	0.0%	19.0%	19.3%	0.0%	0.0%	35.0%
RE	16.8%	0.0%	0.4%	18.4%	22.9%	0.0%	0.0%	18.7%	19.0%	0.0%
GSCI	2.3%	0.0%	29.6%	10.8%	0.1%	5.9%	3.9%	9.9%	11.5%	1.7%
GLD	30.7%	15.6%	0.9%	11.5%	35.0%	21.8%	19.9%	13.4%	9.4%	0.0%
Mean	0.083	0.013	-0.007	0.066	0.084	0.074	0.071	0.067	0.063	0.042
StdDev	0.124	0.428	0.242	0.120	0.142	0.230	0.151	0.117	0.120	0.109
Skew	0.146	1.246	0.201	1.158	-0.410	1.116	1.131	1.114	1.167	0.824
Kurt	-0.543	1.037	-1.456	0.506	-1.443	-0.137	0.047	0.501	0.481	-0.783
SR	0.663	0.030	n.a.	0.548	0.590	0.323	0.468	0.574	0.530	0.382

Table 16 shows the PGP results for the full period. In line with the results from Section 6.1.2, we find volatility exposure in portfolio A with an allocation of 5.0% to the VXZ. Compared to the main period, we find that each portfolio has an allocation to at least one of the VIX ETNs and that the size thereof is considerably higher, a consequence of the effects observed during the backtested period. We can see that the VXX is to be found in portfolios driven by skewness preference, while VXZ exposure tends to be found in portfolios, which include both skewness and kurtosis preferences. To a certain extent, we have already observed this pattern in the main period. Overall, the findings resemble those for the backtested period in that volatility exposure proved beneficial for investors with different types of utility functions.

An interesting technical observation can be made with regards to portfolios B, D and I. All three portfolios differ only slightly, and show nearly identical statistical properties, with skewness maximised, a negative Sharpe Ratio and a substantially positive kurtosis. It seems that other moments are completely ignored, despite preferences for one or two additional moments other than skewness (portfolio D and I respectively). The extent of dominance of the skewness preference has not surfaced for the other two periods and will be further discussed in Section 6.3.2.

Table 16. Polynomial Goal Programming Results Full Period. This table shows the portfolios optimised with the polynomial goal programming model for the full period using quarterly asset returns scaled by a factor of four. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively. SR depicts the Sharpe Ratio (negative values omitted).

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	26.8%	5.7%	24.4%	0.0%	0.7%	0.1%	0.0%	25.0%	0.0%
VXZ	5.0%	0.0%	0.0%	0.0%	1.2%	18.8%	12.7%	11.9%	0.0%	12.3%
S&P500	15.5%	33.9%	0.0%	33.2%	33.8%	14.8%	8.3%	11.0%	34.0%	8.5%
AW	0.0%	0.0%	0.0%	0.0%	0.0%	5.5%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	0.0%	35.0%	0.0%	35.0%	35.0%	35.0%	35.0%	0.0%	35.0%
IGCB	0.0%	0.0%	0.0%	0.0%	0.0%	23.4%	6.1%	3.0%	0.0%	3.8%
HYCB	35.0%	17.0%	0.0%	25.6%	0.0%	0.0%	35.0%	35.0%	24.8%	35.0%
RE	0.0%	10.6%	17.1%	7.4%	0.7%	0.0%	0.0%	0.0%	6.4%	0.0%
GSCI	0.0%	11.7%	17.1%	9.4%	0.0%	1.8%	0.4%	0.0%	9.7%	0.0%
GLD	9.5%	0.0%	25.1%	0.0%	29.3%	0.0%	2.4%	4.1%	0.0%	5.4%
Mean	0.053	-0.053	0.022	-0.040	0.062	0.011	0.034	0.038	-0.044	0.037
StdDev	0.085	0.282	0.151	0.256	0.128	0.111	0.086	0.081	0.264	0.087
Skew	0.139	3.866	-0.167	3.850	-0.162	1.156	0.792	0.656	3.855	0.701
Kurt	0.382	19.534	-0.841	19.434	-0.784	0.490	-0.343	-0.410	19.435	-0.458
SR	0.621	n.a.	0.149	n.a.	0.485	0.098	0.394	0.463	n.a.	0.424

6.3 Critical Discussion of Results

6.3.1 Backtested Data

In the presentation of our results, we extensively analyse the main period, the backtested period and the full period. However, an important point to note is that the backtested period is purely based on hypothetical return data for the VXX and VXZ, as we backcalculated all volatility ETN returns before January 2009. We relied on data on either the respective underlying indices or the VIX futures themselves for the period until availability of index data. This can lead to some of the below listed issues.

Not all first seven month contracts were quoted every day during the first two years after the launch of the VIX futures in 2004. Hence, missing data was interpolated using the methodology suggested by S&P. The interpolation using straddling futures might not perfectly approximate the term structure, and is therefore a potential source of imprecision.

Another drawback of backcalculating return series is that in the first years after introduction the liquidity of VIX futures was relatively low. This could have led to significant price changes caused by very few transactions, and thus deviations in the price from a perceived fair market price.

Further, an important aspect is whether the introduction of the VIX ETPs themselves had a significant influence on VIX futures pricing, and thus the pricing of the VIX ETPs themselves. The descriptive statistics on VIX futures in Section 2.2.1 clearly show a steep increase in VIX futures trading activity from 2009 onwards, the year when the first VIX ETPs were launched. The introduction of VIX ETPs suddenly allowed retail investor as well as institutional investors barred from trading in the futures market to gain exposure to volatility as shown by Whaley (2013). Given that the ETPs offering long exposure to volatility have been significantly more popular than the ones offering short exposure, a substantial net buying demand has been introduced to the VIX futures market. In earlier research, Bollen and Whaley (2004) find that net buying pressure for put options on the S&P 500 affects the implied volatility of index options. Hence, an argument can be made that with a newly introduced net demand for "insurance", the volatility risk premium increased. Therefore, returns of the VXX and VXZ during the backtested period could potentially be overstated, given the relatively lower volatility risk premium due to the previous lack of access for certain investor classes.

6.3.2 Preference Values

An issue that we came across in our PGP model for the full period concerns the dominance of a single preference value over the other preference values. Davies et al. (2006) show that this is related to the high sensitivity of the results to preference values. They show that skewness optimal portfolio allocations, and more so kurtosis optimal portfolio allocations tend to dominate the respective portfolio, if the respective preference value is above a certain threshold. This threshold seems to be very much dependent on the statistical properties of the used data. Hence, a set of preference values for return, skewness and kurtosis that theoretically should represent relatively balanced investor preferences, might not achieve its goal given that it can be dominated by a single preference value.

Besides the high sensitivity of preference values, a more general question emerges regarding preference values: what are the appropriate preference values mirroring the investors' precise utility function? Even though it is established that investors try to increase odd moments and decrease even moments, the exact investor preferences are unknown. This means that we can only draw conclusions about tendencies in asset allocations given certain preferences values, rather than finding a truly optimal portfolio suitable for all investors.

7 Robustness under different Investment Intervals

It is well established that the choice of the investment interval influences the return distribution of assets (see for example Fisher and Lorie (1970), Levy (1972) and Smith (1978)). This effect is called the "intervalling effect". Return variance increases with the investment interval (i.e. the variance of quarterly returns is higher than the variance of monthly returns), while the behaviour of skewness is more complex (see for example Chang et al. (2008), Fisher and Lorie (1970), Lau and Wingender (1989), and Prakash et al. (2003)). Therefore, Levy (1972) argues that if the chosen investment interval does not represent the "true" investment horizon then there will be a systematic bias in the results. Chunhachinda et al. (1997) and Prakash et al. (2003) show empirically that the choice of investment intervals changes consequently not only the portfolio allocation weights but also the number of assets in the optimal portfolio.

We do not claim to capture the "true" investment interval of retail investors by using quarterly returns and therefore test our findings for robustness by looking at different intervals. While a longer investment interval would represent a more reasonable approximation, it would not yield sufficient observations for a proper analysis. Hence, to examine the impact of the intervalling effect on our findings, we rerun our empirical analysis using monthly and weekly returns. Table A - 1 and Table A - 4 in the appendix show the descriptive statistics of monthly and weekly return data. We indeed find that the standard deviation, skewness and kurtosis of the single assets are considerably different than for quarterly return data as in Table 7 in Section 4.3.[11]

The results for the mean-variance spanning tests using monthly and weekly results are reported in Appendix A.VII. It is striking that during the main period none of the tests is statistically significant anymore when using monthly and weekly returns. For the backtested and full period, the results using quarterly returns are confirmed nonetheless. Yet, the tangency portfolios and the global minimum-variance portfolios show significantly lower improvements of the Sharpe Ratio and the standard deviation respectively in all three periods. Gilster (1979) shows that differences in autocorrelation of different assets can be a potential explanation for significantly changed portfolio allocations due to the intervalling effect. As volatility displays a highly pronounced negative autocorrelation due to its strongly mean-reverting characteristic, this result does not completely come as a surprise. It highlights the importance of considering the appropriate return interval that corresponds to the individual investor's investment horizon when making investments in volatility.

[11] Note that standard deviations are based on annual rates of return that are obtained by employing a simple scale factor of 12 and 52 to monthly and weekly returns respectively. The resulting higher magnitude of standard deviations for shorter intervals may therefore seem counterintuitive to the theory at first. However, when rescaling by dividing monthly and weekly standard deviations by 12 and 52 respectively again, we indeed find the correct behaviour. This is in line with Chang et al. (2008) and Prakash et al. (2003). Skewness and kurtosis are scale-invariant.

Table 17 shows the results for the constrained mean-variance portfolio optimisation using the three different return intervals for the main period. We again see changes in volatility allocations: the tangency portfolio for monthly and weekly investment intervals includes the VXZ during the main period, contrary to the portfolio using quarterly intervals. On the other hand, the combined allocation in the global minimum-variance portfolios shrinks slightly in size and favours the VXZ over the VXX. The results for the other two periods as reported in Appendix A.VIII paint a similar picture. Hence, our earlier findings that highly risk-averse investors investing in the global minimum-variance portfolio would have benefitted from including volatility exposure in their portfolio also uphold for investors with shorter investment horizons.

Table 17. Tangency and Minimum-Variance Portfolios Robustness Results Main Period. The table shows the tangency (T) and global minimum-variance (GMV) portfolios for the constrained mean-variance optimisation during the main period using quarterly, monthly and weekly returns scaled by a factor of 4, 12 and 52 respectively. SR depicts the Sharpe Ratio.

	Quarterly Interval		Monthly Interval		Weekly Interval	
	T	GMV	T	GMV	T	GMV
VXX	0.0%	4.1%	0.0%	0.2%	0.0%	0.4%
VXZ	0.0%	4.4%	1.8%	6.4%	2.9%	5.0%
S&P500	30.0%	25.8%	4.2%	3.3%	5.4%	6.7%
AW	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.0%	25.6%	24.0%	29.6%	21.7%	19.5%
HYCB	35.0%	5.1%	35.0%	25.5%	35.0%	33.4%
RE	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GSCI	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GLD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Mean	0.088	0.024	0.061	0.039	0.059	0.050
StdDev	0.095	0.061	0.128	0.108	0.202	0.192
SR	0.922	0.398	0.475	0.366	0.294	0.260

The outcomes of the polynomial goal programming model using monthly returns for the main period can be found in Table 18. Portfolio B and D, which are both skewness driven, show similar or higher volatility exposures when compared to the results using quarterly intervals. This intuitively makes sense, as the skewnesses of the volatility ETNs are considerably higher on a monthly basis than on a quarterly basis, while on average the opposite holds for the other assets. Moreover, we again observe the sensitivity of the preference values as the allocations in portfolio D basically ignore the investors' preference for returns. Portfolios C, E and F are largely kurtosis driven portfolios and exhibit changes in different directions in terms of volatility exposure when employing monthly returns. While portfolios C and F show a significantly lower volatility exposure, portfolio E shows a higher one. The first observation is a direct consequence of elevated levels of kurtosis for the volatility assets when using monthly returns. The latter observation can potentially be explained by the switch from GLD allocations to IGCB allocations and the VIX ETNs' correlations, which are negative with the IGCB and positive with the GLD. The portfolios G to J which include preferences for all moments have constantly higher volatility exposure compared to the quarterly rebalanced portfolios, although portfolio I is affected by the preference value sensitivity problem. This suggests that investors with a shorter investment horizon would benefit from the VIX ETPs to an even greater extent and proves the robustness of our main findings in Section 6.2. The results for the main period using weekly returns can be found in Table A - 21 in the appendix. They are basically identical to the results using monthly returns and hence further substantiate the robustness of our conclusions drawn.

Table 18. Polynomial Goal Programming Results Main Period Monthly Returns. This table shows the portfolios optimised with the polynomial goal programming model for the main period using monthly returns scaled by a factor of 12. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively. SR depicts the Sharpe Ratio (negative values omitted).

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	28.0%	0.0%	23.6%	0.0%	0.0%	0.0%	0.0%	23.7%	0.0%
VXZ	1.8%	0.0%	7.0%	0.0%	5.6%	11.2%	8.4%	5.8%	0.0%	8.0%
S&P500	4.2%	35.0%	0.0%	35.0%	0.0%	4.2%	8.5%	5.7%	35.0%	5.9%
AW	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	9.6%	35.0%	16.8%	35.0%	35.0%	35.0%	35.0%	31.4%	35.0%
IGCB	24.0%	17.9%	15.1%	18.6%	21.2%	0.0%	7.4%	10.2%	4.2%	9.0%
HYCB	35.0%	0.0%	27.2%	0.0%	29.7%	22.4%	28.4%	35.0%	0.0%	27.9%
RE	0.0%	0.0%	3.0%	0.0%	0.0%	6.9%	1.1%	0.0%	0.0%	2.1%
GSCI	0.0%	4.7%	0.4%	1.1%	0.0%	2.0%	0.0%	0.0%	0.0%	0.0%
GLD	0.0%	4.8%	12.3%	4.9%	8.4%	16.3%	11.3%	8.3%	5.6%	12.2%
Mean	0.061	-0.108	0.038	-0.078	0.042	0.029	0.041	0.049	-0.086	0.040
StdDev	0.128	0.522	0.147	0.432	0.127	0.183	0.141	0.127	0.443	0.145
Skew	0.346	2.497	0.321	2.484	0.247	0.486	0.432	0.396	2.478	0.397
Kurt	0.648	9.920	-0.929	9.826	-0.888	-0.784	-0.780	-0.528	9.489	-0.862
SR	0.475	n.a.	0.260	n.a.	0.329	0.159	0.290	0.383	n.a.	0.278

The results for the backtested and full period are reported for both monthly and weekly intervals in Appendix A.IX. Contrary to the main period, the exact results based on monthly and weekly returns now also differ from each other. We can also observe the dominance of the skewness preference over other preferences for portfolios D and I again as the allocations ignore the VXZ and favour the VXX. However, still every single combination of examined preference values yields substantial exposure to volatility. This further proves the suitability of volatility for different investors during the backtested and the full period. Moreover, this stresses again the importance of selecting the appropriate return interval when selecting the optimal portfolio.

All in all, we indeed find different allocations when using monthly or weekly instead of quarterly intervals. However, these allocations still include substantial exposure to the VIX ETPs and prove the robustness of the main findings obtained in Section 6.

8 Conclusion

The staggering amounts lost by investors holding long volatility ETPs led us to study the suitability of those products as portfolio diversifiers for retail investors over different periods. In a first step, we revisit and extend the findings in the existing literature using mean-variance spanning tests and the mean-variance criterion in the context of a typical set of asset classes available to retail investors via ETFs. In a second step, we fill the gap in the literature as we incorporate investor preferences for higher moments and determine optimal portfolios using the PGP model. The findings are then tested for robustness under different investment intervals.

The mean-variance spanning tests show that the VIX ETPs expand the opportunity set of an unconstrained investor. The inclusion of VIX ETPs improves the Sharpe Ratio of the tangency portfolio and reduces the standard deviation of the global minimum-variance portfolio. To provide more practical implications for retail investors, we introduce weight and short-selling constraints and solve for the mean-variance efficient frontier. We find that VIX ETPs are useful instruments for highly risk-averse investors seeking the global minimum-variance portfolio. On the other hand, investors looking to maximize the risk-adjusted return as measured by the Sharpe Ratio will find VIX ETPs less suitable in that they provide performance improvements only during periods of prolonged high levels of volatility. The results are largely robust under different investment intervals.

The PGP analysis considers investor preferences for the first four moments of the portfolio's distribution, thus allowing us to take into account the favourable skewness properties of volatility. In general, the inclusion of higher moments into the optimisation process tends to benefit allocations to VIX ETPs. We find substantial allocations to VIX ETPs for investors who have strong preferences for skewness. Such investors are willing to trade a sizeable part of expected returns in exchange for higher skewness. We also find a similar tendency for investors having an elevated preference for kurtosis, although to a lesser extent. Investors with high preferences for returns on the other hand will find volatility less suitable for their portfolio. These directional findings apply to all periods and are robust under different investment intervals.

Overall, our results suggest that the undiminished popularity of VIX ETPs can be empirically justified from a portfolio diversification perspective. This offers an alternative to the existing notion in the literature that investors in VIX ETPs either lack sophistication or engage in highly speculative trades.

It is important to point out limitations of our study and indicate opportunities for potential future research. Our results from the PGP model are based on different combinations of preference values for the distribution moments. These preference values are however not representative of the "true" investor utility function, which can be at best approximated. Thus, we cannot determine whether the truly optimal portfolio includes VIX ETPs but rather provide directional findings. Furthermore, we have seen that the choice of intervals does have an influence on volatility exposure. Due to limited observations, we examine weekly, monthly and quarterly investment horizons. It is however reasonable to assume that a retail investor's investment horizon might be longer than a quarter. Therefore, it would be interesting to repeat the analysis with longer investment intervals once a larger data sample is available. Another important point to note is that we focus our analysis on U.S. equity volatility in the context of a broad set of asset classes. However, equity volatility futures and ETPs have also been introduced in different geographical markets (e.g. Europe) recently. These assets could be included in a more comprehensive analysis given the selection of international benchmark assets. Moreover, the popularity of volatility is currently restricted to the equity space with recently introduced volatility futures on treasuries failing to gain comparable investor attention. A closer examination of this development and a look into volatility products on other asset classes might appeal to practitioners. Finally, to be able to extract specific practical implications, an ex-ante approach should be carried out. So far, first attempts by Alexander and Korovilas (2012) and Bahaji and Aberkane (2015) have only considered the first three moments. However, we have shown that VIX ETPs have desirable co-kurtosis features. Hence, it would be interesting to carry out an ex-ante analysis using a performance criterion that considers the first four moments.

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Appendix

List of Figures

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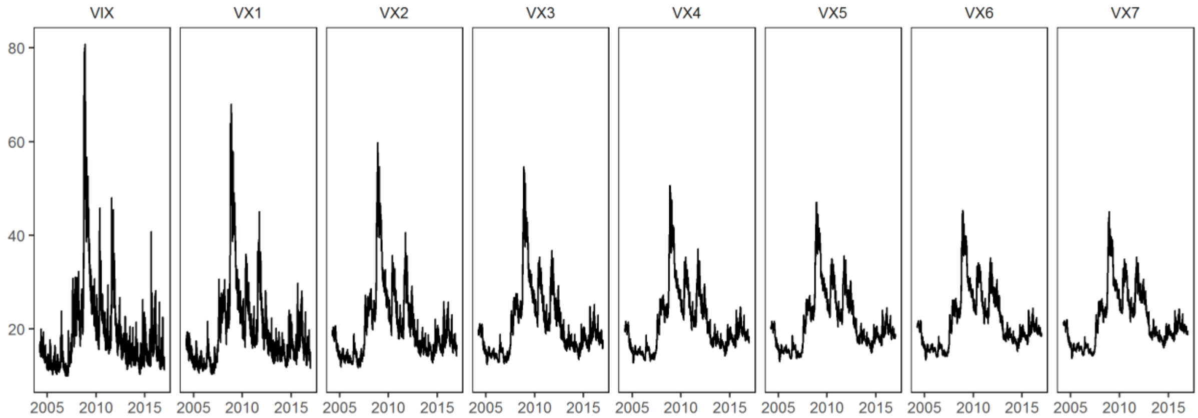
A.I VIX Futures Term Structure over Time

Figure A - 1. VIX and VIX Futures Price Chart. The chart shows the price of the VIX and the first seven month VIX futures from 2004 to 2016. Note that at settlement day, each future will switch to the next calendar month future, i.e. in March the first month future will switch from the March future to the April future. Hence, this future prices are not investable given that the underlying changes every month and no rolling costs are taken into account.



A.II Short-Term Index and Mid-Term Index Calculation

As taken from Standard & Poor's (2016), the returns of the Short-Term Index and the Mid-Term Index are calculated as follows:

$$IndexTR_t = IndexTR_{t-1} \times (1 + CDR_t + TBR_t),$$

where $IndexTR_{t-1}$ is the $IndexTR$ on the preceding business day, CDR_t the Contract Daily Return as determined by

$$CDR_t = \frac{TDWO_t}{TDWI_{t-1}} - 1,$$

where $TDWO_t$ is the Total Dollar Weight Obtained on t, as determined by

$$TDWO_t = \sum_{i=m}^{n} CRW_{i,t-1} \times DCRP_{i,t},$$

and $TDWI_{t-1}$ is the Total Dollar Weight Obtained on $t-1$, as determined by

$$TDWI_{t-1} = \sum_{i=m}^{n} CRW_{i,t-1} \times DCRP_{i,t-1},$$

with $CRW_{i,t}$ being the Contract Roll Weight of the i-th VIX Futures Contract on date t, $DCRP_{i,t}$ being the Daily Contract Reference Price of the i-th VIX Futures Contract on date t, m equal to 1 for the Short-Term Index and equal to 4 for the Mid-Term

Index, and n equal to 2 for the Short-Term Index and equal to 7 for the Mid-Term Index. TBR_t is the Treasury Bill Return, as determined by

$$TBR_t = \left(\frac{1}{1 - \frac{91}{360} \times TBAR_{t-1}} \right)^{\frac{Delta_t}{91}} - 1,$$

where $Delta_t$ is the number of calendar days between the current and previous business days and $TBAR_{t-1}$ is the most recent weekly high discount rate for 91-day US Treasury bills effective on the preceding business day. $CRW_{i,t}$ for the Short-Term Index is defined as

$$CRW_{1,t} = 100 \times \frac{dr}{dt},$$
$$CRW_{2,t} = 100 \times \frac{dt - dr}{dt},$$

and for the Mid-Term Index as

$$CRW_{4,t} = 100 \times \frac{dr}{dt},$$
$$CRW_{5,t} = 100,$$
$$CRW_{6,t} = 100,$$
$$CRW_{7,t} = 100 \times \frac{dt - dr}{dt},$$

where dt is the total number of business days in the current Roll Period beginning with and including, the starting CBOE VIX Futures Settlement Date and ending with, but excluding, the following CBOE VIX Futures Settlement Date and dr is the total number of business days within a roll period beginning with, and including the following business day and ending with, but excluding, the following CBOE VIX Futures Settlement Date.

At the close on the Tuesday, corresponding to the start of the Roll Period, all of the weight is allocated to the first month contract. Then on each subsequent business day a fraction of the first month VIX futures holding is sold and an equal notional amount of the second month VIX futures is bought. The fraction, or quantity, is proportional to the number of first month VIX futures contracts as of the previous index roll day, and inversely proportional to the length of the current Roll Period. In this way the initial position in the first month contract is progressively moved to the second month contract over the course of the month, until the following Roll Period starts when the old second month VIX futures contract becomes the new first month

VIX futures contract. In addition to the transactions described above, the weight of each index component is also adjusted every day to ensure that the change in total dollar exposure for the index is only due to the price change of each contract and not due to using a different weight for a contract trading at a higher price.

The base dates of the Short-Term Index and the Medium-Term Index are 20 December 2005 at base values of 100,000.

A.III Interpolating VIX Futures

Prior to April 2008, not all consecutive first to seventh month VIX futures were listed. For the purpose of the historical S&P 500 VIX Futures Index series calculations, some VIX future prices have to be interpolated. The procedure has been taken from Standard & Poor's (2016).

The following assumptions have been made in interpolating VIX futures contract prices from near-by listed contracts: When the i^{th} future was not listed, but i^{th}+1 and i^{th}-1 futures were listed, the following interpolation has been assumed:

$$DCRP_{i,t}{}^2 = DCRP_{i-1,t}{}^2 + \frac{BDays(T_i - T_{i-1})}{BDays(T_{i+1} - T_{i-1})}\left(DCRP_{i+1,t}{}^2 - DCRP_{i-1,t}{}^2\right).$$

When i^{th} and i^{th}+1 futures were not listed, but i^{th}+2 and i^{th}-1 futures were listed, the following interpolation has been assumed:

$$DCRP_{i,t}{}^2 = DCRP_{i-1,t}{}^2 + \frac{BDays(T_i - T_{i-1})}{BDays(T_{i+2} - T_{i-1})}\left(DCRP_{i+2,t}{}^2 - DCRP_{i-1,t}{}^2\right).$$

When i^{th}, i^{th}+1 and i^{th}+2 futures were not listed, the following interpolation has been assumed:

$$DCRP_{i,t}{}^2 = DCRP_{i-1,t}{}^2 + \frac{BDays(T_i - T_{i-1})}{BDays(T_{i-1} - T_{i-2})}\left(DCRP_{i-1,t}{}^2 - DCRP_{i-2,t}{}^2\right),$$

where T_i is the last trade day of the i^{th} VIX Futures contract and $BDays$ is the number of business days between VIX Futures last trade days.

A.IV VXX and VXZ Calculation

As in Barclays (2016), the VXX and VXZ are calculated by

$$ETN_t = ETN_{t-1} \times \left(\frac{IndexTR_t}{IndexTR_{t-1}} - 0.89\% \times \frac{Delta_t}{365} \right),$$

where ETN_t is the price of the respective VIX ETN, $IndexTR_t$ is the total return index of the Short-Term Index and Mid-Term Index for the VXX and VXZ respectively, and the investor fee amounts to 0.89%.

A.V Descriptive Statistics

Table A - 1. Descriptive Statistics Monthly Return Main Period. The table shows descriptive statistics for monthly asset returns scaled by a factor of 12 over the main period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	93	93	93	93	93	93	93	93	93	93
Mean	-0.625	-0.249	0.153	0.108	0.009	0.064	0.115	0.162	-0.050	0.057
StdDev	2.355	1.173	0.551	0.714	0.027	0.209	0.224	0.751	0.797	0.648
Skew	2.106	1.351	-0.568	0.067	0.280	0.210	0.481	-0.481	0.262	0.263
Kurt	6.583	3.396	1.761	1.027	0.769	0.556	1.912	3.135	1.509	-0.460
SR	-0.265	-0.213	0.278	0.151	0.341	0.305	0.512	0.216	-0.063	0.088
AD	3.389***	1.882***	0.854**	1.11***	0.824**	0.523	0.589	1.043***	0.496	0.473

Table A - 2. Descriptive Statistics Monthly Return Backtested Period. The table shows descriptive statistics for monthly asset returns scaled by a factor of 12 over the backtested period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	57	57	57	57	57	57	57	57	57	57
Mean	-0.120	0.104	-0.012	0.052	0.039	0.020	0.017	0.033	0.014	0.166
StdDev	2.328	1.222	0.500	0.661	0.060	0.255	0.344	0.840	0.989	0.654
Skew	2.682	1.606	-1.143	-0.878	0.437	-1.465	-0.811	-1.553	-0.724	-0.213
Kurt	10.115	3.746	2.252	2.167	0.583	4.480	4.885	4.240	0.359	0.038
SR	-0.051	0.085	-0.024	0.078	0.648	0.078	0.051	0.039	0.014	0.254
AD	3.418***	2.064***	1.76***	1.182***	0.509	1.782***	3.34***	2.437***	0.847**	0.154

Table A - 3. Descriptive Statistics Monthly Return Full Period. The table shows descriptive statistics for monthly asset returns scaled by a factor of 12 over the full period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	150	150	150	150	150	150	150	150	150	150
Mean	-0.431	-0.112	0.086	0.077	0.020	0.047	0.083	0.105	-0.034	0.099
StdDev	2.314	1.186	0.579	0.719	0.045	0.233	0.289	0.837	0.850	0.615
Skew	2.186	1.427	-0.108	-0.004	1.082	-0.731	-0.449	-0.503	-0.395	-0.026
Kurt	6.975	3.249	4.158	2.855	2.913	3.134	5.434	4.931	0.455	0.030
SR	-0.186	-0.095	0.148	0.108	0.450	0.201	0.287	0.126	-0.040	0.161
AD	7.013***	3.555***	2.864***	1.914***	3.096***	1.569***	4.221***	4.046***	0.547	0.110

Table A - 4. Descriptive Statistics Weekly Return Main Period. The table shows descriptive statistics for weekly asset returns scaled by a factor of 52 over the main period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	391	391	391	391	391	391	391	391	391	391
Mean	-0.646	-0.261	0.156	0.107	0.010	0.066	0.116	0.162	-0.057	0.058
StdDev	4.882	2.373	1.180	1.430	0.061	0.386	0.376	1.567	1.595	1.266
Skew	1.240	0.731	-0.056	-0.055	-0.483	0.095	-0.110	0.271	0.114	-0.507
EK	3.508	1.741	2.058	0.784	2.780	0.718	1.907	3.955	1.269	1.447
SR	-0.132	-0.110	0.132	0.075	0.157	0.171	0.309	0.103	-0.035	0.046
AD	6.383***	2.291***	2.987***	0.645*	2.35***	0.621	6.058***	4.269***	1.758***	0.579

Table A - 5. Descriptive Statistics Weekly Return Backtested Period. The table shows descriptive statistics for weekly asset returns scaled by a factor of 52 over the backtested period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	239	239	239	239	239	239	239	239	239	239
Mean	-0.159	0.112	-0.016	0.051	0.041	0.022	0.014	0.028	-0.016	0.173
StdDev	3.663	2.039	1.317	1.561	0.115	0.443	0.643	1.886	1.900	1.508
Skew	1.331	0.677	-1.150	-1.325	0.323	-0.371	-3.636	-0.905	-0.421	0.235
EK	4.107	1.301	11.059	8.452	1.622	2.374	30.056	9.250	0.612	1.922
SR	-0.044	0.055	-0.012	0.033	0.356	0.050	0.022	0.015	-0.008	0.115
AD	5.545***	2.153***	7.368***	6.44***	1.973***	0.933**	17.763***	8.749***	0.537	1.384***

Table A - 6. Descriptive Statistics Weekly Return Full Period. The table shows descriptive statistics for weekly asset returns scaled by a factor of 52 over the full period. Skewness and kurtosis are calculated based on standardised moments. SR depicts the Sharpe Ratio (negative values omitted). AD reflects the test-statistic for the Anderson-Darling normality test. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
N. of obs.	630	630	630	630	630	630	630	630	630	630
Mean	-0.469	-0.124	0.089	0.080	0.021	0.048	0.084	0.106	-0.038	0.104
StdDev	4.474	2.272	1.246	1.496	0.087	0.411	0.509	1.721	1.729	1.376
Skew	1.244	0.668	-0.604	-0.662	0.500	-0.179	-2.701	-0.468	-0.142	-0.089
Kurt	3.901	1.590	6.526	4.389	3.752	1.728	32.918	7.375	0.986	1.811
SR	-0.105	-0.055	0.072	0.053	0.241	0.116	0.165	0.062	-0.022	0.076
AD	10.722***	3.749***	9.254***	4.997***	8.575***	1.372***	26.112***	13.03***	1.55***	1.43***

A.VI Correlations Tables

Table A - 7. Correlation Matrix Backtested Period Quarterly Returns. The table shows the correlation matrix for quarterly asset returns scaled by a factor of 4 over the backtested period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.931	-0.715	-0.640	0.600	-0.177	-0.729	-0.757	-0.457	0.042
VXZ	0.931	1.000	-0.657	-0.600	0.641	-0.085	-0.662	-0.785	-0.416	0.048
S&P500	-0.715	-0.657	1.000	0.932	-0.320	0.475	0.848	0.849	0.254	0.122
AW	-0.640	-0.600	0.932	1.000	-0.328	0.627	0.817	0.734	0.348	0.391
UST	0.600	0.641	-0.320	-0.328	1.000	0.126	-0.182	-0.231	-0.347	0.265
IGCB	-0.177	-0.085	0.475	0.627	0.126	1.000	0.679	0.334	0.230	0.554
HYCB	-0.729	-0.662	0.848	0.817	-0.182	0.679	1.000	0.825	0.382	0.227
RE	-0.757	-0.785	0.849	0.734	-0.231	0.334	0.825	1.000	0.143	0.052
GSCI	-0.457	-0.416	0.254	0.348	-0.347	0.230	0.382	0.143	1.000	0.297
GLD	0.042	0.048	0.122	0.391	0.265	0.554	0.227	0.052	0.297	1.000

Table A - 8. Correlation Matrix Full Period Quarterly Returns. The table shows the correlation matrix for quarterly asset returns scaled by a factor of 4 over the full period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.888	-0.777	-0.732	0.347	-0.289	-0.502	-0.675	-0.434	0.087
VXZ	0.888	1.000	-0.784	-0.703	0.521	-0.259	-0.483	-0.698	-0.439	0.120
S&P500	-0.777	-0.784	1.000	0.857	-0.347	0.447	0.594	0.816	0.401	-0.003
AW	-0.732	-0.703	0.857	1.000	-0.224	0.611	0.684	0.765	0.502	0.226
UST	0.347	0.521	-0.347	-0.224	1.000	0.102	-0.182	-0.173	-0.199	0.346
IGCB	-0.289	-0.259	0.447	0.611	0.102	1.000	0.720	0.528	0.284	0.491
HYCB	-0.502	-0.483	0.594	0.684	-0.182	0.720	1.000	0.652	0.356	0.262
RE	-0.675	-0.698	0.816	0.765	-0.173	0.528	0.652	1.000	0.261	0.143
GSCI	-0.434	-0.439	0.401	0.502	-0.199	0.284	0.356	0.261	1.000	0.274
GLD	0.087	0.120	-0.003	0.226	0.346	0.491	0.262	0.143	0.274	1.000

Table A - 9. Correlation Matrix Main Period Monthly Returns. The table shows the correlation matrix for monthly asset returns scaled by a factor of 12 over the main period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.923	-0.729	-0.690	0.177	-0.406	-0.510	-0.567	-0.385	0.051
VXZ	0.923	1.000	-0.723	-0.686	0.231	-0.420	-0.471	-0.566	-0.395	0.067
S&P500	-0.729	-0.723	1.000	0.916	-0.198	0.535	0.684	0.850	0.565	0.068
AW	-0.690	-0.686	0.916	1.000	-0.105	0.690	0.747	0.866	0.634	0.189
UST	0.177	0.231	-0.198	-0.105	1.000	0.293	0.011	0.038	-0.070	0.415
IGCB	-0.406	-0.420	0.535	0.690	0.293	1.000	0.723	0.684	0.463	0.384
HYCB	-0.510	-0.471	0.684	0.747	0.011	0.723	1.000	0.741	0.476	0.178
RE	-0.567	-0.566	0.850	0.866	0.038	0.684	0.741	1.000	0.445	0.156
GSCI	-0.385	-0.395	0.565	0.634	-0.070	0.463	0.476	0.445	1.000	0.315
GLD	0.051	0.067	0.068	0.189	0.415	0.384	0.178	0.156	0.315	1.000

Table A - 10. Correlation Matrix Backtested Period Monthly Returns. The table shows the correlation matrix for monthly asset returns scaled by a factor of 12 over the backtested period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.883	-0.809	-0.768	0.421	-0.486	-0.776	-0.772	-0.383	-0.222
VXZ	0.883	1.000	-0.763	-0.712	0.460	-0.329	-0.728	-0.779	-0.348	-0.207
S&P500	-0.809	-0.763	1.000	0.903	-0.401	0.458	0.863	0.864	0.267	0.113
AW	-0.768	-0.712	0.903	1.000	-0.366	0.601	0.816	0.829	0.387	0.378
UST	0.421	0.460	-0.401	-0.366	1.000	0.168	-0.222	-0.274	-0.274	0.198
IGCB	-0.486	-0.329	0.458	0.601	0.168	1.000	0.615	0.479	0.246	0.483
HYCB	-0.776	-0.728	0.863	0.816	-0.222	0.615	1.000	0.835	0.349	0.173
RE	-0.772	-0.779	0.864	0.829	-0.274	0.479	0.835	1.000	0.220	0.237
GSCI	-0.383	-0.348	0.267	0.387	-0.274	0.246	0.349	0.220	1.000	0.347
GLD	-0.222	-0.207	0.113	0.378	0.198	0.483	0.173	0.237	0.347	1.000

Table A - 11. Correlation Matrix Full Period Monthly Returns. The table shows the correlation matrix for monthly asset returns scaled by a factor of 12 over the full period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.903	-0.703	-0.701	0.322	-0.391	-0.581	-0.619	-0.382	-0.023
VXZ	0.903	1.000	-0.655	-0.644	0.380	-0.326	-0.545	-0.596	-0.356	0.006
S&P500	-0.703	-0.655	1.000	0.911	-0.300	0.447	0.674	0.875	0.450	0.012
AW	-0.701	-0.644	0.911	1.000	-0.238	0.575	0.677	0.864	0.538	0.172
UST	0.322	0.380	-0.300	-0.238	1.000	0.193	-0.191	-0.150	-0.213	0.266
IGCB	-0.391	-0.326	0.447	0.575	0.193	1.000	0.629	0.556	0.312	0.398
HYCB	-0.581	-0.545	0.674	0.677	-0.191	0.629	1.000	0.703	0.362	0.123
RE	-0.619	-0.596	0.875	0.864	-0.150	0.556	0.703	1.000	0.364	0.135
GSCI	-0.382	-0.356	0.450	0.538	-0.213	0.312	0.362	0.364	1.000	0.257
GLD	-0.023	0.006	0.012	0.172	0.266	0.398	0.123	0.135	0.257	1.000

Table A - 12. Correlation Matrix Main Period Weekly Returns. The table shows the correlation matrix for weekly asset returns scaled by a factor of 52 over the main period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.891	-0.766	-0.744	0.284	-0.093	-0.407	-0.604	-0.423	-0.059
VXZ	0.891	1.000	-0.728	-0.705	0.287	-0.095	-0.423	-0.587	-0.391	-0.065
S&P500	-0.766	-0.728	1.000	0.896	-0.315	0.117	0.520	0.836	0.518	0.090
AW	-0.744	-0.705	0.896	1.000	-0.251	0.279	0.538	0.820	0.609	0.184
UST	0.284	0.287	-0.315	-0.251	1.000	0.468	-0.001	-0.150	-0.165	0.265
IGCB	-0.093	-0.095	0.117	0.279	0.468	1.000	0.481	0.279	0.259	0.371
HYCB	-0.407	-0.423	0.520	0.538	-0.001	0.481	1.000	0.504	0.396	0.153
RE	-0.604	-0.587	0.836	0.820	-0.150	0.279	0.504	1.000	0.475	0.167
GSCI	-0.423	-0.391	0.518	0.609	-0.165	0.259	0.396	0.475	1.000	0.261
GLD	-0.059	-0.065	0.090	0.184	0.265	0.371	0.153	0.167	0.261	1.000

Table A - 13. Correlation Matrix Backtested Period Weekly Returns. The table shows the correlation matrix for weekly asset returns scaled by a factor of 52 over the backtested period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.841	-0.741	-0.687	0.358	-0.195	-0.548	-0.633	-0.153	-0.063
VXZ	0.841	1.000	-0.638	-0.588	0.327	-0.108	-0.428	-0.572	-0.118	-0.063
S&P500	-0.741	-0.638	1.000	0.862	-0.295	0.161	0.620	0.839	0.183	-0.011
AW	-0.687	-0.588	0.862	1.000	-0.212	0.345	0.587	0.821	0.296	0.244
UST	0.358	0.327	-0.295	-0.212	1.000	0.449	-0.115	-0.164	-0.114	0.201
IGCB	-0.195	-0.108	0.161	0.345	0.449	1.000	0.429	0.305	0.225	0.367
HYCB	-0.548	-0.428	0.620	0.587	-0.115	0.429	1.000	0.661	0.223	-0.042
RE	-0.633	-0.572	0.839	0.821	-0.164	0.305	0.661	1.000	0.166	0.060
GSCI	-0.153	-0.118	0.183	0.296	-0.114	0.225	0.223	0.166	1.000	0.375
GLD	-0.063	-0.063	-0.011	0.244	0.201	0.367	-0.042	0.060	0.375	1.000

Table A - 14. Correlation Matrix Full Period Weekly Returns. The table shows the correlation matrix for weekly asset returns scaled by a factor of 52 over the full period.

	VXX	VXZ	S&P500	AW	UST	IGCB	HYCB	RE	GSCI	GLD
VXX	1.000	0.876	-0.748	-0.716	0.297	-0.127	-0.425	-0.603	-0.321	-0.067
VXZ	0.876	1.000	-0.693	-0.658	0.300	-0.102	-0.408	-0.576	-0.288	-0.066
S&P500	-0.748	-0.693	1.000	0.882	-0.303	0.139	0.552	0.839	0.373	0.047
AW	-0.716	-0.658	0.882	1.000	-0.222	0.308	0.532	0.822	0.474	0.209
UST	0.297	0.300	-0.303	-0.222	1.000	0.426	-0.106	-0.163	-0.134	0.214
IGCB	-0.127	-0.102	0.139	0.308	0.426	1.000	0.423	0.284	0.240	0.352
HYCB	-0.425	-0.408	0.552	0.532	-0.106	0.423	1.000	0.567	0.304	0.031
RE	-0.603	-0.576	0.839	0.822	-0.163	0.284	0.567	1.000	0.337	0.122
GSCI	-0.321	-0.288	0.373	0.474	-0.134	0.240	0.304	0.337	1.000	0.305
GLD	-0.067	-0.066	0.047	0.209	0.214	0.352	0.031	0.122	0.305	1.000

A.VII Mean-Variance Spanning with Monthly and Weekly Returns

Table A - 15. Mean-Variance Spanning Test Results Monthly Returns. The table shows the results for the mean-variance spanning test of N-assets (VXX, VXZ) with K-assets (S&P500, AW, UST, IGCB, HYCB, RE, GSCI, GLD) for all three periods using monthly returns scaled by a factor of 12. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	Main Period		Backtested Period		Full Period	
	Value	P-Value	Value	P-Value	Value	P-Value
Lagrange Multiplier	4.612	0.329	9.922	0.042**	10.545	0.032**
Likelihood Ratio	4.712	0.318	10.524	0.032**	10.775	0.029**
Wald	4.815	0.307	11.181	0.025**	11.012	0.026**

Table A - 16. Mean-Variance Spanning Test Results Weekly Returns. The table shows the results for the mean-variance spanning test of N-assets (VXX, VXZ) with K-assets (S&P500, AW, UST, IGCB, HYCB, RE, GSCI, GLD) for all three periods using weekly returns scaled by a factor of 52. ***, **, * denote statistical significance at the 1%, 5%, 10% level.

	Main Period		Backtested Period		Full Period	
	Value	P-Value	Value	P-Value	Value	P-Value
Lagrange Multiplier	3.973	0.410	16.189	0.003***	10.545	0.032**
Likelihood Ratio	3.993	0.407	16.599	0.002***	10.775	0.029**
Wald	4.013	0.404	17.024	0.002***	11.012	0.026**

Table A - 17. Mean-Variance Spanning Portfolios Monthly Returns. The table shows the tangency (T) and global minimum variance (GMV) portfolios for the mean-variance spanning test of N-assets (VXX, VXZ) with K-benchmark-assets (S&P500, AW, UST, IGCB, HYCB, RE, GSCI, GLD) for all three periods using monthly returns scaled by a factor of 12. SR depicts the Sharpe Ratio. Δ denotes the change of the respective metric with regards to the respective metric of the benchmark portfolio.

	Main Period		Backtested Period		Full Period	
	T	GMV	T	GMV	T	GMV
VXX	0.1%	0.5%	-2.7%	-0.1%	-1.6%	0.0%
VXZ	-0.7%	-0.8%	2.2%	-1.3%	1.4%	-0.2%
S&P500	11.1%	2.6%	-11.8%	2.8%	1.9%	1.8%
AW	-6.3%	1.2%	13.9%	5.7%	-1.3%	0.8%
UST	82.3%	101.3%	119.8%	106.6%	96.4%	103.0%
IGCB	1.1%	-4.7%	-20.8%	-5.8%	-7.8%	-8.8%
HYCB	18.5%	3.4%	1.9%	-4.2%	11.7%	4.4%
RE	-4.4%	-2.3%	-1.2%	-1.9%	-1.1%	-1.1%
GSCI	-2.0%	0.1%	0.2%	1.5%	-0.9%	0.6%
GLD	0.2%	-1.3%	-1.6%	-3.1%	1.3%	-0.6%
Mean	0.035	0.010	0.054	0.035	0.033	0.022
StdDev	0.039	0.021	0.056	0.045	0.090	0.073
SR	0.892	0.479	0.972	0.785	0.367	0.296
Δ StdDev		-2.1%		-2.1%		-0.1%
Δ SR	0.5%		12.2%		7.0%	

Table A - 18. Mean-Variance Spanning Portfolios Weekly Returns. The table shows the tangency (T) and global minimum variance (GMV) portfolios for the mean-variance spanning test of N-assets (VXX, VXZ) with K-benchmark-assets (S&P500, AW, UST, IGCB, HYCB, RE, GSCI, GLD) for all three periods using weekly returns scaled by a factor of 52. SR depicts the Sharpe Ratio. Δ denotes the change of the respective metric with regards to the respective metric of the benchmark portfolio.

	Main Period		Backtested Period		Full Period	
	T	GMV	T	GMV	T	GMV
VXX	-1.2%	0.1%	-3.4%	-0.7%	-1.0%	0.4%
VXZ	1.1%	-0.1%	3.4%	0.1%	0.6%	-1.3%
S&P500	7.2%	1.2%	-4.7%	1.2%	5.8%	3.3%
AW	-5.5%	1.1%	3.2%	1.4%	-1.7%	0.8%
UST	75.6%	103.8%	117.1%	107.1%	94.9%	99.7%
IGCB	1.0%	-7.5%	-21.0%	-12.4%	-7.6%	-5.8%
HYCB	25.4%	2.5%	3.8%	4.3%	11.9%	5.7%
RE	-1.2%	-0.7%	0.3%	-1.4%	-3.3%	-2.5%
GSCI	-2.7%	0.5%	0.5%	1.0%	-0.9%	0.7%
GLD	0.3%	-0.8%	0.8%	-0.7%	1.4%	-1.0%
Mean	0.047	0.009	0.057	0.042	0.031	0.021
StdDev	0.114	0.049	0.110	0.094	0.045	0.037
SR	0.413	0.178	0.518	0.443	0.699	0.579
Δ StdDev		-0.1%		-1.3%		-1.5%
Δ SR	2.5%		16.1%		4.0%	

A.VIII Mean-Variance Optimisation with Monthly and Weekly Returns

Table A - 19. Tangency and Minimum-Variance Portfolios Robustness Results Backtested Period. The table shows the tangency (T) and global minimum-variance (GMV) portfolios for the constrained mean-variance optimisation during the backtested period using quarterly, monthly and weekly returns scaled by a factor of 4, 12 and 52 respectively.

	Quarterly Interval		Monthly Interval		Weekly Interval	
	T	GMV	T	GMV	T	GMV
VXX	0.0%	4.7%	0.0%	5.0%	0.0%	4.3%
VXZ	14.0%	2.2%	12.8%	2.8%	8.7%	2.1%
S&P500	0.0%	11.6%	0.0%	16.3%	7.1%	10.5%
AW	0.0%	0.0%	0.6%	0.0%	0.0%	0.0%
UST	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.0%	9.3%	0.0%	20.7%	27.0%	32.0%
HYCB	0.4%	35.0%	30.9%	17.4%	17.4%	15.1%
RE	17.3%	0.0%	3.8%	0.0%	0.0%	0.0%
GSCI	2.4%	2.1%	0.0%	2.4%	0.0%	0.0%
GLD	30.8%	0.0%	16.8%	0.4%	4.7%	1.0%
Mean	0.083	0.015	0.062	0.017	0.040	0.019
StdDev	0.125	0.070	0.157	0.112	0.264	0.247
SR	0.663	0.207	0.395	0.150	0.150	0.077

Table A - 20. Tangency and Minimum-Variance Portfolios Robustness Results Full Period. The table shows the tangency (T) and global minimum-variance (GMV) portfolios for the constrained mean-variance optimisation during the full period using quarterly, monthly and weekly returns scaled by a factor of 4, 12 and 52 respectively.

	Quarterly Interval		Monthly Interval		Weekly Interval	
	T	GMV	T	GMV	T	GMV
VXX	0.0%	1.8%	0.0%	2.2%	0.0%	1.3%
VXZ	5.1%	7.3%	3.6%	4.0%	4.6%	4.4%
S&P500	15.7%	21.0%	4.0%	5.2%	4.6%	8.8%
AW	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.0%	12.8%	14.6%	28.7%	23.7%	30.1%
HYCB	35.0%	20.5%	35.0%	24.1%	29.8%	19.9%
RE	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GSCI	0.0%	1.6%	0.0%	0.7%	0.0%	0.0%
GLD	9.3%	0.0%	7.7%	0.0%	2.3%	0.4%
Mean	0.085	0.071	0.050	0.031	0.045	0.035
StdDev	0.052	0.032	0.142	0.122	0.233	0.225
SR	0.621	0.455	0.351	0.252	0.192	0.155

A.IX Polynomial Goal Programming with Monthly and Weekly Returns

Table A - 21. Polynomial Goal Programming Results Main Period Weekly Returns. This table shows the portfolios optimised with the polynomial goal programming model for the main period using weekly returns scaled by a factor of 52. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively.

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	27.2%	1.6%	22.5%	0.0%	2.0%	1.6%	0.0%	22.5%	0.0%
VXZ	1.1%	0.0%	5.7%	0.0%	8.1%	11.3%	8.9%	4.8%	0.0%	11.1%
S&P500	4.9%	23.0%	0.0%	27.1%	0.0%	0.0%	0.0%	0.0%	17.9%	0.0%
AW	0.0%	7.9%	0.0%	0.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	26.1%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	23.9%	0.0%	35.0%	0.0%	35.0%	35.0%	35.0%	25.2%	0.0%	35.0%
HYCB	35.0%	0.0%	0.1%	0.0%	2.3%	0.0%	7.8%	35.0%	0.0%	3.6%
RE	0.0%	0.0%	0.0%	0.0%	0.0%	1.5%	0.1%	0.0%	2.6%	0.0%
GSCI	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GLD	0.0%	15.9%	22.6%	14.6%	19.6%	15.2%	11.6%	0.0%	21.9%	15.2%
Mean	0.065	-0.119	0.014	-0.090	0.019	-0.005	0.009	0.048	-0.097	0.010
StdDev	0.217	1.079	0.410	0.887	0.373	0.433	0.360	0.203	0.963	0.373
Skew	-0.069	1.434	0.143	1.430	0.133	0.659	0.591	0.403	1.408	0.435
Kurt	1.464	4.419	0.025	4.337	0.018	0.575	0.450	1.040	4.009	0.158
SR	0.298	n.a.	0.035	n.a.	0.052	n.a.	0.024	0.237	n.a.	0.028

Table A - 22. Polynomial Goal Programming Results Backtested Period Monthly Returns. This table shows the portfolios optimised with the polynomial goal programming model for the backtested period using monthly returns scaled by a factor of 12. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively.

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	35.0%	13.4%	35.0%	5.5%	11.1%	0.0%	0.0%	35.0%	4.6%
VXZ	12.8%	0.0%	0.3%	0.0%	0.0%	22.2%	12.0%	12.0%	0.0%	2.5%
S&P500	0.0%	35.0%	18.6%	35.0%	0.0%	0.0%	0.0%	0.0%	35.0%	0.0%
AW	0.5%	0.0%	22.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	3.1%	14.3%	2.8%	35.0%	35.0%	35.0%	35.0%	7.1%	35.0%
IGCB	0.0%	0.0%	13.5%	0.0%	11.9%	0.0%	0.0%	0.0%	0.0%	8.9%
HYCB	31.5%	26.9%	3.0%	27.2%	28.2%	0.0%	35.0%	35.0%	22.9%	31.7%
RE	3.7%	0.0%	3.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GSCI	0.0%	0.0%	9.7%	0.0%	11.1%	3.6%	2.9%	2.5%	0.0%	7.6%
GLD	16.6%	0.0%	0.6%	0.0%	8.2%	28.2%	15.2%	15.6%	0.0%	9.7%
Mean	0.061	-0.040	0.006	-0.040	0.029	0.071	0.058	0.058	-0.039	0.035
StdDev	0.155	0.620	0.204	0.619	0.162	0.509	0.152	0.153	0.630	0.146
Skew	0.489	2.910	-0.062	2.910	0.026	1.435	0.653	0.651	2.905	0.309
Kurt	0.705	11.994	-0.819	11.997	-0.784	2.326	0.446	0.452	11.926	-0.435
SR	0.395	n.a.	0.030	n.a.	0.181	0.139	0.380	0.382	n.a.	0.241

Table A - 23. Polynomial Goal Programming Results Full Period Monthly Returns. This table shows the portfolios optimised with the polynomial goal programming model for the full period using monthly returns scaled by a factor of 12. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively.

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	33.2%	5.0%	27.1%	0.0%	0.0%	0.0%	0.0%	28.0%	0.0%
VXZ	3.5%	0.0%	0.0%	0.0%	9.1%	21.2%	10.5%	10.4%	0.0%	10.1%
S&P500	3.6%	35.0%	17.7%	35.0%	2.3%	0.0%	4.3%	5.7%	35.0%	3.2%
AW	0.0%	6.2%	0.0%	2.9%	0.0%	17.0%	0.0%	0.0%	2.0%	0.0%
UST	35.0%	25.6%	35.0%	35.0%	35.0%	0.0%	35.0%	35.0%	35.0%	35.0%
IGCB	15.4%	0.0%	0.0%	0.0%	0.0%	24.5%	0.0%	0.1%	0.0%	0.0%
HYCB	35.0%	0.0%	0.0%	0.0%	30.2%	0.0%	35.0%	35.0%	0.0%	33.0%
RE	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GSCI	0.0%	0.0%	14.1%	0.0%	4.9%	16.4%	2.2%	0.8%	0.0%	4.0%
GLD	7.4%	0.0%	28.1%	0.0%	18.5%	20.9%	13.0%	13.1%	0.0%	14.7%
Mean	0.050	-0.103	0.024	-0.077	0.041	0.016	0.040	0.042	-0.082	0.039
StdDev	0.142	0.622	0.252	0.499	0.174	0.309	0.152	0.150	0.524	0.160
Skew	-0.091	2.397	0.202	2.392	0.271	0.559	0.455	0.460	2.395	0.381
Kurt	3.138	8.352	-0.353	8.306	-0.273	0.141	-0.019	0.056	8.322	-0.185
SR	0.351	n.a.	0.094	n.a.	0.234	0.052	0.263	0.280	n.a.	0.244

Table A - 24. Polynomial Goal Programming Results Backtested Period Weekly Returns. This table shows the portfolios optimised with the polynomial goal programming model for the backtested period using weekly returns scaled by a factor of 52. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively.

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	15.2%	7.4%	14.5%	3.5%	4.3%	3.2%	1.1%	16.5%	2.8%
VXZ	13.2%	1.8%	12.8%	2.9%	10.2%	9.6%	11.3%	14.6%	1.3%	12.7%
S&P500	0.0%	35.0%	0.0%	35.0%	0.0%	7.8%	6.0%	2.9%	35.0%	2.7%
AW	5.3%	6.5%	0.0%	5.4%	0.0%	4.9%	5.6%	4.4%	4.9%	7.3%
UST	35.0%	35.0%	0.0%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	0.8%	0.0%	29.8%	0.0%	16.3%	35.0%	35.0%	35.0%	0.0%	35.0%
HYCB	28.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
RE	0.4%	6.4%	17.1%	7.2%	11.7%	0.0%	0.0%	2.0%	7.3%	1.7%
GSCI	0.0%	0.0%	19.0%	0.0%	11.5%	0.0%	0.0%	0.0%	0.0%	0.0%
GLD	17.3%	0.0%	14.0%	0.0%	11.8%	3.4%	3.9%	5.0%	0.0%	2.9%
Mean	0.066	-0.008	0.035	-0.006	0.046	0.033	0.038	0.048	-0.012	0.041
StdDev	0.373	0.456	0.658	0.454	0.451	0.323	0.332	0.350	0.465	0.344
Skew	0.138	2.208	0.123	2.205	0.144	0.459	0.443	0.420	2.166	0.410
Kurt	0.957	11.582	-0.507	11.700	-0.446	-0.152	-0.172	-0.147	10.495	-0.228
SR	0.177	n.a.	0.053	n.a.	0.101	0.102	0.115	0.136	n.a.	0.118

Table A - 25. Polynomial Goal Programming Results Full Period Weekly Returns. This table shows the portfolios optimised with the polynomial goal programming model for the full period using weekly returns scaled by a factor of 52. p_1, p_3 and p_4 denote the preference values for mean, skewness and kurtosis respectively.

Portfolio	A	B	C	D	E	F	G	H	I	J
p_1	1	0	0	1	1	0	1	2	1	1
p_3	0	1	0	1	0	1	1	1	2	1
p_4	0	0	0.5	0	0.5	0.5	0.5	0.5	0.5	0.75
VXX	0.0%	18.1%	0.0%	17.3%	0.0%	1.3%	0.8%	0.0%	16.6%	0.0%
VXZ	1.5%	0.0%	13.5%	0.0%	11.0%	11.9%	11.5%	11.7%	0.0%	12.7%
S&P500	2.1%	35.0%	0.0%	35.0%	0.0%	0.0%	0.0%	0.0%	27.3%	0.0%
AW	0.0%	6.1%	0.0%	5.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UST	35.0%	35.0%	23.8%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%	35.0%
IGCB	21.2%	0.0%	32.3%	0.0%	28.4%	35.0%	35.0%	35.0%	0.0%	35.0%
HYCB	35.0%	0.0%	0.0%	0.3%	0.0%	1.2%	3.8%	7.2%	9.9%	0.0%
RE	0.0%	5.8%	9.9%	6.7%	8.5%	5.9%	4.4%	2.3%	6.6%	6.4%
GSCI	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%
GLD	5.2%	0.0%	20.6%	0.0%	17.1%	9.6%	9.5%	8.8%	4.6%	10.9%
Mean	0.052	-0.035	0.035	-0.031	0.034	0.021	0.024	0.027	-0.026	0.026
StdDev	0.258	0.544	0.447	0.522	0.380	0.365	0.349	0.332	0.522	0.356
Skew	-1.500	1.368	0.249	1.364	0.232	0.544	0.522	0.513	1.324	0.446
Kurt	14.777	5.373	0.072	5.532	0.079	0.352	0.315	0.338	4.679	0.184
SR	0.203	n.a.	0.079	n.a.	0.090	0.056	0.068	0.081	n.a.	0.074