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Pricing Options on the Nordic Power Exchange Nord Pool

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Abstract

This thesis studies the options traded at the Nordic power market Nord Pool, which are written on yearly and quarterly forward contracts. The most widely used pricing model at the market is Black 76, a model that assumes the option's underlying variable to be log-normally distributed, and volatility to be constant over time. In order to examine whether these assumptions are true, we firstly perform normality tests for the spot, futures and forward log-returns. We find that neither of them follows a log-normal distribution, but also that the forward log-returns are much closer to being log-normally distributed than the other variables. Secondly, we examine the implied volatility smiles for traded options and the implied volatility term structure at Nord Pool. We find that different options trade at different volatility levels, especially as time to maturity decreases, and hence volatility is not constant over time. Our thesis concludes that there are benefits from using Black 76, but nonetheless are two important model assumptions violated.

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1. INTRODUCTION

In 1995, the Swedish power market was deregulated and the Swedish government decided that the market should be competitive in terms of producing and selling electricity. In the 1990s, the Nordic countries created a common electric power market, and Norway was the first of the Nordic countries to deregulate its power market. In 1996, Norway and Sweden created the power market Nord Pool, and it became the first international exchange for trading electric power. Nord Pool became fully integrated when Finland joined in 1998 and Denmark in 2000.

Electricity producers and consumers meet at Nord Pool, and the electricity prices for the whole Nordic region are set there. At Nord Pool, there is a spot market for daily prices and a financial market, which includes both an exchange of financial instruments and an OTC-market, where buyers and sellers enter contracts on electricity. With futures and forward contracts on electricity, market participants can more easily predict their future costs and revenues, and they are better protected against large future losses. Another benefit from having a Nordic power market, such as Nord Pool, is that the Nordic power plants can be utilized in a more efficient way.

The Nordic electricity market is generally considered to work fairly properly and to be one of the most well-developed electricity markets in the world. However, an electricity market differs from an ordinary stock market. Even if the *market structure* at Nord Pool is well-functioning, this market is different from an ordinary stock market due to the specific characteristics that electricity has as a commodity, e.g. the constraints in the transmission grids and storage problems. Electricity cannot be treated as an ordinary commodity due to the fact that electricity practically is impossible to store once it is produced. Therefore, the pricing of claims, such as forwards and options that are written on electricity, is more difficult than pricing other types of contracts and derivatives, e.g. equity derivatives.

In the Nordic countries, electricity has become a fully commoditized asset. The price model for options that is used today on the Nordic market by most large traders is a model called Black 76, and it is very similar to the Black-Scholes option pricing formula. The model Black 76 was not created especially for electricity derivatives, but for ordinary commodity derivatives and contracts. Therefore, we find it interesting to study how Black 76 is used as an option pricing model and how it is implemented.

1.1 Objectives

The purpose of this thesis is to study the assumptions that are made when options traded at the Nordic electricity market, Nord Pool, are priced. The price model used by most practitioners and market participants is Black 76, but this model was not developed for pricing electricity derivatives, like the ones traded at Nord Pool. In order to able to use Black 76, it is important that the underlying model assumptions are fulfilled.

The options traded at Nord Pool are written on yearly and quarterly forward contracts, and Black 76 assumes that the underlying variable, i.e. the forward price, follows a log-normal distribution. To examine this, we look at the spot, futures and forward prices at the market and how they are distributed and have developed over time. Furthermore, Black 76 uses volatility as a measure of risk, and in the model it is assumed to be constant. In reality, volatility is not constant over time and it also differs for options with different strike prices. Therefore, based on market data, we study the implied volatilities and the volatility term structure. In this thesis we aim to answer the following research questions:

- How is Black 76 implemented as a pricing model at Nord Pool?

- How are the spot, futures and forward prices distributed, and what are the implications from this?

- What does the implied volatility smiles for different options traded at Nord Pool look like?

- What does the volatility term structure at Nord Pool look like, and what are the implications from this?

1.2 Delimitations

In this thesis, we will not compare Black 76 with other pricing models or give any recommendations on what would be the most suitable model to use when pricing options traded at Nord Pool. Furthermore, we will not discuss liquidity or efficiency at Nord Pool, or constraints in the transmission grids that might influence the price. Neither will we discuss the effects from other types of contracts traded at Nord Pool, nor study possible effects from contracts that have been introduced for environmental reasons, such as European Union Allowances and Electricity Certificates.

1.3 Methodology and Structure

In order to examine Black 76 as a pricing model, we consider available price data from Nord Pool. To study Nord Pool and Black 76, we use relevant literature, articles and other relevant research within this area. Our thesis begins with an institutional background and description of the different contracts traded at Nord Pool. This information is mainly retrieved from the Nord Pool website, the Nord Pool Annual Report (2005), Söder and Amelin (2004), Damsgaard and Green (2005), and Bergman et al. (1994) and interviews with Niclas Damsgaard and Markus Hartwig. We present Black 76 and its assumptions as well as other basic theories. Our theory chapter is based on Hull (2003), Björk (1998), Clewlow and Strickland (1998 and 2000), Pilipovic (1998), and Geman (2005). Based on interviews with traders at Vattenfall and Eon, and also employees at Econ and Nord Pool, we examine how Black 76 is implemented as an option pricing model in reality.

Based on the data from Nord Pool we perform a quantitative analysis. The data is collected directly from Nord Pool and consists of the daily average hourly spot prices on electricity, daily futures prices and daily forward prices. To study the data, we perform normality tests and study how the spot, futures and forward log-returns are distributed and behave over time. The sample period used is 1 October 2003 until 1 April 2007. We choose this time period due to the types of contracts having changed several times since Nord Pool first opened. The last major change was made in September 2003.

Since volatility is assumed to be constant over time in Black 76, we study the implied volatilities. The implied volatilities are calculated from call option price data collected from text files on Nord Pool's ftp-server. Since all contracts and options at Nord Pool are traded in euros, we choose to use Euribor as our interest rate in the model. The implied volatilities are solved for through iterations. In order to better understand the implied volatility smiles, we look at the evolution of them. To study the term structure at Nord Pool, the implied volatilities from the option prices are compared against both strike price and time left to maturity. The thesis ends with a conclusion of our results from the data analysis. All results from the data analysis are not included in the text, but can be found in our appendix.

2. INSTITUTIONAL BACKGROUND

2.1 Nord Pool and Market Characteristics

In 1990, the Norwegian Parliament decided to deregulate the power market, and this went into effect in 1991. After two years of trading, the company Statnett Marked AS, since 1996 called Nord Pool, was established as an independent company. Nord Pool is today owned by the two national grid companies, Statnett SF in Norway (50%) and Affärsverket Svenska Kraftnät in Sweden (50%). The headquarters and the actual trading are situated in Oslo, Norway. Futures are traded since 1995 and forwards since 1997. Elbas was launched as a separate market for power balance adjustment in Finland and Sweden in 1999. Options were introduced as tradable products at Nord Pool in late 1999.

Nord Pool is licensed as a regulated exchange and clearinghouse. Nord Pool's spot market activities are organised at a separate company, Nord Pool Spot AS, owned by all of the transmission system operators, such as Svenska Kraftnät, in the Nordic power exchange area and by Nord Pool ASA. The Nordic system is also connected with the German and the Polish systems through high-voltage direct current cables.

The distribution net is divided into three levels. National transmission nets have high voltage to optimize transmission over long distances and are used to optimize the electricity production. Regional sub-transmission nets have the same function as the national nets but over a smaller area. Distribution nets deliver electricity to end-consumers and are always low-voltage. Distribution of electricity is always a natural monopoly, and therefore the owners of the grids and the charged net tariffs in Sweden are supervised by Statens Energimyndighet (STEM).

Once electricity is produced, it is impossible to store to a larger extent and over time. Capacitors and batteries are not efficient for storing large amounts of electricity. It is complicated and expensive to construct capacitors that can endure such high-voltage. The benefit with hydro power is the possibility to store water in reservoirs and then quickly transduce the water into power. This flexibility is not possible in e.g. nuclear or thermal power plants.

Today, futures contracts, forward contracts, Contracts for Difference (CfD), options on forward contracts, Electricity Certificates and European Union allowances (EUAs), so called CO_2-allowences for carbon dioxide emissions, are traded at the market. Through the 14 years of trading, there have been several different contracts, but since 2005 all the above are included. We will later on explain the specific properties of the contracts that we will study more specifically. All contracts at Nord Pool, both at the spot market and at the financial market, are traded in euros. The possible currency risk, which traders in e.g. Sweden face, is taken solely by the traders.

Even though the Nordic electricity market works fairly efficiently, there are problems with the distribution of power to certain regions during high-peak seasons due to constraints in the transmission grid. During a high-peak season in a specific area, there might be a shortage of power due to high consumption, low power generation and not enough capacity to import power to that specific area. This might occur even if there is not a shortage at the market as a whole. This has to be taken into account when structuring the market and the Nordic region is therefore divided into several areas. In practice, this means that the price paid for consumed power in a certain area might be higher or lower than the electricity spot price at Nord Pool. CfDs were therefore introduced as an instrument to be able to hedge against this specific area price risk. However, Sweden is today considered as one region, even if the southern parts of Sweden have more similarities to Denmark and Finland when it comes to power generation and consumption, and the northern parts have similar patterns to the northern parts of Norway.

2.2 Trading and Market Participants

The power supply and demand in the Nordic region are to a large extent dependent on weather constraints, degree of deregulation, capacity constraints, production sources, seasonality and high-peaks/off-peaks during the day or the week. These factors are specific for the power market and especially for a market that has a high influence of hydro power. The participants at Nord Pool are producers, retailers, grid owners, traders, transmission system operators, and end-users. Grid owners have a natural monopoly over a certain area concerning transmission and distribution and are supervised by STEM. The transmission system operators are responsible for the balance between supply and demand on the market. A sudden increase in demand has serious technical consequences that the market might not be able to correct for. This could lead to a decrease in the frequency, which might lead to resonance in the turbine axes. Since the transmission system operators might not have a production of their own, the economic responsibility might be put on someone with a production of regulation power or someone that trades with regulating power at Elbas. End-users use the products at the market as risk management tools, e.g. to hedge, to secure prices and to sell excess power that they have already contracted and cannot consume on their own. Traders profit from volatility at the power market, and contribute to high liquidity and trade activity.

The Nord Pool group consists of three different divisions. Nord Pool ASA is running the financial market; Nord Pool Spot AS is responsible for the physical-delivery spot markets; and Nord Pool Clearing ASA is running the electricity contract clearing services. Nord Pool Clearing clears all contracts traded at Nord Pool, as well as contracts registered for clearing traded at the bilateral financial markets. All trading is run through an electronic system.

2.2.1 Supply

The total generation was about 380 TWh per year in the Nordic power exchange area in 2005. Accordingly, trade in financial contracts is about five times the physical load (not including non-cleared financial contracts). The total production divided between different energy sources and between the different countries is displayed in the table below.

Energy source	Denmark	Finland	Norway	Sweden	Sum	Share (%)
Wind power	6,6	0,2	0,5	0,9	8,2	2,1 %
Refinery gas (Denmark) Fossil fuels (Sweden)	0,2			0,6	0,8	0,2 %
Biofuel	2,9	8,9	0,3	7,4	19,5	4,9 %
Waste	1,3	1,0	0,3	0,9	3,5	0,9 %
Peat		4,5		0,1	4,6	1,2 %
Natural gas	8,6	8,9	0,4	0,7	18,6	4,7 %
Oil	0,3	1,5		1,4	3,2	0,8 %
Coal	14,5	7,0		1,1	22,6	5,7 %
Nuclear power		22,3		69,5	91,8	23,2 %
Hydro power	0	13,6	136,5	72,1	222,2	56,3 %
Total production	34,4	67,9	138	154,7	395	100 %

Table 1: Production in TWh from various energy sources. Source: Nord Pool annual report 2005.

The exchange and the technical system were originally developed for the Norwegian market. As can be shown in *Table 1*, the Norwegian production is dominated by hydro power with its specific characteristics. The market is totally dominated by hydro power and nuclear power. Since thermal power is hard to regulate, according to changes in demand compared to hydro power, the need for a regulating market occurred as Sweden, Finland and Denmark joined the exchange.

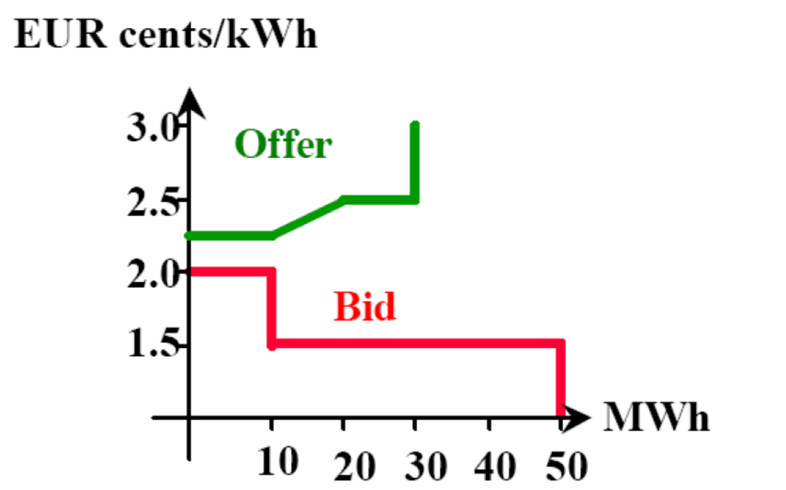

Figure 1: Bid/offer from one player for one specific hour during the day. Source: Nord Pool Spot 2007-04-02.

Figure 1 is an example of either a buying and/or selling bid from one retailer for one hour of operation the upcoming day. It is assumed that the retailer has his/her own hydro power plant. Consequently, for the specific hour during the upcoming day of operation, the following choices of trading and production are available;

- buy the whole amount of the necessary power at the market and therefore save the water in the hydro power reservoir.
- buy some of the power at the market and produce the remaining part in the power plant.
- produce the amount of power that corresponds exactly to the consumers' expected consumption in the specific hour of operation.
- sell power at the market and consequently produce an amount of power which is larger than the consumers' expected consumption for the specific hour of operation.

The supply curve is asymptotic (see *Figure 2* and *Figure 3* below) due to the different marginal cost of production. The curve increases exponentially as the cost of production increases and also shifts depending on the water level in the reservoirs. A decrease in the water level shifts the curve to the left and thus increases the price level. The base load, which is the average load or demand, has a low marginal cost compared to the peak load facilities.

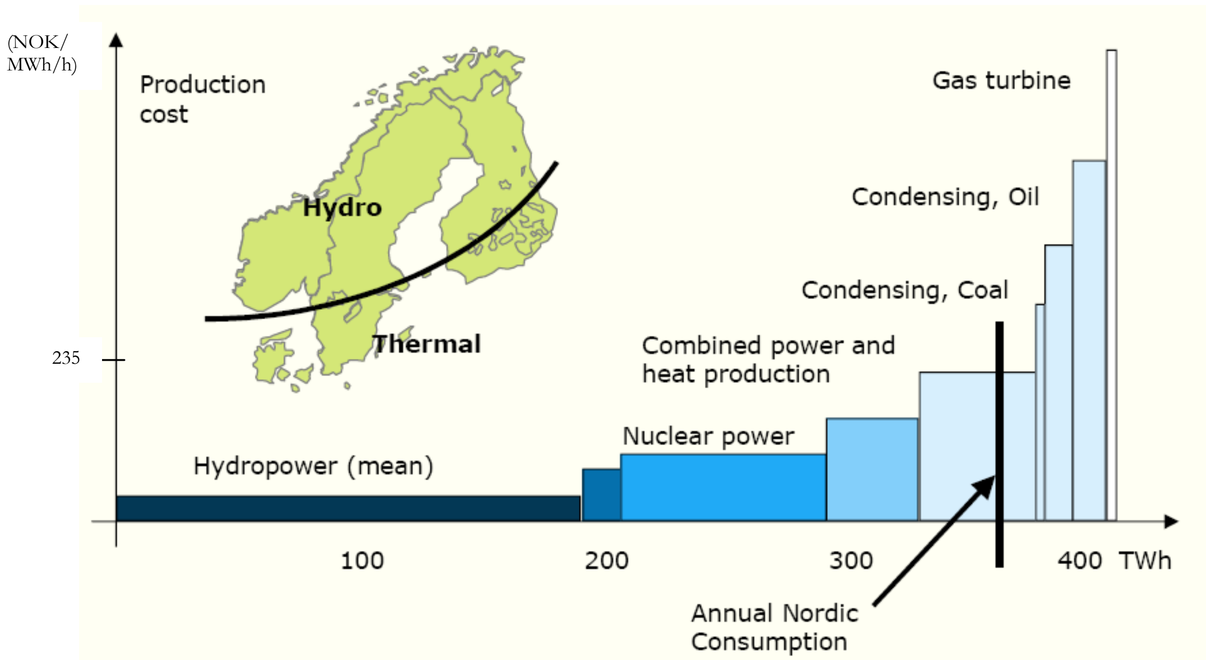

Figure 2: Production capacities from different types of power production in the Nordic countries and the marginal cost of production 2005. Source: Nord Pool 2007-04-02.

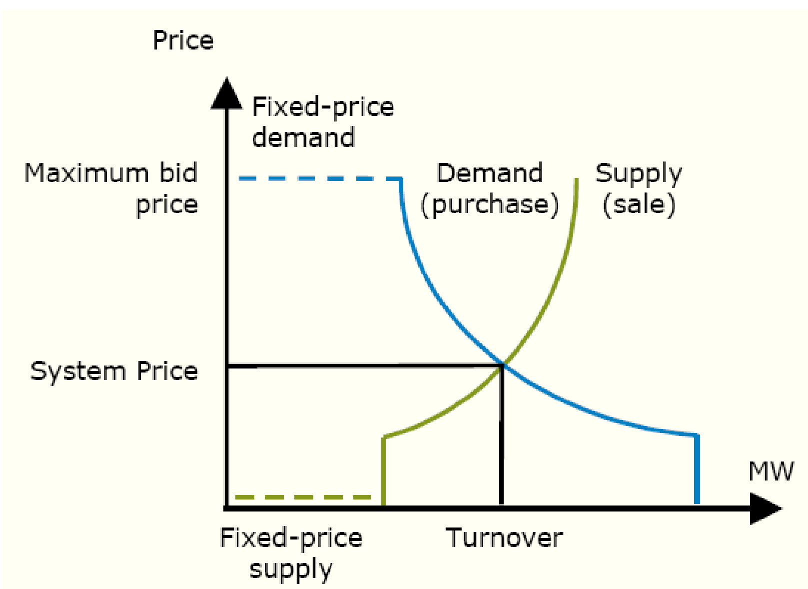

Figure 3: Clearing of the system price at Nord Pool Spot. Source: Nord Pool Spot 2007-04-02.

Country	Number of Participants
Belgium	2
Bermuda	1
Cayman Island	5
Denmark	14
Estonia	2
Finland	37
Germany	3
Great Britain	12
Ireland	1
Italy	1
Lithuania	1
Malta	2
Norway	154
Poland	1
Spain	3
Sweden	95
Switzerland	4
The Netherlands	3
USA	4
Virgin Islands	1
Total	346

Table 2: Source: Nord Pool 2007-04-02.

From *Table 2* above, it can be seen that several other countries except from the Nordic ones are represented. This is a large change compared to just a couple of years ago when the Nordic countries, especially Norway, completely dominated the market.

2.2.2 Demand

The demand for electricity at the Nordic market shifts constantly, and the variations are large. The high-peak versus off-peak seasons makes the production and the price vary to a great extent. The demand differs extremely between different seasons of the year, time of the day, the daily temperature and the day of the week. There is an overall benefit from making sure that the short-term changes in demand are covered. According to Berman et al. (1994), the long-term price elasticity is between -0.3 and -1.0 in Sweden and other countries. This gives specific market characteristics that have to be taken into account. Sudden changes in price are not affecting demand, at least not in the short run.

A bilateral contract is a contract between two parties, where the producer usually takes the price risk, while the end-consumer pays a premium compared to the average price at the market. The bilateral contracts are often not traded at Nord Pool, even though Nord Pool has a clearing function for these types of contracts. Unlike other traded contracts at Nord Pool, bilateral contracts are not influenced by the sudden price changes at the market, due to changes in demand. The end-consumers in Norway have bilateral contracts, just as most end-users in the Nordic region, but have a variable price to a much higher extent than other countries, e.g. bilateral contracts in Sweden. Heating in Norway is mainly dominated by electricity and therefore the Norwegian consumers are highly affected by sudden price changes.

2.3 The Spot Market

The spot market, Elspot, is used for trading electricity for the upcoming 24 hours and the trading horizon is 12-36 hours ahead. The deadline for participants to submit bids to the auction market is at noon and before that the trading capacities and transmissions are known. Each bid specifies price, the specific hour and the volume in MWh/h, which is not the same thing as the electric power, but the electricity volume during one specific hour. The hourly bids might be both fixed in blocks and flexible to meet different demands from the market participants. An example of aggregated curves is displayed in *Figure 3*. After the deadline, all buy and sell orders are matched into two curves for each delivery hour; an aggregate demand curve and an aggregate supply curve is formed to set the system price, which is the market price for a specific hour. The system price is also denoted as the unconstrained market clearing price. This is due to the fact that the trading capacities between the bidding areas have not been taken into account in finding the system price.

When the system price is cleared, Elbas, or the regulating market, is opened and trading proceeds until one hour before the physical delivery starts. Elbas is a complement to Elspot, but does not work as an implicit auction in the same way and is regulated through contracts between the transmission system operators and operators at the markets. The regulating market helps the transmission system operators to regulate both frequency and the time fault that occurs after changes in demand and supply.

The level of liquidity and efficiency is important at the whole Nordic electricity market, and there have been several investigations concerning this matter. The last one was performed by STEM on request by the Swedish government. STEM (2006) concludes that the Nordic electricity market is a well-functioning market, especially compared to other commodity markets. However, there are several problems with transparency at the market, the handling of market sensitive information by governmental authorities, and distinctive and harmonized set of regulation. The report concludes that market participants have great confidence in the functioning of the Nordic electricity market.

2.4 The Financial Electricity Market

All financial power contracts are cash-settled and there is no physical delivery from the traded contracts. The contracts are designed to satisfy the needs of various participants at the market such as producers, retailers, end-users and traders. There are several contracts traded at Nord Pool developed to create a well-functioning market. The different contracts traded do not take technical conditions, such as grid congestion and access to capacity, into account. The contracts are futures, forwards, options, CfDs, Electricity Certificates and EUAs. They are all listed in euros since 2006, which makes it easier to trade across borders. Further on, we solely concentrate on futures, forwards and options, and just give a brief explanation to the function of the other instruments.

The futures and forwards contracts traded at Nord Pool have been restructured several times. The changes in the last couple of years (2003-2005) are mainly changes in the time horizon of the contracts. The time horizon for futures was finally shortened to six weeks. The remaining contracts, for up to four years ahead, are listed as forwards. According to Nord Pool (2006-03-30), the market participants seem to prefer short-term futures close to due date and long-term forwards at the far end of the time horizon. This could be explained by the fact that companies consuming large amounts of electricity in their production, e.g. paper mills and investors in power plants, want to secure prices several years ahead when investing in new plants. However, the lack of really long forwards, e.g. with a time horizon up to ten years, is a problem to many companies.

There are also financial reasons for short-term futures close to due date and long-term forwards at the far end of the time horizon being preferred. Both futures and forwards are settled financially and the futures are marked-to-market daily. This requires the traders having funds available and access to cash accounts, especially for the long period contracts at the far end of the time horizon. The forward contracts on the other hand, are not marked-to-market daily and require posting cash collateral only during the delivery period, starting at the contracts' due date.

2.4.1 Futures Contracts

The futures contracts at Nord Pool are financial and do not consider any physical delivery. The contracts consider the *base load*, which can be described as an average load over a certain time period, 24 hours or 7 days, not considering the seasonality over the day or the week. There are both daily and weekly futures contracts traded at Nord Pool. The daily contracts are traded for five days at most, and the weekly contracts are traded for six weeks.

The settlement of daily futures contracts involves both a daily mark-to-market settlement and a final cash settlement after the contract has reached maturity. At maturity T (see *Figure 4*), the spot price, S_T, on electricity for the upcoming 24 hours $((T+1) - T)$ is known. The final settlement received at T is the difference between the final closing price of the futures contract, which is Euro 55 MWh/h, and the spot price for the delivery period. The spot price varies during the day, and for one specific hour shown in *Figure 4* the spot price equals Euro 58 MWh/h.

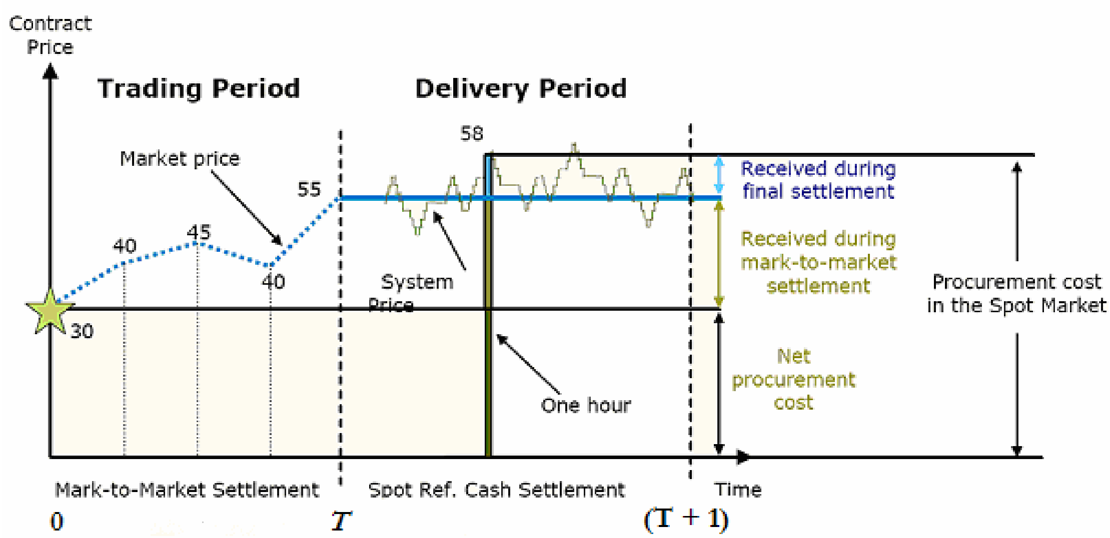

Figure 4: Daily futures contract settlement. Source: Nord Pool 2007-04-02.

In *Figure 4*, a trader is assumed to have entered a futures contract at a price of Euro 30/MWh/h at time 0. The market price for the futures contract increases to Euro 55/MWh/h during the trading period. The cash flow that the trader receives, from the daily mark-to-market settlement, sums up to a total gain of Euro 25/MWh/h.

The trader does not only receive the mark-to-market settlement during the trading period, but also the final settlement X_T at maturity T. This cash flow corresponds to the difference between the last futures price f_T, i.e. the closing price from the previous day, and the spot price, S_T, for each hour that is settled for the upcoming 24 hours. The trader receives an amount of $\left(S_T^n - f_T\right)$, which is Euro 3/MWh/h for a specific hour after maturity as shown in the example in *Figure 4*. The different amounts received from each specific hour during the 24 hours sum up to the final settlement X_T:

$$X_T = \sum_{n=1}^{24} \left(S_T^n - f_T\right)$$

where n = 1, 2, … 24 and corresponds to the different hours during the day.

One large difference with electricity futures compared to ordinary commodity futures, is that the delivery of the cash flows corresponds to a certain time period and not to a specific point in time. The holder of an electricity futures contract receives the cash flow from the mark-to-market settlement and receives/loses the difference between the spot price for each hour and the futures price.

Figure 5 shows the rolling cycle over the six weeks that the weekly futures contracts are traded. For a weekly futures contract, closing time occurs at (T-2), as shown in *Figure 6*. Closing time is two days before the weekly futures contract matures, and the contract is split into a portfolio of daily futures for the upcoming week. The upcoming week begins at T. The daily futures are then marked-to-market during the first trading day ((T-1)-(T-2)) after the weekly contract is split. The contract for the first day ((T+1) – T) is only marked-to-market during one day, ((T-1)-(T-2)) and settled the next day, which is (T – (T-1)), at noon. The other daily futures contracts are marked-to-market in the same way until the day before they are settled.

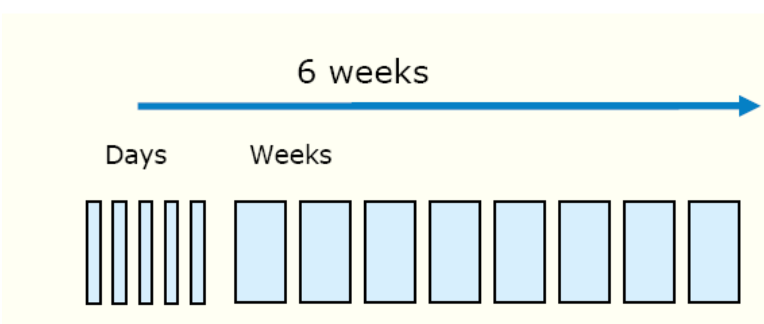

Figure 5: Rolling cycle for the futures contracts over a six week period. Source: Nord Pool 2007-04-02.

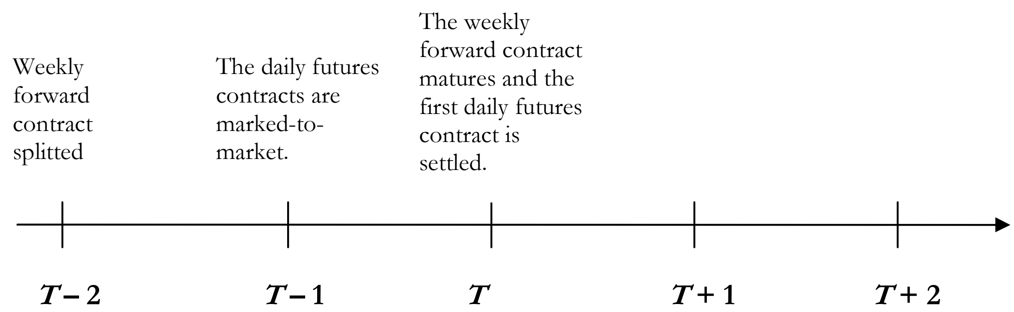

Figure 6: Time line for a weekly forward contract split at $T - 2$.

Nord Pool will reinstate trading with high- and off-peak futures during the late spring of 2007. If the market is efficient, this change will not affect the price, since the seasonality is known and should therefore already be included in the price. Since autumn 2003, the weekly contracts have been listed with eight consecutive contracts based on a rolling cycle. The time period was shortened to six weeks in 2005 to increase liquidity. Before 2003, the cycle implied listing of weeks in groups of 4 after the split of a block contract.

2.4.2 Forward Contracts

The forward contracts traded at Nord Pool are financial and have been changed to a large extent. The previous forward contracts were structured into three seasons or blocks considering the base load; Winter 1, Summer, Winter 2; and yearly forward contracts. The new products introduced in the autumn of 2003 are structured according to each calendar month, quarter, and year, all considering the base load solely. The monthly forward contracts are listed on a six months consecutive rolling basis, and are not subject to splitting. The quarterly forward contracts are split into monthly forward contracts, in the same way as the weekly futures contracts are split into daily futures contracts. The yearly forward contracts are split into quarterly forward contracts. There is no mark-to-market settlement during the trading period prior to maturity, and the daily loss/profit is not realised throughout the trading period. In *Figure 7*, the rolling cycle over a four year time period is shown, and also how the different forward contracts are split.

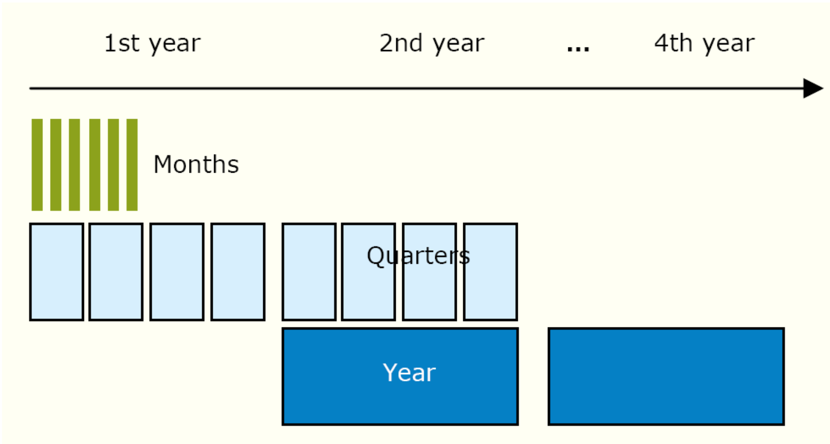

Figure 7: Rolling cycle for the forward contracts over a four year period. Source: Nord Pool 2007-04-02.

In *Figure 8*, an example of a settlement of a monthly forward contract is shown. A forward contract is entered at time 0 for Euro 30 MWh/h. At maturity, the value of the forward contract F_T is Euro 55 MWh/h. To purchase the contract's volume at the spot market for a specific hour during the first day costs Euro 58 MWh/h. The trader receives Euro 3 MWh/h for the specific hour in *Figure 8* from the spot reference cash settlement, i.e. the time period that the forward contract covers.

When the monthly forward contract has reached maturity the spot price is only known for the upcoming 24 hours. Therefore, settlement takes place for each day throughout the delivery period, i.e. the holder of the forward contract receives a cash flow $\left(S_m^n - F_T\right)$ at noon every day, where m = 1, 2, … 30 denotes one specific day during the month, n = 1, 2, … 24 denotes a specific hour during that day and S_m^n is the electricity spot price for a specific day and a specific hour. The cash flow X_T is the sum of all cash flows during the spot reference cash settlement. During the delivery period, the value CF_T accumulated from the trading period, here Euro 25 MWh/h, is realized. The total of all cash flows for a monthly forward contract during the delivery period is:

$$X_T = \sum_{m=1}^{30} \sum_{n=1}^{24} \left(S_m^n - F_T\right) + CF_T$$

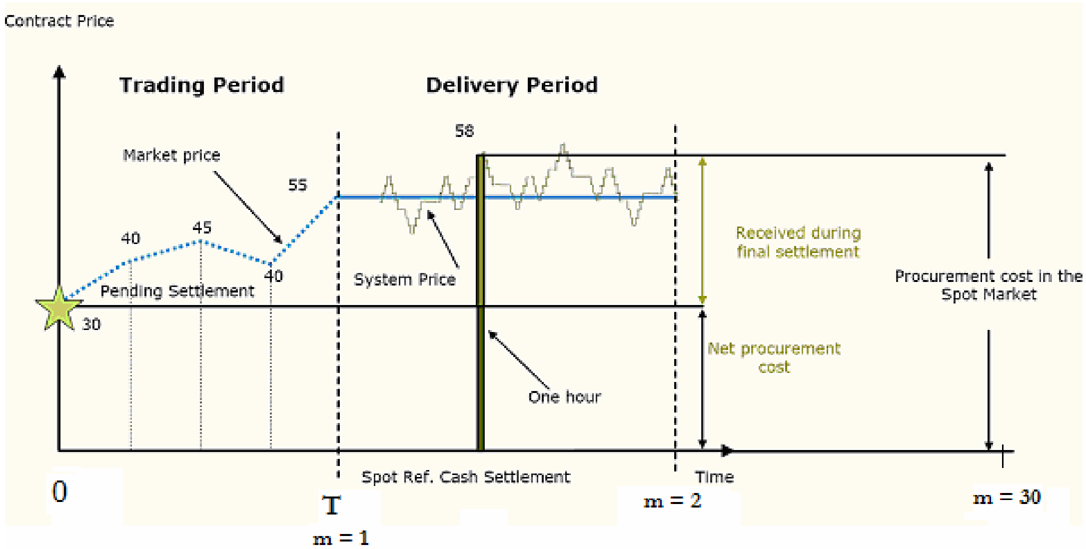

Figure 8: Forward contract settlement. Source: Nord Pool 2007-04-02.

2.4.3 Options

The options traded at Nord Pool are European options that have quarterly and yearly forwards as their underlying instrument. At Nord Pool, five strike prices are set when an options series is initially listed. New strike prices are automatically generated when the traded price or the closing price of the underlying forward contract is at or below (above) the second lowest (highest) strike price. At Nord Pool, the exercise day is always the third Thursday in the month before the delivery period of the underlying forward contract. Since the options are European, the holder can only exercise an option on the exercise day. New series are listed on the expiry day of an old series.

Options, often combined with other contracts, offer the possibility to hedge and manage risk. A call option can be bought if the holder wants to be insured against increases in the forward price on electricity. With the option, the holder is protected against the future electricity price exceeding a certain level. An electricity producer might instead be interested in buying put options, making sure that the future electricity price does not fall beyond a certain level. Options can also be used to insure against volume risk, which is the risk that volumes are low due to dry seasons.

2.4.4 Other Contracts Traded at Nord Pool

At the Nordic electricity market, the price can differ between areas. Therefore, Contracts for Difference were introduced as a listed financial instrument to make it possible to set up a perfect hedge against possible price differences between Elspot and the individual area prices. The price of a CfD can positive or negative, depending on the area price and the system price. If the specific area price is expected to be lower than the system price, then the CfD will have a positive price and v.v. If the specific area price equals the system price, the price of a CfD is zero.

In March 2004, Nord Pool listed the Swedish green certificates, so called Electricity Certificates. The certificates are quoted in SEK and are spot products with physical delivery three days after the trading date. Due to the threat of climate changes, European Union allowances have been introduced all across Europe to handle emissions of carbon dioxide and other greenhouse gases. In 2005, EUAs were introduced at Nord Pool.

3. THEORY

3.1 Forwards and Futures

A forward contract is a direct agreement between two parties and obligates the holder to buy or sell an asset for a predetermined price at a fixed time T in the future. Forwards are not standardized and are traded over the counter (OTC). The forwards are only settled at maturity. A futures contract is similar to a forward contract in the sense that it is an agreement to buy or sell an asset for a predetermined price at fixed time T in the future. However, futures contracts are standardized in terms of the future date and the amount traded. Also, the contracts can be retraded at any point on a futures exchange. Another important difference between forwards and futures is their cash flows. The cash flow from a forward contract occurs only once, which is at maturity. Future contracts are marked-to-market daily, meaning that the contracts are revaluated in order to reflect daily price changes of market valuables. Regardless of the position in the futures contract being long or short, the holder is required to hold a certain amount of money for the daily mark-to-market settlements.

Although there are differences between forward and futures contracts, they can be treated as equal for pricing purposes when future interest rates are deterministic. When interest rates are non-stochastic, the price of a forward contract is the same as the price of a futures contract for the same underlying and maturity. Even if the cash flows from the contracts are different, the correlations of energy prices to interest rates are typically null. Therefore, in energy commodity markets, the prices of futures and forward contracts can be used interchangeably, since they reflect the same value.

It can be shown that the theoretical forward price is given by:

$$F(t, T) = E_t^{Q^T}\left[X\right] \tag{1}$$

and the theoretical futures price by:

$$f(t, T) = E_t^{Q}\left[X\right] \tag{2}$$

T is time at maturity, t is today and $(T–t)$ is time left to maturity. If the world is assumed to be risk-neutral, then the expected return required by all investors on all investments is the risk-free interest rate r. If r is deterministic, then the risk-neutral probability Q, for the future and the risk-neutral probability Q^T for the forward are equal. Hence, expression (1) and (2) are equal and $F = f$. If X in expression (1) is the spot price S_T of a traded financial asset, then the price of a forward contract can be expressed as:

$$F(t, T) = e^{r(T-t)} S_t \tag{3}$$

A financial asset can be defined as an asset whose value derives from a contractual claim and the asset does not necessarily have a physical worth. The financial asset can be converted into cash and be included in a portfolio. Another property of a financial asset is that it can be both bought and short sold. Typical examples of financial assets are stocks and bonds. Non-financial assets have values that are based on their physical property.

3.2 Geometric Brownian Motion

When a variable's value changes randomly over time, it follows a stochastic process. Many asset price models for pricing derivatives have a continuous time framework where a stochastic differential equation (SDE) is assumed. The equation describes the stochastic process that the asset price follows. One example of an assumption of a stochastic process is Geometric Brownian Motion (GBM), which was assumed for the Black-Scholes option pricing formula (1973). GBM is a stochastic process that describes how the price of an asset evolves over time, where the logarithm of the underlying variable follows a Wiener process with constant drift. This can be described by the following expressions:

$$dX_t = \alpha X_t \, dt + \sigma X_t \, dW_t \tag{4}$$

$$Z_t = \ln X_t \tag{5}$$

$$dZ_t = (\alpha - \frac{\sigma^2}{2}) dt + \sigma \, dW_t \tag{6}$$

We thus see that the increment Z_t follows a Wiener process and has a constant drift and variance. Z_t is normally distributed and X_t is log-normally distributed. dW_t is an increment in a standard

Wiener process over the small time interval *dt*, and it can be described as the underlying uncertainty that drives the model.

If the forward contract, as defined in expression (3), is assumed to follow GBM, the stochastic process under the risk-neutral probability *Q* can be written as:

$$dF(t, T) = \sigma F(t, T)\, dW_t \tag{7}$$

3.3 Black 76

An option can be described as a right to buy or sell an underlying asset, e.g. a stock, for a predetermined price at a certain date in the future. The holder of a call option has the right to buy the underlying contract, and the holder of a put option has the right to sell it. *K* is the strike price and σ is the volatility that is constant over time. The underlying, i.e. the stock price S_t, is assumed to follow GBM and to be log-normally distributed. Black-Scholes *assumes* that S_t follows GBM under the real-world probability *P*. Given this assumption, Black-Scholes prove that option pricing is done through taking discounted expected values of payouts under the risk-neutral probability *Q*. From expression (4) the stochastic process for the stock price can be written as:

$$dS_t = \alpha S_t\, dt + \sigma S_t\, dW_t \tag{8}$$

We recall that the Black-Scholes option pricing formula (1973) for a European call option is:

$$C(t) = S \times N(d_1) - Ke^{-r(T-t)} \times N(d_2) \tag{9}$$

where

$$d_1 = \frac{\left(\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right) \times (T-t)\right)}{\left(\sigma\sqrt{(T-t)}\right)}$$

$$d_2 = d1 - \left(\sigma\sqrt{(T-t)}\right)$$

The model Black 76 was derived to calculate the price for commodity derivatives. In the Black-Scholes formula, the underlying is a stock, while the underlying in Black 76 is a forward contract. In the Black-Scholes option pricing formula, it is assumed that the stock price *S* follows GBM under P, while in Black 76 it is assumed that the forward price $F(t, T)$ follows GBM under P. The stochastic process for the forward contract under P can be written as:

$$dF(t, T) = \alpha F(t, T)\, dt + \sigma F(t, T)\, dW_t \tag{10}$$

Having assumed this, a formula for forwards and options written on forwards is derived where expected values under the risk-neutral probability Q are used. It is possible to show that even the forward prices follow GBM under Q, but with a drift term equal to zero, as shown in expression (7).

The Black 76 formula is given by:

$$C^F(t) = e^{-r(T-t)}\left(F^{T^*}(t,T)\times N(d_1) - K\times N(d_2)\right) \tag{11}$$

where

$$d_1 = \frac{\left(\ln\left(F^{T^*}(t,T)/K\right)+\left(\frac{\sigma^2}{2}\right)\times(T-t)\right)}{\left(\sigma\sqrt{(T-t)}\right)}$$

$$d_2 = d1 - \left(\sigma\sqrt{(T-t)}\right)$$

$$T^* > T > t$$

The forward contract matures at T^*, and the options expiry date is at T. $F^{T^*}(t, T)$ denotes the forward price process, assuming no-arbitrage and the dynamics of GBM. The forward contract is assumed to follow the stochastic process (7), and volatility is assumed to be constant over time.

In order to be able to use the Black-Scholes option pricing formula or Black 76, it is important to examine whether the model assumptions are fairly in accordance with real world conditions. When using Black 76, it is relevant to test if the forward price log-returns are normally distributed under P.

If a forward contract is written on a financial asset, then Black 76 can be derived from the Black-Scholes option pricing formula and v.v. However, if the forward contract is written on a non-financial asset, such as the electricity spot price, Black 76 can still be used even though the Black-Scholes option pricing formula is no longer applicable.

3.4 Volatility

Volatility is a measure of uncertainty and can be defined as the annualized standard deviation of price returns and estimated from historical price series. While standard deviations are quite general, volatilities of a price process are used to measure the annualized distribution width of price returns. Standard deviations on their hand measure the width of any distribution chosen. If the process is normally distributed, e.g. a GBM, and the volatility in the process is constant, then the variance of the price returns grows with time.

When using the Black-Scholes option pricing formula, an assumption must be made about the volatility since it cannot be observed. In reality, volatility changes over time and therefore historical volatilities are not applicable to use. Option prices, c*, for each day are observable, including the variables that the option price depends on except for the volatility σ, i.e. today's stock price S_t, the strike price K, time left to maturity (T-t) and the risk-free interest rate r.

$$c^* = c(S_t, t, T, r, \sigma, K)$$

If the option price is known, it is possible to solve for the volatility assumed in the option pricing formula. This is called the implied volatility, and it can be described as the volatility that the market has used implicitly for valuing the options. In other words, what is being solved for is the market expectation of the volatility.

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The implied volatilities can be plotted as a function of the strike prices in order to observe the so called volatility smile. The volatility smile can be described as the variation in the implied volatility in relation to the strike price. In *Figure 9*, a typical volatility smile is displayed, where volatility varies along with the strike price for call and put options respectively.

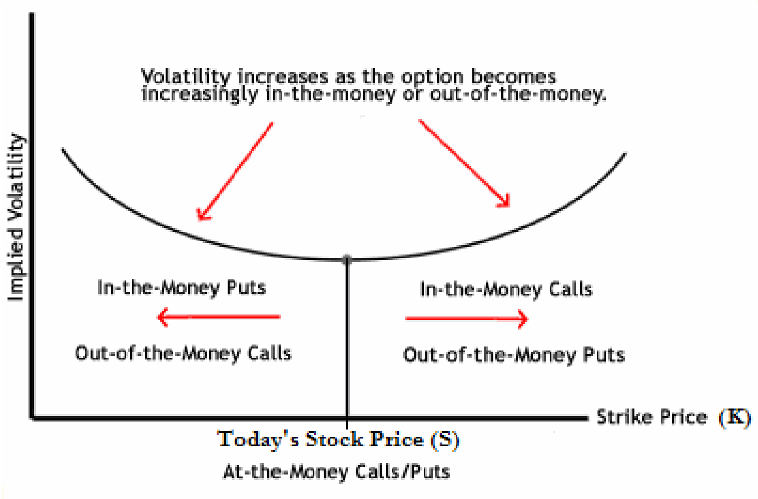

Figure 9: Volatility smile. Source: Investopedia 2007-05-04.

Options that are out-of-the-money trade at different volatility levels than e.g. options that are at-the-money. By plotting the implied volatilities, it is observable that the implied distribution differs from the log-normal distribution that is assumed in the Black-Scholes option pricing formula and Black 76. If the log-normal distribution assumption would be true, then plotting the implied volatilities against the strike price would give a straight line and not a smile, since there is a one-to-one condition between the price and the volatility of a European call option.

The volatility level does not only vary along with different strike prices, but also with time to maturity. This is known as the volatility term structure. In the Black-Scholes option pricing formula and Black 76, volatility is constant over time. This assumption should give a volatility term structure with a "flat" surface, as shown in *Figure 10*.

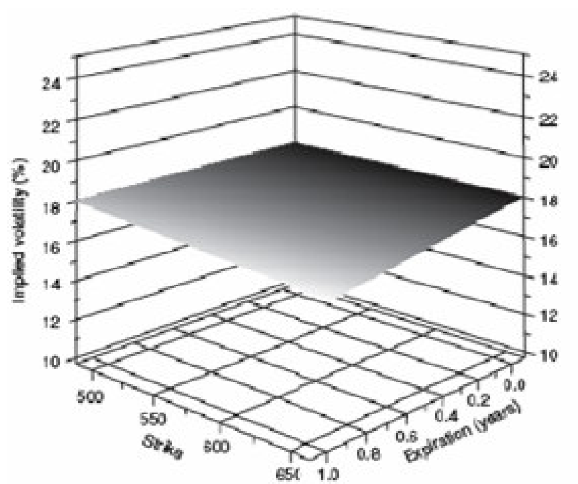

Figure 10: Implied term structure, Black-Scholes. Source: Derman 2006

In reality, the term structure is nothing like a flat surface. The volatility term structure plotted in *Figure 11*, is an example of what the implied volatilities plotted against strike price and time to maturity could look like.

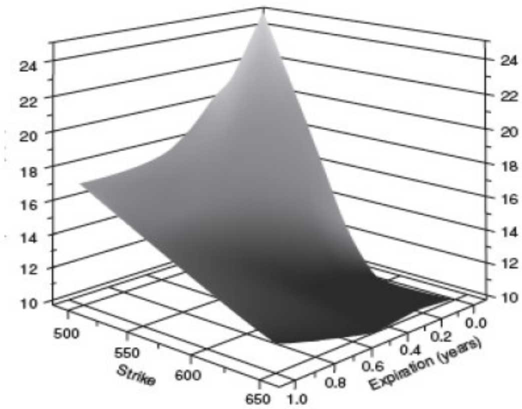

Figure 11: Volatility term structure. Source: Derman 2006.

Hence, important shortcomings of the Black-Scholes model, and also the Black 76 model, are the assumption of constant volatility over time and the log-normal distribution assumption. Before the stock market crash of 1987, the result in the plot was a "smile". The heavy right tail of the "smile" is much less pronounced today, now called a "smirk", and is due to the fact that investors assign a higher probability for higher equity price rather than lower equity prices. The smile and skew that is observed in the implied volatilities from the Black-Scholes option pricing formula and Black 76, is related to fat-tails and skewness in the distributions of spot and forward price returns.

4. BLACK 76 IN REALITY

The electricity options traded at Nord Pool are written on yearly and quarterly forward contracts. The pricing formula for these options, used by most large traders, is Black 76. There are several possible extensions of this model. One example could be that one assumes that the underlying variable is driven by several Wiener processes instead of one, as defined in expression (7). However, Black 76 is a pricing formula for commodity contracts, not a specific formula for pricing electricity options.

There are several ways to use an option pricing model. For example, a model can be used in a *normative* way. This means that certain variables are set by large traders to calculate what the price *should* be. Traders look at e.g. historical volatilities and prices, and make their own assumptions of the current volatility of the option. Thereafter, they trade according to their own calculations of what the option price should be. One example could be that traders at e.g. Vattenfall believe that an option's price is too high because of the market assuming its volatility to be higher than it actually is. The traders then calculate their own volatility measure and a new price for the option. After this, they sell the options that are believed to be overvalued by the market.

Another way of using a price model is for *quoting*. Option prices can be quoted in terms of implied volatility, e.g. options with higher volatility should be more expensive than options with a lower volatility. An option is then said to cost more in terms of volatility. When traders quote options, they use the volatility that is implied from the option prices.

To understand what assumptions are made and for what purposes large traders at Nord Pool use Black 76, we have interviewed traders at Vattenfall, Eon and Econ. From these interviews it is clear that the model used by all large traders is indeed Black 76, and that it is the *standard* version of the model (see description below). All traders that we have interviewed say that they do quote the options according to their implied volatility, and price the options in terms of volatility. They also use their own measures of volatility and trade on these assumptions as described above. However, Black 76 is not used for forecasting purposes. When it comes to forecasting futures volatility, traders use their own so called jump models. A jump model can be explained as a model that takes sudden movements in the underlying variable's price into account.

By standard version of Black 76, we mean the model presented in section 3.3. In this version of the model, the assumptions are made under the risk-neutral probability Q. The two most important assumptions are that the underlying variable, i.e. the forward price, follows GBM and is log-normally distributed, and has a volatility that is constant over time.

5. SPOT, FUTURES AND FORWARD PRICES

5.1 Normal Distribution

To study the distribution of the spot prices and the forward prices we perform normality tests. The options at Nord Pool are written on forwards, but forwards and futures can be used interchangeably if interest rates are assumed to be non-stochastic. Therefore, we also perform a normality test for the daily futures prices with one day left to maturity.

The data that is used is the daily average of the hourly spot prices and the futures prices are the daily futures prices. Both the spot prices and the futures prices are from 1 October 2003 – 1 April 2007. The forward prices are for the forwards that the options at Nord Pool are written on i.e. yearly and quarterly forwards. We choose to include the forward for the second quarter of 2007 in this chapter, and this forward was traded 3 January 2005 – 30 March 2007. The graphs for the other forwards traded during our chosen sample period 1 October 2003 – 1 April 2007 are included in appendix 1. In the graphs below, the spot, futures and forward prices are plotted against time. All prices are in euros.

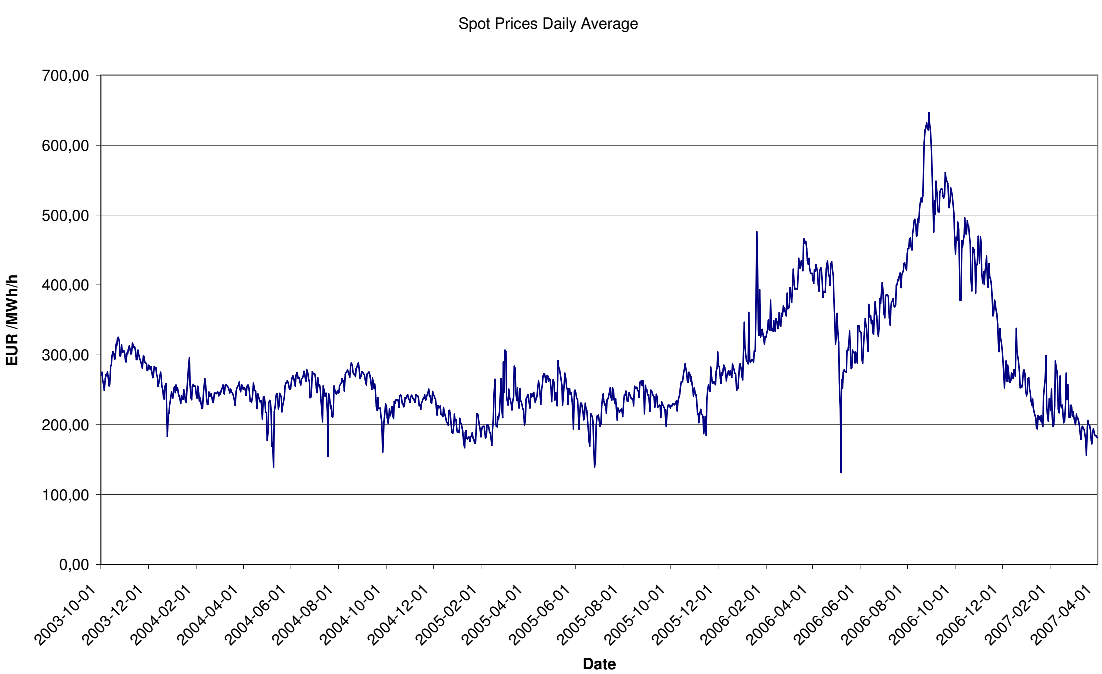

Figure 12: The daily average of the hourly spot prices 1 October 2003 – 1 April 2007.

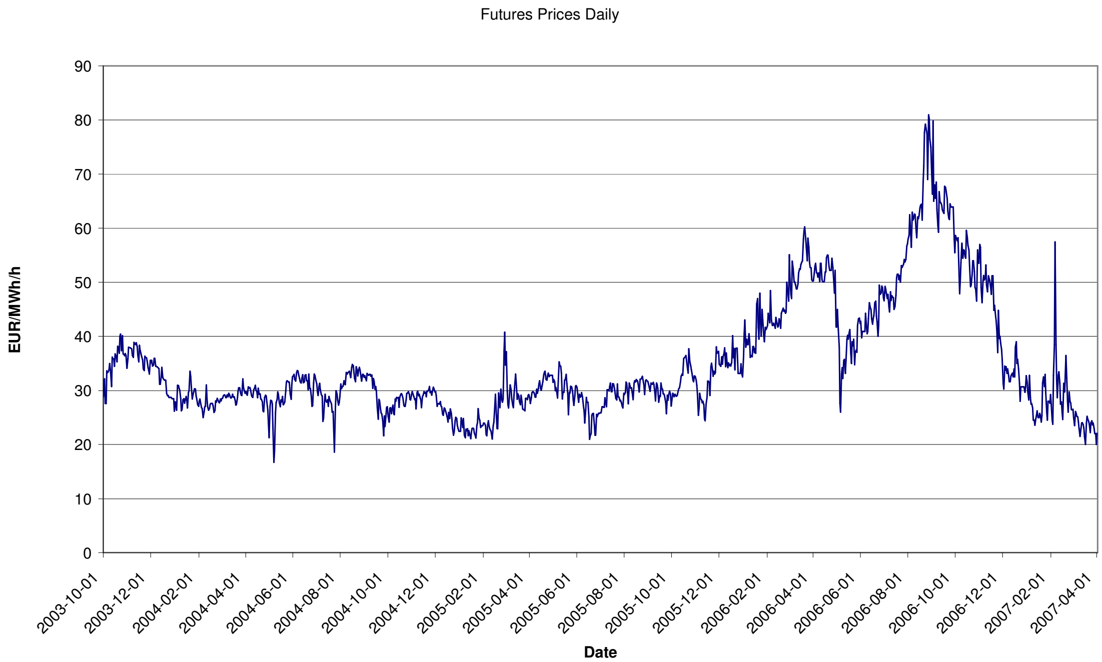

Figure 13: The daily futures prices 1 October 2003 – 1 April 2007 with one day left to maturity.

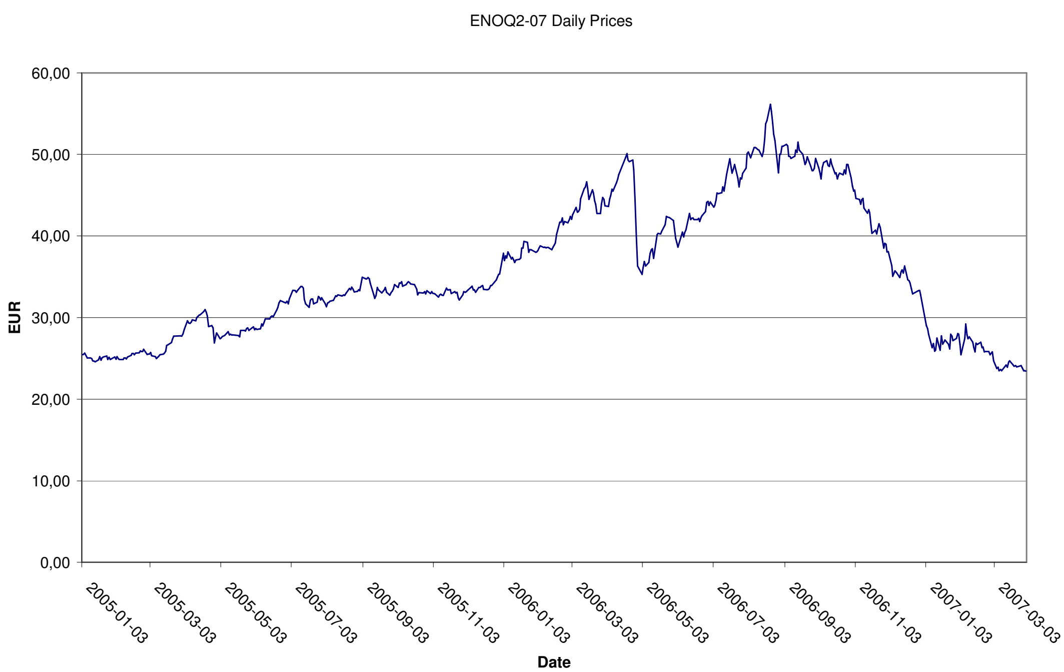

Figure 14: The daily prices for the second quarterly forward 2007. The forward was traded 3 January – 30 March 2007.

In *Figure 12* above, it can be seen that the variations in the spot prices are fairly large and there seem to be many spikes. The futures prices in *Figure 13* follow the same pattern as the spot prices, but seem to have somewhat fewer spikes. *Figure 14* shows the quarterly forward price, beginning in January 2005, and therefore it cannot as easily be seen how it moves in comparison with the spot prices as it can be seen with the futures prices. However, the quarterly forward price does not seem to follow the same pattern as the spot prices and do not have the spikes that are observed in both *Figure 12* and *Figure 13*. One period that contains large fluctuations and high values for the spot prices, futures prices and the forward price is summer and autumn 2006.

Another important observation is that there is a sudden fall in the spot, futures and forward prices during the last days in April 2006. This can be explained by a news release, saying that the level of carbon dioxide within the European Union in 2005 had been proved to bee much lower than earlier expected. The EUA prices fell considerably, and the spot, futures and the forward prices reacted in the same way.

In 3.3, it is mentioned that it should be tested if the forward log-returns are normally distributed. To study to the distributions of the spot prices, futures prices and the forward prices we first log them, and then calculate the first difference of each variable. The reason for taking the first difference and logging the variables is that this corresponds to log-returns.

The first differences of the logged spot, futures and forward prices are shown in histograms below and the normal curve is displayed. The histograms, descriptive statistics and normality tests for the other forwards traded during our chosen sample period are included in appendix 2.

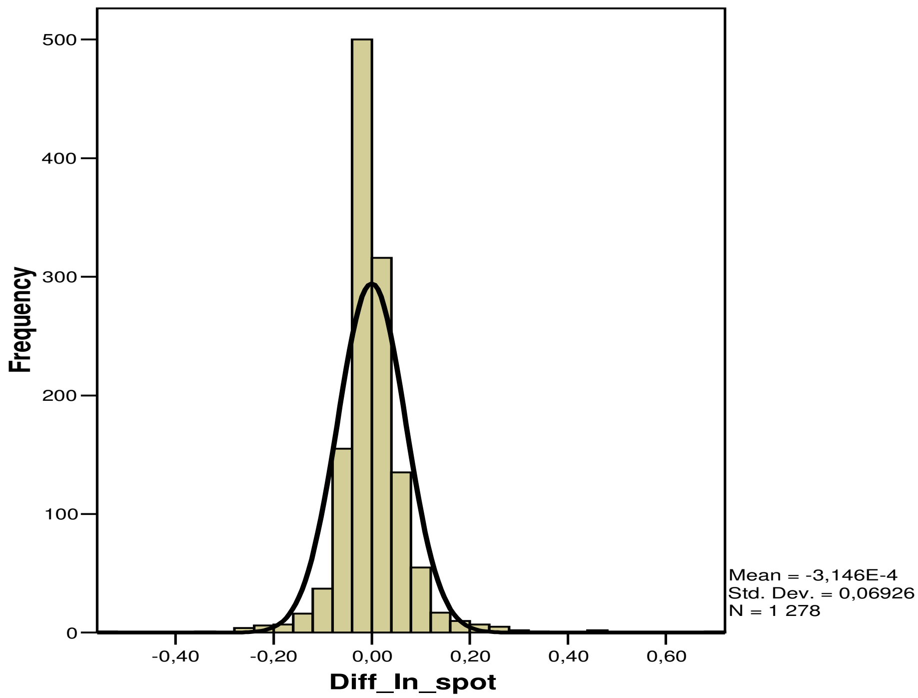

Figure 15: Histogram showing the first difference of the logged daily average hourly spot prices 1 October 2003 – 1 April 2007.

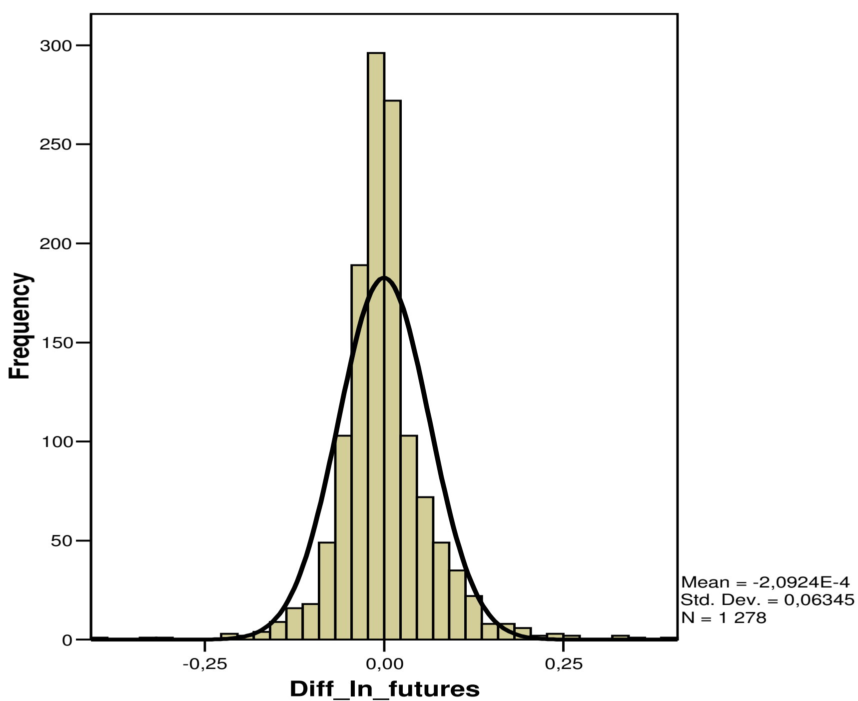

Figure 16: Histogram showing the first difference of the logged daily futures prices 1 October 2003 – 1 April 2007.

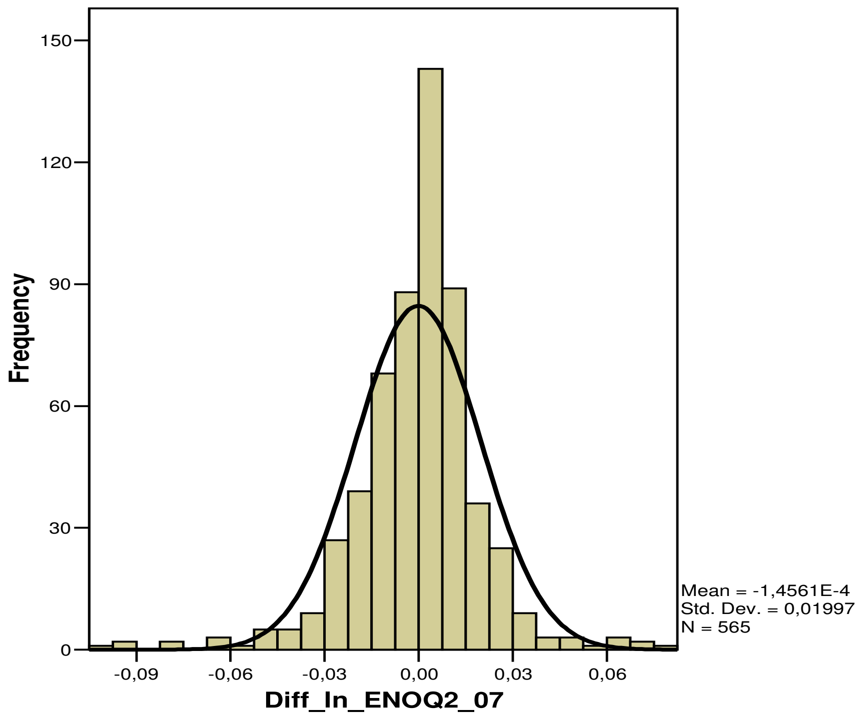

Figure 17: Histogram showing the first difference of the logged daily forward prices for the second quarterly forward 2007.

The histogram for the first difference of the logged daily average hourly spot prices has a lot of kurtosis, but very little skewness. A similar shape can be observed in the histogram for the first difference of the logged daily futures prices, but it contains less kurtosis. The histogram for the first difference of the logged forward prices has more skewness than the other histograms, but much less kurtosis. All the three histograms show skewness in the distributions. The descriptive statistics for the variables are presented in the tables *3*, *4* and *5*.

Descriptive Statistics									
	N	Minimum	Maximum	Mean	Std. Dev.	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Diff ln spot	1278	-0,52691	0,69934	-0,0003146	0,069261	1,106797	0,0684386	16,025536	0,1367709

Table 3: Descriptive statistics for the first difference of the logged daily average hourly spot prices October 2003 – April 2007.

Descriptive Statistics									
	N	Minimum	Maximum	Mean	Std. Dev.	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Diff ln fut	1278	-0,40113	0,4011267	-0,00021	0,0634453	0,5515751	0,0684386	6,280355	0,1367709

Table 4: Descriptive statistics for the first difference of the logged daily futures prices October 2003 – April 2007.

Descriptive Statistics									
	N	Minimum	Maximum	Mean	Std. Dev.	Skewness		Kurtosis	
Diff ln ENOQ2-07	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
	565	-0,10436	0,075650	-0,00015	0,0199695	-0,6494168	0,1027786	4,8524019	0,2051989

Table 5: Descriptive statistics for the first difference of the logged daily forward prices. The forward is the second quarterly forward 2007.

In order to examine if the first differences of the logged spot, futures and forward prices are normally distributed, we perform Jarque-Bera normality tests. The Jarque-Bera test is an asymptotic test for large samples, and has a χ^2-distribution with two degrees of freedom.

The test statistic is:

$JB = n \times [S^2/6 + K^2/24]$,

where S is the skewness statistic value and K is the kurtosis statistic value. The null hypothesis is that the first difference of the logged spot, futures and forward prices are normally distributed respectively. The null hypothesis is rejected if the JB-value is larger than the χ^2 critical value. On the 5%-level, the critical value is 5.991. For the first difference of the logged daily average hourly spot prices, we receive the following value:

$JB = 1278 \times [1.1067974^2/6 + 16.025536^2/24] = 13\,936.47$

Since 13 936.47 > 5.991, the null hypothesis is rejected at the 5%-level. For the daily futures prices, we obtain the following test statistic:

$JB = 1278 \times (0{,}5515751^2/6 + 6{,}280355^2/24) = 2165.13$

Since 2165.13 > 5.991, the null hypothesis is rejected at the 5%-level. For the forward prices, the following value is calculated:

$JB = 565 \times [(-0{,}6494168)^2/6 + 4{,}8524019^2/24] = 594.02$

Since 594.02 > 5.991, the null hypothesis is rejected at the 5%-level.

Even though all of the test statistics are rejected at the 5% significance level, we can conclude that the forward log-returns follow a distribution that is closer to a normal distribution than the futures log-returns do. From appendix 2, it can be seen that this is not always the case. However, the results from studying the forwards log-returns are more interesting than the futures log-returns for two reasons. Firstly, the options at Nord Pool are written on forward contracts. Secondly, the forward contracts are traded over at least six months, while the futures are traded for a much shorter period of time. If the daily futures price for the last day of trading is used, then this is almost the same thing as the spot price itself. The weekly futures are only traded over six weeks. Even though forwards and futures can be used interchangeably for commodity options, the futures contracts are only traded for a very short period of time, and therefore they are not as relevant to compare with the spot price as the forward contracts are.

5.2 Discussion of the Results

From the tests in section 5.1, it can be concluded that neither the spot log-returns nor the forward log-returns are normally distributed. Also, the spot log-returns have a lot more kurtosis in their distribution than the forward log-returns. The distribution of the spot log-returns is far from normal, and this can be said even though the variation in the spot price during one day is not visible since the spot prices we have used are the daily average of the hourly spot prices. From the tests above, it is evident that the assumption of the forward being log-normally distributed is violated. However, the forward log-returns follow a distribution much closer normal distribution than the spot log-returns.

We believe that the results above where somewhat expected. The large variations in the spot price were expected since these can be explained with the differences in the demand for electricity and seasonal differences. We expected the forward prices to be more "stable" and to be much closer to a log-normal distribution than the spot prices. There is no reason to believe that the distribution for the forward prices is the same as for the spot prices or that they would more or less follow the same pattern. This can be explained as an effect of two mean values that the forward price takes into account. By this we mean that while the spot price is immediately affected by hourly and daily changes, the forward has a considerable time left to maturity and is only considering the mean of the spot prices over this time period. In the same way, the forward price considers the mean of the spot prices in the delivery period and not sudden changes.

Another reason for the forward prices not behaving like the spot prices is due to the fact that there is no possible arbitrage relationship between the spot and the forward, as there would be if electricity would have been a financial asset such as an ordinary stock. An ordinary stock can e.g. easily be exchanged for cash at any point in time, which is not true for the spot price on electricity, since the electricity itself at that point in time is not produced or storable. Another implication from this is that since the spot price on electricity is not a financial asset, it cannot be treated as one in a portfolio. In standard arbitrage theory, it is usually assumed that the asset is possible to store, to be sold later on or even short sold. This assumption is not possible to make when electricity is the portfolio asset.

If the asset that a forward contract is written on is a financial asset, then Black 76 can be derived from the Black-Scholes option pricing formula. A great benefit with Black 76 is that it can be used even though the asset that the forward is written on is non-financial asset, such as the spot price on electricity. The forwards that the options at Nord Pool are written on are financial instruments, and our tests in section 5.1 show that the forward prices are much closer to a normal distribution than the spot prices. This could be a motivation for using Black 76.

6. IMPLIED VOLATILITY

6.1 Implied Volatility Smile

The options at Nord Pool are written on forwards and to study the options we collect call option price data. From the option prices we back out the implied volatilities using Black 76 in order to obtain volatility smiles. This is done by plotting the implied volatilities against different option strike prices. The options we use are call options that are written on yearly and quarterly forwards. We choose to include the option written on the yearly forward 2004 and the option written on the first quarterly forward 2007 in the text. The option written on the yearly forward was traded 21 December 2001 – 18 December 2003, and the option written on the quarterly forward was traded 15 June 2006 – 21 December 2006. The volatility smiles for other options traded during our chosen sample period 1 October 2003 – 1 April 2007 are included in appendix 3.

In the graphs below, the volatility smiles for the studied options are shown. To observe the smile, we compare two different days during the option trading period. In *Figure 18*, the option written on the yearly forward 2004 is studied and the two observation dates displayed in the graph are 1 October 2003 and 17 December 2003. The data for the option written on the first quarterly forward 2007 is shown in *Figure 19*, and the observation dates are 28 August 2006 and 5 December 2006.

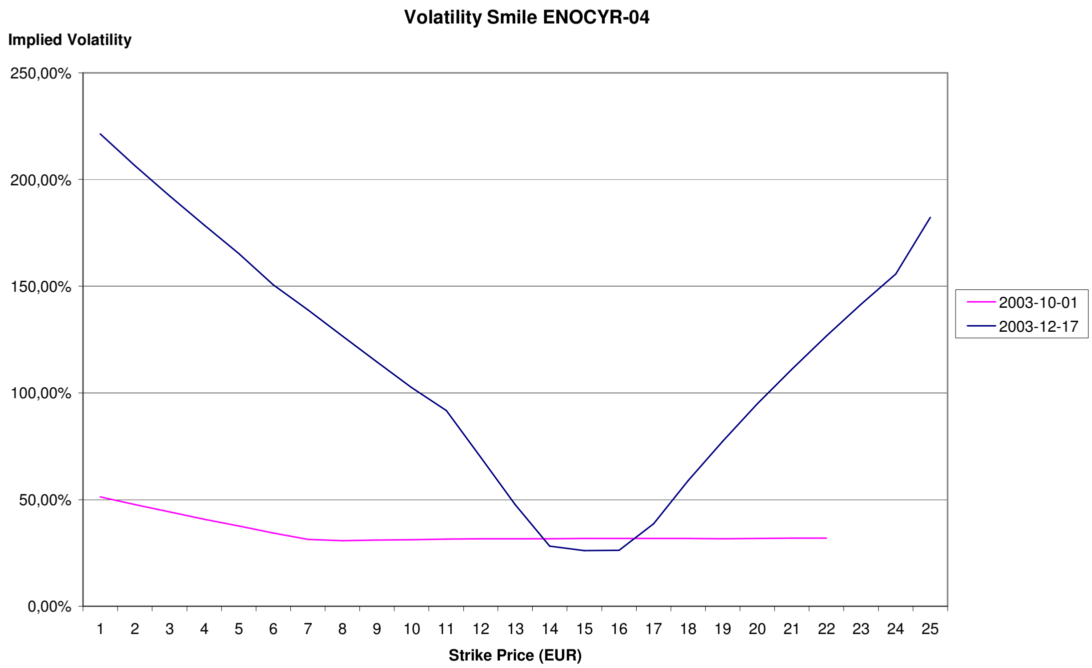

Figure 18: Implied volatility smile of the option written on the yearly forward 2004.

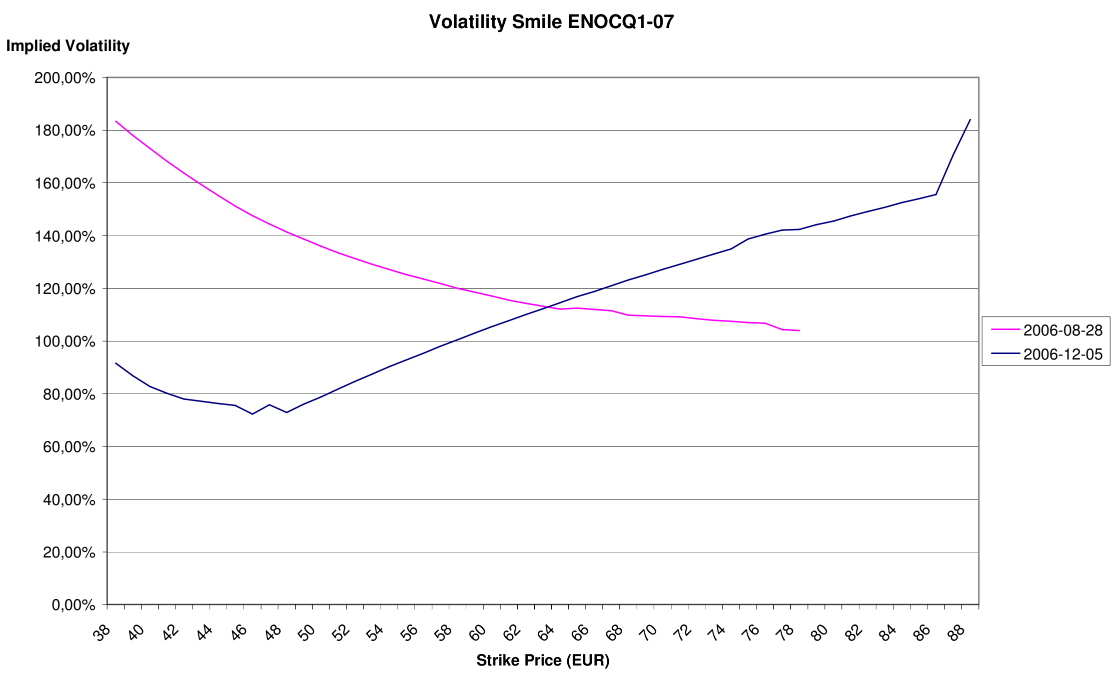

Figure 19: Implied volatility smile of the option on the first quarterly forward 2007.

On the first observation date in *Figure 18*, which is 1 October 2003, the volatility smile is similar to a straight line even though it is observable that the out-of-the money call options have a higher volatility than e.g. in-the-money options. A pronounced volatility smile is observed in *Figure 18* on the observation date that occurs the day before the option expires. In comparison to at-the-money options, options that are deep in-the-money and deep out-of-the-money are relatively illiquid close to maturity. Therefore, a higher liquidity premium is required for these options, which leads to their price being higher in terms of volatility. In *Figure 19*, a ”smirk” is observed on the first observation date almost four months before the option matures. On the second observation date the curve has a more pronounced smile with a heavy right tail.

From *Figure 18* and *Figure 19* it can be concluded that the implied volatilities from the observed option prices plotted against strike prices are nothing like a straight line, especially not close to maturity. This would have been the case if the assumptions of Black 76 were fulfilled. Another important observation is that the implied volatility smiles for electricity options do not have the same shape as the volatility smiles for options written on ordinary stocks. This is due to the large variations in the electricity spot and forward prices. The second observation date in *Figure 19* provides a clear example of this.

6.2 Evolution of the Implied Volatility Smile

To further study the options we look the evolution of the implied volatility smiles. The evolution of the implied volatility smiles is the implied volatility smile plotted against time. This is done in order to test how the price model Black 76 prices the options. In the graphs below, the implied volatility smiles for the options studied in chapter 6.1 are plotted against time. At some points in both graphs, the volatility is equal to zero. This is due to the fact that there are no options prices for those strikes. Evolutions of the implied volatility smiles for other options in our sample period are included in appendix 4.

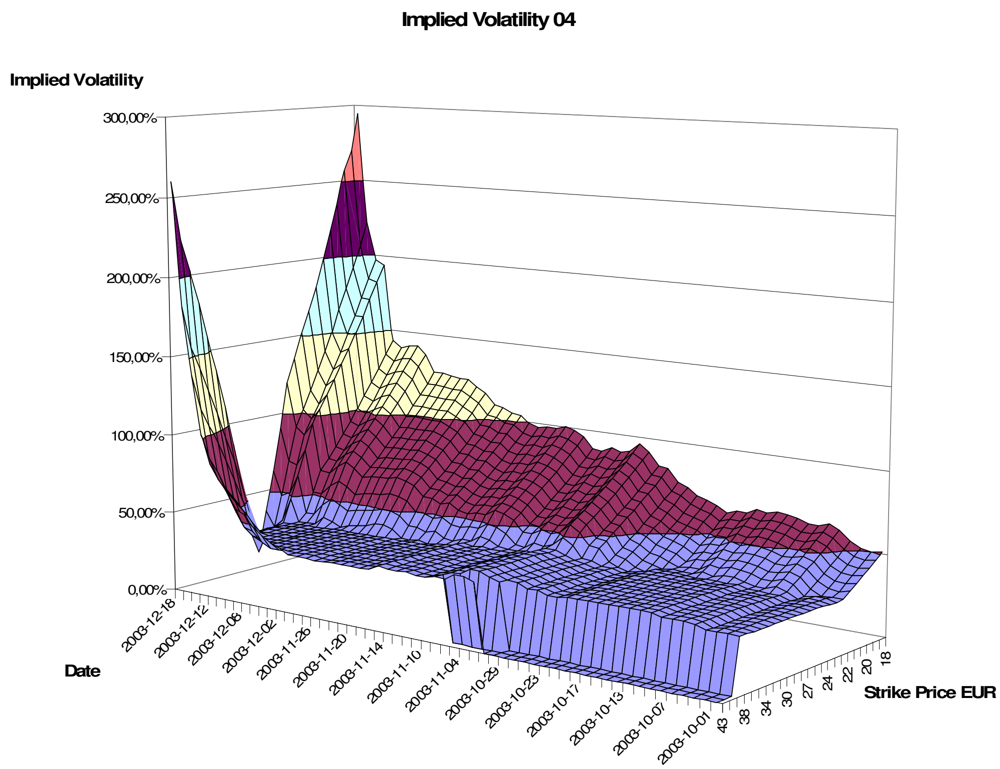

Figure 20: Evolution of the implied volatility smile for the option written on the yearly forward 2004.

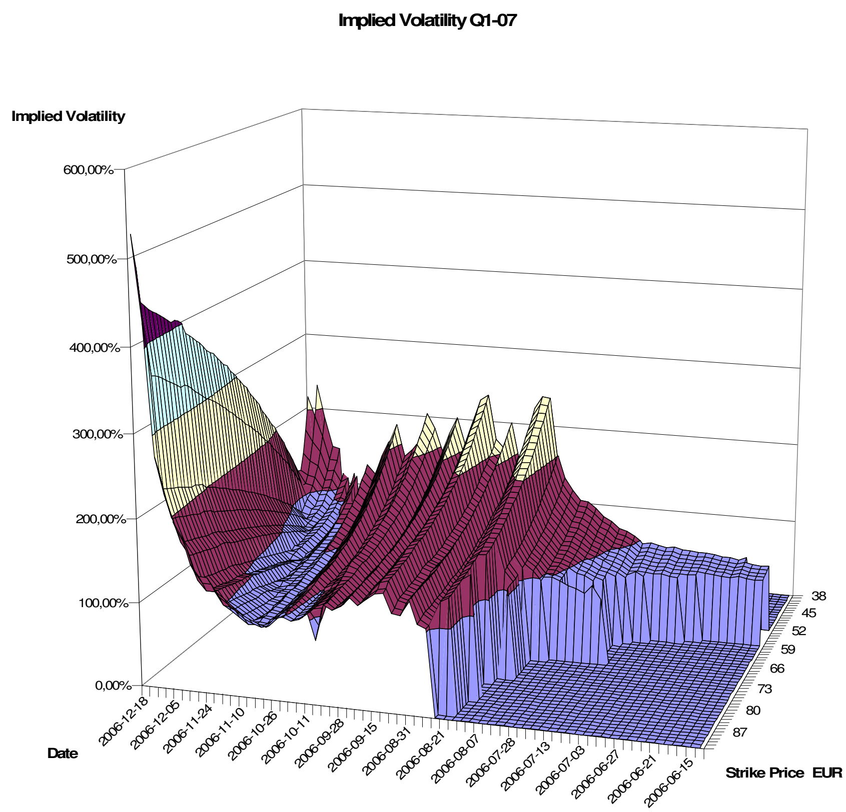

Figure 21: Evolution of the implied volatility smile for the option written on the first quarterly forward 2007.

In *Figure 20*, the evolution of the implied volatility for the option written on the yearly forward 2004 is shown during the last part of the option trading period, which is 10 October 2003 – 18 Dec 2003. At the beginning of this period, with more than two months left to the option expiry date, the volatility surface is fairly flat, and as time gets closer to maturity the smile becomes more pronounced. This it what could have been expected for liquidity reasons as discussed in 6.1.

The evolution of the implied volatility smile for the option written on the first quarterly forward 2007 is shown in *Figure 21*. During the first months of the trading period, the volatility surface is fairly flat, but at the beginning of August, there are changes in the level of the implied volatility smile. This is probably due to the large variations in both spot and forward prices during the third and the last quarter of 2006. The implied volatility smile gets well-pronounced at the very end of the period.

As mentioned in chapter 3, one important assumption in Black 76 is that volatility is constant over time. If this would have been true, the volatility surface in the graphs above should be flat. It is evident from both *Figure 20* and *Figure 21* that this is not the actual scenario. It is clear that the volatility changes as the time to maturity decreases and that Black 76 seems to work fairly well when there is a long time left to maturity. The implied volatility smile gets more pronounced as time to maturity decreases, and Black 76 does not take this fact into account.

In order to understand how large variations in the spot price over a longer time period influence the forward price and hence the option price, we look at the option price curve. The option price curve is the option price plotted against both the option strike price and time. As discussed in 5.2, there is no reason for the forward prices to follow the same pattern as the spot prices. However, if the spot price jumps up and down over a longer period of time this might be the case. The options curves plotted in *Figure 22* and *23* below are for the same options as studied above.

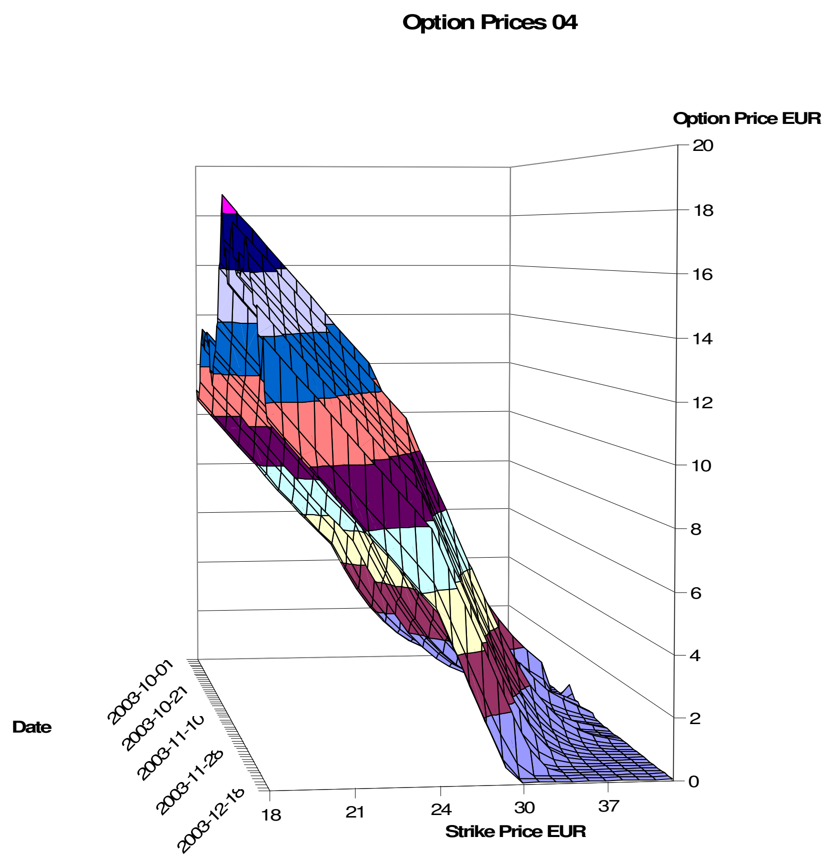

Figure 22: Price Curve for the option written on the yearly forward 2004.

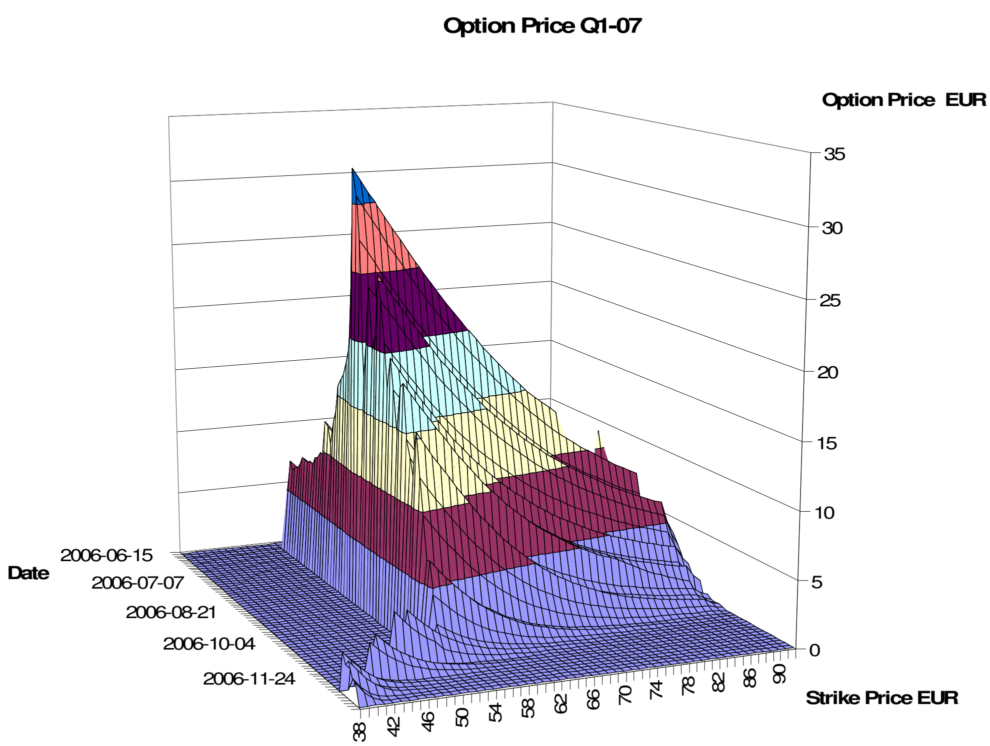

Figure 23: Price curve for the option written on the first quarterly forward 2007.

The price curve for the option written on the yearly forward 2004 is plotted in *Figure 22*. The surface is fairly smooth and shows how the option price decreases as the strike price increases. This is expected for a call option. The price curve shown in *Figure 23*, for the option written on the first quarterly forward 2007, is not as smooth as the price curve in *Figure 22*. This curve has the shape of a tipped cone, which can be explained by the large variations in the spot and forward prices during the third and last quarter of 2006.

6.3 Implied Volatility Term Structure

When studying the volatility smiles and the evolution of the volatility smiles, we look at two options written on different forwards with different strike prices respectively. In order to examine the implied volatility term structure, we plot options with different strike prices and time to maturity and with different underlying forward contracts. The implied volatility term structure shows how the option volatility changes with the option strike price and times left to maturity.

In *Figure 24,* four options written on different forwards and with different time to maturity are plotted. We include one option written on each following forward: the second quarterly forward 2007, the third quarterly forward 2007, the yearly forward 2008, and the yearly forward 2009 respectively. The reason for including only four options is simply because there are no more options traded at the same time at Nord Pool. The option trading dates are shown in *Table 6*. The observation date is 14 March 2007. Other examples of the implied volatility term structure on other observation dates are shown in appendix 5.

Option	First day traded	Last day traded
ENOCQ2-07	2006-09-21	2007-03-14
ENOCQ3-07	2006-12-21	2007-06-20
ENOCYR-08	2005-12-15	2007-12-19
ENOCYR-09	2006-12-21	2007-12-17

Table 6: Trading days for the different options.

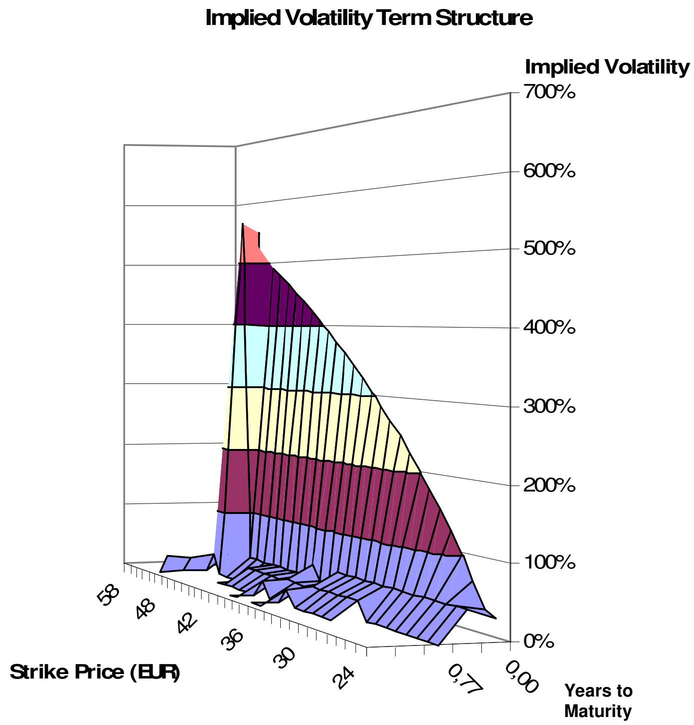

Figure 23: Implied volatility term structure 14 March 2007 at Nord Pool.

Figure 23 implies that the longer time to maturity the lower the volatility. From the implied term structure, it can be concluded that is that the surface is not "flat" as in *Figure 10*, which it should be if the Black-Scholes world assumptions were fulfilled. The implied term structure in *Figure 23* has more similarities with *Figure 11*, but for the surface to be a "smooth" curve as in *Figure 11*, more data is required.

7. CONCLUSION

This thesis studies the options traded at the Nordic power market Nord Pool, and the option pricing model Black 76. From our interviews with different market participants, we conclude that Black 76 is used by most large traders and market makers at Nord Pool. It is used in two ways; normatively, and for quoting options in terms of implied volatility. However, when it comes to forecasting future volatility, traders and marker makers use their own jump models.

The options traded at Nord Pool are written on yearly and quarterly forward contracts. Black 76 assumes that the underlying variable, i.e. the forward, follows a log-normal distribution. From our tests in section 5.1, we draw the conclusion that the forward log-returns do not follow a normal distribution. However, they are much closer to a normal distribution than the spot log-returns, and the forward prices do not contain the spikes that are typical for the spot price. Since there is no simple arbitrage relationship between the forward contracts and the spot price on electricity, and the forward price only is related to the average spot price during the time to maturity and the average spot price during the delivery period, this was expected.

From the call option price data we back out the implied volatilities using Black 76 in order to observe implied volatility smiles. We obtain implied volatility smiles showing that different options trade at different volatility levels. The implied volatility smiles are nothing like the straight line, especially on observation dates close to maturity. We conclude that this would not have been the case if the assumption of constant volatility over time was fulfilled. Due to large variations in the electricity spot price, we can also conclude that the implied volatility smiles for electricity options do not have the same shape as volatility smiles for options written on ordinary stocks.

We plot options with different strike prices and time to maturity and with different underlying forward contracts in order to observe the implied volatility term structure for call options at Nord Pool. We obtain an implied volatility term structure that suggests that the longer time to maturity, the lower the volatility. We can conclude that the implied volatility term structure surface is not flat, which it should be if the assumption of constant volatility for different options and over time was fulfilled. However, our results would have been more robust if we could have studied more options traded at the same time.

In summary, we conclude that a large benefit from using Black 76 as an option pricing model is that it is possible to use even if the asset that the forward contract is written on is a non-financial asset such as the electricity spot price. Black 76 was not derived especially for pricing electricity derivatives, and from our study we find that its assumptions of log-normality and constant volatility over time are violated.

7.1 Further Research

For further studies of the Nordic power market and the traded contracts, it would be interesting to study the following research questions:

- How will the new regulations for pricing derivative contracts change the price and volatility at the market? Is this something that the market already has taken into account or not?

- How do the spot and the financial markets react on different types of information and new governmental regulations? Do the prices of the different contracts react in similar ways?

- Will new innovations, e.g. fuel cells, long-distance heating, and community-owned wind power, change the market conditions?

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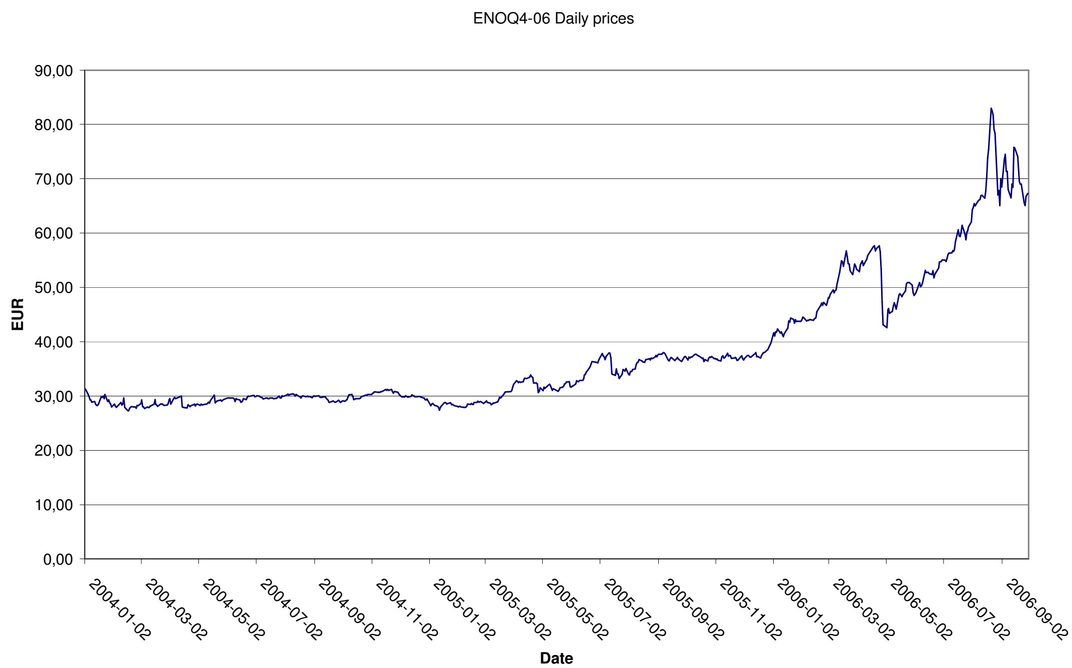
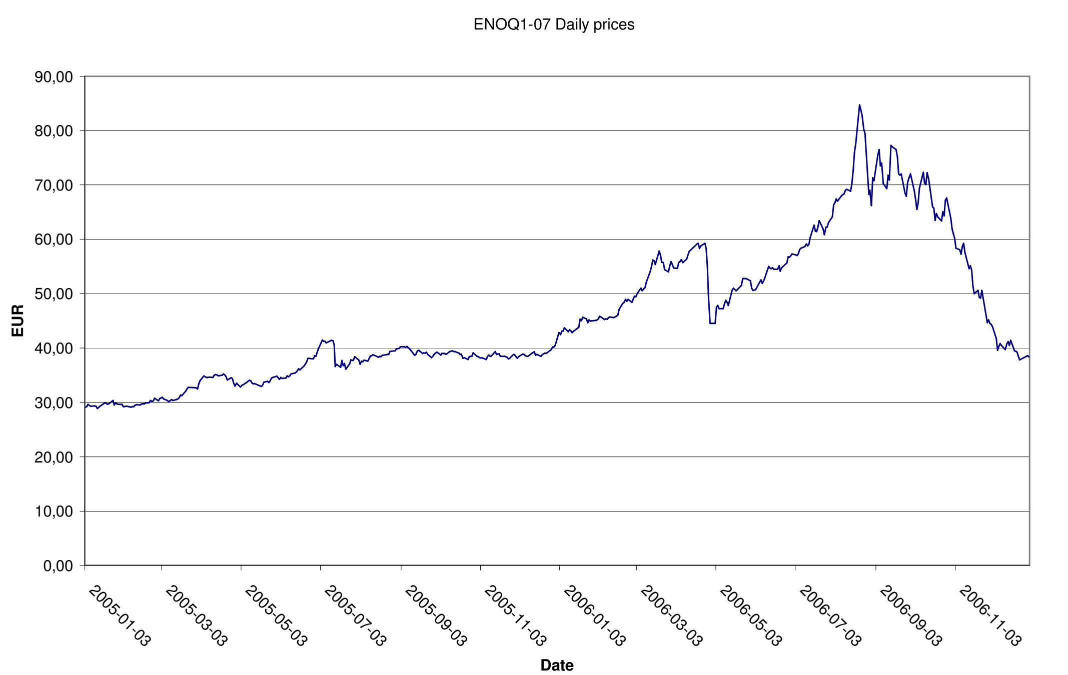
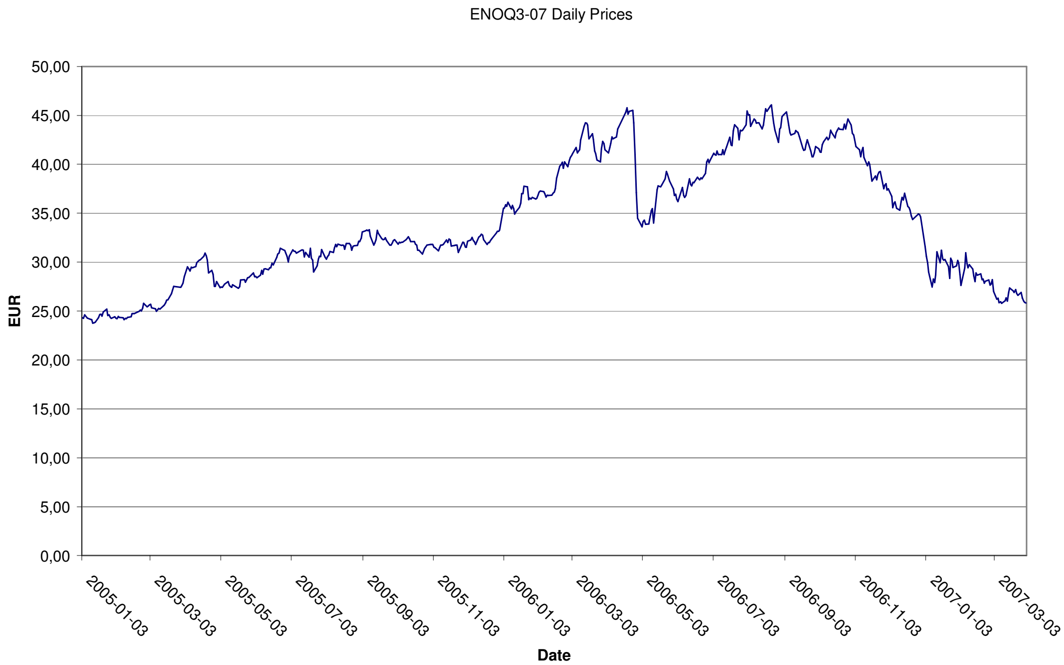
9. APPENDIX 1 PRICE CURVES FORWARDS



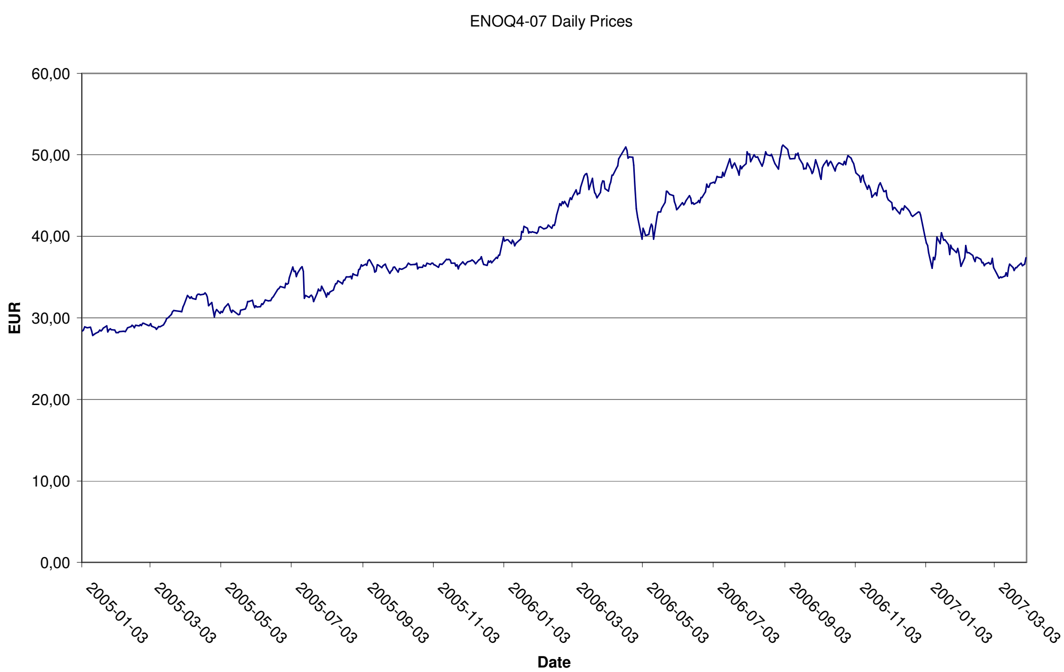
Daily prices for the fourth quarterly forward 2006. The forward was traded 2 January 2004 – 29 September 2006



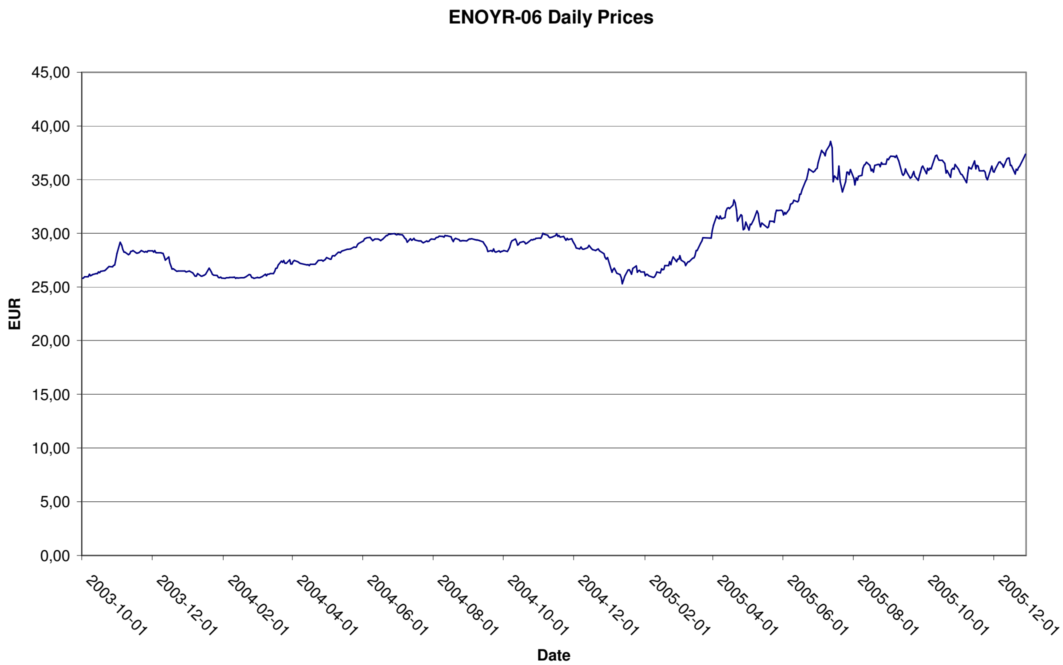
Daily prices for the first quarterly forward 2007. The forward was traded 3 January 2005 – 29 December 2006.



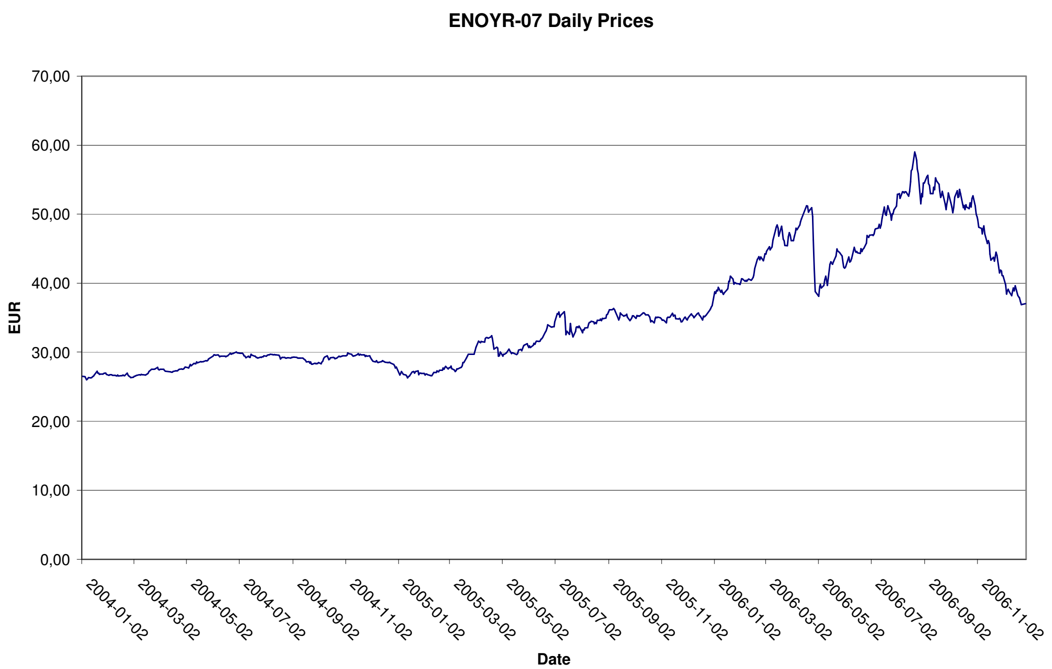
Daily prices for the third quarterly forward 2007. The forward was traded 3 January 2005 – 30 March 2007.



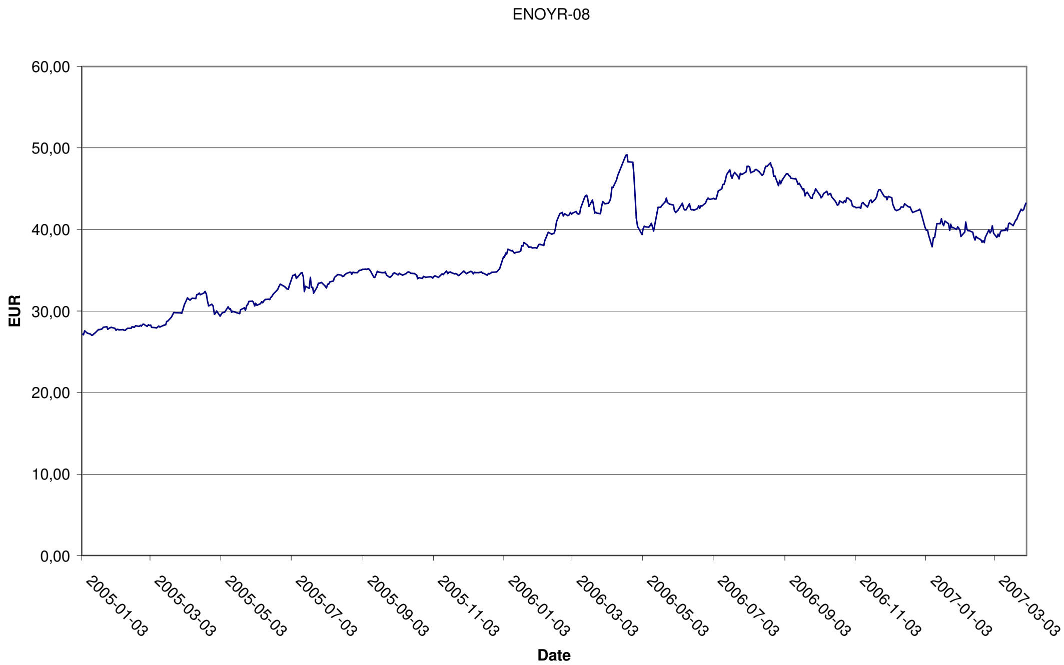
Daily prices for the fourth quarterly forward 2007. The forward was traded 3 January 2005 – 30 March 2007.



Daily prices for the yearly forward 2006. The forward was traded 1 October 2003 – 28 December 2005.

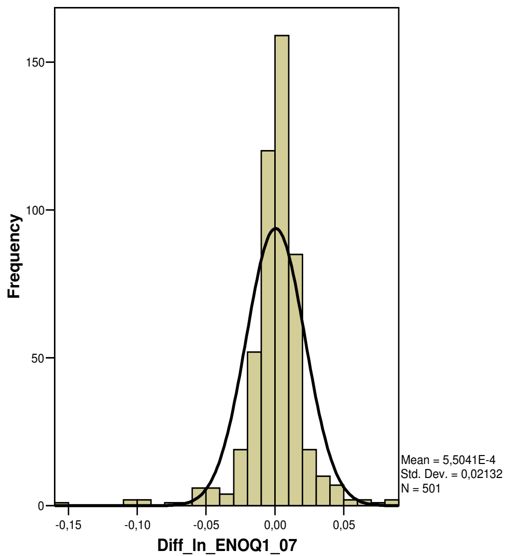


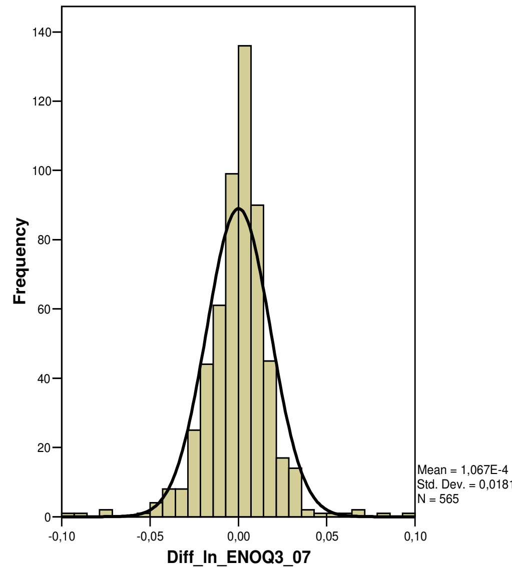
Daily prices for the yearly forward 2007. The forward was traded 2 January 2004 – 27 December 2006

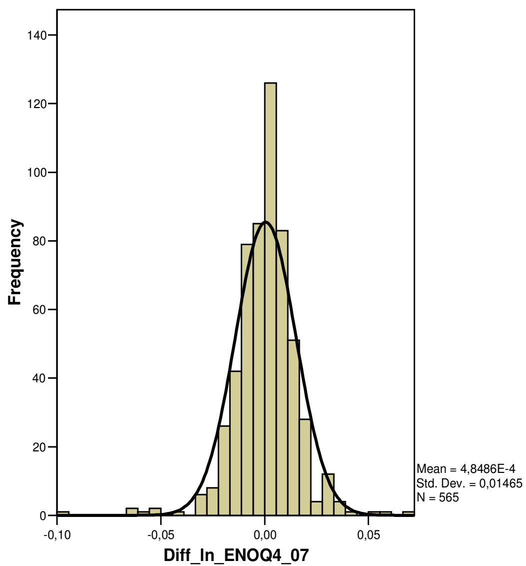


Daily prices for the yearly forward 2008. The forward was traded 3 January 2005 – 30 March 2007.

10. APPENDIX 2 LOG-NORMAL DISTRIBUTION



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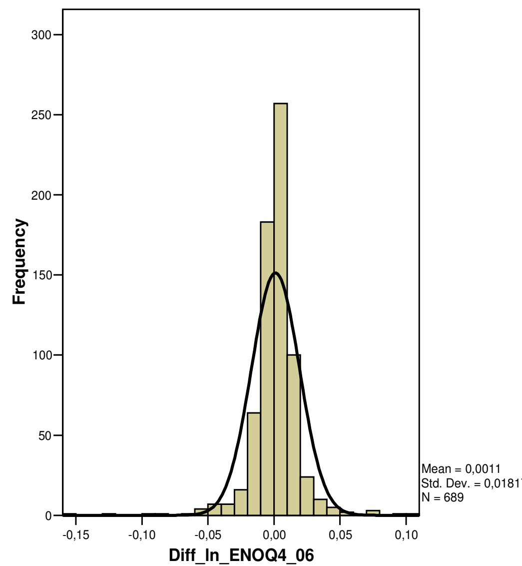
Diff_ln_ENOQ4_07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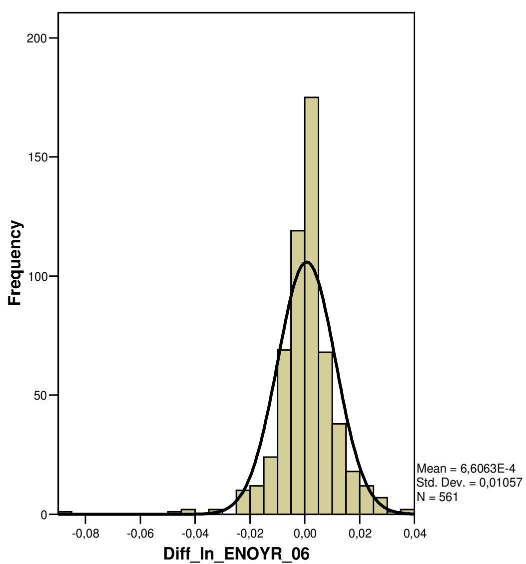
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Std. Dev. = 0,01057
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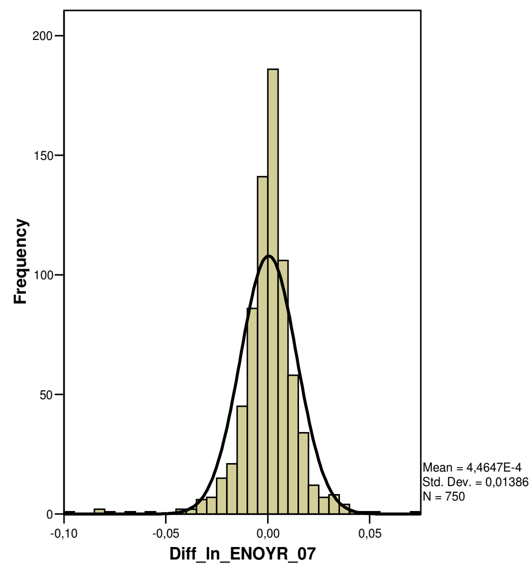
Diff_ln_ENOYR_06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Std. Dev. = 0,01386
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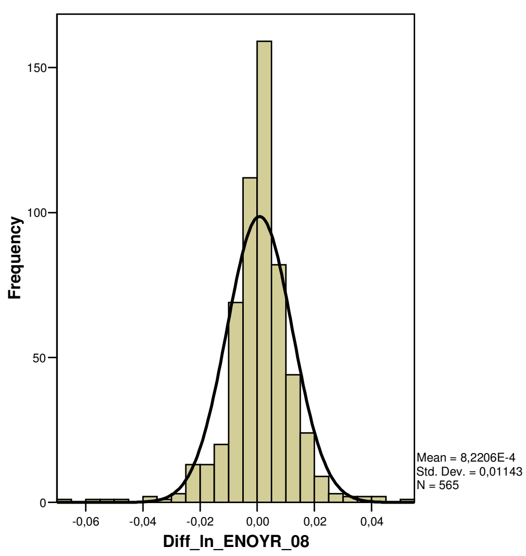
Diff_ln_ENOYR_07


Frequency
Mean = 8,2206E-4
Std. Dev. = 0,01143
N = 565
Diff_ln_ENOYR_08

Descriptive Statistics									
	N	Minimum	Maximum	Mean	Std. Dev.	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Diff ln ENOQ4-06	689	-0,1564933	0,1020656	0,0011145	0,0181695	-1,2374474	0,0931158	15,917172	0,1859647
Diff ln ENOQ1-07	501	-0,1519505	0,0885468	0,0005504	0,0213152	-1,3672972	0,1091094	10,226806	0,2177905
Diff ln ENOQ2-07	565	-0,10436	0,0756504	-0,0001456	0,0199695	-0,6494168	0,1027786	4,8524019	0,2051989
Diff ln ENOQ3-07	565	-0,0999551	0,0953102	0,0001067	0,018097	-0,2815995	0,1027786	5,6347475	0,2051989
Diff ln ENOQ4-07	565	-0,0976097	0,0694131	0,0004849	0,0146523	-0,7075671	0,1027786	6,4599581	0,2051989
Diff ln ENOYR-06	561	-0,0866521	0,0356183	0,0006606	0,0105686	-1,1776821	0,1031425	10,138398	0,2059227
Diff ln ENOYR-07	750	-0,0962826	0,0733516	0,0004465	0,0138587	-1,1594218	0,0892645	9,6098889	0,1782937
Diff ln ENOYR-08	565	-0,0665745	0,0549522	0,0008221	0,0114298	-0,6113023	0,1027786	6,0266522	0,2051989

Jarque-Bera normality tests for different forwards:

ENOQ4-06: $JB = 689 \times [(-1{,}2374474)^2/6 + 15{,}917172^2/24] = 7\ 449.28$

ENOQ1-07: $JB = 501 \times [(-1.3672972)^2/6 + 10.226806^2/24] = 2\ 339.37$

ENOQ3-07: $JB = 565 \times [(-0{,}2815995)^2/6 + 5{,}6347475^2/24] = 754.92$

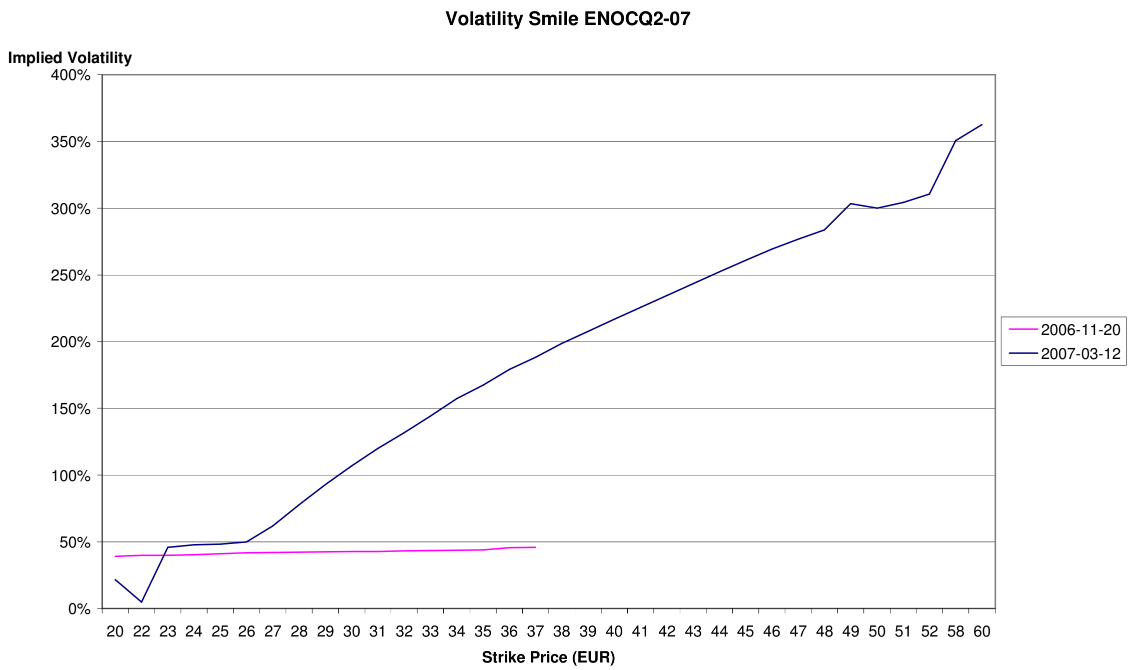
ENOQ4-07: $JB = 565 \times [(-0{,}7075671)^2/6 + 6{,}4599581^2/24] = 1\ 029.56$

ENOYR-06: $JB = 561 \times [(-1{,}1776821)^2/6 + 10{,}138398^2/24] = 2\ 532.33$

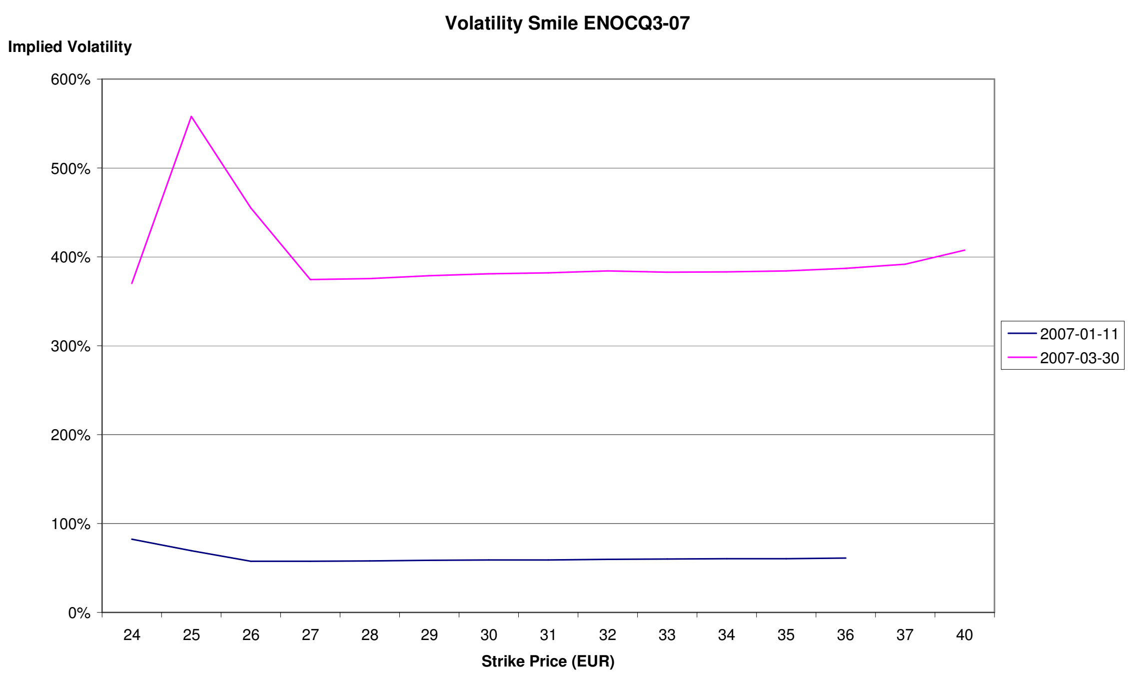
ENOYR-07: $JB = 750 \times [(-1{,}1594218)^2/6 + 9{,}6098889^2/24] = 3\ 053.97$

ENOYR-08: $JB = 565 \times [(-0{,}6113023)^2/6 + 6{,}0266522^2/24] = 890.24$

11. APPENDIX 3 IMPLIED VOLATILITY SMILES

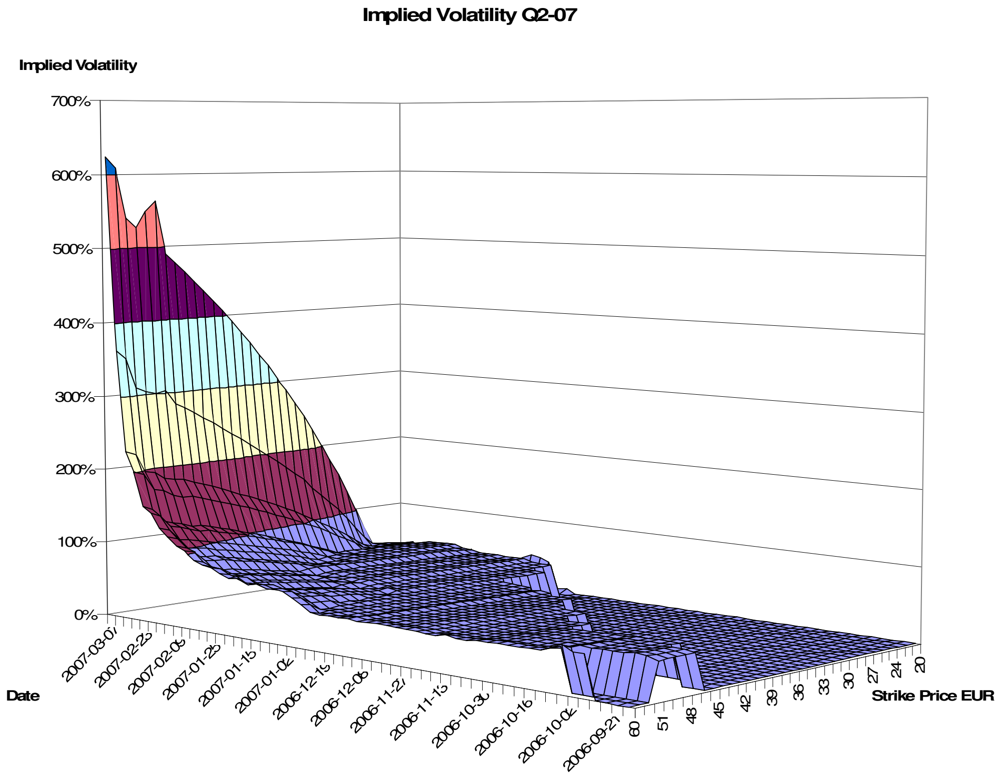


Implied volatility smile of the option written on the second quarterly forward 2007.

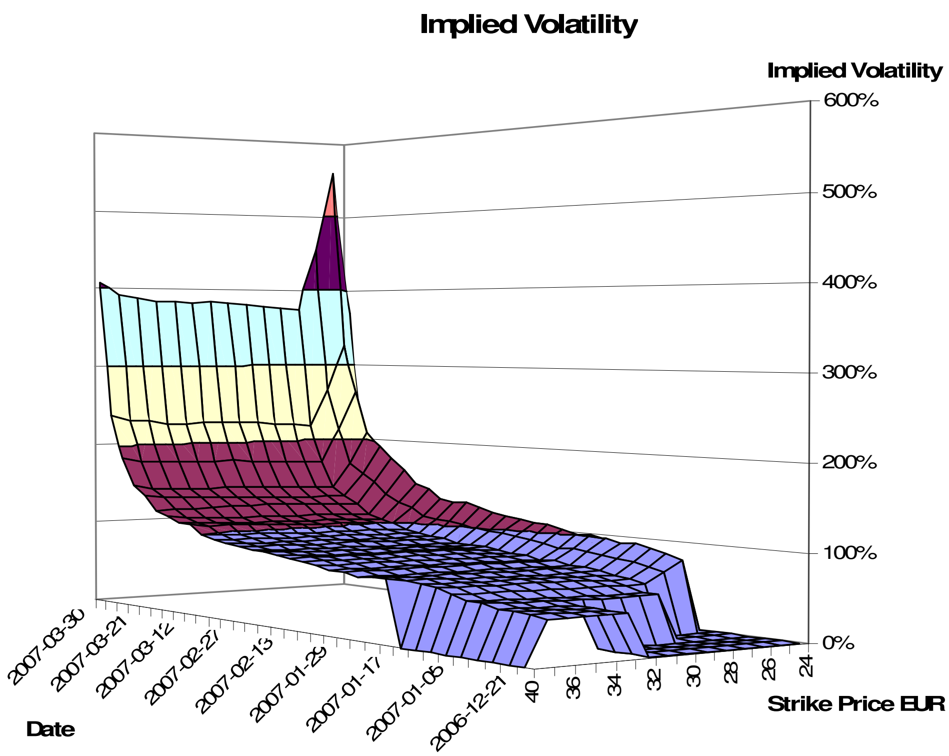


Implied volatility smile of the option written on the third quarterly forward 2007.

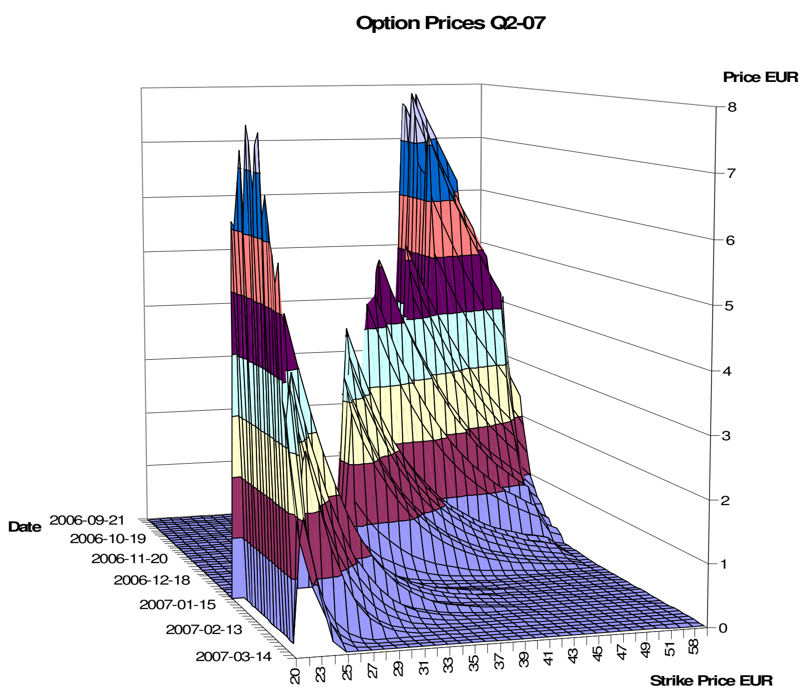
12. APPENDIX 4 EVOLUTION OF THE SMILES



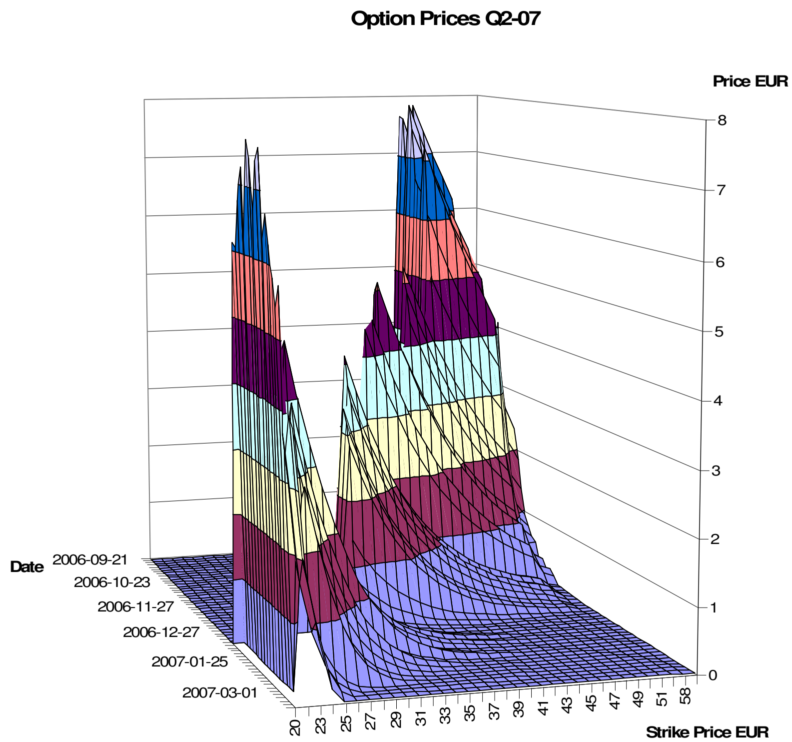
Evolution of the implied volatility smile for the option written on the second quarterly forward 2007.The forward was traded 21 September 2006 – 14 March 2007.



Evolution of the implied volatility smile for the option written on the third quarterly forward 2007.The forward was traded 21 December 2006 – 30 March 2007.

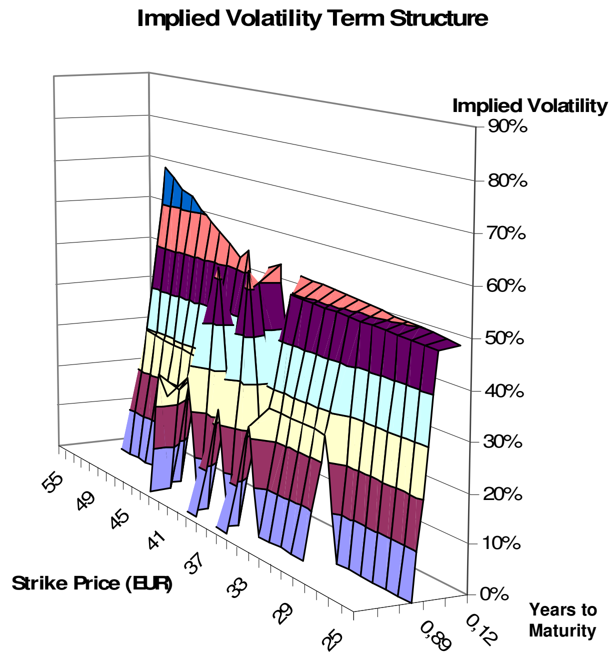


Price curve for the option written on the second quarterly forward 2007. The forward was traded 21 September 2006 – 14 March 2007.

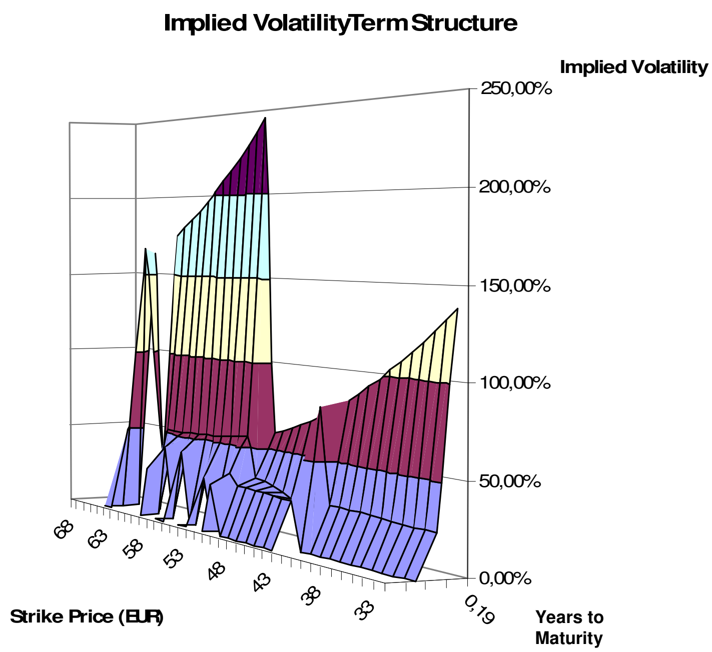


Price curve for the option written on the third quarterly forward 2007. The forward was traded 21 December 2006 – 30 March 2007.

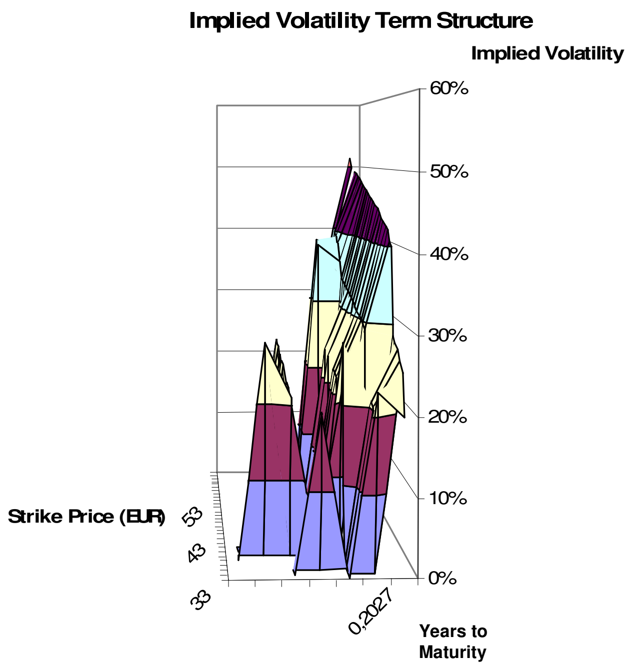
13. APPENDIX 5 VOLATILITY TERM STRUCTURE



Implied volatility term structure 29 January 2007 at Nord Pool.



Implied volatility term structure 11 October 2006 at Nord Pool.



Implied volatility term structure 15 June 2006 at Nord Pool.