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Long term impact of an immigration shock

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Abstract

In this thesis an OLG model is used to estimate long term effects of an low-skilled immigration shock similar to the *Refugee Crisis* faced by the European economies in 2015-2016. The aim is to provide tools and an analytical starting point for rational discussion in the highly politicised debate by evaluating the impacts of the population shock under different policy options. The main finding is that while there are some negative impacts on the economy from the population shock, both in terms of aggregate output and utility, the negative impact is minimal and the different policies mainly differ the outcomes on how the negative impacts manifest themselves on the population.

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1 Introduction

While economic literature has extensively discussed the impact of immigration on the receiving economy, this discussion has had much of its focus on immigration for employment related reasons, where the immigration has in large part been driven by the needs of the labour market. Examples of these kind of immigration patterns come in many: the large scale migration of Europeans to United States in the years following the U.S. Civil War, the Finnish migration to Sweden in the post-WWII decades and more recently the flow of low-skilled labour from the new eastern member states of the European Union to United Kingdom.

However, immigration for other reasons has not caught the attention of mainstream Economics and one of the less discussed themes in the literature is immigration for humanitarian reasons. While refugees have always been present in the modern history, it has only in the past few years became the case that greater flows of asylum seekers have found their way crossing the European borders seeking refuge.

Much of the flow faced by Europe has been focused on few countries, and the number of asylum seekers they have received has had significant change on the economy and the society. At the same time, these countries are faced with the challenge of integrating these newcomers to their economy and must balance these efforts in such manner that the standard of living of the existing population will not decrease significantly to avoid any potential civil unrest.

In an attempt to contribute to this lacking discussion on the newly arisen topic, this thesis outlines an overlapping generation (OLG) model and uses it to study the long term economic impacts of such immigration shock. While the model is not fully calibrated to match any specific European economy, are the parametrisation and calibration done so that the economy resembles an average Western European economy in many of its characteristics. While the intention of this thesis remains to be purely hypothetical, the findings hopefully can later be implemented to analyse real life scenarios and in best case used to resolve the policy dilemma faced by decision-makers in these affected Western European economies.

In order to provide the starting point for a more constructive discussion on a highly politicised topic, a two-fold question is posed; what are the welfare implications of an immigration crisis and specifically, what are the distributional effects between different socio-economic groups i.e. the educated, the non-educated and

the immigrants. The auxiliary question addressed is, whether certain policies yield more preferable[1] outcomes than others. The framework is used to analyse both the immediate shock faced by the economy and the longer term effects of the population influx. This analysis is carried out by using a steady state analysis, meaning that the economy is assumed to be at its steady state at the time of each observation. In order to simplify the analysis of the longer term effects and to quantify the transitional path on which the migrant population is converging to match the characteristics of the native population, the economy is observed at later points of time assuming that it has reached a "transitional steady state"[2]. The analysis is carried out with both assuming the government does not react to the crisis, and with a number of policy tools the government could introduce in order to minimise the welfare impacts of the population shock.

Rest of this section discusses the background of the European Refugee Crisis in more detail and introduces some of the key facts which are later employed in developing the framework and the population shock in particular. After this rest of the thesis is structured such that section 2 looks into the previous literature on the topic, followed by sections 3 and 4 which first establish the model framework and then derive the solutions to the optimisation problem of the agents. Section 5 describes the implementation of the framework on MatLab, providing a schematic overview of the model. Section 6 discusses the parametrisation with section 7 presenting the results of the the initial state of the economy. In section 8 the different responses and long run implications of the population shock are discussed. Finally section 9 provides an overall discussion on the findings of the thesis, elaborates on the shortcomings of the chosen methodology and poses the relevant questions for the future research before concluding on the thesis.

1.1 Background

In the past years the European Union has faced an increased number of asylum seekers entering its area, a phenomenon which has since became known as the European Refugee Crisis. This has ignited a number of heated ongoing debates over the response to the crisis, both on the national and community level, with an aim to finding a solution for solving the crisis while safeguarding the current social and economic cohesion of the European Union. To underline the gravity of this debate, some authors have even taken the bold stance of claiming the Refugee Crisis as one of the underlying causes for the increasing nationalism amongst the

[1] Preferability of an outcome is yet another question, which this thesis does not aim to answer but touches upon.

[2] Imprecision caused by this assumption is discussed at a later point in this thesis.

European nations (Postelnicescu 2016).

Regardless, the crisis constitutes a major challenge, both for the newly arrived and the existing society taking these people in. Not only will the newly arrived face the short term societal exclusion due to reasons like restrictions on their access to the labour market, lack of language skills required or mere prejudice of the locals, but there are also longer lasting effects. According to the OECD (2014), the tertiary educational attainment of the refugees is significantly lower than in Europe. Especially this will have far reaching effects, not only for this generation of immigrants, but also for the future generation, as parental education and societal integration pays significant role in the educational outcomes of the offspring, as will be further elaborated in the section 2.

During the recent crisis, the people who have claimed for asylum have also had a very different age distribution compared to the existing European population; according to the Eurostat (2017a) in 2016 83% of all asylum seekers were under the age of 35, whereas for the Euro-area the Eurostat (2017c) reports the corresponding figure to be 38.3% in 2016. This comes with multisided effects, the younger population may alleviate the looming ageing problem many of the European economies are currently facing, but only if these people are successfully integrated to the society as tax-paying contributors to the society.

2 Previous literature

In this section some of the basic concepts used in this thesis are introduced and discussed to allow for the construction and parametrisation of the model. As the long term impacts of this type of migration shock have not been discussed in the context of academic literature due to recent nature of the events, an overview of discussion regarding the shorter term societal effects is included.

2.1 Overlapping generations model

The overlapping generations model as a term does not refer to any specific economic model, but rather to wide class of different economic models which all can be tracked back to the work of Peter Diamond in 1965, whom in his paper *National Debt in a Neoclassical Growth Model* first described the fundamentals of these models. Common denominator to all different formulations of the overlapping generations model is the utility driven decision making of the economic

agent for solving an inter-temporal allocation problem.

The simple overlapping generations model with two periods was first introduced by Maurice Allais in 1947 (Malinvaud 1987), and what is commonly referred as the most basic overlapping generations model was developed by Paul Samuelson in 1958 (Ljungqvist and Sargent 2012). This very basic model consist of two-period living agents deriving utility from consumption, work for the first period of their life and have the possibility to save some of their consumption good to the next period (Samuelson 1958).

Ideas of Samuelson 1958 were further developed by Peter Diamond, who included neoclassical production function and a government sector to the model, creating a general equilibrium model known as the Diamond OLG-model (Diamond 1965). This fundamental model by Diamond is the one frequently cited as the starting point for using overlapping generations models (McCandless 2008). In the Diamond overlapping generations model agents are introduced with assets which they may purchase in order to save between periods, and the aggregate of these assets forms the physical capital stock employed in the production of the economy (Diamond 1965).

A commonly used and altered variation of overlapping generations model was introduced by Auerbach and Kotlikoff (1987), whose formulation of the model has formed a basis for many modern day overlapping generation models. Their formulation of the model allows for study of number of fiscal issues. (Ljungqvist and Sargent 2012)

From Diamond's work overlapping generations model have developed significantly and are used to analyse a wide array of questions and their macroeconomic impacts. For instance, in academic context this type of models have been used to analysis the public investment in human capital (Annabi, Harvey and Lan 2011) or the impacts of an ageing population on the current pension schemes (Börsh-Supan, Ludwig and Winter 2006). Furthermore, impacts of immigration have also been studied before using overlapping generations model. Shimasawa and Oguro (2010) For a number of more applications of overlapping generations model uses we refer to the list provided by Ljungqvist and Sargent (2012).

Overlapping generations models can also be used a tool to estimate the economic impacts of proposed changes of economic policy. The more elaborate tools used frequently by the public sector in their policy estimation, the DSGE models, such

as the *RAMSES II* of Sveriges Riksbank or MIMER of the Swedish Ministry of Finance, share common foundations in the work done by the early pioneers of overlapping generations models, and can be considered a type of overlapping generations model (Adolfson et al. 2013)(The Swedish Ministry of Finance 2016).

2.2 Immigration

While the impact of immigration has caught a fair amount of discussion in the past, the results of this should be taken with a pinch of salt in the context of this theses, as vast majority of these immigration flows have been a result of voluntary immigration (LaLonde and Topel 1997) whereas this thesis aims to discuss the outcomes of an involuntary immigration.

The economic impacts of immigration have been subject to a range of both empirical and theoretical studies. Several of the empirical studies conclude that while a large number of immigrants entering the economy in short period may cause short term distortions on the labour markets and overall economy as summarised by LaLonde and Topel (ibid.). Dustmann, Fabbri and Preston (2005) in their study on the British labour market also find that there are no overall effects on the economy, but that some localised negative effects on labour markets for low skilled labour come from immigration.

Immigration comes also with long term impact through the integration of the immigrants. While LaLonde and Topel (1997) reports that immigrants who come from relatively similar backgrounds to the host country, they point out that Canadian findings are showing that immigrants of non-European origin integrate significant slower to the population mean compared to those of origin closer to the Canadian society.

Problems of integration are not only constrained to the immigrating generation, but also their descendants who have been shown to have significantly worse economic outcomes. Liebig (2007) has found that in Denmark the lower educational attainment as well as lower labour market participation rates are observed amongst the second generation immigrants than their native-Danish counterparts. His findings show that the lower outcomes are not only a result of the parental educations impact, but also coming from lack of networks which play key role in societal integration. Similar findings are also reported from the United Kingdom, France and Germany, where they also found that not all second generation

immigrants are the same but the persistence of their parental education depends also on the origin of the parent. Especially children of Middle-Eastern origin showed persistence of integration related issues. (Algan et al. 2010)

2.3 Social mobility

In terms of social mobility, the immigrants are in a disadvantaged position as shown by Borjas (2006). According to his research nearly half of the differences observed between groups of immigrants and the native population persist between the generations. The same paper further argues that the lower socio-economic status of some ethnic groups persists for longer if the initial starting point is further away from the native mean due to environmental factors.

Role of parental education and socio-economic status as determinants of the educational outcomes of an individual have been known for long (Sewell and Shah 1967). Both the educational level of the parents, and the disposable income in the family seem to significantly contribute to the offspring's education according to the findings of Solon (2004).

While socio-economic status of offspring is in large part "inherited" from the parents according to Carvalho (2012) as a result of early conditions in childhood, Deary et al. (2005) argue for the importance of improved access to education as a main driver of improvement in individuals socio-economic status.

3 The Model

The framework used in thesis is a straight-forward computable general equilibrium overlapping generations model $CGE - OLG$-model. The model below has been derived from the overlapping generations models described in Auerbach and Kotlikoff (1987) and McCandless (2008) using consumption and leisure as factors of agent utility, with some simplifications made into the model to target the question at hand in the most simple yet effective manner. The model has also been included an endogenous decision to undertake education. In rest of this section, the model framework selected is presented and reasoning for choosing the framework in question are first discussed, followed by detailed description of the model framework.

As in this thesis, our intention is to study the impact of a refugee surge in a model

economy and provide an analytical approach to the heated discussion of the long term impact of the so called *European Refugee Crisis* from the perspective of the receiving economies. In order to study this, an *overlapping generations model* with utility maximising heterogeneous economic agents is used. These economic agents in this modelling framework differ from each other with respect to their abilities and derive utility both from consumption and leisure; consequently facing the decision of optimising their respective consumption and hours worked. Furthermore, at the first period of their lives the agents must decide whether to engage in education for an additional period, or to enter the labour market uneducated with a lower productivity.

We have chosen to use the $CGE-OLG$-model for number of reasons. Firstly, overlapping generation models are widely used to evaluate the implications of different policy options, e.g. Börsh-Supan, Ludwig and Winter (2006), Radulescu and Stimmelmayr (2010), Lledo (2005) and Annabi, Harvey and Lan (2011), especially when addressing the question of how heterogeneous populations are affected by these implications (Zodrow et al. 2013) and are suitable for estimating the long-run equilibrium of the economy(Murphy 2017). Secondly, the overlapping generations framework allows for studying the life-cycle behaviour of the economic agents. While this thesis does not examine the inter-temporal behaviour of the agents, this feature allows for the endogenous determination of education and changes in actualisations of the agents' ability. Furthermore, the $CGE-OLG$-model allows for population dynamics -related effects, which in turn form one of the underlying mechanism through which the immigration shock is expected to effect the economy (Georges et al. 2016).

As the main interest is to model the effect of a refugee surge in an European economy, the best fitting approach is to employ the *small and open economy* assumption, which implies that the interest rates in the economy are exogenously determined and any changes in the domestic capital stock will remain meaningless for the interest rate. Key reasoning behind the use of *small and open economy* assumption is that the member states of the European Union which have faced the greatest surge of immigrants, such as Germany or Sweden (BBC 2016), are either members of the monetary union using Euro or economies having close ties to the monetary union, thus being heavily dependent on the other economies impeding their own ability to have full control over interest rates.

In the model, there are 20 periods, each lasting for three years. With regards to the economic agents, this means that they emerge in the economy at the age of

20 and with full certainty live to see the 80th year of their lives, perishing at the end of that year. The production in the economy is simple, characterised by a standard Cobb-Douglas production function. Alongside the production and the economic agents, the framework has a government with a mechanism in which it collects taxes on the economic activities carried out by the agents and redistributes the proceeds as pension and social security payments to the agents.

The modelled economy is studied at starting from its steady state. The population dynamics of the economy have reached a point where number of people and the age distribution have became static. Each agent has one offspring at the same age resulting in an economy in which the population replenishes itself. Using this simplification of static population size has no impact on the economy itself, as at the steady state trajectory the per capita variables remains constant in time.

The remainder of this section describes each component of the economy in more detail and is structured as follows: first the production side of the economy is discussed in subsection 3.1, followed by discussion on the government and the economic agents in subsections 3.2 and 3.3 respectively. Finally the refugee surge, or the shock, to the economy is introduced in subsection 3.4 which concludes the model framework set up.

3.1 The production side

In the model framework chosen, the production is characterised by a standard Cobb-Douglas production function as presented by Cobb and Douglas (1928) with constant returns to scale and capital and effective labour as the production inputs. The production function is the following:

$$Y_t = \bar{A} K_t^{\theta} H_t^{1-\theta} \tag{1}$$

In which Y_t is the output of the economy, measured in units produced, $\bar{A}$ a total factor productivity multiplier, K_t and H_t are the aggregate stocks of capital and effective labour respectively. The parameter θ is the output elasticity of capital. In our model framework, the aggregate stock of effective labour H_t is further be defined as

$$H_t = \sum_{i=1}^{N} l_{i,t} \sigma_{i,t}$$

Where N is number of agents in the economy, $l_{i,t}$ labour supplied by the agent

and $\sigma_{i,t}$ the productivity of the agent, which in turn takes the value of either one for the uneducated agents or the ability of the educated agent.

The production in the economy is assumed to consist of an infinite number of small producers which in turn implies that none of these producers are unable to exhibit market power. Consequently, the production sector is perfectly competitive and wages are determined by marginal product of labour, which equals to each agents productivity. The rental rate of capital, i.e. the interest rate, in the economy is taken as exogenously given following the assumption of *small and open economy* and not calculated. As the agents are heterogeneous with respect to their ability which in turn translates to efficiency by their educational choice, the wages cannot be derived straight from the Cobb-Douglas production function. Instead the function would have to be set for each individual separately. For reasons of space and interest this step is skipped and the profit maximisation function is shown in its generalised form.

$$\Pi_t = \bar{A} K_t^{\theta} H_t^{1-\theta} - i_t K_t - w_t H_{t,i} \tag{2}$$

In order to solve for the marginal product of labour the first derivative of profit function (2) above is taken with respect to $H_{t,i}$. $H_{t,i}$ represents the effective labour supplied by the agent, equalling to hours worked multiplied with the agents efficiency.

$$\frac{\delta Y_t}{\delta H_t} : (1-\theta)\bar{A} K_t^{\theta} H_t^{-\theta} - w_t = 0$$
$$w_t = (1-\theta)\bar{A} K_t^{\theta} H_t^{-\theta}$$

Wage of the uneducated agent with efficiency equal to one is normalised to equal to one, and accordingly the wages of the educated agents are equal to their respective productivities.

$$w_{t,i} = \begin{cases} \sigma_i & \text{if agent participates in education} \\ 1 & \text{if agent does not participate in education} \end{cases}$$

3.2 The public sector

In the model framework, the role of the government is reduced and the sole purpose of the government in the economy is to act as a redistributive mechanism between the agents. On one hand it has the duty of collecting taxes from labour related income, consumption and capital gains of the economy. On the hand the

public sector is responsible for reallocating the tax income back to the agents in form of pensions and social security contributions.

The public sector faces constraints on both borrowing and savings and must therefore balance its budget at all times. The government income G_t^{inc} must be equal to the expenditure G_t^{exp}. As the income of the public sector comes from the taxes collected, we have the following income equation for the public sector:

$$G^{inc} = \sum_{t=1}^{T} \sum_{i=1}^{N} \{l_{i,t} w_i \tau_w + a_{i,t} i \tau_k + \tau_c c_{i,t}\}$$

Where l, w, a and c are the respective labour supply, wage, assets and consumption of each agent.

The expenditures of the public sector consist of the pension payments and the social security contributions made to the agents. Pension payments are equal to zero for all periods when the agent is an active participant of the labour force, for the periods during which the agents are retired, all agents get an equally large pension payment. Social security contributions are paid to agents who have not taken part in further education and are intended to simulate subsidies paid to low income households and to make the taxation effectively progressive. The social security contributions are paid while the agent, not having engaged in education, is still in employment. This implies the following government expenditure equation:

$$G^{exp} = \sum_{i=1}^{N} \sum_{t=16}^{T} p_{i,t} + \sum_{i=1}^{N_{no\_educ}} \sum_{t=1}^{15} s_{i,t} \tag{3}$$

In which p and s denote pension payments and social security contributions respectively and N_{no_educ} the number of agents lacking further education.

As the government is funded only through the taxation on a Pay-As-You-Go model, an allocation of different sources of tax needs to be set for the government exogenously.

3.3 The economic agent

The economy consists of N utility maximising economic agents, whom are heterogeneous with respect to their ability parameter $\phi_{i,t}$. In all other aspects the agents form a homogeneous population with all having a life span of 80 years after which they all perish in a timely manner. The model only looks at the

agents who have passed their compulsory full-time education, thus having the agents emerging in the economy only at the age of 20, instead of emerging from the birth.

Upon their entry to the economy, the agents have the perfect information regarding the whole economy, including their and everyone else's ability, and are faced with the decision of either engaging in another period, or three years, of education or entering the labour market straight away upon their entry to the economy. The gain from one excess period of education is quantified by increase in productivity, which for the educated agents is set to equal the agent's ability. For the agents not engaging in education efficiency is set equal to one.

The agent behaviour is utility maximising, and each of the agents sets to maximise their own life-time utility which they derive both from consumption and from leisure. Each of the agents is endowed with one unit of time which they may use either use for labour and creating income, or leisure which in turn provides the agent with utility.

$$U(c_{i,t}, 1 - l_{i,t}) = \sum_{t=1}^{20} \beta^{i-1} u(c_{i,t}, 1 - l_{i,t})$$

We have selected utility function of the agent to be iso-elastic both with respect to consumption and leisure, which implies the utility to take the following functional form where A represents consumption c and leisure $1 - l$ respectively :

$$u(A_{i,t}) = \begin{cases} \frac{A_{i,t}^{1-\eta} - 1}{1-\eta} & \text{if } \eta \neq 1 \\ ln(A_{i,t}) & \text{if } \eta = 1 \end{cases}$$

For simplicity we select η equal to one in order to get logarithmic utility for both consumption and leisure. Furthermore, weights α and $(1 - \alpha)$ are used respectively to weight the agents preferences between leisure and consumption derived utility. Thus the following utility function is used:

$$U(c_{i,t}, 1 - l_{i,t}) = \sum_{t=1}^{20} \beta^{t-1} \left[\alpha ln(c_{i,t}) + (1 - \alpha) ln(1 - l_{i,t})\right] \tag{4}$$

In each period, the economic agent faces two constraints. First the agent must always have a balanced budget. Second the agent is unable to work negative hours or more than the one unit of time allocated for each period. The first constraint implies that their expenditure must equal to their streams of income,

alternatively that their net consumption today and assets saved for tomorrow need to equal to net of capital, labour and pension income, and social security payments.

$$(1+\tau_c)c_{t,i} + a_{t+1,i} = [1 + (1-\tau_r)i - \delta]\, a_{t,i} + (1-\tau_w)w_{t,i}l_{t,i} + p_{t,i} + s_{t,i} \qquad (5)$$

Where τ_c, τ_r and τ_w are tax rates for consumption, capital gains and labour income respectively and δ depreciation of capital. Any pension or social security contributions are not subject to taxes. As the government levies taxes on all assets, including the negative ones, this effectively acts as a government subsidy to cover some of the costs of the lending.[3]

The second constraint for working hours simply implies that:

$$1 \geq l_{t,i} \geq 0 \qquad (6)$$

The agents face no credit constraint during their lifetime, which means they may compensate their lack of income by borrowing against their future income. An additional restriction is placed on the agents such that no inheritances or bequests in the economy take place which gives us the following condition:

$$a_1 = a_{21} = 0$$

3.3.1 Educational decision

As mentioned previously, the economic agents are heterogeneous in terms of their ability parameter ϕ_i which measures their aptitude to learn. The decision the agent must make in the first period of their economic activity is whether they should take part in one more period of education, or if they should directly enter the labour market. If they decide to remain in education, they incur utility cost in two different ways. Even though the education is free, there is a direct financial cost of lost income in the first period, and a loss in utility from leisure as most of the agents time will be used by the participation in education. The time it takes for the agent to undertake education is a function of their ability:

$$l_{i,1} = 1/{\phi_i}^4$$

[3]Negative tax on lending can be considered reasonable as many of the European countries, such as Sweden and Finland, allow tax deductibles on interest rates paid and the net effect of direct negative taxes on interest payments of lend assets is equal to the effect of a tax deductible.

[4]Although the variable $l_{i,1}$ is used to denote hours spend on education, it should be noted that this time does not count towards total labour in the economy nor does the agent gain any labour income from time spent in education. The denotion is used for shorthand as leisure is

As previously stated, the agents productivity parameter depends on the educational decision, and is the following:

$$\sigma_i = \begin{cases} \phi_i & \text{if agent participates in education} \\ 1 & \text{if agent does not participate in education} \end{cases}$$

3.4 The refugee surge

As our intention is to provide an analytical approach to exploring the long term impact of a large scale surge of refugees using the framework, the mechanics of constructing this shock need to laid down.

Following from the fact that, we have no evidence of underlying intellectual differences amongst different groups of people, the migrant population will be identical to the native population in terms of the distribution of their ability parameter $\phi_{i,t}$. However, this population differs from the native population in terms of realisation of the educational decision. Their ability parameter is affected by a catch-all parameter ω[5], which will reduce the realisation of the agents' ability and also productivity as consequence of it. The productivity of the migrant agents has the following realisations:

$$\hat{\sigma}_i = \begin{cases} \phi_i \times \omega & \text{if agent participates in education} \\ \omega & \text{if agent does not participate in education} \end{cases}$$

These differences in outcome arise from number underlying characteristics of the economies, for instance differences in efficiency of the educational system, costliness of education or simply lack of organised society capable of providing the agents with facilities needed to fully realise their ability. Despite of the discussion on reasons behind different educational attainments between economies being important, this line of thought is not elaborated any further in this thesis, as the focal point of this thesis are the consequences arising from this rather than the origin of the problem.

The first generation of immigrants enters the economy after having faced the educational decision at their origin, and as a result of their background have lower realisations of educational attainment and productivity from the native population. To reflect the fact that most of the refugees arrived in Europe have been young, with very few individuals of pension age arriving to Europe, the immigrant

measured in terms of hours worked in normal conditions.

[5]For the parameter $\omega \in [0, 1]$

population has an age distribution that differs from the "balanced" distribution of the natives in the economy.

When analysing the longer term impact of the refugee surge to the economy, assimilation of the original surge and their descendants is modelled by gradual approach of $\omega \to 1$ in time. The slow diminishing of the parameter attempts to capture the observed phenomenon of lower educational attainment and labour market participation rates of the later generations of immigrants. The population dynamics within the descendants will also in time converge to the balanced distribution of the native population of the economy.

4 Optimisation problem

Having formulated the model in the previous section, this section solves for the optimisation problem faced by the economic agent. First we formulate the Lagrangian equation, calculate the Karush-Kuhn-Tucker conditions and set up the necessary equations for solving the problem using a backward shooting algorithm.

The Lagrangian multiplier method, in which the Lagrangian equation is set and solved, is a mathematical optimisation strategy for finding maximum and minumum[6] of a function subject to set constraints. For solving the maximisation problem of the economic agents as laid out in equation (4) subject to constaints in equations (5) and (6) we get the follwing Lagrangian function:

$$\mathcal{L} = \sum_{t=1}^{T} \Big[\beta^{i-1} \left[\alpha ln(c_{i,t}) + (1-\alpha) ln(1-l_{i,t}) \right] - \lambda_{1,t} \left[(1 + (1-\tau_r)i - \delta)\, a_{i,t} \right.$$
$$\left. + (1-\tau_w) w_{i,t} l_{i,t} + p_{i,t} + s_{i,t} - (1+\tau_c) c_{i,t} - a_{i,t+1} \right] - \lambda_{2,t} l_{i,t} \Big]$$

By differentiating the Lagrangian function above with respect to the decision variables $c_{i,t}$, $l_{i,t}$ and $a_{t+1,i}$ and Lagrangian multiplier $\lambda_{1,t}$ and $\lambda_{2,t}$ we acquire the necessary Karush-Kuhn-Tucker conditions needed to set up the equations for the

[6] In some cases, several maxima and minima

backward shooting algorithm.

$$\frac{\partial \mathcal{L}}{\partial c_{i,t}} = \beta^{t-1}\frac{\alpha}{c_{i,t}} + \lambda_{1,t}(1+\tau_c) = 0 \tag{I}$$

$$\frac{\partial \mathcal{L}}{\partial l_{i,t}} = -\beta^{t-1}\frac{1-\alpha}{1-l_{i,t}} - \lambda_{1,t}(1-\tau_w)w_{i,t} - \lambda_{2,t} = 0 \tag{II}$$

$$\frac{\partial \mathcal{L}}{\partial a_{i,t+1}} = -\lambda_{1,t+1}\left(1+(1-\tau_r)i-\delta\right) + \lambda_{1,t} = 0 \tag{III}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \lambda_{1,t}} &= \left(1+(1-\tau_r)i-\delta\right)a_{i,t} + (1-\tau_w)w_{i,t} + p_{i,t} \\ &\quad + s_{i,t} - (1+\tau_c)c_{i,t} - a_{i,t+1} = 0\end{aligned} \tag{IV}$$

$$\lambda_{2,t}l_{i,t} = 0 \tag{V}$$

From the set of equations above, we can derive the intertemporal rate of substitution for consumption, the Euler equation and the intratemporal rate of substitution between consumption and leisure. First, we solve equation I for $\lambda_{1,t}$ and $\lambda_{1,t+1}$ at times t and t+1 and divide them by each other.

$$MRS(c_{i,t}, c_{t+1,i}) : \frac{c_{t+1,i}}{c_{i,t}} = \frac{\beta\lambda_{t,1}}{\lambda_{t+1,1}} \tag{7}$$

Then by first solving equation II and then substituting it in equation (7) we get the Euler equation

$$\begin{aligned}\left(1+(1-\tau_r)i-\delta\right) &= \frac{\lambda_{1,t}}{\lambda_{1,t+1}} \\ \frac{c_{t+1,i}}{c_{i,t}} &= \beta\left(1+(1-\tau_r)i-\delta\right)\end{aligned} \tag{8}$$

Last we solve for the intratemporal rate of substitution between consumption and leisure with dividing equation I by equation II

$$MRS(c_{i,t}, 1-l_{i,t}) : \frac{\alpha}{1-\alpha}\frac{1-l_{i,t}}{c_{i,t}} = \frac{-\lambda_{1,t}(1-\tau_c)}{-\lambda_{1,t}(1-\tau_w)w_{i,t} - \lambda_{2,t}}$$

Where the $\lambda_{2,t}$ is equal to zero following from complementary slackness using equation V and assuming that $l_{i,t} > 0$ at all times. Furthermore the $-\lambda_{1,t}$ terms cancel out, leaving us with:

$$MRS(c_{i,t}, 1-l_{i,t}) : \frac{\alpha}{1-\alpha}\frac{1-l_{i,t}}{c_{i,t}} = \frac{(1-\tau_c)}{(1-\tau_w)w_{i,t}} \tag{9}$$

From equation (9) we can solve for the hours worked $l_{i,t}$ which we will then in turn substitute in the budget constraint introduced in equation (5). This substituted equation is then solved for $c_{i,t}$

$$
\begin{aligned}
l_{i,t} &= 1 - \frac{(1-\tau_c)c_{i,t}}{(1-\tau_w)w_{i,t}}\frac{1-\alpha}{\alpha} \\
(1+\tau_c)c_{i,t} + a_{t+1} &= \left[1 + (1-\tau_r)i - \delta\right] a_{i,t} \\
&\quad + (1-\tau_w)w_t\Big(1 - \frac{(1-\tau_c)c_{i,t}}{(1-\tau_w)w_{i,t}}\frac{1-\alpha}{\alpha}\Big) + p_{i,t} + s_{i,t}
\end{aligned}
$$

This substituted equation is then solved for $c_{i,t}$

$$
c_{i,t} = \frac{\alpha}{1+\tau_c}\Big[\left(1 + (1-\tau_r)i - \delta\right) a_{i,t} + (1-\tau_w)w_{i,t} + p_{i,t} + s_{i,t} - a_{i,t+1}\Big] \tag{10}
$$

After this, the equation (10) is substituted into the Euler equation (8) yielding the following,

$$
\begin{aligned}
&\frac{\alpha}{1+\tau_c}\Big[\left(1 + (1-\tau_r)i - \delta\right) a_{i,t} + (1-\tau_w)w_{i,t+1} + p_{i,t} + s_{i,t} - a_{i,t+1}\Big] = \\
&\frac{\frac{\alpha}{1+\tau_c}\Big[\left(1 + (1-\tau_r)i - \delta\right) a_{i,t+1} + (1-\tau_w)w_{i,t+1} + p_{i,t+1} + s_{i,t+1} - a_{i,t+2}\Big]}{\beta\left(1 + (1-\tau_r)i - \delta\right).}
\end{aligned}
$$

To obtain the second order difference equation for asset accumulation, the equation is solved for $a_{i,t}$ and all the terms are collected. From this we get:

$$
\begin{aligned}
a_{i,t} &= \gamma_1 a_{i,t+1} + \gamma_2 a_{i,t+2} + \gamma_3 w_{i,t} + \gamma_4 w_{i,t+1} \\
&\quad + \gamma_5 p_{i,t} + \gamma_6 p_{i,t+1} + \gamma_7 s_{i,t} + \gamma_8 s_{i,t+1} \qquad (11) \\
\gamma_1 &= \frac{1}{\beta\left(1 + (1-\tau_r)i - \delta\right)} + \frac{1}{\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_2 &= -\frac{1}{\beta\left(1 + (1-\tau_r)i - \delta\right)\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_3 &= -\frac{1-\tau_w}{\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_4 &= \frac{1-\tau_w}{\beta\left(1 + (1-\tau_r)i - \delta\right)\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_5 &= -\frac{1}{\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_6 &= \frac{1}{\beta\left(1 + (1-\tau_r)i - \delta\right)\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_7 &= -\frac{1}{\left(1 + (1-\tau_r)i - \delta\right)} \\
\gamma_8 &= \frac{1}{\beta\left(1 + (1-\tau_r)i - \delta\right)\left(1 + (1-\tau_r)i - \delta\right)}
\end{aligned}
$$

The second order difference equation for asset accumulation (11) is used as the backward shooting algorithm on Matlab to solve for each individual's optimal asset holdings for each period. Given this optimal solution for asset holdings, the utility maximising paths of consumption and labour supplied can be derived.

5 Implementation in MatLab

This section describes how different phases of the model are implemented using MatLab.[7] The model consists of three interconnected parts affecting each other. First, in the core of the model there is the backward shooting algorithm which solves for the optimal path of asset holdings for each agent, subject to the current parametrisation of the model. Second, the public sector budget is being balanced by adjusting the tax rates for the given decision path outcomes. Third, the utility of the first agent to attain further education is compared with the utility of non-educated agent and the cut-off for education is adjusted.

As these three parts affect each other simultaneously, the model in MatLab is constructed using three nested loops.The innermost loop consists of the backward shooting algorithm solving for the optimal asset path for each agent by adjusting the guess for last period asset holdings until the optimal asset path is reached and the condition for $a_1 = a_{21} = 0$ is fulfilled. Once the innermost loop has satisfied the asset holding condition, utility, consumption and labour decisions are calculated for all agents. After this the middle loop balances the public sector budget to by adjusting the tax rates until the the budget is at balance for the given matrices of consumption, assets and hours worked. Then the inner loop is optimised again, followed by the middle loop until the optimal vector of taxes satisfies both the optimal asset path condition and the budget balance condition for the public sector. After this the outermost loop checks if the first agent to attain further education (i.e. the marginal agent) has lower utility than the non-educated agent. If the marginal agent has lower utility than the non-educated agent, is the cut-off for education increased and the two inner loops optimised again. This process is repeated until the marginal agent whose utility is higher than the utility of non-educated agents has been found.

The ω-parameter is estimated within the model by using the baseline calibration of the model as described above such that the tax regime and the benefits system of the economy are held constant while the skill distribution is adjusted to reach

[7]The MatLab code used in executing this thesis is available at request from the author*s*.

the desired educational outcome for the migrant employees.

6 Parametrisation

In this section of the thesis we will explain each of the components of the economy and fully parametrise all the exogenous variables in each of the components. The order in which this process takes place follows the order in which the sections of the economy are presented in the previous section of this thesis.

6.1 The production side

For the production side of the economy the following parameters are needed, and then one by one reasoned in this subsection: $\bar{A}, \theta$ and i_t.

To begin with, parameter $\bar{A}$ acts as a scalar multiplier to the economic output and would be useful only in the context of comparing several economies and capturing the technological differences between these. Since in this thesis we are focusing only on one economy possibly at different states, the parameter may as well be normalised to one in order to simplify the process.

θ is the output elasticity of capital and will in line of standard economic literature and empirical findings be set equal to $\frac{1}{3}$ as we see no reason to deviate from this well established standard.

Finally, i_t is the exogenously defined nominal interest rate faced by the economy. As stated above, the parameter is considered exogenous due to the fact that the economy is considered to be small and open which implies that domestic capital stock will have no impact on the interest rate setting decision. Based on modern day nominal interest rates, this long run equilibrium rate could be set relatively low if assuming that there has been a trend change in interest rates some ten years ago. However, we will restrain ourselves from making any such inferences and instead of explicitly defining the nominal interest will for now only give the following:

$$r_t = i_t - \delta_t$$

Where r denotes the real interest rate and δ is the capital depreciation i.e. inflation rate. The real interest rate will be set to an empirically sound level of five percent, as supported by the findings of Damodaran (2015). In order to emulate the existing investment to GDP ratios observed across the advanced economies,

we select the depreciation of capital equal to 0.08 which would yield an investment to output ratio of 20%. Thus the nominal interest rate in the economy is from $5\% = i_t - 8\%$ equal to 13%.

6.2 The public sector

The public sector collects its revenues from taxes and pays out these as pensions and social security contributions. The relative importance of each revenue source is approximated using the latest statistics on tax revenue from Eurostat (2017b) and shown in Table 1.

	Share of total taxes
Capital tax	0.0978
Consumption tax	0.3285
Labour tax	0.5738

Table 1: Relative shares of different sources of taxes in the model framework

Pensions are set to be equal to 50% of the nominal wage of the non-educated labour and the social security contribution paid for the non-educated labour to 20% of the pension paid. The selected 50% is a simplification of pension in line with the reported gross pension replacement rates, as reported by OECD (2015b).

6.3 The economic agent

The economic agent needs the following exogenous parameters to be defined: β, and α. In addition the distribution of skill needs to be addressed.

The β parameter is conventionally defined as $\beta = \frac{1}{1+i}$ but we will deviate from this in order to generate the desired life-cycle consumption effect, as pointed out in Laun and Wallenius (2015, pp. 7-8 footnote 7). Instead β will be defined using the real interest rate and set equal to 0.98. The α parameter will be arbitrarily set equal to 0.55, giving the agents slight preference of consumption over leisure.

To gain the rough correspondence to Northern European levels of further education (OECD 2014), the ability of the population are parametrised to have a normal distribution with mean of 1 and standard deviation of 1.5. The distrubution of the ability is shown below in figure 1. Ability ranges between -4.3459 and 5.2994 with mean 0.9758 and median value of 0.9774.

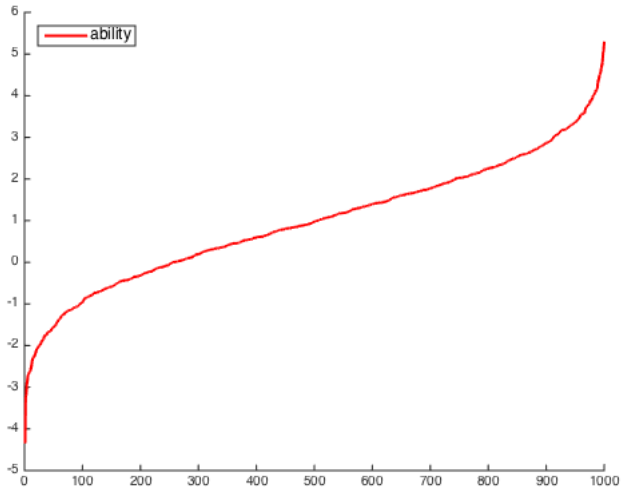

Figure 1: Ability distribution of the economic agents.

6.4 The shock

The initial educational distribution within the migrant population will be determined exogenously using actual data from the most common countries of origin. According to the OECD (2014) approximately one in ten refugees have a higher educational degree. The exogenous educational distribution will then be used to calibrate the magnitude of the ω parameter faced by the immigrants to impede their ability due to the adverse background.

The size of the shock is approximated to be 2% of the existing population size, which is at the upper end of the estimates on numbers of immigrants faced by individual European countries (BBC 2016). Although the estimate falls on the upper end of the distribution, should change in the assumed size of the shock only impact the magnitude of reactions but keep the actual effects and implications the same.

As the long term effects of the immigration shock hinge at the speed at which the migrants are assumed to converge towards the local population, speed of convergence must be decided. To account for the inheritability of education and socioeconomic status, and the observation that even second generation immigrants are empirically shown to have lower educational attainment vis-á-vis the native population(Algan et al. 2010)(Liebig 2007), the speed at which the ω parameter diminishes is equal to 50% of 1-ω.[8]

[8]Changing the speed of convergence will impact the magnitude of effects but not change the effect itself.

Since Eurostat (2017a) shows that the migrants also have a very different age distribution to the existing European population, the migrant shock is modelled to have a different age distribution from the local population. As with the ω-parameter, the age distribution of the migrant population is also expected to converge towards the equal age distribution of population assumed for the native population. The assumed initial distribution is shown in figure 2 below with even population distribution as a reference.

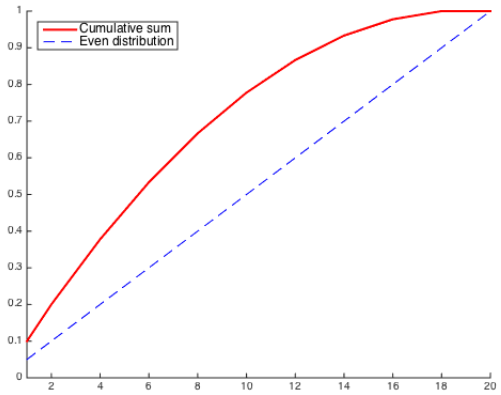

Figure 2: Cumulative age distribution at time of the shock.

Figure 3 below shows the later assumed convergences towards the population mean of even distribution.

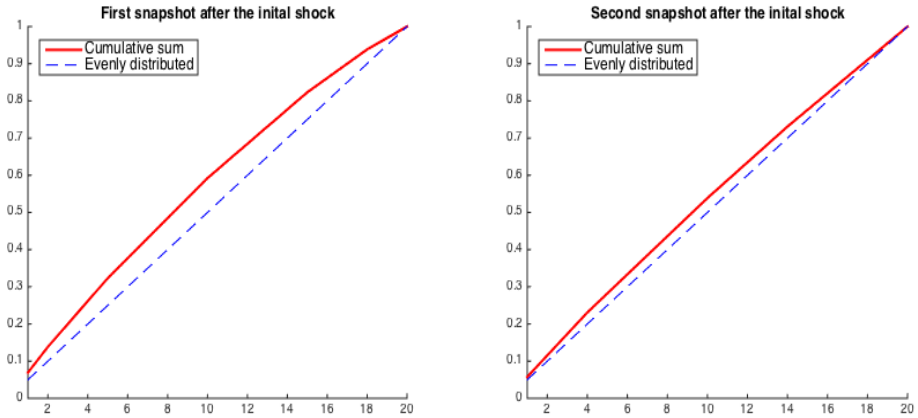

Figure 3: Cumulative age distribution at the first (left) and the second (right) observations after the shock.

On its course to to adjust towards the age distribution, the immigrant population grows in size relative to the native population to account for the fact that women of immigrant background have more children than native women (Mayer and Riphahn 2000). The share of immigrant population goes from the initial 1.9% to

2.91% at the time of the first snapshot observation and 3.84% at the time of the second snapshot.

7 Baseline model

In this section, the baseline model is run to first calibrate the rest of the model as described previously. As part of the calibration, both the share of population acquiring higher education and the tax rates subject to the previously introduced restrictions are determined to solve for the steady state of the economy prior to experiencing an immigration shock. Following this final calibration of the economy, the results of the steady state are presented and discussed, before we move onto the next section where the immigration shock is introduced under different scenario settings to see the potential impacts of the shock on the welfare and output of the economy.

7.1 Calibration of the baseline model

The tax rates in the economy are calibrated subject to the budgetary requirements parametrised previously, and as an outcome of this process the economy at its steady state has 38.6% of higher educated population. As a result of this, the agent productivities[9] compared to their ability are the following as shown in figure 4.

[9]And wages, as the wage rates are a function of agent's productivity

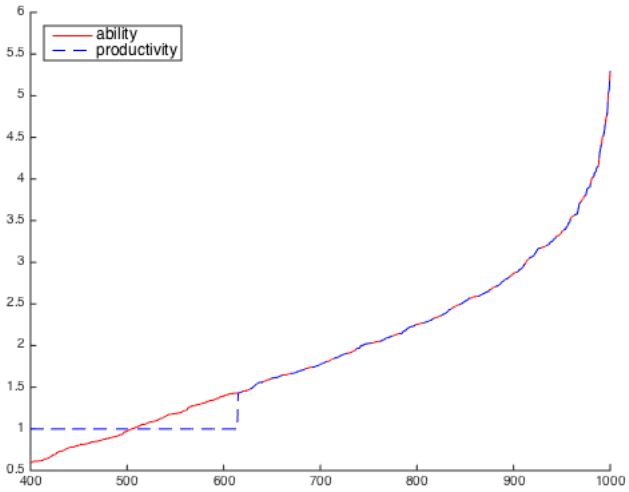

Figure 4: Ability and productivity of the agents. NB: The first 400 agents are omitted from the figure as they all have productivity equal to 1 and their ability has been described before in Figure 1

The tax rates in the baseline calibration of the economy are shown in the Table 2 below.

	Tax rates
τ_k	6.8418%
τ_c	9.2048%
τ_w	14.3169%

Table 2: Tax rates at the steady state of the baseline economy

7.2 Results of the baseline economy

In the baseline economy the agents incur an aggregate utility of -10574.7061, which given the population size gives them an average utility of -10.5747. Table 3 aggregates the utilities and, lifetime capital stock, effective labour supply and consumptions of the uneducated agents and all the educated agents in the economy and shows the total aggregates of these key variables of interest.

From Table 3 below it can be see that much of the capital and labour stocks of the economy originate from those agents, who have attained higher education, although they represent only approximately one third of the total population. To shed more light to differences between the agents concealed by the aggregation,

we are looking into the average uneducated agent and the average educated agent.

	Aggregated		
	Uneducated	**Educated**	**Total**
Utility	-7511.46	-3063.25	-10574.71
Capital stock	16607.74	21046.91	37654.65
Effective labour supply	5090.15	8620.74[10]	13710.90
Consumption	5555.20	6646.14	12201.34
Share of population	61.4%	38.6%	

Table 3: Aggregate values of utlity, capital stock, effective labour supply and consumption.

In Table 4 below the per capita aggregates of the uneducated and the average educated agents are presented. As indicated by the aggregate figures, the average educated agents are better off due to their higher income which allows them to both consume more and to save more for the pension days to sustain the consumption.

	Per capita aggregates		
	Uneducated	**Average educated**	**Total**
Utility	-12.2336	-7.9360	-10.5747
Capital stock	27.0484	54.5257	37.6547
Hours worked	8.2902	9.0736	8.593
Productivity	1	2.455	1.562
Total labour income[11]	8.2902	22.3335	13.7109
Consumption	9.0476	17.2180	12.2013
Share of population	61.4%	38.6%	

Table 4: Per capita aggregates for the uneducated, the marginal and the average educated agents.

The left panel on Figure 5 shows, that all the agents, despite of their educational attainment, collect assets during their working life with their asset holdings peak-

[10]Effective labour supply for the educated agents is equal to the working hours done multiplied by the agents efficiency. The educated agents supplied 3502.4 working hours with average efficiency of 2.461.

[11]Total labour income is also equal to their effective labour supply.

ing at the 16th period, first period in their pension, in order to finance their consumption at retirement. The left panel shows consumption, which increases over time as a result of the deviation from standard β with offsetting interest rate. Furthermore, as logic dictates, the agents with higher incomes through labour and assets have higher consumption.

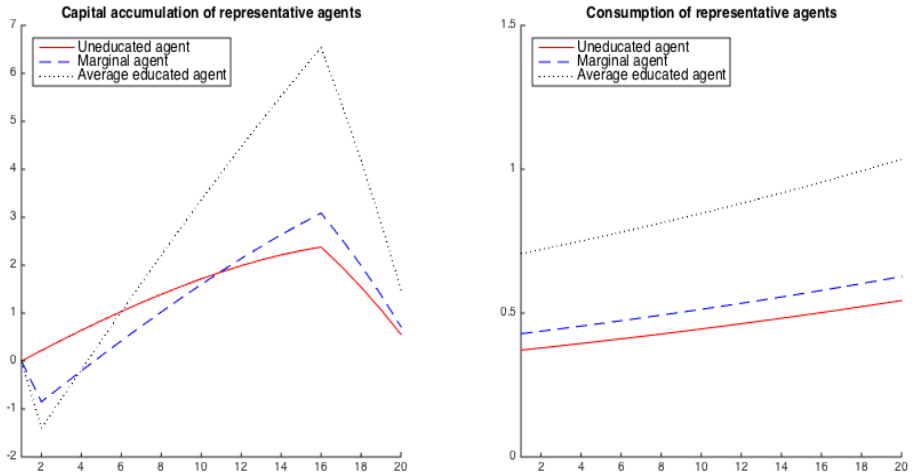

Figure 5: The left panel depicts the asset accumulation of the selected agents and panel on right shows consumption for the same agents.

Agents undertaking education in their first period are not participating members of the labour force but instead spend their time in education, as a result of which the leisure of the agent when undertaking education is determined by their ability as $1-l_{i,1} = 1-\frac{1}{\phi_i}$. For rest of the time prior to forced retirement all agents spend this time in the labour force, as shown by the downward slope in the left panel of Figure 6 below. Over time agents decrease their working hours to offset the effects of discounting their utility from leisure. The right panel of Figure 6 shows the hours worked by the agents increase asymptotically as the productivity of each agent increases.

Given the assumed production function of the economy and the capital stock and effective labour supply accumulated by the agents, the total output of the economy is equal to 19200.684 and 19.200684 measured as GDP per capita. As a measure of equality in the society, a Gini-coefficient has been calculated using utility[12] as a unit of measure. In the baseline economy the Gini-coefficient equals to 0.205792. For comparison, this figure is approximately 0.05 lower than

[12]For calculation of the Gini-coefficient, the utilities are transformed into positive numbers by dividing the mean of utilities by individuals utility.

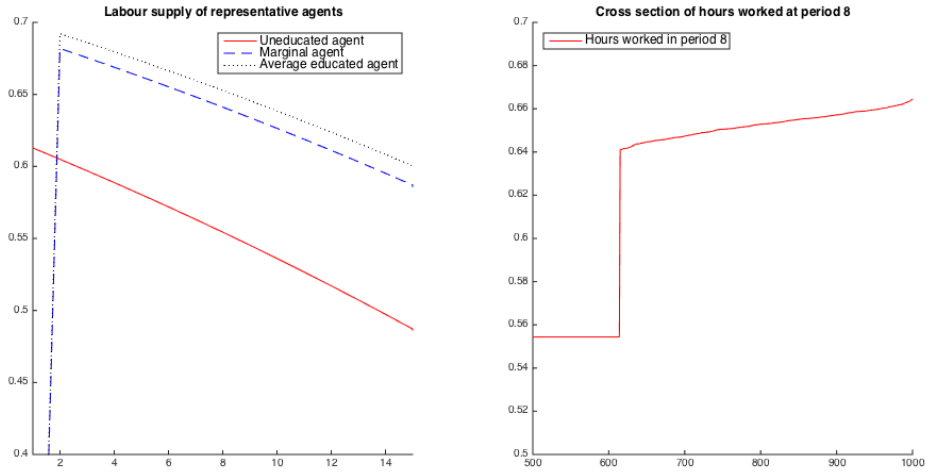

Figure 6: Panel on the left shows the lifetime labour supplies of the selected agents, on the right a cross section of labour supply at period 8 for all agents, omitting first 500 agents as all uneducated agents are homogeneous in labour supply.

income Gini-coefficients after taxes and transfers for the Nordic countries (OECD 2015a).

8 Scenarios

In this section, the different policy reactions of the public sector to the immigration shock and the outcomes of these reactions are discussed. First the parameter ω is defined as described previously in the section 5 regarding the MatLab implementation of the model and then each policy scenario is discussed by first looking at the initial impact of the shock, where the agents may adjust their behaviour in terms of capital accumulation and working hours in response to the new situation, but in which the agents educational decisions remains unaltered from the initial decision. Then the economy is observed at the two stages, which allow the agents to change their educational attainment in response to the changing population dynamics.

As the immigrants in the economy are by the construct of the parameter ω going to be worse off compared to the locals, focus on the analysis will be mostly in the aggregate state of the economy and the within group differences of different policies. Most of the results are reported using differences from the baseline rather than the absolute numbers for easier comparison.

The parameter ω is estimated from the baseline steady state by approximating

how much the skill parameter needs to fall until only 10% of the agents will engage themselves in higher education. The initial value of the ω at the baseline is equal to 0.3503 and as a result of convergence to local population the parameter takes values 0.6102 and 0.7661 in the consecutive times of observation.

8.1 Policy I - Government inaction

The first scenario described is the government inaction, in which the government remains completely passive in any response and maintains the same tax rates as it had in the initial steady state prior to the shock. In this scenario, the pension and social security contributions are allowed to change from their pre-defined levels based on the fluctuations in tax base.

8.1.1 At the time of the shock

The overall utility in the economy at the time of the shock once the agents have adjusted for their behaviour following the incoming immigrant population is equal to -10980.9047 which implies a per capita utility of -10.7656[13]. This means a negative change of -1.8053% from the initial steady state.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-10980.9047	-10.7656	-1.8053%
Capital stock	37837.0917	37.0951	-1.4861%
Effective labour supply	13791.5905	13.5212	-1.3836%
Consumption	12282.3063	12.0414	-1.3105%

Table 5: Aggregate values of utility, capital stock, effective labour supply and consumption after the shock - Policy I

In Table 5 the aggregated and per capita amounts of capital stock, effective labour supply and consumption. While all of these are higher in absolute terms due to the increased population size, in per capita terms the economy is worse off as shown by the percentage changes from the baseline. This result is largely driven by the immigrants who by construct are less wealthy and less well off due to their lower efficiency impeding their incomes.

[13]Population size has risen by 2% as defined by the size of the shock.

Although the native agents are all worse off as the result of the immigration shock, the impact on their well-being is very minimal as they can adjust their savings behaviour much more to smooth out the effects of changing welfare payments. These payments, both the pensions and the social welfare contribution, in the economy drop by 0.653% due to changes in tax base of the economy and due to the agents' response in their behaviour in an attempt to minimise the welfare loss. Since the uneducated agents are more dependent on the welfare system of the economy compared to the educated agents, who become less dependent as ther wage rates increase, they have a greater need to adjust to the change. Table 6 shows the marginal changes for the uneducated and the average educated agents.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Average educated**	**Change**
Utility	-12.2530	-0.1583%	-7.9415	-0.0693%
Capital stock	27.1223	+0.2732%	54.6288	+0.1891%
Hours worked	8.2988	+0.1049%	9.0755	+0.0209%
Effective labour supply[14]	8.2988	+0.1049%	22.3378	+0.0183%
Consumption	9.0359	-0.1293%	17.2119	-0.0354%
Share of native population	61.4%		38.6%	

Table 6: Change in variables for native population after shock - Policy I

The uneducated population increases its capital stock more than the educated agents, and in order to achieve this will also have to work more hours. The educated agents will also increase their working hours but only marginally as their higher wages allow for easier adjustment of assets. Furthermore, the reactions of the educated native population only correspond to the average change, the reactions diminishing in magnitude for the agents with higher productivities and wages. As a consequence of the uneducated agents being nearly three times as much worse off than the average educated agents, the inequality in the economy will increase.

As the immigrant population are by construct worse off and lack comparison values from the baseline economy, only their aggregate and per capita utilities are reported to allow for evaluation of policy from their perspective[15]. Immedi-

[14]The effective labour supply also equals to the pre-tax income of the agents.

[15]Utility for the immigrants is scaled such that it represents 1.9% of the total population.

ately after the shock without government response the aggregate utility of the migrants is equal to -392.1851, or -19.6093 per capita, showing them to be significantly worse off than the native population. However, interpretation of absolute differences in measurements of utility is close to pointless due to the nature of the measurement.

The aggregate output in the economy immediately at the shock is 19307.024 units, or 18.9284 in per capita terms. Compared to the steady state prior to the shock, this means 0.55% increase in absolute and -1.42% decrease in per capita terms. If only looking at the native population's contribution to the output of the economy, their contribution equals to 19221.6307, an increase of 0.11% as a result of their increased economic activity. While the share of the migrant population in the econmy equals to 1.96% of population, their contribution to the GDP is only at 0.46% as a result of their lower productivity. In terms of utility equality, the economy is slightly worse off with a post-shock Gini-coefficient of 0.205957, a 0.00165 increase from baseline[16]. This is driven both by the immigrants, but also by the fact that the changing social security contributions disproportionately hit the uneducated natives.

8.1.2 First snapshot of the economy

Two generations after the initial hit of the shock, the economy is observed again and the agents have had a chance to further adjust to the changing situation. In this new state of the economy, both populations have readjusted their educational attainment in response. The share of native people undertaking education has increased by 0.2% to 38.8%, and on course to assimilation the next generations immigrants have attained 30.1% education, a 20.1% increase from the original exogenously defined 10%.As the tax base of the economy is still undergoing changes, the welfare payments are still below the steady state level, being 0.659% less. The change in tax base is in large part driven by the increase of immigrant population and the convergence of their age structure to the native population's age structure, increasing the number of old and less tax paying immigrants.

On course back to its original steady state, the economy has a total utility -11036.1616, or -10.7147 in per capita terms. This means a percentage change of -1.3239% from the steady state, or alternatively an improvement of 0.47% from

[16]Lower values of Gini-coefficient are better, higher worse.

the level at shock. In Table 7 the aggregate values for the key variables, as well as the per capita values and changes compared to the baseline are shown again. As a result of the adjustments done by the people, the economy overall is converging towards its original steady state and the per capita variables increasing towards these values.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-11036.1616	-10.7147	-1.3239%
Capital stock	38215.4778	37.1024	-1.4667%
Effective labour supply	13958.4293	13.5519	-1.1597%
Consumption	12426.3289	12.0644	-1.1220%

Table 7: Aggregate values of utility, capital stock, effective labour supply and consumption at first snapshot - Policy I

To maintain comparability of the numbers in the table 8 with previous numbers, instead of average educated agent the second comparison figure will be the top 38.6% of the population as the *newly* educated 0.2% of agents have lower productivities decreasing the average productivity of educated agents resulting into non-comparable averages. Because of the social security contributions that have marginally fallen further from the steady state values, the native agents are also marginally worse off than before, having to adjust their behaviour more. The relative behaviour of the two different groups remains similar than at the shock.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Top 38.6%**	**Change**
Utility	-12.2536	-0.1635%	-7.9417	-0.0718%
Capital stock	27.1249	+0.2828%	54.6325	+0.1959%
Hours worked	8.2991	+0.1086%	9.0756	+0.0220%
Effective labour supply	8.2991	+0.1086%	22.3379	+0.0197%
Consumption	9.0355	-0.1337%	17.2117	-0.0366%

Table 8: Change in variables for native population at first snapshot - Policy I

Since the productivity of the immigrants has increased from the initial 0.3503 to 0.6102, their utility increases significantly both as a result of higher wages but

also due to more of the immigrants having attained further education. Their aggregate utility in the first snapshot equals to -448.5507 or -14.9517 per capita. While comparability of these numbers is close to arbitrary, they show the positive impact of integration and intergenerational social mobility.

The aggregate output of the economy has increased to 19542.4992 units, largely driven still by the population growth. The per capita output equals to 18.9733, still below the steady state by 1.18%, or alternatively a 0.24% increase from the state immediately after the shock. The native population is still overrepresented in their share of output with 98.30% of output being produced by them, despite of them being only 97.09% of the population. The measure of utility equality, Gini-coefficient at the first snapshot equals to 0.205945. This implies a marginally increased equality in the society from the state immediately after shock, as the coefficient has fallen by 0.000012. Although the native population, due to small decrease in the welfare payments, experienced higher inequality amongst themselves, the improved situation and growth in relative size of the immigrant population makes the society more equal.

8.1.3 Second snapshot of the economy

Two generations later, following further integration of the immigrant population, the economy is observed and the agents have had a chance to further adjust to the changing situation. Again both of population groups have re-evaluated their educational decisions and the share of people undertaking education among the local population is decreased by 0.1% to 38.7% compared to the previous observation, and on course to assimilation the next generations immigrants have attained 35.7% education, a 5.6% increase from the previous time of observation. Resulting from the changing tax base, the welfare payments of the economy have increased from the previous periods, being still 0.437% below the initial level.

The total utility at the last observation in the economy is equal to -11102.04335, or -10.6750 measured in per capita terms. While this is still -0.9485% below the initial steady state, it represents an increase of 0.37% from the first snapshot. Table 9 shows again the aggregate values for the key variables, as well as the per capita values and changes compared to the baseline. As the immigrant population keeps converging towards the steady state, the per capita values deviate less from the steady state values.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-11102.04335	-10.6750	-0.9485%
Capital stock	38719.6275	37.2304	-1.1168%
Effective labour supply	14140.8793	13.5970	-0.8307%
Consumption	12575.4090	12.0917	-0.8983%

Table 9: Aggregate values of utility, capital stock, effective labour supply and consumption at second snapshot - Policy I

Table 10, as previously, presents only the top 38.6% to maintain comparability with previous tables. As the welfare payments made by the government have increased closer to their initial state, have the changes taken by the agents decreased in magnitude from the previous states of the economy.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Top 38.6%**	**Change**
Utility	-12.2525	-0.1545%	-7.9413	-0.0667%
Capital stock	27.1206	+0.2669%	54.6264	+0.1846%
Hours worked	8.2986	+0.0125%	9.0754	+0.0200%
Effective labour supply	8.2986	+0.0125%	22.3377	+0.0181%
Consumption	9.0362	-0.1260%	17.2121	-0.0343%

Table 10: Change in variables for native population at first snapshot - Policy I

As both the educational attainment and the productivity of the immigrant population have further increased, their average per capita utility has increased close to the level of uneducated native agents being -12.8417.

The total output of the economy at the second snapshot equals to 19783.1146 units, or 19.0222 per capita, which is only 0.9% below the initial level of output per capita in the economy. Relative to the first snapshot this represents an increase of 0.25%. The native population of the economy produced a total of 19224.952, which is still 0.12% more than at the steady state despite of the native population having remained equal in size. Out of the total output of the economy, the immigrant population produces 2.821%, which is close to their relative share

of the population in the economy. In terms of utility equality, the economy has a Gini-coefficient of 0.205852, a further fall from the earlier coefficient towards the original Gini-coefficient at the steady state prior to population change.

8.1.4 Summary

In conclusion, government inaction results in a temporary fall in the welfare payments made in the economy. This hits the uneducated agents disproportionally hard as their own incomes are lower than the educated agents, and they also receive the social security contribution equal to 1/5 of the pension payment for each period in labour. Due to this the disutility impact is also proportionally higher to the uneducated agents which means they will also change their behaviour more than the educated agents.

As the impact of the reduced welfare payments results in more disutility for the uneducated agents, this will also increase the inequality in the economy as measured by the Gini-coefficient. When the share of welfare payments as mean of income decreases, the agents will have to work more and accumulate more assets for the periods when they are not working to maintain as high consumption as possible, without sacrificing too much of their leisure. This implies, that the output contribution of the native population increases.

8.2 Policy II - Government adjusts all taxes to sustain social security

In the second scenario, the government adjusts all taxes to maintain the original distribution of sources of taxation as defined in Table 1[17]. The government will undertake the adjustment at all periods to remain at the targeted pension and social security payments.

8.2.1 At the time of the shock

At the time of the shock, the tax base of the economy experiences a negative shock through injection of less producitive agents as shown in subsection discussing Policy I above. In order to uphold the existing welfare system in the economy, the government reacts immediately by adjusting the tax rates as shown in Table 11 below. To maintain its budgetary balance, the government is forced to increase all tax rates. In relative terms, the tax rate for capital gains τ_k increases the most

[17]Capital taxes 9.79%, consumption taxes 32.83% and labour taxes 57.38%.

despite of capital taxes having smallest contribution to the total tax sum, while the tax rates for consumption and labour income both increase roughly the same in relative terms.

	Tax rate	Δ
τ_k	6.9373%	+0.0955
τ_c	9.2711%	+0.0663
τ_w	14.4216%	+0.1047

Table 11: Changes in tax rates following the shock - Policy II

The impact of increased taxes decreases the total utility to -10981.3793, which is approximately 0.5 units less than under Policy I. In per capita terms this implies utility of -10.7661, which is -1.8100% less than at the steady state prior to any immigration shock. Due to the distortion from the increased taxes, the capital stock, effective labour supply and consumption in the economy are lower than under Policy I as shown in Table 12. The effect is largest in capital stock, with per capita decrease being over 0.5%-points greater than in Policy I, where the agents were encouraged to increase their savings.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-10981.3793	-10.7661	-1.8100%
Capital stock	37605.5952	36.8682	-2.0887%
Effective labour supply	13783.3453	13.5131	-1.4426%
Consumption	12267.3104	12.0268	-1.4302%

Table 12: Aggregate values of utility, capital stock, effective labour supply and consumption after the shock - Policy II

In contrary to Policy I, the average educated agents are worse off than the uneducated agents when measured by change in utility. The average utility loss of an educated agents is nearly double in magnitude compared to the uneducated agents as shown in Table 13. The effect is driven largely by the change in hours worked, which change very little for the educated agents on average, but more for the uneducated. This yields the uneducated agents more utility from leisure. However, the income impact is stronger for the educated agents whose total labour income falls more than uneducated agents despite of changing working hours less. Furhtermore, as a result of the relatively higher taxation of assets,

the educated agents also save clearly less than before and divert this to money to upkeep consumption. As the results below are averages, they hide the fact that negative effects of the tax increase are greater for the more productive agents who suffer from more disutility than the less productive and uneducated agents. It is also noteworthy, that the uneducated agents are better off than under the Policy I regardless of the distortionary taxation.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Average educated**	**Change**
Utility	-12.2478	-0.1161%	-7.9529	-0.2130%
Capital stock	26.9367	-0.4130%	54.3297	-0.3595%
Hours worked	8.2886	-0.0181%	9.0735	-0.0011%
TEffective labour supply	8.2886	-0.0181%	22.3333	-0.0090%
Consumption	9.0302	-0.1923%	17.1819	-0.2097%
Share of native population	61.4%		38.6%	

Table 13: Change in variables for native population after shock - Policy II

At the time of the shock with the government maintaining its welfare system at status quo, the immigrant population has an aggregate utility of -391.3777 or -19.5689 in per capita terms. In comparison to the Policy I, this implies 0.21% higher utilities for these agents on average.

In terms of the aggregate output the economy is worse off than in Policy I, where the distortionary effects of taxes were less, and the total output of the economy equals to 19259.895 units, or 18.8823 in per capita terms. Compared to the steady state prior to the shock, this means 0.31% increase in absolute and 1.66% decrease in per capita terms. This is 0.48%-points less than under the Policy I. If only looking at the native population's contribution to the output of the economy, their contribution equals to 19175.140, a decrease of 0.13% as a result of their decreased economic activity. While the share of the migrant population in the economy equals to 1.96% of population, their contribution to the GDP is only at 0.44% as a result of their lower productivity. In terms of equality in the society, the utility Gini-coefficient 0.204751, which is 0.001041 lower than in the baseline. Due to the tax increase and the larger impact of especially the capital tax on the educated agents, the society becomes more equal in terms of utility.

8.2.2 First snapshot of the economy

Two generations later, the economy is observed again and the agents have had a chance to further adjust to the changing situation. The local population has not deviated from its steady state educational attainment, while on course to their assimilation the next generations of immigrants have attained 30.1% education, a 20.1% increase from the original exogenously defined 10%, just like under the Policy I. In response to the change of population dynamics, as discussed in subsection 8.1.2, the tax rates in the economy are again re-evaluated and shown in the Table 14. Tax rates for capital gains and consumption increase further while the tax rate for labour remains almost at the level of the level following the shock.

	Tax rate	Δ
τ_k	6.9586%	+0.1168
τ_c	9.2856%	+0.0808
τ_w	14.4231%	+0.1082

Table 14: Changes in tax rates at the first snapshot - Policy II

At the first snapshot, the total utility in the economy equals to -11039.8639, or -10.7183 in per capita terms. In comparison to the steady state of the economy, the the per capita utility is 1.36% below the level. In comparison to Policy I, the utility in the economy remains at a slightly lower level, being 0.0036 lower. In Table 15 the aggregate values for the key variables, as well as the per capita values and changes compared to the baseline are reported. As a result of further distortion from the capital taxes in the economy, the per capita stock of capital remains over 2% below the steady state.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-11039.8639	-10.7183	-1.3580%
Capital stock	37972.1929	36.8662	-2.0940%
Effective labour supply	13958.4263	13.5519	-1.1597%
Consumption	12404.8885	12.0436	-1.2925%

Table 15: Aggregate values of utility, capital stock, effective labour supply and consumption at first snapshot - Policy II

As a result of the increased taxes, the native population is even worse off than immediately after the shock as shown in 16. As earlier, the taxes have greater impact on the average educated agent compared to the uneducated agents which results them being relatively worse off vis-á-vis the uneducated..

	Per capita aggregates for native population			
	Uneducated	**Change**	**Top 38.6%**	**Change**
Utility	-12.2494	-0.1292%	-7.9546	-0.2344%
Capital stock	26.9216	-0.4688%	54.3040	-0.4066%
Hours worked	8.2885	-0.0193%	9.0735	-0.0011%
Effective labour supply	8.2885	-0.0193%	22.3333	-0.009%
Consumption	9.0282	-0.2144%	17.1782	-0.2312%

Table 16: Change in variables for native population at first snapshot - Policy II

The utility of the immigrants has increased again following the dynamics of the assimilation. The aggregate utility of the immigrants is equal to -448.2782 at the time of the first snapshot. Measured in per capita terms, the utility of immigrants equals to -14.9426, which in comparison to Policy I is 0.09% higher.

Total output of the economy remains highly distorted by the tax changes in comparison to Policy I. The total output of the economy is 19485.5588 units, which in per capita equals to 18.9180. In comparison to the steady state, the per capita output remains 1.47% below the steady state, remaining 0.29%-points lower than under Policy I. In light of equality, the distortionary effect of the taxes remains positive with the utility Gini-coefficient further decreasing to 0.204652. Again, this development is driven largely by the larger impact of the taxes on the educated agents, whose utility decreases more due to changes in tax.

8.2.3 Second snapshot of the economy

Two generations later the economy is observed again. The educational attainment of the native population remains yet unaltered, with the immigrant population having on their course to assimilation attained 35.9% education, a 5.8% increase from the previous time of observation. In comparison to educational decisions in Policy I, the immigrant population has attained 0.2%-points higher educational level. As a result of further assimilation and change in population structure, the tax base of the economy has altered resulting in changing tax rates as presented in Table 17.

	Tax rate	Δ
τ_k	6.9508%	+0.1090
τ_c	9.2832%	+0.0784
τ_w	14.4201%	+0.1032

Table 17: Changes in tax rates at the second snapshot - Policy II

The aggregate utility of the economy has decreased to equal to -11104.5531, resulting in a per capita utility of -10.6775. In comparison to the steady state it still remains at a level -0.9721% less, and is 0.024%-points less than with Policy I. From previous observation the utility per capita has increased by 0.4%.

The aggregate and per capita values of the key variables shown in Table 18 indicate that the distortionary effect of taxes has less impact on the economy and the convergence of immigrants drives the economy closer to the steady state, but that especially the capital stock remains dwarfed in comparison to both the steady state and the economy under Policy I.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-11104.5531	-10.6775	-0.9721%
Capital stock	38475.3552	35.9955	-1.7506%
Effective labour supply	14128.3331	13.5849	-0.9190%
Consumption	12556.6187	12.0737	-1.0458%

Table 18: Aggregate values of utility, capital stock, effective labour supply and consumption at second snapshot - Policy II

As the tax rates remain still at a higher level than at the steady state, the native agents remain to be worse off as depicted in Table 19. Total effective working hours of the agents have reached nearly the steady state level, but the consumption and capital stock remain considerably below the values at steady state, resulting in lower levels of utility.

Following the assimilation of the immigrants towards the local population, their aggregate utility at the second observation equals to -513.1360, implying a per

	Per capita aggregates for native population			
	Uneducated	**Change**	**Top 38.6%**	**Change**
Utility	-12.2488	-0.1242%	-7.9539	-0.2256%
Capital stock	26.9281	-0.4448%	54.3152	-0.3861%
Hours worked	8.2886	-0.0181%	9.0735	-0.0011%
Effective labour supply	8.2886	-0.0181%	22.3334	-0.0004 %
Consumption	9.0289	-0.2067%	17.1796	-0.2265%

Table 19: Change in variables for native population at second snapshot - Policy II

capita utility of -12.8284. In comparison to Policy I, the average immigrant in the economy remains 0.1% better off.

The total output of the economy at the time of the second observations equals to 19729.7341 units. This equals to 18.9709 units of output per capita, 1.2% below the per capita output at the steady state. Policy II of maintaining the extent of the benefits system results in 0.3% less output at the second when comparing to Policy I. In terms of the equality, the utility Gini-coefficient equals to 0.204631 increasingly better off. While the native agents are becoming slightly less equal due to the lesser distortion from the taxes, the increasing equality of the immigrant population with respect to the natives reduces inequality even further.

8.2.4 Summary

Under the Policy II, the economy performs worse compared to the Policy I, both in terms of the output in the economy but also in terms of aggregate utility. The per capita output of the economy remains approximately 0.3% below the corresponding figures of Policy I, and the aggregate utility in the economy remains on average 0.05%-points below the aggregate utility reached by Policy I. While neither of these objective measurements justify use of Policy I over Policy II in the economy, other aspects are in favour of Policy II.

In Policy I, the loss in utility came almost fully from the uneducated agents and the immigrant population, both of the groups being more heavily impacted by the falling welfare payments rather than the educated agents. However, in Policy II the loss in utility is distributed across the whole population, making the educated agents worse off relative to their state in the steady state, but simultaneously the uneducated agents were experiencing lesser impact on their utility. As the ma-

jority of population remains unemployed and they remained better off compared to the scenario in Policy I at all time periods, majority vote would most likely be in favour of Policy II. Furthermore, even if the relative disutility from Policy II is greater on the educated agents, they still remain better off than uneducated agents, otherwise the marginal agent*s* would have chosen not to educate themselves.

Policy II also saw the overall equality in the economy improve at the cost of the educated agents.

8.3 Policy III - Government education incentive scheme aimed at immigrants

As both policy options presented above leave the immigrant population worse uneducated vis-á-vis the natives, the third policy option considered is a targeted subsidy at the immigrants to promote education amongst them. In this policy plan, the government pays out a grant equal to a single pension payment, or 0.5, to the immigrants to promote education and increase their integration to the society. To maintain the payments balance, the government will use the tax system with equal shares of tax income, just like in Policy II.

The Policy III is chosen to be modelled after the tax system of Policy II due to the egalitarian nature of the policy.

8.3.1 At the time of the shock

Although the educational decision of the population remains fixed at the time of the shock, the government enacts the educational subsidy immediately to the immigrants. As a result of this increase in government expenditure, the government faces different tax rates than under Policy II immediately after the shock as shown in Table 20 below. The tax rates for consumption and income are both higher than in Policy II but the tax rate on capital is lower, due to less lending on on behalf of the immigrants which in turn implies less negative capital taxes paid by the government to subsidise lending.
As a result of the different tax allocation in the economy, the total utility in the economy at the time of the shock equals to -10980.3107, value higher than under either of the other two policies. The per capita utility in the economy as a result of this equals to -10.7650, leaving it -1.7996% below the initial steady state. As shown on Table 21, as a result of lower tax on capital gains, the capital stock of

	Tax rate	Δ
τ_k	6.9356%	+0.0938
τ_c	9.2718%	+0.0670
τ_w	14.4240%	+0.1071

Table 20: Changes in tax rates following the shock - Policy III

the economy is affected less by the shock than under Policy II. The higher taxes on labour result in slightly lower hours worked and income, but this remains compensated by the higher capital income of the agents and consumption remains at 0.0025%-points higher level than in Policy II.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-10980.3107	-10.7650	-1.7996%
Capital stock	37619.3932	36.8818	-2.0526%
Effective labour supply	13782.6950	13.5124	-1.4478%
Consumption	12267.6807	12.0271	-1.4277%

Table 21: Aggregate values of utility, capital stock, effective labour supply and consumption after the shock - Policy III

The marginally higher total utility in the economy is driven by the improved situation of the immigrants and the lower tax rate on assets. On Table 22 showing the marginal changes of the uneducated and average educated agents, it is shown that all natives are worse off compared to Policy II as a result of the new tax regime. However, as with Policy II, the uneducated agents remain better off in comparison to the situation in Policy I.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Average educated**	**Change**
Utility	-12.2481	-0.1185%	-7.9532	-0.2167%
Capital stock	26.9367	-0.4130%	54.3296	-0.3596%
Hours worked	8.2885	-0.0193%	9.0735	-0.0011%
Effective labour supply	8.2885	-0.0193%	22.3333	-0.0090%
Consumption	9.0300	-0.1945%	17.1819	-0.2097%
Share of native population	61.4%		38.6%	

Table 22: Change in variables for native population after shock - Policy III

Following largely from the education subsidy already paid for the educated 10% of the immigrant population, the aggregate utility of the immigrant population has increased to -390.0656 or -19.5033 in per capita terms. In comparison to Policy I this implies a utility 0.54% higher and 0.33% higher utility in comparison to Policy II.

The aggregate output in the economy at the shock equals to 19261.6443 units or 18.8840 in per capita terms. Compared to the steady state prior to the shock, the aggregate output has increased by 0.32% in absolute and decreased by -1.65% in per capita terms, leaving the output performance of Policy III at the time of the shock between the higher output of Policy I and lower output of Policy II. In terms of equality, Policy III outperfoms both Policy I and Policy II with utility Gini-coefficient of 0.19827, resulting from the equality generating taxation and higher utilities of the immigrants in the economy.

8.3.2 First snapshot of the economy

Two generations after the initial hit of the shock, the economy is observed again. While the educational decision of the native agents has again remained unaltered, the immigrants have attained 2% higher education than under either of the Policies I or II, with 32.1% of the immigrants educating themselves as a result of the educational subsidy they have received. However, this increase in public expenditure, as not only targetted for the extra 2% of the immigrants gaining further education, will also result in increased tax burden for all agents as presented in Table 23 below.

	Tax rate	Δ
τ_k	6.9616%	+0.1198
τ_c	9.3010%	+0.0962
τ_w	14.4460%	+0.1291

Table 23: Changes in tax rates at first snapshot - Policy III

The total utility in the economy at the time of the first snapshot equals to -11038.7533 or -10.7172 in per capita terms. In comparison to the steady state of the economy, the per capita utility is -1.3476% lower. In relation to the other policies, this again places Policy III between the higher aggregate utility of Policy I and the lower one of Policy II. The total capital stock in the economy
Table 24 shows again the aggregate values for the key variables, as well as the per capita values and changes compared to the baseline. As a result of the adjustments done by the people, the economy overall is converging towards its original steady state and the per capita variables increasing towards these values. As the higher tax rates are shown to distort the economy further away from its steady state, it is clear that the relative position of Policy III between the other two policy alternatives is a result of the improved stance of the immigrants in the economy.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-11038.7533	-10.7172	-1.3476%
Capital stock	38014.5020	36.9073	-1.9849%
Effective labour supply	13958.7763	13.5522	-1.1575%
Consumption	12267.6807	12404.5581	-1.2949%

Table 24: Aggregate values of utility, capital stock, effective labour supply and consumption at first snapshot - Policy III

In Table 25, the per capita aggregates for the native population are shown. As a result of the higher tax regime to finance the increased social spending in the economy, both the average educated and the uneducated native agents are worse off than in Policy II. In comparison to Policy I, the native agents still remain better off by 0.0998%. However, the average educated are more than 0.2% worse off.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Top 38.6%**	**Change**
Utility	-12.2524	-0.1537%	-7.9581	-0.2785%
Capital stock	26.9093	-0.5143%	54.2816	-0.4477%
Hours worked	8.2882	-0.0229%	9.0734	-0.0022%
Effective labour supply	8.2882	-0.0229%	22.3333	-0.0009%
Consumption	9.0249	-0.2509%	17.1712	-0.2718%

Table 25: Change in variables for native population at first snapshot - Policy III

Driven by both the education subsidy received and the higher educational attainment, the immigrant population has an aggregate utility of -443.9550, translating into per capita utility of -14.7985. In comparison to Policy I, this implies a 1.03% higher utility.

The aggregate output of the economy totals to 19493.1190 units, growth driven by the better situation of the immigrants as the per capita inputs of native population are diminished by the distortion caused by the taxation. The per capita output in the economy equals to 18.9255, a value 1.43% below the steady state placing the effects between the other two policies. Largely driven by the increased utility of the immigrant population and the distortive impacts of taxation, equality in the economy further increases with Gini-coefficient lowering to 0.19657.

8.3.3 Second snapshot of the economy

Following further integration of the immigrant population another two generations later, the second snapshot of the economy takes place. While the educational attainment of the natives remains at the steady state, the educational attainment of the immigrant population has increased to 37.1% being only 1.5% below the educational attainment of the native population. Table 26 summarises the tax rates required to balance the budget to match the additional subsidy to the immigrants.

Table 27 reports the aggregate values of the key variables of the economy. The per capita utility in the economy for the whole population equals to -10.6760, being -0.9579% below the steady state level of utility. As previously, the per capita values of the inputs are dwarfed by the distortionary taxes of the economy below the level of Policy I, but remain higher than under Policy II due to the positive

	Tax rate	Δ
τ_k	6.9528%	+0.1110
τ_c	9.3037%	+0.0989
τ_w	14.4530%	+0.1361

Table 26: Changes in tax rates at the second snapshot - Policy III

effects of Policy III on the immigrant population.

	Whole population		
	Total	**Per capita**	**Change per capita**
Utility	-11103.05340	-10.6760	-0.9579%
Capital stock	38539.6900	36.0574	-1.5863%
Effective labour supply	14126.7814	13.5834	-0.9299%
Consumption	12556.0852	12.0732	-1.0499%

Table 27: Aggregate values of utility, capital stock, effective labour supply and consumption at second snapshot - Policy III

Table 28, shows that due to the higher consumption and labour taxes than at the first snapshot, all natives are worse off than during the first period under the third policy, or the second period in the other two policy option. This provides proof that the subsidy paid for the immigrants in the second period is inefficient and carries a greater cost on the local population than the benefit acquired from it.

	Per capita aggregates for native population			
	Uneducated	**Change**	**Top 38.6%**	**Change**
Utility	-12.2530	-0.1586%	-7.9589	-0.2886%
Capital stock	26.9121	-0.5039%	54.2862	-0.4392%
Hours worked	8.2881	-0.0241%	9.0734	-0.0022%
Effective labour supply	8.2881	-0.0241%	22.3332	-0.0013 %
Consumption	9.0243	-0.2575%	17.1700	-0.2788%

Table 28: Change in variables for native population at second snapshot - Policy III

As a result of higher educational attainment and increasing productivity of the

immigrants their aggregate utility equal to -507.5489, implying a per capita utility of -12.6887 on average. In comparison to the utility of the immigrants under Policy I, the per capita utility is 1.2% higher.

The aggregate output of the economy equals to 19739.2913, or 18.9801 per capita in per capita terms, resulting in 1.15% lower output than at the steady state. Relative to the other policies, this again places the economy in between them. The output gap remains greater than under the Policy I, but lesser than under Policy II. The immigrant contribution to GDP, resulting from their higher educational attainment equals to 2.89% , almost matching to their share of population. The utility Gini-coefficient in the economy increases to 0.199353, but still keeping the Policy III economy the most equal out of the three options due to preferential treatment of the immigrants who otherwise would be worse off.

8.3.4 Summary

Policy III places the economy in aggregate between the Policies I and II. While the economy performs worse under Policy III than Policy II, it outperforms the aggregates of Policy II. However, this comes at a cost of increased disutility to the native agents resulting from sub-optimal taxation arising from the increased spending on the immigrants' educational subsidy. While the subsidy at being equal to a single payment of pension results in better outcomes than Policy II on aggregate, the policy seems to create significant inefficiencies to the taxation resulting in worse outcomes for the native population at the benefit of the immigrants. Better designed policy for educational subsidy would balance between the utility inefficiencies for the natives and utility gains of the immigrants to minimise the cost of the trade off by for instance having the policy targeted only at the marginal immigrants who would not otherwise seek to attain further education

However, in a society thriving for equality the policy as it stands is successful, decreasing the utility Gini-coefficient below 0.2 while still not making the economy worse off overall in comparison to Policy II.

9 Discussion

In general, the model used in estimating effects of different policy options, provides interesting insights in the long term impacts of an immigration shock, the foremost being that impacts are very limited from the perspective of the output of the economy. The greater differences between policies arise when looking at the

distributional effects, as different government policies have different implications for the three groups observed. While government inaction distorts the economy the least, it also has least distributional effects and as a policy *in*action can be considered preferential treatment of the educated, whom as less dependent on the welfare state are impacted the least. The other two policies, where the welfare system is kept intact on the other hand treat the educated more harsh but "distributes" the welfare loss such that it results in a more equal society.

The negative effects are a result of the immigration shock though number of different channels. The most direct impact comes from the immigrants themselves, whom have lower productivity than the local population due to the catch-all parameter ω's impact. This lower productivity results in lower income, and consequently consumption and asset accumulation of these agents which in turn yields lower tax revenue from these agents. The changes in tax base will be root cause though which the effects are carried on to the native agents[18].

Under Policy I, the mechanism through which the immigration shock affects the decision making of all agents is the social security system. As a result lower tax yield per capita and higher proportion of uneducated agents whom get the additional low-income subsidy, the size of social security contributions reduces. This has an impact on the disposable income for all agents during their retirement and for the uneducated agents also during the periods in employment. The falling incomes through social security induce a change in consumption/savings behaviour, but also in the labour decision and as the uneducated are more dependent on the social security, is this effect greater for them. To make up for the changing income, the agents increase working hours which will allow them to save more in an attempt to compensate for the losses.

Under Policies II and III, the social security system carries no effect on the agents' decision making, as these payments remain unchanged. Instead, the tax rate changes bring forth a more deep rooted effects which do not only alter the final asset accumulation behaviour, but also the marginal rates of substitution underlying this decision. The changing tax rates have a direct impact both on the intertemporal rate of substitution for consumption and the intratemporal rate of substitution between consumption and leisure. The former reacts to changes in

[18]In an economy without taxes and subsidies the immigrants would not have an impact on the natives' decision making, limiting the implications of an immigrant shock to lower per capita values of consumption, assets and effective labour. The native behaviour and accordingly their utility would remain unaltered.

tax on capital gains, while the latter changes in labour and consumption taxes. As both policies see the tax on capital gain increase, the marginal rate of substitution decreases meaning the agents' preference for consumption today over future consumption increases and the savings are reduced accordingly i.e. the agents become more impatient and prefer fast gains over long term benefits. The intertemporal rate of substitution changes linearly in capital gains tax. The latter, intratemporal rate of substitution between consumption and leisure, is more complex to interpret, as the relative difference of change in tax rates of determines the agents' preference over consumption and leisure. In both Policies II and III the tax on labour increases more than the tax on consumption, which implies that the agents will prefer leisure over their consumption more due to lesser gains of employment vis-á-vis previous state.

The two effects jointly make the agents more impatient, striving for fast gains from consumption today over savings and leisure over employment. As an outcome, the effect of increased taxes is opposite to decreased social security benefits and the agents will give up saving for contemporaneous consumption.

These changes will ultimately drive the changes in utility seen. Decreasing consumption both directly due to lesser income and changing behaviour through the substitution channels implies less utility. Furthermore, giving up leisure for making up the lost subsidies under Policy I leads into decreasing utility, more so for the uneducated whose gains from increasing working hours are lesser and need for compensation greater. Under the Policies II and III, the employment decreases as a result of lesser benefit of working due to lower real wages.

With respect to differences in the overall utility, the policies are very similar, all resulting in a utility gap of approximately 1% per capita. The differences between the different policies and the utility outcomes of those come at the individuals level. The first policy, which sees the worsening of the welfare system in the economy due to the fixed taxes has the least impact on utility, but the negative impact is almost in its entirety born by the uneducated agents and the immigrants who are worse off in the economy to begin with. Under the second policy impacts are reversed; although the over all utility in the economy is marginally lower than in Policy I, the tax increases required to maintain the welfare system of the economy have far greater impact on the educated population with higher utility than on the uneducated and immigrant population. In the Policy III, the overall utility in the economy is between the other two policies but this time driven by the best stance of the immigrants, whose average utility and educational attainment are

highest out of the three policy options, in other words they are the best integrated to the society if well-being is deemed an appropriate measure of integration.

The only objective measure capable of clearly differentiating between the different policies and their outcome is the utility Gini-coefficient. The first policy increases inequality in the society, as the uneducated agents and immigrants are more dependent on the welfare system of the economy and effected most by the falling contributions from the government. In the second policy, the tax increases have a greater impact on the educated agents whose tax rebate in terms of tax-free welfare payments is relatively less than for the uneducated, thus the tax increases having an equalising impact over the society. However, the immigrants still remain largely unaddressed by the second policy. Under the third policy, the immigrants are targetted with the additional educational subsidy which makes them better off compared to the other policies. While the immigrants are still below the utility levels of the native population due to their lower productivity, Policy III brings them closest to them resulting in a society that is most equal.

The second period gaps in output per capita varied between 0.9% in Policy I and 1.2% in Policy II, the differences being very small between the policies but also in comparison to the steady state value in the economy prior to the shock. Even at the most output distorting Policy II, the magnitude of the impact varied from 1.66% at the time of the shock to 1.2% at the end of the second observation when measured in per capita changes. Distortions caused by the economic cycle can easily outrank this in magnitude and more systemic shocks, such as the financial crisis and the following Euro crisis, may also result in permanent output gap vis-á-vis the pre-shock trend as elaborated by Ollivaud and Turner (2015).

Overall, the question of superiority of one policy over the others remains a question of values represented by the society. In light of objective measures like GDP per capita or overall utility the differences are so menial that they should be disregarded, the stochastic cycles observed in the real economy most likely exceed the outcomes of these differences. If assuming a majority rule in the economy where the less educated are not effectively disenfranchised by social exclusion or other factors diminishing their electoral participation relative to the educated agents, the public pressure results most likely in a policy scenario similar to Policy II where the existing social security regime is maintained. Whether targeted measures to improve the welfare of the immigrants would arise, remains a value question outside the scope of the more traditionalistic view of agents deriving utility only from consumption and leisure.

While the framework used in this thesis allows for experimenting with different policy options in estimation of the long term effects of an immigration shock and provides a starting point for a more constructive discussion on the topic, the model used comes with some caveats that should be kept in mind when interpretting the results from policy experiments.

Most importantly, the model construct will by definition yield negative aggregate outcomes for as long as the convergence of the immigrants is taking place following from the assumption that the average productivity in the society falls temporarily due to the immigrants and their ω parameter. While this does not invalidate the results, needs to be accounted for when contemplating on the results as the steady state acts more as an anchor point than a direct reference. The immediate implication of this is that the different policy outcomes should more be interpretted as differences from each other rather than from the steady state of the economy and evaluate their appropriateness in this light.

Longevity of the impact and the consequent magnitude of it on the economy hinges on the rate of convergence towards the population mean, and in this thesis an extremely cautious stance is taken regarding the convergence. The relatively slow speed of convergence, spanning over several generations of immigrants describes somewhat a worst case scenario most likely overestimating the negative impacts on the economy. While the persistence of the ω parameter is justifiable both under the evidence on intergenerational transmission of education and the patterns observed amongst existing groups of immigrants with respect to their educational and occupational outcomes, the decay of this parameter remains extremely hard to estimate. Endogenisation of the speed of convergence would improve the model by accounting for impact of different policies on the immigrants as success in integrating the first generation should directly translate into better integration of the following generations, even if the policy increasing integration had only fixed term length.

As pointed out previously in parametrisation of the model, the speed of convergence, if altered, does not have a direct impact on the type of reactions that take place in the economy, but rather the magnitude of them. If a faster convergence towards the native population is assumed, the magnitude of actions taken by the native population gets smaller as the distortion diminishes. In light of the literature estimation of the correct speed remains problematic as there are differences across different ethnic groups and backgrounds.

The assumption of the "transitional steady state" used in creating the snapshots of the economy in the periods after the shock, while a convenient tool is a bold simplification of the real world as it assumes that all agents at that time have the same educational attainment. In reality, educational attainment is a dynamic process driven by the cyclical nature of the economy resulting in deviations from the average education and especially for the immigrants assimilating the education should based on real-life findings be distributed in a way in which the younger cohorts living simultaneously with the older ones should have higher educational attainment than their predecessors. However, the problem in our view is mitigated by the mere fact that the aim is to provide a long term overview and while "averaging" the educational outcomes leads to some distortion, should the magnitude of this distortion given the small population size of the immigrants be negligible and discussion regarding policy outcomes especially from the native perspective remain relevant.

There is no economic growth, apart from the increasing population size, in the economy. While this might be considered problematic as economic growth over longer periods of time is usually observed, we believe it is highly unlikely that technological change in the economy would be endogenously driven as Córdoba and Ripoll (2008) show that knowledge, i.e. technology, is transferred between economies and growth differences between countries are driven by factors other than technology. As a consequence of this, the exogenous economic growth in the economy in form of the total factor productivity multiplier would simply reduce to a scalar multiplier without any actual impact between the different scenarios.

The model also overlooks the importance of equality in a society. Utility is only derived from consumption and leisure, and while this in general provides a great measure for well-being of an individual and is commonly used in Economics, it disregards the link between equality and social stability and their importance to individual's well-being completely. This should be especially important in an economy undergoing a socio-economic transition due to a shock it has faced. Delhey and Dragolov (2013) have shown that higher inequality in European societies has a negative impact on happiness (that being utility), primarily driven by increased distrust in unequal societies.

While some reassuring answers have been provided by the framework and experimentation done in this thesis, it remains for the future research in the field to make the estimations more precise and to include more accurate modelling

of the assimilation path of the immigrants through endogeneising this process. Other essential improvements to the model would in our opinion include integration of the equality aspects of the society to the model. Especially following these corrections, the additional calibration to a specific economy rather than to a hypothetical representative economy would yield great extra value.

Albeit several improvements remain to be accounted for in search for a better fitting model, this thesis and the framework used in it serves as a good starting point for discussion. The main conclusion of this thesis remains that while an immigration shock does cause a negative impact on an economy through disturbance to its steady state, the effect is very small and appropriate policy to mitigate for the impacts remains first and foremost a question of the role of the government; whether the government's ultimate goal is to simply maximise the output and minimise any disturbances in the economy, or whether the government's core competence lies in ensuring an equal and balanced society.

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