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Inequality aversion and competition via effort

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Abstract

This paper examines the possible consequences induced by inequality aversion in a competitive setting. By employing the Cournot toolkit, the paper puts forward a competition via effort model. Using the equilibrium for rational players as a benchmark for comparison, the paper identifies the type of suboptimal equilibria that could potentially emerge if agents display a bias toward inequality. The analysis isolates the welfare losses generated by inequality reduction. Based on the findings the paper calls for pragmatism and education to fight against the bias.

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1 Introduction

"If we are concerned about equality of opportunity tomorrow, we need to be concerned about inequality of outcome today." Anthony B. Atkinson

Ever since the publication of Tomas Picketty's book Capital in the Twenty-First Century, inequality has been attacked by research, media and informed citizens alike. The year 2016 politicized the issue in the light of Brexit and the US elections. For a moment, it felt like our bubble was set to burst. On top of this, Atkinson added the moral dimension of intergenerational inequality transfer as "the inequality of outcome today directly affects equality of opportunity—for the next generation" (Atkinson, 2015). While the reality of increasing economic disparities is hard to dismiss (Piketty, 2014), pinpointing the immediate consequences is less straightforward (Mesnard et al., 2001).

Despite the complexity of the matter, the public discourse rarely touches on the subtleties of the issue thus dismissing the multifaceted region-specific dimension of economic disparities. Hills (2014) gives a word of caution on the myth of inequality. Rico et al. (2017) found that anger and feelings of resentment towards unfair situations underpin populist attitudes. One possible justification lies in our subconscious preference for fairness and resistance to inequitable gains which in behavioral economics goes by the name of inequality aversion. Evidence wise, there is a growing body of empirical, experimental and neurological literature that strongly support our bias toward inequality. Fehr and Schmidt (2001) provide an extensive literature review on the topic covering both theoretical and empirical findings. Similarly, Lavergne and Strobel (2004) use simple distribution experiments to supports the inequality aversion hypothesis while Tricomi et al. (2010) provide neurological evidence on the matter.

As the evidence for inequality aversion is growing, it worth exploring the consequences that it can potentially trigger. Although the subcontinent human preference for equality can sometimes generate positive outcomes this is not always the case. From a macro perspective, a strong support for equality can sometimes facilitate constructive populist responses including progressive taxation. Nonetheless, as in the case of South Africa, inequality aversion can also trigger a permanent state of tension which ultimately puts downward pressure on the economic growth of the country (May and Govender, 1998). At a micro level, taking inequality aversion into account can positively impact the job satisfaction (Card et al., 2012). However, in the ultimatum game, the same inequality aversion can leave both players barehanded if the receiver rejects the offer of the dictator. Based on the contradicting findings, the question remains open.

The public narrative and, in certain cases, the reality of the increase in inequality are not the only factors that put upward pressure on our tolerance for disparities. Digitalization, rapid globalization along with the gradual opening of the labour markets change the competitive dimension of our everyday interactions by broadening the type of competitors we encounter (Morsing, 2005). In developed societies, agents are encouraged to compete for the same jobs irrespective of their economic background. Similarly, players can potentially engage in tech contests regardless of their wealth status. Although in a progressive world it is more likely to end up competing against individuals that come from different social-economic backgrounds, the equality of opportunity is still hindered. Therefore, it is worth exploring the way in which inequality aversion affects the outcome of a competition in which the equality of opportunity is constrained (Atkinson, 2015).

Previous studies only looked at the interlink between competition and inequality via the lens of rent-seeking models (Buchanan et al., 1980). Nevertheless, the models are limiting because they exclude value creation. This paper extends the literature on two grounds: First, it explores the dynamics of competition and inequality in a setting which simultaneously allows for value creation and value destruction. Secondly, it investigates the consequences of inequality aversion in a setting where the equality of opportunity is constrained. Explicitly, I use a slightly modified version of the standard asymmetric Cournot model to isolate the potential suboptimalities induced by inequality aversion. The model shows that when the cognitive ability of the agents is impaired, inequality reduction can emerge at the cost of the collective good. Moreover, if both agents are constrained by the bias, there is a significant risk of bringing all gains down to zero. Contrary to the standard intuition, welfare redistribution does not eliminate the possible suboptimalities induced by the existence of a mutual cognitive impairment. Therefore, if the equality of opportunity is constrained, the findings call for pragmatism, education and a reevaluation of the public narrative.

The rest of the paper proceeds as follows: Section 2 provides an overview of the relevant literature and gives the scenario that motivates the paper. Section 3 elaborates on the benchmark case when both players are rational. Section 4 analyzes the behavioral case when the agents are cognitively impaired. Section 5 discusses the results and presents the limitations of the model and Section 6 concludes.

2 Literature Review

Evidence suggests that in certain parts of the world, inequality is galloping (Mesnard et al., 2001). As most economists can agree, inequality in itself is neither bad nor good. Nevertheless, the magnitude matters. Atkinson was among the first that elaborated on the way in which today's increasing inequality negatively feeds into the future of equality of opportunity. The literature on inequalities of opportunities is making progress in bringing evidence on the matter (Ferreira and Gignoux, 2011). Although the developments are applaudable, isolating the difference between the reality of inequalities and the perception of them is challenging (Kluegel and Smith, 1986). Furthermore, keeping a clear distinction between what is acceptable at a macro level and what can be encountered at a micro level is cognitively demanding and can trigger a general bias towards disparities (Rico et al., 2017). Therefore, it is worth exploring the type of consequences brought by such biases at a micro level to help improve the perception of inequality.

2.1 Inequality aversion

Contrasting the homo economicus mantra, a brief overview of the experimental findings on the dictator game and on the ultimatum game indicates that economic agents are willing to give up their gains in order to equalize the outcome (Güth, 1995). The data is strengthened by more recent neurological findings (Boksem and De Cremer, 2010). In terms of the interpretation, one possible hypothesis argues in favor an intrinsic preference for fairness and an inner resistance to unequal gains. In behavioral literature, this is called inequality aversion. Known as one of the pillars of the literature, Fehr and Schmidt (1999) brought forward a paper that shifted the paradigm of a rational selfish agent by introducing a utility function that allows for positive and negative actions towards other players. Simplified, the function takes the form:

$$U_i(x) = x_i - \alpha_i \max(x_j - x_i, 0) - \beta_j \max(x_i - x_j, 0), \beta_j \le \alpha_i, 0$$

Based on the assumption that players are heterogeneous with respect to inequality aversion the authors calibrated the parameters of the function then validated that prediction power of the model in different economic settings. Despite its influence on the behavioral literature, the theory of Fehr and Schmidt (1999) does not lack in criticism on the grounds of numerical manipulations and lack of value added (Binmore and Shaked, 2010). Equally influential but less disputed, Bolton and Ockenfels (2000) provide a different interpretation of the inequality aversion concept, less linked to fairness per se, but rather based on social comparison as the utility function of the players stresses the difference between self-maximizing interests and relative payoffs. Although the interpretation of the bias is different, for two-player interactions, Bolton and Ockenfels (2000) and Fehr and Schmidt (1999) give similar predictions. The qualitative and quantitative differences appear when the number of players increases (Kagel and Wolfe, 2000). Focusing on two players settings, the similarities between Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) offer the ground for an extension which keeps the fairness emphasis of the former and merges it with the competition specific insights of the latter.

2.2 The interlink between competition, effort and inequality

Explicit or not, economic agents engage in competition on a daily basis (e.g. job hunting, promotion seeking or business activity). One way of exploring the contests is by focusing on the effort put in by the agents. At first sight, basing the outcome of a game solely on effort might seem too restrictive as gains also depend on circumstance (Roemer, 2009). Nevertheless, this is a recurring theme in economic literature, closely linked to poverty and inequality (Dalton et al., 2016). Bourguignon (2017) justifies the difference in standards of living based on the difference in hours worked. The public opinion seems to reinforce the idea as in a 2014 American survey, one-third of the respondents coupled poverty with lack of effort (Doherty et al., 2014). In economic literature, the vast majority of papers use rent-seeking models to explore the interlink between effort and competition. Tullock (1967) laid the foundation of the rent-seeking literature which was further elevated by Buchanan et al. (1980). In the standard setting, players engage in competition by making costly investments which in turn affect the probability of winning the prize.

The majority of the rent-seeking papers that allow for inequalities follow the line of Allard (1988) which explores the existence of equilibria and rent dissipation induced by cost disparities. More recently Chaturvedi (2017) showed that inequality affects rent-seeking and that rent-seeking, in turn, affects the wealth distribution. The self-reinforcing mechanism perpetuates wealth disparities thus inducing an inequality trap.

The empirical evidence on rent-seeking suggest that inequality changes the equilibrium effort. For instance, in Fonseca (2009), under financial inequality, the disadvantaged players put in less effort relative to the advanced ones. In contrast, in Kimbrough et al. (2014), high inequality leads to more resources being spent by the less wealthy players. Lastly, Fallucchi and Ramalingam (2017) validates the insights of Fonseca (2009) and find that players are more sensitive to ability inequalities rather than to financial ones.

Although the results are highly relevant for public policy, Buchanan et al. (1980) made a clear distinction between entrepreneurial competition which creates values and traditional rent-seeking competition which destroys it. Therefore, it is worth extending the existing literature by exploring the interlink between inequality and effort in a competitive setting which allows for both value creation and value destruction.

2.3 Cournot Duopoly

In the well-known static Cournot duopoly model, firms compete in quantities, where the products are assumed to be perfect substitutes. The profit function depends on the aggregate demand function, on the inverse demand function and on the player-specific cost function. In a nutshell, the Cournot equilibrium isolates the quantity pairs which exclude unilateral deviations. In line with the inequality concept if the static framework is fixed, one can easily depart from the classic two-agent model by introducing asymmetric cost while keeping the analysis tractable; see Shapiro (1989) and Fudenberg and Tirole (2013) for in-depth proofs. The equilibrium deviations provide valuable insights for understanding the realities of the markets; see Daughety (2005) for an extensive literature review of the topic and Plott (1982) for an initial assessment of the experimental findings.

The Cournot equilibrium properties namely, existence, uniqueness and stability are contingent on the specifications of the model. According to Novshek's existence theorem (Novshek, 1985), if the marginal revenue of one firm declines as the aggregate output of the other firms increase, then the Cournot equilibrium exists; extensions include Szidarovszky and Yakowitz (1977), Kolstad and Mathiesen (1987) and Van Long and Soubeyran (2000). On uniqueness, Gaudet and Salant (1991) provide a widely accepted condition which does not depend on the assumption of non-degeneracy of equilibrium. Hence it is less restrictive than the ones proposed by Szidarovszky and Yakowitz (1977) and Kolstad and Mathiesen (1987). For stability, Cournot brought forward a best-reply dynamics argument. Following his line of tough, the myopic agents engage in a sequential game by best responding in the current period to the existing output levels of the opponents. Although the validity of the proof is broadly disputed, Cournot's dynamic stability argument does not lack in popularity ¹. Nevertheless, for the case of duopoly, Fudenberg and Tirole (1991) provides a more convincing stability condition which easily extends to asymmetric settings.

Welfare wise, an important remark is related to the fact that the firms' total profit is not maximized by the Nash equilibrium outputs of the Cournot's model. Notably, Cournot Nash equilibrium is not Pareto efficient. Furthermore, the welfare comparison between Cournot and Bertrand is debatable. When the asymmetry is strong or when the products are weakly differentiated Zanchettin (2006) shows that the industry profits are higher under Bertrand competition. The results challenge the finding of Singh and Vives (1984), where the ranking of profits is reverse. If the number of players increases, the equilibrium of the Cournot model approaches the Walrasian one; see Daughety (2005) for extensions.

The Cournot model is not generally used to characterize the strategic effort interaction between individuals. The closest model that I could find employs a competition via effort framework to study team behavior (Raab and Schipper, 2009). Nevertheless, Cournot is one of the classic examples of competition in the economic literature which has been previously reinterpreted in less conventional ways; see Goyal and Moraga-Gonzalez (2001) for an example on networks. Despite the lack of popularity, the asymmetric Cournot toolkit could complement the existing literature that explores the interlink between competition, effort and inequality as the Nash setting allows for both value creation and value destruction. Section 3 and Section 4 elaborate on one possible "out of the box"

¹See Zhang and Zhang (1996) and Szidarovszky and Li (2000)

extension.

2.4 Scenario

Before I proceed with the actual formulation of the model, I invite the reader to consider the following hypothetical scenario:

Two agents try to sell two goods which are close to perfect substitutes. The goods are intangible and excludable (e.g. two almost identical set of skills, insights or business ideas). For instance, assume that A and B are two analysts. After long independent work, both players simultaneously cracked the formula that determines the most profitable investments within a specific market. Now both agents try to sell their findings. For simplicity, assume that there is only one potential buyer (i.e. an investor). Despite the investor's interest, the sellers have to convince the potential buyer of the value of the good. The process requires effort. Explicitly, the sellers have to use all necessary means to get in touch with the investor and to produce a convincing argument that maximizes their gains without revealing the formulas. If the players engage in the selling activity separately, they are competitors. Otherwise, they can merge their efforts and split the profits. Note that the buyer is not constrained by a single purchase. If A and B choose to compete, the financial payoff of a player is equal to the individual revenue minus the cost of the selling activity. The revenue derived by a player depends on how much effort he exerts in order to convince the investor. Nevertheless, the revenue is also affected by the way in which the buyer values a unit of effort. Assuming that there exists an upper bound in line with the maximum willingness to pay of the buyer, the price per unit of effort decreases as the total effort put in by both players increases. To understand the intuition behind the changes in the price per unit of effort, recall that the scenario talks about almost identical financial insights. Although to a certain extent the formulas are a silver bullet for good investments, the buyer cannot fully evaluate them until they are revealed (i.e. ex-post purchase). Increasing the total effort put in by the sellers lowers the per unit price because it sends mixed signals. Think about it: if the insight is truly powerful why would someone go that extra mile to sell it and thus lose the exclusivity of the intrinsic gains? In this scenario, one potential explanation could be that A and B do not have enough resources to make full use out of their findings. Nevertheless, this is not known by the investor. Hence, from a buyer's perspective, the risk of a bad purchase is still on the table. Indeed, the price-per-unit feature is only justified by the characteristics of the goods which are both intangible and close to perfect substitutes. Going back to the revenue analysis, the financial gains of player A do not only depend on the effort put in by A but also on the effort put in by B as it feeds into the per unit price of effort. Cost wise, the per unit figure is fixed and reveals what is the cost incurred by a player when he increases his effort of selling the good by a unit. In this scenario, the individual per unit cost of effort can be understood as an weighted average between the corresponding cost of engaging with the investor and the cost of framing a convincing story.

Regarding the sellers, A and B are equally knowledgeable and charismatic. In theory, A and B should be equally persuasive. Nevertheless, based on the background, one of them has one foot in the door. For instance, suppose that A comes from a well-known, respected family which has close ties with the financial market. Compared to player B it is likely that A has a comparative advantage which gives A an easy access to the investor. Furthermore, A can benefit from the good reputation of the family background which could facilitate the process of convincing the buyer. The background of the players is common knowledge. Let's assume that both players display a bias towards inequality (i.e. both players think that the comparative advantage of player A goes against the fair play principle). Hence, the agents are negatively affected by the difference between their total financial payoffs. Therefore, the utility derived from engaging in the selling activity is the difference between the financial gains and the loss in utility incurred due to inequality aversion.

Before the start of the selling activity, both players have to simultaneously choose the level of effort which will maximize their utility, keeping in mind the way in which revenues, costs and inequality aversion feed into each other. Once the efforts are selected, A and B proceed with the selling activity.

This paper tries to narrow down the outcomes of such game. For instance, one could think of several possible options:

a. A and B can compete in the standard way and bear the utility cost of inequality aversion.

b. A and B can decide to merge and split the gains in order to minimize the utility losses induced by inequality aversion.

c. Due to inequality aversion, one player can engage in the selling activity while the other one decides to opt out.

d. While engaging in the selling activity, both A and B can compete either significantly more or less aggressively in order to minimize the utility losses.e. Both A and B can opt out of the game.

By comparing the possible outcomes with the equilibrium ones, I aim to prove that in a setting like the one described above, inequality aversion cannot generate positive outcomes. Going back to the scenario, the results can provide a theoretical justification for the aggressiveness characterizing certain types of competition. For example, one can build up an argument which defends the aggressive behaviour of financial analysts, start-up owners and lobbyists which could otherwise be perceived as immune to inequality per se. At the same time, the findings could justify why in certain cases players opt out of the game, although it does not make sense from a purely financial point of view. To understand why this can be suboptimal one can slightly change the above scenario. Imagine that A and B are trying to pitch their start-up idea which improves the water usage and availability in the less developed parts of the world. If A and B decide to opt out, then they lose the investment funds. Apart from the drop in total gains, by not pitching the insight, the players keep hold of the idea which can be detrimental to the society as a whole. Lastly, to showcase the wide range of applications, the reader could imagine that the investor is a talent seeker and the good is a set of skills. Such a simple twist

can extend the findings to the labour market.

Section 4 provides a generalization of the scenario described above. Nevertheless, before diving into the behavioral analysis, I will elaborate on the case of rational agents, which gives the benchmark for comparison.

3 Theoretical Model

Consider an asymmetric Cournot duopoly game and a set of players i, j. Compared to the standard case in which firms compete through quantities, in this model the players i and j compete via effort.

Each player is characterized by an exogenous initial level of wealth $w_i \in [w_l, w_h]$, where $[w_l, w_h]$ is a bounded subset of \Re^* with $w_l < w_h$. The restriction implies that initial wealth interval is the same for both players. As in the standard framework where each unit comes at a marginal cost, this model assumes that the effort put in by player *i* is also characterized by a marginal cost which in turn, is given by the inverse of his initial wealth w_i (i.e. $mc_i = 1/w_i$).

Similar to classic Cournot case in which the price of the goods depends on the overall quantity produced, the inverse demand function associated with the competition via effort game gives the per unit price of effort. Hence, it depends on the overall effort put in by both players:

$$P(e_i, e_j) = a - b(e_i + e_j) \tag{1}$$

where

$$1/w_i \leq a \text{ and } 1/w_i \leq a$$

Note that the above formulation implies that the efforts e_i and e_j are close substitutes. The assumption along with the condition imposed on the marginal cost of effort put aside the issue of effort productivity. Nevertheless, the restrictions allow for the formulation of the benefit function:

$$b_i(e_i, e_j, w_i) = P(e_i, e_j)e_i - C_i(e_i) = [a - b(e_i + e_j)]e_i - e_i/w_i$$
(2)

The benefit of playing the game depends on the exogenous parameters namely a, b and w_i and on the endogenous variables namely the effort selected by the player and the one put in by his opponent. In plain English, it is the difference between the revenue and the cost associated with the effort e_i . The model mimics the strategic interaction of the standard Cournot setting where the firms strategically interact and profits, in this case, benefits, depend on each other.

To capture the idea of inequality aversion, I assume that players compare their gains with a reference benefit, determined endogenously as the weighted average of the benefits derived through engaging in the competition via effort game 2 :

$$\overline{b_i}(e_i, e_j, w_i, w_j) = \alpha_i b_i(e_i, e_j, w_i) + (1 - \alpha_i) b_j(e_j, e_i, w_j)$$

$$(3)$$

Going back to the previous section, the individual reference benefit is a measure of fair play. The weighted average benefit is contingent on the parameter α_i which in turn reflects the way in which a player internalizes the gains vis-à-vis reference. A high alpha parameter implies that when setting a benchmark, a player cares more about his benefit relative to the one of his opponent. Similarly, a low alpha parameter shows that the weighted benefit average is more in line with the gains of the adversary. From a modelling perspective, the α parameter is similar the one in Fehr and Schmidt (1999). The difference between the player's *i* benefit and the corresponding reference benefits is enclosed in a continuous, differentiable, reference-dependent, zero-bliss point, value function given by ³:

$$v[\frac{b_i(e_i, e_j, w_i) - \overline{b_i}(e_i, e_j, w_i, w_j)}{b_i(e_i, e_j, w_i)}] = -(\frac{b_i(e_i, e_j, w_i) - \overline{b_i}(e_i, e_j, w_i, w_j)}{b_i(e_i, e_j, w_i)})^2 \quad (4)$$

The value function highlights the key feature of the model namely inequality

 $^{^{2}}$ Note:"Reference benefit" and "weighted average benefit" are used interchangeably throughout the paper.

 $^{^{3}}$ The value function is similar to the one introduced by Kahneman and Tversky (1979).

aversion. Similar to the scenario, the value function determines the utility loss suffered by the players when their financial gains depart from the corresponding fair play ones. The minus in front of the quadratic captures the negative impact of the difference between individual benefit and the corresponding benefit reference (i.e. the weighted average of the benefits of the game $\alpha_i b_i + (1 - \alpha_i) b_j$).

Players are interested in maximizing their utilities. Similar to (Dalton et al., 2016), both the benefit function and the value function feed into the utility of a player:

$$u_i(e_i, e_j, w_i, w_j) = b_i(e_i, e_j, w_i, w_j) + v[\frac{(b_i(e_i, e_j, w_i) - \overline{b_i}(e_i, e_j, w_i, w_j))}{b_i(e_i, e_j, w_i)}]$$
(5)

In this model, the utility function depends on the benefit derived from engaging in the competition via effort game and on the loss incurred when the individual benefit does not match the corresponding reference one. Based on the form of the value function, the drop in utility is contingent on the absolute value of the difference, not on the sign. The mechanics of the game proceeds follows. Before the start of the game, the players choose the appropriate level of effort that maximizes their utility. Once the effort is selected, the players engage in competition via effort game which ultimately determines their final wealth. At a superficial look, this effort should always coincide with the one that maximizes the corresponding benefit function. Nonetheless, the second component of the utility function, namely the value function can change the outcome of the game subject to the difference between the individual benefit and the corresponding reference benefit. Section 4 takes a closer look at the possible suboptimalities caused by inequality aversion. Before proceeding with the analysis, I will elaborate briefly on the intuition of the model.

3.1 Intuition of the Model

At this point, the reader might question the underlying of the model. All aside, how credible is a competition via effort game and what are the arguments that support it?

I start by making a brief detour to the literature review section. As mentioned in Section 3, Raab and Schipper (2009) use a similar Cournot competition via effort model to study team behaviors. Buchanan et al. (1980) provides the foundation for numerous examples of a contests contingent on effort. The papers are complemented by an emerging strand of literature that touches upon the interdependence between entrepreneurial activities and effort (Acs et al., 2005). Although limited the existing literature defends the credibility of the model.

To understand the intuition behind the framework, I ask the reader to take the modelling shoes off for a moment and think about real, effort based competition settings. Without a doubt, the benefits of engaging in such games are also affected by other, non-wealth related factors including effort-productivities and know-how. Nevertheless, wealth goes hand in hand with education (Glewwe and Jacoby, 2004), entrepreneurship (Evans and Jovanovic, 1989), group participation (La Ferrara, 2002), health (Schellenberg et al., 2003) and high aspirations (Ray, 2006). Furthermore, empirics suggest that the capacity to strategically use information is in itself contingent on economic background (Duflo et al., 2011). If education, entrepreneurship, aspiration, and group membership positively correlate with wealth it is likely that the effort of a well-off player is less costly compared to one of a less fortunate counterpart. Most importantly, the assumption is in line with the lack of equality of opportunity highlighted by (Atkinson, 2015). As agents internalize the inequality of opportunity, it seems reasonable to assume that before the start of the game, the opponents concentrate solely on wealth, thus basing their decision on a perceived, not a real competition, which otherwise would be subject to efficiency, skills and expertise. In a one-shot game in which the players select their efforts ex-ante, the perception determines the outcome. Admittedly, one could argue that in the face of a repeated interaction, the agents correct themselves. Consequently, after the first round of the game, the perceived competition mirrors the actual one. Although a dynamic approach might capture the problem of perpetuating wealth disparities better, for tractability reasons I stick to a static framework,

which in itself, provides fruitful insights by isolating the suboptimalities induced by inequality aversion. Nevertheless, Section 5 elaborates on this limitation.

3.2 Uncovering the α parameter and the weighted average benefit

In the context of this model, the α parameter is a reflection of the inequality aversion which in turn affects the weighted average benefit that gives the reference relative to which players measure the benefit obtained through engaging in the competition via effort game. In turn $1 - \alpha$ is the magnitude of the behavioral bias. A high alpha parameter shows that the player internalizes his gains relatively well. Therefore, the difference between his benefit and his weighted average benefit is small. If the alpha parameter approaches zero, then the agent values the gains of his opponent more than his own. Therefore, the absolute value of his value function is relatively high. Hence, the player incurs a significant utility loss. As mentioned above the weighted average benefit becomes the reference relative to which a player compares his gains. Genicot and Ray (2014) talk about the way in which the agents set their reference based on their social context. Similarly, Keeping Up with the Joneses (Gali, 1994) preferences bring up the same concept of social comparison. Furthermore, Ray (2016) talks about self-evaluation through the lens of "a limited social cognitive window": I validate my achievements based on my social exposure. In two-players setting, an agent only compares himself with his opponent. As the "cognitive zone" is different from individual to individual, the reference points are different. The model preserves this feature as the weighted average benefit is agent-specific. Consequently, player i and player j might have different reference benefits which in turn affect the outcome of the game. Section 4 elaborates on the interaction between reference benefit and outcome. To fully grasp the findings, I start with a comprehensive analysis of the benchmark case.

3.3 Benchmark case: Rational Players

The outcome of the game depends on the interaction between two key forces namely, competition and inequality aversion. The effort selected depends on the relative advantage (disadvantage) associated with a lower (higher) marginal cost. At the same time, the decision-making process is also constrained by the difference between the individual benefit and the player-specific weighted average benefit derived through the competition via effort game. Given the nature of the interactions the effort selected by players i does not only depend on the effort level put in by his opponent but it is also contingent on the benefit references which are in turn, interdependent. A rational agent fully internalizes the outcome of this four-direction movement. Explicitly, a rational player keeps track of the way in which efforts, benefits and reference benefits feed into each other. To set a benchmark, I start by formally analyzing the case when both players are farsighted.

Definition 1: A solution for a rational player is given by a pair (\hat{e}_i, \hat{a}_i) such that:

$$e_i = f(e_j, w_i, w_j, \alpha_i, \alpha_j) \tag{6}$$

and

$$\hat{\alpha}_i = argmax_{\alpha_i \in [0,1]} u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, w_j, \alpha_i)$$
(7)

According to the first definition, a rational player understands the feedback mechanism between the effort levels and the reference benefits. Therefore, a foresighted player endogenously determines the utility maximizing alpha parameter. For a rational players the value of $\hat{\alpha}_i$ that maximizes the corresponding utility function irrespective of the rest of the parameters is:

$$\frac{\partial u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial \alpha_i} = 0$$
$$\frac{\partial b_i}{\partial \alpha_i} + \frac{\partial v[\frac{b_i - (\alpha_i b_i + (1 - \alpha_i)b_j)}{b_i}]}{\partial \alpha_i} = 0$$

$$(1 - \alpha_i)(b_i - b_j)^2 = 0$$
 i.e $\hat{\alpha}_i = 1$

It is important to understand that a rational player does not exogenously set the value of alpha equal to one. Actually, he determines it via a maximization process which reflects his superior cognitive ability. When both agents are farsighted, the value function hits the zero-bliss point. In other words, the reference benefit of rational players is equal to the benefit they derive by engaging in the competition via effort game. Under this assumption, the first order derivative of the utility function with respect to effort takes a well-defined concave form and has a unique solution which is characterized by:

Proposition 1: The benchmark case in which both players are rational has a unique solution of effort and reference benefit of the form $\overline{e}_i^*, \overline{e}_j^*, \overline{b}_i^*, \overline{b}_j^*$, where \overline{e}_i^* and \overline{e}_j^* are the Nash equilibrium of the Cournot model and \overline{b}_i^* and \overline{b}_j^* are simply the benefits derived through the competition via effort game (i.e. $\overline{b}_i = b_i$ and $\overline{b}_j = b_j$).

Proof. See appendix.

According to Proposition 1, for rational agents, the effort is only contingent on wealth. The difference between the benefits is driven solely by the complementarity between effort and wealth. Therefore, the outcome is justified by the specification of the model. Although this outcome is not insightful per se, it provides the benchmark for understanding the possible behavioral deviation outlined in Section 4.

4 Behavioral Case: Cognitively Impaired Players

In a behavioral setting, the players are not farsighted hence they do not internalize the feedback mechanism between effort, individual benefit and reference benefit. Therefore, the agents do not endogenously determine the alpha parameter that maximizes their corresponding utility function. Contrary to the benchmark case, the individual reference benefit depends on an exogenous parameter $\alpha \in [0, 1)$, which in is in line with the inequality bias and where $1 - \alpha$ is the magnitude of inequality aversion. By assuming that α is exogenous, I follow closely Fehr and Schmidt (1999) and build on the empirical evidence that defends our preferences for fairness and intrinsic resistance to inequality ⁴. More concretely, in a behavioral setting:

$$\exists \alpha_i s.t. \alpha_i \neq argmax_{\alpha_i \in [0,1]} u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, w_j, \alpha_i)$$
(8)

Note that it is not necessary to endogenize α_i and α_j to obtain insightful results. Although α_i and α_j are given, the benchmark relative to which the agents compare their gains remains endogenous as it depends on the efforts put in by both players. For $\alpha_i \in [0, 1)$ the value function is strictly negative. The interval restriction feeds into the utility function and in turn, generates compelling results.

In the context of this model, a behavioral player is aware of his bias towards inequality. Therefore, the agent will adapt his decision-making process accordingly. As a result, the maximization problem becomes:

$$\operatorname{Max}_{e_i \in [0,E]} u(e_i, e_j, w_i, w_j, \alpha_i)$$
$$\operatorname{Max}_{e_j \in [0,E]} u(e_j, e_i, w_j, w_i, \alpha_j)$$

Where the upper bound of the individual effort dependents on set the parameters $\{a, b, w_i, w_j\}$:

$$E = f_E(a, b, w_i, w_j) \tag{9}$$

Note that a behavioral player is not only aware of the parameter that affects his value function but also knows the parameter that plays into the value function of his opponent. Therefore:

Definition 2: In the behavioral case, an equilibrium solution is given by an equilibrium set $\{e_i, e_j\}$ such that

$$e_i = f(e_j, w_i, w_j, \alpha_i, \alpha_j)$$

 $^{^4\}mathrm{See}$ Camerer and Fehr (2006) and Clark and D'Ambrosio (2013) for an in-depth literature review

where

$$e_i^* = argmax_{e_i \in [0,E]} u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, w_j, \alpha_i)$$
(10)

and

$$e_j = f(e_j, w_j, w_i, \alpha_j, \alpha_i)$$

where

$$e_j^* = argmax_{e_j \in [0,E]} u(f(e_j, w_j, w_i, \alpha_j, \alpha_i), e_i, w_j, w_i, \alpha_j)$$
(11)

for $\alpha_i \in [0, 1)$ and $\alpha_j \in [0, 1)$ exogenously given.

Based on Definition 2, in equilibrium, the effort of a player does not only depend on the relative difference between marginal costs but it is also contingent on the alpha parameters of both players (i.e. both α_i and α_j).

Lemma 1: Let the value function take the form $v[\frac{b_i - \overline{b_i}}{b_i}] = -(\frac{b_i - \overline{b_i}}{b_i})^2$ and $v[\frac{b_j - \overline{b_j}}{b_j}] = -(\frac{b_j - \overline{b_j}}{b_j})^2$ where the relative benefit is defined as $\overline{b_i} = \alpha_i b_i + (1 - \alpha_i)b_j$ and $\overline{b_j} = \alpha_j b_j + (1 - \alpha_j)b_i$ and α_i and $\alpha_j \in [0, 1)$ are exogeously given. Then, there is no clear complementarity between the individual effort and the corresponding alpha parameter as:

$$\frac{\partial^2 u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial e_i \partial \alpha_i} \simeq \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} 4(1 - \alpha_1)(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j))$$

$$\frac{\partial^2 u(f(e_i, w_j, w_i, \alpha_j, \alpha_i), e_i, w_j, \alpha_j))}{\partial e_j \partial \alpha_j} \simeq \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} 4(1 - \alpha_1)(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i))$$

Proof. See appendix.

Contrary to the existing literature, there is no clear-cut relationship between a player's effort and the corresponding α parameter. Therefore, the change in marginal net utility as α goes up cannot be easily pinpointed. The finding contrasts Fehr and Schmidt (1999) in part because the setting is different. As players engage in a strategic interaction, it is less likely isolate the interlink between the marginal gains and the corresponding α values. Since the alpha parameter in itself does not bring any insight I will proceed with the equilibrium analysis by stating the result that captures the outcome of the behavioral competition via effort game.

Proposition 2: For behavioral players, the competition via effort game displays multiple pairs of equilibrium effort which either mimic the benchmark one, or generate benefit pairs that are close or equal to (0,0).

Proof. The rest of this section is a step by step proof of this proposition; See Observation 1, Observation 2 and Proposition 3.

Compared to the benchmark, when the cognitive ability of the players is impaired, the equilibrium pair (e_i^E, e_j^E) is not unique. Providing a full characterization of the equilibrium set (e_i^E, e_j^E) is analytically difficult and goes beyond the scope of the model. I will proceed by providing a step-by-step proof for Proposition 2. The analysis disentangles the implications of Proposition 2 and gives a good starting point for understanding the type of equilibria that can potentially emerge in a competition via effort setting when players display a bias towards inequality and are cognitively impaired. I start by isolating the underlying maximization conditions that underpin Proposition 2:

$$\begin{cases} \frac{\partial b_i(e_i,e_j,w_i)}{\partial e_i} = 0\\ \frac{\partial b_j(e_j,e_i,w_j)}{\partial e_j} = 0\\ 3b_i^2(e_i,e_j,w_i) - 2(1-\alpha_i)^2(b_i(e_i,e_j,w_i) - b_j(e_j,e_i,w_j)) = 0\\ 3b_j^2(e_j,e_i,w_j) - 2(1-\alpha_j)^2(b_j(e_j,e_i,w_j) - b_i(e_i,e_j,w_i)) = 0 \end{cases}$$

Proof. See appendix.

If the first two constrains simultaneously hold then the equilibrium of the benchmark case (i.e. $e_i^* = \frac{a-2/w_i+1/w_j}{3b}$ and $e_j^* = \frac{a-2/w_j+1/w_i}{3b}$)⁵ is also an equilibrium in a behavioral case. The emergence of this equilibrium would not bring any changes to the benefit distribution. As mentioned above, this can be seen as an example of perpetuating wealth disparities although the inequality trap is a consequence of the modelling toolkit and it is not insightful per se. Nevertheless, this possible outcome should be noted.

 $^{^5 \}mathrm{See}$ appendix.

Observation 1: $\forall w_i, w_j \in [w_l, w_h]$ and $\forall a_i, a_j \in [0, 1)$ the benefit pair (b_i^*, b_j^*) where $b_i^* = 0$ and $b_j^* = 0$ satisfies the maximization problem corresponding to the behavioral case.

Proof. If the 3^{rd} and the 4^{th} maximization conditions simultaneously hold, then the result follows immediately.

Observation 1 isolates the main suboptimality induced by the inequality bias in a competition via effort setting namely, the existence of the equilibrium benefit pair (0,0). Although the case of equal benefits eliminates the inequality trap, this happens at the cost of the collective good. If both players incur zero gains, then the welfare (i.e. $b_i + b_j = 0$) goes down to zero.

To understand the strength of the constrain, the reader should bear in mind that a zero-restriction imposed on the benefit pair generates numerous effort equilibria of the form:

$$\begin{cases} e_{i} \rightarrow \frac{aw_{i}e_{i}w_{j}e_{j} - \sqrt{w_{i},e_{i}}\sqrt{w_{j}e_{j}}\sqrt{-4bw_{i}e_{i} - 4bw_{j}e_{j} + a^{2}w_{i}e_{i}w_{j}e_{j}}}{2(bw_{i}e_{i}^{2} + 4w_{i}e_{i}w_{j}e_{j})} \\ e_{j} \rightarrow \frac{\frac{aw_{i}e_{i}^{2}w_{2}e_{j}}{bw_{i}e_{i}^{2} + bw_{i}e_{i}w_{j}e_{j}} - \frac{w_{i}e_{i}^{3/2}\sqrt{w_{j}e_{j}}\sqrt{-4bw_{i}e_{i} - 4bw_{j}e_{j} + a^{2}w_{i}e_{i}w_{j}e_{j}}}{2w_{j}e_{j}}}{2w_{j}e_{j}} \\ \begin{cases} e_{i} \rightarrow \frac{aw_{i}e_{i}w_{j}e_{j} + \sqrt{w_{i},e_{i}}\sqrt{w_{j}e_{j}}\sqrt{-4bw_{i}e_{i} - 4bw_{j}e_{j} + a^{2}w_{i}e_{i}w_{j}e_{j}}}{2w_{j}e_{j}} \\ e_{j} \rightarrow \frac{\frac{aw_{i}e_{i}w_{j}e_{j} + \sqrt{w_{i},e_{i}}\sqrt{w_{j}e_{j}}\sqrt{-4bw_{i}e_{i} - 4bw_{j}e_{j} + a^{2}w_{i}e_{i}w_{j}e_{j}}}{2w_{j}e_{j}} \\ e_{j} \rightarrow \frac{\frac{aw_{i}e_{i}w_{j}e_{j} + \sqrt{w_{i},e_{i}}\sqrt{w_{j}e_{j}}\sqrt{-4bw_{i}e_{i} - 4bw_{j}e_{j} + a^{2}w_{i}e_{i}w_{j}e_{j}}}{2w_{j}e_{j}} \\ e_{j} \rightarrow \frac{\frac{aw_{i}e_{i}^{2}w_{2}e_{j}}{bw_{i}e_{i}^{2} + bw_{i}e_{i}w_{j}e_{j}} + \frac{w_{i}e_{i}^{3/2}\sqrt{w_{j}e_{j}}\sqrt{-4bw_{i}e_{i} - 4bw_{j}e_{j} + a^{2}w_{i}e_{i}w_{j}e_{j}}}{2w_{j}e_{j}} \\ \end{cases}$$
(13)

The existence of multiple strictly positive equilibria effort pairs which generate zero gains for both players, means that in equilibrium the players can end up neutralizing the benefit inequality through aggressive competition. This can happen either if one player aggressively over-competes at the expense of his opponent effort or if both players put in more effort than they would have otherwise put in the standard, rational setting case.

Observation 2: The number of equilibrium effort pairs that generate a (0,0) benefit pair is larger than one and strictly depends on the exogenous variables a_i, b_i, w_i, w_j .⁶

 $^{^{6}}$ A numerical analysis shows that the observation extends to the total number of equilibrium

Proof. The proof follows immediately from the form of Equation 17 and Equation 18.

Based on Observation 2 the exogenous parameters support multiple equilibrium effort pairs that generate zero gains for both players. Nevertheless, observation 2 is a mere statement of existence. The equilibria number does not bring any insight into the issue of stability nor does it say anything about the probability of equilibria emergence. Therefore, changing the exogenous parameters namely, a, b, w_i, w_j can only change the number of effort pairs that generate a (0,0) benefit pair without eliminating the potential suboptimality in itself. The finding is particularly interesting with respect to wealth redistribution as it shows that wealth changes cannot eliminate the zero benefit-risk induced by inequality aversion. The way in which wealth changes affect the probability of equilibria emergence remains open to further research.

Up to this point, I have shown that in a behavioral setting a potential equilibrium effort pair can either be equal to the benchmark one or it can induce zero gains for both players. The question that remains unanswered is: What other types of equilibria pairs can emerge and what are their welfare implication? Based on the utility maximizing conditions, all other equilibria pairs satisfy one of the following constraints:

$$\begin{cases} e_i = \frac{a - e_j - 1/w_i}{2}; (b_j(e_j, e_i, w_j) - \frac{2(1 - \alpha_j)^2}{3})^2 = \frac{2(1 - \alpha_j)^2}{3}(\frac{2(1 - \alpha_j)^2}{3} - b_i(e_i, e_j, w_i)) \\ e_j = \frac{a - e_i - 1/w_j}{2}; (b_i(e_i, e_j, w_i) - \frac{2(1 - \alpha_i)^2}{3})^2 = \frac{2(1 - \alpha_i)^2}{3}(\frac{2(1 - \alpha_i)^2}{3} - b_j(e_j, e_i, w_j)) \\ b_i(e_i, e_j, w_i) + b_j(e_j, e_i, w_j) - \frac{2(1 - \alpha_j)^2}{3} - \frac{2(1 - \alpha_i)^2}{3} = 0 \end{cases}$$

Again, narrowing down the formulas for the equilibrium effort pairs that separately satisfy the constraints above is challenging. Nevertheless, none of the conditions imposed on the benefits b_i and b_j dismisses the existence of a positive effort pair that would, in turn, satisfies it. Therefore, the form of the additional equilibria effort pair is not the most relevant. The priority is understanding their welfare implications.

pairs that maximize the utility function of the sophisticated decision makers. Nevertheless, coupling the solution for Equations 14, 15 and 16 is no straightforward. Therefore, I refrain from extending the proof.

4.1 Tradeoff: Inequality vs. Welfare

To understand the tradeoff between inequality and welfare, I take a step backwards to reassess the utility maximization conditions for behavioral players. If the benefit equilibrium is different from the benchmark case then it satisfies one of the following conditions:

$$(b_i(e_i, e_j, w_i) - \frac{2(1 - \alpha_i)^2}{3})^2 = \frac{2(1 - \alpha_i)^2}{3} (\frac{2(1 - \alpha_i)^2}{3} - b_j(e_j, e_i, w_j))$$

or

$$(b_j(e_j, e_i, w_j) - \frac{2(1 - \alpha_j)^2}{3})^2 = \frac{2(1 - \alpha_j)^2}{3} (\frac{2(1 - \alpha_j)^2}{3} - b_i(e_i, e_j, w_i))$$

or

Hence, the welfare maximization condition is either:

$$max(b_i(e_i, e_j, w_i) + b_j(e_j, e_i, w_j)) = \frac{4(1 - \alpha_i)^2}{3}$$
(14)

$$max(b_i(e_i, e_j, w_i) + b_j(e_j, e_i, w_j)) = \frac{4(1 - \alpha_j)^4}{3}$$
(15)

Proof. Follows immediately from the constraints above.

The conditions imposed on the maximum collective benefit allows for a comparison between the maximum welfare achieved in the behavioral setting and the welfare derived in the benchmark case:

$$\frac{4(1-\alpha_i)^2}{3} \text{ vs } \frac{4(1-\alpha_j)^4}{3} \text{ vs } \frac{(a-2/w_i+1/w_j)^2 + (a-2/w_j+1/w_i)^2}{9b}$$

Therefore, keeping in mind that the α_i and α_j parameter are small (i.e. $\alpha_i, \alpha_j \in [0, 1)$), it follows:

Proposition 3: The behavioral competition via effort game admits effort equilibria that simultaneously reduce the inequality and increase the welfare only when the pie is small. Otherwise, from a welfare perspective, all equilibrium effort pairs that can potentially emerge are suboptimal. Proposition 3 has strong implications. If the pie is big, the benchmark effort equilibrium welfare dominates all other equilibria that could potentially emerge in the behavioral setting. In contrast, the only setting in which the inequality aversion can potentially generate positive outcomes by reducing the inequality benefit between the players while increasing the welfare relative to the benchmark case is when the pie is already small. This brings into question issue of certainty vs potential. If the overall gains are significant is the sacrifice in welfare justified by a possible reduction in inequality? If the answer is no then what can be done to mitigate the effect of bias?

4.2 Asymmetry in Cognitive Capacity

Consider the case when one of the players is rational and the other is one is not. In other words, one of the agents has the cognitive ability to endogenously determine the corresponding maximizing α while the other is constrained by a α parameter which is exogenously given. Using the same formulations as above, the maximizing utility constraints are:

$$\begin{cases} e_i = \frac{a - e_j - 1/w_i}{2} \\ and \\ (b_j(e_j, e_i, w_j) - \frac{2(1 - \alpha_j)^2}{3})^2 = \frac{2(1 - \alpha_j)^2}{3} (\frac{2(1 - \alpha_j)^2}{3} - b_i(e_i, e_j, w_i)) \end{cases}$$

Based on the insights derived, when only one agent displays a cognitive impairment toward inequality, the set of equilibria effort pairs does not include any pair that generates zero gains for both agents. Therefore, the asymmetry in cognitive ability eliminates the main suboptimality induced by inequality aversion. Nevertheless, along with the equilibrium benchmark solution, there are still multiple equilibrium effort pairs that could potentially emerge which approach the (0, 0) benefit case. As outlined previously, contingent on the size of the pie, all the other potential equilibrium effort pairs generate a cumulative benefit loss.

5 Discussion

The findings of this paper are centered around the utility maximizing conditions of behavioral decision makers. In this sense, the paper provides a step-by-step descriptive proof rather than a full-blown analysis of the best response functions characterizing the corresponding Cournot Nash equilibria of the competition via effort game. Nevertheless, these initial findings provide interesting insights. Most importantly, I narrow down the types of equilibria benefit pairs that can potentially emerge in a competition via effort setting when players are constrained by inequality aversion. In the best-case scenario, the players end up with the same benefit distribution as in the benchmark case. In the secondbest scenario, inequality aversion reduces the benefit inequality by generating small positive gains for both agents. Nevertheless, contingent on the exogenous parameters which in turn determine the size of the pie, the benefit inequality reduction comes at the cost of the collective good. Lastly, the behavioral competition via effort game display multiple effort pairs that generates zero gains for both players. Going back to the initial scenario, my findings say that in equilibrium, the two inequality averse financial analysts will never cooperate. In order to avoid possible deviations, the agents can play their cards as they are or be significantly more or less aggressive at the cost of their total gains. The only other equilibrium option is to stay out of the game.

Apart from the equilibria analysis, the paper also captures a striking result related to wealth redistribution. Explicitly I motivate the fact that a change in the exogenous parameters, including w_i and w_j does not eliminate the suboptimalities cause by inequality aversion. Therefore, wealth redistribution alone cannot eliminate the risk of pushing both benefits down to zero. The idea grows in popularity among inequality and poverty researchers including Atkinson (2015) and Appadurai (2004) who look for different avenues to mitigate the dangerous effects of wealth disparities.

The findings call for pragmatism. If the benefit pie is big it is worth exploring the possible ways of mitigating the potential welfare loss induced by inequality aversion. Conversely, if the size of the pie is small, then there exist behavioral equilibrium benefit pairs that welfare dominates the benchmark one. Nonetheless, as equilibria existence is not equivalent to equilibria emergence one should look deeper into the possible ways of securing the emergence of the optimal equilibrium benefit pair.

5.1 Policy Implication

The main policy implication of this paper is related to education. As seen in section 4, if one side is educated the risk of zero gains is eliminated. Indeed, it is difficult to broaden the cognitive ability of the players by eliminating their inequality aversion. Nevertheless, making people aware of the possible suboptimalities generated by their inequality aversion can be the first step in eliminating the bias. As technical theoretical models cannot educate masses one way to explicitly go about it is by changing the uniformity of the public narrative and by fostering public debate (Krugman, 2005).

Another way to tackle the issue is to work from within. Keep in mind that this model is based on the assumption of inequality of opportunity or at least on the perception that players have regarding the issues. If the perception changes then the input can change which in turn can mitigate the effects. According to Brock (2010) one of the barriers for success is university enrolment. Davies and Guppy (1997) found that social economic status affects the enrolment into elitist universities and increases the likelihood of landing selective jobs. Carnevale and Strohl (2013) brings evidence that competent students from lower economic background do not give themselves the chance to apply for the elitist universities that would otherwise match their capabilities. The decision is based on a so-called "misfit" (i.e. the students do not see any point in applying as they do not identify with the schools). One possible way of fighting against this misconception is by increasing the quotas for lower-income students. If less privileged students could identify themselves more with highly selective institutions, then the misconception could disappear which could increase the gains of the agents.

5.2 Limitations and Further Research

One limitation of the model is the lack of stability analysis. Although I show that the behavioral setting admits multiple equilibrium pairs, I do not provide any insights on the issue of stability. If agents are not rational, the utility function takes the form of a polynomial of degree two which in turn depends on two benefit function namely b_i and b_j . Therefore, I cannot easily isolate the best response function as in the standard Cournot setting. Hence, I cannot test the stability conditions given by (Fudenberg and Tirole, 1991). Similarly, I do not provide an analysis of the probability of equilibria emergence. The shape of the utility function allows for little manipulation. Nevertheless, future research is invited to dive into the issue of probability especially in connection to wealth changes as it could provide more insight into the possible added value of wealth redistribution.

Another limitation relates to the distinction between rational and behavioral players. In the context of this model, a rational player is farsighted. Hence, he can determine optimal value of the corresponding alpha parameter. In contrast, the cognitively impaired one takes the alpha parameter as given. This can be seen as a striking difference. Ideally, the behavioral player should also be capable of endogenizing the value of his alpha parameter subject to certain constraints. Nonetheless, for tractability reasons, I follow Fehr and Schmidt (1999) by operating with an exogenous alpha parameter. Further research should provide more input on the matter.

As mentioned in Section 3, one can argue that in the face of multiple interactions the bias disappears. Although this might be the case, a static framework is in itself insightful as certain competitive interaction can happen only once. Going back to the initial scenario, the likelihood of recurrence is relatively small. Nevertheless, in the face of a repeated interaction, the agents can potentially correct themselves as long as the equilibria emerging in the previous rounds is different from the standard rational one. Otherwise, all future interactions are subject to suboptimalities. Admittedly, this is more of an intuitive explanation rather than an actual proof. Further research is invited to investigate the interlink between inequality aversion and competition in a dynamic setting.

Regarding the issue of multiple players, I employ the same argument as Fehr and Schmidt (1999) who justified the two-agent model by assuming that in a setting with multiple players, each interaction is considered separately. The main reason why I employ the same justification is to avoid approaching the Walrasian equilibrium. Nevertheless, the case of multiple players deserves further attention.

Last but not least, this model assumes that equality of opportunity is constrained. Future research is invited to relax this assumption.

6 Conclusion

This paper builds from scratch a competition via effort model in which players are constrained by inequality aversion. I start by analyzing the outcome when players are both rational, meaning they can endogenously determine the utility maximizing inequality aversion parameter. I use the equilibria findings as my benchmark for understanding what are the implication brought by a mutual cognitive impairment with respect to inequality. The analysis of the behavioral competition via effort game isolates the possible equilibria that can emerge due to inequality aversion. The welfare comparison shows that most likely a reduction in the inequality of gains triggers a significant drop in the common gains. Furthermore, it shows that a welfare redistribution does not eliminate the potential suboptimalities caused by the bias. Based on the findings I call for pragmatism and for education. Future research is invited to supplement the initial findings by investigating the probability of equilibria emergence, to understand better the magnitude of the risks induced by inequality aversion.

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Appendix

Proof of Proposition 1: The optimal α_i parameter is given by:

$$\frac{\partial u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial \alpha_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial \alpha_i} + \frac{\partial v[\frac{b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i, w_j)}{b_i(e_i, e_j, w_i)}]}{\partial \alpha_i} = 0$$

i.e. $(1 - \alpha_i)(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j))^2 = 0$ hence $\hat{\alpha_i} = 1$

Similarly, the optimal α_i parameter is given by:

$$\frac{\partial u(f(e_i, w_j, w_i, \alpha_j, \alpha_i), e_i, w_j, \alpha_j))}{\partial \alpha_j} = \frac{\partial b_j(e_j, e_i, w_j)}{\partial \alpha_j} + \frac{\partial v[\frac{b_j(e_j, e_i, w_j) - b_j(e_j, e_i, w_j, w_i)}{b_j(e_j, e_i, w_j)}]}{\partial \alpha_j} = 0$$

i.e. $(1 - \alpha_j)(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j))^2 = 0$ hence $\hat{\alpha_j} = 1$

Therefore, the maximization problem becomes:

$$\frac{\partial u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial e_i} = \frac{\partial P(e_i, e_j)e_i}{\partial e_i} - \frac{\partial C(e_i)}{\partial e_i} = 0$$
$$\frac{\partial u(f(e_i, w_j, w_i, \alpha_j, \alpha_i), e_i, w_j, \alpha_j))}{\partial e_j} = \frac{\partial P(e_j, e_i)e_j}{\partial e_j} - \frac{\partial C(e_j)}{\partial e_j} = 0$$
where $e_i = \frac{a - be_j - 1/w_i}{2b}$ and $e_j = \frac{a - be_i - 1/w_j}{2b}$

thus $e_i = \frac{a-2/w_i+1/w_j}{3b}$ and $e_i = \frac{a-2/w_j+1/w_i}{3b}$ i.e. $e_i > e_j$ hence the equilibrium benefit pair is $b_i = \frac{(a-2/w_i+1/w_j)^2}{9b}$ and $b_j = \frac{(a-2/w_j+1/w_i)^2}{9b}$

Proof of Lemma 1: Consider the following steps:

1. Given the initial wealth level w_i and w_j , with $w_i < w_j$, the marginal net utilities of effort are:

For player i

$$\frac{\partial u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i))}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i))}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i))}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i))}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i))}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i))}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{v[\frac{(b_i(e_i, e_j, w_i) - b_i(e_i, e_j, w_i)}{b_i(e_i, e_j, w_i, w_j)}]}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{\partial b_i(e_i, e_j, w_i, w_j)}{\partial e_i} + \frac{\partial b_i(e_i, e_j, w_$$

$$=\frac{\partial b_i^3(e_i,e_j,w_i)}{\partial e_i}-(1-\alpha_i)^2\big(\frac{\partial b_i^2(e_i,e_j,w_i)}{\partial e_i}-2\frac{\partial b_i(e_i,e_j,w_i)b_j(e_j,e_i,w_j)}{\partial e_i}+\frac{\partial b_j^2(e_j,e_i,w_j)}{\partial e_i}\big)=0$$

$$3b_i^2(e_i, e_j, w_i) \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} - 2(1 - \alpha_i)^2 (b_i(e_i, e_j, w_i) \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} - b_j(e_j, e_i, w_j) \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i})$$
$$-2(1 - \alpha_1)^2 (b_i \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_i} - b_j \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_i}) =$$
$$\simeq \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} [3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2 (b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j)]$$

$$\simeq \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} [3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j))]$$

For player j

$$=\frac{\partial b_j^3(e_j,e_i,w_j)}{\partial e_j}-(1-\alpha_j)^2(\frac{\partial b_j^2(e_j,e_i,w_j)}{\partial e_j}-2\frac{\partial b_j(e_j,e_i,w_j)b_i(e_i,e_j,w_i)}{\partial e_j}+\frac{\partial b_i^2(e_i,e_j,w_i)}{\partial e_j})=$$

$$\simeq \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} [3b_j^2(e_j, e_i, w_j) - 2(1 - \alpha_i)^2(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i)],$$

Given
$$\frac{\partial b_j}{\partial b_i} = 0$$

2. The marginal net utility of effort as α goes up:

$$\frac{\partial^2 u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial e_i \partial \alpha_i} \simeq \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} 4(1 - \alpha_1)(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j))$$

$$\frac{\partial^2 u(f(e_i, w_j, w_i, \alpha_j, \alpha_i), e_i, w_j, \alpha_j))}{\partial e_j \partial \alpha_j} \simeq \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} 4(1 - \alpha_1)(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i))$$

Maximization constrains for Proposition 2: The F.O.C for player i is given by :

$$\frac{\partial u(f(e_j, w_i, w_j, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial e_i} = \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} + \frac{\partial v[\frac{b_i - (\alpha_i b_i + (1 - \alpha_i) b_j)}{b_i}]}{\partial e_i} = 0$$

Based on Lemma 1, I get:

$$\frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} [3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2 (b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j) - \frac{\partial b_j(e_j, e_i, w_j)}{\partial b_i(e_i, e_j, w_i)} (b_i - b_j))] \simeq 0$$

$$\frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} [3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2 (b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j)] \simeq 0$$

Similarly, the maximization condition for player j is:

$$\frac{\partial u(f(e_i, w_j, w_i, \alpha_i, \alpha_j), e_j, w_i, \alpha_i))}{\partial e_i} = \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} + \frac{\partial v[\frac{b_j - (\alpha_i b_j + (1 - \alpha_i)b_i)}{b_j}]}{\partial e_j} = 0$$

Following the same logic as above I get:

$$\frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} [3b_j^2(e_j, e_i, w_j) - 2(1 - \alpha_i)^2(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i)] \simeq 0$$

Bruteforcing the maximization conditions can be rewritten as:

$$\begin{aligned} \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} &= 0\\ \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} &= 0\\ 3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j))0\\ 3b_j^2(e_j, e_i, w_j) - 2(1 - \alpha_j)^2(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i))0 \end{aligned}$$

Therefore an equilibrium effort pair has to satisfy one of the following constraints:

$$\begin{aligned} \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} &= \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} = 0\\ \frac{\partial b_j(e_j, e_i, w_j)}{\partial e_j} &= 3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j)) = 0\\ \frac{\partial b_i(e_i, e_j, w_i)}{\partial e_i} &= 3b_j^2(e_j, e_i, w_j) - 2(1 - \alpha_j)^2(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i)) = 0\\ 3b_i^2(e_i, e_j, w_i) - 2(1 - \alpha_i)^2(b_i(e_i, e_j, w_i) - b_j(e_j, e_i, w_j)) = 0\\ 3b_j^2(e_j, e_i, w_j) - 2(1 - \alpha_j)^2(b_j(e_j, e_i, w_j) - b_i(e_i, e_j, w_i)) = 0 \end{aligned}$$