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Dynamic access pricing of Google Maps's user data

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Abstract

The present thesis examines interactions on the mobile cartography market where firms use big data as a competitive advantage. I develop a dynamic access pricing model suggesting a regulatory scheme to maximise social welfare in which a regulator sets a price for user data that is made accessible to a potential entrant to a side market, the mobile advertising market. An optimal access price is rising over time. The stronger an incumbent's accumulated advantage of serving a higher number of end users, the less socially desirable it is to have a second firm in the market. This effect counteracts the incentive of a new firm to enter the market when capturing wealthier end users by employing an own map.

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1 Introduction

The present thesis examines interactions of firms on the mobile cartography market, the market of mapping services like Google's or Apple's "Maps" , in a situation where firms use big data as a competitive advantage. The aim of the analysis is to develop a regulatory scheme to maximise social welfare by setting a price for data.

According to Acemoglu and Johnson (2017), the effects of the technological development in the 21st century, which led to the *era of big data*, need to be incorporated in a redesign of institutions that may assure a well-functioning economic and political system. By breaking down the interactions on the mobile cartography market to a simple dynamic model, I introduce an approach to analyse the effects of the *era of big data* on the organization of the tech industry. The tech industry is subject to changes in market proportions which Acemoglu and Johnson (2017) attribute to the era of big data: few players like Google, Apple or Microsoft dominate the industry. The main driver of their market power is said to be *software and increasing returns to data*. The model I propose utilises big data to explain changes in the organization and dynamics of the tech industry.

Mobile cartography is dominated by Google Maps, and serves as an exemplifying market. The firm was the first mover in the market of maps with web interface, which enabled it to collect data and use its databases for refining the map over its years of operation: Since 2008, Google Maps has been operating independent of external data. (Fisher, 2013) The dominant position of a firm in this setting leads to an irreconcilable imbalance between the amount of data available for entrants and the dominanting firm (Acemoglu and Johnson, 2017). Potential entry firms, therefore, may face their lack of data as a barrier of entry due to non-competitivity. Apple's failure to provide a competitive mapping service when first introduced its own map on iPhones in 2012 (Wingfield and Chen, 2012) demonstrates the difficulties of overcoming this entry barrier.

The occurrance of entry barriers, leading to imperfect competition, gives rise to market failures as a result of the violation of the first fundamental welfare theorem in modern welfare economics (Stiglitz, 1991). Hence, with the rise of big data, sharing data seems to be necessary in order to *level the playing field* (Acemoglu and Johnson, 2017) between operating and potentially operating firms on a dominated industry. I suggest a regulatory approach to incorporate data sharing in the tech industry and prevent market failure.

When analyzing the structural effects of the era of big data on the mobile cartography market, the analysed firms' business models that are shaped by the gathering, processing and selling of big data, need to be incorporated. The gathering of user data depends on the cartography service users. The service providers' revenue source, however, is the mobile advertising market. In order to account for this business model, the effects of the mobile advertising market must therefore be considered.

The operation systems Android and iOS, owned by Google and Apple, respectively, dominate the mobile advertising market with a joint market share higher than 98 percent (StatCounter, 2018). Therefore, market characteristics are mainly based on the interaction of advertisers with one of these two firms. Data on mobile advertising spending (Smaato Inc., 2017) shows that advertisement prices on iOS devices are around 30 percent higher, in comparison to the price of advertisement on an Android device. On the other hand, Google controls the global mobile advertising market through devices operated by Android with close to 70 percent market share, whereas Apple's share is slightly above 28 percent (StatCounter, 2018).

Generalizing these insights, Apple may be seen as offering higher quality advertisement, as higher prices can be expected to be an indicator for advertisers valuing

Apple's advertising space higher in comparison to Google's. Conversely, Google is leading in the quantitative aspect through its higher market share. The model I provide incorporates each firm's competitive advantage on the mobile advertising market.

To my knowledge, there is no economic literature incorporating the structure of business models that is based on big data. A novel contribution of my work is, therefore, to account for these business models and, in line with this, allow the demand of advertisers to be based on two dimensions, quantity and quality.

The question addressed in this thesis is twofold: One aspect is whether a system in which data is shared needs to be regulated. A second aspect is how the price of data can be accurately set by a regulator. To examine these questions, I use a dynamic access price model representing the mobile cartography market and develop a time dependent regulation scheme for the price of data. An entering firm has the option to invest in its own mobile map to offer independent advertising services. I propose a theoretical model that describes this scenario.

The results show that in a two-period game, a regulator optimally sets access prices in the first period at the cost of offering the mobile advertising service, and incentivizes a potential entrant to enter the market by setting a sufficiently low access price in the second period. The value of this price depends on the afore mentioned quantity and quality aspects of the mobile advertising services. As the quantity advantage of data collected by the first-mover firm is irreconcilable, from a social welfare perspective, a larger quantity advantage should weaken the incentive for the entrant to enter. Additionally, a higher quality advantage of the entrant increases the equilibrium access price, as the firm's entry does not have to be incentivized by the regulator.

2 Literature Review

I use the theory of access pricing[1] to model the effects of the age of big data on the organization of the mobile cartography industry, in particular by analysing entry decisions in an oligopoly setting. Access price theory has been analysed extensively on the market of telecommunications[2]. The literature relies on the general assumption that a regulator's only instrument is an access price regulation for the input an incumbent sells to an entrant (Foros, 2004).

My model builds on Avenali et al.'s (2010) paper on dynamic access pricing in the telecommunications market. I reinterpret their model in the context of big data on the mobile cartography market and add a quantity aspect to the vertical service differentiation.

Avenali et al. (2010) model a two-period game with perfect information where the entrant decides whether to invest in infrastucture. In the paper, such an investment eventually leads to offering an enhanced service, as an entering firm is assumed to be able to capture consumers with high willingness to pay for the telecommunications service. The authors find that a rising access price over time encourages the entrant firm to invest in infrastructure, which is socially optimal.

This result corresponds to the results of my model. Additionally, my model provides insight of how a socially optimal access price depends on the competing firms' performance on a side market, which is relevant in terms of the underlying business model.

My finding that a higher quantity advantage of the incumbent counteracts the social value addition of a second firm's entry to the market gives new insights on how a market influenced by the age of big data affects social welfare. These insights may lead to more effective policy decisions on how to regulate the market

[1] See Laffont and Tirole (1994) and Armstrong et al. (1996) for a general introduction to the theory.

[2] See for example Laffont and Tirole (1994), Armstrong et al. (1996), Avenali et al. (2010) and Manenti and Scialà (2013)

power of an incument firm driven by the unique access to user data.

In Avenali et al.'s (2010) model, vertical service differentiation is subject to the willingness to pay for the firms' service, and depends on how end customers perceive the telecommunication service's quality. I change two aspects in my model.

First, I account for the structure of the business model behind the mobile cartography market, where monetary transactions occur on the mobile advertising (side) market rather than on the cartography (main) market. Specifically, this leads to a model where the willingness to pay of agents on a side market, rather than of the end customers on the main market, is decisive for the vertical service differentiation. The first change is therefore the determination of vertical service differentiation on a side market. The role of the end users in my model is restricted to serving as an input for data gathering.

Second, I add a quantity aspect to the determination of the willingness to pay. Similar to Vandenbosch and Weinberg (1995), both differentiation dimensions I use are vertical. Advertising space service in my model is differentiated by the profitability (quality aspect) of the end users that see the advertisement and by the number (quantity aspect) of users that are reached. End users of the mobile map have no active role in the determination of the equilibrium in the model. The addition of a quantity aspect to the vertical service differentiation not only gives insight to the severity of first-mover advantages, reflected in the quantity aspect of the service, but also provides a dynamic solution for a regulator to level the market power of players in the market when it is socially desirable.

A second paper that is closely related to mine is Manenti and Scialà's (2013). Starting from a similar structure as in Avenali et al. (2010), they additionally distinguish between the infrastructure investment and a type of research and development investment for the service's enhancement. Similar to Avenali et al. (2010), the authors find that an unregulated market leads to entry foreclosure of the entrant firm. In my model, I capture the investment in developing an enhanced service through the data input that needs to be incorporated in the map before the service can be perceived as enhanced by advertisers. As in Avenali et al. (2010), my model assumes a time dimension here. The process of data incorporation takes time and so an enhanced service can only be offered in the second period.

Foros (2004) also focuses on analyzing the source of an entrant's market power. The author finds that a firm's relative ability to transform input to output is decisive for the effect of the access price regulation on social welfare. By including a relative advantage of each firm in one of the mentioned vertical service differentiation dimensions, I consider the entrant's and the incumbent's source of market power, which critically depend in the quality and quantity parameters on the advertising market.

In terms of the demand structure of my model, the work of Katz and Shapiro (1985) is particularly relevant. The authors provide a model of network competition with positive consumption externalities. For this purpose, they include the network size as a factor that shifts consumer's willingness to pay.

I borrow the underlying idea of the utility function's construction. Other papers that are related to my work, like Foros (2004) and Avenali et al. (2010), use similar utility functions. In addition to the baseline structure provided by Katz and Shapiro (1985), I account for partial market participation of the advertisers on the mobile advertisement market as it is done in Avenali et al.'s (2010) model. The authors mention that allowing for partial market participation is based on the assumption that the demand for new services is not mature. This setting is applicable to the advertising market, as there is a wide range of advertising channels and therefore it is likely that demand for a single channel is not utilised at its full capacity. This holds especially for mobile advertising, since the channel is based on technological solutions that are still developing at the time (Thompson, 2018).

In contrast to Avenali et al.'s (2010) interpretation, it is not the product differentiation itself that attracts more advertisers on the mobile advertising market. However, it is the evolution of the mobile cartography market that defines the attractiveness of marketing via mobile channels. A larger number of potential customers using mobile cartography may attract further advertisers on the mobile advertising market. This means that the results derived from my model depend on the interaction of players on a side market, whereas when considering the telecommunications industry, there is only one relevant market. The role of the end user as a bare data provider is accounted for in my model.

The setting provided in my work is a novel economic approach modeling the interaction in markets where big data is an essential input for product or service differentiation.

3 Mobile Cartography Market

One of the major contributions of my model is to incorporate big data business models to the economic analysis. The specific market I use is the mobile cartography market using big data as a competitive advantage. Compared to the telecommunications market which is analysed by most of the presented literature in Section 2, there are two major differences.

First is the role of the players in the market. While seller and buyer of telecommunication service are interacting on a sole market, cartographic service providers use the data from its users to enhance their services' quality and make revenue on a side business which is the mobile advertisement market. Advertisers pay for having placed their advertisement on one of the channels of a mobile phone. The pricing structure of an advertisement placement in the technology industry is based on various factors and uses auction formats or algorithms in the advertising system. For example, Google uses AdWords, its advertising program, with a cost-per-click system (Google, 2018). The interaction between the two markets is necessary in the model as mobile cartographic services like Google maps are of no charge for end users.

Second is the source of market power. Telecommunication network owners are monopolists due to high costs of building the infrastructure. The advantage of first movers in data based competition, however, is the amount of collected data which is unlikely to be caught up by later entrants. Competition in mobile cartography can be differentiated again by the two markets considered. On the main market, the mobile cartography market, firms compete for users, which are in turn attracting more advertisers. This competition relies on data, as the services use "data to create data" (O'Beirne, 2017), meaning that the map can be improved by using the users' location data and incorporating it to the algorithms creating the map. On the side market, the mobile advertising market, the competition lies in attracting advertisers with the advertising service. Hereby, not only a qualitative aspect of the service but also quantity, meaning the number of users that can be reached, may play a role. The competitive environment on the mobile cartography and advertisement market is restricted due to Google and Apple's dominance, controlling the global mobile advertisement market with 98 percent of market share (StatCounter, 2018). Other players are therefore not considered in this setting.

4 Model

In the following section, I introduce the model's underlying assumptions and its dynamics. I model the mobile cartography market in a dynamic access pricing model and suggest a regulatory pricing scheme.

4.1 Market Organization and Entry

Advertising space is offered by two firms that compete à la Bertrand in the mobile advertising market. In line with the big data business models, this market may be considered as a side market of the mobile cartography service (main) market. The main market is where the firms compete for end users, and where there are no monetary transactions. My model considers this main market as a necessary input. However, the model's oligopolistic competition is subject to the side market.

Each firm offers several mobile advertising channels. In the channel of advertisement via mobile mapping, the firms are an incumbent i and an entrant e. Through its operation on the mobile cartography market, the incumbent constantly creates data describing user behavior, the access data. It may be an essential input for innovative activities, like the precision of the map, that ensures a service's competitivity on the mobile cartography market, attracting end users that are in turn attracting advertisers on the mobile advertising market.

If e does not enter the mobile map advertising market, it can still offer advertisement spaces on other channels. Mobile devices offer generally a high degree of personalisation, with different applications being individually selected on each device. Mapping services, however, are a default service that are installed via the respective operator. Based on this characteristic of mobile mapping services, I use the simplifying assumption that mobile map advertisement spaces are the only source of service differentiation in the mobile advertising market, and also the preferred way to place advertisement for advertisers.

Similar to Avenali et al. (2010) and Manenti and Scialà (2013), e can enter the mobile map advertising market in two ways. Either by employing i's mapping service or by creating its own mobile map. In the former case, e needs access to the incumbent's datasets to be able to operate on the advertising market. If e enters, the firms compete on the mobile advertising market via *service based* competition. In the latter case, by incurring sunk cost R, e can process the accessed datasets and use them for creating its own map. This allows the entrant to operate independently in the mobile mapping advertisement market. In this scenario, following Avenali et al. (2010) and Manenti and Scialà (2013), advertisement placement market competition will be called *facility based*.

As a corollary from the assumption of complementarity between the access data and its processing, e can only choose facility based entry when having incurred cost R in advance. Hence, facility based entry can only occur in period two.

There are two types of costs to be modeled. First is the production cost of providing the advertising space service, consisting of a fixed and a variable part. Without loss of generality (Katz and Shapiro, 1985), I set both costs to zero for both the entrant and the incumbent. Second is a sunk setup cost R for a new mobile map that occurs due to the complementarity assumption.

4.2 Vertical Service Differentiation

A major novel contribution of my model consists in the inclusion of a two-dimensional vertical service differentiation. To do this, I generally follow the conceptual framework of Vandenbosch and Weinberg (1995) and apply it to advertising service characteristics on the mobile cartography channel. Advertising services encompass nonnegative valuations of two characteristics, represented by γ and β. The characteristics are perceptual dimensions of reach quality and reach quantity, respectively.

Parameter γ represents the reach quality defined by the expected return on the advertisement investment per reached end user, which may be correlated with the wealth of the end users. Wealthier mobile users are expected to respond with a higher willingness to pay for advertised products.

Parameter β represents the reach quantity defined as the number of reached mobile phone users. Advertisers prefer higher quality and quantity to less, meaning that they always prefer reaching wealthier mobile device users and a higher number of end users.

As I model a game with complete information, advertisers make there entry decision with complete knowledge about service characteristics and prices. In contrast to Vandenbosch and Weinberg (1995), the advertisers' reservation price allows for partial market participation. Advertising in the tech industry may be characterised by partial market participation of advertisers, as there are many alternative advertising channels, and new possibilities of advertising arise with new technologies that have yet to be exploited. However, due to technological advancements, I expect the mobile advertising channel to grow at a high speed and hence increasingly attract advertisers. For example, video advertisement has quickly reached the position of the "fastest-growing mobile ad format" (Smaato Inc., 2017) after being incorporated in programmatic advertising, a technology processing user data for targeted mobile advertisement (Thompson, 2018).

Service positioning competes via asymmetric characteristics, meaning that each firm has a relative advantage in a specific characteristic (Vandenbosch and Weinberg, 1995). In the context of mobile mapping, I assume that i has a relative advantage in reach quantity due to its position as the first-mover, and e has a relative advantage in reach quality, as it is tied to high-end users. Therefore, the advertisers' willingness to pay for a mobile advertising service is determined by two aspects: the wealth (quality) and the number (quantity) of the mobile device users that are reached. Based on the price differences of advertisement displayed on devices operated by Apple versus Google operated devices, I assume the entrant to serve high-end mobile device users, leading to a high reach quality.

Additionally, the reach quality is assumed to be perceived by advertisers as the superior characteristic. Intuitively, advertisers prefer reaching high-end users, even if they reach fewer. This assumption reflects the substantially higher prices on advertising via iOS devices compared to Android devices. Due to Google's leading position in terms of market shares, the incumbent in my model is assumed to have a comparative advantage in the size of the user network, leading to a high reach quantity. However, if there are considerably less users to be reached, advertisers are assumed to prefer reaching a higher number of low-end users. Formally, advertisers strictly prefer the set (γ_h, β_l) to the set (γ_l, β_h). The superiority holds up to a point where the quantity advantage of the incumbent is too high and the preference relation switches. This happens at the threshold level of the quantity characteristics, with $\beta_{\overline{h}}$ for the incumbent and $\beta_{\overline{l}}$ for the entrant, where advertisers prefer the set $(\gamma_l, \beta_{\overline{h}})$ to the set $(\gamma_h, \beta_{\underline{l}})$.

These characteristics shape the model as following. Under service based entry, marketers have a higher willingness to pay for i's advertisement space, as it is the only map that is available. Conversely, under facility based entry, as e is perceived to offer high reach quality, advertisers' willingness to pay for e's service is higher, due to the assumption of the reach quality being the superior advertising service characteristic.

4.3 Demand Structure

Consumers' choices are modeled by unit demands, meaning that they consume either of the firms' services. Additionally, the demand functions used, similar to the ones in Avenali et al. (2010), allow for partial market competition. Hence, advertisers with reservation price for the firms' services, ϕ, at which willingness to pay does not exceed utility yielded from buying the service, are allowed to stay out of the market.

There is a mass one of each of two different cohorts of heterogeneous advertisers of the side business. Advertisers' types are heterogeneous in their willingness to pay for the advertisement placement. As in Katz and Shapiro (1985), w is interpreted as an advertiser's basic willingness to pay for the service, which varies across advertisers and is uniformly distributed over the unit interval $[0, 1]$ with uniform density. Hence, each value of w represents a type of advertiser.

In contrast to Katz and Shapiro (1985) and following Avenali et al. (2010), I do not allow for negative w. However, I account for the possibility of advertisers' non-participation by setting a threshold value for the participating types at the point where utility from e's service is zero. Advertiser w yields utility $u_{kt}(w)$ from using firm $k \in \{i, e\}$'s service in period $t \in \{1, 2\}$.

4.4 Two-Period Game Dynamics

I define a two-period game with complete information that is similar to Avenali et al.'s (2010) model. The entrant's decisions are *whether* and in *how* to enter. The incumbent's decision is how to set the access price.

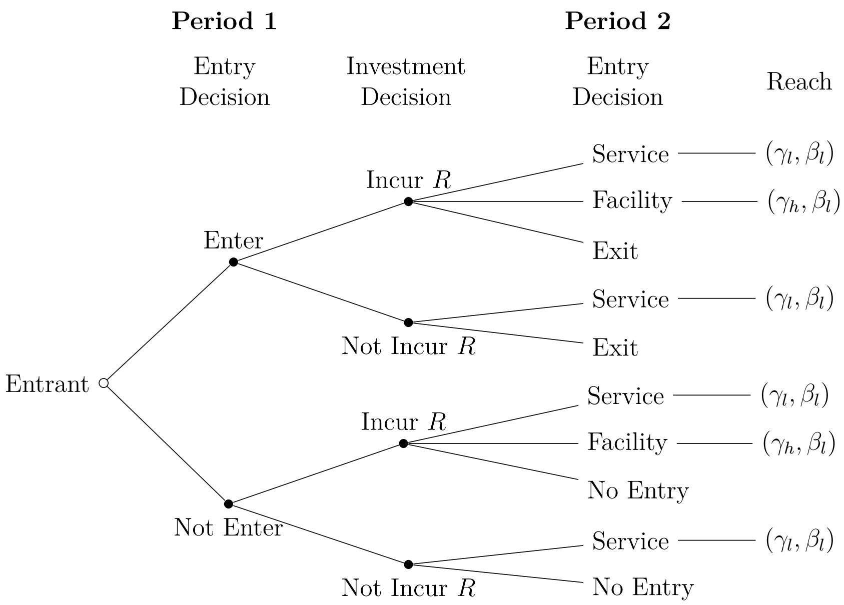
The timing of the firms' decisions is modeled as follows. Each period consists of two stages. In period one, the incumbent sets the acccess price a_1 at stage one. The entrant simultaneously decides whether to enter the mobile mapping market[3] and whether to buy access data. In the case of entering, e always buys access data. The reason for this is that user data is complementary information sold alongside the advertising service that is essential information for advertisers. For example, it may indicate the users' demographics or preferences and therefore may be used by the advertisers for strategic advertisement positioning and customer segmentation. As it is common in access pricing theory (Laffont and Tirole, 1994), one unit of output is assumed to require one unit of access data. At the second stage, e may decide to employ i's mapping service or to incur sunk cost R associated with processing the data and using it in its own map. In any scenario, firms compete à la Bertrand with vertically integrated services and i offers the only available

[3]Note that by assumption, if e does not enter, the marketers will always prefer to use i's services.

service in the first period with the advertisement reach characterised by bundle (γ_l, β_h). In period two, i sets the access price a_2 at stage one, and e again decides whether to enter or not in the case it had not entered previously. In the case where it has entered and incurred sunk cost R in period one, the entrant has an additional option for entering, which is to offer the high reach quality advertising service characterised by γ_h.

Fig. 1 displays firm e's decisions at stage two in the respective period and the advertising service's reach parameters γ and β. The investment decision in period two is omitted as investing in the last period is not reasonable, due to the assumption that building a map needs time. In light of the preference relations defined in Section 4.2, Fig. 1 shows that facility based entry is the only way for e to offer an overall preferred advertising service compared to i, as it leads to a high quality service. The decisions of a profit maximising entrant firm depend on both the demand, in terms of advertiser preferences, and the cost, in terms of access prices, set by the incumbent firm or the regulator.

Figure 1: Entrant firm's decisions



4.5 Regulation

The crucial part of my model is determining a, the access price for data paid by e to i on the mobile advertising market. This transfer occurs whenever e decides to enter the market by using its own map. Both a regulated and an unregulated market are analysed.

Similar to Avenali et al. (2010) and Manenti and Scialà (2013), I find that the unregulated incumbent sets a_1 to foreclose the entrant in period one. The equilibrium outcome is therefore a monopolistic outcome. In case of access prices being regulated, the regulator sets a such that social welfare is maximised. As assumed by Manenti and Scialà (2013), the regulator cannot set access prices below the cost of providing access, hence by the zero cost assumption in 4.1, I use $a \geq 0$. The regulator is assumed to be fully committed to his decisions.

Distinguishing between the two regimes allows to demonstrate market failure in the unregulated case and to highlight the social welfare improvements an optimal regulation implies.

5 Market Entry

In the following section I present the formal description of the baseline model's demand and supply side for both possibilities of the entrant firm e's entry to the mobile advertising market. Service based entry occurs when the entrant serves advertisers by using the incumbent's map. Facility based entry means that the entrant sets up its own map. By assumption, facility based entry is only possible in the second period if the entrant has incurred the sunk setup cost of the mobile map in the first period.

5.1 Service based entry

Under service based entry, formally denoted S, the entrant firm does not undertake the investement R to set up its own mobile map. Therefore, both firms offer mobile map advertising via the incumbent's map. For any unit of advertising output sold, the entrant pays the access price of user data to the incumbent. Advertisement is possible either via i's map or via other channels. The incumbent offers high quantity advertising on all channels, whereas the entrant offers high quality advertising on all except the mapping channel. These advantages shape advertiser w's utilities of e and i's advertising service in period $t \in \{1, 2\}$ as following:

$$u_{it}^S(w) = \beta_h w - p_{it}^S$$
$$u_{et}^S(w) = \gamma_l w - p_{et}^S \ ,$$

where $\beta_h w$ and $\gamma_l w$ measure advertiser w's willingness to pay of using i's or e's mobile advertising, respectively. $\beta_h > 1$ and $\gamma_l \in (0, 1)$ represent the demand shifts for the firms' services due to their advertisement reach quality and quantity attributes. p_{kt}^S is the service's price. Katz and Shapiro (1985) use a similar type of utility functions. Avenali et al. (2010) add the shifting parameter indicating quality perception, which is similar to the parameter γ I use for reach quality. I add the shifting parameter for a second dimension of vertical differentiation, reach quantity β.

As advertisers prefer advertising via the mapping channel, i serves the market first and skims advertisers with high willingness to pay. Advertisers that have not been served by the incumbent are either placing their advertisement via e's service on other channels or not entering the mobile advertising market at all. The reservation price $w = \phi^S$ for the non-participating advertisers is at $u^S_{et}(w) = 0 \Leftrightarrow \phi^S = \frac{p^S_{et}}{\gamma_l}$. Hence, only the advertisers with $w \geq \phi^S$ enter the market. Given the uniform distribution of w, there are $1 - \phi^S$ advertisers that do not enter. The firms are selling $Q_t \equiv \sum^2_{k\in\{i,e\}} q_{kt}$ units of their services. Therefore, their prices must be set such that $1 - p^S_{it} = Q_t$ for the incumbent and $1 - \frac{p^S_{et}}{\gamma_l} = Q_t$ for the entrant. The incumbent and the entrant firm set p_{it} and p_{et} to maximise their respective profits

$$\Pi^S_i = \Pi^S_{i1} + \Pi^S_{i2} = (p^S_{i1}q^S_{i1} + a_1 q^S_{e1}) + (p^S_{i2}q^S_{i2} + a_2 q^S_{e2}) \tag{1}$$

$$\Pi^S_e = \Pi^S_{e1} + \Pi^S_{e2} = ((p^S_{e1} - a_1)q^S_{e1}) + ((p^S_{e2} - a_2)q^S_{e2}) \,. \tag{2}$$

Betrand outcomes with partial market participation of the advertisers are therefore

$$q^S_{it} = 1 - \frac{p^S_{it} - p^S_{et}}{\beta_h - \gamma_l}$$
$$q^S_{et} = \frac{p^S_{it} - p^S_{et}}{\beta_h - \gamma_l} - \frac{p^S_{et}}{\gamma_l}$$

for $t \in \{1, 2\}$. Detailed derivations of the optimal quantities are provided in Appendix A. Consumer (advertiser) surplus under service based entry, CS^S, is the sum of the advertisers' positive utilities given that they are first served by the incumbent. Mass q^S_{it} of the advertisers uses i's service, whose service characteristics shift their willingness to pay up by β_h. Mass q^S_{et} uses e's service, with advertisers' willingness to pay shifted down by γ_l. Formally, consumer surplus is as follows.

$$CS^S = \sum_{t\in\{1,2\}} CS_t^S = \sum_{t\in\{1,2\}} \left(\int_{\frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l}}^{1} (\beta_h w - p_{it}^S) dw + \int_{\frac{p_{et}^S}{\gamma_l}}^{\frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l}} (\gamma_l w - p_{et}^S) dw \right)$$

Further, using Eq. (1) and Eq. (2) as well as the consumer surplus CS^S, social welfare under service based entry is as follows.

$$W^S = W_1^S + W_2^S = (\Pi_{i1}^S + \Pi_{e1}^S + CS_1^S) + (\Pi_{i2}^S + \Pi_{e2}^S + CS_2^S)$$

5.2 Facility based entry

As facility based entry can only occur in period two (see 4.1), under facility based entry, formally denoted F, advertisers' first period utilities are the same as under service based entry. In case of entry, e depends on i's map and is therefore perceived as the low quality and high quantity map advertising service provider in period one. Utilities of buying data from e or i in period one are

$$u_{i1}^F(w) = \beta_h w - p_{i1}^F$$
$$u_{e1}^F(w) = \gamma_l w - p_{e1}^F \ ,$$

where $p_{k1}^F = p_{kt}^S$ for $k \in \{i, e\}$. Similar argumentation as in the service based entry case in Section 5.1 yields the following firms' profit maximising quantities under Bertrand competition.

$$q_{i1}^F = 1 - \frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l}$$
$$q_{e1}^F = \frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l} - \frac{p_{e1}^F}{\gamma_l}$$

In period two, if the entrant has accessed data and incurred cost R in period one and subsequently decides to employ its own map, e's reach quality will be perceived as high. Hence, e's advertising service is characterised by bundle (γ_h, β_l). Advertiser w's utility of e's service is shifted upwards by $\gamma_h > 1$. Reach quantity is still high for the incumbent, but is assumed to not have reached the threshold level $\beta_{\overline{h}}$. Hence, i's advertising service is characterised by (γ_l, β_h). Due to the preference relations defined in Section 4.2, the entrant's service under facility based entry in period one is preferred. The utility functions of an advertiser accessing both firms' services are

$$u^F_{i2}(w) = \beta_h w - p^F_{i2}$$
$$u^F_{e2}(w) = \gamma_h w - p^F_{e2} \ .$$

The entrant serves the market first and skims advertisers with high willingness to pay. Advertiser type w's reservation price is at $w = \phi^F \Leftrightarrow \phi^F = \frac{p^F_{i2}}{\beta_h}$. By maximising their profits

$$\Pi^F_i = \Pi^F_{i1} + \Pi^F_{i2} = (p^F_{i1} q^F_{i1} + a_1 q^F_{e1}) + (p^F_{i2} q^F_{i2}) \tag{3}$$
$$\Pi^F_e = \Pi^F_{e1} + \Pi^F_{e2} = ((p^F_{e1} - a_1) q^F_{e1} - R) + (p^F_{e2} q^F_{e2}) \ , \tag{4}$$

in a Bertrand competition with partial market entry of the advertisers, the firms have the following optimal response quantities.

$$q^F_{i2} = \frac{p^F_{i2} - p^F_{e2}}{\beta_h - \gamma_h} - \frac{p^F_{i2}}{\beta_h}$$
$$q^F_{e2} = 1 - \frac{p^F_{i2} - p^F_{e2}}{\beta_h - \gamma_h}$$

Consumer (advertiser) surplus under facility based entry, CS^F, is the sum of the advertisers' positive utilities given that in period one, they are first served by the incumbent, and in period two, they are first served by the entrant. In period one, mass $q_{i1}^F = q_{i1}^S$ of the advertisers uses i's service, and the remaining mass $q_{e1}^F = q_{e1}^S$ uses e's service. In period two, mass q_{e2}^F uses e's service, and the remaining mass q_{i2}^F uses i's service. Formally, CS^F is as follows.

$$\begin{aligned} CS^F &= CS_1^F + CS_2^F \\ &= \left(\int_{\frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l}}^{1} (\beta_h w - p_{i1}^F) dw + \int_{\frac{p_{e1}^F}{\gamma_l}}^{\frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l}} (\gamma_l w - p_{e1}^F) dw \right) \\ &+ \left(\int_{\frac{p_{i2}^F}{\beta_h}}^{\frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h}} (\beta_h w - p_{i2}^F) dw + \int_{\frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h}}^{1} (\gamma_h w - p_{e2}^F) dw \right) \end{aligned}$$

Similar to Section 5.1, using Eq. (3), Eq. (4) as well as the consumer surplus CS^F, social welfare under facility based entry is as follows.

$$W^F = W_1^F + W_2^F = (\Pi_{i1}^F + \Pi_{e1}^F + CS_1^F) + (\Pi_{i2}^F + \Pi_{e2}^F + CS_2^F)$$

6 Dynamic Access Pricing

This section examines the determination of unregulated and regulated access prices. The incumbent collects user data from its mobile cartography service and uses the data as a competitive advantage. A potential entrant buys access to the data. In a market with unregulated access pricing, the access price is set by the incumbent who maximises its profit. In a market with regulated access pricing, a regulator sets socially optimal access charges.

6.1 Unregulated Market

With unregulated access prices, the incumbent forecloses the entrant's entry to the mobile advertising market. Therefore, the game's outcome is a monopolistic eqilibrium. Hence, according to the first principle of the fundamental welfare theorem, the introduction of a regulatory scheme is justified.

Proposition 1 (Entry foreclosure). *Let* $0 < \gamma_l < 1 < \gamma_h$, $0 < \beta_l < 1 < \beta_h$. *There exists a level* $R > R_{min}$ *where the incumbent forecloses the entrant in both periods. The game's outcome corresponds to the level under monopoly, with equilibrium access prices* $a_1^m = a_2^m = \frac{\gamma_l(10\beta_h+\gamma_l-2)}{2(\gamma_l+8\beta_h)}$.

Proof. From Eq. (1) it is possible to show that i's first period profit increases in a_1. As there are no regulations on the access price determination, the incumbent sets a by maximising its profit, which leads to entry foreclosure of the entrant. Formally, i maximises its profit by setting $\frac{\partial \Pi_{i1}^K}{\partial a_1} = 0$. In both entry cases $K = \{S, F\}$, the maximisation yields the following incumbent's equilibrium access price in period one.

$$a_1^* = \frac{\gamma_l(10\beta_h + \gamma_l - 2)}{2(\gamma_l + 8\beta_h)}$$

The entrant's equilibrium output sold, $q_{e1}^K = \frac{\beta_h(\gamma_l - 2a_1^*)}{\gamma_l(4\beta_h - \gamma_l)}$, is derived by maximising

Eq. (2)[4]. Given the incumbent's optimal level of the access price, the entrant's best response is not to enter, as $q_{e1}^{K}|_{a_1=a_1^*} < 0$. Therefore, by setting an unregulated access price through maximising profits in period one, i deters e's entry to the market.

As the entrant does not offer a high quality map advertising service channel, its reach quantity, by assumption, drops to the threshold value $\beta_{\underline{l}}$. This leads to a situation where advertisers prefer i's service, as the drop in quantity neutralizes the quality advantage.

I assume for simplicity that e's perceived mobile advertising service quantity and quality in period two when not having entered in period one is the same as in the service entry scenario discussed in Section 5.1. Avenali et al. (2010) use the same assumption. The service's reach quality level γ_l, however, could be anywhere inbetween $[0, 1]$. Hence utility functions are $u_{i2}(w) = \beta_{\overline{h}} w - p_{i2}$ and $u_{e2}(w) = \gamma_l w - p_{e2}$. The entrant's decision at stage two of period one is restricted to deciding whether or not to invest in the sunk cost R. In case of investing, e's profit is

$$\Pi_e^F = -R + \Pi_{e2}^F|_{a_2=0} \ .$$

$\Pi_e^F < 0$ when $R > R_{min} = \frac{H(\gamma_h,\beta_h)}{2\gamma_h(1+\beta_h)-(\beta_h(2\beta_h-1))^2}$, where

$$H(\gamma_h, \beta_h) = \left[\gamma_h(\gamma_h(1+\beta_h) - 2\beta_h^2) - \beta_h^2(1-\beta_h)\right]\left[\beta_h(1-\beta_h) + \gamma_h(1+\beta_h)\right] \ .$$

Given the low quantity and low quality perception of the entrant's service on the advertising market, R_{min} is a threshold of the fixed set up cost measuring the highest possible value of R such that the e's profit is nonnegative after having entered.

When advertisers perceive the entrant's advertising service via its own map

[4] Detailed calculations are provided in Appendix A.

by quality, γ_l, and quantity, $\beta_{\underline{l}}$, meaning that the quality advantage is balanced through the low quantity dimension, then for all $R > R_{min}$, it is not profitable for e to invest in R in period one and enter the market via facility in period two.

In the case of not incurring the cost R in period one, the entrant can enter the market in period two only by using the incumbent's map, hence service based. At stage one of period two, the incumbent maximises profit Π^S_{i2} by setting a_2. Similar to period one, the access price forecloses the entrant also in period two. The maximisation problem

$$\frac{\partial \Pi^S_{i2}}{\partial a_2} = 0$$

yields $a_2^* = \frac{\gamma_l(10\beta_h + \gamma_l - 2)}{2(\gamma_l + 8\beta_h)}$. Given this access price, e is foreclosed also in period two, as it sets $q^S_{e2}|_{a_2 = a_2^*} < 0$.

This leads to an equilibrium where the incumbent sets access charges sufficiently high to deter entry in both periods. Consequently, the incumbent sets monopolistic prices for its data. The unregulated game's equilibrium is defined by the following monopolistic (m) access prices, prices, quantities and profit[5].

$$\begin{aligned}
a_t^m &= \frac{\gamma_l(10\beta_h + \gamma_l - 2)}{2(\gamma_l + 8\beta_h)} \\
p_{it}^m &= \frac{\beta_h}{2} + \frac{3\gamma_l(\beta_h - 1)\beta_h}{(4\beta_h - \gamma_l)(\gamma_l + 8\beta_h)} \\
q_{it}^m &= \frac{1}{2} - \frac{\gamma_l(\beta_h - 1)}{(4\beta_h - \gamma_l)(\gamma_l + 8\beta_h)} \\
\Pi_i^m &= \frac{\beta_h}{2} + \frac{2\gamma_l\beta_h(\beta_h - 1)}{(4\beta_h - \gamma_l)(\gamma_l + 8\beta_h)}\left(1 - \frac{3\gamma_l(\beta_h - 1)}{(4\beta_h - \gamma_l)(\gamma_l + 8\beta_h)}\right)
\end{aligned}$$

□

Many papers in the economic literature on access pricing find similar dynamics in their models[6]. By introducing β as a second dimension on vertical service

[5] See calculations in Appendix C.

[6] Avenali et al. (2010) and Manenti and Scialà (2013) also find that the incumbent uses the access charge to foreclose the entrant's entry. Foros (2004) does not allow for facility based entry,

differentiation, the result derived in my model depends on the parameter's specification. In the following, I show that the monopolistic market entails an additional restriction on β.

Corollary 1. *As the entrant's entry is foreclosed, reach quantity of the incumbent must be $\beta_h = 1$. Therefore, the monopolistic outcome is not affected by the reach characteristics γ and β.*

Proof. The reason for the restriction on the parameter β can be seen in the definition of consumer surplus.

$$CS_t^m = \underbrace{\int_{\frac{p_{it}^m - p_{et}^m}{\beta_h - \gamma_l}}^{1} (\beta_h w - p_{it}^m) dw}_{\text{advertisers' surplus from } i\text{'s service}} + \underbrace{\int_{\frac{p_{et}^m}{\gamma_l}}^{\frac{p_{it}^m - p_{et}^m}{\beta_h - \gamma_l}} (\gamma_l w - p_{et}^m) dw}_{\text{advertisers' surplus from } e\text{'s service}}$$

For the second term to be zero, the fraction of advertisers that buy i's service must be equal the threshold $\phi^m = \frac{p_{et}^m}{\gamma_l}$ of advertisers not participating in the market.

$$\frac{p_{it}^m - p_{et}^m}{\beta_h - \gamma_l} = \frac{p_{et}^m}{\gamma_l}$$
$$\beta_h \overset{!}{=} 1$$

By Corollary 1, the monopoly outcome of the game with access prices, prices, quantities, profits and social welfare is as follows.

$$a_t^m = \frac{\gamma_l}{2} \qquad p_{it}^m = \frac{1}{2} \qquad q_{it}^m = \frac{1}{2}$$
$$\Pi_i^m = \frac{1}{2} \qquad W^m = \frac{3}{4} \quad \square$$

but finds a similar dynamic.

Corollary 1 shows that an initial advantage in a factor shifting the advertisers' willingness to pay for the incumbent's service on the mobile advertisement market does not affect the monopolistic outcome in the unregulated access price scenario. The social welfare on the access price is not affected by its quantitative user advantage.

The monopolistic market outcome generally corresponds to the results found by Avenali et al. (2010) on the telecommunications market. Similar findings for a one-period game are presented by Manenti and Scialà (2013). The monopoly outcome does not depend on the access price. Therefore, the access price can be interpreted as a transfer from the entrant to the incumbent which does not affect social welfare directly, but only indirectly via the firms' optimal quantities sold (Manenti and Scialà, 2013).

The first principle of the fundamental welfare theorem states that an equilibrium tends to be Pareto efficient only if markets are complete and firms are price-takers. By conveying that the outcome of the analyzed market with unregulated access prices is a monopolistic equilibrium, Proposition 1 shows that the first welfare theorem is violated. This finding justifies the introduction of a regulative scheme on access pricing. An alternative approach to demonstrate inefficiency on the market is provided by Manenti and Scialà (2013). The authors compare the welfare level of the monopolistic market to a market where the entrant has entered.

Throughout the following section, I develop a dynamic scheme to regulate the price of access data that is based on Avenali et al.'s (2010) regulatory scheme for a regulator in a the telecommunications market.

6.2 Regulated Market

Equilibrium access prices in the regulated market depend on the level of sunk cost related to set up a map and the advertising service's perceived reach quality and quantity. In period one, access prices are set at cost in equilibrium. In period two, for low sunk costs, any arbitrarily high access price promotes the entrant's entry, whereas the access price cannot promote entry for high sunk costs. For intermediate costs, the higher the entrant service's quality perception, the higher the minimal equilibrium access price. The advertisers' positive shift in willingness to pay is sufficient to induce the entrant to enter the market using its own map and therefore the firm does not need to be incentivized additionally with a low access price. Further, a higher quantity advantage of the incumbent increases the access price, counteracting the entry incentives for the entrant. In the following sections, I will first derive the formal description of these findings and second specify parameter values and provide comparative statistics.

6.2.1 Dynamic Access Price Schedule

Proposition 2. *Let $0 < \gamma_l < 1 < \gamma_h$ and $0 < \beta_l < 1 < \beta_h$ and $R > R_{min}$. When $R_{min} < R \leq R_{res}$, e enters at any a_2. When $R_{res} < R \leq R_{max}$, the regulator can incentivize e to enter via facility with a sufficiently high a_2. When $R > R_{max}$, e's entry is never profitable and the regulator sets $a_2 = 0$. Socially efficient facility based entry is incentivized through the following equilibrium access price schedule,*

$$a_1 = 0$$

$$a_2 = \begin{cases} \geq 0 & \text{if } R_{min} < R \leq R_{res} \\ \geq a_2^F = \frac{\gamma_l}{2} - \frac{4\beta_h - \gamma_l}{2}\sqrt{\frac{(\Pi_{e2}^F - R)\gamma_l}{\beta_h(\beta_h - \gamma_l)}} & \text{if } R_{res} < R \leq R_{max} \\ 0 & \text{if } R > R_{max} \end{cases},$$

where $R_{res} = \frac{H(\gamma_h, \beta_h)}{(\beta_h(1-2\beta_h)+2\gamma_h(1+\beta_h))^2} - \frac{(\beta_h - \gamma_l)\gamma_l\beta_h}{(4\beta_h - \gamma_l)^2}$ *and* $R_{max} = \Pi_{e2}^F = \frac{H(\gamma_h, \beta_h)}{(\beta_h(1-2\beta_h)+2\gamma_h(1+\beta_h))^2}$ *for* $H(\gamma_h, \beta_h) = [\gamma_h(\gamma_h(1+\beta_h) - 2\beta_h^2) - \beta_h^2(1-\beta_h)]\,[\beta_h(1-\beta_h) + \gamma_h(1+\beta_h)]$.

Proof. Regulation may introduce the possibility for the entrant to enter the market in period one. As $R > R_{min}$, by Proposition 1, the entrant always prefers facility based entry when having entered the market in period one.

Up until the point where facility based entry yields no additional profit compared to service based entry, setting up its own map and serving independently on the mobile cartographic advertising market is profitable. The point at which profits in both entry cases are equal is at $R = R_{res}$. Therefore, when $R_{min} < R \leq R_{res}$, e's profit is always non-negative. The entrant responds to the access price set by the regulator by choosing the entry type that yields the highest profit. Therefore, to find R_{res}, I start by defining how the regulator optimally sets the access price.

The regulator sets a_t by maximising social welfare $W^K = \Pi_e^K + \Pi_i^K + CS^K$ where $K \in \{S, F\}$. In the first period, welfare is maximised by setting the lowest access price possible, as

$$\frac{\partial W^K}{a_1} = -\frac{a_1(5\gamma_l + 4\beta_h)\beta_h + \gamma_l(3\beta_h + 1 - 4\gamma_l)\beta_h}{\gamma_l(4\beta_h - \gamma_l)^2} < 0\ .$$

Hence, the socially optimal access price in period one is $a_1 = 0$ for both entry scenarios. This means that the regulator sets the access price at the cost of offering a mobile advertising service (which is assumed to be zero) in order to incentivize the entrant's entry. The entrant's optimal, this is, profit maximising, response depends on its investment decisions. Two scenarios are to be differentiated: the firm has or has not set up its own map in period one.

Assume the entrant has not set up its own map and hence not incurred cost R in the first period. By Proposition 1, e opts for service based entry in period two. Similar to the first period, as $\frac{\partial W^S}{a_2} < 0$, the regulator optimally sets the access price at cost, hence $a_2 = a_1 = 0$. The entrant's overall profit is

$$\Pi_e^S|_{a_t=0} = \frac{2\gamma_l(\beta_h - \gamma_l)\beta_h}{(4\beta_h - \gamma_l)^2}$$

and social welfare is

$$W^S|_{a_t=0} = \frac{2(\beta_h - \gamma_l)(4\beta_h + \gamma_l)\beta_h + (r\gamma_l + 4\beta_h)\beta_h^2}{(4\beta_h - \gamma_l)^2} \ .$$

Assume e has set up its own map and hence incurred cost R in the first period. By Proposition 1, e opts for facility based entry in period two. As W^F does not vary with a_2, e's overall profit is

$$\Pi_e^F = \frac{\gamma_l(\beta_h - \gamma_l)\beta_h}{4\beta_h - \gamma_l)2} - R + \frac{H(\gamma_h, \beta_h)}{(\beta_h(1 - 2\beta_h) + 2\gamma_h(1 + \beta_h))^2} \ ,$$

where $H(\gamma_h, \beta_h) = [\gamma_h(\gamma_h(1+\beta_h) - 2\beta_h^2) - \beta_h^2(1-\beta_h)]\,[\beta_h(1-\beta_h) + \gamma_h(1+\beta_h)]$ and social welfare is

$$W^F|_{a_1=0} = \frac{1}{2}\left(\frac{T(\gamma_h, \beta_h)}{2(\beta_h(1 - 2\beta_h) + 2\gamma_h(1 + \beta_h)} - \frac{\beta_h(2\gamma_l^2 + \gamma_l\beta_h - 12\beta_h^2)}{(4\beta_h - \gamma_l)^2}\right) \ ,$$

where $T(\gamma_h, \beta_h) = \gamma_h^3(3 + \beta_h(2 + 3\beta_h) + \gamma_h^2(2 - \beta_h^2(2 + \beta_h)) + \gamma_h(5\beta_h + 3\beta_h^2(1 - \beta_h) - 4) - (\beta_h(4 - 5\beta_h + 1)\beta_h^3$.
The entrant compares the profits of both entry scenarios when deciding on how to enter. Its decision ultimately depends on the sunk cost R. e prefers facility based entry if $\Pi_e^F > \Pi_e^S$. Formally, this occurs when $R \leq R_{res}$, with

$$R_{res} = \frac{H(\gamma_h, \beta_h)}{(\beta_h(1 - 2\beta_h) + 2\gamma_h(1 + \beta_h))^2} - \frac{\gamma_l(\beta_h - \gamma_l)\beta_h}{(4\beta_h - \gamma_l)^2} \ .$$

For $a_1 = a_2 = 0$, if $R > R_{res}$, the entrant prefers entering via service. As the entrant's profit does not depend on a_2, the firm incurs the setup cost R at any a_2 and hence the regulator can incentivize e's facility based entry through a sufficiently high access price in the second period.

There is a threshold value R_{max} such that whenever $R > R_{max}$ it is never profitable for e to enter. R_{max} is found at the point where the investment in R does not increase overall profit from both periods, such that $\Pi_e^F - \Pi_e^S \leq 0$. As

$\Pi^S_{e2} > 0$ and $\Pi^F_{e1} = \Pi^S_{e1} - R$, the threshold value is at

$$R > R_{max} = \Pi^F_{e2} = \frac{H(\gamma_h, \beta_h)}{(\beta_h(1-2\beta_h) + 2\gamma_h(1+\beta_h))^2}$$

This leads to the only room for the regulator to incentivize e via the access price at $R_{res} < R \leq R_{max}$. As for $a_2 \in [0, \frac{\gamma_l}{2})$, the entrant's relative profit of facility based compared to service based entry increases in a_2: $\frac{\partial(\Pi^F_e - \Pi^S_e)}{\partial a_2} > 0$. To incentivize e's facility based entry, the regulator must set a_2 such that $(\Pi^F_e - \Pi^S_e)$ is nonnegative. This value can be derived as following.

$$\begin{aligned} \Pi^F_{e2} - R &\geq \Pi^S_{e2} \\ a_2 \geq a^F_2 &= \frac{\gamma_l}{2} - \frac{4\beta_h - \gamma_l}{2}\sqrt{\frac{(\Pi^F_{e2} - R)\gamma_l}{\beta_h(\beta_h - \gamma_l)}} \end{aligned}$$

Therefore, the socially optimal equilibrium access price in period two is $a_2 \geq a^F_2$.
$\square$

As shown in Proposition 2, the socially optimal access price is rising over time. In general terms, this result corresponds to the finding of Avenali et al. (2010). The access price in period one is set at the cost of offering the mobile advertising service, leading to zero profit for the incumbent from selling its user data. In period two, the access price depends on the level of sunk cost R. For low sunk cost levels $R_{min} < R \leq R_{res}$, the regulator can set the access price arbitrarily high, as the entrant firm's entry does not have to be incentivized further. For high sunk cost levels $R > R_{max}$, e's entry cannot be incentivized by the regulator's sole instrument of access pricing. For intermediary sunk cost levels, $R_{res} < R \leq R_{max}$, the equilibrium access price in period two that maximises social welfare depends on the parameters γ_h, γ_l and β_h, representing high and low reach quality, and high reach quantity, respectively. I find that the higher the incumbent's reach quantity, the less socially desirable it is to incentivize the entrant to enter the market by setting a low access price. As the number of the incumbent service's users is not reconciled by the entrant within the two periods, the higher quantity parameter β_h can be interpreted as the incumbent's quantity advantage. Further, the higher the shift in the advertisers' willingness to pay induced by the entrant's reach quality, the higher the access price, as a less strong incentive of the regulator via a lower access price is necessary to induce the entrant to enter.

The regulator's best response for given reach characteristics of the firms on the mobile advertising market is further analysed in Section 6.2.2. Note that by Proposition 2, given any parameter values, in the case of the sunk cost being exactly at its maximum level, $R = R_{max}$, the equilibrium access price is at least as high as the access price of the monopolistic equilibrium in the unregulated case derived in Section 6.1: $a_2 \geq a_t^m = \frac{\gamma_l}{2}$.

6.2.2 Access Price for Intermediate Sunk Cost Levels

In the following section, I analyse the equilibrium access price of period two with sunk cost at the level $R_{res} < R \leq R_{max}$ with respect to its parameters γ_h, γ_l and β_h. A common practice of comparative statistics is based on applying the implicit function theorem on first order conditions (Milgrom and Shannon, 1994). I will not provide a full mathematical proof of whether the assumptions validating the proper use of this method are met. Rather, I approximate the total differential that is commonly used by applying partial derivatives of the variable $a_2^F(\gamma_h, \gamma_l, \beta_h)$ with respect to the parameters to be chosen. Hereby, I assume that the total differential can be expressed as the sum of partial differentials, which is not necessarily the case. However, the approximation I provide gives a general understanding of how the equilibrium access price changes with the parameter inputs and is a common approach of comparative statistics in microeconomic research.

A prior restriction on the parameters is that the assumption in Section 4.5 of $a \geq 0$ must hold. Additionally, when choosing the parameters, I account for the assumption that quality is the superior characteristic of an advertisement's reach compared to the quantity aspect (see Section 4.2). This leads to the parameter relations $\gamma_h > \beta_h$ and $\gamma_l + 1 > \beta_h$, where I use $1 + \gamma_l$ as, per definition, $\gamma_l < 1$ and this arrangement makes the magnitude in the negative shift of willingness to pay of an advertiser due to the low quality reach characteristic γ_l comparable to the positive shift due to high quantity characteristic β_h.

A possible set of parameters such that the equilibrium access price in period two is nonnegative and the model's assumptions are met is $\{\gamma_h, \gamma_l, \beta_h\} = \{1.5, 0.4, 1.3\}$. The value for γ_h represents a 50 percent increase in the advertisers willingness to pay induced by the high quality perception of the service, whereas a low quality perception, represented by γ_l in the advertisers' utility function, lowers their willingness to pay by 60 percent. The value of β_h, finally, increases the willingness to pay by 30 percent.

It follows from Proposition 2 that the equilibrium access price in period two at $R_{res} < R \leq R_{max}$, is increasing in γ_h. Formally, this can be seen by $\frac{\partial a_2^F}{\partial \gamma_h} > 0$. Representing an upward shift in the advertisers' willingness to pay by 50 percent, the high quality characteristic arising from higher valued end users, γ_h, increases firm e's profits in period two. The regulator can set the access price higher (which increases i's profit Π_i) as the entrant can bear a higher access charge.

In line with the first finding, a_2^F is decreasing in γ_l. Formally, this is shown by $\frac{\partial a_2^F}{\partial \gamma_l} < 0$. Representing a downward shift in the advertisers' willingness to pay, the low quality characteristic arising from lower end users that are expected to be less wealthy, γ_l, decreases e's profits. Hence, a sufficiently low access price a_2^F is needed to induce the entrant firm to enter.

Further, a_2^F is increasing in β_h. Formally, this is shown by $\frac{\partial a_2^F}{\partial \beta_h} > 0$. Fig. 1 illustrates that the entrant cannot reach the position of being peceived as the high quantity advertising service provider. According to the model's equilibrium in Proposition 2, the higher the incumbent's quantity advantage, β_h, the less a regulator incentivizes the entrant to enter the market in period two. This result shows that in terms of the aggregate value for advertisers and firms, it is socially more appealing to restrict a second firm's entry if the irreconcilable advantage of the incumbent is high. The additional value for an advertiser to use the entrant's high quality map plus the additional access profits of the incumbent may not exceed the social loss occuring through the sunk cost of building a new map and the purchase of user data that decreases an entrant's profit. This finding is supported by the fact that the magnitude of the access price increase is significantly higher for the quantity effects compared to the quality effects. An increase in one unit of quantity therefore increases the access price more than an increase in one unit of quality.

By Proposition 2, for intermediate sunk costs $R_{res} < R \leq R_{max}$, any access price a_2 that is higher than a_2^F is effective in terms of promoting the entrant's facility based entry in period two.

The partial differentials derived above illustrate the mechanism in the regulated equilibrium. The chosen parameters represent the case of $\gamma_h < 1 + \gamma_l$, hence a negative shift in the advertisers' willingness to pay for mobile advertising service induced by γ_h is larger than a positive shift induced by γ_l. In the following, I compare these results to other potential parameter values.

When switching to the case of $\gamma_h > 1 + \gamma_l$, a positive shift in the advertisers' willingness to pay due to a high quality service is larger than a respective negative shift. The results, hence the direction of the change in the access price when increasing the parameters, are similar to the baseline case and differ only slightly in magnitude.

In the case of equal positive and negative shifts in willingness to pay, the parameter space reduces to two dimensions, as γ_h can be expressed by γ_l. The results in the quantity aspect are similar to the previously mentioned cases, with a_2^F increasing in β_h. However, the results may change with respect to the quality aspect. In all the mentioned cases, the high quantity parameter is bounded, i.e. $\beta_h \in (1.29, 1.5)$. The lower bound assures the access price to be positive, whereas the upper bound assures that the model's assumption on the superiority of the quality are met.

7 Discussion and Outlook

The presented model is a novel approach to modeling the economics of big data as a competitive advantage in the tech industry. I account for the business model of big data driven companies like Google or Apple, which offer services in exchange for data and sell the data on a side market. The inclusion of this side market, being the mobile advertising market, reveals novel insights on how the firms' service characteristics shape the equilibrium access price on user data that is sold from the incumbent to a potential entrant.

The model shows that private and social interests are not aligned in all periods when an incumbent is setting the access price. Private interests are represented by the incumbent's goal of preserving its dominant position. The incumbent is controlling the industry by exercising market power arising from its sole access to user data. In my model, its goal can be achieved by hindering the potential entrant from investing in its own mobile map. In an unregulated access price market, the incumbent can do so by setting a sufficiently high access price, where access to the incumbent's data is essential for a competitive service on the advertising market. The outcome of the unregulated access price model is therefore a monopolistic incumbent.

Social interests are represented by the joint value of advertisers and firms. An entry foreclosure of the entrant in the first period is not socially desirable. However, in the second period, an exclusion of the entrant may be desired both, privately and socially. This result differs from Avenali et al. (2010), where private and social interests are not aligned. If socially desirable, dynamic access price regulation can incentivize the entrant to invest in building an independent mobile map. A regulator therefore needs to apply an access price schedule that maximises social welfare. I find that the value of the equilibrium access price crucially depends on the mobile advertising service parameters, and the regulator incentivizes the entrant's market entry less when observing a higher quantity advantage of the incumbent. An increase of the perceived value of end users also leads to a higher equilibrium access price, but in a significantly lower magnitude. It is therefore socially optimal to not induce a second entry in period two if the incumbent is capturing too many end users.

My model serves as a novel approach of taking into account changing market dynamics in the age of big data from an economic perspective. I have used the market of mobile cartography as an exemplifying industry. The value of data is derived by its value on the mobile advertising market, the revenue source of the firms owning the data. To address a broader range of industries as well as to account for more characteristics of the markets, there are many possible extensions. Some of those I want to mention in the following.

In the regulated market equilibrium of my model, access price in period one is set at cost of entering the mobile advertising market, and hence the incumbent does not yield any revenue from selling the data. This finding gives rise to a further investigation of the regulation's effects on the incumbent's willingness to be the first-mover. Such investigations are addressed in some of the access price literature on telecommunications, as for example in Laffont and Tirole (1994). The analysis of first-mover incentives may be interesting for a regulator when deciding on policies affecting other industries where big data is expected to drive competitiveness in the future. However, markets where the dominant players are already set, as the mobile advertising market, are not subject to this further analysis.

Additionally, in my model, arbitrarily high access prices in period two are effectively promoting the entrant firm's entry. As pointed out by Avenali et al. (2010), the reason for this result may be that there is only one entrant firm in the model. The authors extend their model to a three-firm case. An extension of my model accounting for several entrants may be interesting when applying to other markets beyond the mobile cartography market. In the specific market addressed in my work, however, the analysis with one incumbent (Google) and one entrant (Apple) is most reasonable as they are the dominant players in the mobile advertising market with their market shares adding up to more than 98 percent (StatCounter, 2018).

Further, throughout my analysis, the regulator is assumed to have full commitment power. Other authors, however, account for the possibility of the regulator having no commitment[7]. However, as illustrated by the findings of Avenali et al. (2010), the commitment issue vanishes in the two-firm case.

Lastly, the end user of mobile cartography services like Google's or Apple's Maps do not play an active role in my model's calculations. I suggest the inclusion of the end users' added value resulting from being able to choose between alternative cartography services in a potential future extension of the model.

[7] Foros (2004), Avenali et al. (2010) and Manenti and Scialà (2013) model the effects of a regulator's limited commitment as an extension of the basic access pricing model.

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Appendices

A Service based entry

Profits. The incumbent's (i) and entrant's (e) profits are:

$$\Pi_i^S = \Pi_{i1}^S + \Pi_{i2}^S = (p_{i1}^S q_{i1}^S + a_1 q_{e1}^S) + (p_{i2}^S q_{i2}^S + a_2 q_{e2}^S)$$
$$\Pi_e^S = \Pi_{e1}^S + \Pi_{e2}^S = ((p_{e1}^S - a_1) q_{e1}^S) + ((p_{e2}^S - a_2) q_{e2}^S)$$

Utility functions. Advertisers' utilities of using e's or i's mobile advertising service are

$$u_{it}^S(w) = \beta_h w - p_{it}^S$$
$$u_{et}^S(w) = \gamma_l w - p_{et}^S \ ,$$

where $t = \{1, 2\}$, $0 < \gamma_l < 1$ and $\beta_h > 1$. After having served the market, there are $\phi^S + q_{et}^S$ marketers left for e to serve. $\phi^S + q_{et}^S$ is found at the intersection $u_{it}^S(w) = u_{et}^S(w)$:

$$\beta_h w - p_{it}^S = \gamma_l w - p_{et}^S$$
$$w = \frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l} = \phi^S + q_{et}^S$$

Hence, using total demand $Q_t \equiv q_{it}^S + q_{et}^S = 1 - \phi^S$, the quantity demanded from i is

$$q_{it}^S = 1 - \frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l} \ .$$

e serves the market until $u_{et}^S(w) = 0$:

$$\gamma_l w - p_{et}^S = 0$$
$$w = \frac{p_{et}^S}{\gamma_l} = \phi^S$$

ϕ^S is said to be the reservation price at which no type of advertiser can yield positive utility from buying a firm's service. Hence, advertisers whose willingness to pay is lower than $\frac{p_{et}^S}{\gamma_l}$ stay out of the market. Quantity demanded from e can be expressed as following.

$$q_{et}^S = \frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l} - \frac{p_{et}^S}{\gamma_l}$$

Optimal prices. We can find the optimal prices by the setting the first order conditions of the firms' profit function in each period to zero.

$$\frac{\partial \Pi_{it}^S}{\partial p_{it}^S} = q_{it}^S + \frac{\partial q_{it}^S}{\partial p_{it}^S} p_{it}^S + a_t \frac{\partial q_{et}^S}{\partial p_{it}^S} = 0$$
$$\frac{\partial \Pi_{et}^S}{\partial p_{et}^S} = (p_{et}^S - a_t) \frac{\partial q_{et}^S}{\partial p_{et}^S} + q_{et}^S = 0$$

This leads to the following relations.

$$p_{et}^S = 2p_{it}^S - \beta_h + \gamma_l - a_t$$
$$p_{et}^S = \frac{\beta_h a_t + p_{it}^S \gamma_l}{2\beta_h}$$

Therefore, optimal prices are:

$$p_{it}^S = \frac{3\beta_h a_t - 6\beta_h^2}{4 - \gamma_l} + 2\beta_h$$
$$p_{et}^S = \frac{6\beta_h a_t - 12\beta_h^2}{4\beta_h - \gamma_l} + 3\beta_h + \gamma_l - a_t \ .$$

Optimal quantities. Using optimal prices, quantities for the firms' services yield:

$$q_{it}^S = \frac{2\beta_h - a_t}{4\beta_h - \gamma_l}$$
$$q_{et}^S = \frac{\beta_h(\gamma_l - 2a_t)}{\gamma_l(4\beta_h - \gamma_l)}$$

Optimal profits.

$$\begin{aligned}
\Pi_{it}^S &= p_{it}^S q_{it}^S + a_t q_{et}^S \\
&= \left(\frac{2\beta_h^2 - 2\gamma_l\beta_h + 3\beta_h a_t}{4\beta_h - \gamma_l}\right)\left(\frac{2\beta_h - a_t}{4\beta_h - \gamma_l}\right) + a_t\left(\frac{\beta_h(\gamma_l - 2a_t)}{\gamma_l(4\beta_h - \gamma_l)}\right) \\
&= \frac{4\gamma_l(\beta_h - \gamma_l)\beta_h^2 + \gamma_l a_t(10\beta_h + \gamma_l - 2)\beta_h - a_t^2(\gamma_l + 8\beta_h)\beta_h}{\gamma_l(4\beta_h - \gamma_l)^2}
\end{aligned}$$

$$\begin{aligned}
\Pi_{et}^S &= (p_{et}^S - a_t)q_{et}^S \\
&= \left(\frac{2\gamma_l a_t - 2\beta_h a_t + \gamma_l\beta_h - \gamma_l^2}{4\beta_h - \gamma_l}\right)\left(\frac{\beta_h(\gamma_l - 2a_t)}{\gamma_l(4\beta_h - \gamma_l)}\right) \\
&= \frac{(\beta_h - \gamma_l)(\gamma_l - 2a_t)^2\beta_h}{\gamma_l(4\beta_h - \gamma_l)^2}
\end{aligned}$$

Consumer surplus.

$$\begin{aligned}
CS^S = \sum_{t\in\{1,2\}} CS_t^S &= \sum_{t\in\{1,2\}}\left(\int\limits_{\frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l}}^{1} (\beta_h w - p_{it}^S)dw + \int\limits_{\frac{p_{et}^S}{\gamma_l}}^{\frac{p_{it}^S - p_{et}^S}{\beta_h - \gamma_l}} (\gamma_l w - p_{et}^S)dw\right) \\
&= \frac{2\gamma_l(5\gamma_l + 4\beta_h)\beta_h^2 - 2\gamma_l(a_1 + a_2)(\gamma_l + 9\beta_h - 1)\beta_h + (a_1^2 + a_2^2)(5\gamma_l + 4\beta_h)\beta_h}{2\gamma_l(4\beta_h - \gamma_l)^2}
\end{aligned}$$

B Facility based entry

Profits. The incumbent's (i) and entrant's (e) profits are:

$$\Pi_i^F = \Pi_{i1}^F + \Pi_{i2}^F = (p_{i1}^F q_{i1}^F + a_1 q_{e1}^F) + (p_{i2}^F q_{i2}^F)$$
$$\Pi_e^F = \Pi_{e1}^F + \Pi_{e2}^F = ((p_{e1}^F - a_1) q_{e1}^F - R) + (p_{e2}^F q_{e2}^F)$$

Utility functions. Assume that e's reach quality is perceived as low in period one and as high in period two, and that i's reach quantity is perceived as high in both periods. In period one, the incumbent serves the market first, and advertiser type w's utility derived from the firms' services are the same as in the service based entry scenario.

$$u_{i1}^F(w) = \beta_h w - p_{i1}^F$$
$$u_{e1}^F(w) = \gamma_l w - p_{e1}^F \ ,$$

where $t \in \{1, 2\}$, $0 < \gamma_l < 1$ and $\beta_h > 1$, and $p_{k1}^F = p_{kt}^S$ for $k \in \{e, i\}$. Similar to the argumentation of the service based entry case in Appendix A, demand functions are as follows.

$$q_{i1}^F = 1 - \frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l}$$
$$q_{e1}^F = \frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l} - \frac{p_{e1}}{\gamma_l}$$

where again $p_{k1}^F = p_{kt}^S$ for $k \in \{e, i\}$. Optimal prices, quantities and profits of period one are the same as in the service based entry scenario.

Optimal prices.

$$p_{i1}^F = \frac{3\beta_h a_1 - 6\beta_h^2}{4 - \gamma_l} + 2\beta_h$$
$$p_{e1}^F = \frac{6\beta_h a_1 - 12\beta_h^2}{4\beta_h - \gamma_l} + 3\beta_h + \gamma_l - a_1 \ .$$

Optimal quantities. Using optimal prices, quantities for the firms' services yield:

$$q_{i1}^F = \frac{2\beta_h - a_1}{4\beta_h - \gamma_l}$$
$$q_{e1}^F = \frac{\beta_h(\gamma_l - 2a_1)}{\gamma_l(4\beta_h - \gamma_l)}$$

Optimal profits.

$$\Pi_{i1}^F = \frac{4\gamma_l(\beta_h - \gamma_l)\beta_h^2 + \gamma_l a_1(10\beta_h + \gamma_l - 2)\beta_h - a_1^2(\gamma_l + 8\beta_h)\beta_h}{\gamma_l(4\beta_h - \gamma_l)^2}$$

$$\Pi_{e1}^S = \frac{(\beta_h - \gamma_l)(\gamma_l - 2a_1)^2\beta_h}{\gamma_l(4\beta_h - \gamma_l)^2} - R$$

In period two, assuming that reach quality is the superior characteristic, e serves the market first at price p_{e2}^F. After having skimmed the advertisers with highest willingness to pay, i serves the market at price p_{i2}^F. Advertisers who yield non-positive utility at the given price stay out of the market. The threshold of their willingness to pay (reservation price) is ϕ^F. Advertiser w's utility of i and e's service are

$$\begin{aligned} u_{i2}^F(w) &= \beta_h w - p_{i2}^F \\ u_{e2}^F(w) &= \gamma_h w - p_{e2}^F \ , \end{aligned}$$

where $\gamma_h > 1$ and $\beta_h > 1$. After having served the market, e leaves $\phi^F + q_{i2}^F$ for i to serve. This threshold of marketers is at the intersection $u_{i2}^F(w) = u_{e2}^F(w)$:

$$\begin{aligned} w - p_{i2}^F &= \gamma_h w - p_{e2}^F \\ w &= \frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h} = \phi^F + q_{i2}^F \end{aligned}$$

Hence, using total demand $Q_2 = 1 - \phi^F = q_{i2}^F + q_{e2}^F$, e's optimal quantity is

$$q_{e2}^F = 1 - \frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h}$$

i serves the market until $u_{i2}^F = 0$:

$$\begin{aligned} \beta_h w - p_{i2}^F &= 0 \\ w &= \frac{p_{i2}^F}{\beta_h} = \phi^F \end{aligned}$$

Hence, i's optimal quantity can be expressed as following.

$$q_{i2}^F = \frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h} - \frac{p_{i2}^F}{\beta_h}$$

advertisers whose willingness to pay is lower than p_{i2} stay out of the market.

Optimal prices. Again, we can find the optimal prices by the first order conditions of the firms' profit function. In contrast to period one, in period two, profits do not depend on the access price. The first order conditions of the firms' profit functions are

$$\frac{\partial \Pi_{i2}^F}{\partial p_{i2}^F} = q_{i2}^F + \frac{\partial q_{i2}^F}{\partial p_{i2}^F} p_{i2}^F = 0$$
$$\frac{\partial \Pi_{e2}^F}{\partial p_{e2}^F} = q_{e2}^F + \frac{\partial q_{e2}^F}{\partial p_{e2}^F} p_{e2}^F = 0 \ ,$$

leading to the following optimal prices.

$$p_{i2}^F = \frac{\beta_h(\gamma_h - \beta_h)}{2\gamma_h(1+\beta_h) + \beta_h(1-2\beta_h)}$$
$$p_{e2}^F = \frac{\gamma_h(\gamma_h(1+\beta_h) - 2\beta_h^2) - \beta_h^2(1-\beta_h)}{2\gamma_h(1+\beta_h) + \beta_h(1-2\beta_h)}$$

Optimal quantities. Using optimal prices, quantities for the firms' services yield:

$$q_{i2}^F = \frac{\beta_h(\gamma_h - \beta_h + 1)}{\beta_h(1-2\beta_h) + 2\gamma_h(1+\beta_h)}$$
$$q_{e2}^F = \frac{\beta_h(1-\beta_h) + \gamma_h(1+\beta_h)}{\beta_h(1-2\beta_h) + 2\gamma_h(1+\beta_h)}$$

Optimal profits. Optimal profits in period two are:

$$\begin{aligned}\Pi_{i2}^F &= p_{i2}^F q_{i2}^F \\ &= \frac{\beta_h^2(\gamma_h - \beta_h)(\gamma_h - \beta_h + 1)}{(\beta_h(1-2\beta_h) + 2\gamma_h(1+\beta_h))^2}\end{aligned}$$

$$\begin{aligned}\Pi_{e2}^F &= p_{e2}^F q_{e2}^F \\ &= \frac{[\gamma_h(\gamma_h(1+\beta_h) - 2\beta_h^2) - \beta_h^2(1-\beta_h)]\,[\beta_h(1-\beta_h) + \gamma_h(1+\beta_h)]}{(\beta_h(1-2\beta_h) + 2\gamma_h(1+\beta_h))^2}\end{aligned}$$

Consumer surplus.

$$
\begin{aligned}
CS^F &= CS_1^F + CS_2^F \\
&= \left(\int_{\frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l}}^{1} (\beta_h w - p_{i1}^F) dw + \int_{\frac{p_{e1}^F}{\gamma_l}}^{\frac{p_{i1}^F - p_{e1}^F}{\beta_h - \gamma_l}} (\gamma_l w - p_{e1}^F) dw \right) \\
&+ \left(\int_{\frac{p_{i2}^F}{\beta_h}}^{\frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h}} (\beta_h w - p_{i2}^F) dw + \int_{\frac{p_{i2}^F - p_{e2}^F}{\beta_h - \gamma_h}}^{1} (\gamma_h w - p_{e2}^F) dw \right) \\
=& \frac{2\gamma_l(5\gamma_l + 4\beta_h)\beta_h^2 - 2\gamma_l a_1(\gamma_l + 9\beta_h - 1)\beta_h + a_1^2(5\gamma_l + 4\beta_h)\beta_h}{2\gamma_l(4\beta_h - \gamma_l)^2} \\
&+ \frac{\gamma_h^2\beta_h(2 + \beta_h(2 - \beta_h)) + \gamma_h(7\beta_h(1 + \beta_h) - 10\beta_h^3) + \gamma_h^3(1 + \beta_h(2 + \beta_h)) + 3\beta_h^3(1 - \beta_h(2 - \beta_h))}{2(\beta_h(1 - 2\beta_h) + 2\gamma_h(1 + \beta_h))^2}
\end{aligned}
$$

Note that $CS_1^F = CS_1^S$.

C Unregulated Market

The monopolistic equilibrium is calculated as following. By using a_t^m from Proposition 1, the monopoly price for i's service is

$$p_{it}^m = p_{it}^S|_{a_t=a_t^m} = \frac{\beta_h}{2} + \frac{3\gamma_l(\beta_h - 1)\beta_h}{(4\beta_h - \gamma_l)(\gamma_l + 8\beta_h)}$$

and optimal quantity is

$$q_{it}^m|_{a_t=a_t^m} = \frac{1}{2} - \frac{\gamma_l(\beta_h - \gamma_l)}{(\gamma_l + 8\beta_h)(4\beta_h - \gamma_l)} \ .$$

Using these results in the profit function for $t \in \{1, 2\}$ yields

$$\Pi_{it}^m = \frac{\beta_h}{4} + \frac{\gamma_l(\beta_h - 1)\beta_h}{(\gamma_l + 8\beta_h)(4\beta_h - \gamma_l)} \left(1 - \frac{3\gamma_l(\beta_h - 1)}{(\gamma_l + 8\beta_h)(4\beta_h - \gamma_l)}\right)$$

and hence total profit of both periods is

$$\Pi_i^m = \frac{\beta_h}{2} + \frac{2\gamma_l\beta_h(\beta_h - 1)}{(\gamma_l + 8\beta_h)(4\beta_h - \gamma_l)} \left(1 - \frac{3\gamma_l(\beta_h - 1)}{(\gamma_l + 8\beta_h)(4\beta_h - \gamma_l)}\right)$$