# International Volatility-Managed Equity Factors

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Master Thesis in Finance

Spring 2018

**Abstract**

Recent studies show that volatility timing works well across a number of different US asset pricing factors and for 20 developed market indices. Our study expands the literature by testing the same strategy across seven equity factors on an aggregate international level as well as for five equity factors on a country level in 24 developed markets. We test volatility timing strategies on the market (MKT), size (SMB), value (HML), momentum (MOM) and betting-against-beta (BAB) factors on the country level and also add the profitability (RMW) and investment (CMA) factors on the international level. For the international factors, we find similar results to previous US studies for the profitability, momentum and betting-against-beta factors. However, our country-level results show that volatility timing yields poor to mixed results for all factors except for momentum. Consistent with the momentum crashes literature, we find large benefits from volatility timing for the country-level momentum factors with positive and significant alphas in 21/24 countries. Hence, our results bring the cross-factor benefits of volatility timing into question and might suggest that the benefits are primarily related to momentum.

**Supervisor:** Riccardo Sabbatucci

**Keywords:** international asset pricing, volatility-managed portfolios, volatility timing, momentum crashes

**Acknowledgements:** We would like to express our gratitude to Assistant Professor Riccardo Sabbatucci for his valuable inputs and guidance throughout the process of writing this paper.

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# 1 Introduction

Traditional asset pricing theory suggests that investors are compensated in the long-run for investing in assets that perform poorly in bad times such as recessions or financial crises, when volatility tends to be high. However, Moreira and Muir (2017) show that volatility-managed portfolios, that take less risk when volatility is high and more risk when volatility is low, outperform non-managed portfolios. These results are observed across a wide set of standard asset pricing factors in the US and for 20 developed market indices, but no study has been performed in a broader international setting to our knowledge. Our study sheds light on the performance of volatility timing for international factors as well as for a broad set of country-level factors in 24 developed markets.

Our study finds that the benefits of volatility timing for international asset pricing factors are similar to previous US results. The alphas for the profitability, momentum and betting-against-beta factors are significant and in the range of $1.81\% - 11.61\%$ per annum. However, we are unable to find any statistically significant improvement for the international market factor. While our international-level findings follow previous US results, our country-level results show poor to mixed benefits of volatility timing across all factors, except momentum. The regression alphas for momentum are significant in 21/24 countries and are in the range of $6.50\% - 18.77\%$ per annum, suggesting that there are large and persistent benefits of volatility timing. Further analysis shows that the benefits of volatility timing are evident in an international multifactor setting as well as robust to transaction costs and leverage constraints. Further analysis also indicates that the benefit of volatility timing is partly explained by a successful reduction in drawdowns. Contrary to ideas put forward previously in the literature, our study indicates that volatility timing is primarily suitable for sophisticated investors.

This paper is organized as follows. Section 2 discusses theory and related literature. Section 3 describes our data and empirical approach in detail. Section 4 presents our results, together with a closer look at the empirical relationships required for a successful implementation of volatility timing. Section 5 tests the robustness of our results by considering the impact of transaction costs and leverage constraints. Section 6 discusses implications for investors and finally section 7 concludes our study.

# 2 Theory & Related Literature

## 2.1 Traditional Asset Pricing Factors

### 2.1.1 Fama-French Factors

Fama and French (1993) observe that the market factor in combination with the size (SMB) and value (HML) factors perform well in explaining the cross-section of average US stock returns. The Fama-French three factors have since been used as standard factors in controlling for systematic risk in the asset pricing literature. The market factor is the return on all CRSP firms incorporated in the US minus the 1-month Treasury bill rate collected from Ibbotson Associates. The SMB (Small Minus Big) factor captures the tendency for stocks with low market capitalization to outperform stocks with high market capitalization. The HML (High Minus Low) factor is a proxy for value and is constructed by buying a portfolio of stocks with high book-to-market ratios (value stocks) and selling a portfolio of stocks with low book-to-market ratios (growth stocks).

After growing empirical evidence of additional anomalies, Fama and French (2015) present a five-factor model. In addition to the FF3-model, the robust minus weak profitability (RMW) and the conservative versus aggressive investment (CMA) factors are added. The profitability factor is the difference in returns between portfolios of stocks with high versus low operating profitability. The investment factor is the difference in returns between portfolios of stocks with low versus high investment. The FF5-model is outlined below:

$$r_{it}^{e} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \beta_i^{RMW} RMW_t + \beta_i^{CMA} CMA_t + \varepsilon_t \quad (1)$$

where the left-hand side represents the excess return of each security where $r_{it}^{e}$ is defined as $r_{it} - r_{Ft}$.

### 2.1.2 Momentum

Jegadeesh and Titman (1993) find that strategies which buy stocks with strong recent performance and sell stocks with weak recent performance generates significant positive returns. Furthermore, the authors find that the momentum effect is not captured by exposure to systematic risk. Carhart (1997) evaluates mutual fund performance with a four factor model to account for the momentum effect, by adding the momentum factor to the FF3-factors. The factor is rebalanced on a monthly basis and constructed by investing in a long-short portfolio of winners (high past returns) minus losers (low past returns). In addition to the momentum factor for individual stocks, there exists a vast literature on additional specifications for momentum across asset classes, industries and countries (see Novy-Marx (2012)). The most cited theories for momentum are behavioural and focuses on investor over- and underreaction (see Ilmanen (2011) and Ang (2014)).

### 2.1.3 Betting Against Beta

In standard asset pricing models such as the CAPM, the equilibrium result is that investors invest in the portfolio with the maximum Sharpe ratio. However, the CAPM requires a number of strong assumptions, one of which is the absence of leverage constraints. To investigate this issue, Frazzini and Pedersen (2014) develop a model with leverage constraints and show that sophisticated investors (with access to leverage) lever-up undervalued low-beta assets and less sophisticated investors (with restricted access to leverage) invest in overvalued high-beta assets in equilibrium. Furthermore, there is empirical evidence suggesting that the Security Market Line (SML) is too flat in relation to the CAPM (see Jensen, Black, and Scholes (1972)). The betting-against-beta strategy is a zero-beta, self-financing strategy that is long the low-beta assets and short the high-beta assets.

## 2.2 Volatility-Managed Portfolios

A central notion in the asset pricing literature is the compensation for risk, investors who have higher exposure to risk factors should in the long-run be compensated through higher expected returns. Contrary to this notion, Moreira and Muir (2017) find that a strategy which takes *less* risk when risk is high and *more* risk when risk is low actually increases Sharpe ratios for several US factor portfolios. The strategy builds upon the empirical observation of a weak relationship between the previous period's volatility and the current period's return but a stronger persistence in period-to-period volatility. In a mean-variance framework, the observation implies that investors should reduce exposure when volatility is high (mean-variance trade-off weakens) and increase exposure when volatility is low (mean-variance trade-off strengthens). Moreira and Muir (2017) test volatility timing strategies in the US on the local FF5, momentum (MOM) and betting-against-beta (BAB) factors. In addition, the authors also investigate performance for a FX carry factor, a return on equity (ROE) factor and an additional investment (IA) factor. So far, the volatility timing literature has primarily been focused on factors in the US or market factors internationally.

### 2.3 Momentum Crashes

Historically, momentum strategies have delivered high risk premia but have suffered from large but infrequent drawdowns. The drawdowns of momentum strategies can make them an unappealing choice for risk-averse investors despite their high average returns. Barroso and Santa-Clara (2015) investigate the momentum-specific risks and propose a volatility timing strategy that drastically reduces drawdowns and improves overall performance. In terms of international evidence, Barroso and Santa-Clara (2015) test the volatility timing strategy for country-level momentum factors in the UK, France, Germany and Japan. Furthermore, Daniel and Moskowitz (2016) observe that the momentum crashes occur in the late stages of a bear market during strong market rebounds. Taking into account the partial predictability of momentum crashes, Daniel and Moskowitz (2016) construct a dynamic momentum strategy based on forecasts of momentum's conditional mean and variance. The results are similar to those found in Barroso and Santa-Clara (2015) with significant improvement in performance and sharply reduced drawdowns. For out-of-sample testing, Daniel and Moskowitz (2016) examine the performance of the dynamic momentum strategy across markets in the US, the UK, Continental Europe and Japan.

# 3 Data & Empirical Methodology

## 3.1 Data

### 3.1.1 Data Sources

We obtain daily and monthly international as well as country-level equity factor series from two data sources, the websites of Kenneth French and Andrea Frazzini (through *AQR Capital Management*). The international Fama-French five factors, being the market excess return (MKT), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA) of Fama and French (2017), along with the international momentum factor (MOM) are all obtained from K. French's website.[1] The international betting-against-beta (BAB) factor of Frazzini and Pedersen (2014) is instead obtained from A. Frazzini. Similarly, we obtain the daily and monthly country-level series for the Fama-French three factors, the momentum factor and the betting-against-beta factor in 24 developed markets from the latter source.

### 3.1.2 Data Descriptives

Correlation and summary statistics on a monthly basis for our international factors can be found in Table I. As presented in Panel A, the pairwise correlation between our international factors and the corresponding US factors remain high in the range of 0.72 – 0.90. The correlations are highest for the market and momentum factors both with correlations at 0.90. The overall high correlations are most likely driven by the scope and the size of the US investment universe relative to the other markets.[2]

In Panel B we present the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK), excess kurtosis (EKurt) and number of observations (N) for our international factors. All means for the factors are positive and close to zero, with the betting-against-beta factor being the highest with an average monthly return of 0.77%. In terms of minimum and maximum, the momentum factor display the largest extremes. The low minimum return of -24.26% on a monthly basis captures the large drawdowns of the momentum factor. For higher moments, we see exceptionally high excess kurtosis at 7.09 for the international momentum factor, suggesting that its

[1] *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*

[2] The international-US factor correlations are calculated using the US factors provided by K. French. Hence, they differ to a lesser degree compared with the international-US factor correlations reported in Appendix B, which are calculated using the US factors of A. Frazzini.

distribution has relatively fatter tails.

The cross-correlation for our international factors is presented in Panel C. The correlations are low or negative in general, however, there are exceptions. In the case of the international betting-against-beta factor we see positive correlation coefficients in the range of $0.36 - 0.41$ for the size, profitability and investment factors. Another notably high correlation exists between the investment and value factors at 0.71.

Descriptive statistics for the country-level factors are presented in Appendix B. In addition to the statistics presented for the country-level factors we also include the correlation to the corresponding international factor ($\rho_{i,INT}$). The market factors have positive means in all countries, are mostly negatively skewed and exhibit correlation to the international market factor in the range of $0.42 - 0.91$. However, the correlations are centered towards the higher end of the range.

For the size factors, most means are negative and the series appear less negatively skewed compared to the market factors. Correlations to the international size factor range from $0.08 - 0.71$. The value factors have positive mean returns in all countries except Denmark and New Zealand where the average monthly returns are slightly negative but still very close to zero. Correlations for the size factors range from -$0.03 - 0.85$, where Israel exhibits the lowest correlation and the US exhibits the highest correlation to the international factor.

The unconditional means of the momentum series are positive and high in relation to the other factors for all countries, with Japan being the country where momentum has the lowest mean consistent with previous studies (see Asness, Moskowitz, and Pedersen (2013)). The correlation to the international momentum factor ranges from $0.24 - 0.91$, where Greece has the lowest correlation and the US has the highest correlation to the international momentum factor.

The country-level unconditional means for the betting-against-beta series are positive and relatively high for all countries. The correlation between the country-level betting-against-beta series and the international counterpart ranges from $0.01 - 0.88$ where New Zealand has the lowest correlation to the international factor and the US has the highest.

### 3.1.3 Details on Factor Construction

The international factors are constructed in a similar way as the country-level factors but with a pooled universe of equities from 23 developed markets for the international Fama-French five factors and the momentum factor. The international betting-against-beta factor is obtained through A. Frazzini and includes Israel in addition to the 23 previously mentioned markets. The market (MKT) factor, or the market excess return, is constructed using the aggregate international or country-level value-weighted market return minus the US 1-month Treasury bill rate.

The size (SMB) factor is the average return on three small portfolios minus the average return on three big portfolios. The portfolios are value-weighted with book-to-market (B/M) and size breakpoints updated in June each calendar year. Lastly, the portfolios are rebalanced at the end of each month to maintain value weights. More formally, the SMB factor is constructed as follows:

$$\begin{aligned} SMB = \tfrac{1}{3}(SmallValue + SmallNeutral + SmallGrowth) \\ -\tfrac{1}{3}(BigValue + BigNeutral + BigGrowth) \end{aligned} \tag{2}$$

The value (HML) factor follows the construction in Fama and French (1993) where two value portfolios are traded against two growth portfolios. The HML portfolios are value-weighted and the breakpoints are updated in June of each calendar year with monthly rebalancing. The B/M ratios are calculated as the total market value of equity at the fiscal year-end. For firms with fiscal years not ending in December, the prices prevailing at the respective firm's year-end is used. More formally, the value factor is constructed as follows:

$$HML = \tfrac{1}{2}(SmallValue + BigValue) - \tfrac{1}{2}(SmallGrowth + BigGrowth) \tag{3}$$

The profitability (RMW) and investment (CMA) factors are constructed in the same way as the value factor by $2 \times 3$ sorts but the ranking is instead done on operating profitability or investment respectively.

The momentum (MOM) factor, also known as winners-minus-losers (WML), is the average return between two value-weighted portfolios with winners minus losers. The indicator for the momentum factor is determined by the security's prior 12-month return, skipping the most recent month. Size and book-to-market breakpoints are updated every calendar month. More formally, the momentum factor is constructed as follows:

$$MOM = \tfrac{1}{2}(SmallWinner + BigWinner) - \tfrac{1}{2}(SmallLoser + BigLoser) \tag{4}$$

Following the intuition of the leverage-constrained model, Frazzini and Pedersen (2014) create the betting-against-beta (BAB) factor which is a beta-neutral portfolio consisting of low-beta longs and high-beta shorts. The betas are estimated from rolling regressions of daily excess stock returns on daily market excess returns. The market returns are defined as the value-weighted returns for all stocks in each country. The estimation of beta is given by the following formula $\hat{\beta}_i^{TS} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m}$ where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ represents the individual stock's volatility and the volatility of the market. The volatilities are estimated on a 1-year rolling basis. The correlation, $\hat{p}$, between security $i$'s individual return and the local market index $m$, is estimated on a rolling 5-year basis. For returns, the betting-against-beta series is constructed with 1-day log-returns for volatilities and 3-day log-returns for correlations to account for non-synchronous trading. Furthermore, 120 trading days of non-missing data is required for the volatility estimation and 750 trading days with non-missing data for correlations. To reduce the influence of outliers, an adjustment of the betas is made towards the cross-sectional mean, $\hat{\beta}^{XS}$. The beta is constructed in the following way:

$$\hat{\beta}_i = w\hat{\beta_i^{TS}} + (1-w)\hat{\beta^{XS}} \tag{5}$$

where $w = 0.6$ and $\beta^{XS} = 1$, following the procedure in Frazzini and Pedersen (2014). The adjustments do not affect the relative ranks within the portfolios. However, the adjustments will have an impact on the betting-against-beta factor, since the estimated betas are used for sizing the long and short legs of the portfolios. The factor construction can be described more formally as:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^L - r_f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r_f) \tag{6}$$

where the factor returns are computed as $r_{t+1}^H = w_t^{H'} r_{t+1}$ and $r_{t+1}^L = w_t^{L'} r_{t+1}$. The weights are determined by the following formulas $w_t^H = k_t(z_t - \bar{z}_t)^+$ and $w_t^L = k_t(z_t - \bar{z}_t)^-$ where $x^+$, $x^-$ indicates the positive and negative elements of vector $x$. The inputs, $z$, are given by the rank for each security's estimated beta and so $z_{it} = rank(\hat{\beta_{it}})$. For the calculation of weights, a normalization factor is used in the following way, $k_t = \frac{2}{1'_N|z_t - \bar{z}_t|}$, in order to make each portfolio weight sum to one (i.e $1'_N w_H = 1$ and $1'_N w_L = 1$). Lastly, both the high-beta and the low-beta portfolios are rescaled to have a beta of one according to the following formulas: $\beta_t^H = w_t^{H'}\beta_t$ and $\beta_t^L = w_t^{L'}\beta_t$ .

**Table I**
**Correlation & Summary Statistics**

In this table, we present the pairwise correlation between international and US factor returns, summary statistics for international factor returns as well as the cross-correlation of the international factor returns. The factors covered in our study are the Fama-French five factors, being the market excess return (MKT), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA); the momentum factor (MOM) as well as the betting-against-beta factor (BAB). The sample period for the international factor returns ranges from 1990 to 2017 for MKT, SMB, HML, RMW and CMA; from 1990 to 2017 for MOM; and from 1987 to 2017 for BAB. In Panel A, we present the correlation between the international and US asset pricing factors. In Panel B, we present the summary statistics for the international asset pricing factors. We report the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK), excess kurtosis (EKurt) and number of observations (N) for the factor returns. In Panel C, we present the cross-correlation of international asset pricing factors.

| Panel A: International-US Correlation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | (1) $MKT_{INT}$ | (2) $SMB_{INT}$ | (3) $HML_{INT}$ | (4) $RMW_{INT}$ | (5) $CMA_{INT}$ | (6) $MOM_{INT}$ | (7) $BAB_{INT}$ |
| $MKT_{US}$ | 0.90 | -0.11 | -0.23 | -0.41 | -0.47 | -0.20 | -0.24 |
| $SMB_{US}$ | 0.19 | 0.72 | -0.09 | -0.42 | -0.17 | 0.12 | -0.06 |
| $HML_{US}$ | -0.12 | -0.02 | 0.85 | 0.16 | 0.63 | -0.27 | 0.31 |
| $RMW_{US}$ | -0.41 | -0.29 | 0.47 | 0.81 | 0.46 | -0.05 | 0.35 |
| $CMA_{US}$ | -0.29 | 0.10 | 0.62 | 0.08 | 0.80 | -0.04 | 0.30 |
| $MOM_{US}$ | -0.26 | 0.03 | -0.14 | 0.21 | 0.09 | 0.90 | 0.27 |
| $BAB_{US}$ | -0.24 | 0.14 | 0.44 | 0.41 | 0.39 | 0.21 | 0.88 |

| Panel B: Summary Statistics | | | | | | | |
|---|---|---|---|---|---|---|---|
| | (1) $MKT_{INT}$ | (2) $SMB_{INT}$ | (3) $HML_{INT}$ | (4) $RMW_{INT}$ | (5) $CMA_{INT}$ | (6) $MOM_{INT}$ | (7) $BAB_{INT}$ |
| Mean | 0.48 | 0.12 | 0.31 | 0.34 | 0.24 | 0.59 | 0.77 |
| Median | 0.94 | 0.07 | 0.22 | 0.34 | 0.05 | 0.74 | 0.98 |
| Max | 11.41 | 7.97 | 11.64 | 6.10 | 9.62 | 17.81 | 10.44 |
| Min | -19.52 | -8.42 | -9.54 | -5.41 | -6.48 | -24.26 | -12.48 |
| SD | 4.27 | 1.95 | 2.27 | 1.45 | 1.87 | 3.87 | 2.87 |
| SK | -0.73 | -0.35 | 0.55 | -0.05 | 0.69 | -0.99 | -0.58 |
| EKurt | 1.76 | 2.29 | 5.23 | 1.99 | 4.03 | 7.09 | 2.62 |
| $N$ | 330 | 330 | 330 | 330 | 330 | 326 | 371 |

| Panel C: International Cross-Factor Correlation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | (1) $MKT_{INT}$ | (2) $SMB_{INT}$ | (3) $HML_{INT}$ | (4) $RMW_{INT}$ | (5) $CMA_{INT}$ | (6) $MOM_{INT}$ | (7) $BAB_{INT}$ |
| $MKT_{INT}$ | 1.00 | | | | | | |
| $SMB_{INT}$ | -0.08 | 1.00 | | | | | |
| $HML_{INT}$ | -0.17 | 0.06 | 1.00 | | | | |
| $RMW_{INT}$ | -0.42 | -0.25 | 0.17 | 1.00 | | | |
| $CMA_{INT}$ | -0.39 | -0.03 | 0.71 | 0.18 | 1.00 | | |
| $MOM_{INT}$ | -0.23 | 0.12 | -0.25 | 0.16 | -0.05 | 1.00 | |
| $BAB_{INT}$ | -0.21 | 0.26 | 0.41 | 0.37 | 0.36 | 0.25 | 1.00 |

### 3.2 Portfolio Construction

To construct international volatility-managed equity factors, we follow the procedure in Moreira and Muir (2017) by forming new portfolios that rebalance monthly and are scaled by the inverse of a proxy for conditional variance. The volatility-managed portfolios can be described as:

$$f_{t+1}^{\sigma} = \frac{c}{\hat{\sigma_t}^2(f)} f_{t+1} \tag{7}$$

where $f_{t+1}$ is the factor's non-managed return and $\hat{\sigma_t}^2(f)$ represents a proxy for the non-managed factor's conditional variance. Similar to Moreira and Muir (2017), we set $c$ so that the managed and the non-managed series have the same unconditional variance. However, $c$ will not affect the Sharpe ratio of the strategy. The variance proxy $\hat{\sigma_t}^2(f)$ is calculated as the previous month's realized variance:

$$\hat{\sigma}_t^2(f) = RV_t^2(f) = \sum_{d=1/22}^{1} \left( f_{t+d} - \frac{\sum_{d=1/22}^{1} f_{t+d}}{22} \right)^2 \tag{8}$$

As shown in Equation 8, we calculate the monthly variance at the end of each month using 22 daily return observations. The monthly variance is subsequently used as a proxy for the next month's conditional variance.

### 3.3 Multifactor Framework

We extend our analysis to a multifactor approach by applying the multifactor framework of Moreira and Muir (2017). In this framework, portfolios are formed on factors using a mean-variance optimization and later scaled with the same approach as for the single factors. Hence, the volatility timing shifts the conditional beta of the strategy while the relative weights across factors remain the same.

We also control for risk parity (RP) when evaluating the benefits of volatility timing for multifactor portfolios. Risk parity is a strategy where investors allocate capital through diversification of *risk* rather than *dollar amounts*. We construct risk parity portfolios following Asness, Frazzini, and Pedersen (2012):

$$RP_{t+1} = b_t' f_{t+1} \tag{9}$$

with $b_{i,t} = \frac{1/\tilde{\sigma}_t^i}{\Sigma_i 1/\hat{\sigma}_t^i}$ and $f$ being a vector of factors. In this risk parity setting, each asset is weighted each month according to its relative contribution to the overall risk of the portfolio. The risk is measured by the assets realized standard deviation in the previous month.

### 3.4 Testing the Benefits of Volatility Timing

The main goal of this paper is to test whether international volatility-managed equity factors outperform their non-managed counterparts. To test the performance, we follow the approach presented in Moreira and Muir (2017) and thus consider the following regression:

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \varepsilon_{t+1} \tag{10}$$

where $f_{t+1}^{\sigma}$ is defined as in Equation 7. In Equation 10, the sign of the intercept, $\alpha$, will tell us if volatility-managed portfolios increase or decrease Sharpe ratios relative to its non-managed version. Positive (negative) alphas indicate higher (lower) Sharpe ratios. Theoretically, the relationship between the change in Sharpe ratios and the scaled alphas $\frac{\alpha}{\sigma_\epsilon}$, also known as the appraisal ratios, is defined as:

$$SR_{new} = \sqrt{SR_{old}^2 + \left(\frac{\alpha}{\sigma_\epsilon}\right)^2} \tag{11}$$

### 3.5 Volatility Modeling & Forecasting

While the majority of our study uses realized variance and volatility as inputs for the volatility timing, we also consider forward-looking estimates of the same variables in Section 5. The volatility-forecasting literature has developed a number of different models over time. Our study focus on three standard models which have been used previously in the literature.

Engle (1982) introduces one of the earliest conditional variance models, where the return on an asset is modeled as follows:

$$r_t = \mu + \sigma_t \varepsilon_t \tag{12}$$

where $\varepsilon_t$ is a sequence of N(0,1) i.i.d random variables and the residual return is defined as $a_t = \sigma_t \varepsilon_t$. A simple ARCH(1)-model for the variance of the residual return is then given by the following equation:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 \tag{13}$$

where the parameters have to satisfy the following constraints to ensure positive, finite variance and stationarity of the series: $\alpha_0 > 0$ and $0 \leq \alpha_1 < 1$. In order to account for the persistence in volatility, Bollerslev (1986) develops an extension to the ARCH(1)-model which also include the past period's forecasted variance. The GARCH(1,1)-model can be represented as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{14}$$

In the GARCH(1,1)-model, the unconditional variance is given by $\frac{\alpha_0}{1-\alpha_1-\beta_1}$.

Furthermore, empirical studies have shown asymmetric repsonses in conditional variance to negative and positive return innovations. To account for the asymmetry, Glosten, Jagannathan, and Runkle (1993) propose a threshold GARCH-model, known as the GJR-model, which introduces an interaction term between an indicator variable and the past squared innovation. The GJR(1,1,1)-model then becomes:

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_1 I_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{15}$$

where

$$I_{t-1} = \begin{cases} 1, & \text{if } a_{t-1} < 0 \\ 0, & \text{if } a_{t-1} \geq 0 \end{cases} \tag{16}$$

# 4 Empirical Results

## 4.1 Single-Factor Portfolios

### 4.1.1 International Single-Factor Portfolios

Our first step in the investigation of volatility timing for international factors is a regression analysis of the volatility-managed series on the factors themselves without volatility timing. Panel A in Table II reports the results from our tests, we see mixed results with positive and economically meaningful alphas for international profitability, momentum and betting-against-beta factors. Moreover, only the results for the momentum and betting-against-beta factors are significant at the 1%-level.

Starting with the market factor, we see an alpha of 2.64% per annum, however the results are not significant at any standard significance level. Turning to the additional international Fama-French five factors, the pattern is similar, with positive but insignificant alphas. The exception is the profitability factor, with an alpha of 1.70% per annum significant at the 5%-level. We find considerably stronger evidence for performance improvement in the international momentum and betting-against-beta factors with significant results at the 1%-level. The results indicate an annualized alpha for the volatility-managed international momentum factor of 12.21% per annum representing both an economically and statistically significant benefit over the non-managed strategy. A similar pattern is found in the betting-against-beta factor with an annualized alpha of 9.19%. All volatility-managed series appear to be different from the original series, having an $\mathrm{R}^2$ close to or lower than 50%, suggesting that collinearity is not an issue.

In Panel B and C we control for Fama-French three and five factors respectively. As can be seen in Panel B, controlling for Fama-French three factors, generally lowers the alphas for the international volatility-managed factors. The exception is profitability which increases from an alpha of 1.70% to 1.81% per annum. The aforementioned factor remains statistically significant at the 5%-level and the momentum as well as the betting-against-beta factors remain statistically significant at the 1%-level. Adding further layers of control with the Fama-French five factor factors in Panel C, we see slightly different results. While the results for the market, size, profitability and momentum factors remain rather unchanged, the results are different for the other factors. For the size and investment factors we see positive alphas of 2.98% and 2.00% respectively, significant at the 5%-level. In the case of the betting-against-beta factor we see a substantial reduction in the magnitude of the alpha, indicating that parts of the benefit from volatility timing is priced out by exposure to other asset pricing factors.

The difference in time sample do not allow us to conduct a direct comparison between the international results and the US results of Moreira and Muir (2017). Our results are nevertheless similar to those observed by the authors, with the profitability, momentum and betting-against-beta factors having large and statistically significant alphas in both samples. This is to a certain degree expected due to the high international-US factor correlations, as can be seen in Panel C in Table I. A notable difference is the results of the volatility-managed market portfolios. Moreira and Muir (2017) observe a large and significant annualized alpha of 4.86% for the US market portfolio in the full sample analysis and an annualized alpha of 4.22% in a more recent sub-sample ranging from 1986-2015. As our time sample to a certain degree matches the later sub-sample, the international results bring into question the performance improvement of volatility timing for less sophisticated investors. Controlling for standard asset pricing factors seems to reduce the market alphas both in terms of economic and statistical significance.

**Table II**
**International Volatility-Managed Factor Alphas**

In Panel A, we present the results from running a univariate time-series regression for each international volatility-managed factor on the non-managed counterpart according with $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. The managed factor is constructed by scaling the factor returns with the inverse of realized variance of the month before according with $f_t^\sigma = \frac{c}{RV_{t-1}^2} f_t$. The factors covered in our study are the Fama-French five factors, being the market excess return (MKT), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA); the momentum factor (MOM) as well as the betting-against-beta factor (BAB). In Panel B and C, we add the Fama-French three factors and five factors respectively as control variables in the regressions. Alphas are annualized by multiplying monthly alphas by 12 and presented as percentages. Volatility-managed returns used in the regressions are monthly with the international sample period of 1990 to 2017 for MKT, SMB, HML, RMW and CMA; 1991 to 2017 for MOM; and 1987 to 2017 for BAB. All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Panel A: Univariate Regressions | | | | | | | |
| | $\text{MKT}^\sigma_{\text{INT}}$ | $\text{SMB}^\sigma_{\text{INT}}$ | $\text{HML}^\sigma_{\text{INT}}$ | $\text{RMW}^\sigma_{\text{INT}}$ | $\text{CMA}^\sigma_{\text{INT}}$ | $\text{MOM}^\sigma_{\text{INT}}$ | $\text{BAB}^\sigma_{\text{INT}}$ |
| $\text{MKT}_{\text{INT}}$ | 0.67 | | | | | | |
| t | 7.64*** | | | | | | |
| $\text{SMB}_{\text{INT}}$ | | 0.74 | | | | | |
| t | | 7.69*** | | | | | |
| $\text{HML}_{\text{INT}}$ | | | 0.60 | | | | |
| t | | | 3.16*** | | | | |
| $\text{RMW}_{\text{INT}}$ | | | | 0.68 | | | |
| t | | | | 5.94*** | | | |
| $\text{CMA}_{\text{INT}}$ | | | | | 0.59 | | |
| t | | | | | 4.24*** | | |
| $\text{MOM}_{\text{INT}}$ | | | | | | 0.49 | |
| t | | | | | | 4.79*** | |
| $\text{BAB}_{\text{INT}}$ | | | | | | | 0.59 |
| t | | | | | | | 6.11*** |
| Alpha ($\alpha$) | 3.41 | 1.07 | 1.72 | 1.70 | 0.72 | 12.21 | 9.19 |
| t | 1.47 | 1.28 | 1.16 | 2.03** | 0.68 | 4.58*** | 5.60*** |
| $N$ | 328 | 328 | 328 | 328 | 328 | 323 | 369 |
| $R^2$ | 0.44 | 0.55 | 0.36 | 0.47 | 0.34 | 0.24 | 0.35 |
| RMSE | 38.07 | 15.72 | 21.77 | 12.69 | 18.15 | 40.50 | 27.71 |
| Panel B: Alphas Controlling for Fama-French Three Factors | | | | | | | |
| Alpha ($\alpha_{FF3}$) | 3.60 | 0.94 | 1.17 | 1.81 | 0.29 | 11.61 | 9.42 |
| t | 1.52 | 1.11 | 0.85 | 2.23** | 0.30 | 4.30*** | 5.77*** |
| $N$ | 328 | 328 | 328 | 328 | 328 | 323 | 330 |
| Panel C: Alphas Controlling for Fama-French Five Factors | | | | | | | |
| Alpha ($\alpha_{FF5}$) | 2.39 | 0.54 | 2.98 | 1.98 | 2.00 | 10.22 | 1.66 |
| t | 0.98 | 0.62 | 2.30** | 2.40** | 2.01** | 4.03*** | 6.00*** |
| $N$ | 328 | 328 | 328 | 328 | 328 | 323 | 330 |

### 4.1.2 Country-Level Single-Factor Portfolios

We expand our study of the volatility-managed factors to a full-sample analysis in 24 developed markets for the market, size, value, momentum and betting-against-beta factors.[3] The results in terms of regression alphas and t-statistics across countries and factors can be seen in Table III. While our US results are similar to those found by Moreira and Muir (2017), the country-level results paint a different picture of the benefits of volatility timing. The results show considerable support for the benefit of volatility timing in the momentum strategy across countries, but poor to mixed effects for the market, size, value and betting-against-beta factors. The results are persistent after controlling for the Fama-French three factors.

In the case of the market factor we see mixed benefits of volatility timing. The alphas are positive and significant in 12/24 countries, with 4/24 being significant at the 1%-level. The benefits in terms of annualized alphas are visible in the cases of Hong Kong (12.56%), Denmark (7.67%), the Netherlands (6.81%) and the US (4.52%). For the two other Fama-French three factors, the size and value factors, the benefits of volatility timing are rather poor. For the size factor, alphas are negative in 12/24 cases and almost none of the alphas are statistically significant at any standard threshold. A notable exception is Switzerland with a positive annualized alpha of 2.03%, which is significant at the 1%-level. The results for the value factors are similarly poor across countries. While there are few negative alphas for the value factors, only 5/24 are positive and significant, with 2/24 being significant at the 1%-level. The two countries are Austria (4.32%) and Japan (4.27%). Hence, the results across the Fama-French three factors are similar to those of their international counterparts.

Contrary to the results of the Fama-French factors, volatility timing works well for the momentum factor across countries. Our regression alphas are positive and significant in 21/24 countries, with 20/24 of the alphas being significant at the 1%-level. The results are in line with our findings in the international sample. The three countries where we find weak support for volatility timing are Greece, Ireland and Japan. The negative results in Japan carries meaning, since the momentum strategy is known to underperform in the country as shown by Asness, Moskowitz, and Pedersen (2013).

[3] We use three letter abbreviations for our 24 countries as well as our international factors. The countries included are Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Switzerland (CHE), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), Great Britain (GBR), Greece (GRC), Hong Kong (HKG), Ireland (IRL), Israel (ISR), Italy (ITA), Japan (JPN), Netherlands (NLD), Norway (NOR), New Zealand (NZL), Portugal (PRT), Singapore (SGP), Sweden (SWE), USA and our international factors (INT).

For our final factor, betting-against-beta, we see similar results as for the market factor, with mixed results across countries. In 13/24 countries the alphas are positive and statistically significant, with 6/24 countries having results significant at the 1%-level. The alpha is exceptionally high in Hong Kong (18.78%), which constitutes the highest alpha across countries and factors. Together with the market excess return, the betting-against-beta factor constitutes one of the factors Moreira and Muir (2017) find to have both positive and significant alphas in the US data. Similar results can also be found in our analysis, however, our country-level evidence seems to suggest that the benefits of volatility timing is far from persistent across borders.

In Table IV, we report our annualized alphas controlling for the Fama-French three factors. The results remain largely robust for the control variables, with the benefits for the betting-against-beta factors weakening the most. More specifically, the number of positive and significant country alphas drop from 13/24 countries (with 6/24 significant results at the 1%-level) to 11/24 countries (with 5/24 significant results at the 1%-level).

The lack of statistical significance for many country-level factors might be driven by our relatively short sample compared to Moreira and Muir (2017), however, there are still results worth highlighting. The 12/24 negative signs of alphas for the size factor indicate that volatility timing provides little to no benefit for an investor with a high exposure to the size factor. For such investors volatility timing have even proved to be harmful historically in some markets. On the contrary, strong benefits of volatility timing are evident across markets for the momentum factor. These country-level momentum results are similar to previous findings in the momentum crashes literature (see Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016)). Looking across country-level factors, our results bring the factor-wide benefits of volatility timing into question and might even suggest that the benefits are uniquely related to momentum.

**Table III**
**Country-Level Volatility-Managed Factor Alphas**

In this table, we present the results from running a univariate time-series regression for each volatility-managed factor on the non-managed factor in 24 developed markets as well as on an international level. This is done according with $f_t^{\sigma} = \alpha + \beta f_t + \epsilon_t$. The managed factor is constructed by scaling the factor returns with the inverse of realized variance of the month before according with $f_t^{\sigma} = \frac{c}{RV_{t-1}^2} f_t$. The factors covered in our study are the Fama-French three factors, being the market excess return (MKT), the size factor (SMB), the value factor (HML); the momentum factor (MOM) as well as the betting-against-beta factor (BAB). Monthly alphas are annualized by multiplying monthly alphas by 12 and presented as percentages. Returns used in the regressions are monthly with varying start dates in different countries and factors, however, all samples end in year-end 2017. All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level.

| | $MKT_i$ | | $SMB_i$ | | $HML_i$ | | $MOM_i$ | | $BAB_i$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | $\alpha$ | t | $\alpha$ | t | $\alpha$ | t | $\alpha$ | t | $\alpha$ | t |
| AUS | 1.51 | 0.44 | -1.96 | -1.37 | -0.93 | -0.63 | 9.48 | 4.10*** | 7.36 | 3.71*** |
| AUT | 2.98 | 0.83 | -0.83 | -0.55 | 4.32 | 2.99*** | 8.79 | 2.52** | -3.74 | -1.04 |
| BEL | 6.33 | 2.26** | 1.73 | 1.51 | -1.26 | -0.96 | 10.25 | 3.36*** | 4.18 | 2.16** |
| CAN | -3.17 | -1.00 | -0.63 | -0.64 | 3.68 | 2.17** | 10.94 | 3.59*** | 4.23 | 1.44 |
| CHE | 3.06 | 1.52 | 3.03 | 2.73*** | 0.52 | 0.35 | 8.25 | 3.45*** | 3.09 | 1.82* |
| DEU | 4.05 | 1.59 | -0.67 | -0.53 | 3.23 | 1.90* | 8.80 | 3.18*** | 2.69 | 1.22 |
| DNK | 7.67 | 3.02*** | 1.14 | 0.87 | 1.40 | 0.85 | 6.81 | 3.02*** | 1.40 | 0.79 |
| ESP | 4.26 | 1.51 | -1.59 | -0.91 | 0.33 | 0.19 | 8.07 | 3.19*** | 4.58 | 2.40** |
| FIN | 9.82 | 2.50** | 0.43 | 0.24 | 0.07 | 0.02 | 8.88 | 3.17*** | 0.43 | 0.14 |
| FRA | 3.28 | 1.24 | -0.36 | -0.32 | 2.39 | 1.32 | 13.23 | 5.76*** | 6.60 | 3.13*** |
| GBR | 4.31 | 1.85* | 1.50 | 1.07 | 0.78 | 0.46 | 16.94 | 5.51*** | 7.81 | 3.22*** |
| GRC | 6.89 | 1.33 | 4.29 | 0.98 | -4.62 | -1.02 | 2.73 | 0.96 | 1.75 | 0.35 |
| HKG | 12.56 | 3.73*** | -2.26 | -0.94 | 1.39 | 0.69 | 15.63 | 5.02*** | 18.78 | 4.25*** |
| IRL | 10.35 | 2.53** | -0.26 | -0.12 | 2.06 | 0.69 | 3.43 | 0.67 | -2.35 | -0.56 |
| ISR | -1.29 | -0.34 | 1.81 | 0.77 | 4.21 | 1.47 | 8.81 | 2.84*** | 5.45 | 2.19** |
| ITA | 4.21 | 1.37 | -0.02 | -0.01 | 3.74 | 1.99** | 7.46 | 3.29*** | 0.65 | 0.37 |
| JPN | -0.58 | -0.23 | 0.46 | 0.44 | 4.27 | 3.16*** | 3.33 | 1.23 | 2.87 | 1.19 |
| NLD | 6.81 | 2.67*** | 1.18 | 0.92 | -2.53 | -1.47 | 8.88 | 3.46*** | 6.98 | 2.51** |
| NOR | 6.15 | 1.72* | 0.31 | 0.21 | -0.93 | -0.45 | 9.00 | 3.28*** | 5.37 | 1.97** |
| NZL | 6.35 | 2.37** | -2.14 | -1.23 | 0.53 | 0.21 | 7.14 | 3.85*** | 0.54 | 0.23 |
| PRT | 7.15 | 2.34** | 0.04 | 0.02 | 1.24 | 0.58 | 5.47 | 2.41*** | -4.62 | -1.22 |
| SGP | 3.70 | 1.10 | -1.44 | -0.67 | 2.80 | 1.47 | 13.65 | 4.08*** | 5.83 | 2.78*** |
| SWE | 6.92 | 2.27** | 0.71 | 0.55 | 0.51 | 0.17 | 15.22 | 5.75*** | 4.54 | 1.73* |
| USA | 4.52 | 2.75*** | -1.35 | -1.47 | 1.47 | 1.39 | 12.05 | 6.21*** | 6.43 | 4.86*** |
| INT | 3.41 | 1.47 | 1.07 | 1.28 | 1.72 | 1.16 | 12.21 | 4.58*** | 9.19 | 5.60*** |

**Table IV**
**Country-Level Volatility-Managed FF3 Alphas**

In this table, we present the results from running a time-series regression for each volatility-managed factor on the non-managed factor and the Fama-French three factors as control. This is done in 24 developed markets as well as on an international level in accordance with $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. The managed factor is constructed by scaling the factor returns with the inverse of realized variance of the month before according with $f_t^\sigma = \frac{c}{RV_{t-1}^2} f_t$. The factors covered in our study are the Fama-French three factors, being the market excess return (MKT), the size factor (SMB), the value factor (HML); the momentum factor (MOM) as well as the betting-against-beta factor (BAB). Alphas are annualized by multiplying monthly alphas by 12 and presented as percentages. Returns used in the regressions are monthly with varying start dates in different countries and factors, however, all samples end in year-end 2017. All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level.

| | $MKT_i$ | | $SMB_i$ | | $HML_i$ | | $MOM_i$ | | $BAB_i$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | $\alpha_{FF3}$ | t | $\alpha_{FF3}$ | t | $\alpha_{FF3}$ | t | $\alpha_{FF3}$ | t | $\alpha_{FF3}$ | t |
| AUS | 1.43 | 0.37 | -1.99 | -1.30 | -0.96 | -0.64 | 7.74 | 2.96*** | 6.52 | 2.85*** |
| AUT | 1.87 | 0.51 | -0.80 | -0.57 | 4.51 | 2.98*** | 9.26 | 2.53** | -1.55 | -0.45 |
| BEL | 6.02 | 1.95* | 1.52 | 1.23 | -1.34 | -0.95 | 8.30 | 2.65*** | 3.54 | 1.83* |
| CAN | 0.00 | 0.14 | -0.42 | -0.40 | 3.46 | 1.99** | 11.51 | 3.70*** | 3.95 | 1.42 |
| CHE | 3.84 | 1.79* | 2.89 | 2.59*** | 0.30 | 0.21 | 7.62 | 3.08*** | 2.11 | 1.16 |
| DEU | 4.86 | 1.64 | -0.89 | -0.69 | 2.64 | 1.55 | 10.23 | 3.82*** | 1.85 | 0.84 |
| DNK | 6.18 | 2.26** | 1.51 | 1.08 | 0.84 | 0.52 | 7.02 | 3.20*** | 1.84 | 1.02 |
| ESP | 3.94 | 1.23 | -2.06 | -1.17 | -0.01 | -0.01 | 6.51 | 2.28** | 2.93 | 1.65* |
| FIN | 6.55 | 2.04** | 0.46 | 0.24 | -1.73 | -0.52 | 8.82 | 2.94*** | -0.08 | -0.02 |
| FRA | 3.04 | 1.05 | -0.56 | -0.47 | 1.75 | 1.03 | 13.44 | 5.78*** | 5.01 | 2.81*** |
| GBR | 5.40 | 2.25** | 1.63 | 1.18 | 0.60 | 0.35 | 18.38 | 5.73*** | 7.99 | 3.30*** |
| GRC | 6.05 | 0.92 | 4.97 | 1.00 | -4.27 | -1.03 | 5.50 | 1.01 | 6.13 | 1.13 |
| HKG | 12.6 | 3.62*** | -2.16 | -0.86 | 1.16 | 0.59 | 16.86 | 4.98*** | 19.33 | 4.23*** |
| IRL | 13.96 | 3.12*** | -0.06 | -0.03 | 1.95 | 0.65 | -0.36 | -0.07 | -1.76 | -0.40 |
| ISR | -4.69 | -1.09 | 2.06 | 0.96 | 4.04 | 1.41 | 8.65 | 2.56** | 5.27 | 2.13** |
| ITA | 4.52 | 1.29 | 0.05 | 0.03 | 3.79 | 2.06** | 8.17 | 3.45*** | 0.20 | 0.12 |
| JPN | -2.87 | -1.21 | 0.17 | 0.16 | 4.16 | 3.20*** | 2.63 | 1.04 | 2.29 | 1.03 |
| NLD | 7.51 | 2.65*** | 1.15 | 0.86 | -2.25 | -1.28 | 9.08 | 3.36*** | 5.88 | 2.15** |
| NOR | 5.11 | 1.34 | 0.23 | 0.16 | -0.97 | -0.45 | 9.26 | 3.17*** | 5.76 | 2.05** |
| NZL | 6.70 | 2.30** | -2.30 | -1.29 | 1.06 | 0.43 | 4.76 | 2.84*** | -1.91 | -0.78 |
| PRT | 6.28 | 2.14** | -0.65 | -0.32 | 1.25 | 0.59 | 6.50 | 2.65*** | -3.41 | -0.84 |
| SGP | 3.03 | 0.81 | -0.12 | -0.06 | 2.69 | 1.42 | 14.47 | 4.28*** | 5.60 | 2.56** |
| SWE | 5.35 | 1.75* | 0.23 | 0.18 | -0.60 | -0.21 | 16.02 | 5.75*** | 4.40 | 1.60 |
| USA | 4.87 | 2.98*** | -1.40 | -1.54 | 2.05 | 1.93* | 10.02 | 6.02*** | 6.00 | 4.83*** |
| INT | 3.60 | 1.52 | 0.94 | 1.11 | 1.17 | 0.85 | 11.61 | 4.30*** | 9.42 | 5.77*** |

## 4.2 International Multifactor Portfolios

In addition to our single factor analysis, we also conduct a multifactor analysis. Following Moreira and Muir (2017), we utilize a mean-variance framework to construct mean-variance efficient (MVE) portfolios. Volatility timing is then implemented on the MVE portfolios to investigate if it expands the efficient frontier for investors. An expansion would be captured by positive and significant regression alphas. Our results show that such an expansion can be seen for most international factor combinations, however, there is one exception in the Fama-French three factor combination. The results on a country level are similar to our international-level findings, with strong benefits for all combinations except the Fama-French three factor combination.

As can be seen in Panel A of Table V, our international data show strong support for the Fama-French five factor combination, the Fama-French factor and momentum combination as well as an all-factor combination. The all-factor approach includes all Fama-French five factors, the momentum factor as well as the betting-against-beta factor. All alphas for these combinations are positive and significant at the 1%-level, moreover, the appraisal ratios are in the range of 0.82 – 0.99. The results for the Fama-French three factor MVE portfolio, however, are low both in terms of economic and statistical significance with an annualized alpha of 1.37% and an appraisal ratio of 0.37.

Similar to Moreira and Muir (2017), we add a risk parity factor for control purposes which is constructed in accordance with Asness, Frazzini, and Pedersen (2012). The results, presented in Panel B, are largely consistent with our earlier findings. This suggests that our findings on the volatility-managed MVE portfolios are different from the cross-sectional low risk anomaly. Extending our test to a country level, as presented in Appendix C, we see similar results. For the Fama-French three factor MVE portfolios, 12/24 countries have significant results with 3/24 significant at the 1%-level. These results are persistent after controlling for risk parity, which brings into question the general benefits of volatility timing for mean-variance portfolios.

**Table V**
**International Mean-Variance Efficient Factors**

In Panel A, we present the results from the univariate time series regression on the volatility-managed mean-variance efficient (MVE) portfolios on their non-managed counterparts. The MVE portfolios are formed on an international level with the full sample of different factor combinations. These MVE portfolios are then evaluated in accordance with $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. In Panel B, we add a risk parity factor constructed using the same underlying factors as for the MVE portfolios for control. The risk party factor is given by $RP_{t+1} = b_t' f_{t+1}$, with $b_{i,t} = \frac{1/\hat{\sigma}_t^i}{\Sigma_i 1/\hat{\sigma}_t^i}$ and $f$ being a vector of factors. The definition follows the approach suggested by Asness, Frazzini, and Pedersen (2012). We add the volatility-managed market factor (MKT) in both panels for comparability. Alphas are annualized by multiplying monthly alphas by 12 and presented as percentages. Returns used in the regressions are monthly with varying start dates, however, all samples end in year-end 2017. All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level.

Panel A: Mean-Variance Efficient Portfolios

| | (1) $\text{MKT}_{\text{INT}}$ | (2) $\text{FF3}_{\text{INT}}$ | (3) $\text{FF5}_{\text{INT}}$ | (4) $\text{FF3\&MOM}_{\text{INT}}$ | (5) $\text{FF5\&MOM}_{\text{INT}}$ | (6) $\text{ALL}_{\text{INT}}$ |
|---|---|---|---|---|---|---|
| Alpha ($\alpha$) | 3.41 | 1.37 | 1.87 | 2.91 | 2.21 | 2.75 |
| t | 1.47 | 1.60 | 3.54*** | 3.30*** | 3.70*** | 3.90*** |
| $N$ | 328 | 328 | 328 | 323 | 323 | 323 |
| $R^2$ | 0.44 | 0.47 | 0.34 | 0.41 | 0.29 | 0.39 |
| RMSE | 38.07 | 12.71 | 7.61 | 12.24 | 8.13 | 9.56 |
| Original Sharpe | 0.42 | 0.72 | 1.41 | 1.08 | 1.51 | 1.59 |
| Vol-Managed Sharpe | 0.51 | 0.77 | 1.51 | 1.33 | 1.61 | 1.77 |
| Appraisal Ratio | 0.31 | 0.37 | 0.85 | 0.82 | 0.94 | 0.99 |

Panel B: Controlling for Risk Parity

| | (1) $\text{MKT}_{\text{INT}}$ | (2) $\text{FF3}_{\text{INT}}$ | (3) $\text{FF5}_{\text{INT}}$ | (4) $\text{FF3\&MOM}_{\text{INT}}$ | (5) $\text{FF5\&MOM}_{\text{INT}}$ | (6) $\text{ALL}_{\text{INT}}$ |
|---|---|---|---|---|---|---|
| Alpha ($\alpha_{RP}$) | 3.41 | 1.79 | 1.97 | 2.60 | 2.40 | 3.01 |
| t | 1.47 | 2.16** | 3.43*** | 2.82*** | 3.80*** | 3.86*** |
| $N$ | 328 | 293 | 293 | 293 | 289 | 289 |
| $R^2$ | 0.44 | 0.46 | 0.31 | 0.40 | 0.27 | 0.39 |
| RMSE | 38.07 | 12.25 | 7.57 | 11.83 | 7.83 | 9.37 |

### 4.3 International Business Cycles

One suggested benefit of volatility timing is reduced risk-taking in bad times. In our business cycle analysis for the international factors, reduced risk-taking is visible to some extent, but more so in our analysis of rolling drawdowns. The benefit of the volatility timing is particularly evident for the momentum factor which follows the results found in previous studies on momentum crashes.

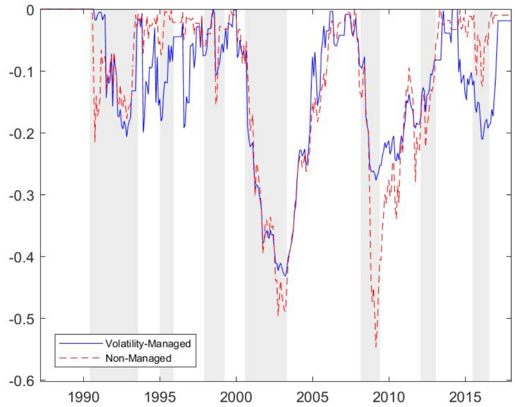
The efficacy of volatility timing to reduce risk in bad times can to a certain degree be seen in our business cycle analysis. The study is conducted in a similar fashion to Moreira and Muir (2017), by running a regression of our volatility-managed factors on their non-managed counterparts and an interaction term. The interaction term is constructed by interacting our non-managed factor with an OECD recession dummy. Hence, the coefficient of the interaction term should indicate whether there is a difference in the volatility-managed strategy between the expansion and recession periods. As can be seen in Table VI, there is some evidence for this among our factors as all signs are negative. However, only 4/7 factors are significant, with only the betting-against-beta factor being significant at the 1%-level.

Stronger support for the case of volatility timing can be found in Figure 2 where we plot the rolling drawdowns of each international volatility-managed strategy and its non-managed counterpart. As can be seen for the size, profitability, investment and momentum factors, volatility timing is able to avoid the large drawdowns around the time of the dot-com bubble. For the investment, momentum and betting-against-beta factors the strategy also avoids the crashes following the financial crisis of 2007. The benefit of volatility timing by avoiding crashes is consistent over time for momentum in our sample, which is in line with earlier findings of Daniel and Moskowitz (2016).

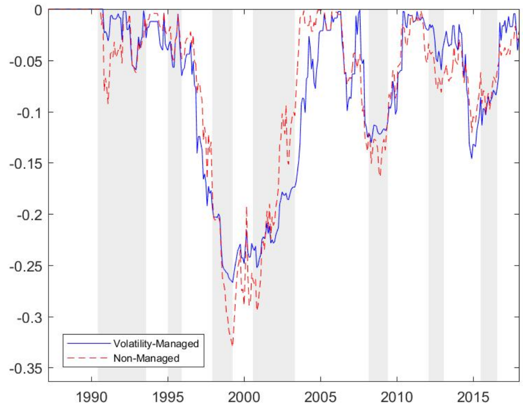
**Table VI**
**International Recession Betas by Factor**

In this table, we regress each volatility-managed factor or the non-managed counterpart and OECD recession dummies $1_{rec,t}$, the recession dummies are also interacted with the original factor, yielding the following regression $f_t^{\sigma} = \alpha_0 + \alpha_1 1_{rec,t} + \beta_0 f_t + \beta_1 1_{rec,t} \times f_t + \epsilon_t$. The betas for the interaction terms provides the relative beta of the volatility-managed factor conditional on a recession compared to the unconditional case. Hence, a negative beta suggests that the betas for a factor is relatively lower in recessions. The factors covered in our study are the Fama-French five factors, being the market excess return (MKT), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA); the momentum factor (MOM) as well as the betting-against-beta factor (BAB). All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level. All OECD recession indicators are obtained from the St. Louis Federal Reserve's website.

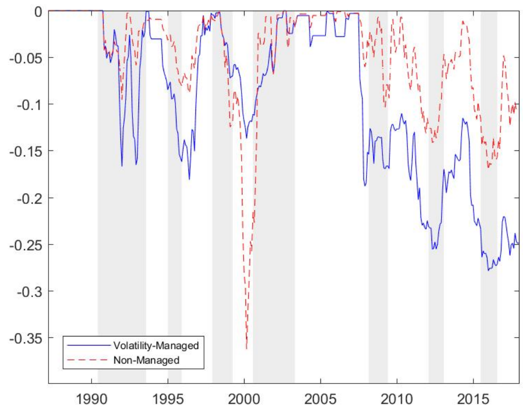
| | $\text{MKT}^{\sigma}_{\text{INT}}$ | $\text{SMB}^{\sigma}_{\text{INT}}$ | $\text{HML}^{\sigma}_{\text{INT}}$ | $\text{RMW}^{\sigma}_{\text{INT}}$ | $\text{CMA}^{\sigma}_{\text{INT}}$ | $\text{MOM}^{\sigma}_{\text{INT}}$ | $\text{BAB}^{\sigma}_{\text{INT}}$ |
|---|---|---|---|---|---|---|---|
| $\text{MKT}_{\text{INT}}$ | 0.93 | | | | | | |
| t | 6.74*** | | | | | | |
| $\text{MKT}_{\text{INT}} \times 1_{rec}$ | -0.41 | | | | | | |
| t | -2.51** | | | | | | |
| $\text{SMB}_{\text{INT}}$ | | 0.92 | | | | | |
| t | | 4.60*** | | | | | |
| $\text{SMB}_{\text{INT}} \times 1_{rec}$ | | -0.40 | | | | | |
| t | | -1.94* | | | | | |
| $\text{HML}_{\text{INT}}$ | | | 0.78 | | | | |
| t | | | 3.14*** | | | | |
| $\text{HML}_{\text{INT}} \times 1_{rec}$ | | | -0.32 | | | | |
| t | | | -1.08 | | | | |
| $\text{RMW}_{\text{INT}}$ | | | | 0.86 | | | |
| t | | | | 5.43*** | | | |
| $\text{RMW}_{\text{INT}} \times 1_{rec}$ | | | | -0.30 | | | |
| t | | | | -1.58 | | | |
| $\text{CMA}_{\text{INT}}$ | | | | | 0.99 | | |
| t | | | | | 3.59*** | | |
| $\text{CMA}_{\text{INT}} \times 1_{rec}$ | | | | | -0.59 | | |
| t | | | | | -2.00** | | |
| $\text{MOM}_{\text{INT}}$ | | | | | | 0.67 | |
| t | | | | | | 2.90*** | |
| $\text{MOM}_{\text{INT}} \times 1_{rec}$ | | | | | | -0.31 | |
| t | | | | | | -1.24 | |
| $\text{BAB}_{\text{INT}}$ | | | | | | | 1.00 |
| t | | | | | | | 7.26*** |
| $\text{BAB}_{\text{INT}} \times 1_{rec}$ | | | | | | | -0.65 |
| t | | | | | | | -4.19*** |
| $N$ | 328 | 328 | 328 | 328 | 328 | 323 | 369 |
| $R^2$ | 0.48 | 0.58 | 0.38 | 0.49 | 0.42 | 0.26 | 0.45 |




**(a)** MKT

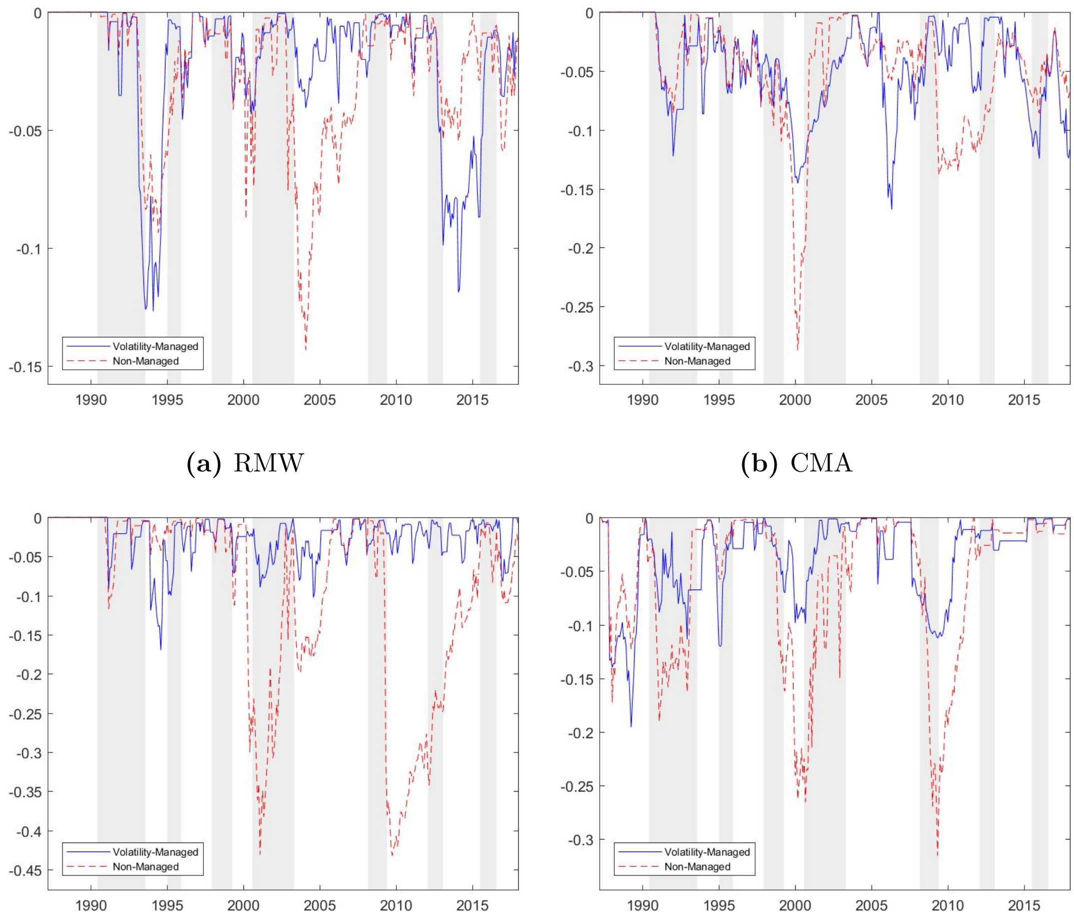



**(b)** SMB




**(c)** HML

**Figure 1: Drawdown Plots for International Market, Size and Value Factors.** In this figure, we present the rolling drawdown (peak-to-trough) for the international market (MKT), size (SMB) and value (HML) factors. The rolling drawdowns are computed as the current price over the previous rolling high and is an indication of the maximum loss an investor would have experienced in a given period. The shaded regions indicate OECD recessions (downloaded from the St. Louis Federal Reserve's website).




**(a)** RMW **(b)** CMA

**(c)** MOM **(d)** BAB

**Figure 2: Drawdown Plots for International Profitability, Investment, Momentum and Betting-Against-Beta Factors.** In this figure, we present the rolling drawdown (peak-to-trough) for the international profitability (RMW), investment (CMA), momentum (MOM) and betting-against-beta (BAB) factors. The rolling drawdowns are computed as the current price over the previous rolling high and is an indication of the maximum loss an investor would have experienced in a given period. The shaded regions indicate OECD recessions (downloaded from the St. Louis Federal Reserve's website).

## 4.4 International Cross-Factor Comparison

In this section, we examine the time-series relationship between risk and return for the international factors. A closer examination of these relationships reveals why volatility timing is successful for some factors but not for others. Volatility timing works well for series which show a clear downward slope, from left to right, for the risk-return plots for each factor.[4] By scaling the exposure dynamically by the inverse of variance, a volatility-managed strategy will have high (low) risk-exposure in periods with high (low) Sharpe ratios. Hence, the benefit is not only related to the potential reduction of drawdowns, but also due to the strategy's increasing exposure to the factor in good states of the world.

In order to highlight these empirical relationships, we construct bucket plots similar to Moreira and Muir (2017). Firstly, we group and compute the average monthly return for each international factor given the previous month's realized variance. Secondly, we compute the average volatility in each bucket sorted on the previous month's variance. Thirdly, we show the average risk-reward trade-off for each bucket sorted on previous month's variance. (The trade-off shows the relative attractiveness of investing in each period for a risk-averse investor.) Lastly, we compute the probability of an OECD recession as the mean of a recession dummy conditional on the level of previous month's variance. For each variable, the data is organized into a quintile sort on the previous month's variance (see Equation 8). The low (high) volatility bucket represents 20% of the months with the lowest (highest) previous month's variance.

The bucket plots in Figure 3 to 6 show evidence consistent with the results obtained from the volatility-managed series presented in Table II. The international profitability, momentum and betting-against-beta factors, with positive and statistically significant alphas, show downward sloping risk-return plots. On the other hand, the international market, size, value and investment factors without statistically significant alphas show less of a downward sloping shape.

The bucket plot for the momentum factor further highlights the momentum crashes and shows why volatility timing work so well. Average returns are positive and high for

[4]Volatility timing strategies work well for series which exhibit a negative covariance between the risk-return trade-off, $\frac{\mu_t}{\sigma_t^2}$, and variance. Moreira and Muir (2017) show that the alpha obtained from volatility timing can be expressed by the following relationship $\alpha = -cov(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2)\frac{c}{E[\sigma_t^2]}$. Note that the time-subscript of the variance variables differs from the one shown in the bucket plots. However, the plots to the top right show that the persistence in volatility is quite high from period to period indicating that last period's variance could be used as a proxy for the current period's variance.

all buckets except for the high volatility state where average returns become negative. By reducing risk in these high-volatility states, the performance of momentum improves meaningfully (see Table II and Table III). As shown in Figure 2, the performance improvement is related to the reduction of the severe but infrequent drawdowns that the momentum factor suffers from.

It is important to stress that a drawdown reduction in itself is insufficient to improve the long-term performance of the factor. For instance, the volatility timing for the investment factor seems to be able to reduce some large drawdowns (see Figure 2 and Figure 8). However, the bucket plot does not show a clear, downward sloping pattern which is in line with the lack of performance improvement.

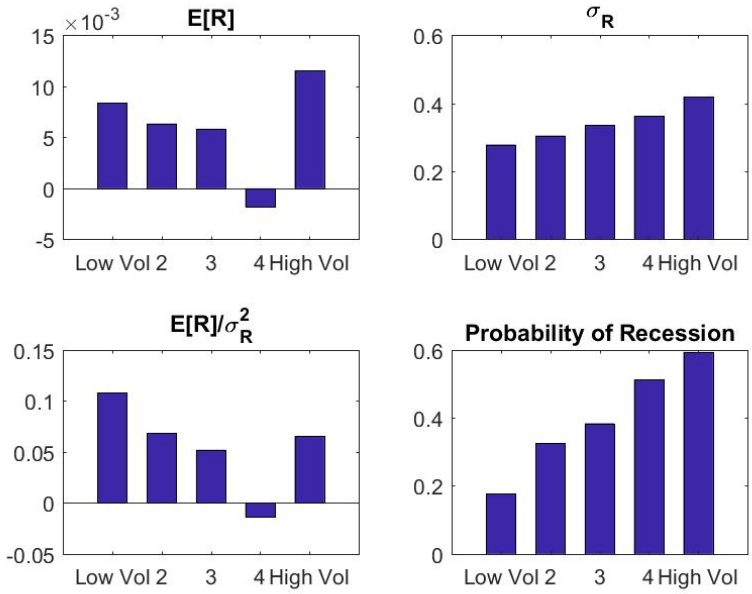
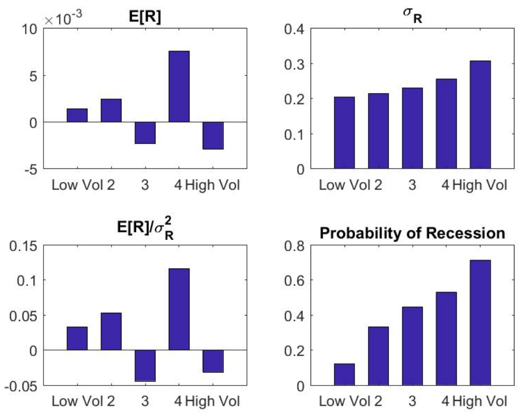
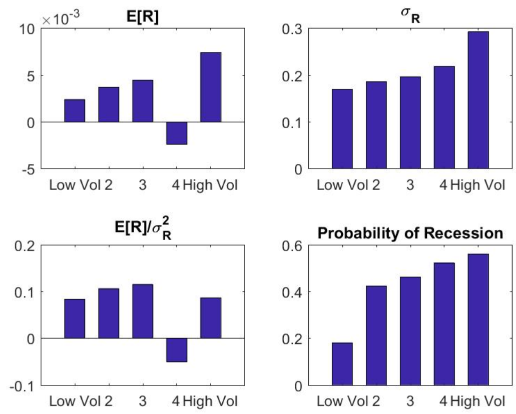



**Figure 3: Bucket Plot for International Market Factor.** The bucket plots show the sorts on previous month's realized volatility for the market factor's returns, realized volatility, the risk-return trade-off and the probability of recession. The Low Vol (Highest Vol) bucket shows the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We first show the average return following different buckets of realized volatility. Then we show the realized volatility in the following month given where realized volatility was in the previous month. Thirdly, we show the average return per unit of variance as a representation of the risk-reward trade-off the investor faces conditional on previous month's realized volatility. Lastly, we show the probability of entering into an OECD recession given previous month's realized volatility. The probability of recession is simply calculated as the mean of a binary (0,1) recession dummy for each bucket.

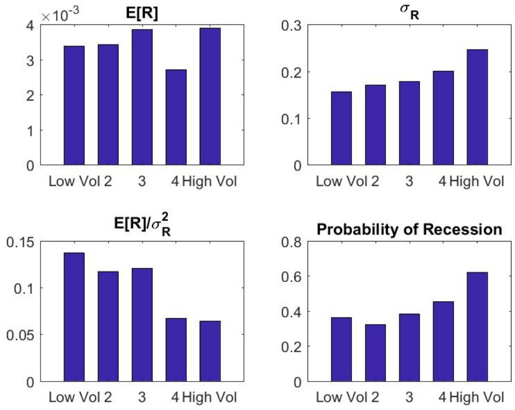



**(a)** SMB

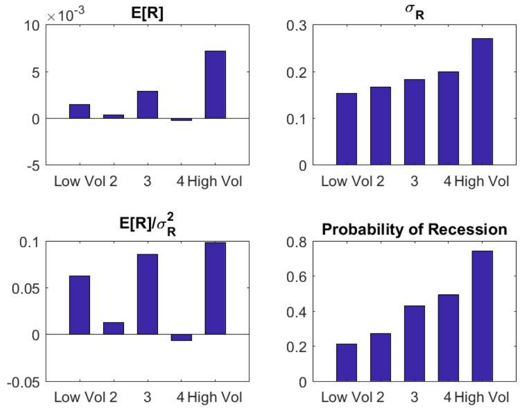



**(b)** HML

**Figure 4: Bucket Plots for International Size and Value Factors** The bucket plots show the sorts on previous month's realized volatility for the international size (SMB) and value (HML) factor's returns, realized volatility, the risk-return trade-off and the probability of recession. The Low Vol (Highest Vol) bucket shows the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We first show the average return following different buckets of realized volatility. Then we show the realized volatility in the following month given where realized volatility was in the previous month. Thirdly, we show the average return per unit of variance as a representation of the risk-reward trade-off the investor faces conditional on previous month's realized volatility. Lastly, we show the probability of entering into an OECD recession given previous month's realized volatility. The probability of recession is simply calculated as the mean of a binary (0,1) recession dummy for each bucket.

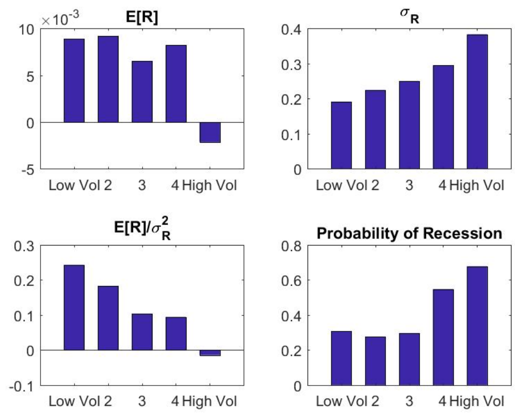



**(a)** RMW

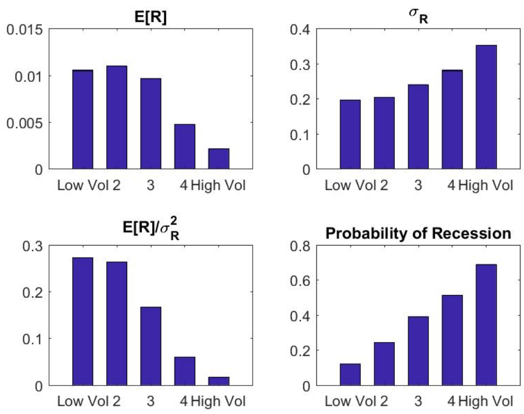



**(b)** CMA

**Figure 5: Bucket Plots for International Profitability and Investment Factors** The bucket plots show the sorts on previous month's realized volatility for the international profitability (RMW) and investment (CMA) factor's returns, realized volatility, the risk-return trade-off and the probability of recession. The Low Vol (High Vol) bucket shows the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We first show the average return following different buckets of realized volatility. Then we show the realized volatility in the following month given where realized volatility was in the previous month. Thirdly, we show the average return per unit of variance as a representation of the risk-reward trade-off the investor faces conditional on previous month's realized volatility. Lastly, we show the probability of entering into an OECD recession given previous month's realized volatility. The probability of recession is simply calculated as the mean of a binary (0,1) recession dummy for each bucket.




**(a)** MOM




**(b)** BAB

**Figure 6: Bucket Plots for International Momentum and Betting-Against-Beta Factors.** The bucket plots show the sorts on previous month's realized volatility for the international momentum (MOM) and betting-against-beta (BAB) factor's returns, realized volatility, the risk-return trade-off and the probability of recession. The Low Vol (High Vol) bucket shows the properties of returns over the month following the lowest (highest) 20% of realized volatility months. We first show the average return following different buckets of realized volatility. Then we show the realized volatility in the following month given where realized volatility was in the previous month. Thirdly, we show the average return per unit of variance as a representation of the risk-reward trade-off the investor faces conditional on previous month's realized volatility. Lastly, we show the probability of entering into an OECD recession given previous month's realized volatility. The probability of recession is simply calculated as the mean of a binary (0,1) recession dummy for each bucket.

## 5 Robustness

### 5.1 Transaction Costs

In this section, we investigate the impact of trading costs on the volatility timing strategies. Specifically, we focus our analysis on the market, profitability, momentum and betting-against-beta factors. We use the market factor as a benchmark given its ease of implementation for investors and choose profitability, momentum and betting-against-beta due to the statistically significant alphas produced by volatility timing (see Table II). Furthermore, since volatility timing strategies can be implemented in several ways, we employ seven different approaches to determine the weights and assess the impact of trading costs under each alternative approach.

For the variance input, we investigate different proxies for variance based on raw measures of variance, conditional variance models and finally we also consider leverage constraints. The first two measures are simply the realized variance (see Equation 8) and standard deviation (square root of variance). The measures are calculated daily on a rolling basis and thereafter collapsed to a monthly frequency. For the conditional variance estimates we employ a simple AR(1)-model, a GARCH(1,1)-model and a GJR(1,1,1)-model. All models are fitted full-sample on a monthly frequency, then the conditional variance is estimated. Lastly, we present two different leverage-constrained strategies based on the realized variance. The first constrained strategy employs no leverage (weights ranging from 0 to 1.00) and the second constrained strategy is allowed to use 50% leverage (weights ranging from 0 to 1.50). Lastly, estimates for transaction costs vary both for different agents and under different market environments. In specifying the actual trading costs, we follow the procedure in Moreira and Muir (2017) and consider 1 bp trading costs from Fleming, Kirby, and Ostdiek (2003) as well as 10 bps and 14 bps from Frazzini, Israel, and Moskowitz (2012). The 14 bps scenario represents more adverse conditions following a shift in VIX from 20% to 40% (98th percentile).

Table VII presents our findings for the transaction cost analysis. For each factor, we show the mean of monthly changes in weights for the strategy, the annualized return before trading costs, the annualized alphas before trading costs as well as the impact on annualized alphas after applying 1 bp, 10 bps and 14 bps in trading costs. Furthermore, we would like to stress that these trading costs are primarily related to the market factor and that actual trading costs for the factor strategies are likely to be higher.

In Panel A, the results for the market factor are presented for comparison. In Panel B, we present the results for the profitability factor. The results indicate a drop in alphas to relatively low levels which could present a potential concern. In contrast, while the results presented for the momentum factor in Panel C show a decrease in alphas, the magnitude of the net alphas are still high. We find similar results for the betting-against-beta factor, presented in Panel D, where alphas decrease in magnitude but remain on a high level.

**Table VII**
**Transaction Costs of Volatility Timing**

In this table, we show the impact of trading costs on alphas for the international factors with different variations of variance proxies. By using different proxies for variance, we test the robustness of the notion of volatility timing in a broader sense. Different proxies will lead to different weights and changes in weights, which furthermore will have an impact on trading costs. The first two proxies are simply the realized volatility and variance for the previous period. For the next three proxies, we employ conditional variance models (AR(1), GARCH(1,1) and GJR(1,1,1)) to estimate conditional variance with all models estimated full-sample. Lastly, we present two series with leverage constraints where the first strategy allows no leverage and the second strategy allows for 50% leverage. In the estimation of trading costs, we follow Moreira and Muir (2017) where 1 bp cost comes from Fleming, Kirby, and Ostdiek (2003), the 10 bps comes from Frazzini, Israel, and Moskowitz (2012) under the estimate of trading about 1% of daily volume and the last column represents higher costs experienced in high-volatility episodes using another model from the same paper. The extra 4 bps comes from a shift in VIX from 20% to 40% (98th percentile). Panel A shows the results for the international market (MKT) factor, Panel B shows the results for the international profitability (RMW) factor, Panel C shows the results for international momentum (MOM) factor and Panel D shows the results for the international betting-against-beta (BAB) factor.

Panel A: Market (MKT)

| | | | | | $\alpha$ After Trading Costs | | |
|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\overline{\lvert\Delta w_t\rvert}$ | $E[R]$ | $\alpha$ | 1bps | 10bps | 14bps |
| $\frac{1}{RV_t^2}$ | Realized Variance | 0.57 | 7.59 | 3.41 | 3.03 | 2.42 | 2.15 |
| $\frac{1}{RV_t}$ | Realized Vol | 0.31 | 7.71 | 2.16 | 1.87 | 1.53 | 1.38 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 0.46 | 5.74 | 0.76 | 1.02 | 0.53 | 0.31 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 0.12 | 7.12 | 1.95 | 1.94 | 1.81 | 1.75 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 0.12 | 7.28 | 2.28 | 2.28 | 2.15 | 2.09 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 0.17 | 4.51 | 1.35 | 1.11 | 0.92 | 0.83 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 0.28 | 5.83 | 2.21 | 1.91 | 1.61 | 1.47 |

Panel B: Profitability (RMW)

| | | | | | $\alpha$ After Trading Costs | | |
|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\overline{\lvert\Delta w_t\rvert}$ | $E[R]$ | $\alpha$ | 1bps | 10bps | 14bps |
| $\frac{1}{RV_t^2}$ | Realized Variance | 0.50 | 4.53 | 1.70 | 1.65 | 1.11 | 0.87 |
| $\frac{1}{RV_t}$ | Realized Vol | 0.27 | 4.70 | 1.01 | 0.98 | 0.68 | 0.55 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 0.49 | 3.73 | 0.28 | 0.23 | -0.30 | -0.54 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 0.11 | 4.12 | 0.62 | 0.61 | 0.49 | 0.43 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 0.11 | 4.16 | 0.66 | 0.65 | 0.53 | 0.48 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 0.15 | 2.92 | 0.75 | 0.73 | 0.57 | 0.50 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 0.26 | 3.52 | 1.04 | 1.01 | 0.73 | 0.61 |

Panel C: Momentum (MOM)

| | | | | | $\alpha$ After Trading Costs | | |
|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\overline{\lvert\Delta w_t\rvert}$ | $E[R]$ | $\alpha$ | 1bps | 10bps | 14bps |
| $\frac{1}{RV_t^2}$ | Realized Variance | 0.88 | 15.79 | 12.21 | 12.43 | 11.49 | 11.07 |
| $\frac{1}{RV_t}$ | Realized Vol | 0.46 | 14.82 | 8.91 | 9.18 | 8.69 | 8.47 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 0.48 | 12.28 | 6.55 | 6.66 | 6.14 | 5.91 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 0.35 | 14.61 | 10.02 | 10.01 | 9.63 | 9.46 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 0.34 | 14.76 | 10.19 | 10.19 | 9.82 | 9.66 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 0.14 | 6.91 | 4.43 | 4.68 | 4.53 | 4.46 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 0.25 | 9.05 | 6.22 | 6.53 | 6.26 | 6.14 |

Panel D: Betting-Against-Beta (BAB)

| | | | | | $\alpha$ After Trading Costs | | |
|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\overline{\lvert\Delta w_t\rvert}$ | $E[R]$ | $\alpha$ | 1bps | 10bps | 14bps |
| $\frac{1}{RV_t^2}$ | Realized Variance | 0.58 | 14.61 | 9.19 | 9.15 | 8.54 | 8.27 |
| $\frac{1}{RV_t}$ | Realized Vol | 0.32 | 13.94 | 6.26 | 6.25 | 5.91 | 5.76 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 0.46 | 9.52 | 2.05 | 1.92 | 1.43 | 1.21 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 0.16 | 10.94 | 3.70 | 3.69 | 3.52 | 3.44 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 0.14 | 11.50 | 4.30 | 4.30 | 4.15 | 4.08 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 0.13 | 7.79 | 3.85 | 3.85 | 3.72 | 3.66 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 0.22 | 10.25 | 5.66 | 5.65 | 5.42 | 5.32 |

## 5.2 Leverage Constraints

In this section, we explore the effect of leverage constraints on the volatility timing strategies. Since far from all investors have access to the sometimes high amount of leverage required in the volatility timing strategies, an important question from a practical standpoint is whether the benefits of volatility timing still exist after imposing leverage constraints. Our results, presented in Table VIII, show that the benefits of volatility timing strategies do not seem to be materially impacted by leverage constraints. While alphas are affected by leverage, the impact on Sharpe ratios is typically assumed to be zero in theory, since returns and variances increase proportionally. In practice, given the dynamic leverage profile implied by the volatility timing strategies, constraints can have an impact on Sharpe ratios depending on when the constraint becomes binding. In Panel A, the results for the international market factor are shown for consistency even though volatility timing does not generate statistically significant alphas in our sample.

The results for the profitability factor are presented in Panel B. Firstly, we observe that the alphas from volatility timing are still significant under both versions of the leverage-constrained strategies. A similar pattern can be seen for the profitability factor as Sharpe ratios do not appear to be materially affected by volatility timing.

As can be seen in Panel C, volatility-managed momentum alphas are positive and statistically significant at the 1%-level for all seven specifications of variance. Our results confirm previous findings in the momentum crashes literature of Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). The benefits of volatility timing seem to be robust to alternative specifications of variance. Moreover, the annualized alphas are relatively high with the highest alpha generated by the realized variance input yielding an annualized alpha of 12.21% compared to the non-managed momentum strategy. The high alphas are partly driven by the high leverage employed in the unconstrained realized variance-strategy. Interestingly, restricting maximum leverage to 50% for the realized variance-strategy does not materially impact the Sharpe ratio.

In Panel D, we present results for the betting-against-beta factor. The positive alphas are statistically significant for all alternative specifications of variance for the betting-against-beta factor. In addition, all specifications except for the AR(1)-model are statistically significant at the 1%-level. The results of the AR(1)-specification are still significant, but only at the 10%-level. Restricting leverage to 50% does not reduce the Sharpe ratio meaningfully for the betting-against-beta strategy. However, we see that the alpha is reduced compared to the unconstrained strategy which is expected given the lower leverage. Looking at the distribution of weights in Panel D, we can see that the leverage constraint is binding in at least 25% of the time. Similar to the results for the momentum strategy, the Sharpe ratio decreases somewhat for the strategy where no leverage is allowed.

## Table VIII
## Volatility Timing & Leverage

In this table, we show several different volatility-managed strategies for the international factors. In each panel, the alphas, Sharpe ratios (SR), appraisal ratios ($\frac{\alpha}{\sigma_\epsilon}$) and distributions of weights is shown for each of the volatility-managed strategies. The alternative strategies are composed of realized volatility (square root of the realized variance), conditional variance models and two strategies which employ leverage restrictions of no leverage ($0-100\%$) and 50% leverage ($0-150\%$). The conditional variance models are composed of an AR(1)-model (see Equation 13), a GARCH(1,1)-model (see Equation 14) and a GJR(1,1,1)-model (see Equation 15) to allow for different approaches towards estimating conditional variances. All conditional variance models are fitted full-sample. Panel A shows the results for the international market (MKT) factor, Panel B shows the results for the international profitability (RMW) factor, Panel C shows the results for international momentum (MOM) factor and Panel D shows the results for the international betting-against-beta (BAB) factor.

Panel A: Market (MKT)

| | | | | | | Distribution of Weights $w$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\alpha$ | t | SR | $\frac{\alpha}{\sigma_\epsilon}$ | P50 | P75 | P90 | P99 |
| $\frac{1}{RV_t^2}$ | Realized Variance | 3.41 | 1.47 | 0.31 | 0.51 | 0.82 | 1.55 | 2.20 | 4.61 |
| $\frac{1}{RV_t}$ | Realized Vol | 2.16 | 1.42 | 0.32 | 0.52 | 1.11 | 1.53 | 1.82 | 2.63 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 0.76 | 0.48 | 0.10 | 0.39 | 1.07 | 1.36 | 1.52 | 1.54 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 1.95 | 1.32 | 0.29 | 0.48 | 1.08 | 1.54 | 1.81 | 2.12 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 2.28 | 1.28 | 0.30 | 0.49 | 1.20 | 1.61 | 1.77 | 1.97 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 1.35 | 1.21 | 0.27 | 0.50 | 0.82 | 1.00 | 1.00 | 1.00 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 2.21 | 1.49 | 0.32 | 0.53 | 0.82 | 1.50 | 1.50 | 1.50 |

Panel B: Profitability (RMW)

| | | | | | | Distribution of Weights $w$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\alpha$ | t | SR | $\frac{\alpha}{\sigma_\epsilon}$ | P50 | P75 | P90 | P99 |
| $\frac{1}{RV_t^2}$ | Realized Variance | 1.70 | 2.03** | 0.46 | 0.90 | 0.85 | 1.47 | 2.25 | 4.33 |
| $\frac{1}{RV_t}$ | Realized Vol | 1.01 | 2.08** | 0.45 | 0.93 | 1.10 | 1.45 | 1.80 | 2.49 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 0.28 | 0.58 | 0.10 | 0.74 | 1.06 | 1.49 | 1.63 | 1.66 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 0.62 | 1.19 | 0.23 | 0.82 | 1.14 | 1.47 | 1.79 | 2.60 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 0.66 | 1.30 | 0.25 | 0.83 | 1.15 | 1.47 | 1.78 | 2.58 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 0.75 | 2.18** | 0.43 | 0.92 | 0.85 | 1.00 | 1.00 | 1.00 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 1.04 | 2.11** | 0.44 | 0.91 | 0.85 | 1.47 | 1.50 | 1.50 |

Panel C: Momentum (MOM)

| | | | | | | Distribution of Weights $w$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\alpha$ | t | SR | $\frac{\alpha}{\sigma_\epsilon}$ | P50 | P75 | P90 | P99 |
| $\frac{1}{RV_t^2}$ | Realized Variance | 12.21 | 4.58*** | 0.25 | 0.52 | 0.82 | 1.33 | 1.92 | 3.94 |
| $\frac{1}{RV_t}$ | Realized Vol | 8.91 | 4.50*** | 0.25 | 0.54 | 1.06 | 1.36 | 1.63 | 2.33 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 6.55 | 3.39*** | 0.09 | 0.47 | 1.03 | 1.32 | 1.43 | 1.46 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 10.02 | 4.79*** | 0.16 | 0.47 | 1.03 | 1.32 | 1.43 | 1.46 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 10.19 | 4.99*** | 0.26 | 0.55 | 1.19 | 1.36 | 1.43 | 1.47 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 4.43 | 4.13*** | 0.17 | 0.50 | 0.82 | 1.00 | 1.00 | 1.00 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 6.22 | 4.54*** | 0.24 | 0.53 | 0.82 | 1.33 | 1.50 | 1.50 |

Panel D: Betting-Against-Beta (BAB)

| | | | | | | Distribution of Weights $w$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $w_t$ | Description | $\alpha$ | t | SR | $\frac{\alpha}{\sigma_\epsilon}$ | P50 | P75 | P90 | P99 |
| $\frac{1}{RV_t^2}$ | Realized Variance | 9.19 | 5.60*** | 1.15 | 1.47 | 0.91 | 1.74 | 2.57 | 4.79 |
| $\frac{1}{RV_t}$ | Realized Vol | 6.26 | 5.67*** | 1.15 | 1.40 | 1.23 | 1.70 | 2.06 | 2.69 |
| $\frac{1}{E[RV]_{t+1}}$ | AR(1) | 2.05 | 1.67* | 0.35 | 0.96 | 1.16 | 1.51 | 1.64 | 1.67 |
| $\frac{1}{E[RV]_{t+1}}$ | GARCH(1,1) | 3.70 | 2.67*** | 0.60 | 1.10 | 1.36 | 1.78 | 2.09 | 2.69 |
| $\frac{1}{E[RV]_{t+1}}$ | GJR(1,1,1) | 4.30 | 3.11*** | 0.69 | 1.16 | 1.37 | 1.85 | 2.11 | 2.57 |
| $min(\frac{c}{RV_t^2}, 1)$ | No Leverage | 3.85 | 4.78*** | 1.05 | 1.38 | 0.91 | 1.00 | 1.00 | 1.00 |
| $min(\frac{c}{RV_t^2}, 1.5)$ | 50% Leverage | 5.66 | 5.48*** | 1.13 | 1.45 | 0.91 | 1.50 | 1.50 | 1.50 |

# 6 Investor Implications

A natural question to ask from a practical point of view is: who benefits from volatility timing? While volatility timing itself might be a simple strategy to implement, the knowledge about factor construction or access to short-selling facilities required to create the factors in the first place might not be equally accessible for all types of investors. Hence, we use the results on the market factor as an indicator of the performance improvement for less sophisticated investors and the results from the advanced factor strategies as proxies for the benefits obtained by sophisticated investors.

The results on the international level presented in Table II indicate benefits for the profitability, momentum and betting-against-beta factors but less so for the market factor. Turning to the country-level findings in Table III, we similarly obtain mixed results for the market factor and only find broad support for the momentum factor. In total, our results indicate that it is primarily sophisticated investors who benefit from volatility-managed strategies. An interesting topic would be to examine if the benefits of volatility timing persist for long-only implementations of the factor strategies, but we leave these investigations for future research.

Our results show that an important part of the benefits of volatility timing comes from a reduction in drawdowns, primarily for the momentum factor. However, increased following of the volatility timing strategies could introduce new risks. A coordinated increase in the usage of volatility timing strategies among investors, for instance on the momentum factor, raises the risk of magnifying rare but occasional factor unwinds. A study by Khandani and Lo (2011) show that coordinated investing can lead to tight liquidity situations referred to as unwinds with temporarily very weak factor performance. Given that volatility timing leads to both non-negligible changes in target weights as well as periods of high target leverage, such unwinds could become increasingly severe with volatility timing. The amplification mechanism highlights the risk of a potential Peso problem when assessing the long-run profitability of the strategy (see Ilmanen (2011)).

# 7 Conclusion

Our study investigates the benefits of volatility timing for equity factors on an international level and for 24 developed markets on a country level. With mixed results on both international and country levels, our findings cast doubt on the cross-factor benefits of volatility timing. Moreover, with the momentum factor being the only factor showing strong benefits on both international and country levels, our results may suggest that the benefits of volatility timing are momentum-related. While our results provide additional support for the previous findings of the momentum crashes literature, our cross-factor analysis also highlights the importance of the distribution of the risk-reward relationships across volatility buckets.

The results on an international level are similar to the US results of Moreira and Muir (2017), with the regression alphas being positive and significant for the profitability, momentum and betting-against-beta factors. The multifactor results of the international factors also follow previous findings with the benefits of volatility timing being evident across factor combinations. However, our country-level results show weak to mixed benefits of volatility timing for most factors, with the momentum factor being the major exception. On a country level, the momentum factor is positive and significant in 20/24 countries. Moreover, on an international level, the benefits of the momentum factor is consistent after controlling for standard asset pricing factors as well as trading costs and leverage constraints.

While our study is performed across a broader set of markets, it should be noted that most of our non-US factors begin in the 1980's making the time-sample shorter and thus providing less power to our tests. For further research, we propose an expanded robustness test of the benefits of volatility-managed portfolios on the country level. Furthermore, an investigation into the performance benefits from the long versus the short legs for the factor portfolios could contribute to a deeper understanding of volatility timing. Previous US-focused studies have emphasized that the benefits of volatility-managed portfolios should be available to less sophisticated investors, however, our study performed on the international level indicates that the benefits mainly exist for sophisticated investors. Lastly, we note that widespread utilization of the volatility-managed strategies among sophisticated investors might amplify risks related to herding behavior, thus concealing potential crash risks not seen in our sample.

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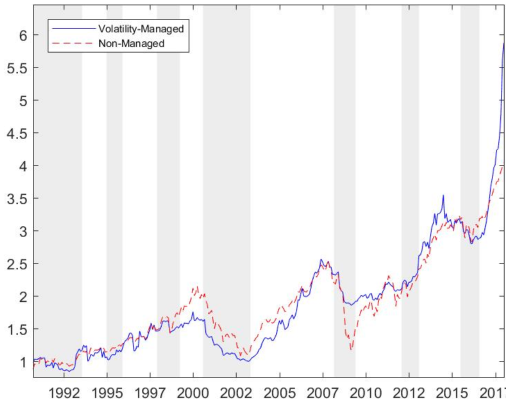
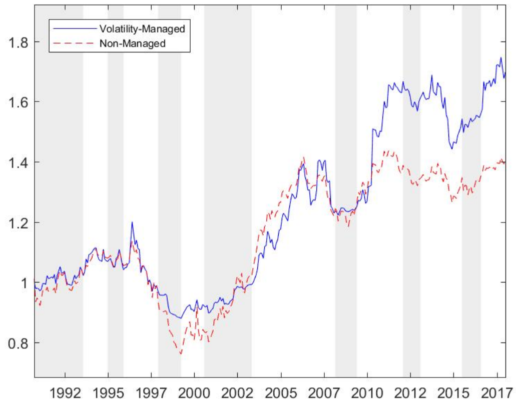
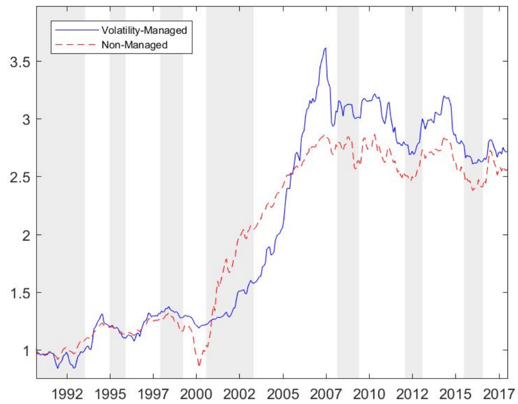
# A Cumulative International Factor Plots




**(a)** MKT

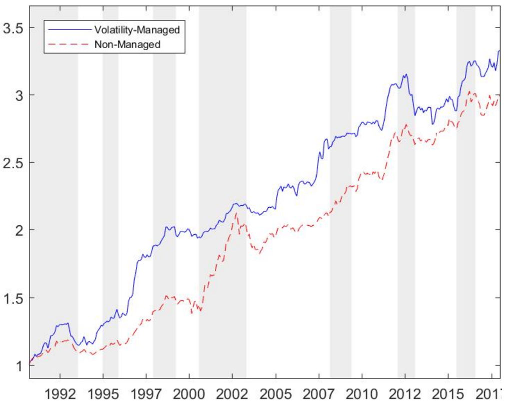



**(b)** SMB

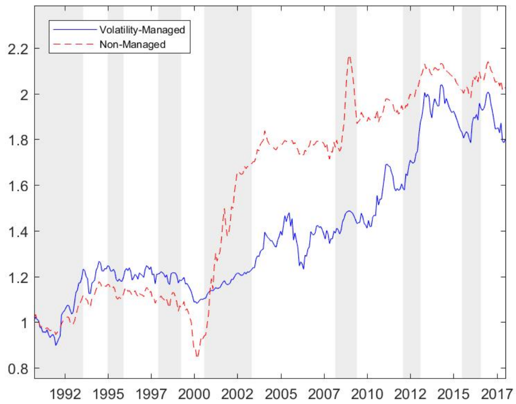



**(c)** HML

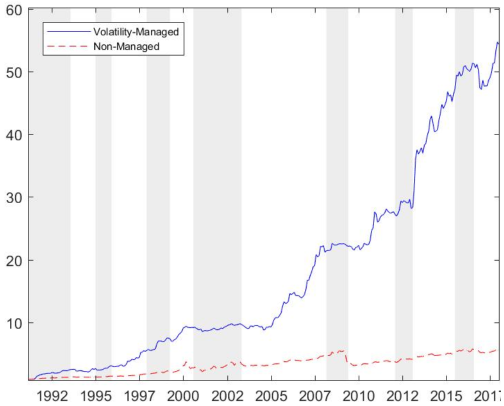
**Figure 7: Cumulative Return Series for International Factors.** In this figure, we show the cumulative return series for the international market (MKT), size (SMB) and value (HML) factors versus their volatility-managed counterparts. The non-managed series are constructed by compounding the monthly managed and non-managed returns from the beginning of the respective samples. The volatility-managed series are computed by scaling the factor returns with the inverse of the previous month's realized variance according to the following formula $f^{\sigma}_{t+1} = \frac{c}{\hat{\sigma_t}^2(f)} f_{t+1}$. In the preceding formula, we set $c$ such that the variance of the volatility-timed series is equal to the variance of the non-managed series for comparability. The shaded regions indicate OECD recessions (downloaded from the St. Louis Federal Reserve's website).



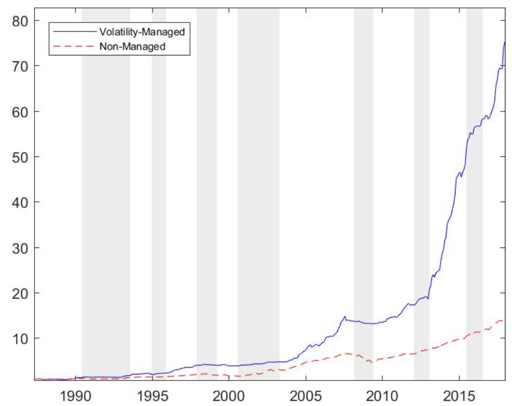

**(a)** RMW




**(b)** CMA

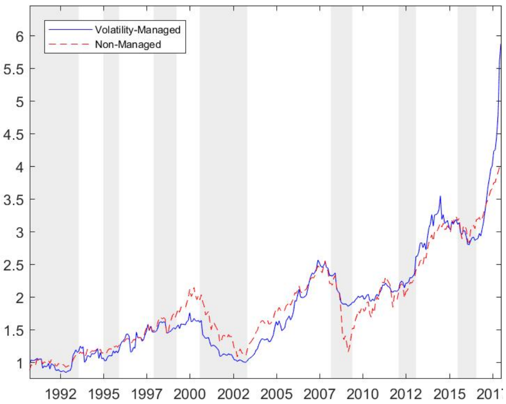



**(c)** MOM

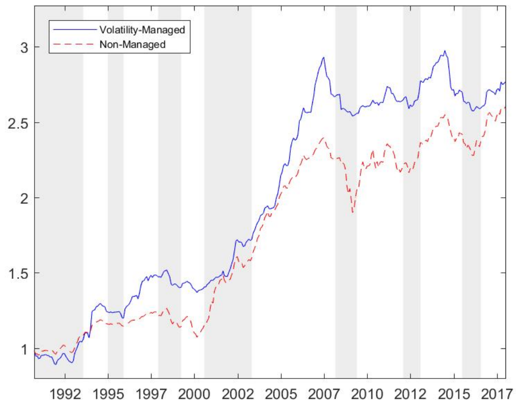



**(d)** BAB

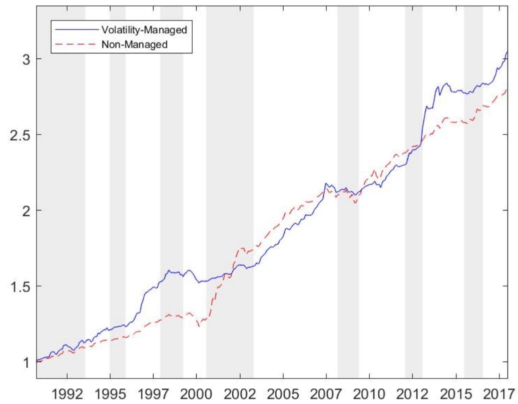
**Figure 8: Cumulative Return Series for International Factors.** In this figure, we show the cumulative return series for the international profitability (RMW), investment (CMA), momentum (MOM) and betting-against-beta (BAB) factors versus their volatility-managed counterparts. The non-managed series are constructed by compounding the monthly managed and non-managed returns from the beginning of the respective samples. The volatility-managed series are computed by scaling the factor returns with the inverse of the previous month's realized variance according to the following formula $f^{\sigma}_{t+1} = \frac{c}{\hat{\sigma}_t{}^2(f)} f_{t+1}$. In the preceding formula, we set $c$ such that the variance of the volatility-timed series is equal to the variance of the non-managed series for comparability. The shaded regions indicate OECD recessions (downloaded from the St. Louis Federal Reserve's website).




**(a)** MKT

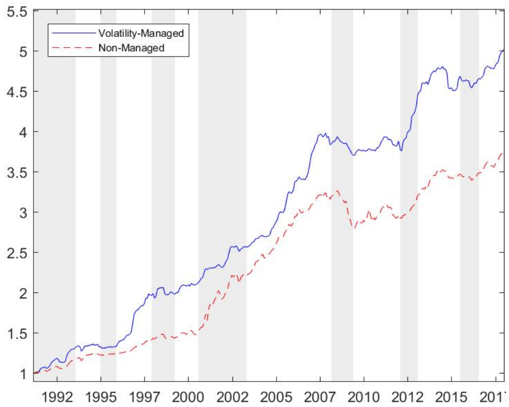



**(b)** FF3

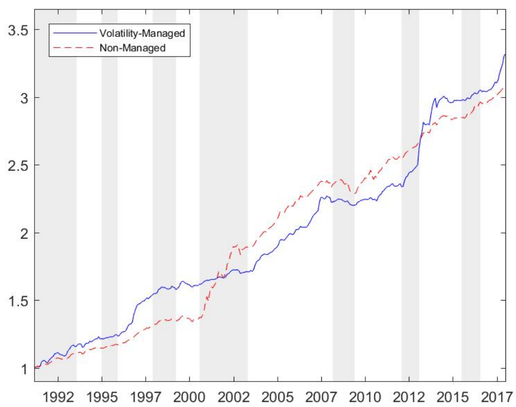



**(c)** FF5

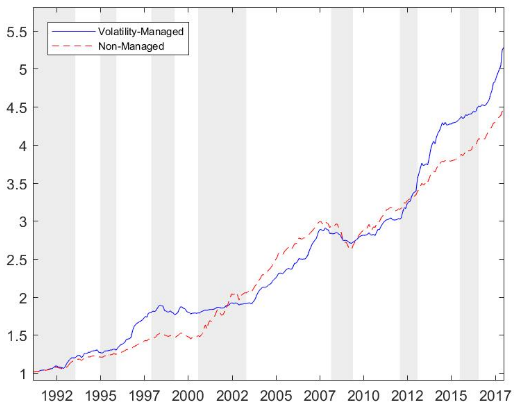
**Figure 9: Cumulative Return Series for International Mean-Variance Efficient Portfolios.** In this figure, we show the cumulative return series for the international market, the mean-variance efficient (MVE) portfolio of the Fama-French three (FF3) and five (FF5) factors versus their volatility-managed counterparts. The non-managed series are constructed by compounding the monthly managed and non-managed returns from the beginning of the respective samples. The MVE portfolios are formed on an international level with the full sample of different factor combinations. These MVE portfolios are then volatility managed in accordance with $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. In the preceding formula, we set $c$ such that the variance of the volatility-timed series is equal to the variance of the non-managed series for comparability. The shaded regions indicate OECD recessions (downloaded from the St. Louis Federal Reserve's website).




**(a)** FF3 & MOM




**(b)** FF5 & MOM




**(c)** ALL

**Figure 10: Cumulative Return Series for International Mean-Variance Efficient Portfolios.** In this figure, we show the cumulative return series for the mean-variance efficient (MVE) portfolios of the momentum factor combined the Fama-French three (FF3 & MOM) and five (FF5 & MOM) as well as an all-factor combination (ALL) versus their volatility-managed counterparts. The non-managed series are constructed by compounding the monthly managed and non-managed returns from the beginning of the respective samples. The MVE portfolios are formed on an international level with the full sample of different factor combinations. These MVE portfolios are then volatility managed in accordance with $f_t^{\sigma} = \alpha + \beta f_t + \epsilon_t$. In the preceding formula, we set $c$ such that the variance of the volatility-timed series is equal to the variance of the non-managed series for comparability. The shaded regions indicate OECD recessions (downloaded from the St. Louis Federal Reserve's website).

# B Summary Statistics of Country-Level Factors

**Table IX**
**Summary Statistics for Country Market Factors**

In this table, we present the summary statistics for the market (MKT) factor for each of the 24 countries in our sample. We report the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK), excess kurtosis (EKurt), number of observations (N) and the pairwise correlation to the international MKT factor $\rho_{i,INT}$ for the monthly factor returns. The country MKT factors have varying start dates, however, all samples end in year-end 2017. On the last row in the table, we add the same summary statistics for the international MKT factor for comparability.

| $i$ | $\text{Mean}_i$ | $\text{Median}_i$ | $\text{Max}_i$ | $\text{Min}_i$ | $\text{SD}_i$ | $\text{SK}_i$ | $\text{EKurt}_i$ | $\text{N}_i$ | $\rho_{i,INT}$ |
|---|---|---|---|---|---|---|---|---|---|
| AUS | 1.00 | 1.27 | 36.90 | -42.98 | 6.82 | -0.47 | 6.90 | 386 | 0.75 |
| AUT | 0.64 | 0.92 | 23.09 | -35.56 | 6.69 | -0.47 | 3.49 | 384 | 0.73 |
| BEL | 0.80 | 1.01 | 23.78 | -31.99 | 5.50 | -0.74 | 4.72 | 384 | 0.78 |
| CAN | 0.51 | 0.44 | 20.39 | -28.00 | 5.06 | -0.76 | 3.86 | 430 | 0.80 |
| CHE | 0.67 | 0.96 | 16.55 | -18.53 | 4.97 | -0.41 | 1.12 | 384 | 0.80 |
| DEU | 0.56 | 0.87 | 19.05 | -20.80 | 5.91 | -0.46 | 1.38 | 384 | 0.85 |
| DNK | 0.85 | 1.17 | 21.99 | -25.43 | 5.39 | -0.39 | 2.23 | 384 | 0.78 |
| ESP | 0.88 | 0.96 | 23.88 | -23.09 | 6.73 | -0.07 | 1.15 | 384 | 0.78 |
| FIN | 0.92 | 0.84 | 30.28 | -29.20 | 7.81 | 0.11 | 1.64 | 384 | 0.71 |
| FRA | 0.71 | 0.83 | 19.52 | -22.15 | 5.81 | -0.39 | 1.04 | 384 | 0.87 |
| GBR | 0.57 | 0.68 | 15.01 | -22.18 | 5.00 | -0.44 | 1.82 | 384 | 0.88 |
| GRC | 0.43 | 0.29 | 55.01 | -33.74 | 10.73 | 0.83 | 4.20 | 352 | 0.56 |
| HKG | 1.01 | 1.51 | 27.56 | -46.61 | 7.72 | -0.65 | 4.76 | 384 | 0.65 |
| IRL | 0.83 | 1.23 | 62.01 | -24.55 | 7.86 | 1.10 | 10.49 | 384 | 0.69 |
| ISR | 0.25 | 1.05 | 17.63 | -96.16 | 8.83 | -5.16 | 51.30 | 277 | 0.42 |
| ITA | 0.47 | 0.50 | 27.42 | -23.42 | 6.93 | 0.12 | 0.76 | 384 | 0.73 |
| JPN | 0.33 | 0.38 | 25.53 | -17.97 | 6.08 | 0.30 | 0.97 | 384 | 0.69 |
| NLD | 0.71 | 1.11 | 16.49 | -28.98 | 5.33 | -0.96 | 3.50 | 384 | 0.88 |
| NOR | 0.88 | 1.31 | 20.07 | -30.93 | 7.18 | -0.67 | 1.99 | 384 | 0.75 |
| NZL | 0.77 | 0.94 | 30.20 | -37.81 | 6.65 | -0.21 | 3.84 | 384 | 0.65 |
| PRT | 0.21 | 0.43 | 26.60 | -28.61 | 6.39 | -0.07 | 1.70 | 359 | 0.68 |
| SGP | 0.85 | 0.83 | 27.66 | -37.88 | 6.99 | -0.31 | 4.45 | 384 | 0.72 |
| SWE | 0.94 | 1.11 | 25.31 | -26.65 | 7.01 | -0.27 | 1.33 | 384 | 0.83 |
| USA | 0.65 | 0.97 | 37.36 | -28.31 | 5.28 | 0.12 | 7.20 | 1097 | 0.91 |
| INT | 0.48 | 0.94 | 11.41 | -19.52 | 4.27 | -0.73 | 1.76 | 330 | 1.00 |

**Table X**
**Summary Statistics for Country Size Factors**

In this table, we present the summary statistics for the size (SMB) factor for each of the 24 countries in our sample. We report the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK),excess kurtosis (EKurt), number of observations (N) and the pairwise correlation to the international SMB factor $\rho_{i,INT}$ for the monthly factor returns. The country SMB factors have varying start dates, however, all samples end in year-end 2017. On the last row in the table, we add the same summary statistics for the international SMB factor for comparability.

| $i$ | $\text{Mean}_i$ | $\text{Median}_i$ | $\text{Max}_i$ | $\text{Min}_i$ | $\text{SD}_i$ | $\text{SK}_i$ | $\text{EKurt}_i$ | $\text{N}_i$ | $\rho_{i,INT}$ |
|---|---|---|---|---|---|---|---|---|---|
| AUS | -0.06 | -0.16 | 13.18 | -9.69 | 3.40 | 0.19 | 0.55 | 330 | 0.25 |
| AUT | 0.29 | 0.37 | 12.01 | -10.08 | 3.82 | -0.02 | 0.36 | 330 | 0.11 |
| BEL | -0.15 | 0.00 | 11.59 | -18.60 | 3.52 | -0.74 | 3.17 | 330 | 0.10 |
| CAN | -0.03 | -0.10 | 14.03 | -9.66 | 2.62 | 0.43 | 3.52 | 402 | 0.53 |
| CHE | 0.03 | 0.13 | 14.33 | -14.73 | 2.80 | 0.14 | 5.09 | 330 | 0.29 |
| DEU | -0.44 | -0.26 | 10.43 | -11.75 | 3.03 | -0.02 | 1.13 | 330 | 0.28 |
| DNK | -0.04 | -0.06 | 10.49 | -11.29 | 3.43 | -0.15 | 0.51 | 330 | 0.16 |
| ESP | -0.31 | -0.26 | 17.38 | -15.68 | 3.82 | -0.11 | 2.39 | 330 | 0.18 |
| FIN | -0.02 | -0.03 | 20.20 | -13.64 | 4.48 | 0.12 | 1.81 | 330 | 0.26 |
| FRA | -0.02 | 0.04 | 12.61 | -9.84 | 3.05 | 0.25 | 0.94 | 330 | 0.49 |
| GBR | -0.16 | 0.03 | 13.25 | -14.10 | 3.49 | -0.31 | 1.95 | 354 | 0.60 |
| GRC | 0.31 | 0.14 | 32.09 | -18.77 | 6.06 | 0.76 | 3.88 | 258 | 0.08 |
| HKG | -0.03 | -0.29 | 16.54 | -14.72 | 4.83 | 0.40 | 1.37 | 330 | 0.35 |
| IRL | -0.08 | -0.12 | 25.70 | -25.63 | 5.85 | -0.14 | 2.97 | 330 | 0.20 |
| ISR | 0.23 | 0.04 | 25.75 | -14.63 | 4.13 | 0.63 | 5.94 | 258 | 0.13 |
| ITA | -0.17 | -0.13 | 14.00 | -9.54 | 3.46 | 0.30 | 1.12 | 330 | 0.26 |
| JPN | 0.02 | 0.08 | 10.75 | -10.67 | 2.68 | -0.08 | 1.19 | 354 | 0.34 |
| NLD | 0.08 | 0.22 | 15.86 | -8.55 | 3.24 | 0.23 | 1.28 | 330 | 0.21 |
| NOR | -0.03 | -0.13 | 10.57 | -12.09 | 3.63 | -0.15 | 0.68 | 330 | 0.10 |
| NZL | 0.09 | 0.03 | 13.68 | -12.67 | 3.67 | 0.24 | 2.29 | 330 | 0.06 |
| PRT | -0.10 | 0.01 | 15.54 | -24.99 | 5.12 | -0.73 | 2.55 | 305 | 0.06 |
| SGP | -0.05 | -0.16 | 24.81 | -18.29 | 4.09 | 1.40 | 9.32 | 330 | 0.17 |
| SWE | -0.22 | -0.26 | 11.65 | -21.37 | 3.63 | -0.48 | 3.94 | 330 | 0.21 |
| USA | 0.23 | 0.01 | 36.14 | -13.29 | 3.01 | 2.04 | 20.72 | 1097 | 0.71 |
| INT | 0.12 | 0.08 | 7.97 | -8.42 | 1.95 | -0.35 | 2.29 | 330 | 1.00 |

**Table XI**
**Summary Statistics for Country Value Factors**

In this table, we present the summary statistics for the value (HML) factor for each of the 24 countries in our sample. We report the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK), excess kurtosis (EKurt), number of observations (N) and the pairwise correlation to the international HML factor $\rho_{i,INT}$ for the monthly factor returns. The country HML factors have varying start dates, however, all samples end in year-end 2017. On the last row in the table, we add the same summary statistics for the international HML factor for comparability.

| $i$ | Mean$_i$ | Median$_i$ | Max$_i$ | Min$_i$ | SD$_i$ | SK$_i$ | EKurt$_i$ | N$_i$ | $\rho_{i,INT}$ |
|---|---|---|---|---|---|---|---|---|---|
| AUS | 0.78 | 0.73 | 9.55 | -10.03 | 2.95 | -0.01 | 0.55 | 330 | 0.41 |
| AUT | 1.04 | 1.14 | 16.13 | -13.91 | 4.47 | 0.19 | 1.25 | 330 | 0.02 |
| BEL | 0.46 | 0.40 | 12.09 | -12.18 | 3.93 | 0.05 | 0.46 | 330 | 0.18 |
| CAN | 0.49 | 0.52 | 21.94 | -18.49 | 3.71 | 0.04 | 5.42 | 402 | 0.58 |
| CHE | 0.27 | 0.11 | 15.49 | -11.85 | 3.49 | 0.23 | 1.77 | 330 | 0.26 |
| DEU | 0.65 | 0.68 | 23.83 | -15.05 | 3.63 | 1.20 | 9.17 | 330 | 0.56 |
| DNK | -0.10 | 0.13 | 12.15 | -11.80 | 4.29 | -0.06 | 0.09 | 330 | 0.26 |
| ESP | 0.28 | 0.41 | 17.37 | -12.20 | 3.92 | 0.21 | 0.86 | 306 | 0.34 |
| FIN | 0.60 | 0.49 | 31.65 | -19.26 | 6.60 | 0.65 | 3.20 | 330 | 0.45 |
| FRA | 0.35 | 0.28 | 14.81 | -22.09 | 3.38 | -0.43 | 7.25 | 330 | 0.63 |
| GBR | 0.41 | 0.46 | 17.45 | -14.97 | 3.30 | 0.07 | 4.49 | 354 | 0.64 |
| GRC | 0.33 | -0.38 | 20.81 | -18.12 | 5.16 | 0.52 | 1.93 | 246 | 0.10 |
| HKG | 0.43 | 0.45 | 13.81 | -20.45 | 4.09 | -0.05 | 2.69 | 330 | 0.35 |
| IRL | 0.20 | 0.32 | 20.02 | -19.73 | 6.52 | -0.22 | 0.86 | 306 | 0.13 |
| ISR | 0.49 | 0.27 | 16.34 | -18.76 | 4.88 | 0.17 | 1.69 | 246 | -0.03 |
| ITA | 0.21 | 0.32 | 12.57 | -11.34 | 3.91 | -0.01 | 0.56 | 330 | 0.27 |
| JPN | 0.50 | 0.54 | 10.28 | -10.04 | 2.64 | 0.15 | 1.54 | 354 | 0.53 |
| NLD | 0.47 | 0.51 | 15.54 | -12.46 | 4.14 | 0.05 | 0.52 | 330 | 0.37 |
| NOR | 0.21 | 0.12 | 16.97 | -15.50 | 5.12 | 0.11 | 0.74 | 330 | 0.40 |
| NZL | -0.02 | -0.10 | 28.26 | -19.09 | 5.11 | 0.48 | 4.63 | 318 | 0.16 |
| PRT | 0.22 | 0.00 | 22.46 | -14.41 | 4.93 | 0.59 | 2.30 | 270 | 0.10 |
| SGP | 0.65 | 0.52 | 25.94 | -16.87 | 3.83 | 0.77 | 8.15 | 330 | 0.28 |
| SWE | 0.36 | 0.10 | 23.39 | -23.63 | 5.38 | 0.16 | 3.44 | 330 | 0.53 |
| USA | 0.32 | 0.13 | 37.53 | -11.18 | 3.39 | 2.68 | 23.27 | 1097 | 0.85 |
| INT | 0.31 | 0.22 | 11.64 | -9.54 | 2.27 | 0.55 | 5.23 | 330 | 1.00 |

**Table XII**
**Summary Statistics for Country Momentum Factors**

In this table, we present the summary statistics for the momentum (MOM) factor for each of the 24 countries in our sample. We report the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK), excess kurtosis (EKurt), number of observations (N) and the pairwise correlation to the international MOM factor $\rho_{i,INT}$ for the monthly factor returns. The country SMB factors have varying start dates, however, all samples end in year-end 2017. On the last row in the table, we add the same summary statistics for the international MOM factor for comparability.

| $i$ | Mean$_i$ | Median$_i$ | Max$_i$ | Min$_i$ | SD$_i$ | SK$_i$ | EKurt$_i$ | N$_i$ | $\rho_{i,INT}$ |
|---|---|---|---|---|---|---|---|---|---|
| AUS | 1.62 | 1.47 | 21.64 | -17.10 | 4.36 | -0.18 | 2.56 | 374 | 0.55 |
| AUT | 0.51 | 0.56 | 21.08 | -39.32 | 5.43 | -1.41 | 9.59 | 372 | 0.40 |
| BEL | 0.86 | 1.01 | 19.17 | -30.90 | 4.73 | -1.06 | 7.38 | 372 | 0.54 |
| CAN | 1.35 | 1.46 | 22.48 | -27.01 | 5.32 | -0.67 | 3.67 | 396 | 0.72 |
| CHE | 0.84 | 0.84 | 22.63 | -25.02 | 4.52 | -0.59 | 5.91 | 372 | 0.67 |
| DEU | 1.13 | 1.01 | 26.95 | -23.23 | 5.26 | -0.32 | 6.43 | 372 | 0.65 |
| DNK | 1.10 | 1.53 | 13.10 | -22.03 | 4.52 | -0.77 | 2.62 | 372 | 0.49 |
| ESP | 0.67 | 0.88 | 16.42 | -28.14 | 5.09 | -0.71 | 3.32 | 372 | 0.56 |
| FIN | 1.02 | 1.34 | 26.17 | -34.09 | 6.19 | -0.32 | 3.76 | 372 | 0.46 |
| FRA | 0.72 | 0.76 | 20.69 | -26.61 | 4.63 | -0.68 | 6.17 | 372 | 0.75 |
| GBR | 1.02 | 1.21 | 13.53 | -34.18 | 4.46 | -1.74 | 11.19 | 372 | 0.84 |
| GRC | 0.73 | 1.10 | 33.96 | -33.35 | 7.46 | -0.77 | 4.16 | 340 | 0.24 |
| HKG | 0.49 | 1.08 | 21.79 | -28.28 | 5.40 | -1.20 | 5.09 | 372 | 0.49 |
| IRL | 1.12 | 1.10 | 30.61 | -56.77 | 9.48 | -0.94 | 6.62 | 335 | 0.39 |
| ISR | 1.16 | 1.35 | 13.07 | -24.64 | 4.96 | -0.92 | 3.21 | 263 | 0.45 |
| ITA | 0.73 | 1.03 | 20.08 | -21.16 | 4.88 | -0.27 | 2.43 | 372 | 0.62 |
| JPN | 0.14 | 0.48 | 13.60 | -24.84 | 4.61 | -0.74 | 3.51 | 372 | 0.61 |
| NLD | 0.42 | 0.60 | 19.04 | -29.52 | 5.08 | -0.72 | 4.60 | 372 | 0.68 |
| NOR | 1.07 | 1.00 | 20.11 | -22.44 | 6.18 | -0.25 | 0.73 | 372 | 0.49 |
| NZL | 1.05 | 1.13 | 18.71 | -15.11 | 4.46 | -0.07 | 1.55 | 372 | 0.27 |
| PRT | 1.13 | 1.36 | 18.02 | -23.85 | 5.72 | -0.58 | 2.14 | 346 | 0.34 |
| SGP | 0.42 | 1.12 | 12.97 | -46.72 | 5.39 | -3.04 | 19.68 | 372 | 0.39 |
| SWE | 0.78 | 0.99 | 20.71 | -27.09 | 5.91 | -0.54 | 3.74 | 372 | 0.63 |
| USA | 0.70 | 0.91 | 17.03 | -48.41 | 4.54 | -3.04 | 26.56 | 1092 | 0.91 |
| INT | 0.59 | 0.74 | 17.81 | -24.26 | 3.87 | -0.99 | 7.09 | 326 | 1.00 |

**Table XIII**
**Summary Statistics for Country Betting-Against-Beta Factors**

In this table, we present the summary statistics for the betting-against-beta (BAB) factor for each of the 24 countries in our sample. We report the mean, median, maximum (Max), minimum (Min), standard deviation (SD), skewness (SK), excess kurtosis (EKurt), number of observations (N) and the pairwise correlation to the international BAB factor $\rho_{i,INT}$ for the monthly factor returns. The country SMB factors have varying start dates, however, all samples end in year-end 2017. On the last row in the table, we add the same summary statistics for the international BAB factor for comparability.

| $i$ | Mean$_i$ | Median$_i$ | Max$_i$ | Min$_i$ | SD$_i$ | SK$_i$ | EKurt$_i$ | N$_i$ | $\rho_{i,INT}$ |
|---|---|---|---|---|---|---|---|---|---|
| AUS | 1.60 | 1.19 | 23.69 | -10.22 | 4.86 | 1.07 | 3.08 | 349 | 0.16 |
| AUT | 0.93 | 0.35 | 26.84 | -19.54 | 6.97 | 0.67 | 1.70 | 347 | 0.17 |
| BEL | 0.64 | 0.65 | 23.16 | -18.75 | 4.61 | 0.17 | 2.28 | 347 | 0.29 |
| CAN | 1.73 | 1.37 | 17.07 | -13.10 | 4.35 | 0.03 | 1.63 | 371 | 0.39 |
| CHE | 0.71 | 0.50 | 36.34 | -13.85 | 4.99 | 1.55 | 9.01 | 347 | 0.43 |
| DEU | 0.89 | 1.14 | 17.18 | -21.32 | 4.96 | -0.24 | 1.51 | 347 | 0.49 |
| DNK | 0.76 | 0.58 | 21.07 | -20.72 | 4.99 | 0.20 | 2.17 | 347 | 0.38 |
| ESP | 0.61 | 0.58 | 21.42 | -20.76 | 4.74 | 0.06 | 2.62 | 347 | 0.43 |
| FIN | 1.09 | 0.34 | 31.35 | -20.45 | 6.47 | 0.35 | 2.66 | 347 | 0.35 |
| FRA | 1.30 | 1.17 | 24.39 | -16.83 | 4.78 | 0.28 | 1.83 | 347 | 0.57 |
| GBR | 0.50 | 0.47 | 16.30 | -19.75 | 4.56 | -0.55 | 2.81 | 347 | 0.62 |
| GRC | 0.95 | 0.59 | 38.95 | -34.71 | 8.97 | 0.66 | 3.86 | 315 | 0.11 |
| HKG | 1.72 | 1.33 | 31.31 | -31.03 | 6.63 | 0.64 | 5.52 | 347 | 0.29 |
| IRL | 0.85 | 0.41 | 50.58 | -44.58 | 10.39 | 0.01 | 2.79 | 347 | 0.18 |
| ISR | 1.40 | 1.09 | 24.07 | -11.12 | 4.36 | 0.95 | 3.81 | 239 | 0.06 |
| ITA | 0.75 | 0.47 | 22.44 | -14.27 | 4.07 | 0.61 | 3.10 | 347 | 0.34 |
| JPN | 0.38 | 0.45 | 15.09 | -11.66 | 4.24 | 0.10 | 0.60 | 347 | 0.42 |
| NLD | 0.98 | 1.07 | 21.73 | -19.36 | 4.54 | -0.23 | 3.15 | 347 | 0.44 |
| NOR | 1.02 | 0.76 | 22.48 | -18.70 | 6.06 | 0.12 | 1.17 | 347 | 0.34 |
| NZL | 1.27 | 0.76 | 25.83 | -17.63 | 6.00 | 0.72 | 2.14 | 347 | 0.01 |
| PRT | 1.39 | 0.77 | 51.30 | -23.35 | 9.61 | 1.47 | 4.86 | 322 | 0.16 |
| SGP | 1.18 | 0.97 | 29.68 | -13.46 | 4.01 | 1.23 | 9.43 | 347 | 0.21 |
| SWE | 1.16 | 1.01 | 23.33 | -20.70 | 5.59 | -0.22 | 2.74 | 347 | 0.49 |
| USA | 0.67 | 0.77 | 18.65 | -21.95 | 3.17 | -0.81 | 7.20 | 1045 | 0.88 |
| INT | 0.77 | 0.98 | 10.44 | -12.48 | 2.87 | -0.58 | 2.62 | 371 | 1.00 |

# C Country-Level Mean-Variance Efficient Portfolios

**Table XIV**
**Country-Level Mean-Variance Efficient Factor Alphas**

In this table, we present the results from the univariate time series regression on the volatility-managed mean-variance efficient (MVE) portfolios on their non-managed counterparts. The MVE portfolios are formed on a country level with the full sample of different factor combinations in 24 developed markets. These MVE portfolios are then evaluated in accordance with $f_t^{\sigma} = \alpha + \beta f_t + \epsilon_t$. We add the market factor (MKT) and international MVE factor results for comparability. Alphas are annualized by multiplying monthly alphas by 12 and presented as percentages. Returns used in the regressions are monthly with varying start dates, however, all samples end in year-end 2017. All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level.

| | $MKT_i$ | | $FF3_i$ | | FF3&$MOM_i$ | | $ALL_i$ | |
|---|---|---|---|---|---|---|---|---|
| $i$ | $\alpha$ | t | $\alpha$ | t | $\alpha$ | t | $\alpha$ | t |
| AUS | 1.51 | 0.44 | 0.38 | 0.40 | 3.93 | 3.65*** | 3.57 | 4.10*** |
| AUT | 2.98 | 0.83 | 1.23 | 1.35 | 0.67 | 0.63 | 1.01 | 1.02 |
| BEL | 6.33 | 2.26** | 2.10 | 1.29 | 3.04 | 2.85*** | 3.29 | 3.15*** |
| CAN | -3.17 | -1.00 | 0.80 | 0.75 | 4.74 | 3.63*** | 4.11 | 1.67** |
| CHE | 3.06 | 1.52 | 1.22 | 0.99 | 4.29 | 3.67*** | 4.47 | 4.06*** |
| DEU | 4.05 | 1.59 | 4.41 | 1.95* | 8.01 | 3.58*** | 4.64 | 1.91* |
| DNK | 7.67 | 3.02*** | 4.18 | 2.40** | 3.93 | 3.63*** | 3.65 | 3.25*** |
| ESP | 4.26 | 1.51 | 4.47 | 1.88* | 3.26 | 2.83*** | 3.06 | 2.63*** |
| FIN | 9.82 | 2.50** | 3.60 | 2.47** | 3.31 | 3.20*** | 2.21 | 2.14** |
| FRA | 3.28 | 1.24 | 1.54 | 1.64 | 3.72 | 4.55*** | 3.63 | 2.79*** |
| GBR | 4.31 | 1.85* | 1.01 | 0.93 | 6.24 | 4.65*** | 6.53 | 4.69*** |
| GRC | 6.89 | 1.33 | -3.79 | -1.09 | 2.34 | 0.99 | 0.32 | 0.07 |
| HKG | 12.56 | 3.73*** | 3.63 | 2.14** | 5.86 | 3.60*** | 9.04 | 5.74*** |
| IRL | 10.35 | 2.53** | 9.67 | 3.09*** | 6.03 | 2.68*** | 5.16 | 2.47** |
| ISR | -1.29 | -0.34 | 2.40 | 1.32 | 4.81 | 2.68*** | 3.66 | 2.56** |
| ITA | 4.21 | 1.37 | 7.33 | 2.66*** | 6.26 | 3.86*** | 3.81 | 2.51** |
| JPN | -0.58 | -0.23 | 2.55 | 2.50** | 2.56 | 2.90*** | 1.75 | 2.37** |
| NLD | 6.81 | 2.67*** | 1.93 | 1.39 | 2.94 | 2.70*** | 2.42 | 2.16** |
| NOR | 6.15 | 1.72* | 2.15 | 1.56 | 4.27 | 3.58*** | 3.65 | 2.92*** |
| NZL | 6.35 | 2.37** | 2.69 | 1.51 | 1.74 | 2.40** | 1.80 | 2.35** |
| PRT | 7.15 | 2.34** | 3.08 | 2.54*** | 3.24 | 3.14*** | 2.62 | 2.38** |
| SGP | 3.70 | 1.10 | 3.36 | 1.78* | 3.30 | 2.33** | 5.12 | 3.88*** |
| SWE | 6.92 | 2.27** | 2.86 | 1.69* | 3.63 | 2.93*** | 5.07 | 2.95*** |
| USA | 4.52 | 2.75*** | 2.10 | 2.39** | 3.69 | 5.41*** | 3.20 | 4.24*** |
| INT | 3.41 | 1.47 | 1.37 | 1.60 | 2.91 | 3.30*** | 3.67 | 3.90*** |

## Table XV
## Country-Level Mean-Variance Efficient Factor Alphas with Risk Parity Control

In this table, we present the results from the time series regression on the volatility-managed mean-variance efficient (MVE) portfolios on their non-managed counterpart and a risk parity factor for control. The MVE portfolios are formed on a country level with the full sample of different factor combinations in 24 developed markets. These MVE portfolios are then evaluated in accordance with $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. The risk parity factor added is constructed using the same underlying factors as for the MVE portfolios. The risk party factor is given by $RP_{t+1} = b_t' f_{t+1}$, with $b_{i,t} = \frac{1/\hat{\sigma}_t^i}{\Sigma_i 1/\hat{\sigma}_t^i}$ and $f$ being a vector of factors. The definition follows the approach suggested by Asness, Frazzini, and Pedersen (2012). We add the market factor (MKT) and international MVE factor results for comparability. Monthly alphas are annualized by multiplying monthly returns by 12 and presented as percentages. Returns used in the regressions are monthly with varying start dates, however, all samples end in year-end 2017. All t-statistics are calculated using White-adjusted standard errors to account for heteroscedasticity. The number of stars, ranging from one to three, indicate if the results are statistically significant at a 10%, 5% or 1%-level.

| | MKT$_i$ | | FF3$_i$ | | FF3&MOM$_i$ | | ALL$_i$ | |
|---|---|---|---|---|---|---|---|---|
| $i$ | $\alpha_{RP}$ | t | $\alpha_{RP}$ | t | $\alpha_{RP}$ | t | $\alpha_{RP}$ | t |
| AUS | 1.51 | 0.44 | 0.82 | 0.80 | 4.52 | 4.07*** | 4.04 | 4.42*** |
| AUT | 2.98 | 0.83 | 1.34 | 1.39 | 0.72 | 0.59 | 1.24 | 1.11 |
| BEL | 6.33 | 2.26** | 2.29 | 1.29 | 3.15 | 2.94*** | 3.55 | 3.19*** |
| CAN | -3.17 | -1.00 | 0.68 | 0.62 | 4.61 | 3.42*** | 5.52 | 2.65*** |
| CHE | 3.06 | 1.52 | 1.10 | 0.84 | 4.24 | 3.31*** | 4.48 | 3.73*** |
| DEU | 4.05 | 1.59 | 5.16 | 2.08** | 9.47 | 4.12*** | 6.99 | 3.58*** |
| DNK | 7.67 | 3.02*** | 5.33 | 2.86*** | 3.68 | 2.99*** | 4.06 | 3.16*** |
| ESP | 4.26 | 1.51 | 6.00 | 2.40** | 4.44 | 3.76*** | 4.18 | 3.35*** |
| FIN | 9.82 | 2.50** | 4.29 | 2.85*** | 3.49 | 3.06*** | 1.68 | 1.46 |
| FRA | 3.28 | 1.24 | 1.85 | 1.85* | 4.00 | 4.57*** | 3.85 | 2.67*** |
| GBR | 4.31 | 1.85* | 1.35 | 1.20 | 6.41 | 4.93*** | 6.97 | 4.88*** |
| GRC | 6.89 | 1.33 | -3.56 | -1.02 | 2.54 | 1.00 | 2.22 | 0.50 |
| HKG | 12.56 | 3.73*** | 3.82 | 2.00** | 6.35 | 3.55*** | 8.80 | 5.33*** |
| IRL | 10.35 | 2.53** | 8.49 | 2.57** | 4.79 | 2.00** | 4.89 | 2.11** |
| ISR | -1.29 | -0.34 | 2.97 | 1.35 | 5.88 | 3.45*** | 2.54 | 1.56 |
| ITA | 4.21 | 1.37 | 7.33 | 2.51** | 6.58 | 3.71*** | 4.47 | 2.81*** |
| JPN | -0.58 | -0.23 | 2.83 | 2.55** | 2.72 | 2.80*** | 1.90 | 2.36** |
| NLD | 6.81 | 2.67*** | 2.85 | 2.01** | 3.52 | 3.44*** | 3.11 | 2.82*** |
| NOR | 6.15 | 1.72* | 1.62 | 1.05 | 4.25 | 3.30*** | 3.38 | 2.59*** |
| NZL | 6.35 | 2.37** | 2.47 | 1.29 | 2.08 | 2.96*** | 2.05 | 2.62*** |
| PRT | 7.15 | 2.34** | 2.94 | 2.21** | 2.54 | 3.22*** | 1.93 | 1.79* |
| SGP | 3.70 | 1.10 | 2.84 | 1.33 | 3.44 | 2.16** | 4.98 | 3.52*** |
| SWE | 6.92 | 2.27** | 3.20 | 1.72* | 3.68 | 2.79*** | 5.02 | 2.72*** |
| USA | 4.52 | 2.75*** | 2.13 | 2.34** | 3.79 | 5.43*** | 2.57 | 3.76*** |
| INT | 3.41 | 1.47 | 1.79 | 2.16** | 2.81 | 3.15*** | 3.97 | 3.88*** |