Solvency Capital Requirement Coverage Ratio at Risk

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Abstract
The Solvency II regulation is an important part of a property insurance company's reality. There is a need to complement the risk management focus on value changes and the financial result with a focus on the regulatory consequences of the value changes. The most important measure in the regulation is the Solvency Capital Requirement coverage ratio. This thesis investigates the risk of a changed Solvency Capital Requirement coverage ratio due to risk in the investment portfolio. To do this a new key ratio, the Solvency Capital Requirement coverage ratio at risk (SCRCRaR), is developed. The SCRCRaR measure is strongly influenced by the value at risk approach. Instead of calculating the negative value change that will be surpassed with a certain probability, the SCRCRaR calculates the negative Solvency Capital Requirement coverage ratio change that will be surpassed with a certain probability. The results presented show that the SCRCRaR is significantly different from the value at risk, but that big negative changes in the Solvency Capital Requirement coverage ratio can be accurately estimated from value changes of the investment portfolio. If a property insurance company wants to improve the risk profile of its Solvency Capital Requirement coverage ratio, diversification among the risks in the Solvency II regulation's Standard Formula is key.

JEL Classification: C15, G32
Keywords: Solvency Capital Requirement Coverage Ratio at Risk, Monte Carlo Simulation, Value at Risk, Solvency II, Risk Management, Mathematical Finance, Financial Engineering

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1 Introduction

1.1 Background

1.1.1 Financial risk management

In the aftermath of the Great Recession, risk management has become a hot topic all over the financial world. The crisis shed light on the importance of having control over how much risk you take as a firm[1]. The trend of firms focusing more on risk management is also a result of regulatory demands such as the Basel regulation in the banking industry and the Solvency regulation in the insurance industry.

There are currently multiple methods used to model financial risk. The simplest one is to use standard deviations as a measure of risk. This approach, however, has the important drawback that it treats positive and negative deviations equally. In most risk management settings, what you care about are the potential losses. As a response to this and to the financial disasters of the early 1990s, the value at risk (VaR) approach was developed (Jorion, 2007). The VaR method calculates a negative value change of the investment portfolio that will be surpassed with a certain probability. If the 99% 1-year VaR is SEK 1 million, you can be 99% sure that the investigated portfolio will not lose more than SEK 1 million in one year[2].

1.1.2 The property insurance industry

The focus in this thesis is on the property insurance industry. This industry is strongly influenced by the Solvency II regulation, that is implemented by all insurance companies in the European Union ('Solvency II', 2018). As the market risk calculations in the regulation are standardized, their precision when it comes to predicting the potential losses of a particular company's investment portfolio is not very high. For example, all listed stocks in the OECD or the European Economic Area are stressed by 39% and it is not controversial to state that they are not equally risky[3]. A stress of 39% means a calculated value change of -39%, so the stressed asset value is 61% of the current value. The impreciseness of the Standard Formula shows the need

[1] Notice, however, that bad performance during the recession was not necessarily a result of bad risk management. The bad performing firm may have been aware of the risk it took and probably thought it was a risk that was worth to take.

[2] Given that the VaR model is well-calibrated.

[3] A symmetric adjustment of up to +/-10%, depending on the stock market development in the European Union is also added to the 39%.

for a more accurate way to measure the potential losses of the investment portfolios.

Nevertheless, the Solvency II regulation is an important part of a property insurance company's reality. The Solvency Capital Requirement coverage ratio (SCRCR) is calculated as own funds divided by the stress on the assets (European Commission, 2015). Own funds is the value of the company's assets minus its liabilities. A SCRCR of at least 100% is considered acceptable and at least 130% is recommended to guarantee an acceptable value after a shock (Morales, 2012). Accordingly, the property insurance companies need to complement their focus on value changes and the financial result with a focus on the regulatory consequences of the changes in for example value. Despite its importance, this aspect of risk management has not got much attention in the academic literature so far.

1.1.3 Dina Försäkring AB

This master thesis is written in collaboration with Dina Försäkringar (Dina). Dina is currently Sweden's sixth largest property insurer ('Dina Försäkringar', 2018). 31 December 2017 the company group had a market share of 2.73% (*Dina Försäkring AB Årsredovisning 2017*, 2018). Dina consists of eleven mutually owned and locally based insurance companies that together cover all of Sweden. The eleven local companies together own Dina Försäkring AB (Dina AB), a company focused on service, development, and reinsurance ('Dina Försäkring AB', 2018). 31 December 2017 the investment management division at Dina AB was responsible for a portfolio of approximately SEK 3.2 billion. This portfolio is in focus of this thesis. The asset allocation is shown in Figure 1, where fully owned property companies are placed in the property category despite the fact that they are owned through stocks. In the Appendix, Section 8.1, the separate pieces of this diagram are divided into subcategories.

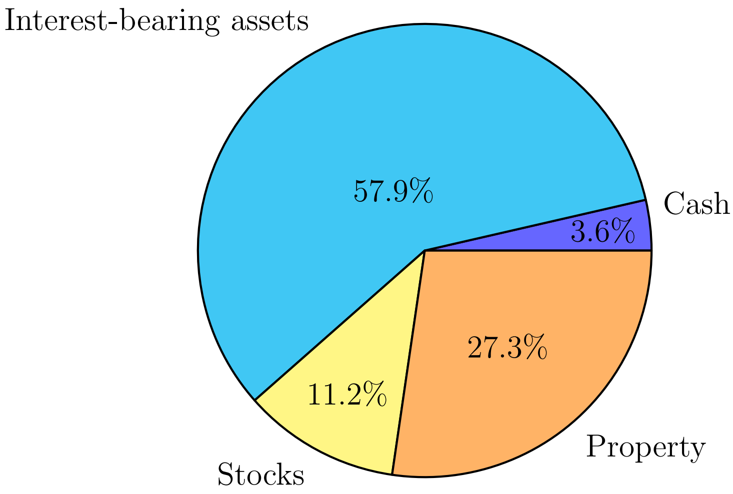

Figure 1: Dina Försäkring AB's asset allocation 31 December 2017.

Currently the investment management division at Dina AB focuses only on the Solvency II Standard Formula in its market risk management, so there is a need for a more sophisticated way of modelling market risk.

1.2 Purpose

Dina AB serves as an example of a property insurance company and the academic purpose of this thesis is to find a way to assess the risk of a changed SCRCR due to risk in the investment portfolio.

The ambition is also to develop a model for risk assessment that can help Dina AB to improve in terms of market risk management. The developed model will hopefully lead to a better understanding of market risk at the company.

1.3 Problem specification

The research question that is going to be answered in this thesis is:
How can property insurance companies evaluate the risk for large drops in their Solvency Capital Requirement coverage ratios due to risk in their investment portfolios?

More concretely, the ultimate goal of this thesis is to calculate the Solvency Capital Requirement coverage ratio at risk (SCRCRaR). SCRCRaR is a new key ratio developed in this thesis, that is strongly influenced by the VaR approach. Instead of calculating the negative value change that will be surpassed with a certain probability, SCRCRaR calculates the negative SCRCR change that will be surpassed with a certain probability. After discussions with the executives at Dina AB, the SCRCRaR that will be in focus of this thesis is the 99.5% 1-year SCRCRaR.

1.4 Structure

The way towards the goal of calculating SCRCRaR goes through five steps, described in Table 1, with their own unique challenges. The table also shows in which sections the steps are discussed.

Step	Description	Section
1	Import current positions and risk factors	Data
2	Generate scenarios	Method
3	Calculate new values of the assets	Method, Results and discussion
4	Calculate VaR and backtest the model	Method, Results and discussion
5	Calculate SCRCRaR	Method, Results and discussion

Table 1: Steps in the development of the Solvency Capital Requirement coverage ratio at risk model and the sections in which they are discussed.

The demanding part of the first step is to import the risk factors. This is associated with two main challenges. Firstly, the import procedure must be made as dynamic as possible, so that the model will be easy to use in the ongoing business at Dina AB. As this is a thesis in finance and not in programming, this aspect is only discussed very briefly. Secondly, there must be enough good data to be able to draw conclusions. This is key to the quality of this thesis, as the data sets the limit for how accurate the SCRCRaR calculation can be. In this step all data about the current positions is also gathered, from Dina's internal systems.

The second step is to generate future scenarios. The imported risk factors are an important input to this process. This step is not limited by external factors in the same sense as the first step, so here it is up to the author of this thesis to find a reasonable approach.

The third step is to calculate new values of the assets, based on the scenarios generated in step 2. Data limitations from step one will force the author of this thesis to spend a lot of time on figuring out how to value different types of assets.

The fourth step tests the quality of the scenarios generated (step 2) and how the scenarios are translated to new asset values (step 3). This is done by calculating VaR and by running backtests to assess its accuracy. Focus here is on the 99.5% 1-year VaR. This is the VaR that the Standard Formula in the Solvency II regulation calculates (*'SOLVENCY II': Frequently Asked Questions (FAQs)*, n.d.).

The fifth and final step is to calculate SCRCR in the different scenarios generated in the first three steps and to use these calculations to compute the 99.5% 1-year SCRCRaR. The SCRCR calculations are affected by the new market values and credit ratings of the financial assets and by the new values of a couple of input parameters. In an ideal world the SCRCRaR model would have been backtested in a similar way as the VaR model. However, restrictions in terms of data and computational power make such an approach impossible.

The amount of data that this process handles requires an efficient programming language. Therefore SAS is used, as SAS is the programming language used for heavier numerical analysis at Dina AB. A modular approach to programming is used, to make parts of this framework easier to decompose and use in different settings in the future.

2 Review of literature

2.1 Risk budgeting

A key to successful investment management in the property insurance industry is to understand risk. Taking the right kind and amount of risk at the right time is crucial for the long term result. For this, a central concept is risk budgeting. Risk budgeting means to decompose the risk of an investment portfolio into its constituents by using risk measures and to base the asset allocation on these measures (Pearson, 2011). No consideration is taken to the returns of the assets (Bruder & Roncalli, 2012). For example, stocks may be allowed to represent 50% of the total risk in the portfolio. The big challenge is to define how risk is measured. Successful risk budgeting is dependent on accurate risk measures and one popular measure used in risk budgeting is VaR (Culp, Mensink, & Neves, 1998). This thesis has its starting point in the VaR measure.

2.2 Value at risk

2.2.1 Basics of value at risk

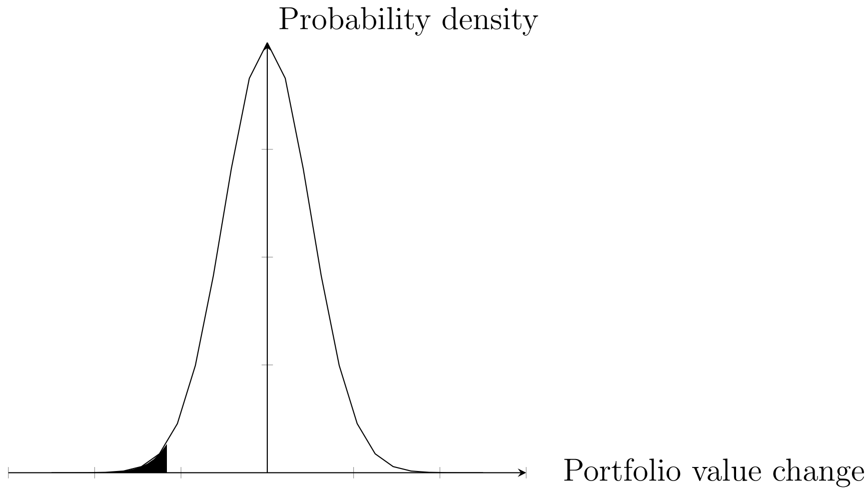

Figure 2: Illustration of the 99% value at risk in the case of a normal distribution. The total area under the graph is 100% and the worst percentage is marked.

The VaR approach was developed as a response to the financial disasters of the early 1990s (Jorion, 2007). The method calculates a negative value change of the investment portfolio that will be surpassed with a certain probability. If the 99% 1-year VaR is SEK 1 million you can be 99% sure that the investigated portfolio will not lose more than SEK 1 million in one year[4]. In Figure 2, the right part of the marked area corresponds to a portfolio value change of SEK -1 million. As you can see, the outcome can be worse than SEK -1 million, but the probability for this is only 1%. To calculate VaR, three things need to be decided (Papaioannou, 2006):

1. Whether the portfolio value change is determined in a currency or in percentage (SEK in the example above).

2. The VaR horizon (one year in the example above). The VaR for the horizon can be calculated by multiplying the 1-day VaR with the square root of the horizon in trading days. The underlying assumption is that VaR is proportional to the standard deviations of the risk factors (the variance increases in proportion to the elapsed time, and the variance is the square of the standard deviation). For this to hold, there must be no autocorrelation (one daily risk factor value does not affect the next day's risk factor value) and the data series must fit a standard normal distribution (Nath, 2003). The standard is to assume that a year has 252 trading days.

3. The confidence level (99% in the example above). This is a measure of how sure you can be that the portfolio losses over the chosen horizon do not exceed the VaR.

2.2.2 Different kinds of value at risk models

The risk in the investment portfolio is measured by statistical models or simulation models starting from some chosen risk factors and the portfolio positions (Jorion, 2007). Common risk factors are the rates of return for the assets in the portfolio. The historical values of the risk factors and distribution assumptions are used to calculate new values of the risk factors. The new values of the risk factors affect the values of the assets in the portfolio and thereby the value of the entire portfolio. This process is determined by a valuation model. So all VaR methods can be categorized in two dimensions:

1. Valuation methods for the assets.

2. Distribution assumptions for the risk factors.

[4]Given that the VaR model is well-calibrated.

When it comes to the valuation methods, there are two categories. Local-valuation methods revalue the portfolio by using local derivatives (Jorion, 2007). The effects of risk factor value changes are estimated by the linear asset value changes due to risk factor value changes, given the current asset values and a risk factor value of 0 (see Figure 3). This is a good method if the value changes of the assets due to value changes of the risk factors are approximately linear. When the relationship is not linear, the model works worse. Therefore, this approach is less suitable for modelling credit risk. The main advantage of this approach is the speed. A less complicated valuation requires less computational power. The most popular method within the local-valuation category is the variance-covariance model that is also called the delta-normal model. The distribution assumption for the variance-covariance model is that the risk factors are normally distributed. Sometimes, if the number of risk factors is small, second-order derivatives can also be used in local-valuation methods.

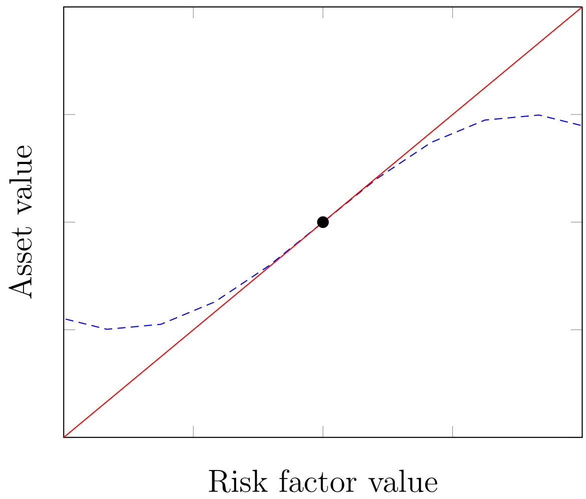

Figure 3: Illustration of the local-valuation method. The black point shows the value of the asset if the risk factor value is 0. If the asset value is affected by the risk factor in the way shown in the dashed blue curve, the linear approximation shown in the solid red line is inaccurate for large positive or negative risk factor values. If the asset value is affected by the risk factor in a way similar to the solid red line, the solid red line is a good prediction.

The second category consists of full-valuation methods. Full-valuation methods revalue the portfolio completely over a range of scenarios. Of course, these valuation methods are more accurate, but they also require more computational power than local-valuation methods. The most popular methods within this category are historical simulation and Monte Carlo simulation methods (Jorion, 2007). The historical simulation approach is a nonparametric method, which means that no assumptions about the distributions of the risk factors are required and no covariance matrix needs to be calculated. The historical simulation method instead uses the historical risk factor outcomes to revalue the portfolio. One historical risk factor outcome is drawn for each simulation. This fact makes the method easier to explain and to justify, as you can go back and analyze the circumstances leading to such a loss as the VaR calculates. Overall, this model has many appealing characteristics, but one important drawback is that the sampling variation is high. A 99.5% VaR means one tail event in 200 days, so the VaR measure is highly affected by a small number of historical days, which is negative for the robustness. If there was no extreme negative value change in the time window from which risk factor values were drawn, the model may underestimate the risk for large losses. If, on the other hand, there were a couple of really extreme negative events, the model may overstate the probability of such events. Very much historical data needs to be used to deal with this problem, meaning that more old and probably outdated data is used. Furthermore, this method is not suitable for measuring credit risk. Basing future credit rating changes on historical changes is not a reasonable approach, as this would imply no credit risk in bonds that never have been downgraded before.

The Monte Carlo simulation method is parametric, which means that random movements of the risk factors are generated, based on estimated distributions (Jorion, 2007). The estimated distributions in this stochastic process are based on the risk factors' historical movements and their correlations with each other. The Monte Carlo simulation approach is the most flexible method, meaning that it allows more comprehensive modelling (Ammann & Reich, 2001). This approach has the highest potential in terms of accuracy and it is most suitable for measuring credit risk. The method, however, also has a number of disadvantages. The most important one is that it is the most computationally intensive one and it requires a lot of computational power. Therefore, the trend of faster computers has made the method increasingly popular (Förster, 1997). Another important disadvantage is that the method relies on assumptions when the stochastic process is modelled (Jorion, 2007). Furthermore, the estimates are affected by sampling variation, if the number of simulations is not high. A 99% VaR means one tail event in 100 simulations, so the VaR measure is highly affected by a small number of simulations. The problem is, however, smaller here than for historical simulations as even a low number of simulations (such as 1000) means that at least a few unique observations worse than the VaR are expected (10 for 1000 simulations).

There are also methods that combine the mentioned methods. A grid

Monte Carlo approach means that exact valuations are calculated at a number of grid points (Jorion, 2007). The portfolio value in each simulation is then approximated by linear interpolation from the values at the adjacent grid points. In this way, the calculations can be made simpler and faster than in a regular Monte Carlo simulation, which is why this method was invented.

2.2.3 Evaluation of existing value at risk literature

VaR has a number of advantages. Even though the calculations behind can be difficult, the result boils down to one number that is easy to understand and to communicate. Furthermore, VaR can be calculated for different kinds of assets and portfolios of assets and comparisons are easy to make[5]. These characteristics have made VaR popular among firms and the measure is included in many financial softwares.

Nevertheless, the approach also has a number of disadvantages. Nassim Taleb has pointed out a number of them (Taleb, 1997). His main critique is that the probability estimations are so imprecise that it is not sensible to rely on them, especially for tail events (infrequent and really large losses). He emphasizes that the model risk, the risk of using a misspecified model, can outweigh the fact that the model helps you understand the risk you take. A model is a simplification of the reality and not the reality itself, so if the model is too far from the reality it loses its value. Therefore, Taleb is overall utterly skeptical towards the field of financial engineering. The quotation "nothing predictable can be truly harmful and nothing truly harmful can be predictable" illustrates Taleb's general view of quantitative methods for risk management. The author of this thesis thinks that quantitative models can be really useful if the models are well-calibrated, but agrees that relying on a measure that appears to be very precise can be dangerous when the measure is not that exact. This is the most important objection against the VaR framework. A sense of false certainty can be created (Hoppe, 1998). To minimize this risk, solid backtesting is essential (Nieppola et al., 2009).

More concretely, Taleb points out that "the one week volatility of volatility is generally between 5 and 50 times higher than the one week volatility", which makes VaR predictions based on volatility very volatile and imprecise (Taleb, 1997). As a response to this kind of critique, GARCH models have been included in the VaR framework, to model volatilities that change over time (Jorion, 1997).

Much of the critique towards VaR is also centered around the fact that the model does not calculate how bad outcomes might be in a worst case scenario.

[5]Comparisons are easy to make given that the same VaR model is used. Comparing VaR numbers calculated in different ways is much harder.

However, the VaR framework has never had an ambition to calculate how bad outcomes might be and therefore the framework should not be the only risk management tool used (Jorion, 1997). The author of this thesis thinks that criticizing VaR based on the fact that the model does not calculate how bad outcomes might be in a worst case scenario is like criticizing stocks based on the fact that they, one-by-one, are more volatile than stock funds (see for example Figure 5 in Section 3.3). No sensible investor holds only one stock and no sensible risk manager uses only one risk measure. To measure the outcomes in the cases of outcomes worse than the VaR, measures such as expected shortfall (or conditional value at risk) have been developed (Acerbi & Tasche, 2002). Stress tests can also be useful for this.

To conclude this section, the author of this thesis interprets the existing VaR literature in the following way:

1. VaR is a measure with a number of positive characteristics and should be included as a part of any sensible risk management framework.

2. VaR is a measure with a number of weaknesses that makes it inappropriate to use as the only measure in a risk management framework.

3. VaR is a measure with a potential for development.

This thesis wants to contribute to the field of VaR literature by extending the model and applying it in a slightly different context, where SCRCR changes are in focus instead of value changes.

2.3 Solvency II

The Solvency II regulation has had a decisive impact on Swedish insurance companies (Andersson & Lind, 2016). Overall, the regulation has increased the desire from companies to create sound risk cultures and the risk awareness among employees. The requirements on both organizations and employees have increased. On the negative side, the extensive workload and the system requirements associated with Solvency II have lead to increased costs.

The Standard Formula and the SCRCR is the main tool for calculating a company's solvency level in the Solvency II regulation (Andersson & Lind, 2016). A SCRCR of at least 100% is considered to be acceptable and at least 130% is recommended to guarantee an acceptable value after a shock (Morales, 2012). Having a SCRCR over these numbers is accordingly crucial for all insurance companies. Despite this fact, very little research has been undertaken to investigate the risk of decreases in the SCRCR. Even the European Insurance and Occupational Pensions Authority (EIOPA) base their stress tests on the difference between assets and liabilities rather than on changes in the SCRCR[6] (*2016 EIOPA Insurance Stress Test Report*, 2016). There is a need for new risk measures focusing on the SCRCR and the SCRCRaR measure developed in this thesis is a first step. Hopefully more research will follow later on.

[6]EIOPA is the financial regulatory institution of the European Union focused on the insurance and occupational pensions sector ('About EIOPA', 2018).

3 Data

3.1 Current positions

Figure 1 showed Dina AB's asset allocation 31 December 2017. To model the riskiness of the investment portfolio, a slightly different division is made. Before the new division is made, three holdings are excluded from the portfolio. Dina Försäkringar Mälardalen AB will be merged with Dina AB during 2018 and as the acquisition price is already decided, the executives at Dina AB see no reason to spend time on modelling the riskiness of this stock. It constitutes 1.31% of Dina AB's investment portfolio. LFS Invest AB is a life-insurance product that constitutes 0.15% of Dina AB's investment portfolio. This product has performed disastrously and the ambition is to sell it as fast as possible and the company has no plan of holding any similar assets in the future. A corporate bond in Mobylife Holding has defaulted and trades at 5% of face value and this bond is also excluded (0.01% of the portfolio). These three assets are together 1.47% of Dina AB's investment portfolio and they are excluded in all calculations from now on.

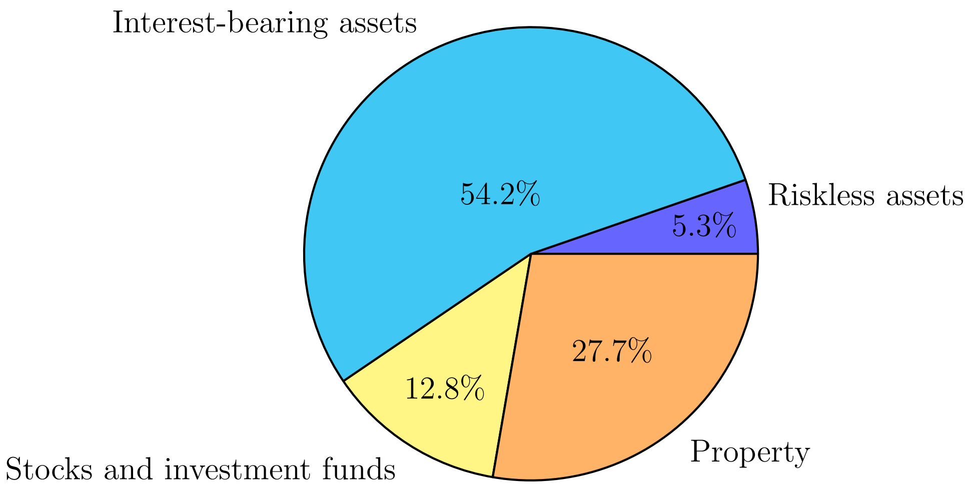

Figure 4: Dina Försäkring AB's calculation category allocation 31 December 2017.

After the three assets have been excluded, the portfolio is divided into four calculation categories. These categories are different in terms of how their risks are modelled, rather than in terms of the size or the nature of their risks. For example, bond funds have more in common with bonds than with stocks, but the method that will be used to model bond fund risk and equity risk is the same. The calculation categories are described below and the calculation category allocation is shown in Figure 4.

1. *Riskless assets*, consisting of assets that are not revalued in the model. This category consists of cash, intra-group loans, and of the unlisted stock SOS International. For cash, there is of course a risk that a bank defaults, but modelling such a risk is out of this thesis' scope. The intra-group loans are loans given to the property companies Diana Skog AB and Dina Palaisbacken AB and as the cash flows of these companies are much higher than the payments associated with the loans, the loans are considered to be risk-free in a 1-year horizon. Furthermore, both companies hold a lot of cash (Dina Palaisbacken AB plan to repay their loan during 2018). SOS International is a collaboration among Nordic insurance companies and the purpose of the company is to assist Nordic citizens that get into emergency situations abroad ('SOS International', 2018). This stock is valued to the acquisition value and it is never expected to be sold. No risk factors are needed for this category.

2. *Stocks and investment funds*, consisting of assets whose new values are calculated directly from imported daily price changes of the assets and of the currencies in which they are traded. This category includes all stocks and investment funds, except stocks in fully owned companies that are exposed to property risk.

3. *Property*, consisting of assets whose new values are calculated by regression models. The reason why the values are calculated in this way is that daily price observations are not available for these assets. This category consists of real estate (fully owned through stocks and directly owned) and a fully owned forest company.

4. *Interest-bearing assets*, consisting of all assets whose new values are calculated from cash flows, risk-free interest rates, and credit spreads. These assets are exposed to interest rate risk, spread risk, and credit risk. This category consists of all directly owned interest-bearing assets, except the intra-group loans. Unfortunately, the look-through information from the investment funds is not good enough to enable this method, so all investment funds are included in the stocks and investment funds category instead. This means that bond funds will be valued based on their previous value changes rather than on their interest rate risk, spread risk, and credit risk.

Information about the current positions at the calculation date, such as market values, credit ratings, maturities, coupon dates, etcetera is received from Dina's internal systems.

3.2 Calculation date and data period

Five years of data are used for all risk factors. There are two reasons why such a long period is used. Firstly, Dina AB has an investment horizon of approximately five years in the investment portfolio. Secondly, it is necessary to use a long period to take advantage of the few available valuations of the property. For the same reason, the portfolio is evaluated 31 December 2017. An earlier evaluation date would have made backtesting out-of-sample easier, but data for stocks and investment funds is only available from 1 January 2013 and onwards and five years of data is necessary to get enough valuations of the property. So, to summarize, the calculation date is set to 31 December 2017 and data from 1 January 2013 to 31 December 2017 is used for the risk factors.

3.3 Stocks and investment funds risk factors

The risk factor $r_{a,t}$ of asset a at day t is calculated from the price of the asset P_a, as the logarithmic return between day t and the day before, $t-1$:

$$r_{a,t} = ln\left(\frac{P_{a,t}}{P_{a,t-1}}\right) \tag{1}$$

The reason why logarithmic returns are used is that some calculations in this thesis require that returns can be added to each other in time series. Overall logarithmic returns make calculations easier, which is a big advantage when much data is handled.

Almost all prices are imported dynamically from Dina's Millistream database, with data from 2013 and onwards. The only exception is the prices of Danske Invest European Corporate Sustainable Bond Class A-sek h, that are missing in the database and are found at Danske Invest's webpage ('Danske Invest', 2018). The fact that data is only available from 2013 and onwards is limiting for this thesis, but for the application of the model this fact will make a smaller and smaller impact as time passes.

There is, for natural reasons, very little historical data for the subsription rights in Karo Pharma. Therefore, the logarithmic return of the Karo Pharma stock is instead used as the risk factor for the subscription rights. For the same reason, the logarithmic return of the SCA B stock is used as risk factor for Essity B. Essity is a spin-off from SCA and 80% of the former

SCA B value was attributable to the current Essity B stocks (SCA, 2017a). The spin-off occurred 15 June 2017 (SCA, 2017b).

Exchange rates are also taken into account in this category, even though only one stock (Protector Forsikring) is affected by this. Some investment funds have holdings in different currencies, but this risk is reflected in the price changes of the investment funds themselves. The exchange rates are imported dynamically from Riksbanken Web services and Formula 1 is used once again, but this time with FX instead of a as subscript ('Riksbanken Web services', 2018).

To make sure that the quality of the data is good, the daily values of all risk factors are plotted. By doing so it is possible to adjust for some stock splits that have taken place. It is not possible to identify and adjust for all dividends, but the effect of this is not expected to be big. After the adjustments have been made, the daily risk factor values are plotted in Figure 5. As you can see in the graphs, the value changes for investment funds are much smaller than for individual stocks. Holding one investment fund means less risk than holding one stock, which makes sense. You can also see that the currency risk is smaller than the risk of price changes in the assets. The same picture can be seen in Table 2 with descriptive statistics for all stocks and investment funds risk factors and a number of subcategories.

Statistic	N	Mean	Std. Dev.	Min	Max
All stocks and investment funds	69	0.047%	1.632%	-26.149%	28.030%
Stocks in SEK	62	0.046%	1.675%	-26.149%	28.030%
Stocks in other currencies	1	0.154%	1.954%	-10.736%	15.345%
Stock funds	2	0.055%	0.861%	-8.339%	4.885%
Fixed income funds	4	0.012%	0.536%	-3.626%	2.119%
NOK	1	-0.012%	0.475%	-2.288%	2.843%

Table 2: Descriptive statistics of the daily values of the stocks and investment funds risk factors. N is the number of series.

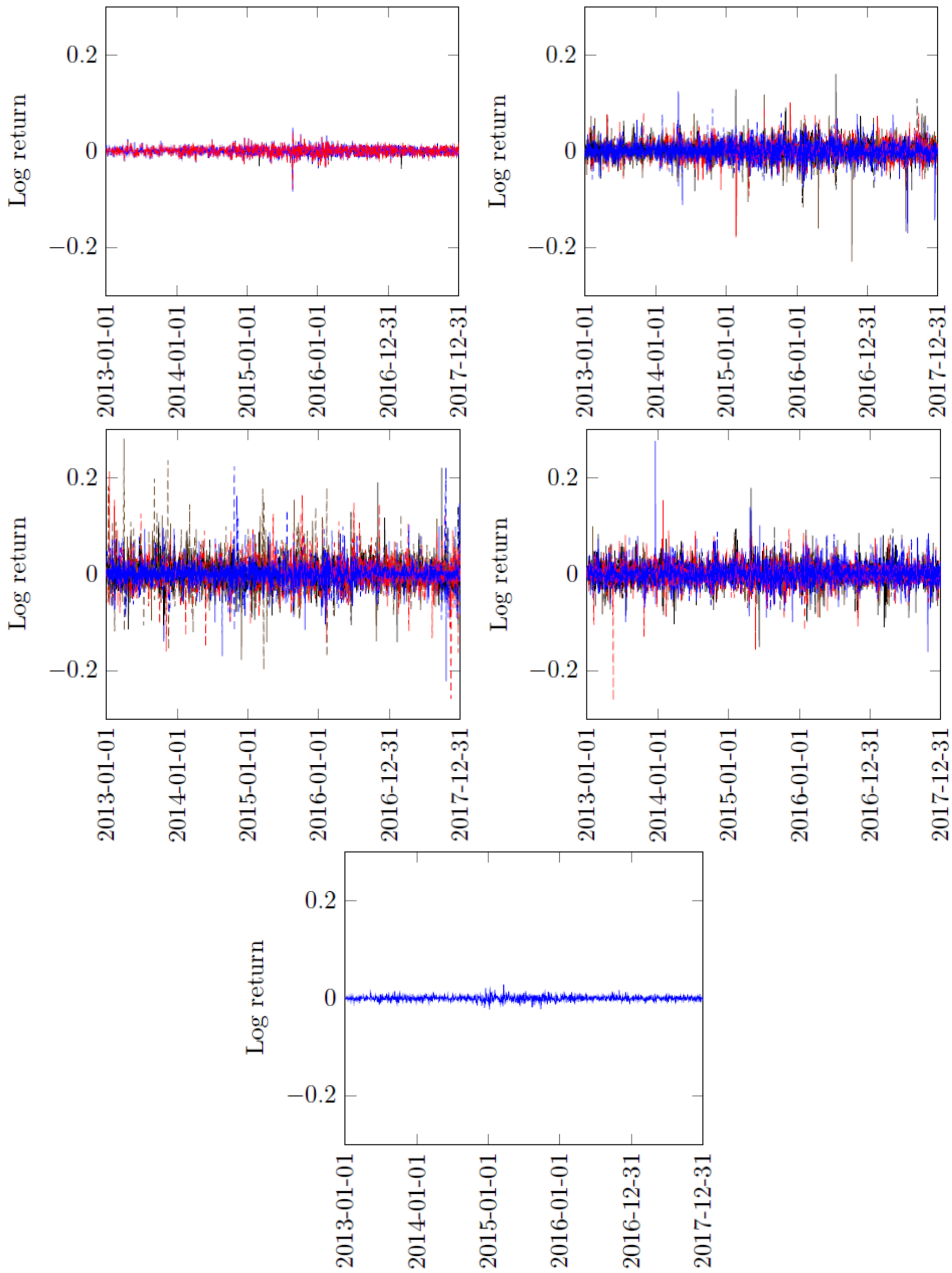

Figure 5: Daily values of the stocks and investment funds risk factors. The log returns of the investment funds are plotted up to the left and the following three graphs represent one third of the stocks each. The last graph shows the log return of the exchange rate SEK/NOK.

3.4 Property risk factors

The property holdings are described in Table 3. All real estate is commercial real estate located in the Old Town in Stockholm and the forest is located in Småland in southern Sweden. Market values of the real estate are received once a year, while the forest is revalued twice a year. Accordingly, the daily value changes need to be modelled. Linear regressions are used for this modelling. The regressions are described in more detail in the Method section (Section 4.3.2). Four risk factors are used in the regressions. The first one is the 10-year swap rate, that is used as risk-free rate for both real estate and forest. The reason why a swap rate is used as the risk-free rate is that the implied risk-free rates are much closer to swap rates than to Treasury rates (Hull, 2015). The reason why the 10-year rate is used is that ten years is the longest horizon available and the property investments are expected to be held for more than ten years. The 10-year swap rate is found at Nasdaq OMX's webpage ('Nasdaq OMX', 2018). The risk factor value is calculated as the daily change in interest rate (see Formula 2 and the motivation for it in the Interest-bearing assets risk factors section that is Section 3.5).

Name	Market value	Ownership	Subcategory
Dina Palaisbacken AB	485 000 000	Fully owned through stocks	Real estate
Diana Skog AB	138 800 000	Fully owned through stocks	Forest
Diana 2	96 000 000	Directly owned	Real estate
Juno 9	55 000 000	Directly owned	Real estate

Table 3: Information about property holdings.

The other risk factors are dependent on the calculation subcategory. For real estate, the thought is that Carnegie Real Estate Index can predict value changes. Carnegie Real Estate Index is a value-weighted index consisting of pure real estate companies from all the different lists at the Stockholm Stock Exchange ('Fastighetsvärlden', 1998). The reason why this index is chosen as a risk factor is that it has daily prices and that it is a common benchmark for investment funds that focus on real estate. Of course this index could be sensitive to the sentiment at the stock market, but the hypothesis is that it will predict the value changes of the real estate well enough. Data is only available for the last five years and since the index values were gathered 26 January 2018, data is missing for some days in the beginning of the data period ('Carnegie Real Estate Index', 2018). The value of this risk factor is calculated with Formula 1.

For forest, it is harder to find reasonable risk factors. A number of different risk factors are investigated and those with the best prediction values are chosen. It is reasonable to believe that factors such as the wood price could have an impact on the price of forest. It is also likely that forest company stocks could correlate with the value of Diana Skog AB (the reasoning is the same as with the Carnegie Real Estate Index for real estate). Thus, the wood price (converted to SEK with the exchange rates data described in the Stocks and investment funds risk factors section that is Section 3.3), found at VA Finans ('VA Finans', 2018), and SCA B, that is already among the stocks and investment funds risk factors, are chosen. The values of these risk factors are calculated with Formula 1.

The returns of the property risk factors are plotted in Figure 6 and descriptive statistics are shown in Table 4. The risk-free rate is, as expected, much less volatile than the other property risk factors. Among the more volatile risk factors, the wood price is most volatile.

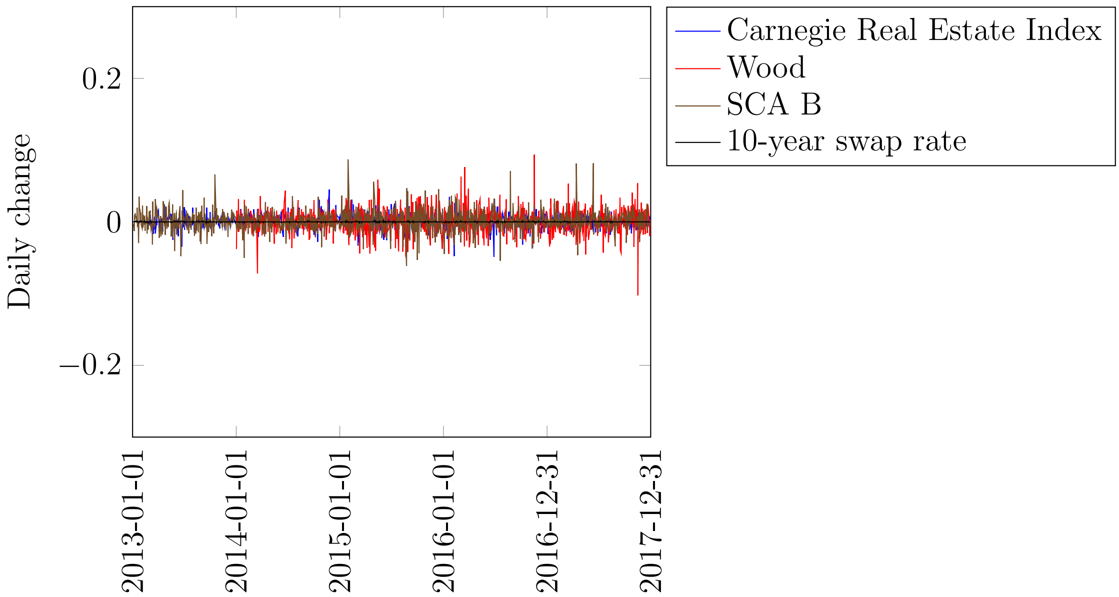

Figure 6: Daily values of the property risk factors.

Statistic	N	Mean	Std. Dev.	Min	Max	Level
Carnegie Real Estate Index	1197	0.064%	0.987%	-4.867%	4.512%	1515.15
Wood	952	0.032%	1.716%	-10.243%	9.379%	3688.03
SCA B	1236	0.095%	1.384%	-6.119%	8.706%	84.45
10-year swap rate	1237	-0.001%	0.035%	-0.134%	0.230%	1.2050%

Table 4: Descriptive statistics of the daily values of the property risk factors and the levels 31 December 2017.

3.5 Interest-bearing assets risk factors

3.5.1 Subcategories

Two discount rates affect the values of interest-bearing assets: the risk-free interest rates (interest rate risk factors) and the credit spreads (spread risk factors). Credit spreads represent the additional yield that investors require to hold an investment with credit risk compared to an investment without credit risk. The credit spread is strongly affected by the credit rating of the financial instrument, as the credit rating is an assessment of the credit risk.

The two subcategories of interest-bearing assets risk factors are discussed in the following two subsections.

3.5.2 Interest rate risk factors

STIBOR rates are used as the risk-free rates for horizons shorter than one year (1 week, 1 month, 2 months, 3 months, and 6 months). The STIBOR rates are imported dynamically from Riksbanken Web services ('Riksbanken Web services', 2018). For horizons longer than one year, the swap rates are imported from Nasdaq OMX ('Nasdaq OMX', 2018). Yearly horizons up to ten years are available. The returns of the risk-free interest rates can not be calculated with Formula 1, as the risk-free interest rates are close to zero. There are four problems associated with using Formula 1:

1. A minimal change means an enormous percentage change if the denominator is close to 0. For example $ln(0.000001/0.0000001) \approx 230\%$.

2. If the risk-free interest rate at time $t - 0$ was 0, no value can be calculated as the denominator is not allowed to be 0.

3. If the risk-free interest rate in one of the times $t - 1$ and t is negative and the other is positive, no value can be calculated as ln can not be calculated for a negative value (e raised to any number can not be negative).

4. If the risk-free interest rate at both $t - 0$ and t is negative, a positive risk-free interest rate change is calculated as a negative return and a negative risk-free interest rate change is calculated as a positive return. It would of course be possible to adjust the formula to take this into account by adding a minus sign before the formula, but due to the previous three problems this is not a reasonable approach.

Because of the reasons listed above, it is more reasonable to look at the level changes in this case, so the formula used here is:

$$r_{h,t} = P_{h,t} - P_{h,t-1} \tag{2}$$

$r_{h,t}$ is the change in the risk-free rate with horizon h, at day t. $P_{h,t}$ is the risk-free rate with horizon h at day t, while $P_{h,t-1}$ was the same rate the day before $(t-1)$.

As you can see in Figure 7, the values of the interest rate risk factors are really small. This should not be interpreted as if these changes are unimportant, as they affect the discounting of very large cash flows. As you can see in Table 5, the 1-week interest rate risk factor is much more volatile (higher standard deviation and higher absolute values of max and min) than the other interest rate risk factors. As expected, you can also see that the yield curve is upward sloping (levels are higher the longer the horizon is).

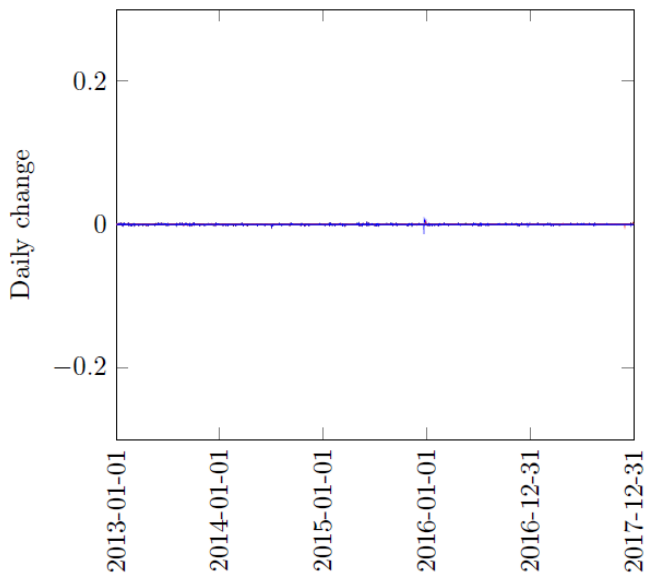

Figure 7: Daily values of the interest rate risk factors.

Statistic	N	Mean	Std. Dev.	Min	Max	Level
1-week	1245	-0.0013%	0.0491%	-1.3200%	0.7460%	-0.5250%
1-month	1245	-0.0014%	0.0180%	-0.2710%	0.2460%	-0.5070%
2-month	1245	-0.0014%	0.0130%	-0.2370%	0.1290%	-0.4800%
3-month	1245	-0.0014%	0.0120%	-0.2400%	0.0860%	-0.4690%
6-month	1245	-0.0014%	0.0109%	-0.2210%	0.0420%	-0.3620%
1-year	1228	-0.0013%	0.0125%	-0.1530%	0.0640%	-0.3600%
2-year	1239	-0.0011%	0.0184%	-0.1370%	0.1000%	-0.1510%
3-year	1239	-0.0010%	0.0221%	-0.1140%	0.1170%	0.0760%
4-year	1239	-0.0009%	0.0265%	-0.1040%	0.1300%	0.2950%
5-year	1239	-0.0009%	0.0297%	-0.1030%	0.1410%	0.4950%
6-year	1239	-0.0008%	0.0313%	-0.1020%	0.1620%	0.6740%
7-year	1239	-0.0008%	0.0325%	-0.1130%	0.1780%	0.8320%
8-year	1239	-0.0007%	0.0337%	-0.1190%	0.1960%	0.9720%
9-year	1239	-0.0007%	0.0346%	-0.1260%	0.2130%	1.0960%
10-year	1237	-0.0006%	0.0354%	-0.1340%	0.2300%	1.2050%

Table 5: Descriptive statistics of the daily values of the interest rate risk factors and the levels 31 December 2017.

3.5.3 Spread risk factors

When it comes to credit spreads, the portfolio is divided into credit quality steps in the way shown in Table 6. This replicates the method in the Solvency II regulation's Standard Formula and the only difference is that Swedbank's ratings are also used (Swedbank's ratings are not allowed to be used in the Standard Formula) (European Commission, 2016). The reason why Swedbank's ratings are used is that Swedbank's assessments of the credit quality of unrated bonds are likely to be more accurate than the option to view them all as equally risky.

Credit quality step	Moody's rating	S&P rating
0	Aaa	AAA
1	Aa	AA
2	A	A
3	Baa	BBB
4	Ba	BB
5	B	B
6	Caa, Ca, C	CCC, CC, R, SD/D

Table 6: Credit quality steps and their equivalents among Moody's and S&P ratings.

For all the credit quality steps, the indices in Table 7 are gathered from FRED Economic Data and their daily changes are used as the spread risk factors ('FRED Economic Data', 2018). Formula 2 is used for these risk factors as well. The fact that these spreads are based on American data is problematic, but due to lack of access to Swedish data, this is the best data that has been found. The reason why this is problematic is that the market conditions in Sweden and in the US are different. For example the risk-free rates are higher in the US. How this problem is handled is discussed more in the Method section (Section 4.3.3.4).

Credit quality step	Risk factor
0	ICE BofAML US Corporate AAA Option-Adjusted Spread
1	ICE BofAML US Corporate AA Option-Adjusted Spread
2	ICE BofAML US Corporate A Option-Adjusted Spread
3	ICE BofAML US Corporate BBB Option-Adjusted Spread
4	ICE BofAML US High Yield BB Option-Adjusted Spread
5	ICE BofAML US High Yield B Option-Adjusted Spread
6	ICE BofAML US High Yield CCC or Below Option-Adjusted Spread

Table 7: Spread risk factors for the different credit quality steps. The value of the risk factor is calculated with Formula 2.

Figure 8, with the spread risk factors, shows a similar pattern as Figure 7 did for interest rate risk factors. The daily changes are small, but not unimportant. Table 8 shows that the worse credit quality, the higher spreads and the higher changes in spreads. Assets with low credit quality have a higher spread risk than more creditworthy assets, as expected.

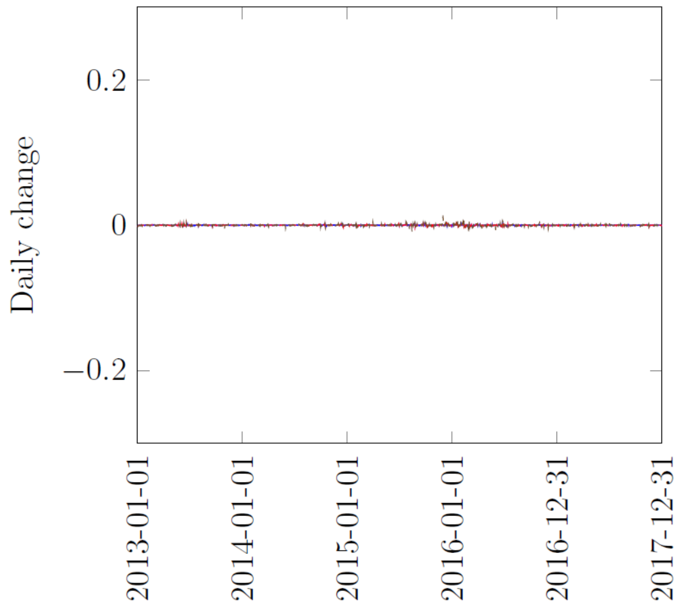

Figure 8: Daily values of the spread risk factors.

Statistic	N	Mean	Std. Dev.	Min	Max	Level
Step 0	1302	-0.0001%	0.0089%	-0.0400%	0.0600%	0.5300%
Step 1	1302	-0.0003%	0.0087%	-0.0300%	0.0600%	0.5500%
Step 2	1302	-0.0003%	0.0095%	-0.0300%	0.0700%	0.7600%
Step 3	1302	-0.0006%	0.0153%	-0.1100%	0.1000%	1.2800%
Step 4	1302	-0.0013%	0.0665%	-0.2700%	0.4300%	2.1200%
Step 5	1302	-0.0012%	0.0830%	-0.3700%	0.5600%	3.6100%
Step 6	1302	-0.0009%	0.1279%	-0.8600%	1.3500%	8.6100%

Table 8: Descriptive statistics of the daily values of the spread risk factors and the levels 31 December 2017.

4 Method

4.1 Overview of the Method section

The Method section describes the process of transforming the data about positions and historical risk factor values into an estimation of the SCRCRaR. While the Data section was focused on the past, this section is forward looking and past outcomes are just an input to estimate future scenarios. This process goes through the steps described in Section 1.4.

The first step is to generate future scenarios. In this process, the historical risk factor values are an essential input. Both how volatile the risk factors are on their own and how they vary in relation to each other must be considered. The motivation for which kind of method to use in this step is found in Section 4.2.1 and the method and the underlying assumptions are described in more detail in Section 4.2.2-4.2.4.

The second step is to calculate new values of the assets in the generated scenarios. The valuation is based on valuation parameters that were generated in the previous step. Different valuation methods are used for the different calculation categories and the methods are described in Section 4.3.1-4.3.3.

The third step is to test the quality of the scenarios generated (step 1) and how the scenarios are translated to new asset values (step 2). This is done by calculating the VaR and by running backtests to assess its accuracy. Focus here is on the 99.5% 1-year VaR, as the decided SCRCRaR is the 99.5% 1-year SCRCRaR[7]. Section 4.4.1 is a description of the VaR calculation. Section 4.4.2 motivates the backtests used and Section 4.4.3-4.4.5 are devoted to the chosen backtests.

The fourth and final step is to calculate the SCRCR in the generated scenarios (with the new asset values calculated in step 2) and to use these calculations to compute the 99.5% 1-year SCRCRaR. The SCRCR calculations are affected by the new values and credit ratings of the assets in the generated scenarios and also by the new values of a couple of input parameters (risk-free yield curves published by EIOPA and a symmetric equity adjustment from the same organization). Section 4.5.2 is devoted to describe how new values of the EIOPA yield curves are calculated for the generated scenarios and Section 4.5.3 is used to describe how new values of the symmetric adjustment parameter are calculated. After all inputs have been determined, Section 4.5.4-4.5.9 are used to describe how the Solvency Capital Requirements (SCR) for the different market risks are calculated. The

[7]The 99.5% 1-year VaR is the VaR that the Standard Formula in the Solvency II regulation calculates (*'SOLVENCY II': Frequently Asked Questions (FAQs)*, n.d.).

new values of the market risks are used to calculate the total market risk after diversification benefits. Section 4.5.10 describes this process. The same section continues by using the new market risk to calculate the new total Solvency Capital Requirement. Here, other risks facing an insurance company are considered, but they are kept fixed as this thesis only investigates risks facing the investment portfolio. Section 4.5.11 is used to describe the calculation of the SCRCR. The SCRCR is the value of own funds (assets minus liabilities) divided by the SCR, so in this section the calculation of new own funds in the scenarios is also described. Finally, Section 4.5.12 is devoted to describe how the SCRCRaR is calculated.

In an ideal world the SCRCRaR model would have been backtested in a similar way as the VaR model. However, restrictions in terms of data (daily data of risk-free yield curves and symmetric adjustments) and computational power make such an approach impossible. This is the reason why the VaR model is backtested instead. If the VaR is accurate, one can draw the conclusion that the new market values calculated by the simulated risk factors and the valuation methods are reliable. If the new market values are accurate, only the modelling of the risk-free yield curves and the symmetric adjustment may be inaccurate in the SCRCRaR model.

Table 9 shows an overview of the structure of the Method section. Section 4.2 generates new scenarios and the scenarios are interpreted and analyzed in the following three subsections.

Subsection	Description
4.2	Generate scenarios
4.3	Calculate new values of the assets
4.4	Calculate VaR and backtest the model
4.5	Calculate SCRCRaR

Table 9: Overview of the Method section.

4.2 Generate scenarios

4.2.1 Motivation of method

A number of decisions have to be made when constructing a VaR model and the model developed in this thesis is an extension of a VaR model. The first one is whether a local-valuation method or a full-valuation method should be used (Jorion, 2007). Section 2.2.2 describes these concepts and the main trade-off that has to be considered is the one between speed and accuracy. Speed is not a very important issue for a property insurance company and the fact that accurate information about the positions in the investment portfolio is only available on a monthly basis makes it unwise to focus on building a really fast model. Accuracy is more important and this is one reason why a full-valuation method is chosen. Another reason is that a local-valuation model is not suited for credit risk modelling, where value changes due to rating changes or defaults are highly non-linear.

After having decided that a full-valuation method is going to be used, the next step is to determine how the risk factor distribution should be modelled. Jorion suggests three alternatives: historical simulation, Monte Carlo simulation, and grid Monte Carlo simulation (Jorion, 2007). The historical simulation model is easy to explain and to justify, as you can go back and analyze the circumstances leading to the VaR loss. Despite this advantage, a historical simulation approach will not be used in this thesis. The main reason for this is that the method is not suitable for measuring credit risk. Basing future credit rating changes on historical changes is not a reasonable approach. For example, this would imply no credit risk for bonds that never have been downgraded. Combining a historical simulation with another approach would mean that the historical simulation loses its biggest advantage, as it does not represent a historical outcome any more.

The Monte Carlo simulation approach is the most flexible method, meaning that it allows more comprehensive modelling (Ammann & Reich, 2001). This approach has the highest potential in terms of accuracy and it is most suitable for measuring credit risk. This is why a Monte Carlo simulation approach is used in this thesis. The method, however, also has a number of disadvantages. The most important one is that it is the most computationally intensive method and requires a lot of computational power. As mentioned earlier, speed is not very important in the property insurance setting so this disadvantage is not devastating here. Furthermore, this disadvantage will be smaller as the time goes and the computers become more powerful. The trend of faster computers has already made the method increasingly popular (Förster, 1997). The most important disadvantage in this setting is that the method relies on assumptions in the stochastic process of generating risk factors (Jorion, 2007). The ambition in this thesis is, however, to make good assumptions and to make the model similar to reality. Another disadvantage is that the estimates are affected by sampling variation, if the number of simulations is not high. A 99.5% VaR means one tail event in 200 simulations, so the VaR measure is highly affected by a small number of simulations. The problem is, however, smaller here than for historical simulations as even a low number of simulations (such as 1000) means that at least a few observations worse than the VaR are expected (5 for 1000 simulations). Given that a full Monte Carlo simulation approach is doable in terms of computational power and more accurate, such a method is preferable over a grid Monte Carlo approach. Furthermore, getting close to a full-valuation in a grid Monte Carlo method would require a lot of grid points and it would become really complex to construct in this setting.

To decide the number of simulations to run, the same trade-off between accuracy and speed as above has to be considered. The entire model constructed in this thesis takes approximately four hours to run for 1000 simulations. Four hours is acceptable, but much more than that is not if the framework is going to be practically useful. The program must be possible to run in a working day and the gains in precision of running more simulations is not worth the cost in time. As you can see in Section 5.2.5, 1000 simulations is enough to get acceptable results in terms of robustness. 1000 simulations means a good balance between accuracy and speed. To evaluate the robustness of the model when 1000 simulations are run, all the calculations in the Method section are repeated five times and the outcomes in the five iterations are compared. Theoretically, recalculating the framework a multiple times and constructing confidence intervals for the SCRCRaR would have been an ideal approach, but this is not possible given how long time one run takes.

The Monte Carlo simulation process is described in the following subsections.

4.2.2 Stochastic model for valuation parameters

The risk factors described in Section 3 are used to generate new values of the assets in the investment portfolio. The first step in this process is the simulation of new valuation parameters, the scenario generation. The valuation parameters are the levels of the asset prices, exchange rates, interest rates, and credit spreads for which the risk factors are daily changes. If the daily logarithmic return on SCA B is the risk factor, the new price of SCA B that is calculated by using the risk factor is the corresponding valuation parameter. If the daily change in the 10-year swap rate is the risk factor, the new 10-year swap rate that is calculated by using the risk factor is the corresponding valuation parameter.

To generate scenarios, a stochastic model has to be chosen. A standard assumption is that the values of the valuation parameters follow a geometric brownian motion[8] (Jorion, 2007). An underlying assumption of this model is that changes in the valuation parameters are uncorrelated over time. The geometric brownian motion is described by the following formula (Marathe

[8]It is geometric in the sense that all parameters are scaled by the current value ($V_{a,0}$) and brownian as its variance decreases continuously with the time interval.

& Ryan, 2005):

$$P_t = P_0 \cdot e^{\bar{r}t+\sigma U\sqrt{t}} \tag{3}$$

P_t is a vector with the values of the valuation parameters at day t and P_0 consists of their values 31 December 2017. The standard is to assume that a year, which is the SCRCRaR (and VaR) horizon, has 252 trading days so t is 252 (Nath, 2003). $\bar{r}$ is a vector with the average daily risk factor values and σ is a vector with the standard deviations of the daily risk factor values. $ln\left(\frac{P_t}{P_0}\right)$ is normally distributed with mean $t\bar{r}$ and variance $t\sigma^2$ (standard deviation $\sqrt{t}\sigma$). Both how the risk factors move on their own and how they move in relation to each other is considered in the formula. How the risk factors move on their own is determined by $\bar{r}$ and σ and Section 4.2.3 describes how these values are determined. How the risk factors vary in relation to each other is incorporated in U, a vector of correlated random numbers (the shock parameters). The calculation of U is described in Section 4.2.4. Notice that everything except the value of U is the same in all scenarios.

In Formula 3, $\bar{r}t + \sigma U\sqrt{t}$ is the return of the asset. For interest rates and credit spreads, the risk factors are changes in the rates and not logarithmic returns (see Formula 2). For these assets, the formula below is used instead of Formula 3:

$$P_t = P_0 + \left(\bar{r}t + \sigma U\sqrt{t}\right) \tag{4}$$

$(P_t - P_0)$ is normally distributed with mean $t\bar{r}$ and variance $t\sigma^2$ (standard deviation $\sqrt{t}\sigma$).

4.2.3 The risk factors' movements on their own

The standard deviations of the risk factors, σ, are assumed to be the daily standard deviations based on the data period (1 January 2013 to 31 December 2017). This is the standard assumption, used by for example Jorion (Jorion, 2007). No volatility adjustments, through for example GARCH models, are made. The reason for this is that it is unreasonable to believe that the volatilities over a 1-year period can be modelled accurately from recent daily observations. If the SCRCRaR (and VaR) horizon had been shorter, such an approach would have made more sense. The standard deviations are calculated with the following formula (Hull, 2015):

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}(r_n - \bar{r})^2} \tag{5}$$

n is the observation number and N is the last observation number or the sample size (the number of observations between 1 January 2013 and 31 December 2017). r_n is the values of the risk factors for the observation number. $\bar{r}$ is the average values of the risk factors. When risk factor values are missing for a certain observation number, no calculations are made for the missing risk factors.

The average risk factor values, $\bar{r}$, are assumed to be 0. For exchange rates, interest rates, and credit spreads it is reasonable to believe that the average long-term drift is 0. For asset prices, the assumption is less obvious as asset prices tend to increase over time. The reason why this assumption still is made is that it is unreasonable to believe that the price movements 2013-2017, the data available, are representative for the long-term average movements. From a risk management perspective it is dangerous to assume that prices will continue to increase in the way they have done 2013-2017. No bull market lasts forever. Another approach would have been to base average daily increases in asset prices on return assumptions for different asset categories (for example based on volatilities). This would, however, require strong assumptions about returns that are hard to justify and to make accurate. The conservative assumption of an average daily return of 0 makes the estimates of the SCRCRaR (and VaR) slightly upward biased, but the effect of this is expected to be small. Dina AB expects a yearly return of 2.5% on the total asset portfolio, so the expected value change due to asset price changes is smaller than 2.5%. The 0 mean assumption means that Formula 3 can be simplified to:

$$P_t = P_0 \cdot e^{\sigma U \sqrt{t}} \tag{6}$$

$ln\left(\frac{P_t}{P_0}\right)$ is normally distributed with mean 0 and standard deviation $\sqrt{t}\sigma$.

Formula 4 can, for the same reason, be simplified to:

$$P_t = P_0 + \sigma U \sqrt{t} \tag{7}$$

$(P_t - P_0)$ is normally distributed with mean $t\bar{r}$ and standard deviation $\sqrt{t}\sigma$.

In both Formula 6 and Formula 7 the distributions of changes in the valuation parameters depend on the standard deviations of their risk factors in the way shown in Figure 9. Notice, however, that the standard deviations are a lot smaller than in the simplified example below, so the changes are much more centered around 0. The implication of the distribution assumptions made in this subsection is that a positive and a negative valuation parameter change is equally likely and that most of the changes are centered around 0.

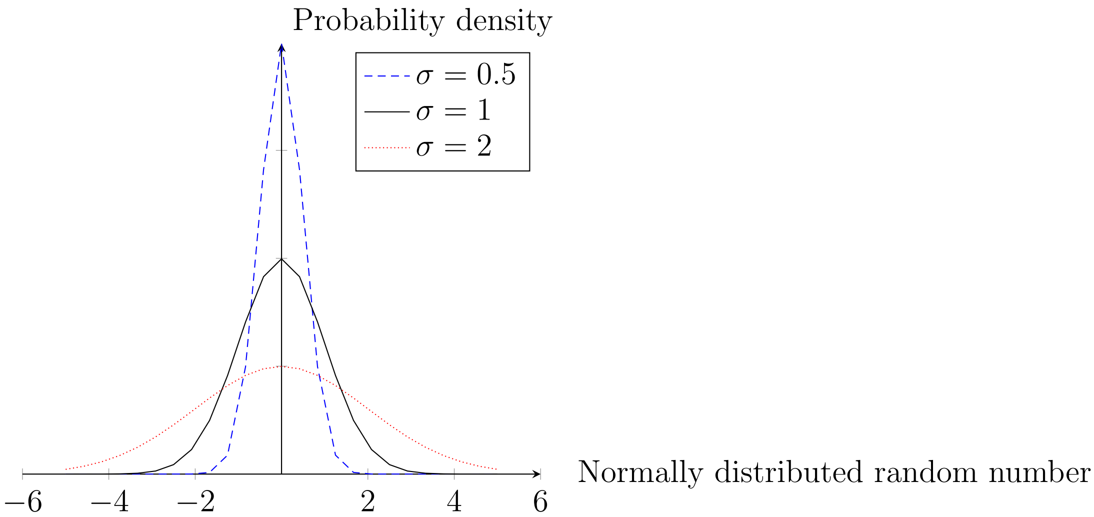

Figure 9: Normal distributions with means 0 and different standard deviations.

4.2.4 The risk factors' movements in relation to each other

To get realistic scenarios when new values of the valuation parameters are simulated, the risk factors' movement in relation to each other also have to be considered. This process requires a number of steps. The first one is to calculate the historical correlation coefficients between all risk factors (Jorion, 2007). The historical data is from 1 January 2013 to 31 December 2017. The Pearson correlation, ρ, between risk factor j and k is calculated with the following formula (Vořechovský & Novák, 2009):

$$\rho_{j,k} = \frac{\sigma_{j,k}}{\sigma_j \sigma_k} = \frac{\sum_{n=1}^{N}(j_n - \bar{j})(k_n - \bar{k})}{\sqrt{\sum_{n=1}^{N}(j_n - \bar{j})^2}\sqrt{\sum_{n=1}^{N}(k_n - \bar{k})^2}} \tag{8}$$

$\sigma_{j,k}$ is the covariance between the risk factors, while σ_j and σ_k are the standard deviations of them. A correlation matrix is formed based on the correlations between all risk factors. The correlation matrix is shown in Table 10. The diagonal ($\rho_{j,j}$, $\rho_{k,k}$, $\rho_{l,l}$,...) represent each risk factor's correlation with itself and all these values are by definition 1.

	j	k	l	...
j	$\rho_{j,j}$	$\rho_{j,k}$	$\rho_{j,l}$	...
k	$\rho_{k,j}$	$\rho_{k,k}$	$\rho_{k,l}$	...
l	$\rho_{l,j}$	$\rho_{l,k}$	$\rho_{l,l}$	...
...	...	...	...	...

Table 10: Correlation matrix ρ for risk factors j, k, l,...

The correlation matrix is used in the generation of new valuation parameters. To generate correlated random variables, U in Formula 6-7, a technique called Cholesky decomposition is used (Higham, 1990). Cholesky decomposition means to manipulate the correlation matrix, to incorporate the desired correlation structure on randomly generated variables. The correlation matrix, ρ, can be decomposed into triangular matrices in the following way:

$$\rho = L^T L \tag{9}$$

L is the upper triangular matrix and L^T the lower triangular matrix. As you can see, the lower triangular matrix is a transposed version of the upper triangular matrix. The Appendix (Section 8.2) shows an example of the Cholesky decomposition in a two-variable case. The Cholesky decomposition only works when the correlation matrix is positive definite (Jorion, 2007). This is the case here.

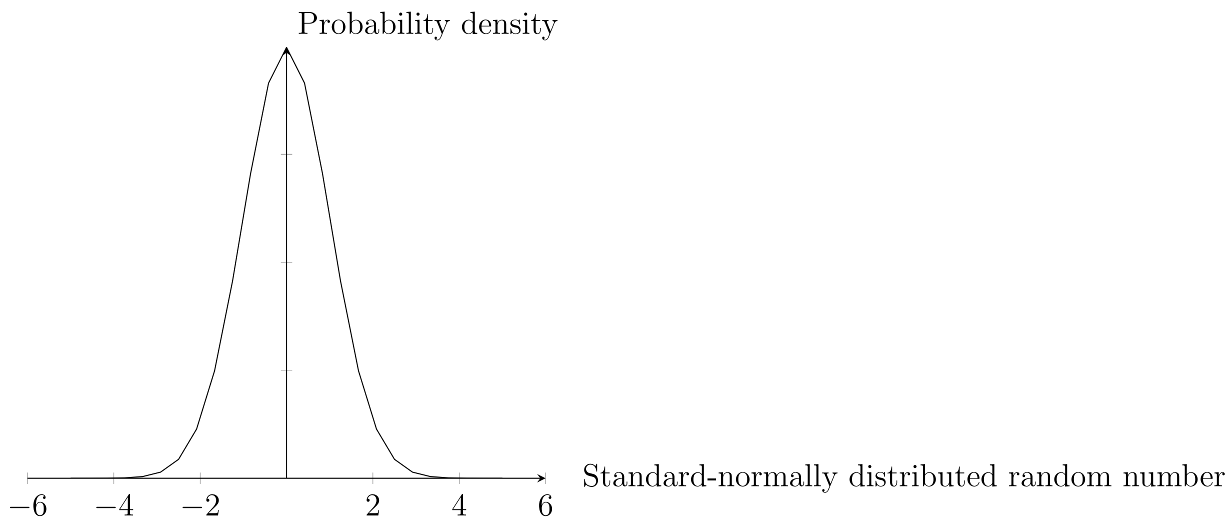

Figure 10: A standard normal distribution, the assumed distribution in a Gaussian copula.

To take the next step and calculate correlated random variables, U in

Formula 6-7, uniformly distributed random numbers between 0 and 1 are generated[9]. One random number is generated for each risk factor and together these numbers form vector u. More mathematically expressed, the marginal probability distributions are uniform. The marginal probability distributions must be transformed into a multivariate probability distribution, that gives the probability of all the random variables. This transformation goes through a copula (Romano, 2002). A copula maps the random variables onto a common distribution. The distribution assumption here is a standard normal distribution (a Gaussian copula). This is the standard assumption and it is easiest to combine with the Cholesky decomposition. As the results from this distribution are good, which you can see in Section 5.2.1, no other copula is investigated. Figure 10 shows this distribution. The uniformly distributed random numbers, u, are used as probabilities and the probabilities represent the fractions of the area under the standard normal distribution, starting from the left, that are covered for the standard-normally distributed random numbers, x (the x-axis values). One standard-normally distributed random number is calculated for each risk factor and together these standard-normally distributed random numbers form vector x. So the Gaussian copula has converted the uniformly distributed random numbers into standard-normally distributed random numbers. Mathematically, this is expressed by the following formula (Romano, 2002):

$$x = \Phi^{-1}(u) \tag{10}$$

The lower triangular matrix from the Cholesky decomposition is multiplied with the standard-normally distributed random numbers to generate a vector of correlated standard-normally distributed random variables, A (Romano, 2002):

$$A = L^T x \tag{11}$$

Section 8.3 in the Appendix proves that this formula generates numbers with the desired correlation structure.

The numbers in vector A are converted into uniformly distributed random variables (probabilities) by using the standard normal distribution once again. This time the value on the x-axis is converted to a probability by considering the fraction of the area under the graph that it corresponds to. This vector is called U and consists of correlated random numbers. Mathematically, this is expressed by the following formula (Romano, 2002):

$$U = \Phi(A) \tag{12}$$

[9] Any number between 0 and 1 is equally likely.

U can be inserted in Formula 6 and 7, that are used to calculate the new valuation parameters. This procedure is repeated 1000 times, to generate 1000 scenarios.

4.3 Calculate new values of the assets

4.3.1 Stocks and investment funds

The values of the stocks and the investment funds are calculated from the valuation parameters with the following formula:

$$V_{a,i} = Number\,of\,shares_a \cdot P_{a,i} \cdot P_{FX,i} \tag{13}$$

$V_{a,i}$ is the market value of asset a in scenario i. $P_{a,i}$ is the valuation coefficient for the stock or investment fund asset and $P_{FX,i}$ is the currency valuation coefficient (SEK/foreign currency).

4.3.2 Property

The purpose of this subsection is to describe how new values of the property are calculated from the values of the property risk factors generated by the Monte Carlo simulation described in Section 4.2. For property, the way from simulated risk factor values to new property values goes through regression models instead of through calculation of valuation parameters with Formula 6 or 7. The reason why the property returns are not directly used as risk factors is that property values are not available on a daily basis (see Table 11).

The regression models are inspired by the capital asset pricing (CAPM) model in the sense that they build on excess returns, in this case of the property risk factors (Sharpe, 1964). The assumption made in the regressions is that the property returns and the risk factors co-move in the same way on a daily basis as they do in the time between the valuations. The regressions used to calculate property returns are based on the following formula:

$$r_{a,\Delta n} - r_{f,\Delta n} = b_1 \cdot (r_{1,\Delta n} - r_{f,\Delta n}) + b_2 \cdot (r_{2,\Delta n} - r_{f,\Delta n}) + b_3 \cdot (r_{3,\Delta n} - r_{f,\Delta n}) + c \tag{14}$$

Δn is the number of days between a property valuation and the previous property valuation. $r_{a,\Delta n}$ is the average daily logarithmic return of property a. The average daily returns are calculated by dividing the logarithmic returns (see Formula 1) by the number of days between the observations. To take into account that values are not calculated during weekends, the number of days in the denominator is multiplied by 252/365.25, assuming 252 observations per year. The same method is used to calculate the values of the risk factors $r_{1,\Delta n}$, $r_{2,\Delta n}$, and $r_{3,\Delta n}$. The parentheses in which the risk-free rate, $r_{f,\Delta n}$, is subtracted represent the excess returns due to the risk taken. To calculate $r_{f,\Delta n}$, the first step is to get the accumulated risk-free rates for all days days in the sample. This is done by summing the daily values of the (yearly) risk-free rate divided by 252. The division by 252 is done to get daily values and the underlying assumption is continuous compounding. The accumulated returns of the risk-free rate are then calculated by taking e to the power of the accumulated risk-free rate and subtracting 1. The value of $r_{f,\Delta n}$ is finally calculated by subtracting the accumulated return at the time of the property valuation before from the accumulated return at the time of the property valuation. This number is also divided by Δn times 252/365.25. b_1, b_2, and b_3 are coefficients estimated by the regression. They indicate how much one unit of excess return affects the return on the property holding. c is a constant and represents the return if the values of all risk factors are 0.

For real estate, the regression model can be written as follows:

$$r_{real\,estate,\Delta n} - r_{10\text{-}year\,swap\,rate,\Delta n} = b_{Carnegie\,Real\,Estate\,Index} \cdot (r_{Carnegie\,Real\,Estate\,Index,\Delta n} - r_{10\text{-}year\,swap\,rate,\Delta n}) + c \tag{15}$$

For forest, the regression model can be written as follows:

$$r_{forest,t} - r_{10\text{-}year\,swap\,rate,\Delta n} = b_{wood} \cdot (r_{wood,\Delta n} - r_{10\text{-}year\,swap\,rate,\Delta n}) + b_{SCA\,B} \cdot (r_{SCA\,B,\Delta n} - r_{10\text{-}year\,swap\,rate,\Delta n}) + c \tag{16}$$

The regressions are based on yearly values for real estate and for forest there are two values per year. The first yearly value for Carnegie Real Estate Index is from the end of 2013, so the first return is calculated 31 October 2014 ('Carnegie Real Estate Index', 2018). The valuations and their dates are shown in Table 11. There are five values and four returns for real estate and nine values and eight returns for forest.

Date	Dina Palaisbacken AB	Diana 2	Juno 9	Diana Skog AB
December 31, 2013	382 000 000	149 854 546	36 000 000	123 600 000
July 31, 2014				123 300 000
October 31, 2014	404 000 000	159 000 000	38 400 000	
December 31, 2014				122 200 000
July 31, 2015				126 000 000
September 30, 2015	449 000 000	177 000 000	43 000 000	
December 31, 2015				124 300 000
July 31, 2016				126 400 000
December 31, 2016	480 000 000	188 000 000	46 500 000	124 300 000
July 31, 2017				129 800 000
December 31, 2017	485 000 000	192 000 000	48 800 000	138 000 000

Table 11: Property valuations.

The estimates of b_1, b_2, b_3, and c in the regressions are used to estimate the daily returns of the property in the simulated scenarios. Hats are added to denote estimates.

$$r_{a,i} = r_{f,i} + \hat{b}_1 \cdot (r_{1,i} - r_{f,i}) + \hat{b}_2 \cdot (r_{2,i} - r_{f,i}) + \hat{b}_3 \cdot (r_{3,i} - r_{f,i}) + \hat{c} \tag{17}$$

To convert this expression into a value for the property, the geometric brownian motion with 0 average risk factor values is used. Formula 6, but for the value V of asset a in iteration i is written below:

$$V_{a,i} = V_{a,0} \cdot e^{\sigma_a U_{a,i} \sqrt{t}} \Rightarrow \sigma_a U_{a,i} \sqrt{t} = ln\left(\frac{V_{a,i}}{V_{a,0}}\right) \tag{18}$$

The property returns are calculated with Formula 1. This formula is rewritten for value, V, in iteration i:

$$r_{a,i} = ln\left(\frac{V_{a,i}}{V_{a,0}}\right) \tag{19}$$

The right side of the second part (after the arrow) of Formula 18 is the same as the right side of Formula 19. Then the left sides must be the same as well:

$$r_{a,i} = \sigma_a U_{a,i} \sqrt{t} \tag{20}$$

This relationship can be used in the first part (before the arrow) of Formula 18:

$$V_{a,i} = V_{a,0} \cdot e^{r_{a,i}} \tag{21}$$

This relationship can, in turn, be used to calculate the property value from Formula 17:

$$V_{a,i} = V_{a,0} \cdot e^{r_{f,i} + \hat{b}_1 \cdot (r_{1,i} - r_{f,i}) + \hat{b}_2 \cdot (r_{2,i} - r_{f,i}) + \hat{b}_3 \cdot (r_{3,i} - r_{f,i}) + \hat{c}} \tag{22}$$

Formula 20 is used to calculate the values of the risk factors that are independent variables (this was done in Section 4.2):

$$V_{a,i} = V_{a,0} \cdot e^{P_{f,0} + \sigma_f U_{f,i} \sqrt{t} + \hat{b}_1 \cdot (\sigma_1 U_{1,i} \sqrt{t} - P_{f,0} - \sigma_f U_{f,i} \sqrt{t}) + \hat{b}_2 \cdot (\sigma_2 U_{2,i} \sqrt{t} - P_{f,0} - \sigma_f U_{f,i} \sqrt{t}) + \hat{b}_3 \cdot (\sigma_3 U_{3,i} \sqrt{t} - P_{f,0} - \sigma_f U_{f,i} \sqrt{t}) + \hat{c}} \tag{23}$$

For real estate, the valuation model can be written as follows:

$$V_{real\,estate,i} = V_{real\,estate,0} \cdot exp\Big[P_{\text{10-}year\,swap\,rate,0} + \sigma_{\text{10-}year\,swap\,rate} U_{\text{10-}year\,swap\,rate,i} \sqrt{t} + \hat{b}_{Carnegie\,Real\,Estate\,Index} \cdot (\sigma_{Carnegie\,Real\,Estate\,Index} U_{Carnegie\,Real\,Estate\,Index,i} \sqrt{t} - P_{\text{10-}year\,swap\,rate,0} - \sigma_{\text{10-}year\,swap\,rate} U_{\text{10-}year\,swap\,rate,i} \sqrt{t}) + \hat{c}\Big] \tag{24}$$

For forest, the valuation model can be written as follows:

$$V_{forest,i} = V_{forest,0} \cdot exp\Big[P_{\text{10-}year\,swap\,rate,0} + \sigma_{\text{10-}year\,swap\,rate} U_{\text{10-}year\,swap\,rate,i} \sqrt{t} + \hat{b}_{Wood} \cdot (\sigma_{Wood} U_{Wood,i} \sqrt{t} - P_{\text{10-}year\,swap\,rate,0} - \sigma_{\text{10-}year\,swap\,rate} U_{\text{10-}year\,swap\,rate,i} \sqrt{t}) + \hat{b}_{SCAB} \cdot (\sigma_{SCAB} U_{SCAB,i} \sqrt{t} - P_{\text{10-}year\,swap\,rate,0} - \sigma_{\text{10-}year\,swap\,rate} U_{\text{10-}year\,swap\,rate,i} \sqrt{t}) + \hat{c}\Big] \tag{25}$$

4.3.3 Interest-bearing assets

4.3.3.1 Interest rates

The interest rates between the horizons of the interest rate valuation parameters (1 week, 1 month, 2 months, 3 months, 6 months, and 1 year to 10 years) are calculated with linear interpolation. The value, $P_{h_{middle},i}$, for a horizon, h_{middle}, between h_{short} and the longer horizon, h_{long}, is calculated with the following formula:

$$P_{h_{middle},i} = P_{h_{short},i} + \frac{h_{middle} - h_{short}}{h_{long} - h_{short}} \cdot (P_{h_{long},i} - P_{h_{short},i}) \tag{26}$$

This is a simple and generally accepted method and it is described by for example Hagan and West (Hagan & West, 2008).

4.3.3.2 Cash flows

Since the maturity dates and coupon dates are known from before, most of the cash flows are easily estimated for all interest-bearing assets. The only tricky thing is to calculate the amounts of the coupons for Floating Rate Notes (FRNs). The coupons of FRNs consist of one fixed and one floating part. The fixed parts are known, while the floating parts depend on the prevailing risk-free rate, in this case the 3-months STIBOR rate. The 3-months STIBOR rate is known today, but it is much harder to know the level in the future. To solve this, a bootstrapping technique is used (Röman, 2017). The technique starts from the following relationship:

$$h_{long} \cdot P_{h_{long},0} = h_{short} \cdot P_{h_{short},0} + (h_{long} - h_{short}) \cdot P_{h_{long-short},short} \tag{27}$$

The left side of the formula represents the value today of investing in an interest rate with long horizon. The first term of the right side of the formula represents the value today of investing in an interest rate with short horizon. The second term of the right side of the formula represents the value of a reinvestment at horizon *short* (the maturity of the short investment) that lasts until horizon *long*. The idea is that the market assumes that you get an interest rate based only on the total horizon of your investment. Investing in the 10-year rate now gives the same return in ten years as ten yearly investments in the 1-year rate. The formula can be rearranged to the following:

$$P_{h_{long-short},short} = \frac{h_{long} \cdot P_{h_{long},0} - h_{short} \cdot P_{h_{short},0}}{(h_{long} - h_{short})} \tag{28}$$

$long - short$ is set to three months and *long* is the number of days until the coupon. This interest rate, divided by four as the coupon is paid quarterly, is the floating coupon rate.

4.3.3.3 Credit risk simulation

The values of interest-bearing assets are affected by credit risk. If an interest-bearing asset defaults or is downgraded in terms of credit quality, the value of it decreases. In the case of default, the payments from the issuer are cancelled, which results in a drop in value. In the case of downgradings, investors require a higher compensation as they take a higher risk[10].

To be able to model the default risk and the risk of downgradings in terms of credit quality step, a transition matrix from Moody's is used (*Annual*

[10]It is of course not the downgrading as such that makes an interest-bearing asset more risky, but downgradings make investors aware of a higher risk.

Default Study: Corporate Default and Recovery Rates, 1920-2017, 2018). The matrix is based on the years 1970-2017 and can be seen in Table 12. In this thesis the values in the unrated column are withdrawn and the probability is split proportionally to the other columns. An advantage of using data of such a long period as 1970-2017 is that the matrix is based on data from both recessions and economic booms. Furthermore, the probabilities of really unlikely changes are more accurately estimated with more data. If the data had been based on only one year, there is a big chance that a transition with 0.01% probability does not incur and then the value in the matrix is 0.00%. There is also a matrix released based on the years 1920-2017, but the advantage of having more data is in that case more than outweighed by the fact that more old and possibly outdated data is used. Ideally, the matrix should have been updated in every simulated scenario to reflect a changed macroeconomic environment, but such an approach would have made the model too complex for a master thesis (Trück, 2008).

	0	1	2	3	4	5	6	*Unrated*	*Default*
0	87.71%	7.94%	0.58%	0.07%	0.02%	0.00%	0.00%	3.67%	0.00%
1	0.82%	85.15%	8.51%	0.42%	0.06%	0.03%	0.02%	4.95%	0.02%
2	0.05%	2.46%	86.78%	5.37%	0.48%	0.11%	0.04%	4.64%	0.05%
3	0.03%	0.14%	4.12%	85.72%	3.79%	0.69%	0.17%	5.17%	0.17%
4	0.01%	0.04%	0.42%	6.12%	76.32%	7.17%	0.82%	8.22%	0.88%
5	0.01%	0.03%	0.14%	0.45%	4.78%	73.49%	7.14%	10.70%	3.27%
6	0.00%	0.01%	0.04%	0.04%	0.45%	4.40%	59.53%	18.24%	17.31%

Table 12: Average one-year credit quality step transition matrix, 1970-2017 (*Annual Default Study: Corporate Default and Recovery Rates, 1920-2017*, 2018). The ratings at the start of the year are listed on the vertical axis and the ratings at the end of the year on the horizontal axis. Therefore all the rows sum to 100% (the numbers shown are rounded to two decimals so the sum may differ slightly from 100%).

From the transition probabilities, the cumulative probabilities are calculated in the way shown in Table 13. After this is done, the new matrix is shown in Table 14.

Probability	Cumulative probability
$R_{e\,to\,default}$	$R_{e\,to\,default}$
$R_{e\,to\,6}$	$R_{e\,to\,default} + R_{e\,to\,6}$
$R_{e\,to\,5}$	$R_{e\,to\,default} + R_{e\,to\,6} + R_{e\,to\,5}$
$R_{e\,to\,4}$	$R_{e\,to\,default} + R_{e\,to\,6} + R_{e\,to\,5} + R_{e\,to\,4}$
$R_{e\,to\,3}$	$R_{e\,to\,default} + R_{e\,to\,6} + R_{e\,to\,5} + R_{e\,to\,4} + R_{e\,to\,3}$
$R_{e\,to\,2}$	$R_{e\,to\,default} + R_{e\,to\,6} + R_{e\,to\,5} + R_{e\,to\,4} + R_{e\,to\,3} + R_{e\,to\,2}$
$R_{e\,to\,1}$	$R_{e\,to\,default} + R_{e\,to\,6} + R_{e\,to\,5} + R_{e\,to\,4} + R_{e\,to\,3} + R_{e\,to\,2} + R_{e\,to\,1}$
$R_{e\,to\,0}$	$R_{e\,to\,default} + R_{e\,to\,6} + R_{e\,to\,5} + R_{e\,to\,4} + R_{e\,to\,3} + R_{e\,to\,2} + R_{e\,to\,1} + R_{e\,to\,0}$

Table 13: Probability and corresponding cumulative probability.

	0	1	2	3	4	5	6	$Default$
0	100.00%	8.94%	0.70%	0.09%	0.02%	0.00%	0.00%	0.00%
1	100.00%	99.14%	9.53%	0.58%	0.14%	0.07%	0.07%	0.04%
2	100.00%	99.95%	97.37%	6.35%	0.71%	0.21%	0.09%	0.05%
3	100.00%	99.97%	99.82%	95.48%	5.08%	1.09%	0.36%	0.18%
4	100.00%	99.99%	99.95%	99.49%	92.82%	9.66%	1.85%	0.96%
5	100.00%	99.99%	99.96%	99.80%	99.29%	93.94%	11.66%	3.66%
6	100.00%	100.00%	99.99%	99.94%	99.89%	99.34%	93.96%	21.17%

Table 14: Cumulative transition matrix used in the credit risk simulation.

To calculate the new credit quality step (or default), a new simulation is run. The rating 31 December 2017 decides which credit quality step, e, that an interest-bearing asset starts from. To decide the new credit quality step (including default), a random number between 0.00% and 100.00% is generated. The new credit quality step is the worst step for which the random number is not higher. If the credit quality step 31 December 2017 is 3 and Table 14 is used, a random number over 99.97% means that the new credit quality step is 0. The probability for this is 0.03%, which you can see in Table 12. If the random number is higher than 99.82%, but lower or equal to 99.97%, the new credit quality step is 1. This is repeated for all transitions and a random number lower or equal to 0.18% means default. One random number is generated for each interest-bearing asset and the assumption is that they are uncorrelated. The main reason for this simplifying assumption is lack of data. It is reasonable to believe that macroeconomic factors have an impact on the transitions. Furthermore, industry-specific factors may have an impact and creating transition correlations between bonds. The CreditMetrics Technical Document proposes three methods to take transition correlations into account (Morgan, 1997). The first method is to examine credit ratings time series across firms and no such data is publicly available. The second method is to use bond spreads and data is missing in this case as well. The third method is to use equity values to estimate transition correlations and the problem is the same once again. The lack of data makes a zero correlation assumption inevitable. This may lead to downward biased SCRCRaR (and VaR) estimates, but the effect is not expected to be big, as transition and (even more importantly) default correlations are small on a one-year horizon (Zhou et al., 1997).

The process described in this section is in line with the CreditMetrics methodology and the process is repeated 1000 times for all the interest-bearing assets (Morgan, 1997).

4.3.3.4 Valuation

If the interest-bearing assets have defaulted, their values are calculated with the following formula:

$$V(a) = Nominal\,amount_a \cdot P_{recovery} \tag{29}$$

The average issuer-weighted recovery rate (how much of the invested amount that is recovered in the case of a default), $P_{recovery}$, is gathered from Moody's (*Annual Default Study: Corporate Default and Recovery Rates, 1920-2017*, 2018). The data is based on the years 1983-2017, which is the longest period available. The fact that the same recovery rate, 37.74%, is used for all interest-bearing assets is a simplification.

If the interest-bearing assets have not defaulted, their values are calculated as the discounted values of all their cash flows. For each cash flow, the following formula is used to calculate the current value, $V(CF_{t,i})$:

$$V(CF_{\tau,i}) = CF_{\tau,i} \cdot e^{-(P_{\tau,i}+P_{s,i}+P_a)\cdot\tau/360} \tag{30}$$

$CF_{\tau,i}$ is the cash flow in time τ (that is converted to a number of years by dividing by 360) in scenario i, $P_{\tau,i}$ is the risk-free rate with horizon τ, $P_{s,i}$ is the credit spread, and P_a is an extra spread. The extra spread is unique to each interest-bearing asset, a, and it is added to adjust the theoretical value to the actual market value 31 December 2017 (when all risk factor values are 0). The extra spread is not necessarily positive and not necessarily negative and it makes sure that the valuation is not biased in any direction. It is used in all scenarios. There are three reasons why the market value may be different from the theoretical value. Firstly, the spread data is American, so it is not expected to be exact in a Swedish context. Secondly, the ratings of some interest-bearing assets are from Swedbank. The spreads of the assets with Swedbank ratings could be slightly higher, as many institutions require higher returns to invest in officially unrated assets. Thirdly, all interest-bearing assets with the same credit quality step are not necessarily seen as exactly equally risky by the market and this motivates that they have different spreads. The first two reasons are due to shortcomings of the data, while the third reason is a problem for this way of modelling risk.

Although necessary, this adjustment could be problematic. As the extra spread is the same in all scenarios, it is considered to be risk-free. If the extra spread is volatile (the assumption that it is fixed is inaccurate) and it is a large fraction of the total discount rate ($P_{\tau,i}+P_{s,i}+P_a$), a large fraction of the risk in interest-bearing asset is not modelled. This is the biggest weakness in the valuation method used, but a necessary evil to handle the shortcomings in the data. As you can see in Section 5.2.1, the results with this extra spread are acceptable.

4.4 Calculate VaR and backtest the model

4.4.1 Value at risk

In each scenario, the total market value of the investment portfolio is calculated with the following formula:

$$V_i = \sum_{a=1}^{A} V_{i,a} \tag{31}$$

a is the asset numbers and A is the total number of assets. V_i is calculated for all 1000 iterations and ΔV_i is the change in value ($\Delta V_i = V_i - V_0$). The 1000 values of ΔV_i form the discrete cumulative distribution function $F_{\Delta V}$. VaR is calculated in the way described in Section 2.2.1. Mathematically, this is described by the following formula:

$$VaR_\alpha(V) = -F_{\Delta V}^{-1}(1-\alpha) \tag{32}$$

α is the confidence level 99.5%. The minus sign before the formula aims to convert a loss into a positive number.

4.4.2 Motivation of backtests

To test that the VaR model is well-calibrated, it must be backtested against historical data. The VaR model estimates one loss larger than the VaR in 200 observations, as 99.5% is the confidence level. A loss larger than the VaR is called a failure. To test that the model is accurate, the share of failures must be reasonably close to 99.5%. If the number of failures is too low, the model overestimates the risk. This could be costly, as too little risk is taken. If the number of failures is too high, the model underestimates the risk, which means that the company takes more risk than it is aware of. To check that the number of failures is reasonable, a proportion of failures test is run. This test is described in Section 4.4.3.

One underlying assumption of all calculations in this thesis is that the outcome one day can not be estimated by the outcome the day before. To check whether this assumption is reasonable, an independence test is run. If all failures are realized in a limited period, the risk is varying in a way that is not predicted by the model. It is higher when many failures are realized and lower in other periods. This test is described in Section 4.4.4.

The proportion of failures and the independence tests are finally combined in a conditional coverage test, that is described in Section 4.4.5.

The VaR model is primarily evaluated in-sample. Historical data from 1 January 2013 to 31 December 2017 is used (the same data as the model was based on). The reason for this is that there is not enough data available outside the sample. The number of failures out-of-sample is calculated to see if the pattern is reasonable, but no formal test can be run. This is a weakness in the approach, but a necessary evil, as all the data needs to be used for the model specification. The lack of data before 1 January 2013 is also one reason why a new VaR can not be calculated for each day. Another reason is that such an approach would have taken many days for the program to complete, as it would have been extremely computationally demanding.

All the daily outcomes are evaluated against the VaR 31 December 2017, but some of the financial instruments are excluded from the VaR 31 December 2017 when compared to the outcomes for some days. The reason for this is that there are not values available for all assets all days. For example, there are not daily values of the property available and all bonds have not existed for the entire period that the method is evaluated against. Therefore the property values are excluded from the evaluated VaR and all bonds are not included in all days. This is done to make the VaR comparable to the portfolio it is evaluated against. For example, no daily property value changes were available 18 December 2017. Therefore, the VaR 31 December 2017 is recalculated with property excluded and this VaR is compared with the outcome 18 December 2017.

One potential problem in evaluating the portfolio 31 December 2017 is survivorship bias. If a company has defaulted, stocks or bonds issued by the company can not be in Dina AB's portfolio 31 December 2017. This leads to systematically higher returns of the assets in the portfolio. The problem is, however, expected to be small as only one holding, a bond with a face value of SEK 2.1 million, has defaulted in the history of Dina AB.

4.4.3 Proportion of failures

The method used to investigate the proportion of failures is called the Kupiec test. It was developed by Paul Kupiec and presented in the paper "Techniques for verifying the accuracy of risk measurement models" (Kupiec, 1995). For all historical outcomes a dummy variable for failures is created:

$$I_t = \begin{cases} 1, \, if \, \Delta V_t < -VaR_\alpha(V_t) \\ 0, \, if \, \Delta V_t \geq -VaR_\alpha(V_t) \end{cases} \tag{33}$$

The number of observations with $I_t = 1$ is denoted q_1 and the number of observations with $I_t = 0$ is denoted q_0. The null hypothesis that is tested is that:

$$\frac{q_1}{q_0 + q_1} = \alpha \tag{34}$$

The likelihood function can be constructed in the following way, assuming a binomial distribution for the number of failures:

$$L\left(\frac{q_1}{q_0+q_1}\right) = \left(\frac{q_0}{q_0+q_1}\right)^{q_0}\left(\frac{q_1}{q_0+q_1}\right)^{q_1} = \left(1-\frac{q_1}{q_0+q_1}\right)^{q_0}\left(\frac{q_1}{q_0+q_1}\right)^{q_1} \tag{35}$$

The likelihood function under the null hypothesis can accordingly be written in the following way:

$$L(\alpha) = (1-\alpha)^{q_0}(\alpha)^{q_1} \tag{36}$$

The likelihood ratio test is used to compare the two models. The test statistic LR_{POF} is calculated with the following formula:

$$LR_{POF} = -2ln\left(\frac{L\left(\alpha\right)}{L(\frac{q_1}{q_0+q_1})}\right) = -2ln\left(\frac{(1-\alpha)^{q_0}(\alpha)^{q_1}}{\left(1-\frac{q_1}{q_0+q_1}\right)^{q_0}\left(\frac{q_1}{q_0+q_1}\right)^{q_1}}\right) \tag{37}$$

Under the null hypothesis that $\frac{q_1}{q_0+q_1}$ equals α, the test statistic follows a chi-squared distribution with 1 degree of freedom. The p-value can be calculated with the following formula[11]:

$$p\text{-}value = 1 - F_{\chi^2_1}(LR_{POF}) \tag{38}$$

If the p-value is under 5%, the null hypothesis and the model is rejected.

[11] A p-value represents the probability under a specified statistical model that a statistical summary of the data would be equal to or more extreme than its observed value (Wasserstein & Lazar, 2016).

4.4.4 Independence

To test the independence, a test developed by Peter Christoffersen is used (Christoffersen, 1998). The test starts by dividing the potential outcomes into four groups, representing the probabilities of each outcome:

$$\pi = \pi_{00} + \pi_{01} + \pi_{10} + \pi_{11} \tag{39}$$

π_{00} is the probability of not observing a failure given that the previous outcome was not a failure, π_{01} is the probability of observing a failure given that the previous outcome was not a failure, π_{10} is the probability of not observing a failure given that the previous outcome was a failure, and π_{11} is the probability of observing a failure given that the previous outcome was a failure. This method investigates if failures tend to follow failures or if outcomes are independent on a daily basis. The method does not investigate dependence over longer horizons than one day. If the observed failures are independent, it does not matter whether the observed outcome in the period before was a failure or not. The numbers of outcomes for each group are denoted by q and the subscripts 00, 01, 10, and 11 in the same way as for π. The likelihood function can be written in the following way:

$$L(\pi) = (\pi_{00})^{q_{00}}(\pi_{01})^{q_{01}}(\pi_{10})^{q_{10}}(\pi_{11})^{q_{11}} = (1-\pi_{01})^{q_{00}}(\pi_{01})^{q_{01}}(1-\pi_{11})^{q_{10}}(\pi_{11})^{q_{11}} = \left(1 - \frac{q_{01}}{q_{00}+q_{01}}\right)^{q_{00}} \left(\frac{q_{01}}{q_{00}+q_{01}}\right)^{q_{01}} \left(1 - \frac{q_{11}}{q_{10}+q_{11}}\right)^{q_{10}} \left(\frac{q_{11}}{q_{10}+q_{11}}\right)^{q_{11}} \tag{40}$$

If all observations are independent, the null hypothesis is that:

$$\pi_{01} = \pi_{11} \tag{41}$$

A new variable is created:

$$\pi_1 = \left(\frac{q_{01}+q_{11}}{q_{00}+q_{01}+q_{10}+q_{11}}\right) \tag{42}$$

If the null hypothesis holds:

$$\pi_1 = \pi_{01} = \pi_{11} \tag{43}$$

If the null hypothesis of independence holds, Formula 40 can be simplified by using Formula 41-43 and the following relationship holds:

$$L(\pi_{H0}) = (\pi_{00})^{q_{00}}(\pi_{01})^{q_{01}}(\pi_{10})^{q_{10}}(\pi_{11})^{q_{11}} = (1-\pi_{01})^{q_{00}}(\pi_{01})^{q_{01}}(1-\pi_{11})^{q_{10}}(\pi_{11})^{q_{11}} = (1-\pi_1)^{q_{00}}(\pi_1)^{q_{01}}(1-\pi_1)^{q_{10}}(\pi_1)^{q_{11}} = (1-\pi_1)^{q_{00}+q_{10}}(\pi_1)^{q_{01}+q_{11}} = \left(1 - \frac{q_{01}+q_{11}}{q_{00}+q_{01}+q_{10}+q_{11}}\right)^{q_{00}+q_{10}} \left(\frac{q_{01}+q_{11}}{q_{00}+q_{01}+q_{10}+q_{11}}\right)^{q_{01}+q_{11}} \tag{44}$$

The test statistic for the likelihood ratio test is calculated with the following formula:

$$LR_{IND} = -2ln\left(\frac{L(\pi_{H0})}{L(\pi)}\right) =$$
$$-2ln\left(\frac{\left(1-\frac{q_{01}+q_{11}}{q_{00}+q_{01}+q_{10}+q_{11}}\right)^{q_{00}+q_{10}}\left(\frac{q_{01}+q_{11}}{q_{00}+q_{01}+q_{10}+q_{11}}\right)^{q_{01}+q_{11}}}{\left(1-\frac{q_{01}}{q_{00}+q_{01}}\right)^{q_{00}}\left(\frac{q_{01}}{q_{00}+q_{01}}\right)^{q_{01}}\left(1-\frac{q_{11}}{q_{10}+q_{11}}\right)^{q_{10}}\left(\frac{q_{11}}{q_{10}+q_{11}}\right)^{q_{11}}}\right) \tag{45}$$

The test statistic is approximately chi-squared with one degree of freedom. The p-value can be calculated with the following formula:

$$p\text{-}value = 1 - F_{\chi_1^2}(LR_{IND}) \tag{46}$$

If the p-value is under 5% the null hypothesis and the model is rejected.

4.4.5 Conditional coverage

A way to jointly test the proportion of failures and the independence of the VaR model is to use another test developed by Christoffersen (Christoffersen, 1998). The null hypothesis is that:

$$\pi_{01} = \pi_{11} = \alpha \tag{47}$$

The test statistic is the sum of the test statistics in the previous two tests, as the same two null hypotheses are tested jointly:

$$LR_{CC} = -2ln\left(\frac{L(\alpha)\,L(\pi_{H0})}{L(\frac{q_1}{q_0+q_1})L(\pi)}\right) = -2ln\left(\frac{L(\alpha)}{L(\frac{q_1}{q_0+q_1})}\right) - 2ln\left(\frac{L(\pi_{H0})}{L(\pi)}\right) =$$
$$LR_{POF} + LR_{IND} \tag{48}$$

In this case, the test statistic is approximately chi-squared with two degrees of freedom, as two null hypotheses are tested jointly. The p-value can be calculated with the following formula:

$$p\text{-}value = 1 - F_{\chi_2^2}(LR) \tag{49}$$

If the p-value is under 5%, the null hypothesis and the model is rejected.

4.5 Calculate SCRCRaR

4.5.1 Overview of the SCRCRaR calculation process

After the previous steps described in the Method section have been completed, everything needed to calculate the SCRCRaR is known, except from three yield curves from EIOPA that are used to calculate interest rate risk and the symmetric equity adjustment released by the same organization, that is used in the equity risk calculation. Section 4.5.2 describes how new values of the EIOPA yield curves are calculated and Section 4.5.3 describes the calculation of new symmetric adjustment parameters.

After all inputs have been prepared, the next step is to calculate the different market risks in the Standard Formula. Section 4.5.4-4.5.9 are used to describe the calculations of Solvency Capital Requirements for the different market risks. In all calculations, the allocation inside investment funds is assumed to remain the same as 31 December 2017.

After that, Section 4.5.10 is devoted to describe the calculation of total market risk after diversification benefits have been taken into account. In the same section, this market risk and the other risks facing an insurance company are used to calculate the total Solvency Capital Requirement. Only risks affecting the investment portfolio are changed and all other risks are treated as fixed.

After the total Solvency Capital Requirement has been calculated, Section 4.5.11 is used to describe the calculation of the SCRCR. Here, the effects on own funds (the value of assets minus liabilities) are considered, as SCRCR is calculated by dividing own funds by the SCR.

The final step is to use the SCRCR values to calculate the SCRCRaR and this is described in Section 4.5.12.

The descriptions in the following subsections should not be seen as a complete description of the regulation. The ambition is only to describe the calculations that need to be made in this particular case. Therefore, the subsections describe Dina's interpretation of the regulation (including simplifications) rather than the actual regulation or the author of this thesis' interpretation of the regulation. Dina's interpretation of the regulation is approved by Finansinspektionen, Sweden's financial supervisory authority, and serves as an example of a typical interpretation of the regulation.

Four things differ in the 1000 scenarios generated by the Monte Carlo simulation described in Section 4.2: the values of the assets (see Section 4.3), the credit quality steps of the interest-bearing assets (see Section 4.3.3.3), the symmetric adjustment (see Section 4.5.3), and the yield curves used to calculate interest rate risk (see Section 4.5.2).

4.5.2 EIOPA yield curve

EIOPA has published risk-free yield curves once a month since December 2015 ('EIOPA', 2018). The curves are used for the calculation of interest rate risk. Three curves are published each time: the current yield curve, a yield curve with increased interest rates, and a yield curve with decreased interest rates. The curves consist of one yield per year (1-year, 2-year,...,150-year). In this thesis, daily data is necessary and the monthly data can not be used directly. The daily values of the yields are modelled with linear regressions. The assumption made is that the regression results based on monthly data can be used to model the daily values of the EIOPA yield curves. The regressions for the different horizons are based on the swap rates with the same horizons and for horizons over ten years the 10-year swap rate is used as independent variable. This is done for the unchanged interest rates, the increased interest rates, and the decreased interest rates.

$$R_{EIOPA,h,n} = b \cdot P_{\tilde{h},n} + c \tag{50}$$

$R_{EIOPA,h,n}$ is the EIOPA yield with horizon h in observation n. The change between one n and the following is one month, as the EIOPA yield curves are published monthly. $P_{\tilde{h},n}$ is the swap rate with horizon $\tilde{h}$ (see above how these horizons relate to h). b is a coefficient estimated by the regression. It indicates how one unit of increase in the swap rate affects the EIOPA yield. c is a constant and it represents the EIOPA yield with horizon h is if the swap rate is 0.

The regression results are used to calculate $R_{EIOPA,h,i}$. This is done with the following formula ($P_{\tilde{h},i}$ is calculated with a formula corresponding to Formula 7):

$$R_{EIOPA,h,i} = \hat{b} \cdot \left(P_{\tilde{h},0} + \sigma_{\tilde{h}} U_{\tilde{h},i} \sqrt{t}\right) + \hat{c} \tag{51}$$

4.5.3 Symmetric adjustment

The symmetric adjustment, SA, that is used to calculate equity risk, is based on the following formula (European Commission, 2015):

$$SA = \frac{1}{2} \cdot \left(\frac{CI - AI}{AI} - 8\%\right) \tag{52}$$

CI is the current level of EIOPA Equity Index. AI is the average daily level the last 36 months. The symmetric adjustment can not be lower than -10%

and not higher than 10%. This formula can also be expressed in the following way:

$$SA = \frac{1}{2} \cdot \left(\frac{CI}{AI} - 1.08 \right) \tag{53}$$

It is accordingly possible to solve for $\frac{CI}{AI}$:

$$\frac{CI}{AI} = 2SA + 1.08 \tag{54}$$

The quota 31 December 2017 can thus be calculated by this formula:

$$\frac{CI_0}{AI_0} = 2SA_0 + 1.08 \tag{55}$$

Assuming that the index change in the scenario investigated is x_i% and that the change is immediate (it does not affect AI), the symmetric adjustment can be calculated with the following formula:

$$SA_i = \frac{1}{2} \cdot \left(\frac{CI_0}{AI_0} \cdot (1 + x_i) - 1.08 \right) \tag{56}$$

A simplifying assumption made here is that the changes in the EIOPA Equity Index, x, follow the development of Dina's portfolio of listed stocks in local currency (so Protector Forsikring is not affected by exchange rates). The validity of this assumption is evaluated by investigating the historical correlation between Dina's stock portfolio and the EIOPA Equity Index. The reason why an assumption is made is to simplify and to deal with the fact that enough data for EIOPA Equity Index is not available. The index has only been calculated since 2015 ('EIOPA', 2018). Otherwise, the index change could have been used as a risk factor.

4.5.4 Interest rate risk

All interest-bearing assets, including those in investment funds, are subject to interest rate risk. Dina has made the simplifying assumption to base the interest rate risk on modified durations rather than on cash flows[12]. The horizon of the discount rates (no change, increased rates, and decreased rates) for each asset are determined by rounding the modified duration of the asset

[12] Modified duration is the relationship between proportional changes in a bond price and actual changes in its yield (Hull, 2015). Modified durations are included in the current positions data, so in this thesis they are just an input.

upwards to an integer. So if the modified duration is 2.01, the 3-year interest rates are used. The interest rate horizons used are named h in the formulas below.

The discounted present value of an asset, $V_{a,i,0}$, is calculated with the following formula:

$$V_{a,i,0} = Face\,value \cdot \left(e^{-r_{EIOPA,h,i,0} \cdot d} + \frac{1 - e^{-r_{EIOPA,h,i,0} \cdot d}}{r_{EIOPA,h,i,0}} \cdot r_{fixed} + \frac{1 - e^{-r_{EIOPA,h,i,0} \cdot d}}{r_{EIOPA,h,i,0}} \cdot r_{floating} \right) \tag{57}$$

This means that the value consists of three parts: the face value discounted at the modified duration $\left(Face\,value \cdot e^{-r_{EIOPA,h,i,0} \cdot d}\right)$, an annuity of continuous fixed coupon payments until the modified duration $\left(Face\,value \cdot \frac{1 - e^{-r_{EIOPA,h,i,0} \cdot d}}{r_{EIOPA,h,i,0}} \cdot r_{fixed}\right)$, and an annuity of continuous floating coupon payments until the modified duration $\left(Face\,value \cdot \frac{1 - e^{-r_{EIOPA,h,i,0} \cdot d}}{r_{EIOPA,h,i,0}} \cdot r_{floating}\right)$. d is the modified duration. For investment funds, the face value is replaced by the market value of the interest-bearing part of the fund, as the face value is not available.

The discounted value in the case of increased rates is calculated with the following formula:

$$V_{a,i,+} = Face\,value \cdot \left(e^{-r_{EIOPA,h,i,+} \cdot d} + \frac{1 - e^{-r_{EIOPA,h,i,+} \cdot d}}{r_{EIOPA,h,i,+}} \cdot r_{fixed} + \frac{1 - e^{-r_{EIOPA,h,i,0} \cdot d}}{r_{EIOPA,h,i,0}} \cdot r_{floating} \right) \tag{58}$$

This is the same formula as when the interest rates were unchanged, but with the increased discount rate for the face value and the fixed coupons.

The discounted value in the case of decreased rates is calculated with the following formula:

$$V_{a,i,-} = Face\,value \cdot \left(e^{-r_{EIOPA,h,i,-} \cdot d} + \frac{1 - e^{-r_{EIOPA,h,i,-} \cdot d}}{r_{EIOPA,h,i,-}} \cdot r_{fixed} + \frac{1 - e^{-r_{EIOPA,h,i,0} \cdot d}}{r_{EIOPA,h,i,0}} \cdot r_{floating} \right) \tag{59}$$

This is also the same formula as when the interest rates were unchanged, but with the decreased discount rate for the face value and the fixed coupons.

The total value of the assets is calculated in the three scenarios:

$$V_{i,0} = \sum_{i=1}^{Iterations} V_{a,i,0} \tag{60}$$

The same formula, but with + (-) instead of 0 is used to calculate the value in case of increased (decreased) interest rates. The total value change due to a decrease or an increase in interest rates is calculated with the following formula (replace the + subscripts with - in the case of a decreased interest rate):

$$\Delta V_{i,+} = V_{i,+} - V_{i,0} \tag{61}$$

These value changes are combined with the value changes of the liabilities, to calculate the total change of assets minus liabilities. The effects on the liability side are assumed to be fixed, as this thesis only investigates changes in the investment portfolio.

$$\Delta Total_{i,+} = \Delta V_{i,+} - \Delta L_{i,+} \tag{62}$$

The absolute value of the lowest value of $\Delta Total_i$ (with increased or decreased interest rate) is the capital requirement for interest rate risk.

4.5.5 Equity risk

Stocks that are unlisted or listed outside both the OECD and the European Economic Area are considered to be type 2 equity. All type 2 equities, except strategic participations, are stressed by 49% plus the value of the symmetric adjustment. All other stocks are type 1 equity and stressed by 39% plus the value of the symmetric adjustment. The total equity risk SCR is calculated with the following formula:

$$Equity_i = \sqrt{\begin{bmatrix} Type\,1_i & Type\,2_i \end{bmatrix} \begin{bmatrix} 0.75 & 1 \\ 1 & 0.75 \end{bmatrix} \begin{bmatrix} Type\,1_i \\ Type\,2_i \end{bmatrix}} \tag{63}$$

$Type\,1$ is the SCR for type 1 equity and $Type\,2$ is the SCR for type 2 equity. The shares of investment funds consisting of equity are handled in the same way.

The fully owned property companies are seen as equity in this case. The values considered are the values of their assets minus their liabilities. They are considered to be strategic participations and are stressed by 22% and the symmetric adjustment is not used for strategic participations. The fully owned property companies are included in the type 2 equities category despite the low stress, since the stocks are unlisted. All other stocks are non-strategic.

4.5.6 Property risk

Property risk is calculated as 25% of the market values of directly owned property. Accordingly, this category only applies to Diana 2 and Juno 9.

$$Property_i = MV_{directly\,owned\,property,i} \cdot 25\% \tag{64}$$

$Property_i$ is the property risk for iteration i. $MV_{directly\,owned\,property,i}$ is the market value of the directly owned property in iteration i.

4.5.7 Spread risk

The credit quality steps used to calculate spread risk are the credit quality steps calculated in the simulation, if the credit quality steps were based on an official rating. If the rating was set by Swedbank, the asset is considered to be unrated.

The first step in the spread risk calculation is to divide the interest-bearing assets into four groups:

1. Government bonds (including supra-national organizations) in the European Economic Area
2. Investment funds
3. Covered bonds
4. Other bonds

For these groups, the spread risk stress is dependent on modified durations and credit quality steps. Table 15 shows the groups and the calculations, while Table 16-21 show the values of the parameters. For government bonds there is no stress.

Duration	Stress
Up to 5	$b \cdot duration$
More than 5 and less than 10	$a + b \cdot (duration - 5)$
More than 10 and less than 15	$a + b \cdot (duration - 10)$
More than 15 and less than 20	$a + b \cdot (duration - 15)$
More than 20	$min[a + b \cdot (duration - 20); 1]$

Table 15: Duration groups and stress calculations for spread risk.

Duration	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5 and 6	Unrated
Up to 5	-	-	-	-	-	-	-
More than 5 and less than 10	4.5%	5.5%	7.0%	12.5%	22.5%	37.5%	15.0%
More than 10 and less than 15	7.0%	8.4%	10.5%	20.0%	35.0%	58.5%	23.5%
More than 15 and less than 20	9.5%	10.9%	13.0%	25.0%	44.0%	61.0%	29.5%
More than 20	12.0%	13.4%	15.5%	30.0%	46.5%	63.5%	35.5%

Table 16: the a's in the calculations in Table 15 for investment funds.

Duration	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5 and 6	Unrated
Up to 5	0.9%	1.1%	1.4%	2.5%	4.5%	7.5%	3.0%
More than 5 and less than 10	0.5%	0.6%	0.7%	1.5%	2.5%	4.2%	1.7%
More than 10 and less than 15	0.5%	0.5%	0.5%	1.0%	1.8%	0.5%	1.2%
More than 15 and less than 20	0.5%	0.5%	0.5%	1.0%	0.5%	0.5%	1.2%
More than 20	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%

Table 17: the b's in the calculations in Table 15 for investment funds.

Duration	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5 and 6	Unrated
Up to 5	-	-	-	-	-	-	-
More than 5 and less than 10	3.5%	4.5%	7.0%	12.5%	22.5%	37.5%	15.0%
More than 10 and less than 15	6.0%	7.0%	10.5%	20.0%	35.0%	58.5%	23.5%
More than 15 and less than 20	8.5%	9.5%	13.0%	25.0%	44.0%	61.0%	29.5%
More than 20	11.0%	12.0%	15.5%	30.0%	46.5%	63.5%	35.5%

Table 18: the a's in the calculations in Table 15 for covered bonds.

Duration	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5 and 6	Unrated
Up to 5	0.7%	0.9%	1.4%	2.5%	4.5%	7.5%	3.0%
More than 5 and less than 10	0.5%	0.5%	0.7%	1.5%	2.5%	4.2%	1.7%
More than 10 and less than 15	0.5%	0.5%	0.5%	1.0%	1.8%	0.5%	1.2%
More than 15 and less than 20	0.5%	0.5%	0.5%	1.0%	0.5%	0.5%	1.2%
More than 20	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%

Table 19: the b's in the calculations in Table 15 for covered bonds.

Duration	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5 and 6	Unrated
Up to 5	-	-	-	-	-	-	-
More than 5 and less than 10	0.0%	0.0%	5.5%	7.0%	12.5%	22.5%	15.0%
More than 10 and less than 15	0.0%	0.0%	8.4%	10.5%	20.0%	35.0%	23.5%
More than 15 and less than 20	0.0%	0.0%	10.9%	13.0%	25.0%	44.0%	29.5%
More than 20	0.0%	0.0%	13.4%	15.5%	30.0%	46.5%	35.5%

Table 20: the a's in the calculations in Table 15 for other bonds.

Duration	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5 and 6	Unrated
Up to 5	0.0%	0.0%	1.1%	1.4%	2.5%	4.5%	3.0%
More than 5 and less than 10	0.0%	0.0%	0.6%	0.7%	1.5%	2.5%	1.7%
More than 10 and less than 15	0.0%	0.0%	0.5%	0.5%	1.0%	1.8%	1.2%
More than 15 and less than 20	0.0%	0.0%	0.5%	0.5%	1.0%	0.5%	1.2%
More than 20	0.0%	0.0%	0.5%	0.5%	0.5%	0.5%	0.5%

Table 21: the b's in the calculations in Table 15 for other bonds.

4.5.8 Concentration risk

Cash is not considered in any of the concentration risk calculations. The stocks in the fully owned property companies are valued to the amount of their assets minus their liabilities. The credit quality steps are the credit quality steps calculated in the simulation if there is an official rating. If the rating was set by Swedbank, the asset is considered to be unrated.

The exposures are summed per issuer. The credit quality step of each issuer is calculated as the value-weighted average of all the exposures to that issuer, rounded to the closest integer. For concentration risk, all assets (except cash) are divided into categories, in the way shown in Table 22. The concentration thresholds (exposures higher than the concentration threshold are stressed) and risk factors (the stress applied to exposures higher than the concentration threshold) for the different categories are shown in Table 23-27. Investment funds are assumed to not contribute to any concentration risk. The capital requirements are taken to the power of 2. These numbers are summed for all the issuers. The square root of the sum is the total capital requirement for concentration risk.

Category	Subcategory	Asset type	Rating	Issuer	Country of issuer
Rated	1	Covered bonds	Rated	All	All
Rated	2	Uncovered bonds	Rated	Government	Outside EEA
Rated	Other	Uncovered bonds	Rated	All	All
S2	1	All	Unrated	Insurer	Within EEA
Other	1	Property	Unrated	All	All
Other	2	All but property	Unrated	Government	Within EEA
Other	3	All but property	Unrated	MFI	All
Other	4	All but property	Unrated	Insurer	Outside EEA
Other	Other	All but property	Unrated	Other	All

Table 22: Calculation classes in the concentration risk calculations. EEA is an abbreviation for European Economic Area and MFI is an abbreviation for Monetary Financial Institution.

Subcategory	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
1	15.0%	15.0%	3.0%	1.5%	1.5%	1.5%	1.5%
2	1.5%	1.5%	1.5%	3.0%	3.0%	3.0%	3.0%
Other	1.5%	1.5%	1.5%	3.0%	3.0%	3.0%	3.0%

Table 23: Concentration thresholds for rated bonds (category Rated).

Subcategory	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
1	12.0%	12.0%	21.0%	27.0%	73.0%	73.0%	73.0%
2	0.0%	0.0%	12.0%	21.0%	27.0%	73.0%	73.0%
Other	12.0%	12.0%	21.0%	27.0%	73.0%	73.0%	73.0%

Table 24: Risk factors for rated bonds (category Rated).

Subcategory	SCRCR≥196%	≥175%	≥122%	≥100%	<100%
1	3.0%	3.0%	3.0%	1.5%	1.5%

Table 25: Concentration thresholds for insurance companies subject to Solvency II (category S2).

Subcategory	SCRCR≥196%	≥175%	≥122%	≥100%	<100%
1	12.0%	21.0%	27.0%	64.5%	73.0%

Table 26: Risk factors for insurance companies subject to Solvency II (category S2).

Subcategory	Concentration threshold	Risk factor
1	10.0%	12.0%
2	0.0%	0.0%
3	1.5%	64.5%
4	1.5%	64.5%
Other	1.5%	73.0%

Table 27: Concentration thresholds and risk factors for unrated issuers that are not subject to Solvency II (category Other).

4.5.9 Currency risk

Currency risk is calculated as 25% of the market values of assets denoted in foreign currency (not SEK). This stress is used no matter if the asset is directly owned or owned through investment funds. All the investment funds are denoted in SEK, but if they had been denoted in foreign currency the stress of 25% would have been applied on 100% of their market value. This can be described by the following formula:

$$Currency_i = (MV_{all,foreign\,currency,i} + MV_{investment\,funds,SEK,i} \cdot f_{investment\,funds,SEK}) \cdot 25\% \tag{65}$$

$Currency_i$ is the currency risk for iteration i. $MV_{all,foreign\,currency,i}$ is the market value of all assets denoted in foreign currency in iteration i. $MV_{investment\,funds,SEK,i}$ is the value of all investment funds denoted in SEK in iteration i and $f_{investment\,funds,SEK}$ is the fraction of them that is invested in foreign currency.

4.5.10 Solvency Capital Requirement

The total Solvency Capital Requirement for market risk is calculated by the following equation:

$$Market_i = \sqrt{\begin{bmatrix} Interest\,rate_i \\ Equity_i \\ Property_i \\ Spread_i \\ Concentration_i \\ Currency_i \end{bmatrix}^T \begin{bmatrix} 1 & A & A & A & 0 & 0.25 \\ A & 1 & 0.75 & 0.75 & 0 & 0.25 \\ A & 0.75 & 1 & 0.5 & 0 & 0.25 \\ A & 0.75 & 0.5 & 1 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 & 1 \end{bmatrix} \begin{bmatrix} Interest\,rate_i \\ Equity_i \\ Property_i \\ Spread_i \\ Concentration_i \\ Currency_i \end{bmatrix}} \tag{66}$$

A is 0 (0.5) when the capital requirement for interest rate risk is given by an upward (downward) shock in the interest rate term structure (Thoren, 2015).

The total Solvency Capital Requirement is calculated with the following formula:

$$SCR_i = \sqrt{\begin{bmatrix} Market_i \\ Counterparty \\ Life \\ Health \\ Non\text{-}life \end{bmatrix}^T \begin{bmatrix} 1 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 1 & 0.25 & 0.25 & 0.5 \\ 0.25 & 0.25 & 1 & 0.25 & 0 \\ 0.25 & 0.25 & 0.25 & 1 & 0 \\ 0.25 & 0.5 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Market_i \\ Counterparty \\ Life \\ Health \\ Non\text{-}life \end{bmatrix}} \tag{67}$$

All risk modules except the market risk module are assumed to be unchanged, as only changes in the investment portfolio are considered in this thesis.

4.5.11 Solvency Capital Requirement coverage ratio

Own funds, O, are calculated with the following formula (the tax rate is 22%):

$$O_i = O_0 + \Delta V_i \cdot (1 - tax\,rate) \tag{68}$$

The SCRCR can be calculated with the following formula:

$$SCRCR_i = \frac{O_i}{SCR_i} \tag{69}$$

4.5.12 Solvency Capital Requirement coverage ratio at risk

$\Delta SCRCR_i$ is the change in SCRCR ($\Delta SCRCR_i = SCRCR_i - SCRCR_0$). The 1000 values of $\Delta SCRCR_i$ form the discrete cumulative distribution function $F_{\Delta SCRCR}$.

SCRCRaR is calculated in the same way as VaR was calculated for value changes. Mathematically, this is described by the following formula:

$$SCRCRaR_\alpha(SCRCR) = -F^{-1}_{\Delta SCRCR}(1-\alpha) \tag{70}$$

α is the confidence level 99.5%. The minus sign before the formula is there to turn negative changes into positive numbers.

5 Results and discussion

5.1 Findings

5.1.1 Solvency Capital Requirement coverage ratio at risk for Dina Försäkring AB and difference from value at risk

The structure of the Results and discussion section is that Section 5.1 presents the main findings. Section 5.1.1 presents the calculated values of the SCRCRaR and shows that the measure is different from VaR. Section 5.1.2 presents the main drivers of the SCRCRaR and the causes of the worst outcomes. Section 5.2 motivates why the results are trustworthy.

Table 28 shows that the SCRCRaR is fairly stable between the five iterations. Dina AB can be 99.5% confident that the SCRCR will not drop more than approximately 17.5-19.6 percentage points or 11.2-12.6%. The company has a strategic goal to never let the SCRCR fall below 130%. Given that the SCRCR was 165.4% 31 December 2017, Dina AB can be 99.5% confident that the SCRCR will not be lower than 145.8-147.9% 31 December 2018 due to risk in the investment portfolio[13]. Accordingly, Dina AB can be very confident that the SCRCR will not drop below 130% due to risk in the investment portfolio.

Iteration	VaR (SEK)	VaR (%)	SCRCRaR (pp)	SCRCRaR (%)
1	291 555 041	9.4%	19.1pp	12.3%
2	281 987 670	9.1%	18.4pp	11.8%
3	294 267 389	9.4%	19.6pp	12.6%
4	282 590 831	9.1%	17.5pp	11.2%
5	289 253 313	9.3%	18.3pp	11.7%

Table 28: 99.5% 1-year value at risk and Solvency Capital Requirement coverage ratio at risk in the five iterations. pp is an abbreviation for percentage points.

In Figure 11, the SCRCR changes in the different scenarios are divided into bins and a histogram is formed. A line showing the implied probability density function is added. The distributions for all five iterations are very similar, which you will see in Figure 12. As you can see in Figure 11, there is almost no upside and quite a big downside in the SCRCR measure for Dina AB. The analysis in Section 5.1.2 will show that SCRCR changes are mainly caused by value changes and that value changes are mainly caused by value changes of the property. In Dina AB's case, the dominant market

[13] 31 December 2017 is the calculation date for this prediction.

risks in the Standard Formula are concentration risk, that is caused by the holdings in Dina Palaisbacken AB and Diana Skog AB, and equity risk, that is also strongly influenced by the value of the fully owned property companies. These two holdings represent 72% of the property portfolio. From this, one can draw the conclusion that positive value changes of the property lead to positive value changes of the investment portfolio, increased concentration risk, and increased equity risk. Negative value changes of the property have the opposite impact. Formula 66 from Section 4.5.10, that is used to calculate the market risk, implies a diversification benefit. Therefore, increases in the highest market risks, concentration risk and equity risk, have a stronger effect on the total market risk than decreases have. Furthermore, market risk is the dominant risk module for Dina AB, so an increased market risk has a stronger effect on the Solvency Capital Requirement than a decrease has, for the same reason (see Formula 67 in Section 4.5.10). Thus, the effect on the Solvency Capital Requirement is higher for positive value changes of the investment portfolio than for negative value changes. The effect on own funds is the same, so the effect on SCRCR is stronger for decreases in value of the investment portfolio than for increases (see Formula 68-69 in Section 4.5.11). Therefore, the upside potential of the SCRCR is much smaller than the downside.

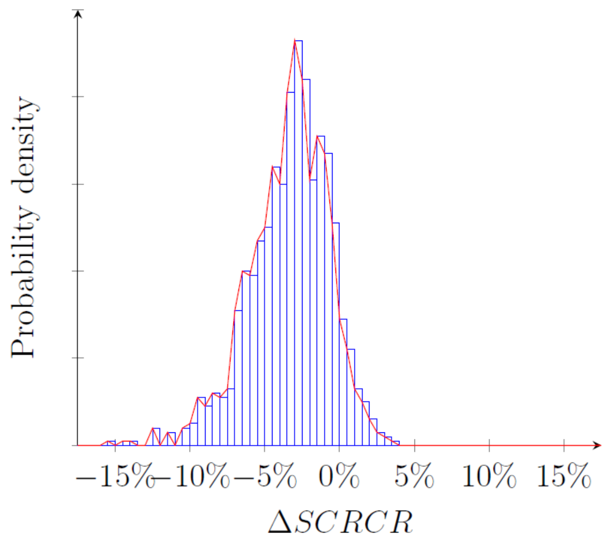

Figure 11: Histogram over SCRCR changes and the implied probability density function in iteration 1. The first bin from the left consists of all returns higher or equal to -17.5%, but lower than -17.0%. The second bin consists of all returns higher or equal to -17.0% but lower than -16.5%, the third bin consists of all returns higher or equal to -16.5% but lower than -16.0%, and so on.

As you could see in Table 28, the SCRCRaR in % is overall higher than the corresponding VaR[14]. Figure 12 shows that the risk profile of Dina AB is much worse for SCRCR changes than for value changes. The reason is the one mentioned in the previous paragraph: Dina AB is punished very hardly for the lack of diversification in the Standard Formula. At a first glance this may seem unreasonable, but if the regulator wants to promote diversification, the configuration of the Standard Formula is rational. If Dina AB wants to improve the risk profile of its SCRCR, the way goes through increased diversification. Notice, however, that the relationship between SCRCRaR and VaR (or between SCRCR changes and value changes of the investment portfolio) is unique for Dina AB and not general. Hopefully further research can investigate the relationship in different cases to be able to draw a more general conclusion of how it looks. The conclusion that can be drawn here is that the measures are different from each other and there is accordingly a need for the SCRCRaR measure that has been developed in this thesis. If a property insurance company wants to improve the risk profile of its SCRCR, diversification among the risks in the Solvency II regulation's Standard Formula is key.

[14] The measures are only comparable in their % versions.

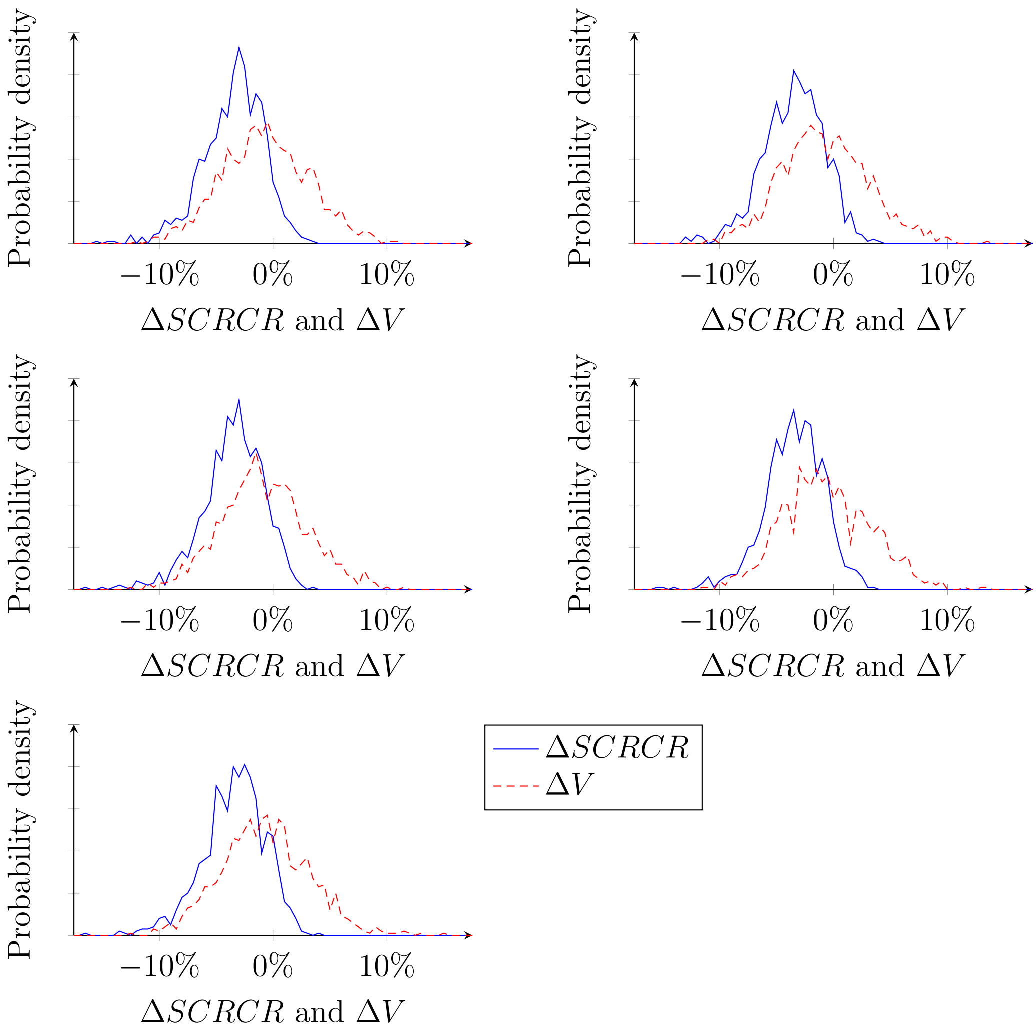

Figure 12: Implied probability density functions for Solvency Capital Requirement coverage ratio changes ($\Delta SCRCR$) and value changes (ΔV) in the five iterations.

5.1.2 Key drivers of the Solvency Capital Requirement coverage ratio at risk

Four things change in the SCRCR calculations for the 1000 simulations:

1. Values of the assets.
2. Credit ratings of the assets.
3. Symmetric adjustment parameter.

4. Risk-free yield curves.

The results show that value changes have, by far, the biggest impact. Figure 13 shows the relationship between changes in values of the investment portfolio and changes in SCRCR. The correlation is approximately 0.7 and for large decreases, the relationship is almost linear. This relationship is not surprising given that the value changes have a direct impact (100%-tax rate, see Formula 68) on the own funds (the numerator in the SCRCR formula, see Formula 69). The relationship is very similar in the five iterations.

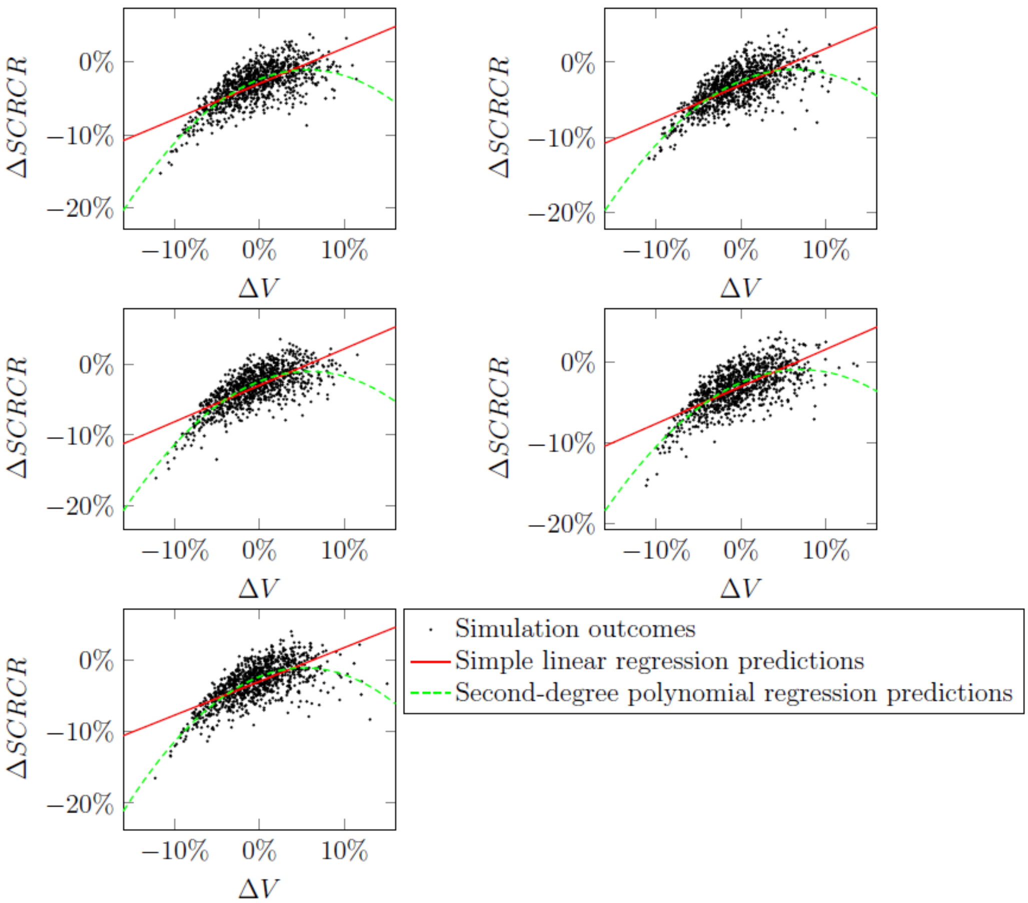

Figure 13: Percentage change in the Solvency Capital Requirement coverage ratio as a function of the corresponding percentage change in value of the investment portfolio for the five iterations.

It is possible to model SCRCR changes directly from value changes of the investment portfolio. By running regressions of changes in SCRCR based on changes in value of the investment portfolio, fairly strong results are obtained (see Table 29 and 30). All the results are statistically significant on any reasonable significance level. There is accordingly strong evidence that value changes of the the investment portfolio can be used to model SCRCR changes. The coefficient of determination is on average 48% for the simple linear regression and 57% for the second-degree polynomial regression[15]. Accordingly, value changes explain quite a big part of the changes in SCRCR. A coefficient of determination of 25% is considered to be weak, a coefficient of determination of 50% is considered to be moderate, and a coefficient of determination of 75% is considered to be substantial (Hair, Ringle, & Sarstedt, 2011). Notice that the coefficients of determination are based on the entire distribution. Figure 13 shows that the relationship is stronger for the worst outcomes, so the coefficient of determination would have been higher if only the worst outcomes had been investigated. The worst outcomes in terms of SCRCR changes are often the same observations as the worst value changes of the investment portfolio.

Iteration	ΔV	p ΔV	c	p c	R^2
1	0.49	0.00%	-0.03	0.00%	48.75%
2	0.48	0.00%	-0.03	0.00%	47.59%
3	0.51	0.00%	-0.03	0.00%	50.56%
4	0.46	0.00%	-0.03	0.00%	47.02%
5	0.47	0.00%	-0.03	0.00%	44.29%

Table 29: Simple linear regression results for Solvency Capital Requirement coverage ratio changes ($\Delta SCRCR$) as a function of value changes of the investment portfolio. ΔV denotes the effect of value changes of the investment portfolio, p denotes the p-values of the t-statistics, and c denotes the constant.

[15] A coefficient of determination indicates how much of the variation in the dependent variable that is explained by the regression model (Harel, 2009).

Iteration	ΔV	$p\ \Delta V$	ΔV^2	$p\ \Delta V^2$	c	p c	p F	R^2
1	0.46	0.00%	-4.12	0.00%	-0.02	0.00%	0.00%	58.23%
2	0.48	0.00%	-3.76	0.00%	-0.02	0.00%	0.00%	56.03%
3	0.48	0.00%	-4.13	0.00%	-0.02	0.00%	0.00%	59.37%
4	0.46	0.00%	-3.31	0.00%	-0.03	0.00%	0.00%	53.96%
5	0.47	0.00%	-4.41	0.00%	-0.02	0.00%	0.00%	58.23%

Table 30: Second-degree polynomial regression results for Solvency Capital Requirement coverage ratio changes ($\Delta SCRCR$) as a function of value changes of the investment portfolio. ΔV denotes the effect of value changes of the investment portfolio, ΔV^2 denotes the effect of squared value changes of the investment portfolio, p denotes the p-values of the t-statistics, and c denotes the constant.

From a risk management perspective, the big negative SCRCR changes mentioned in the end of the last paragraph are most interesting and these changes are not accurately modelled by linear regressions based on all outcomes (see the solid red lines in Figure 13). Second-degree polynomial regressions, the dashed green curves in Figure 13, do a much better job for the worst outcomes. The prediction abilities of the second-degree polynomial regressions are more accurate for big negative SCRCR changes than for the better outcomes, as Figure 13 shows. Thus, big negative changes in the SCRCR can be accurately modelled from value changes. Higher order polynomial regressions mean no significant changes in the prediction ability overall, but improve the prediction ability for the worst outcomes further. It is, however, not obvious that a certain kind of regression model performs better for all property insurance companies. The relationship between value changes of the investment portfolio and the SCRCR is strongly affected by the composition of the Solvency Capital Requirement and of the risk profile, in terms of value changes, of the investment portfolio. The composition of the Solvency Capital Requirement is known for Dina AB and what is left to explore is what causes the value changes in the portfolio and how the risk profile in terms of value changes of the portfolio looks. For this, VaR is a key measure.

The implied probability density functions for the asset categories *Stocks*, *Property*, and *Interest-bearing assets* are shown in Figure 14. Stock funds are considered to be *Stocks*, bond funds are considered to be *Interest-bearing assets*, and the fully owned property companies are considered to be *Property*. The distributions for all five iterations are very similar (see Section 8.4 in the Appendix). It is likely that the zigzag patterns of the curves would disappear with more simulations than 1000, but the overall pattern is clear also with

1000 simulations. As you can see, *Interest-bearing assets* are least risky and *Stocks* are most risky. There is, however, a significant diversification benefit from having all these asset categories. Table 31 shows that the total VaR is on average 12.3% lower than the sum of the VaR for the asset categories. You can also see in the table that the VaR is stable between the iterations. As the table indicates, the worst outcomes are mainly due to large declines in the property values[16]. The average property contribution to the five worst value changes is 53%, while it is 28% for stocks and 19% for interest-bearing assets. Notice that the drivers of value changes of the investment portfolio are very specific for Dina AB, so these results are only valid in this particular case.

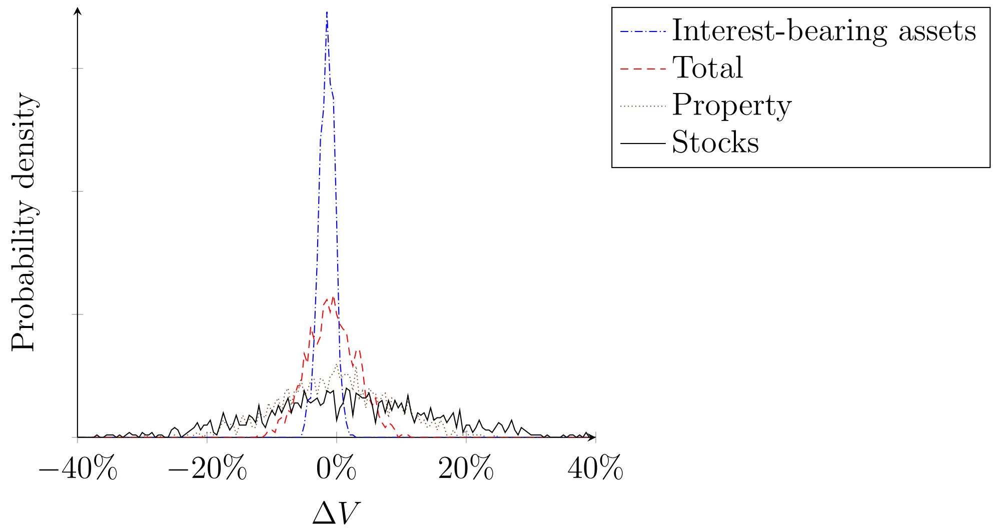

Figure 14: Implied probability density functions for the different asset categories in iteration 1.

[16]Notice, however, that the worst observations for property are not necessarily exactly the same observations as the worst outcomes on aggregate.

Iteration	VaR Stocks	VaR Property	VaR Interest-bearing assets	VaR Total	VaR Total (%)
1	84 764 137	166 282 455	77 982 513	291 555 041	9.4%
2	94 161 471	167 559 247	71 498 265	281 987 670	9.1%
3	81 379 963	167 135 119	79 636 084	294 267 389	9.4%
4	81 528 921	168 583 376	75 735 473	282 590 831	9.1%
5	88 126 570	159 095 062	79 037 206	289 253 313	9.3%

Table 31: 99.5% 1-year value at risk for the different asset classes in the five iterations.

As the property holdings have a significant impact on the total value changes of the portfolio, their impact on the SCRCR needs to be investigated further. In Dina AB's case, the dominant market risks in the Standard Formula are concentration risk, that is caused by the holdings in Dina Palaisbacken AB and Diana Skog AB, and equity risk, that is also strongly influenced by the value of the fully owned property companies. These two holdings represent 72% of the property portfolio. From this, one can draw the conclusion that positive value changes of the property lead to positive value changes of the investment portfolio, increased concentration risk, and increased equity risk. Negative value changes of the property have the opposite impact. Formula 66 from Section 4.5.10, that is used to calculate the market risk, implies a diversification benefit. Therefore, increases in the highest market risks, concentration risk and equity risk, have a stronger effect on the total market risk than decreases have. Furthermore, market risk is the dominant risk module for Dina AB, so an increased market risk has a stronger effect on the Solvency Capital Requirement than a decrease has, for the same reason (see Formula 67 in Section 4.5.10). Thus, the effect on the Solvency Capital Requirement is higher for positive value changes of the investment portfolio than for negative value changes. The effect on own funds is the same, so the effect on SCRCR is stronger for decreases in value of the investment portfolio than for increases (see Formula 68-69 in Section 4.5.11). This explains the concave pattern with little upside for the SCRCR in Figure 13. It also explains why the second-degree polynomial regressions are more accurate than the linear regressions. Notice, however, that the conclusions in this paragraph are specific to Dina AB and not general. A property insurance company with a more diversified risk composition would have had a more linear relationship between value changes of the investment portfolio and changes in the SCRCR. The main conclusion from this subsection is therefore that changes in SCRCR are mainly caused by value changes of the investment portfolio and that it is possible to model big negative changes in the SCRCR accurately from value changes of the investment portfolio[17].

5.2 Validity

5.2.1 Values of the assets

The aim of Section 5.2 is to show that the findings are reliable. Section 5.2.1-5.2.4 show that the four things that differ among the 1000 SCRCR simulations are accurately modelled (see the list in Section 5.1.2). To further validate the SCRCRaR framework, its robustness is evaluated. It is important that the results are similar between iterations and otherwise a higher number of simulations must be considered. The robustness is discussed in Section 5.2.5.

The 99.5% SCRCRaR focuses on the change in the SCRCR for the fifth to sixth worst SCRCR observations out of 1000. Table 32 shows that the average rank of the value change (from worst to best) for the fifth worst SCRCR outcome is 5.4. For the sixth worst SCRCR outcome, the number is 6.2. Accordingly, the observations that affect the SCRCRaR are the same as or very similar to the observations that affect the VaR. From this, it is possible to draw the conclusion that an accurate VaR model is crucial for the quality of the SCRCRaR. If the VaR model is accurate, the new values that affect the SCRCRaR can be seen as accurate. The information in Table 32 is also consistent with the conclusion that big negative changes in the Solvency Capital Requirement coverage ratio can be accurately estimated from value changes.

Iteration	Fifth worst SCRCR change	Sixth worst SCRCR change
1	5	7
2	6	7
3	5	2
4	6	9
5	5	6

Table 32: The value change rank (from worst to best) of the fifth and sixth worst Solvency Capital Requirement coverage ratio change.

As you can see in Table 33, the VaR model consistently passes all the tests. The failures occur at almost the same dates. The dates of the four failures in iteration 2 are among the dates of the five failures in iteration 1

[17]Notice that new values of the assets have an impact on the calculation of SCRCR, so the order of the causality chain is easily specified.

and 5, which are the same except for one. All these failure dates are among the ten failure dates in iteration 4, which are among the eleven failure dates in iteration 3. Overall, the failures are associated with large declines in the stock market. Notice that the property is not included in the backtests, as daily data is not available for the property. Section 8.5 in the Appendix shows that the valuation models used for property are accurate.

Iteration	q_0	q_1	p-value POF	q_{00}	q_{01}	q_{10}	q_{11}	p-value IND	p-value CC
1	1246	5	60.2%	1240	5	5	0	-	-
2	1247	4	33.3%	1242	4	4	0	-	-
3	1240	11	8.6%	1229	10	10	1	8.2%	5.1%
4	1241	10	16.7%	1231	9	9	1	6.6%	7.1%
5	1246	5	60.2%	1240	5	5	0	-	-

Table 33: P-values for the backtests of the 99.5% 1-year value at risk. POF is an abbreviation for the proportion of failures test (the Kupiec test), IND is an abbreviation for the independence test (the Christoffersen test), and CC is an abbreviation for the conditional coverage test. - means that the independence test was impossible to conduct, since there were no consecutive failures and accordingly no signs of dependence. The conditional coverage test is not possible to run in these cases, so the model passes the tests if it passes the proportion of failures test.

A perfectly accurate model would render 6.3 failures, while the average among the five iterations is 7, with values from 4 to 11. All these numbers are close enough to 6.3 to pass the proportion of failures test (the Kupiec test).

There are only two failures in a row in two out of the five iterations and even in these two cases the model passes the independence test (the Christoffersen test). The p-values are not far from the significance level of 5%, but one should consider that 3/5 models have no consecutive failures at all. Accordingly, there are no signs of dependence. It is, however, worth to notice that the lack of consecutive failures in 3/5 cases could indicate that a too small sample is used. Only 6.3 failures are expected and this means that really strong dependence is required to make it likely that many consecutive failures show up. This potential problem makes it unwise to run tests based on subsamples of the data, which otherwise could have been a way to investigate the performance of the model on data that is not exactly the same as the data used for the construction of the model.

The conditional coverage tests are only undertaken when there are consecutive failures. Otherwise, only the proportion of failures test (the Kupiec test) is used to evaluate the quality of the model. The conclusion for these tests are the same as for the independence tests (the Christoffersen tests). The p-values in the two cases for which the test is undertaken are not far from the significance level of 5%, but one should consider that 3/5 models have no consecutive failures at all and all iterations except the third have high p-values in the proportion of failures tests (the Kupiec tests). Accordingly, the results indicate that the model is accurately specified.

When backtests are undertaken for 1 January to 31 March 2018 (out-of-sample), no failures occur in any of the cases and the predicted number of failures is 0.3. As mentioned in Section 4.4.2, this is to little data to base a formal test on. The conclusion can be drawn is, however, that there are no signs that the VaR model performs worse out-of-sample. So, to summarize, the backtests indicate that the specified VaR model is accurate. Thus, the results indicate that the values of the assets in the SCRCRaR framework are accurately modelled.

5.2.2 Credit ratings of the assets

The credit ratings generated in the 1000 simulations contribute to the new values of the assets. Therefore, the results from the backtests in Section 5.2.1 indicate that the modelling of credit ratings is reasonably accurate. In addition to that, the quality should be evaluated on the basis of the validity of the underlying assumptions (the recovery rate, transition matrix, and correlation structure). New credit ratings have a small impact on the SCRCRaR and given that the valuation model passes all backtests, the credit ratings are considered to have been modelled accurately enough.

5.2.3 Symmetric adjustment parameter

The historical daily correlation between EIOPA Equity Index and Dina AB's portfolio of listed stocks is 0.89. This is a strong correlation and a clear indication that modelling the symmetric adjustment from the return on Dina AB's portfolio of listed stocks is accurate. To further strengthen the validity of the modelling, one should consider that the symmetric adjustment can never be lower than -10%. In all the outcomes worse than or equal to the SCRCRaR, the symmetric adjustment takes its minimum value and has a margin up to any level over -10%. Accordingly an even simpler approach could have been used: to assume that the symmetric adjustment is -10%. The disadvantage with such an approach is that it would not have been possible to investigate the the entire distribution (like in Figure 11-13).

5.2.4 Risk-free yield curves

As Table 34-36 show, the simple linear regression results of the risk-free rates as a function of swap rates are extremely strong. All the results are statistically significant on any reasonable significance level and the coefficients of determination are very substantial[18]. A coefficient of determination over 75% is considered to be substantial and here all are over 95% (Hair et al., 2011). Accordingly, the risk-free yield curves are modelled very accurately.

[18] A coefficient of determination indicates how much of the variation in the dependent variable that is explained by the regression model (Harel, 2009).

Horizon	Swap rate	p Swap rate	c	p c	R^2
1	0.98	0.00%	-0.11	0.00%	98.92%
2	0.98	0.00%	-0.11	0.00%	99.57%
3	0.99	0.00%	-0.10	0.00%	99.10%
4	1.00	0.00%	-0.10	0.00%	99.33%
5	1.01	0.00%	-0.10	0.00%	99.28%
6	1.01	0.00%	-0.10	0.00%	99.25%
7	1.02	0.00%	-0.11	0.00%	99.18%
8	1.03	0.00%	-0.11	0.00%	99.16%
9	1.03	0.00%	-0.11	0.00%	99.13%
10	1.05	0.00%	-0.13	0.00%	99.18%
11	1.03	0.00%	0.06	0.00%	99.03%
12	0.98	0.00%	0.30	0.00%	98.91%
13	0.92	0.00%	0.54	0.00%	98.82%
14	0.86	0.00%	0.77	0.00%	98.79%
15	0.81	0.00%	0.98	0.00%	98.77%
16	0.76	0.00%	1.17	0.00%	98.76%
17	0.72	0.00%	1.34	0.00%	98.75%
18	0.68	0.00%	1.49	0.00%	98.75%
19	0.65	0.00%	1.63	0.00%	98.75%
20	0.62	0.00%	1.76	0.00%	98.74%
21	0.59	0.00%	1.87	0.00%	98.77%
22	0.56	0.00%	1.98	0.00%	98.75%
23	0.54	0.00%	2.07	0.00%	98.76%
24	0.51	0.00%	2.16	0.00%	98.75%
25	0.49	0.00%	2.24	0.00%	98.75%
26	0.47	0.00%	2.32	0.00%	98.77%
27	0.46	0.00%	2.39	0.00%	98.75%
28	0.44	0.00%	2.45	0.00%	98.77%
29	0.43	0.00%	2.51	0.00%	98.75%
30	0.41	0.00%	2.57	0.00%	98.76%
31	0.40	0.00%	2.62	0.00%	98.76%
32	0.39	0.00%	2.67	0.00%	98.76%
33	0.38	0.00%	2.71	0.00%	98.73%
34	0.36	0.00%	2.76	0.00%	98.78%
35	0.35	0.00%	2.80	0.00%	98.78%

Table 34: Regression results for the unchanged yield curve. p denotes the p-values of the t-statistics and c denotes the constant.

Horizon	Swap rate	p Swap rate	c	p c	R^2
1	0.98	0.00%	0.89	0.00%	98.92%
2	0.98	0.00%	0.89	0.00%	98.57%
3	0.99	0.00%	0.90	0.00%	99.10%
4	1.00	0.00%	0.90	0.00%	99.33%
5	1.01	0.00%	0.90	0.00%	99.28%
6	1.01	0.00%	0.90	0.00%	99.25%
7	1.02	0.00%	0.90	0.00%	99.18%
8	1.03	0.00%	0.89	0.00%	99.16%
9	1.03	0.00%	0.89	0.00%	99.13%
10	1.05	0.00%	0.87	0.00%	99.18%
11	1.03	0.00%	1.06	0.00%	99.03%
12	0.98	0.00%	1.30	0.00%	98.91%
13	0.92	0.00%	1.54	0.00%	98.82%
14	0.86	0.00%	1.77	0.00%	98.79%
15	0.81	0.00%	1.98	0.00%	98.77%
16	0.76	0.00%	2.17	0.00%	98.76%
17	0.72	0.00%	2.34	0.00%	98.75%
18	0.68	0.00%	2.49	0.00%	98.75%
19	0.65	0.00%	2.63	0.00%	98.75%
20	0.62	0.00%	2.76	0.00%	98.74%
21	0.59	0.00%	2.87	0.00%	98.77%
22	0.56	0.00%	2.98	0.00%	98.75%
23	0.54	0.00%	3.07	0.00%	98.76%
24	0.51	0.00%	3.16	0.00%	98.75%
25	0.49	0.00%	3.24	0.00%	98.75%
26	0.47	0.00%	3.32	0.00%	98.77%
27	0.46	0.00%	3.39	0.00%	98.75%
28	0.44	0.00%	3.45	0.00%	98.77%
29	0.43	0.00%	3.51	0.00%	98.75%
30	0.41	0.00%	3.57	0.00%	98.76%
31	0.40	0.00%	3.62	0.00%	98.76%
32	0.39	0.00%	3.67	0.00%	98.76%
33	0.38	0.00%	3.71	0.00%	98.73%
34	0.36	0.00%	3.76	0.00%	98.78%
35	0.35	0.00%	3.80	0.00%	98.78%

Table 35: Regression results for the increased yield curve. p denotes the p-values of the t-statistics and c denotes the constant.

Horizon	Swap	p Swap rate	c	p c	R^2
1	0.98	0.00%	-0.11	0.00%	98.92%
2	0.98	0.00%	-0.11	0.00%	98.57%
3	0.97	0.00%	-0.11	0.00%	98.84%
4	0.81	0.00%	-0.12	0.00%	95.83%
5	0.66	0.00%	-0.10	0.00%	97.77%
6	0.60	0.00%	-0.07	0.00%	99.27%
7	0.62	0.00%	-0.06	0.00%	99.18%
8	0.66	0.00%	-0.07	0.00%	99.16%
9	0.69	0.00%	-0.08	0.00%	99.13%
10	0.72	0.00%	-0.09	0.00%	99.18%
11	0.72	0.00%	0.04	0.00%	99.03%
12	0.69	0.00%	0.21	0.00%	98.93%
13	0.66	0.00%	0.39	0.00%	98.82%
14	0.62	0.00%	0.55	0.00%	98.79%
15	0.59	0.00%	0.71	0.00%	98.77%
16	0.55	0.00%	0.84	0.00%	98.77%
17	0.52	0.00%	0.96	0.00%	98.74%
18	0.49	0.00%	1.08	0.00%	98.77%
19	0.46	0.00%	1.16	0.00%	98.75%
20	0.44	0.00%	1.25	0.00%	98.73%
21	0.42	0.00%	1.33	0.00%	98.77%
22	0.40	0.00%	1.41	0.00%	98.73%
23	0.38	0.00%	1.48	0.00%	98.77%
24	0.37	0.00%	1.55	0.00%	98.75%
25	0.35	0.00%	1.61	0.00%	98.74%
26	0.34	0.00%	1.66	0.00%	98.75%
27	0.33	0.00%	1.72	0.00%	98.75%
28	0.32	0.00%	1.77	0.00%	98.74%
29	0.31	0.00%	1.81	0.00%	98.75%
30	0.30	0.00%	1.86	0.00%	98.76%
31	0.29	0.00%	1.90	0.00%	98.76%
32	0.28	0.00%	1.93	0.00%	98.75%
33	0.27	0.00%	1.97	0.00%	98.76%
34	0.27	0.00%	2.01	0.00%	98.80%
35	0.26	0.00%	2.04	0.00%	98.78%

Table 36: Regression results for the decreased yield curve. p denotes the p-values of the t-statistics and c denotes the constant.

5.2.5 Robustness

The SCRCRaR approach produces fairly similar results in all five iterations. In terms of accuracy, the model consistently produces results that pass all the quality checks. For passing all tests five times in a row with 50% chance, the probability of passing all tests must be over 87%[19]. Accordingly, the model constructed is likely to have a high probability of passing all the tests.

The VaR ranges from SEK 281 987 670 to SEK 294 267 389 or 9.1% to 9.4% in the five iterations. This shows that the VaR measure is fairly stable.

The SCRCRaR ranges from 17.5 percentage points to 19.6 percentage points or from 11.2% to 12.6%. This measure is accordingly less robust than the VaR. The additional calculations compared to the VaR introduce more things that may vary between iterations, so the SCRCRaR is more dependent on the number of simulations run. Nevertheless, the measure is fairly stable and robust enough to draw the conclusions that are drawn in this thesis. The reader can be confident that the SCRCRaR is different from the VaR, that large decreases in the SCRCR can be modelled accurately from decreases in the value of the investment portfolio, and that a property insurance company can improve the risk profile of its SCRCR by diversifying among the risks in the Solvency II regulation's Standard Formula.

[19] $50\%^{1/5} > 87\%$

6 Conclusions and implications

6.1 Potential for deeper research

Property insurance companies can evaluate the risk for large drops in their Solvency Capital Requirement coverage ratios due to risk in their investment portfolios with the SCRCRaR measure, that has been developed in this thesis and is significantly different from the VaR measure. It has been shown that big negative changes in the SCRCR can be accurately estimated from value changes in the investment portfolio. Given how clear this pattern is for Dina AB, it is reasonable to expect that a relationship exists for other property insurance companies as well. What is left to explore is how this relationship works, in a more general context. How is the relationship affected by the composition of own funds, different market risks, and other risks facing a property insurance company? What impact has the asset allocation? The potential for more research in this area is huge, but this thesis has taken a first step.

The analysis in this thesis can also be extended to involve other risks facing a property insurance company. Only the risks facing the investment portfolio have been considered here and to get a more complete picture of the risk for a decreased SCRCR, all risks facing a property insurance company must be considered.

6.2 Potential for broader research

The main contribution of this thesis is an extension of the view on financial risk. In a more and more regulated environment, the risk management focus on value changes needs to be complemented by a focus on the regulatory consequences of the value changes. In the property insurance industry, a sensible start on this journey is to adopt current risk measures on SCRCR changes rather than on value changes. The SCRCRaR has been developed to be a VaR equivalent and the results show that the SCRCRaR and the VaR differ significantly, so there is a need for new risk measures. The SCRCRaR is a first step and more risk measures connected to the SCRCR will hopefully be developed in the future. From a scenario generator, like the one used in this thesis, other similar measures can be constructed. For example, a conditional Solvency Capital Requirement coverage ratio at risk (CSCRCRaR) could be a conditional value at risk (CVaR) equivalent in the same way as SCRCRaR is a VaR equivalent. There is, of course, also room for brand new risk measures, but existing risk measures are reasonable starting points.

The extension of the view on financial risk may also be useful in other industries. For example, life insurance companies could use a modified version of the SCRCRaR. Banks, that are affected by the Basel regulations, could also construct similar risk measures.

6.3 Practical usage

This thesis has not only taken a first step towards a more regulation focused view of risk and paved the way for further research. It has also produced results that are practically useful for firms. It has been shown that the risk management focus on value changes must be complemented by a focus on the regulatory consequences of the value changes. Measures such as the SCRCRaR should be calculated on a regular basis as a natural part of the risk management at a property insurance company. Spontaneously, extending the view of risk probably sounds like a huge job. The results presented have, however, shown that big negative changes in the SCRCR can be accurately estimated from value changes of the investment portfolio. Accordingly, only minor extensions of current scenario generators and VaR models are required to model the risk of a changed SCRCR. If a property insurance company wants to improve the risk profile of its SCRCR, diversification among the risks in the Solvency II regulation's Standard Formula is key.

It can, however, not be stressed enough that a risk management framework needs to include many measures and the SCRCRaR should not replace any of the existing risk measures. An effective risk management framework calculates many measures and looks at risk from many perspectives to achieve understanding. Understanding of the complex risk concept is key to achieve good investment management results in the long run. No one has explained this better than Theodore Roosevelt:

> Risk is like fire: If controlled it will help you; if uncontrolled it will rise up and destroy you.

7 References

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8 Appendix

8.1 Dina Försäkring AB's asset allocation 31 December 2017

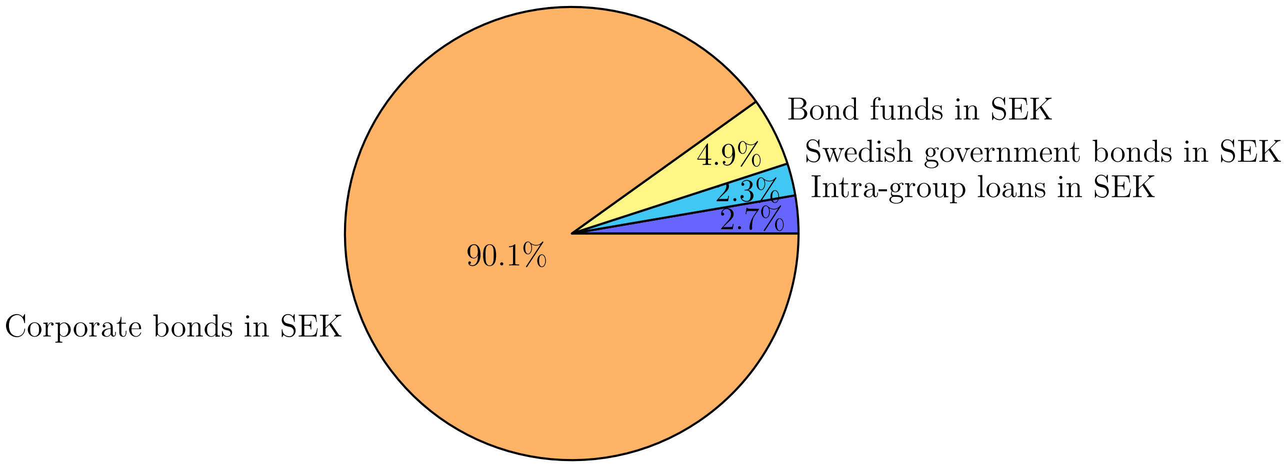

Figure 15: Dina Försäkring AB's interest-bearing assets 31 December 2017. The assets in all categories except bond funds in SEK are directly owned.

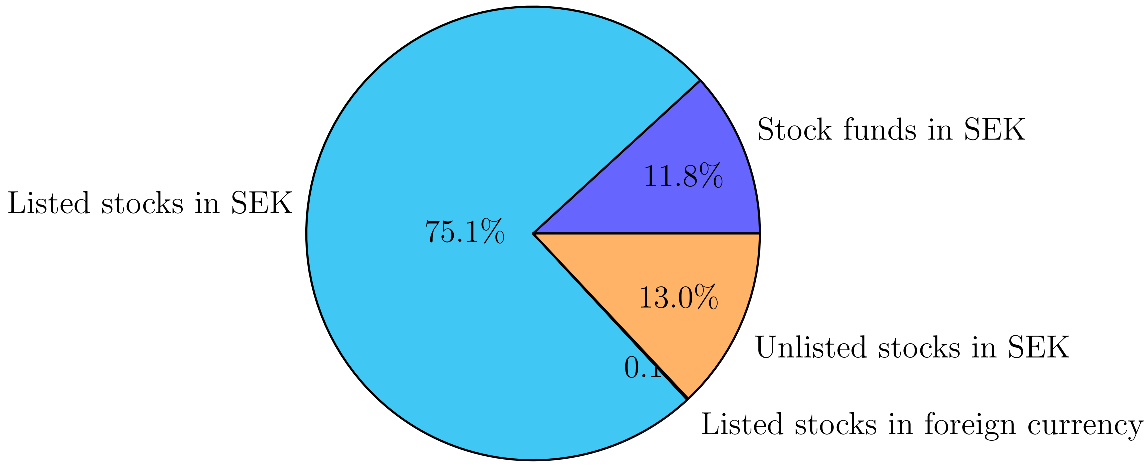

Figure 16: Dina Försäkring AB's stocks 31 December 2017. The assets in all categories except stock funds in SEK are directly owned.

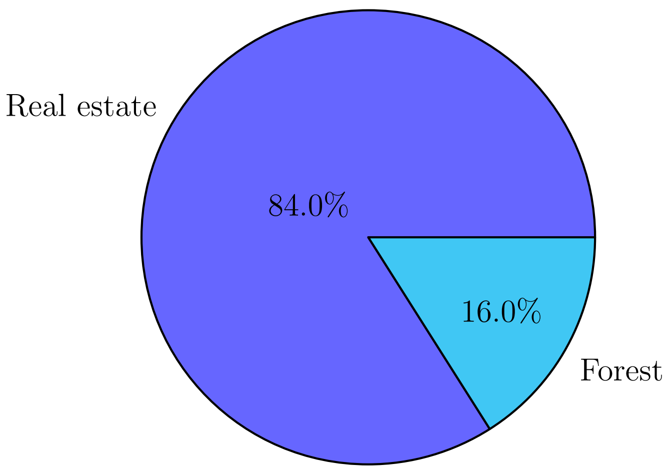

Figure 17: Dina Försäkring AB's property 31 December 2017. The forest is located in Småland in southern Sweden and is fully owned through stocks. The real estate is commercial real estate and it is located in the Old Town in Stockholm. It is owned directly or is fully owned through stocks.

8.2 Cholesky decomposition example

In a two-variable case the correlation matrix is written (Jorion, 2007):

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{71}$$

This matrix can be decomposed in a lower and an upper triangular matrix in the following way:

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \tag{72}$$

The triangular matrices are multiplied with each other and this gives the following formula:

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{12} \\ a_{11}a_{12} & a_{12}^2 + a_{22}^2 \end{bmatrix} \tag{73}$$

Thus:

$$a_{11}^2 = 1 \Rightarrow a_{11} = 1 \tag{74}$$

$$a_{11}a_{12} = \rho \Rightarrow a_{12} = \rho/a_{11} = \rho/1 = \rho \tag{75}$$

$$a_{12}^2 + a_{22}^2 = 1 \Rightarrow a_{22} = \sqrt{(1 - a_{12}^2)} = \sqrt{(1 - \rho^2)} \tag{76}$$

These expressions are used to reformulate Formula 72:

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{(1 - \rho^2)} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ 0 & \sqrt{(1 - \rho^2)} \end{bmatrix} \tag{77}$$

Formula 77 is the Cholesky decomposition (Formula 9) in a two-variable case. Notice that ρ can not be 1, as this means that the correlation matrix is not positive definite (Jorion, 2007).

8.3 Proof that the Cholesky decomposition implies the desired correlation structure

The correlation matrix of a random vector is calculated as the expected value of the random vector times the transposed value of the random vector (Park, Simeone, Sahin, & Shamai, 2013). This relationship is applied on the vector of correlated standard-normally distributed random variables, A:

$$E\left(AA^T\right) \overset{\text{Formula 11}}{=} E\left(L^T x (L^T x)^T\right) = E\left(L^T x L x^T\right) = L^T E\left(xx^T\right) L = L^T I L = L^T L \overset{\text{Formula 9}}{=} \rho \tag{78}$$

This shows that the vector of correlated standard-normally distributed random variables, A, has the desired correlation, ρ. L^T is the lower triangular matrix and L is the upper triangular matrix. x is the vector of standard-normally distributed random numbers. $E(xx^T)$ is an expression for the correlation matrix of x (see the first sentence in this subsection). As the numbers in x are expected to be uncorrelated with all the numbers in the vector except themselves (where the correlation per definition is 1), the correlation matrix is an identity matrix, I.

8.4 Implied probability density functions for the different asset categories in iteration 2-5

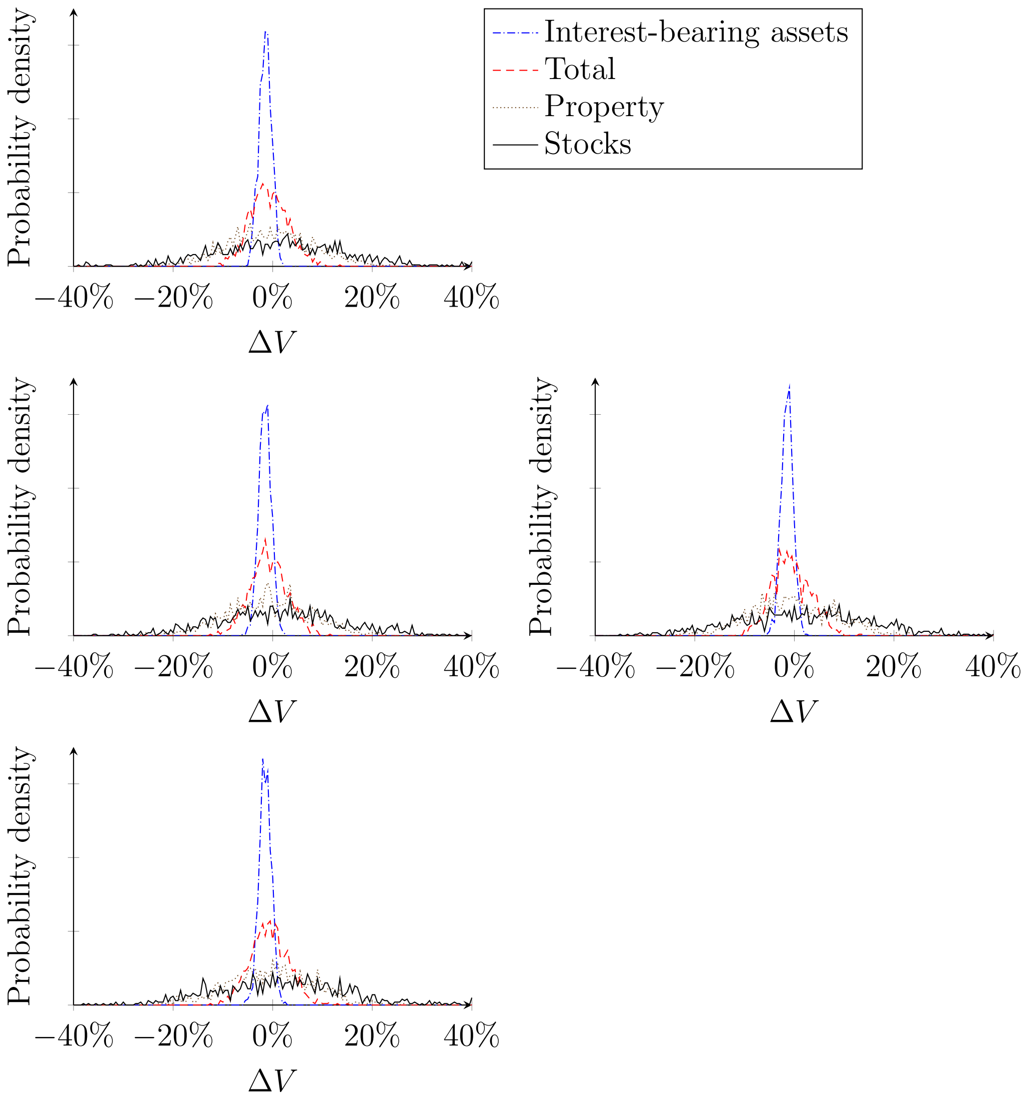

Figure 18: Implied probability density functions for the different asset categories in iteration 2-5.

8.5 Property valuation model

8.5.1 Real estate

For Dina Palaisbacken AB, one unit of excess return of the Carnegie Real Estate Index is associated with an excess return of 0.63 units. For Diana 2 the number is 0.60 and for Juno 9 it is 0.46. Accordingly, Juno 9 is less sensitive to risk factor changes than the other real estate holdings, so Juno 9 is considered to be least risky. One possible reason why all real estate holdings are considered to be less risky than the Carnegie Real Estate Index is that the values of the real estate have been determined by (possibly conservative) valuators rather than by the market itself. Furthermore, Carnegie Real Estate Index could be sensitive to the sentiment at the stock market. The constants are not statistically significant and of negligible size in all the three regressions in Table 37.

Name	CRI	p CRI	c	p c	R^2
Dina Palaisbacken AB	0.63	5.01%	-0.00	15.73%	90.23%
Diana 2	0.60	3.31%	-0.00	12.95%	93.50%
Juno 9	0.46	8.53%	-0.00	67.05%	83.67%

Table 37: Regression results for real estate. p denotes the p-values of the t-statistics, CRI denotes the effect of excess returns of the Carnegie Real Estate Index, and c denotes the constant.

To evaluate the quality of the regression results in Table 37, two measures are used. The t-statistics of the effect of excess returns of Carnegie Real Estate Index are all statistically significant on any significance level over 8.53%, as the highest p-value is 8.53%[20]. This shows that the excess return of Carnegie Real Estate Index can be used to estimate the excess returns of the real estate. The coefficients of determination, R^2, of all three regressions are over 83.67%, which means that 83.67% or more of the variations in the excess returns of the real estate are explained by the regression model (Harel, 2009). A coefficient of determination over 75% is considered to be substantial (Hair et al., 2011). Accordingly, the excess return of Carnegie Real Estate Index explains a substantial fraction of the excess return of the real estate and the simple linear regression model works well.

[20]A p-value represents the probability under a specified statistical model that a statistical summary of the data would be equal to or more extreme than its observed value (Wasserstein & Lazar, 2016).

8.5.2 Forest

One unit of excess return of the wood price is associated with an excess return of 0.09 units for Diana Skog AB. One unit of excess return of the SCA B stock is associated with an excess return of 0.17 units for Diana Skog AB. The constants are not statistically significant and of negligible size in the regression (see Table 38).

Name	Wood	p Wood	SCA B	p SCA B	c	p c	p F	R^2
Diana Skog AB	0.09	7.57%	0.17	0.46%	-0.00	9.66%	0.94%	84.54%

Table 38: Regression results for forest. p denotes the p-values of the t- and F-statistics and c denotes the constant.

To evaluate the quality of the regression results in Table 38, four measures are used. The t-statistic of the effect of excess return of the wood price is statistically significant on any significance level over 7.57% (the p-value[21]). The t-statistic of the effect of excess return of SCA B is statistically significant on any significance level over 0.46%. Thus, both the excess return of the wood price and the excess return of SCA B can be used to estimate the excess return of Diana Skog AB. The F-statistic of the regression model is statistically significant on any significance level over 0.94%. This shows that the linear regression model as a whole can be used to estimate the excess return of Diana Skog AB. The coefficient of determination, R^2, means that 84.54% or more of the variation in the excess returns of Diana Skog AB is explained by the regression model (Harel, 2009). A coefficient of regression over 75% is considered to be substantial (Hair et al., 2011). Accordingly, the linear regression model explains a substantial fraction of the excess return of Diana Skog AB.

[21] A p-value represents the probability under a specified statistical model that a statistical summary of the data would be equal to or more extreme than its observed value (Wasserstein & Lazar, 2016).

8.5.3 Prediction ability

Figure 19 shows that the regression models have very good prediction abilities in-sample. Data limitations make it impossible to do the same kind of analysis out-of-sample at this stage, but in the future such an approach will be feasible.

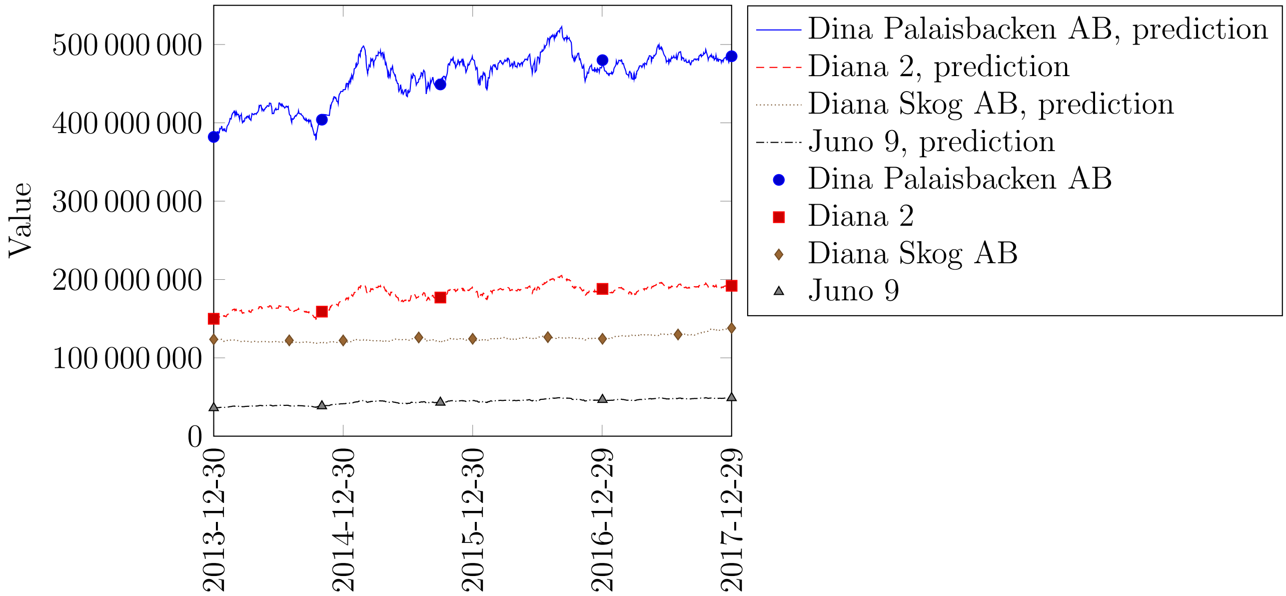

Figure 19: In-sample prediction of property values.

Overall, Section 8.5 has shown that the regression models give acceptable results that are possible to use as a part of the SCRCRaR framework.