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You Say You Want a Revolution

A Model on the Implications of Prospect Theory on Voting Behaviour

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Abstract

In this thesis we create a voting model with a prospect theory foundation, introducing reference-dependent risk preferences, loss aversion, and probability weighting in the individual utility function. Classical voting models represent perfectly rational voters, we argue that voter behaviour can be better modelled by introducing prospect theory preferences, especially recent political trends. We consider a voting strategy for voters, and a campaigning strategy for parties; we find stable aggregate equilibria in a stochastic setting where the policy platform will yield either a positive or a negative outcome, with a 50-50 chance. We show that these equilibria depend on the distribution of the voters, on the degree of risk aversion, on the valuation function and on probability weighting. We explore the dynamics of our model, where we allow our variables to evolve over time. We find that when expectations outpace reality, disruption becomes inevitable. An extension is explored where we vary probabilities, finding that voters prefer disruption to the status quo even if disruption represents a less-than-fair bet. We show how introducing prospect theory characteristics into the decision making of voting can help explain the success of disruptive parties and shed light on some empirical anomalies in recent political trends.

Keywords: Prospect Theory; Voting Model; Uncertainty; Expectations; Disruption

JEL: D72; D78; D80; D81; D84; D91

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1. Introduction

Two electoral results in 2016 upended commonly held expectations, seemingly marking a shift from the political status quo. On the 23^{rd} of June 51.9% of British voters that participated in the Brexit referendum chose to leave the European Union. Earlier that year political newcomer Donald J. Trump was declared the Republican candidate for President of the United States and ultimately went on to win the general election against Hillary Rodham Clinton on the 8^{th} of November.

Though these are arguably two of the most striking examples of a departure from establishment politics, they can be viewed as part of a larger trend already experienced in Europe. Many European political arenas had already seen the genesis of completely new parties, oftentimes led by personalities with hitherto little to no experience in politics. A common feature of such parties has been the denigration of the elite and the promise to disrupt the status quo by introducing a better form of governance.

This is probably best characterised by the Italian political party Movimento 5 Stelle ("5 Star Movement"), co-founded by comedian Beppe Grillo and entrepreneur Gianroberto Casaleggio in 2009, which claims to have no political ideology. The results of the 2018 general election confirmed them as the largest political party in Italy. These parties can be found across the political spectrum: radical-left Syriza (abbreviation for "Coalition of the Radical Left") was founded in 2009 and started leading the Greek government in 2015; left-wing Podemos ("We Can") skyrocketed to 21% of votes in Spain in 2016, after two years of being established. On the opposite end of the spectrum we have seen the emergence of AfD ("Alternative for Germany"), the first extreme right party in over half a century to enter German parliament, and the Dutch PVV ("Party for Freedom") founded in 2006, which is now the second largest party in the House of Representatives in the Netherlands.

It is notable that many of these break-out parties have succeeded at establishing themselves in countries whose political forum had been dominated by the same parties throughout the post-war period. Their programmes include a range of disruptive, often untested, policies offering a shake-up. Such policies range from restructuring the national debt to leaving the Eurozone or the European Union, and have arguably difficult-to-predict consequences with opaque repercussions.

Similarly, we have observed previously peripheral parties gathering popular support and becoming institutionalized. Among these are Front National ("National Front") in France, in Austria the FPÖ ("Freedom Party of Austria"), in Sweden the Sverigedemokraterna ("Swedish Democrats"), Lega Nord ("Northern League") in Italy, and Jobbik in Hungary. Their programmes often combine strong anti-immigration or anti-Islam arguments with a narrative going against the corrupt, distant establishment (as argued, among others, by Inglehart and Norris, 2016).

Another noteworthy observation that emerges when looking at the data is that a considerable amount of voters move across the spectrum. This seems to be difficult to justify with changes in ideology, given that the political platforms differ dramatically, with the main overlapping theme being a disruption from the status quo. In France, 23% of voters that identify themselves with an "extreme left" ideology voted for Le Pen in the 2017 second round presidential elections, according to Ipsos (Ipsos France, 2017).[1] In the US 12% of American citizens who supported Bernie Sanders in the primaries eventually voted for Donald Trump in the general election, as shown by the Cooperative Congressional Election Study led by YouGov (Ansolabehere and Schaffner, 2017).[2]

Several explanations have been given for this growing support for populist and radical parties, ranging from emotional to irrational to misinformed voters (Stroud et al., 2005; Reedy et al., 2014; Schill and Kirk, 2017). Several empirical studies have been carried out to analyse these events, but the existing theoretical framework in the literature is largely lacking (Bendor et al., 2011, p. 2–11). Classical voting models in fact represent perfectly rational voters; we argue that voter behaviour can be better modelled by introducing prospect theory preferences.

Behavioural characteristics have been applied in several fields, such as insurance or finance, but their application in political economy, and specifically in voting models, is relatively rare, though growing (Schnellenbach and Schubert, 2015). Prospect theory, in particular, has never been introduced *in toto* in the literature on elections. We argue that, especially examining recent political trends, voting decisions can be considered as a risky gamble, and hence introducing prospect theory, which deals with decision under uncertainty, may offer some novel insight. We therefore ask:

How does introducing prospect theory into a voting model help explain voter behaviour?

To answer this, we build a voting model made of two parts. First, we define individual voters' preferences using a behavioural framework, by adapting Tversky and Kahneman (1992)'s model of decision under uncertainty. Second, we define a strategy for parties, who maximise the probability of obtaining a majority. This allows us to, finally, characterise an equilibrium outcome for our model.

We find that, with the introduction of a prospect theory foundation, seeking uncertainty while being aware of potential downsides can be fully rational and can help explain some

[1]Similarly, 7% of voters that supported extreme left candidate Jean-Luc Mélenchon in the first round of the elections turned to Marine Le Pen in the second round. Analogous patterns were experienced in other countries. In Austria 25.5% of new voters that the far-right FPÖ gained came from the centre-left (SORA, 2017). In Germany 1 in 5 new voters the AfD gained in the 2017 elections came from the centre-left or the far-left (Blickle et al., 2017).

[2]This percentage, though apparently small, was decisive in some states where the margin of victory was narrow, such as in perennial swing states Pennsylvania, Michigan and Wisconsin. In Pennsylvania, 116,000 Sanders supporters turned to Trump, where the margin of victory was 44,000, in Michigan it was 47,000 with a margin of 11,000, and in Wisconsin it was 51,000 with a margin of 22,000 (Ansolabehere and Schaffner, 2017).

empirical puzzles. We claim that when parties do not live up to voters' expectations, there is a higher demand for candidates proposing radical policies, since they can potentially offer, with some unpredictability, a better prospect. Although there is some evidence showing that *unsatisfied* people are indeed more prone to voting for unorthodox parties (e.g. Amory and Mark, 2017; Mutz, 2018), the data does not provide a strong justification of why it is the case. Our model provides a theoretical framework to analyse it.

Prospect theory offers novel results with respect to 'traditional' voting models with preferences defined by expected utility theory. As will be clearer when looking at the model in more detail, prospect theory allows us to convey these novel characteristics in voter preferences: loss aversion, diminishing sensitivity in both the realm of losses and the realm of gains, reference dependence – implying the notion that agents base their evaluation of reality on their situation relative to a reference point rather than on its absolute level – and probability weighting in the evaluation of outcomes. We argue that there is an element of risk over the policy platform which translates into electoral choice. We make this element explicit in our model via the use of prospect theory, and this distinguishes it from 'traditional' voting models where this intuition is largely lacking.

Moreover, in our model, we used a two-party competition framework *à la* Downs, but we consider a different policy space than the right-left one, by defining the policy proposal as the departure from the status quo political good, and by allowing the outcomes to be stochastic. This definition implies that the magnitude ω is also a measure of the policy's inherent risk. In 'traditional' voting models this distinction would not be as meaningful as it is in our framework, since reference-dependent risk preferences are a defining characteristic of prospect theory but not of expected utility theory. As we argue in future sections, in depth in Section 4 and 5, these characteristics allow us to have novel results that can help shed light to mechanisms that the existing literature to a great extent fails to justify.

This thesis is structured as follows: in Section 2 we present our model, outlining the individual preferences of the voters, and defining the strategies of the parties. In Section 3 we find the aggregate equilibria for both a basic and a general case and we describe the dynamics. We also present an extension to this framework by allowing the probability of success of a policy to vary. In Section 4 we present a survey of the literature that is relevant to our research question, pointing out our contribution. Next, in Section 5 we discuss a possible application of our model where we analyse how our results can shed light into some empirical anomalies; finally, in Section 6 we discuss the implications of our model, examining reference point formation and the nature of the political good, and we consider its limitations. In Section 7 we conclude.

2. The Model

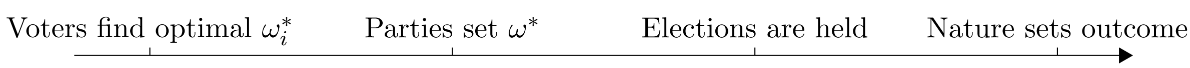
2.1. *Setup and Timing*

There are two identical parties, j and k, which coexist in a non-cooperative system. There are n voters, with n an odd number, and each voter i for $i \in I = \{1, 2, ..., n\}$ casts her vote. Each party's sole motivation is to hold office, and each voter's only goal is to maximise her utility. There is perfect information where voters are aware of the range of policies that parties can implement, and parties are aware of the voters' preferences and voter distribution. Every policy ω is unidimensional and defined in $\mathbb{R}_0^+$, existing in a stochastic environment: it yields a good outcome $+\omega$ with probability p and a bad outcome $-\omega$ with probability $1 - p$.

The voting model defines how voters find their optimal policy, as well as how parties finally set their proposed policy platform in order to win the election.

This is the timing of events:

1. Each individual i finds her optimal ω_i^* that maximises her utility.
2. Parties set ω^* so as to maximise the likelihood of winning the election.
3. Elections are held and one party is elected.
4. Nature decides which outcome, $+\omega$ or $-\omega$, occurs.



2.2. *The Utility Function*

Voters maximise this utility function, as in the framework of prospect theory (Kahneman and Tversky, 1979):

$$\sum_{h=1}^{m} W(\pi_h) v(x_h)$$

where $v(\cdot)$ is the valuation of outcome x_h associated with event h and $W(\cdot)$ describes the probability weights attached to event h which occurs with probability π_h. Note that these probability weights are not necessarily equal to the objective probabilities, and in fact differ as a general rule.

Prospect theory proposes that economic agents value prospects over two periods, an "editing phase", when agents formulate the prospects in terms of gains and losses relative to a reference point, and an "evaluation phase". In the second phase the prospects are valuated according to the valuation function $v(x)$ and weighted by their perceived probability via $W(\pi)$, which allows for overweighting of small probabilities and underweighting of large probabilities. Note that we drop the subscript h for the sake of legibility without loss of generality to our model.

We use the following valuation function for each agent $i \in I$, as proposed by Tversky and Kahneman (1992, p. 309):

$$v(x) = \begin{cases} (x)^{\theta} & x \geq 0 \\ -\lambda(-x)^{\theta} & x < 0 \end{cases} \tag{2.2.1}$$

$x \in \mathbb{R}$ is the prospect relative to some reference point, θ and λ are parameters where $\theta < 1$ and $\lambda > 1$. Graphically, the value function can be represented as shown in Figure 1.

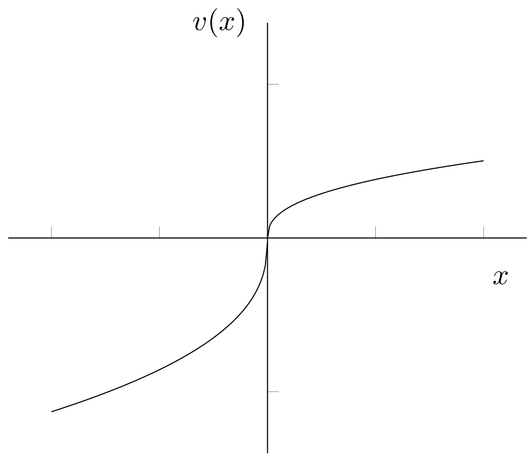


Figure 1: A visual representation of the value function

The outcome x that is being valued by the agents is given by its distance from the reference point. At the initial state of the model, x is defined with certainty for each respective agent $i \in I$, as follows:

$$x_i = e_i - r_i \tag{2.2.2}$$

Definition 1 *e_i is the level of governmental good each individual $i \in I$ currently receives.*

For an elaboration on the governmental good see Section 6.1.

Definition 2 *r_i is the level of governmental good each individual $i \in I$ expects to receive* *in the period. It is the individual reference point.*

Given this, we can calculate the outcome x_i of each individual $i \in I$ at the initial state of the model. The outcome is perceived by the agents to be what they receive from the government compared to their expectations. Note that it can take on positive and negative values or zero.

The value of each prospect is also weighted by its perceived probability. We choose to employ a version of the neo-additive function as described by Wakker (2010), which allows for a greater degree of parsimony than other parametric forms while retaining the most important features of the original one. It is described as follows:

$$W(\pi) = \begin{cases} 0 & \pi = 0 \\ a + b\pi & \pi \in (0,1) \\ 1 & \pi = 1 \end{cases} \tag{2.2.3}$$

Where $a \geq 0$, $b \geq 0$, and $a + b \leq 1$. Probability is $\pi = \{p, 1-p\}$, referring respectively to the probability of success and the probability of failure of the policy platform proposed by the party. Note that π does not vary between agents. For the sake of generality, we allow a and b to be different in the realm of gains and in the realm of losses. To differentiate between the two functions, we will use the following notation: $W^+(\pi)$ in the realm of gains, where $(a,b) = (a^+, b^+)$ and $W^-(\pi)$ in the realm of losses, where $(a,b) = (a^-, b^-)$.

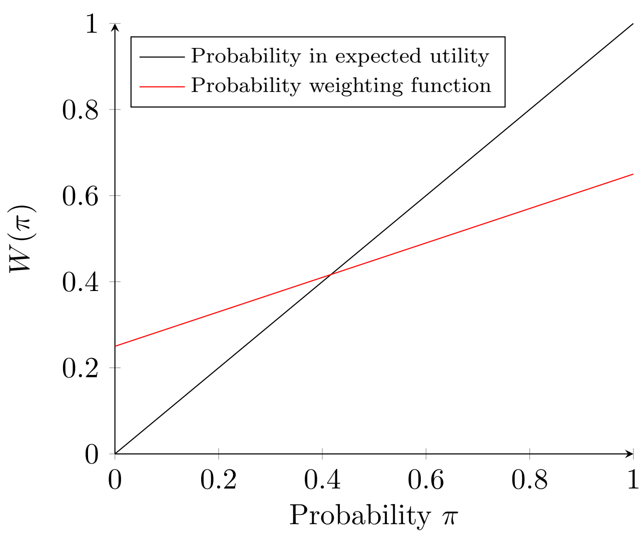


Figure 2: A neo-additive weighting function

The model, composed of the two separate functions, endeavours to better model real-world behaviour than expected utility theory. The valuation function allows for risk-aversion in the realm of gains and risk-seeking in losses, loss aversion, reference dependence, and the quality of diminishing sensitivity both in gains and losses. The probability weighting function allows for underweighting of large probabilities and overweighting of small probabilities.

Reference Dependence Under prospect theory, agents derive utility from the consumption of a good relative to a reference point, rather than from absolute levels of consumption: outcomes are formulated as gains or losses[3]. Note that reference points can increase or decrease, which affects the utility derived from the same level of absolute consumption by the same agent, at different times.

Reference dependence is expressed in the valuation function (Equation 2.2.1) by the variable x, because it denotes changes in the consumption of the governmental good e_i relative to the reference point r_i, as stated in the Identity 2.2.2. On the contrary, in expected utility theory, utility is a function of absolute wealth (using our notation, it would only be a function of e_i). In the graph in Figure 1, this characteristic is represented by the scale of the horizontal axis: the reference point coincides with $x = 0$, positive values of x correspond to gains, and negative values of x correspond to losses.

The reference point has traditionally been treated as the status quo, however there is a growing literature on how it is formed. Models have been developed where the reference point is generated by rational expectations, and the most prominent contribution is that of Kőszegi and Rabin (2006), who are the first authors to formalise this concept. For the purposes of our model we follow their intuition and allow expectations to inform the reference point.

Loss Aversion Loss aversion is explained by Tversky and Kahneman (1991) by the intuition that "losses loom larger than gains", which means that agents are more sensitive to losses than to gains of the same magnitude (Kahneman and Tversky, 1984, p. 346).[4] Loss aversion is expressed via the parameter λ in the valuation function (Equation 2.2.1), which is larger than one. In the graph, this characteristic is illustrated by a kink of the value function at the reference point – the origin.

Loss aversion is also used to explain what has been termed the "endowment effect",

[3]This intuition is not necessarily restricted to constant levels of consumption but also to growth rates, such as wage growth. For an agent with a reference point of a yearly wage growth of 2%, receiving an increase of 5% would be perceived as a gain, and therefore positive utility would be derived, whereas receiving an increase of just 1% would be seen as a loss, and therefore negative utility would be derived. Compare this to an economic agent with a reference point of no wage growth, who would generate positive utility from an increase of just 1%. The same level of growth, 1%, generates entirely different levels of utility depending on the reference point of the agent.

[4]This explains why many agents turn down fair bets, such as a 50/50 bet of gaining X and losing X, given that less utility is derived by gaining X than disutility generated by losing X. Depending on the degree of loss aversion, this can extend to turning down significantly more-than-fair-bets (Tversky and Kahneman, 1991).

named as such by Thaler (1980). People are found to demand much more to give up an object than they are willing to pay for it: the loss of parting with something is greater than the gain received from any trade. An example used by Thaler to describe the phenomenon is that of a wine collector who refuses to sell a bottle of wine that has increased in value but would not buy any wine at that price. This effect is also tested and found empirically in Kahneman, Knetsch, and Thaler's famous coffee mug experiment (Kahneman et al., 1991)[5]. It should be noted however that List (2003) finds the endowment effect to diminish if not disappear with market experience. This apparent anomaly is cleared when modelling reference point formation with expectations rather than the status quo: experienced traders can *rationally expect* to trade and hence they adjust their reference point towards trading (as argued by Kőszegi and Rabin, 2006).

Diminishing Sensitivity Diminishing sensitivity indicates the generally accepted quality of diminishing returns to utility from increased consumption, which is also contained in classical economic theory. This feature is however mirrored in the realm of losses, where disutility diminishes from increased losses.[6] Diminishing sensitivity also implies a pivotal aspect of prospect theory, that agents have reference-dependent risk attitudes. That is, individuals tend to be *risk-averse* for prospects that they code as a gain relative to their reference point and *risk-seeking* for prospects that they code as a loss. Formally, this characteristic is exhibited in the value function by the fact that θ is smaller than one. This translates into the function being concave in the realm of gains and convex in the realm of losses.

Probability Weighting Finally, small probabilities are overweighted (the "possibility effect") and large probabilities are underweighted (the "certainty effect") in the decision-making process of individuals[7]. This fourth feature of prospect theory is represented by the probability weighting function, which enters the valuation phase of the outcome for the agent. In fact, differently from expected utility theory in which objective probabilities are considered, prospect theory posits that probabilities are weighted by a function $W(\cdot)$. We employ the neo-additive function described in Equation 2.2.3.

2.3. *The Voting Decision*

Each agent $i \in I$ faces a choice between two alternative political parties j and k, each offering a policy ω that affects the agent's value function. The agent eventually chooses

[5] Participants were endowed with an object – a coffee mug or a chocolate bar – and were then offered to exchange it. Significant undertrading was observed.

[6] Thus, increasing (decreasing) wages from \$1,200 to \$1,700 (vice versa) has a considerable effect on utility, whereas increasing (decreasing) wages from \$11,200 to a \$11,700 (vice versa) has a relatively modest influence on utility.

[7] Probability weighting potentially helps explain why agents simultaneously buy insurance as well as lottery tickets, as these small probabilities are overweighted by agents.

the party from which she derives higher utility. Formally, each voter $i \in I$ will vote for party j over party k if and only if

$$U(x_i, \omega_j) \geq U(x_i, \omega_k)$$

In case $U(x_i, \omega_j) = U(x_i, \omega_k)$, the voter breaks her indifference by randomising between the two parties. This implies that agents always resolve who to vote for, and elections are never inconclusive, even when $\omega_j = \omega_k$.

The policy ω enters the utility function as follows:

$$u(x_i, \omega) = \begin{cases} (x_i \pm \omega)^\theta & x_i \pm \omega \geq 0 \\ -\lambda(-(x_i \pm \omega))^\theta & x_i \pm \omega < 0 \end{cases} \tag{2.3.1}$$

Where x_i is the difference between actual and expected consumption of the political good at the initial state, ω is the policy proposed by each party, $\theta < 1$, and $\lambda > 1$.

Definition 3 *ω is the magnitude of the policy that the party proposes; its sign is only determined after the election by nature. It is homogeneous across all agents in the economy.*

Note that, formally, this means that the value of ω is not determined in the equilibrium, only its magnitude is. Whether it is negative or positive is in fact set only *after* the election and voters are aware of this gamble. Given the definition above, $\omega > 0$ can be understood as an untested policy yielding either positive or negative disruption, as it represents the degree of deviation from the initial governmental good e_i (Identity 2.2.2 and Equation 2.3.1). The larger the ω, the more extreme the deviation from the initial state. When $\omega = 0$, the governmental good is equivalent to its initial value, since no change is generated in x_i.

Implicitly then, since e_i is a determined value, the magnitude of the policy ω is also a measure of its risk. Agents face a gamble with two outcomes: a positive change $+\omega$ occurring with probability p and a negative change $-\omega$ occurring with probability $1 - p$. Note that the degree of deviation ω does not depend on individual preferences (it has no subscript i). We alternatively term ω as the "level of risk" and, when it is larger than zero, as the "disruptive policy".

This definition of policy is novel, as it does not use any of the traditional criteria that are most commonly used in voting models. Policies are usually defined based on their positioning on a scale that looks at the content of the policy, *e.g.* on a simple left-right scale, or on a matrix accounting for economic and social aspects of political ideology (Downs, 1957, and its extensions). We, instead, define policies by the amount of risk that they bear, *i.e.* as the magnitude of disruption from the status quo policy e_i that they offer. The graph in Figure 3 provides an illustration of the possible effect of $\omega > 0$ on the agent's value function, for a hypothetical initial positive x and a hypothetical initial negative x.

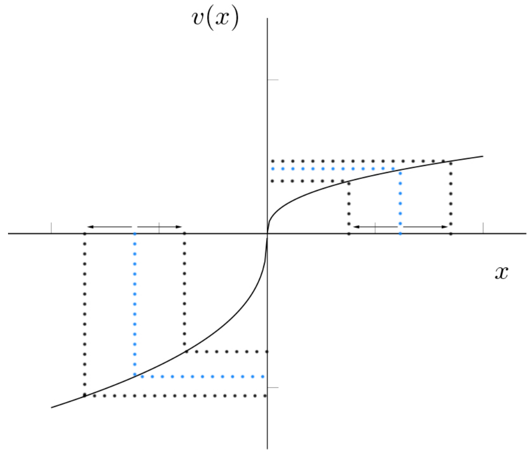


Figure 3: The effect of $\omega > 0$ on utility via the value function

As can be seen in the graph, the same policy ω has different effects on the valuation of the outcome depending on the initial value of x_i. The effects (both positive and negative) of the policy on utility are amplified in the realm of losses.

After valuating the outcomes, agents weight the probability of each outcome using the neo-additive weighting function described in Equation 2.2.3. It follows that the *overall utility* that agents derive from the policy is represented by the following function.

$$U(x_i, \omega) = \begin{cases} (a^+ + b^+\pi) \cdot (x_i \pm \omega)^\theta & (x_i \pm \omega) \geq 0 \\ (a^- + b^-\pi) \cdot -\lambda[-(x_i \pm \omega)]^\theta & (x_i \pm \omega) < 0 \end{cases} \tag{2.3.2}$$

As we described in the rest of this section, both the probability weighting function and the value function vary depending on where the agent is located with respect to the reference point. The function is defined differently depending on the initial x_i – whether the voter starts off in the realm of gains or losses – and on the final outcome – similarly, where the voter ends up after the policy has been implemented.

2.4. *Types of Voters*

The population of voters is distributed along the x axis according to a normal distribution $\psi \sim \mathcal{N}(\mu, \sigma^2)$ where μ is not fixed. This means that voters are found both in the realm of losses and gains, with different proportions depending on μ.

Each type of voter solves a different maximisation problem because of how we defined the overall utility. As shown in Equation 2.3.2, in fact, the function changes depending on where the agent starts off (x_i) and on where she ends up after the policy has been implemented (considering both possible outcomes $x_i + \omega$ and $x_i - \omega$). To capture every possible prospect, we will solve separate maximisation problems for each type.

Within the framework of our model voters differ continuously across the distribution, however they can be grouped into three distinct categories: those in the realm of gains where $x_i > 0$, those in the realm of losses where $x_i < 0$, and those who are neutral where $x_i = 0$. Mathematically, this means that all voters initially in the realm of losses solve the same maximisation problem, as do those initially in the realm of gains, and neutrals.

3. Results

We present three sets of results for our model: the simple case, where we included some simplifying assumptions that help clarify the mechanisms behind our results, a general case where we drop such assumptions, and a dynamic case, where we illustrate how the results change when we allow the variables to evolve over time. We then present an extension to our results, by allowing p, the probability of success of the disruptive policy, to vary.

3.1. *The Simple Case*

In the simple case, we introduce some assumptions on the type of voters. We impose that the population is formed of three types of voters that are located around the reference point. There is a proportion α_G of agents that are in the realm of gains, *i.e.* they have $x_i = x_G > 0$, a proportion α_L of agents located in the realm of losses, *i.e.* they have $x_i = x_L < 0$, and a proportion α_N of agents whose expectations match their reality, where $\alpha_G + \alpha_L + \alpha_N = 1$ meaning they make up the entire population. There is symmetry around the reference point such that $x_L = -x_G$. Additionally, we assume that $p = 0.5$.

3.1.1 *Optimal Policies by Type*

Each voter type finds its optimal ω_i^* by maximising its value function, given a constraint on ω. As shown in Equation 2.3.2, the function changes depending on where the agent ends up after the policy has been implemented. We introduce the following constraint:

$$\omega \leq x_G \text{ [8]}$$

This means that ω cannot be large enough that any of the groups can trespass the reference point: regardless of the outcome, those in the realm of gains will remain in the realm of gains, and vice versa for those in the realm of losses.

Proposition 1 $\omega_i^* = 0$ *when* $x_i \geq 0$ *for all* $i \in I = \{1, 2, ..., n\}$.

This proposition follows from the solution of these problems[9]:

$$\begin{aligned} \underset{\omega}{\text{maximise}} \quad & W^+(\pi)(x_G + \omega)^\theta + W^+(\pi)(x_G - \omega)^\theta \\ \text{subject to} \quad & \omega \in \left[0, x_G\right] \end{aligned}$$

for those in the realm of gains ($x_i = x_G > 0$).

[8] This constraint also implies that $-\omega \geq x_L$, because we have assumed symmetry around the reference point for the simple case.

[9] A discussion of the proof is provided in Appendix A.1

$$\begin{aligned} \underset{\omega}{\text{maximise}} \quad & W^{+}(\pi)(x_N+\omega)^{\theta} - \lambda W^{-}(\pi)[-(x_N-\omega)]^{\theta} \\ \text{subject to} \quad & \omega \in \left[0, x_G\right] \end{aligned}$$

for those that are at their reference point ($x_i = x_N = 0$).

Intuitively, those that are in the realm of gains are risk-averse (negative concavity of the value function for $x > 0$), so their utility will be maximised when the uncertainty over the outcome of the policy is null, *i.e.* when disruption is zero. The optimal policy is the same for agents that are at the reference point because of loss aversion: losses loom larger than gains, so the agent's utility will be maximised when uncertainty is minimal.

Thinking of the policy outcome as a fair bet between a good and a bad outcome, those at least at their reference point will always prefer a policy that allows them to remain at the *status quo*, with $\omega = 0$. One can interpret this as a form of "endowment effect" for the governmental good. The intuition of this proposition is that when people are satisfied with the state of their government they do not seek disruption from a challenger to the incumbent, but rather pursue continuity.

Proposition 2 $\omega_i^* = |x_L|$ *when* $x_i = x_L < 0$ *for all* $i \in I = \{1, 2, ..., n\}$.

Agents that are in the realm of losses maximise the following function. Note that the constraint can be rewritten as $-\omega \in \left[x_L, 0\right]$ given that we imposed symmetry.

$$\begin{aligned} \underset{\omega}{\text{maximise}} \quad & -\lambda W^{-}(\pi)[-(x_L+\omega)]^{\theta} - \lambda W^{-}(\pi)[-(x_L-\omega)]^{\theta} \\ \text{subject to} \quad & \omega \in \left[0, x_G\right] \end{aligned}$$

This is solved when the constraint is binding, *i.e.* for $\omega_i^* = x_G$, because the utility function increases in ω. What this tells us is that, in our basic model, agents of this type maximise their utility by "gambling" to reach the consumption of the political good that reflects what they expect to have (*i.e.* they gamble to reach their reference point). Note that agents would ideally *surpass* their reference point, so the constraint is binding[10].

Voters in the realm of losses would like to vote for a party that promises to fulfil their expectations about the state of the government, even with the knowledge that it is equally likely these promises fail and in fact produce a worse outcome. Intuitively, this gamble is preferred given that voters in the realm of losses function under the idea that "things cannot get much worse".

[10]See Appendix A.1 for proof.

3.1.2 *Aggregate Equilibria*

Aggregate equilibria are identified by finding the parties' optimal strategy. Policies are set in pursuit of winning the election, *i.e.* of gaining more than half of the votes. Since there are only two parties, the probability of victory is defined by the probability of obtaining more votes than the opponent. Therefore, party j maximises:

$$Pr(U(x_i, \omega_j) > U(x_i, \omega_k))$$

The opposing party, k, is defined symmetrically.

It follows that the policies the parties set depend on the preferences of the voter types, and specifically on how the population is distributed across them. Two scenarios are possible.

Proposition 3 *If $\alpha_G + \alpha_N > \alpha_L$, there is a single Nash equilibrium where parties propose policy platforms setting ω^* equal to zero.*

If the majority of the population is in the realm of gains or exactly at their reference point, more than 50% of voters will prefer the policy to be in line with "politics as usual", without any risky deviation. Any party that deviates from this equilibrium would gain the votes from a proportion α_L of agents but lose all votes from α_G and α_N agents, therefore also the election. Hence, setting $\omega^* = 0$ is the dominant strategy. This is what happens in most elections, when the majority of voters are content with the government and disruptive challengers are only endorsed by a minority.

Proposition 4 *If $\alpha_L > \alpha_G + \alpha_N$, there is a single Nash equilibrium where parties propose policy platforms setting ω^* equal to $|x_L|$.*

If the majority of the population finds itself in the realm of losses, then again there will be a single equilibrium. However this time both parties set their ω^* so as to gain the votes of all agents in the realm of losses, whose $\omega_i^* = |x_L|$. Deviating from this equilibrium would, again, mean losing the election. The proposed ω^* depends exclusively on x_L, and therefore on how deep in the realm of losses the agents perceive themselves to be. The further away they are from the reference point, the larger will be the deviation offered by parties. We represent these equilibria in Figure 4.[11]

The figure shows how voters in the realm of gains, with $x_i = x_G$, prefer no disruption, *i.e.* $\omega_G^* = 0$; conversely, those in the realm of losses, with $x_i = x_L$, prefer to gamble and reach their reference point, given by $\omega_L^* = |x_L|$.

[11] A formal proof is provided in Appendix A.1 for both Proposition 3 and Proposition 4.

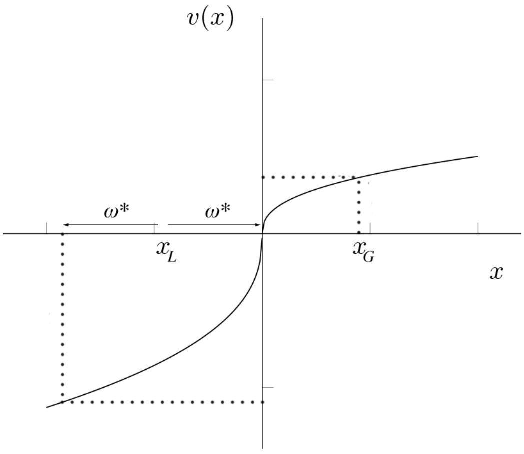


Figure 4: Equilibrium ω^* for the different voters in the Basic Case

3.2. *The General Case*

We now drop the assumption of symmetry of voter types around the reference point and the constraint on the policy. The policy can now take any value on $\mathbb{R}_0^+$, and the voters are distributed along the x axis according to a normal distribution $\psi \sim \mathcal{N}(\mu, \sigma^2)$ where μ is not fixed. Dropping the simplifying assumptions implies that there exist more possible outcomes to consider when finding the equilibria. For voters whose initial consumption of the political good is *not* equal to their reference point, two additional cases arise and must be accounted for when finding ω_i^*. These correspond to the outcomes in which agents shift from one realm to the other. In the next section, we describe the aggregate equilibria of the model.

3.2.1 *Aggregate Equilibria*

There are two equilibria, depending on where the median of the population, μ, lies.

Proposition 5 *When $\mu \geq 0$, there is a unique Nash equilibrium where both parties set $\omega^* = 0$.*

Proof The individual voter's preference is found by solving a series of *unconstrained* maximisation problems. In particular, two cases can be distinguished, depending on where the median voter is located with respect to her reference point[12].

- The ω_i^* for voters initially in the realm of gains is found by solving:

$$\underset{\omega}{\text{maximise}} \quad U(x_i, \omega) = W^+(\pi)(x_i + \omega)^\theta + W^+(\pi)(x_i - \omega)^\theta; \qquad x_i \geq \omega > 0$$
$$\underset{\omega}{\text{maximise}} \quad U(x_i, \omega) = W^+(\pi)(x_i + \omega)^\theta - \lambda W^-(\pi)(-(x_i - \omega))^\theta; \quad \omega > x_i > 0$$

[12]See Appendix A.2 for an elaboration of this proof, with more detailed reasoning.

The solution $\omega_i^* = 0$ is found for the above maximisation. Note that the utility function is strictly decreasing in ω, hence this agent will vote for the party offering the lowest ω.

- The ω_i^* for voters initially at the reference point ($x_i = 0$) is found by solving:

$$\begin{aligned} &\underset{\omega}{\text{maximise}} \quad W^+(\pi)(x_i + \omega)^\theta - \lambda W^-(\pi)(-(x_i - \omega))^\theta \qquad \Longrightarrow \\ &\underset{\omega}{\text{maximise}} \quad [W^+(\pi) - \lambda W^-(\pi)]\omega^\theta \end{aligned}$$

The maximisation problem in this case does not change from the one in the basic model, since the constraint was never binding. The solution $\omega_i^* = 0$ is again found. Note, again, that the utility function is strictly decreasing in ω, hence agents at the reference point will vote for the party offering the lowest ω.

We fully disprove the existence of Nash equilibria other than that of setting the policy that is preferred by the agent with $x_i \geq 0$. We divide the proof into six cases:

1. $\omega_j^* = 0$ and $\omega_k^* > 0$

   All voters in the realm of gains or at the reference point will vote for part j, given that they propose their optimal policy as opposed to party k. Given that more than 50% of the voters are in the realm of gains or at the reference point, party j receives more than 50% of the votes and therefore wins the election. Party k will be strictly better off by deviating and setting ω_k^* equal to zero and therefore will be increasing their chances of success from 0% to 50%. Therefore, this is not a Nash equilibrium.

2. $\omega_j^* > 0$ and $\omega_k^* = 0$

   By symmetry, consistent to (1) this is again not a Nash equilibrium.

3. $\omega_j^* = \omega_k^* > 0$

   In this case voters randomise between both parties and each has a 50% chance of winning. However, they each have an incentive to deviate, reducing their respective ω by any small value ϵ, therefore gaining the entire vote of all voters whose $x_i \geq 0$. Given that this proportion is larger than one half, the party would also win the election. Therefore this is not a Nash equilibrium.

4. $\omega_j^* > \omega_k^* > 0$

   In this case party k wins the election, given that all voters who have $x_i \geq 0$ vote for the party offering the lower ω, constituting more than one half of the electorate. Party j has the incentive to reduce its proposed ω to one lower than ω_k^* by any small value ϵ, thereby winning the election. Hence, this is not a Nash equilibrium.

5. $0 < \omega_j^* < \omega_k^*$

By symmetry, consistent to (4) this is again not a Nash equilibrium.

6. $\omega_j^* = 0$ and $\omega_k^* = 0$

   In this case voters randomise between both parties, giving each a chance of electoral success equal to 50%. Deviating from this proposal would reduce the respective party's chances from 50% to 0%, there is therefore no incentive to deviate. This is a Nash equilibrium.

It follows from the cases above that $\omega_j^* = \omega_k^* = 0$ is the unique Nash equilibrium if the median voter is located above or at her reference point. ■

Proposition 5 asserts that parties must target the median (pivotal) voter, who is at or above her reference point, where $\mu \geq 0$. Since the median voter in a normal distribution is the one whose x value is equal to μ (henceforth x_μ), the equilibrium party platform for both parties is that preferred by such voter's individual preference.

Intuitively, dropping the binding constraint for ω strengthens the earlier results found in the basic model: given loss aversion, agents will *a fortiori* prefer avoiding all uncertainty and remain at the *status quo*, since they risk falling below their expectations. Note that this holds for *all* agents in the realm of gains, regardless of the magnitude of their respective x_i. The same is true for those in line with their expectations. It follows that most voters are satisfied with the state of the government and therefore do not want to engage in any gamble that would upset the status quo. Establishment parties continue to be elected.

An illustration of this equilibrium is provided in Figure 5, for a hypothetical normal distribution with $\mu > 0$. The figure shows the median voter's position on the x axis along with the level of utility she derives from the equilibrium policy $\omega^* = 0$.

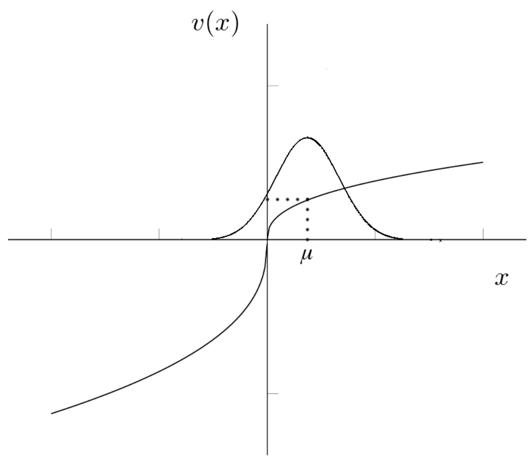


Figure 5: Optimal policy when $\mu > 0$

Proposition 6 *When $\mu < 0$, there exists a unique Nash equilibrium where both parties set ω^* as follows:*

$$\omega^* = x_\mu \frac{1 + R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}} - 1} \tag{3.2.1}$$

where R is given by

$$R = \lambda \frac{W^-(\pi)}{W^+(\pi)}$$

The proof of this proposition is analogous to the one for the scenario in which $\mu \geq 0$; we go over it in Appendix A.2.

Similarly to the previous case, the majority of voters dictate ω^*, however this time they find themselves in the realm of losses and seek disruption. The status quo does not fulfil their expectations and a gamble is utility maximising. They therefore choose the gamble even though they are aware that the situation can deteriorate exactly as much as it can improve.

Figure 6 illustrates this Nash equilibrium for a hypothetical normal distribution with negative μ. We see that the median voter sets the level of ω^*, which increases in magnitude the further she finds herself in the realm of losses and vice versa. The dotted lines represent the valuation of the good and the bad outcomes. Overall utility is found when these outcomes are weighted by the probability weighting function.

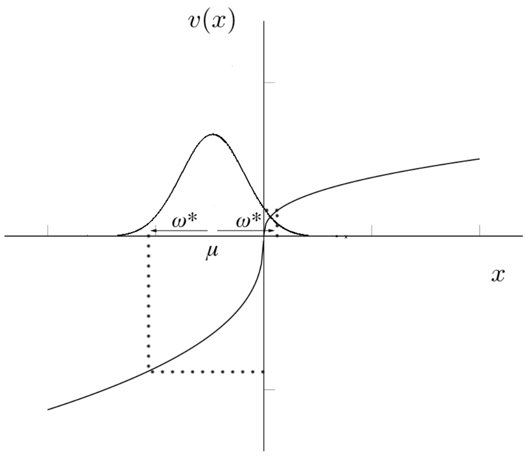


Figure 6: Optimal policy when $\mu < 0$

Note that since $R > 1$ and $\theta \in (0, 1)$, the multiplier of x_μ will be larger than 1, implying that the equilibrium policy will be larger than in the simple case[13]. We see that ω^* increases in $|x_\mu|$ – the deeper the agents perceive themselves to be in the realm of losses, the more they will want to gamble. This implies that anyone on the left of x_μ has a larger ω_i^*, so will always prefer any $\omega^* > 0$ to $\omega^* = 0$. This can be seen in Figure 6, where the

[13]This depends on $\frac{W(p^-)}{W(p^+)} > \frac{1}{\lambda}$, an uncontroversial assumption given that we keep the probability p equal to 0.5. The terms a^-, a^+ and b^-, b^+ should be respectively similar, and therefore their ratio close to one – larger than $\frac{1}{\lambda}$ by construction.

good outcome allows the agent to reach the realm of gains. Moreover, ω^* depends on the relative risk parameter R, on θ, and on the population distribution and specifically μ. It increases in the relative risk parameter because as λ increases so does the agent's optimal ω^*. It increases similarly if the ratio between the probability weighting of the bad outcome to the good outcome increases $\left(\uparrow \frac{W^-(\pi)}{W^+(\pi)}\right)$. This means that as agents attach lower probability weighting to the bad outcome, they demand higher ω^*, and vice versa.

3.3. *Dynamics*

In the dynamic setting, we introduce the temporal dimension by allowing the level of governmental good that is offered e_i and the reference point r_i to evolve over time. We define the growth rates of the governmental good and the reference point before analysing how the results change.

Definition 4 $e_{i,g} = e_{i,t} - e_{i,t-1}$ *is the absolute change in the governmental good received by individual* i*, where* $e_{i,g} = e_{-i,g}$ *for all* $i \in I$*.*

Definition 5 $r_{i,g} = r_{i,t} - r_{i,t-1}$ *is the absolute change in the expectations of the governmental good received by individual* i*, where* $r_{i,g} = r_{-i,g}$ *for all* $i \in I$*.*

We define the respective growth rates to be equal across all voters, therefore we can drop the i notation. We can concisely refer to e_g as the growth rate of the governmental good e for the population as a whole and to r_g as the growth rate of the expected governmental good r, again for the population as a whole, although they represent absolute rather than proportional changes. These definitions serve to make the analysis and its interpretation more intelligible, while not changing the intuition.

Changes to the growth rates e_g and r_g do not affect the population variance, but are now translated as shifts in the entire population distribution, and its median, keeping all other properties constant. Note that in this discussion, we set the initial state of the world to be one where $\mu \geq 0$. This can be interpreted as the situation where the majority of the population is at least content with their government, and mainstream parties win at elections, arguably the ordinary state of Western democracies.[14]

Normal Times In what can be considered normal, stable times with full employment, regular growth, little or no perceived decreases in quality of life regarding safety, the growth rate of the expectations for the public sector are met exactly by the government.

[14]We only do this to not make the discussion burdensome, but the mechanisms are unaltered.

We term the following case to be normal times, where:

$$e_g = r_g$$

In this state the equilibrium is such that ω^* is equal to zero and remains so given the initial state of the world. There are no shifts in the population distribution along the gains/losses axis. Our model also predicts policy convergence towards no disruption. Note that this does not imply that there are no voters in the realm of losses. However, it does not serve the political parties to appeal to them, given that they are not the majority. For a graphical representation of this state see Figure 5.

Golden Times In a situation where expectations are growing at an exceptionally low pace or the growth of the government performance has been unusually high, there is a period that we term "Golden Times". Potential factors leading to this are exceptional economic growth and prosperity, technological advances, or national events. Using our notation, this would be expressed with the following condition:

$$e_g > r_g$$

In this case μ continuously shifts rightwards along the outcome axis, in the realm of gains. We see this graphically in the figure below, where the median voter shifts rightwards every period, and her respective utility derived from the governmental good increases at a marginally decreasing rate.

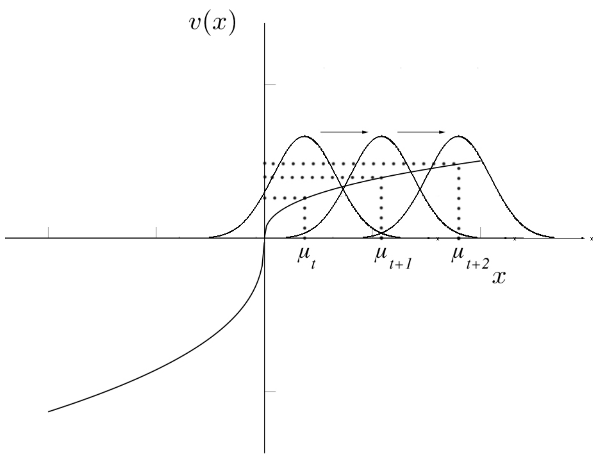


Figure 7: Optimal policy evolution when $\mu > 0$

There is again no shift in equilibrium, since the dynamics confirm the support for the status quo represented by $\omega^* = 0$ and mainstream parties continue to achieve electoral success. This is similar to "Normal Times", with the exception that the median voter, and the voter distribution as a whole, is consistently shifting rightwards.

Times of Disappointment When the performance of the government in providing the political good is disappointing and falls behind the course of the voter's expectations, the following condition holds:

$$e_g < r_g$$

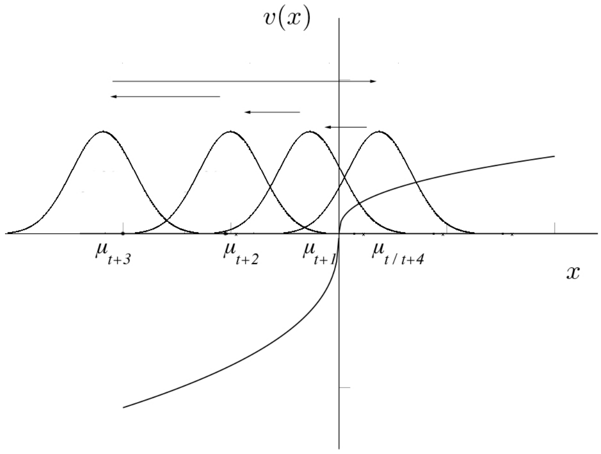


Figure 8: An example of optimal policy evolution when $\mu < 0$

Given that we imposed that initially $\mu \geq 0$, voters start by electing establishment parties and $\omega^* = 0$. However, the population continues to shift to the left and, assuming no change to the growth rates in e and r, the distribution will eventually reach $\mu < 0$. Graphically, this is represented by the first shift in Figure 8, where the median voter moves from μ_t to μ_{t+1}.

At this point equilibrium ω^* changes from zero to that described in Equation 3.2.1, a positive value. This continues to be the case until disruption succeeds, μ shifts back to the positives, and re-aligns expectations r with actual performance e. In Figure 8, disruption fails twice, moving the median voter from μ_{t+1} to μ_{t+2}, then μ_{t+3}. Note that optimal ω^* increases the further the median voter is in the realm of losses, and the shifts in the distribution increase in tandem. Finally, in the example presented in the figure, there is disruptive success in the next perod and the median voter shifts rightwards to μ_{t+4}.

It should be noted, however, that if the initial condition $e_g < r_g$ continues to hold, μ will continue to shift leftwards again until it becomes negative. The cycle repeats itself, providing a sustained pattern of establishment followed by disruption. In this scenario it is possible for the government to continue increasing the voter's welfare in absolute terms, and yet still be perceived as disappointing expectations, if these develop more rapidly.

3.3.1 *An Exogenous Shock*

Here we consider a parallel case and how it plays out within the framework described, presenting a situation where an exogenous negative shock occurs to e_i for all $i \in I$. In practical terms this could be characterised as a global financial crisis in the context of a small country, which the respective government arguably played no role in creating.

We look at the case where the two growth rates are in line, *i.e.* "Normal Times": $e_g = r_g$. In the case where the exogenous shock is small enough such that μ_t does not shift into the realm of losses, we see no effect. This can be seen below in Figure 9 as the movement from μ_t to $\mu_{t+1,a}$.

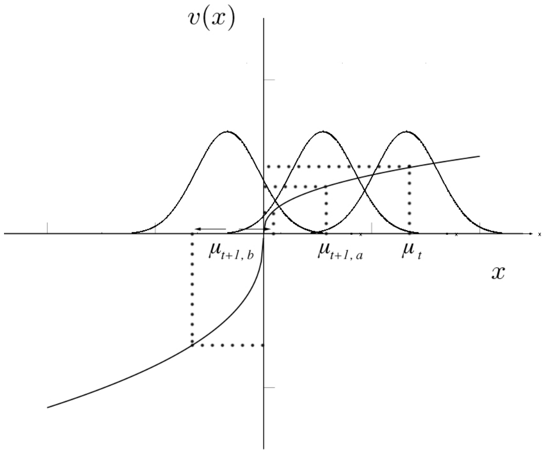


Figure 9: Effects of an exogenous shock

However, in the case where the exogenous shock is large enough to move μ_t such that it is smaller than 0, as in the move from μ_t to $\mu_{t+1,b}$, we see a change in equilibrium: disruptive policies become the equilibrium outcome as ω^* changes from zero to a positive value (described in Equation 3.2.1). Then, depending on the policy performance, and on which outcome is produced, the dynamics shift.

Assuming the positive outcome occurs, μ shifts into the realm of gains and optimal ω^* returns to zero, *i.e.* disruption ceases being desirable. Assuming instead that the negative outcome occurs, further disruption becomes appealing at a new, higher, rate. The median voter has shifted to $\mu_{t+1} - \omega^*_{t+1}$; the new optimal policy ω^*_{t+2} will satisfy the preferences of the pivotal voter at μ_{t+2}, which is a higher level of disruption. This cycle continues until there is success from disruption, or there comes a positive exogenous shock which re-aligns e and r again. When that occurs, ω^* changes back to zero and remains there, as before the shock.

In the case where $e_g > r_g$, *i.e.* the "Golden Times", the dynamics are similar to the ones described above for the "Normal Times".[15] The effects of the negative shock on

[15]There is a case where e_g exceeds r_g enough that it shifts the median voter to the realm of gains even with disruptive failure. In this case return to non-disruption occurs within the first period.

e_i, however, are different when $e_g < r_g$, *i.e.* the "Times of Disappointment". In this scenario, in fact, the shock might *speed up* the entry to the realm of losses. The rest of the evolution unfolds as without the shock, described above.

3.4. *An Extension: Varying the Probability of Success*

In this section we extend our findings by varying p, the probability of success of the disruptive policy, thereby introducing another level of generality. We restrict our focus to the range where $p < 0.5$ to investigate the threshold until which voters prefer disruption to the status quo. By decreasing p, the gamble turns to a less-than-fair bet. The threshold is the lowest probability of success at which agents are still willing to take the gamble, to the nearest full integer. Note that this threshold refers to the objective probability, rather than the probability weights. We then investigate how the threshold interacts with the parameters θ, λ, and x_μ.

In order to test how the different variables can affect the equilibrium policy, we have to make some assumptions to define the baseline case. We impose that x_μ is less than zero to describe the most interesting mechanisms of our model. As baseline values, we set θ and λ to be 0.88 and 2.25 respectively, to match the values found in Tversky and Kahneman (1992) for the purposes of demonstration. These values also fit the predictions of several experimental studies on the topic (see e.g. Schmidt et al., 2008).

To calibrate the neo-additive probability weighting function we impose the condition that the probability weighting function crosses the 45^o line when probability is equal to $\frac{1}{3}$. This is a typically accepted stylised fact for probability functions, which is found in the vast majority of experimental evidence on the subject (the most authoritative work on the topic is that in Camerer and Ho, 1994; Wu and Gonzalez, 1996, and in the literature reviewed by Wakker, 2010). In other words we want the following condition to hold:

$$a + b\frac{1}{3} = \frac{1}{3}$$

At baseline, we choose the values of 0.2 and 0.4 for a and b respectively, satisfying the condition above for our weighting function. For the purposes of this illustration we use these same values both in the realm of gains and in the realm of losses as is done by Schmidt et al. (2008).

In the following tables we illustrate some descriptive results for the threshold p, until which voters prefer disruption. We show the baseline results in the first row, and then vary one parameter at a time: in Table 1 we vary x_μ, in Table 2 we change λ, and finally in Table 3 we vary θ.

Varying x_μ By varying the location of the median voter, we test how the threshold p changes the further the median voter is in the realm of losses. We begin by considering a value of x_μ that is only marginally negative, and then look at increasingly more negative values.

Version	$\boldsymbol{x_\mu}$	$\boldsymbol{\theta}$	$\boldsymbol{\lambda}$	**Threshold p**
Baseline	-0.1	0.88	2.25	15%
I	-0.5	0.88	2.25	15%
II	-2	0.88	2.25	15%
III	-10	0.88	2.25	15%

Table 1: Shifts in the threshold p with changes in x_μ

We see two, potentially surprising, results. First we note that the threshold p is independent of the x value, which means that the cutoff probability of success, at which the status quo is preferred to disruption, does not depend on how far in the realm of losses the median voter is. Note that this independence refers to the threshold p and not the optimal ω^*, which *does* depend on x_μ, as the deeper in the losses the agent is located, the larger her optimal ω.

Secondly, we see that, when the parameters are at their baseline values, the median voter prefers to gamble with disruption rather than stick with the establishment up to the threshold p of 15%. This means that, even knowing that there is an 85% chance of failure from disruptive policies, and therefore an 85% chance that her utility will decrease, the median voter derives more utility from the gamble than from the status quo. This follows from the curvature of the valuation function in the realm of losses as well as from the probability weighting introduced by prospect theory, where agents overweight small probabilities and underweight large ones.

Varying $\boldsymbol{\lambda}$ Next we change the value of λ, *i.e.* the level of loss aversion, and see how it affects the threshold p. In general, higher loss aversion implies a larger reluctance to incur in further disutility, and is seen graphically by the kink at the origin. Most empirical studies find values of $\lambda \in (1, 2.5]$ – as reviewed by Schmidt et al. (2008) – therefore we choose values in that range.

Version	$\boldsymbol{x_\mu}$	$\boldsymbol{\theta}$	$\boldsymbol{\lambda}$	**Threshold p**
Baseline	< 0	0.88	2.25	15%
I	< 0	0.88	1.1	15%
II	< 0	0.88	1.5	15%
III	< 0	0.88	2.5	15%

Table 2: Shifts in the threshold p with changes in the loss aversion parameter λ

Similarly to the previous case, we see that variations in λ do not affect the threshold p. Changes in λ translate instead to changes in ω^*, as can be seen in Equation 3.2.1: loss aversion affects the degree of the gamble rather than the required probability of success.

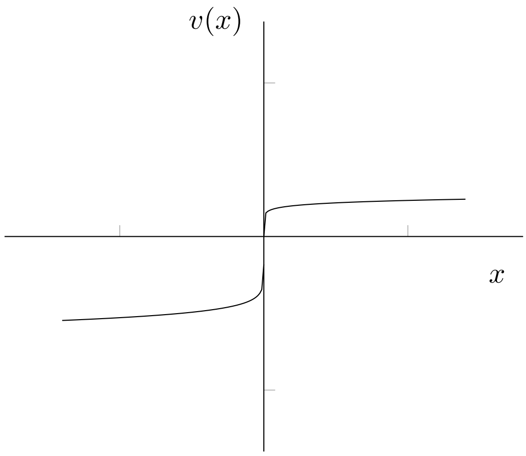
Varying $\boldsymbol{\theta}$ Finally, we see how varying θ influences the threshold p. We choose three values around our baseline value 0.88, varying in increments of 0.1.

Version	$\boldsymbol{x_\mu}$	$\boldsymbol{\theta}$	$\boldsymbol{\lambda}$	**Threshold $\boldsymbol{p}$**
Baseline	< 0	0.88	2.25	15%
I	< 0	0.75	2.25	2%
II	< 0	0.85	2.25	12%
III	< 0	0.95	2.25	21%

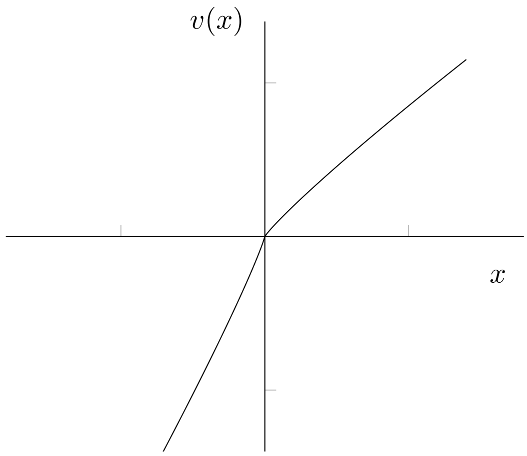
Table 3: Shifts in the threshold p with changes in θ

In this case, we see that θ *does* affect the threshold p; the two variables have a positive relationship. Nonetheless, even when θ has a value of 0.95, generating valuation functions that are almost linear, the threshold p is a relatively low 21%. The median voter prefers disruption to the status quo even if she believes that failure is almost four times as likely as success is.

One can argue that θ can represent the level of government intervention. The lower the value of θ, the higher the curvature of the valuation function, providing both an effective lower band for the disutility beyond which it is difficult to fall (*i.e.* unemployment benefits, universal healthcare) and a maximum level of utility that can be obtained from the government (*i.e.* progressive taxation).



(a) A more interventionist government



(b) A more free-market government

Figure 10: Example comparison between different levels of government

Lowering θ has the effect of a safety net: it makes any further losses less costly and hence it acts as an incentive for voters to gamble[16]. This can be clearly seen in Figure 10a. Increasing θ, on the other hand, makes gambling more risky, since the utility function becomes more linear, as can be seen in Figure 10b.

[16]This is not one of the main phenomena that this work aims at analysing, however these mechanisms of our framework may go some way in explaining why wealthy, social countries such as Austria, Denmark, France, or Sweden find persistent support for fringe parties despite comparatively high standards of living.

4. Literature Review

Our work fits into the literature on voting models, since it expands on them by introducing a novel utility function. In Section 4.1 we review the theoretical foundations on voting that we employ in our model. Our work also belongs to the strand of the literature on applications of behavioural economics, and specifically prospect theory, to other fields; the most relevant work is described in Section 4.2. Finally, since we apply our model to recent political developments, in Section 4.3 we review the studies that have been done on the topic of disruptive political choice, focusing on the most pertinent ones to our framework.

4.1. *Theoretical Foundations on Voting*

The theoretical foundations on which we rely to build our model come from two main blocks of economic research: models on utility functions, which we use to define individual preferences, and voting models, which we employ to aggregate individual preferences and find an equilibrium. In this section we go over the voting literature that is most relevant for our own model, first reviewing rational agent models, and then turning to voting models that incorporate behavioural features.

4.1.1 *Rational Agent Voting Models*

The diffusion of rational choice theories has generated a drastic change in political science research: since Downs (1957) presented his seminal paper with a two-party competition model with deterministic voting, rational agent models have been at the core of the research in the subfield of electoral choice. Downs' model builds on the pioneering work on market competition by Hotelling (1929); it posits that policies can be represented on a left-right scale, on which voters identify themselves. The scale can be interpreted as the degree of government intervention, the further right being more free-market and the further left being more interventionist. Because of this definition of the policy, Downs' model and the ones that build on it are defined as 'spatial models': the political ideology *affinity* can be evaluated in terms of *proximity* on the policy unidimensional scale.

The model is both "a theory of voter choice [and] a theory of party positioning" (Stokes, 1963, p. 368). Its main result is the definition of an equilibrium where parties converge towards the same policy platform, positioned at the median voter's optimal location. Policy convergence is the main conclusion that is reached, and is also the prediction of the model that attracted the most criticism from the academic world. Since it was first published, Downs' model has been progressively extended, revised and improved to provide a more accurate depiction of reality by relaxing some of its assumptions.

Some of the most prominent extensions of this framework include the depiction of a multiparty election system (e.g. Schofield et al., 1998; Shepsle, 2002) and the extension to a multidimensional policy space that allows parties to compete on different programmes (e.g. Shepsle, 1979; Budge et al., 1987). Dropping the axiom of policy unidimensionality and the assumption of a binary party choice allows the authors to predict policy divergence to some extent. Grofman (2004) offers a thorough survey of the different variations that have been suggested, looking at how predictions change when, for instance, politicians are not solely interested in holding office (Wittman, 1983). The results that follow are heterogeneous, and oftentimes allow the authors to depart from the standard Downsian prediction of convergence (Grofman, 2004).

In our model, we used a two-party competition framework *à la* Downs, with a unidimensional policy platform. We considered a different policy space than the right-left one, by defining ω as the departure from the status quo political good, and by allowing the outcomes to be stochastic. We chose this framework because it allows us to make relevant predictions that depart from the standard Downsian ones, while not overcomplicating these components of the analysis. Overall, we aimed our attention to voter preferences, which do not follow expected utility theory.

4.1.2 *Voting Models with Behavioural Features*

The introduction of all elements of prospect theory into a voting model is the main contribution of our work. Therefore, one of the strands of the literature in which our thesis fits is the one that attempts to challenge the hypothesis of perfect rationality of agents by introducing behavioural components in voting models.

Quite some attempts at introducing a behavioural approach in political science and political economy have been made in the past decades, with many encouraging assessments of such developments (e.g. Schnellenbach and Schubert, 2015). A large part of this research has aimed attention at the solution of the "voter turnout paradox", the incapacity of rational choice models to provide a solid justification of the individual decision to vote or not to vote (Ferejohn and Fiorina, 1974, p. 525). Schnellenbach and Schubert (2015) provide a comprehensive review of such attempts to solve it using behavioural economic principles, including the introduction of intrinsic motivations into the utility function, such as social welfare (Riker and Ordeshook, 1968, pp. 27–28), or social image (DellaVigna et al., 2016) concerns, and the introduction of a cognitive bias (notably, the "voter's illusion" described by Quattrone and Tversky, 1988).

Note that, despite the growing popularity of behavioural applications to voting, the most important model in behavioural economics and, arguably, the most meaningful change of paradigm that has been introduced in classical economic theory in the last thirty years, has never been featured *in toto* in a voting model. As noted by Barberis (2013), in fact, *prospect theory* has stood out for not having been applied very extensively "even though it has the potential to offer useful insights" (p. 190), particularly in the field of voter preferences and elections. One partial exception to this trend is the recent working paper by Alesina and Passarelli (2015), where the authors extrapolate loss aversion from the prospect theory framework and they analyse how results differ in a majority voting model when this feature is added to the voter's expected utility. Even in this case, however, the authors only introduce elements of prospect theory in a standard taxation choice model, but keep the expected utility function largely unchanged otherwise.

Our main contribution is the introduction of *all* elements of prospect theory – reference dependence, loss aversion, diminishing sensitivity, and probability weighting – into a stochastic voting model.

4.2. *Applications of Prospect Theory*

A further strand of the literature our work fits in is the one attempting to apply prospect theory to different fields. Prospect theory can be understood as a critique of expected utility theory, which oftentimes predicts behaviour in contradiction to what can be reasonably expected by agents. A famous example is the Allais paradox presented by Allais (1953), which provides one such inconsistency regarding choices under risk.

Notably, elements of prospect theory have been imported into the finance literature, which, as argued by Barberis (2013), has been its most fruitful area of application. Some notable examples are that of Benartzi and Thaler (1995), who try to explain the equity premium puzzle, Odean (1998), who investigates some stock selling behaviour anomalies, and Barberis and Huang (2008), who look at asset pricing. Other fields in which it has been applied include that of insurance – unsurprisingly, given that risk attitudes are decisive –, industrial organisation – where prospect theory informs price setting strategies – and labour economics, where it mainly looks at the reaction of labour supply to wages (Barberis, 2013).

Finally, some authors have attempted to import prospect theory intuitions into political theory, even though, as we mentioned, there are not as many papers. The most successful examples involve international relations and conflicts (Levy, 1992; Stein, 1992; List, 2003). With our work, we contribute to this strand of the literature by importing all aspects of prospect theory into a voting model.

4.3. *Drivers of Disruptive Political Choices*

Since we apply our model as a framework to understand disruption (Section 5), we review the existing empirical literature on the topic, which has been analysed under many different lights – particularly in the fields of political economy, political sciences, sociology, and media economics. The literature hence offers a vast panorama of theories and considerable empirical evidence; we offer a brief overview of the most influential hypotheses that are relevant to the application we present in Section 5.

Relative Deprivation Relative deprivation is the perception that one is in a state of disadvantage compared to others or to some standard; this perception is accompanied by a sense of anger and animosity, and can be developed at the interpersonal level or at the intergroup level, depending on whether the individual perceives that she herself is deprived or the group she belongs to is deprived (Runciman, 1966, p. 33–34). There is a large, well-established literature that tested the relative deprivation hypothesis, for instance investigating the occurrence of political unrest and violence in concert with relative deprivation (Vanneman and Pettigrew, 1972; Guimond and Dubé-Simard, 1983; Walker and Mann, 1987), or analysing intergroup prejudice and social identity (e.g. Pedersen and Walker, 1997; Pettigrew et al., 2008).[17]

Feelings of entitlement and injustice are hence crucial in the relative deprivation framework. As underlined by Smith et al. (2012), this is the substantial difference from behavioural theories, like anchoring or prospect theory, which pose the focus on the comparison to *specific* reference points. This is also the fundamental distinction between our framework and the relative deprivation hypothesis: perception of unfairness and resentment do play a role in the intuition behind our model, but the *driver* of the attitude change is not emotional, but is rather based on a "rational" change in risk attitudes. An additional difference of our model is in the way that the comparison is developed: the vast majority of papers find evidence in favour of fraternalistic relative deprivation, which looks at intergroup relations, while in our model the agent forms an *individual* reference point.

Economic Insecurity Hypothesis The economic insecurity theory associates adverse economic conditions to the rise of populist candidates and the support for radical politicians, and is one of the most prominent theories of disruptive political choices. This branch of the literature, in particular, stresses the consequences that profound economic changes – structural technological innovations, growing exposure to international trade, crises – have on a country's political architecture. Several authors, for instance, have shown that the increase in import competition is related to a shift away from moderate ideologies (e.g. Autor et al., 2016; Jensen et al., 2017, who offer an interesting analysis of the 2016 US presidential election). The Great Recession, notably, was identified as a sizable driver of political skepticism and of the popularity of non-mainstream candidates

[17] Note that this segment of academic work is closely affiliated to social identity theory, pivotal in the sociological psychology literature, which finds the roots of racial discrimination and segregation in the mechanisms of ingroup favoritism and outgroup bias (see for example Tajfel and Turner, 1979; Mummendey et al., 1999)

in Europe (see Kriesi and Pappas, 2015, who put together a thorough survey on individual countries).

Cultural Backlash Hypothesis This hypothesis suggests that the roots of the popularity of anti-establishment and disruptive parties lie in the response to cultural change of the once-predominant social group. More specifically, the argument maintains that since the postwar period a gradual change in the cultural paradigm has taken place, shifting from traditional to liberal, progressive values – multiculturalism, gender equality, environmentalism – so that a 'silent revolution' has taken place (Inglehart, 1997). The thesis goes on to say that the social groups that were once predominant are reacting to this erosion of their status and set of values by changing their political attitudes and preferences (Inglehart and Norris, 2016).

The Role of the Media The role of the media in encouraging political polarisation as a source of disruption has been at the centre of recent news after the Facebook data leak scandal (Kleinman, 2018), but had already attracted the attention of researchers in the past years. Several studies have been conducted to investigate how the consumption of homogeneous sources of news and limited interactions with opposing opinions has impacted polarisation (Iyengar and Hahn, 2009; Conover et al., 2011; Prior, 2013). Mechanisms of intentional or unintentional selective exposure are deemed to foster political polarisation and partisanship because users encounter highly homogeneous streams of opinions and fewer valid opposing views; this leads them to reinforce their pre-existing ideas and hence to develop more drastic attitudes (see also Sunstein, 2002).

In our model, both the economic and the cultural hypotheses can fit in the framework of our model by informing the reference point: financial circumstances and cultural change both feed into one's perception of the current state of the world and their place in it (e_i and r_i), and as such contribute to forming the risk attitudes of individuals during elections. Social media is also a channel through which agents can set their reference point, for instance via comparisons with their peer group and their larger social network.[18]

[18] We discuss some of these mechanisms that are relevant for the reference point formation in Section 6.

5. An Application: Disruptive Politics

Recent developments in the political arenas of many Western democracies show how introducing prospect theory into voting models can provide a powerful analytical tool. Our model in fact provides a solid intuition for understanding the rise of radical and populist parties in recent years, and for the untangling of other empirical puzzles, where traditional majority voting models fail to do so. Classical economic theory, in fact, features perfectly rational agents with expected utility preferences. Standard voting models of this kind conclude that the pivotal voter is the median one, found in terms of political ideology, where expectations are usually irrelevant (Stokes, 1963). Using these tools, the growing support for radical policies associated with negative expected utility cannot be rationalised, and agnostic voting streams remain a puzzling anomaly. Our framework, on the contrary, can shed light on such instances.

5.1. *Brexit, Agnostic Voting Streams, and Disruption*

Brexit and Disruptive Votes The most striking example of an event for which standard voting models have low explanatory power is that of the Brexit referendum. The choice of the British to leave the European Union could be explained, using a classical economic framework, only if benefits outweighed costs for more than half of voters. Yet, the majority of experts argued that the potential gains of leaving the European Union were not comparable to the expenses associated with it, both in a 'pessimistic' and in an 'optimistic' scenario (this has been shown mainly in terms of foreign investments and trade, as illustrated, among others, by Kierzenkowski et al., 2016; Dhingra et al., 2016). While the long-term consequences on the British economy will depend significantly on how the final deal will materialise, there have nevertheless been immediate costs in terms of the plunge of the British Pound Sterling and the ensuing political turmoil.[19]

In our framework, on the contrary, it is completely plausible that voters would seek out uncertainty while being aware of potential downsides associated with a given policy platform. Voters would seek disruption as opposed to continuity as a tool to gamble for a better prospect, when one accounts for expectations from the government r_i, and one's perception of the status quo, e_i. Specifically, we argue that establishment parties have not lived up to voters' expectations – *i.e.* that the reference point r_i of many citizens has outdone the actual offer of governmental good e_i – and hence the safe choice of endorsing cautious parties has lost its appeal to a segment of the population. On the contrary, candidates proposing untested policies offer, with some degree of uncertainty, a *potentially* favourable shake-up. The riskiness entailed by the uncertain consequences of such policies, once the shortcoming of radical and populist parties, became their strength.

[19]GBP became one of the most volatile currencies of 2016 against the dollar (Verma, 2017).

This has been argued by some political figures themselves, who have repeatedly asserted their disruptive features and have stressed how they represent a fracture from the elite. Notably, the portrayal of Donald Trump given by U.S. senator Ted Cruz for Time magazine is an accurate illustration of this tendency. He in fact describes President Trump as the herald of such a political fracture, doing "what he was elected to do: disrupt the status quo". Cruz also defined the disorientation surrounding his statements and his decisions as "not a bug but a feature" (Cruz, 2018).

Voting Streams Another observation taken from recent political outcomes that represents an empirical puzzle is one concerning voting streams. According to traditional voting models, in fact, electoral outcomes ultimately depend on the preferences of the median voter, for whom it would be irrational to take into consideration anything else than the absolute level of consumption related to each policy platform. As such, it would not be rational to go from one extreme of the political spectrum to the other, unless the policies offered are analogous – which has not been the case (as discussed in Section 1).

Again, this empirical puzzle confirms the mechanisms involved in our framework: when certain conditions are met in the population distribution around the reference point, disruption is in high demand, and extreme and radical parties provide it. In these instances of seemingly politically agnostic voting streams it is not explicit policies or ideology driving voting intention but rather the degree of disruption. It follows that switching from one end of the political spectrum to the opposite one may be a more coherent decision than shifting towards a moderate, centrist party of the same political colour. Disruption, and the uncertainty it entails, are fully in line with the paradigm described by prospect theory, and are motivated by reference-dependent risk attitudes.

5.2. *Application of our Framework*

Adaptation to our Terminology The mechanisms explained above are translated in the jargon of our model as follows: every voter faces a choice between two alternative political parties, each offering a policy that affects the agent's value function. An issue dominates the political competition, with each party proposing their political platform with respect to that good. An intuitive way to think about the political good would be the vote over the permanence in the EU, in which case ω equal to zero would be the non-disruptive choice of remaining, bearing no uncertainty over the resulting outcome.

Another example that could provide intuition for this political good is the standard policy question used in most political economy models (e.g. the ones detailed in Persson and Tabellini, 2002) – the level of taxation. Leaving the taxation *system* essentially unchanged while only varying the taxation *levels* would correspond to setting ω equal to zero. On the contrary, a disruptive change could be the proposal to move from progressive taxation towards a constant marginal rate – a flat tax (such a proposal was the core of the economic reform plan presented by the centre-right coalition in the Italian elections last March, see for instance Politi, 2018). The consequences are less predictable, ω is different from zero and will ultimately effect positive or negative change. Other applications, in this sense, could be seen with respect to immigration – as the proposal of building a wall advanced by Donald Trump or that of closing the borders as furthered by Lega Nord.

Convergence In our model, the equilibrium is that of convergence to ω^*. This is admittedly not an exact representation of reality, where disruptive parties have to a great extent remained anchored to radical policies that were rarely endorsed by moderate parties. However, we argue that a degree of convergence has indeed taken place.

The equilibrium of ω^* being zero is arguably what characterises the state of Western democracies for most of the post-war period. Policy convergence in this case does not necessarily mean that mainstream parties become exactly alike, but rather that centre-left and centre-right parties borrow policies from each other in order to not introduce too much disruption. This could arguably be seen in the rise of “Third Way” politics, adopted by British Prime Minister Tony Blair, US President Bill Clinton, and German Chancelor Gerhard Schröder (Campbell and Rockman, 2001; Bastow et al., 2002), and the pejorative term “Republicrat”, meant to indicate the apparent likeness between the two main parties in the US.

On the other hand, there also exist examples of policy convergence when ω^* is non-zero, where the appeal of fringe, or disruptive parties informs both the political discourse and policy of mainstream parties. In the UK the Conservative Party, who in 1973 campaigned to join the European Community, in 2013 promised to hold a referendum on the permanence in the EU, under pressures from UKIP in this direction (Winnett and Mason, 2013). Similarly, we have seen immigration and integration becoming an increasingly important topic for mainstream parties, even before the European migrant crisis.

Another form of convergence towards a non-zero ω^*, or appeal of disruption, is one in which policy does not necessarily play a role. This is the case in which politicians whose *policy proposals* are conventional and quite different from those of their radical opponents, but their *political persona* is portrayed as that of an outsider, as was the case of Italian Prime Minister Matteo Renzi. While being part of an establishment party, and endorsing orthodox approaches, he made “rottamazione” (literally, “car scrapping”) the core of his narrative. He identified himself with the need to completely discard the old ruling class, reinvent the Italian parliament, and transform the system. Similarly, Emmanuel Macron in France benefited from the newness of his political persona, despite the ‘conventional’ policies he promoted, as he became, at the age of 39, the youngest French president. With a business background he made the battle against “the ‘vacuous’ and failing French political system" his trademark (Chrisafis, 2017).

Two-Party System We argue that having a two-party system does not undermine the insights our model can offer, and on the contrary can be a valid representation of reality. In multi-party democratic systems, we have increasingly observed the bundling of parties between 'establishment' and 'disruptive' in the public perception.

This can be seen in practice when governments are formed along "grand coalition" lines with often explicit intent of not involving disruptive or populist parties. Examples of this are the German coalition negotiations of 2018 that ruled out AfD participation, and similarly the Dutch government of 2017 that avoided PVV participation. Similarly to our model, the median voter decided the level of disruption in these elections, as over 50% voted for "establishment" parties. The mirror case is seen in Italy where the median voter preferred disruption as more than half voted for Movimento 5 Stelle and Lega Nord.

6. Discussion

In our framework we chose a single political good and the respective reference point as the *expected* consumption of said good. In this section we expand on the intuition behind the political good and we discuss more in detail the formation of reference points.

6.1. *The Political Good*

In previous sections, we provided some examples of 'political goods' that one could consider. However insightful, such examples of *individual* political goods (referendum, taxation, immigration regulation) are flawed: the degree of deviation ω that parties can propose is limited, allowing only for restrained disruption. In reality the policy platform is multi-dimensional as there exists a large basket of matters that parties can reform. We believe, however, that the simplification of having a unidimensional policy is not too stringent and that moving towards a multi-dimensional policy platform would not affect the mechanisms of our model.

To see this, assume that there exists a consumption bundle of goods that agents believe depend at least partially on the actions of the government, and through which individual utility is affected. These can be financial in nature (such as income, total wealth, wage growth), social (such as income inequality, employment, relative wealth, social status) or of different kind (such as crime rates, perceived safety, worker rights, civil rights).

A way to represent multidimensional policy effects is to allow each agent to have a "vector" of issues in the utility function, similar to Kőszegi and Rabin (2006). There exist two vectors, one reflecting what the government *actually provides* and another reflecting what each agent *expects* from the government. The agent then derives utility from the difference between her "actual vector" and her "reference vector" (again, in line with Kőszegi and Rabin, 2006). This characterisation provides a framework that better reflects the choice between two alternative party platforms. However, we argue that this intuition can be carried over into our model.

Voters assign each vector a single value that reflects their overall assessment of said vector of issues, the intuition being that people tend to evaluate the government's general performance. Hence, any multidimensional effect on the vector is translated into a unidimensional one in this unitary assessment. This simplification allows us to find results for the unidimensional case that are analogous to the multidimensional one.

6.2. *Reference Point Formation*

A fundamental element of prospect theory, which has provoked ample discussion in the literature over the applicability of its framework, is the definition of how reference points are formed. In Kahneman and Tversky (1979), in fact, the authors only broadly outline the concept, and refer to the status quo – the current state of affairs – as a synonym for 'reference point' throughout the whole paper. They however acknowledge the possibility that "an expectation or an aspiration level that differs from the status quo" (*ibid*, p. 286) could be the measure of the coding of gains or losses.

In our model, we chose to provide that the reference point is formed based on expectations, following a growing tradition of theoretical and experimental papers that argue in favour of this interpretation (e.g. Bell, 1985; Loomes and Sugden, 1986; Shalev, 2000; Kőszegi and Rabin, 2006). When taking on the multidimensional perspective and introducing the 'reference vector', however, reference points could be defined differently depending on the nature of the good.

The reference entry can depend on or be completely set by past values, *i.e.* by the historical consumption of the good, such as for income, or social status. For example, a once-predominant social group, being aware of its status and privilege being eroded to equality may perceive the current state as a loss. This is also present in the cultural backlash hypothesis developed mainly by Inglehart and Norris (2016). For income, historical consumption would serve as a comparison platform in terms of its average growth: a generation that experienced steady increases in income could set a high reference point that would leave it in the realm of losses for any lower rate of growth, however positive. This has been the case for several countries, where the median income *growth* rate deteriorated in 2007, after a long period of regular increases (as it can be seen when looking at data from the U.S. Bureau of the Census, 2016).

The reference entry can also depend on or be completely set by expectations derived from external cues, *i.e.* general or social media, such as for luxury goods. Being exposed to the highlights of your peers via social media may overly inflate your point of reference, encouraging a perception of underachievement compared to a network (Denti et al., 2012). There is a growing number of studies and surveys which have in fact shown that social media usage can negatively impact one's life satisfaction and increase one's feelings of inadequacy: constantly seeing updates on others' successes, holidays, and nights out can set unrealistic expectations expressed by the reference point, thereby creating disutility for a level of consumption of luxury goods that could have otherwise been positive (Krasnova et al., 2013).

6.3. *Limitations and Possible Extensions*

As we have outlined in the discussion of our application on the appeal of disruption (Section 5), introducing prospect theory into a voting model can provide useful insights to interpret some empirical facts. This is true despite the simplifications we have introduced to the voting mechanisms, such as the ones we discussed in Sections 6.1 and 6.2 of having an individual political good and a two-party system. The assumptions of a unidimensional policy space and the two-party system represent the two main limitations of our model. We previously discussed how they do not strictly reflect reality and require some interpretation in order to carry the intuition of our model over to a real-world setting. Dropping these assumptions could thus generate some interesting variations in our results.

As we have discussed in Section 4.1, the main criticism that the original Downsian model (Downs, 1957) received was on the prediction of policy convergence. Given that we also predict policy convergence, we acknowledge that those criticisms can carry over to our model. In Section 5.2 we discussed how policy convergence can be interpreted. Nevertheless, it could be interesting to extend our model by constraining the level of risk that can be reasonably proposed by a party. This could be done, for instance, by defining ω_t as a function of ω_{t-1}, thus introducing a 'credibility constraint', effectively imposing a maximum variation in ω. Therefore, a party that sets $\omega = 0$ in one period would not be able to arbitrarily shift to a high ω^* in the following one without losing part of its appeal. Introducing this constraint, while eventually still leading to convergence, could temporarily result in divergent policies.

By assumption, the probability of success p is independent of ω. This may be a simplification of reality, as more disruptive policies, given by a higher ω, could arguably have a lower probability of success. This could be achieved by defining p as a function of ω, imposing a negative relationship between the two. This would be an interesting extension to our model, with potentially novel results. Another suitable way to define p endogenously would be by relating it to previous success rates. For instance, by imposing that p decreases every time the disruptive policy is unsuccessful. We have illustrated in Section 3.4 how a lower probability of success p can affect the equilibrium of our model, but this extension could also potentially alter its dynamics. An equilibrium where disruption is appealing would in fact remain so only for some time, unless it proved successful: after a series of failures, the status quo would again be the preferred alternative, even if the population distribution had shifted deeper into the realm of losses.

Finally, another way to extend our findings could be to introduce some refinements in the utility function, for instance by incorporating political ideology, voter turnout, or an exogenous shock to account for candidate popularity, thus including an additional stochastic element in the environment. These extensions could be produced using a probabilistic approach such as the one proposed by Lindbeck and Weibull (1987) and may be a better approximation of reality. Alternatively, changing the population distribution might bring further insights, such as allowing skewness or changing the overall type of the distribution. This would naturally affect the placement of the median voter and therefore the results of the model.

7. Conclusion

In our thesis we use prospect theory to explain voting behaviour. We portrayed individual preferences with the four main features that distinguish them from expected utility theory: reference dependence, implying that outcomes are evaluated with respect to a reference point, rather than in absolute terms, and are hence coded as gains or losses; diminishing sensitivity, which translates into varying concavity in gains and in losses and results in reference-dependent risk attitudes; loss aversion, entailing a larger sensitivity to losses than to gains of the same magnitude; and probability weighting, which allows us to portray the certainty and the possibility effects.

When introducing these elements from prospect theory into a two-party majority voting model, we are able to create a framework with novel results. We find stable equilibria where voters define their optimal level of deviation from the status quo and elect either "establishment" parties or "disruptive" parties. Contrary to rational agent voting models, which amount to the vast majority of existing models on electoral choice, we find that expectations play a pivotal role in the equilibrium, and that the distribution of the population around the reference point determines the equilibrium policy. When exploring the potential dynamics of our model, we show the different scenarios that arise when allowing the reference point and the governmental good to evolve over time.

We find that most of these results extend to the case in which the gamble becomes a *less-than-fair* bet: when varying the probability of disruptive success, we find that voters in the realm of losses find disruption more appealing than the status quo even when the perceived success rate is significantly lower than the failure rate.

We finally proposed an application of our framework as a tool to explain some empirical puzzles. Recent political developments in the Western world have in fact presented a conundrum for experts. Various explanations have been posited to try and reconcile the facts of near-consistent progress in social development, economic growth, and enduring peace with a seeming disappointment at the establishment that was integral in providing for these advancements. The model as presented then provides a framework that explains recent political upheavals without resorting to giving voters explicit emotional, irrational, or uninformed motivations. It allows us to interpret political trends dictated by expectations rather than absolute results. For these reasons, we believe that it would be an interesting exercise to further develop the model, for example along the lines we suggested.

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A. Appendix

A.1. *Proofs for the simple case equilibria*

Proof for Proposition 1

Realm of gains For $x_i = x_G > 0$, the maximisation problem is the following one:

$$\begin{aligned} \underset{\omega}{\text{maximise}} \quad & W^{+}(\pi)(x_G + \omega)^{\theta} + W^{+}(\pi)(-(x_G - \omega))^{\theta} \\ \text{subject to} \quad & \omega \in \big[0, x_G\big] \end{aligned}$$

The first order condition is:

$$\frac{\partial U}{\partial \omega} = \theta W^{+}(\pi)\big[(x_G + \omega)^{\theta-1} - (x_G - \omega)^{\theta-1}\big] = 0$$

The only solution is $\omega^* = 0$ for a fair bet $\big(\pi = \frac{1}{2}\big)$. The second order derivative at that point is unequivocally smaller than zero, hence this point is a global maximum. The proof goes as follows:

$$\frac{\partial^2 U}{\partial \omega^2} = \theta(\theta - 1)W^{+}(\pi)\big[(x_G + \omega)^{\theta-2} + (x_G - \omega)^{\theta-2}\big] < 0$$

$$2(x_G)^{\theta-2}\theta(\theta - 1)W^{+}(\pi) < 0$$

The terms θ, $W^{+}(\pi)$, and $(x_G)^{\theta-2}$ are unambiguously positive and, since $\theta < 1$, the term $(\theta - 1)$ must be negative. It follows that the whole expression is unambiguously negative, and $\omega_i^* = 0$ is indeed a maximum. The policy platform that is closest to zero will be preferred. ■

At the reference point For $x_i = x_N = 0$, the maximisation problem is the one that follows. Note that the constraint is not needed for $x_i = x_N$, since for any level of $\omega > 0$ the agent enters one of the two realms and hence her utility function is already defined differently for positive and negative ω. We specify it nevertheless, for consistency with the other maximisation problems:

$$\begin{aligned} \underset{\omega}{\text{maximise}} \quad & W^{+}(\pi)(x_N + \omega)^{\theta} - \lambda W^{-}(\pi)[-(x_N - \omega)]^{\theta} \\ \text{subject to} \quad & \omega \in \big[0, x_G\big] \end{aligned}$$

The first order condition is:

$$\frac{\partial U}{\partial \omega} = \theta \omega^{\theta-1} \big[W^{+}(\pi) - \lambda W^{-}(\pi) \big] = 0$$

The only solution is $\omega^* = 0$ for a fair bet $\left(\pi = \frac{1}{2}\right)$. Since $x_i = 0$ is a point of inflection by construction, the second order derivative will be null, and hence employing the second order condition would not contribute to this proof. Instead, we check that the agent's utility is decreasing in ω, by looking at the sign of the first derivative:

$$\frac{\partial U}{\partial \omega} = \theta \omega^{\theta-1} \big[W^{+}(\pi) - \lambda W^{-}(\pi) \big] < 0$$

$$\frac{W^{+}(\pi)}{W^{-}(\pi)} < \lambda.$$

The inequality then holds as long as this condition is satisfied. Given that we keep the probability p equal to 0.5, this assumption is non-controversial. The terms a^-, a^+ and b^-, b^+ are in fact respectively similar, and therefore their ratio close to one - smaller than λ by construction. The agent's utility will then be maximised for $\omega_i^* = 0$; the utility decreases in ω, so the voter will choose the lowest ω. ■

Proof for Proposition 2

Realm of Losses For $x_i = x_L < 0$, the maximisation problem is the one that follows. Note that the constraint can be rewritten as $-\omega \in \left[x_L, 0\right]$ because we imposed symmetry.

$$\begin{aligned} \underset{\omega}{\text{maximise}} \quad & -\lambda W^-(\pi)[-(x_L+\omega)]^\theta - \lambda W^-(\pi)[-(x_L-\omega)]^\theta \\ \text{subject to} \quad & \omega \in \left[0, x_G\right] \end{aligned}$$

The first order condition is:

$$\frac{\partial U}{\partial \omega} = \theta \lambda W^-(\pi)\left[(-x_L-\omega)^{\theta-1} - (-x_L+\omega)^{\theta-1}\right] = 0$$

The only point of interest is $\omega^* = 0$ for a fair bet $\left(\pi = \frac{1}{2}\right)$. We prove that it is a point of global minimum: the second order derivative at that point is unequivocally larger than zero, as shown below.

$$\frac{\partial^2 U}{\partial \omega^2} = -(\theta-1)\theta\lambda W^-(\pi)\left[(-x_L)^{\theta-2} + (-x_L)^{\theta-2}\right] > 0$$

The terms θ, λ, $W^-(\pi)$, and $(-x_L)^{\theta-2}$ are all unambiguously positive. Given that $\theta < 1$, $(\theta-1)$ must be negative. The negative sign then makes the whole expression unambiguously positive.

The other potential point of interest is the upper bound of the constraint we imposed on ω, the variable we are optimising over. We find that the utility function is in fact increasing in ω: $\frac{\partial U}{\partial \omega} > 0$. The proof is as follows:

$$\frac{\partial U}{\partial \omega} = \theta \lambda W^-(\pi)\left[(-x_L-\omega)^{\theta-1} - (-x_L+\omega)^{\theta-1}\right] > 0$$

$$(-x_L-\omega)^{\theta-1} > (-x_L+\omega)^{\theta-1}$$

Simplifying this expression, considering the range of values that each parameter can assume, this boils down to:

$$-\omega < \omega$$

The inequality holds trivially, therefore the utility of the agents in the realm of losses is strictly increasing in ω. This means that the utility is maximised when ω is at its upper bound, *i.e.* $\omega_i^* = x_G = |x_L|$. ■

Proof for Proposition 3

We fully disprove the existance of Nash equilibria other than that of setting the policy that is preferred by the agents in the realm of gains or at the reference point. We divide the proof into five cases:

1. $\omega_j^* = 0$ and $\omega_k^* > 0$

   In this case all voters in the realm of gains and at the reference point vote for party j given that their respective optimal ω^* is zero, while all voters in the realm of losses vote for party k. Given that $\alpha_G + \alpha_N > \alpha_L$, party j will win the election. Party k can increase their chances of winning from 0% to 50% by changing ω_k^* to zero. Therefore, this is not a Nash equilibrium.

2. $\omega_j^* > 0$ and $\omega_k^* = 0$

   By symmetry, consistent to (1) this is again not a Nash equilibrium.

3. $\omega_j^* > \omega_k^* > 0$

   In this case party k receives the votes of all voters in the realm of gains and at the reference point, since they strictly prefer a smaller ω. Given that $\alpha_G + \alpha_N > \alpha_L$, party k wins the election. Party j has the incentive to change its policy so that it is any small amount ϵ smaller than ω_k^*, thereby capturing more than half of the electorate and winning the election. Hence this is not a Nash equilibrium.

4. $\omega_j^* < \omega_k^* > 0$

   By symmetry, consistent to (3) is also not a Nash equilibria.

5. $\omega_j^* = \omega_k^* = 0$

   In this case voters randomise between either party, giving each a 50% chance of winning. Were one party to deviate by increasing their proposed ω they would gain all votes of agents in the realm of losses while losing all votes from those in the realm of gains or at the reference point. Given that $\alpha_G + \alpha_N > \alpha_L$, this would result in losing the election, deceasing their chances of electoral success from 50% to 0%. Therefore there is no incentive to deviate, this is a Nash equilibrium.

It follows from the cases above that $\omega_j^* = \omega_k^* = 0$ is the unique Nash equilibrium if $\alpha_G + \alpha_N > \alpha_L$. ■

Proof for Proposition 4

This proof follows a similar pattern as the proof for Proposition 3, however the parties must now target the voters in the realm of losses. We fully disprove the existance of Nash equilibria other than that of setting the policy that is preferred by the agents in the realm of losses by considering all five possible cases, as follows:

1. $\omega_j^* = |x_L|$ and $\omega_k^* < |x_L|$

   In this case all voters in the realm of gains and at the reference point vote for party k while all voters in the realm of losses vote for party j . Given that $\alpha_L > \alpha_G + \alpha_N$, party j will win the election. Party k can increase their chances of winning from 0% to 50% be changing ω_k^* to $|x_L|$. Therefore, this is not a Nash equilibrium.

2. $\omega_j^* < |x_L|$ and $\omega_k^* = |x_L|$

   By symmetry, consistent to (1) this is again not a Nash equilibrium.

3. $\omega_j^* < \omega_k^* < |x_L|$

   In this case party k receives the votes of all voters in the realm of losses, since they derive higher utility from higher ω. Given that $\alpha_L > \alpha_G + \alpha_N$, party k receives more than half the votes and therefore wins the election. Party j has the incentive to change its policy so that it is any small amount ϵ higher than ω_k^*, thereby capturing more than half of the electorate and winning the election. Hence this is not a Nash equilibrium.

4. $\omega_j^* > \omega_k^* < |x_L|$

   By symmetry, consistent to (3) is also not a Nash equilibria.

5. $\omega_j^* = \omega_k^* = |x_L|$

   In this case voters randomise between either party, giving each a 50% chance of winning. Were one party to deviate by decreasing their proposed ω they would gain the votes of agents in the realm of gains and at the reference point. Given that $\alpha_L > \alpha_G + \alpha_N$ this would result in losing the election, deceasing their chances of electoral success from 50% to 0%. Therefore there is no incentive to deviate, this is a Nash equilibrium.

It follows from the cases above that $\omega_j^* = \omega_k^* = |x_L|$ is the unique Nash equilibrium if $\alpha_L > \alpha_G + \alpha_N$. ■

A.2. *Proofs for the general case equilibria*

Elaboration of the proof for Proposition 5

Here, we discuss the individual equilibrium for $x_i \geq 0$. The individual voter's preference is found by solving a series of *unconstrained* maximisation problems. In particular, two cases can be distinguished, depending on where the median voter is located with respect to her reference point.

At the reference point For $x_i = 0$, the ω_i^* is found by solving:

$$\begin{aligned} &\underset{\omega}{\text{maximise}} \quad W^+(\pi)(x_i+\omega)^\theta - \lambda W^-(\pi)(-(x_i-\omega))^\theta \qquad \Longrightarrow \\ &\underset{\omega}{\text{maximise}} \quad [W^+(\pi) - \lambda W^-(\pi)]\omega^\theta \end{aligned}$$

This is similar to the maximisation as already explained in A.1, the difference being only in notation, changing x_N to x_i. It therefore follows that the same proof holds, and that for voters at the reference point ω_i^* is equal to zero.

Realm of gains For $x_i > 0$, we again find a similar proof as in A.1, however we also consider the additional case of voters moving from the realm of gains into the realm of losses, because in the general case ω_i^* is unconstrained. Therefore there exist two maximisation problems which need to be considered:

$$\begin{aligned} &\underset{\omega}{\text{maximise}} \quad U(x_i,\omega) = W^+(\pi)(x_i+\omega)^\theta + W^+(\pi)(x_i-\omega)^\theta; \qquad x_i \geq \omega > 0 \\ &\underset{\omega}{\text{maximise}} \quad U(x_i,\omega) = W^+(\pi)(x_i+\omega)^\theta - \lambda W^-(\pi)(-(x_i-\omega))^\theta; \quad \omega > x_i > 0 \end{aligned}$$

- The first order condition for the first maximisation, where voters do not shift into the realm of losses, is given as follows:

$$\frac{\partial U}{\partial \omega} = \theta W^+(\pi)\big[(x_i+\omega)^{\theta-1} - (x_i-\omega)^{\theta-1}\big] = 0$$

  Which gives us the solution $\omega_i^* = 0$. To check that this is a maximum we look at the second order derivative at this point, given as follows:

$$\frac{\partial^2 U}{\partial \omega^2} = (\theta-1)\theta W^+(\pi)\big[2x_i^{\theta-2}\big] < 0$$

  Given that $(\theta-1)$ is negative, and that θ, $W^+(\pi)$, and $2x_i^{\theta-2}$ are all positive the overall expression is negative. The second order condition at $\omega_i^* = 0$ is therefore unambiguously negative meaning that it is indeed a maximum.

- To confirm that this is the only maximum , we consider the solution to the second maximisation problem, when voters in the realm of gains can gamble such that they move into the realm of losses. This gives us the following first order condition:

$$\frac{\partial U}{\partial \omega} = \theta W^{+}(\pi)(x_i + \omega)^{\theta-1} - \lambda\theta W^{-}(\pi)\big[-(x_i - \omega)\big]^{\theta-1} = 0$$

With the solution:

$$\omega = \frac{x_i(1 + R^{\frac{1}{\theta-1}})}{R^{\frac{1}{\theta-1}} - 1}$$

This solution forces a negative value for ω, which is outside of its domain. It follows that the only optimal ω^* for those in the realm of gains is equal to zero.

We have shown in Section 3.2.1 how to translate the equilibrium from the individual to the aggregate level and we have shown that when $\mu \geq 0$, the unique Nash equilibrium is $\omega_j^* = \omega_k^* = 0$ ■

Proof for Proposition 6

This proof follows a similar pattern as the proof for Proposition 5 explained above, however the parties must now target the median (pivotal) voter who is in the realm of losses, since $\mu < 0$. To find the optimal policy for the median voter, we solve these maximisation problems:

$$\underset{\omega}{\text{maximize}} \quad U(x_i, \omega) = -\lambda W^-(\pi)(-(x_i + \omega))^\theta - \lambda W^-(\pi)(-(x_i - \omega))^\theta \quad \omega < |x_i|; x_i < 0$$

$$\underset{\omega}{\text{maximize}} \quad U(x_i, \omega) = -\lambda W^+(\pi)(-x_i + \omega)^\theta + W^+(\pi)(x_i + \omega)^\theta \quad \omega \geq |x_i|; x_i < 0$$

Maximising the above gives us two solutions, depending on the value of ω_i.

- For $\omega_i < x_i$, we get the following first order condition:

  $$\frac{\partial U}{\partial \omega_i} = \lambda\theta W^-(\pi)\left[(-x_i - \omega_i)^{\theta-1} - (-x_i + \omega_i)^{\theta-1}\right] = 0$$

  This boils down to this solution:

  $$\omega_i^* = x_i \frac{1 + A}{A - 1}$$

  where A is given by

  $$A = \left[\frac{W^-(\pi)}{W^+(\pi)}\right]^{\frac{1}{\theta-1}}$$

  This solution, however, forces a value of ω_i that is smaller than 0 or a value of ω_i that is larger than $|x_i|$. So this is not a feasible equilibrium: $\omega < 0$ is outside domain of the variable, and if $\omega > x_i$ the maximisation problem changes, invalidating this solution.

- For $\omega \geq x_i$ we get the following first order condition:

  $$\frac{\partial U}{\partial \omega_i} = \lambda\theta W^-(\pi)(x_i - \omega_i)^{\theta-1} + \theta W^+(\pi)(x_i + \omega_i)^{\theta-1} = 0$$

  This boils down to this solution:

  $$\omega^* = x_i \frac{1 + R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}} - 1}$$

  where R is given by

  $$R = \lambda \frac{W^-(\pi)}{W^+(\pi)}$$

  To prove that it is indeed a maximum, we show that the second order condition is unequivocally negative:

$$\frac{\partial^2 U}{\partial \omega_i^2} = \theta(\theta-1)\big[W^+(\pi)(x_i+\omega)^{\theta-2} - \lambda W^- \pi(x_i-\omega)^{\theta-2}\big] < 0$$

This boils down to the following condition, where R is the one defined above:

$$R^{\theta-1} - R^{\theta-2} < 0$$

Given that parties will pursue the voter at the median we then replace x_i with μ in Proposition 6. We show that not only is it a Nash equilibrium for both parties to offer this equilibrium but also that it is unique. There exist nine possible combinations of strategies, which we divide into 5 cases.

1. $\omega_j^* = \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$ and $\omega_k^* \neq \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$

   When $\omega_k^* < \omega_j^*$ (in the mirror case, $\omega_k^* > \omega_j^*$), all voters $i \in I$ with $x_i \geq \mu$ (in the mirror case, $x_i \leq \mu$) will vote for party j, while the rest vote for party k. It follows logically that party k receives less than half the electorate and therefore loses the election. They have an incentive to deviate by setting ω_k^* equal to ω_j^* and thus increasing their chances of success from 0% to 50%. Therefore, neither of these two cases are a Nash equilibrium.

2. $\omega_j^* \neq \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$ and $\omega_k^* = \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$

   By symmetry, consistent to (1) the two cases above are again not a Nash equilibrium.

3. $\omega_j^* > \omega_k^* > \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$

   In this case party k receives the votes of all voters with an x_i larger than or equal to the one to which ω_k is optimal for. Given that this includes the voter at the median, this means party k receives more than half the votes and therefore wins the election. Party j has the incentive to change its policy so that it is ϵ smaller than ω_k^* thereby capturing more than half of the electorate and winning the election. Hence this is not a Nash equilibrium.

4. $\omega_j^* < \omega_k^* > \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$; $\omega_j^* < \omega_k^* < \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$; $\omega_j^* > \omega_k^* < \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$

   By symmetry, consistent to (3) the ones above are not Nash equilibria.

5. $\omega_j^* = \omega_k^* = \mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$

   In this case voters randomise between either party, giving each a 50% chance of winning. Were one party to deviate by increasing their proposed ω they would gain all votes of agents whose x_i implies a larger optimal ω. Given that $\mu\frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$ reflects the optimal ω of the median voter, the proportion of voters gained would be less than half, by definition, while losing more than half of voters, with certainty.

By similar logic decreasing their proposed ω would lead to the same less-than-fair trade-off. Therefore there is no incentive to deviate, this is a Nash equilibrium.

It follows from the cases above that $\omega_j^* = \omega_k^* = \mu \frac{1+R^{\frac{1}{\theta-1}}}{R^{\frac{1}{\theta-1}}-1}$ is the unique Nash equilibrium if the median voter is located below her reference point. ■