Lower boundary violations and market efficiency: Evidence from the Swedish subscription rights market

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August 2007

Abstract

The purpose of this thesis is to study the market efficiency of the Swedish market for subscription rights between 1993 and 2006. Very few studies have examined the market for rights issues and no study has attempted to provide explanations for market inefficiency evidence of these markets. We use ex-post and ex-ante tests of lower-boundary violations in order to test the weak form market efficiency. Our main findings are that the Swedish market for subscription rights is characterized by remarkable arbitrage opportunities. Even after considering transaction costs and other frictions such as the bid-ask spread and allowing for one day execution lag, the profit opportunities are not eliminated, not even for the individual investor. However, this market is characterized by poor liquidity, and therefore the role of liquidity for the violation of the boundary conditions is examined by regression analysis. The results indicate that liquidity risk is a considerable obstacle to the role of arbitrage in pricing assets and that observed violations to some extent reflect a premium for illiquidity.

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Tutor:	Professor Clas Bergström
Presentation:	September 4, 2007, 10:15 am
Venue:	Room 343, Stockholm School of Economics

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1. Introduction

In financial theory it is often claimed that there is no such thing as a "free lunch" due to the fact that markets are assumed to be efficient. However, in real life this is not always the case and in this thesis we aim to investigate whether this assumption holds regarding the Swedish subscription rights market.[1] The question of whether these subscription rights are efficiently priced is of great importance, both from a theoretical and practical viewpoint. We believe that the results of this study is of great interest and importance for scholars interested in the case of market efficiency and for investors involved in options trading, hedging or arbitrage strategies.

When a publicly traded company proceeds with a new equity offering, it can do it in several ways, namely through a public offering, a rights offering or a private placement. Almost all US equity issues are organized through public offers, where the public has the right to sign up for the new shares in the issue. However, in Sweden and in most European countries public offerings are virtually non-existent, new shares are instead more often issued through a private placement or a rights procedure. Private placements are targeted to specific investors that do not necessarily need to be existing shareholders. In a rights offering new shares are pre-emptively offered to the existing shareholders on a pro rata basis. Subscription rights are then distributed to the existing stockholders. These owners have the choice to either exercise the rights or sell them in the market. Hence, contrary to other procedures, a rights offering creates a market where investors can trade in these pre-emptive rights. The objective of this thesis is to look further into the Swedish subscription rights market and test the efficiency of this market by using ex-post and ex-ante tests of lower-boundary violations.

The empirical research concerning subscription rights is very limited. To our knowledge, only two studies of rights markets have been performed. Both are based on the same data on the Finnish market between 1977 and 1981. Hietala (1994) analyses the efficiency for rights issues using the lower boundary condition as a benchmark, whereas Berglund and Wahlroos (1985) examine deviations from Black and Scholes' Option Pricing Model (OPM). Berglund and Wahlroos find that, after accounting for transaction costs, there is no evidence of market inefficiency. Hietala, on the other hand, finds evidence of an astounding amount of violations of the lower boundary conditions. While high transaction costs prevent ordinary investors from gaining on these violations, stockbrokers seem to be able to earn arbitrage profits in the Finnish subscription rights market. Moreover, Hietala finds evidence of persistence of these violations. However, it should be noted that these studies leave plenty of room for further research and improvement. Berglund and Wahlroos apply Black and Scholes' OPM in a market where no institutional framework for short-selling exists and ignore dividend payments during the life of the rights. Both of the studies make use of weekly data which can be argued not to be sufficiently

[1] The terms rights, subscription rights and pre-emptive rights are used analogously in this paper.

specific to give accurate results and may underestimate the findings regarding arbitrage opportunities. Furthermore, they do not consider implicit transaction cost such as the bid-ask spread, which on the other hand may overestimate the possibilities for arbitrages. In this study, we use more recent data of daily closing prices that cover a longer time period and we also incorporate implicit transaction costs. Doing this we hope to contribute to the research of this seemingly unexploited research area. In addition, we will attempt to explain potential violations and arbitrage opportunities by examining the effect of liquidity on the magnitude of violations. The role of liquidity has never been considered in a rights market study and our paper is therefore a significant contribution to the existing research on the pricing of subscription rights.

2. Theoretical Foundation and Hypotheses

In this section we describe some characteristics of the Swedish rights market and define the concept of efficiency. We will also examine the valuation of Swedish subscription rights and its similarities to call options and warrants, and present our hypotheses.

2.1 The Swedish Rights Market

A rights offering is an equity issue where all the existing shareholders of the issuing company have the right to sign up for new shares on a pro rata basis. A specified number of these publicly traded rights gives the owner the right to buy a new share from the company at a fixed price in a predetermined time period.[2] In most cases the existing stockholders receive one right for every share they own. Since all rights through this procedure are distributed to the current stockholders, companies can set the price of new shares considerably below the current stock price while protecting the existing shareholders from the effects of dilution. If a holder of rights for any reason does not wish to exercise her right to subscribe to new shares, she can always sell these publicly traded rights in the market.

The subscription rights allocated in the rights offering normally have a life of two to four weeks and are freely transferable. While in most cases one existing share entitles the stockholder to one right, the number of rights needed to buy a share of a new issue, usually referred to as the subscription ratio, varies and is determined by the company in advance of an offering. The holder may subscribe to the new shares anytime during the life of the subscription rights by using the required number of rights and paying the offering price of the new shares to the company.

The regulations and routines of rights issues vary between countries. The following figure illustrates the time line of a Swedish rights issue.

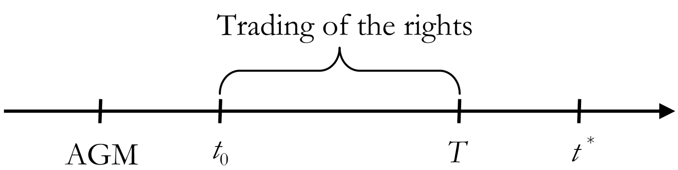

Figure 1 Time line illustrating the events in a rights issue

At the annual general meeting (AGM) the equity issue and its conditions are decided. Between t_0 and T the rights are trading and the holders need to decide whether to exercise their rights and participate in the rights issue by paying the exercise price or to sell them over the market. At

[2] The conditions of rights issues are not standardized and thus vary between issues.

t^* the new shares are allocated, which usually occurs approximately a month following the trading of the rights.

Call options or warrants are option contracts giving the owner the right, but not the obligation, to buy a specified amount of an underlying security at a specified price within a specified time. In comparison, it is therefore evident that the characteristics of Swedish rights closely resemble call options or warrants.

2.2 The Efficiency Concept

Since we are conducting a study on efficiency of a specific market, we need to define what we mean with efficiency. In our study we follow Jensen's (1978) definition of the efficient-market hypothesis, which states that a market is efficient if it is impossible to make risk-adjusted profits that, after adjusting for taxes and transaction costs, exceed the risk-free rate by trading on the basis of a given information set. This is also the definition the majority of other studies follow.

In the literature a number of versions of the efficient-market hypothesis have been proposed and examined. The variations mainly concern the definition of the information set. The three categories of market efficiency which have been developed are: (i) weak form efficiency, (ii) semi-strong efficiency and (iii) strong form efficiency. In the weak form of the efficient-market hypothesis the given information set is simply the information contained in the past price history of the market. Thus, if the weak form efficiency holds, a strategy based on past information cannot generate economic profits. In the second form, the semi strong form efficiency, the information set is all publicly available information, naturally including information on historical prices. Finally, the information set associated with the strong form of efficiency includes all information known to anyone.

In our study, we will test the efficiency of the Swedish market for subscription rights using historical information of asset prices, and we are therefore testing the weak form efficiency. Thus, when we in this study refer to efficiency, we mean the weak form efficiency.

2.3 Testing Market Efficiency

There are several possible ways of testing the efficiency of the Swedish subscription rights market. One strategy is to compare transaction prices observed in the market with prices implied by a theoretical pricing model, such as the Black and Scholes' (1973) Option Pricing Model

(OPM) or a similar method, adjusted for the characteristics of Swedish subscription rights.[3] However, this approach is associated with several significant disadvantages. The main problem is that it implicates a joint test of several hypotheses, i.e. that (i) the market for options is efficient, (ii) the pricing model is valid, (iii) the parameters of the model are correctly specified, and (iv) the data are synchronized (Mittnik and Rieken, 2000). In order to distinguish between the hypotheses, three of these have to be taken as an assumption. Another disadvantage of this method associated with (iii) is that we have to estimate standard deviations of the underlying stock price. This is not observable in the market and we therefore need to make a choice on the best estimate of the stock price volatility, a highly debated and unresolved question.

As an alternative it is possible to use tests based on the requirement of no systematic arbitrage opportunities. One example would be using the boundary conditions that the right needs to satisfy in order to prevent arbitrage. This method avoids the problem of jointly testing the hypotheses of market efficiency and model specification, a dilemma prevalent in many efficiency studies. The number of jointly tested hypotheses is reduced to two, i.e. (i) and (iv). In addition, when using this approach it is not necessary to impose restrictions on the stochastic process underlying the stock and all the variables required are observable in the market. Moreover, the vast majority of the Swedish rights are deep-in-the-money warrants; the average ratio of the stock price to the exercise price in the data set is 1.56. As this ratio increases, the Black and Scholes' solution and the lower boundary condition approach each other asymptotically (Hull, 2003). Thus, our results should not drastically differ whether we use the Black and Scholes' OPM or the boundary conditions for our tests. For the reasons mentioned above, all tests in this thesis are model independent and performed using the boundary conditions as a benchmark. We therefore do not examine the Black and Scholes' OPM or any other option valuation model.

In order to derive the boundary conditions for subscription rights, we first look into the arbitrage bounds of call options and warrants. As mentioned earlier, the two have large similarities to subscription rights and therefore constitute a good starting point.

2.4 Boundary Conditions of a Call Option

To further analyse the boundary conditions of a call option will help us derive the boundary conditions for a Swedish right. A Swedish subscription right may be exercised any time before maturity, however, all new shares are allocated at the same time, approximately a month following the trading of the rights. Thus, an investor who exercises her shares before maturity

[3] Black and Scholes (1973) showed that a risk-less hedge portfolio can be created and that the fair price of an option can be derived from this portfolio. Examples of other models include the Constant-elasticity-of-variance model (Cox and Ross, 1976), Jump-diffusion model (Merton, 1976 and Naik and Lee, 1990), Variance-gamma option pricing model (Madan and Milne, 1991) and Bivariate diffusion model (Hull and White, 1987).

will not receive the new shares until about a month after the trading period, i.e. at the same time as all other investors. In addition, the rights are not eligible for any dividends paid before the distribution of the new shares. As a result, the rights have a European feature as regardless of when the investor decides to pay the exercise price, the allocation day is fixed. Hietala (1994) does not take this fact into account when he discusses the valuation of American call options. It should also be mentioned that, for the same reasons, it is not beneficial for the investor to pay the exercise price early.

Using simple dominance arguments, Merton (1973) demonstrated that under very general conditions the price of an option must be placed inside a specific upper and lower bound. For a European call option on an underlying stock that does not pay dividends during the call option's lifetime, the boundary conditions for the option price, given that the market is efficient, are:

$$C_t(S,T,X) \leq S_t \tag{1a}$$

$$C_t(S,T,X) \geq \max\left(0, S_t - X\mathrm{e}^{-r(T-t)}\right) \tag{2a}$$

where $C_t(S,T,X)$ is the value at time t of a call option which is written on stock S with an exercise price of X, S_t equals the value of the stock at time t, T denotes the expiration date of the call option and r is the riskless interest rate.

In the following sections, we will refer to condition (1) as the upper boundary condition and condition (2) as the lower boundary condition. To see how it is possible to earn arbitrage profits from a violation of the boundary conditions, see Appendix A.

2.4.1 Boundary Conditions of Call Options on Dividend-Paying Stocks

If the underlying stock is expected to pay dividends during the life of the option, some adjustments to the boundary conditions have to be made. A holder of a European option is not entitled to dividends and therefore this value should be deducted. If we know that the stock in question will pay dividends D_{t_j} at time t_j, j = 1, - - - , n over the life of the option, then Equation (2a) has to be modified as follows for a European call option:

$$C(S,T,X) \geq \max\left(0, S_t - X\mathrm{e}^{-r(T-t)} - \sum_{j=1}^{n} D_{t_j}\mathrm{e}^{-r(T_j-t)}\right) \tag{2b}$$

where $\sum_{j=1}^{n} D_{t_j}\mathrm{e}^{-r(T_j-t)}$ is the present value of the known dividends per share to be paid in the interval t to T. Correspondingly, the upper boundary condition for a European call option changes to:

$$C(S,T,X) \leq S_t - \sum_{j=1}^{n} D_{t_j}\mathrm{e}^{-r(T_j-t)} \tag{1b}$$

If short selling is allowed and any of the above illustrated boundary conditions are violated, then in perfect capital markets investors would be able to earn arbitrage profits. For markets where short selling is not allowed, arbitrage profits cannot be earned even if the option price breaches one of the boundary conditions. These equations are in this case stochastic dominance bounds. A violation then implies that one portfolio of securities dominates another portfolio, which means that the expected return on the first is higher than the expected return on the latter. In Sweden, restrictions on short-selling were removed on August 1, 1991. Thus, after this date, the boundary conditions presented above reflect arbitrage bounds. Due to the restrictions on short-selling in Finland at the time of Hietala's (1994) study, the boundary conditions in his study constitute stochastic bounds, whereas the conditions in our study are arbitrage bounds.

2.5 Boundary Conditions of Warrants

A warrant constitutes, just as a call option, a right to buy one stock of a certain company at a specified price during a specified time period. The difference between a warrant and a call option is that a warrant is issued by the company and its proceeds are a part of the company's equity. The outstanding shares of the company in question will therefore increase if a warrant is exercised and dilute the equity of the company. Call options, on the other hand, are exchange instruments and are not issued by the company. Since Swedish rights are distributed by the issuing companies and the subscription price is set well below the price of the stock, it is apparent that Swedish subscription rights in fact are deep-in-the-money warrants. It has been shown that the boundary conditions for call options also apply for warrants, which we demonstrate in Appendix B. Consequently, all boundary conditions developed in Sections 2.4-2.4.1 are also valid for warrants.

2.6 Boundary Conditions of a Swedish Right

In this section we will consider the special features associated with Swedish subscription rights and modify the results developed for a call option, and hence also for a warrant, in order to derive boundary conditions of a Swedish right.

2.6.1 Payment of the Exercise Price and Dividends

As previously mentioned, the shares exercised by the rights are not distributed until after the trading period of the subscription rights. These new shares are therefore not entitled to any dividends paid during the lifetime of the right, nor during the time period between the expiration date of the rights and the allocation of shares.[4] This implies that there is no incentive for the owners of Swedish rights to exercise their rights prior to the dividend payment date, as might be the case with American call options.

[4] For the time period between the exercise of the rights and the allocation of shares, "paid subscribed shares" are usually allocated. The main difference between these securities and the underlying stocks is that its holders are not entitled to any dividends paid on the underlying share during their lifetime.

The owner of Swedish rights who subscribes to new shares is eligible for the full dividend following the issue.[5] Since Swedish companies pay dividends once a year and no of the issues in our sample lasted more than 6 weeks, there may be at most one dividend payment on the underlying stock to which the owners of the new shares are not entitled.

Taking these features into consideration, we obtain the following formula for valuing a Swedish subscription right:

$$S'_t \geq F(S,T,X) \geq \max\left(0, S'_t - X\mathrm{e}^{-r(T-t)}\right) \tag{3}$$

where

$$S'_t = \begin{cases} S_t - D\mathrm{e}^{-r(t_D - t)} & \text{when} \quad t < t_D < t^* \\ S_t & \text{when} \quad t_D < t < t^* \end{cases}$$

where $F(S,T,X)$ is the value of a Swedish right, t^* is the date of the allocation of shares, $t^* > T$, and D is any dividend payment during the life of the right at time t_D. In Sweden, the due date for the payment of the subscription price coincides with the maturity of the right, and we therefore use the expiration date of the right, T, in our formulas when calculating the net present value of the exercise price. In addition, the ex-dividend dates and dividend payment dates coincide. Having incorporated the issue of dividends, we now turn to how the subscription ratio affects Equation (3).

2.6.2 Number of Old Shares Needed to Exercise One New Share

In a rights offering, the company states how many stocks (later converted to rights) that are required to exercise one new share. Together with the exercise price, these depend on how much money the company wishes to gather and the number is therefore not standardized. In order to correctly value a Swedish right one has to know the number of shares needed prior to the rights issue, N, to exercise one new stock. This information is normally found from the conditions of the rights issue, and after having incorporated this feature into our analysis, the boundary conditions for a subscription right are given by:

$$\frac{S'_t}{N} \geq F(S,T,X) \geq \frac{\max\left(0, S'_t - X\mathrm{e}^{-r(T-t)}\right)}{N} \tag{4}$$

Thus, we have showed that, in perfect capital markets, investors are able to earn arbitrage profits on the Swedish subscription rights market if Equation (4) is violated. However, in the actual market transaction costs and other market frictions exist, which may restrict the possibility to earn riskless profits due to the existence of these violations.

[5] This is not the case in some markets, for example in Finland, cf. Hietala (1994).

2.7 Liquidity Risk

One significant restriction to earn arbitrage profits from a violation of Equation (4) is the potential illiquidity of the market. The trading in the Swedish subscription rights market is relatively thin and for a real investor it might not be possible to purchase the desired volumes at the closing price, thus the market impact costs to investors are amplified. Second, there is a risk involved as a trading strategy designed to exploit any violation of the boundary condition includes several simultaneous transactions. As the Swedish market for subscription rights is a very small market with relatively poor liquidity, having multiple buy or sell orders creates an additional risk. In order to apply the trading rule, after having observed a violation, the investor must short the stock and then purchase the necessary number of rights and treasury bills. This means that once the stock is sold short, an unfavourable change in the price may take place before the option is purchased, and the investor may therefore decide to abort the strategy but be left with the results of the short position. To sum up, the low liquidity creates extra risk both regarding market impact and the possibility that an order is not executed as expected. This could be a part of explaining some of the violations of the boundary conditions and violations might therefore reflect a premium for illiquidity. The risks associated with poor liquidity must be compensated.

Some studies on option markets have examined what impact liquidity has on the magnitude of the violations. Kamara and Miller (1995), for example, examined whether deviations from the put-call parity are reflected in a premium for liquidity risk rather than market inefficiency. Ackert and Tian (2001) replicate Kamara and Miller's study by examining the determinants of violations of the boundary conditions, the put-call parity and the box spread. Both of these studies find evidence that violations reflect a premium for liquidity risk.

2.8 Hypotheses

Based on the theoretical foundation presented above, we have designed the following hypothesis that will be tested in order to draw conclusions related to the efficiency of the Swedish subscription rights market:

H_1: *There are not significant violations from the lower boundary condition.*

Failing to reject H_1, signals market efficiency, given that the data is synchronized. To overcome the problem of nonsimultaneity, we will also test the following hypothesis:

H_2: *It is not possible to create profitable trading rules that exploit violations from the boundary conditions.*

These two hypotheses assume that no arbitrage opportunities exist and that arbitrageurs are unable to consistently generate abnormal returns after transaction costs have been considered. If we find evidence of market inefficiency from our tests, we will attempt to provide an explanation based on the relatively small market of subscription rights in Sweden, and additionally test a third hypothesis:

H_3: Any deviations from market efficiency reflect a liquidity risk premium.

These hypotheses will be tested in Section 5. The conclusion of market efficiency will be drawn from the results of these tests.

3. Methodology

In this section, we will introduce the methodology for analysing the hypotheses presented in Section 2. First, we will present the different transaction cost scenarios that will be applied in our tests. Thereafter, we will present an outline of the tests performed in Section 5.

3.1 Transaction Cost Scenarios

As transaction costs are difficult to determine with certainty, the tests regarding market efficiency are carried out in multiple scenarios both with and without transaction costs. More specifically, in this study we will assume three different transaction cost alternatives:

(1) No transaction costs. Trades are executed at the prices implied from historical transaction data. This enables us to compare our results with studies not assuming any transaction costs.

(2) Transaction costs are equal to costs faced by members and brokers of the exchanges, i.e. the least-cost traders. We take into account the bid-ask spread as well as direct transaction costs such as trading and clearing fees.

(3) Transaction costs are equivalent to those faced by the individual investor that apart from the bid-ask spread include brokerage commissions.

We will assume identical transaction costs for buying and selling stocks. An alternative approach could be to assume that shorting incurs higher costs than buying the stocks. This is noted by Puttonen (1993), who comments that as stocks needed for a short hedge must be borrowed, a premium may have to be paid by the arbitrageur to the owners of the stocks. Following this reasoning, transaction costs for shorting should be assumed to be higher than for buying the stocks. However, as specifying this premium is difficult, we will assume that the trading costs are equal in both kinds of transactions. We are conservative in all other aspects and this assumption should have a negligible impact on our results.

3.2 Testing the Boundary Conditions

In order to test whether the market for Swedish subscription rights is efficient and whether it is synchronous with the Swedish stock market, two kinds of empirical tests will be conducted. We will conduct a test of the boundary conditions as well as performing trading rule tests.

The tests of the boundary conditions examine, as mentioned above, the joint hypothesis that the market is efficient and synchronized with the stock market. Therefore, these types of tests cannot reject market efficiency; deviations from the boundary conditions merely indicate that the two markets are either not synchronized or that at least one of them is inefficient (Galai, 1978). These types of test are referred to as *ex-post tests,* since a profit-making strategy executed due to violations of the boundary conditions presume that transactions could have been made at the same prices that signalled these violations.

However, in practice, a certain time period passes between the observation of the irrational pricing and the execution of an arbitrage strategy. Based on information at time $t-1$, a trading strategy is created and the position is established at time t at prices that are unknown to the investor at time $t-1$. Therefore we also ask ourselves whether risk-adjusted profits can be revealed and exploited and develop ex-ante trading rules that would be able to earn such profits. Thus, using *ex-ante tests* will allow us to test the efficiency of the Swedish subscription rights market.[6] According to the efficient-market hypothesis irrational pricing should disappear over time. In an ex-post test any violations can be fully exploited without incurring any risk, whereas in an ex-ante test profits are affected by potential price movements between $t-1$ and t, and the strategy can even result in losses.[7]

3.2.1 Ex-post Test of the Lower Boundary Condition

In our first test we examine whether actual trading prices of subscription rights and the underlying equity satisfy the boundary condition given in Equation (4) above. The following tests of the boundary condition will examine the joint hypothesis that the market is efficient and synchronous with the Swedish stock market.

As described in Section 2, if the price of any subscription right in our sample is above the upper boundary condition or below the lower boundary condition, there exists an arbitrage opportunity that allows arbitrageurs to extract risk-free profits. Since the upper boundary condition in Equation (4) is satisfied by all price pairs in the data set, we will focus on the lower boundary condition. If the subscription right and stock markets are synchronous and efficient, then testing the efficient-market hypothesis in terms of the lower boundary condition amounts to testing $\varepsilon(t) \leq 0 \quad \forall t$. We perform the tests in three scenarios assuming different transaction costs. In Scenario (1) we assume no transaction costs:

$$\varepsilon_1(t) = S'(t) - X\mathrm{e}^{-r(T-t)} - N \cdot F(t) \leq 0 \qquad (5)$$

N denotes the number of rights required to exercise one share. In Scenario (2), we consider the bid-ask spread as well as direct transaction costs such as trading and clearing fees:

$$\varepsilon_2(t) = S'_{bid}(t) - X\mathrm{e}^{-r(T-t)} - N \cdot F_{ask}(t) - TC_F - TC_S \leq 0 \qquad (6)$$

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[6] This is the type of test Galai (1978) proposed for testing market efficiency.

[7] The risk incurred by price movements during the execution lag is also referred to as the immediacy risk, cf. Kamara and Miller (1995).

where subscript *ask* denotes ask prices and *bid* bid prices. TC_F and TC_S denote the direct transaction costs for the right and the stock, respectively. An arbitrage strategy involves simultaneously buying N rights and shorting the stock. At expiry, the stock is bought back and the position closed. Therefore, we need to consider a round-trip transaction in the stock and costs for buying N rights. In Scenario (3) we consider the bid-ask spread and commissions faced by individual investors:

$$\varepsilon_3(t) = S'_{bid}(t) - X\mathrm{e}^{-r(T-t)} - N \cdot F_{ask}(t) - TC_K \leq 0 \qquad (7)$$

where TC_K denote the sum of the commissions of buying the rights and the round-trip cost in trading the stocks. Since the empirical distribution of $\varepsilon(t)$ has been found to be considerably skewed, a conventional t-test may be misleading. Therefore, when reporting mean violation and mean profit in our ex-post and ex-ante test we apply the modified t-test for skewed data of Johnson (1978), which we denote by t^J, and perform a two-sided test against zero for the mean of $\varepsilon(t)$.[8]

3.2.2 Trading Rule Test

Following the ex-post tests, we turn to considering the ex-ante tests that more closely resemble the scenarios and opportunities faced by the real world trader or hedge fund manager. Since it is far from certain that an investor can set up a position at the prices observed ex-post, market efficiency can never be rejected using ex-post tests. Thus, by attempting to create ex-ante trading rules which would be able to earn risk-adjusted profits, we are able to analyse whether the Swedish market for subscription rights is efficient or not. If our trading rules are able to systematically exploit violations from the boundary conditions, it follows that the market is not efficient.

The ex-ante tests can be described as follows: Lower boundary violations at time $t-1$, based on observations of F_{t-1} and S_{t-1}, are regarded as mispricing signals that initiate a formation of a hedge position at time t. Given that the data set consists of daily closing prices, we consider a time lag of one day between the event of a mispricing signal and the transactions creating the hedge. At $t + \tau$ the position is closed, $\tau > 0$. The trading rule used in order to decide τ is established at $t-1$. Since it cannot be guaranteed that the prices used in the trading strategy equal the prices that signalled the formation of the portfolio, nonnegative returns cannot be assured.

[8] Johnson proposed that the conventional t-test, which assumes a symmetrical distribution, could be modified to assess the mean of asymmetrical distributions for any parent distribution ranging from the normal to a distribution as asymmetric as the exponential distribution for sample sizes as small as 13. The modified t statistic is given by: $t^J = (\bar{x} + \mu_3 / 6\sigma^2 n + \mu_3 \bar{x}^2 / 3\sigma^4)(s^2 / n)^{-1/2}$, where $\bar{x}$ is the sample mean and s^2 denote the sample variance, σ^2 equals the variance of X, μ_3 equals the third central moment of X and n denotes the sample size. Under the null hypothesis of a zero population mean, t^J is Student t distributed with $n-1$ degrees of freedom. Generally, μ_3 and σ^2 are changed to their sample counterparts $\hat{\mu}_3$ and s^2. Given that the sample distribution is symmetric, $\hat{\mu}_3$ is zero and the modified t statistic t^J collapses to the usual t statistic $t = \sqrt{n}\bar{x} / s$.

In our tests we apply an ex-ante trading rule that takes the following actions at time t if the price of the right at time $t-1$ was below the lower boundary condition by more than the transaction costs faced by the investor:

(1) Buy N subscription rights.
(2) Short the underlying stock.
(3) Invest an amount equal to the sum of the present values of the exercise price and the expected dividend, if any, in treasury bills.

This portfolio is held until expiration. If the stock price at T is below the exercise price X, then the subscription rights are not exercised; the stock is bought and the portfolio is liquidated. If the stock price at the right's maturity exceeds the exercise price, the subscription rights are exercised and the portfolio liquidated at time t^*, when the shares are allocated. Table I shows the different cashflows at t, T and t^*, depending on whether the subscription rights are exercised or not.

Table I
Trading Rule for Violations of Equations (6) and (7)

		$S_T \le X$		$S_T > X$	
Action at t	at t	at T	at t^*	at T	at t^*
Buy N rights	$-N \cdot F_t$	-	-	-	S_{t^*}
Short stock	S_t	$-S_T$	-	-	$-S_{t^*}$
Lend PV(X)	$-X\mathrm{e}^{-r(T-t)}$	X	-	$X-X$	-
Payoff	$S_t - N \cdot F_t - X\mathrm{e}^{-r(T-t)}$	$X - S_T \ge 0$	-	0	0

Since we assume an execution lag of one day, the cash balance at t may be negative, due to movement in the stock or right prices during the lag. It is problematical to measure the return of a portfolio where the net investment is zero or negative. In this study we simply use the net SEK payoff per underlying share of the arbitrage portfolio as a measure of the return. The net proceeds resulting from our trading strategy at t is denoted CF and, from the perspective of members of the exchanges, defined by:

$$CF = S'_{bid}(t) - X\mathrm{e}^{-r(T-t)} - N \cdot F_{ask}(t) - TC_F - TC_S \tag{8}$$

For individual investors, we define the net proceeds by:

$$CF = S'_{bid}(t) - X\mathrm{e}^{-r(T-t)} - N \cdot F_{ask}(t) - TC_K \tag{9}$$

The ex-ante tests are designed to be a more realistic scenario faced by traders and hedge fund managers. We therefore do not include Scenario (1), i.e. the no transaction cost scenario, in these tests. As noted, CF may be either positive or negative. If CF is positive, the portfolio is self-financing and the investor lends the excess. On the other hand, if CF is negative, the portfolio is not self-financing, and the investor needs to finance the deficit by borrowing. When the right matures, the net value of the portfolio equals

$$CF \cdot e^{r(T-t)} + \begin{Bmatrix} 0 & \text{when } S_T > X \\ X - S_{bid}(T) & \text{when } S_T \le X \end{Bmatrix} \tag{10}$$

Thus, if the portfolio created implementing the ex-ante trading strategy is self-financing, the net value of the portfolio will be nonnegative at maturity. However, there is a risk of having to finance this portfolio by borrowing, and as a consequence, repayment of debt may be necessary at T and the portfolio may result in a loss at the maturity of the right. Therefore, it is apparent that profits earned from this strategy are not pure arbitrage profits, and that it is problematical to separate the arbitrage component of the profit due to market inefficiency from the portion associated with illiquidity and drift of the stock price. For the same reasons as for $\varepsilon(t)$ in the ex-post tests, the modified t-test for skewed data of Johnson (1978) is applied for testing the significance of the net proceeds.

Halpern and Turnbull (1985) note that the problem of nonsimultaneity of the data may bias the results towards finding that the market is efficient. Even though an option is priced within the rational bounds, a recording of a violation may occur because of the nonsimultaneity of the data. If this false violation is used as trading rule signal, this decreases the possibility of earning a positive payoff.

3.3 Examining the Role of Liquidity

In order to examine our third hypothesis, i.e. to what extent liquidity risk is a determinant of the magnitude of violations in the pricing relationships, we use regression analysis. This has also been done in studies of options markets, such as for example Kamara and Miller (1995) and Ackert and Tian (2001). We will find inspiration from the option market literature and follow the method of these studies when examining the role of liquidity.

To approximate the liquidity risk in the subscription rights market we measure how far, as a percentage of the exercise price, an option is from at-the-money. This is expressed by $\mathrm{LN}(|S - X| / X)$ where, as previously defined, S represents the price of the stock and X is the strike price. The reason for choosing this variable as a proxy for liquidity is based on the data in Figure 2 that reveals that most of the trading volume in subscription rights during 1993 to 2006 was concentrated in near-the money and to some extent in-the-money rights. The more in-the-money or out-of-the-money a right is, the smaller is the trading volume. Thus, our chosen variable that measures how far an option is from at-the-money should be a good proxy for liquidity.

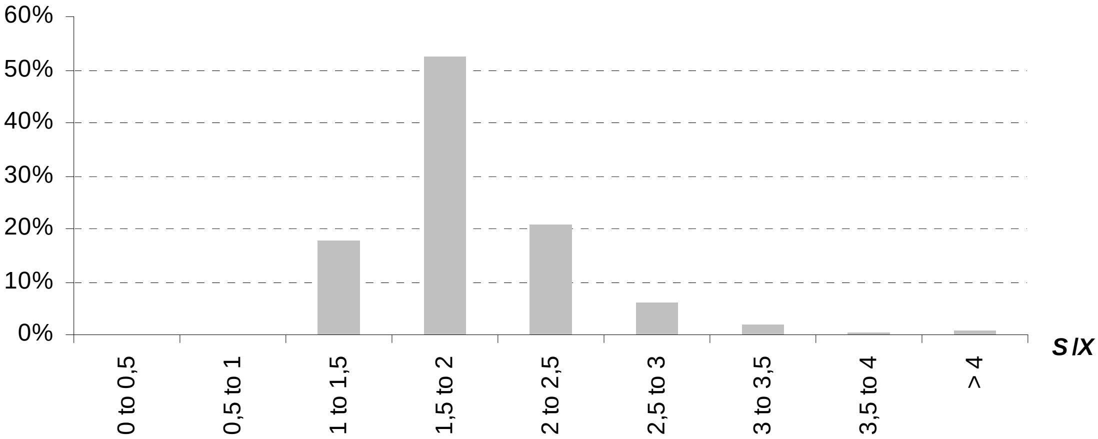

Figure 2 Percentage of Trading Volume by Moneyness

Thus, the time to transact and the liquidity risk increase with $\text{LN}(|S-X|/X)$, which is our independent variable. We would expect more severe violations for highly illiquid series than for illiquid ones and the expected sign of the coefficient is therefore positive. The effect of moneyness on violations from rational option pricing has also been examined in other studies, such as Halpern and Turnbull (1985) and Puttonen (1993), by splitting the sample in different categories based on the ratio of the stock price to the exercise price and comparing the results. However, in these studies the role of liquidity was not directly considered and moneyness itself was used as determinant of violations. While we use moneyness as a proxy for liquidity, it is possible that moneyness alone might be a factor in violations of the boundary condition.

There are several other potential liquidity proxies we could use. Kamara and Miller (1995) argue that the higher the volatility of the underlying asset, the larger liquidity risk traders carry between the decision to trade and the completion of the transaction. The expected length of time required to complete a transaction is another proxy for liquidity, since the greater the expected time needed to carry out a transaction, the larger the risk of adverse changes in asset prices. However, these two measures are not independent, as liquidity risk is an increasing function of the asset's volatility times the expected transaction time. Since we do not observe the time between transactions in our data set, we do not use this proxy in our test. We also wish to avoid overfitting the model by including several proxies for liquidity, and therefore choose to only include $\text{LN}(|S-X|/X)$. In addition, the effect of moneyness has been used in previous studies of option markets and is therefore a familiar variable when examining determinants of violations of rational option pricing.

To measure the violation and the dependent variable in the regressions we use Equations (5), (6) and (7). When a pricing relationship holds it means that the dependent variable is censored or unobservable and we therefore have chosen to use a Tobit censored regression model. In the standard Tobit model it is assumed that the regression residuals are identically distributed and independent (Gujarati, 2003). It should be noted that the independence assumption does not hold in our case due to the fact that some options overlap across expiration months and exercise prices. This results in downward-biased standard errors. We use a bootstrap procedure in order to overcome the dependence of the residuals across expiration months and exercise prices. Using this method, we allow for unequal variance over exercise months and exercise prices and assume equal variance across exercise prices and exercise months within each trading day. The Tobit regression and the sampling process have been repeated 500 times and the standard deviation of the vector of the estimates of the Tobit coefficients is the standard error.

Our model can be expressed as follows:

$$\varepsilon_{1,i} = \beta_1 + \beta_2 LN(|S - X| / X)_i + u_i \qquad \text{if RHS} > 0$$
$$= 0 \qquad \text{otherwise}$$
$$\varepsilon_{2,i} = \beta_1 + \beta_2 LN(|S - X| / X)_i + u_i \qquad \text{if RHS} > 0$$
$$= 0 \qquad \text{otherwise}$$
$$\varepsilon_{3,i} = \beta_1 + \beta_2 LN(|S - X| / X)_i + u_i \qquad \text{if RHS} > 0$$
$$= 0 \qquad \text{otherwise}$$

We have now defined the methods we intend to use in order to test our hypotheses. In the next section we will present the data set on which we apply these procedures.

4. The Data Set

Including rights issues on the Stockholm Stock Exchange (SSE) as well as the two smaller exchanges Nordic Growth Market (NGM) and Aktietorget, which started and ended between 1993 and 2006, i.e. a period of 14 years, there have been 474 rights issues.[9] The Stockholm Stock Exchange includes companies from the Large Cap (former A-list), Mid Cap and Small Cap list (former O-list) and First North (former Nya Marknaden). 332 of the rights issues were issued by companies belonging to the SSE, 110 were issued by firms on NGM and 32 were issued by firms on Aktietorget. Figure 3 shows the yearly distribution of rights issues from 1993 to 2006.

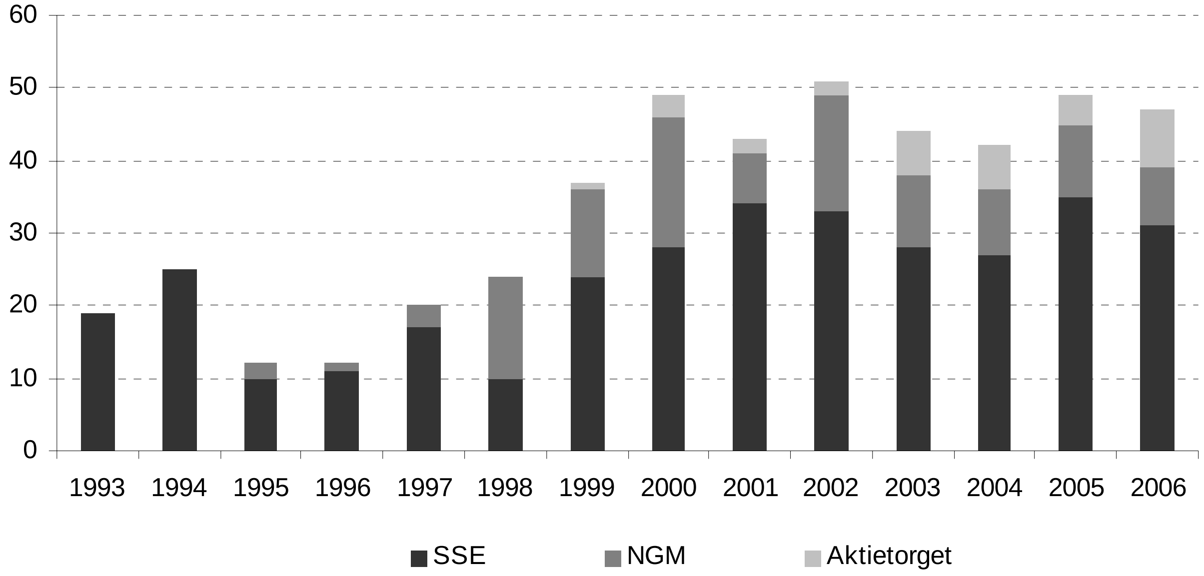

Figure 3 The total number of rights issues over time

The average number of issues per year during the period was 34. The highest number was observed in 2002 with 51 issues and the lowest in 1995 and 1996, when there were only 12 rights issues. In Sweden some companies have shares of different voting rights outstanding. Thus, when a company with several classes of shares issues new equity, separate subscription rights for every class are distributed. The data set therefore includes the rights for all the different classes of shares. However, when we refer to the number of rights issues, we count an issue of several classes of shares by one company as one rights issue. During the sample period, 22 of the rights issues comprised two classes of shares, i.e. there were 496 different types of subscription rights trading between 1993 and 2006.

In order to perform the proposed tests of market efficiency, daily closing prices for subscription rights have been gathered as well as subscription prices and subscription ratios. This data have

[9] Trading of shares on Aktietorget commenced in 1997. NGM was prior to 2001 known as SBI and IM.

been matched with data on the underlying securities, such as daily closing prices for the shares and dividend payments. In order to consider implicit transaction costs, bid and ask prices of both the rights and the shares were collected. Trading days when no trading on either the underlying share or the subscription right occurred were removed from our sample. In addition, when information about the exercise price or subscription ratio was missing, the rights issue had to be removed from the sample.

The data on the subscription rights and the underlying shares have been collected from the SIX Trust database. The closing prices of the subscription rights and the exercise prices collected from the SIX Trust database are not adjusted for subsequent splits or rights issues. However, the database does not contain any unadjusted historical stock prices. It is important that we match the unadjusted data of the subscription rights with unadjusted stock price data. Therefore, for those issues where unadjusted stock prices were needed (i.e. for those issues that have been followed by another issue or split), these were obtained directly from the Stockholm Stock Exchange, NGM and Aktietorget.

In total, the amount of observations in the data set after data drops sum up to 4011 trading days over 411 different rights issues of which 20 contained two classes of shares. Table II displays the distribution of the trading days over the years studied.

Table II

Summary Statistics of the Data Set

Number of Trading Days, by Year								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
No. of trading days	305	279	123	114	142	192	279	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
No. of trading days	361	343	357	350	315	420	431	**4011**

Table III displays the 63 rights issues between 1993 and 2006 that we did not have sufficient information about in order to include in the final sample divided by year and reason.

Table III

Excluded Rights Issues by Year and Reason

Excluded Rights Issues, by Year								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
Excluded obs.	1	2	3	2	7	3	9	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
Excluded obs.	12	7	9	2	3	2	1	**63**

Excluded Rights Issues, by Reason				
Reason	*No trading in rights*	*No trading in stocks*	*No unadjusted prices*	*Total*
Excluded obs.	33	20	10	**63**

In our tests we also need estimates of the risk-free interest rate. The STIBOR, i.e. the interest rate banks face when borrowing from each other, has been collected from the Riksbank, the Swedish Central Bank, and is frequently used in Sweden as the analogue of the short-term Treasury bill rate in the U.S. The average lifetime of the subscription rights in the data set is 14 days, ranging between 5 and 55 days, and the median is 13 days. If the time to maturity is equal or less than 18 days, we use the weekly STIBOR as an estimate of the risk-free interest rate. For maturities exceeding 18 days, we use the monthly rate.[10]

4.1 Estimating Dividends

If the ex-dividend date of a company occurs within the trading period of the company's subscription rights, it will affect the boundary condition, as shown in Equation (5). There are several ways of estimating the future dividend D. First, it can be assumed that the dividend per share D is equal to the previous dividend paid, i.e. a naive forecast from the past. Alternatively, we can estimate the dividend payments as if they had been perfectly forecasted, i.e. D is equal to the dividend actually paid.

Swedish companies pay dividends once a year. As the mean lifetime of the rights in our data set is 14 days, in most cases of our sample the dividend payment will already be known at the time the rights are trading, which supports the use of the second method. The same will be true for the ex-dividend date t_D, which therefore can be assumed to be known with certainty. Moreover, virtually all Swedish companies have the policy to pay dividends at roughly the same calendar time each year. Thus, we can comfortably make this assumption. In addition, the ex-dividend dates in our data sample take place within the trading period of the matching companies' subscription rights or before shares are allocated in only 9 rights issues. Of these only four companies had a positive dividend payment either (i) during the trading period of the rights or (ii) on the preceding dividend payment date, corresponding to 44 trading days. Thus, only 1.1 % of the data in our sample is affected by the D in the lower boundary condition in Equation (6), and we can safely assume that the various ways of estimating D will have a negligible impact on the test results of this study. We therefore choose to estimate both future dividend payments as well as the ex-dividend date as though they had been forecasted perfectly.

There is evidence that the stock price on average drops less than the amount of the dividend per share on the ex-dividend date.[11] Most studies, however, assume this drop as a proportion of the dividend (α) to be 1. In order to check the impact of this assumption on our results, we could

[10] This division is set arbitrarily, but is not likely to have any noteworthy effect on the results.
[11] See for example Elton and Gruber (1970) and Frank and Jagannathan (1997). For evidence from the Stockholm Stock Exchange, see Hedman and Moll (2006).

test for different values of α. However, since dividend payments do not affect the boundary conditions in the vast majority of cases, we will assume an α of 1 in this study.

4.2 Transaction Costs

An important issue that is critical in the analysis of market efficiency is the treatment of transaction costs. These include direct transaction costs such as commission fee and trading and clearing fee, as well as indirect costs, both of which will be further outlined in the following paragraphs.

4.2.1 Direct Transaction Costs

Trading at the SSE, NGM and Aktietorget is primarily conducted in the SSE's automated trading system, SAX, which only members of the exchanges have access to. Members include banks and stock broking firms. Members are faced with trading and clearing fees. Other investors, that are not members, have to go through a bank or stock broking firm to be able to trade shares on the SSE, NGM or Aktietorget. Therefore, these investors are faced with commission charges which increase the transaction costs. As the transaction costs differ between members and other investors we will perform the analysis from both perspectives.

Table IV displays the trading and clearing fees and how they have changed over the sample period. The fees for subscription rights and stocks are the same. This study assumes that fixed costs of exchange seats and other opportunity costs are sunk costs and thus do not affect any of the market participants' actions. The trading fees of the NGM are assumed to be the same as those of the SSE. We assume a brokerage commission of 0.50% of the contract value for traded stocks and subscription rights for individuals. This is a rather high estimate, which makes our approach conservative.

Table IV

Value-based Trading and Clearing Fees for Equities and Rights

Stockholm Stock Exchange	
Year	*Value-based Trading and Clearing Fees*
1993	0.0040%
1994	0.0036%
1995 – 1999	0.0035%
2000 – 2006	0.0039%
Aktietorget	
Year	*Value-based Trading and Clearing Fees*
1997 – 2006	0.1%

4.2.2 Indirect Transaction Costs

In addition to the explicit transaction costs involved in buying or selling stocks or subscription rights on the SSE, NGM or Aktietorget, there are implicit costs involved, which perhaps are even more important. Phillips and Smith (1980) and Bhattacharya (1983) were the first who highlighted that financial assets in general are bought at the ask price and sold at the bid price, which implies an indirect source of transaction costs.

A common assumption is that the bid-ask spread, i.e. the difference between the highest price that a buyer is willing to pay for an asset and the lowest offer to sell, is constant. Using this assumption the transaction cost can be derived from a sample of bid-ask quotations (Phillip and Smith, 1980) or originated from the movements of the transaction prices (Roll, 1984, Stoll, 1989, Smith and Whaley, 1994). However, as Kamara and Miller (1995) note, options that are more deep-in-the-money are more likely to have wider spreads. Therefore, we choose to use the quoted bid- and ask prices in our analysis instead of assuming a constant bid-ask spread.

According to Phillips and Smith (1980), trades also occur inside the spread, which implies that including the bid-ask spread might overestimate the actual transaction costs. Since there is no reliable way of correcting for this error, we have chosen to use bid- and ask prices in our analysis as this is the best estimation and reflection of these indirect transaction costs. Thus, our method is again conservative.

4.3 Data Synchronization

The problem of data synchronization is the most difficult to overcome. This problem means that the data on the stock price and corresponding subscription right price might not be synchronous, i.e. the timing of the closing price for the right and the stock may differ (Bookstaber, 1981). Since some of our tests involve trading strategies that require different positions in both the rights and the underlying stocks, these values should be synchronized in time. However, last transacted prices of stocks and subscription rights are very likely to happen at different times, especially for less liquid assets. Our data set consists of daily data and in order to minimize the problem of nonsimultaneity, one could use intraday data, which has been done in some studies of the efficiency of certain options markets. It has not been possible for us to obtain intraday data, and in reality, a large number of studies in option market efficiency are still using daily prices.[12] In addition, previous studies of subscription rights market such as Hietala (1994) and Berglund and Wahlroos (1985) have used weekly data. It should be noted that the synchronization between the rights and stock market in these studies sometimes has been severely affected, since the authors

[12] See for example Mittnik and Rieken (2000) and Cavallo and Mammola (2000). Evnine and Rudd (1985) demonstrate very similar results using intraday data and closing price data for S&P 100. This is also supported by Kamara and Miller (1995) who use data on the S&P 500.

have chosen to replace missing data on a particular day with price information from either the following or previous trading day. Thus, in some cases the time interval between the trades of the rights and the underlying stock can be up to two days. In our study we do not replace any absence of trading with prices from another day and therefore all trades in our study occur on the same day. As a result, compared to the previous studies on subscription rights, we minimize the data synchronization problem and consider the data to be specific enough in order to draw reliable conclusions. However, with daily data we cannot detect any inefficiencies in the market which exist for less than a day.

5. Empirical Results

In this section we test our efficient-market hypothesis and discuss the empirical results. We will test the lower boundary condition, which is model independent. In addition, we will examine the possibility to generate profits by using ex-ante trading strategies in which an abnormal return will indicate an inefficient market. Following the results of these tests, we will examine whether violations merely represent a premium for liquidity.

5.1 Results from Ex-post Test of the Lower Boundary Condition

The ex-post tests are performed under the different transaction-costs scenarios discussed in Section 3.1. Tables V-VII display the test results for the Equations (5), (6) and (7). The second to sixth rows in the tables show the means, standard deviations, maximums, minimums and medians of ε_1, ε_2 and ε_3. In the subsequent rows we show the absolute numbers, percentages, means and standard deviations (in SEK) of these violations.

Table V

Results from Ex-post Tests, Scenario 1

Scenario 1, No Transaction Costs								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
Sample size	305	279	123	114	142	192	279	
Mean (ε_1)	2.05‡	1.73‡	1.53‡	0.32	3.51‡	0.49	1.19‡	
Std (ε_1)	4.63	1.64	3.07	4.43	7.07	4.69	2.96	
Max (ε_1)	37.00	8.56	11.36	10.72	31.42	13.19	15.47	
Min (ε_1)	-8.00	-3.37	-2.08	-17.90	-10.15	-29.98	-18.96	
Median (ε_1)	1.03	1.61	0.31	0.24	2.03	0.93	1.00	
Violations	277	244	99	92	126	137	248	
Violation %	91%	87%	80%	81%	89%	71%	89%	
Mean(violation)	2.65	2.05	2.14	1.84	4.47	2.20	1.67	
Std (violation)	4.3	1.47	3.11	2.25	6.80	2.52	2.28	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
Sample size	361	343	357	350	315	420	431	**4011**
Mean (ε_1)	3.36‡	0.32‡	0.56‡	0.24†	0.47‡	0.44‡	1.12‡	**1.17‡**
Std (ε_1)	7.47	1.70	1.89	2.01	0.89	0.94	5.10	**3.99**
Max (ε_1)	50.94	9.02	11.15	8.59	4.77	10.01	38.54	**50.94**
Min (ε_1)	-4.29	-10.01	-6.56	-12.07	-0.96	-1.50	-13.38	**-29.98**
Median (ε_1)	0.70	0.16	0.05	0.21	0.08	0.10	0.15	**0.32**
Violations	310	250	230	270	259	349	343	**3234**
Violation %	86%	73%	64%	77%	82%	83%	80%	**81%**
Mean(violation)	4.01	0.84	1.23	0.70	0.60	0.56	1.67	**1.78**
Std (violation)	7.87	1.29	1.82	0.98	0.93	0.98	5.41	**3.94**

Note: *, † and ‡ denote significance of the mean against zero at the 10, 5, and 1% level, respectively.

Table V shows evidence of an astounding amount of violations. The average violation percentage is 81% in the zero transaction cost scenario and the mean violation is 1.78 SEK. For a trader or a hedge fund manager this implies that, given the assumption that it was possible to buy or sell without any time lag at prices observed ex-post, on average 1.78 SEK of profits could be made per underlying share sold short. The maximum and minimum size of the mean violations is 4.47 SEK and 0.56 SEK, occurring in 1997 and 2005, respectively. As previously mentioned, it is problematical to measure this profit as some form of return on investment, since the net investment may be zero or negative. It is also difficult to assess the real world opportunity, since we do not know how many rights or stocks that can be bought without having an impact on the price, thus the total possible profit is ultimately decided by the liquidity of the market.

Table VI

Results from Ex-post Tests, Scenario 2

Scenario 2, Transaction Costs of Members of the Exchanges								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
Sample size	305	279	123	114	142	192	279	
Mean (ε_2)	0.20	0.76‡	0.87‡	-1.19†	2.18‡	-1.08‡	0.04	
Std (ε_2)	4.49	1.96	3.07	6.49	7.00	4.97	3.21	
Max (ε_1)	18.06	5.56	10.68	8.20	30.90	19.99	10.40	
Min (ε_1)	-23.90	-13.59	-4.77	-35.41	-10.61	-30.53	-22.97	
Median (ε_1)	0.07	1.01	0.04	0.07	0.71	-0.09	0.12	
Violations	177	202	69	74	102	94	163	
Violation %	58%	72%	56%	65%	72%	49%	58%	
Mean(violation)	2.16	1.58	2.37	1.49	4.05	1.50	1.34	
Std (violation)	4.04	1.03	3.30	1.77	7.22	2.78	2.10	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
Sample size	361	343	357	350	315	420	431	**4011**
Mean (ε_2)	1.61‡	-0.16*	0.14	0.05	0.24‡	0.19‡	0.36†	**0.31‡**
Std (ε_2)	6.37	1.63	1.71	1.99	0.78	0.88	2.93	**3.59**
Max (ε_1)	32.06	4.30	11.01	8.58	3.72	4.42	20.39	**32.06**
Min (ε_1)	-56.49	-10.11	-7.84	-12.17	-2.59	-4.90	-14.22	**-56.49**
Median (ε_1)	0.20	0.04	-0.01	0.09	0.03	0.01	0.03	**0.05**
Violations	257	188	166	232	200	243	248	**2415**
Violation %	71%	55%	46%	66%	63%	58%	58%	**60%**
Mean (violation)	3.01	0.59	1.11	0.62	0.53	0.55	1.24	**1.45**
Std (violation)	5.90	0.88	1.62	0.96	0.75	0.88	2.96	**3.22**

Note: *, † and ‡ denote significance of the mean against zero at the 10, 5, and 1% level, respectively.

Table VI displays the results from Scenario (2). When we take transaction costs of members of the different exchanges into account, the average violation percentage drops to 60% and the average violation reduces to 1.45 SEK. The maximum and minimum size of the mean violations is now 4.05 SEK and 0.53 SEK, occurring in 1997 and 2004.

Table VII

Results from Ex-post Tests, Scenario 3

Scenario 3, Transaction Costs of Individual Investors								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
Sample size	305	279	123	114	142	192	279	
Mean (ε_3)	-0.87‡	0.34‡	0.30	-1.96‡	1.16†	-2.13‡	-0.80‡	
Std (ε_3)	4.73	2.30	3.25	6.72	6.80	4.93	3.53	
Max (ε_1)	17.02	5.19	10.30	4.92	29.22	16.39	9.24	
Min (ε_1)	-24.75	-16.18	-6.65	-37.11	-10.76	-31.05	-23.62	
Median (ε_1)	-0.61	0.73	-0.06	0.00	-0.04	-0.87	-0.19	
Violations	108	178	54	58	69	48	125	
Violation %	35%	64%	44%	51%	49%	25%	45%	
Mean(violation)	2.65	1.53	2.65	1.24	4.65	1.44	1.31	
Std (violation)	4.60	0.95	3.42	1.37	8.01	2.85	2.00	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
Sample size	361	343	357	350	315	420	431	**4011**
Mean (ε_3)	0.40	-0.37‡	-0.05	-0.06	0.07	-0.10*	0.03	**-0.21‡**
Std (ε_3)	5.65	1.64	1.69	1.97	0.85	1.28	2.82	**3.54**
Max (ε_1)	24.12	4.00	10.61	8.23	3.26	4.32	17.43	**29.22**
Min (ε_1)	-57.07	-10.21	-8.16	-12.17	-4.76	-8.19	-15.47	**-57.07**
Median (ε_1)	0.09	-0.03	-0.04	0.05	0.01	-0.01	-0.01	**-0.01**
Violations	211	153	145	209	169	178	208	**1913**
Violation %	58%	45%	41%	60%	54%	42%	48%	**48%**
Mean (violation)	2.30	0.50	1.01	0.59	0.46	0.62	1.17	**1.35**
Std (violation)	4.52	0.76	1.57	0.91	0.65	0.93	2.68	**2.96**

Note: *, † and ‡ denote significance of the mean against zero at the 10, 5, and 1% level, respectively.

The results from Table VII reveal that even for an individual investor that is confronted with all of our conservative transaction cost estimates, a staggering 48% of the observations violate the lower boundary condition. The average violation amounts to 1.35 SEK per underlying share, with a maximum of 4.65 SEK in 1997 and a minimum of 0.46 SEK in 2004. A decreasing frequency of the violations cannot be observed over the 14 years considered in either of the transaction cost scenarios. However, the magnitude of the average violation seems to have decreased implying an improvement in market efficiency over time.

To sum up, Tables V-VII show evidence of an astounding amount of violations. The average violation percentage is 81% in the zero transaction cost scenario. This number drops when accounting for transaction costs, but even for an individual investor an astounding 48% of the observations violate the lower boundary condition. The results are further supported by the significance tests, which on average show that option prices traded significantly below their lower bounds in the period examined. As a result, Tables V-VII undoubtedly entitle us to reject the joint hypothesis that the subscription right and stock markets in Sweden are efficient and synchronous. However, it is not yet possible to separately reject either part of this joint hypothesis. It is assumed in the definition of the violations that all relevant data are recorded at the same point in time. Any empirical test of this kind is faced with the problem of nonsimultaneity given the absence of continuous trading in the stock and option markets. This is something that must be taken into consideration when interpreting the results from this section.

5.1.1 Relative Efficiency

The concept of *relative* efficiency, defined as the efficiency of one market measured against another, may be a more useful approach than the all-or-nothing view that much of the traditional market-efficiency literature is taking. Next, we therefore compare these empirical results regarding the efficiency of the Swedish subscription rights market to evidence from other markets.

Various studies have been performed regarding the efficiency of different options markets. Most of these have focused on U.S. markets. The approaches have varied from studying violations of boundary conditions and the put-call parity to examining deviations from a theoretical option pricing model. Studies that have examined option price violations of lower bound constraints include Galai (1978) and Bhattacharya (1983) who analysed stock options traded on the Chicago Board Options Exchange, and Evnine and Rudd (1985) and Chance (1988) who tested options on S&P 100.[13]

Studies that have examined lower boundary violations on non-U.S. options markets include Halpern and Turnbull (1985), who studied Canadian stock options, Mittnik and Rieken (2000) who investigated options on the German DAX-index and Chesney et al. (1994) who analysed Swiss index options. Examples of Nordic contributions are Berg, Brevik and Sættem (1996) and Puttonen (1993) who studied the Norwegian and Finnish index options, respectively. In comparison, studies on European markets are relatively few and, in particular, studies on the Swedish security markets have been subject to only a few efficiency tests.[14]

The results of these studies of option markets demonstrate some indications of market inefficiencies. However, taking transaction costs and problems associated with data synchronization into consideration, the frequency and magnitude of the violations seem to decrease. In addition, evidence suggests that the possibility to exploit irrational option pricing is limited, since such irrationality tends not to be very long-lasting.

[13] Among prominent articles that used other methods for evaluating market efficiency than the lower boundary conditions are Klemkoski and Resnick (1980) and Wagner, Ellis and Dubofsky (1996) who tested the put-call parity on options on S&P 100. Bharadwaj and Wiggins (2001) used put-call parity and the box-spread on S&P 500. In his classical article Galai (1977) applied Black and Scholes' OPM on CBOE. Similar European contributions are Cavallo and Mammola (2000) and Brunetti and Torricelli (2003) who examined the put-call parity on the Italian option market, and Deville (2004) who studied the put-call parity on the French index option market.

[14] Andersson (1995) tested the Swedish call option market using put-call parity and a dynamic hedging strategy applying Black and Scholes' (1973) OPM and Byström (2000) tested pricing bias in the Swedish index option market with volatility implied from Black and Scholes' OPM as well as stochastic volatility.

Table VIII
Violation Percentages in Call Option Samples in Five Studies (No Transaction Costs)

Study	*Time period*	*Exchange*	*Violation percentage*
Puttonen (1993)	1988-1990	Helsinki	7.6%[a]
Berg, Brevik and Sættem (1996)	1990-1991	Oslo	5.1%
Mittnik and Rieken (2000)	1992-1995	German	3.3%
Hietala (1994)	1977-1981	Helsinki	62%[b]
Bennerholm and Berglund (2006)	1993-2006	Stockholm	81%[b]

[a] Transaction costs are those faced by the market maker.
[b] Subscription rights market.

In Table VIII we present a comparison of the results from the most recent studies that have examined violations from the lower boundary conditions. In comparison with the studies of Puttonen (1978), Berg, Brevik and Sættem (1996), Mittnik and Rieken (2000) and our study the main difference is the fact that nearly all rights in our study are in-the-money, while the other studies mentioned also have included observations from out-of-the-money options. As the results differ a great deal depending on whether all observations are included or not, we have only compared our results with the in-the-money sample from the reports in order to be consistent except in the case of Mittnik and Rieken's study, where the results are not divided by moneyness. The results are reported with no transaction costs assumed apart from Puttonen's, where transaction costs are assumed to be those of a market maker. Most of Hietala's sample consists of observations where a violation merely is a breach of stochastic dominance bounds. We therefore compare our results with the part of Hietala's sample where the boundary conditions constitute arbitrage bounds.

From Table VIII it can be concluded that the violation percentage in our study is much larger than in previous research. It should be noted that Puttonen (1978), Berg, Brevik and Sættem (1996) as well as Mittnik and Rieken (2000) have performed their studies on option markets while the latter two, including ours, are focused on subscription rights. However, as the rights market is defined as part of the option market this alone cannot explain the difference of the results. If our data were to be less synchronised than the comparable studies this could be a potential explanation. However, in comparison, our study has minimized the problem of nonsimultaneity of the data compared to other studies of subscription rights. Especially since other authors, e.g. Hietala (1994) and Berglund and Wahlroos (1985), have replaced missing information in one market with data from adjacent trading days without making the same changes in the corresponding stock or rights market.

Following this reasoning, it seems implausible that the astounding number of violations against the lower boundary condition can be explained barely by the potential presence of non-synchronous data. Hence, given the assumption of synchronized data we have illustrated that market inefficiencies are present. The market inefficiency is more thoroughly tested in the following section, where we investigate whether it is possible for an investor to create trading rules in order to exploit the observed violations found in this section.

5.2 Results from Trading Rule Test

In the ex-ante tests we examine if it is possible to make profits when assuming a time lag of one day between a mispricing signal and the execution of the strategy. The results from applying our ex-ante trading rule are reported in Table IX. In the first row the number of investments is reported. If there was no trading in the shares or the rights on the trading day following the mispricing signal, the strategy was aborted. Hence, the number of investments will be lower than the number of violations reported in the ex-post tests. The mean, standard deviation, maximum, minimum and median of the net payoff in SEK per underlying share on the arbitrage portfolio are also reported.

Table IX

Results from Ex-ante Tests, Scenario 2

Scenario 2, Transaction Costs of Members of the Exchanges								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
No of inv.	157	175	55	67	94	85	142	
% of signals	89	87	80	91	92	90	87	
Mean (profit)	1.81‡	1.55‡	2.39‡	1.22‡	3.57‡	0.71*	0.91‡	
Std (profit)	4.31	1.12	3.71	2.11	7.12	3.46	2.16	
Max (profit)	18.06	5.06	10.68	8.20	26.89	19.99	10.39	
Min (profit)	-4.97	-2.97	-3.21	-7.89	-4.94	-12.46	-4.68	
Median (profit)	0.45	1.65	0.59	0.18	1.20	0.14	0.64	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
No of inv.	232	174	143	198	164	215	211	**2112**
% of signals	90	93	86	85	82	88	85	**87**
Mean (profit)	2.20‡	0.47‡	0.85‡	0.62‡	0.45‡	0.39‡	1.02‡	**1.18‡**
Std (profit)	7.10	1.02	1.55	1.03	0.86	1.02	3.06	**3.55**
Max (profit)	32.06	4.30	7.31	8.58	3.72	4.42	18.89	**32.06**
Min (profit)	-56.49	-4.39	-4.38	-1.91	-2.23	-4.90	-12.09	**-56.49**
Median (profit)	0.49	0.15	0.24	0.34	0.15	0.10	0.21	**0.33**

Note: *, † and ‡ denote significance of the mean against zero at the 10, 5, and 1% level, respectively.

Table IX displays the results from Scenario (2). From the perspective of the members of the different exchanges, the mean profit per underlying share amounts to 1.18 SEK. The maximum and minimum size of the mean violations is 3.57 SEK and 0.39 SEK, occurring in 1997 and 2005. All years exhibit positive and statistically significant mean violations, indicating that in each of the years in the sample deviations from the lower bound could be exploited. The findings also indicate that there was persistence in the violations, which implies that nonsimultaneity of the data alone cannot explain the deviations from rational pricing.

Table X

Results from Ex-ante Tests, Scenario 3

Scenario 3, Transaction Costs of Individual Investors								
Year	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	
No of inv.	93	155	42	53	63	42	107	
% of signals	86%	87%	78%	91%	91%	88%	86%	
Mean (profit)	2.33‡	1.46‡	2.85‡	1.15‡	4.08‡	0.03	0.90‡	
Std (profit)	4.97	1.04	3.85	1.37	7.96	3.27	2.20	
Max (profit)	17.02	4.70	10.30	4.92	25.24	16.39	9.24	
Min (profit)	-7.80	-3.76	-4.94	-0.25	-3.73	-6.26	-5.80	
Median (profit)	0.38	1.55	0.64	0.17	0.77	-0.05	0.61	
Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*Total*
No of inv.	187	141	124	178	137	154	175	**1651**
% of signals	89%	92%	86%	85%	81%	87%	84%	**86%**
Mean (profit)	1.44‡	0.38‡	0.68‡	0.58‡	0.42‡	0.52‡	1.04‡	**1.10‡**
Std (profit)	6.46	0.83	1.59	1.01	0.65	1.00	2.65	**3.39**
Max (profit)	20.68	4.00	6.77	8.23	3.26	4.32	15.99	**25.24**
Min (profit)	-57.07	-1.80	-4.86	-2.86	-0.82	-1.76	-3.07	**-57.07**
Median (profit)	0.32	0.13	0.13	0.29	0.20	0.18	0.22	**1.10**

Note: *, † and ‡ denote significance of the mean against zero at the 10, 5, and 1% level, respectively.

The results from Table X illustrate that even an individual investor can profit from our trading rule. The mean profit in this scenario is 1.10 SEK per underlying share. Thus even with conservative transaction cost estimates and a time lag of one day, violations of the lower boundary condition could be exploited. The maximum of the mean violations is 4.08 SEK in 1997 and the minimum is 0.03 SEK in 1998. The mean profits are statistically significant at the 1% level in all years except in 1998.

Tables IX-X clearly show that both least-cost traders and individual investors can exploit the violations against the lower boundary. The results show that the arbitrage signals are persistent and that investors are able to benefit from the original arbitrage signal using an ex-ante strategy with a time lag of one day. As stated previously, we have been conservative in all of our assumptions except when assuming identical transaction costs for buying and selling stocks. However, this assumption can only explain a small part of the profit opportunities, since these are of such a considerable size and number.

To sum up, we can draw several conclusions as a result from the ex-ante tests. To start with, even after taking the role of transaction costs and other frictions such as bid-ask spreads into consideration, it is evident that the Swedish rights market is characterized by remarkable arbitrage opportunities. Second, the efficiency of the Swedish subscription rights market seems to have improved over time, which is consistent with the results of the ex-post tests. This can be seen from the test results that indicate that mean profits tend to be lower for the second half of the sample and is supported by the statistical significance.

The ex ante tests indicate that significant abnormal profit opportunities existed even the day following the violation of a pricing relationship. It should be noted, however, that it is not entirely certain that investors can earn substantial profits in this market due to a market friction that we up to this point have not analysed, namely low liquidity. It is possible that violations against the lower boundary condition may reflect a premium for liquidity, a matter we will look further into in the following section.

5.3 Liquidity Premium

Although the results in the previous sections tend to indicate that the Swedish subscription rights market is inefficient, we have to treat this conclusion with caution due to two reasons. First, large volumes might not be available at the closing ask price. Second, there is a risk of having multiple orders in the market, as it is possible that all transactions are not completed as expected. Both these risks are related to the liquidity premium. It is interesting to note that the role of liquidity has never been considered as a determinant of violations in a rights market study.

To calculate the liquidity premium we use a bootstrapped Tobit regression and the results are reported in Table XI.[15]

Table XI
Bootstrapped Tobit Model Regression Results

	Dependent Variable		
	$\max(0, \varepsilon_1(t))$	$\max(0, \varepsilon_2(t))$	$\max(0, \varepsilon_3(t))$
$\mathrm{LN}(\lvert S - X\rvert / X)$	1.106‡	0.882‡	0.697‡
Constant	2.174‡	0.752‡	-0.132

Note: *, † and ‡ denote significance at the 10, 5, and 1% level, respectively.

Our results are consistent with the findings of Kamara and Miller (2000) and Ackert and Tian (2001) from options markets, who reach the conclusion that liquidity clearly is a significant determinant of the size of violations in option pricing relationships. As expected, the coefficient of liquidity is positive, which means that the magnitude of violations are larger in the cases when a subscription right is further from at-the-money and hence when the liquidity is low. The coefficient is statistically significant at the 1% probability level. Thus, underpricing decreases with an increase in liquidity. In this section we have seen that our results strongly support our hypothesis that deviations from the lower boundary condition are reflected in a liquidity risk premium.

[15] The inferences remain unchanged if we use Ordinary Least Squares analysis instead.

These results could imply that the Swedish subscription rights market actually is efficient after taking a liquidity premium into account, since the theoretical profit opportunities observed in the ex-ante tests partly or fully could reflect a premium for the risk the investor has to bear when investing in assets with limited liquidity. It is difficult to determine to what extent a liquidity premium explains the results from the ex-post and ex-ante tests. However, we can conclude that there exists a correlation.

6. Conclusion

The purpose of this study has been to study the efficiency of the Swedish market for subscription rights between 1993 and 2006 and subsequently investigate the impact of liquidity on the magnitude of the violations. The question of whether these subscription rights are efficiently priced is of great importance, both from a theoretical and practical viewpoint.

The findings are based on both ex-post and ex-ante tests of lower-boundary violations. Our measures of transaction costs have been conservative. We conclude that the Swedish market for subscription rights is characterized by remarkable arbitrage opportunities. We report that an astounding percentage of the sample violates the no-arbitrage conditions derived to accommodate Swedish rights. Even after considering transaction costs and other frictions such as the bid-ask spread and allowing for one day execution lag, the profit opportunities are not eliminated, not even for the individual investor. Thus, we have found support for neither our first hypothesis of no significant violations from the lower boundary condition, nor our second hypothesis that it is not possible to create profitable trading rules that exploit violations from the boundary conditions.

However, our analysis has proven that liquidity plays an important role for the violation of the boundary conditions. The results indicate that liquidity risk is a considerable obstacle to the role of arbitrage in pricing assets and support our third hypothesis that violations reflect a premium for illiquidity. The inefficiencies we have observed could therefore partly or even fully be explained by a liquidity premium, which diminish the opportunities for investors to make arbitrage profits.

Very few studies have examined the market for rights issues and no study has considered the poor liquidity in these markets as a determinant of violations. Our results therefore constitute a valuable contribution to the present research of this unexploited research area by significantly improving previous studies and suggesting an explanation of the large amount of violations. It is possible that this also could explain the different results between studies of rights and option markets, as option markets tend to be more liquid than rights markets.

7. Suggestions for Further Research

New questions arise when investigating an interesting and little examined subject. It is very difficult within the scope of a master thesis to elaborate on every aspect of the chosen subject, and a suggestion for further research in line with the hypotheses and in the same field might be to use intra-day data instead of daily data in order to minimize the data synchronization problem.

As we have observed that liquidity and moneyness play an important role for violations of the lower boundary condition, another suggestion for further research would be to more closely investigate this fact. The observation that deep in-the-money rights tend to violate the boundary conditions in an overwhelming majority of the cases also opens a road for future research, and a comparison between Sweden and other countries would be of value.

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Appendix

A. Violation of Condition (2a) - Arbitrage Strategy

A violation of the boundary conditions indicates an immediate arbitrage opportunity and, thus, irrational option pricing. To see this, consider a violation of condition (2a), i.e. $C(S,T,X) \leq S_t - X\mathrm{e}^{-r(T-t)}$. An arbitrage profit can be made by buying the undervalued call, shorting the stock and lending the present value of the exercise price at the risk-free rate. At expiration the position is liquidated with two possible outcomes. The cash flows are depicted in Table XII.

Table XII

Violation of Condition (2a) - Arbitrage Strategy

	at t	at T	
Action at t		$S_T \leq X$	$S_T > X$
Buy call	$-C$	0	$S_T - X$
Short stock	S_t	$-S_T$	$-S_T$
Lend PV(X)	$-X\mathrm{e}^{-r(T-t)}$	X	X
Payoff	$S_t - C - X\mathrm{e}^{-r(T-t)} > 0$	$X - S_T \geq 0$	0

Since this trading strategy yields a positive payoff at time t and a nonnegative payoff at expiry, violations of condition (2a) would allow arbitrage profits.

B. Boundary Conditions of Warrants

Galai and Schneller (1978) derive the value of a warrant with respect to the value of a call option on the stock of an, *ceteris paribus,* identical firm with no warrants. The authors start their analysis by assuming a firm that has n shares outstanding and is expected to be liquidated at time t_1. The stochastic liquidation value of the firm's equity is denoted by $\tilde{E}_1$. The firm is depicted in Table XIII, rows (a) to (c). The value of a share of a firm that has not distributed warrants at t_0 is denoted S_0^* at t_0 and S_1^* at t_1. S_0 and S_1 refer to share prices of the firm that has issued warrants.

Next, we assume that the firm issues warrants at t_0 with an exercise price X. The ratio of the number of warrants to the number of shares is denoted by q, i.e. if the warrants are exercised at t_1, the value of the firm will rise from $\tilde{E}$ to $\tilde{E} + nqX$. The holders of the warrants will then exercise the warrants in the event that the stock price after the exercise (S_1) is greater than the exercise price. Thus, the condition for exercising the warrant is:

$$S_1 = \frac{\tilde{E}_1 + nqX}{n + nq} = \frac{nS_1^* + nqX}{n + nq} = \frac{S_1^* + qX}{1 + q} > X \tag{11}$$

This equation is equivalent to $S_1^* > X$, which allows us to, in row (d)-(f) of Table XIII, illustrate a firm issuing nq warrants at t_0.

Table XIII

Number of Shares, Values of Shares and Warrants at t_0 ant t_1

	at t_0	at t_1	
		$S_1^* \le X$	$S_1^* > X$
Firm with no warrants			
(a) Number of shares	n	n	n
(b) Value of the firm's equity	E_0	$\tilde{E}_1$	$\tilde{E}_1$
(c) Value of a share	$S_0^* = E_0 / n$	$S_1^* = \tilde{E}_1 / n$	$S_1^* = \tilde{E}_1 / n$
Firm with warrants			
(d) Number of shares	n	n	$n(1+q)$
(e) Value of a warrant	W_0	0	$\frac{S_1^* + qX}{1+q} - X = \frac{S_1^* - X}{1+q}$
(f) Value of a share	S_0	$S_1 = S_1^*$	$S_1 = \frac{S_1^* + qX}{1+q}$

Comparing the terminal values of the warrant, i.e. row (e), to the terminal values of a European call option on the firm with no warrants, i.e. 0 if $S_1^* \le X$ and $S_1^* - X$ if $S_1^* > X$, it is evident that the value of the warrant, in order to prevent arbitrage, must be:

$$W_0 = \frac{C_0}{1 + q} \tag{12}$$

Thus, the value of a warrant is always $1/(1 + q)$ the value on the call option of an identical firm without warrants. However, more importantly it can be shown that the boundary conditions stated for call options are also valid for warrants. In the case of a firm with warrants, the financial market already reflects the potential dilution effect, and we observe S and not S^*. We know that the terminal value for a call option on the firm with no warrants is $\max(0, S_1^* - X)$, correspondingly the terminal value for a warrant must be $\max(0, S_1 - X)$. This can also be seen by expressing S_1^* in terms of S_1, q and X from row (f) and inserting these expressions into the equations of the warrant value in row (e). Allowing for dividends will not change the relative values of the options and warrants, and consequently, all boundary conditions developed for call options are also valid for warrants.