STOCKHOLM SCHOOL OF ECONOMICS
Department of Economics
5350 Masters Thesis in Economics
Academic Year 20172018

Who Gets Which Seat And Why

SSE's Master Exchange Program from a Matching Theory Perspective

Francesco Caslini (41035)

Abstract
In this thesis, we analyze the master exchange program of the Stockholm School of Economics from a matching theory perspective. We develop a theoretical framework, the *student exchange problem*, within which we define the desirable properties we are interested in. We present and analyze the system currently in use, whose main flaw is to force students to truncate their preference list. We propose some easytoimplement modifications which would not alter significantly the way the program is managed. Finally, we make a more articulate proposal.

Keywords: Matching theory, Student placement, Stability, Strategyproofness, Truncation

JEL classification: D70, D63, D47, D61

Supervisor: Albin Erlanson
Date submitted: 13052018
Date examined: 29052018
Discussant: Sara Davidsson
Examiner: Maria Perrotta Berlin

Contents

1 Introduction **2**

2 Literature review **4**

3 The model **6**

3.1 Environment 6

3.2 Matchings, mechanisms and their properties 7

4 The system in use at the Stockholm School of Economics **11**

4.1 Priority formation procedure 11

4.1.1 An example 13

4.2 Allocation of seats 14

4.3 Properties of the mechanism 15

4.4 Discussion on truncation 16

5 Alternative mechanisms **19**

5.1 Longer list 19

5.2 Reorganizing priority factors 20

5.3 Different rankings with composed valuation of motivation letter . 22

5.3.1 New priority formation mechanism 22

5.3.2 An example 25

5.3.3 Proposed matching mechanism 26

5.3.4 Analysis 26

6 Conclusion **30**

References **31**

1 Introduction

Each year, the Stockholm School of Economics has a fixed number of exchange seats to be allocated among its Master students. Students willing to participate in the program are required to apply, submitting a motivation letter and a list of preferences. The school elaborates a priority ranking of students and then, based on this ranking and the students' preferences, some students are offered a seat and some are not. This apparently simple procedure, carried on by most universities in the world, is in reality a delicate optimization problem, with several stakeholders involved. Students constitute the first category. They are ultimately going to accept or reject the seats proposed to them and, in case they accept, they are going to commit a semester of their student life to their destination university. Second, SSE, which faces a tradeoff over two goals: to satisfy its students as much as possible, and to select the best possible students to represent it. Third, partner universities, which might have specific requirements.

In this thesis, we will try to answer the following research question:

Is is possible to find implementable improvements on the current system used by SSE to allocate exchange seats?

We will search potential areas of improvement by analyzing the system in use. When making our suggestions, we will try to be as little invasive as possible. As specified in the research question, implementability will be key for us. This will bear consequences, posing several constraints. First, we will not change the format of the applications required to the students. Second, we will make sure that the school's administrative staff will not be required additional work. To this point, we will also write a software to compute the allocations, effectively relieving the staff from the burden of computation. Overall, we believe that it is possible to improve the system for the school, making sure that it reaches its goals effectively, and for the students, allowing them to report their true preferences more freely and obtain better allocations.

The theoretical part of this thesis makes use of concepts from matching theory, which belongs to the field of mechanism and market design. Mechanism and market design is a field of economics that takes an engineering approach to the design of mechanisms and incentives in strategic settings where the players act rationally. This approach consists of formulating a theoretical problem, which we will call the *student exchange problem*, to model the analyzed situation. Within this framework, we will define some desirable theoretical properties, and then check if the current system possesses them. Later, we will propose two modifications that would help the current mechanism satisfy such properties. Finally, we will present a more ambitious and articulated proposal.

In practice, matching theory investigates how to match agents from two sides of a nonmonetary market, based on each agent's preference rankings over the agents on the other side of the market. Here is a practical example, in which students represent one side of the market and schools represent the other. There are two students (i_1, i_2) and two destinations (s_1, s_2), each with one available seat. Simply put, a matching assigns students to schools in an operable way: a student is assigned to maximum one seat, no school is assigned more than its available seats and students can be assigned to no school. For example, a possible matching is $((i_1\ s_2),\ (i_2\ s_1))$. In the situation considered, there are seven valid matchings possible. However, two matchings are not necessarily qualitatively equivalent. For instance, intuitively, a matching in which one or more seats go unassigned is a waste. Or again, suppose that both students prefer s_1 to s_2, and that the schools have a shared priority ranking, in which i_1 has priority over i_2. Then, a matching which assigns i_1 to s_1 will be fair given the priority ranking, and consequently more desirable than a matching that assigns i_2 to s_1. In this example, we have used the words *waste* and *fair* according to their conventional meaning, in order to convey the intuition. Later, *(non)wastefulness* and *fairness* will be defined formally.

Since it is not possible to model all aspects of the problem theoretically, our mechanism design approach will be complemented by practical considerations. For what concerns the rankingformation procedure, for example, we will analyze data from the past year to show how effective tiebreaking criteria are, and how well they serve the school's purposes.

The remainder of this thesis is organized as follows. In the next section, we will review the literature concerning similar applications of matching theory. In Section 3, we will introduce our model. In Section 4, we will analyze the system currently in use. In Section 5, we will make our proposals. In Section 6, we will conclude.

2 Literature review

This work, finalized to a practical implementation, builds on wellestablished theoretical foundations in a delimited field. For this reason, we drew deeply from the three categories of matching problems defined in the following paragraphs. Due to the specificity of the field, all our references come from it.

The *student exchange problem* defined and analyzed in this thesis is closely related to three other categories of matching problems present in the literature. The first one is the *college admission problem*, defined by David Gale and Lloyd Shapley (Gale & Shapley, 1962). In it, each college is an agent, with a strict preference list over students. This problem has received a lot of attention in the literature. For example, Roth & Sotomayor, 1989, Gale & Sotomayor, 1985 and Dubins & Freedman, 1981 expand Gale and Shapley's results theoretically. Moreover, Gale and Shapley's findings have been successfully applied to a number of entry level job markets, such as that for medical interns in the United States. Roth's work (1984, 1991) is central in such research. Another significant application is that to kidney exchange (Roth, Sönmez, & Ünver, 2004; Roth, Sönmez, & Ünver, 2005)

The second related framework is the *school choice problem* (Abdulkadiroğlu & Sönmez, 2003), which differs from the college admission problem in that it models the schools' seats as mere objects to be consumed by students. The authors applied their framework to the public school districts in New York City (Abdulkadiroğlu, Pathak, & Roth, 2005) and in Boston (Abdulkadiroğlu, Pathak, Roth, & Sönmez, 2005).

The third category is that of *student placement problems*, introduced by Balinski & Sönmez (1999). This model is similar to that of the school choice problem, but internalizes the formation of the schools' priority rankings (Gale & Shapley, 1962). Our analysis does take this step into consideration, but separately from the matching model.

The analysis of SSE's current matching mechanism draws on the literature on the specific mechanism, sequential priority (also called serial dictatorship), and on the topic of truncation. Such mechanism has been most notably analyzed with regards to housing allocation problems (Abdulkadiroğlu & Sönmez, 1998).

In Balinski & Sönmez (1999) a variant of the mechanism, multicategorical serial dictatorship, is analyzed in the student placement problem.

For what concerns truncation, the main contribution comes from Roth & Rothblum (1999), in which the authors prove that in a lowinformation environment, it is never profitable to state preferences that reverse the true priority ordering. The subject of constrained school choice has also been openly analyzed by Haeringer & Klijn (2009), with an equilibrium approach.

Finally, our last proposal aims to enable students to reveal their preferences openly and, separately, allows them to signal their preference intensities, via a motivation letter. A similar mechanism was proposed in Abdulkadiroğlu, Che, & Yasuda (2015). In this paper's proposed mechanism, students simply named one school; such signal will give them an advantage in the final tiebreaking process, so to avoid randomness at that step. Conversely, in our solution the signal comes into play before the final tiebreaking. In fact, we do not even consider it part of the matching mechanism, but rather as part of the ranking formation procedure.

Along with this procedure, we propose the adoption of the deferred acceptance algorithm, proposed in (Gale & Shapley, 1962). When selecting the mechanism to propose, one faces a tradeoff between stability, efficiency and strategyproofness. This tradeoff has been researched deeply, as in Abdulkadiroğlu, Pathak, & Roth (2009), or in the discussion in Balinski & Sönmez (1999).

3 The model

In this section, we will introduce the theoretical framework that will be applied to answer our research question: the *student exchange model*. First, we will present the environment: students, exchange destination, their attributes and preference relations. Then, we will define formally what we mean by *matching* and *mechanism* in this context. Finally, we will define some properties of matchings and mechanisms.

3.1 Environment

A *student exchange problem* consists of:

1. A set of students $I = \{i_1, i_2, ..., i_n\}$.

2. A set of exchange partner universities $S = \{s_1, s_2, ..., s_m\}$. We will indicate the outside option being assigned no destination as s_0. For convenience, we also define $\hat{S} = S \cup \{s_0\}$, the union of S and the outside option.

3. A capacity vector $q = (q_{s_0}, q_{s_1}, q_{s_2}, ..., q_{s_m})$. Each q_s is a natural number indicating how many students can be accepted by destination s, how many available seats s has: $q_s \in \mathbb{N}$. For the outside option, $q_{s_0} = |I|$ because virtually every student in I could be assigned to it.

4. A list of strict (not allowing ties) student rankings over exchange destinations $P = (P_{i_1}, P_{i_2}, ..., P_{i_n})$. Each P_i is a complete, transitive and acyclical binary relation over $\hat{S} \times \hat{S}$.

5. A list of strict (not allowing ties) priority rankings over students, one for each partner university: $\succ = (\succ_{s_1}, \succ_{s_2}, ..., \succ_{s_m})$. Each $\succ_i$ is a complete, transitive and acyclical binary relation over $I \times I$.

Both n and m are natural numbers and there is no specific relation among them. We will use R_i for students and $\succeq_s$ for schools to indicate a weak preference (least as good). We will use P_i for students and $\succ_s$ for schools to indicate a strict preference. To indicate the set of rankings by all students except student i (i.e. $P \setminus P_i$), we will use the notation P_{-i}. Here follows an example to introduce the usage of such notation.

Example 1. Student i has a weak preference for s_1 over s_2: $s_1 R_i s_2$.
Student j has a strict preference for s_1 over s_2: $s_1 P_j s_2$.
School s_1 has a weak preference for student i over student j: $i \succeq_{s_1} j$.
School s_2 has a strict preference for student j over student i: $j \succ_{s_2} i$.

A tuple of $(I, S, q, P, \succ)$ is a student exchange problem. In our analysis, I, S, q and $\succ$ are fixed; thus, each problem corresponds to a profile of preferences P.

3.2 Matchings, mechanisms and their properties

The concept of matching is central for our analysis. In essence, a matching is an allocation of resources, such that no more resources are allocated than there are available, and that no agent is allocated more resources than she can actually consume. In most applied cases, including the one that we analyze, a given procedure is used to create a matching: such procedure is a mechanism.

Formally, a matching μ is a function that maps I to $\hat{S}$ (which includes s_0), so that each student is assigned at most one partner university and each school is assigned a number of students less than or equal to its capacity. In mathematical notation: $\mu : I \to \hat{S}$ such that $|\mu^{-1}| \leq q_s, \forall s \in \hat{S}$.

A mechanism φ is a function that *selects a matching* for each student exchange problem. Therefore, it maps the set of all possible student exchange problems to the set of all possible matchings. The space of all possible student exchange problems equals the set of all possible preference profiles, which we will indicate with $\mathscr{P}$. We will indicate the set of all possible μ with $\mathscr{M}$. In notation, $\varphi : \mathscr{P} \to \mathscr{M}$: a mechanism is a mapping from $\mathscr{P}$ to $\mathscr{M}$.

To complete the model, we define five fundamental properties of matchings and mechanisms.

1. A matching μ is **individually rational** if $\mu(i) P_i s_0$, for all i in I.

The condition for individual rationality is that no student is matched to a destination to which she prefers being unmatched. In other words, that at an individual level it is rational to accept the matching.

2. A matching μ is **nonwasteful** if, for all $s \in S$ such that $sP_i\mu(i)$ for some i, it holds that $|\mu^{-1}(s)| = q_s$.

Nonwastefulness requires that no seat preferred by at least one student to her current matching is unassigned. If a student prefers a destination s to the one she is matched with, s needs to have no free seats. We define **wasteful** a matching that does not satisfy nonwastefulness.

3. A matching μ is **fair** if there is no pair $(i, s) \in I \times S$ such that: $sP_i\mu(i)$ and $i \succ_s j$ for some $j \in \mu^{-1}(s)$.

Fairness requires that if two or more students compete for one seat, that seat is assigned to the student with the higher priority. If a student i prefers a school s to her current matching, it must be that all students assigned to s have higher priority than i. If there is a student j assigned to s despite having lower priority than i, we say there is **justified envy** (of student i towards student j).

Properties 1 to 3 allow us to define stability.

4. A matching μ is **stable** if and only if it is individually rational, nonwasteful and fair.

Stability of a matching is a fundamental concept in the literature, because it reflects the likelihood for the matching to be successful in practice. It has been proved (Roth & Rothblum, 1999; Alcalde, 1996) that mechanisms implementing stable matchings succeed much more often than those that do not. RomeroMedina (1998) presents a similar finding applied to college admission.

In the marriage model (Gale and Shapley, 1962), stability ensures that no coalition of agents with the power of altering the matching has an incentive to do so. In our context, the meaning is similar, although less direct. In our context, SSE overlooks the process, as destinations are not active agents (unlike individuals in the marriage model, or colleges in college admission models). In other words, one side of the market is controlled by a social planner, and it is this social planner alone who has the possibility to alter the matching (with the only exception of matchings that do not satisfy individual rationality, in which the student simply drops out). Still, the social planner has an incentive to propose stable matchings, which avoid wastes and eliminate justified envy.

The last property we define is Pareto efficiency:

5. A matching μ Pareto dominates another matching μ' if: $\mu R_i \mu'$ for all $i \in I$ and $\mu P_j \mu'$ for some $j \in I$. A matching μ is **Pareto efficient** if it is not Pareto dominated by any other matching.

In a Student Exchange Problem, Pareto efficiency implies individual rationality and non wastefulness. However, the converse is not true.

Lemma 1. *Any Pareto efficient matching is individually rational and non-wasteful.*

Proof. Consider a matching μ that is not individually rational because a student i does not find $\mu(i)$ acceptable. Let us call μ' the allocation in which i is unmatched and all other students in I have the same matching as in μ. Trivially, those students will have the same utility in the two matchings. Student i, however, will be better off in μ'; hence, μ' dominates μ, that cannot be Pareto efficient. Similarly, consider a matching μ which is wasteful because student i prefers the unassigned seat s to $\mu(i)$. Let us call μ' the allocation in which i is assigned s and all other agents keep their matchings: μ' dominates μ. Those examples show how a violation of individual rationality or non wastefulness entails a violation of Pareto efficiency. This happens because if nonwastefulness is violated, a student can be made better off by assigning her a previously unassigned seat. All other students keep their matchings, so they will not be worse off: the wasteful matching is Pareto dominated by its nonwasteful version and therefore it is not efficient. □

Our definition of Pareto efficiency considers only the students' utility, an implementation which is generally associated with the other side of the market (in our case, exchange seats) being objects to consume. This raises a question: why worrying about something else than efficiency? In fact, since efficiency implies individual rationality and non wastefulness, this question corresponds to: why worrying about fairness? Should fairness be discarded, in case it contrasts with efficiency? Our answer is that fairness has indeed a place in the model, for two reasons. The first reason is that, as argued by Abdulkadiroğlu and Sönmez (2003), fairness eliminates justified envy. Justified envy is not captured by efficiency because it is related to students whose assigned destination does not change when switching from a matching μ to the matching μ' which Pareto dominates it, but it does affect students' *perception* of their allocated seats. Therefore, it is desirable to eliminate it, although it does not affect utility as we define it. The second reason is that SSE has an interest in choosing the best possible representation for the school, and the ranking reflects this. So, an unfair assignment is a loss from this point of view.

Individual rationality, non wastefulness and fairness are also properties of mechanisms; so is stability.

6. A mechanism φ is **individually rational** if, for any $P \in \mathscr{P}$, it selects an individually rational allocation $\mu \in \mathscr{M}$.

7. A mechanism φ is **nonwasteful** if, for any $P \in \mathscr{P}$, it selects a non wasteful allocation $\mu \in \mathscr{M}$.

8. A mechanism φ is **fair** if, for any $P \in \mathscr{P}$, it selects a fair allocation $\mu \in \mathscr{M}$.

9. A mechanism φ is **stable** if, for any $P \in \mathscr{P}$, it selects a stable matching $\mu \in \mathscr{M}$.

Similarly, we define efficient a mechanism that always selects a Pareto efficient matching.

10. A mechanism φ is **efficient** if, for any $P \in \mathscr{P}$, it selects a Pareto efficient allocation $\mu \in \mathscr{M}$.

Finally, a mechanism is strategyproof if, under any circumstances, no student can ever make herself better off by misreporting her true preferences. We will refer to such behaviour as *strategic behavior* or as *strategizing*. Formally:

11. Consider a matching mechanism φ. Let $\varphi_i(P)$ be i's matching in the allocation induced by preference profile P. φ is **strategyproof** if for all $P_i \in \mathscr{P}_i$, for all $P_{-i} \in \mathscr{P}_{-i}$ and for all $i \in I$, it holds that $\varphi_i(P_i, P_{-i}) R_i \varphi_i(P'_i, P_{-i})$.

Strategyproofness is a desirable property for three reasons. The first one is that its absence introduces an element of randomness, since players are forced to strategize based on incomplete information. Secondly, it could bring about potential unfairness, because the ability to play strategically becomes a factor. This has been empirically observed in several cases. One of them is the well-known case of the Boston school system, in which organized groups of parents suggested strategies, giving an advantage to their members (Pathak & Sönmez, 2008). The third reason is that SSE might be interested in knowing its students' true preferences, in order to collect reliable data and possibly expand or modify the program in the future.

4 The system in use at the Stockholm School of Economics

This section will be dedicated to describing and analyzing the system used by the Stockholm School of Economics to allocate seats for the student exchange program. Such system does not correspond to a mechanism as we defined it before, because it does not just generate an allocation. Rather, it has a first part whose purpose is to generate a priority ranking of students. We will call this part *priority formation procedure*. The priority formation procedure takes some attributes of students and creates a priority ranking $\succ$ of students according to them. The second part is the proper mechanism: φ^{SSE}. Such mechanism selects an allocation for each student exchange problem and $\succ$, product of the ranking formation procedure, is one of its inputs. The other inputs to φ^{SSE} are the students' shortlists of 5 preferences and the schools' capacities. The two parts of the system will be described in order: then we will move on to the analysis. Finally, we will compare φ^{SSE} to a similar mechanism defined in the literature (Abdulkadiroğlu & Sönmez, 1998; Svensson, 1999), the sequential priority mechanism [1] ($\varphi^{\succ}$).

4.1 Priority formation procedure

Students apply by submitting a motivation letter, along with their preference list which is not considered at this step. Additionally, the school observes the students' study pace and grades. First, motivation letters are graded on a scale from 1 to 5. The priority ranking is then formed by taking into account study pace (as number of courses passed, 0 to 4), motivation letter's score (1 to 5) and grades (which can give 0 to 3 bonus points). We will refer to these as *raw points*.

Let us clarify straight away a possible source of confusion: the students' study pace score and motivation letter score are used *twice*. The first time, they are used in steps 3 and 4 respectively. Then, they are used a second time in step 5, jointly and along with bonus points given by grades. Here is a formal description of the algorithm used to generate the priority ordering of students $\succ$:

[1] Also called *serial dictatorship* mechanism

Step 0: Each student submits an application: a motivation letter and an ordered list of 5 destinations (his preferences). His study pace (number of courses passed) and GRE/GMAT/GPA are observable by the school.

Step 1: Each student's motivation letter is graded, from 1 to 5.

Step 2: Each student i's score, ξ_i, is initialized to a value of 0.

Step 3: For each student i, 50 points are added to ξ_i if she passed 3 or 4 courses, 0 if she passed 2. If i passed less than 2 courses, she is not eligible for exchange.

Step 4: For each student i, 10 to 40 points are added to ξ_i, according to the student's motivation letter score, as follows: - 40 points if the motivation letter was awarded 4 to 5 points. - 30 points if the motivation letter was awarded 3 points - 20 points if the motivation letter was awarded 2 points - 10 points if the motivation letter was awarded 1 point.

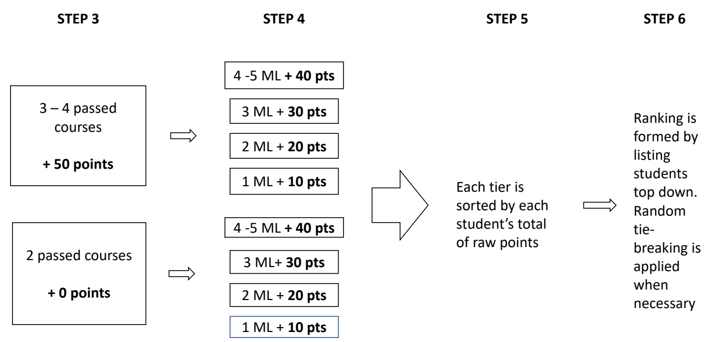
Step 5: For each student, the total of raw points is added to the previous subtotal ξ_i. These raw points are: number of courses passed, motivation letter score, and a bonus in case their bachelor's GPA (for students who did their bachelor at SSE) or GMAT/GRE score (for students from other universities) qualifies them as top 25% students. This bonus ranges from 0 to 3 raw points. In total, 3 to 12 points are added.

Step 6: A unique priority ranking $\succ_s$ (valid for all $s \in S$) is constructed by ranking students according to their total score ξ, in descending order. If two students have the same total score, the tie is broken randomly. Formally, given any two students i and j ranked in $\succ_s$, it holds that $i \succ_s j$ if and only if $\xi_i \geq \xi_j$.

This procedure produces one priority ranking $\succ$, which will be valid for all schools $s \in S$. We call it *unique* to emphasize this, which is not required by our model. In fact, as defined in Section 3, the model accounts for a priority ranking $\succ_s$ for each destination s.

Let us note that at steps 3, 4, 5 and 6 a sequential tiebreaking operation is done.

At step 3, students are divided in two main tiers. No student from the bottom tier could ever end up ranked above any student from the top tier, for simple numerical reasons. Then, step 4 divides in four tiers the two main tiers of step 3. Step 5 ranks the students within each tier based on their total score. [2] Finally, step 6 randomly breaks the remaining ties. Here is a visual representation of the procedure.



4.1.1 An example

Consider students i_1, i_2, i_3, i_4, i_5, who have the following raw points.

Student	**Study pace** Courses passed	**Motivation Letter** score 05	**GPA/GMAT/GRE** 03 bonus points
i_1	3	5	1
i_2	4	3	3
i_3	2	5	1
i_4	2	3	0
i_5	3	4	0

Step 3: Students are split into those who have passed 3 or more courses, who

[2]It is very important to note that step 5 can never subvert the ordering established by the previous steps. After step 4, we are left with eight tiers; any two of them are separated by at least 10 points. Now, as the maximum number of points addable in step 5 is 12, one could think that it is indeed possible for a student to end up above another who is in a superior tier. But one should remember that there is also a minimum number of addable points, that is 3. So, the maximum possible difference, 9, is not big enough to subvert the tier hierarchy.

will end up forming the upper part of the ranking, and those who have passed 2. i_1, i_2 and i_5, having passed 3 or more courses, are assigned 50 points. i_3 and i_4's scores remain 0.
Step 4: Now, in each of those groups, students are ranked according to their total score. i_1, i_5 and i_3 whose motivation letter was graded 4 or higher are assigned 40 points. i_2 and i_4 are assigned 30 points. Note that the maximum amount of points receivable at this step is 40, so that it could never overturn the partial ordering created in the previous one.
Step 5: For each student, the sum of scores is added to the points received in the first two steps: 9 for i_1, 10 for i_2, 8 for i_3, 5 for i_4 and 7 for i_5.

The resulting ranking (total score in parentheses) will be: $i_1(99)$, $i_5(97)$, $i_2(90)$, $i_3(48)$, $i_4(35)$. Formally, $\succ = (i_1, i_5, i_2, i_3, i_4)$.

4.2 Allocation of seats

This second part has the first as starting point. In the first part, students were asked to submit a shortlist of 5 schools. We define *quota* this maximum number of rankable destinations. In this analysis, we will consider the generic case with a quota k, as the exact value of k makes no difference. We will call the shortlist of schools A_i. An untruthful report will be indicated by A_i'. By truthful we mean that A_i reports elements 1 to k of P_i, in the same order.

The algorithm's functioning is simple: each student starting from the topranked one in $\succ$ is simply assigned her most preferred destination among those ranked in her shortlist. Formally:

Step 0 The unique priority ranking $\succ_s$ is considered, along with each student i's shortlist A_i.

Step 1 The topranked student i_1 in the ordering $\succ_s$ is assigned her preferred available seat, according to A_{i_1}.

Step h The student i ranked hth in the ordering $\succ_s$ is assigned his preferred available seat, according to A_i. If no seat in A_i is available, the student is assigned no seat (that is s_0).

Let us note that this mechanism, just as a sequential priority mechanism without a quota, requires that there is one unique priority ranking for all schools. In other words, schools are not allowed to have different preferences.

4.3 Properties of the mechanism

We will now analyze φ^{SSE} using the framework presented in Section 2. Of the five properties presented, the mechanism satisfies one, individual rationality. The proof is selfexplanatory.

Proposition 1. *The mechanism φ^{SSE} is individually rational.*

Proof. A student can only be assigned to a seat in a university he ranked in A_i. If a student prefers s_0 to a given destination, he never includes that destination in his list, and hence he is never assigned to it. □

On the other hand, φ^{SSE} lacks all other desirable properties. Here follows a brief explanation of the intuition. Everything stems from the fact that a student, having to truncate her preference list, might end up excluding her first achievable matching. If that seat goes unassigned, there is a waste, and a loss in efficiency. If it does not go unassigned, but goes to a student with lower priority, fairness is violated. In any of these cases, our student would have benefited from including this destination, which is strategic behavior: strategyproofness is violated. Formal proofs can be found below.

Proposition 2. *The mechanism φ^{SSE} is wasteful.*

Proof. Consider a student i and a destination s, which is acceptable to the student, and namely her sixth favourite option. The student does not report s in A_i. Now consider matching μ, selected by φ^{SSE} in which i is assigned s_0, and not all the seats at s are assigned; this is possible since we posed no constraints on the other agents' preferences and on i's position in $\succ$. Non wastefulness is violated because $sP_i\mu(i)$ and $|\mu^{-1}(s)| < q_s$. □

Proposition 3. *The mechanism φ^{SSE} is not efficient*

Proof. Given that φ^{SSE} is wasteful, this follows from Lemma 1 in Section 2. □

Proposition 4. *The mechanism φ^{SSE} is not fair*

Proof. Consider i, s and μ from the examples above. Now take an agent j that is ranked below i in $\succ$, but ranks s as first choice, and thus reports it in A_j. Following the same assumptions as above, j will be assigned s, whereas i will be assigned s_0. This represents a violation of fairness, because it holds that $i \succ_s j$, $s = \mu(j)$, and $sP_i\mu(i)$. $\square$

Proposition 5. *The mechanism φ^{SSE} is not strategyproof*

Proof. Let us take, once again, student i and school s from the examples above. If i had reported s in his shortlist, it would have been assigned s and would be better off. In formal terms, there is a A'_i s.t. $\varphi^{SSE}(A'_i, P_{-i})P_i\varphi^{SSE}(A_i, P_{-i})$. $\square$

4.4 Discussion on truncation

So far, we have analyzed the sequential priority mechanism with fixed quotas, pointing out its limits. However, it is worth noting that such limits are not inherent to the mechanism itself, but are a consequence of imposing a quota on the preferences that it is possible for students to list. This imposes a trade-off between the attractiveness of a destination and the expectation that it is achievable. If a student fails to strategize effectively, there is a potential waste, which implies a loss in efficiency. A pure sequential priority mechanism would indeed be stable, strategyproof, and efficient. Here are all the properties of a pure sequential priority mechanism without quotas. We will refer to it as $\varphi^{\succ}$.

Proposition 6. *The mechanism $\varphi^{\succ}$ is individually rational.*

Proof. Similarly to what happens in φ^{SSE}, in $\varphi^{\succ}$ a student can only be assigned to a seat in a university he ranked in A_i. If a student prefers s_0 to a given destination, he never includes that destination in his list, and hence he is never assigned to it. $\square$

Proposition 7. *The mechanism $\varphi^{\succ}$ is nonwasteful.*

Proof. Let us suppose that $\varphi^{\succ}$ selects an allocation for which nonwastefulness is violated: some student i is not matched to any destination, although school s, which she finds acceptable, has a free seat. In $\varphi^{\succ}$, student i, who ranks k-th in $\succ$, is assigned his topranked available destination at step k. So, if i is not assigned s, given that s has a free seat, it must be that i has not ranked s. This contradicts the fact that she finds it acceptable. $\square$

Proposition 8. *The mechanism $\varphi^{\succ}$ is efficient.*

Proof. Let us suppose that $\varphi^{\succ}$ selects an allocation μ that is not Paretoefficient: μ is Paretodominated by μ'. This means that some student i is better off under allocation μ', while all other students are at least as well off. However, if student i is better off under μ', it means that she is assigned a destination, which we will call s, that was not available when she was assigned $\mu(i)$ by $\varphi^{\succ}$. If s was not available, it is because it was the best available one for some student j ranked above i. Student j can either be worse off, which would be a contradiction, or be better off by being assigned a destination that she prefers to s and that was unavailable because assigned to a third student: in any case, when the cycle ends, a contradiction is reached. □

Proposition 9. *The mechanism $\varphi^{\succ}$ is fair.*

Proof. Suppose that fairness is violated in the allocation selected by $\varphi^{\succ}$. Namely, suppose that there are two students i and j and a school s such that i has higher priority than j, j is matched to s, i is matched to a destination to which he strictly prefers s. Since i has higher priority, $\varphi^{\succ}$ assigns her the highestranked available choice before j. If s is not available at the step when i was allocated her matching, it cannot be available later, when j is allocated hers: this is a contradiction. □

Proposition 10. *The mechanism $\varphi^{\succ}$ is strategyproof.*

Proof. Each student is assigned her matching at a step independent from what she reports. At that step, the student is assigned her topranked available option. A student could misreport her true preferences by omitting one or more destinations or by changing their ordering. Omitting a destination can never be profitable. If the student is matched with a betterranked destination, or if the destination would not be available anyway, it has no consequence. On the other hand, if the destination would have been the matching, omitting it leads to the student being assigned a less preferred destination. Changing the ordering of preferences can only be harmful too. If it affects the allocation, it needs to be because the student is assigned to a less preferred destination which has been put before a better one, with a loss in utility. □

As we have shown, imposing a quota deprives the sequential priority mechanism of four desirable properties. However, this mechanism is not alone. All the most common ones lose such properties under this condition: an example is the deferred acceptance algorithm (Gale & Shapley, 1962). Generally speaking, most mechanisms require to scan the whole list of preferences to guarantee fairness and nonwastefulness, and consequently efficiency. Also, to reduce a preference list implies making choices that can affect a player's final outcome, at the expenses of strategyproofness.

5 Alternative mechanisms

In this section, we will present and analyze some alternatives to the current system. The first two are tweaks that can be applied to the ranking formation procedure, also jointly, without changing the substance of the system. A more articulate proposal will follow, allowing each destination to have its own ranking over students. This aims to enable SSE to account for the fact that a student might be a better fit for a destination than for another.

5.1 Longer list

We have seen before (see Section 3.4, *Discussion on truncation*) that the sequential priority mechanism has several desirable properties which go lost when students are required to submit a truncated list of preferences: strategyproofness, fairness, nonwastefulness and efficiency. Truncating preferences creates the need for strategizing, balancing a tradeoff between attractiveness of a destination and the expectation that it is achievable. Also, it makes the resulting matching potentially wasteful and inefficient. The intuition for this is that a student i, when reporting her preferences, might miscalculate and exclude his first achievable destination s, listing five unachievable ones. If this happens and at least one seat at s remains unassigned, there is a violation of nonwastefulness and (consequently) of efficiency. If the seat is assigned to a student with lower priority than i, there is a violation of fairness.

This problem exists as long as students are not allowed to submit their whole list of preferences. However, it is evident that the lower k is, the more students will have to strategize and inefficient or unfair allocations will happen. Our first suggestion is therefore to raise the quota k, without necessarily eliminating it.

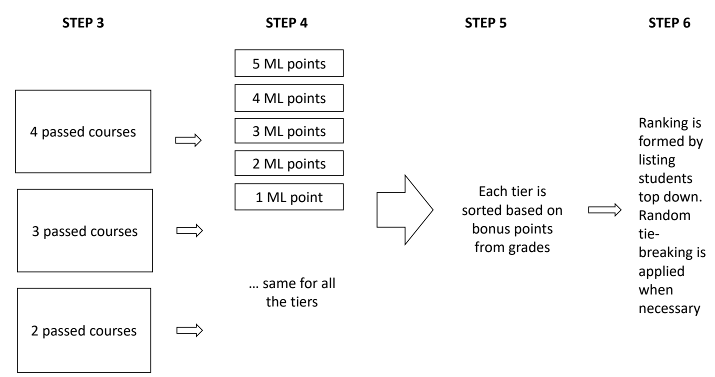
A potential implementation issue for this proposal is that having a longer list of preferences requires more time and effort to the program administration. With more options listed, the computation and check of matchings will take more time. To address this issue, we have built a software to do the computation. The software takes as input the students' information, in an Excel file, and outputs the allocation.

5.2 Reorganizing priority factors

The priority ranking used to assign seats is computed on the basis of three factors: study pace, motivation letter and grades from tests such as GMAT or GRE or, for former SSE students, their BSc's GPA. These three factors have decreasing weight, but are not used straightforwardly. The study pace is used as primary factor, distinguishing between students who passed 3 or 4 courses and students who passed 2, with all students in the first group (3 or 4 courses passed) having priority over all students in the second group (2 courses passed). In the final step, the number of courses passed will contribute to the sum of scores, so that there will be a distinction between students who passed 3 courses and students who passed 4, but such distinction will be marginal. A similar thing happens with the motivation letter. Students with 4 or 5 motivation letter points are assigned to the same tier. Only later, the number of points will contribute to the sum of scores, so that students with 5 points will get one more than students with 4 a rather marginal distinction. This system manages to advantage students with good study pace and motivation letter, without being too strict on any of them. However, it also introduces significant complexity and does not get much tiebreaking power in return.

Looking at data from year 2017 (exchange program of academic year 2017/2018), we find support for this argument. Firstly, out of 165 students, only 5 passed 2 courses, questioning the necessity of the first clustering. The motivation letter clustering seems more effective: among the top tier (students with 3 or 4 courses), 58 people were given five or four points for their motivation letter, 75 people were given three and 25 were given two. So, students are quite balanced among those clusters, but these are too big to hope for the sum of scores to break ties effectively. For example, the 58 people in the first tier can have sum of raw points between 7 and 11. Since there are five potential values and more than 50 students, the best it can be hoped for is tiers with no less than ten students. In reality, twentyfive of them have a sum of raw points of 8. In the second tier, 33 out of 75 people have a sum of raw points of 7 (the possible range for that group is from 6 to 10). This seems to be a consequence of the role of grades: they only allow students in the top percentiles to gain bonus points. As a matter of fact, when they come to play a role there are less ties. Therefore, extending the bonus points system to the lower end of the grade distribution might solve the problem.

Using the same data, we tried a stricter tiebreaking criteria, which directly used the study pace as primary tiebreaking factor and motivation letter score as secondary one, with no grouping. Instead of two primary tiers, we have three of them, each of which is divided into five subtiers, instead of four. Finally, since all the information on study pace and motivation letter score is accounted for in the first and second step respectively, only the grades are used in the third step, instead of the sum of scores. A visual representation can be found below.



The results are encouraging, and give clear indications. First, the maximum number of students with a tied score is reduced to 21. Second, in the two tiers with the most students (21 and 17), they did not receive bonus points for grades. Namely, the two tiers had students with four passed courses, three motivation letter points and zero bonus points (430), and four passed courses, four motivation letter points and zero bonus points (440).

5.3 Different rankings with composed valuation of motivation letter

In this paragraph we will describe a more articulate proposal. This proposal targets a specific limit of the current rankingformation procedure: that the motivation letter contributes to the score for all the destinations. This contradicts the fact that such letters are used more effectively to state one's motivation to go in exchange at one or more specific destinations, rather than a generic motivation to join the exchange program. Most students apply to the program to experience a new culture, a new city and a new university. Such desire is better communicated by discussing one or two destinations, and arguing why they would be good matches. As a matter of fact, this is what most students do. Besides, it seems reasonable that a student could be considered a better match for a destination rather than for another. Possible reasons are language, background and interest of the student.

With the current system, there is no way for the school to promote a student-destination matching; it can only be done as a side product of rewarding the student's motivation letter. For example, it is virtually impossible to guarantee that a student is matched with a destination believed to be her perfect fit if this student ranks it as her second preferred option. The proposed solution manages to create different rankings for different destinations, requiring virtually no extra effort by the program administration. It consists of a modification in the ranking formation procedure, which needs to be followed by one in the matching mechanism. A new mechanism will be necessary because the *serial priority mechanism* requires one unique priority ranking.

5.3.1 New priority formation mechanism

Our proposal is to expand the grading system of the motivation letter. Instead of giving one unique score, from 1 to 5, in our proposal the letter is assigned one base score plus some bonus points for specific destinations. For example, a student with a generally good motivation letter and a wellmotivated preference for Sciences Po in Paris could be assigned 3 base points plus 2 bonus points for that destination.

After grading the letter, a priority ranking will be computed for each exchange destination in S, using a ranking formation procedure similar to that already in use. The only difference is that their motivation letter points will possibly vary from destination to destination. Namely, their motivation letter score will be the same as their base score for all destinations but the ones for which they received bonus points, in which case the score will amount to the sum of base score and bonus points. So, if student i has a base score of 3, plus 2 bonus points for destination s, when computing priority rankings for s his motivation letter score will be 5, while for all other destinations it will be 3.

In order to get the best from this system, as we will see later, the letter's purpose should be twofold. First, it should be to express motivation to participate in the program. Second, it should be to signal one or two destinations that the student believes to be a good fit for. A crucial point is that students should not be supposed to mention their top (or top two) choices in the letter; it is legitimate to mention any of the ranked destinations. For this reason, we will not consider doing this as misreporting or strategic behavior.

Moreover, we will adopt the reorganization of priority factors proposed above. Here is a formal definition of the rankingformation process.

Step 0: Each student submits an application: a motivation letter and an ordered list of all the destinations that she finds acceptable. Her study pace and GRE/GMAT/GPA are observable by the school.

Step 1: Each student's motivation letter is graded as follows. First, it is assigned a base score from 0 to 3; then, it is assigned 1 or 2 additional points for zero or more destinations for which she expressed a particularly motivated interest.

Step 2: For each student i in I, a score vector ξ_i is initialized, with m elements. Each element ξ_{i_s} represents the score of student i for school s. The initial value for all ξ_{i_s} is zero.

Step 3 For each student i, the number of courses passed, multiplied by a factor of 100, is added to all ξ_{i_s}. Multiplying it by a factor of 100 gives it priority over the other tiebreaking factors.

Step 4: For each student i and destination s, i's motivation letter base score, plus possible bonus points for destination s, multiplied by a factor of 10, are added to ξ_{i_s}.

Step 5 For each student i, the GPA/GMAT/GRE bonus points are added to all ξ_{i_s}.

Step 6: For each school s, $\succ_s$ is formed by ranking all students according to their score ξ_{i_s}.

We will illustrate how the mechanism works via a simple example.

5.3.2 An example

Let us consider two destinations, s_1 and s_2, and three students: i_1, i_2 and i_3. Here are their raw points, collected in steps 0 and 1:

Student	**Study pace** Courses passed	**ML** s_1 Base + Bonus	**ML** s_2 Base + Bonus	**GPA/GMAT/GRE** 05 points
i_1	3	3 + 1	3 + 0	2
i_2	4	2 + 2	2 + 0	4
i_3	3	3 + 0	3 + 1	5

The mechanism, starting from step 2, would work as follows:

Step 2 All students start from a score of zero for both destinations. We will refer to the scores with the notation ξ_{i_s}; for example, i_1's score for s_1 will be $\xi_{i_{1_{s_1}}}$.

Step 3 To i_2's scores, 400 points are added. To i_1 and i_3's scores, 300 points are added.

Step 4 For each student i, the ML s_1 raw points, multiplied by 10, are added to $\xi_{i_s 1}$ and the ML s2 raw points, multiplied by 10, are added to $\xi_{i_s 2}$. For instance, let us take i_2: a score of 40 is added to $\xi_{i_{2_{s_1}}}$, while a score of 20 is added to $\xi_{i_2]_{s_2}}$. After this step,the scores are: $\xi_{i_{2_{s_1}}} = 440$ and $\xi_{i_{2_{s_2}}} = 420$.

Step 5 For each student, their bonus points are added to both scores. For i_2, $\xi_{i_2]_S 1} = 444$ and $\xi_{i_{2_S} 2} = 424$.

Step 6 $\succ_{S1}$ is formed considering $\xi_{i_S 1}$ for all students i; similarly $\succ_{S2}$ is formed considering $\xi_{i_S 2}$.

At the end of the process, the rankings will look like this (again, scores in parentheses):
$\succ_{S1} = (i_2\ (444)\ ,\ i_1\ (342)\ ,\ i_3\ (335))$
$\succ_{S2} = (i_2\ (424)\ ,\ i_3\ (345)\ ,\ i_1\ (332))$

5.3.3 Proposed matching mechanism

The need to introduce a different mechanism arises from the presence of different priority orderings, not compatible with a sequential priority mechanism. We will replace it with the StudentProposing Deferred Acceptance (SPDA) algorithm, in which students and school are tentatively matched repeatedly, following the students' preferences, until a matching is reached such that each student either is assigned to an acceptable destination or does not find acceptable any of the remaining ones. Here is a formal description.

Step 0 The set of priority rankings $\succ$ is considered, along with each student i's reported preference list A_i.

Step 1: Each student i is tentatively matched to her preferred destination in A_i. If a destination s has more students assigned than it can accept (q_s), it keeps the q_s which are the highest ranked in $\succ_s$ and rejects the others.

Step h: Each unmatched student i is tentatively matched to her next preferred destination in A_i, if there is any remaining. If there is no next most preferred destination, student i is assigned s_0. If a destination s has more students assigned than it can accept (q_s), it keeps the first q_s in $\succ_s$ and rejects the others, regardless of the step at which they were tentatively assigned to it. If there is no unmatched student with at least one next acceptable destination, the process terminates.

5.3.4 Analysis

This system creates a priority system that captures the students' inclinations better than the current one, and then computes a matching that privileges stability but also guarantees efficiency among stable matchings. Once again, our analysis will be split. First, we will present some practical advantages of the ranking formation procedure. Secondly, we will analyze the theoretical properties of our mechanism.

We have already explained the main advantages of switching to a composed valuation of the motivation letter. Namely, it will enable the administration to reward a student's motivation to go in exchange to a certain school without the side effect of boosting her ranking for others. Now, we would like to focus on another interesting feature of the proposed rankingformation procedure: that switching to it would require no commitment or renounces. The administration would have the option to promote some specific studentdestination matchings, but would not be obliged in any way to do so. It might decide, for example, to give bonus points exclusively in outstanding cases, with an only marginal change in its way of grading motivation letters. In an extremecase scenario, it could decide not to use bonus points at all. Although we certainly do not hope for this to happen, in such case the system would still work, and exactly as it does now, or with some chosen tweaks. Thus, choosing this mechanism would not require commitment to radically alter the way motivation letters are graded, or to putting in a given amount of extra time and effort.

The StudentProposing Deferred Acceptance (SPDA) algorithm with no quota is stable, strategyproof and efficient among stable matchings.

Proposition 11. *The SPDA algorithm is individually rational.*

Proof. In the SPDA, each student i is tentatively matched to the schools in her reported preference ranking A_i, starting from the topranked one and descending, if necessary. When the ranking is exhausted, the student is assigned s_0. It is therefore impossible that a student is assigned to a destination not listed in A_i, and thus by assumption impossible that she is assigned to an unacceptable destination. If no student can ever be assigned to a school she finds unacceptable, individual rationality holds. □

Proposition 12. *The SPDA algorithm is nonwasteful.*

Proof. For a matching μ to be wasteful, it is necessary that a student i is assigned s_0 while a school s that she finds acceptable has a free seat. Suppose this happens. Since students are assigned s_0 only after being tentatively matched to all their acceptable schools, it must be that i has been tentatively matched to s, prior to being assigned at s_0. So, she had either been rejected in the first place, or tentatively accepted and rejected later. The first case contradicts the fact that s has a free seat, as schools reject students only when they are at capacity. Similarly, a school rejects a student it has tentatively accepted only when there are enough better qualified students to fill all seats. In all cases, we come to a contradiction. □

Proposition 13. *The SPDA algorithm is fair.*

Proof. Suppose that fairness is violated: in a matching μ, a student i prefers school s to her matching $\mu(i)$, while there is a student j, to which is assigned s, who is lower than i in the priority ranking of s. If i prefers s to $\mu(i)$, it must have been tentatively matched to s before being assigned to $\mu(i)$. If she was rejected immediately, it is because there were already q_s betterranked students matched to s. But then, the same students (or others with ever better ranking) should have been matched to s when j was tentatively matched to s, which contradicts the fact that j was accepted. Similarly, if i was tentatively accepted and rejected at a later step, at that later step there were q_s students with a higher ranking than j, which contradicts the facts that j was assigned s. $\square$

Proposition 14. *The SPDA algorithm is strategyproof.*

Proof. Suppose that a student i profits from reporting an untruthful preference profile A'_i, obtaining a matching $\mu'(i)$ which she prefers to the matching $\mu(i)$ that would have been obtained by truthfully reporting A_i. The fact that i gets a better match via a misreport is selfcontradictory. If $\mu'(i)$ is achievable under a reported preference profile, and we assume it is under A'_i, it needs to be under all reported preference profiles. If it is preferred to $\mu(i)$, it needs to be ranked above it in the truthful one. Therefore, if i truthfully reports her preferences, it will be tentatively matched to $\mu'(i)$ before being matched to $\mu(i)$, and since $\mu'(i)$ is achievable, it should be her matching. $\square$

Proposition 15. *The SPDA algorithm is efficient in the set of stable matchings. However, it is not efficient outside of it.*

Proof. Suppose that a matching μ, produced by the algorithm, is Paretodominated by a matching μ'. This could happen for three reasons:

1. a student in μ' is assigned to s_0 instead of an unacceptable destination he was matched with in μ,
2. a student is μ' is assigned to a seat that was unassigned in μ,
3. two or more students switch the seats they were assigned in μ.

The first two cases cannot happen because they would violate respectively individual rationality and non wastefulness, and we have proved that the SPDA algorithm is individually rational and nonwasteful. It remains to be proved that if two students switch seats with a gain in efficiency, the resulting matching is necessarily unstable. Let us take two students i and j, and their matchings in μ,

$\mu(i)$ and $\mu(j)$. Since our model admits no indifferences, we need to assume strict preferences for both students (instead of only one strict preference and all other weak ones): $\mu(j)P_i\mu(i)$ and $\mu(i)P_j\mu(j)$. A matching μ' such that the students switch their seats (in notation: $\mu'(i) = \mu(j)$ and $\mu'(j) = \mu(i)$) Pareto dominates μ. Now, both students prefer the other student's assignment, so, in μ, they must have been tentatively matched to it before being definitively matched to their assignment. In other words, i must have been tentatively matched to $\mu(j)$ before being matched to $\mu(i)$; the same is true for j. These tentative matchings must have been blocked by some other student k, so that at least one between i and j was forced to move on, eventually getting respectively $\mu(i)$ or $\mu(j)$. If there was no such student k, i and j would have kept their preferred choices, without blocking each other's matching, so k must exist. But if k exists, μ' is unfair to k, thus unstable. Hence, a matching μ' can be found to Pareto dominate μ, proving that the SPDA algorithm is not efficient. However, μ' can never be stable: efficiency holds in the set of stable matchings. $\square$

As it emerges from the proofs above, we chose stability over efficiency. Efficiency is granted only if stability allows it; if it does not, then an efficient matching among the stable ones is chosen. The value of stability has been discussed in Section 3.2, when defining matchings' properties. The main points for choosing stability over efficiency are two. First, stability eliminates justified envy, possibly improving the students' perception of how the exchange program is managed. Second, a stabilityprivileging approach gives more weight to the destinations' rankings over students, eventually selecting students which are better fits for their matched destinations.

6 Conclusion

In this thesis, we analyzed the allocation of seats in the Stockholm School of Economics's exchange program for Master students. We did so by adopting a matching theory perspective. Our goal was to suggest some modifications, as simple as possible, to increase the students' satisfaction with the program and the school's effectiveness in administering it.

Drawing on the literature covering the *school choice problem*, the *student placement problem* and the *college admission problem*, we formulated a *student exchange problem*. This theoretical framework allowed us to assess the mechanism currently in use, which is individually rational but loses all other desirable properties because it imposes to the students the truncation of their preferences. Consequently, our first suggestion is to allow students to report more than five preferences, and possibly as many as they wish.

On a more practical note, we examined the efficiency of the tiebreaking criteria adopted by SSE. The school chooses to tiebreak based firstly on study pace and secondly on motivation letter score. However, it does so in a loose way, keeping students who passed 3 courses and students who passed 4 in the same tier, and similarly for students with 4 and 5 as motivation letter raw points. The tiebreaking power of this procedure proves insufficient, leaving a lot to randomness. A second suggestion is then to tiebreak with the same hierarchy between criteria, but more strictly.

In order to facilitate the implementation of these proposals, we built a software to compute the matchings. Our software takes an Excel file as input, along with information on column names, scrapes the students' information and outputs the allocation, given the chosen criteria.

Finally, we make a proposal that aims to enable the school to rank students differently for different destinations. This proposal allows the school administration to create different rankings for different destinations, giving to each student motivation letter raw points for different destinations. At the same time, it allows students to report their preferences without the burden of strategizing.

References

Abdulkadiroğlu, A., Che, Y.-K., & Yasuda, Y. (2015). Expanding "choice" in school choice. *American Economic Journal: Microeconomics*, *7*(1), 1–42.

Abdulkadiroğlu, A., Pathak, P. A., & Roth, A. E. (2005). The New York City high school match. *American Economic Review*, *95*(2), 364–367.

Abdulkadiroğlu, A., Pathak, P. A., & Roth, A. E. (2009). Strategy-proofness versus efficiency in matching with indifferences: Redesigning the NYC high school match. *American Economic Review*, *99*(5), 1954–78.

Abdulkadiroğlu, A., Pathak, P. A., Roth, A. E., & Sönmez, T. (2005). The Boston public school match. *American Economic Review*, *95*(2), 368–371.

Abdulkadiroğlu, A., & Sönmez, T. (1998). Random serial dictatorship and the core from random endowments in house allocation problems. *Econometrica*, *66*(3), 689–701.

Abdulkadiroğlu, A., & Sönmez, T. (2003). School choice: A mechanism design approach. *American economic review*, *93*(3), 729–747.

Alcalde, J. (1996). Implementation of stable solutions to marriage problems. *Journal of Economic Theory*, *69*(1), 240–254.

Balinski, M., & Sönmez, T. (1999). A tale of two mechanisms: student placement. *Journal of Economic theory*, *84*(1), 73–94.

Dubins, L. E., & Freedman, D. A. (1981). Machiavelli and the Gale-Shapley algorithm. *The American Mathematical Monthly*, *88*(7), 485–494.

Gale, D., & Shapley, L. S. (1962). College admissions and the stability of marriage. *The American Mathematical Monthly*, *69*(1), 9–15.

Gale, D., & Sotomayor, M. (1985). Some remarks on the stable matching problem. *Discrete Applied Mathematics*, *11*(3), 223–232.

Haeringer, G., & Klijn, F. (2009). Constrained school choice. *Journal of Economic theory*, *144*(5), 1921–1947.

Pathak, P. A., & Sönmez, T. (2008). Leveling the playing field: Sincere and sophisticated players in the Boston mechanism. *American Economic Review*, *98*(4), 1636–52.

Romero-Medina, A. (1998). Implementation of stable solutions in a restricted matching market. *Review of Economic Design*, *3*(2), 137–147.

Roth, A. E. (1984). The evolution of the labor market for medical interns and residents: a case study in game theory. *Journal of political Economy*, *92*(6), 991–1016.

Roth, A. E. (1991). A natural experiment in the organization of entry-level labor markets: regional markets for new physicians and surgeons in the united kingdom. *American economic review*, 415–440.

Roth, A. E., & Rothblum, U. G. (1999). Truncation strategies in matching marketsin search of advice for participants. *Econometrica*, *67*(1), 21–43.

Roth, A. E., Sönmez, T., & Ünver, M. U. (2004). Kidney exchange. *The Quarterly Journal of Economics*, *119*(2), 457–488.

Roth, A. E., Sönmez, T., & Ünver, M. U. (2005). Pairwise kidney exchange. *Journal of Economic theory*, *125*(2), 151–188.

Roth, A. E., & Sotomayor, M. (1989). The college admissions problem revisited. *Econometrica: Journal of the Econometric Society*, 559–570.

Roth, A. E., & Sotomayor, M. (1992). Two-sided matching. *Handbook of game theory with economic applications*, *1*, 485–541.

Svensson, L.-G. (1999). Strategy-proof allocation of indivisible goods. *Social Choice and Welfare*, *16*(4), 557–567.