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Dependence structure and risk spillovers between real estate and stock markets: An application of VMD based time-varying copula approach

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Abstract

In this thesis, we combine copulas with the variational mode decomposition (VMD) method to explore the dependence structure between real estate and stock market in three countries, namely China, U.S. and Australia. We explore the static and dynamic symmetric and asymmetric copulas, and investigate the time-varying dependence structures in the short-term and long-term time horizons. Our empirical results provide strong evidence of tail dependence between the stock market and real estate market in all countries. Furthermore, lower tail dependence is generally higher than upper dependence in all time horizons. We then quantify risk measures, namely value at risk (VaR), conditional VaR (CoVaR) and the delta CoVaR (ΔCoVaR) to analyze both upside and downside risk spillovers at different time horizons for each country. We find significant bidirectional risk spillovers between the stock market and the real estate market. The risk spillover effect between real estate-stock pair is the strongest in U.S and the weakest in Australia. In addition, we observe that downside risk spillovers are significantly stronger than the upside spillovers, and the systemic risk contribution of real estate to stock markets is larger than that of its opposite direction in all three countries.

Keywords: Time varying copula, Variational mode decomposition, Risk spillovers, CoVaR, delta CoVaR

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1.Introduction

Real estate is important not only for homebuyers, but also constitutes an important asset class for mutual funds both locally and internationally (Higgins, 2007). Hudson et al. (2003), point out that one of the primary benefits of including real estate assets in a portfolio is the reduction on the overall risk of the portfolio due to its low correlation with other assets. Because of these diversification gains, real estate has already become an attractive investment choice as a part of a mixed-asset portfolio (Worzala and Sirmans, 2003). Thus, studying the linkages between real estate returns and other asset returns especially stock returns is beneficial for asset allocation and portfolio risk management.

There are three main mechanisms that interpret the linkages between the stock market and the real estate market. The first mechanism is the ‘wealth effect’, which is a unidirectional relationship from the stock market to the real estate market. This theory suggests that a rise in stock prices increases the wealth of investors, and thus boosts real estate consumption and prices (Okunev and Wilson, 1997; Okunev et al., 2000). The second mechanism is the ‘credit price effect’. It states that a rise in real estate prices can lead to an increase in the credibility of the firms and thereafter allows them to increase investment and profits, which will in turn lead to a rise in the firms’ stock prices (Kapopoulos and Siokis, 2005). The last mechanism called ‘substitution effect’ describes a bidirectional relationship between the two markets. It arises from the modern portfolio theory (Markowitz, 1952) and suggests that investors adjust their allocation in real estate and stock when their respective market values change. The existing literature has studied a lot on the linkages between the two markets, but whether the transmission mechanism is unidirectional or bidirectional is still inconclusive.

A large amount of the empirical literature models the linkages between the stock market and the real estate market by applying cointegration test and dependence analysis, but little is known about how the two markets co-move at different market conditions and at different investment horizons. Based on the concept of a range of symmetric and asymmetric copulas, we utilize Conditional Value at Risk (CoVaR) and delta Conditional Value at Risk (ΔCoVaR) risk measures to quantify the upside and downside risk spillover effects from the stock market to the real estate market and vice versa. By applying time-varying copulas in addition to the static ones, we are able to investigate the dynamic dependence structure of the two markets. To distinguish the stock and real estate co-movements at different time horizons, we apply an advanced multiresolution decomposition method called variational mode decomposition (VMD) to decompose the time series into short and long-term components. The dependence in different time horizons is thus captured by using time-varying copulas on the decomposed components.

In our study, we use the performance of REITs to represent the performance of real estate market. A real estate investment trust (REIT) is a company that owns or finances income-producing real estate. Unlike direct investment in private real estate market to own properties, REITs are more accessible to individuals and institutions, with lower capital requirement, higher liquidity, and a wide range of property types. As tradable financial products, REITs have constant price discovery mechanism and daily liquidity, which are essential characteristics for our study. Last but not least, REITs co-move with the private real estate market under most situations, as they are exposed to similar commercial, residential, public or other types of real estate assets.

We focus on three countries in total, namely China, U.S. and Australia. The rapid growth of China's economy and the continuous rise in real estate asset prices over the past decade have attracted investors' interest. The policy and regulation changes are in favor of the growth of rental market. Within the next one or two years, China is likely to launch the first authentic real estate investment trust (REIT), which might open the way for an estimated $1.9 trillion worth of issuance (Wildau and Jia, 2018). Moreover, Chinese stock market is seeing more investment opportunities as well. In June 2017, MSCI, a leading provider of global equity indexes, announced that it would include China A shares into the MSCI Frontier Emerging Markets Index and the MSCI ACWI index beginning in June 2018. As more institutions and individuals would leverage REITs to diversify their portfolios and invest in real estate markets, it is worth studying the dynamic dependence between the Chinese securitized real estate and stock market.

On the other hand, United States and Australia have a long history of REITs market and are probably the two most mature public real estate markets in the world. The co-movement of their stock and real estate markets often gathers the world's attention. In United States, REITs were established in 1960 and by the end of 2017, there were 222 listed REITs with market capitalization of more than $1.1 trillion. With its first REIT introduced in 1971, Australian REITs market is also very mature. REITs offer a good degree of diversification in terms of tenant diversity, geographic diversification and diversification by property asset class. The difference between the mature financial markets and developing markets concerning the real estate-stock relationship deserve our attention.

To the best of our knowledge, our paper is the first study to investigate the cross-country difference in time-varying dependence between real estate and stock markets at different time horizons. By combining VMD and copula methods and utilizing risk measures including VaR, CoVaR and ΔCoVaR, we contribute to the literature by adding strong evidence of time-varying tail dependence between the real estate market and stock market. We find that lower tail dependence is in general higher than upper tail dependence, and both dependence increase sharply during crisis. We find strong evidence of bidirectional risk spillovers both from the stock market to the real estate market and vice versa, but the credit price effect is higher in those three countries. Moreover, U.S. has the largest risk spillover effects, and its short-term systemic risk effect is significantly stronger than its long-term counterpart.

This paper is organized as follows. Section 2 presents the literature related to this study. Section 3 describes the methodology used. Section 4 presents the data and descriptive statistics. Section 5 shows the empirical results obtained. Section 6 presents the risk implications. Section 7 presents the robustness tests and Section 8 concludes the paper.

2. Literature review

The linkages between real estate and stock markets are important for asset allocation and portfolio management. A large volume of empirical literature has examined the linkages between those two markets, using different methods such as correlation tests, vector autoregressive (VAR) model and copulas. However, they don't always draw the same conclusions.

Okunev and Wilson (1997) use a non-linear cointegration test to REITs and the S&P 500 indices and conclude that real estate and local stock markets are fractionally integrated. Maurer et al. (2004) perform a multivariate correlation analysis and conclude that German open-end real estate funds returns show more correlation with bond and money markets than with the stock market. The financial characteristics of open-end real estate funds are in many aspects similar to those reported for direct real estate investments.

Liow and Yang (2005) implement a fractional integrated vector error correction model (FIVECM) on the securitized real estate and stock markets of Singapore. Their study implies that securitized real estate and stocks in Singapore are fairly substitutable assets over the long run and those assets may not be held together in a portfolio for diversification purpose. Also by focusing on cointegration and partial cointegration relations, Lin and Lin (2011) show that real estate investment and stocks are substitutable in China, Hong Kong, Japan, and Taiwan, while providing diversification potential for investment portfolios in South Korea and Singapore. Using quantile causality tests, Ding et al., (2014) find a significant causal relationship between real estate and stock markets in China, especially in the tail quantile.

Employing vector autoregressive models, Glascock et al., (2000) explore the causality and long run linkages between REIT and stock returns. Their study points out that REITs behave more like stocks and less like bonds after the structural changes in the early 1990s. Sim and Chang (2006) apply a vector autoregressive model on Korean market data, but find no evidence of converse causation from stock to real estate markets, indicating no wealth effect between the two markets. Using threshold error-correction model, Su (2011) finds the existence of both wealth and credit price effects in the real estate markets and stock markets of Western European countries.

Simon and Ng (2009) analyze the diversification effect of REITs in U.S. market by applying a mixed-copula framework to measure the asymmetric tail dependence. They find that investing in REITs provides better protection against severe downturns of the U.S. stock market than by investing in a foreign common stock index. Using different models including the Gaussian, Student-t, Clayton and Gumbel copulas in their work, Rong and Trück, (2010) show that copula functions provide a powerful tool for modelling the dependence structure between financial asset variables and accurate measurement of the Value-at-Risk (VaR) for portfolios that contain investment in real estate. Chang et al., (2011) apply various static copulas to study portfolio Value-at-Risk for estimations of joint distribution of a portfolio consisted of REITs and Russell 2000 in different time periods. They find that time-varying risk is a more important driver in the results than model specification. Chen et al., (2014) find that among their estimated static copulas in their study of the co-movement among financial markets, static SJC copula performs best and is able to capture asymmetric characteristics of the tail dependence structure.

Even though the studies cited above have mainly focused on the static correlation between real estate markets and stock markets, various other literature addresses the importance of examining the dynamic aspect of the links by applying more advanced tools such as time varying regression analysis and dynamic copulas.

By using time-varying regression techniques called flexible least squares, Clayton and MacKinnon, (2001) show that the relationship between REIT returns and returns to bonds, small cap stocks, large cap stocks and unsecuritized real estate has changed over time. The authors show further that variance decomposition for REIT returns can be separated into components directly related to major stock, bond, and real estate-related return indices, as well as idiosyncratic or sector-specific effects.

Based on U.S data from 1999 to 2003, Cotter and Stevenson (2006) examine the time-varying conditional volatilities and correlations in the daily REITs and equity return series using multivariate VAR-GARCH techniques. Their study recommends investors to incorporate time-varying volatilities and correlations in portfolio selection.

Huang and Zhong (2006) apply Engle's (2002) Dynamic Conditional Correlation (DCC) model to portfolio constructing with REITS. They find that the DCC model, outperforms other correlation structures such as rolling, historical and constant correlations. Case et al., (2011) use the Dynamic Conditional Correlation model with Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) to examine dynamics in the correlation of returns between REITs and non-REIT stocks. Their results suggest that REIT-stock correlations form three distinct periods over year 1972 to 2008. Liow (2012) uses the Asymmetric Dynamic

Conditional Correlation (ADCC) model, which allows for asymmetric and time varying effects in estimating dynamic market conditions, on data from eight Asian securitized real estate markets over year 1995 to 2009. He finds that conditional correlations between real estate markets and stock markets are subject to regime changes caused by the global financial crisis. Heaney and Sriananthakumar, (2012) find that conditional correlations between Australia REITs and share market returns are quite high and increased further during both the Wall Street Crash and the global financial crisis. The authors argue that A-REITs behave more like shares than the underlying assets that they purport to mimic.

Liow et al., (2011) analyze the dynamics and transmission of conditional volatilities across five major securitized real estate markets, by developing a multivariate regime-dependent asymmetric dynamic covariance model that allows the conditional matrix to be both time- and state-varying. They point out that it is likely the optimal portfolio for the long-term would be different from that of the short term given different volatility dynamics.

Li et al., (2015) examine the relationship between the U.S. housing and stock markets by considering a wavelet analysis, which allows the simultaneous examination of co-movement and causality between the two markets in both the time and frequency domains. Their findings provide robust evidence that co-movement and causality vary across frequencies and evolve over time.

Utilizing an ARMA-GARCH model for the marginal distributions and a copula for the joint distribution, Sun et al., (2009) analyze the co-movement of global stock markets. Among the alternative models investigated their study, they find that Student-t copula ARMA(1, 1)-GARCH(1, 1) is good at capture the long-run dependence and tail dependence between global capital markets.

Hoesli and Reka (2011) apply time varying SJC copula on U.S., U.K. and Australia markets and state that rather important tail dependence coefficients are observed. And they also point out that the strongest volatility spillovers between the stock and the securitized real estate markets are found in the U.S. Mensi et al., (2017) add to the evidence that time-varying SJC copula and the CoVaR based on SJC copula is the best among their estimated time-varying copulas in estimating the dependence structure and tail dependence for financial markets. By applying Variational Mode Decomposition (VMD) method, they further point out dependence changes over different time horizons.

Kiohos et al., (2017) examine the long co-memory process between real estate and stock market returns in U.K. and Germany. Their error correction auto regressive fractionally integrated moving average (ECM-ARFIMA) model capture the fractional difference and the long-term error effect of the initial non-linear fractional integration model in Germany and the short–term effects (ARMA coefficients) in the UK. Their results provide support to the 'wealth effect' and to the fractional integration process in both countries.

Our study complements the existing literature by adding evidence of time-varying tail dependence between regional real estate markets and stock markets in China, U.S. and Australia. We also add further discussions about dependence variation and risk spillovers in different time horizons. By quantifying the risk measures, we analyze the asymmetric systemic risk effect for short and long positions in short-term and long-term.

3. Methodology

3.1. The marginal distribution model

We estimate the parameters of the marginal distribution models and copula models following a two-step procedure, proposed by Joe and Xu (1996). We first estimate a marginal model for the stock return and real estate return series respectively by applying the maximum likelihood (ML) method. We then transform the standardized residuals of the marginal models into their uniform marginal through probability integral transform. Using the transformed residuals pair, the copula models can be finally estimated by ML method.

For the marginal distribution models, we employ the ARMA-GARCH model on the return series and use statistical tests to choose the best fitted models. The autoregressive moving average mean equation is expressed as ARMA(p, q):

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t, \tag{1}$$

where δ is a constant term, ϕ_i is the *i*-th autoregressive coefficient, θ_j is the *j*-th moving average coefficient, and ε_t is the independently distributed error term at time *t*. *p* and *q* are the orders of autoregressive and moving average terms, respectively.

The GARCH variance model addresses the heteroskedastic effects of the time series. The Generalized Autoregressive Conditional Heteroskedastic (GARCH) function models the ε_t terms in the ARMA mean equation, or alternatively, the innovations of the time series process. Flowing Bollerslev (1986), a GARCH(p, q) process is expressed as:

$$\varepsilon_t = z_t \sigma_t$$

$$z_t \sim D_\vartheta(0, 1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_t^2 + \sum_{i=1}^{q} \beta_j \sigma_t^2, \tag{2}$$

where σ_t^2 is the conditional variance, and z_t is an i.i.d. process with zero mean and unit variance.

Considering the possibility of long memory in the return series, we also test ARFIMA-GARCH models. Introduced by Granger and Joyeux (1980), fractional integrated autoregressive moving average model is appropriate for the statistical analysis of a univariate time series with long memory. Fractionally integrated series are slowly mean-reverting and display significant persistence in the long term. Mathematically, Granger and Joyeux (1980) define a time series r_t is said to follow an ARFIMA (p, d, q) process if

$$\Phi(L)(1-L)^d(r_t-\mu)=\Theta(L)\varepsilon_t$$

$$\varepsilon_t = z_t\,\sigma_t\ , \tag{3}$$

where ε_t is the independently distributed error term with variance σ_t at time *t* as explained in the GARCH variance model, *d* is the fractional differencing parameter that measures the degree of long memory, and L is the lag operator. For the autoregressive (AR) part, $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$. And for the moving average part, $\Theta(L) = 1 + \Theta_1 L + \Theta_2 L^2 + \cdots + \Theta_p L^p$.

The usefulness of a fractional filter $(1-L)^d$ is that it produces hyperbolic decaying autocorrelations, the long memory property. If d < 0.5, the process is covariance stationary and we can process with the ARFIMA model. When 0 < d < 0.5, the process demonstrates a long memory. When d=0, the process demonstrates a short memory and it becomes an ARMA process.

For GARCH model, we consider three different distribution for the innovation process z_t, namely Gaussian distribution, Student-t distribution and skewed Student's t distribution. The GARCH model was initially combined with normal distributed errors by Bollerslev (1986), and later combined with Student distributed errors by Bollerslev (1987). The Bollerslev (1987) Student-t distribution has the probability density function given by:

$$g_v(y) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1-\frac{y^2}{v}\right)^{-\frac{v+1}{2}}, \tag{4}$$

where v is the degrees of freedom parameter and $\Gamma(\ )$ is the gamma function. When $v \rightarrow \infty$, it specializes to the standard normal distribution.

The skewed Student's t distribution innovation is based on the transformations introduced by Fernandez and Steel (1998) and developed by Lambert and Laurent (2000). Following their studies, the skewed Student-t density distribution is expressed as:

$$f(z_t|\,\xi, v) = \begin{cases} \frac{2}{\xi+1/\xi} g_v\big(\xi(sz_t+m)\big)\,, & z_t < -\frac{m}{s} \\ \frac{2}{\xi+1/\xi} g_v\big((sz_t+m)/\xi\big)\,, & z_t \geq -\frac{m}{s} \end{cases}, \tag{5}$$

where $g_v(\ )$ is the symmetric Student-t density distribution function with the degrees of freedom parameter v, and ξ is the asymmetry parameter ($\xi > 0$). And the parameter m and s^2 are respectively the mean and variance of the skewed Student-t distribution, described in Fernandez and Steel (1998) as follows:

$$m(\xi,\gamma) = \frac{\Gamma\left(\frac{\gamma-1}{2}\right)\sqrt{\gamma-2}}{\sqrt{\pi}\,\Gamma\left(\frac{\gamma}{2}\right)}\left(\xi - \frac{1}{\xi}\right);\ s^2 = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2.$$

If $\xi = 1$, This skewed Student-t distribution specializes to symmetric Student-t distribution. In order to get the true skewness of the skewed Student-t distribution, we need to take the log of the parameter ξ.

3.2 Copula approach

In order to examine the average and tail dependence between the real estate and stock markets, we employ different copulas to the pseudo-sample observations given by the probability integral transformation of the standardized residuals for each marginal model. According to Sklar's (1995) theorem, a joint distribution can be written as univariate marginal distribution functions of the variables and a copula that describes the dependence structure between the variables:

$$F_{r_t^s, r_t^r}(r_t^s, r_t^r) = C\left(F_{r_t^s}(r_t^s), F_{r_t^r}(r_t^r) \right), \tag{6}$$

where $u = F_{r_t^s}(r_t^s)$ and $v = F_{r_t^r}(r_t^r)$, which are the marginal distribution functions for stock return and real estate return respectively, and $F_{r_t^s, r_t^r}(r_t^s, r_t^r)$ is their joint distribution.

The joint probability density function of stock return and real estate return can be derived from Eq. (6), $c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$, as

$$f_{r_t^s, r_t^r}(r_t^s, r_t^r) = c(u,v)\, f_{r_t^s}(r_t^s)\, f_{r_t^r}(r_t^r), \tag{7}$$

where $f_{r_t^s}(r_t^s)$ and $f_{r_t^r}(r_t^r)$ are the marginal density functions of stock return and real estate return, respectively.

Copulas offer more flexibility in separate modelling the marginal distribution and dependence. We employ different copula specifications with different dependence structures as shown in **Table 1.** Restrictions for the parameters and the functions of tail dependence are also reported in the table. The symmetric copulas include the Gaussian copula, the Frank copula. The asymmetric copulas are the Gumbel copula with an upper tail dependence, the Clayton copula with a lower tail dependence, and the symmetrized Joe-Clayton copula (SJC), which is modified by Patton (2006) from the original Joe-Clayton copula. SJC copula is flexible at asymmetric dependence in either direction and nests symmetric dependence as a special case, that is when $\lambda_U = \lambda_L$. One feature of SJC is that even though it can nest symmetric dependence as a special case, it does not impose symmetric dependence restriction like the Gaussian copula.

Table 1 Bivariate copula functions

Copula Name	Formula	Parameter	Tail dependence
Gaussian (N)	$C_N(u,v,\rho) = \phi(\phi^{-1}(u), \phi^{-1}(v))$	$\rho \epsilon [-1,1]$	Zero tail dependence: $\lambda_L = \lambda_U = 0$
Gumbel (G)	$C_G(u,v,\delta) = \exp(-[(-\log u)^\delta + (-\log v)^\delta]^{1/\delta})$	$\delta \epsilon [1,\infty)$	Asymmetric tail dependence: $\lambda_L = 0, \lambda_U = 2 - 2^{1/\delta}$
Clayton (C)	$C_C(u,v,\delta) = \max\{(u^{-\delta} + v^{-\delta} - 1)^{-\frac{1}{\delta}}, 0\}$	$\delta \epsilon [1,\infty)$ \{0}	Asymmetric tail dependence: $\lambda_L = 2^{-1/\delta}, \lambda_U = 0$
Frank (F)	$C_F(u,v,\delta) = -\frac{1}{\delta}\log\left(\frac{(1-e^{-\delta}) - (1-e^{-\delta u})(1-e^{-\delta v})}{(1-e^{-\delta})}\right)$	$\delta \epsilon (-\infty,\infty)$ \{0}	Zero tail dependence: $\lambda_L = \lambda_U = 0$
SJC	$C_{SJC}(u,v,\lambda_U,\lambda_L) = 0.5(C_{JC}(u,v;\ \lambda_U,\lambda_L) + C_{JC}(1-u,1-v;\lambda_L,\lambda_U) + u + v - 1)$ where $C_{JC}(u,v,\lambda_L,\lambda_U)$denotes the Joe-Clayton copula defined as: $C_{JC}(u,v,\lambda_U,\lambda_L) = 1 - (1 - \{[1-(1-u)^k]^{-\gamma} + [1-(1-v)^k]^{-\gamma} - 1\}^{-1/\gamma})^{1/k}$ where $k = \frac{1}{log_2(2-\lambda_U)}, \gamma = -\frac{1}{log_2(\lambda_L)}$	$\lambda_U \epsilon (0,1)$ $\lambda_L \epsilon (0,1)$	$\lambda_L = 2^{-1/\gamma}$ $\lambda_U = 2 - 2^{1/k}$

In order to capture the time variation in the dependence structure, we construct time varying copulas by varying the parameters of the copulas according to some evolution equation, while fixing the functional form over the sample. Alternatively, we can use a switching-parameter copula model proposed by Rodriguez (2003) to introduce time variation in the functional form. We do not explore the latter time varying copulas here.

We estimate the time varying Gumbel, Clayton and SJC copulas in our study. Following the practice of the previous literature such as Patton (2006), Mensi et al., (2017), we assume that copula parameters are evolving in an ARMA(1,q)-type pattern. The current-period parameter is affected by one-period lagged parameter and the average difference between the transformed residuals of the stock and real estate marginal models. The time varying tail dependence parameters of Gumbel and Clayton copulas are as follows:

$$\delta_t = \omega + \beta\delta_{t-1} + \alpha\frac{1}{q}\sum_{j=1}^{q}|u_{t-j} - v_{t-j}|, \tag{8}$$

where δ_t is the copula parameter, ω, β , α are the parameters of the evolution function, u and v are the transformed residuals from marginal models. Following the common practice, we assume that the copula parameters follow a restricted ARMA(1, 10) process and thus take q=10 in the equation.

According to Patton (2006), the time varying tail dependence parameters of SJC copula follow a similar evolution function:

$$\lambda_t^U = \Delta\left(\omega_U + \beta_U\lambda_{t-1}^U + \alpha_U\frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|\right)$$

$$\lambda_t^L = \Delta\left(\omega_L + \beta_L\lambda_{t-1}^L + \alpha_L\frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|\right), \tag{9}$$

where$\lambda_t^U$and λ_t^L are the copula parameters of SJC, representing the upper tail dependence and lower tail dependence respectively, ω_U , β_U , α_U are the parameters of the upper tail dependence evolution function, ω_L, β_L , α_L are the parameters of the lower tail dependence evolution function, u and v are the transformed residuals from marginal models. $\Delta(x) = (1 + e^{-x})^{-1}$ is the modified logistic transformation used to keep λ_t^U and λ_t^L in (0, 1).

3.3 Variational mode decomposition (VMD)

To distinguish between short- and long-term variations of the time series that we study, we use a non-recursive decomposition technique known as the Variational Mode Decomposition (VMD), proposed by Dragomiretskiy and Zosso (2014), to decompose the returns series. The low (high) frequency modes obtained through VMD represent the long (short) term dynamics of the original time series. Thus, this mode-by-mode decomposition technique enables us to examine the change in real estate markets and stock markets dependence on different scales.

The general idea of VMD is to decompose a time series f into k discrete number of sub-series (known as modes) u_k, which has limited bandwidth in the spectral domain. Each decomposed mode k is required to be compressed around a center pulsation ω_k, which is determined along with the decomposition. The algorithm to determine the bandwidth of a time series requires the following steps: (i) obtain a unilateral frequency spectrum for each mode u_k by computing associated analytic series through Hilbert transform; (ii) for each mode u_k, shift the mode's frequency spectrum to a baseband by mixing with an exponential tuned to the respective estimated center frequency; (iii) compute the bandwidth of the demodulated series through Gaussian smoothness (Dragomiretskiy and Zosso, 2014)

Then, the constrained variational problem is given as:

$$min_{\{u_k\},\{\omega_k\}=} \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\} \quad (10)$$
$$s.t. \ \sum_k u_k = f,$$

where f is the time series and u is its mode while ω, δ, and $*$ represent the frequency, the Dirac distribution and convolution, respectively. The mode u with high-order k represents low frequency components. Thus, $\{u_k\} \coloneqq \{u_1, \ldots, u_k\}$ and $\{\omega_k\} \coloneqq \{\omega_1, \ldots, \omega_k\}$ are the sets of all variational modes and their central frequency, respectively. The solution to Eq. (10) is the saddle point of the following augmented Lagrange ($\mathcal{L}$) expression:

$$\mathcal{L}(u_k, \omega_k, \lambda) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] \right\|_2^2 + \|f - \sum u_k\|_2^2 + \langle \lambda, f - \sum u_k \rangle, \quad (11)$$

where $\mathcal{L}$ is the Lagrange multiplier, α denotes the balancing parameter of the data-fidelity constraint, and $[\![\, . \,]\!]_p$ denotes the usual vector ℓ_p norm where p=2. The solution to Eq. (11) is found in a sequence of k iterative sub-optimizations. Finally, the solutions for u and ω are found in Fourier domain and are given by:

$$u_k^{n+1} = \left(f - \sum_{j \neq k} u_i + \frac{\lambda}{2} \right) \Big/ (1 + 2\alpha(\omega - \omega_k)^2)$$
$$\omega_k^{n+1} = \left(\int_0^\infty \omega |u_k(\omega)|^2 \, d_\omega \right) \Big/ \left(\int_0^\infty |u_k(\omega)|^2 \, d_\omega \right), \quad (12)$$

where n is the number of iterations.

And for λ it's updated as

$$\hat{\lambda}^{n+1} \leftarrow \hat{\lambda}^n + \tau\left(\hat{f} - \sum_k \hat{u}_k^{n+1}\right) \quad (13)$$

until convergence: $\sum_k \|\hat{u}_k^{n+1} - \hat{u}_k^n\|_2^2 / \|\hat{u}_k^n\|_2^2$.

Following Lahmiri (2015), Shahzad et al., (2016), Mensi et al., (2017) etc., we set the number of modes k to ten.

3.4 VaR risk measure

The value at risk (VaR) is one of the most common and widely employed risk measures in risk management. The q%-VaR is the maximum loss within a specific time horizon and with the q%-confidence interval. In our study, we quantify both the downside and upside VaRs.

The downside VaR is the maximum loss that an investor may incur by holding a long position in the underlying market for a confidence level *1-α* and a time horizon *t.* In other words, it measures the α quantile of the return series r_t that Pr($r_t \leq VaR^D_{\alpha,t}$)= α. Assuming a skewed Student-t distribution for the return series, one-day-downside VaR can be computed from the marginal models as

$$VaR^D_{\alpha,t} = \mu_t + t^{-1}_{\eta,\lambda}(\alpha)\sigma_t, \tag{14}$$

where μ_t and σ_t are the conditional mean and standard deviation of the ARFIMA-GARCH model chosen for the return series, and $t^{-1}_{\eta,\lambda}(\alpha)$ is the α quantile of the skewed Student-t distribution expressed as Eq. (5).

Similarly, the one-day-upside VaR quantifies the maximum loss that an investor may incur by holding a short position in the underlying market and it is expressed by Pr($r_t \geq VaR^U_{1-\alpha,t}$)= α, which could be calculated by

$$VaR^U_{1-\alpha,t} = \mu_t + t^{-1}_{\eta,\lambda}(1-\alpha)\sigma_t. \tag{15}$$

After obtaining the time varying value at risk series, we use backtesting technique to determine the accuracy of the predicted VaR model. Backtesting in value at risk compares the predicted losses from the calculated VaR with the actual losses realized at the end of the specified time horizon. It is called a breach of VaR when the actual loss is greater than the estimated VaR loss. The acceptable frequency of breaches is decided by the confidence level for VaR. We use two-sided backtesting, which means that for a 95% VaR, the frequency of breaches should neither be significantly more or less than 5%. In other words, the value of VaR is neither underestimated or overestimated.

3.5 CoVaR and ΔCoVaR risk measures

VaR measures the risk of one market, but fails to consider the potential spillover effects that its financial distress may have on other markets. In order to study the possible risk spillovers between the real estate market and stock market, we further estimate the conditional VaR (CoVaR) developed by Girardi and Ergün (2013) and Adrian and Brunnermeier (2016). The CoVaR of the underlying market is the VaR of that market conditional on the fact that a given market experiences financial distress.

We quantify CoVaR both from stock market to real estate market (wealth effect) and from real estate to stock market (credit price effect). CoVaR from real estate market to stock market measures the VaR of stock market conditional on the distressed state of real estate market. Mathematically, this concept requires calculating the quantile of a conditional distribution. Following Girardi and Ergün (2013) and Adrian and Brunnermeier (2016), for a confidence level *1-β*, downside conditional VaR , denoted as $CoVaR_{\beta,t}^{s,D}$, could be expressed as

$$\Pr(r_t^s \leq CoVaR_{\beta,t}^{s,D} \mid r_t^r \leq VaR_{\alpha,t}^{r,D}) = \beta, \tag{16}$$

where r_t^s and r_t^r stand for return for stock and return for real estate respectively, and $VaR_{\alpha,t}^{r,D}$ represents the downside VaR for real estate return for a confidence level $1\text{-}\alpha$ and a specific time horizon *t*.

From the conditional probability concept, we could derive the quantile of the unconditional bivariate distribution:

$$\frac{\Pr(r_t^s \leq CoVaR_{\beta,t}^{s,D}, r_t^r \leq VaR_{\alpha,t}^{r,D})}{\Pr(r_t^r \leq VaR_{\alpha,t}^{r,D})} = \beta$$

Given that $\Pr(r_t^r \leq VaR_{\alpha,t}^{r,D}) = \alpha$, we can get:

$$\Pr(r_t^s \leq CoVaR_{\beta,t}^{s,D}, r_t^r \leq VaR_{\alpha,t}^{r,D}) = \alpha\beta \tag{17}$$

As explained in the copula part, we can describe the joint distribution of r_t^s and r_t^r by their marginal models and the copula C, as:

$$C(F_{r_t^s}(CoVaR_{\beta,t}^{s,D}),\ F_{r_t^r}\big(VaR_{\alpha,t}^{r,D}\big)) = \alpha\beta, \tag{18}$$

where $F_{r_t^s}$ and $F_{r_t^r}$ are the marginal models for stock return and real estate return respectively.

Similarly, the upside CoVaR for stock return ($CoVaR_{1-\beta,t}^{s,U}$) could be expressed as:

$$\Pr(r_t^s \geq CoVaR_{1-\beta,t}^{s,U} \mid r_t^r \geq VaR_{1-\alpha,t}^{r}) = \beta, \tag{19}$$

where $VaR_{1-\alpha,t}^{r}$ represents the upside VaR for real estate return for a confidence level 1-α and a specific time horizon *t*.

Then we could derive the quantile of the unconditional bivariate distribution for upside CoVaR as:

$$\Pr(r_t^s \geq CoVaR_{1-\beta,t}^{s,U}, r_t^r \geq VaR_{1-\alpha,t}^r) = \alpha\beta \tag{20}$$

And its corresponding copula form would be:

$$1 - F_{r_t^s}(CoVaR_{1-\beta,t}^{s,U}) - F_{r_t^r}\left(VaR_{1-\alpha,t}^{r,U}\right) + C\left(F_{r_t^s}(CoVaR_{1-\beta,t}^{s,U}),\ F_{r_t^r}\left(VaR_{1-\alpha,t}^{r,U}\right)\right) = \alpha\beta, \tag{21}$$

where $F_{r_t^s}$ and $F_{r_t^r}$ are the marginal models for stock return and real estate return respectively.

We follow a two-step procedure to compute CoVaR proposed by Reboredo and Ugolini (2015). Firstly, given a copula function, and a confidence level of 1-β for CoVaR of stock returns, we could obtain the value of $F_{r_t^s}(CoVaR_{\beta,t}^{s,D})$ and $F_{r_t^s}(CoVaR_{1-\beta,t}^{s,U})$ by solving Eq. (16) and Eq. (17) respectively, as we know that $F_{r_t^r}\left(VaR_{\alpha,t}^{r,D}\right) = \alpha$ and $F_{r_t^r}\left(VaR_{1-\alpha,t}^{r,U}\right) = 1-\alpha$. Secondly, we could obtain the value of $CoVaR_{\beta,t}^{s,D}$ and $CoVaR_{1-\beta,t}^{s,U}$ by inversing the marginal distribution $F_{r_t^s}$, after computing the cumulative probability from step one.

We use the Kolmogorov–Smirnov boot strapping test (KS test) proposed by Abadie(2002) to test for the significance of risk spillover between real estate market and stock market.

$$KS_{mn} = \left(\frac{mn}{m+n}\right)^{1/2} sup_x|F_m(x) - G_n(x)|\,, \tag{22}$$

where $F_m(x)$ and $G_n(x)$ are the cumulative CoVaR and VaR distribution functions, respectively, and *m* and *n* are the size of the two return series samples. Here, we test the null hypothesis of no risk spillovers between real estate and stock market in both upside and downside tail, respectively. The null hypothesis for the stock market can be written as $CoVaR_{\beta,t}^{s,D} = VaR_{\beta,t}^{s,D}$, $CoVaR_{1-\beta,t}^{s,U} = VaR_{1-\beta,t}^{s,U}$ respectively and the alternative is $CoVaR_{\beta,t}^{s,D} \neq VaR_{\beta,t}^{s,D}$, $CoVaR_{1-\beta,t}^{s,U} \neq VaR_{1-\beta,t}^{s,U}$ respectively. And the null hypothesis and alternative are similar for real estate market.

In addition, the systemic risk contribution of a particular market can be described by delta conditional value-at-risk (ΔCoVaR) proposed by Adrian and Brunnermeier (2016) and Girardi and Ergün (2013). The delta CoVaR of real estate market represents the difference between the VaR of the stock market conditional on the distressed state of real estate market, expressed as $\mathrm{CoVaR}_{\beta,t}^{s|r}$, and the VaR of the stock market conditional on the median state of real estate market (that is the VaR value when α=0.5), expressed as $\mathrm{CoVaR}_{\beta,t}^{s|r,\alpha=0.5}$. Thus, the delta CoVaR of real estate to stock market, expressed as $\Delta\mathrm{CoVaR}_{\beta,t}^{s|r}$, can be written as:

$$\Delta\text{CoVaR}_{\beta,t}^{s|\text{r}} = (\text{CoVaR}_{\beta,t}^{s|\text{r}} - \text{CoVaR}_{\beta,t}^{s|\text{r},\alpha=0.5}) / \text{CoVaR}_{\beta,t}^{s|\text{r},\alpha=0.5} \tag{23}$$

Thus, ΔCoVaR could be used to capture the marginal contribution of real estate market to the stock market, and vice versa.

3.6 Implementation R packages

In our paper, we use "rugarch" package which is an R package developed by Alexios Ghalanos (2014) to estimate our marginal ARMA-GARCH models. The "rugarch" package supports a range of univariate distributions including the Normal, Student and their skew variants based on the transformations described in Fernandez and Steel (1998) and Ferreira and Steel (2006). When estimating VMD model, we use "VMD" package developed by Nicholas Hamilton (2017) who ports and extents the original Matlab code developed by Dragomiretskiy & Zosso (2013). We fit static copulas using the "VineCopula" package developed by Ulf Schepsmeier et al., (2016). "BiCopSelect" function gives the maximum Likelihood Estimation of Bivariate Copula Families.

4. Data and descriptive statistics

The daily indices of the stock and real estate markets from December 2007 to March 2018, covering the 2008-2009 global financial crisis and 2015-2016 Chinese stock market turbulence, were collected from Yahoo finance. Given that China has not introduced real estate investment trusts (REITs) yet, and thus we look for a substitute, which is ideally a passively managed fund that follows the performance of Chinese real estate index and is accessible to international investors. We use the Guggenheim China Real Estate ETF (ticker: TAO), which tracks the AlphaShares China Real Estate Index, to represent the real estate prices. Accordingly, we use SPDR® S&P® China ETF (ticker: GXC), which follows the S&P® China BMI Index, to represent the stock market performance. We use all the data available and the time range of the data is from 12.18.2007 to 26.03.2018.

For U.S. market, we use the daily closing prices of S&P 500 and S&P United States REIT. For Australian market, we use the daily closing prices of ASX-Australian stock exchange All Ordinaries and S&P/ASX 200 REIT. The data were sourced from investment.com. The U.S. data is from 3.31.2008 to 26.03.2018 due to data availability. The Australian data has the same time rage as that of Chinses data.

Fig. 1 depicts the daily level series over the sample period in Chinese market, United States market and Australian market, respectively. We could see that Australian real estate market is the worst among all series, with its value not recovering back to its pre-crisis level yet. Australian stock market is recovering very slowly, with value slightly above the pre-crisis level. On the other hand, Chinese real estate market and stock market are reaching a new high recently. Uniquely, Chinese real estate market's performance matches very closely with stock market and sometimes outperforms the stock market. U.S. stock market has climbed up steadily from 2009. Its real estate market follows and becomes relatively stable in recent years. The data period is marked by some global events. We can see all series slump badly during the 2008 global financial crisis and slip when the European debt crisis worsened in the end of 2011. 2015 Chinese stock market turbulence, which begins with popping of stock market bubble in June 2015, can also be seen clearly from the graph. During this turbulence, stock market starts to recover at the end of 2015, but tumbles again at the beginning of 2016 after the introduction of the circuit-breaker mechanism by the China Securities Regulatory Commission. The circuit-breaker mechanism was aimed to halt losses and help stabilize stocks, but the market responded to this mechanism by triggering the break two days in the first four trading days and the

Shanghai Composite fell 13.8 percent. As a result, the China Securities Regulatory Commission had to give up this mechanism due to the concern of the volatility.

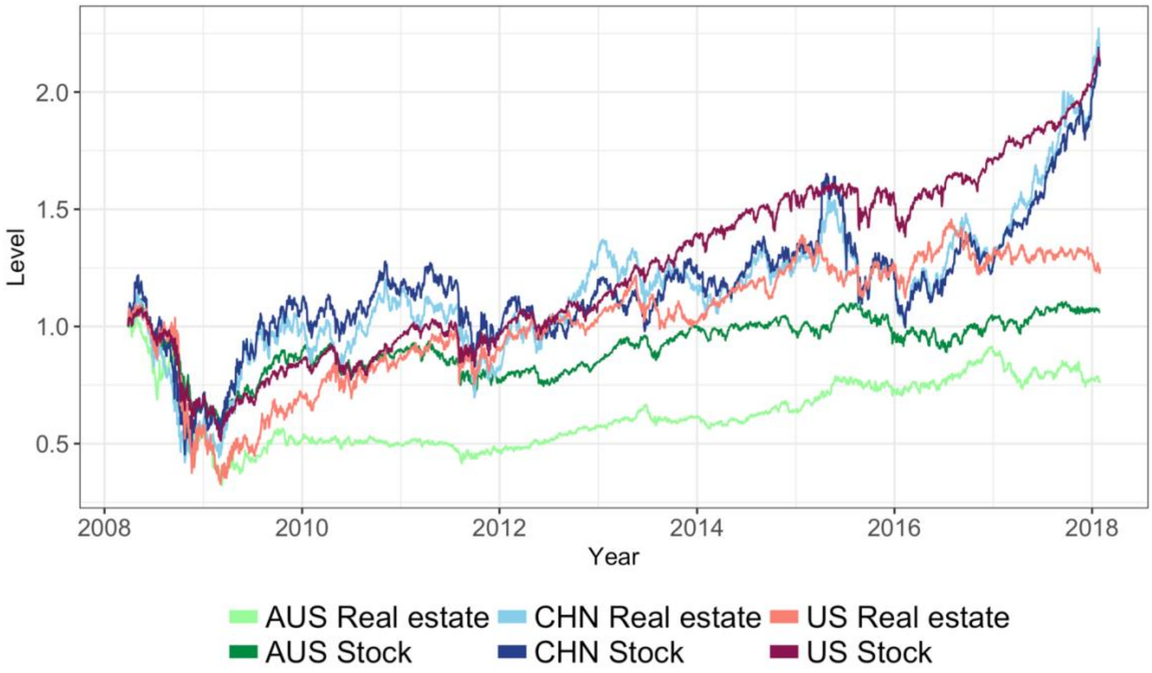
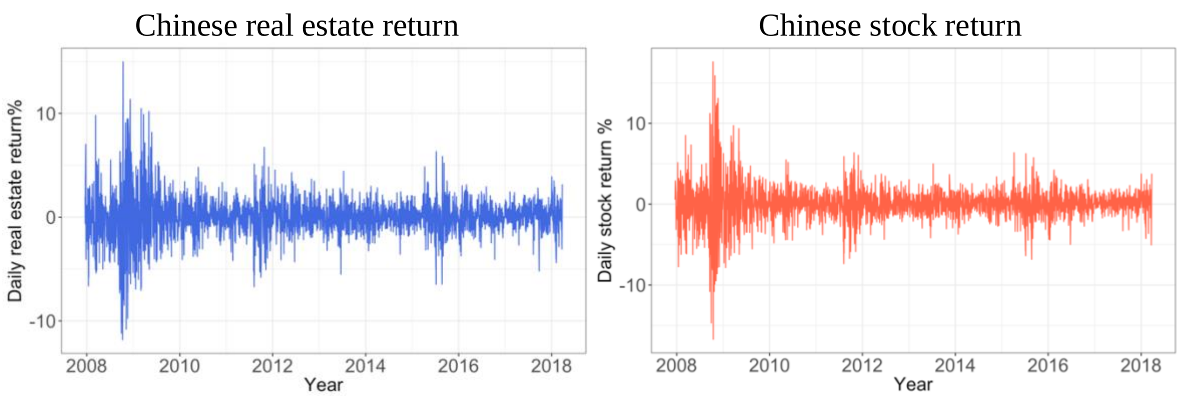


Fig. 1. Daily price series in Chinese market, United States market and Australian market

We then calculate the continuously compounded daily returns of each series by taking the difference in the natural logarithm of two consecutive prices, and multiply them by 100. **Fig. 2** depicts the daily return series of stock market and real estate market for each country. All series have similar patterns with daily returns fluctuate around zero. We could also see volatility clustering in all series and returns become especially volatile during the 2008 financial crisis. Among the three countries, Australia has the lowest volatilities in both stock market and real estate market during the crisis.



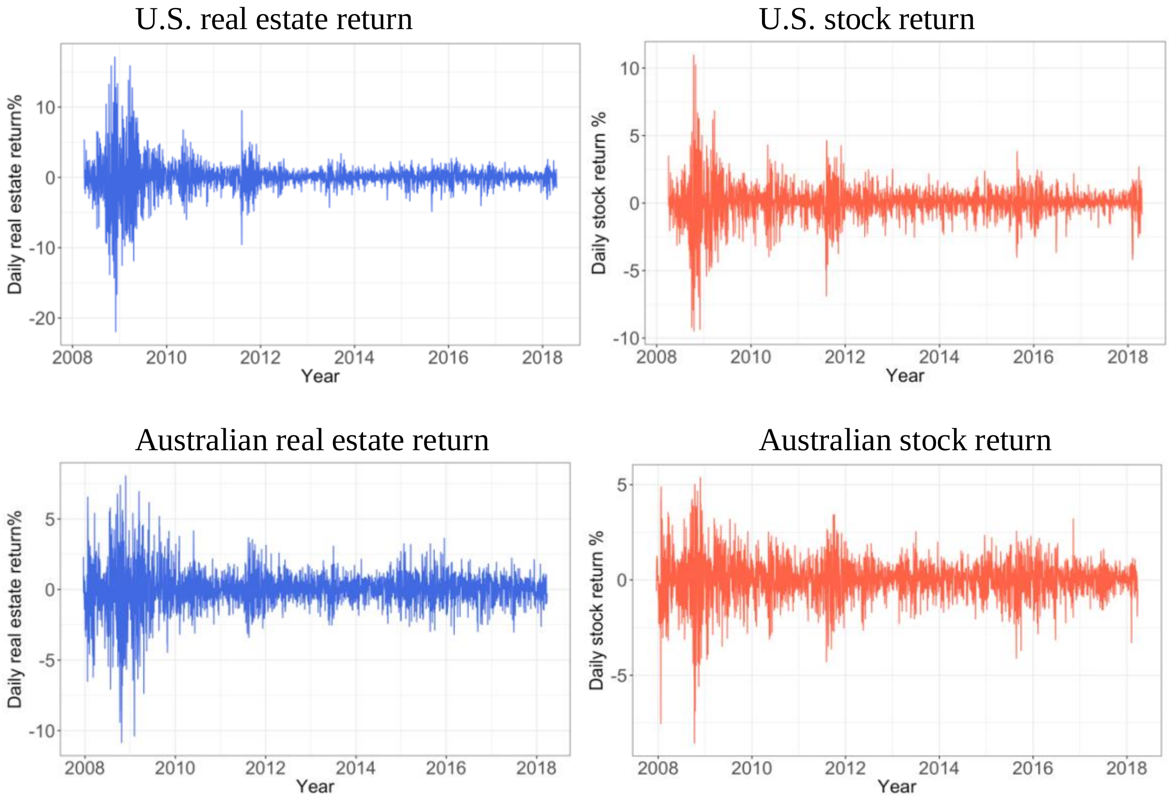


Fig. 2. Daily return series in Chinese market, United States market and Australian market.

Table 2 shows the descriptive statistics of stock and real estate return series (expressed in %) in three regional markets. The average daily returns are slightly positive for all the series except for Australian series which has a -0.0178% return in real estate market an -0.0024% return in stock market. This relatively bad performance may due to Australia's slow pace in recovering the drop happened in the financial crisis. Australian real estate market, however, is the least volatile among the three real estate markets, which does not experience extreme upper and lower values as Chinese and U.S. do. Australian stock market, in the same time, is the least volatile among all series. Sharpe (1994) ratio, which is calculated under the assumption of a zero-return risk-free asset, characterizes how well the return of the underlying asset compensates the investor for the risk taken. It shows that U.S. stock market (0.0217) is the best choice to invest in, while Australian real estate market (-0.0122) is the worst. Moreover, only Chinese real estate market has a higher Sharpe ratio (0.0104) than that of its stock market's (0.0083), suggesting the unique opportunity in Chinese market. Unlike U.S. and Australian markets, Chinese real estate and stock market exhibit positive skewness, which might be a favorable feature from an investment point of view. US real estate return series has the highest standard deviation and kurtosis, which might be caused by the extreme values during the financial crisis.

Serial correlation is found in return series and squared return series by LjungBox test, suggesting the use of ARMA and GARCH models. The ARCH test also indicates strong ARCH effects in all series. Both Shapiro-Wilk test and KPSS test reject the null of Gaussian distribution for all the series, suggesting that return series do not follow simple normal distributions as what are usually simplified in the textbook. ADF tests shows that all the series are stationary and thus ARMA model without differencing can be implemented.

Table 2

Descriptive analysis of stock and real estate return series

	CHN real estate	U.S. real estate	AUS real estate	CHN stock market	U.S. stock market	AUS stock market
Mean	0.0201	0.0061	-0.0178	0.0174	0.0277	-0.0024
Std. Dev.	1.9381	2.2726	1.4633	2.1014	1.2794	1.0702
Sharpe ratio	0.0104	0.0027	-0.0122	0.0083	0.0217	-0.0022
Median	0.0000	0.0688	0.0027	0.0453	0.0602	0.0435
Minimum	-11.8137	-21.9450	-10.8489	-16.7112	-9.4695	-8.5536
Maximum	14.9745	17.1235	8.0506	17.6152	10.9572	5.3601
Skewness	0.1723	-0.2084	-0.5823	0.2198	-0.3787	-0.5061
Kurtosis	6.5100	15.0938	7.1481	10.2998	11.3457	5.4611
Ljung-Box	89.54***	218.21***	105.26***	193.03***	112.34***	30.615*
Ljung-Box^2	3384.2***	6075.5***	4692.8***	5500.3***	4106.4***	3051***
ARCH	3381***	6048***	4677***	5489***	4098***	3039***
ShapiroWilk	0.9219***	0.784***	0.905***	0.8817***	0.8635***	0.9427***
KPSS	0.101	0.123	1.14	0.0851	0.424	0.26
ADF	-10.1***	-10.6***	-11.7***	-10.7***	-11.2***	-12.3***

Notes: Returns in all series are % log returns. Ljung-Box and Ljung-Box^2 are the Ljung-Box autocorrelation test statistics for return series and squared return series, respectively, computed with 20 lags. ARCH is the test statistics of Portmanteau-Q test for the ARCH effect, computed with 20 lags. Shapiro-Wilk and KPSS are the test statistics of the Shapiro Wilk (1965) normality test and Kwiatkowski et al., (1992) unit root test for no drift and no trend type. ADF is Augmented Dickey Fuller (1979) stationarity test statistics for no drift and no trend type, setting lag equal 20. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

5. Empirical results

5.1 Marginal model results

This analysis estimates the ARMA-GARCH or ARFIMA-GARCH models for each return series, using Gaussian, Student-t and skewed Student-t distribution respectively. When choosing the most adequate model, we consider the Ljung-Box, Hosking and ARCH test results of the residuals, the value of Akaike Information Criteria (AIC), and the significance of each parameter. We found that the fitted models with skewed Student-t distribution or Student-t distribution tend to have the minimum value of AIC, while fitted models with normal distribution pass all the diagnostic tests except for Australian real estate return series. Considering the descriptive statistics described above, which indicates that the return series are skewed with excess kurtosis and fail to pass the normality tests, we choose the most adequate models with skewed Student-t distribution. We also estimate the fractional integrated parameters in the ARFIMA model, and found d=0 most of the time, which indicates a short memory as ARMA model instead of a long range memory.

Following Sun et al., (2009) and Mensi et al., (2017), we consider different combinations of the lag parameters for ARMA and GARCH ranging from zero to maximum 2. The most adequate model with skewed Student-t distribution is shown in **Table 3**, while the fitted models with normal distribution and Student-t distribution could be found in the appendix. Specifically, ARMA(1,1)-GARCH(1,2) model is the best model for US and Australian real estate return series, while ARMA(1,0)-GARCH(2,2) model is the best for Chinese real estate return series. For stock market, the best model for China, United States and Australia are ARMA(2,2)-GARCH(2,2), ARMA(2,1)-GARCH(2,1) and ARMA(1,1)-GARCH(1,2) respectively. These fitted models are compatible with those in Sun et al., (2009) and Mensi et al., (2017).

From the table, we could see that the ARCH components for almost all the series are significant at the 5% level, meaning that one-period lagged squared innovations affect the current-period volatility. Meanwhile, current-period volatility is related to the variance of the previous innovations, as indicated by the GARCH components significant at the 1% level for almost all the series are.

It can also be seen that all the series exhibit significant skew and shape parameters at the 1% significant level, suggesting that the fat tails of skewed Student-t distribution characterize the distribution of stock and real estate return series, and all the return series are negatively skewed. It justifies the use of skewed t distribution for the innovation process in the GARCH model. It also shows the possibility of tail dependence, which would be discussed in the later section.

Table 3

Marginal model estimations ARFIMA-GARCH with skewed t innovations

	CHN real estate	U.S. real estate	AUS real estate	CHN stock market	U.S. stock market	AUS stock market
Cst(M)	0.054411*	0.040197**	0.027824*	0.067084***	0.065364***	0.0267*
	(0.029882)	(0.016937)	(0.014719)	(0.024318)	(0.011493)	(0.015108)
AR(1)	0.032059*	0.721844*	0.723662***	0.46105	0.778038***	-0.603452***
	(0.019451)	(0.422386)	(0.116117)	(0.028672)	(0.08507)	(0.182985)
AR(2)				0.248683	0.024055	
				(0.110972)	(0.02761)	
MA(1)		-0.771257**	-0.777163***	-0.496332	-0.867285***	0.590437***
		(0.386701)	(0.107216)	(0.025725)	(0.080248)	(0.184168)
MA(2)				-0.26252		
				(0.101899)		
Cst(V)	0.043292***	0.012959**	0.014463***	0.037873***	0.017847***	0.007909**
	(0.016087)	(0.005905)	(0.005299)	(0.012812)	(0.006556)	(0.003717)
Alpha1	0.069265***	0.104683	0.091719***	0.040863**	0.06686**	0.080666***
	(0.025101)	(0.025859)	(0.01767)	(0.017755)	(0.028528)	(0.017959)
Alpha2	0.039083			0.062572**	0.090918**	
	(0.028525)			(0.020884)	(0.043472)	
Beta1	0.545755***	0.762107***	0.515795***	0.509434*	0.838014***	0.912395***
	(0.103776)	(0.097287)	(0.073917)	(0.149617)	(0.031803)	(0.010619)
Beta2	0.332273***	0.129701*	0.38221***	0.376228		0.000196
	(0.094806)	(0.073567)	(0.07119)	(0.136444)		(0.008579)
Skewness	-0.00151***	-0.14591***	-0.04082***	-0.06001***	-0.14352***	-0.12146***
	(0.028157)	(0.025556)	(0.02708)	(0.027323)	(0.030629)	(0.022853)
Shape	9.207612***	8.962336***	12.294875***	7.71313***	5.509945***	10.878683***
	(1.463394)	(1.410321)	(2.466515)	(1.090988)	(0.646272)	(2.057582)
AIC	3.7031	3.2989	3.0213	3.7329	2.6148	2.5865
Ljung-Box	21.135	19.894	23.214	24.174	27.324	20.149
Ljung-Box^2	29.16*	31.228	32.069*	32.007*	23.559	19.076
Hosking	21.1182607	19.877832	23.197178	24.155301	27.302506	20.133755
Hosking^2	28.0381156	30.17253*	30.967547	30.736868*	22.875028	17.726458
ARCH	28	30.1*	30.9	30.63*	22.78	17.66

Notes: This table reports the ML estimates and the robust standard deviations in parenthesis for the parameters of the marginal distribution model. Ljung-Box and Ljung-Box^2 are Ljung-Box autocorrelation test statistics for standardized residuals and squared standardized residuals, respectively, computed with 20 lags. Hosking and Hosking^2 are the Hosking (1980) autocorrelation test statistics standardized residuals and squared standardized residuals, respectively, computed with 20 lags. ARCH is the test statistics of Portmanteau-Q test for the ARCH effect in the standardized residuals, computed with 20 lags. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Concerning the diagnostic test, the values of the Hosking (1980) tests for serial correlation in the standardized residuals and the squared standardized residuals do not reject the null of no serial correlation for Chinese and Australian real estate return series, and U.S. and Australian stock return series. And the four series show no ARCH effects in the standardized residuals. On the other hand, the standardized residuals of Chinese stock return and U.S. real estate return fitted models with skewed Student-t distribution demonstrate slight ARCH effects, while models with normal distribution accept the null of no ARCH effects. Thus, we would continue with those models and discuss the effect of different distribution models on our later analysis.

5.2 Copula model results

Table 4 presents the estimation of static copulas for each market pair using the probability integral transform of the standardized residuals from the marginal models. Based on the AIC values, our results show that static SJC copulas offer a good fit for the U.S and Australian pairs, the choice of SJC copula is agree with Chen et al., (2014). Static Gaussian copula only offers a good fit for Chinese pair. Gaussian copula indicates that tail independence to some extent exists between the Chinese stock market and real estate market. Nevertheless, static Gumbel and Joe Clayton copulas have very similar AICs with the Gaussian copula. This table indicates that all real estate markets and stock markets co-move in the same direction.

Table 4

Bivariate static copula estimates

Copulas	CHN	US	AUS
Gaussian			
ρ	0.8203	0.6770	0.6318
	(.0018)	(.0038)	(.0042)
AIC	-2874.1	-1542.6	-1360.5
Clayton			
α	2.1908	1.2782	1.0903
	(.1082)	(.0989)	(.1569)
AIC	-2459.0	-1283.2	-1131.5
Gumbel			
α	2.4976	1.8618	1.7128
	(.0515)	(.0344)	(.0292)
AIC	-2870.9	-1543.5	-1331.0
Frank			
δ	8.3334	5.5331	4.7227
	(1.3853)	(.4488)	(.3229)
AIC	-2664.3	-1484.6	-1258.7

Table 4 (continued)

Copulas	CHN	US	AUS
SJC			
λu	2.0837	1.6765	1.5106
	(.0604)	(.0465)	(.0404)
λl	1.7614	0.9601	0.8456
	(.07)	(.0512)	(.0451)
AIC	-2862.9	-1577.7	-1361.8

Note: This table reports the ML estimates for the parameters of different static copulas as well as their respective AICs. Standard errors are reported in the parentheses.

Considering that the average and tail dependence may changes over time reacting to dynamic market condition, we further examine time-varying parameter (TVP) copulas on the return series of each country. **Table 5** shows the estimation of time-varying copulas we use. The time-varying Gumbel and Clayton copulas reflect dynamic upper tail and lower tail dependence respectively, while time-varying SJC copula reflects both upper and lower tail dependence. We find that time-varying copulas are generally doing better than their static copula peers, suggesting that the dependence structure between the real estate-stock pair is time varying. Our result is consistent with studies by Huang & Zhong, (2006), Case et al., (2011), Mensi et al., (2017) etc., who suggest dynamic dependence models outperform the others. Moreover, the dependence is well captured by SJC copulas, indicating to evidence of both upper tail and lower tail dependence in each pair. This finding is consistent with the finding from Hoesli and Reka (2011), Han et al., (2016) who evidence the better performance of SJC copula in estimating tail dependence.

Table 5

Bivariate time varying copula estimates

Copulas	CHN	US	AUS
TVP Gumbel			
ω	0.1805	0.3952	0.1443
	(.0576)	(.1373)	(.058)
β	0.9470	0.8421	0.8915
	(.0188)	(.063)	(.0622)
α	-0.8191	-1.8412	-0.6533
	(.2509)	(.6104)	(.2413)
AIC	-2750.75	-1622.34	-1261.69
TVP Clayton			
ω	0.2629	0.1811	0.7970
	(.2032)	(.0531)	(.2494)
β	0.8818	0.9186	-0.5123
	(.0612)	(.0286)	(.2728)
α	-1.2039	-0.9914	-5.0000
	(1.4214)	(.2848)	(1.341)
AIC	-2525.26	-1450.19	-1154.08

Table 5 (continued)

Copulas	CHN	US	AUS
TVP SJC			
ωu	0.0568	0.0962	0.0302
	(.0308)	(.0288)	(.0171)
βu	0.9770	0.9583	0.9857
	(.0153)	(.0119)	(.0084)
αu	-0.3601	-0.6218	-0.1944
	(.1876)	(.1899)	(.1081)
ωl	1.0420	0.4612	1.2446
	(.5307)	(.1693)	(.3546)
βl	0.4272	0.7433	-0.7076
	(.3065)	(.0984)	(.1361)
αl	-4.5812	-2.5967	-7.8437
	(2.3074)	(.9842)	(1.9622)
AIC	-2953.59	-1754.13	-1383.37

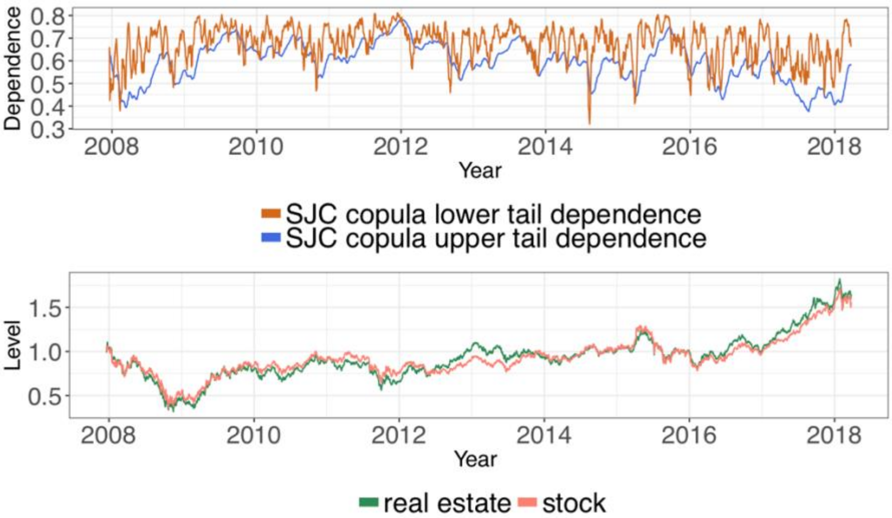
Note: This table reports the ML estimates for the parameters of different static copulas as well as their respective AICs. Standard errors are reported in the parentheses.

Fig. 3. offers graphical insights into dynamic tail dependence values obtained through time-varying SJC copulas for each pair. Tail dependence formulas are represented in **Table 1.** in Section 3.2. The dependence trajectory for all countries differs in terms of the time, suggesting the time varying nature of dependence. Especially, lower tail dependence is in general higher than upper tail dependence and dependence level quickly rises to a high level during the 2008 global financial crisis, consistent with the findings from the existing literature (e.g. Knight et al., (2006), Liow (2012), Heaney and Srinanthakumar, (2012)).

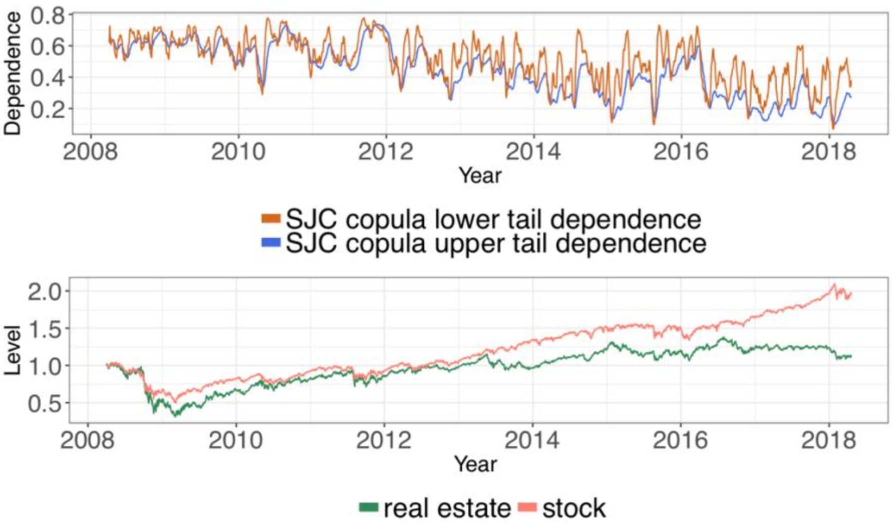
For Chinese market, we observe strong and high tail dependence levels, which complies with the findings of Ding et al., (2014). To be more detailed, there is a greater variation in the lower tail than upper tail dependence between the stock market and the real estate market. Tail dependence rise quickly during the global financial crisis and they remain quite high even after the stock market starts to recovery in the Spring of 2009. From 2011 to 2012, tail dependence especially the upper tail dependence, increase steadily as stock and real estate markets fall. From the start of February 2016 to the beginning of 2018, Chinese stock market experienced a bull market. CSI 300 stock market index was up from around 2900 to above 4000. Upper dependence, however, has a descending trend in this period. The major dip in the upper tail dependence from 2017 corresponds to the stock market surge. Throughout this data period, we see a generally inverse relation between tail dependence and market conditions. Tail dependence increase when the market conditions deteriorate and vice versa.

On the other hand, both upper tail and lower tail dependence for U.S. have a downward trend after the financial crisis, pointing to a declining dependence for the real estate-stock pair as the market conditions for real estate and stocks improve. This downward trend in dependence suggests a potential for increasing portfolio diversification. However, we need to point out that tail dependence remains at a high level during the financial crisis and dependence tends to increase when there are turbulences in the markets. For Australia, lower tail dependence remains relatively stable around 0.45, while upper tail dependence fluctuates in a range of 0.6 to 0.2. Moreover, Australian pair's dependence level is the lowest among the three countries.

Panel A: Tail dependence and price level of real estate-stock pair for China



Panel B: Tail dependence and price level of real estate-stock pair for United States



Panel C: Tail dependence and price level of real estate-stock pair for Australia

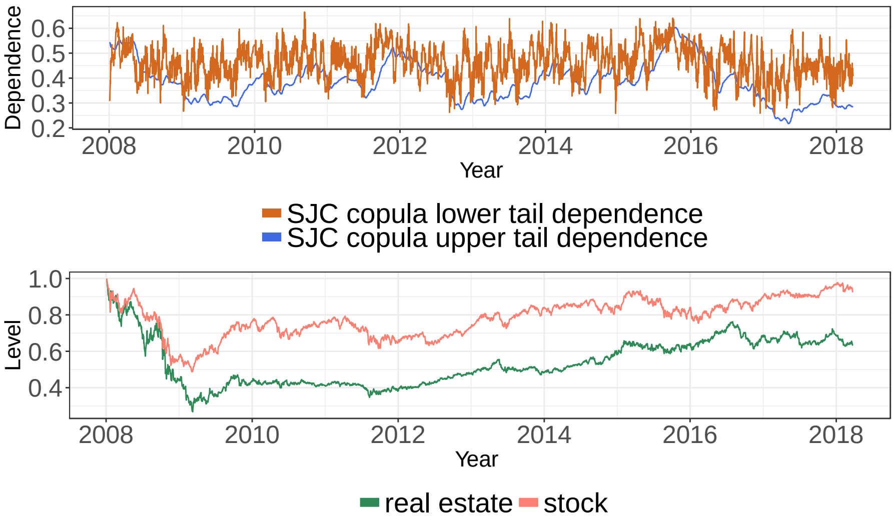


Fig. 3. SJC Time-varying copula tail dependence of real estate-stock market. Notes: The time-varying tail dependence is based on the fitted copula parameters λ_U, λ_L. For better comparative analysis with the market condition, the level graph for each country is combined with its tail dependence graph.

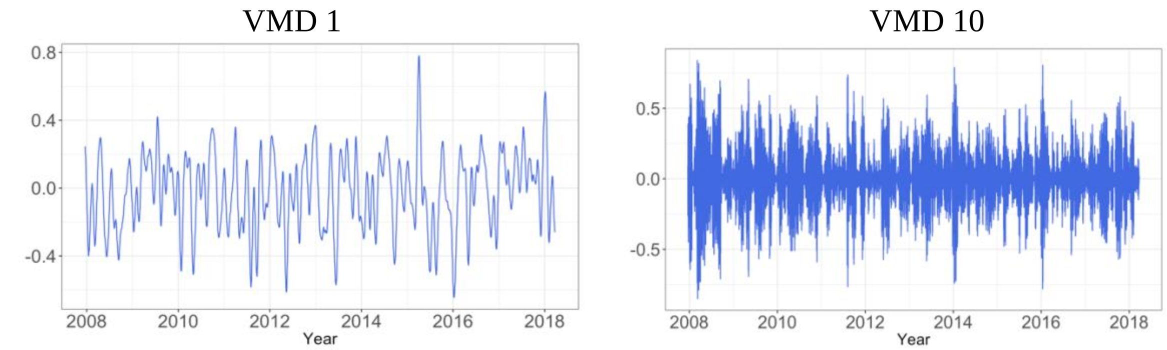
5.3 Variational mode decomposition results

In order to further analyze the short-term and long-term dependence structure and risk spillovers between real estate and stock market, we apply the mode-by-mode VMD decomposition on the standardized residuals of the fitted marginal models. **Panel A** and **B** of **Fig. 4** demonstrates the VMD for mode 1 (long-term) and 10 (short-term) for Chinese stock and real estate series. VMD decomposition graphs for other series are included in the appendix.

For both the stock series and real estate series, the VMD figures show a volatility clustering and short term volatility is larger. For the long-term (mode 1) stock series, we find asymmetry with more downside extreme values. We also observe a clear upside peak in the mid 2015 which reflects the 2015 Chinese stock market crash. In the short-term (mode 10), we observe more extreme values in 2011 which reflects the European debt crisis.

For the real estate series, we observe more downside extreme values during the sample period in the long-term (mode 1), with upside peaks at the start of 2015 and 2018 and the lowest value at the start of 2016. In the short-term (mode 10), we observe more extreme values in 2011 as in the stock series.

Panel A: Variational mode decomposition for the stock return series



Panel B: Variational mode decomposition for the real estate return series

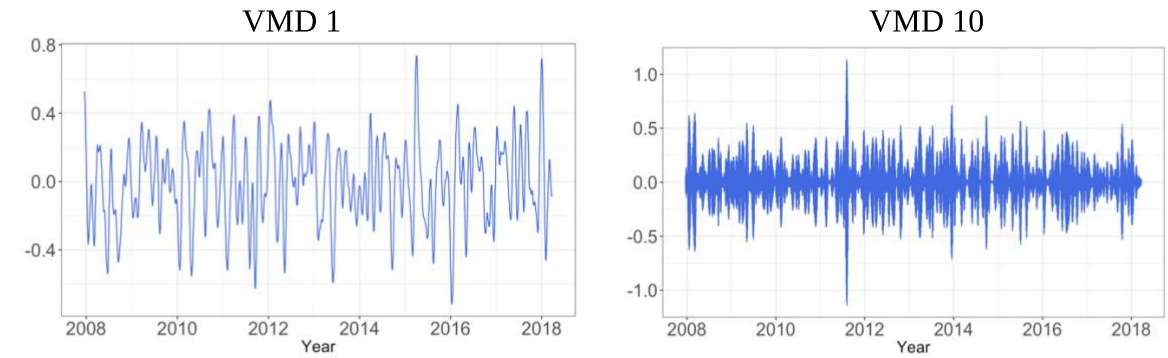


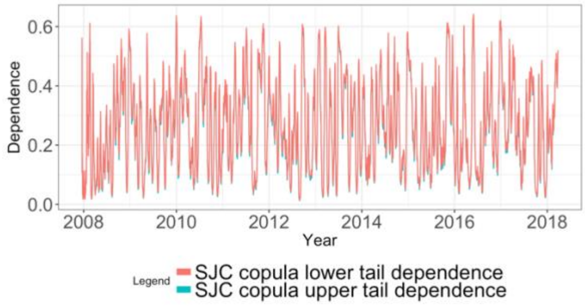
Fig. 4 Variational mode decomposition for mode 1 (long-term) and mode 10 (short-term) for Chinese stock and real estate series. Note: Following the common practice from the finance literature, we set the maximum variational models equal to 10.

5.4 Long term and short term dependence

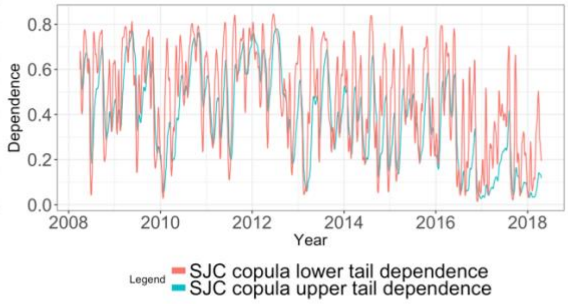
Panel A and **B of Fig. 5** show the time-varying short-term and long-term tail dependence using SJC copulas for each market, respectively. For space saving, estimation for copula parameters are shown in **Table 14** in the appendix. For Chinese market, there are not many difference between upper tail and lower tail dependence in the short-term and they both have a lot of variation. In the long-term horizon, however, there are a lot more variations in the upper tail dependence, while the lower tail dependence is high and smooth. We see no big difference in the pattern between the long-term and short-term tail dependence for U.S., except for the drop in long-term tail dependence from 2013 to 2016. For Australia, there are a lot more variations in the lower tail dependence than in the upper tail dependence both in the short- and long-term, and the lower tail dependence is more volatile in the short-term. Also, the short-term upper tail dependence is much lower than its long-term counterpart since 2013.

Panel A: SJC Time varying copula short-term tail dependence

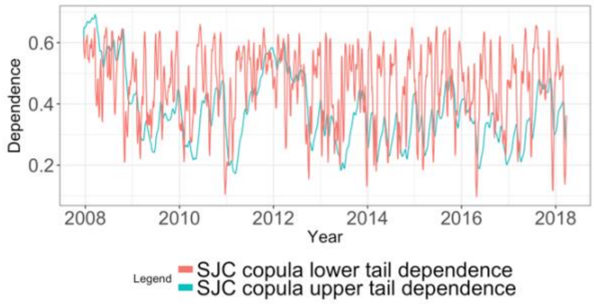
China



United States

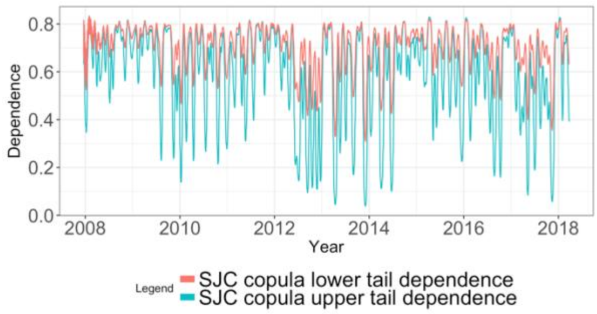


Australia

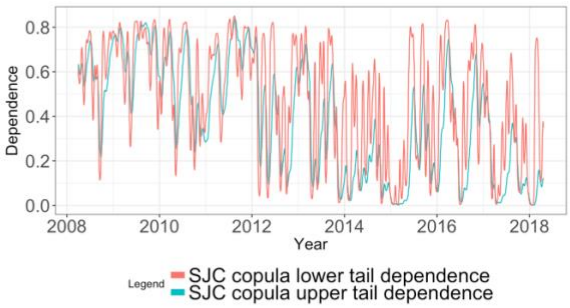


Panel B: SJC Time varying copula long-term tail dependence

China



United States



Australia

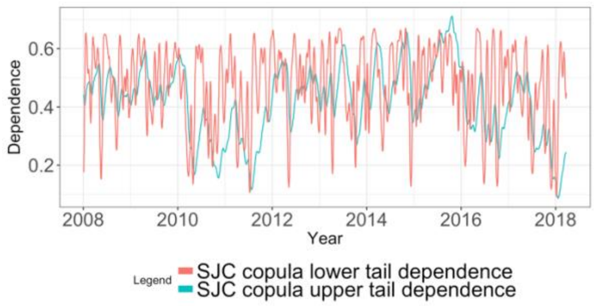


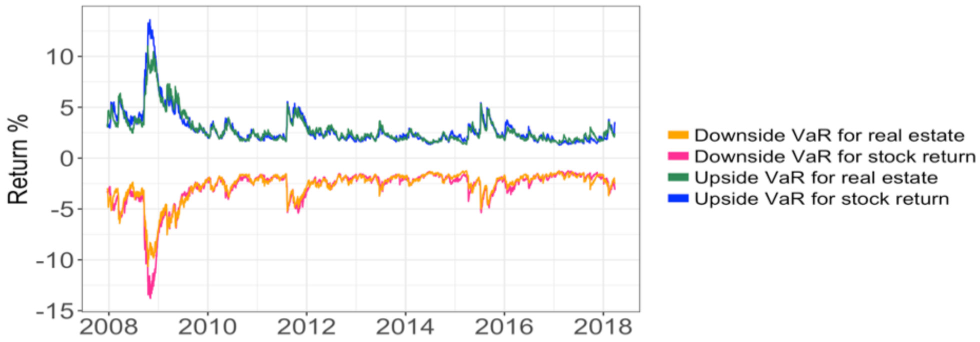
Fig. 5. SJC Time varying copula short-term and long-term tail dependence of real estate-stock market. Notes: The time-varying tail dependence is based on the fitted copula parameters λ_U, λ_L from each short-term and long-term series, which is obtained through the VMD decomposition.

6. Risk implications

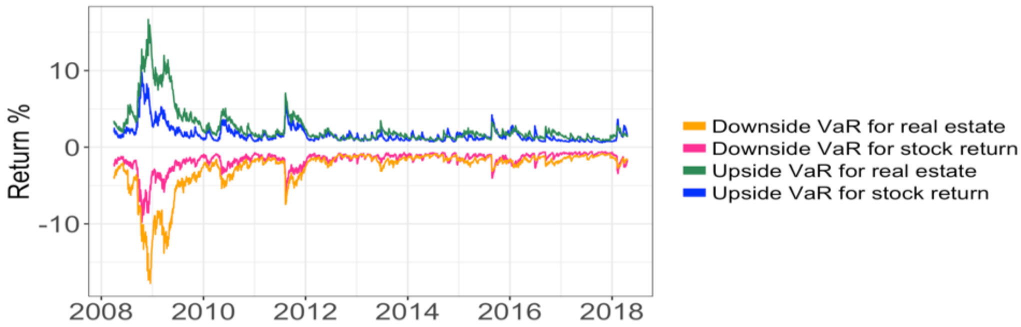
6.1 Value at Risk analysis

We first examine the upside and downside 95% VaRs of real estate-stock pairs presented in **Fig. 6**. It is shown that during financial distress period, Chinese stock market has significantly higher absolute values of downside and upside VaRs than their counterparts of Chinese real estate market. But for U.S. and Australian markets, the absolute values of downside and upside VaRs for real estate are significant higher than those for stock.

Panel A: Upside and downside VaR for China



Panel B: Upside and downside VaR for United States



Panel C: Upside and downside VaR for Australia

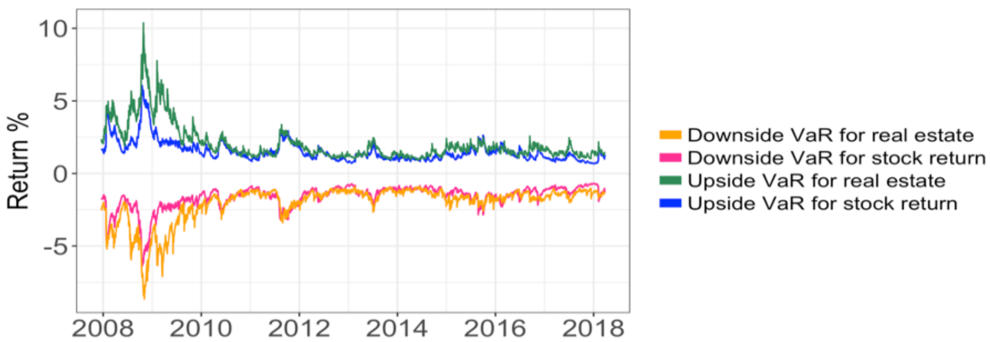


Fig. 6. Upside and downside VaR for real estate markets and stock markets. Notes: Downside and Upside VaRs are calculated using Eqs. (14) and (15), respectively.

The descriptive statistics shown in **Table 6** also indicates higher average and standard deviations of both upside and downside VaRs for Chinese stock market than its real estate market, suggesting that real estate investment is safer than equity investment in China. On the contrary, U.S. and Australian real estate markets have significantly higher average and standard deviations of VaRs than those of their stock markets'. When comparing across regions, we find that Australian stock market and real estate markets are significantly safer than other markets, with the lowest average and standard deviations of VaRs among the three countries. In general, the maximum possible one-day loss with 5% possibility is the largest in Chinese stock markets, indicating high risk for investors.

Table 6

Descriptive statistics of value at risk (VaR) and conditional value at risk (CoVaR)

	Upside		Downside	
	VaR	CoVaR	VaR	CoVaR
Panel A: VaR of stock markets and CoVaR from real estate to stock market				
CHN	2.8517	5.6928	-2.8345	-6.0734
	(1.7097)	(3.4355)	(1.7639)	(3.7323)
US	1.6635	3.3769	-1.7015	-4.1149
	(1.1513)	(2.4121)	(1.1823)	(2.9073)
AUS	1.5197	2.7467	-1.6085	-3.2111
	(.7232)	(1.3308)	(.7915)	(1.5871)
Panel B: VaR of real estate markets and CoVaR from stock to real estate market				
CHN	2.8077	5.5423	-2.7044	-5.4661
	(1.4474)	(2.8799)	(1.4477)	(2.8958)
US	2.5282	4.7022	-2.7296	-5.7018
	(2.3609)	(4.525)	(2.606)	(5.4942)
AUS	2.0193	3.6812	-2.0000	-3.8467
	(1.2226)	(2.2191)	(1.203)	(2.31)

Notes: This table presents the mean and the standard deviation (in parenthesis) of the VaR calculated using all the data, upside and downside CoVaRs.

We then conduct backtesting on VaR and the results are presented in **Table 7**. The upside and downside VaRs for the return series do not reject the null of correct exceedances except for the downside VaR for U.S. stock return series. It suggests that in general, the VaR models and the marginal distribution models are well designed and efficient. The excessive number of downside VaRs breaches for the U.S. stock market is caused by too many uncommon negative returns during the global financial crisis, which is hard to model. In times of extreme financial distress, the returns for many financial products may not follow the distribution that they used to do.

Table 7

Results of value at risk (VaR) backtesting

	For stock return		For real estate return	
	Upside	Downside	Upside	Downside
CHN	0.2232	1.1031	0.3706	0.6178
	(.6366)	(.2936)	(.5427)	(.4319)
US	0.1818	9.065***	0.1818	0.0227
	(.6698)	(.0026)	(.6698)	(.8802)
AUS	0.2399	0.6911	0.3358	0.8424
	(.6243)	(.4058)	(.5622)	(.3587)

Notes: This table reports the unconditional coverage test Likelihood Ratio statistics for the null hypothesis of correct exceedances. and their corresponding p values in parenthesis. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

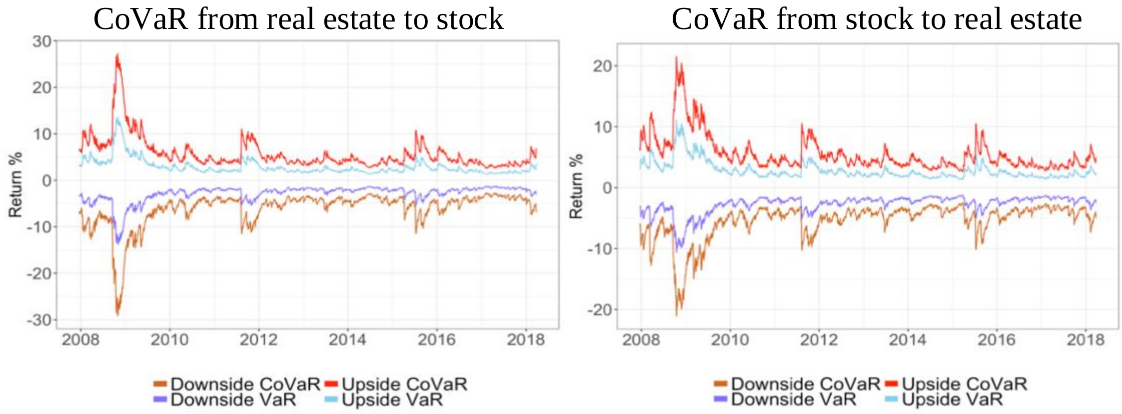
6.2 Asymmetric bidirectional risk spillovers

Using our time-varying SJC copula relation from Section 5.2, we quantify the downside and upside CoVaR value for stock (real estate) returns at the 95% confidence level (ß=0.05) conditional on the VaR value for real estate (stock) returns at the 95% confidence level (∂=0.05). **Fig. 7.** illustrates that the upside and downside VaR and CoVaR series exhibit a similar trend with difference in magnitude. And the absolute values of both upside and downside CoVaRs are significantly larger than those of their corresponding VaRs. This observation is confirmed by the results of the K-S test in **Table 8**, which indicates significant differences between all VaR-CoVaR pairs. It implies that there are significant risk spillovers both from stock to real estate and from real estate to stock in all markets. To be more specific, an extreme upside (downside) movement in the real estate market would have a positive (negative) impact on stock market, and vice versa. This finding is similar with Su (2011) who finds the bidirectional effects in Western European countries, while different with Chang (2006) who finds no wealth effect in Korea.

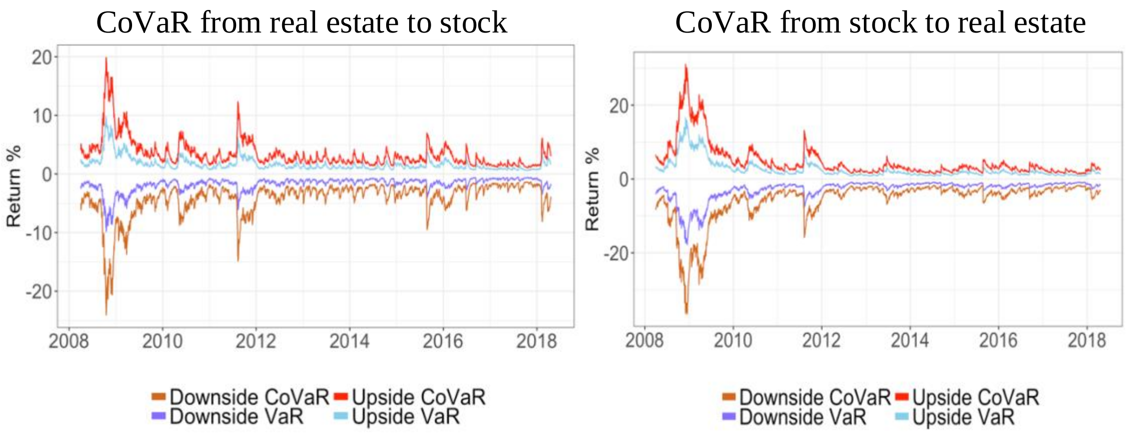
The impact of the global financial crisis is reflected in the sudden surge in the absolute values of CoVaRs and the gaps between VaR and CoVaR. In the real estate-to-stock direction, Chinese stock market is the most affected by the global financial crisis. Its downside CoVaR jumps from the average of -6% to near -30% since mid 2008. The absolute values of downside and upside CoVaR also increase significantly in mid 2011 and in 2015 when the stock market experienced quite some turbulences.

In the stock-to-real estate direction, U.S. real estate market is the most affected by the extreme movement in the stock market. Its downside CoVaR drops from the average of -5.7% to around -28% during the global financial crisis. It also experiences sharp decrease in mid 2011. The risk spillover effect from U.S. stock to real estate market lessens since then.

Panel A: Upside and downside VaRs and CoVaRs for China



Panel B: Upside and downside VaRs and CoVaRs for United States



Panel C: Upside and downside VaRs and CoVaRs for Australia

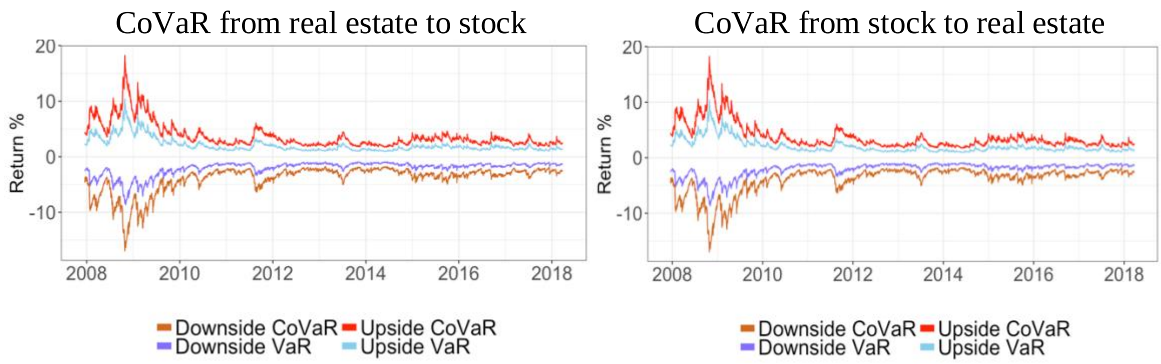


Fig. 7. Upside and downside VaRs and CoVaRs for Chinese stock and real estate market. Notes: For the sake of comparative analysis, CoVaRs are illustrated with their VaRs. Downside and Upside VaRs are calculated using Eqs. (14) and (15), respectively. Downside and Upside CoVaRs are calculated using Eqs. (16) and (19), respectively.

In addition, asymmetries in upside and downside risk spillovers exist both in stock and real estate market. We use the K-S bootstrapping test (**Table 9**) and find significant differences between the upside CoVaRs normalized by the upside VaRs and the downside CoVaRs normalized by the downside VaRs. On average, the downside risk spillovers measured by normalized CoVaRs are larger than the upside spillovers.

Table 8

Tests of equalities of VaR and CoVaR in upside and downside conditions

	CoVaR from real estate markets to stock markets		CoVaR from stock markets to real estate markets	
	Upside	Downside	Upside	Downside
CHN	0.6792	0.7140	0.7190	0.7187
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
US	0.5472	0.6569	0.4860	0.5756
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
AUS	0.5877	0.6558	0.6502	0.6926
	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests the null hypothesis of no systemic impact between the stock markets and real estate markets. The p-values for the KS statistic are in the parentheses.

Table 9

Upside and downside CoVaR asymmetry from stock markets to real estate markets and vice versa

$$H_0: \frac{CoVaR}{VaR}\ (D) = \frac{CoVaR}{VaR}\ (U),\ H_1: \frac{CoVaR}{VaR}\ (D) > \frac{CoVaR}{VaR}\ (U)$$

	from real estate markets to stock markets	from stock markets to real estate markets
CHN	0.9706	0.7837
	(0.0000)	(0.0000)
US	0.9199	0.9139
	(0.0000)	(0.0000)
AUS	0.9981	0.7245
	(0.0000)	(0.0000)

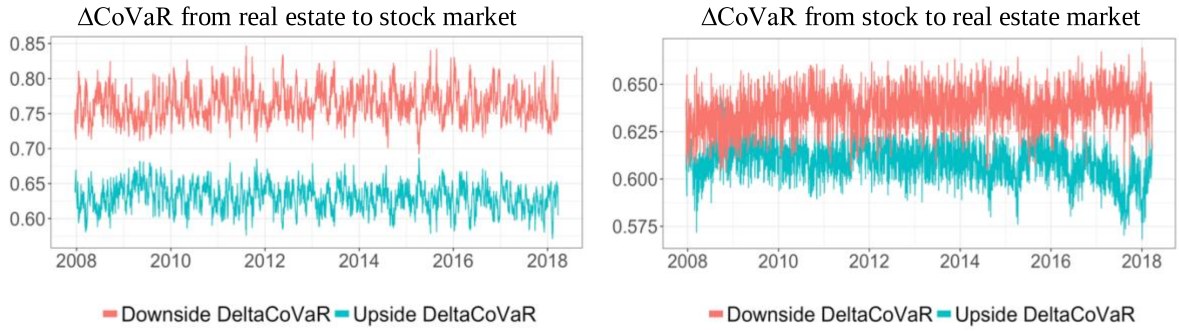
Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests test the null hypothesis of no difference between the downside and upside systemic risk contribution. The p-values for the KS statistic are in the parentheses.

6.3 ΔCoVaR and systemic risk

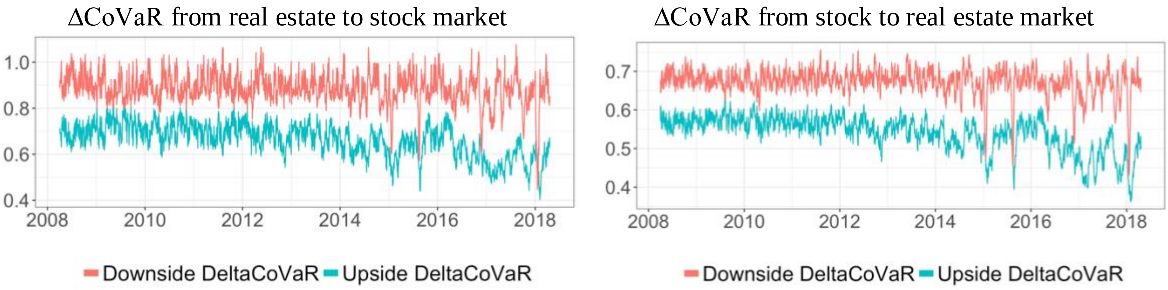
To further study the risk spillover effect and systemic risk contribution of the stock and real estate markets, we compute the time varying delta conditional value-at-risk (ΔCoVaR) from real estate to stock market and from stock to real estate market, as shown in **Fig. 8**. The averages and standard deviations of ΔCoVaR are displayed in **Table 10**.

Considering the bidirectional effect, U.S. stock and real estate markets are the most correlated under both upside and downside market conditions. The average downside and upside ΔCoVaRs from real estate to stock market are 0.88 and 0.69, respectively, and those of the opposite direction are 0.67 and 0.56, respectively. This observation is in line with the studies by Hoesli & Reka (2011) and Hui & Chan (2014), who suggest that contagion between U.S. equity and real estate markets is most significant among the countries they observe. Meanwhile, Australian stock and real estate markets are the least correlated among the three. And the systemic risk contribution of real estate-stock pair in China is the most stable one.

Panel A: Upside and downside VaRs and CoVaRs for China



Panel B: Upside and downside VaRs and CoVaRs for United States



Panel C: Upside and downside VaRs and CoVaRs for Australia

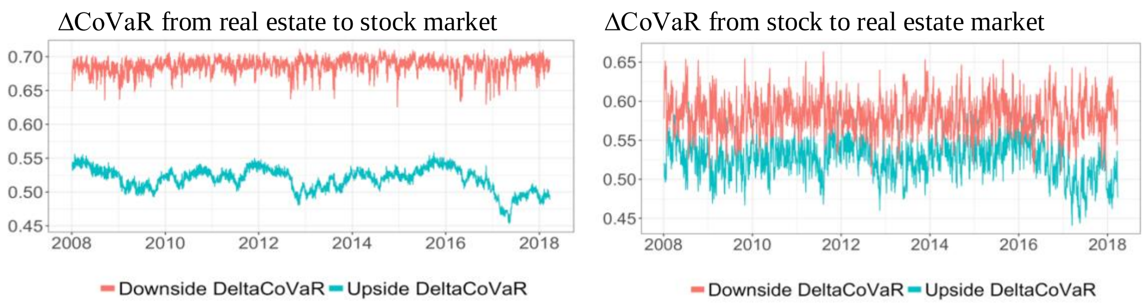


Fig. 8. Upside and downside delta conditional value-at-risk (ΔCoVaR). Notes: ΔCoVaRs are calculated using Eqs. (23).

Table 10
Descriptive statistics of ΔCoVaR

	ΔCoVaR from real estate markets to stock markets		ΔCoVaR from stock markets to real estate markets	
	Upside	Downside	Upside	Downside
CHN	0.6304	0.7635	0.6100	0.6369
	(.0187)	(.0227)	(.0112)	(.0107)
US	0.6890	0.8840	0.5581	0.6728
	(.0499)	(.0667)	(.0286)	(.0347)
AUS	0.5341	0.6868	0.5432	0.5820
	(.007)	(.0146)	(.0205)	(.0245)

Notes: This table presents the mean and the standard deviation (in parenthesis) of the ΔCoVaR. Panel A presents ΔCoVaR from real estate markets to stock markets, and Panel B presents ΔCoVaR from stock markets to real estate markets.

Regarding the systemic risk contribution of real estate markets to stock markets, the average and standard deviations for the downside ΔCoVaRs are much larger than their counterparts for the upside ΔCoVaRs. It implies that stock markets react more strongly to extreme downside movement than to extreme upside movement in the real estate markets. Specifically, U.S. stock market is highly affected by the extreme movement in the real estate market, with few drops in ΔCoVaR in 2015 and at the start of 2018.

Similarly, concerning the systemic risk contribution of stock markets to real estate markets, the average and standard deviations for the downside ΔCoVaRs are much larger than those of the upside ΔCoVaRs. The gap is the largest in the U.S market. This observation is confirmed by the K-S test results in **Table 11**.

Table 11
Upside and downside ΔCoVaR asymmetry

	H_0: Downside ΔCoVaR = Upside ΔCoVaR H_1: Downside ΔCoVaR ≠ Upside ΔCoVaR	
	from real estate markets to stock markets	from stock markets to real estate markets
CHN	0.9737	0.7678
	(0.0000)	(0.0000)
US	0.9439	0.9526
	(0.0000)	(0.0000)
AUS	0.9989	0.6286
	(0.0000)	(0.0000)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests test the null hypothesis of no difference between the downside and upside systemic risk contribution. The p-values for the KS statistic are in the parentheses.

We also observe stronger risk contribution of the real estate to stock market than the risk contribution of the stock to real estate market, except for the downside risk spillover in

Australian markets. Considering the results mentioned above, the downside risk contribution of real estate to stock market is worth extra attention.

6.4 Short-term and long-term risk spillovers

In order to distinguish between the short-term and long-term risk spillovers, we employ the VMD results to re-estimate CoVaR and ΔCoVaR risk measures. The averages and standard deviations of the CoVaRs based on all, short-term and long-term data are presented in **Table 12**. As indicated by the KS test in **Table 13**, there are significant differences between long-term and short-term downside and upside CoVaRs in the U.S. markets, and between long-term and short-term downside CoVaRs in Chinese markets. However, the U.S. and Chinese markets have opposite results at different time horizons. The bidirectional long-run risk spillovers are significantly higher than short-run risk spillovers in the U.S. markets. On the contrary, the downside risk spillover effect is stronger in the short-term for Chinese markets, as Su et al. (2018) suggest that Chinese stock and real estate markets are generally segmented in the short run but are integrated in the long run.

Table 12

Descriptive statistics of raw, short-term and long-term CoVaR

	Upside			Downside		
	CoVaR (all data)	CoVaR (short term)	CoVaR (long term)	CoVaR (all data)	CoVaR (short term)	CoVaR (long term)
Panel A: VaR of stock markets and CoVaR from stock markets to real estate markets						
CHN	5.6928	5.5968	5.6508	-6.0734	-5.9389	-6.0823
	(3.4355)	(3.4101)	(3.4872)	(3.7417)	(3.6868)	(3.7445)
US	3.3769	3.2767	3.2234	-4.1149	-3.9840	-3.9346
	(2.4121)	(2.3515)	(2.4315)	(2.9073)	(2.814)	(2.9483)
AUS	2.7467	2.7356	2.7488	-3.2111	-3.1940	-3.1962
	(1.3308)	(1.3569)	(1.3346)	(1.5871)	(1.5692)	(1.5884)
Panel B: VaR of real estate markets and CoVaR from real estate markets to stock markets						
CHN	5.5423	5.4521	5.4980	-5.4661	-5.3548	-5.4729
	(2.8799)	(2.8651)	(2.9239)	(2.8958)	(2.8692)	(2.902)
US	4.7022	4.5946	4.5654	-5.7018	-5.5719	-5.5389
	(4.525)	(4.4524)	(4.565)	(5.4942)	(5.4257)	(5.5296)
AUS	3.6812	3.6706	3.6962	-3.8467	-3.8258	-3.8335
	(2.2191)	(2.2613)	(2.268)	(2.31)	(2.2907)	(2.3319)

Notes: This table presents the mean and the standard deviation (in parenthesis) of the CoVaR calculated using all data, short-term and long-term data. CoVaR (short term) and CoVaR (long term) represent CoVaR in the short term and long term, respectively.

Table 13

Tests of equalities of long-term and short-term CoVaR

	H0: Long-term CoVaR = short-term CoVaR H1: Long-term CoVaR ≠ short-term CoVaR			
	CoVaR from real estate markets to stock markets		CoVaR from stock markets to real estate markets	
	Upside	Downside	Upside	Downside
CHN	0.0163	0.0503	0.0190	0.0484
	(.8844)	(.0029)	(.7416)	(.0047)
US	0.0580	0.0454	0.0640	0.0604
	(.0004)	(.0108)	(.0001)	(.0002)
AUS	0.0335	0.0178	0.0223	0.0086
	(.0985)	(.7852)	(.5151)	(1.0)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests the hypothesis of no difference between the long-term and short-term CoVaR. The p-values for the KS statistic are in the parentheses.

Given that U.S. has both different upside and downside CoVaRs at different time horizons, we continue to explore its ΔCoVaR at different horizons. **Fig. 9.** presents the comparison between long-term and short-term ΔCoVaR from real estate to stock market and vice versa. For the U.S. market, we could observe stronger systemic risk contribution of real estate to stock market than that of the opposite direction despite different time horizons. In the long-run, the bidirectional systemic risk spillovers decrease with increasing volatility since 2012, but it only happens after 2016 in the short-run horizon.

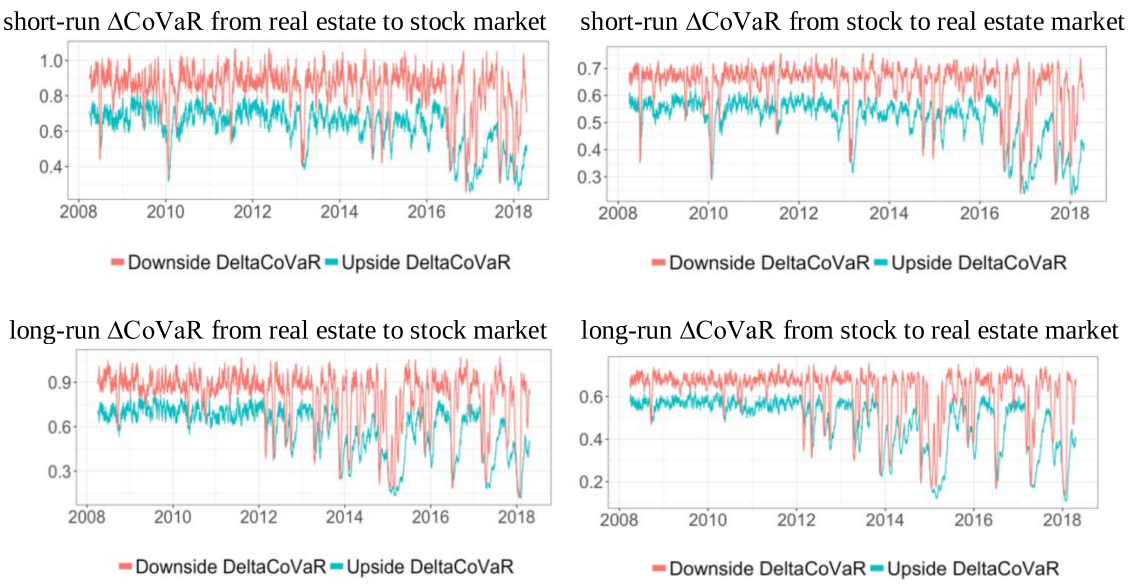


Fig .9. Short-term and long-term upside and downside delta conditional value-at-risk (ΔCoVaR) from U.S. real estate to U.S. stock market and vice versa. Notes: ΔCoVaRs are calculated using Eqs. (23).

7. Robustness tests

7.1 Alternative copulas for risk spillovers

We calculate upside and downside CoVaRs based on the estimated SJC copulas, which are capable to capture the upper and lower tail dependence of the data. We check the robustness of our findings on the CoVaR by applying time-varying Gumbel and Clayton copulas on the marginal models. Gumbel copula only has upper tail dependence while Clayton copula only has lower tail dependence. The estimated parameters for these two copulas are already shown in **Table 5** in Section 5.2, and the dynamic tail dependence is shown in **Fig. 12** in the appendix. We then rerun the upside and downside CoVaRs as well as ΔCoVaR and their respective K-S tests for each country. The results are shown in the **Table 15-17** in the appendix. We find very similar results with those we get by using SJC copulas. It suggests significant differences between the values of VaR-CoVaR pairs in all situations, which indicates significant bidirectional upside and downside risk spillovers. We also observe that downside risk spillovers are significantly larger than upside spillovers, measured by normalized downside and upside CoVaR and ΔCoVaR. From the descriptive statistics of ΔCoVaR in **Table 18**, we also find that risk spillover effect from real estate to stock markets is stronger than that of the opposite direction. Those conclusions are in line with what we observe with the CoVaRs and ΔCoVaR based on SJC copulas.

7.2 Alternative innovation assumptions for marginal models

In addition to the original assumption that innovations are following a skewed Student-t distribution, we further test our models on Student-t distributed and normal distributed innovations. The fitted marginal models are included in **Table 19** and **Table 20** in the appendix. Note that, the best fitted model for Australian real estate market is an ARFIMA(1, d, 1)-GARCH(1, 1) under both Student-t and normal distribution assumptions. Even though the fraction terms are larger than zero, indicating the existence of long memory or long-range dependence, these terms are not statistically significant. Therefore, we conclude that the markets we study do not have apparent fractionally-integrated relationship.

For real estate market, the best marginal model estimation yields similar results with normal distributed errors and with Student-t distributed errors. Specifically, ARMA(1,0)-GARCH(2,2), ARMA(2,2)-GARCH(1,2), ARMA(1,1)-GARCH(1,2) models are the best model for Chinese, U.S. and Australian real estate markets. For the stock market, the best marginal model estimation with normal distributed innovations and with Student-t distributed innovations are different. All the shape parameters of the best model with Student-t distributed errors are significant at 1% confidence level, indicating significant excess kurtosis.

Using models with Student-t innovations and models with normal innovations, we re-conduct our analysis on the copulas and CoVaRs for each country. The SJC time varying copula estimates are presented in **Table 21** and **Fig. 13**. The upper and lower tail dependences between the stock and real estate market are very similar despite the distribution form of the innovations for marginal models.

Because of the differences in marginal distribution models with normal distributed errors and Student-t distributed errors, the VaR and CoVaR results are different as presented in **Table 22-23**. The absolute values of upside and downside VaRs are slightly larger with normal distributed models, while the absolute values of upside and downside CoVaRs are significantly larger with Student-t distributed models, which account for excess kurtosis and stronger tail dependence. Taking the negative skewness into accounts, we would get more extreme downside values but better upside counterparts, as shown in Section 6.1 with skewed Student-t distributed models. Based on the VaR backtesting results in **Table 24**, we reject the null of correct exceedances for VaRs in more than half of the cases when applying models with normal distributed innovations and Student-t distributed innovations, indicating that those VaR models are not convincing.

K-S test results in **Table 25** indicates significant differences between all VaR-CoVaR pairs regardless of the marginal models used. It implies that an extreme upside (downside) movement in the real estate market would have a positive (negative) impact on stock market, and vice versa.

Fig. 14-15 present the time varying delta CoVaR (ΔCoVaR) results for models with normal distributed and Student-t distributed innovations. It could be clearly seen that the systemic risk effect measured by ΔCoVaR is significantly larger when using Student-t distributed models. However, bidirectional systemic risk spillovers can still be observed in spite of different scales, and the main conclusions remain unchanged.

In general, the stock markets react more strongly to extreme downside movement than to extreme upside movement in the real estate markets. Specifically, U.S. stock market is highly affected by the extreme movement in the real estate market, but there is a declining trend in the systemic risk spillovers with high volatility since 2014. Similarly, the real estate markets react more strongly to extreme downside movement than to extreme upside movement in the stock markets. Those results are confirmed by the K-S tests in **Table 26**.

By comparing the bidirectional systemic impact, we can also conclude that the systemic risk contribution of real estate to stock markets are higher than the systemic impact of the opposite direction.

8. Conclusions and future research

In this paper, we examine the tail dependence as well as risk spillovers between stock markets and real estate markets in three countries. We begin with ARMA-GARCH models which capture the dynamics of the return series. We apply both static and time-varying copulas to study the co-movements of the real estate markets and stock markets in the short and long term. To distinguish the short- and long-term effects, we combine the copula with the variational mode decomposition (VMD) method. We obtain the dependence structures from copula analysis and utilize them to quantify the upside and downside risk spillovers measured by conditional value-at-risk (CoVaR) from real estate to stock markets and vice versa. We also assess the systemic risk contribution of real estate-stock pair in each country.

Our study provides strong evidence of dynamic tail dependence between real estate markets and stock markets in China, U.S., and Australia. Real estate markets commove closely with stock markets. For China and U.S., upper tail dependence and lower tail dependence clearly decrease during bull markets while increase sharply during shocks and crises. These results are important for portfolio managers when considering the limitation of risk diversification effect from real estate asset.

By estimating risk measures for each market, we are able to have a deeper discussion on the topic of portfolio risk management. We observe that U.S. and Australian real estate markets are much riskier than the stock markets in both bearish and bullish market conditions, while Chinese real estate markets are not. This difference may due to the fact that Chinese real estate market is in a stage of constant growth due to relatively strong economic growth and high speed of urbanization.

The estimate results of CoVaR and ΔCoVaR show that there are significant risk spillovers both from stock to real estate and from real estate to stock in all three markets. In general, the correlation and risk spillover effect between real estate-stock pair is the strongest for U.S, reflecting the fact that U.S. is the center of shock of the mortgage crisis and global financial crisis. The risk spillover effect is the weakest in Australia, showing that A-REITs is a relatively successful indirect real estate investment vehicle in terms of portfolio risk diversification.

In addition, we observe asymmetricities in systemic impact between stock and real estate markets. First, the risk spillovers, measured by CoVaR, for long positions are significantly larger than the risk spillovers for short positions. In other words, stock markets react stronger to extreme downside movement in real estate markets than to extreme upside movement, and vice versa. Second, in spite of the bidirectional risk spillovers, the systemic risk effect measured by ΔCoVaR from real estate to stock markets is larger than the systemic risk effect of the opposite direction in all three markets. It suggests that when investing in both markets, investors should remain especially vigilant of the downside risk spillovers from the real estate markets. Lastly, by applying the VMD result series, we are able to analyze the difference between the short- and long-term systemic risk spillover. For U.S. markets, the bidirectional short-run systemic risk effect is significantly stronger than its long-run counterpart, while for Chinese markets, the long-run downside risk spillover effect is stronger. No significant difference between short- and long-run systemic impact is found for Australian markets.

There are several implications for individual and institutional investors as well as portfolio managers. Investors should be aware of the bidirectional risk spillover effect between the real estate and stock market for both long and short positions, especially during the crisis period when the tail dependences tend to increase. The credit price effect for long position is the most significant compared with wealth effect or credit price effect for short positions. However, investors need to take the time horizons into consideration. Long-run and short-run risk spillovers have different implications in different markets. Additionally, country specific characteristics are also worth attention.

We suggest that future research should investigate dependence of real estate markets and stock markets in a longer time period by including more data. By doing so, structural break test can also be implemented to test the effects of extreme events such as subprime mortgage crisis and global financial crisis on the dependence structure. In addition to VMD, applying wavelet decomposition can provide further examination of co-movement and causality between markets in both the time and frequency domains.

Other marginal models besides ARFIMA-GARCH types models could be employed to better capture the characteristics of the return series. For example, various asymmetric GARCH models could be explored to capture the asymmetry in the impact of positive shocks and negative shocks, as in real world, unexpected bad news often increases conditional volatility more than unexpected good news.

Further research could also address the similarities and differences in risk spillover effect of real estate-stock pairs by including more emerging markets and developed markets, to see whether the systemic impact varies across regions.

References

Abadie, A., 2002. Bootstrap tests for distributional treatment effects in instrumental variables models, Journal of the American Statistical Association 97, 284–292.

Adrian, Tobias, and Markus K. Brunnermeier, 2016. CoVaR, American Economic Review, 106 (7), 1705-41.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity Journal of Economics 31, 307-327.

Bollerslev, T., 1987. A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return, Review of Economics and Statistics 69, 542-547.

Case, B., Y. Yang and Y. Yildirim, 2011. Dynamic Correlations among Asset Classes: REIT and Stock Returns. The Journal of Real Estate Finance and Economics 44, 298-318.

Chang, M., Salin, V. and Jin Y., 2011. Diversification effect of real estate investment trusts: Comparing copula functions with kernel methods, Journal of Property Research 28(3), 189-212.

Chen, W., et al. 2014. Financial market volatility and contagion effect: A copula–multifractal volatility approach, Physica A 398, 289-300.

Clayton, J., and MacKinnon, G., 2001. The Time-Varying Nature of the Link between REIT, Real Estate and Financial Asset Returns. Journal of Real Estate Portfolio Management 7, 43-54.

Cotter, J. and S. Stevenson. 2006. Multivariate Modeling of Daily REIT Volatility. The Journal of Real Estate Finance and Economics 32, 305–325.

Dickey, D. and Fuller W., 1979. Distribution of the Estimators for Autoregressive Time Series with a Unit Root, Journal of the American Statistical Association 74, 427-431.

Ding, H., Chong T.T., Park S.Y., 2014, Nonlinear dependence between stock and real estate markets in China, Economics Letters, 124 (3), 526-529.

Dragomiretskiy, K., & Zosso, D., 2014. Variational mode decomposition. IEEE Transactions on Signal Processing 62, 531–544.

Engle, R., 2002. Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models, Journal of Business and Economic Statistics 20, 339-350.

Fernandez, C., Steel, M., 1998. On Bayesian modelling of fat tails skewness. Journal of the American Statistical Association 93, 359–371.

Girardi, G., Ergün, A.T., 2013. Systemic risk measurement: multivariate GARCH estimation of covar, Journal of Banking and Finance 37, 3169–3180.

Glascock, J., C. Lu and R. So. 2000. Further Evidence on the Integration of REIT, Bond and Stock Returns, Journal of Real Estate Finance and Economics 20(2), 177–194.

Granger, C. and R. Joyeux, 1980. An Introduction to Long Memory Time Series Models and Fractional Differencing, Journal of Time Series Analysis 1(1), 15-39.

R. Heaney, S. Sriananthakumar, 2012. Time-varying correlation between stock market returns and real estate returns, Journal of Empirical Finance 19 (4), 583-594.

Hoesli, M. & Reka, K. J, 2013. Volatility Spillovers, Comovements and Contagion in Securitized Real Estate Markets, The Journal of Real Estate Finance and Economics 47(1): 1-35.

Hosking, J.R.M., 1980. The multivariate portmanteau statistic, Journal of the American Statistical Association 75, 602–608.

Huang, J.Z. and Zhong, Z.D., 2006. Time-variation in diversification benefits of commodity, REITs, and TIPS, paper presented at the 2006 China International Conference in Finance, Xi'an, July 17-20.

Hudson, S., Fabozzi, F.J. and Gordon, J.N., 2003. Why real estate? Journal of Portfolio Management Special Real Estate 29 (5), 12-25.

Hui E.C.M., Chan K.K.K., 2014. The global financial crisis: Is there any contagion between real estate and equity markets? Physica A 405, 216-225.

Joe, H., Xu, J.J., 1996. The estimation method of inference functions for margins for multivariate models. Technical Report No. 166. Department of Statistics, University of British Columbia.

Kapopoulos,P. and Siokis,F., 2005. Stock and real estate prices in Greece: wealth versus ‘credit-price’ effect, Applied Economics Letters 12, 125-128

Kiohos, A., Babalos, V. and Koulakiotis, A., 2017. Wealth effect revisited: novel evidence on long term co-memories between real estate and stock markets, Finance Research Letters 20(2), 217-222.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., Shim, Y., 1992. Testing the null hypoth- esis of stationarity against the alternative of a unit root: how sure are we that economic time series are non-stationary? Journal of Economics 54, 159–178.

S. Lahmiri, 2015. Long memory in international financial markets trends and short movements during 2008 financial crisis based on variational mode decomposition and detrended fluctuation analysis, Physica A 437, 130-138.

Lambert, P., Laurent, S., 2000. Modelling Skewness Dynamics in Series of Financial Data, Discussion Paper. Institut de Statistique, Louvain-la-Neuve.

X-L. Li, T. Chang, S. Miller, M. Balcilar, R. Gupta, 2015. The co-movement and causality between the U.S. housing and stock markets in the time and frequency domains, International Review of Economics & Finance 38, 220-233.

T.C. Lin, Z.-H. Lin, 2011. Are stock and real estate markets integrated? An empirical study of six Asian economies, Pacific-Basin Finance Journal 19(5), 571-585.

Liow, K.H. 2012. Co-movements and correlations across Asian securitized real estate and stock markets, Real Estate Economics 40(1), 97-129.

Liow, K.H., Z. Chen and J. Liu. 2011. Multiple Regimes and Volatility Transmission in Securitized Real Estate Markets, The Journal of Real Estate Finance and Economics 42(3), 295–328.

Liow, K. H., Yang, H. S., 2005. Long-term co-memories and short-run adjustment: securitized real estate and stock markets, The Journal of Real Estate Finance and Economics 31(3), 283-300.

Markowitz, H. 1952. Portfolio Selection. The Journal of Finance 7(1), 77-91.

Maurer, R., Reiner, F., Rogalla, R., 2004. Return and risk of German open-end real estate funds, Journal of Property Research 21 (3), 209-233.

Okunev, J., & Wilson, P. J., 1997. Using Nonlinear Tests to Examine Integration Between Real Estate and Stock Markets, Real Estate Economics 25(3), 487-503.

Okunev,J., R. Zurbruegg,P. Wilson, 2000. The causal relationship between real estate and stock markets, Journal of Real Estate Economics 21, 251-261.

Patton, Andrew J., 2006. Modelling Asymmetric Exchange Rate Dependence. International Economic Review 47(2), 527-556.

Reboredo, J.C., Ugolini, A., 2015. Systemic risk in European sovereign debt markets: A CoVaR-copula approach, Journal of International Money and Finance 51, 214–244.

Rodriguez, J.C., 2007. Measuring Financial Contagion: A Copula Approach, Journal of Empirical Finance 14(3), 401-423.

Rong, N., Trück, S., 2010. Returns of REITS and stock markets: Measuring dependence and risk, Journal of Property Investment & Finance 28 (1), 34-57.

Shahzad S.J.H., Kumar R.R., Ali S., Ameer S., 2016. Interdependence between Greece and other European stock markets: A comparison of wavelet and VMD copula, and the portfolio implications, Physica A 457, 8-33.

Shapiro, S. S., Wilk, M. B., 1965. An analysis of variance test for normality (complete samples), Biometrika 52 (3–4), 591–611.

Sharpe, William F., 1994. The Sharpe Ratio, The Journal of Portfolio Management 21 (1), 49–58.

Sim, S.-H., & Chang, B.-K., 2006. Stock and real estate markets in Korea: wealth or credit–price effect, Journal of Economic Research 11, 99–122.

Simon, S., Ng, W.L., 2009. The Effect of the Real Estate Downturn on the Link between REITs and the Stock Market, The Journal of Real Estate Portfolio Management 15(3), 211-220.

Sklar, A., 1959. Fonctions de r´epartition `a n dimensions et leurs marges, Publications de l'Institut de Statistique de l'Universit´e de Paris 8, 229–231.

Su, CW., 2011. Non-linear causality between the stock and real estate markets of Western European countries: Evidence from rank tests, Economic Modelling 28, 845-851.

Su, CW., Yin, XC., Chang, HL. et al. 2018. Are the stock and real estate markets integrated in China? Journal of Economic Interaction and Coordination 1–20.

Sun, W., Rachev, S., Fabozzi, F. J., & Kalev, P. S. 2009. A new approach to modeling co-movement of international equity markets: evidence of unconditional copula-based simulation of tail dependence, Empirical Economics, 36 (1), 201-229.

Wilson, P., Okunev, J., 1997. Using nonlinear tests to examine integration between real estate and stock markets, Real Estate Economics 25(3), 487-503.

Worzala, E., Sirmans, C.F., 2003.Investing in international real estate stocks: a review of the literature, Urban Studies 40, 1115-1149.

Higgins, D., 2007. Placing commercial property in the Australian capital market, RICS research paper series 7, 9-32

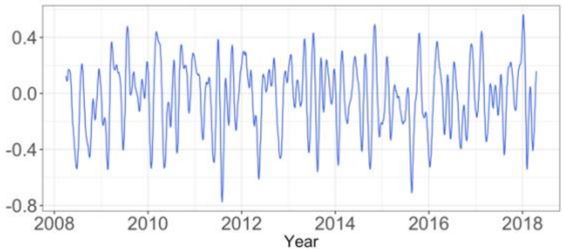
Wildau, G., Jia, Y., 2018. China looks to Reits to ease housing woes, Financial Times. Retrieved from https://www.ft.com/

Mensi, W., Hammoudeh, S., Shahzad, S.J.H., Shahbaz, M., 2017. Modeling systemic risk and dependence structure between oil and stock markets using a variational mode decomposition-based copula method, Journal of Banking and Finance 75, 258-279.

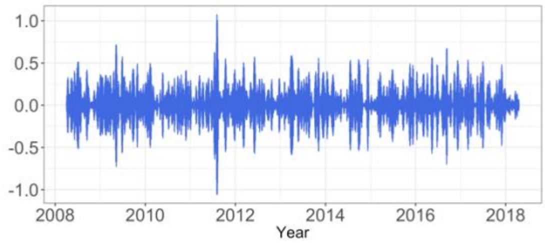
Appendix

Panel A: Variational mode decomposition for U.S. stock return series

VMD1

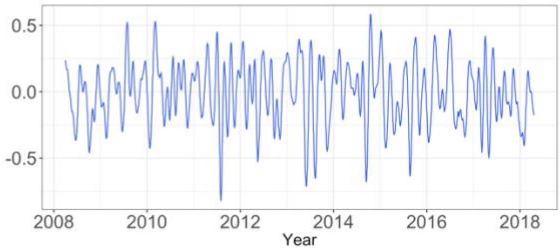


VMD 10



Panel B: Variational mode decomposition for U.S. real estate return series

VMD1



VMD 10

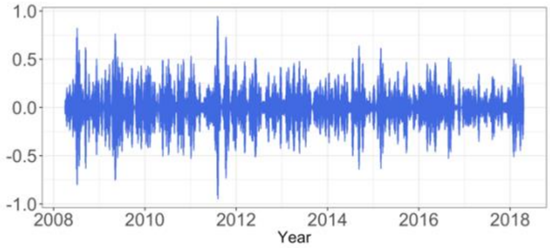
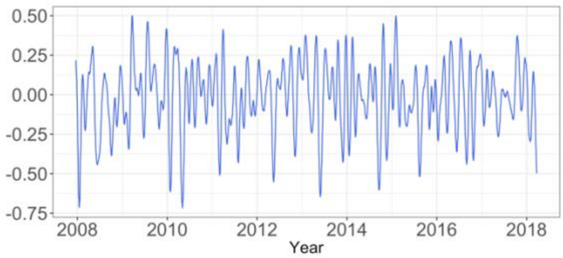


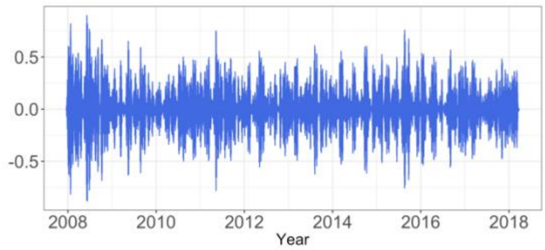
Fig. 10. Variational mode decomposition for mode 1 (long-term) and mode 10 (short-term) for U.S. stock and real estate return series

Panel A: Variational mode decomposition for Australian stock return series

VMD1

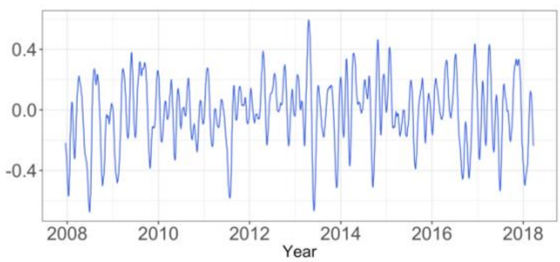


VMD 10



Panel B: Variational mode decomposition for Australian real estate return series

VMD1



VMD 10

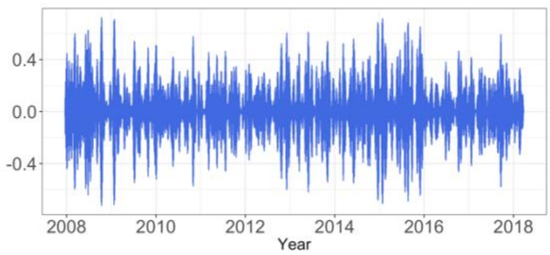


Fig. 11. Variational mode decomposition for mode 1 (long-term) and mode 10 (short-term) for Australian stock and real estate return series

Table 14

SJC time varying copula estimates with skewed Student-t distributed models

SJC Copula	CHN			US			AUS		
	All	Short term	Long term	All	Short term	Long term	All	Short term	Long term
ωu	0.057	2.425	2.160	0.096	1.642	0.939	0.030	1.905	1.561
	0.031	0.240	0.009	0.029	0.239	0.164	0.017	0.331	0.291
βu	0.977	-0.877	-0.261	0.958	-0.270	0.186	0.986	-1.001	-0.157
	0.015	0.076	0.023	0.012	0.176	0.199	0.008	0.209	0.198
αu	-0.360	-13.574	-13.563	-0.622	-10.530	-6.894	-0.194	-14.848	-14.817
	0.188	1.765	0.022	0.190	1.725	0.126	0.108	2.643	2.591
ωl	1.042	2.723	2.093	0.461	2.395	1.422	1.245	0.831	1.716
	0.531	0.183	0.046	0.169	0.112	0.005	0.355	0.156	0.275
βl	0.427	-0.994	-0.685	0.743	-0.654	0.211	-0.708	-0.309	-0.567
	0.306	0.004	0.045	0.098	0.107	0.004	0.136	0.057	0.234
αl	-4.581	-14.964	-4.534	-2.597	-15.000	-7.067	-7.844	-7.161	-7.763
	2.307	1.246	0.568	0.984	0.327	0.004	1.962	0.906	1.245
AIC	-2953.6	-2331.6	-3613.2	-1754.1	-1805.2	-1991.0	-1383.4	-1446.5	-2026.5

Note: This table reports the ML estimates for the parameters of different static copulas as well as their respective AICs, based on all, short-term and long-term models. Standard errors are reported in the parentheses.

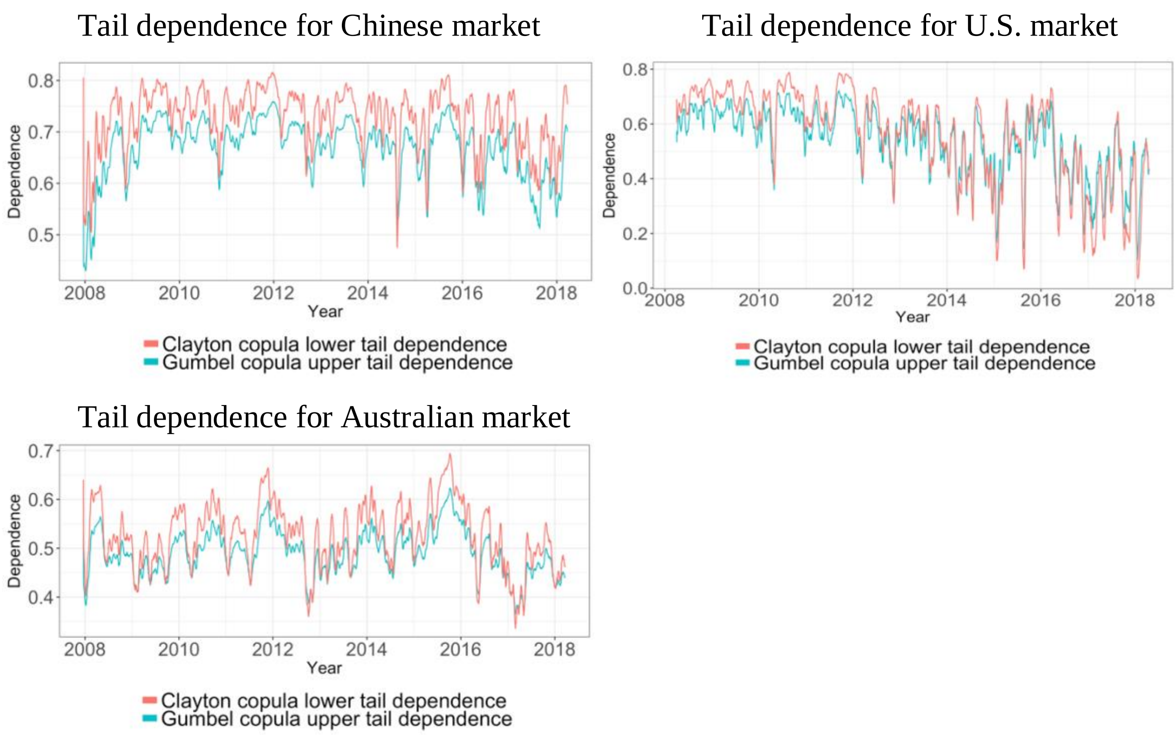


Fig. 12. Gumbel (upper) and Clayton (lower) tail dependence in Chinese, U.S. and Australian markets

Table 15

Tests of equalities of VaR and CoVaR using Gambel and Clayton copulas

	CoVaR from real estate markets to stock markets		CoVaR from stock markets to real estate markets	
	Upside	Downside	Upside	Downside
CHN	0.6838	0.7167	0.7198	0.7210
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
US	0.5788	0.6668	0.5128	0.5843
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
AUS	0.6078	0.6078	0.6662	0.7015
	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests the hypothesis of no systemic impact between the stock markets and real estate markets. The p-values for the KS statistic are in the parentheses.

Table 16

Upside and downside CoVaR asymmetry from stock markets to real estate markets and vice versa using Gambel and Clayton copulas

$$H_0: \frac{CoVaR}{VaR}\ (D) = \frac{CoVaR}{VaR}\ (U),\ H_1: \frac{CoVaR}{VaR}\ (D) > \frac{CoVaR}{VaR}\ (U)$$

	from real estate markets to stock markets	from stock markets to real estate markets
CHN	0.9729	0.7860
	(0.0000)	(0.0000)
US	0.9250	0.9364
	(0.0000)	(0.0000)
AUS	0.9993	0.7164
	(0.0000)	(0.0000)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests test the null hypothesis of no difference between the downside and upside systemic risk contribution. The p-values for the KS statistic are in the parentheses.

Table 17

Upside and downside ΔCoVaR asymmetry from stock markets to real estate markets and vice versa using Gambel and Clayton copulas

	H0: Downside ΔCoVaR = Upside ΔCoVaR H1: Downside ΔCoVaR ≠ Upside ΔCoVaR	
	from real estate markets to stock markets	from stock markets to real estate markets
CHN	0.9737	0.7678
	(0.0000)	(0.0000)
US	0.9412	0.9574
	(0.0000)	(0.0000)
AUS	0.9996	0.7204
	(0.0000)	(0.0000)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests test the null hypothesis of no difference between the downside and upside systemic risk contribution. The p-values for the KS statistic are in the parentheses.

Table 18

Descriptive statistics of ΔCoVaR using Gambel and Clayton copulas

	ΔCoVaR from real estate markets to stock markets		ΔCoVaR from stock markets to real estate markets	
	Upside	Downside	Upside	Downside
CHN	0.6325	0.7105	0.6120	0.6379
	(.0198)	(.0204)	(.0126)	(.0107)
US	0.6890	0.8811	0.5581	0.5581
	(.0499)	(.0637)	(.0286)	(.0286)
AUS	0.5341	0.6327	0.5432	0.5889
	(.007)	(.0077)	(.0205)	(.0224)

Notes: This table presents the mean and the standard deviation (in parenthesis) of the ΔCoVaR. Panel A presents ΔCoVaR from real estate markets to stock markets, and Panel B presents ΔCoVaR from stock markets to real estate markets.

Table 19

Marginal model estimations ARFIMA-GARCH with normal distribution errors

	CHN real estate	U.S. real estate	AUS real estate	CHN stock market	U.S. stock market	AUS stock market
Cst(M)	0.054264**	0.045769***	0.036883**	0.066968***	0.069569***	0.032633**
	(0.029092)	(0.015774)	(0.020402)	(0.016913)	(0.013233)	(0.015284)
AR(1)	0.03556*	0.823158***	0.688915***	0.199663***	-0.409993***	-0.660619***
	(0.020798)	(0.022482)	(0.090114)	(0.013592)	(0.023671)	(0.169292)
AR(2)		0.098914***		0.744327***	0.559657***	
		(0.021914)		(0.014217)	(0.017467)	
MA(1)		-0.873528***	- 0.781095***	-0.219688***	0.349854***	0.649458***
		(0.00303)	(0.066637)	-0.000214	(0.005587)	(0.17089)
MA(2)		-0.068259***		-0.74496***	-0.622519***	
		(0.001675)		(0.000045)	(0.000246)	
Arfima			0.044017			
			(0.069583)			
Cst(V)	0.060359**	0.021866***	0.013826**	0.050652***	0.027973***	0.011984**
	(0.025175)	(0.007956)	(0.005456)	(0.017689)	(0.007952)	(0.004888)
Alpha1	0.077106***	0.133648***	0.090336***	0.04997***	0.09998**	0.092698***
	(0.025718)	(0.030099)	(0.017738)	(0.012107)	(0.040934)	(0.020201)
Alpha2	0.053478			0.092827***	0.059475	
	(0.036444)			(0.019573)	(0.049861)	
Beta1	0.449746	0.701925***	0.52815***	0.019535	0.821121***	0.89697***
	(0.223768)	(0.161398)	(0.066546)	(0.027664)	(0.031365)	(0.021122)
Beta2	0.400021	0.158309	0.37168***	0.823418***		
	(0.200095)	(0.148947)	(0.064277)	(0.029174)		
AIC	3.722	3.3337	3.0305	3.7587	2.6868	2.6111
Ljung-Box	20.764	18.067	23.043	22.295	23.306	20.206
Ljung-Box squared	23.482	23.146	32.481**	25.507	13.127	16.544
Hosking	20.7476288	18.0530181	23.025723	22.2781661	23.2878591	23.025723
Hosking squared	22.6326759	22.408046	31.282166	24.9587222	12.626294	31.282166
ARCH	22.57	22.32	33.2*	24.86	12.56	31.2

Notes:This table reports the ML estimates and the robust standard deviations in parenthesis for the parameters of the marginal distribution model. Ljung-Box and Ljung-Box squared are Ljung-Box autocorrelation test statistics for standardized residuals and squared standardized residuals, respectively, computed with 20 lags. Hosking and Hosking squared are the Hosking (1980) autocorrelation test statistics standardized residuals and squared standardized residuals, respectively, computed with 20 lags. ARCH is the test statistics of Portmanteau-Q test for the ARCH effect in the standardized residuals, computed with 20 lags . K–S is the test statistics of Kolmogorov–Smirnov test on the adequacy of normal distribution. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Table 20

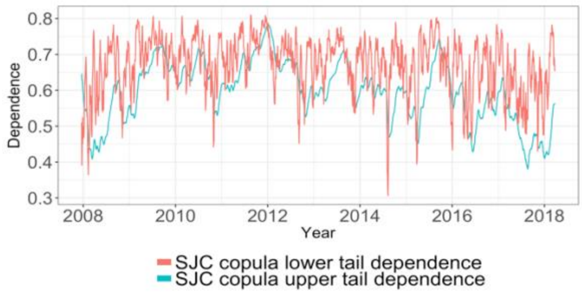
Marginal model estimations ARFIMA-GARCH with Student-t distribution errors

	CHN real estate	U.S. real estate	AUS real estate	CHN stock market	U.S. stock market	AUS stock market
Cst(M)	0.054765**	0.058914***	0.034742*	0.080981***	0.084169***	0.040202***
	(0.027669)	(0.015339)	(0.018001)	(0.022972)	(0.009277)	(0.015112)
AR(1)	0.03209*	0.771036***	0.697503***	0.78916***	0.902257***	-0.425647**
	(0.019448)	(0.021882)	(0.106434)	(0.11558)	(0.01291)	(0.178346)
AR(2)		0.13021***				
		(0.021231)				
MA(1)		-0.818457***	-0.772352***	-0.820671***	-0.974719***	0.430095**
		(0.002345)	(0.085793)	(0.107537)	(0.009519)	(0.179765)
MA(2)		-0.105833***			0.039529***	0.027897
		(0.001775)			(0.003532)	(0.020041)
Arfima			0.026789			
			(0.063077)			
Cst(V)	0.043279***	0.014853***	0.014278***	0.038587**	0.020702***	0.008056*
	(0.015984)	(0.005271)	(0.005328)	(0.014293)	(0.007857)	(0.004145)
Alpha1	0.069211***	0.11492***	0.092518***	0.041496**	0.071697**	0.081843***
	(0.02503)	(0.021316)	(0.017985)	(0.01818)	(0.032134)	(0.020082)
Alpha2	0.039173			0.067392**	0.101793**	
	(0.028717)			(0.023438)	(0.047556)	
Beta1	0.54524***	0.739195***	0.524641***	0.456421	0.82551***	0.903202***
	(0.105171)	(0.111814)	(0.07446)	(0.266016)	(0.034218)	(0.011523)
Beta2	0.332775***	0.143111	0.37308***	0.424645		0.008996
	(0.095964)	(0.106049)	(0.071663)	(0.245571)		(0.009696)
Shape	9.200094***	8.279293***	12.257666***	7.450393***	4.973168***	9.750381***
	(1.491851)	(1.215832)	(2.467459)	(1.019325)	(0.490551)	(1.745374)
AIC	3.7024	3.3079	3.0219	3.7324	2.6238	2.5933
Ljung-Box	21.128	17.856	22.434	22.479	21.741	16.982
Ljung-Box squared	29.169*	28.878	32.02**	30.923*	21.455	19.556
Hosking	21.1117161	17.8419762	22.417312	22.4614284	21.7237429	16.9695472
Hosking squared	28.0365237	27.170347	30.880257	29.523485*	20.845328	17.651917
ARCH	28	27.1	30.8	29.42*	20.76	17.59

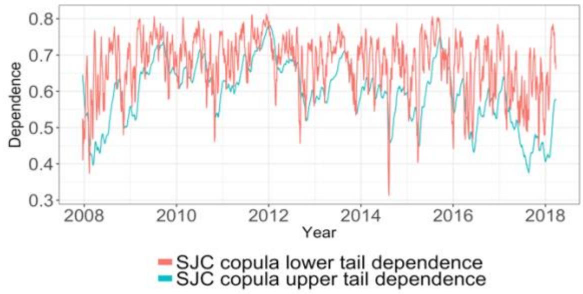
Notes:This table reports the ML estimates and the robust standard deviations in parenthesis for the parameters of the marginal distribution model. Ljung-Box and Ljung-Box squared are Ljung-Box autocorrelation test statistics for standardized residuals and squared standardized residuals, respectively, computed with 20 lags. Hosking and Hosking squared are the Hosking (1980) autocorrelation test statistics standardized residuals and squared standardized residuals, respectively, computed with 20 lags. ARCH is the test statistics of Portmanteau-Q test for the ARCH effect in the standardized residuals, computed with 20 lags . K–S is the test statistics of Kolmogorov–Smirnov test on the adequacy of student-t distribution. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Panel A: SJC copulas for Chinese market

with normal distributed innovations

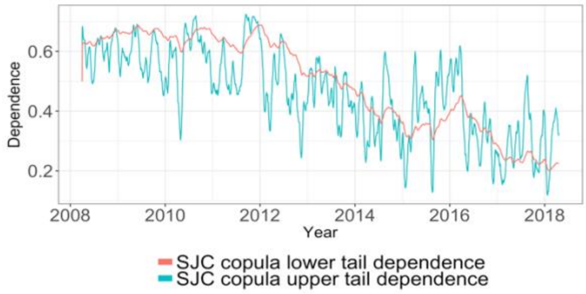


with Student-t distributed innovations

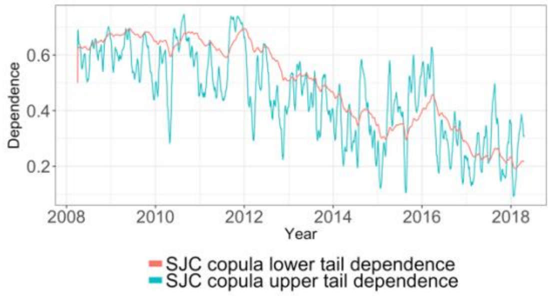


Panel B: SJC copulas for U.S. market

with normal distributed innovations

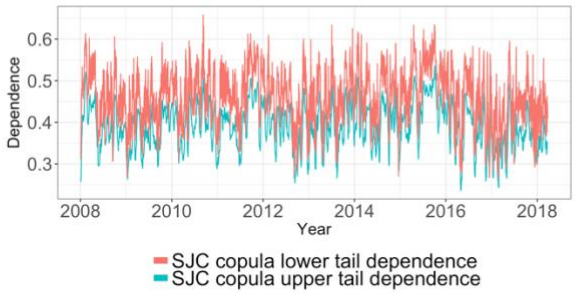


with Student-t distributed innovations



Panel C: SJC copulas for Australian market

with normal distributed innovations



with Student-t distributed innovations

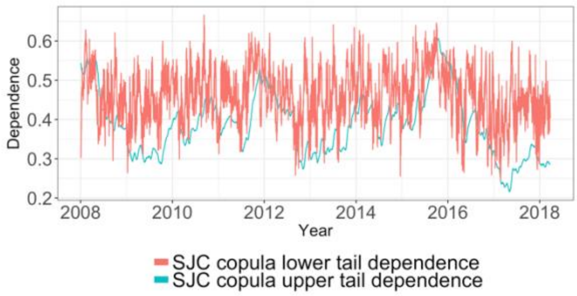


Fig. 13. SJC time varying copula of models with normal distributed innovations and models with Student-t distributed innovations

Table 21

SJC time varying copula estimates with normal distributed models and Student-t distributed models

Copulas	Normal innovations			Student-t innovations		
TVP SJC	CHN	US	AUS	CHN	US	AUS
ωu	0.0499	0.1539	0.1717	0.0548	0.1570	0.0296
	(.045)	(.0563)	(.185)	(.0305)	(.0476)	(.0178)
βu	0.9805	0.9172	0.5711	0.9780	0.9240	0.9861
	(.0222)	(.0345)	(.3309)	(.0151)	(.0253)	(.0088)
αu	-0.3174	-0.9754	-1.9136	-0.3473	-1.0037	-0.1895
	(.2704)	(.364)	(1.5613)	(.1852)	(.3129)	(.1129)
ωl	1.2416	0.0154	1.1951	1.0685	0.0166	1.2762
	(.5353)	(.0026)	(.3686)	(.5368)	(.0032)	(.3551)
βl	0.3053	0.9950	-0.7098	0.4139	0.9946	-0.7168
	(.3133)	(.0007)	(.1437)	(.3088)	(.0009)	(.1328)
αl	-5.5052	-0.0941	-7.5163	-4.7258	-0.1005	-8.0452
	(2.3784)	(.015)	(2.0206)	(2.3519)	(.018)	(1.967)
AIC	-2928.43	-1808.28	-1378.87	-2953.28	-1786.89	-1391.96

Note: This table reports the ML estimates for the parameters of different static copulas as well as their respective AICs. Standard errors are reported in the parentheses.

Table 22

Descriptive statistics of VaR and CoVaR based on models with normal distributed errors

	Upside		Downside	
	VaR	CoVaR	VaR	CoVaR
Panel A: VaR of stock markets and CoVaR from real estate to stock market				
CHN	3.0068	5.0465	-2.8154	-4.8652
	(1.8617)	(3.121)	(1.7448)	(3.01)
US	1.8137	2.9999	-1.6603	-2.8518
	(1.1664)	(1.9796)	(1.1349)	(1.948)
AUS	1.6229	2.6899	-1.5571	-2.6493
	(.7615)	(1.2773)	(.7608)	(1.2873)
Panel B: VaR of real estate markets and CoVaR from stock to real estate market				
CHN	2.8488	4.8077	-2.7428	-4.7115
	(1.4626)	(2.4841)	(1.461)	(2.4885)
US	2.7404	4.5808	-2.6189	-4.4676
	(2.5645)	(4.3554)	(2.4905)	(4.2834)
AUS	2.0672	3.4276	-1.9934	-3.3866
	(1.2365)	(2.0495)	(1.213)	(2.0467)

Notes: This table presents the mean and the standard deviation (in parenthesis) of the VaR calculated using all the data, upside and downside CoVaRs.

Table 23

Descriptive statistics of VaR and CoVaR based on models with Student-t distributed errors

	Upside		Downside	
	VaR	CoVaR	VaR	CoVaR
Panel A: VaR of stock markets and CoVaR from real estate to stock market				
CHN	2.9442	6.0309	-2.7601	-5.8682
	(1.7622)	(3.6468)	(1.7367)	(3.6349)
US	1.8045	4.0041	-1.5793	-3.8067
	(1.2328)	(2.8542)	(1.1175)	(2.7342)
AUS	1.6108	3.0611	-1.5322	-3.0247
	(.7599)	(1.4777)	(.7624)	(1.4911)
Panel B: VaR of real estate markets and CoVaR from stock to real estate market				
CHN	2.8098	5.5497	-2.7026	-5.4608
	(1.4484)	(2.8838)	(1.447)	(2.8934)
US	2.7176	5.3722	-1.5793	-5.2467
	(2.5356)	(5.1753)	(2.4725)	(5.1091)
AUS	2.0509	3.7820	-1.5322	-3.7571
	(1.2308)	(2.2687)	(1.1991)	(2.2686)

Notes: This table presents the mean and the standard deviation (in parenthesis) of the VaR calculated using all the data, upside and downside CoVaRs.

Table 24

Results of value at risk (VaR) backtesting

	For stock return		For real estate return	
	Upside	Downside	Upside	Downside
Panel A: models with normal innovations				
CHN	2.8280*	0.9262	0.0263	0.2703
	(.0926)	(.3358)	(.8712)	(.6031)
US	10.3264***	10.1091***	8.4895***	1.4344
	(.0013)	(.0015)	(.0036)	(.231)
AUS	15.8242***	3.1418*	3.4606*	1.0082
	(.0001)	(.0763)	(.0628)	(.3153)
Panel B: models with Student-t distribution				
CHN	2.5190	5.5039**	0.3706	0.7644
	(.1125)	(.019)	(.5427)	(.382)
US	12.3642***	29.1093***	7.3736***	5.4268**
	(.0004)	(0.0000)	(.0066)	(.0198)
AUS	10.1849***	5.9717**	2.8036*	2.3043
	(.0014)	(.0145)	(.0941)	(.129)

Notes: This table reports the unconditional coverage test Likelihood Ratio statistics for the null hypothesis of correct exceedances. and their corresponding p values in parenthesis. ***, ** and * denote 1%, 5% and 10% significance levels, respectively.

Table 25

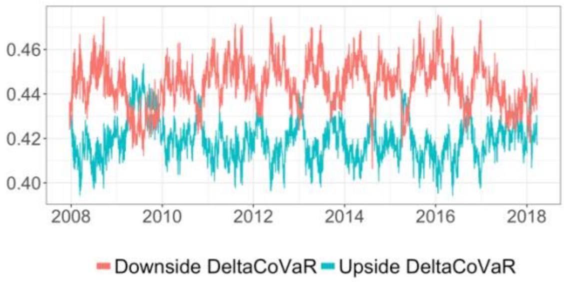
Tests of equalities of VaR and CoVaR in upside and downside conditions

	CoVaR from real estate markets to stock markets		CoVaR from stock markets to real estate markets	
	Upside	Downside	Upside	Downside
Panel A: models with normal innovations				
CHN	0.5770	0.5964	0.6188	0.6231
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
US	0.4603	0.4682	0.4323	0.4449
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
AUS	0.5413	0.5509	0.5740	0.6015
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Panel B: models with Student-t distribution				
CHN	0.6935	0.7055	0.7190	0.7187
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
US	0.6076	0.6443	0.5326	0.5515
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
AUS	0.6301	0.6468	0.6561	0.6833
	(0.0000)	(0.0000)	(0.0000)	(0.0000)

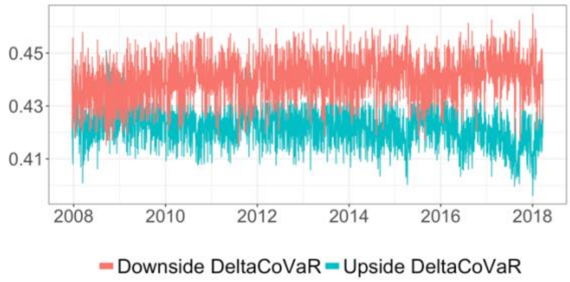
Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests the null hypothesis of no risk spillovers between the stock markets and real estate markets. The p-values for the KS statistic are in the parentheses.

Panel A: ΔCoVaR for China markets with normal distributed models

ΔCoVaR from real estate to stock market

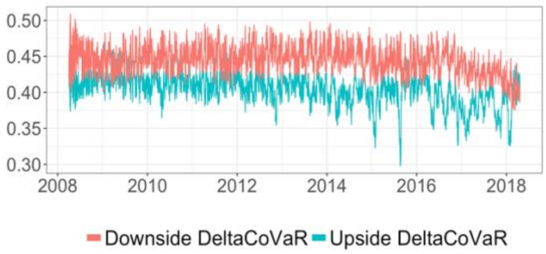


ΔCoVaR from stock to real estate market

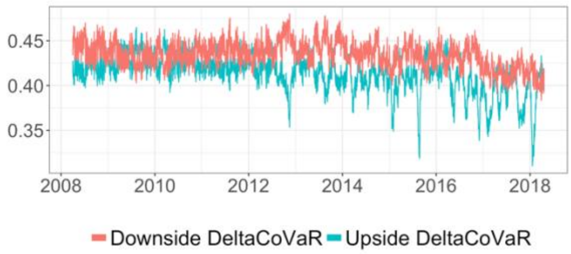


Panel B: ΔCoVaR for U.S. markets with normal distributed models

ΔCoVaR from real estate to stock market



ΔCoVaR from stock to real estate market



Panel C: ΔCoVaR for Australia markets with normal distributed models

ΔCoVaR from real estate to stock market ΔCoVaR from stock to real estate market

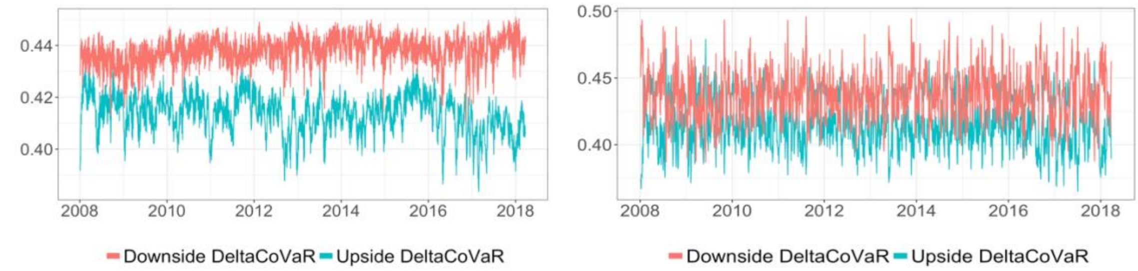
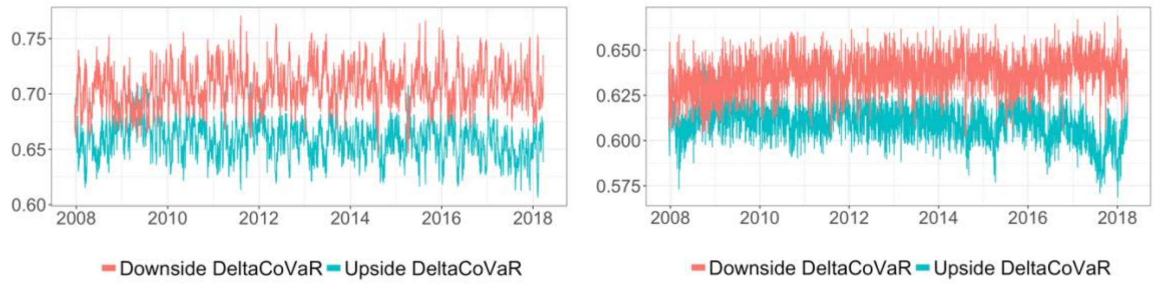


Fig. 14. Upside and downside delta conditional value-at-risk (ΔCoVaR) for models with normal distributed innovations

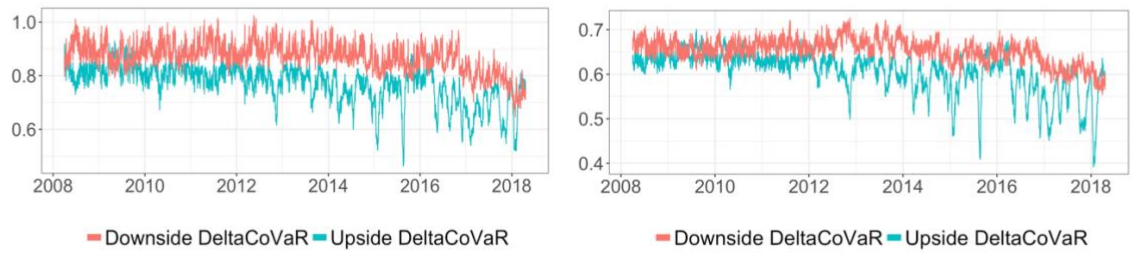
Panel A: ΔCoVaR for China markets with Student-t distributed models

ΔCoVaR from real estate to stock market ΔCoVaR from stock to real estate market



Panel B: ΔCoVaR for U.S. markets with Student-t distributed models

ΔCoVaR from real estate to stock market ΔCoVaR from stock to real estate market



Panel C: ΔCoVaR for Australia markets with Student-t distributed models

ΔCoVaR from real estate to stock market ΔCoVaR from stock to real estate market

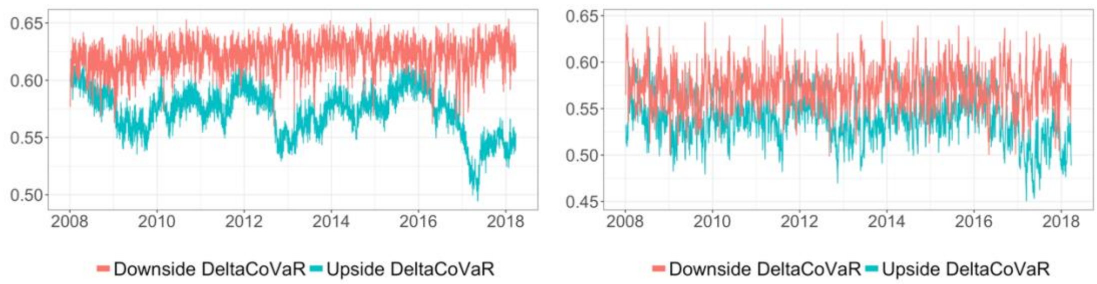


Fig. 15. Upside and downside delta conditional value-at-risk (ΔCoVaR) for models with Student-t distributed innovations

Table 26

Upside and downside ΔCoVaR asymmetry from stock markets to real estate markets and vice versa

	H0: Downside ΔCoVaR = Upside ΔCoVaR H1: Downside ΔCoVaR > Upside ΔCoVaR	
	from real estate markets to stock markets	from stock markets to real estate markets
Panel A: normal innovations		
CHN	0.7767	0.7217
	(0.0000)	(0.0000)
US	0.6621	0.4358
	(0.0000)	(0.0000)
AUS	0.9454	0.4595
	(0.0000)	(0.0000)
Panel B: Student-t distribution		
CHN	0.8228	0.7732
	(0.0000)	(0.0000)
US	0.6155	0.5026
	(0.0000)	(0.0000)
AUS	0.8435	0.5182
	(0.0000)	(0.0000)

Notes: This table presents the results of the Kolmogorov–Smirnov (KS) test. The KS tests test the null hypothesis of no difference between the downside and upside systemic risk contribution. The p-values for the KS statistic are in the parentheses.