The predictive power of stock style: size, value-growth orientation and the shape of the future return distribution

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Abstract

While there has been extensive research trying to explain the variability in the cross-section of expected stock returns, the predictability of other shape characteristics of the future return distribution is a less studied subject. This thesis investigates the relationship between the size and value-growth orientation of stocks (as measured according to Morningstar) and the shape parameters of their future returns. Two particularly interesting results stand out: (i) future volatility tends to be higher for both deep-value and high-growth stocks compared to more moderately valued stocks; (ii) skewness of future returns becomes more negative with stock size. To our knowledge, these empirical findings are not adequately explained by existing theories and therefore highlight the need for further research in this field. Additionally, the predictive model developed in this thesis is tested in a portfolio application exercise. Across different size- and value-growth-specific stock universes, the model correctly identifies portfolios of stocks whose realized performance is consistent with their forecasted risk profile. The results of this exercise suggest the potential practical importance of our empirical findings for portfolio and risk management.

Keywords: Return predictability, stock size, stock valuation, skewness, Morningstar

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1 Introduction

There has been extensive research into explaining the variability observed in the cross-section of expected stock returns. Indeed, by 2012, 316 factors had been identified as having explanatory power for this purpose (Harvey, Liu, & Zhu, 2016). However, there has been less research seeking to explain the variability in higher order moments of the distribution of stock returns. This is despite the fact that there is a large literature documenting the asymmetry of the distribution of stock returns (see, for example, Chen et al. (2001), Hueng and McDonald (2005), Silvennoinen et al. (2005)). Several explanations have been put forward to explain this observation, including the leverage effect, funding liquidity and liquidity spirals, and stochastic rational bubbles. However, it has not been fully addressed. Understanding the factors that determine this asymmetry is therefore a pertinent and important line of inquiry in academia. Moreover, it has significant implications for portfolio and risk management, as an understanding of what determines higher order moments of stock returns may enable more precise estimation of risk measures such as volatility and Value at Risk (VaR).

In this thesis, we investigate how stock style influences the shape of the predictive return distribution. To do this, we take inspiration from Morningstar's Style Box, which is a tool used for evaluating a stock or portfolio investment style. The Morningstar Style Box considers a company's size, value orientation and growth orientation, and assigns scores that allow stocks to be classified according to these criteria. We use the Morningstar methodology to classify stocks in our dataset accordingly, and so construct scores for size and value-growth orientation with which we then seek to explain the asymmetry in the distribution of future stock returns. Our consideration of size and value-growth orientation is also influenced by the seminal three-factor model introduced by Fama and French (1992).

To this end, we use data on all US listed companies from 1962-2018. We consider US companies for two reasons related to the availability of data. Firstly, a sample period should be sufficiently long to attain significant results. Secondly, we need to have a large number of companies in our sample with sufficient variation in size and value-growth orientation. US data satisfies these requirements due to the good availability of historic stock price and accounting data.

To construct the dependent variables for our analysis, we look at the distribution of future (12-month) returns. In order to allow for enough flexibility to capture the skewness and fat tails present in stock returns, we fit returns data using the skew-t distribution of Azzalini (2014). In this way, we can account for both of these features. As we separately demonstrate, the alternative normal fit on the returns data is strongly rejected whereas assumed skew-t consistently fails to be statistically rejected, which strengthens the argument for the chosen skew-t framework. The parameters from the fitted skew-t distributions are further used as the dependent variables.

Our analysis of the relationships between company size and value-growth orientation and the distribution parameters of future returns is conducted in a predictive econometric framework, an approach influenced by Halling and Giordani (2018). Scores for size and value-growth orientation measured according to the methodology of the Morningstar Style Box are used to create size- and value-growth-specific portfolios of stocks in each month. Returns of these portfolios are calculated over time and skew-t distributions are fitted on the portfolio returns series. Each of the resulting skew-t parameters is regressed on the size and value-growth scores of the portfolios using standard OLS. By doing so, we test the hypothesis that company size and value-growth orientation have a significant impact on the shape parameters of the distribution of future 12-month returns. The resulting coefficients indicate strong relationships and thus enable us to accept above hypothesis. Two particularly interesting regression results stand out. Firstly, that both deep-value and high-growth stocks exhibit heightened future volatility; secondly, that skewness of future returns becomes more negative with company size.

Finally, the regression results are tested by applying our estimated models to portfolio construction from 2004-2018. In doing so, we test the hypothesis that our regression model has predictive power. More specifically, we test that the regression model incorporating the relationships between size and value-growth orientation and the shape of the future return distribution can identify portfolios of stocks whose realized future performance exhibits certain forecasted risk and risk-reward characteristics. In order to incorporate the model's ability to predict higher order moments of the return distribution, we select stocks using two criteria. Firstly, we select stocks with the bottom quartile forecasted VaR and compare their realized VaR with that of stocks with the top quartile forecasted VaR. Secondly, we select stocks with the top quartile forecasted Reward-to-VaR (defined as return per unit of tail risk) and compare their realized Reward-to-VaR with that of stocks with the bottom quartile forecasted Reward-to-VaR. We find that our model has good predictive power for both of these criteria, particularly in the case of VaR. For illustrative purposes, we also compare the performance of portfolios selected using these criteria to that of relevant benchmark indices.

The thesis continues as follows. Section 2 outlines the theoretical and empirical research on this topic and places the thesis within that literature. Section 3 describes the dataset used. Section 4 provides a detailed description of the methodology used in our empirical analysis and portfolio application. Section 5 presents the results. Section 6 offers concluding remarks and recommendations for future research.

2 Literature Review

Several theories have sought to rationalize the negative asymmetry observed in stock returns, beginning with Black (1976) and Christie (1982) who introduced the notion of leverage effects. In this framework, a drop in market valuations leads to an increase in operating and financial leverage, which in turn increases the volatility of subsequent returns. As this effect only takes place when valuations fall, it leads to a negative asymmetry in stock returns. However, Schwert (1989) and Bekaert and Wu (2000) found that this explanation lacks the quantitative power to explain the data, particularly in the case of high frequency returns.

As an alternative to this, the "volatility feedback" mechanism was proposed by Pindyck (1984), French et al. (1987) and Campbell and Hentschel (1992). The idea of this mechanisms is that when good news is made available, this signals an increase in market volatility which in turn increases the risk premium. The increased risk premium offsets the direct effect of the good news. When bad news is made available, however, the two forces work in the same direction, and so the increase in the risk premium amplifies the direct effect of the bad news. This asymmetry leads to overall negative skewness in stock returns. Once again, however, there is a lack of supporting quantitative evidence for this theory. In particular, Poterba and Summers (1986) find that the shocks to market volatility are generally so short-lived that they cannot reasonably be expected to have a large effect on risk premia.

A third explanation for the asymmetry in stock returns comes in the form of stochastic rational bubbles, as laid out by Blanchard and Watson (1982). This model proposes that at any given time, stock returns are composed of a rational and a bubble component. This bubble component is in turn dependent on valuation levels. At high valuations, when the bubble component is incorporated, the return distribution is blend of the symmetric rational component and a bubble component. At low valuations the distribution of stock returns is made up only of the symmetric, rational component. This leads to the conclusion that the distribution of stock returns should become more negatively skewed at higher valuation levels, in agreement with our results.

Brunnermeier and Pedersen (2009) offer a fourth explanation. Here, assets in which speculators invest can exhibit negative skewness resulting from funding constraints and consequent liquidity spirals. When speculators face losses they can encounter funding constraints, i.e. margin calls, that can in turn lead them to unwind their positions. This can further depress prices and raise additional funding constraints, leading to a liquidity spiral. Speculators' gains, on the contrary, are not amplified by this mechanism, resulting in an asymmetry in returns. Brunnermeier et al. (2008) find supporting evidence for this in currency markets, where crash risk is associated with funding constraints for carry traders.

Finally, there is the theory put forward by Hong and Stein (2003) that investor heterogeneity and differences of opinion generate negatively skewed stock returns. Key to this theory is the idea that investors have differences of opinion about fundamental value, and some of those investors face short-sale constraints. When bearish investors face short-sale constraints, they will be forced to sit out of the market. As a consequence, their information will not be fully incorporated in prices. If previously bullish investors have a change of opinion and exit the market, leading to a fall in price, bearish investors may enter the market at a lower price. This revelation of hidden information tends to occur during market declines, generating negative skewness. However, Hueng and McDonald (2005) find no evidence to support this theory in the case of aggregate stock market returns.

Empirical evidence of the dependence of the distribution of stock returns on value and size characteristics is relatively limited, mainly due to a lack of research on this topic. However, there are several studies, which provide evidence that size and value characteristics influence not only expected returns, but also higher order moments of the return distribution. Chen et al. (2001) investigate daily returns of individual stocks and find that return skewness becomes more negative with firm size, i.e. market capitalization. Size in this paper is included only as a control variable, so no strong theoretical interpretation of this result is made. Similarly, Dennis and Mayhew (2002) find evidence from stock options that skewness becomes more negative with firm size. Once again, however, size is included only as a control variable. Despite this, the repeated finding that size influences higher order moments of stock return distributions justifies our decision to include this characteristic as an explanatory variable in our analysis.

Evidence that the return distribution of stocks is dependent on valuation is stronger. Gormsen and Jensen (2017) look at higher-order moments of quarterly and monthly returns using option data. They find that higher-order moments increase in good times, i.e. when returns are high, which qualitatively corroborates the theory that valuation affects not only the expected return of stocks, but volatility and skewness as well. This finding is, however, limited by the lack of availability of data on long-horizon stock options and the small sample used. Similarly, Greenwood et al. (2017) find that a large increase in stock prices significantly increases the probability of a crash. This, again, provides qualitative evidence to support the notion that the shape of the return distribution is influenced by valuation, and that crash risk (i.e. negative skewness) increases with valuation.

The paper most closely related to our thesis is by Halling and Giordani (2018). In this, they investigate how the distribution of returns on the S&P 500 depends on Shiller's cyclically adjusted price-earnings ratio. They find that at high valuations returns are more negatively skewed. This goes some way to explaining the mean-reversion behaviour of stock prices, which are commonly observed to go "up the stairs, down the elevator". We broadly follow their methodology, extending it in two ways. Firstly, we base our analysis on individual stocks rather than aggregate stock indices. Secondly, we include size and value-growth orientation, rather than simple valuation ratio, as factors determining the distribution of stock returns.

3 Data

Two types of data are needed in our analysis:

1) Stock Return Data (data necessary to calculate future 12-month returns);
2) Accounting Data (data necessary to calculate value and growth characteristics).

We use the CRSP/Compustat Merged Database provided by Wharton Research Data Services (WRDS) for both return and accounting data. The data is for all US traded companies from January 1962 to December 2018.

3.1 Stock Return Data

The following variables are collected on a monthly basis:

Table 1. **Variables from price dataset**

CRSP name	Description	Units
-	Company Name	-
PERMNO	Company Identifier	-
LINKPRIM	Primary Issue Marker	-
PRCCM	Close Price	USD
CSHOQ	Common Shares Outstanding (quarterly data)	million
AJEXM	Cumulative Adjustment Factor (ex-distribution date)	Units
DVPSXM	Dividends per Share (ex-distribution date)	USD

The following adjustments and data cleaning are necessary. Firstly, we consider only primary issues of stocks, meaning that only observations corresponding to *LINKPRIM = J* or *N* are selected. This was done in order to obtain a one-to-one match between each security and company (each *PERMNO*).

Secondly, we adjust prices and dividends for stock splits in order to correctly measure the returns over time. For that purpose, *AJEXM* variable was used. *AJEXM* is equal to 1 for the most recent price datapoint for a given company, and for historical datapoints incorporates the cumulative split ratio. For example, if a given company made the most recent stock split with ratio 2:1 in February 2014 (2 new shares for 1 old share), earlier made another stock split with ratio 3:1 in June 2010 (3 new shares for 1 old share), and made no more stock splits, the factor *AJEXM* then equals 1 for the period from February 2014 until the most recent observation, equals 2 for the period between June 2010 until January 2014, and equals 6 for the period between the first price observation until May 2010. Price adjusted for stock splits is therefore calculated as

$$PRCCM_{\ split-adjusted} = \frac{PRCCM}{AJEXM}. \tag{1}$$

Prices adjusted in this way enable us to correctly compute capital gains of stocks over time. We adjust dividends (*DVPSXM*) in a similar way in order to correctly compute dividend returns over time:

$$DVPSXM_{\ split-adjusted} = \frac{DVPSXM}{AJEXM}. \tag{2}$$

Adjustment for common shares outstanding (*CSHOQ*) is inversely related to the adjustment of per-share values and, therefore, implies multiplication:

$$CSHOQ_{\ split-adjusted} = CSHOQ \times AJEXM. \tag{3}$$

Since, according to CRSP methodology, split events are applied on the ex-distribution date, we used the ex-date convention for dividends. This means that information on dividends in a given quarter for a given company is recorded at the ex-date rather than on the pay-date.

We use adjusted prices and dividends to calculate 1-month and cumulative 12-month returns, as explained in greater detail in the Methodology section.

3.2 Accounting Data

The following variables were collected on an annual basis:

Table 2. **Variables from accounting dataset**

Compustat name	**Description**	**Units**
CSHO	Common Shares Outstanding	million
BKVLPS	Book Value per Share	USD
DVC	Dividends Common/Ordinary	mUSD
DVP	Dividends – Preferred/Preference	mUSD
EBITDA	Earnings Before Interest, Tax, Depreciation, and Amortization	mUSD
NI	Net Income (Loss)	mUSD
REVT	Revenue – Total	mUSD
OANCF	Operating Activities Net Cash Flow	mUSD

Although we obtain common shares outstanding from the most recent quarter in the price dataset, we also need shares outstanding as of the end of fiscal year (*CSHO*) for the purpose of re-calculating the book value per share, which is discussed later.

Finally, we use CRSP Monthly Stock database to obtain Share Code variable, which is a two-digit code describing the type of security traded. For the purpose of studying only ordinary common shares, we select only the observations with corresponding share codes – code 10 (ordinary common shares, which have not been further defined) and code 11 (ordinary common shares, which need not be further defined).

3.3 Dataset Summary

The resulting dataset comprises monthly price observations and annual accounting data observations for all companies traded in the US since January 1962 until December 2018, for which at least both prices and common shares outstanding were available. In total, there are 19,456 companies in the dataset. Comparing the total market capitalization of the companies from the dataset to the statistics from the World Bank on all US traded companies[1], we conclude that the completeness of the dataset is sufficient:

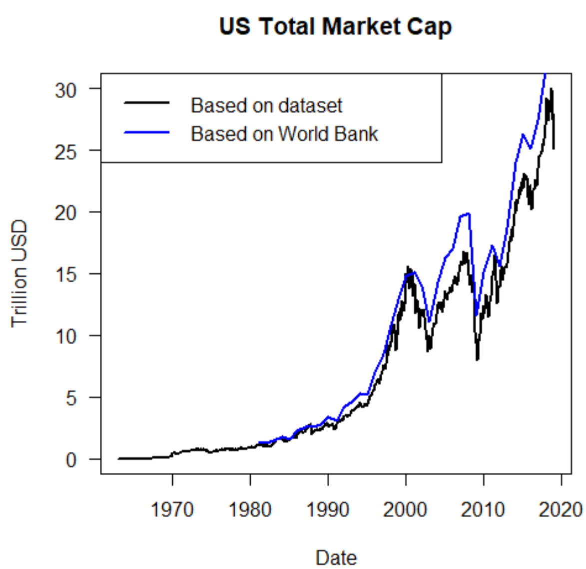

Figure 1. **Total market capitalization of US traded companies**

[1] Source: https://data.worldbank.org/indicator/CM.MKT.LCAP.CD?locations=US

4 Methodology

The methodological part of the thesis continues as follows:

1) Introducing the Morningstar Style Box framework (section Morningstar Style Box);
2) Determining the size, value, and growth characteristics of the stocks (sections Size, Value Orientation, and Growth Orientation, respectively)
3) Determining the shape characteristics of the future 12-month return distribution using the skew-t distribution fit (section Shape of Future Returns Distribution)
4) Determining the relationship between the size, value and growth characteristics and the shape of future 12-month return distribution (section Relationship Between Size/VCG Characteristics and Future Returns);
5) Applying the resulting relationships from 4) to the size- and value-growth-specific portfolios (section Portfolio Application).

4.1 Morningstar Style Box

Developed by Morningstar in 1992, the Morningstar Style Box is a nine-square grid that classifies securities by size along the vertical axis and by value and growth characteristics along the horizontal axis:

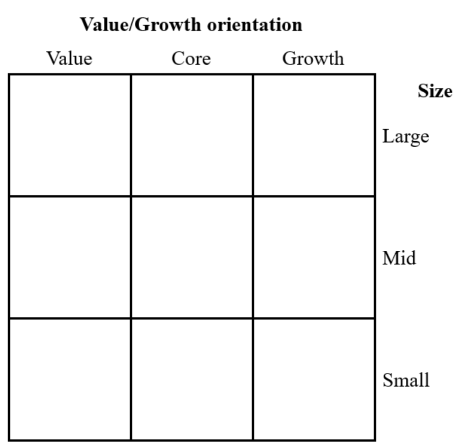

Figure 2. **Illustration of the Morningstar Style Box**

According to Morningstar, different investment styles often have different levels of risk and these differences can lead to differences in returns. Indeed, Schadler and Eakins (2001) find that the risk of cells in the Morningstar Style Box is consistent with the risk expectations published by Morningstar.

The most common use of the Morningstar Style Box is to measure overall portfolio or fund style. The style (size and value-growth orientation) is first determined at the stock level, and stock attributes can be then "rolled up" to determine the overall investment style of a fund or portfolio. For the purpose of this thesis, we consider the style of individual stocks and aggregate portfolios of stocks from different style categories.

The Morningstar Style Box captures three of the major considerations in equity investing: size, security valuation and security growth. First, the size of the stock is measured. Next, the value- and growth-orientation of a stock are measured. In short, a stock's value orientation reflects the price that investors are willing to pay for the stock's anticipated per-share earnings, book value, revenues, cash flow, and dividends. A high price relative to these measures indicates that a stock's value orientation is weak. A stock's growth orientation is independent of its price and reflects the growth rates of fundamental variables such as earnings, book value, revenues, and cash flow.

With the Morningstar Style Box methodology, a stock can be classified into a size category (small-cap, mid-cap, and large-cap) and a value-growth category (value, core, and growth). Identification of categories uses certain numerical measurements, which we call the "scores". As explained in the sections below, the definitions of scores in this thesis differ from those offered by the Morningstar Style Box but are nevertheless closely related to the latter.

4.2 Size

4.2.1 Size Category

There are three size categories according to the Morningstar Style Box: small-cap, mid-cap, and large-cap. In each month, all stocks are ordered in descending order by their market capitalization. For the resulting ordered sample, the cumulative capitalization is calculated as a percentage of total sample capitalization as each stock is added to the list. The stock that causes cumulative capitalization to equal or exceed 70% of the total cap is the final one assigned to the large-cap category. The largest of the remaining stocks are assigned to the mid-cap category until the cumulative capitalization equals or exceeds 90%. The remaining stocks are assigned to the small-cap category.

This procedure is performed every month for the period between January 1962 and December 2018. The above split by size categories results in 70% of total market cap represented by large-cap stocks, 20% by mid-cap stocks, and 10% by small-cap stocks every month:

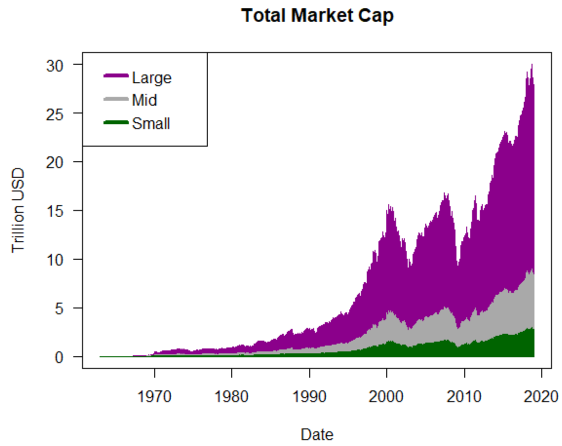

Figure 3. **Total market capitalization of companies from the dataset**

To understand the size of the small-cap, mid-cap, and large-cap stocks, the graph below shows the log market cap of the largest stock within each size category over time. As of December 2018, market capitalization of small-cap stocks is approximately equal or below USD 4.7bn, mid-cap stocks' market cap is between USD 4.7bn and USD 24.5bn, and large-cap stocks' market cap is between USD 24.5bn and USD 780bn.

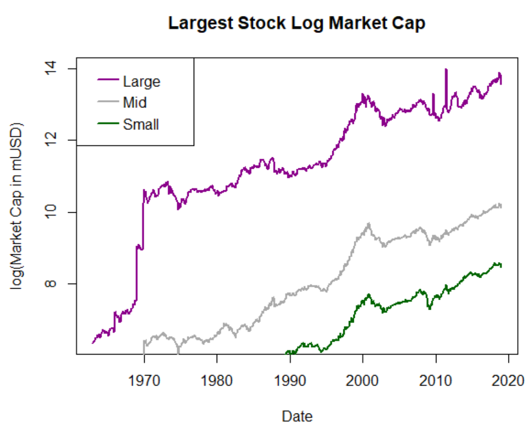

Figure 4. **Log market capitalization of the largest stocks within respective size categories**

We acknowledge that there are different definitions of small-cap, mid-cap, and large-cap stocks. For example, large-caps are sometimes considered companies with market capitalization above USD 10bn (versus USD 24.5bn in December 2018 as defined according to the methodology), small-caps are often considered companies with market cap below USD 2bn (versus USD 4.7bn in December 2018 as defined according to the methodology). We do not claim the method chosen in the thesis to be better than other ways of determining the size categories but believe that it reflects the overall size characteristic of stocks sufficiently well.

4.2.2 Size Scores

In order to investigate the relationship between size characteristics and return distributions, we define the size score in such a way that it satisfies several criteria. The size score should be stock-specific and easily interpretable. It should also have a sufficient range of values and should not be heavily concentrated around a certain value. Finally, the selected size measure should be universal over time meaning that it should not be tied to the size of the market at any given point in time.

Considering these three criteria, we define the size score as the natural logarithm of the share of a stock's market cap in the total market cap – or, in other words, the log of a stock's market share. We use this measure for three reasons. Firstly, market share is the most straightforward and easily interpretable way to define the size of a specific company. Secondly, although for almost all companies (especially for the smallest ones) the market share is very close to zero and only for the largest stocks like Apple, the share in the total market can reach up to 3%, taking logarithm of the market share helps solve two problems. Namely, it helps reduce the concentration of the score values around 0 thus widening the distribution (see Figure 5 below) and enables us to interpret the market share in terms of percentage changes rather than absolute changes, which will be important for the purposes of interpretation in the following regression analysis. Thirdly, although the natural logarithm of dollar market cap would also satisfy the requirement of having a sufficiently wide distribution, this measure would be tied to the size of the market and thus, given exponential growth of the market, as shown in Figure 3, is inadequate for using across time. Hence the chosen measure of size satisfies the three criteria.

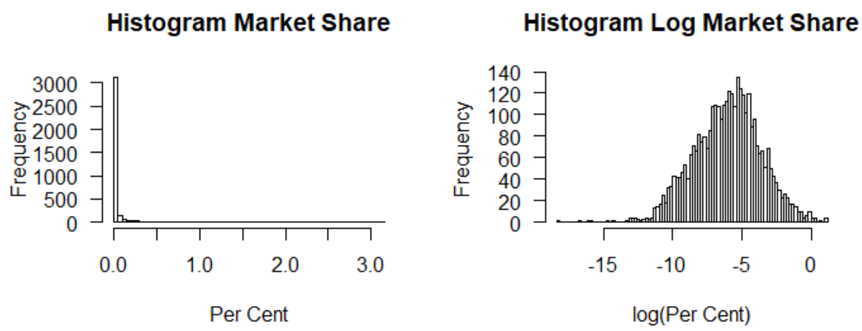

Figure 5. **Histograms of stocks' market shares and log market shares as of December 2018**

A potential disadvantage of the chosen definition of the size score is that it is not directly related to the size categories defined using the cumulative market capitalization. However, size score based on cumulative market capitalization and the changes in such score would be more difficult to interpret. Moreover, cumulative market capitalization is not readily available for a company and requires the information on all traded companies smaller than the given company to calculate.

The Morningstar Style Box methodology offers another definition of the size score (raw Y score) using the following formula for the stock *i*:

$$raw\ Y_i = 100 \times \left(1 + \frac{log(cap_i) - log(cap_1)}{log(cap_2) - log(cap_1)}\right), \quad (4)$$

where cap_i is the market capitalization of stock *i*, cap_1 is the market capitalization that corresponds to the breakpoint value between mid-cap and small-cap stocks, and cap_2 is the market capitalization that corresponds to the breakpoint between large-cap and mid-cap stocks. Defined in this way, size scores range between 100 and 200 for mid-cap stocks, between $-\infty$ and 100 for small-cap stocks, and between 200 and $+\infty$ for large-cap stocks. This scaled score has an advantage over size score defined as a market share in that it incorporates the size categories. However, the disadvantage of difficult interpretability of such a score and of the changes in such a score, in our view, outweighs that benefit.

4.3 Value Orientation

4.3.1 Notation and Caveats

A stock's value orientation reflects the price investors are willing to pay for a share of some combination of the stock's prospective earnings, book value, revenue, cash flow, and dividends. Morningstar measures a stock's value orientation in relation to its size category as defined in the previous section. The methodology works with the fundamentals expressed per share. Earnings per share, book value per share, revenue per share, cash flow per share, and dividends per share are denoted *e, b, r, c,* and *d*, respectively.

All values needed to calculate *e, b, r, c*, and *d*, are taken from the price and accounting datasets described above in the respective sections and, where needed, we perform adjustments as described below using the notation introduced in Table 1 and Table 2.

Earnings per share *e* is calculated as

$$e = \frac{NI - DVP}{CSHOQ_{\ split-adjusted}}, \tag{5}$$

where $CSHOQ_{\ split-adjusted}$ is the most recent available quarterly observation.

Book value per share *b* is calculated as

$$b = \frac{BKVLPS \times CSHO}{CSHOQ_{\ split-adjusted}}. \tag{6}$$

Revenue per share *r* is calculated as

$$r = \frac{REVT}{CSHOQ_{\ split-adjusted}}. \tag{7}$$

As availability of operating cash flow data is poor prior to June 1986, for all previous months we approximate this with *EBITDA*. It has to be noted that *EBITDA* does not account for changes in net working capital and other non-cash items compared to operating cash flow, but we assume that it serves a reasonable proxy for the latter. Hence, we calculate cash flow per share in and after June 1986 as

$$c = \frac{OANCF}{CSHOQ_{\ split-adjusted}}, \tag{8}$$

and before June 1986 approximate as

$$c \approx \frac{EBITDA}{CSHOQ_{\ split-adjusted}}. \tag{9}$$

Dividends per share are calculated as

$$d = \frac{DVC}{CSHOQ_{\ split-adjusted}}. \qquad (10)$$

Certain details and caveats must be discussed. For a given company in a given month, all values from the accounting dataset (*NI, DVP, BKVLPS, REVT, OANCF,* and *EBITDA*) are the values from the most recent fiscal year end. For example, a fiscal year 2014 for MICROSOFT CORP is from June 2013 to June 2014, and fiscal year 2015 is from June 2014 to June 2015. Therefore, in 2015 for January to May, the fundamentals data are taken from fiscal year 2014, and starting from June 2015 – from fiscal year 2015. This approach implies that only the most recent available accounting data are considered in the given month. At the same time, it also assumes that as soon as the fiscal year ends (for example, FY2014 ends in June 2015), the data for that fiscal year are instantly available. Typically, it would take several months before the numbers for the last fiscal year are released after the fiscal year ended. However, we continued with the above approach for several reasons. Firstly, we do not know exactly how long the period between the fiscal year end and corresponding numbers release for different companies lasts. Secondly, shifting data accordingly would be difficult to perform given the large size of the dataset. Finally, considering accounting data from the last fiscal year as soon as that fiscal year ends realistically implies that market participants have some expectations about them even a few months before their release. Implicitly, this approach assumes that those expectations are correct on average.

Dividends per share are calculated based on the last fiscal year's dividends (*DVC*) rather than the annual sum of most recent quarterly dividend amounts (sum of *DVPSXM*). Although the latter approach would be a better reflection of the current dividends per share, for consistency reasons we consider last fiscal year's values (*DVC*), as is done for per-share transformations of other accounting variables.

Value orientation is based on historical as well as prospective per-share accounting fundamentals. The formulas introduced above are discussed as the per-share values for the most recently finished fiscal year (year 0), and later notation for them will be e_0, b_0, r_0, c_0, and d_0. Earnings per share values for the fiscal year prior to year 0 (year -1) is denoted e_{-1}, for the fiscal year prior to year -1 (year -2) is denoted e_{-2}, for the fiscal year prior to year -2 (year -3) is denoted e_{-3}, and for the fiscal year prior to year -3 (year -4) is denoted e_{-4}. Forecasted earnings per share for the current fiscal year (year 1) is denoted e_1. The method to determine the forecasted values is discussed later. The same notation applies for *b, r, c,* and *d*.

4.3.2 Steps to Determine Value Orientation

A stock's value orientation is determined using the following steps:

1) Calculate up to five prospective yields (e_1/p, b_1/p, r_1/p, c_1/p, and d_1/p: as many as are available) for each stock;
2) Calculate a market-cap-weighted[2] percentile score (0-100) for each available yield factor for each stock within each size category;
3) Calculate the overall value score (0-100) for each stock. This is a weighted average of the individual percentile scores for each of the five value factors. The overall value score represents the strength of the stock's value orientation.

The above steps are described in detail below.

4.3.3 Prospective Yields

Yield factors are calculated by dividing forecasted per share values (e_1, b_1, r_1, c_1, and d_1) by the month-end stock price p. Price p corresponds to $PRCCM_{split-adjusted}$ taken from the price dataset and adjusted for stock splits.

According to Morningstar's methodology, the forecasted value e_1 is based on both third-party forecasts and historical dynamics of e. Since third-party estimates in our case are not available for the majority of stocks and years, our forecasts for e_1 are based only on historical values of e. For that purpose, Morningstar determines growth measure $g(e_1)$, which is applied to e_0, in the following way. First, calculate as many as possible of four periodic growth rates:

$$g(e)_{-4} = \left(\frac{e_0}{e_{-4}}\right)^{\frac{1}{4}} - 1 \quad (11)$$

$$g(e)_{-3} = \left(\frac{e_0}{e_{-3}}\right)^{\frac{1}{3}} - 1$$

$$g(e)_{-2} = \left(\frac{e_0}{e_{-2}}\right)^{\frac{1}{2}} - 1$$

$$g(e)_{-1} = \left(\frac{e_0}{e_{-1}}\right) - 1$$

If e_{-1}, e_{-2}, e_{-3}, or e_{-4} is negative, no periodic growth rate is calculated using that data point. A minimum of one periodic growth rate must be available to determine $g(e_1)$. Because this growth rate is used

[2] Morningstar uses float-weighted percentile scores. Due to the lack of information on free float, we approximate the original approach with market-cap-weighted approach.

to estimate the current year earnings per share, e_0 must be positive and e_0 serves as the numerator for calculating growth.

When all available growth rates have been calculated, average the results to arrive at $g(e_1)$:

$$g(e_1) = Average[g(e)_{-4},\ g(e)_{-3},\ g(e)_{-2},\ g(e)_{-1}]. \tag{12}$$

As can be noted, in calculating $g(e_1)$, recent growth rates are included in more of the averaged terms than are older growth rates; recent growth rates are therefore weighted more heavily than older growth rates.

After $g(e_1)$ has been obtained, it is applied to positive e_0 to obtain the forecast e_1:

$$e_1 = e_0 \times \left(1 + g(e_1)\right). \tag{13}$$

In this way the prospective earnings yield e_1/p can be obtained. Prospective book value, revenue, cash flow, and dividend yields are calculated in the same way.

For stocks that do not pay dividends ($d_0 = 0$), dividend yield is still calculated and thus 0% dividend yield ($d_1/p = 0\%$) is considered a valid data point.

If the stock has value factor data available only for forecasted dividend yield or no information at all, the stock is eliminated from the scoring group for calculating value factor percentile scores.

4.3.4 Percentile Scores for Each Value Factor

Once we have calculated one or more of e_1/p, b_1/p, r_1/p and c_1/p values, with or without d_1/p, each stock is assigned a market-cap-weighted percentile score for each relevant factor. The percentile scores are calculated within each stock's size category.

To calculate an earnings yield score (0-100) for each stock in a size category:

1) Rank all stocks in the size category by e_1/p yields in ascending order.
2) Determine the total market capitalization of all stocks in the category.
3) Starting with the lowest observations, trim all stocks that sum up to 5% of total market capitalization. Then, trim 5% of the market capitalization from the highest observations. When a stock "straddles" the 5^{th} percentile point or 95^{th} percentile point, remove it from the sample.
4) Calculate the market-cap-weighted average e_1/p for the remaining stocks.

5) Add the trimmed stocks back to the sample. Calculate the ratio of each stock's e_1/p to the market-cap-weighted average e_1/p.
6) Assign each stock to an *e/p* "bucket" as follows:
    a. If the stock's e_1/p is equal to or less than 0.75 times the market-cap-weighted average e_1/p ("the lower value cutoff"), the stock is assigned to the low *e/p* bucket.
    b. Or, if the stock's e_1/p is equal to or less than the market-cap-weighted average e1/p, the stock is assigned to the mid-minus *e/p* bucket.
    c. Or, if the stock's e_1/p is equal to or less than 1.25 ("the upper value cutoff") times the market-cap-weighted average e_1/p, the stock is assigned to the mid-plus *e/p* bucket.
    d. Or, the stock is assigned to the high *e/p* bucket.

When each stock has been assigned to an *e/p* bucket, it is then scaled relative to other stocks in the same bucket. The low e/p bucket is used as an example here:

1) Order the stocks within the low *e/p* bucket by their raw e_1/p scores, from lowest to highest.
2) Within the low *e/p* bucket, assign each stock a value equal to the cumulative market capitalization represented by that stock and all stocks with a lower e_1/p. Thus, the stocks in the low *e/p* bucket have values ranging from 0.00+ (the stock with the lowest e_1/p in the low *e/p* bucket) to 100 (the stock with the highest e_1/p in the low *e/p* bucket).
3) Where two or more stocks have the same e_1/p, they are assigned a value that represents the cumulative float of all stocks with a lower e_1/p plus one-half of the total float of the stocks that share the same e_1/p.
4) Re-scale the scores in the low *e/p* bucket between 0.00+ and 33.33.

Repeat the four steps immediately above for each of the mid-minus, mid-plus and high *e/p* buckets; and re-scale the values as follows:

Table 3. **Ranges of e/p scores for different e/p buckets**

Bucket	**Minimum Score**	**Maximum Score**
Low *e/p*	0.00+	33.33
Mid-minus *e/p*	33.34	50.00
Mid-plus *e/p*	50.01	66.66
High *e/p*	66.67	100.00

All the steps in this section are then repeated for each of b_1/p, r_1/p, c_1/p and d_1/p. For stocks that do not pay dividends, 0% dividend yield is considered a valid data point and is given a dividend yield score. Repeat all the steps above for each size category.

For financial stocks, price-to-cash flow is not used for the value factor calculation because cash flow from operations data is not meaningful for banks and insurance companies. To identify financial stocks, we use variable *GSECT*OR from the CRSP/Compustat Merged Database – Fundamentals Annual database, which is the item representing the first level in the hierarchy of the Global Industry Classification Standard (GICS). The Sector is represented by the leftmost 2 digits of the total GICS code. Value 40 of *GSECTOR* corresponds to the financial companies.

4.3.5 Overall Value Scores

Once all of the five value factors have been scored from 0-100, we calculate a weighted average overall value score for each stock. If available, *e/p* scores are assigned a weight of 50% in the overall value score; each of the other value factors is assigned an equal share of the remaining weight (either 50% or, if *e/p* is unavailable, 100%).

For example, if all five value factors are available, the weights are:

Table 4. **Assigning weights to components of overall value score: all factors present**

Scores	***e/p***	***b/p***	***r/p***	***c/p***	***d/p***	**Overall Value Score**
Weights	50%	12.50%	12.50%	12.50%	12.50%	
Stock A	41	78	73	88	81	61

Or, for example, if *b/p* is missing, the weights are:

Table 5. **Assigning weights to components of overall value score: one factor missing**

Scores	***e/p***	***b/p***	***r/p***	***c/p***	***d/p***
Weights	50%	-	16.7%	16.7%	16.7%

Or, for example, if *e/p* and *b/p* are both missing, the weights are:

Table 6. **Assigning weights to components of overall value score: two factors missing**

Scores	***e/p***	***b/p***	***r/p***	***c/p***	***d/p***
Weights	-	-	33.3%	33.3%	33.3%

If only forecasted dividend yield, or no information, is available for a given stock, the stock is not given an overall value score. However, the stock may still receive an overall growth score, which is discussed in the next section.

The procedure of calculating overall value score is performed for all stocks in the dataset for each month. More value-oriented stocks, whose valuation in terms of earnings, book value, revenue cash flow, and dividend is low (and thus corresponding yields are high), have higher overall value scores.

4.4 Growth Orientation

A stock's growth orientation reflects the rate at which its earnings, book value, revenue, and cash flow are expected to grow. Dividend growth rates are not used in determining a stock's growth orientation. A stock's growth orientation is measured in relation to stocks in its size category.

Growth orientation is determined using the following three steps:

1) Calculate up to four average growth rates $g'(e)$, $g'(b)$, $g'(r)$ and $g'(c)$ for each stock, using the process described in the next section[3].
2) Calculate a market-cap-weighted percentile score (0-100) for each available growth rate for each stock within each size category.
3) Calculate the overall growth score (0-100) for each stock. This is a weighted average of the individual percentile scores for each of the five growth factors. The weighting scheme is described below. The weighted average score represents the strength of the stock's growth orientation.

Details of each of these steps are provided below.

4.4.1 Calculating Stock Growth Rates

We calculate as many as possible of $g'(e)$, $g'(b)$, $g'(r)$ and $g'(c)$ for each stock. The example historical growth rate calculation below uses $g'(e)$, but the process is identical for $g'(b)$, $g'(r)$ and $g'(c)$.

If e_0 and e_{-1} are both negative, then $g'(e)$ is not calculated. If e_0 or e_{-1} is positive, then $g'(e)$ is calculated as follows.

[3] According to Morningstar methodology, in addition, third-party estimates for long-term projected earnings growth rate $g(e_5)$ should also be collected. Due to lack of such information for most stocks, we omit this step and use $g'(e)$, $g'(b)$, $g'(r)$ and $g'(c)$ based only on historical data.

First, we calculate as many as possible of four periodic growth rates:

$$g'(e)_{-4} = \left(\frac{e_n}{e_{-4}}\right)^{\frac{1}{n+4}} - 1 \tag{14}$$

$$g'(e)_{-3} = \left(\frac{e_n}{e_{-3}}\right)^{\frac{1}{n+3}} - 1$$

$$g'(e)_{-2} = \left(\frac{e_n}{e_{-2}}\right)^{\frac{1}{n+2}} - 1$$

$$g'(e)_{-1} = \left(\frac{e_n}{e_{-1}}\right)^{\frac{1}{n+1}} - 1$$

where n is the most recent period (0 or -1) in which e is positive.

If e_{-1}, e_{-2}, e_{-3} or e_{-4} is negative, no periodic growth rate is calculated using that data point. A minimum of two periodic growth rates must be available to determine $g'(e)$. If $n = 0$, up to four rates are calculated; and if $n = -1$, up to three growth rates are calculated.

When all available growth rates have been calculated, we average the results:

$$g'(e) = Average[g'(e)_{-4}, g'(e)_{-3}, g'(e)_{-2}, g'(e)_{-1}]. \tag{15}$$

If $n = 0$, $g'(e)$ is the same as the growth rate used in the calculation of the stock's value orientation, $g(e_1)$.

Book value, revenue, and cash flow growth rates are calculated in the same way.

If the stock has no growth factor data available, we eliminate the stock from the size category when calculating growth factor percentile scores.

4.4.2 Percentile Scores for Each Growth Factor

As with the value factors, percentile scores are assigned to each of the five growth factors. The percentile scores are calculated within each stock's scoring group.

To calculate an earnings growth rate score (0-100) for each stock within a scoring group:

1) Rank all stocks in the scoring group by their $g'(e)$ growth rates in ascending order.
2) Determine the total market capitalization of all stocks in the group.
3) Starting with the lowest observations, trim all stocks that sum up to 5% of the total market capitalization of all stocks in the group. Then, trim 5% of the float from the highest observations.

When a stock "straddles" the 5th percentile point or 95th percentile point, remove it from the sample.

4) Calculate the share-weighted[4] average growth rate for the remaining stocks. See Appendix A for a description of the share-weighted average.
5) Add the trimmed stocks back to the sample. Calculate the ratio of each stock's *g′(e)* to the share-weighted average *g′(e)*.
6) Assign each stock to a *g′(e)* "bucket" as follows:
    a. If the stock's *g′(e)* is equal to or less than 0.75 times the share-weighted average *g′(e)* ("the lower growth cutoff"), the stock is assigned to the low *g′(e)* bucket.
    b. Or, if the stock's *g′(e)* is equal to or less than the share-weighted average *g′(e)*, the stock is assigned to the mid-minus *g′(e)* bucket.
    c. Or, if the stock's *g′(e)* is equal to or less than 1.25 times the share-weighted average *g′(e)* ("the upper growth cutoff"), the stock is assigned to the mid-plus bucket.
    d. Or, the stock is assigned to the high *g′(e)* bucket.

When each stock has been assigned to a *g′(e)* bucket, it is then scaled relative to other stocks in the same bucket. The low *g′(e)* bucket is used as an example here:

1) Order the stocks within each bucket by raw *g′(e)* score, from lowest to highest.
2) Within the low *g′(e)* bucket, assign each stock a value equal to the cumulative market capitalization represented by that stock and all stocks with a lower *g′(e)*. Thus, the stocks in the low *g′(e)* bucket have values ranging from 0.00+ (the stock with the lowest *g′(e)* in the low *g′(e)* bucket) to 100 (the stock with the highest *g′(e)* in the low *g′(e)* bucket).
3) Where two or more stocks have the same *g′(e)*, they are assigned a value which represents the cumulative float of all stocks with a lower *g′(e)*, plus one-half of the total float of the stocks that share the same *g′(e)*.
4) Re-scale the scores in the low *g′(e)* bucket between 0.00+ and 33.33.

Repeat the four steps immediately above for each of the mid-minus, mid-plus and high *g′(e)* buckets; and re-scale the values as follows:

[4] Share-weighted approach is used in the growth rate calculation instead of market-cap-weighted approach because the growth orientation should not reflect the price characteristics of stocks.

Table 7. **Ranges of g′(e) scores for different g′(e) buckets in calculating growth scores**

Bucket	**Minimum Score**	**Maximum Score**
Low $g'(e)$	0.00+	33.33
Mid-minus $g'(e)$	33.34	50
Mid-plus $g'(e)$	50.01	66.66
High $g'(e)$	66.67	100

All of the steps in this section are then repeated for the other four growth orientation factors. Repeat all of the steps above for each size category.

For financial stocks, cash flow growth is not used for the growth factor calculation because cash flow from operations data is not meaningful for banks and insurance companies.

4.4.3 Overall Growth Scores

When all of the five growth factors have been scored from 0-100, we calculate a weighted average overall growth score for each stock. In the overall growth score calculation, we do not consider $g(e_5)$, as Morningstar methodology suggests. Growth rate $g(e_5)$ is a projected long-term (5-year) growth rate for earnings per share (the same applies to *b*, *r*, *c*, and *d*). These rates should be the third-party estimates, which are not available for majority of stocks and years in our dataset and therefore are not used. Growth rate $g(e_5)$, if available, would be assigned weight 50% with remaining growth rates ($g'(e)$, $g'(b)$, $g'(r)$, and $g'(c)$) having equal weights. Instead, we assign 50% weight to $g'(e)$ in the overall value score; each of the other value factors is assigned an equal share of the remaining weight (either 50% or, if $g'(e)$ is unavailable, 100%).

For example, if all four growth factors are available, the weights are:

Table 8. **Assigning weights to components of overall growth score**

Scores	$g'(e)$	$g'(b)$	$g'(r)$	$g'(c)$
Weights	50%	16.7%	16.7%	16.7%

In this way, we consistently overweigh the earnings yield (in overall value score) and the growth in earnings per share (in overall growth score) over other factors and other factors' growth rates, respectively. The same overweighting of earnings is also imbedded in the Morningstar methodology because both *e/p* and $g(e_5)$ constitute the largest weights in the calculation for respective overall scores.

If no growth rates are available for a given stock, the stock is not given an overall growth score or a net VCG style score (discussed in the next section).

4.5 Value-Core-Growth (VCG) Score and Orientation

Each stock now has an overall value score and an overall growth score; both of these range from 0 to 100. Following Morningstar's methodology, we calculate a final value-core-growth (VCG) score for each stock by subtracting the stock's overall value score from its overall growth score.

As to value and growth orientation, value-oriented stocks (later called value stocks), growth-oriented stocks (later called growth stocks), and core stocks are each assumed to account for one-third of the total capitalization of each scoring group. Hence assignment of value, core, and growth orientation is based on the percentiles of cumulative market capitalization of stocks ranked in ascending order by VCG score at the given date: stocks below 1/3 of the cumulative market capitalization are assigned value orientation, stocks between 1/3 and 2/3 of cumulative market capitalization are assigned core orientation, and the remaining stocks are assigned growth orientation.

We directly consider the VCG score as the score measuring the stock's value-growth orientation. There is no need to take the natural logarithm of VCG score values because their distribution is already sufficiently wide for the purposes of further analysis. As an illustration of this distribution property, below is the histogram of VCG scores calculated according to above steps for the stocks in December 2017.

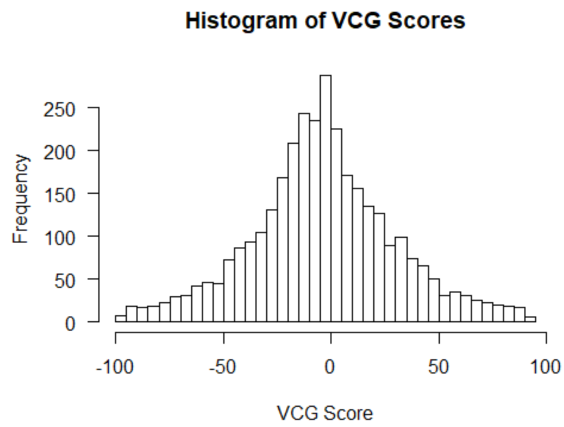

Figure 6. **Histogram of VCG scores for all companies as of December 2017**

Although for different months in different years, the shape of VCG scores distribution will vary somewhat with the overall market being more or less value- or growth-oriented depending on the business cycle, the overall property of VCG scores being sufficiently disperse holds.

As the assignment of size scores (log of market share) and VCG scores is completed, the next step is to determine the parameters explaining the shape of the future return distributions and estimate the relationships between the latter and the two types of scores. These next steps are discussed in the following sections.

4.6 Shape of Future Returns Distribution

4.6.1 Defining Future Returns

Since the goal is to study how well size and VCG characteristics predict the shape of the return distribution, for each stock in month *t* we measure the current value of size and VCG scores and future realized 12-month return. In our thesis, we use the log convention in determining stock returns and thus further in the text use the term "returns" meaning the log returns. For a company with $PERMNO = p$ in month *t*, we define future 1-month total return $logret1m_t^p$ as

$$logret1m_t^p = log\left(\frac{P_{t+1}^p + D_{t+1}^p}{P_t^p}\right), \tag{16}$$

where P corresponds to the split-adjusted price of the stock $PRCCM_{split-adjusted}$ and D corresponds to split-adjusted dividend per share $DVPSXM_{split-adjusted}$. Then the corresponding future 12-month return $logret12m_t^p$ is defined as

$$logret12m_t^p = \sum_{m=0}^{11} log\left(\frac{P_{t+m+1}^p + D_{t+m+1}^p}{P_{t+m}^p}\right). \tag{17}$$

4.6.2 Skew-t Distribution Fit

As opposed to the large finance literature investigating the predictability of expected return (first moment of the distribution), in this thesis, apart from distribution mean, we also focus on higher order moments. To account for the asymmetry and fat tails present in financial data, we use the skew-t distribution. Below, the theory behind skew-t distribution is described in detail.

We use a theoretical framework for skew-t distribution from Azzalini (2014). The derivation of the skew-t distribution starts with the usual symmetrical Student's t-distribution whose density function is

$$t(x;\nu) = \frac{\Gamma(\frac{1}{2}(\nu+1))}{\sqrt{\pi\nu}\,\Gamma(\frac{1}{2}\nu)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{1}{2}(\nu+1)}, x \in \mathbb{R}, \tag{18}$$

where $\nu > 0$ denotes degrees of freedom, and Γ denotes gamma function

$$\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx. \tag{19}$$

It can be shown that the following transformation of this density function produces the density of the asymmetric version of the Student's t-distribution

$$2t(x;\nu)T(\alpha x;\nu), \tag{20}$$

where T(. ; ν) denotes symmetrical Student's t-distribution function, and α is a slant parameter. Define Z as

$$Z = \frac{Z_0}{\sqrt{V}}, \tag{21}$$

where $Z_0 = SN(0, 1, \alpha)$ and $V = \chi_\nu^2/\nu$ are independent variates. *SN* denotes skew-normal distribution $SN(\xi, \omega^2, \alpha)$, where ξ denotes location parameter, and ω denotes dispersion parameter[5]. χ_ν^2 denotes Chi-Square distribution with ν degrees of freedom. In this case, if $h(\cdot)$ denotes the density function of *V*, then it can be shown that density function of *Z* is

$$t(x;\alpha,\nu) = 2t(x;\nu)T\left(\alpha x\sqrt{\frac{\nu+1}{\nu+x^2}};\nu+1\right). \tag{22}$$

If $\alpha = 0$, (22) reduces to the usual Student's density (18). If $\nu \to \infty$, then (22) converges to the $SN(0, 1, \alpha)$ density.

From the graphical representation below we can see how Student's skew-t density function behaves given different values for degrees of freedom ν and slant parameter α:

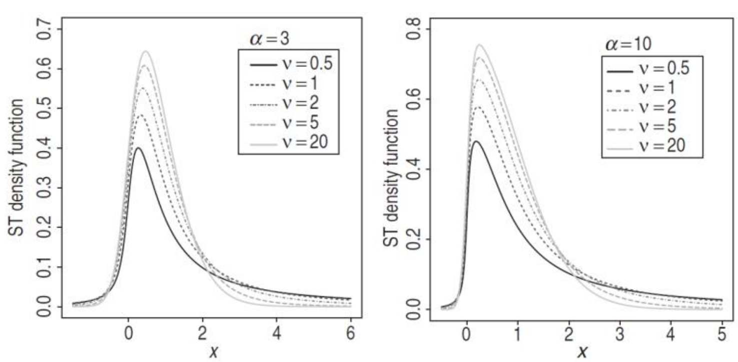

Figure 7. **Shape of skew-t density for different values of slant parameter (α) and degrees of freedom (ν)**

Low values of ν correspond to heavier tails while greater (more negative) α correspond to a higher probability that observations fall to the right (left) side from the mode, respectively.

[5] Details of the derivation of the skew-normal distribution can be found in Azzalini (2014).

The distribution family represented by (22) is further extended to include the location and scale parameter.

Consider $Y = \xi + \omega Z,$ leading to a four-parameter family of distributions whose density function at x is $\omega^{-1} t(z; \alpha, \nu)$, where $z = \omega^{-1}(x - \xi)$.

Hence, we can say that Y follows a skew-t distribution and write

$$Y \sim ST(\xi, \omega^2, \alpha, \nu). \tag{23}$$

The skew-t distribution in (23) is the final version of the distribution used in the thesis for further empirical analysis. The parameters describe the shape of the distribution and are discussed in greater detail in the next section.

To fit the skew-t distributions on the returns from our dataset, we use the function "st.mple" from package "sn" in R developed by Azzalini (2018) based on the theory described above. The algorithm of fitting skew-t uses maximum likelihood for parameter estimation and is described in detail by Azzalini (2014). As the output, estimates of parameters of the fitted skew-t are obtained $(\xi, \omega^2, \alpha, \nu)$.

4.6.3 Skew-t Parameters of Interest

After fitting the skew-t distribution, a set of parameters $(\xi, \omega^2, \alpha, \nu)$ is obtained. Below we discuss what exactly each of the parameters says about the shape of the distribution.

The location parameter ξ, as it is defined by Azzalini (2014), very closely approximates the distribution mode, or most likely value of the distribution. In other words, it shows where the peak of the distribution is located and is used in our further analysis as one of the four parameters of interest.

The second parameter used in our further analysis is the dispersion parameter ω, which measures the volatility of the variable.

The slant parameter α indicates to which side and to what extent the distribution is skewed. A drawback of this measure, as discussed by Halling and Giordani (2018), is that its estimates may be rather unstable. For this reason, we instead use a different measure of skewness, namely, the percentage of area below the density function located to the left of the location parameter, further denoted as π. The higher the π, the more negatively skewed the distribution is. Estimates of this measure of skewness are expected to be more stable. In addition, π has a rather intuitive interpretation: it indicates the percentage probability that the future returns will fall below the most likely future return.

Finally, the fourth parameter of interest is degrees of freedom ν, which indicates how heavy-tailed the distribution is. The higher the ν, the less leptokurtic a distribution is. In extreme case, when $\nu \to \infty$, the skew-t distribution converges to the skew-normal distribution with no leptokurtosis.

Based on above, for the purposes of further analysis, we select four parameters of the skew-t distribution that describe different features of its shape – location ξ, dispersion ω, skewness as percentage π, and degrees of freedom ν. It should be noted that the parameters of the skew-t distribution $ST(\xi, \omega^2, \alpha, \nu)$, or direct distribution parameters, can be used to calculate so called centered parameters. These are the mean μ (first central moment), variance σ^2 (second central moment), skewness γ_1 (third central moment), and excess kurtosis γ_2 (fourth central moment minus 3). On the one hand, central moments may seem to be more intuitive measures of the distribution shape. On the other hand, as can be seen from the formulas[6], each centered parameter is dependent on the whole set of direct parameters. In other words, using the centered parameters instead of direct ones, we would not be able to disentangle different features that determine the distribution shape.

4.7 Testing Goodness of Skew-t Fit

In order to test the goodness of skew-t fits used in the thesis, we perform Chi-Square goodness of fit test. This test is widely used to find out whether the observed value of given phenomena is significantly different from the expected value. The term "goodness of fit" is used to compare the observed sample distribution with the expected probability distribution and determines how well theoretical distribution fits the empirical distribution. In the Chi-Square goodness of fit test, sample data is divided into intervals. Then the numbers of points that fall into the interval are compared with the expected numbers of points in each interval.

The *null hypothesis (H_0)* of this test states that, at a given significance level, there is no significant difference between the observed sample distribution and the expected, or assumed, probability distribution. The *alternative hypothesis (H_1)* states that there is significant difference between the observed sample distribution and the expected, or assumed, probability distribution. The p-value of the test helps indicate whether the null hypothesis can or cannot be rejected.

In Section 5.1, for resulting series of returns, we test the goodness of skew-t fit and compare it to the goodness of the alternative normal fit in order to demonstrate that the former fit is superior to the latter one and to have more confidence that the parameter estimates of the fitted skew-t distributions are reliable. In Section 5.3, we test the goodness of skew-t fits and find that those cannot be rejected for vast majority of

[6] Formulas for calculating the centered parameters can be found in Appendix B.

the return series used. The alternative normal fit is already strongly rejected in Section 5.1 and there is clearly no need to confirm these results again. In the portfolio application part of the thesis, Chi-Square test is not performed because skew-t fits used there are based on the same returns that have been tested in the previous sections and are hence also assumed to be "good".

4.8 Relationship Between Size/VCG Characteristics and Future Returns

Previous sections describe how to measure the size and VCG characteristics of stocks (explanatory variables), how to identify the main parameters explaining the shape of future return distributions (dependent variables), and how to make sure these parameters are reliable, i.e. the corresponding skew-t distribution fits are "good". The current section explains the framework used to measure the relationships between these explanatory and dependent variables. As the first step, we consider the issue from the broader perspective of size and VCG categories based on the Morningstar Style Box.

4.8.1 Size and VCG Category Perspective

This part of analysis aims to provide a broader view of the relationship between size/VCG characteristics and the shape of future returns. In each month, stocks are split into nine size/VCG categories based on the Morningstar Style Box (large value, large core, etc.). For each category c, we consider an equally weighted portfolio of stocks and in each month t of the period between January 1962 and December 2017 obtain future 12-month portfolio returns

$$logret12m_t^c = Average[logret12m_t^p], \qquad (24)$$

where p denotes the stocks from category c in month t, and $logret12m_t^p$ is calculated as in (17). After the series of future returns are obtained for all categories, estimate parameters of interest for the corresponding nine portfolio return distributions – ξ_c, ω_c, π_c and ν_c.

A market-cap-weighted approach delivers essentially very similar results and therefore is not considered in the results section. This part of analysis based on the category perspective provides a convenient way to demonstrate the resulting distributions graphically and gives a more general idea about the relationships, which are analyzed in greater detail later within the regression framework.

4.8.2 Regression Analysis

The size of the dataset (56 years of monthly observations, ~8000 companies as of December 2018, in total ~2.4m observations) enables us to make use of both time-series and cross-sectional dimensions to conduct deeper analysis of the relationships between size/VCG characteristics and shape of future return distributions.

Sufficient cross-sectional variation in stocks' size scores achieved by log transformation and in stocks' VCG scores enables us to consider different portfolios of stocks based on their size and VCG scores. In each month, we split the companies cross-sectionally into 10 equally sized portfolios by size score. The borderline values of the size score (size scores splitting the 10 size-score-based portfolios) vary over time. However, this variance is sufficiently low, and therefore, in order to obtain single values of borderline size scores applicable to all months, we calculate the time-series median of borderline size scores. These median values determine the ends of the size intervals i, where $i = 1,2,\dots,10$. We apply the same procedure to VCG and thus VCG intervals are defined as j, where $j = 1,2,\dots,10$. In each month, we assign stocks to 100 portfolios based on different combinations of i and j. It should be noted that size and VCG intervals must be fixed over time to make sure that stocks falling into resulting portfolios in all periods have size and VCG characteristics from exact known size and VCG ranges.

After assigning stocks to portfolios based on size and VCG intervals, we calculate the equally weighted average cross-sectional future return within each portfolio. Then, $logret12m_t^{i,j}$ is the average 12-month future return of stocks from portfolio corresponding to size interval i and VCG interval j. We consider simple average returns instead of market-cap-weighted average returns because the latter would overweight returns of larger stocks, which is not desirable given that we assume stocks within a given portfolio to be homogenous by size.

After calculating $logret12m_t^{i,j}$ for all t, i and j, we make use of the time-series dimension and obtain 100 series of future average 12-month returns of the portfolios based on size and VCG intervals. The table below illustrates this idea.

Table 9. **Portfolio forward-looking 12-month returns for different combinations of size intervals *i* and VCG intervals *j***

	Portfolio returns				
Month (*t*)	i = 1; j = 1	i = 1; j = 2	…	i = 10; j = 9	i = 10; j = 10
Jan 1962	$logret12m^{1,1}_{Jan\ 1962}$	$logret12m^{1,2}_{Jan\ 1962}$	$\cdots$	$logret12m^{10,9}_{Jan\ 1962}$	$logret12m^{10,10}_{Jan\ 1962}$
Feb 1962	$logret12m^{1,1}_{Feb\ 1962}$	$logret12m^{1,2}_{Feb\ 1962}$	$\cdots$	$logret12m^{10,9}_{Feb\ 1962}$	$logret12m^{10,10}_{Feb\ 1962}$
…	…	…	…	…	…
Nov 2017	$logret12m^{1,1}_{Nov\ 2017}$	$logret12m^{1,2}_{Nov\ 2017}$	$\cdots$	$logret12m^{10,9}_{Nov\ 2017}$	$logret12m^{10,10}_{Nov\ 2017}$
Dec 2017	$logret12m^{1,1}_{Dec\ 2017}$	$logret12m^{1,2}_{Dec\ 2017}$	$\cdots$	$logret12m^{10,9}_{Dec\ 2017}$	$logret12m^{10,10}_{Dec\ 2017}$

It should be noted that for certain combinations of size and VCG intervals in some months portfolios are empty. This issue mostly appears in the earliest years (1960s and 1970s), when the number of companies present in the dataset is not large enough to contain stocks from all 100 combinations of *i* and j. Therefore, further we consider only those months *t* at which returns $logret12m_t^{i,j}$ are present for all 100 portfolios. In this way, we make sure that return series for each of the 100 portfolios contains returns strictly from the same months. The resulting returns series start and contain almost no gaps from December 1982.

It should be noted that for any given portfolio, there is most likely a large variation in companies across time. We do not consider this as a serious problem because the main principle for portfolio construction is based on the size/VCG scores, and so the persistence of companies within portfolios over time is not crucial.

For all 100 resulting series of returns, we fit skew-t distributions and obtain 100 sets of parameters of interest $(\xi_{i,j}, \omega_{i,j}, \pi_{i,j}, \nu_{i,j})$. These portfolio-specific values of the distribution parameters are the dependent variables for the four regression equations discussed later.

In order to obtain portfolio-specific explanatory variables capturing the portfolio's size characteristics, we consider a median size score (across time and stocks) denoted further as $size_{i,j}$:

$$size_{i,j} = Median[size\ scores\ of\ stocks\ from\ portfolio\ i,j\ across\ time]. \qquad (25)$$

For example, $size_{1,1}$ represents a single estimate of the size score for the companies with the smallest size and VCG scores. Similar interpretation applies for $i, j = 2,3, \ldots, 10$. The same procedure is performed for VCG scores and hence $VCG_{1,1}$ is obtained as a single estimate of the VCG score for the companies with the smallest size and VCG scores, with similar interpretation for $j = 2,3, \ldots, 10$:

$$VCG_{i,j} = Median[VCG\ scores\ of\ stocks\ from\ portfolio\ i,j\ across\ time]. \qquad (26)$$

Taking median instead of averaging the scores is justified by the fact that for the portfolios of the smallest stocks (when *i* = 1), averaging the size scores would lead to significantly lower estimates of $size_{1,j}$ compared to the case if median-based approach is used. This is because averaging the scores would imply assigning unreasonably high weight to companies with extremely low size scores. Such companies in turn are heavily underrepresented in the sample. Thus, median-based approach helps obtain estimates of $size_{1,j}$ more adequately representing the size score of companies from respective portfolios. Above issue also concerns portfolios of largest stocks (when *i* = 10).

Firstly, to obtain a more general understanding of the relationships between portfolio-specific size/VCG scores and distribution parameters of the portfolios, we estimate the simple linear OLS models shown below. Each equation is estimated using 100 portfolio-specific observations.

$$\xi_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{VCG} VCG_{i,j} + \varepsilon_{i,j} \tag{27}$$

$$\omega_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{VCG} VCG_{i,j} + \varepsilon_{i,j} \tag{28}$$

$$\pi_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{VCG} VCG_{i,j} + \varepsilon_{i,j} \tag{29}$$

$$\nu_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{VCG} VCG_{i,j} + \varepsilon_{i,j} \tag{30}$$

Secondly, some adjustments to the regression specifications are made. In particular, for cases when location parameter and dispersion parameter are dependent variables, a visual inspection of the corresponding scatterplots motivates the addition of quadratic terms, as discussed in more detail in the Results section. Moreover, the lack of clear relationships between size/VCG and degrees of freedom motivates the exclusion of explanatory variables for $\nu_{i,j}$. The resulting final regression specifications have the following functional form:

$$\xi_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{size^2} size_{i,j}^2 + \beta_{VCG} VCG_{i,j} + \varepsilon_{i,j} \tag{31}$$

$$\omega_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{VCG} VCG_{i,j} + \beta_{VGC^2} VGC_{i,j}^2 + \varepsilon_{i,j} \tag{32}$$

$$\pi_{i,j} = \beta_0 + \beta_{size} size_{i,j} + \beta_{VCG} VCG_{i,j} + \varepsilon_{i,j} \tag{33}$$

$$\nu_{i,j} = \beta_0 + \varepsilon_{i,j} \tag{34}$$

It should be emphasized that even though size and VCG scores are available for every individual stock, the dependent variables (distribution parameters) are only available for 100 obtained portfolios. Therefore, we consider medians of the size and VCG scores of stocks within these portfolios across time in order to obtain 100 values of portfolio-specific explanatory variables – $size_{i,j}$ and $VCG_{i,j}$. An alternative approach would be to consider the number of observations in the regressions equal to the number of all available stocks in the dataset, consider stock-specific size and VCG scores as explanatory variables, and, as before, 100 portfolio-specific values of dependent variables $parameter_{i,j}$. In this case, every stock from portfolio i,j would be assigned the same value of $parameter_{i,j}$. Even though such an approach would considerably increase the number of observations, it would not help add value in identifying the relationships between size/VCG and future return. The variability in dependent variables would still be the same as in the case of portfolio-based regressions. In other words, the resulting relationships would be essentially the same but the t-statistics for the coefficients would be artificially inflated due to the large number of observations. This, in turn, would make it difficult to judge the "true" significance of the relationships.

Overall, by performing the regression analysis described above, we test the following hypothesis:

Hypothesis 1: Size and value-growth orientation measured in month *t* have a significant impact on the shape parameters of the distribution of future 12-month returns.

Results from the regression analysis are further used in the application part of the thesis, which is discussed in the next section.

4.9 Portfolio Application

The goal of this section of the thesis is to test whether the regression model described in the previous section has the predictive power. By this we mean testing whether the model can identify portfolios of stocks whose future performance exhibits certain forecasted risk and risk-reward characteristics. The general idea of this test is to construct "best" and "worst" portfolios of stocks based on the estimated criteria (VaR and Reward-to-VaR), rebalance the portfolios every period based on the re-estimated criteria values, and compare the realized risk characteristics (in case of VaR used as criterion) and risk-reward characteristics (in case of Reward-to-VaR used as criterion) of portfolios' performance. For illustrative purposes, we also investigate how the performance of such portfolios compares to the performance of the benchmark indices.

The methodological steps of this test are as follows:

1) Determine the test period (see section Test Period);
2) Re-estimate distribution parameters of stocks every month over the test period (see section Re-estimation of Distribution Parameters)
3) Determine and calculate the criteria for stock selection (see section Criteria for Stock Selection);

4) Select portfolios of "best" and "worst" stocks every month based on these criteria and compare the risk and risk-reward characteristics of the performance of the selected portfolios (see section Portfolio Construction).
5) Compare the cumulative performance of the "best" and "worst" stocks with the cumulative performance of the benchmark market indices.

Each of these steps is discussed in greater detail below.

4.9.1 Test Period

In choosing a test period, it is important to consider a sufficiently long timeframe that contains both periods when the market was growing and periods when it was declining. According to the National Bureau of Economic Research[7], the trough following the 2001 recession in the United States was registered in November 2001. However, the economic cycle in that period did not coincide with the cycle of the stock market, which in 2001 continued to decline as the dotcom bubble was bursting. October 2002 is believed to mark the end of the dotcom bubble as the US stock market reached the bottom, however fluctuations continued well into 2003. Therefore, we choose January 2004 as the beginning of the test period, when the stock market was generally in the upturn, as can be seen in Figure 1. Thus, we consider the period from January 2004 to December 2018 (180 months) including the steady growth of early and mid-2000's, crisis of 2008-2009, and following growth until 2018.

4.9.2 Re-estimation of Distribution Parameters

Before determining the criteria for stock selection, we first discuss how the parameters of the future return distributions are re-estimated every month within the test period. Essentially, re-estimation of parameters implies performing the Regression Analysis described in the previous section for every month t of the test period, as is explained in greater detail below.

Firstly, in each month t of the test period, we re-estimate the coefficients of the regression equations based on the portfolio returns from Table 9 considering only months before month t. For example, when t is January 2004, in Table 9 we consider returns of 100 portfolios until January 2003[8], fit 100 skew-t distributions on returns from these months, obtain estimates of distribution parameters ($\xi_{i,j}^{t}$, $\omega_{i,j}^{t}$, $\alpha_{i,j}^{t}$, and

[7] Source: https://www.nber.org/cycles.html

[8] One-year lag relative to January 2004 comes from the fact that $logret12m_t^{i,j}$ are 12-month *future* returns, so the last available future 12-month return observable in January 2004 is the one from January 2003.

$\nu_{i,j}^t$) and estimate regressions. The total of 180 sets (equal to the number of months *t*) of 4 regressions (for four skew-t parameters) are re-estimated:

$$\xi_{i,j}^t = \beta_0^t + \beta_{size}^t size_{i,j} + \beta_{size^2}^t size_{i,j}^2 + \varepsilon_{i,j} \tag{35}$$

$$\omega_{i,j}^t = \beta_0^t + \beta_{size}^t size_{i,j} + \beta_{VCG}^t VCG_{i,j} + \beta_{VCG^2}^t VCG_{i,j}^2 + \varepsilon_{i,j} \tag{36}$$

$$\alpha_{i,j}^t = \beta_0^t + \beta_{size}^t size_{i,j} + \beta_{VCG}^t VCG_{i,j} + \varepsilon_{i,j} \tag{37}$$

$$\nu_{i,j}^t = \beta_0^t + \varepsilon_{i,j} \tag{38}$$

There are two differences between above regression specifications and those from the full sample fit described in the previous section. The first is that we do not include the coefficient for VCG in the equation for $\xi_{i,j}^t$. This is because the relationship between VCG and location parameter is not found to be significant in the full sample fit and, when tested, is generally not significant when re-estimated over the test period. Secondly, we use the slant parameter $\alpha_{i,j}^t$ as the measure of skewness instead of skewness as percentage $\pi_{i,j}^t$ used in the full sample fit. The reason is that $\alpha_{i,j}^t$ and $\pi_{i,j}^t$ behave in the identical way, and for the purposes of calculating the criteria values for stock selection, as is described further in this section, it is more convenient to obtain estimates of $\alpha_{i,j}^t$, which is the direct parameter of the skew-t distribution.

The significance of the regression specifications from (35) – (38) holds firmly over time, which is separately tested (for details see Appendix C).

The obtained portfolio-based coefficients are applied to size and VCG scores of every individual stock p in month *t* and hence estimates of stock-specific distribution parameters are estimated as ξ_p^t, ω_p^t, α_p^t, and ν_p^t. We apply portfolio-specific coefficients to stock-specific size and VCG scores assuming that relationships between distribution parameters and size and VCG characteristics are the same in case of both size/VCG-specific portfolios and individual stocks. The estimated stock-specific distribution parameters form the basis to compute the values of criteria for stock selection, which is discussed in the next section.

We apply the above procedure to all stocks in month *t* and, in the same way, to all remaining months of the test period until December 2018.

4.9.3 Criteria for Stock Selection

For stock selection we introduce two criteria:

- Value at Risk (VaR), which captures the risk characteristics of stocks;
- Reward-to-VaR (RtVaR), which captures the risk-reward characteristics of stocks.

Value at Risk is a standard measure used to assess tail risk. It estimates the potential loss in value of a stock over defined period for a given confidence level. We prefer this measure of risk over volatility or dispersion because VaR better captures both the negative skewness and leptokurtosis of returns. Both these phenomena are evident in our data, as shown in the Regression Results of this thesis. In our case, 12-month forward-looking 5% VaR for stock *p* in month *t* is estimated as

$$VaR_p^t = -qst(0.05, \xi_p^t, \omega_p^t, \alpha_p^t, \nu_p^t\ ). \quad (39)$$

In other words, VaR_p^t is a 5% (0.05) quantile (applied using function "qst" in R) implied by skew-t distribution with parameters $\xi_p^t, \omega_p^t, \alpha_p^t$ and ν_p^t. We add the negative sign in order for VaR_p^t to be a positive number.

In order to capture the risk-reward characteristics of stocks, as the second criteria we introduce Reward-to-VaR (RtVaR), which for stock *p* in month *t* determines the stock's future most likely return per unit of future tail risk captured by VaR:

$$RtVaR_p^t = \frac{\xi_p^t}{VaR_p^t}. \quad (40)$$

We choose most likely future return, or location parameter ξ_p^t, in the numerator instead of expected return because the estimates of expected returns based on our model are negative for a large number of smallest companies. This, in turn, would make the estimates of RtVaR negative, in which case the changes in RtVaR lack interpretability.

4.9.4 Portfolio Construction

Firstly, we determine different stock universes, within which the stock selection is implemented. Since the methodology of the thesis is to a large extent based on the Morningstar Style Box, we consider universes corresponding to each of the nine combinations of size and VCG categories. Within a certain universe (for example, small value stocks) we consider a benchmark portfolio. The most intuitive benchmark portfolio is a corresponding market-cap-weighted index, which is a type of benchmark commonly used in practice (for example, MSCI US Small Cap Value Index[9] for small value stocks). In our case, we compose these indices ourselves based on the available dataset and track the cumulative performance on a monthly basis. The monthly realized return in month *t+1* of every constituent stock *p* included in the index in month *t* is calculated as

[9] Source: https://www.msci.com/documents/10199/25244741-85f5-4851-98f4-872d4fd82423

$$ret1m_t^p = e^{logret1m_t^p} - 1, \tag{41}$$

where $logret1m_t^p$ is calculated as in (16). Market capitalization of stock p in month t, or $mcap_t^p$, is defined as

$$mcap_t^p = PRCCM_{split-adjusted}{}_t^p \times CSHOQ_{split-adjusted}{}_t^p, \tag{42}$$

where the price and common shares outstanding are split-adjusted, as defined in (1) and (2). Market-cap-weighted average return of the index realized in month $t+1$ is calculated as

$$ret1m_t^{index} = \frac{\sum_{p \in index\ in\ month\ t} ret1m_t^p \times mcap_t^p}{\sum_{p \in index\ in\ month\ t} mcap_t^p}. \tag{43}$$

As the rule for stock selection in month *t*, we consider the "best" and "worst" quartiles of stocks based on the selection criteria determined in section Criteria for Stock Selection. Namely,

- for Value at Risk criterion, the "best" ("worst") quartile corresponds to the 25% of stocks with the lowest (highest) values of VaR in a given month;
- for Reward-to-VaR criterion, the "best" ("worst") quartile corresponds to the 25% of stocks with the highest (lowest) values of respective criteria.

Further, we consider value-weighted portfolios of stocks from the "best" and "worst" quartiles and track their performance in the same fashion as is done for the benchmark index. The selection of "best" and "worst" stocks implemented in our thesis is essentially equivalent to reducing the value-weighted benchmark index by keeping only the constituents within the "best" and "worst" quartiles based on the two criteria without any further change of relative weights.

As the next step, we estimate the risk and risk-reward characteristics implied by the performance of the "best" and "worst" stocks. For "best" portfolio and "worst" portfolio based on the VaR criteria, we calculate the realized portfolio Value at Risk implied by its performance over the whole test period calculated as the 5% percentile of the distribution of annual portfolio returns realized every month. For "best" portfolio and "worst" portfolio based on the RtVaR criterion, we calculate the portfolio Reward-to-VaR implied by its performance over the test period. Realized portfolio RtVaR is calculated as the most likely realized portfolio annual return estimated by the skew-t fit divided by the 5% percentile of the distribution of annual portfolio returns realized every month.

The same analysis is conducted for the remaining eight combinations of size and VCG categories. In addition, we consider three stock universes based solely on VCG category (value, core and growth), three stock universes based solely on size category (small, mid and large), as well as the total stock universe. Hence 16 universes are considered in total.

As was noted, our stock selection implies a simple exclusion of stocks from the benchmark index without any further weights optimization. We choose this simplified approach because a proper optimization routine aimed at minimizing or maximizing portfolio VaR or RtVaR of "best" or "worst" stocks would be computationally infeasible given the large number of stocks in each of the stock universes. Moreover, we admit that correlations between individual stocks should ideally be taken into consideration when implementing a proper portfolio optimization routine. In other words, by selecting the "best" and "worst" stocks in the criteria-based portfolios, we may not necessarily obtain the "best" and "worst" portfolios but can only approximate "best" and "worst" portfolios. However, the goal of this section is to test the relationships obtained by analyzing the whole US stock market. For that reason, we find it necessary to consider all companies in the portfolio construction. More rigorous portfolio optimization procedure as well as incorporating correlations between individual stocks may be suggested as a future line of inquiry.

If the realized VaR of the portfolios of "best" stocks selected by VaR criterion (stocks with lowest forecasted VaR) is consistently below realized VaR of the portfolios of "worst" stocks (stocks with highest forecasted VaR) across different stock universes, we can conclude that the VaR criterion incorporating the shape of predictive return distributions driven by stocks' size and value-growth orientation tends to correctly identify the stocks with higher or lower forecasted future tail risk. Similarly, if the realized RtVaR of the portfolios of "best" stocks selected by RtVaR criterion (stocks with highest forecasted RtVaR) is consistently below RtVaR of the portfolios of "worst" stocks (stocks with lowest forecasted RtVaR) across different stock universes, we can conclude that the RtVaR criterion incorporating the shape of predictive return distributions driven by stocks' size and value-growth orientation tends to correctly identify the stocks with higher or lower implied future reward to tail risk. In other words, if this consistency is observed, it will ultimately support the significance of relationships between size/VCG and future return distributions found in the regression analysis and enable us to accept the Hypothesis 2 formulated below.

Hypothesis 2: If Hypothesis 1 is accepted, the regression model incorporating the relationships between size/VCG and the shape of future returns distribution has the predictive power, i.e. it can identify portfolios of stocks whose actual future performance exhibits certain forecasted risk and risk-reward characteristics.

As outlined in the beginning of this section, the secondary goal is to compare the cumulative performance of "best" and "worst" portfolios based on VaR and RtVaR with that of benchmark indices for the respective stock universes. This goal of the portfolio application has a more illustrative purpose, and therefore we do not test any hypothesis.

5 Results

5.1 Testing Goodness of Skew-t Fit from Category Perspective

It should be highlighted that skew-t indeed provides a much better fit for the empirical future return distributions compared to normal fit, as can be seen from Figure 8 below.

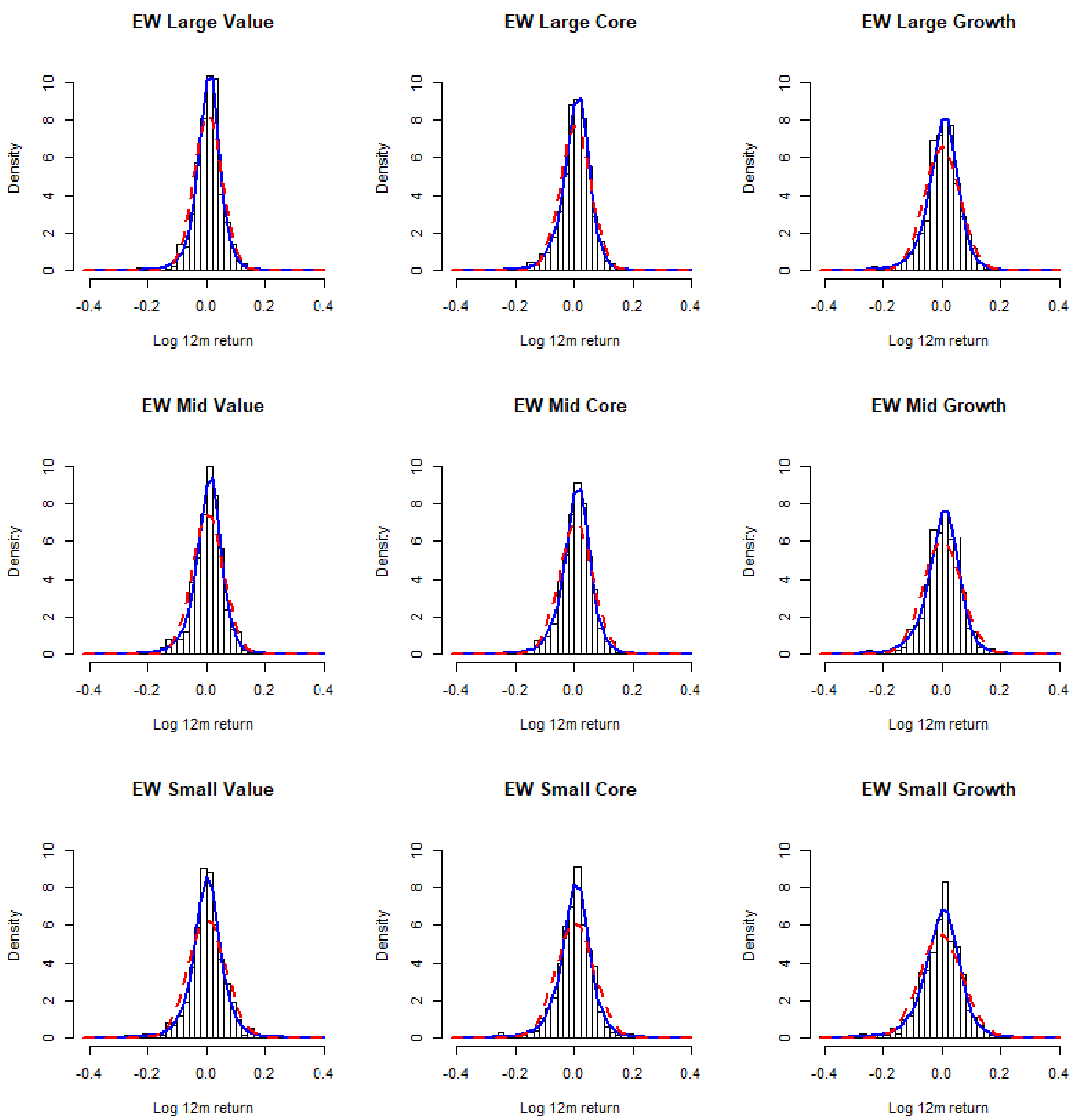

Figure 8. **Histograms of future 12-month returns of equally weighted portfolios of stocks corresponding to different combinations of size and VCG categories**. Blue lines illustrate skew-t fits, red lines illustrate normal fits. Distributions are fit on monthly observations of annual future 12-month log returns from January 1962 to December 2017.

The same conclusion about higher accuracy of skew-t fits is obtained by performing the Chi-Square goodness of fit test. At 5% significance level, this fails to reject assumed skew-t fits for all 9 portfolios, and rejects assumed normal fits for all 9 portfolios (for details see Table 10 below).

Table 10. **P-values of Chi-Square test for skew-t and normal distribution fits for future 12-month returns of equally weighted portfolios based on size and VCG categories.** Goodness of fit is tested using the Chi-Square test. H_0 hypothesis states that empirical return samples are drawn from the distributions of assumed type. At 5% significance level, p-values higher than 0.05 indicate that assumed fit cannot be rejected, p-values below 0.05 (highlighted) indicate that assumed fit is rejected. Distributions are fit on monthly observations of annual future 12-month log returns from January 1962 to December 2017.

Chi-Square test p-values (skew-t fit assumed)			
	Value	Core	Growth
Large	0.1854	0.7686	0.1774
Mid	0.3328	0.5277	0.0555
Small	0.1964	0.0700	0.3408

Chi-Square test p-values (normal fit assumed)			
	Value	Core	Growth
Large	< 0.0005	< 0.0005	< 0.0005
Mid	< 0.0005	< 0.0005	< 0.0005
Small	< 0.0005	< 0.0005	< 0.0005

Visual comparison of the quality of skew-t and normal fits, as well as output of the Chi-Square test, firstly, clearly indicate that skew-t fit is superior to normal fit for returns data in our analysis, and secondly, strongly suggest that the estimates of skew-t parameters of the nine distributions are reliable and can be used in further analysis.

5.2 Size/VCG Category Perspective Results

Inspecting Figure 8 further, we see that the distributions show a clear pattern. Namely, moving from small-caps to large-caps and from growth orientation to value orientation, distributions become significantly more peaked and narrower. This suggests that large value stocks are least volatile while small growth stocks are most volatile, in agreement with Schadler and Eakins (2001).

However, based only on visual inspection, it is hard to see any other patterns related to our distribution parameters of interest. Therefore, below we consider individual estimates of the parameters for respective fitted skew-t future return distributions.

Table 11. **Parameters of the fitted skew-t distributions of the future 12-month returns of equally weighted portfolios of stocks from different size and VCG categories.** Distributions are fit on monthly observations of annual future 12-month log returns from January 1962 to December 2017.

Location ξ			
	Value	Core	Growth
Large	2.1%	3.4%	3.1%
Mid	3.0%	2.8%	3.3%
Small	0.6%	1.7%	3.0%

Dispersion ω			
	Value	Core	Growth
Large	3.7%	4.8%	5.2%
Mid	4.4%	4.6%	5.5%
Small	4.3%	4.6%	6.1%

Skewness as percentage π			
	Value	Core	Growth
Large	64%	73%	71%
Mid	70%	68%	71%
Small	53%	61%	69%

Degrees of freedom ν			
	Value	Core	Growth
Large	4.29	5.39	4.93
Mid	4.33	4.47	4.54
Small	3.49	3.75	4.72

Looking at the location parameter, we clearly see that more growth-oriented stocks tend to realize the highest most likely future returns. Another useful observation is that while growth stocks from all categories demonstrate approximately similar most likely future returns, value stocks perform relatively well only in case of large-caps.

The pattern related to varying volatility across categories observed in Figure 8 is also clearly reflected in values for dispersion parameter in the table above – dispersion grows consistently as we move to the right bottom corner of the Style Box to small growth category.

Interesting observations can be made by looking at the values of skewness. We can observe that the distribution of future returns of smaller and more value-oriented stocks tends to be more symmetric than that of larger and more growth-oriented stocks, which exhibit more negative skewness. For the equally weighted index of small value stocks, based on the period from 1962 until December 2018, future 12-month returns are almost equally likely to exceed or fall behind future most likely return ($\pi = 53\%$), while future returns of large growth stocks index exceeds its most likely future return only in 29% of cases ($\pi = 71\%$) implying significantly more negative skewness.

Finally, the degrees of freedom parameter, determining how heavy-tailed a distribution is, suggests that as we move from small to large and from value to growth, distributions tend to become less heavy-tailed.

Below we present the summary of the results of this section.

Table 12. **Summary of the results based on size and VCG categories**

Type of parameters	Results summary
Location	• Most likely future returns increase with size • Most likely future returns increase with VCG (with growth orientation)
Dispersion	• Dispersion of future returns decreases with size • dispersion of future returns increases with VCG (with growth orientation)
Skewness	• Skewness of future returns becomes more negative with size • Skewness of future returns becomes more negative with VCG (with growth orientation)
Degrees of freedom	• Future returns tend to become less heavy-tailed with size • Future returns tend to become less heavy-tailed with VCG (with growth orientation)

5.3 Testing Goodness of Skew-t Fit from Regression Perspective

The results from the previous section provide useful insights the significance of which should be further investigated within a more rigorous regression framework. Firstly, however, as in case of size and VCG category perspective, the reliability of the skew-t fits used in the regression analysis has to be separately discussed.

The Chi-Square test performed for the returns series of 100 size/VCG-specific portfolios shows that for 84 of them, at the 5% significance level, the hypothesis that returns are drawn from the skew-t distributions cannot be rejected (for details see Table 13).

Table 13. **P-values of Chi-Square test for skew-t distribution fits for future 12-month returns of portfolios based on size and VCG intervals.** Rows "size i" correspond to size intervals *i=1,2,…,10*. Columns "VCG j" correspond to VCG intervals *j=1,2,…,10*. Goodness of fit is tested using the Chi-Square test. H_0 hypothesis states that empirical return samples are drawn from the distributions of assumed type. At 5% significance level, p-values higher than 0.05 indicate that assumed fit cannot be rejected, p-values below 0.05 (highlighted) indicate that assumed fit is rejected. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Chi-Square test p-values (skew-t fit assumed)										
	VCG 1	VCG 2	VCG 3	VCG 4	VCG 5	VCG 6	VCG 7	VCG 8	VCG 9	VCG 10
size 10	0.178	0.158	0.172	0.136	0.018	0.040	0.151	0.021	0.279	0.104
size 9	0.232	0.190	0.568	0.168	0.363	0.042	0.347	0.067	0.056	0.036
size 8	0.141	0.335	0.360	0.137	0.052	0.392	0.008	0.060	0.039	0.039
size 7	0.070	0.263	0.094	0.197	0.082	0.073	0.315	0.123	0.215	0.206
size 6	0.145	0.636	0.083	0.222	0.075	0.096	0.121	0.313	0.182	0.335
size 5	0.226	0.080	0.388	0.102	0.394	0.615	0.127	0.284	0.036	0.304
size 4	0.338	0.046	0.119	0.072	0.340	0.078	0.043	0.076	0.056	0.264
size 3	0.203	0.061	0.107	0.190	0.159	0.143	0.278	0.357	0.205	0.043
size 2	0.063	0.037	0.215	0.299	0.493	0.121	0.004	0.051	0.229	0.284
size 1	0.191	0.039	0.216	0.150	0.005	0.066	0.664	0.158	0.225	0.257

This result indicates that the distributions of future returns of the vast majority of portfolios based on size and VCG intervals are accurately enough fitted by the skew-t distribution and overall enables us to rely on the estimated skew-t parameters in the further discussion of regression results.

5.4 Regression Results

Below we present and interpret the main results of the regression analysis for the four distribution parameters of interest.

5.4.1 Location

Table 14. **Regression results for location parameter.** Skew-t location parameters (most likely future 12-month return) of the 100 size- and VCG-specific portfolios are dependent variables, respective size and VCG scores are explanatory variables. Three stars (***) and two stars (**) indicate 0.1% and 1% significance level, respectively. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Location ξ						
	Linear model			**Quadratic model**		
	Coefficient	t-value		Coefficient	t-value	
(Intercept)	0.053	17.56	***	0.017	2.89	**
β_{size}	0.004	8.81	***	-0.010	-4.43	***
β_{size^2}				-0.001	-6.48	***
β_{VCG}	0.00005	1.46		0.00003	1.24	
F-test p-value	1.586E-13			< 2.2E-16		
Adj. R^2	0.4475			0.613		

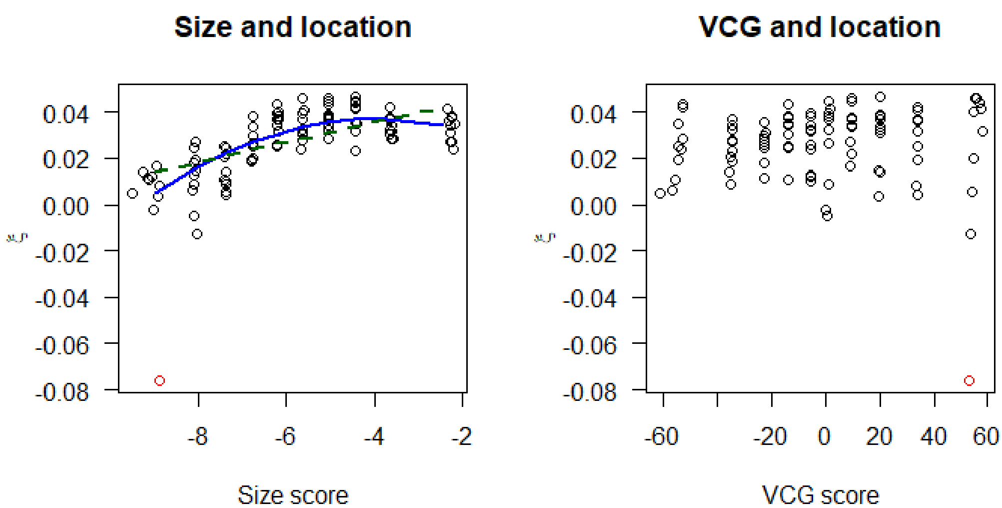

Figure 9. **Portfolio size and VCG scores and corresponding location parameters.** Black dots show the observed location parameters (most likely future 12-month return) corresponding to 100 portfolios with respective size score points $size_{i,j}$ and VCG score points $VCG_{i,j}$ as defined in (25) and (26), respectively. Red dots indicate outliers excluded from the regression samples. The blue line shows quadratic regression fit and the green dashed line indicates linear fit. When plotting a regression curve showing the relationship between size (VCG) score and location parameter, VCG (size) score is fixed at the average level of $VCG_{i,j}$ ($size_{i,j}$) points. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Analyzing the relationships between size and location parameter, we observe that most likely future returns indeed tend to be higher for larger stocks. However, in addition, the scatterplot suggests that the increase in location slows down and location even slightly declines for the largest stocks, which is also confirmed by a significant quadratic fit. The overall positive relationship between size and location can be interpreted using the dashed green linear fit, which also exhibits rather high significance and percentage of explained variance. Ceteris paribus, an increase in market share by one percent[10] tends to correspond to an increase in future 12-month most likely return by 0.4 p.p. It should be noted that the negative coefficients for size and size squared in the quadratic model should not be confusing since their direct interpretation is not possible. The reason is that the ceteris paribus assumption in this case cannot hold (a change in size always leads to a change in size squared and vice versa).

As to VCG and location, regression analysis does not indicate any significant relationship between the two.

5.4.2 Dispersion

Table 15. **Regression results for dispersion parameter.** Skew-t dispersion parameters (dispersion of the future 12-month returns) of the 100 size- and VCG-specific portfolios are dependent variables, respective size and VCG scores are explanatory variables. Three stars (***) indicate 0.1% significance level. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

	Dispersion ω					
	Linear model			**Quadratic model**		
	Coefficient	t-value		Coefficient	t-value	
(Intercept)	0.042	16.33	***	0.037	18.15	***
β_{size}	-0.002	-5.50	***	-0.002	-7.32	***
β_{VCG}	0.0001	3.70	***	0.0001	4.93	***
β_{VCG^2}				0.000005	8.282	***
F-test p-value	2.85E-08			< 2.2E-16		
Adj. R^2	0.2892			0.5829		

[10] For example, an increase of market share from 0.1% to $0.1\% \times (1 + 1\%) = 0.101\%$.

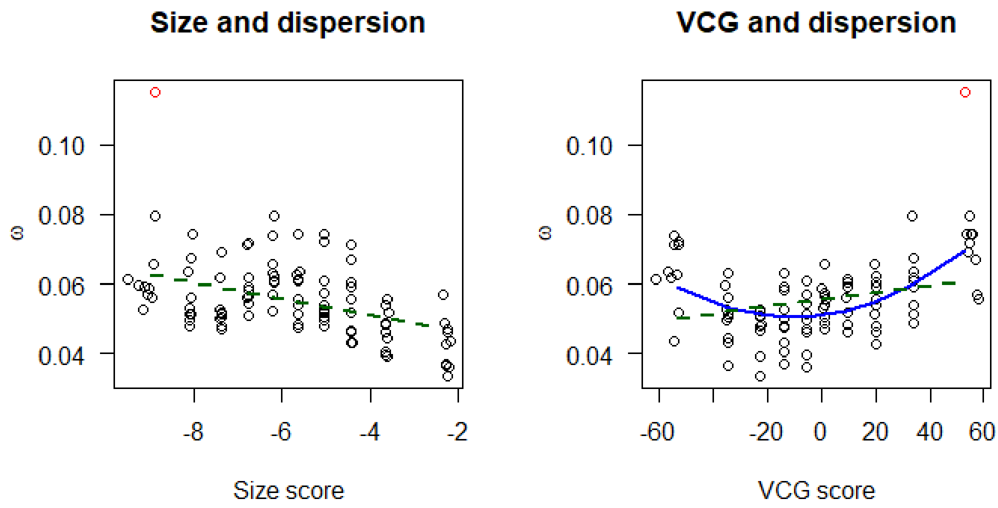

Figure 10. **Portfolio size and VCG scores and corresponding dispersion parameters.** Black dots show the observed dispersion parameters (dispersion of the future 12-month returns) corresponding to 100 portfolios with respective size score points $size_{i,j}$ and VCG score points $VCG_{i,j}$ as defined in (25) and (26), respectively. Red dots indicate outliers excluded from the regression samples. Blue line shows optimal quadratic regression fit and green dashed lines indicate linear fits. When plotting a regression curve showing the relationship between size (VCG) score and dispersion parameter, VCG (size) score is fixed at the average level of $VCG_{i,j}$ ($size_{i,j}$) points. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

A highly significant linear fit for the relationship between size and dispersion indicates a negative relationship between the two, as was also observed in the histograms for categories in the previous section. Namely, all else equal, a 1 per cent increase in market share corresponds to a decrease in dispersion of future 12-month returns by 0.2 p.p.

We interpret an overall positive relationship between VCG and dispersion in the following way: ceteris paribus, an increase in VCG score by 1 unit leads to an increase in future 12-month most likely returns by 0.01 p.p. However, significant non-linear fit revealed that these relationships go beyond linearity, which is one of the main results of the thesis. Namely, the squared fit indicates that both deep-value and high-growth stocks (stocks with lowest and highest VCG scores, respectively) exhibit particularly heightened future volatility relative to stocks in the middle of the VCG spectrum. The potential reasons for this phenomenon are discussed in the Conclusions section.

5.4.3 Skewness

Table 16. **Regression results for skewness as percentage parameter.** Skew-t skewness as percentage parameters (likelihood of the future 12-month returns to fall behind future 12-month most likely return) of the 100 size- and VCG-specific portfolios are dependent variables, respective size and VCG scores are explanatory variables. Three stars (***) indicate 0.1% significance level. VCG denotes value-core-growth orientation scores. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Skewness as percentage π			
	Linear model		
	Coefficient	t-value	
(Intercept)	79.497	62.19	***
β_{size}	1.564	7.50	***
β_{VCG}	0.063	4.72	***
F-test p-value	1.18E-13		
Adj. R^2	0.4509		

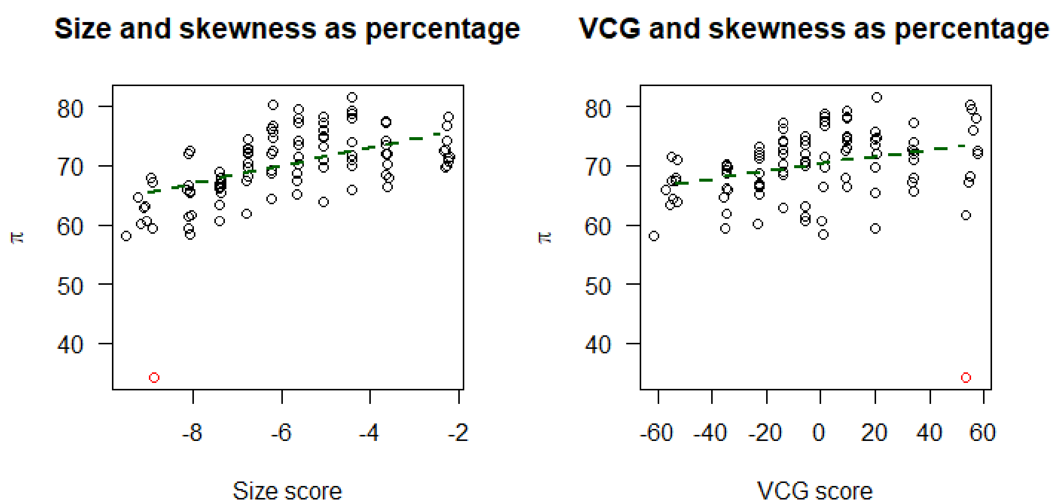

Figure 11. **Portfolio size and VCG scores and corresponding skewness as percentage parameters.** Skewness as percentage is defined as likelihood of the future 12-month returns to fall behind future 12-month most likely return. Black dots show the observed skewness parameters corresponding to 100 portfolios with respective size score points $size_{i,j}$ and VCG score points $VCG_{i,j}$ as defined in (25) and (26) respectively. Red dots indicate outliers excluded from the regression samples. Green dashed lines indicate linear fits. When plotting a regression curve showing the relationship between size (VCG) score and skewness parameter, VCG (size) score is fixed at the average level of $VCG_{i,j}$ ($size_{i,j}$) points. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Considering the relationship between size/VCG and skewness, first, it should be noted that regression analysis has confirmed a common observation in finance literature that equity returns consistently exhibit negative skewness. Indeed, in our case, future return distributions of all 100 portfolios, except only one outlying portfolio, are negatively skewed.

As to size and skewness, we observe the second most significant result of our regression analysis. Namely, future returns indeed become more negatively skewed with company size. This is consistent with the previous observations from the size and VCG category perspective. We can interpret obtained linear regression coefficients in the following way: ceteris paribus, a 1 per cent increase in market share makes future 12-month returns 1.564 p.p. more likely to fall behind most likely future return.

As to relationships between VCG and skewness, as can be seen in the plot on the right in Figure 11, skewness becomes more negative with VCG (or, increase in growth orientation), This is in agreement with the results from the size/VCG category perspective shown in Table 11. The regression fit provides the following interpretation: ceteris paribus, an increase in VCG score by 1 unit makes future 12-month returns 0.063 p.p. less likely to exceed a future mode return. This negative relationship between skewness and VCG corroborates the conclusions from Halling and Giordani (2018) in that high valuation levels corresponds to more negative skewness of future returns. In our case, more growth-oriented stocks are those with relatively high overall growth score (see Growth Orientation section) and relatively low overall value score (for details see Value-Core-Growth (VCG) Score and Orientation section). Low value scores imply low yields, high multiples (as inverses of yields) and hence high valuation.

5.4.4 Degrees of Freedom

Table 17. **Regression results for degrees of freedom parameter.** Skew-t degrees of freedom of the future 12-month returns of the 100 size- and VCG-specific portfolios are dependent variables, respective size and VCG scores are explanatory variables. Three stars (***) and one star (*) indicate 0.1% and 5% significance level, respectively. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Degrees of freedom ν						
	Linear model			**Constant model**		
	Coefficient	t-value		Coefficient	t-value	
(Intercept)	3.770	14.18	***	4.346	50.13	***
β_{size}	-0.099	-2.29	*			
β_{VCG}	0.001	0.51				
F-test p-value	0.0702					
Adj. R^2	0.0338					

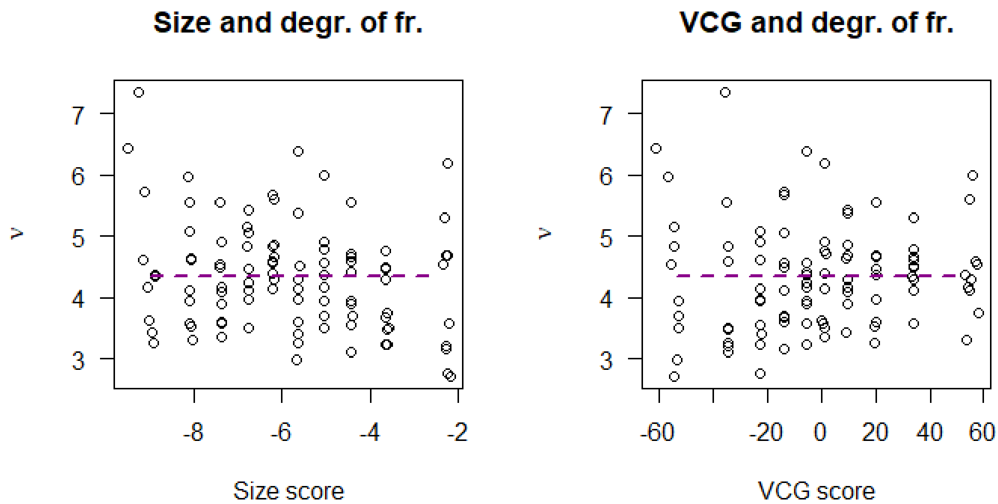

Figure 12. **Portfolio size and VCG scores and corresponding degrees of freedom parameters.** Black dots show the observed degrees of freedom of the future 12-month returns corresponding to 100 portfolios with respective size score points $size_{i,j}$ and VCG score points $VCG_{i,j}$ as defined in (25) and (26), respectively. Purple dashed lines illustrate the fit on constant. VCG denotes value-core-growth orientation score associated with the portfolios, size score measures median log market share of the stocks in the portfolios. Distributions are fit on monthly observations of annual future 12-month log returns from December 1982 to December 2017.

Finally, as opposed to highly significant results for the previous parameters, we find no strong relationships between degrees of freedom and size or VCG. Although the linear fit has revealed some significance for the size regressor, a high p-value of the F-test for this linear fit indicates that the simpler model on constant cannot be rejected. An extremely low R-squared for the linear model also clearly indicates that size and VCG have almost no explanatory power for degrees of freedom parameter. Therefore, we consider a model on constant to be optimal, which provides an average estimate of degrees of freedom parameter equal to 4.346. This estimate is obviously smaller than infinity highlighting an overall propensity of stock returns to exhibit leptokurtosis.

Below we present the summary of the relationships obtained in the regression analysis.

5.4.5 Regression Results Summary

Table 18. **Summary of the regression results.** As the basis, the summary from the size/VCG category perspective is considered when the same relationships were found to hold. New insights gained thanks to the regression analysis are stated in bold.

Type of parameters	Results summary
Location	• Most likely future returns increase with size **but somewhat decline for the largest stocks** • Most likely future returns **are not significantly explained by the value-growth orientation**
Dispersion	• Dispersion of future returns decreases with size • Dispersion of future returns **increases for both deep-value and high-growth stocks**
Skewness	• Skewness of future returns becomes more negative with size • Skewness of future returns becomes more negative with VCG (with growth orientation)
Degrees of freedom	• **No significant relationships are found between heavy-tailedness of future returns and size or value-growth orientation**

Overall, based on the highly significant results of the regression analysis, we accept Hypothesis 1 that size and value/growth characteristics do explain the shape of the future 12-month return distributions.

The next section discusses the results of the portfolio application part of the thesis.

5.5 Portfolio Application Results

The results relating to the first goal of the portfolio application part of the thesis are summarized in the following table.

Table 19. **Comparison of realized risk and risk-reward characteristics of portfolios.** The table includes the comparison of realized annual VaR of the portfolios of "worst" and "best" stocks selected by VaR criterion and comparison of realized annual Reward-to-VaR (RtVaR) of the portfolios of "worst" and "best" stocks selected by RtVaR criterion. Realized VaR and RtVaR is based on the performance over the test period from January 2004 to December 2018.

	Stock selection criteria					
	VaR			**Reward-to-VaR**		
Stock universe	Realized annual VaR of stocks from high forecasted VaR ("worst") quartile	Realized annual VaR of stocks from low forecasted VaR ("best") quartile	% decrease in realized annual VaR for "best" quartile	Realized annual RtVaR of stocks from low forecasted RtVaR ("worst") quartile	Realized annual RtVaR of stocks from high forecasted RtVaR ("best") quartile	% increase (decrease) in realized annual RtVaR for "best" quartile
Small Value	23%	13%	46%	-0.037	0.302	highly positive
Small Core	13%	10%	20%	0.296	0.516	74%
Small Growth	18%	12%	31%	0.427	0.571	34%
Mid Value	19%	11%	43%	0.233	0.431	85%
Mid Core	12%	10%	18%	0.659	0.442	-33%
Mid Growth	16%	11%	33%	0.439	0.585	33%
Large Value	18%	9%	51%	0.358	0.337	-6%
Large Core	11%	8%	30%	0.625	0.462	-26%
Large Growth	13%	8%	34%	0.687	0.613	-11%
Value	31%	12%	63%	0.008	0.328	4250%
Core	13%	9%	33%	0.499	0.464	-7%
Growth	14%	8%	39%	0.458	0.634	39%
Small	14%	11%	25%	0.147	0.466	217%
Mid	12%	10%	15%	0.570	0.450	-21%
Large	11%	10%	12%	0.654	0.410	-37%
Total	12%	9%	22%	0.361	0.446	24%
Median			**32%**			**24%**

Firstly, we consider the results related to portfolios based on the risk measure – VaR. We clearly see that within all stock universes, choosing the stocks in each month of the test period from the bottom ("best") quartile based on estimated forward-looking 12-month 5% VaR delivered consistently lower realized portfolio VaR than stocks from the top ("worst") quartile based on estimated forward-looking 12-month VaR. This means that the regression model incorporating the relationships between size/VCG and shape parameters of the future return distribution enabled us to identify portfolios of stocks whose actual performance is in agreement with forecasted risk profile. It is of particular interest that the greatest decrease in realized VaR for "best" VaR-based quartile of stocks compared to "worst" VaR-based quartile of stocks is observed for value-related stock universes – small value (46% decrease), mid value (43% decrease), large value (51% decrease), and overall value (63% decrease). This particular observation strongly suggests that the functional form of the relationships between VCG and distribution parameters, most importantly implying the heightened dispersion for deep-value stocks, have been correctly identified.

Looking at the results related to the portfolios based on the risk-reward measure RtVaR, we do not observe as consistent outcomes as in case of VaR. However, several observations stand out. Firstly, for all small-related universes, choosing the stocks in each month of the test period from the top ("best") quartile based on estimated forward-looking 12-month RtVaR delivered consistently higher realized portfolio RtVaR than stocks from the bottom ("worst") quartile based on estimated forward-looking 12-month RtVaR. This observed consistency for the small-related universes can be explained by the fact that for small stocks, the size scores, as they are defined in the thesis, have the greatest variation and thus the relationships between size and future return distribution parameters are identified with the highest accuracy compared to the case of mid or large stocks. This higher accuracy in estimation of the forecasted parameters, in turn, clearly translated into desired actual performance. Even though there is inconsistency in results for RtVaR across different universes, for the total market universe overall, we can still see that future risk-reward characteristics of stocks from "best" and "worst" portfolios have been correctly identified (24% increase in realized RtVaR for stocks with "best" forecasted RtVaR compared to the case of stocks with "worst" forecasted RtVaR).

We can make the conclusions regarding Hypothesis 2, which we raise in the Methodology section. The hypothesis assumes that the regression model incorporating the relationships between size/VCG and the shape of future returns distribution has the power to identify portfolios of stocks whose realized performance exhibits certain forecasted risk and risk-reward characteristics. Based on above results, we can conclude that our model identifies stocks with desired future risk characteristics based on VaR consistently within all stock universes. This is particularly evident in the case of stocks within value-related universes. The model identifies portfolios of stocks with desired future risk-reward characteristics based on RtVaR consistently only within small-related universes and for the overall market.

Finally, for illustrative purposes, we look at the performance of "best" and "worst" portfolios in comparison to respective benchmark indices. Two observations particularly stand out. First, the consistency in above results for RtVaR observed in case of all small stock universes (overall small, small value, small core and small growth) clearly translates into consistently better performance of "best" RtVaR-based stocks not only compared to "worst" stocks but also compared to benchmark indices, as shown below in Figure 13.

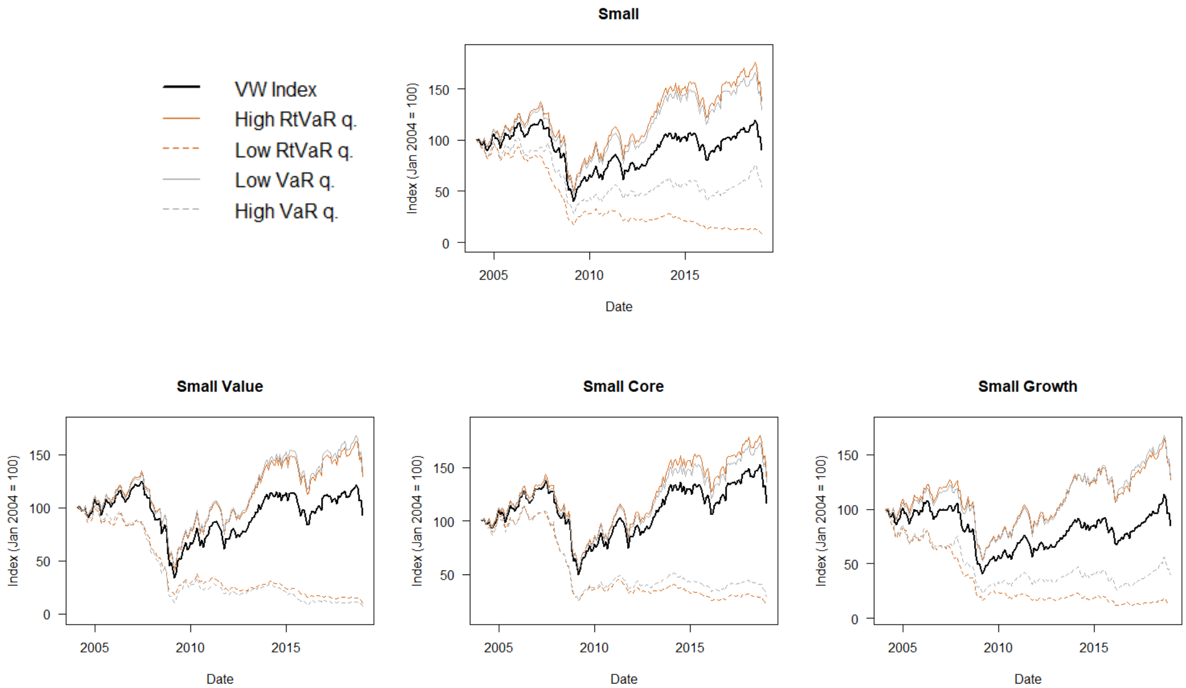

Figure 13. **Performance of value-weighted portfolios in small-related stock universes**. Namely, following portfolios are considered: the benchmark market portfolio (VW Index), "best" stocks from top RtVaR quartile (High RtVaR q.), "worst" stocks from bottom RtVaR quartile (Low RtVaR q.), "best" stocks from bottom VaR quartile (Low VaR q.), and "worst" stocks from top VaR quartile (High VaR q.).

Secondly, particularly strong results for VaR-based portfolios within value-related universes, as highlighted above, translates into consistent outperformance of "best" VaR-based portfolios not only relative to "worst" stocks but also relative to benchmark indices. This means that even though VaR as a criterion estimates only the future risk, it clearly helps identify "best" or "worst" performing stocks as well. This is especially pronounced in case of large value universe, as shown below in Figure 14, which can be a particularly useful observation for value investors focusing on large caps given a well-known difficulty to succeed in this type of investing.

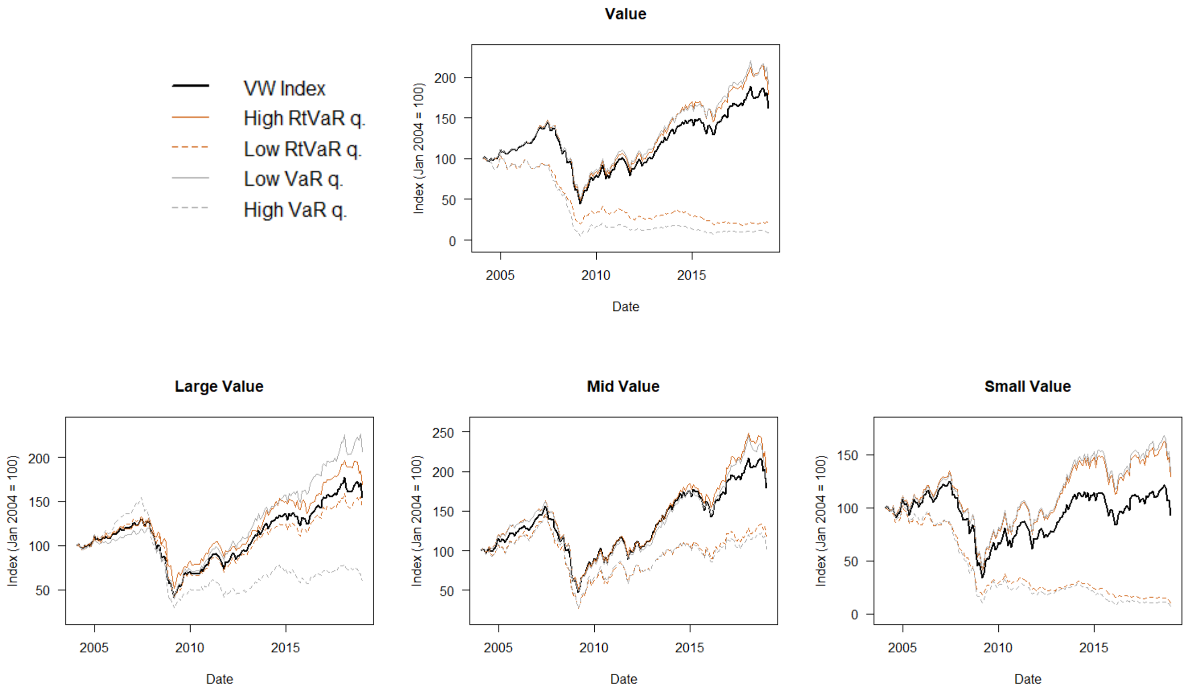

Figure 14. **Performance of value-weighted portfolios in value-related stock universes**. Namely, following portfolios are considered: the benchmark market portfolio (VW Index), "best" stocks from top RtVaR quartile (High RtVaR q.), "worst" stocks from bottom RtVaR quartile (Low RtVaR q.), "best" stocks from bottom VaR quartile (Low VaR q.), and "worst" stocks from top VaR quartile (High VaR q.).

In Appendix D, we present the plots illustrating the performance of the benchmark portfolios and the portfolios of "best" and "worst" stocks based on VaR and RtVaR criteria for all stock universes.

6 Conclusions

In our thesis, we provide evidence to support our two hypotheses that (i) company size and value-growth orientation have a significant impact on the shape parameters of the distribution of future 12-month returns and that (ii) our model has predictive power, i.e. that the model incorporating the relationships between size/VCG and the shape of future returns distribution can identify portfolios of stocks whose realized performance exhibits certain forecasted risk and risk-reward characteristics.

Two particularly striking findings stand out in our regression results. Firstly, the relationship between value-growth orientation and dispersion shows clear signs of non-linearity. Specifically, it appears to have a quadratic functional form, with future volatility increasing for both deep-value and high-growth companies. For high-growth companies, this observation is consistent with the theory of leverage effects as proposed by Black (1976) and Christie (1982). Growth stocks are stocks with high valuations relative to the rate at which earnings, book value, cash flow and revenue are expected to grow. As such, there is more capacity for valuations to drop for growth stocks than for stocks with low valuations. In the framework of the leverage effect theory, this drop in valuation would in turn lead to an increase in operating and financial leverage, and so an increase in volatility, as we see in our results.

However, we find no formal theory that rationalizes our result that future volatility increases for deep-value companies. We propose the informal hypothesis that there is a "low base" effect in play. Deep-value stocks, which have very low valuations relative to prospective growth in earnings, book value, revenue, cash flow and dividends, are likely those that are performing or expected to perform very poorly and are possibly close to bankruptcy. As a result, their stock price may be more sensitive to news, whether good or bad. This in turn may explain the increased volatility we see for deep-value stocks.

Secondly, we find that skewness of future returns becomes more negative with company size. This is in agreement with the empirical findings of Chen et al. (2001) and Dennis and Mayhew (2002). However, we are also not aware of any formal framework that rationalizes this finding. Chen et al. (2001) offer the informal theory that (i) firms tend to be quick to release good news while bad news "dribbles out" and (ii) small firms tend to have more capacity to hide bad news. For these reasons, return distributions of larger companies become more negatively skewed. This explanation seems reasonable, but further research is certainly needed to make a more confident interpretation of this result.

We also explore an application of our regression analysis that highlights the potential value of our findings to portfolio managers and risk management more generally. We find that our model has some ability to identify portfolios of stocks whose realized VaR-based risk profile is consistent with that forecast by our model. Importantly, this result is obtained for all different stock universes based on the Morningstar

Style Box. This suggests our model incorporating the relationship between the shape of the future return distribution and company size and value-growth orientation can be of benefit to portfolio construction when VaR is an important consideration. Our results indicate that for small caps, the model can also help choose stocks whose realized risk-reward profile is consistent with that forecast by our model. Furthermore, by identifying stocks that are expected to have low VaR and high Reward-to-VaR according to our model, we are able to generate some outperformance over benchmark indices. This is particularly evident for stocks within different small size and value-related stock universes. In the latter case, this result can be of particular interest for value investors given a well-known difficulty to succeed in this type of investing.

We measure company value-growth orientation following Morningstar's value-core-growth score and find that this is a powerful explanatory variable for the shape of the future return distribution. However, we do not consider alternative measures of value-growth orientation, and this is the first avenue for future research that we suggest. Secondly, while we find that value-growth orientation and size are powerful variables for explaining the future return distribution for US stocks, we are unaware of any papers analyzing this topic for other countries. Thirdly, we did not consider to what extent the relationships we identify are industry-specific. Finally, we were unable to explore more sophisticated portfolio optimization techniques utilizing the relationships we identified as we lack the computational power to do so. Conducting further research with a focus on above issues would provide a valuable contribution to the literature.

7 Appendices

Appendix A

Share-weighted average

The example below uses earnings, but the procedure is the same for book value, revenue, or cash flow growth.

Step 1: Calculate a 4-year earnings growth rate

$$egr(0,-4) = \left(\frac{earn(0)}{earn(-4)}\right)^{\frac{1}{4}} - 1, \tag{44}$$

where

egr(0,i) = portfolio earnings growth rate from year *i* to year 0

$$earn(i) = \sum_{j=1}^{n} CSHOQ_{\ split-adjusted} \times e_{ij}$$

e_{ij} = earnings per share for fiscal year-end *i* for stock *j*

n = the number of stocks in the peer group that were not trimmed and for which e_0 and e_1 are > 0.

Step 2: Calculate a 3-year earnings growth rate

$$egr(0,-3) = \left(\frac{earn(0)}{earn(-3)}\right)^{\frac{1}{3}} - 1. \tag{45}$$

Step 3: Calculate a 2-year earnings growth rate

$$egr(0,-2) = \left(\frac{earn(0)}{earn(-2)}\right)^{\frac{1}{2}} - 1. \tag{46}$$

Step 4: Calculate a 1-year earnings growth rate

$$egr(0,-1) = \frac{earn(0)}{earn(-1)} - 1. \tag{47}$$

Step 5: Calculate the historical share-weighted average for this growth factor as

$$Average[egr(0,-4), egr(0,-3), egr(0,-2), egr(0,-1)\,]. \tag{48}$$

Appendix B

Calculating centered parameters of the skew-t distribution based on the direct parameters

Define b_ν as

$$b_\nu = \frac{\sqrt{\nu}\Gamma\left(\frac{1}{2}(\nu-1)\right)}{\sqrt{\pi}\Gamma\left(\frac{1}{2}\nu\right)}, \qquad \text{if } \nu > 1, \qquad (49)$$

and δ as

$$\delta = \delta(\alpha) = \frac{\alpha}{\sqrt{1+\alpha^2}}, \qquad \delta \in (-1,1). \qquad (50)$$

Define μ as the mean of Y (first central moment), σ^2 as variance of Y (second central moment), γ_1 as skewness of Y (third central moment), and γ_2 as excess kurtosis of Y (fourth central moment minus 3). It can be shown that convenient closed-form formulas for central moments of skew-t distribution can be obtained based on corresponding direct distribution parameters $(\xi, \omega^2, \alpha, \nu)$ as follows

$$\mu = \mathbb{E}\{Y\} = \xi + \omega b_\nu \delta, \qquad \text{if } \nu > 1, \qquad (51)$$

$$\sigma^2 = Var\{Y\} = \omega^2\left[\frac{\nu}{\nu-2} - (b_\nu\delta)^2\right] = \omega^2\sigma_z^2, \text{ say,} \qquad \text{if } \nu > 2, \qquad (52)$$

$$\gamma_1 = \frac{b_\nu\delta}{\sigma_z^{3/2}}\left[\frac{\nu(3-\delta^2)}{\nu-3} - \frac{3\nu}{\nu-2} + 2(b_\nu\delta)^2\right], \qquad \text{if } \nu > 3, \qquad (53)$$

$$\gamma_2 = \frac{1}{\sigma_z^4}\left[\frac{3\nu^2}{(\nu-2)(\nu-4)} - \frac{4(b_\nu\delta)^2\nu(3-\delta^2)}{\nu-3} + \frac{6(b_\nu\delta)^2\nu}{\nu-2} - 3(b_\nu\delta)^4\right] - 3, \qquad \text{if } \nu > 4. \qquad (54)$$

Appendix C

Significance of the regression coefficients over the test period for the portfolio application part

Table 20. **Summary of the significance of the regression coefficients over the test period.** Average t-stat measures the average t-statistic for the respective coefficient for regressions (35) – (38) re-estimated every month *t* over the test period. Percentage of months with 5% significance indicates how frequently a regression coefficient is significant at 5% significance level when regressions are re-estimated every month *t* over the test period.

Coefficient	**Average t-stat**	**% of months with 5% significance**
β_0^{ξ}	3.93	100%
β_{size}^{ξ}	-2.53	72%
$\beta_{size^2}^{\xi}$	-4.57	100%
β_0^{ω}	20.61	100%
β_{size}^{ω}	-6.72	100%
β_{VCG}^{ω}	6.88	100%
$\beta_{VCG^2}^{\omega}$	7.77	100%
β_0^{α}	-17.23	100%
β_{size}^{α}	-8.27	100%
β_{VCG}^{α}	-3.02	92%
β_0^{ν}	47.23	100%

Appendix D

Performance of portfolios

Below is shown the performance of value-weighted portfolios, namely, the benchmark portfolios (VW Index), “best” stocks from top Reward-to-VaR quartile (High RtVaR q.), “worst” stocks from bottom RtVaR quartile (Low RtVaR q.), “best” stocks from bottom VaR quartile (Low VaR q.), and “worst” stocks from top VaR quartile (High VaR q.).

Both Size- and Value/Growth-Specific Stock Universes

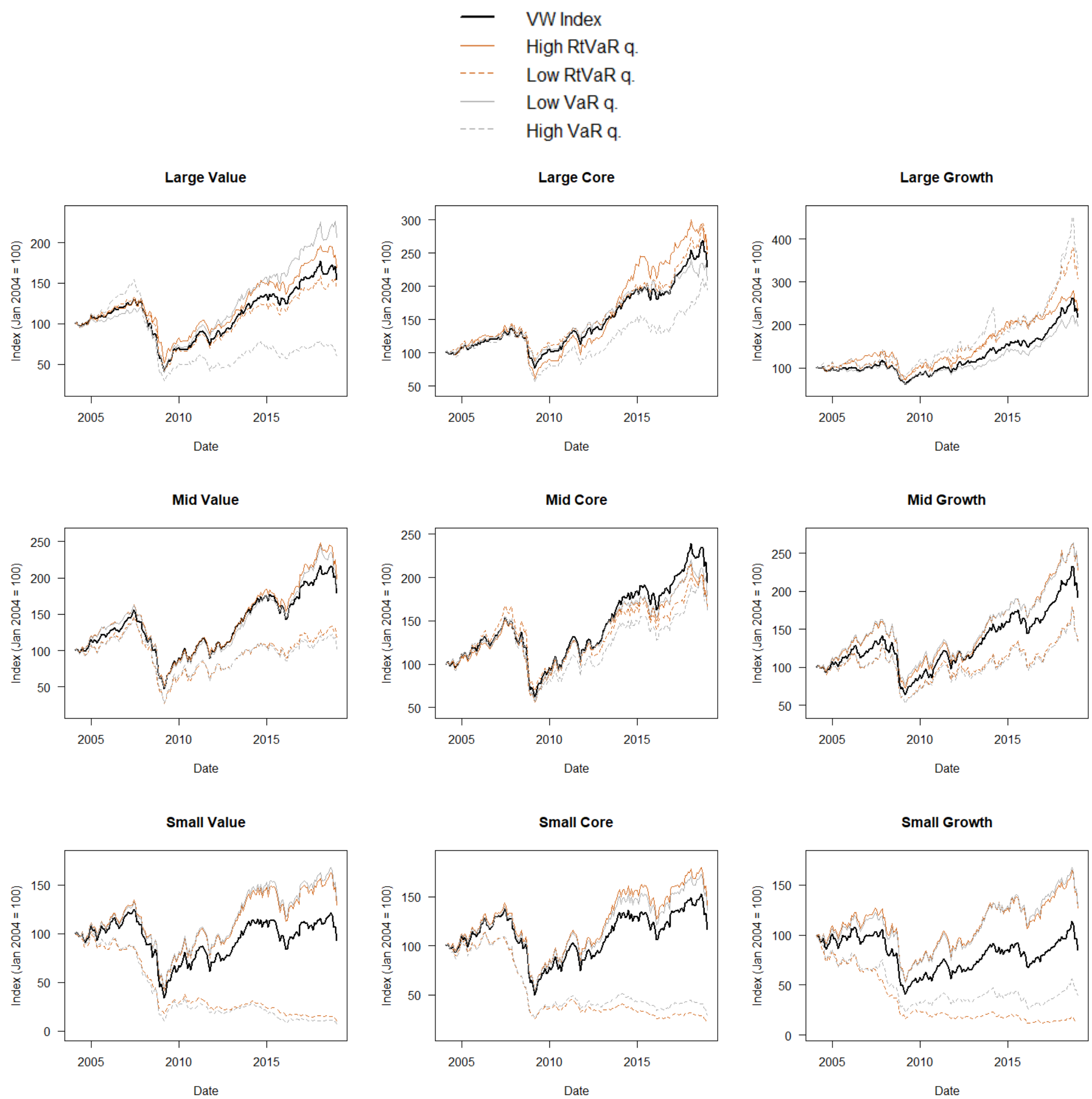

Figure 15. **Size and value-growth specific universes**

Value/Growth-Specific Universes

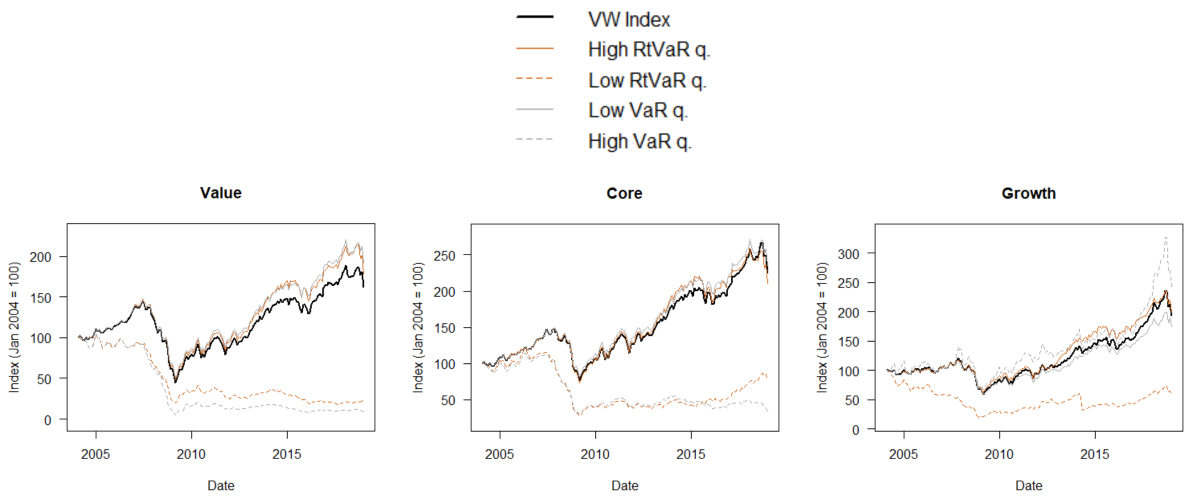

Figure 16. **Value, core, and growth universes**

Size-Specific Stock Universes

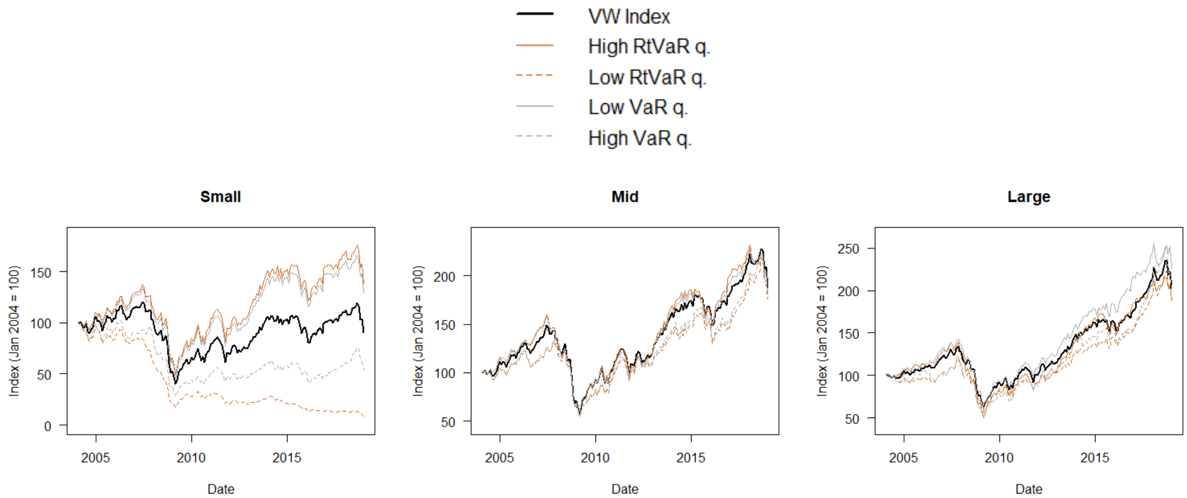

Figure 17. **Small, mid, and large stock universes**

Total Stock Market Universe

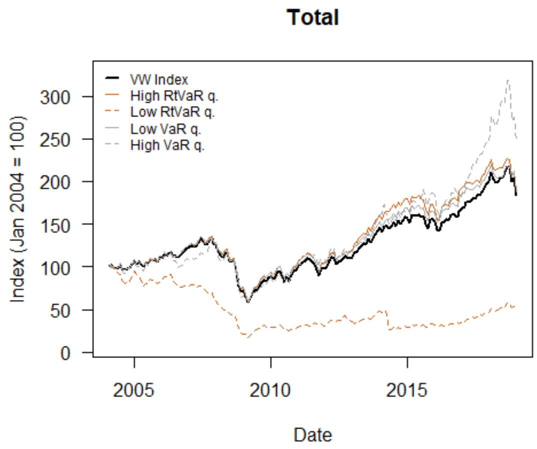

Figure 18. **Total stock market universe**

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