SOLVING THE VALUATION PARADOX

Applying Hedonic Valuation to Paradoxical Companies

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Abstract:

Traditional valuation models that are widely used by investors and scholars, for instance, residual income valuation (RIV) model, do not yield accurate results when valuing paradoxical companies, i.e. companies with the market value significantly above the book value and high growth, despite not generating positive payoffs. With the increasing number of paradoxical companies entering the market, there is a need to identify them and estimate their equity value with better accuracy. We expand prior research by defining four simple quantitative approaches to identify paradoxical companies. Also, the approaches can be easily adapted to different markets and economic cycles. This thesis aims to improve the accuracy of pricing of paradoxical companies by applying hedonic regression, an empirical valuation technique previously used on other asset classes, such as real-estate and art. We develop multiple hedonic models that yield promising results for valuation of paradoxical companies, both regarding accuracy and applicability. Our models rely solely on short term historical data and avoid complex calculations (e.g. discount rates), hence are easy-to-use even for investors with limited analytical skills. However, considering the empirical nature of our models, which are based on a specific time period and Nordic sample, the results can vary for different markets and time periods.

Keywords:

equity valuation, paradoxical company, hedonic regression, residual income valuation (RIV) model, pricing errors

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Contents

1 Introduction

Traditional valuation approaches, such as discounted dividend (DDM), free cash flow (FCF), residual income valuation (RIV), and abnormal earnings' growth (AEG) models, often come short when it comes to valuation of paradoxical companies, i.e. companies with market value significantly above the book value and high growth, despite not generating positive payoffs[1]. Attempts have been made to adjust the traditional models in order to accommodate at least one of the three aforementioned companies' characteristics (e.g. Cornell and Damodaran 2014, Simon 2016), however, the valuation results still drastically underestimate companies' market value[2]. Since the setting of traditional company valuation does not seem to provide sufficient methods for paradoxical firms, other valuation techniques need to be explored. Hedonic valuation that relies on quantifying objects' characteristics and defining an econometric model, provides an alternative pricing method (e.g. Rosen 1974, Lucas 1975). Hedonic models are often applied to asset classes such as real-estate (e.g. Englund et al. 1998, Fisher et al. 2007) and art (e.g. Scorcu et al. 2011, Kundu and Raza 2016), that match the paradoxical characteristics, given the owners do not rent them out.

The purpose of the thesis is to investigate whether the empirically estimated hedonic pricing model can improve the valuation accuracy of paradoxical companies. Unlike traditional valuation models, hedonic pricing model does not rely on payoffs as main drivers of a firm's value. Instead, hedonic regressions derive the value from various quantifiable characteristics of the object, in case of equity valuation, the financial and non-financial indicators of a company.

Building on prior research, our contribution stems from the focus on paradoxical companies. First, since previous studies do not combine all of the three characteristics that define paradoxical companies, we provide four approaches specifying how to quantify and implement the characteristics in practice. Second, we estimate 14 hedonic valuation models using different statistical methods, namely enter and stepwise regression, in order to identify the most accurate and most applicable one. Finally, as a robustness check, we verify our results by applying the RIV model to the paradoxical companies. Often, the companies

[1] e.g. Tesla

[2] "Theoretically, companies like Tesla should have been bankrupt for years." - Kenth Skogsvik, Professor, Department of Accounting, Stockholm School of Economics

are excluded from the papers' samples, due to the application issues given their short-term history, negative values of ROE, and analyst forecasts not being available.

We develop multiple hedonic valuation models that yield promising results for paradoxical companies, both regarding accuracy and applicability. The main advantages of our models compared to traditional approaches are that they rely on short term financial data, no forecasts are required, and they avoid complex calculations such as discount rates. However, our models are empirical and based on a specific time period and Nordic sample. Hence, the conclusions can vary for different markets and time periods.

The rest of the thesis is structured as follows. In section 2, we first combine the studies focusing on each one of the paradoxical characteristics. Then we discuss prior research on the accuracy of traditional valuation models and on the application of hedonic models. Section 3 specifies four approaches on paradoxical company identification, defines variables entering the regressions, describes steps taken to arrive at final models, and defines accuracy metrics used to compare the models. The empirics are summarised in section 4. Subsections 4.1 and 4.2 describe the whole sample, while subsection 4.3 focuses on the paradoxical companies. The estimated models are described in subsection 4.4 and they are evaluated in subsection 4.5. Section 5 concludes our thesis with limitations, suggestions for further research and a short summary.

2 Literature Review

2.1 Paradoxical companies

Before proceeding with our research, it is important to know which companies are paradoxical and how to identify them. Unfortunately, there is a lack of consensus on the concept in previous literature. There are numerous studies focusing on firms with one of the features of paradoxical companies (e.g. Damodaran 2009, Simon 2016), however, current research does not provide much insight into companies that have all the features.

Damodaran has made an important contribution when it comes to the valuation of companies that are not generating positive payoffs. Instead of proposing a new valuation model, his main focus is to improve a traditional valuation model, namely FCF, to increase valuation accuracy, especially for companies in early-stage (Damodaran, 2009).

When it comes to high market value, Simon (2016) documents the phenomenon of 'unicorns' in his report and identifies the characteristics of those companies. He further discusses the reason behind the birth of unicorns and examines the external environment for unicorns development in the European Union (EU) by qualitative global sample investigation.

Another feature that usually comes with both negative payoff and high market value is high growth. Eurostat and OECD (2007) bring some insight into the definition of high growth companies in their methodological manual that provides both practical and theoretical guidance for business demography in EU and OECD.

We combine problematic characteristics of companies discussed in prior literature and define paradoxical companies as companies with:

- negative payoff,
- high market value, and
- high growth.

2.1.1 Negative Payoffs

In his paper, Damodaran (2009) proposed a better (than current venture capital approach) FCF framework in evaluation young, start-up and growth companies that barely generate positive payoffs and lack historical financial data. He argues that the current venture capital approach is flawed due to the usage of exit multiples, short-term forecast horizon, and the lack of details in forecasting cash flows. The suggestions to improve the valuation model include estimating the payoffs in details and with a long-term focus, using real intrinsic value instead of exit multiples, adjusting the discount rate for survival, and application of real options. Yet, when applying an improved FCF model to Tesla, Cornell and Damodaran (2014) still get a high deviation of the valuation price from the share price.

2.1.2 High Market Value

Simon's (2016) study on 'unicorns', the companies that have market valuation over 1 billion USD, points out the paradoxical phenomenon: it is dumbfounding to see the discrepancy between actual payoffs and market capitalisation for some of the 'unicorns'. For instance, Twitter had impressive losses of 2 billion USD (Molla, 2018) since its IPO until February 8, 2018, while the market capitalization was over 8 billion USD on the day of IPO and increased to over 39 billion within a month. But this paradoxical phenomenon is not Simon's (2016) focus. In his empirical study, by observing the growth models, the role of the business environment, and the R&D environment of 23 global unicorns, Simon (2016) concludes that the technological trend that allows unicorns to surf on the internet mobile wave and the economic trends of globalization mainly contribute to the unicorn phenomenon. In his sample, 39% of the unicorns generate negative profit.

2.1.3 High Growth

Another type of research that relates to one of the features of the paradoxical companies are studies on high-growth firms (HGFs). Eurostat and OECD (2007) recommend that all firms with a greater than 20% average annualized growth (in terms of the number of employees or turnover) over a three year period should be defined as HGFs. Coad et al. (2014) point out that HGFs tend to be in early stage of the business cycle, but not necessarily small in size. High growth rates in HGFs are not persistent over time, and the prediction of HGFs is difficult. In an empirical study that examines firm growth dynamics in ten OECD countries, Biosca et al. (2013) conclude that legislation on employment protection, the tightness of bankruptcy regime, development of financial institutions and R&D incentives shed light on the drivers of firm growth.

2.2 Traditional Valuation Models

The usefulness of accounting numbers has been shown by many researchers. For instance, Ball and Brown (1968) find a connection between earnings and the subsequent stock prices. Based on the relation between accounting information and prices, numerous valuation models have been designed. Among the most commonly used valuations are the discounted dividend (DDM), the residual income (RIV), free cash flow (FCF), and the abnormal earnings' growth (AEG) valuation models. The basic formulas for the models with infinite horizon are presented in table 1. Even though the accounting models are in theory supposed to yield the same result when infinite forecast horizon is used, in practice this is not the case, as truncation is applied (Penman and Sougiannis, 1998, p. 346). Several research papers aim to compare these models based on different aspects including the accuracy, variance of the results, and their explainability.

Table 1. Traditional Valuation Models

The table shows the formulas for the most commonly used traditional models. The formulas are shown for infinite forecast horizon. $V(E)_0$ is the value of equity at valuation date $t = 0$ and $V(IC)_0$ is the value of invested capital at valuation date $t = 0$. NI is net income, DVT is total proposed dividend, FCF is free cash flow, BV is total book value of equity, r_E is cost of equity capital, and r_{WACC} is the weighted average cost of capital.

Model	Name	Formula
AEG	abnormal earnings' growth	$V(E)_0 = \frac{NI_1}{r_E} + \sum_{t=1}^{\infty} \frac{(NI_{t+1}+DVT_t r_E - NI_t(1+r_E))/r_E}{(1+r_E)^t}$
DDM	discounted dividend	$V(E)_0 = \sum_{t=1}^{\infty} \frac{DVT_t}{(1+r_E)^t}$
FCF	discounted free cash flow	$V(IC)_0 = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+r_{WACC})^t}$
RIV	residual income valuation	$V(E)_0 = BV_0 + \sum_{t=1}^{\infty} \frac{NI_t - r_E BV_{t-1}}{(1+r_E)^t}$

Penman and Sougiannis (1998) provide a comparison of all four previously mentioned models. Their results illustrate some of the drawbacks of the theoretical models when the payoffs are forecasted for a finite horizon. The DDM and FCF models yield valuation with high positive errors for short horizons, however, the precision increases as the forecast horizons become longer. The RIV and AEG models generate results closer to the market value and outperform the other two models. While Penman and Sougiannis (1998) use realised historical values for the predictions, Francis et al. (2000) base their estimates on analyst forecasts. They find corroborating evidence of the RIV model dominating the FCF and DDM value estimates, based both on their accuracy and explainability. The authors explain the superiority of the RIV model by the ability of the book value of equity to sufficiently capture the intrinsic value of a company.

In his more recent study, Penman (2005) focuses on the comparison between the RIV and the AEG valuation models and finds that the RIV model outperforms AEG both when it comes to the accuracy and the variance of the estimated values. Similarly as Francis et al. (2000), he argues that book value based models like RIV have a great advantage, as with fair value accounting the valuation is straightforward. Moreover, using book value as an anchor for the model is more reliable than trusting the forecast of future earnings, which are used as the anchor for the AEG model. Brief (2007) discusses the prior results and estimates the standard deviation for both of the models' distributions using interquartile range method. He provides supporting evidence of the results in Penman (2005), as the AEG distribution variability is four times as high as the one of the RIV model.

All the aforementioned studies use different types of U.S. data to evaluate the models. On the other hand, a study by Anesten et al. (Forthcoming) tests the accuracy of the AEG, DDM and RIV models in the Nordic market and finds that the RIV model is most applicable to the sample of Nordic companies. In addition, they test various specifications of the models, from the most parsimonious one to specifications taking the probability of failure and transitory items into account. Interestingly, the results show higher accuracy for the DDM and RIV model, compared to the U.S. studies. Overall, the DDM model yields better results when the analyst forecasts are available and the RIV model performs best when using inputs based on historical data.

Table 2 provides an overview of the mentioned studies with the focus on the operationalisation of the models and the results evaluation.

2.3 Hedonic valuation

Despite the existence of a large number of traditional valuation models, the valuation of some asset classes utilises an alternative approach, such as hedonic valuation. Hedonic pricing model is developed from Lancaster's (1966) consumer theory that lays the microeconomic foundation for value estimation based on utility characteristics. Rosen (1974) extends the hedonic pricing model to the residential market, sketches a standard hedonic pricing model based on the hypothesis that goods are valued at their characteristics, and anticipates that this hedonic framework will have many applications to cross-sectional data involving market equilibrium. Based on Rosen's (1974) theory, Edmonds Jr (1984) integrates several strands of hedonic pricing function (HPF) into a single theoretical model. Edmonds Jr (1984) discusses three main topics when designing a hedonic regression valuation, (1) the proper selection of characteristic (2) the dummy variable characteristics, and (3) the regression intercept and the functional form. He points out that (1) only the utility affecting characteristics that are components of the purchased consumption' (p. 80) should be included as independent variables of HPF; (2) the dummy variable characteristics act as an intercept shift in the hedonic regression and should thus be interpreted accordingly; and (3) the approximation of the consumer possibility frontier might not be linear, hence there might be a need to move to non-linear regression. He further confirms that hedonic regression can be applied to any goods in the market where the framework is appropriate. Both Rosen (1974) and

Table 2. Studies comparing the traditional valuation models

The table shows an overview of papers comparing the performance of traditional valuation models: abnormal earnings' growth model (AEG), discounted dividend model (DDM), discounted free cash flow model (FCF), and residual income valuation model (RIV). The table includes the sample description, type of predictions, forecasting horizon (T), computation of terminal value and cost of equity capital (r_E), performance measures, and conclusions. SPE refers to signed prediction error, APE to absolute prediction error, CT to central tendency (definitions can be found in section 3.3.2), r_f to the risk-free rate, and FF to Fama-French.

Authors (Year)	**Data**	**Models**	**Operationalisation**				**Evaluation of the Results**	**Main Conclusions**
			Predictions	*Horizon (T)*	*Terminal value*	r_E		
Penman, Sougiannis (1998)	NYSE, AMEX NASDAQ firms; 1973-90; portfolios formed	AEG DDM FCF RIV	realised historical data	1-10 years	Gordon growth model, alternating growth rate (g)	(1) r_f (3-year T-bond) + risk premium (6%) (2) CAPM: r_f as above, firms-specific βs (3) industry r_E, based on FF 3-factor model (4) 10%	mean portfolio SPE; compared with market errors	DDM+FCF: high positive errors declining with increasing T; RIV+AEG: lower errors for all horizons
Francis et al. (2000)	NYSE, AMEX, NASDAQ firms; 1989-93; individual securities	DDM FCF RIV	annual forecast data	5 years	Gordon growth model: (1) g = 0 (2) g = 4% (3) if fundamental negative at T=5, assume TV = 0	CAPM: r_f = (intermediate T-bond yield) - (hist. premium on T-bonds over T-bills); β = industry β (mean of firm-specific β), firm β calculated using daily returns over fiscal year $t-1$; market premium = 6%	mean SPE, CT, R^2 for univariate and multivariate regressions of market price on estimated values	RIV superior, regardless of the specification and performance metric; valuation based on analyst forecast more accurate than historical data
Penman (2005)	U.S. traded firms; 1975-2002	AEG RIV	annual forecast data	2 years	Gordon growth model, g = 4%	$r_E = 10\%$	value-to-price ratio: $V(model)_0/P_0$	AEG consistently worse results, longer T might be beneficial
Brief (2007)	U.S. traded firms; 1975-2002	AEG RIV	annual forecast data	2 years	Gordon growth model, g = 4%	$r_E = 10\%$	st. dev. of estimated values using IQR method	AEG results 4-times more variable than RIV; AEG more complex
Anesten et al. (Forthcoming)	Nordic firms traded NASDAQ; 2005-14	AEG DDM RIV	forecast and historical data	3/5 years	AEG: Gordon growth model, g = 0% DDM: Gordon growth model, g = 4% RIV: q-value from Runsten (1998)	CAPM: r_f = 10-year government bond; company specific βs estimated over 60 months; market premium = 5.5%	mean SPE, median SPE, mean APE, CT, AM-score	DDM: best with forecasted data; RIV: best with historical data, more applicable; DDM+RIV: perform well with parsimonious specification $p_f ail$ beneficial; prolonging T not useful

Edmonds Jr (1984) elaborate their theory based on the assumption that consumers, i.e. price-takers, are competitive.

2.3.1 Functional Forms of Hedonic Regression

In general, regression analysis is applied to data to analyse the relationship between an independent variable and one or more explanatory variables. Hedonic regression aims to explain the price of a good as a function of its characteristics. The functional form of the hedonic regression depends on the good that is priced and its attributes. In this subsection, we discuss the most commonly used functional forms of hedonic regression and their application in different settings.

Linear Regression Model

The linearity of the regression model can stem from two sources. First, the model can be linear in the variables (LIV), which means that the dependent variable is a linear function of the explanatory variables. Second, the function can be linear in the parameters (LIP), which on the other hand implies that the dependent variable is a linear function of the regression coefficients (Gujarati, 1999, p. 133). When both of the features apply, the general multivariate model looks as follows

$$y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni} + e_i, \tag{1}$$

where y_i is the dependent variable, β_0 is intercept, $\beta_1, \ldots, \beta_n$ are the regression coefficients, $X_{1i}, \ldots, X_{ni}$ are the independent or explanatory variables, and e_i is the random error. In a multivariate model, the coefficients $\beta_1, \ldots, \beta_n$ measure the effect of one explanatory variable on the mean value of the dependent variable, given the remaining explanatory variables do not change (Gujarati, 1999, p. 200). The regression coefficients (including the intercept) are most commonly estimated using the ordinary least square (OLS) method that minimises the sum of squared errors (e_i^2), given the the Gauss-Markov assumptions hold (for details see e.g. Gujarati 1999, p.201).

The hedonic model in linear form is suitable when both the priced item and the buyers are heterogeneous and the supply for the product is uninterrupted and continuous (Sopranzetti, 2015, p. 2122). The model was applied for instance by Sopranzetti (2015), who compares multiple functional forms of the hedonic model on the U.S. real-estate data. Hence, the dependent variable y is the most recent transaction price of the property and the explanatory variables $X_1, \ldots, X_n$ are the house features, including structural characteristics of the home and property, neighbourhood characteristics, location within a given market, and contract conditions.

Semi-logarithmic Regression Model

As was mentioned before, the basic linear model is both LIV and LIP. When one of these conditions is however not satisfied, different functional forms of the model can be derived. The semi-logarithmic, or semilog model relaxes the linearity in the variables and can be written in general as

$$\ln(y_i) = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni} + e_i, \tag{2}$$

where ln is the natural logarithm. The model can be still estimated using the OLS method, even though the dependent variable is in the logarithmic form, as long as the needed assumptions hold. The coefficients $\beta_1, \ldots, \beta_n$ in the semilog model capture how, ceteris paribus, the unit change in the independent variable affects the change in the average logarithm of y (Gujarati, 1999, p.251-252).

Selim (2011) presents an analysis of the determinants of house prices in Turkey applying a semilog form of hedonic regression. The independent variables include housing location, type of house and building, utility systems, number of rooms, and other structural characteristics. The motivation behind the choice of semilog form in her valuation is threefold: (1) it is the most common form of hedonic regression, (2) the form fits the data well, and (3) the coefficient can be interpreted as the proportion of a good's price that is directly linked to the characteristics. She also points out that the hedonic estimation in housing can be used for making inferences about non-observable values, for instance, neighbourhood amenities and noises.

Double-log Regression Model

A regression model in which both the dependent and explanatory variables appear in the model in logarithmic form is called the double-log model and can be generally expressed in the multivariate form as

$$\ln(y_i) = \beta_0 + \beta_1 \ln(X_{1i}) + \cdots + \beta_n \ln(X_{ni}) + e_i. \tag{3}$$

In this case, the regression coefficients $\beta_1, \ldots, \beta_n$ measure the percentage change in the dependent variable for a respective percentage change in an explanatory variable, given the other variables remain constant. As for the other models, if the assumptions are satisfied, the OLS method can be used to estimate the regression parameters.

Double-log is also a widely used functional form in hedonic regression (Ginsburgh et al., 2005). In their thesis attempting to identify the value drivers of art, Kundu and Raza (2016) create a hedonic price index with Swedish art prices. They assess how the value of art, i.e. the dependent variable, is driven by the explanatory variables including artist, size, signature, date of creation, and the auction house. Interestingly, the authors conclude that even though different auction houses can have significant differences in the pricing for the same painting, the pure love of art is the only justification for investing in paintings.

2.4 Contribution

Even though many of the traditional valuation models have proven to fairly value equity, some companies are still too problematic for the traditional models to provide correct enough valuation (e.g. Cornell and Damodaran 2014). Problematic are, for instance, companies with the market value significantly above the book value and high growth, despite not generating positive payoffs. Various papers have studied one of the mentioned attributes (e.g. Simon 2016), we, however, focus on the companies with all of the three mentioned features and define them as paradoxical companies. An alternative valuation method for paradoxical companies can be the hedonic regression, which has so far been used mostly for real estate and art (e.g. Englund et al. 1998). The basic idea of the hedonic model is that the object's characteristics can explain the market value of given object (Rosen, 1974). Similarly to the paradoxical companies, real estate and art have inexplicably high market values, but do not generate positive payoffs unless rented out. We draw on the similarities and investigate whether hedonic valuation can yield a more accurate valuation model for paradoxical companies. We aim to fill the current gap in the literature when it comes to the valuation of paradoxical companies.

3 Methodology

In this section, we discuss the approach to our analysis. First, we discuss how paradoxical companies are identified. Second, we focus on the hedonic models, specifically the dependent variable, choice of independent variables and the process of model specification. Last, we discuss how the results from the estimated models are evaluated, in particular, we introduce the RIV model and different means of models' performance measurement.

3.1 Identification of Paradoxical Companies

As we mentioned before, prior literature does not provide explicit guidance on identifying paradoxical companies. Therefore, we combine different sources to collect characteristics of companies that tend to be difficult to value using traditional valuation. These characteristics are high market value (MV), negative payoffs, and high growth. We use multiple ways to choose the best cut-off values for the category of paradoxical companies. Table 3 shows a summary of the different approaches.

First, we apply absolute values as cut-offs for the three characteristics. Following Simon (2016), companies with MV over 1 billion USD are defined as 'unicorns'. But the concept of unicorns is first addressed based on the U.S. companies (Aileen Lee, 2013). Due to the structural difference between U.S. and Nordic market, we translate market value accordingly. We use the ratio of the average market value of all companies listed on NYSE to the average market value of all companies listed on NASDAQ Nordic Exchanges at the end of 2017 as a benchmark

$$\text{Nordic cut-off} = \frac{\text{Average MV in NASDAQ Nordic Exchange}_{2017}}{\text{Average MV in NYSE}_{2017}} \times 1 \text{ bil USD.}^{3} \tag{4}$$

Hence, the cut-off market value for paradoxical companies is 208 million USD. For annual growth in sales, we use the limit of 20%, as Eurostat and OECD (2007) suggest. Generally, paradoxical companies have short histories, which is why we only take the most recent year into consideration for the growth characteristic. Finally, we choose net income (NI) as a representation of payoffs and take companies with negative values.

However, we believe that high market value by itself is not a determinant of a paradoxical

[3] Data source: Statista (2018a,b,c,d)

Table 3. Approaches to identifying paradoxical companies
The table summarises the approaches to identifying paradoxical companies. MV is total market value of equity, MV/BV is total market value of equity 3 months after the financial year-end over total book value of equity at fiscal year end, NI is net income, and growth refers to growth in sales.

Approach	**Variables**	**Cutoff values**
absolute limits (a)	MV	>208 mil USD
	growth	>20%
	NI	<0
absolute limits (b)	MV/BV	>4.3
	growth	>20%
	NI	<0
quartiles	MV/BV	>75th percentile
	growth	>75th percentile
	NI	<25th percentile
combination	MV/BV	>75th percentile
	growth	>75th percentile
	NI	<0

company. If MV is explained by the firm's book value (BV), the MV is rather a determinant of company's size (Fama and French, 1992). Therefore, we search for companies with high market value to book value (MV/BV) ratio, instead of just high MV. For the MV/BV ratio, we choose the limit of 4.3 based on the lowest decile of book-to-market value in Sundqvist (2017), who replicates the study of Fama and French (1992) on the Nordic stock market.

The third approach sorts the data into quartiles and identifies companies in the most extreme quartile for each characteristic. In other words, we determine the top 25% of companies according to MV/BV and growth, and bottom 25% of companies according to the net income in a given year. Then paradoxical companies are the ones that are in all three extreme quartiles for a given year. We believe the quartile approach better reflects the current conditions on the given market.

However, when applying the quartile approach, the cut-off value for NI can be negative. We argue that any values of NI below zero are highly contradictory to high MV/BV, hence we define a fourth approach. We combine absolute and quartile limits, and find companies within the top MV/BV and growth quartiles that have simultaneously negative income.

3.2 Hedonic model

This subsection defines the specifications of our hedonic model. We follow prior applications where the dependent variable is the value of the good, in our case the company. In the hedonic valuation, the independent variables are the characteristics of the valued object. Since firms are commonly described by using different ratios, we define a set of financial and non-financial indicators that are used as explanatory variables in our model. Finally, we discuss different specifications of the model.

3.2.1 Dependent Variable

The aim of our model is to provide a more accurate valuation for paradoxical companies than can be achieved by traditional valuation. In order to apply hedonic regression, we define the dependent variable as the ratio of market value to book value computed as

$$\frac{MV}{BV} = \frac{\text{Total market value of equity}_{t+3m}}{\text{Total book value of equity}_t}, \tag{5}$$

where t refers to the end of a fiscal year. We take the market value 3 months after the fiscal year-end since that is usually the time when the annual report is published. Working under the assumption of semi-strong market efficiency, the information contained in the annual report should affect the stock prices immediately after the report is published.

We choose MV/BV ratio as the dependent variable instead of only MV following the same reasoning as Fama and French (1992), who document the significant relationship between firm size, book-to-market ratio, and average stock returns for non-financial firms. The reasoning is twofold: (1) MV is normally served as an indicator of firm size; (2) MV/BV is a scaled version of price, which is in line with the essence of hedonic pricing function.

3.2.2 Independent Variables

We identify four sets of appropriate explanatory variables for our model. First, we introduce ratios that are in prior literature often labelled as "value drivers" and therefore are expected to be important determinants of the firm's capitalisation. Second, we rely on statistical tools that are commonly used to narrow down a larger set of ratios to describe certain aspects of a company. Third, we follow literature on characteristics potentially important for valuation of paradoxical companies and identify additional variables to include in our model. Last, to capture the effects common for whole industries, we define a group of industrial dummy variables. All abbreviations of the used variables are in appendix A.

Value Drives

Stemming from the RIV model, Penman (2013) refers to the return on equity (ROE) and the book value of equity (BV) at the beginning of the given period as the drivers of firm's value. He explains that apart from increasing their value by growth in the book value, firms also raise their value above the book value by increasing their ROE above the cost of equity capital (p. 148). ROE is defined as

$$ROE_t = \frac{NI_t}{BV_{t-1}}, \tag{6}$$

and can be further decomposed using DuPont identity (Berk and DeMarzo, 2014, p. 44) into three components

$$\textbf{Decomposition 1: } ROE_t = \underbrace{\frac{NI_t}{\mathrm{SALE}_t}}_{\text{Net profit margin}_t} \times \underbrace{\frac{\mathrm{SALE}_t}{\mathrm{AT}_{t-1}}}_{\text{Asset turnover}_t} \times \underbrace{\frac{\mathrm{AT}_{t-1}}{BV_{t-1}}}_{\text{Equity multiplier}_t}. \tag{7}$$

ROE can be also expressed as a function of return on total assets (ROA), cost of total debt (COD) and debt to equity ratio (D/E) (Johansson, 1998, p. 28) as

$$\textbf{Decomposition 2: } ROE_t = ROA_t + \underbrace{(ROA_t - COD_t)}_{\text{Yield margin}_t} \times \left(\frac{D}{E}\right)_{t-1}. \tag{8}$$

We include ROE and its drivers in our model as explanatory variables and the list of the variables together with the definition of each variable can be found in appendix B.

Statistical Methods

One approach to choosing the most suitable financial ratios for an analysis is to "let the data speak" and rely on the statistical tools. Several researchers chose this way to identify which ratios are the most relevant for the respective models. Two important contexts for such studies were the prediction of corporate failure (e.g. Altman 1968) and testing for market

Table 4. Overview of studies using statistical methods to identify relevant financial ratios

The table shows an overview of papers using statistical methods to identify relevant financial ratios for their respective analysis. Panel A includes studies forecasting corporate failure and panel B includes papers testing market efficiency. The table contains the sample description, the initial set of accounting ratios, steps in selecting the most suitable ratios, and the final sample size of ratios.

Authors (Year)	**Data**	**Initial Set of Accounting Ratios**	**Selection Process**	**Final Sample Size of Accounting Ratios**
Panel A: Studies on corporate failure prediction				
Altman (1968)	U.S. manufacturing companies; 1946-65	22 ratios in 5 groups (liquidity, profitability, leverage, solvency, activity); based on popularity in literature, potential relevancy	(1) relative contribution (2) inter-correlations between ratios (3) predictive accuracy of various profiles (4) judgement of the analyst	5 ratios
Skogsvik K. (1990)	Swedish large industrial companies; 1966-80	approx. 70 ratios in 7 groups (profitability, cost structure, capital turnover, liquidity, asset structure, financial structure, growth)	(1) principal component analysis (2) univariate tests (3) iterative re-estimation process	4-6 ratios, depending on the prediction horizon
Panel B: Studies on market efficiency				
Ou, Penman (1989)	U.S. companies traded on AMEX or NYSE; 1965-1983	68 acc. ratios	(1) univariate LOGIT prediction model (2) significance in multivariate model (3) stepwise inspection	16-18 ratios, depending on the estimation window
Holhausen, Larcker (1992)	U.S. companies traded on NYSE, AMEX, and OTC firms; 1978-88	68 acc. ratios, following Ou, Penman (1989); exclude 8 due to missing observations	stepwise logistic regression	4 different logit models, 8-9 ratios in a typical model
Skogsvik S. (2008)	Swedish manufacturing companies listed on the Stockholm Stock Exchange; 1970-94	117 acc. ratios based on prior empirical studies	(1) principal component analysis (2) stepwise backward selection in multivariate logit models	8-12 ratios, depending on the estimation period

efficiency (e.g. Ou and Penman 1989). Table 4 provides an overview of the papers from the two streams of literature and the respective methods applied to choose the accounting ratios for the analysis.

A common tool used to select the most relevant ratios is the principal component analysis (PCA). The technique aims to narrow down a big sample of variables by transforming the sample into a new set of uncorrelated variables - principal components. To retain as much variation of the original set as possible, the principal components are ordered so that the first few contain most of the variation of the original sample (Jolliffe, 2011, p.1).

Due to the limited scope of our thesis, we do not perform PCA ourselves. Instead, we choose to follow Skogsvik S. (2002), who identifies 22 dimensions using the financial information of Swedish manufacturing companies to analyse the relevance of financial information to the investors and the efficiency of the Swedish market. Since the author applies PCA to three overlapping subperiods, we pick the ratios from the most recent subperiod available. Furthermore, from each dimension, we use the ratio with the highest correlation with the underlying dimension in the given subperiod. Since the accounting standards are constantly changing and IFRS was not in use when Skogsvik S. (2002) was published, we adjust the definition of some of the ratios, to match current financial reporting. In addition, the dimension of growth in untaxed reserves is excluded, as the item of untaxed reserves is not reported by the companies anymore. The full list of used ratios with the definitions are presented in appendix B.

Additional variables

Many scholars have tried to identify the linkage between the valuation of the firm and research and development (R&D) expenditures. For instance, Chan et al. (2001) use two R&D intensity measurements: R&D expenditures to sales and R&D expenditures to market value of equity. R&D expenditures to sales is used as another independent variable in our model because (1) market value of equity is used as a part of the dependent variable in our model, (2) R&D expenditures to sales is widely used as an indicator of how much resources a firm dedicated to R&D activities (Chan et al., 2001), and (3) the aim of R&D expenditures is to develop new products, which are mainly reflected in sales.

Moreover, Davila et al. (2003) identify IPO, acquisition, or going-out-of-business as the three main exit options for a company to leave the private stage. Even though the companies we analyse have successfully exited through IPO, we believe that the analysis of companies that become targets for acquisition can still provide us insight on the value-adding characteristics. Davis and Stout (1992) identify the characteristics of takeover targets for a large sample of firms in the U.S. market. Compared to many other papers that study takeover targets, Davis and Stout (1992) bring several non-financial variables into their hypotheses, such as the age of the firm, the proportion of tenured workforce, the ownership structure, the level of interlocking directorate, and the functional background of the chief executive officer. Based on Davis and Stout's (1992) insight, we include two non-financial variables in our model.

Since permanent contracts are often signed with full-time employees in most of the Nordic countries, the proportion of the tenured workforce is not a viable option. Therefore, we include the total number of employees in one of the explanatory variables. Eurostat and OECD (2007) define high growth rate both in terms of number of employees and turnover, and average revenue per employee is often used as an indicator for firm performance in past papers that study worker productivity (Chowdhury et al., 2014). Although not many studies link worker productivity to equity valuation, with the suspicion that worker productivity could be one of the explaining characteristics for the high valuation of paradoxical companies, we include the natural logarithm of revenue to the number of total employees as one of the independent variables.

Another indicator mentioned by Davis and Stout (1992) is the age of the firm. They hypothesize that firms with longer history will face a greater risk of takeover. We also include the age of the firm as an additional indicator, by collecting the year the company was founded and assuming that all companies are founded at the beginning of the calendar year. Following Davis and Stout (1992) we add natural logarithm of the age of the firm in our model.

Industrial Dummies

Dummy variables are used in the regression models to identify categories of qualitative characteristics (Gujarati, 1999, p.275). We include industrial dummy variables in our models to capture the potential effect of different level of conservative accounting bias in different

Table 5. Industrial categories

The table shows the industrial categories and respective dummies. Categorisation is based on the SIC codes, which refer to the industries included column. Number of observations refer to the estimation sample consisting of 1828 firm-year observations.

Category	Industries included	SIC code	Number of observations	Dummy
Manufacturing	Manufacturing	2000-3999	970	-
Mining	Mining	1000-1499	104	D_{mining}
Services	Services	7000-8999	394	$D_{service}$
Trade	Wholesale and retail trade	5000-5999	98	D_{trade}
Transport	Transportation, communications, electric, gas and sanitary service	4000-4999	180	$D_{transport}$
Other	Agriculture, forestry, fishing, construction, and non-classifiable	0100-0999, 1500-1799, 9900-9999	82	D_{other}

industries. The different industries with respective categories and the number of observations are shown in table 5.

The measurement bias of conservative accounting is defined as $q = (MV/BV) - 1$, and indicates how market value deviates from the book value of a company in competitive equilibrium. We identify MV/BV as the dependent variable in our model, therefore we believe that different levels of accounting conservatism are reflected in the dependent variable in different industries. Runsten (1998) uses Swedish companies to estimate q values for various industries, thus supporting the validity of industrial dummies in a valuation model.

We categorise the companies into 6 industry groups shown in the first column of table 5. Since the manufacturing companies are represented the most, we choose this category as the base case and introduce 5 dummies to our model shown in the last column of the table. The category Other combines industries with similar q coefficients estimated in Runsten (1998). We hope that including the industry dummies will also help capture investors' preferences in specific industries.

3.2.3 Model Specification

In the previous subsection, we identify 31 explanatory variables to include in our valuation model. To arrive at a more parsimonious model, we follow different procedures to eliminate irrelevant independent variables. The process is visualised in appendix C. Considering the heterogeneity of the investors in Nordic market and the continuous availability of traded shares, based on Sopranzetti (2015), we apply the hedonic regression in its linear functional form.

Step 1: Univariate regression First, we run univariate regressions, in which we regress our dependent variable MV/BV on each explanatory variable separately. Following prior studies (e.g. Ou and Penman 1989), we reduce the sample of potential explanatory variables based on the significance of the estimated coefficients. Hence, before estimating any multivariate regression, we exclude variables that are not significant at the 5% level from the set of potential explanatory variables.

Step 2a: Enter regression As a first alternative of the multivariate regression we apply the enter method, in which all the explanatory variables enter the estimation simultaneously. In our case, we differentiate three main versions of the enter model, depending on which value drivers are included in the model. Since we define ROE and its two decompositions as explanatory variables, we want to avoid including multiple variables that in essence reflect the same characteristic. Hence, we incorporate separately ROE itself, ratios from the first decomposition in equation 7, and eventually variables from the second decomposition in equation 8, given they are deemed important based on the univariate regressions. The value drivers are always complemented by all the univariately significant variables from the statistical methods and additional indicators, and models E1, E2, and E3 are estimated. To finesse the models, to each of the three specifications we also add the industrial dummies and estimate models E1d, E2d, and E3d.

Step 2b: Stepwise regression Additional alternative of the multivariate estimation used is the stepwise regression, a technique used in multiple accounting studies, including Holthausen and Larcker (1992) and Skogsvik S. (2008). Stepwise regression begins by including the most significant variable from the univariate regressions. Then, the remaining indicators are one-by-one tested in combination with the first chosen variable. After testing each newly defined 2-variable models, the predictor with the lowest p-value is added to the model. However, if the addition of a new variable turns any of the previously included variables insignificant at a chosen significance level (α), they have to be removed from the model. The process continues by iteratively adding and removing predictors to and from the model until no new variable can be added given the value of α (Draper and Smith, 1998, p.335).

Under the stepwise procedure, we estimate two groups of models - general and tailored. While general models do not take into account the amount of missing explanatory variables, tailored models exclude independent variables that are most commonly not available for paradoxical companies and therefore are designed to be more suitable for the companies' valuation. Following the same logic as in the enter regression models, under each group of stepwise models we differentiate which value drivers are included in the set of tested explanatory variables. Hence, we arrive at three general models G1, G2, and G3, and three tailored models T1, T2, and T3. Last, we add industrial dummies and define models G1d, G2d, G3d, T1d, T2d, and T3d.

3.3 Result Evaluation

As summarised in subsection 2.2, former research clearly shows the superiority of the RIV model compared to the other traditional models when it comes to valuation accuracy. Therefore, we choose to evaluate the results of our hedonic models based on the performance of the RIV model. Hence, in this section, we introduce the RIV model and define the measures of performance which we use to compare the results of the models.

3.3.1 Residual Income Valuation Model

Even though the RIV model generally outperforms the DDM, it can be derived from

$$V(E)_0 = \sum_{t=1}^{\infty} \frac{DVT_t}{(1+r_E)^t}, \tag{9}$$

where $V(E)_0$ is the intrinsic value or owner's equity at time 0, DVT_t is the expected total dividend at time t, and r_E is the cost of equity capital. Despite the assumption of the company surviving 'forever', when it comes to the application of the model, infinite forecasting becomes cumbersome, hence truncation is applied

$$V(E)_0 = \sum_{t=1}^{T} \frac{DVT_t}{(1+r_E)^t} + \frac{V(E)_T}{(1+r_E)^T}, \tag{10}$$

where $V(E)_T$ is the intrinsic value of the company at time T. By utilizing the clean surplus relation (CSR) $BV_t = BV_{t-1} + NI_t - DVT_t$ (where BV_t is the book value of equity at time t, and NI_t are firm's net income at time t), the dividends can be substituted with NI and BV in the equation 10 (for detailed derivation see e.g. Skogsvik K., 2002), and arrive to the RIV formula

$$V(E)_0 = BV_0 + \sum_{t=1}^{T} \frac{BV_{t-1}\left(ROE_t - r_E\right)}{(1+r_E)^t} + \frac{q_T BV_T}{(1+r_E)^T}, \tag{11}$$

where ROE_t is the return on owner's equity at time t and q_T is the accounting measurement bias of owners' equity at competitive equilibrium. As was mentioned before, the model's anchor is the book value of a firm's equity at the valuation date. The second term in the equation 11 uses the residual income as the value relevant variable and discounts it to the valuation date. Finally, the third term represents the present value of a firm's goodwill/badwill (Skogsvik K., 2002).

One great advantage of the RIV model is that only the CSR needs to hold for the model to be valid, regardless of the accounting practices (Skogsvik K., 2002). Moreover, unlike the DDM, RIV is consistent with Modigliani and Miller (1958) dividend irrelevance proposition that states the dividend payout policy does not affect the firm's value. Ohlson (1995) shows that since the CSR holds, the paid out dividends decrease current book value and the firm value obtained using the RIV model is not influenced by the payout policy.

3.3.2 Accuracy Measurement

To assess the performance of our models, we compute multiple measures analysing different aspects of the model. We follow the metrics used in Anesten et al. (Forthcoming), who apply three traditional valuation models including the RIV model to a Nordic data sample. The authors evaluate the models based on signed and absolute prediction errors, central tendency, and AM-score.

Signed prediction error (SPE) is defined as

$$SPE_{0,k} = \frac{V(Model)_{0,k} - MV_{0,k}}{MV_{0,k}}, \tag{12}$$

where $V(Model)_{0,k}$ is the intrinsic value of the company k computed from a model and $MV_{0,k}$ is the observed market value for the given company, both estimated at the valuation date

$t = 0$. The SPE measures the bias of the model and provides information on whether the model tends to over- or underestimate the price of the company. Absolute prediction error (APE) is defined similarly as SPE, however, the numerator is in absolute value

$$APE_{0,k} = \frac{\mid V(Model)_{0,k} - MV_{0,k} \mid}{MV_{0,k}}. \tag{13}$$

This measure therefore ranks the models based on accuracy (Francis et al., 2000). In line with Anesten et al. (Forthcoming), we compute the mean and median SPE ($\overline{\text{SPE}}_0$) and Med(SPE_0), respectively) and mean APE ($\overline{\text{APE}}_0$) for the estimated hedonic model.

Central tendency (CT) also tests the accuracy of the results, however not focusing on the precise measurement of the error, but rather on the spread between the estimated values and the true market values (Francis et al., 2000). The CT assesses how many of the estimated values lie within 15% of the observed market value of the company and is therefore defined as

$$CT_{0,k} = \frac{1}{N} \sum_{k=1}^{N} I_k, \tag{14}$$

where N is the sample size and I_k takes values 0 or 1 for each company k, depending on the size of the APE

$$I_k = \begin{cases} 0 & \text{if } APE_k > 15\% \\ 1 & \text{if } APE_k \leq 15\%. \end{cases} \tag{15}$$

Finally, we also consider the AM-score (AM) which Anesten et al. (Forthcoming) define as

$$AM_0 = \frac{(1/Iqr(SPE)_0)}{\overline{APE}_0}. \tag{16}$$

The metric utilises the previously defined $\overline{\text{APE}}_0$ and interquartile range (Iqr) of SPE, which is the difference between the third and the first quartile of the error measure. Hence, this measure combines both the precision and the spread metrics and evaluates the overall performance of the models.

4 Analysis and Discussion

4.1 Sample

In order to keep our sample homogeneous and at the same time maximise the number of paradoxical companies, we focus our analysis on Nordic countries, excluding financial and real-estate sector. Hence, the sample consists of Swedish, Finnish, Norwegian, and Danish companies. For the analysis, we need financial statement data available from COMPUSTAT and market values that were obtained from FinBas for Sweden and from Datastream for Denmark, Finland, and Norway. Our sample covers the years between 2011 and 2017, where 2011-2014 is the estimation period during which we estimate the hedonic models, and 2015-2017 is verification period during which we test the accuracy of the estimated models.

Since we use multiple data sources, we first merge our data into one file based on the ISIN code. Therefore, from the original data downloaded from COMPUSTAT, we exclude observations that are missing this identification code. We also restrict our sample to companies with fiscal year-end in December, to keep the valuation dates always at the end of the calendar year. Moreover, to be able to identify paradoxical companies in our sample, we exclude all companies that are missing any of the variables needed for paradoxical categorisation, i.e. MV/BV, NI, and growth in sales. After this exclusion, some observations with infinite sales growth still remain, which are also deleted. Finally, we exclude observations that have missing SIC codes, to ensure our sample does not include financial and real-estate companies. The number of eliminated observations by each step is shown in table 6. The final sample includes of 3346 firm-year observations, of which most are from Sweden (47.5%), followed by Finland (20.0%) and Norway (19.5%), and the fewest observations are from Denmark (13.0%).

4.2 Descriptive Statistics

Table 7 shows the summary statistics for estimation and verification period. The estimation period includes 1828 firm-year observations, with firms' median market value 692 mil SEK and sales 856 mil SEK. The mean values are high above the middle value, which implies that our sample contains some extreme values. We verified the extreme values by comparing the data to the annual reports and find that they are correct and that the outliers are

Table 6. Sample reduction
The table presents the steps of the sample reduction separately for estimation and verification period.

	Estimation period (2011-2014)	Verification period (2015-2017)
Number of observations from COMPUSTAT	5052	3225
(Excluding financial and real-estate sector)		
- Observations missing ISIN	- 50	-23
- Companies with fiscal year-end not in December	-451	-265
Sum	4551	2937
Paradoxical characteristics		
- Missing MV/BV	-2301	-1254
- Missing NI	-117	-38
- Missing growth in sales	-245	-72
- Infinite growth in sales	-19	-20
Sum	1869	1553
Other criteria		
- Missing SIC	-4	-2
Number of firm-years in final sample	1828	1518

mostly accountable to the companies we identify as paradoxical. Paradoxical characteristics (MV/BV, NI, growth in sales) show the same trend. The middle value of MV/BV is 2.0, which implies that the company's total market value of equity is twice as high as its total book value of equity. However, on average the companies show a market value of 3.6, which is almost twice as high. The NI median (13 mil SEK) and mean (623 mil SEK) values are positive, while the 25th percentile (-8 mil SEK) is negative, which suggests that our sample includes paradoxical companies. This belief is supported also by the statistics of growth in sales, where the middle value is 4.4%, but the mean is more than 13-times higher (57.6%).

The verification period comprises of 1518 firm-years and comparing the two subperiods, they show similar trends in the values presented. Both market value and sales have similar median values (965 mil SEK and 823 mil SEK, respectively) as in the earlier years, with mean values significantly above the median. The biggest difference is in MV/BV that averages to 5.6 in the verification period, compared to 3.6 in the estimation period and the middle value is 2.4, slightly above the 2.0 value from the previous subperiod. The 25th percentile of NI is again negative (-12 mil SEK) and growth in sales averages to 58.8%, which is close to the mean value in the estimation period. However, the median value of sales growth is

Table 7. Descriptive statistics

The table presents the summary statistics for the market value, sales, MV/BV, net income and growth in sales. For each variable and subperiod, the number of observations, mean, 25th percentile, median and 75th percentile are shown.

	Observations	Mean	25th percentile	Median	75th percentile
	Estimation period (2011-2014)				
Market value (in mil SEK)	1828	13 132	161	692	4 993
Sales (in mil SEK)	1828	11 970	110	856	5 209
MV/BV	1828	3.6	1.1	2.0	3.8
Net income (in mil SEK)	1828	623	-8	13	169
Growth in sales	1828	57.6%	-4.2%	4.4%	17.2%
	Verification period (2015-2017)				
Market value (in mil SEK)	1518	15 190	211	965	5850
Sales (in mil SEK)	1518	10 705	105	823	5 510
MV/BV	1518	5.6	1.4	2.4	4.4
Net income (in mil SEK)	1518	518	-12	19	218
Growth in sales	1518	58.8%	-3.3%	7.2%	21.6%

7.2%, which is considerably higher than the middle value from the estimation period. The more extreme values of paradoxical characteristics in the verification period suggest that even though there are fewer observations, the amount of paradoxical companies might be sufficient for the analysis (see subsection 4.3). In general, we believe the two subperiods are fairly similar and therefore the model estimated during the first period can be tested during the verification subperiod.

4.3 Paradoxical Companies

To identify paradoxical companies, we apply all four paradoxical company identification approaches mentioned in section 3.1. The approaches are separately employed for estimation and verification period and the number of paradoxical companies' observations under each approach in both subperiods is reported in panel A of table 8. As the table shows, in both subperiods, absolute limit (a) approach generates the least number of observations, which confirms our concern that the absolute market value may not be a suitable cut off for paradoxical companies. In general, absolute limits (b) and combination approaches present a similar number of observations and yield most observations in both periods, while absolute limits (b) gives a slightly higher number of observations in verification period.

Table 8. Paradoxical companies - statistics

The table shows the statistics of paradoxical companies. Panel A presents the number of firm-year observations for each identification approach and subperiod. Panel B reports the industries to which the paradoxical companies most often belong. The analysis is performed based on the companies' SIC codes.

Panel A: Number of observations per approach

Approach	Estimation period (2011-2014)	Verification period (2015-2017)
absolute limits (a)	21	18
absolute limits (b)	82	89
quartiles	58	44
combination	92	78

Panel B: Number of observations per industry

Industry	Estimation period (2011-2014)	Verification period (2015-2017)
Biological, pharmaceutical and medical products	33 (0.36)	17 (0.22)
Electromedical, orthopedic, surgical and dental equipment	17 (0.19)	8 (0.10)
Computer and software service	9 (0.10)	13 (0.17)
Non-computer related electrical equipment	9 (0.10)	9 (0.12)
Laboratory equipment	6 (0.07)	5 (0.06)

The industries with the highest concentration of the paradoxical companies are shown in table 8 panel B. The industrial compositions are similar among observations under all four approaches, hence we further focus on one approach. In particular, we analyse the industry structure by SIC codes under the combination approach of the 92 and 78 observations in the estimation period and verification period, respectively. The industrial segmentations in both periods are comparable, where observations that are in biological, pharmaceutical, medical and computer service industries account for over 70% of the total paradoxical companies in each period.

The industrial structure of the paradoxical companies is not surprising. Investors often have high expectations on the prospect of the above-mentioned industries, especially when the companies claim to cure severe diseases, provide more accurate and convenient diagnose approaches, or offer life-changing IT solutions. When the investors are convinced by the company's claims, they are willing to invest, despite high cash burn rate (mainly due to R&D and marketing activities) and the company not generating any positive payoff for the shareholders. Hence, the investors' expectations drive the high share price and lead to high MV/BV ratio.

4.4 Model Estimation

In this subsection, we focus on the estimation of the final valuation models. We discuss the results of the univariate regressions, which is followed by the estimation of multivariate hedonic models, namely enter and stepwise models.

4.4.1 Univariate Regressions

Table 9 shows the results of univariate linear regressions, where the dependent variable MV/BV is regressed on each of the 31 explanatory variables discussed in subsection 3.2.2. The table shows how many of the observations are available for each descriptor (N), the estimated coefficient and the respective p-value. Following Ou and Penman (1989), we run the regressions without excluding any firms because of missing some of the variables, in order to avoid any selection bias at this stage of the analysis. At this stage of the analysis, we do not discuss whether the signs of the coefficients in panel A and B are economically intuitive, they will be analyses once the final models are estimated.

Panel A shows the results for the value drivers. ROE and most of its components have p-values less than the significance level of 0.05 and the estimated coefficients are negative. This implies that the higher ROE or its components are, the lower the MV/BV is. The one inconsistent exception among the value drivers is AssTO, which has a coefficient greater than zero, which however is not significantly different from zero based on its p-value above the 0.05 level.

Panel B includes the estimated regressions for variables identified using statistical methods. From the 21 variables tested, only 6 have p-values below 0.05. From these, Prof, S, and FinStr have negative coefficients, which means that the higher they are, the closer MV is to BV. However, dProf, dCapInt, and Liq have estimated coefficients greater than zero, implying a positive relationship between the values of these variables and the MV/BV ratio.

From the results for additional variables reported in panel C, it is obvious that all three identified characteristics are statistically significant. The negative relationship between Age and MV/BV can be explained by older companies being less agile compared to young ones,

Table 9. Estimated univariate regressions

The table reports the results of univariate regressions for each explanatory variable. Column N shows the number of observations available for a given variable, the next column shows the coefficient obtained from the regression and the p-value is reported in the last column.

Panel A: Value drivers

Variable	N	Coefficient	p-value
ROE_t	1826	-0.226	0.004
$NProfMarg_t$	1808	-0.006	0.000
$AssTO_t$	1826	0.206	0.210
$EqMult_t$	1826	-0.363	0.000
ROA_t	1826	-3.330	0.000
$YielMar_t$	1781	-2.916	0.000
$(D/E)_{t-1}$	1826	-0.363	0.000

Panel B: Statistical methods

Variable	N	Coefficient	p-value
$Prof_t$	1805	-0.006	0.000
$dProf_t$	1822	2.138	0.005
$CapInt_t$	1808	-0.004	0.920
$dCapInt_t$	1808	0.003	0.000
CaF_t	1819	0.024	0.727
Liq_t	1808	0.007	0.000
$dLiq_t$	1809	-0.005	0.752
dWC_t	1825	-0.005	0.733
S_t	1828	-0.527	0.000
Gr_t	1767	0.031	0.183
$AssStr_t$	1828	-0.886	0.452
$gTanAss_t$	1826	5.630	0.096
$FinStr_t$	1828	-3.371	0.000
$gIntLiab_t$	1520	-0.002	0.820
$TaxCost_t$	1828	-0.013	0.766
$IntCost_t$	1782	-0.570	0.439
Inv_t	1603	-0.793	0.793
$PayOut_t$	836	0.010	0.656
$gDiv_t$	713	-0.698	0.247
Div/CaF_t	837	0.006	0.698
NI/CaF_t	1826	-0.004	0.509

Panel C: Additional variables

Variable	N	Coefficient	p-value
$Empl_t$	1508	-1.009	0.000
Age_t	794	-0.836	0.006
$R\&D_t$	852	0.020	0.000

making them unlikely to have dramatic strategic changes and have poorer ability to seek future business opportunities. The estimated coefficient for R&D is, on the other hand, positive, suggesting that companies that spend more on research and development have also higher MV/BV ratio. This also does not come as a surprise since in section 4.3 we identified that industries which often have high research costs like pharmaceuticals, medical equipment and IT are most common industries with paradoxical companies. However, the negative coefficient for Empl does not seem as reasonable. The sign implies that the higher the worker productivity the lower the discrepancy between MV and BV, which goes against the expectation that companies like startups, that do not have many employees but generate high revenue will have unjustifiably high MV.

Missing Explanatory Variables

When it comes to the number of available observations for each variable, PayOut, gDiv and Div/CaF have the most missing data points from the statistical methods. To find out the reason why the data is missing, we randomly pick 20 observations with missing dividend information and check the annual reports. We find that among the 20 picked firms, 12 firms have zero proposed dividend, 5 firms do not disclose any information on dividend, or have no available annual report, mainly due to take-overs or not being listed anymore, and only 3 firms have proposed dividend. Thus, we assume the proposed dividend on all shares and ordinary shares to be 0 for all observations with missing data on dividends. After conducting univariate linear regression again, we find that the coefficients for PayOut, gDiv and Div/CaF still have p-value high above 0.05.

The issue of the high amount of missing values also appears in additional variables, specifically for Age and R&D. We apply the same approach to identify the reason behind the missing R&D data. We randomly pick 20 observations with missing R&D information and check their annual report. Among the 20 firms, 10 are listed on First North Nasdaq or Spotlight Stock Market (Aktietorget), which are for smaller businesses with less extensive reporting requirements. In the annual reports of the companies that are listed on First North Nasdaq or Spotlight Stock Market, only R&D amortization for capitalized R&D assets (if any), instead of total R&D expenditure, is disclosed in their annual reports. We suspect the poor financial information disclosure quality to be the main explanation for the missing data. For the remaining 10 firms, we find R&D expenditure to be 0 or not disclosed at all in their annual reports. Hence, putting the missing R&D observations equal to zero would not be an accurate approximation of reality. We believe that Age is missing mainly due to (1) the limited information provided in smaller stock exchange markets, and (2) the founding year is not usually collected by the databases.

4.4.2 Multivariate Models

After analysing the importance of each variable in the univariate regressions, in this section, we move on to estimating multivariate models. First, based on the results in the previous subsection, we exclude the variables that are not significant at the 5% level, as is visualised in appendix C. To arrive at a multivariate model, we employ two approaches - enter and stepwise regression.

4.4.2.1 Enter Regression

Table 10 shows the estimated enter multivariate regression models, with variables significant at the 5% level. Panel A shows the result of models E1, E2, and E3 which do not include industrial dummies in the regression, while panel B shows the result of models E1d, E2d, and E3d when all industrial dummies are included in the estimation process. It is intriguing that industrial dummies do not seem to play an important role in our enter regression models, and the significant variables are the same for the models in panel A and panel B, where the coefficients have the same sign and only have small differences in the value. One thing to note regarding the enter models is that the models are estimated on a small fraction of the firm-year observations, since all the explanatory variables entering the model need to be available for each firm-year. Thus, the validity of the models can be questioned.

Among all the models in Table 10, at least one of the ROE drivers or ROE itself is significant in the final models. Interestingly, the measures of management efficiency, namely ROE and ROA, are both negatively related to MV/BV. One potential explanation is that even though ROE and ROA are still among the most well-known indicators to measure the performance of a company, a low ROE or ROA in the current year can be considered by investors as a potential for future growth. Furthermore, we consider mean reversion to be another potential explanation for this phenomenon. Interestingly, YielMar has a positive coefficient, which is in line with the expectation we have on the ratios that reflect measures of management efficiency, however, contradicts the negative coefficient of ROA.

The negative coefficient of EqMult is also surprising since it implies that the higher total asset the lower market value, which is also supported by the positive coefficient of FinStr. The phenomenon could be explained by the trend of the rise of successful light-assets companies, for instance, with shared economy business models (e.g. Uber and Airbnb), that positively outsource, or rely heavily on intellectual property.

When it comes to D/E, scholars have spent decades in discussing the relationship among business risk, leverage ratio and firm valuation (e.g. Brennan and Schwartz 1978). In the

Table 10. Enter Regression Models

The table shows the final models estimated using enter regression. panel A reports the models without industrial dummies and panel B presents the models with industrial dummies included in the estimation. For each model the estimated intercept, coefficients for given variables and p-values (in brackets) are shown. The last two rows refer to the number of observations included in the model estimation (N) out of the whole sample of 1828 firm-years, and R^2 respectively.

	Panel A: Without dummies			**Panel B: With dummies**		
	Model E1	Model E2	Model E3	Model E1d	Model E2d	Model E3d
Intercept	9.210	15.763	13.070	10.540	16.152	14.134
	(0.001)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)
ROE_t	-1.888			-1.880		
	(0.001)			(0.001)		
$EqMult_t$		-3.774			-3.821	
		(0.000)			(0.000)	
ROA_t			-97.889			-95.826
			(0.008)			(0.010)
$YielMarg_t$			103.287			101.785
			(0.004)			(0.005)
D/E_t			-4.115			-4.164
			(0.000)			(0.000)
S_t			-0.517			-0.541
			(0.017)			(0.019)
$FinStr_t$		12.930	22.332		13.731	23.121
		(0.000)	(0.000)		(0.000)	(0.000)
N	402	402	393	402	402	393
R^2	0.041	0.330	0.358	0.032	0.327	0.357

model E3 and E3d, the negative coefficient of D/E suggests that the majority of our sample has D/E ratio above the value-adding limit and therefore the more the companies increase their leverage, the lower the MV/BV becomes. The negative coefficient of S is in line with its definition, where the higher the BV, the higher the indicator, and the lower the MV/BV.

4.4.2.2 Stepwise Regression

General Stepwise Models

The estimated general models G1, G2, and G3 are shown in table 11 panel A. The coefficients of EqMult, D/E and S show the same patterns as in enter multivariate models. Similar to enter multivariate models, in each one of the general models, at least one ROE driver or ROE itself appears in the final models, while dProf, Liq and S are the remaining final explanatory variables in both models G1 and G3. The positive coefficient of dProf implying that high growth in profitability has a positive effect on MV/BV supports our above conclusion that investors put emphasis on the potential future growth. The measure of liquidity, Liq, is positively related to MV/BV, which aligns with our expectations.

Surprisingly, when including NProfMarg and EqMult in the initial set of potential predictors in model G2, Age eliminates most of the variables and becomes one of the only two variables in the final model. However, this model is estimated only on approximately half of the observations from our sample, due to many missing data points. Therefore, this model is not as useful and widely applicable as the previous stepwise models, which can be applied almost to the whole sample. It is also interesting to see that except Age, none of the other additional ratios remains in the final model, suggesting that R&D and work productivity may not be weighted as important by investors after all.

Tailored Stepwise Models

Since our aim is to define a model that can be easily applied to value paradoxical companies, in the second approach we start by analysing which of the dependent variables are often not available for paradoxical companies and exclude them from the potential value indicators before running a stepwise regression.

Table 11. Stepwise regression models without industrial dummies

The table shows the final models estimated using stepwise regression without including the dummy variables. Panel A reports the general models when all ratios significant at 5% level in the univariate regression are included. Panel B presents the models tailored to data available for paradoxical companies. For each model, the estimated intercept, coefficient for given variable and p-value (in brackets) are shown. The last two rows refer to the number of observations included in the model estimation (N) and R^2 respectively.

	Panel A: General models			**Panel B: Tailored models**		
	Model G1	Model G2	Model G3	Model T1	Model T2	Model T3
Intercept	6.394	8.151	6.396	6.394	7.317	6.396
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ROE_t	-0.394			-0.394		
	(0.017)			(0.017)		
$EqMult_t$		-0.719			-0.375	
		(0.000)			(0.000)	
ROA_t			-2.035			-2.035
			(0.000)			(0.000)
$(D/E)_{t-1}$			-0.366			-0.366
			(0.000)			(0.000)
$dProf_t$	1.590		2.306	1.590	2.043	2.306
	(0.031)		(0.001)	(0.031)	(0.005)	(0.001)
Liq_t	0.007		0.006	0.007	0.007	0.006
	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)
S_t	-0.460		-0.387	-0.460	-0.463	-0.387
	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)
Age_t		-0.851				
		(0.004)				
N	1802	792	1802	1802	1802	1802
R^2	0.080	0.065	0.115	0.080	0.103	0.115

When analysing the paradoxical companies, we focus on the subsample identified using the combination approach. We believe this approach both takes into consideration the market conditions thanks to using the quartile approach, and at the same time includes all the observations that have negative NI, which is a crucial characteristic of paradoxical companies. Out of the 92 paradoxical observations identified in the estimation period using the combination approach, 80 are missing Age, 55 R&D, and 33 Empl. The missing data is mainly due to the markets where most of the paradoxical companies are listed, in particular, Spotlight and First North Nasdaq, as explained in subsection 4.4.1. Therefore, we exclude these explanatory variables from the stepwise regression, despite being significant predictors in the univariate regression, and estimate three new models that we believe will be more suitable for pricing of paradoxical companies. The tailored estimated models T1, T2 and T3 are shown in panel B table 11.

First thing to notice is that models T1 and T3 are the same with models G1 and G3, respectively. Since neither one of the excluded variables entered the model in stepwise regression, the final model does not change compared to using the whole set of significant explanatory variables. However, as soon as we exclude Age from the set of potential indicators, the model tailored to the paradoxical companies G2 differs from the general one T2.

Model T2 includes 4 explanatory variables, namely EqMult, dProf, Liq and S, all of which appear in the previously estimated models and their coefficient signs are consistent with the prior explanation. Thanks to excluding Age, the model can be applied on 1802 firm-years from the total sample of 1848 observations, which makes it much more relevant to our sample than model G2.

Stepwise Models Including Industrial Dummies

Prior research shows that including multiple dummy variables describing one characteristic in stepwise regression might lead to inaccurately defined models (e.g. Cohen 1991). Since we define 5 dummies categorising the companies into industries, we include them after arriving at the final models estimated above, namely G1, G2, G3, and T2. To each model we add the categorical variables defined in subsection 3.2.2, re-estimate the coefficients and

Table 12. Stepwise regression models including industrial dummies

The table shows the final models estimated using stepwise regression when including the dummy variables. The general models G1d, G2d and G3d, and the distinct tailored model T2d are reestimated. For each model the estimated intercept, coefficients for given variables and p-values (in brackets) are shown. The last two rows refer to the number of observations included in the model estimation (N) out of the whole sample of 1828 firm-years, and R^2.

	Model G1d	Model G2d	Model G3d	Model T2d
Intercept	7.002	8.849	6.865	7.883
	(0.000)	(0.000)	(0.000)	(0.000)
ROE_t	-0.346			
	(0.037)			
$EqMult_t$		-0.705		-0.364
		(0.000)		(0.000)
ROA_t			-1.863	
			(0.000)	
$(D/E)_{t-1}$			-0.357	
			(0.000)	
$dProf_t$	1.491		2.192	1.926
	(0.042)		(0.003)	(0.008)
Liq_t	0.007		0.006	0.007
	(0.000)		(0.000)	(0.000)
S_t	-0.474		-0.402	-0.481
	(0.000)		(0.000)	(0.000)
Age_t		-0.886		
		(0.004)		
D_{mining}	-1.480			-1.365
	(0.026)			(0.037)
$D_{service}$	-1.080		-0.772	-1.076
	(0.005)		(0.044)	(0.005)
D_{trade}	-1.552			-1.412
	(0.020)			(0.032)
N	1802	792	1802	1802
R^2	0.085	0.069	0.118	0.108

keep the ones with p-value below 0.05. The re-estimated models G1d, G2d, G3d, and T2d are presented in table 12.

Out of the 5 dummy variables defined, only three of them remain significant in at least one of the models, in particular, D_{mining}, $D_{service}$, and D_{trade}. All of the coefficients are negative which means that, on average, the measurement bias is lower for these industries than for manufacturing. It is not surprising, since one of the main components creating measurement bias is R&D expenses, which are usually generated more in manufacturing companies than in the ones operating in mining, service and trade.

The fact that D_{other} and $D_{transport}$ are not significant could imply that the accounting measurement bias for industries included in these categories is in expectation the same as for our benchmark industry - manufacturing. However, there can be other factors than the accounting conservatism that can affect differences between industries and therefore potentially cancel the effect of measurement bias.

4.5 Evaluation of the Hedonic Models

We use the estimated models to value the companies in the verification period and compute the accuracy measures defined in section 3.3.2. The accuracy metrics for the 6 enter models (E1, E2, E3, E1d, E2d, and E3d) are shown in table 13, and table 14 presents the results of 8 distinct stepwise models (G1, G2, G3, T2, G1d, G2d, G3d, and T2d). To be able to better analyse the results, we split the observations into two groups: paradoxical and normal companies, where the latter includes all companies that are not paradoxical based on the combination approach. The results for normal and paradoxical companies are presented in panels A and panels B, respectively. Consistent with Anesten et al. (Forthcoming), we only retain valuation results for firm-year observations for which our models yield positive MV.

We use the accuracy results for parsimonious RIV with projected historical data in Anesten et al. (Forthcoming) as our benchmark, since the authors use recent[4] Nordic sample, and we believe that applying the traditional valuation model in a parsimonious form mostly corresponds to the simple application of our models. In addition, the authors use projected data based on historical information as input for the model, which is in line with the required information for our models. The results of RIV are reported in the last column of both tables 13 and 14.

4.5.1 Normal Companies

The accuracy metrics of enter models applied to normal companies are shown in panel A of table 13 and suggest that the models yield strongly biased results. Despite being widely applicable, the $\overline{\mathrm{SPE}_0}$ varies between 4.25 and 7.87, which suggests strong overvaluation based on our models. Other accuracy metrics also support the bad performance: Med(SPE_0) is above 2.0 for all models, $\overline{\mathrm{APE}_0}$ lies between 4.4 and 7.9, only 2-5% of observations have APE_0 lower than 15%, and AM-score is close to zero. Interestingly, adding industrial dummies the

[4] 2005-2014

Table 13. Accuracy measures of enter regression models

The table presents the accuracy metrics for the enter models. The last column reports the results of parsimonious RIV specification based on projected historical data from Anesten et al. (Forthcoming). Panel A contains the results for normal companies, while panel B shows the results for paradoxical companies. SPE refers to signed pricing error, APE to absolute pricing error, CT is central tendency, AM is AM score. The definitions of the measures can be found in section 3.3.2. N(frac) refers to the fraction of observations from the verification period to which each model could be applied.

	Model E1	**Model E2**	**Model E3**	**Model E1d**	**Model E2d**	**Model E3d**	**RIV**
				Panel A: Normal companies			
$\overline{\mathrm{SPE}}_0$	5.16	6.07	4.25	6.04	6.39	7.87	0.12
Med(SPE_0)	3.02	3.26	2.07	3.59	3.44	4.11	-0.16
$\overline{\mathrm{APE}}_0$	5.21	6.14	4.39	6.07	6.46	7.91	0.57
CT_0	0.03	0.04	0.05	0.02	0.03	0.03	0.21
AM_0	0.04	0.03	0.05	0.03	0.03	0.02	2.70
N(frac)	1.00	0.96	0.89	1.00	0.96	0.97	0.75
				Panel B: Paradoxical companies			
$\overline{\mathrm{SPE}}_0$	0.64	0.46	0.08	0.80	0.51	0.21	
Med(SPE_0)	0.34	0.37	-0.15	0.51	0.41	0.08	
$\overline{\mathrm{APE}}_0$	0.96	0.78	0.69	1.07	0.82	0.72	
CT_0	0.09	0.11	0.15	0.12	0.08	0.10	
AM_0	0.96	1.11	1.33	0.78	1.04	1.08	
N(frac)	0.97	0.91	0.77	0.97	0.91	0.87	

Table 14. Accuracy measures of stepwise models

The table presents the accuracy metrics for the stepwise models. The last column reports the results of parsimonious RIV specification based on projected historical data from Anesten et al. (Forthcoming). Panel A contains the results for normal companies, while panel B shows the results for paradoxical companies. SPE refers to signed pricing error, APE to absolute pricing error, CT is central tendency, AM is AM score. The definitions of the measures can be found in section 3.3.2. N(frac) refers to the fraction of observations from the verification period to which each model could be applied.

	Model G1	**Model G2**	**Model G3**	**Model T2**	**Model G1d**	**Model G2d**	**Model G3d**	**Model T2d**	**RIV**
	Panel A: Normal companies								
$\overline{\mathrm{SPE}}_0$	1.10	1.25	1.12	1.10	1.17	1.68	1.29	1.16	0.12
Med(SPE_0)	0.34	0.38	0.36	0.36	0.38	0.66	0.47	0.40	-0.16
$\overline{\mathrm{APE}}_0$	1.43	1.54	1.45	1.42	1.48	1.88	1.58	1.47	0.57
CT_0	0.11	0.12	0.10	0.11	0.12	0.11	0.11	0.11	0.21
AM_0	0.38	0.35	0.37	0.38	0.36	0.25	0.33	0.37	2.70
N(frac)	0.99	0.37	0.98	0.98	0.99	0.37	0.98	0.98	0.75
	Panel B: Paradoxical companies								
$\overline{\mathrm{SPE}}_0$	-0.34	-0.49	-0.33	-0.41	-0.32	-0.40	-0.32	-0.38	
Med(SPE_0)	-0.36	-0.41	-0.29	-0.37	-0.31	-0.33	-0.26	-0.32	
$\overline{\mathrm{APE}}_0$	0.47	0.49	0.39	0.42	0.45	0.40	0.38	0.40	
CT_0	0.18	0.13	0.25	0.20	0.19	0.38	0.31	0.24	
AM_0	4.67	4.09	5.21	5.15	5.01	4.44	5.15	5.41	
N(frac)	0.99	0.10	0.96	0.95	0.99	0.10	0.96	0.95	

enter models worsens models' performance, and the trend also extends to the stepwise models analysed in panel B of table 14.

Overall, the stepwise models not including industrial dummies yield the best results for normal companies among all the estimated models, as shown in panel A of table 14. Even though the models still overvalue the companies, models G1, G3, and T2 show best accuracy with $\overline{\text{SPE}}_0$ of approximately 1.1 and Mean(SPE_0) around 0.35. The large difference between mean and median of the SPE suggests that our sample of companies is skewed towards large SPE values. $\overline{\text{APE}}_0$ is somewhat larger than 1.4 and approximately 11% of observations' APE_0 does not exceed 15%. The AM-score is considerably higher than for enter models and reaches almost the value of 0.4. Finally, the three models G1, G3, and T2 are not only yielding accurate results but also are widely applicable to the sample. Model G2's performance is slightly worse compared to other stepwise models, however, we believe the biggest problem with this model is the applicability due to including Age variable and only being suitable for about 37% of the observations.

In general, the results of our models for normal companies are significantly worse than those reported in Anesten et al. (Forthcoming). However, we find that the highest pricing error values are accountable to companies in the offshore business, which suffered large share price decreases due to the drop in oil price in 2015. We observe that the affected companies have the smallest MV/BV in our sample and naturally lead to the largest positive SPE, and they seem to be highly represented in our sample.

4.5.2 Paradoxical Companies

When enter regression models are applied to paradoxical companies, they yield better results than when applied to normal companies, however including industrial dummies in the estimation still worsens the performance, as shown in panel B of table 13. The most striking is the accuracy of model E3, that shows $\overline{\text{SPE}}_0$ of only 0.08 and Med(SPE_0) of -0.15. The $\overline{\text{APE}}_0$ is 0.69, 15% of estimated MVs lie within 15% of the observed MV, and the AM-score reaches the value of 1.33. However, this model can be applied to the smallest fraction of the paradoxical companies out of the enter models. Moreover, when the model is estimated, only 5 out of 92 paradoxical observations have all the variables needed to enter the regression, hence we believe the results could be coincidental and the model should be used with caution.

Contrary to the enter regression models, including industrial dummies in the stepwise models increases the accuracy for paradoxical companies and models G1d, G3d, and T2d produce comparably positive results, as shown in panel B of table 14. Among the models, model G3d yields the least biased results with $\overline{\mathrm{SPE}}_0$ equal to -0.32 and Med(SPE_0) -0.26. The negative values imply that the model manages to explain the high MV of the companies only to a certain extent. Model G3d also has the lowest $\overline{\mathrm{APE}}_0$ of only 0.38, with almost one-third of the APE_0 below 15%, and high AM-score of 5.15. The model can also be applied to almost all the paradoxical companies in the sample.

We deem model G2d as not suitable to estimate the value of paradoxical companies, mainly due to the low applicability of the model. As we mention before, Age is available only for a small fraction of paradoxical companies and therefore a model including this variable does not have the potential to be widely applied to the subsample.

When comparing the results with Anesten et al. (Forthcoming), the accuracy of model G3d is close to their RIV model. The authors report the RIV model to yield results with $\overline{\mathrm{SPE}}_0$ (Med(SPE_0)) of 0.12 (-0.16). Even though the performance based on the SPE measure is more positive for the RIV model, given the simplicity of our model, we believe our results with $\overline{\mathrm{SPE}}_0$ (Med(SPE_0)) equal to -0.32 (-0.26) are quite promising. Furthermore, Anesten et al. (Forthcoming) get values of $\overline{\mathrm{APE}}_0$ 0.57, which is higher than the mean absolute error for G3d. In addition, our model yields better CT_0 compared to RIV and the AM score is high above the 2.7 value reported in Anesten et al. (Forthcoming). Even with the lower value of $\overline{\mathrm{APE}}_0$, the better results for CT_0 and large difference in the AM scores imply that our SPEs have lower spread. Lower spread in the signed errors, however, follows from the defining characteristics of paradoxical companies, i.e. high MV/BV which always leads to the model underestimating the observed market value.

Based on the aspects of applicability and accuracy of our models, we believe models G3d is the most suitable option for pricing of paradoxical companies and can serve as a starting point for further studies. Despite slightly more negative results, we suggest not to dismiss models G1d and T2d, given the high applicability.

4.5.3 Application of the RIV model to Paradoxical Companies

In the previous subsections, we compare the accuracy of our models with the results from Anesten et al. (Forthcoming). They, however, mostly focus on the companies that would not be categorised as paradoxical following our approaches. Hence, to further strengthen the validity of our results for paradoxical companies, we follow the parsimonious projected history information approach of RIV in Anesten et al. (Forthcoming) to apply RIV to paradoxical companies.

To avoid application issues, Anesten et al. (Forthcoming) exclude observations by following criteria: (1) less than 2 historical values of ROE or payout ratio (pr), and (2) the four-year-average value of ROE is lower than -25%. Among the 78 firm-year observations of paradoxical companies, following Anesten et al.'s (Forthcoming) exclusion rules, 46 observations of paradoxical companies are excluded due to less than 2 historical values of ROE, and additional 28 observations are excluded due to below -25% average ROE. Thus, only observations of 4 firms remain, which suit Anesten et al.'s (Forthcoming) conditions to apply RIV with the projected history information setting. The reason behind the large amount of elimination of observations in paradoxical companies is that the exclusion criteria are naturally contradictory to the characteristics of paradoxical companies, and that is also the main reason why we do not see RIV as a viable approach for the valuation of such companies.

To improve the comparability of our results, we follow Anesten et al.'s (Forthcoming) approach to apply RIV on the 4 remaining paradoxical observations and specify the model as follows:

- projected ROE and pr are computed as four-year historical average;
- assuming the CSR holds, we predict BV based on forecasted ROE and pr together with the known BV_0 at the valuation date;
- the forecasting horizon is 3 years;
- the accounting measurement bias (q) is obtained from Runsten (1998);
- capital asset pricing model is applied to estimate r_E, which is assumed constant over time;
- risk-free rate is the yield for 10-year government bond of the country where the company is listed at the valuation time;

Table 15. Accuracy of RIV model applied to paradoxical companies

	Aker BP ASA	**Zealand Pharma AS**	**Erria AS**	**EasyFill AB**
Year	2015	2016	2017	2017
RIV SPE_0	-0.93	-0.99	-0.94	-0.81
G3d SPE_0	-0.90	-0.62	-1.00	0.02

- The market risk premium is set to 5.5%;
- company specific betas are estimated through regressions over up to 60 months of company stock and market excess returns, where the market return is taken from the market index where the company is listed. Since the monthly data of the market index of Spotlight Stock Market is not available, we apply First North 25 index instead considering the similar size of the listed companies.

We compare the outcome of parsimonious RIV with the projected history information setting and one of our model G3d, which generally yields the best result for paradoxical companies, as shown in table 15. Among the 4 companies, model G3d generates better results compared to RIV for 3 of the firms. The only company for which G3d yields less accurate results than RIV is Erria AS, a company operating in shipping and shipping related activities. We believe that the negative result of our model was mainly due to Erria's financial performance and share price being under the severe impact of the oil price drop in 2015, a macroeconomic event that is not captured by our model. However, such macroeconomic event is also not captured by RIV, since it is a strictly accounting based valuation model.

4.5.4 Application Advantages of Hedonic Models

One great advantage of our models is that the application issues that often appear in traditional valuation models are avoided. As discussed above, under Anesten et al. (Forthcoming) criteria, RIV can be applied to only 5% of the paradoxical observations in the verification period while model G3d can be applied to 96%. Furthermore, traditional valuation models often rely on assumptions that may not accurately reflect the reality (e.g. CSR, steady state, and constant cost of capital), while our empirical based models do not require these assumptions to hold.

The hedonic models also eliminate the issues caused by data projection which is required by all traditional valuation models. Forecasting of financial information demands comprehensive analytical skills and business knowledge, yet can still be biased depending on different perspectives of individual analysts, and in essence, remains uncertain. In addition, traditional valuation models demand the user to undertake rather complicated calculations of parameters such as cost of equity and accounting measurement bias. In our models, all needed information can be found directly in companies' annual reports without complex computations.

The short time frame for historical information required is another advantage of our models, as only data for two years prior to valuation date is needed. There are two main reasons why we do not widen the time frame for explanatory variables. First, a significant amount of paradoxical companies have short histories and second, considering how dynamic the financial performance can be in paradoxical companies which have high growth, including earlier historical data can distort the valuation of the company.

5 Further Research and Conclusion

5.1 Limitations and Further Research

Considering the fact that hedonic models are based on empirics instead of being embedded in valuation theory, potential shortcomings should be noted. First, variables with a significant amount of missing observations, as mentioned in section 4.4.1, could constrain the accuracy of our models. The missing data is mainly due to the poor quality of financial information disclosure for companies listed on smaller stock exchange markets, and the systematic problem cannot be solved unless the smaller stock exchange markets apply more restrictive financial information disclosure rules. We consider using proxies for the most commonly missing variables, for instance, the IPO date instead of the founding date of a company. However, we do not believe that the IPO date and founding date are significantly correlated due to the complexity of strategic decisions regarding an IPO.

Moreover, our set of variables under the statistical methods solely relies on the analysis conducted by Skogsvik S. (2002), whose sample includes only Swedish manufacturing companies in 1972-1985. If the scope of a future study on paradoxical companies allows, conducting new PCA on a more recent and broader sample could be beneficial for the final model estimation.

The hedonic models can be further extended to include ownership structure as one of the independent variables, as suggested by Davila et al. (2003), however, such information on Nordic companies is not collected by the commonly available databases. We believe it could shed more light on the study of paradoxical companies if given data is available.

Finally, the impact of the macroeconomic environment also draws our attention, especially when we examine the companies with the highest pricing errors. We observe that the black swan events, such as a dramatic drop in oil price, impact companies that take part in the whole relevant supply chain, for instance, all companies that relate to oil and energy supply. However, due to the cross industry effect, such events are not captured by industry dummies in our models. We suggest further research to incorporate macroeconomic variables to improve the accuracy of hedonic pricing models.

5.2 Conclusion

We make two main contributions in the thesis. First, we define paradoxical companies and develop four different identification approaches that can be adapted to different markets and economic cycles. Second, we develop new valuation models for paradoxical companies based on hedonic regressions by treating financial and non-financial indicators as the characteristics of a company. Our models not only yield comparable accuracy results with parsimonious RIV specification but also avoid the application issues that usually appear for paradoxical companies when traditional models are utilized.

In general, our parsimonious models yield a rather decent accuracy compared to traditional valuation models when applied to paradoxical companies in the Nordic market. In practice, our models allow even investors who are not equipped with profound business knowledge to make better investment decisions, especially for companies in the early stages. Our findings on variables' significance also provide insight into what investors may truly put the emphasis on when valuing companies. We hope to inspire further study on the paradoxical companies and on the phenomenon of the rise of such companies.

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A List of Variables

Panel A: Pro-forma income statement	
SALE	Revenue
-COGS	-Cost of goods sold
-RDX	-Research and development costs
-OPEX	-Operating expenses(excluding RDX)
EBITDA	Earning before interest, tax, and depreciation and amortisation
-DP	-Depreciation and amortization
EBIT	Earnings before interest and tax
-XINT	-Financial costs
+IDIT	+Financial income
EBTX	Earnings after financial income and expenses
-CGA	-Capital gains
-XI	-Extraordinary items
EBT	Earnings before tax
-TXC	-Current tax expense
-TXDI	-Deferred tax expense
NI	Net income
Panel B: Other variables	
AT	Total assets
ACT	Current assets
CHEE	Cash and cash equivalents
IVST	Short term investment
INVT	Inventory
LT	Total liabilities
LCT	Short term liabilities
DLC	Short term loans
TXP	Tax liabilities
TXDB	Deferred tax liabilities
DLTT	Long term liabilities
BV	Total book value of equity
MV	Total market value of equity
CAPX	Capital expenditures
OPCF	Cash flow from operations
DVT	Total proposed dividend on all shares
DVC	Total proposed dividend on ordinary shares
EMP	Number of full time employees

B Model Variables

Label	Name	Definition
	Panel A: Dependent Variable	
$(\text{MV/BV})_t$	Market value to book value	$\frac{MV_{t+3m}}{BV_t}$
	Panel B: Explanatory variables - Value drivers	
ROE_t	Return on equity	$\frac{NI_t}{BV_{t-1}}$
NProfMarg_t	Net profit margin	$\frac{NI_t}{SALE_t}$
AssTO_t	Asset turnover	$\frac{SALE_t}{AT_{t-1}}$
EqMult_t	Equity multiplier	$\frac{AT_{t-1}}{BV_{t-1}}$
ROA_t	Return on total assets	$\frac{EBT_t}{AT_{t-1}}$
YielMar_t	Yield margin	$ROA_t - COD_t$
$(\text{D/E})_{t-1}$	Debt to equity	$\frac{LT_{t-1}}{BV_{t-1}}$
	Panel C: Explanatory variables - Statistical methods	
Prof_t	Profitability	$\frac{EBIT_t}{SALE_t}$
dProf_t	Change in profitability	$\Delta\left(\frac{EBITDA}{AT}\right)_t$
CapInt_t	Capital intensity	$\frac{INVT_t}{SALE_t}$
dCapInt_t	Change in capital intensity	$\Delta\left(\frac{ACT}{SALE}\right)_t$
CaF_t	Cash flow	$\Delta\left(\frac{OPCF}{LCT+DLTT}\right)_t$
Liq_t	Liquidity	$\frac{ACT_t-INVT_t}{SALE_t}$
dLiq_t	Change in liquidity	$\Delta\left(\frac{CHEE+IVST}{LCT}\right)_t$
dWC_t	Change in working capital	$\Delta\left(\frac{ACT}{LCT}\right)_t$
S_t	Size	$\ln(AT_t - LCT_t)$
Gr_t	Growth	$\frac{\Delta DP_t}{DP_{t-1}}$
AssStr_t	Asset structure	$\frac{INVT_t}{AT_t}$
gTanAss_t	Growth in tangible assets	$\Delta\left(\frac{INVT}{AT}\right)_t$
FinStr_t	Financial structure	$\frac{DLC_t+DLTT_t}{AT_t}$
gIntLiab_t	Growth in interest-bearing liabilities	$\frac{\Delta(DLC+DLTT)_t}{(DLC+DLTT)_{t-1}}$
TaxCost_t	Tax costs	$\frac{TXC_t+TXDI_t}{EBT_t}$
IntCost_t	Interest costs	$\frac{XINT_t}{LCT_t+DLTT_t}$
Inv_t	Investments	$\Delta\left(\frac{CAPX}{AT}\right)_t$
PayOut_t	Pay out ratio	$\frac{DVC_t}{EBT_t+TXC_t-TXDI_t}$
gDiv_t	Dividend growth	$\Delta\left(\frac{DVC}{BV-DVT}\right)_t$
Div/CaF_t	Dividend/cash flow	$\frac{DVT_t}{OPCF_t}$
NI/CaF_t	Net income/cash flow	$\frac{NI_t}{OPCF_t}$
	Panel D: Explanatory variables - Additional variables	
Empl_t	Employee productivity	$\ln\left(\frac{SALE_t}{EMP_t}\right)$
Age_t	Age	ln(Years since the company was founded)
$\text{R\&D}_t$	Research and development expenditure	$\frac{RDX_t}{SALE_t}$

C Model Specification