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Up and Down the Quality Ladder: A Macroeconomic Model of Innovation and Growth under Demand-Side Inequality

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Abstract

I study an economic growth model with non-homothetic preferences and demand-side inequality that induces innovation along three dimensions: Firms undertake product innovation to enter the market, they improve upon the production process of existing products, or they upgrade their quality. Departing from models of homothetic preferences, process and quality innovation are no longer isomorphic choices for firms, and the demand side dictates incentives to innovate. If economic growth is mainly driven by quality or process innovation, a lower income gap between rich and poor consumers encourages growth while a higher income concentration has an ambiguous effect. If the economy relies on product innovation, a higher income gap has positive effects. Finally, the model accounts for a set of empirical regularities: (i) many products are initially only affordable to affluent consumers, (ii) product cycles are characterized by decreasing prices, (iii) the variety of products is expanding in a firm's lifetime, and (iv) the average quality of products increases over time.

Keywords: Endogenous growth; Inequality; Innovation; Technical change; Non-Gorman preferences; Structural change;

JEL: D30; D40; L16; O30; O41;

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The luxury of today is the necessity of tomorrow. Every advance first comes into being as the luxury of a few rich people, only to become, after time, the indispensable necessity taken for granted by everyone. Luxury consumption provides industry with the stimulus to discover and introduce new things. (...) To it we owe the progressive innovations by which the standard of living of all strata of the population has been gradually raised.

Liberalism, von Mises (1928, p.32)

1 Introduction

Many innovations follow a similar pattern: A new good is introduced as a luxury to a narrow market of affluent consumers. After some time, it becomes affordable to a wide range of consumers. This happens either through firms selling older vintages of the same product at a reduced price, or through dedicated mass production. Regardless of how they emerge, these broadly affordable versions are typically of lower quality than the luxury product sold at the top end of the market. An example are consumer durables. Cars, televisions, and mobile phones were all initially luxury consumption goods, but nowadays they have become necessities for many consumers. For instance, as of writing, there exist five different versions of Apple's latest iPhone, at least six core-versions of the VW Golf, and more than twenty different variations of the same television produced by Samsung Electronics.[1] Empirically, innovation is not always purely vertical or horizontal but a cross-over of product, process and quality innovation. Rationalizing these observations, this thesis develops a model of endogenous long-run growth through three distinct types of innovation: Firms introduce new goods via product innovation to a narrow market of rich consumers (horizontal innovation). After some time, firms can conduct process innovations that cut quality and cost of a given product making it affordable to a wider range of consumers (process innovation). Alternatively, they develop high-quality alternatives to upgrade existing products and sell older vintages at a discount (quality innovation). The driving force behind such innovation patterns is demand-side inequality.

Besides these empirical observations, there is an additional theoretical motivation for this thesis. The literature on innovation and endogenous economic growth has struggled to formalize process and quality innovation in a joint framework. For instance, in the standard Schumpeterian model of growth, the two types of innovation are isomorphic choices for firms up to prices. That is, whether a firm increases the quality-to-cost ratio of existing products through a change in marginal cost (process innovation) or through a change in product quality (quality innovation) does not matter for profits. I break this isomorphism through the introduction of consumer heterogeneity which

[1] See the respective model catalogues on manufacturers' websites. For more evidence see Varian et al. (2004, Ch.5).

induces the two innovation types to differ non-trivially in their impact on resource efficiency. Naturally, this leads to the question of how inequality affects innovation and in particular the choice between process and quality innovation.

Mechanisms, findings and contribution

I develop a two-sector model of innovation and growth. It combines (Schumpeterian) vertical innovation, horizontally expanding variety and quality-and-cost-decreasing (process) innovation. There are rich and poor consumers who have identical, intra-temporally non-Gorman preferences over an aggregated consumption good from the two production sectors. Within each sector there is an expanding measure of differentiated and indivisible products. Consumers decide for every product whether or not to consume. If they choose to consume, they need to select the associated product quality level. In this setting, income inequality affects both the incentive to develop new products and the incentive to introduce new vintages of exiting products. Firms pay a fixed cost to set up operation in either of the two sectors. This is a product innovation. From then on sectors differ in terms of the prescribed innovation pattern. While one sector is associated with quality innovation, the other is associated with process innovation. That is, after entry in either of the two sectors firms produce an initially expensive luxury product. In the quality innovation sector, they can upgrade the quality of that product. In the process innovation sector, they can introduce a low-cost-low-quality version of it. In either case, a firm attains a second quality vintage of an existing product. This new vintage is used to price-discriminate consumers based on their income. To do so, firms employ the optimal non-linear pricing strategy known from monopolistic screening (Mirrlees 1971). A consumer's type is private information, and the willingness to pay is always higher for rich types. Hence, if firms have only one quality at their disposal and if the share of rich types is sufficiently high, firms set prices such that they price-out poor consumers from luxury consumption. This captures the notion that more affluent households consume both a larger variety and a higher average quality of goods. However, as soon as firms have attained a second quality vintage through innovation, they devise an optimal non-linear contract and sell the higher quality to rich consumers while selling the lower quality to poorer consumers at a discount. Fundamentally, firms innovation activities are intended to improve on their effectiveness in price discrimination. This mechanism is commonly known as the *surplus appropriation effect*. Given the innovation choice, quality innovation predominantly happens at the top end of the product range while process innovation is conducted on mass markets.

In this setting, I investigate the demand-side incentives for firms to choose either innovation regime and I determine the optimal mix of innovation. The answer depends on two crucial issues. First, it is important whether higher inequality is due to a larger income gap or to higher income concentration. Second, the intensity of technological spillovers from the various innovation activities matters. If growth is mainly driven by making goods affordable in mass markets, a smaller income gap between rich and poor households encourages growth while a weaker income concentration has an ambiguous effect. This is due to the interaction of a *market size* and a *price effect*. If growth is mainly driven by introducing new luxuries to the rich, a smaller income gap and a weaker income concentration discourage growth. In general, growth is dictated by the applicability of innovation-specific knowledge in the various research and development (R&D) activities and subsequent production. This introduces a trade-off between cost savings from process innovation and knowledge spillovers from quality upgrading, and there exists an optimal mix of these innovation activities. In equilibrium, the relative prevalence of product, process and quality innovation then depends on consumer preferences for quality, the distribution of consumer incomes, and the size of consumer groups. The model suggests that the sectoral allocation of resources in R&D and production might be driven by demand-side incentives rather than supply-side firm decisions. Moreover, I show that the endogenous innovation pattern in this model matches empirical regularities on product cycles previously unaccounted for by both vertical and horizontal models of endogenous growth. I extend my model to account for continuous quality growth and deterministic product cycles with falling prices. Finally, the model lends itself to the analysis of structural change and Baumol's cost disease.

The contribution of my thesis is two-fold. First, it introduces a setting where process and quality innovation are no longer isomorphic. In most vertical innovation models of homothetic utility and homogenous households, process and quality innovation are isomorphic with respect to firms' decision making. I investigate the conceptual differences between quality and process innovation in a setting where the two innovation types differ in their impact on resource efficiency. To be precise, conducting a quality innovation implies a higher efficiency gain in production than conducting a process innovation. This is a novelty that addresses a theoretical weakness found in Schumpeterian growth models. Second, beyond the separation of process and quality innovation, this thesis is the first to model product, process and quality innovation in a joint framework of endogenous growth. In a novel approach, I combine (Schumpeterian) vertical innovation, horizontally expanding variety and quality-and-cost-decreasing (process) innovation in a two-sector model.

Literature

The literature on endogenous growth via R&D can be divided broadly along the lines of horizontal and vertical innovations. Starting with Romer (1990), horizontal innovation models explain the growth process along an ever expanding set of product varieties. Conversely, vertical innovation models focus on quality ladders, that is, the repeated improvement on a closed set of products (Segerstrom, Anant & Dinopoulos 1990, Grossman & Helpman 1991, Aghion & Howitt 1992). A third branch of the literature combines vertical and horizontal innovation in order to eliminate the strong scale effect of first-generation endogenous growth, for instance Segerstrom (1998), Howitt (1999) or Peretto (2018). Moreover, a series of recent papers analyzes supply-side rationales for the joint occurrence of horizontal and vertical innovation - typically in connection with heterogenous firm size. Acemoğlu & Cao (2015), for example, study a growth model where entrants add new products while incumbents conduct vertical innovation because of scale effects to productivity. Akcigit & Kerr (2018) analyze a model where incumbents can conduct internal (vertical) innovation on existing products or diversify business lines through external (horizontal) innovations. Additionally, there is entry of new firms. Firms are heterogenous and innovation prowess decreases in firm size. In contrast, Peretto & Connolly (2007) combine a horizontal innovation model with fixed operating costs where vertical innovation implies higher production efficiency. In this model, fixed costs render horizontal innovation to be constrained by the economy's resources such that long-run growth through stand-alone product innovation is infeasible. However, firms' returns to innovation scale in their operations such that vertical innovation spills over to make horizontal innovation sustainable. Crucially, the authors define vertical innovation as an improvement in efficiency. Yet, this can entail both an improvement in quality or the production process.[2] As in most vertical innovation models of homothetic utility and homogenous households, process and quality innovation are isomorphic. In fact, Acemoğlu (2009) demonstrates that they are not only qualitatively equivalent but mathematically very similar. That is, for the incentives to innovate, it does not matter whether a firm improves the quality of an existing product holding marginal cost constant or decreases marginal cost while holding quality constant. In contrast, under non-homothetic preferences, the distinction between product and process innovations becomes non-trivial. Foellmi, Wuergler & Zweimueller (2014) introduce a model of product and process innovation. They model process innovation as the systematic decrease in quality of existing products and in their associated cost in order to produce at a higher quality-to-cost-ratio. Their mechanism departs from homothetic preferences and household equality. Process and product innovations have different implications for consumption choice;

[2] An earlier version of what I refer to as *product transformation* can be found in Peretto (1999).

such that the innovation mix itself is, in a non-trivial way, affected by the distribution of income. Allowing for non-homothicities therefore introduces a third dimension of innovation. My thesis generalizes Foellmi et al. (2014) by allowing for the co-existence of process, product and quality innovations through a two-sector setup. In particular, I use Foellmi et al. (2014) specification of a process innovation and nest it in a two-sector endogenous growth model that additionally features quality innovation. To the best of my knowledge, this thesis is the first to jointly allow for product, process and quality innovation. The main mechanism of innovation in my model is firms' desire to improve on price discrimination by introducing new quality versions of existing products. While this mechanism can be found in Foellmi et al. (2014) for process innovation and in Latzer (2018) for quality innovation, in this thesis it drives both process and quality innovation. Therefore one can understand this thesis as a combination of these two aforementioned papers in a two-sector setup. Finally, although Foellmi et al. (2014) also capture that products start out as luxuries before becoming necessities over time, their model can only explain this pattern for process innovation but not for quality innovation. In contrast, this thesis can explain such product cycles under both process and quality innovation. Besides the above literature on multi-dimensional economic growth through R&D, my analysis relates to the literature along at least three strands.

First, it investigates demand-side incentives for technical progress via the composition of aggregate consumer demand. In particular, it belongs to the class of contributions generating consumption difference via non-Gorman preferences. Foellmi & Zweimueller (2006) analyze the impact of inequality on growth for a model with non-homothetic, hierarchic preferences. They identify a price effect that allows firms to charge higher prices if inequality is high, and a market size effect where higher inequality renders smaller markets. In this expanding variety model, an increase in inequality unambiguously elevates growth rates such that a redistribution from the poor to the rich may be Pareto improving for low levels of inequality. Zweimueller (2000) and Zweimueller & Brunner (2005) study a model of quality innovation allowing for the coexistence of multiple vintages of a given product in an oligopolistic market. In a two-class society, the quality leader sells to rich consumers while the challenger supplies the second highest quality to poorer consumers. In general, they find that a more equal distribution of income incentivizes innovation but the nature of redistribution matters. Matsuyama (2002) provides a first model of mass production with non-Gorman preferences where sectoral productivity improvements trigger the successive transformation of luxuries into necessities. In Matsuyama's horizontal innovation model, the economy remains stuck in a poverty trap if inequality is too low while a too unequal society halts growth prematurely. Finally, Foellmi & Zweimueller (2017) study market size and price effects for a setting where product innovation entails a process innovation. The price effect dominates and inequality increases growth if innovators have a large productivity edge, and hence high mark-ups. Latzer (2018) introduces a Schumpeterian model with multi-product firms under non-Gorman preferences. Similar to Foellmi et al. (2014), there is positive investment into R&D due to a *surplus appropriation effect* - that is, more effective price discrimination due to a vertically expanding product set - offsetting the *replacement effect* usually found in models of vertical innovation.[3]

Second, the thesis loosely connects to the study of biased technical change, a literature that analyzes the forces skewing knowledge accumulation towards a particular production factor (e.g. Acemoğlu 1998, 2002, 2009, Ch.15). Similar to my model, this literature emphasizes the counteracting forces of market size and price effects for the allocation of resources. However, while directed technical change determines the allocation of resources through the relative demand for production factors, this thesis works along a the relative demand for consumption goods. Factor allocation and R&D-spending are therefore determined by demand-side rather than supply-side incentives.

Third, the thesis relates to the literature on demand-side rationals of structural change. Foellmi & Zweimueller (2008) study a model where (asymptotically non-homothetic) hierarchic preferences and non-linear Engel-curves generate endogenous consumption cycles. That is, a product's income elasticity is initially high and gradually decreases with growing income. Hence, goods start out as luxury needs but gradually become necessities. The model satisfies the Kaldor facts on the aggregate while allowing for continuous sectoral reallocation and changing sectoral composition. In contrast to Foellmi & Zweimueller (2008) who focus on income effects, Boppart (2014) discusses structural change for the case of two composite goods explicitly allowing for relative price effects through Muellbauer price-independent generalized linear preferences (PIGL). Expanding on this, Comin, Lashkari & Mestieri (2018) introduce a variation of non-homothetic CES preferences allowing for heterogeneity in income elasticity among the goods within each nested composite. This heterogeneity amplifies the role that demand-side non-homotheticity plays for structural change.

The remainder of the thesis is organized as follows: Section 2 presents historical evidence motivating five empirical regularities of innovation and growth. Section 3 provides a primer on utility theory in macroeconomic models. Section 4 outlines the formal framework and derives the balanced growth equilibrium. Section 5 sketches three possible extensions to the core model, and section 6 discusses mechanism and findings. Section 7 concludes.

[3]There is some misunderstanding surrounding the notion of the *replacement effect* versus the *business stealing effect*. See for example Acemoğlu (2009).

2 Motivating evidence

I present motivating evidence on consumption and innovation. First, I describe three empirical regularities on macroeconomic innovation patterns. In particular, I present evidence that innovation is both vertical and horizontal. Second, I argue that these patterns can be related to Engel's law suggesting that more affluent consumers spend a larger share of their income on luxury consumption goods. I support this theory with empirical and anecdotal evidence.

Empirical regularities. *Empirically, the following regularities can be observed:*

(i) The number of products supplied by a firm is increasing in its lifetime, and the number of firms increases over time.

(ii) For the majority of products, quality increases over time.

(iii) The price for a product of a given quality is decreasing over time.

Moreover, I argue that these regularities can be related to the following casual observations on luxury consumption and innovation:

(iv) Many new products are initially affordable only to affluent consumers. After some time, cheaper, low-quality-low-price versions are introduced which are sold to the broader public.

(v) Many new products are initially affordable only to affluent consumers but as their quality improves, the initial versions are sold at a discount to the broader public.

Empirical regularity one

There is an expanding variety of products both on the aggregate and within firms over time. To put it differently, the number of products supplied by a firm is increasing in its lifetime, and the number of firms increases over time. A common starting point used in many contributions that document this empirical regularity is a figure similar to figure 1. It shows the logarithm of the total number of patents registered in the United States since 1790. The growth rate in patent registration has been remarkably constant over the last two hundred years. Now obviously not every new patent refers to a new product and not every new product is patented, but it illustrates that there has been considerable innovation activity over the last two hundred years. Many of these innovations were in fact new consumer goods or intermediate inputs. Let me focus on the variety of goods consumed. In a seminal paper, Bils & Klenow (2001*a*) exploit consumer spending patterns to measure growth in product variety between 1960 and 2000 in the United States. They document that the range of

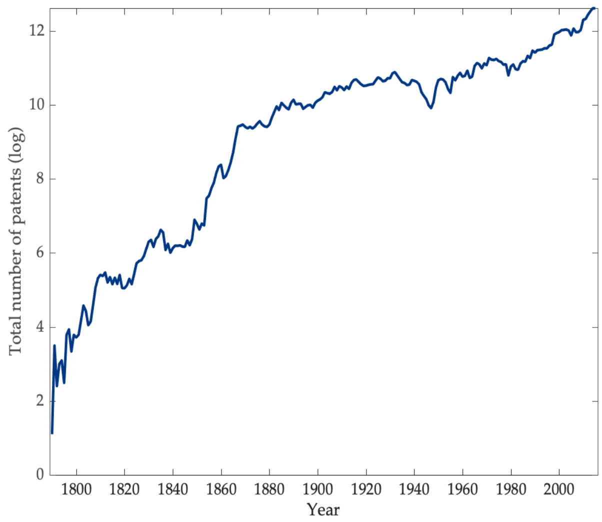

Figure 1: Log number of patents in the United States, 1790-2015.
Source: Author's rendering of U.S. Patent and Trademark Office data (2019).

consumer goods has been growing by approximately 1% year-on-year. Moreover, their evidence suggests that variety growth has accelerated in the second part of the sample with growth rates up to 2.2%. Besides the *aggregate* growth in product variety, there is compelling evidence that also the variety *by firm* expands in its lifetime. Argente, Lee & Moreira (2019) use firm-level barcode data to document the number of products supplied by a firm over time for a sample of U.S. consumer retail products. I reproduce their findings in figure 2. The horizontal axis denotes an average firm's age in quarters, and the vertical axis denotes the log-number of products supplied. The grey area are 95%-confidence bands. One can see that in particular in the early stages of operation, the av-

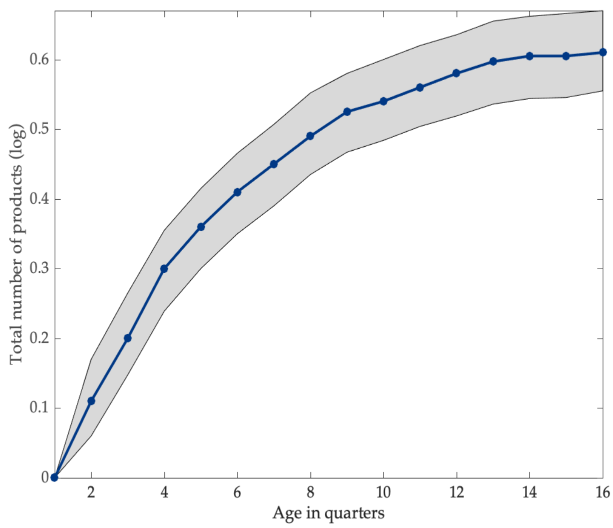

Figure 2: Total number of products over firm life cycle.
Source: Author's replication of Argente, Lee & Moreira (2019, p.49, Fig.14).

erage number of products supplied is expanding rapidly by about 5% to 10% quarter-on-quarter. As firms get older (or more established), the growth rate decreases. Now if one thinks of the *kind* of new products that are introduced, there is strong evidence that firms offer a wide range of versions and qualities within given product lines (their core business). Akcigit & Kerr (2018) indicate that the overwhelming majority of innovations conducted by incumbent firms are incremental, that is, improving the efficiency in production of existing products. This can entail an improvement in quality or a less resource-intensive process of production. Evidence by Garcia-Macia, Hsieh & Klenow (2019) suggests that firms innovate along multiple dimensions but the majority of innovation are incremental changes to existing product lines within incumbent firms.[4] Garcia-Macia et al. also find that the overwhelming majority of innovations happens within incumbent firms (up to 87% from 2006-2013) and these innovations are typically within their own product line, that is, process or quality innovations (up to 77% during the same time). Loosely speaking, firms stick to their product lines. I interpret this evidence to corroborate the notion that firms offer a large variety of products within a product line such that the set of products is expanding in a firm's lifetime.

Empirical regularities two and three

The last century has seen an astonishing increase in the quality of many products. I argue there are two empirical regularities: (i) For the majority of products quality increases over time; and (ii) the price for a product of a given quality is decreasing over time. Let me start by going back to figure 5. Over the last 100 years, not only has the share of people owning motor vehicles increased, but so has product quality. A bottom-of-the-range car today is more durable, more convenient, more economical, faster and generally more refined than the Ford Model T of 1928; the hardware of even a new bottom-of-the-range computer improves every year; and so on. This anecdotal evidence documents the quality progress of even the most inferior versions of many products. On a sample of durable goods, Bils & Klenow (2001*b*) estimate average quality growth of 3.7% per annum for 1980-1996. Quality increases account for about 40% of observed price increases over time with the residual 60% attributed to inflation.[5] Bils (2009) estimates average yearly quality growth of consumer durables at 2.5%, with higher rates of 3.3% for motor vehicles and 4.4% for consumer electronics.[6] Similarly,

[4]The authors use employment churn and labor reallocation to proxy for creative destruction. The idea is that if innovation is vertical, that is creatively destructive, a new product increases labor employment by the innovator at the expense of its competitors. Conversely, if innovation is expanding variety, new products should leave employment relatively unaffected.

[5]The sample in Bils & Klenow (2001*b*) comprises durables accounting for 80% of consumer durable spending in the U.S. and 12% of consumer price index goods (CPI). Of course, there are some products for which there has been no quality improvement or cost saving. I will touch upon this in section 5 where I discuss an application to the cost disease first described by Baumol (1967).

[6]With even higher quality growth rates for quality consumed if one accounts for model changing by consumers. This

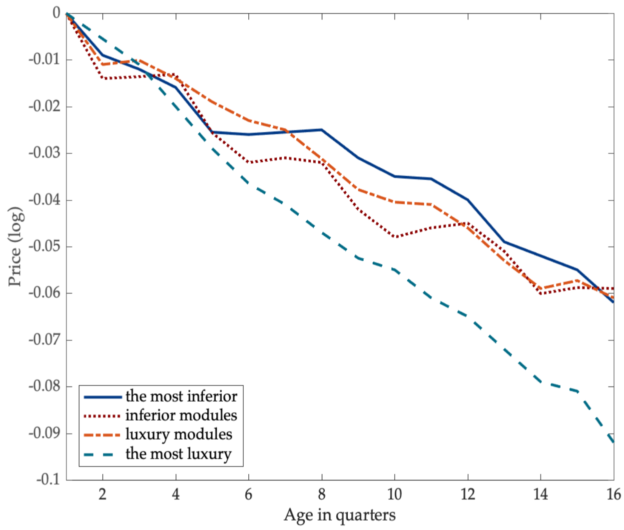

Figure 3: Price of product over the lifecycle by category's target income groups.
Source: Author's replication of Argente, Lee & Moreira (2019, p.47, Fig.12).

this implies that a substantial part of observed price increases is due to quality improvements with quality-adjusted inflation to be adjusted by two percentage points. Finally, this begs the question what happens to products that do not improve their quality. Figure 3 is adapted from Argente, Lee & Moreira (2019). It shows the log-price change of an average product in its first sixteen quarters of marketing (the average lifecycle of a new product) while holding quality constant. I display the price change for the most luxurious and the most inferior product of a product line, as well as two intermediate versions. For all qualities, the price is falling in its lifetime. The price decrease, however, is most pronounced for luxury goods where prices fall at an almost constant rate. Conversely, the decrease is weaker at the lower end of the quality range.

Observations on innovation and luxury consumption

In this thesis I argue that the majority of innovation follows one of two patterns: (i) Many new products are initially affordable only to affluent consumers. After some time, cheaper, low-quality-low-price versions are introduced which are sold to the broader public; (ii) Many new products are initially affordable only to affluent consumers but as their quality improves, the initial versions are sold for a lower price to the broader public. To rationalize these observations, let me first discuss the importance of luxury consumption. It is well documented that the expenditure share of luxury products rises as households become more affluent. This regularity is widely known as Engel's law.[7] For example, figure 4 plots the average annual expenditure of the top and bottom 20% of U.S.

may also include forced quality changes such as old models become obsolete.

[7] As Foellmi & Zweimueller (2008) elaborate, Engel originally observed that the share of food in a consumer's expen-

households' income distribution from 1984 to 2017. Goods are classified in accordance with the Bureau of Labor Statistic's categorization of luxury and necessity consumption. While an average

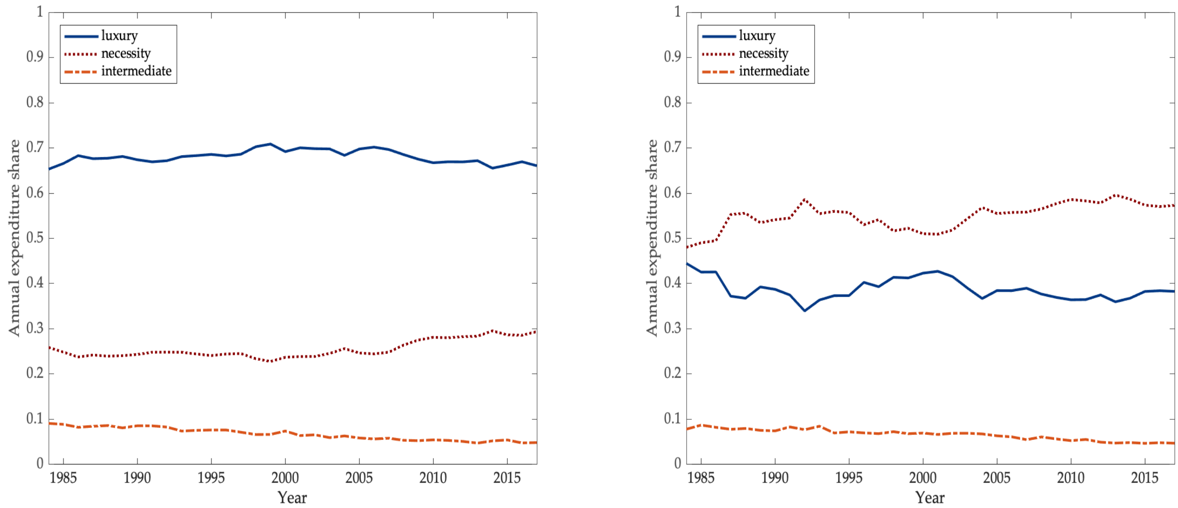

Figure 4: Luxury and necessity expenditure in the United States, 1984-2017, for top 20% quintile (left) and bottom 20% quintile (right).
Expenditure classification according to Bureau of Labor Statistics.
Source: Author's rendering of Bureau of Labor Statistics Consumer Expenditure Survey (2019).

household in the bottom quintile spends roughly 40% of its annual income on luxury consumption, for a household in the highest quintile it is 65%. These shares have been remarkably stable over time suggesting that this pattern is in fact driven by consumer preferences. Similar evidence is provided by Aguiar & Bils (2015) who estimate income elasticities and associated Engel curves for various categories of consumer spending. They find income elasticities ranging from 0.4 for some necessities (home food consumption, utilities) to 1.7 for some luxury goods (entertainment spending, education). Hence, the empirical evidence corroborates non-linear Engel curves.[8] I argue in this thesis that many innovations are driven by the higher willingness to pay of rich households for new luxury consumption goods. A good example are consumer durables. Figure 5 shows the per-capita ownership of various consumer durables in the United States in the last 100 years. At introduction, most types of consumer durables are only consumed by a fraction of the households. The figure also shows that levels of penetration rise over time. Many new products are initially only affordable to a small subset of affluent consumers but after some time become more affordable to a broad range of consumers. Penetration rates for automobiles in the early 20^{th}-century, for instance, had been as low as 1% but with the introduction of the Ford Model T in 1908 per capita ownership reached almost 20% within fifteen years. By 1924, 50% of households and 22% of Americans had

diture basket decreases in income. The definition by Houthakker (1987) encompasses the decreasing share of necessities.

[8] And hence the non-homotheticity of demand. I discuss the implications in the subsequent section.

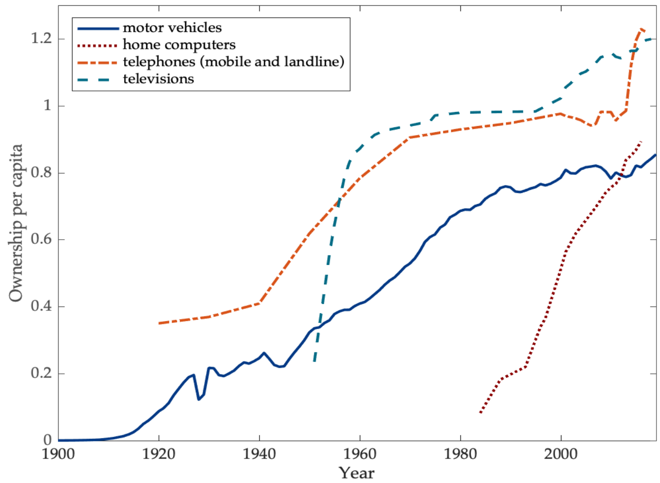

Figure 5: Consumer durable ownership in the United States, 1900-2019.
Source: Author's rendering of Bureau of Labor Statistics Consumer Expenditure Survey (2019).

access to a car (Bowden & Offer 1994). While the automobile might be the prime example, the same pattern can be observed for many consumption goods, durables and services. As the technological frontier advances and incomes grow, former luxuries become broadly affordable. Matsuyama (2002) refers to this as the *Flying Geese pattern*. Besides plain income effects of rising living standards, key elements of such product cycles are cost-saving process innovations or quality upgrades - both inherently improving the quality-to-cost ratio. After a product has been invented, initial manufacturing costs are relatively high, and sales volumes low, as the good can only be afforded by a few rich households. A process innovation spurs the potential for mass production. Similarly, quality upgrades often entail learning by doing such that the initial versions can now be produced more efficiently and sold at lower prices while the new (high quality) upgraded product remains exclusive and expensive. Reconsider figure 5. As established, (durable) goods such as cars, TVs and mobile phones were initially after their introduction only affordable to a narrow segment of affluent consumers but over the course of the product lifecycle, have been rolled out on mass markets. To do so, firms can improve on the existing quality and sell older vintages of the same product at a cheaper price such that they become affordable to less affluent consumers. Alternatively, many firms introduce low-cost version of their bestsellers. Such versioning is particularly common for consumer electronics but also occurs for many other durable and non-durable consumption goods, and even many services. Varian, Farrell & Shapiro (2004, Ch.5) provide an abundance of anecdotal evidence on versioning - manly for information goods.

3 Preferences and the theory of economic growth

The model in this thesis relies on the following mechanism: Firms introduce new vintages of existing products in order to price-discriminate between consumers based on their wealth. More affluent consumers are willing to pay higher prices, so it might be profit maximizing to price poorer consumers out of the market. This implies that consumption across consumers can be heterogenous at both the intensive and extensive margin. From a theory point of view this poses a challenge. For this mechanism to work, preferences need to be such that consumers purchase differentiated consumption baskets depending on their wealth. The majority of growth models, however, relies on preferences where rich and poor consumers differ only in the intensive margin (and possibly the quality) of consumption but not the variety consumed. For example, the (quasi-homothetic) Gorman class of utility functions requires that marginal utility of wealth be a function of the price vector but not of wealth itself. Allowing for distinct consumption bundles implies that marginal utility is no longer independent of wealth levels. In fact, the majority of contributions in the growth literature restricts preferences not only to be Gorman but additionally to lie in the set of inter-temporally additively-separable and homothetic preferences, that is, to be represented by the constant relative risk aversion (CRRA) utility function. In order to understand what drives the results in my model, I want to further illustrate the common restrictions on preferences. Consider definition 1.

Definition 1 (Homothetic preferences)**.** *A preference relation $\succcurlyeq$ defined on a cone $K \subset \mathbb{R}^n$ is (weakly) homothetic if $\forall \boldsymbol{x}, \boldsymbol{y} \in K$, $\forall \lambda \in \mathbb{R}_{++}$: $\boldsymbol{x} \succcurlyeq (\sim) \boldsymbol{y} \Rightarrow \lambda \boldsymbol{x} \succcurlyeq (\sim) \lambda \boldsymbol{y}$.*

Weakly homothetic and continuous preferences are homothetic. Additionally, if they are continuous and increasing, one can represent them by a utility function $u : \mathbb{R}^n_+ \to \mathbb{R}$ that is homogenous of degree one (Dow & Ribeiro 1992). A popular way to illustrate this relationship is to say that the slope of indifference curves remains constant along rays through the origin, or that the marginal rate of substitution is invariant to rescaling by a positive scalar. Figure 6 illustrates. Following from that, quasi-homothetic utility functions are simply affine transformations on a homothetic utility function. This is the Gorman (polar) class. For illustration, assume a continuum of consumers $i \in I = [0, 1]$ with quasi-homothetic preferences and wealth $\mathcal{I}^i$. The budget set is $\{\boldsymbol{x} : \langle \boldsymbol{p}, \boldsymbol{x} \rangle \leqslant \mathcal{I}^i\}$ and the indirect utility function of consumer i is

$$v^i(\boldsymbol{p}, \mathcal{I}^i) = a^i(\boldsymbol{p}) + b(\boldsymbol{p})\mathcal{I}^i, \tag{3.1}$$

where $a^i(\cdot)$ and $b(\cdot)$ are functions of the price vector $\boldsymbol{p}$. It is easy to see that (3.1) is in fact homogenous of degree one in wealth up to the constant $a^i(\boldsymbol{p})$.[9] Via Roy's identity the (Marshallian) uncompensated demand function of consumer i for good j is given by $x_j^i(\boldsymbol{p}, \mathfrak{I}^i) = -\frac{\partial v^i(\cdot)/\partial p_j}{\partial v^i(\cdot)/\partial \mathfrak{I}^i}$ and the marginal utility of wealth is

$$\frac{\partial x_j^i(\boldsymbol{p}, \mathfrak{I}^i)}{\partial \mathfrak{I}^i} = -\frac{\frac{\partial b(\boldsymbol{p})}{\partial p_j}}{b(\boldsymbol{p})} \equiv -\frac{b_j(\boldsymbol{p})}{b(\boldsymbol{p})}, \tag{3.2}$$

implying that the wealth expansion path for two goods j and k is affine with slope $\frac{b_k(\boldsymbol{p})}{b_j(\boldsymbol{p})}$, and that marginal utility of wealth is independent of one's wealth level. Moreover, the marginal utility $\frac{b_j(\boldsymbol{p})}{b(\boldsymbol{p})}$ is the same for all consumers which means that their Engel curves are parallel and can be aggregated to represent aggregate demand as a function of aggregate wealth (Mas-Colell et al. 1995, 4.B.1). Consider the aggregate demand for good j,

$$D_j(\boldsymbol{p}, \mathfrak{I}) = \int\limits_{i \in I} x_j^i(\boldsymbol{p}, \mathfrak{I}^i) di = -\int\limits_{i \in I} \frac{\partial v^i(\boldsymbol{p})/\partial p_j}{\partial v^i(\boldsymbol{p})/\partial \mathfrak{I}^i} di = -\frac{1}{b(\boldsymbol{p})} \int\limits_{i \in I} a_j^i(\boldsymbol{p}) di - \frac{b_j(\boldsymbol{p})}{b(\boldsymbol{p})} \int\limits_{i \in I} \mathfrak{I}^i di.$$

This result is often used in macroeconomics because via Gorman's aggregation theorem it suffices for the existence of a (normative) representative consumer. Heuristically speaking, because income expansion paths are parallels, there does not exist a redistribution of wealth that can change aggregate demand. One may well treat the demand of a continuum of household as if it was a representative consumer's demand. Note that Gorman forms are sufficient for the existence of a representative consumer but there exist various other classes, for example the widely used price-independent generalized linearity (PIGL), that allow for aggregation (Boppart 2014, p.2174). For

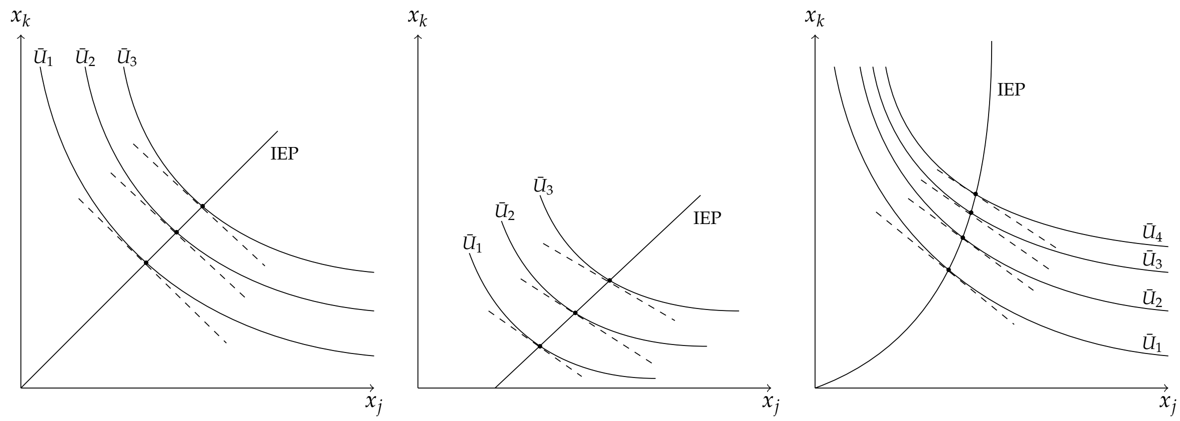

Figure 6: Income expansion paths
Homothetic preferences (left), Gorman preferences (center), and non-homothetic preferences (right)

Gorman forms, (3.2) implies that the only difference in consumption between rich and poor is the

[9]By Euler's theorem the derivative of a homogenous function of m is homogenous of $m-1$ (Acemoğlu 2009, p.29).

intensive margin of consumption (a poor consumer's budget set is just a subset). Any impact of income distribution on the various products' demand curves is assumed away. The standard way to break this relationship is to introduce non-homothetic preferences. In the next section, I specify utility over binary consumption where only the extensive margin of consumption can be chosen. Before that, there is a second aspect of preferences to be discussed. Until now I have focused on the intra-temporal aspect of consumption but in many dynamic models, preferences are typically restricted to be of constant relative risk aversion (CRRA). Proposition 1 illustrates.

Proposition 1. *The CRRA utility function is the most general in the class of utility functions satisfying (i) homotheticity, (ii) additively-separable utility, and (iii) balanced growth.*

Proof. Acemoğlu (2009, pp.308-9,323,E8.27). □

In the same way that the Gorman property describes an intra-temporal allocation, one can think of it in an inter-temporal context. A common restriction placed on utility is that it be additively-separable across time implying a linear path of consumption growth - which is the same as to say that the income expansion path be linear. The most general class of utility functions that admits additively-separable and Gorman preferences is of hyperbolic-absolute risk aversion, that is, the Arrow-Pratt measure is hyperbolic (HARA or Pollak preferences, Mrázová & Neary 2017, App.1, p.21). The HARA class can be characterized by its marginal utility

$$u'(x) = \left(\frac{\Xi x}{\sigma} - \bar{x}\right)^{-\sigma}, \tag{3.3}$$

where $\Xi > 0, \sigma, \bar{x}$ are preference parameters. With HARA preferences, there is no inter-temporal impact of income distribution on aggregate demand. The individual's Euler-Lagrange equation also characterizes optimal aggregate behavior. This discussion illustrates that HARA is a subset of the Gorman class. In the same way that there are other classes besides Gorman that allow for aggregation, there are other classes besides HARA that allow for inter-temporal aggregation. The most general can be found in Alder, Boppart & Müller (2019, p.22-3). Finally, many macroeconomic models are concerned with balanced growth. For a model to admit this feature, there needs to be inter-temporal homotheticity. In the same way that homothetic utility requires that the marginal rate of substitution between two goods be constant within a period, HARA is only strictly homothetic across time if the intertemporal elasticity of substitution is constant. This is the case if one restricts (3.3) with $\bar{x} = 0$ and $\sigma \in \mathbb{R}_{++}$, yielding CRRA.

4 Model

4.1 Setup

Consumers, endowment, and distribution

Consider an economy with a measure $L \in \mathbb{R}_{++}$ of ex-ante heterogenous consumers. A share $\beta \in (0,1)$ is poor (P) and the remainder $1-\beta$ is rich (R). Rich and poor types $i \in \{P,R\}$ have identical preferences but unequal labor endowments. While a P-type's endowment is low and given by $\theta \in (0,1)$, a R-type has a high endowment of $\frac{1-\beta\theta}{1-\beta} > \theta$.[10] Labor supply is inelastic such that every consumer supplies their respective endowment taking the wage $w(t)$ as given. Therefore, incomes from work grow with the wage rate, aggregate labor income for all P-types is $\beta\theta w(t)L$ and aggregate labor income of R-types is $(1-\beta\theta)w(t)L$. Additional to earning labor incomes, consumers

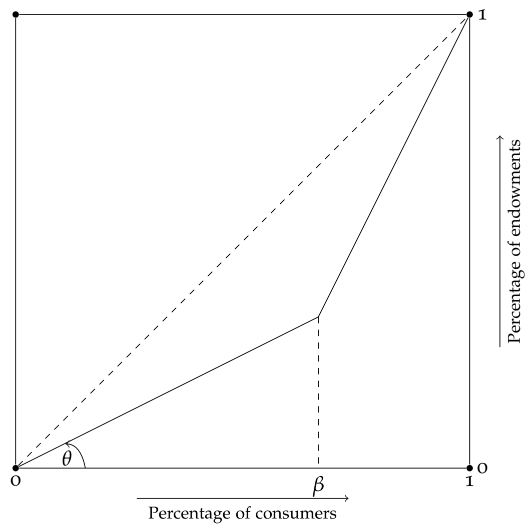

Figure 7: Lorenz curve.

have a shareholding in firms. Let the total value of firms be $v(t)$. Different to labor endowments, consumers can change their shareholding through saving. Assume that at time zero a P-type holds a fraction $\xi(0)$ in firms that amounts to the total value $v_P(t) = \xi(t)v(t)$, while for an R-type holdings are $v_R(t) = \frac{1-\beta\xi(t)}{1-\beta}v(t)$. As time passes by, the distribution of firm shares therefore changes. Following Foellmi et al. (2014), I make two simplifying assumptions. First, in this thesis I restrict myself to the analysis of a balanced growth path (henceforth BGP). As discussed in section 3, savings rates are independent of type on such a BGP if preferences feature a constant inter-temporal elasticity of substitution, that is, if they are homothetic across time. In fact, below I assume additive and logarithmic inter-temporal preferences generating equal optimal savings rates for all consumers. Therefore, the

[10]Notice that $\beta\theta + (1-\beta)\frac{1-\beta\theta}{1-\beta} = 1$.

wealth distribution is stationary at $\xi(t) = \xi$. Second, I follow Foellmi et al. (2014, p.625) who argue that it is without loss of generality to assume that on a BGP the wealth distribution is identical to (initial) labor endowments. Formally, this means that $\xi = \theta$. While this is clearly a rather special case, it keeps the analysis simple. This assumption, together with an absence of income shocks, ensures that the initial distribution persists over time. From these two assumptions it follows that the tuple $(\beta, \theta) \in (0,1)^2$ fully characterizes the economy's distribution on such an equilibrium path. Figure 7 illustrates via a Lorenz curve. Notice that inequality increases in β and decreases in θ. I interpret changes in θ as changes in the *income gap* and changes in β as changes in the *income concentration*.

Preferences

Consumer preferences are independent of type and inter-temporally additively-separable with respect to an aggregated consumption good $X_i(t)$. For consistency with balanced growth, I assume CRRA and to make the exposition as clear as possible I set the inverse inter-temporal elasticity of substitution to one. Consumer i's total discounted utility is

$$U_i(0) = \int_0^\infty \ln X_i(t)\, e^{-\rho t}\, dt, \tag{4.1}$$

with subjective discount rate $\rho \in \mathbb{R}_{++}$. I introduce non-homotheticity via the aggregation of the consumption good. For tractability, I rely on binary preferences as suggested in Gabszewicz & Thisse (1979) and Shaked & Sutton (1982). The aggregated consumption good $X_i(t)$ is defined over two sectors $s \in S = \{e, m\}$ each containing a measure $N^s(t)$ of goods, respectively. Let $x_i(t, j^s)$ denote i's consumption of good $j^s \in N^s(t)$. It is indivisible, so consumers choose the extensive margin of consumption only, that is, $x_i(t, j^s) \in \{0, 1\}$. As soon as one unit has been consumed, there is satiation for that good and an additional unit yields no marginal utility. Note that although binary preferences are not part of the HARA class in (3.3), they can be derived as a limiting case with $\bar{x} = -1$, $\Xi = -\sigma$ and $\sigma \to \infty$. In addition to this take-it-or-leave-it consumption choice, a consumer selects the associated discrete quality level at which to consume a good. There are at most two qualities available, high (h) and low (l), so $k \in \{h, l\}$. I denote the quality of j^s consumed by i as $q_i(t, j^s) \in \{q_l^s, q_h^s\}$, where $q_h^s > q_l^s$. The consumption aggregator is

$$X_i(t) = [X_i^e(t)]^\phi\, [X_i^m(t)]^{1-\phi}\,, \tag{4.2}$$

where $\phi \in [0,1]$ is the elasticity with respect to an aggregated sector-consumption good of the form

$$X_i^s(t) = \int_0^{N^s(t)} x_i(t,j^s)q_i(t,j^s)dj^s, \tag{4.3}$$

for $s \in S$. This setup has two implications. First, the (inverse) elasticity of substitution across time is equal to the elasticity of substitution across sectors. From Cobb-Douglas aggregation and log-preferences, this is unity. Second, goods within each sector are treated as perfect substitutes.[11] To complete the setup of the economy's demand side, consider i's lifetime budget constraint

$$\int_0^{\infty}\left[\int_0^{N^e(t)} p(t,j^e,q_i^e)x_i(t,j^e)dj^e + \int_0^{N^m(t)} p(t,j^m,q_i^m)x_i(t,j^m)dj^m\right]e^{-R(t)}dt \leqslant \int_0^{\infty}\mathcal{I}_i(t)e^{-R(t)}dt, \tag{4.4}$$

where the price of j^s depends on the quality it is being consumed in, as well as time. I abbreviate the price by $p(t,j^s,q_i^s) \equiv p[t,j^s,q_i(t,j^s)]$. Moreover, $R(t,\tau) \equiv \int_\tau^t r(s)ds$ denotes the cumulative market discount factor between dates τ and t where $r(s)$ is the real interest rate. Hence, $R(t) \equiv R(t,0)$ is the market discount factor between time zero and t. Finally, $\mathcal{I}_i(t)$ is instantaneous income.

Consumer optimization

Since utility is separable across time, consumer's utility maximization problem consists of two independent parts: (i) Choosing the optimal mix of consumption within an instant of time (intra-temporal problem), and (ii) determining the optimal allocation across time (inter-temporal problem).[12] For the intra-temporal problem, suppose a consumer is deciding on consuming good j^s. A good will be consumed if the marginal utility of consumption exceeds its price. For example, j^s is being consumed in high quality if the marginal utility of that good at quality level h exceeds its price. But it also has to (weakly) dominate all other alternatives: Consuming the good at the low quality level l and paying a lower price, or not consuming the good at all. Let me formalize this optimality condition. Denote by $u_{i,k}^s$ i's surplus of consuming quality $k \in \{h,l\}$ and by $u_{i,\neg k}^s$ the surplus from the other quality. Then, a good is being consumed in k if and only if $u_{i,k}^s \geqslant (u_{i,\neg k}^s)^+ \equiv \max\{0, u_{i,\neg k}^s\}$.

[11] Preempting the model mechanism, choosing only the extensive margin generates differences in the variety consumed by the types. Consumption is such that R-types choose a larger variety of goods while P-types cannot consume the full range of goods. The drivers behind this result are (i) the extensive margin choice and (ii) incomplete information coupled with monopolistic screening, and (iii) the assumption of a sufficiently large income gap between types.

[12] Notice that the binary structure of consumption choice implies that the optimal control is a piecewise continuous function. Hence, it requires some additional attention. Seierstad & Sydsaeter (1987, p.362-3) provide the sufficient and necessary conditions. In appendix A.1, I solve the problem for general HARA preferences and then derive the limiting case of binary preferences. Apart from that, CRRA inter-temporal preferences dictate that the inter-temporal allocation is governed by the standard Euler-Lagrange equation.

The full set of conditions is

$$
\begin{aligned}
&\{x_i(t,j^s), q_i(t,j^s)\} \\
&= \begin{cases}
\{1, q_h^s\} & \text{if } q_h^s \mu_i^s(t) - p(t,j^s,q_h^s) \geqslant \max\{0, q_l^s \mu_i^s(t) - p(t,j^s,q_l^s)\} \\
\{1, q_l^s\} & \text{if } q_l^s \mu_i^s(t) - p(t,j^s,q_l^s) \geqslant \max\{0, q_h^s \mu_i^s(t) - p(t,j^s,q_h^s)\} \\
\{0, \cdot\} & \text{else}
\end{cases} \\
&\text{for } s \in \{m,e\}, \\
&\text{with } \mu_i^e(t) \equiv \frac{\phi}{\lambda_i(t) X_i^e(t)}, \text{ and } \mu_i^m(t) \equiv \frac{1-\phi}{\lambda_i(t) X_i^m(t)},
\end{aligned}
$$

where $\lambda_i(t)$ is the marginal utility of wealth at date t (the current value multiplier on the budget constraint) which is independent of sector. Define a variable $\mu_i^s(t)$ that captures i's willingness to pay per unit of quality for a good in sector s. One might think of it as an adjusted marginal utility of wealth. Figure 8 shows the demand curve for good j^s. Hence, good j^s is being consumed in quality k if i's marginal willingness to pay is sufficiently larger than the associated price $p(t,j^s,q_i^s)$ such that choosing another quality level or not consuming the good at all are (weakly) dominated. Notice that marginal willingness to pay for the rich is always higher than for the poor.

Lemma 1. *For $s \in \{e,m\}$, it holds that $\mu_R^s(t) > \mu_P^s(t)$.*

Proof. Foellmi & Zweimueller (2017, p.364) for the continuous case. □

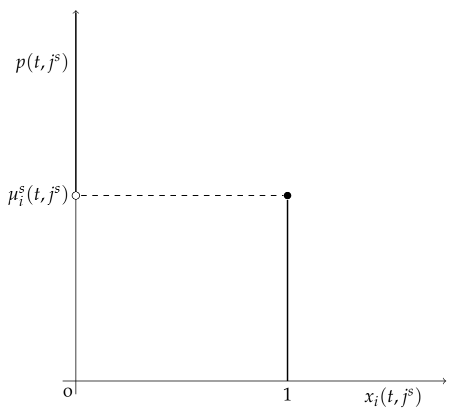

Figure 8: Individual demand.

Finally, consider the inter-temporal aspect of the consumer problem. From equation (4.1) the utility function is additively-separable and homothetic across time guaranteeing that consumption growth follows the Euler-Lagrange equation

$$\frac{\dot{X}_i^s(t)}{X_i^s(t)} + \frac{\dot{\mu}_i^s(t)}{\mu_i^s(t)} = r(t) - \rho. \tag{4.5}$$

which holds for both sectors $s \in S$ and consumer types $i \in \{R, P\}$.[13]

Firms and innovation

The economy contains two sectors $s \in S$ with $N^s(t)$ firms, respectively. To start production in s an entrant has to invest a specific fixed cost of $\tilde{F}^s(t)$ units of labor that deterministically yields the blueprint to a new product. This is a *product innovation*. Having entered, the firm obtains an infinite patent on the marketing of its product. Such a firm is a *stage-one producer*. Labor is the sole production factor, the labor market is perfectly competitive and clears at wage $w(t)$. Firms produce at an increasing-returns-to-scale technology. The subsequent innovation pattern is sector-specific:

(i) **m-sector**: A new product has quality q_h^m and requires high labor unit cost of $\tilde{a}_h^m(t)$. After a successful product innovation, the firm has the option to undertake a *process innovation* that cuts both the quality of the product and its cost. More precisely, I assume that after a further investment of $\tilde{G}^m(t)$ labor units, the product can also be supplied in lower quality $q_l^m < q_h^m$ at cost $\tilde{a}_l^m(t) < \tilde{a}_h^m(t)$ such that the lower quality carries a higher quality-to-cost ratio $\frac{q_l^m}{\tilde{a}_l^m(t)} > \frac{q_h^m}{\tilde{a}_h^m(t)}$. I call a firm that has conducted such a process innovation a *stage-two producer in the m-sector*. Denote the number of stage-two producers in this sector by $Z^m(t)$ and its percentage by $z^m(t) \equiv \frac{Z^m(t)}{N^m(t)}$.

(ii) **e-sector**: A new product has quality q_l^e and requires low labor unit cost of $\tilde{a}_h^e(t)$. After a successful product innovation, the firm has the option to conduct a *quality upgrade* elevating quality at the same cost. More precisely, I assume that after a further investment of $\tilde{G}^e(t)$ labor units, the product can also be supplied in higher quality $q_h^e > q_l^e$ for the same cost $\tilde{a}_h^e(t)$ as before. At the same time, it also becomes cheaper to produce the low quality q_l^e, which can now be supplied at low costs $\tilde{a}_l^e(t) < \tilde{a}_h^e(t)$. Crucially, quality to cost ratios are such that

[13] The Euler-Lagrange equation does not feature ϕ or $1-\phi$. Consider the optimality condition $-\frac{\dot{\lambda}(t)}{\lambda(t)} = r(t) - \rho$. See appendix A.1. Now, using the first-order conditions $\lambda_i(t) = \frac{\phi}{\mu_i^e(t) X_i^e(t)}$, and therefore

$$-\frac{\dot{\lambda}_i(t)}{\lambda_i(t)} = \phi \left[\frac{\dot{\mu}_i^e(t)}{(\mu_i^e(t))^2 X_i^e(t)} + \frac{\dot{X}_i^e(t)}{\mu_i^e(t)(X_i^e(t))^2} \right] \left[\frac{\phi}{\mu_i^e(t) X_i^e(t)} \right]^{-1} = \frac{\dot{\mu}_i^e(t)}{\mu_i^e(t)} + \frac{\dot{X}_i^e(t)}{X_i^e(t)}$$

which holds for both $i \in \{R, P\}$ and *mutatis mutandis* for m. This result is driven by the separability of the utility function in the two sectors which in turn comes from log-utility and Cobb-Douglas weighting. In the same way, one can get this sector-independence if the inverse inter-temporal elasticity of substitution equals the elasticity of substitution between goods (which is both unity for the log-Cobb-Douglas case, so trivially fulfilled).

$\frac{q_l^e}{\tilde{a}_l^e(t)} > \frac{q_h^e}{\tilde{a}_h^e(t)}$. I call a firm that has conducted such a quality upgrade a *stage-two producer in the e-sector*. The number of stage-two producers is $Z^e(t)$ and its percentage is $z^e(t) \equiv \frac{Z^e(t)}{N^e(t)}$.

This discussion illustrates the fundamental difference in the pattern of innovation between the sectors. The assumption of two sectors can be made without loss of generality. In fact, it is straightforward to interpret the production sectors as separated undertakings by one firm, potentially even over the same product, such that one can think of the ascending sector as the *luxury* or high-quality line and the digressive sector as the *mass* or low-quality line of the same product line or product family. Hence, the differentiation between e and m is a formal, not a conceptual one. The crucial point is: In e, innovation is vertical and can be compared to a Schumpeterian growth model with multi-quality firms (e.g. Latzer 2018). In m, however, quality innovation is digressive (e.g. Foellmi et al. 2014). To put it in a more stylized way: In e a firm innovates by climbing up the quality ladder, while in m it innovates by sliding down.

Prices

A consumer's type is private information and firms cannot make inference about it. In a first-best setting, firms would set prices such that $u_{i,k}^s = (u_{i,\neg k}^s)^+ = 0$ but under incomplete information they can only achieve the second-best outcome. To do so, they resort to a non-linear pricing strategy. To be more precise, the setting described leads to the classical problem of optimal non-linear monopolistic screening with hidden valuation as introduced by Mirrlees (1971). The basic intuition is the following: A firm offers different schedules of qualities and prices as to maximize profits. From these schedules consumers pick the one that maximizes their utility. For a profit-maximizing stage-one producer, there are only two possible contracts that can be offered. In sector e, the first-stage firm faces the choice of setting a price at the marginal willingness to pay of P-types and sell to the entire market at $q_l^e \mu_P^e(t)$, or set a price $q_l^e \mu_R^e(t)$ at the marginal willingness to pay of R-types and exclude P-types from consuming a particular good. Similarly, for a firm in m, the choice is between selling q_h^m to the entire market or only to R. Monopolists choose the second option in both sectors if and only if assumption 1 holds.

Assumption 1 (Income concentration)**.** *Assume parameters are such that*

$$(1-\beta)[q_h^m \mu_R^m(t) - \tilde{a}_h^m(t)w(t)]L \geqslant [q_h^m \mu_P^m(t) - \tilde{a}_h^m(t)w(t)]L,$$

$$(1-\beta)[q_l^e \mu_R^e(t) - \tilde{a}_l^e(t)w(t)]L \geqslant [q_l^e \mu_P^e(t) - \tilde{a}_l^e(t)w(t)]L.$$

So if the share of R-types is sufficiently large, firms set the price $p_1^s(t)$ at the marginal willingness to pay of R-type consumers. Lemma 1 and assumption 1 imply that P-types are priced out of the market for stage-one goods in both sectors. Pricing at marginal willingness to pay gives

$$p_1^m(t) = q_h^m \mu_R^m(t) \quad \text{and} \quad p_1^e(t) = q_l^e \mu_R^e(t). \tag{4.6}$$

Second, a stage-two producer has two quality vintages at disposal. Here I draw on a result from mechanism design, the *revelation principle* (Myerson 1981). Following from that it suffices to analyze the firm's problem for the contract (direct mechanism) that consumers truthfully choose (Mas-Colell et al. 1995, p.868). Hence, I can restrict attention to a small set of contracts to be outlined below. As the firm's problem for a stage-two producer is perfectly symmetric, I only give the generic options for sector s. In Appendix A.2 I analyze sufficient incentive compatibility and individual rationality constraints for truth-telling to be a Bayes-Nash equilibrium. Under the revelation principle, a firm essentially has the following pricing options: (i) serve R-types at qualities $k \in \{h, l\}$ by setting prices equal to $\mu_R^s(t) q_k^s$ and exclude P-types; (ii) pool independently of type and serve both types at the marginal utility of P-types at one of the two qualities $k \in \{h, l\}$; or (iii) perfectly price-discriminate P-types and allow R-types an information rent in the size of the relative quality increment to separate the types. This third option deters R-types from mimicking P-types such that they self-select into the high quality. Following Foellmi et al. (2014), I analyze a separating equilibrium where stage-two firms in both sectors choose option (iii). Lemma 2 states the conditions under which this pricing strategy is optimal.

Lemma 2. *Monopolists choose separating pricing in $s \in S$ if and only if (i) $\frac{q_h^s - q_l^s}{q_l^s} p_1^s(t) > \tilde{a}_h^s(t) - \tilde{a}_l^s(t)$, (ii) $\frac{q_l^m}{\tilde{a}_l^m(t)} > \frac{q_h^m}{\tilde{a}_h^m(t)}$, (iii) $a_h^e > a_l^e$, and (iv) the Spence-Mirrlees single-crossing condition $q_h^s > q_l^s$ is satisfied.*

Proof. Appendix A.2. □

To achieve this separation, the monopolist must allow R-types an information rent corresponding to the quality increment. Let $p_l^s(t)$ denote the price for the low quality in s in stage two. Equivalently let $p_h^s(t)$ be the price for high quality. That is

$$p_l^s(t) = q_l^s \mu_P^s(t) \quad \text{and} \quad p_h^s(t) = q_l^s \mu_P^s(t) + (q_h^s - q_l^s)\mu_R^s(t). \tag{4.7}$$

The restriction to a separating equilibrium means that in sector m prices are such that rich consumers purchase all $N^m(t)$ goods at quality q_h^m and poor consumers only obtain the $Z^m(t)$ goods that made the process innovation at quality q_l^m. In sector e, rich types consume the $Z^e(t)$ goods that have reached stage two at quality q_h^e and consume the remainder at q_l^e. Consequently, poor types consume $Z^e(t)$ goods at quality q_l^e and are priced out of all stage-one goods. Consumption is

$$\begin{aligned} X_R^m(t) &= q_h^m N^m(t) \quad \text{and} \quad X_R^e(t) = q_l^e\,[N^e(t) - Z^e(t)] + q_h^e Z^e(t), \\ X_P^m(t) &= q_l^m Z^m(t) \quad \text{and} \quad X_P^e(t) = q_l^e Z^e(t). \end{aligned} \tag{4.8}$$

At this point a few comments are in order. First, if there is one quality the choice is between selling only to the rich or selling to the entire market. The co-existence of multiple qualities in the market is therefore due to firms attempting to screen heterogenous consumers. Second, for the purpose of this thesis the source of heterogeneity is irrelevant. That is, whether an unequal distribution of labor endowments or differing saving rates lead to unequal wealth distribution does not matter for firms' incentives. It is, however, crucial to assume non-homothetic preferences. Under homothetic utility, the only difference between R and P-types is the intensive margin of consumption while average consumption as a fraction of income as well as consumption variety are identical. The assumption of binary preferences in turn reduces the complexity in consumer choice. Third, allowing for differing consumption baskets between types yields an incentive for product innovation whereas allowing for different quality motivates process and quality innovation.

R&D-timing and labor markets

Entry into the R&D sector is free. Innovators make zero economic profits in equilibrium. Inventing a new firm is attractive if the present value of future cash flows offsets the initial R&D cost of $\tilde{F}^s(t)$ labor units or $w(t)\tilde{F}^s(t)$ units of output. Having entered the market, a firm realizes the stage-two innovation if the present value of innovation exceeds its cost $w(t)\tilde{G}^s(t)$. Present value and cost depend on the timing of the innovation. It is attractive for a firm to postpone an innovation because fixed costs are inversely proportional to the aggregate stock of technology, $A(t)$. That is, $\tilde{F}^s(t) = \frac{F^s}{A(t)}$ and $\tilde{G}^s(t) = \frac{G^s}{A(t)}$. Let $\Delta(j^s)$ denote the point in time when a stage-one producer moves to stage two. One might think of $\Delta(j^s)$ as the lifetime of a stage-one firm in sector s. Moreover, let $\pi_1(t, j^s)$ and $\pi_2(t, j^s)$ denote profits of some firm j^s in stages one and two. Firms solve for the present value

$$V(j^s) = \sup_{\Delta(j^s)} \left[\int_0^{\Delta(j^s)} \pi_1(t, j^s) e^{-R(t)} dt + \int_{\Delta(j^s)}^{\infty} \pi_2(t, j^s) e^{-R(t)} dt - w(t)\tilde{G}^s(t) e^{-R[\Delta(j^s)]} \right]. \tag{4.9}$$

As firms are homogenous within a sector, all choose the same $\Delta(j^s)$. Finally, I discuss the aggregate resource constraint in the economy. The labor market is perfectly competitive, so the economy's resources are at capacity L. Labor is employed in R&D where $\dot{N}^s(t)\tilde{F}^s(t)$ and $\dot{Z}^s(t)\tilde{G}^s(t)$ units of labor are being used for producing stage-one and stage-two products, respectively. Moreover, labor is needed for the production of high- and low-quality output $Y_k^s(t)$ with $k \in \{h, l\}$. The associated labor costs are $Y_k^s(t)\tilde{a}_k^s(t)$. The aggregate resource constraint is

$$\sum_{s\in S} \dot{N}^s(t)\tilde{F}^s(t) + \sum_{s\in S} \dot{Z}^s(t)\tilde{G}^s(t) + \sum_{s\in S}\sum_{\forall k} Y_k^s(t)\tilde{a}_k^s(t) \leqslant L.$$

Technical progress is driven by product, process and quality innovation. Endogenous growth is achieved by knowledge spillovers from past research activities on current productivity levels. I assume that not only fixed costs but all labor requirements are inversely related to the stock of technology generating the desired increasing returns to technology, so $\tilde{a}_k^s(t) = \frac{a_k^s}{A(t)}$. Hence, one can rewrite the resource constraint as

$$\sum_{s\in S} \dot{N}^s(t)F^s + \sum_{s\in S} \dot{Z}^s(t)G^s + \sum_{s\in S}\sum_{\forall k} Y_k^s(t)a_k^s \leqslant A(t)L. \tag{4.10}$$

Now it becomes apparent that an increase in $A(t)$ directly expands the feasible set of labor allocations (the production possibility frontier). The stock of technology is linked to product, quality, and process innovations via a spillover function $\Psi : \mathbb{R}^4_{++} \to \mathbb{R}_{++}$ where

$$A(t) = \Psi\left[\{N^s(t), Z^s(t)\}_{s\in S}\right]. \tag{4.11}$$

To generate balanced growth, the stock of knowledge $A(t)$ needs to be a linearly homogeneous function of the range of product varieties $\{N^s(t)\}_{s\in S}$, and the subset of varieties that underwent stage-two innovations, $\{Z^s(t)\}_{s\in S}$. Note that $\Psi(\cdot)$ need not be quasi-concave because it is not a production function. I elaborate on the functional of (4.11) when deriving the equilibrium.

4.2 A purely-separating equilibrium

I now derive equilibrium. Recall lemma 2 and assumption 1 establish that the share of rich consumers in the economy is sufficiently high such that firms separate types by pricing. That is, they serve poor and rich consumers with different qualities and - if they have only one quality at their disposal - sell exclusively to the rich. Hence, the equilibrium analyzed in this thesis is what I refer to as a *purely-separating balanced growth equilibrium* (PSBGE). As the name implies, it is a balanced growth equilibrium where all key variables grow at a common constant rate. Additionally, it is *purely separating* exactly because firms in both sectors separate in both stages.[14]

Definition 2 (Purely-separating balanced growth equilibrium)**.** *The economy is in a purely-separating balanced growth equilibrium (PSBGE) if: (i) all stage-one producers sell at prices given by (4.6) and all stage-two producers sell at (4.7); (ii) labor markets clear at wage $w(t)$; (iii) the stock of technology $A(t)$, the wage rate $w(t)$, and the number of products in both sectors $\{N^s(t), Z^s(t)\}_{s\in S}$ grow at the common and constant growth rate of the aggregate stock of technology $g \equiv \frac{\dot{A}(t)}{A(t)}$ defined by (4.11), and consumption for $i \in \{R, P\}$ given by (4.8) grows at that rate as well; (iv) the fraction of stage-two firms in each sector $\{z^s\}_{s\in S} = \left\{\frac{Z^s(t)}{N^s(t)}\right\}_{s\in S} \in [0,1]^2$ is constant; (v) $\{\mu_i^s\}_{i\in\{R,P\}, s\in S}$ are constant; (vi) consumers' saving rate is constant and the wealth distribution is stationary at (β, θ); and (vii) the real interest rate r is constant and pinned down by g in (4.5).*

To demonstrate the key mechanism of quality improvement in the most parsimonious setting, I impose two symmetry conditions. The equilibrium derivations are algebraically somewhat involved. Below I focus on the intuition. All calculations are in appendix A.3.

Derivations

To find equilibrium, I check that all seven requirements of definition 4.2 are fulfilled. I start by exploiting property (ii): Both wages $w(t)$ and knowledge $A(t)$ grow at the same rate g which is why I define the wage per efficiency unit as the numeráire. Variable cost decrease with the stock of technology such that labor unit costs are constant in equilibrium at $a_k^s = \frac{a_k^s w(t)}{A(t)}$. The same holds for fixed cost at $F^s = \frac{F^s w(t)}{A(t)}$ and $G^s = \frac{G^s w(t)}{A(t)}$. Moreover, from (v) it follows that in the PSBGE marginal willingnesses to pay for all goods are constant, and therefore prices are constant as well. Given (ii) and (v), firms' profits are $\pi_1^s = (1-\beta)(p_1^s - a_h^s)L$ in stage one, and at $\pi_2^s = \beta(p_l^s - a_l^s)L + (1-\beta)(p_h^s - a_h^s)L$ in stage two. As there is free entry, there are no arbitrage opportunities. Hence, a firm's monopoly profit in any stage must equal the cost of attaining this monopoly. I denote the present value of market entry by V_N^s, and the no-arbitrage condition in stage one reads $V_N^s = \frac{\pi_1^s}{r} = F^s$. After having entered the market, firms can conduct the stage-two innovation.[15] The cost of this upgrade is G^s and the additional flow of profits compared to staying a stage-one producer are $\frac{\pi_2^s - \pi_1^s}{r}$. Therefore,

[14] Apart from PSBGE, there might also exist other equilibria with pooling in one or both sectors or on the initial stage. I relegate their discussion to appendix A.2 because only the PSBGE replicates the empirical regularities.

[15] Apart from the associated costs of becoming a stage-two producer, the value such of an innovation also depends on its timing. The optimal timing of proceeding to stage two can be derived from the Hamilton-Jacobi-Bellman problem (4.9) where the real interest rate is pinned down by consumers' inter-temporal problem in (4.5). This is

$$V_Z^s = \max_{\Delta^s} \left[\int_0^{\Delta^s} \pi_1^s e^{-rt} dt + \int_{\Delta^s}^{\infty} \pi_2^s e^{-rt} dt - G^s e^{-r\Delta^s} \right].$$

As it turns out, the timing of a stage-two innovation on the balanced growth path is in-determined. See Foellmi et al.

the no-arbitrage condition in stage two reads $V_Z^s = \frac{\pi_2^s}{r} = F^s + G^s$. At this point, let me impose the first regularity condition that stipulates cost symmetry across sectors.

Assumption 2 (Cost symmetry)**.** *Let costs be symmetric across sectors by a scalar $\delta \in \mathbb{R}_{++}$. That is, fixed costs are such that $(F^e, F^m) = (\delta F, F)$ and $(G^e, G^m) = (\delta G, G)$. Variable costs are $(a_h^e, a_l^e) = (a, b)$ and $(a_h^m, a_l^m) = \left(\frac{a}{\delta}, \frac{b}{\delta}\right)$ where $a > b > 0$.*

To illustrate this, suppose $\delta > 1$ such that market entry and unit costs are higher in e than in m. Recall that sector e is characterized by vertical quality innovation. It can be thought of as an *exclusive* sector that caters to luxury needs. Developing such products is expensive and requires trained employees or a large time investment. And after having introduced them to the market, firms incur high variable costs per unit. Conversely, think of m as the sector associated with process innovation or *mass* production. Innovation happens by improving on production technology rather than quality. Introducing a product in m is relatively cheap and unit costs are lower making them more affordable. Hence, assumption 2 formalizes the notion that quality innovation predominantly happens at the top end of the product range while process innovation is conducted on mass markets. Returning to the derivations, cost symmetry and free entry imply that profits between the sectors are scaled by the same factor as fixed costs, so $\pi_1^e = \delta\pi_1^m$, and therefore stage-one prices are proportional by the same wedge $p_1^e = \delta p_1^m$. Equivalently, stage two profits are $\pi_2^e = \delta\pi_2^m$. To infer stage-two prices, the next assumption concerns product quality.

Assumption 3 (Symmetric quality increments)**.** *Let $\alpha \in (0,1)$ and $\gamma \in \mathbb{R}_{++}$ and assume that $q_l^e = k$, $q_h^e = (1+\alpha)k$, $q_h^m = \gamma k$, $q_l^m = (1-\alpha)\gamma k$, and $k = 1$.*

Assumption 3 requires that the quality increments - that is, the percentage by which a firm can change the quality of a product - be symmetric in both sectors. Conducting a quality upgrade in sector e increases quality by α while a process innovation in m decreases quality by α relative to the baseline level. This assumption simplifies the solvability of the model considerably. Using the price setting rule in equation (4.7), assumption 3 implies that stage two prices for the high quality are $p_h^s = p_l^s + \alpha p_1^s$. Moreover, the second part of assumption 2 ensure that stage two prices are also scaled by the wedge δ, so $p_l^e = \delta p_l^m$ and $p_h^e = \delta p_h^m$. This collapses the set of prices from seven to three; two for consumption goods (p_1^m, p_l^m), and the real wage (which is the numeráire). Prices are set at marginal willingness to pay. The ratio of prices therefore reflects quality adjusted marginal willingnesses to pay independent of the shadow value of wealth. I exploit this to find relative

(2014) for a discussion of an equivalent HJB-problem. However, this is not a concern for equilibrium derivations because free entry into R&D ensures that the value of a stage-two innovation has to satisfy the no-arbitrage condition.

consumption as

$$\frac{\mu_R^m}{\mu_R^e} = \frac{p_1^m/q_h^m}{p_1^e/q_l^e} = \frac{1}{\delta\gamma} = \frac{1-\phi}{\phi}\frac{X_R^e(t)}{X_R^m(t)} \quad \text{and} \quad \frac{\mu_P^m}{\mu_P^e} = \frac{p_l^m/q_l^m}{p_l^e/q_l^e} = \frac{1}{(1-\alpha)\delta\gamma} = \frac{1-\phi}{\phi}\frac{X_P^e(t)}{X_P^m(t)}.$$

This yields an expression for the relative consumption bundles chosen by rich and poor consumers as $X_P^m(t) = \frac{(1-\alpha)(1-\phi)\gamma\delta}{\phi}X_P^e(t)$ and $X_R^m(t) = \frac{(1-\phi)\gamma\delta}{\phi}X_P^e(t)$. In equilibrium, the set of products $\{N^s(t), Z^s(t)\}_{s\in S}$ grows at rate g, so the percentages of stage-one-to-stage-two products $\{z^s\}_{s\in S}$ are constant. One can therefore plug (4.8) in the above equations to find $z^m = \frac{1}{\alpha}\left(1 - \frac{1-\phi}{\phi}n\right)$ and $z^e = \frac{1}{\alpha}\left(\frac{\phi}{1-\phi}\frac{1}{n} - 1\right)$. The next step is to determine z^s. Monotonicity of the utility function and optimality of the solution require that consumers' budget constraints (4.4) be met. Moreover, requirement (vi) states that (β, θ) describes the stationary wealth distribution. Hence, a R-type holds a share $\frac{1-\beta\theta}{1-\beta}$ of the wealth, while θ is held by a poor type. Consequently, R's instantaneous income is scaled up relative to P's by $\frac{1-\beta\theta}{(1-\beta)\theta}$. From (4.8) consumption bundles are known. Taking the ratio of P's and R's budget constraints gives

$$\frac{1-\beta\theta}{(1-\beta)\theta} = \frac{[N^e(t) - Z^e(t)]p_1^e + Z^e(t)p_h^e + [N^m(t) - Z^m(t)]p_1^m + Z^m(t)p_h^m}{Z^e(t)p_l^e + Z^m(t)p_l^m}. \tag{4.12}$$

Using the expressions for z^s as well as the price function $p_h^s = p_l^s + \alpha p_1^s$, this reduces to

$$\frac{1}{1-\beta}\frac{p_l^m}{p_1^m} = \frac{\theta}{1-\theta}\left[\frac{1}{z^m} - \alpha\phi - (1-\alpha)\right], \tag{4.13}$$

depending on z^m and (p_1^m, p_l^m). Via the no-arbitrage condition and the present values of innovation (V_N^s, V_Z^s), one can express $\frac{p_l^m}{p_1^m}$ in terms of the real interest rate r. That is, use $p_1^m = \frac{rF}{(1-\beta)L} + \frac{a}{\delta}$ and $p_l^m = \frac{rG}{L} + \beta\frac{b}{\delta} + (1-\alpha)(1-\beta)p_1^m$, and plug into (4.13) to obtain an equation depending on r and z^m. This gives

$$\frac{\delta Gr + \beta bL}{\delta Fr + (1-\beta)aL} = \frac{\theta}{1-\theta}\left[\frac{1}{z^m} - \alpha\phi - \frac{1-\alpha}{\theta}\right]. \tag{4.14}$$

For later use, I define the ratio of stage-two-to-stage-one prices as the right-hand side of (4.14)

$$\chi(z^m) := \frac{\theta}{1-\theta}\left[\frac{1}{z^m} - \alpha\phi - \frac{1-\alpha}{\theta}\right].$$

Notice that this price ratio can be thought of in terms of resources deployed in the production of high and low quality: $\beta\frac{b}{\delta}L$ are total labor cost for the production of stage-two products in low quality, and G are the associated fixed cost of developing these products. Similarly, $(1-\beta)\frac{a}{\delta}L$ are total labor cost of producing high quality goods, and F are associated fixed costs. This ratio is the same in both sectors because $\frac{p_l^m}{p_1^m} = \frac{\delta p_l^e}{\delta p_1^e}$ by the assumption of cost symmetry.[16] Hence, the price ratio inherently reflects the economy's resource allocation between high and low quality output. Solving this equation for r in terms of $\chi(z^m)$ gives

$$r(z^m) = \frac{\chi_Z(z^m) - \frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F} - \chi_Z(z^m)} \cdot \frac{(1-\beta)aL}{\delta F}.$$

The last step is to express z^m in terms of g. This is the solution to the intra-temporal problem. In equilibrium, it indicates market clearing within each instant of time. To find the equilibrium ratio z^m in terms of g, I need to take a stance on the functional form of technological progress. Assume that knowledge accumulates entirely through spillovers. For generating balanced growth, the stock of technology $A(t)$ needs to evolve via a homogenous function $\Psi : \mathbb{R}^4_{++} \to \mathbb{R}_{++}$ of degree one in the set of all products $\{N^s(t), Z^s(t)\}_{s\in S}$. The accumulation of knowledge follows a Cobb-Douglas functional of

$$A(t) = \psi\,[N^e(t)]^{\zeta_{N^e}}\,[N^m(t)]^{\zeta_{N^m}}\,[Z^e(t)]^{\zeta_Z}\,[Z^m(t)]^{1-\zeta_{N^m}-\zeta_{N^e}-\zeta_Z}\,, \tag{4.15}$$

where $\{\{\zeta_{N^m}, \zeta_{N^e}, \zeta_Z\} \in [0,1]^3 | \zeta_{N^m} + \zeta_{N^e} + \zeta_Z \leqslant 1\}$, and $\psi \in \mathbb{R}_{++}$. Given (4.15), the resource constraint pins down z^m as a function of g and r. In the following, I discuss two polar cases: In section 4.3, $A(t)$ evolves only through stage-two innovation, that is $\zeta_{N^e} = \zeta_{N^m} = 0$, and in section 4.4, $A(t)$ evolves only through stage-one innovation, that is $\zeta_{N^e} + \zeta_{N^m} = 1$. All cases where both stage-one and stage-two innovation affect $A(t)$ are (arbitrary) convex combinations of the polar cases.

4.3 Technical progress through quality and process innovation

I start with the case where quality and process innovation advance the stock of technology but product innovation does not - so only second-stage innovation matters. Assume that $\zeta_{N^e} = \zeta_{N^m} = 0$ in (4.15), such that the equation reduces to

$$A(t) = \psi_Z\,[Z^e(t)]^{\zeta_Z}\,[Z^m(t)]^{1-\zeta_Z}\,, \tag{4.16}$$

where $\zeta_Z \in [0,1]$ and $\psi_Z \in \mathbb{R}_{++}$. I choose this parsimonious setting to isolate the comparative statics effects of stage-two innovation from the (counteracting) impact of new firms entering the market.

[16] Cost symmetry is in fact a necessary condition to admit a closed-form solution of the model.

Now using (4.16) to determine z^m in terms of g yields a function $\chi_Z(g)$. The exact expression is equation (A.17). As I have already mentioned in the preceding sections, concavity and separability of the instantaneous utility function imply that the intra-temporal problem and the inter-temporal problem are independent. Given this expression $\chi_Z(g)$, the solution to the intra-temporal problem is

$$r_Z^R(g) = \frac{\chi_Z(g) - \frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F} - \chi_Z(g)} \cdot \frac{(1-\beta)aL}{\delta F}, \tag{4.17}$$

where $\chi_Z : \mathcal{G} \to \left(\frac{G}{F}, \frac{\beta}{1-\beta}\frac{b}{a}\right) \subset \mathbb{R}_+$ is a function of g. The precise expression is (A.17). It equals the ratio of stage-two and stage-one prices, $\frac{p_l^m}{p_1^m}$, and hence the quality adjusted marginal willingnesses to pay of R and P, $\frac{p_l^m}{p_1^m} = \frac{q_l^m \mu_P^m}{q_h^m \mu_R^m}$. Subsequently I refer to (4.17) as the intra-temporal market clearing line. Alternatively, because it also indicates full utilization of resources, I label it $r_Z^R(g)$. Notice that it is increasing and convex in the growth rate. I elaborate on the reason for this in section 4.4. Apart from solving the consumer problem at every instant of time, (4.17) also reflects that consumers meet their budget constraints with equality, the goods and the labor market clear, and all resources are used up, which is why I refer to it as the resource curve $r_Z^R(g)$. Besides the intra-temporal problem, the inter-temporal problem is governed by the standard Euler-Lagrange equation. To see this, notice that marginal willingnesses to pay are constant in equilibrium, that is, $\frac{\dot{\mu}_i^s(t)}{\mu_i^s(t)} = 0$ for all s. Moreover, by definition of the equilibrium, the number of products grows with rate g. Using this in (4.5) yields

$$r^E(g) = \rho + g. \tag{4.18}$$

This is the standard Euler-Lagrange equation known from neoclassical growth, hence I label it $r^E(g)$. The growth rate in (4.18) is the usual difference between market discount r and subject discount rate ρ. Equations (4.17) and (4.18) define a system of two equations in two unknowns - the growth rate g and the real interest rate r - describing the economy. I show in proposition 2 that there exists an equilibrium characterized by these two equations. The intuition of the existence proof is a simple intermediate value theorem. Under a weak parameter restriction, (4.17) is convexly and monotonically increasing on the domain of the growth rate. The Euler-Lagrange equation is increasing and affine. Figure 9 illustrates. There exists a unique equilibrium where the two market-clearing curves intersect. This point is labeled E corresponding to the equilibrium tuple (g^*, r^*).

Proposition 2 (Existence of equilibrium I)**.** *Technical progress follows* (4.16)*. Let parameters satisfy lemma* 2*, assumption* 5*, and let* $\frac{\beta}{1-\beta}\frac{b}{a} > \frac{G}{F}$*. Moreover, let* $\psi_Z \in \mathcal{K}_Z$ *and* $\rho \in \mathcal{P}$*. Then, the solution to the intra-*

temporal problem (4.17) *is strictly increasing, continuous and strictly convex for all* $g \in \mathcal{G} = [0,1]$ *with boundary points* $0 < r_Z^R(0) < \rho$ *and* $r_Z^R(g) - 1 > \rho$. *The solution to the inter-temporal problem* $r^E(g)$ *given by* (4.18) *is strictly increasing, continuous and affine on* $\mathbb{R}$. *Via the (Bolzano) intermediate value theorem there exists a unique equilibrium growth rate* $g^* \in (0,1)$. *It satisfies definition* 2.

Proof. Appendix A.4.1. □

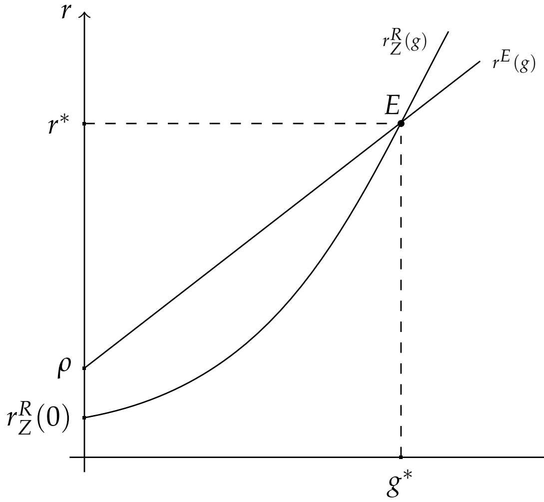

Figure 9: Separating equilibrium with stage-two innovation.

I now proceed to analyzing the comparative static effects of changes in the demand-side structure of the economy. Firms base their innovation decisions not only on consumer preferences, but also on income distribution and the efficiency gain (loss) from innovation. Therefore, I discuss the effect of a change in three key parameters: (i) Consumers' taste for e-type products captured by an increase in ϕ, (ii) a decrease in the income gap captured by an increase in θ, and (iii) a decrease in income concentration captured by an increase in β.

Consumer tastes

Consider a change in consumer tastes, namely an increase in the elasticity of the utility aggregator with respect to e-type goods. Increasing ϕ raises marginal utility from e-type goods while lowering marginal utility for m. As the marginal willingness to pay rises in e, so do prices and it becomes more lucrative for firms to enter. The number of products consumed, $X_i^e(t)$, increases. The reverse is true in m. Formally, the effect on $X_i^s(t)$ can be obtained using monotone comparative statics as described by Milgrom & Shannon (1994).[17] An increase in ϕ increases $X_i^e(t)$ (and decreases $X_i^m(t)$). The composition of consumption shifts. Consumers move consumption towards e and away

[17]It can easily be verified that the CES (Cobb-Douglas) utility aggregator is quasi-supermodular in the consumption goods $X_i^e(t)$ and the budget set is indeed a lattice. Hence, monotonicity is guaranteed.

from m. For economic growth, there is an interesting implication. A shift towards more e-type production has two effects: A positive one through the expansion in e, and a negative effect through the contraction in m. The overall effect on the growth rate depends on the (relative) importance of sectors e and m.

Proposition 3 (Consumer tastes)**.** *In the PSBGE described in definition 2 and proposition 2, stronger preference for e-type goods captured by an increase in ϕ increases g if $\zeta_Z > \phi$ and decreases g if $\zeta_Z < \phi$. Growth is maximized at g^{**} if $\zeta_Z = \phi$.*

Proof. Appendix A.4.2. □

The effect is unambiguously positive if $\zeta_Z > \phi$. Recall that ζ_Z is the elasticity of $A(t)$ with respect to $Z^e(t)$, the number of products that underwent a quality upgrade. Conversely, $1 - \zeta_Z$ is the elasticity of $A(t)$ with respect to $Z^m(t)$, the number of products that underwent a process innovation. To understand why the effect is positive if $\zeta_Z > \phi$, a good heuristic is to think in terms of marginal products of new goods in e and m: As described, increasing ϕ leads to a re-allocation of resources from m to e. Now these resources are put to better use (they cause stronger spillovers) if the marginal product of goods in the sector is high, that is if ζ_Z is high. Conversely, if ζ_Z is low relative to ϕ there is already high production in e and the economy would gain in terms of spillovers from producing more m-type products. A further increase in ϕ shifts even more resources away from m and thus

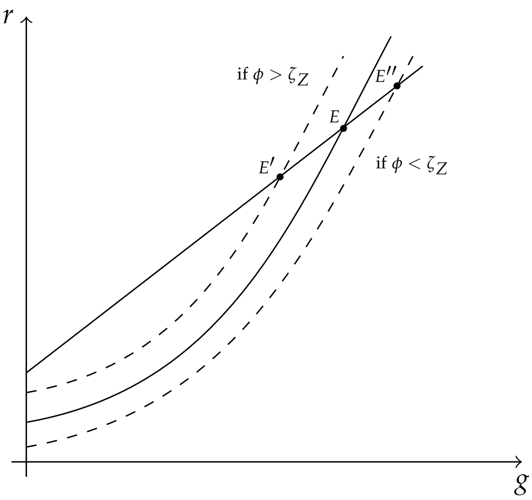

Figure 10: Increase in preference for e-type products (increase in ϕ).

reduces growth.[18] Hence, growth decreases if $\phi > \zeta_Z$. Naturally, both ϕ and ζ_Z cannot be chosen. They are *deep* parameters of the economy. However, this particular implication of the model might

[18]There is a second part to this proposition that is a bit more subtle because it can be shown that an increase in ϕ not only affects the percentage of stage-two firms z^m and z^e but also the relative sector size n. This can be done by applying the total differential on (A.10) or (A.14).

explain why some countries show high production in ineffective sectors, and do not reallocate resources to production in more lucrative sectors with high marginal products. Figure 10 illustrates. Finally, in light of this discussion, one can anticipate that growth is maximized at rate g^{**} if $\zeta_Z = \phi$. This is in fact true, and I prove it in appendix A.4.2.

Price and market size effects

Firms' incentives for innovation are affected by the economy's distributional characteristics. Consumers are ex-ante heterogenous with non-homothetic preferences, and they consume different baskets of goods between them. A change in income distribution then affects demand and thus production. Because of binary choice, consumers only choose at the extensive margin of consumption and changes in income distribution do not channel through increasing quantities. That is, changes in the income distribution channel solely through prices. Therefore, such a change in income distribution fully affects the number of goods in each sector, the incentives to develop further stage-two products and thus growth. The properties of the income distribution that characterizes

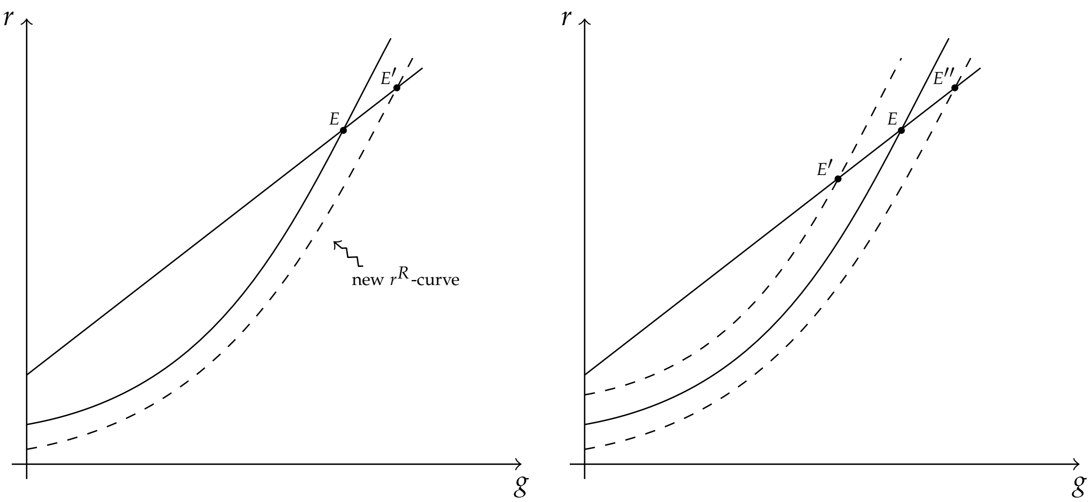

Figure 11: Decrease in the income gap (increase in θ, left), and decrease in income concentration (increase in β, right).

the economy are summarized in the tuple (β, θ). A higher β implies that there are more poor and less rich consumers holding the endowment θ constant. Income becomes more concentrated. Common measures of inequality such as the Gini index indicate an increase in inequality. Conversely, an increase in θ raises the endowments of poor relative to rich consumers holding the share of consumers constant. It therefore corresponds to a decrease in inequality. Whether a change in inequality fosters growth depends on two effects: The *market size* and the *price effect*. These two effects emerge from the interplay of non-homothetic preferences and asymmetric distribution. On the one hand, the market size effect literally captures that firms make higher profits if the market is larger. Clearly, it implies that a more equal distribution of income is favorable for innovation and growth. On the other hand, there is the price effect. A very unequal distribution implies that the richest consumers have a very high willingness to pay for new goods, so profit margins are comparably high in the early phases of a product life. Inequality benefits growth. As described by Bertola et al., whether or not the price effect dominates the market size effect depends not only on the distribution but also an innovator's scope of price setting. If competition is strong, innovators' market power is small and the market size effect dominates. Conversely, if market power is high, the price effect might prevail (2005, pp.276-7).

In the present model, an increase in θ has an unambiguously positive effect on growth. This is because product innovations do not affect the stock of technology. If technical progress is based on making products affordable to the masses and upgrading quality, then a smaller income gap is good for growth. Notice that in this specification of technical progress, it does not matter whether firms make new products available through quality or process innovation. However, if one would set $\zeta_Z = 1$ ($\zeta_Z = 0$) then only quality (process) innovation would cause spillovers but nevertheless firms would devote considerable resources to the respective other type of innovation because they do not take into account the technological externality of their production. The effect of an increase in θ will be weaker but still unambiguously positive. Different to that, for an increase in β, the total effect remains ambiguous. A higher β reduces profits from selling to the rich in the stage two but also increases profits from a larger market of poor consumers. Whether the market size or the price effect dominates depends on the parameterization. Running a series of numerical simulations shows that for an economy with a low share of poor consumers an increase might encourage growth. For an already large share of poor consumers a further increase usually reduces growth but also might entail a break-down of equilibrium if pooling becomes too attractive. Finally, in this numerical exercise the effect of a change in the income concentration also depends on income gaps: If poor consumer's endowment is very low, an increase might discourage growth even if there are only few poor types initially. Proposition 4 summarizes.

Proposition 4 (Changes in inequality)**.** *In the PSBGE described in definition 2 and proposition 2, a smaller income gap captured by an increase in θ increases growth while a lower income concentration captured by an increase in β has an ambiguous effect.*

Proof. Appendix A.4.3. □

4.4 Technical progress through product innovation

Let me discuss the case where only product innovation drives growth. To be more specific, assume that only e-type product innovation augments the stock of technology. Let $\zeta_{N^e} = 1$ in (4.16) such that neither quality innovation, process innovation nor product innovation in m cause spillovers. Technological progress follows

$$A(t) = \psi_{N^e} N^e(t), \tag{4.19}$$

where $\psi_{N^e} \in \mathbb{R}_{++}$ is a weight or shifter.[19] Given $A(t)$, equilibrium can be obtained along the same lines as in the previous section. Since only the solution to the intra-temporal problem changes, the two characterizing equations are (4.18) and

$$r^R_{N^e}(g) = \frac{\chi_{N^e}(g) - \frac{\beta}{1-\beta}\frac{b}{a}}{\frac{\mathcal{G}}{F} - \chi_{N^e}(g)} \cdot \frac{(1-\beta)aL}{\delta F}, \tag{4.20}$$

where $\chi_{N^e} : \mathcal{G} \to \left(\frac{\mathcal{G}}{F}, \frac{\beta}{1-\beta}\frac{b}{a}\right) \subset \mathbb{R}_{++}$ is a non-linear function in g that characterizes the price ratio between stage one and two. Proposition 5 summarizes and figure 12 illustrates.

Proposition 5 (Existence of equilibrium II)**.** *Technical progress follows (4.19). Let parameters satisfy lemma 2 and $\frac{\mathcal{G}}{F} < \frac{\beta}{1-\beta}\frac{b}{a}$. Moreover, let $\psi_{N^e} \in \mathcal{K}_{N^e}$. Then, the solution to the intra-temporal problem (4.20) is decreasing, continuous and strictly convex on $\mathcal{G}$, and the solution to the inter-temporal problem (4.18) is affine and strictly increasing on $\mathbb{R}_{++}$. For any $0 < \rho < r^R_{N^e}(0)$ a unique equilibrium exists via the (Bolzano) intermediate value theorem. It satisfies definition 2.*

Proof. Appendix A.4.4. □

[19] An alternative specification would be to let the sum of products across both sectors add to the stock of technology. This would stress the notion that spillovers are caused by product innovation rather than sector-specific knowledge. Moreover, it would allow for the separation of stage-one and stage-two innovation. However, I choose to use the Cobb-Douglas specification in (4.16) for three reasons. First, given that I discuss the cases where product innovation in m and e as well as the case where stage-two innovation cause spillovers, it is straightforward to see that all other cases are convex combinations. Setting the weights $\zeta_{N^m} = \zeta_{N^e} \neq 0$ implies equal weighing of the two spillovers from product innovation. Second, inducing linearity in (4.16) would imply that the model is no longer solvable in the (g, r)-space. Additionally, most of the comparative statics results would be driven by the linearity. For example, the critical determinant of spillovers would then be ψ_{N^s} relative to δ rather than the elasticities of accumulation. Third, in section 5 I sketch an extension to unbalanced growth where only sector-specific knowledge reduces cost. For this extension, (4.19) is a nice benchmark.

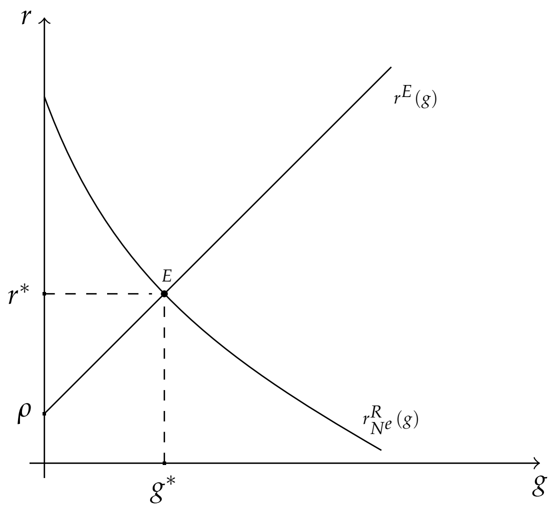

Figure 12: Separating equilibrium with stage-one innovation.

I have already mentioned above that the intra-temporal market-clearing condition is convex increasing when growth follows stage-two innovation (equation $r^R_Z(g)$ in (4.17)) but convex decreasing when growth follows from stage-one innovation (equation $r^R_N(g)$ in (4.20)). This seems surprising at first glance. To phrase it differently, why are high real interest rates positively related to growth if spillovers come from process and quality innovation but inversely related if they stem from product innovation? The reason for this is not trivial to see and touches on the core mechanism of the model: The behavior of the relative price function $\chi(\cdot)$. Let me illustrate via an example. Suppose there is an increase in $Z^m(t)$ implying a ceteris paribus increase in z^m, the ratio of stage-one to stage-two products. First, there is a *demand effect* which implies that an increase in z^m is associated with higher consumption of the poor. There is additional demand, and more employment in production is needed to satisfy it. The economy's resources are shifted towards production and away from research. As less labor is available in R&D, the growth rate drops. Second, there is a *productivity effect*. A higher z^m implies more efficient production and therefore more resources are available for innovation and growth. The effect of an increase in z^m on the intra-temporal market clearing curve are as follows: If technical progress follows stage-two innovation, the productivity effect is strong and it offsets the demand effect.[20] Stronger spillovers stimulate the economy and growth increases. Holding inequality (β, θ) constant, it follows from (4.13) that p_1^m must rise.[21] This ensures that the no-arbitrage condition in stage one holds (keeping (β, θ) constant). Similarly, p_l^m has to go up as well, such that the stage-two arbitrage condition holds, but it does less so than p_1^m because part of the increase in p_1^m feeds through p_h^m as can be seen from (4.7).[22] Hence, the relative price function

[20] Notice that it a situation where the demand effect eventually dominates arises if $\zeta_{N^e} + \zeta_{N^m} > 0$.
[21] Proposition 6 shows that p_1^m in fact goes up (and that it is not just the ratio $\frac{p_1^m}{p_l^m}$ going up).
[22] This is because the firm keeps R-types' information rent constant.

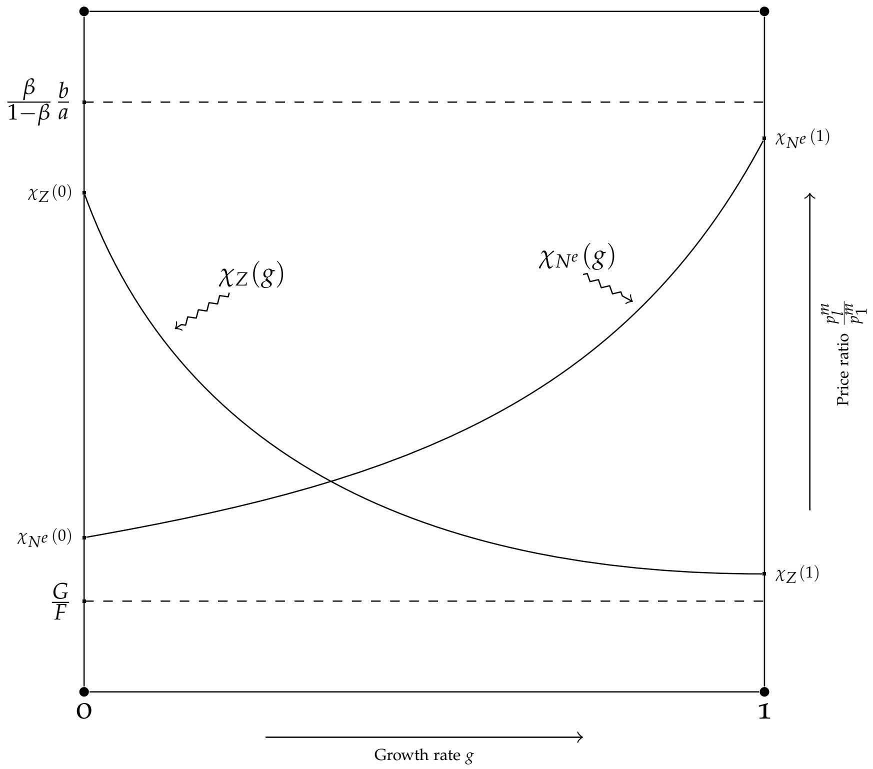

Figure 13: Behavior of the relative price function.

$\chi_Z(\cdot) = \frac{p_l^m}{p_1^m}$ decreases in g. Conversely, if technical progress follows stage-one innovation, the productivity effect from a higher z^m is zero but the demand effect persists. The growth rate drops and the relative price function has to decrease as well. Thus, $\chi_{N^e}(\cdot)$ increases in g. Now, for the behavior of the real interest rate, suppose that there is an increase in r. Again to ensure that no arbitrage opportunities can arise, p_1^m (and p_l^m) have to increase. By the same mechanism as before, if technical progress follows stage-two innovation, the relative price $\chi_Z(\cdot)$ has to drop, so growth rises. Therefore g and r are positively related (the r_Z^R-curve is upward-sloping). If technical progress follows stage-one innovation, following an increase in the real interest rate, the relative price χ_{N^e} drops and growth falls. Therefore, g and r are negatively related (the r_N^R-curve is downward-sloping). Figure 13 illustrates relative prices as a function of g for the case with only stage-one innovation ($\chi_{N^e}(\cdot)$) and only stage-two innovation ($\chi_Z(\cdot)$). The upper and lower bounds are given by the existence condition $\frac{G}{F} < \frac{\beta}{1-\beta}\frac{b}{a}$. Proposition 6 summarizes.

Proposition 6 (Behavior of the relative price function)**.** *Assume $\frac{G}{F} < \frac{\beta}{1-\beta}\frac{b}{a}$. If technical progress follows* (4.16)*, no-arbitrage requires that $\chi_Z(\cdot)$ decreases in g. In that case, $r_Z^R(g)$ increases in g. Conversely, if technical progress follows* (4.19)*, no-arbitrage requires that $\chi_{N^e}(\cdot)$ decreases in g. In that case, $r_N^R(g)$ decreases in g.*

Proof. Appendix A.4.5. □

Now let me turn to the comparative static effects of changes in the demand side in this new equilibrium. These are somewhat different. First, the effect of an increase in θ is now negative, so a smaller income gap reduces growth. It directly reduces the willingness to pay by R-types μ_R^s and therefore discourages market entry in stage one. Firms conduct more stage-two innovation shifting resources away from stage-one innovation. The externality from product innovation weakens and growth rates fall. Intuitively, if innovation is based on catering to exclusive needs of a few affluent consumers a more egalitarian society - measured by a smaller income gap - discourages growth. Figure 14 illustrates. Second, for β the effect remains ambiguous. The reason is that the underlying market and price effects are unaffected. The price effect still discourages entry into stage two while the market size effect provides opposing incentives. The source of the growth externality does not matter. Nevertheless, a negative effect of lower income concentration can emerge. This is because product innovation targets new rich consumers. It can be shown that if the quality gap between stage one and two is small, that is, if quality innovation and process innovation are less lucrative, an increase in β can decrease growth. If income is less concentrated, entering the market becomes less attractive and therefore growth slows down. Third, consider an increase in ϕ. If one assumes that the quality gap between stages one and two is not unrealistically large (which is equivalent to saying that the PSBGE exists), an increase in ϕ increases growth.[23] This result is intuitive as well: The elasticity of (4.19) with respect to e-type products is unity. Hence, there is no trade-off as in the previous section. Proposition 7 summarizes.

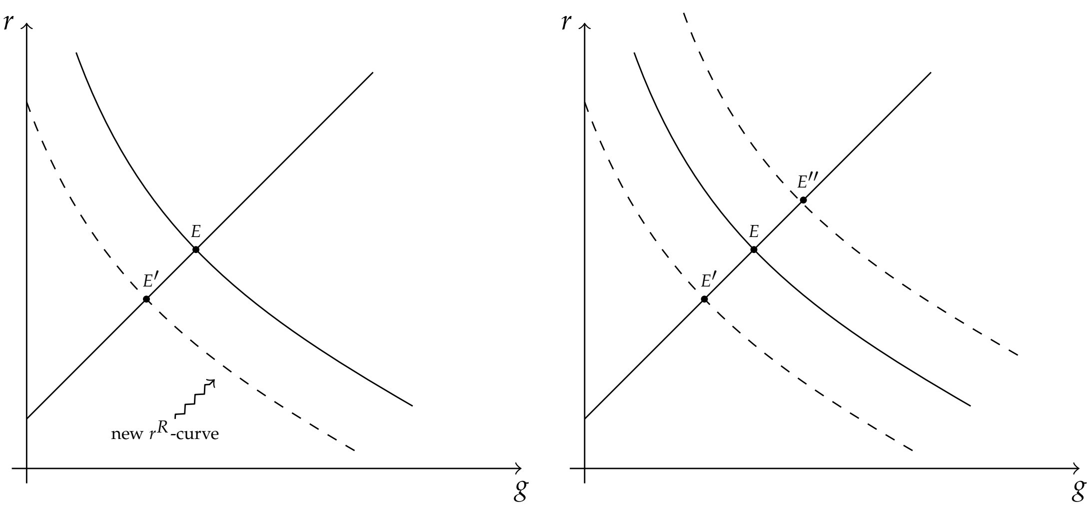

Figure 14: Decrease in the income gap (increase in θ, left), and decrease in income concentration (increase in β, right).

[23]The conditions is $\alpha\phi < \frac{1}{2}$. If this condition is not fulfilled it can be that for very large growth rates an increase in ϕ has a negative effect on growth. This curious result emerges because if the quality gap $\alpha \to 1$ in the limit stage-two innovation would introduce products of quality zero. Obviously these products will not be purchased and hence there is no reason for a firm to move to stage-two in the first place. Hence, $\alpha\phi > \frac{1}{2}$ is associated with breakdown of the PSBGE and the emergence of a degenerate equilibrium.

Proposition 7 (Comparative statics under stage-one innovation)**.** *In the PSBGE described in definition 2 and proposition 5, a smaller income gap captured by an increase in θ decreases growth while a lower income concentration captured by an increase in β has an ambiguous effect. Moreover, stronger preference for e-type goods captured by an increase ϕ increases growth.*

Proof. Appendix A.4.6. □

Before concluding this subsection, let me sketch the case where (4.16) follows only product innovation in m, so $\zeta_{N^m} = 1$, and

$$A(t) = \psi_{N^m} N^m(t).$$

where $\psi^m \in \mathbb{R}_{++}$. In terms of the equilibrium derivations as well as predictions, this case is isomorphic to (4.19). The only difference is the comparative statics effect of a change in ϕ which is now negative. The growth-maximizing sectoral weight would be $\phi = 0$ as one would expect. Finally, if technical progress follows both m and e-type innovation, that is $\{\{\zeta_{N^e}, \zeta_{N^m}\} \in (0,1)^2 | \zeta_{N^e} + \zeta_{N^m} = 1\}$, one can generate the same pattern of falling and rising growth rates as a result of an increase in ϕ as in the previous section. If $\phi < \zeta_{N^e}$ the growth rate rises, and if $\phi > \zeta_{N^e}$ it falls. However, in this case the model is only solvable by numerical approximation.

4.5 Alternative equilibria

Until now I have not discussed the case of spillovers along sectoral lines. These are the cases when (i) $\zeta_{N^e} = \zeta_Z = 0$ or (ii) $\zeta_{N^e} + \zeta_Z = 1$. The reason for this is that case (i) corresponds to Foellmi et al. (2014) when the knowledge-accumulation function is Cobb-Douglas. Hence, my model nests the model T-version. Case (ii) yields a Schumpeterian growth model with non-homothetic preferences, horizontal and vertical innovation, and innovation depth one. All other cases where one or more types of innovation affect the aggregate stock of technology can just be seen as convex combinations of the two (three) polar cases that I have presented above. In particular, the overall strength of market size and price effect will depend on the elasticity of the respective spillovers in the functional form of the aggregate stock of technology (4.15). For example, a decrease in inequality through an increase in θ will have a positive effect on growth if quality-innovation spillovers are strong and a negative effect if product innovation dominates.

Another class of equilibria are those where the PSBGE does no longer hold. That is, there exist other equilibria with pooling in one or multiple sectors and stages as well as degenerate equilibria where no stage-two innovation happens. I touch on these equilibria in appendix A.2 where I derive the parameter conditions on the PSBGE. For instance, if G is prohibitively high, one can imagine a case where no second-stage innovation is conducted. In such a case, the model becomes equivalent to a model with expanding product varieties such as Foellmi & Zweimueller (2006). In such equilibria, higher inequality unambiguously increases growth. The reason why I do not discuss these equilibria in detail is because only the PSBGE matches the pattern of luxury goods becoming necessities over time that I have outlined in the section on empirical regularities.

5 Beyond the core model

Quality growth

In the previous section I have assumed constant quality over time. As summarized in the section on empirical regularities, the quality of products satisfying the same need grow over time. Early cars or mobile phones were not only exclusive and affordable to a small subset of the population. They were also of very low quality compared to the mobile phones and cars purchased by the general population today. To account for quality growth I assume that quality at a point in time grows with the stock of technology through some spillover, so $q_k(t, j^s) = q_k^s A(t) = q_k^s e^{gt}$ for $k \in \{h, l\}$. Thus, it is possible to account for a situation where the low quality of a product exceeds the initial high quality after some time, that is, $q_l(\hat{t}, j^s) > q_h(t, j^s)$ for $\hat{t} > t$. As for balanced growth, $\mu_i^s q_k^s$ needs to be constant, so one can define a de-trended marginal willingness to pay $\hat{\mu}_i^s$ and proceed as before. The only difference is the Euler Lagrange equation where balanced growth now requires consumption to grow at $2g$ instead of g. Hence the equilibrium interest rate is higher at $r = 2g + \rho$.

Sectoral divergence

This extension covers sector-specific growth. Until now, the stock of technology was assumed to be common cross sectors. Assume now, however, that knowledge becomes sector specific, so firms can use insights from new mass products (m-sector) only for process innovation purposes or for the development of new m-type products. Equivalently, in the quality sector (e-sector) the only knowledge that can be used in R&D stems from past activities in e. For simplicity, assume that technical progress depends only on stage-one innovation (product innovation), the sector specific stock of knowledge evolves by

$$A^s(t) = \psi^s N^s(t) \quad \forall s \in S, \tag{5.1}$$

where $\psi^s \in \mathbb{R}_+$. Let costs be inversely related to $A^s(t)$, so $\tilde{F}^s(t) = \frac{F^s}{A^s(t)}$ and equivalently for $\tilde{G}^s(t)$, $\tilde{a}_k^s(t)$ with $k \in \{h, l\}$, and let the new numeráire be the efficiency wage in m. Thus, $\frac{w(t)}{A^m(t)} \equiv 1$, and therefore in e the efficiency wage is $\frac{w(t)}{A^e(t)} = \frac{A^m(t)}{A^e(t)}$. Let me denote the sector-specific growth rate as $g^s(t) = \frac{\dot{A}^s(t)}{A^s(t)}$ and consider a simplified example. Assume that initial stocks of technology in sectors s are normalized at unity $A^m(0) = A^e(0) = 1$, and that $\psi^e = 0$ while $\psi^m > 0$, so the mass production sector becomes increasingly productive while the quality sector does not. This setting can formalize the cost disease described by Baumol (1967). That is, there is a rise in wages and prices in a sector that has experienced low or no productivity growth in response to rising wages and prices in another sector that has experienced high labor productivity growth. Given initial technology stocks normalized to unity, the cost faced by a stage-one producer in e is given by $\tilde{a}_h^e(t)w(t) = a_h^e A^m(t)$. Then, instantaneous profits in e are $\pi_1^e(t) = (1-\beta)[q_l^e \mu_R^e(t) - a_h^e A^m(t)]L$ so the real variable cost of production are growing in e with the same rate that they are falling in m (the growth rate of $A^m(t)$). For the no-arbitrage condition to hold prices $p_1^e = q_l^e \mu_R^e(t)$ must be growing. This is not only despite but because there is no productivity improvement. Relative to m, sector e becomes ever less cost-efficient.

Structural change

This setting can be extended to allow for structural transformation of the economy. To do so, one has to abandon the assumption of constant sectoral expenditure shares implied by the Cobb-Douglas aggregator. A tractable way to allow for changing sectoral weights is introducing a CES-framework. If consumption from both sectors enter the utility aggregator as complements, one can generate an interesting pattern of structural change. Consider the following modification of equation (4.2):

$$X_i(t) = \left[\phi[X_i^e(t)]^{\frac{\sigma-1}{\sigma}} + (1-\phi)[X_i^m(t)]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{5.2}$$

where the Cobb-Douglas aggregator in the main text can be obtained by letting the elasticity of substitution $\sigma \to 1$. Now for the following discussion consider the case where $\sigma < 1$.[24] Taking the relative price ratio $\frac{p_1^e(t)}{p_1^m(t)}$ we know that $\frac{d[p_1^e(t)/p_1^m(t)]}{dt} > 0$. This implies that the expenditure share attributed by rich and poor households towards m is decreasing. For example, let me define the expenditure share of consumer of type R and P attributed to s-type products as

$$\eta_R^s(t) = \frac{p_1^s(t)N^s(t) + p_h^s(t)Z^s(t)}{\mathfrak{I}_R(t)} \quad \text{and} \quad \eta_P^s(t) = \frac{p_l^s(t)Z^s(t)}{\mathfrak{I}_P(t)} \tag{5.3}$$

where the expenditure identity $\eta_i^m(t) + \eta_i^e(t) = 1$ has to hold for all t and i. It is straightforward to show that in fact $\sigma < 1$ implies decreasing expenditure shares for m-type products over time. For instance, consider the relative expenditures between the two sectors, $\frac{\eta_P^e(t)}{\eta_P^m(t)} = \frac{p_l^e(t)Z^e(t)}{p_l^m(t)Z^m(t)}$. To isolate the relative price effect (substitution effect) take the derivative with respect to relative prices

$$\frac{\partial[\eta_P^e(t)/\eta_P^m(t)]}{\partial[p_l^e(t)/p_l^m(t)]} = \frac{Z^e(t)}{Z^m(t)} + \frac{\partial[Z^e(t)/Z^m(t)]}{\partial[p_l^e(t)/p_l^m(t)]} \cdot \frac{p_l^e(t)}{p_l^m(t)} = (1-\sigma)\frac{Z^e(t)}{Z^m(t)}. \tag{5.4}$$

[24]For illustrative purposes one might think of a Leontief-aggregator $X_i(t) = \min\left\{\frac{\phi}{1-\phi}X_i^e(t), X_i^m(t)\right\}$.

Equation (5.4) is clearly positive if $\sigma < 1$.[25] For the growth path, since in general with CES-aggregation the optimality conditions for both sectors are no longer independent, the aggregate growth rate is now a weighted average of the sectoral growth rates - or equivalently a weighted average of expenditure shares on products of each type.[26] As the share of e-type products in the budget grows, the growth rate of the economy asymptotically moves towards the growth rate in e which is $g^e = 0$. Baumol's cost disease leads to a halt in economic growth.[27] Figure 15 illus-

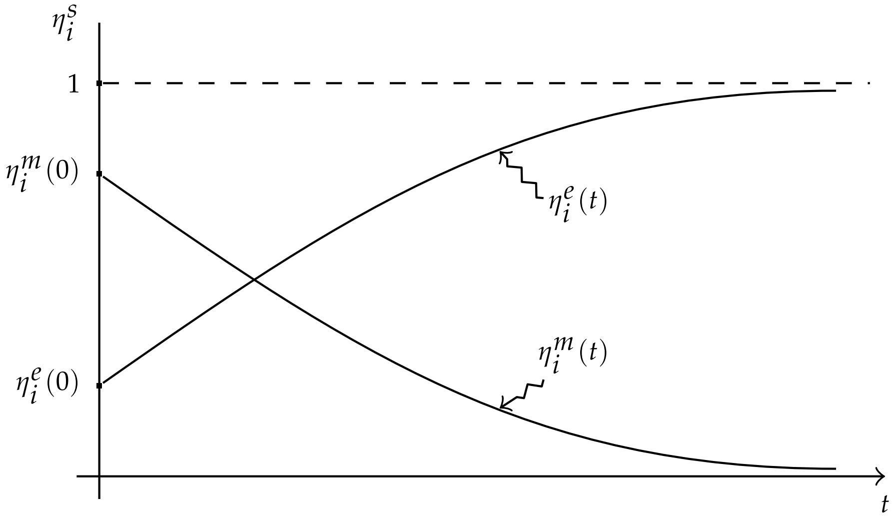

Figure 15: Expenditure shares, asymmetric growth, and structural change.

trates the structural transformation. In order to reconcile structural change with balanced growth on the aggregate, some special attention is needed. Boppart (2014) discusses the conditions under which structural change is consistent with balanced growth using non-homothetic utility and PIGL. Foellmi & Zweimueller (2008) rely on hierarchic preferences such that demand is asymptotically non-homothetic while there is balanced growth. Finally, Comin et al. (2018) propose a specification with a non-homothetic CES function. However, as argued in Alder et al. (2019), Comin et al.'s preferences are not inter-temporally aggregable.

[25] $\sigma = -\frac{\partial \ln[Z^e(t)/Z^m(t)]}{\partial \ln[p_i^e(t)/p_i^m(t)]}$ is a reduced-form measure of isoquant curvature. It is sometimes called the (inverse) Morishima elasticity of substitution (IME), and in general it need not be symmetric. Consider Comin et al. (2018, p.7) for a discussion. From a mathematical point of view, IME requires some special attention under non-homothetic utility (or non-homogenous production) functions. Yet, it is straightforward to define a *pseudo Morishima elasticity* in the same way. This pseudo elasticity then corresponds to the IME whenever the degree of homogeneity is one (For a discussion consider Baqaee & Farhi 2017, p.9).

[26] A way to generate sectoral independence is to modify the instantaneous utility in (4.1) by

$$U_i(0) = (1-\varepsilon)^{-1} \int_0^{\infty} [X_i(t)]^{1-\varepsilon} \, e^{-\rho t} \, dt,$$

to get the standard CRRA case. Sectoral independence can be obtained if one sets $\varepsilon = \frac{1}{\sigma}$.

[27] Note that for $\sigma > 1$, the economy shifts resources away from the cost-inefficient sector e. It relies increasingly on mass production. Growth accelerates until it asymptotically reaches the growth rate of productivity in m.

Falling prices and expanding product portfolios

The final extension is a model with more than two types and qualities. It accounts for an increasing number of products within each firm and simultaneously decreasing prices of given qualities. For simplicity, set $\phi = 0$ (Model-T specification). Assume there is a set $I = (1, \ldots, n)$ of discrete types with distribution characterized by $\beta_i \in (0, 1)$ and $\sum_{i=1}^{n} \beta_i = 1$, and a vector of endowments $\vec{\theta} = (\theta_1, \ldots, \theta_n)$ with $0 < \theta_1 < \cdots < \theta_i < \cdots < \theta_{n-1} < \frac{1-\sum_{i=1}^{n-1} \beta_i \theta_i}{1-\sum_{i=1}^{n-1} \beta_i} = \theta_n$.[28] Firms enter the market as before and sell the highest quality q_K to the richest group of consumers. After having entered, there is the possibility to conduct $K \leqslant n$ process innovations. Let $\vec{q} = (q_K, \ldots, q_1)$ be an ordered vector of qualities with $q_K > \cdots > q_k > \cdots > q_1$ and $k \in \{1, \ldots, K\}$. The associated cost vector is $\vec{a}$ such that $\frac{q_K}{a_K} < \cdots < \frac{q_1}{a_1}$ implying that efficiency improves with every step. Marginal willingnesses to pay are $\mu_1 < \cdots < \mu_i < \cdots < \mu_n$ by lemma 1. Moreover, assume that the shares of types are such

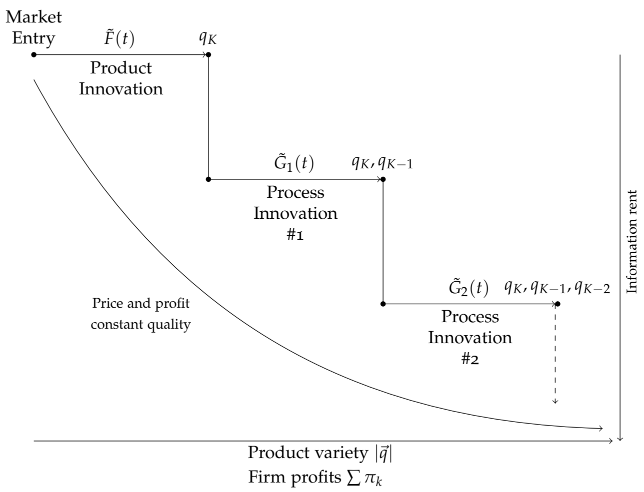

Figure 16: Falling prices and expanding product portfolios.

that separating is profitable in every stage, that is for example, if a firm has two qualities K and $K-1$ available, it sells the highest to type n and the other to $n-1$ while pricing all other types out of the market. For every new quality the monopolist introduces, the information rent attributed to higher types increases. For instance, suppose that the economy has the three highest qualities $\{K, K-1, K-2\}$ available. Comparing prices between the stages gives

$$q_K \mu_n > q_{K-1}\mu_{n-1} + (q_K - q_{K-1})\mu_n > q_{K-2}\mu_{n-2} + (q_{K-1} - q_{K-2})\mu_{n-1} + (q_K - q_{K-1})\mu_n.$$

which can be shown to be true by lemma 1 and $\mu_1 < \cdots < \mu_i < \cdots < \mu_n$. Interpreting different qualities as distinct (perfectly substitutable) products, the set of products supplied by each firm expands and the price for a product of given quality decreases over time as new vintages arrive.

[28] Foellmi & Zweimueller (2017) discuss a similar model with a continuum of types.

6 Discussion

6.1 Accounting for empirical regularities

The first empirical regularity is that *the number of products supplied by a firm is increasing in its lifetime, and the number of firms increases over time.* This is in fact the case. In the PSBGE, the number of firms increases at constant rate g through horizontal innovation. For each firm, the number of qualities is restricted to two but if one interprets qualities as sufficiently different products, the number of products is expanding in a firm's lifetime. This restriction to two qualities is without loss of generality. As illustrated in the extension, the setup can be generalized for K qualities. Moreover, regarding the expanding set of products by firms, recall that the sector separation is a formal not a conceptual one. In particular, if aggregation is Cobb-Douglas such that the elasticity of substitution is one and qualities are $q_h^m = q_l^e$ (that is $\gamma = 1$), one might as well interpret the setup as admitting a firm to supply two different product lines (e and m) of the same product family.

Additionally, the evidence suggests that *many products are initially only affordable to affluent consumers* before they become broadly available through quality innovation or mass production. Matching these observation in facts (ii) and (iii) is the reason why I introduce the PSBGE (and the necessary parameter restrictions). If the share of rich households is sufficiently high, firms will price poorer consumers out of the market for new products and later conduct a process or quality innovation to appropriate the idle surplus. Note that the symmetry of the timing problem in (4.9) implies that the product cycle (the timing of an innovation) remains in-determined. Introducing asymmetries either through learning-by-doing (supply-side) or hierarchic preferences (demand-side) can yield deterministic product cycles (Foellmi et al. 2014). This offers a pathway to matching empirical regularities of the product cycle with a quantitative model.

Regardless of the indeterminacy of the product cycle, the model qualitatively captures that *the price for a product of a given quality is decreasing over time* holding quality constant. Different to the conjecture by Argente et al. (2019, p.15), however, it is not consumer tastes for *newness* that cause decreasing prices. Rather, prices decrease because monopolists appropriate higher information rents to wealthy types in order to keep them from mimicking. Again, this becomes particularly apparent in the extension to multiple qualities.[29]

The final empirical regularity poses that *for the majority of products quality increases over time* such

[29]Given that prices decrease with the information rent, so do profits on the balanced growth path (holding quality constant). This is because costs in efficiency units are constant while with every new version (quality) of an existing product, the price effect eats away part of a firm's margin on the higher (more luxurious) qualities.

that even the most inferior product eventually surpasses the initial luxury product. This can be achieved by including a simple trend in the quality level through a spillover. For firms' innovation decisions, a trend in quality does not change incentives - which is why I have abstracted from it.

6.2 Mechanisms of the model

Innovation within each firm is driven by the *surplus appropriation effect*. Firms introduce new qualities of existing products to price-discriminate consumers based on their income (wealth). Whether a firm relies on process or quality innovation does not matter insofar as incentives to innovate are determined by the market size effect and the price effect. In particular, asymmetric information implies that by widening the market, the firm forgoes part of its profits from rich consumers in favor of selling to a wider market. However, if the surplus appropriation effect drives intra-firm innovation, what is the conceptual difference between quality and process innovation?

First, the two innovations place different restrictions on implementability and hence on firms' price-setting behavior. Recall that with a process innovation, marginal costs need to decrease more than quality such that $\frac{q_l^m}{a_l^m} > \frac{q_h^m}{a_h^m}$. For sector m, moving to stage-two implies that the fraction of goods produced in low quality put less pressure on the economy's resources than the high-quality production. There are productivity gains but they are limited to the low quality. For sector e, however, there is a productivity improvement inherent in the production of both qualities. In stage one the quality-to-cost ratio is $\frac{q_l^e}{a_h^e}$, but in stage two it is $\frac{q_h^e}{a_h^e} > \frac{q_l^e}{a_h^e}$ and $\frac{q_l^e}{a_l^e} > \frac{q_h^e}{a_h^e}$. This is because quality innovation entails a process innovation in this model. If this were not the case, there would not exist a separating equilibrium. That is, if it is optimal to price out poor types of the market in stage one where the monopolist manufactures $\frac{q_l^e}{a_h^e}$, it surely will be the case as well in stage two where $\frac{q_h^e}{a_h^e}$. Thus, if quality innovation does not induce cost savings in the production of the low quality, it may never become affordable to poor consumers.

Second, process and quality innovation have different implications for resource utilization. For firms conducting process innovations, costs have to fall at a higher rate than quality while for firms conducting quality upgrading, quality has to grow faster than costs. If one only focuses on the quality-to-cost ratio as the main driver to innovation, there is still some isomorphism between the types of innovation in this model. However, in terms of the resource utilization, e is more efficient than m. Quality innovation implies an efficiency gain for both products while process innovation only does so for the low quality.[30]

[30]The standard vertical growth model, process innovation (reduction in marginal costs) is passed through to prices through a falling mark-up. In my model, it is again the price-discrimination behavior that is responsible for falling prices.

7 Conclusion

I study a long-run growth model with non-homothetic preferences and demand-side inequality that induces innovation along three dimensions: Firms undertake product innovation to enter the market, they improve upon the production process of existing products, or they upgrade their quality. Unlike in models of quasi-homothetic preferences, process and quality innovation are no longer isomorphic, and the demand side dictates incentives to innovate. If economic growth is mainly driven by quality or process innovation, a lower income gap encourages growth while a higher income concentration has an ambiguous effect. If the economy relies on product innovation, a higher income gap has positive effects. Finally, the model accounts for a set of empirical regularities: (i) many products are initially only affordable to affluent consumers and then become more broadly affordable either through process or quality innovation, (ii) product cycles are characterized by decreasing prices, (iii) the variety of products is expanding in a firms lifetime, and (iv) the average quality of products satisfying a particular need increases over time. The model lends itself to the study of structural change, mass production, and endogenous product cycles. In particular, developing a model of structural change with quality and process innovation seems a promising route for future research.

A Mathematical appendix

A.1 Consumer optimization

In this appendix, I discuss the consumer's problem in more general form. In particular, assume that the instantaneous utility is no longer binary but some function $v(\cdot)$ of the HARA class as specified in (3.3). I show how to obtain the optimality conditions in the text. It follows Seierstad & Sydsaeter (1987, p.362-3). Consumer i maximizes

$$U_i(0) = \int_0^\infty \left[\phi \ln \left(\int_0^{N^e(t)} v[x_i(t,j^e)q_i(t,j^e)]dj^e \right) + (1-\phi) \ln \left(\int_0^{N^m(t)} v[x_i(t,j^m)q_i(t,j^m)]dj^m \right) \right] e^{-\rho t}dt,$$

subject to the non-negativity constraint $x_i(t,j^s) \geqslant 0 \ \forall j^s, t, s$ and the inter-temporal budget constraint $\int_0^\infty E_i(t)e^{-R(t)}dt \leqslant v_i(0) + \int_0^\infty \theta_i w(t)e^{-R(t)}dt$, where $E_i(t) = \sum_{s\in S} \int_0^{N^s(t)} p(t,j^s,q_i^s)x_i(t,j^s)dj^s$ corresponds to total expenditure as on the right-hand side of (4.4). For P one needs to use $\theta_P = \theta$ and for R one needs $\theta_R = \frac{1-\beta\theta}{1-\beta}$. Additionally, impose the no-Ponzi-game condition $\lim_{t\to\infty} \exp[-R(t)]v(t) \geqslant 0$ on the inter-temporal budget constraint. The first-order conditions with respect to $x_i(t,j^s)$ are

$$e^{-\rho t}\frac{\phi v'(\cdot)}{X_i^e(t)}q_i(t,j^e) - \Lambda_i e^{-R(t)}p(t,j^e) + \vartheta(t,j^e) = 0 \quad \forall j^e, t,$$
$$e^{-\rho t}\frac{(1-\phi) v'(\cdot)}{X_i^m(t)}q_i(t,j^m) - \Lambda_i e^{-R(t)}p(t,j^m) + \vartheta(t,j^m) = 0 \quad \forall j^m, t,$$

where Λ_i denotes the present value Lagrange multiplier on the inter-temporal budget constraint, and $\vartheta(t,j^s)$ the Lagrange multiplier on the non-negativity constraints. Note that the current value multiplier (shadow value of wealth) is $\lambda_i(t) = \Lambda_i \exp[-R(t)+\rho t]$. Clearly, $\vartheta(t,j^s) \geqslant 0$ by complementary slackness. Moreover, the derivative of the HARA subutility is $v'(\cdot) = \left(\frac{\Xi x}{\sigma} - \bar{x}\right)^{-\sigma}$ as in (3.3). Now I can use the fact that binary preferences limit the HARA class for $\bar{x} = -1$, $\Xi = -\sigma$ and $\sigma \to \infty$. Evaluating at $v'(0) \equiv v'(1) - v'(0)$, the optimality conditions in the text can be obtained (exploiting there are only two qualities). That is,

$$\frac{\phi}{\lambda_i(t)X_i^e(t)}q_i(t,j^e) - p(t,j^e) \geqslant 0 \quad \forall j^e, t,$$
$$\frac{1-\phi}{\lambda_i(t)X_i^m(t)}q_i(t,j^m) - p(t,j^m) \geqslant 0 \quad \forall j^m, t.$$

Now define $\mu_i^e(t) \equiv \frac{\phi}{\lambda_i(t)X_i^e(t)}q_i(t,j^e)$ and $\mu_i^m(t) \equiv \frac{1-\phi}{\lambda_i(t)X_i^m(t)}q_i(t,j^m)$ for types $i \in \{R,P\}$. Finally for the Euler equation (4.5), differentiate the present value multiplier $\Lambda_i = \lambda_i(t)\exp[R(t)-\rho t]$ with respect to time to get $-\frac{\dot{\lambda}(t)}{\lambda(t)} = r(t) - \rho$. Using $\lambda_i(t) = \frac{\phi}{\mu_i^e(t)X_i^e(t)}$, and therefore

$$-\frac{\dot{\lambda}_i(t)}{\lambda_i(t)} = \frac{\phi\left[\frac{\dot{\mu}_i^e(t)}{(\mu_i^e(t))^2 X_i^e(t)} + \frac{\dot{X}_i^e(t)}{\mu_i^e(t)(X_i^e(t))^2}\right]}{\frac{\phi}{\mu_i^e(t)X_i^e(t)}} = \frac{\dot{\mu}_i^e(t)}{\mu_i^e(t)} + \frac{\dot{X}_i^e(t)}{X_i^e(t)} = r(t) - \rho$$

which holds *mutatis mutandis* for m and obviously for both $i \in \{R, P\}$. Notice that the Euler-Lagrange equation is sector-specific and does not feature any weights ϕ or $1-\phi$. This is because the inverse inter-temporal elasticity of substitution equals the rate of substitution between sectors.[31] Finally, the conditions above are in fact sufficient. The budget set is closed and bounded from above and below, hence compact by Heine-Borel. The objective is upper-semicontinuous, (strictly) concave and increasing. By Weierstrass' theorem a unique maximum exists.

A.2 Existence

The following proof is equivalent to Bolton & Dewatripont (2005). Let me first show equilibrium existence in m. Recall that $\frac{w(t)}{A(t)} = 1$ is the numeráire. First, a stage-one firm chooses between only selling to R or pooling. Assumption 1 requires $(1-\beta)(q_h^m\mu_R^m - a_h^m) \geqslant q_h^m\mu_P^m - a_h^m$. Rearranging gives

$$\begin{aligned} (\mu_R^m - \mu_P^m) + \beta\frac{a_h^m}{q_h^m} &\geqslant \beta\mu_R^m \\ (1-\beta)\mu_R^m q_h^m &\geqslant \frac{q_h^m}{q_l^m}\mu_P^m q_l^m - \beta a_h^m. \end{aligned} \tag{A.1}$$

Second, by the revelation principle a stage-two firm's program reads

$$\begin{aligned} (\mathcal{P}) \quad & \max_{(p_h^m, p_l^m)} \{\beta(p_l^m - a_l^m)L + (1-\beta)(p_h^m - a_h^m)L\} \\ \text{s.t. (IR.1)} \quad & p_h^m \leqslant q_h^m\mu_R^m \\ \text{(IR.2)} \quad & p_l^m \leqslant q_l^m\mu_P^m \\ \text{(IC.1)} \quad & p_h^m - \mu_R^m q_h^m \leqslant p_l^m - q_l^m\mu_R^m \\ \text{(IC.2)} \quad & p_l^m - q_l^m\mu_P^m \leqslant p_h^m - q_h^m\mu_P^m. \end{aligned}$$

[31] To see this consider the specification

$$U_i(0) = (1-\varepsilon)^{-1}\int_0^\infty \exp(-\rho t)\left(\left[\phi X_i^e(t)^{\frac{\sigma-1}{\sigma}} + (1-\phi)X_i^m(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)^{1-\varepsilon} dt$$

where utility is CRRA and the sectoral aggregator is CES. Setting $\frac{1}{\sigma} = \varepsilon$ gives sector-independence, and $\frac{1}{\sigma} = \varepsilon = 1$ is the case in this thesis.

(IR.1) and (IR.2) are individual rationality constraints and (IC.1) and (IC.2) are incentive compatibility constraints that ensure self-selection into the separating equilibrium. Now, consider the recursive solution strategy: (IR.2) must be active otherwise firms could increase profits from P without encouraging R to mimic P. But then (IC.1) must be active, too. Otherwise a firm could increase profits without violating (IR.1) and (IC.2). This implies together with $\mu_R^m > \mu_P^m$ that $q_h^m \mu_R^m - p_h^m \geqslant q_l^m \mu_R^m - p_l^m > q_l^m \mu_P^m = 0$ and therefore $q_h^m \mu_R^m - p_h^m > 0$, so (IR.1) is slack. But then (IC.2) cannot be binding either. To see this, rewrite the binding (IC.1) as $p_h^m - p_l^m = (q_h^m - q_l^m)\mu_R^m > (q_h^m - q_l^m)\mu_P^m$ which is obviously true as $\mu_R^m > \mu_P^m$. The constraint set on problem ($\mathcal{P}$) is therefore a subset of a constraint problem ($\mathcal{P}'$) where only (IR.2) and (IC.1) constrain the objective. The reduced program reads

$$
\begin{aligned}
(\mathcal{P}') \quad & \max_{(p_h^m, p_l^m)} \{\beta(p_l^m - a_l^m)L + (1-\beta)(p_h^m - a_h^m)L\} \\
\text{s.t. (IR.2)} \quad & p_l^m \leqslant q_l^m \mu_P^m \\
\text{(IC.1)} \quad & p_h^m - \mu_R^m q_h^m \leqslant p_l^m - q_l^m \mu_R^m.
\end{aligned}
$$

Setting up the virtual surplus equation confirms that this problem is indeed strictly concave (affine) on the constraint set defined by (IR.2) and (IC.1). Moreover, it is also closed and bounded from above and below, hence compact by the Heine-Borel theorem. Via Weierstrass' (extreme value) theorem a unique maximum exists and the Karush-Kuhn-Tucker conditions yield both necessary and sufficient condition for a maximum. Solving the Lagrangian problem yields prices (4.7). I refer to Bolton & Dewatripont (2005) for the steps. The final step is to prove that a stage-two producer does not want to choose a strategy other than separating pricing. First, note that $\mu_R^m > \mu_P^m$ implies that a firm can never sell l to R while selling h to P. Second, I denote the profits of the remaining alternative options by $\pi_{k,k}$ where the first subscript refers to the quality sold to R and the second to P. These options are:

(i) Serve R-types at quality h by setting prices equal to $\mu_R^m(t)q_h^m$ and exclude P-types; Profits are $\pi_{h,0} = (1-\beta)(q_h^m \mu_R^m - a_h^m)L$.

(ii) Serve R-types at quality l by setting prices equal to $\mu_R^m(t)q_l^m$ and exclude P-types; Profits are $\pi_{l,0} = (1-\beta)(q_l^m \mu_R^m - a_l^m)L$.

(iii) Serve both types at the marginal utility of P-types at quality h; Profits are $\pi_{h,h} = (q_h^m \mu_P^m - a_h^m)L$.

(iv) Serve both types at the marginal utility of P-types at quality l; Profits are $\pi_{l,l} = (q_l^m \mu_P^m - a_l^m)L$.

These pricing options translate into the following set of inequalities:

$$\beta(p_l^m - a_l^m) + (1-\beta)(p_h^m - a_h^m) > (1-\beta)(p_1^m - a_h^m)$$
$$\beta(p_l^m - a_l^m) + (1-\beta)(p_h^m - a_h^m) > (1-\beta)(q_l^m \mu_R^m - a_l^m)$$
$$\beta(p_l^m - a_l^m) + (1-\beta)(p_h^m - a_h^m) > q_h^m \mu_P^m - a_h^m$$
$$\beta(p_l^m - a_l^m) + (1-\beta)(p_h^m - a_h^m) > q_l^m \mu_P^m - a_l^m.$$

Notice that in equilibrium one can replace $p_h^m = p_l^m + \frac{q_h^m - q_l^m}{q_h^m} p_1^m$. The last two inequalities are easy to check. Because of (A.1) pooling is not optimal on stage one. It is therefore never optimal in stage two. Hence, the last two inequalities hold given assumption 1. For the first inequality, rewriting the constraint gives

$$\beta(p_l^m - a_l^m) + (1-\beta)(p_h^m - a_h^m) > (1-\beta)(p_1^m - a_h^m)$$
$$\beta(p_l^m - a_l^m) + (1-\beta)\left[p_l^m + \frac{q_h^m - q_l^m}{q_h^m} p_1^m - a_h^m\right] > (1-\beta)(p_1^m - a_h^m)$$
$$p_l^m - (1-\beta)\frac{q_l^m}{q_h^m} p_1^m - \beta a_l^m > 0,$$

and for the second inequality,

$$\beta(p_l^m - a_l^m)L + (1-\beta)(p_h^m - a_h^m) > (1-\beta)(q_l^m \mu_R^m - a_l^m)$$
$$\frac{q_h^m - q_l^m}{q_h^m} p_1^m > a_h^m - a_l^m.$$

Hence, one obtains two additional constraints. And the full set of conditions is

$$(1-\beta)p_1^m - \frac{q_h^m}{q_l^m} p_l^m + \beta a_h^m > 0 \tag{A.2}$$
$$p_l^m - (1-\beta)\frac{q_l^m}{q_h^m} p_1^m - \beta a_l^m > 0 \tag{A.3}$$
$$\frac{q_h^m - q_l^m}{q_h^m} p_1^m > a_h^m - a_l^m. \tag{A.4}$$

Now equations (A.2) and (A.3) can be written as

$$(1-\beta)\frac{q_l^m}{q_h^m} p_1^m + \beta a_l^m < p_l^m < \frac{q_l^m}{q_h^m}(1-\beta)p_1^m + \beta \frac{q_l^h}{q_h^m} a_h^m$$
$$\frac{q_h^m}{a_h^m} < \frac{q_l^m}{a_l^m} \tag{A.5}$$

Now for sector e, the proof works *mutatis mutandis*. Notice that in stage one separation requires that $(1-\beta)(q_l^e\mu_R^e - a_l^e) > q_l^e\mu_P^e - a_l^e$. Crucially in stage two, the cost of producing quality h increases as well as the quality. The program for a stage-two firm in e is therefore exactly the same as for a stage-two firm in m. The profit constraints are

$$\begin{aligned}
&\beta(q_l^e\mu_P^e - a_l^e) + (1-\beta)[q_l^e\mu_P^e + (q_h^e - q_l^e)\mu_R^e - a_h^e] > (1-\beta)(\mu_R^e q_h^e - a_h^e)\\
&\beta(q_l^e\mu_P^e - a_l^e) + (1-\beta)[q_l^e\mu_P^e + (q_h^e - q_l^e)\mu_R^e - a_h^e] > (1-\beta)(q_l^e\mu_R^e - a_l^e)\\
&\beta(q_l^e\mu_P^e - a_l^e) + (1-\beta)[q_l^e\mu_P^e + (q_h^e - q_l^e)\mu_R^e - a_h^e] > q_h^e\mu_P^e - a_h^e\\
&\beta(q_l^e\mu_P^e - a_l^e) + (1-\beta)[q_l^e\mu_P^e + (q_h^e - q_l^e)\mu_R^e - a_h^e] > q_l^e\mu_P^e - a_l^e.
\end{aligned}$$

Condition one is fulfilled if

$$\begin{aligned}
q_l^e\mu_P^e - \beta a_l^e &> (1-\beta)q_l^e\mu_R^e\\
p_l^e - (1-\beta)p_1^e - \beta a_l^e &> 0.
\end{aligned}$$

Given that this inequality holds,[32] the second condition can be characterized by $(1-\beta)(\mu_R^e q_h^e - a_h^e) > (1-\beta)(q_l^e\mu_R^e - a_l^e)$ which is true if

$$\frac{q_h^e - q_l^e}{q_l^e}p_1^e > a_h^e - a_l^e$$

and therefore by transitivity parameters fulfill the second inequality as well. As above options (iii) and (iv) can be ruled out because pooling is never optimal. So, the existence conditions are

$$(1-\beta)p_1^e - p_l^e + \beta a_h^e > 0 \tag{A.6}$$

$$p_l^e - (1-\beta)p_1^e - \beta a_l^e > 0 \tag{A.7}$$

$$\frac{q_h^e - q_l^e}{q_l^e}p_1^e > a_h^e - a_l^e. \tag{A.8}$$

And as above, (A.6) and (A.7) can be condensed to $a_h^e > a_l^e$. Therefore I have derived the full set of conditions for the existence of a separating equilibrium. For completeness, it is known form mechanism design that the Spence-Mirrlees single crossing condition must be satisfied. This is the case if $q_h > q_l$ (Bolton & Dewatripont 2005). Lemma 2 summarizes.

[32] Notice that if $p_l^e - (1-\beta)p_1^e - \beta a_l^e > 0$ this implies $\beta(q_l^e\mu_P^e - a_l^e) + (1-\beta)[q_l^e\mu_P^e + (q_h^e - q_l^e)\mu_R^e - a_h^e] > (1-\beta)(\mu_R^e q_h^e - a_h^e)$

A.3 Equilibrium derivation

Starting with the the intra-temporal problem, I derive (4.17). Stage one profits are

$$\pi_1^e = (1-\beta)\,(p_1^e - a)\,L = \delta r F \quad \text{and} \quad \pi_1^m = (1-\beta)\left(p_1^m - \frac{a}{\delta}\right) L = rF. \tag{A.9}$$

From assumption 2 costs are $F^e = \delta F^m = \delta F$. Then, $\delta\pi_1^m = \delta r F = \pi_1^e$ and thus $\delta p_1^m = p_1^e$. Prices in stage one are (4.6). Using this with assumption 3 gives $\delta\mu_R^m\gamma = \mu_R^e$. From the optimality conditions, the ratios of marginal willingnesses are $\mu_i^e(t) \equiv \frac{\phi}{\lambda_i(t)X_i^e(t)}$ and $\mu_i^m(t) \equiv \frac{1-\phi}{\lambda_i(t)X_i^m(t)}$. Note that $\lambda_i(t)$ is independent of s. Therefore, for the (constant) marginal willingnesses of R-types one gets

$$\frac{\mu_R^m}{\mu_R^e} = \frac{1}{\delta\gamma} = \frac{1-\phi}{\phi}\frac{X_R^e(t)}{X_R^m(t)} \quad \Leftrightarrow \quad X_R^e(t) = \frac{\phi}{(1-\phi)\delta\gamma}X_R^m(t).$$

Plug in (4.8) and replace qualities as specified in assumption 3 to get

$$(1+\alpha)kZ^e(t) + [N^e(t) - Z^e(t)]k = \frac{\phi}{(1-\phi)\delta\gamma}\gamma k N^m(t) \quad \Leftrightarrow \quad z^e = \frac{1}{\alpha}\left[\frac{\phi}{(1-\phi)\delta}\frac{1}{n} - 1\right], \tag{A.10}$$

where $(n, z^e) = \left(\frac{N^e(t)}{N^m(t)}, \frac{Z^e(t)}{N^e(t)}\right)$. Given (A.10), proceed to stage-two profits and prices. These are

$$\begin{aligned}
\pi_2^e &= \beta(p_l^e - b)L + (1-\beta)(p_h^e - a)L = \delta r(F+G) \\
\pi_2^m &= \beta\left(p_l^m - \frac{b}{\delta}\right)L + (1-\beta)\left(p_h^m - \frac{a}{\delta}\right)L = r(F+G).
\end{aligned} \tag{A.11}$$

Multiplying π_2^m by δ and equating gives

$$\begin{aligned}
\beta(p_l^e - a) + (1-\beta)(p_h^e - a) &= \delta\beta\left(p_l^m - \frac{a}{\delta}\right) + \delta(1-\beta)\left(p_h^m - \frac{a}{\delta}\right) \\
\beta p_l^e + (1-\beta)p_h^e &= \delta\beta p_l^m + \delta(1-\beta)p_h^m.
\end{aligned} \tag{A.12}$$

Stage-two prices are defined in (4.7). Plugging quality increments from assumption 3 into pricing equation gives an expression for p_h^s in terms of (p_1^s, p_l^s). Moreover, from above one can replace p_1^e with δp_1^m. Price equations are

$$\begin{aligned}
p_h^e &= p_l^e + \left(\frac{q_h^e - q_l^e}{q_l^e}\right)q_l^e\mu_R^e = p_l^e + \left(\frac{(1+\alpha)k - k}{k}\right)q_h^e\mu_R^e = p_l^e + \alpha p_1^e = p_l^e + \alpha\delta p_1^m \\
p_h^m &= p_l^m + \left(\frac{q_h^m - q_l^m}{q_h^m}\right)q_h^m\mu_R^m = p_l^m + \left(\frac{\gamma k - (1-\alpha)\gamma k}{\gamma k}\right)q_h^m\mu_R^m = p_l^m + \alpha p_1^m.
\end{aligned} \tag{A.13}$$

Plugging prices into (A.12) gives

$$p_l^e + \alpha(1-\beta)\delta p_1^m = \delta p_l^m + \alpha(1-\beta)\delta p_1^m,$$

and therefore $p_l^e = \delta p_l^m$. Go back to the ratios of marginal willingness to pay, now for P. Prices are again set at $\mu_i^e(t) \equiv \frac{\phi}{\lambda_i(t)X_i^e(t)}$ in e and $\mu_i^m(t) \equiv \frac{1-\phi}{\lambda_i(t)X_i^m(t)}$ in m., so $p_l^m = \gamma(1-\alpha)k\mu_P^m$ and $p_l^e = k\mu_P^e$. Plugging in as above renders

$$\frac{\mu_P^m}{\mu_P^e} = \frac{1}{\gamma(1-\alpha)\delta} = \frac{1-\phi}{\phi}\frac{X_P^e(t)}{X_P^m(t)} \quad \Leftrightarrow \quad q_l^m Z^m(t) = \frac{(1-\phi)(1-\alpha)\gamma\delta}{\phi} q_l^e Z^e(t) \quad \Leftrightarrow \quad z^m = \frac{(1-\phi)\delta}{\phi} z^e n, \tag{A.14}$$

where $z^m \equiv \frac{Z^m(t)}{N^m(t)}$. Equations (A.10) and (A.14) are two equations in (n, z^m, z^e) and these are the ones to carry forward. Now consider the budget constraint of R and P in (4.4). The tuple (β, θ) characterizes the distribution on the BGP. As an R-type's endowment is $\frac{1-\beta\theta}{1-\beta}$ and a P-type's is θ, the ratio of incomes between R and P-types must be $\frac{1-\beta\theta}{(1-\beta)\theta}$. Hence, not only incomes are scaled by $\frac{1-\beta\theta}{(1-\beta)\theta}$ but also expenditures. Use this with (4.8) to get

$$\frac{1-\beta\theta}{(1-\beta)\theta} = \frac{[N^e(t) - Z^e(t)]p_1^e + Z^e(t)p_h^e + [N^m(t) - Z^m(t)]p_1^m + Z^m(t)p_h^m}{Z^e(t)p_l^e + Z^m(t)p_l^m},$$

which corresponds to (4.12). Expressing $(p_1^e, p_h^e, p_l^m, p_h^m)$ in terms of (p_1^m, p_l^m):

$$\begin{aligned}
\frac{1-\beta\theta}{(1-\beta)\theta} &= \frac{[\delta[N^e(t) - Z^e(t)] + N^m(t) - Z^m(t)]\, p_1^m + [Z^e(t)\delta + Z^m(t)](p_l^m + \alpha p_1^m)}{[Z^e(t)\delta + Z^m(t)]p_l^m} \\
\frac{1-\beta\theta}{(1-\beta)\theta} - 1 &= \left[\frac{\delta N^e(t) + N^m(t)}{\delta Z^e(t) + Z^m(t)} - (1-\alpha)\right]\frac{p_1^m}{p_l^m} \\
\frac{1-\theta}{(1-\beta)\theta} &= \left[\frac{\delta\frac{N^e(t)}{N^m(t)} + 1}{\delta\frac{Z^e(t)N^e(t)}{N^e(t)N^m(t)} + \frac{Z^m(t)}{N^m(t)}} - (1-\alpha)\right]\frac{p_1^m}{p_l^m}.
\end{aligned}$$

The next step is to derive (4.13). Plugging in the product percentages derived in (A.10) and (A.14):

$$\begin{aligned}
\frac{1-\theta}{(1-\beta)\theta} &= \left[\frac{\delta\left[\frac{\phi}{(1-\phi)\delta}N^m(t) - \alpha Z^e(t)\right] + N^m(t)}{\delta\frac{\phi}{(1-\phi)\delta}Z^m(t) + Z^m(t)} - (1-\alpha)\right]\frac{p_1^m}{p_l^m} \\
\frac{1-\theta}{(1-\beta)\theta} &= \left[\frac{\left(1+\frac{\phi}{1-\phi}\right)N^m(t) - \alpha\delta\left(\frac{\phi}{(1-\phi)\delta}\right)Z^m(t)}{\left(1+\frac{\phi}{1-\phi}\right)Z^m(t)} - (1-\alpha)\right]\frac{p_1^m}{p_l^m} \\
\frac{1-\theta}{(1-\beta)\theta} &= \left[\frac{N^m(t)}{Z^m(t)} - \alpha\phi - (1-\alpha)\right]\frac{p_1^m}{p_l^m} \\
\frac{1-\theta}{(1-\beta)\theta}\frac{p_l^m}{p_1^m} &= \frac{1}{z^m} - \alpha\phi - (1-\alpha),
\end{aligned}$$

where the last line corresponds to (4.13). This is an equation in z^m and the two remaining unknown prices (p_1^m, p_l^m). These prices can be expressed in terms of the (unknown) interest rate r. Reconsider (A.9). Solving π_1^m for p_1^m yields $p_1^m = \frac{rF}{(1-\beta)L} + \frac{a}{\delta}$, and solving π_2^m for p_l^m gives $p_l^m = \frac{r(F+G)}{L} + \left[\beta\frac{b}{\delta} + (1-\beta)\frac{a}{\delta}\right] - \alpha(1-\beta)p_1^m$. Taking the ratio

$$\frac{p_l^m}{p_1^m} = \frac{\frac{r(F+G)}{L} + \left[\beta\frac{b}{\delta} + (1-\beta)\frac{a}{\delta}\right] - \alpha(1-\beta)p_1^m}{p_1^m} = (1-\beta)\left[1 - \alpha + \frac{\delta rG + \beta bL}{\delta rF + (1-\beta)aL}\right].$$

Substitute into the expenditure share expression to eliminate relative prices,

$$\begin{aligned}
\frac{1-\theta}{(1-\beta)\theta}\frac{p_l^m}{p_1^m} &= \frac{1}{z^m} - \alpha\phi - (1-\alpha) \\
\frac{1-\theta}{(1-\beta)\theta}(1-\beta)\left[1 - \alpha + \frac{\delta rG + \beta bL}{\delta rF + (1-\beta)aL}\right] &= \frac{1}{z^m} - \alpha\phi - (1-\alpha) \\
\frac{1-\theta}{\theta}\left[\frac{\delta rG + \beta bL}{\delta rF + (1-\beta)aL}\right] &= \frac{1}{z^m} - \alpha\phi - \frac{1-\alpha}{\theta} \qquad \text{(A.15)}
\end{aligned}$$

where (A.15) is an equation in two unknowns (z^m, r). The final piece of the derivation is the aggregate resource constraint. From (4.10) it is

$$\begin{aligned}
A(t)L &= \dot{N}^e(t)F^e + \dot{N}^m(t)F^m + \dot{Z}^m(t)G^m + \dot{Z}^e(t)G^e + Y_h^e(t)a_h^e + Y_l^e(t)a_l^e + Y_h^m(t)a_h^m + Y_l^m(t)a_l^m \\
\frac{A(t)L}{N^m(t)} &= \left[\frac{\dot{N}^e(t)}{N^e(t)}\frac{N^e(t)}{N^m(t)}\delta F + \frac{\dot{N}^m(t)}{N^m(t)}F + \frac{\dot{Z}^m(t)}{Z^m(t)}\frac{Z^m(t)}{N^m(t)}G + \frac{\dot{Z}^e(t)}{Z^e(t)}\frac{Z^e(t)}{N^e(t)}\frac{N^e(t)}{N^m(t)}\delta G\right] + \frac{Y_h^e(t)a_h^e + Y_l^e(t)a_l^e + Y_h^m(t)a_h^m + Y_l^m(t)a_l^m}{N^m(t)}.
\end{aligned}$$

To find output costs, recall that in any sector there are $N^s(t)$ stage-one products and $Z^s(t)$ stage-two products which also equal output because of the binary choice structure of demand.

$$\begin{aligned}
&\text{for } m: \quad Y_h^m(t) = (1-\beta)[N^m(t) - Z^m(t)]\frac{a}{\delta}L + (1-\beta)Z^m(t)\frac{a}{\delta}L = (1-\beta)N^m(t)\frac{a}{\delta}L \quad \text{and} \quad Y_l^m(t) = \beta Z^m(t)\frac{b}{\delta}L \\
&\text{for } e: \quad Y_h^e(t) = (1-\beta)Z^e(t)aL \quad \text{and} \quad Y_l^e(t) = Y_{l,1}^e(t) + Y_{l,2}^e(t) = (1-\beta)[N^e(t) - Z^e(t)]aL + \beta Z^e(t)bL.
\end{aligned}$$

Summing within both sectors, quantities produced are $Y^s = (1-\beta)N^s a^s L + \beta Z^s b^s L$ where a^s is a or $\frac{a}{\delta}$, and the same for b^s. Plugging into the resource constraint and using (n, z^m, z^e) gives

$$\begin{aligned}
\frac{A(t)L}{N^m(t)} &= [n\delta F + F + z^m G + z^e n\delta G]\, g + [(1-\beta)\delta an + \beta\delta bz^e n + (1-\beta)a + \beta bz^m]\,\frac{L}{\delta} \\
&= (1 + n\delta)\left[Fg + (1-\beta)\frac{a}{\delta}L\right] + (z^m + \delta z^e n)\left[Gg + \beta\frac{b}{\delta}L\right] \\
&= \left[1 + \delta\left(\frac{\phi}{(1-\phi)\delta} - \frac{\alpha\phi}{(1-\phi)\delta}z^m\right)\right]\left[Fg + (1-\beta)\frac{a}{\delta}L\right] + \left(z^m + \delta\frac{\phi}{(1-\phi)\delta}z^m\right)\left[Gg + \beta\frac{b}{\delta}L\right] \\
&= \frac{1 - \alpha\phi z^m}{1-\phi}\left[Fg + (1-\beta)\frac{a}{\delta}L\right] + \frac{z^m}{1-\phi}\left[Gg + \beta\frac{b}{\delta}L\right] \\
\delta(1-\phi)\frac{A(t)L}{N^m(t)} &= \delta Fg + (1-\beta)aL + [\delta(G - \alpha\phi F)g + (\beta b - \alpha\phi(1-\beta)a)L]\, z^m.
\end{aligned}$$

The missing piece is the de-trended technology stock $\frac{A(t)L}{N^m(t)}$. I will do the derivation for the case where technology follows (4.16) but it is analogous for (4.15) and (4.19). Use $\frac{A(t)L}{N^m(t)} = \psi_Z \left[\frac{\phi}{(1-\phi)\delta}\right]^{\zeta_Z} z^m L$ on the left-hand side of the resource constraint. This yields the desired expression for z^m depending only on g and model parameters.

$$
\begin{aligned}
\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}z^m L &= \delta F g + (1-\beta)aL + [\delta(G-\alpha\phi F)g + (\beta b - \alpha\phi(1-\beta)a)L]\, z^m \\
z^m &= \frac{\delta F g + (1-\beta)aL}{\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z} + (\alpha\phi(1-\beta)a - \beta b)\right] L + \delta(\alpha\phi F - G)g}.
\end{aligned} \tag{A.16}
$$

Substitute the result for z^m into (A.15) to get

$$
\begin{aligned}
\frac{1-\theta}{\theta}\frac{\delta r G + \beta b L}{\delta r F + (1-\beta)aL} &= \frac{\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z} + (\alpha\phi(1-\beta)a - \beta b)\right] L + \delta(\alpha\phi F - G)g}{\delta F g + (1-\beta)aL} - \alpha\phi - \frac{1-\alpha}{\theta} \\
\frac{1-\theta}{\theta}\frac{\delta r G + \beta b L}{\delta r F + (1-\beta)aL} &= \frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z} L - \beta b L - \delta G g}{\delta F g + (1-\beta)aL} - \frac{1-\alpha}{\theta}.
\end{aligned}
$$

Notice that the left-hand side depends only on the interest rate r and the right-hand side only on the growth rate g. Define a function $\chi_Z(g)$ equal to the latter and solve for r. This gives

$$
\chi_Z(g) = \frac{\theta}{1-\theta}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z} L - \beta b L - \delta G g}{\delta F g + (1-\beta)aL} - \frac{1-\alpha}{1-\theta}, \tag{A.17}
$$

and thus

$$
\begin{aligned}
\delta r G + \beta b L &= \chi_Z(g)[\delta r F + (1-\beta)aL] \\
r_Z^R(g) &= \frac{\chi_Z(g) - \frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F} - \chi_Z(g)} \frac{(1-\beta)aL}{\delta F}.
\end{aligned} \tag{A.18}
$$

(A.18) corresponds to (4.17). This equation is by derivation consistent with the definition of the PSBGE. Together with the solution to the inter-temporal problem (4.18) this defines equilibrium. To get (4.18) use $\dot{\mu}_i^s(t) = 0$ and $\dot{X}_i^s(t)/X_i^s(t) = g$ in (4.5) which gives

$$
r_Z^E(g) = g + \rho. \tag{A.19}
$$

A.4 Proofs

A.4.1 Proof of proposition 2

The proof consists of three parts. Let $g \in \mathcal{G} \subset \mathbb{R}_+$. Show that: (i) $r_Z^R(g)$ is increasing and strictly convex $\forall g \in \mathcal{G}$, (ii) $r_Z^R(g) > 0$ $\forall g \in \mathcal{G}$; and (iii) $\exists g^* \in \mathcal{G}$ that is a fixed point of $H_Z : \mathcal{G} \to \mathbb{R}$ where

$H_Z(g) := r_Z^R(g) - \rho$, so $r_Z^R(g^*) = r_Z^E(g^*)$ and thus $g^* = r_Z^R(g^*) - \rho$. Before starting, I want to provide two comments. First, I restrict the domain of the growth rate to $g \in \mathcal{G} = [0,1]$. The proof below also works for larger domains of the growth rate, but for $g \to \infty$ the separating equilibrium can break down. Additionally, anticipating a quantitative example, it seems apt to require $g \in \mathcal{G} = [0,1]$. Second, given that $g^* \in [0,1]$, I replace requirement (iii) by a simple application of the intermediate value theorem: (iii') Define $\tilde{H}_Z[0,1] \to \mathbb{R}$ where $\tilde{H}_Z(g) := r_Z^E(g) - r_Z^R(g) = g + \rho - r_Z^R(g)$. Show that $0 < r_Z^R(0) < r_Z^E(0) = \rho$ and $r_Z^R(1) > r_Z^R(1) = 1 + \rho$. Third, numerical simulations show that an equilibrium can only be constructed if the following condition holds:

Assumption 4 (Equilibrium condition I)**.**

$$\frac{G}{F} < \frac{\beta}{1-\beta}\frac{b}{a}$$

The reasons for this will become clear during the proof. This condition is also implicitly present in Foellmi et al. (2014, p.646). Albeit it is not mentioned in the necessary conditions. The intuition behind it is roughly this: $\frac{\beta}{1-\beta}$ is the ratio of R and P-type consumers in the economy, while $\frac{a}{b}\frac{G}{F}$ is the relative costs in the two stages. When it is relatively much more expensive to conduct a stage-two innovation (high G) and there would be very few P-types (low β), the separating equilibrium would break down. One would either end up in a degenerate equilibrium with no innovation at all or in a case where the model is unsolvable with separating pricing. Now, turning to the proof:

(i) I start by proving that $r_Z^R(g)$ is increasing and strictly convex on $\mathcal{G} = [0,1]$. I impose the ad-hoc constraint

$$\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L - \left[\frac{\beta}{1-\beta}\frac{b}{a} - \frac{G}{F}\right](1-\beta)aL > 0. \tag{A.20}$$

This constraint becomes redundant when I move to proving (ii) but for now assume it holds. A sufficient condition for a twice (Fréchet) differentiable function to be increasing and strictly convex is that $\frac{\partial r_Z^R(g)}{\partial g} > 0$ and $\frac{\partial^2 r_Z^R(g)}{\partial g^2} > 0$. Taking the derivatives gives

$$\frac{\partial r_Z^R(g)}{\partial g} = \frac{\overbrace{\frac{G}{F} - \frac{\beta}{1-\beta}\frac{b}{a}}^{<0}}{[G - \chi_Z(g)F]^2} \cdot \frac{(1-\beta)aLF}{\delta} \cdot \frac{\partial \chi_Z(g)}{\partial g}.$$

Moreover, $\frac{\partial \chi_Z(g)}{\partial g}$ is given by

$$\begin{aligned}\frac{\partial \chi_Z(g)}{\partial g} &= -\frac{\theta}{1-\theta}\frac{[\delta Fg+(1-\beta)aL]\,\delta G+\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg\right]\delta F}{[\delta Fg+(1-\beta)aL]^2}\\ &= -\frac{\theta}{1-\theta}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}-\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)a}{[\delta Fg+(1-\beta)aL]^2}\delta FL<0.\end{aligned}$$

Negativity follows from the ad-hoc constraint (A.20). This clearly implies that $\frac{\partial r_Z^R(g)}{\partial g}>0$, so $r_Z^R(g)$ is in fact increasing in g. Similarly, for convexity take the second derivative

$$\frac{\partial^2 r_Z^R(g)}{\partial g^2}=\frac{(1-\beta)FaL}{\delta}\frac{\partial^2\chi_Z}{\partial g^2}\frac{\overbrace{\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}}^{<0}}{[G-\chi_Z(g)F]^2}+(-2)(-F)\underbrace{\frac{\overbrace{\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}}^{<0}}{[G-\chi_Z(g)F]^3}}_{<0}\left(\frac{\partial\chi_Z(g)}{\partial g}\right)^2\frac{(1-\beta)FaL}{\delta}$$

As the second summand is unambiguously positive, curvature depends on $\frac{\partial^2\chi_Z}{\partial g^2}$:

$$\begin{aligned}\frac{\partial^2\chi_Z(g)}{\partial g^2} &= -\frac{\theta}{1-\theta}\frac{[\delta Fg+(1-\beta)aL]\,\delta G+\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg\right]\delta F}{[\delta Fg+(1-\beta)aL]^3}(-2\delta F)\\ &= -\frac{2\delta F}{\delta Fg+(1-\beta)aL}\underbrace{\frac{\partial\chi_Z(g)}{\partial g}}_{<0}>0\end{aligned}$$

So,

$$\begin{aligned}\frac{\partial^2 r_Z^R(g)}{\partial g^2} &= \frac{(1-\beta)FaL}{\delta}\frac{\overbrace{\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}}^{<0}}{[G-\chi_Z(g)F]^2}\left[\frac{\partial^2\chi_Z}{\partial g^2}+\frac{2F}{[G-\chi_Z(g)F]}\left(\frac{\partial\chi_Z(g)}{\partial g}\right)^2\right]>0\\ &\Leftrightarrow \frac{\partial^2\chi_Z}{\partial g^2}+\frac{2F}{[G-\chi_Z(g)F]}\left(\frac{\partial\chi_Z(g)}{\partial g}\right)^2<0\\ &\Leftrightarrow -\frac{2\delta F}{\delta Fg+(1-\beta)aL}\underbrace{\frac{\partial\chi_Z(g)}{\partial g}}_{<0}+\frac{2F}{[G-\chi_Z(g)F]}\left(\frac{\partial\chi_Z(g)}{\partial g}\right)^2<0\\ &\Leftrightarrow -\frac{\delta}{\delta Fg+(1-\beta)aL}+\underbrace{\frac{\frac{\partial\chi_Z(g)}{\partial g}}{G-\chi_Z(g)F}}_{<0}>0\\ &\Leftrightarrow \frac{\partial\chi_Z(g)}{\partial g}<\frac{\delta[G-\chi_Z(g)F]}{\delta Fg+(1-\beta)aL}\\ &\Leftrightarrow -\frac{\theta}{1-\theta}\frac{[\delta Fg+(1-\beta)aL]\,\delta G+\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg\right]\delta F}{[\delta Fg+(1-\beta)aL]^2}<\frac{\delta[\frac{G}{F}-\chi_Z(g)]F}{\delta Fg+(1-\beta)aL}\end{aligned}$$

and thus

$$
\begin{aligned}
-\frac{\theta}{1-\theta}\left\{[\delta F g+(1-\beta)aL]\,\delta G+\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg\right]\delta F\right\}&\\
<\delta\left[\frac{G}{F}-\left(\frac{\theta}{1-\theta}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg}{\delta Fg+(1-\beta)aL}-\frac{1-\alpha}{1-\theta}\right)\right][\delta Fg+(1-\beta)aL]F&\\
-\frac{\theta}{1-\theta}[\delta Fg+(1-\beta)aL]\,\delta G-\frac{\theta}{1-\theta}\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg\right]\delta F&\\
<\delta\left[\frac{G}{F}+\frac{1-\alpha}{1-\theta}\right][\delta Fg+(1-\beta)aL]F-\frac{\theta}{1-\theta}\left(\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg\right)\delta F&
\end{aligned}
$$

Cancel terms to get

$$
\begin{aligned}
-\frac{\theta}{1-\theta}[\delta Fg+(1-\beta)aL]\,\delta G&<\delta\left[\frac{G}{F}+\frac{1-\alpha}{1-\theta}\right][\delta Fg+(1-\beta)aL]F\\
0&<\frac{1}{1-\theta}\frac{G}{F}+\frac{1-\alpha}{1-\theta}
\end{aligned}
$$

Therefore $r^R(g)$ is strictly convex and increasing on its domain. I have proved requirement (i).

(ii) The second part is to prove that $r^R_Z(g)>0$ for $\mathcal{G}=[0,1]$. Reconsider (A.18) and assumption 4. These two together imply that any admissible $\chi_Z(g)$ in (A.17) must satisfy $\frac{G}{F}<\chi_Z(g)<\frac{\beta}{1-\beta}\frac{b}{a}$ and therefore it must hold that $\chi_Z(g)>0$. As I have shown in (i), $\chi_Z(g)$ is strictly decreasing in g. Therefore to show $\frac{G}{F}<\chi_Z(g)$ it is enough to show that the smallest admissible value of $\chi_Z(g)$ satisfies this requirement. Hence, I need to show that $\frac{G}{F}<\chi_Z(1)$, which requires

$$
\begin{aligned}
\frac{G}{F}&<\chi_Z(1)\\
\frac{G}{F}&<\frac{\theta}{1-\theta}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta G}{\delta F+(1-\beta)aL}-\frac{1-\alpha}{1-\theta}\\
\psi_Z&>\frac{\delta[G+(1-\alpha)F]+\left[(1-\theta)\frac{G}{F}+(1-\alpha)+\frac{\beta}{1-\beta}\frac{b}{a}\theta\right](1-\beta)aL}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L}
\end{aligned}
\tag{A.21}
$$

Equivalently, for $\frac{\beta}{1-\beta}\frac{b}{a}>\chi_Z(g)$ it suffices to show that the largest admissible value of $\chi_Z(g)$ satisfies this requirement. Hence, I need to show that $\frac{\beta}{1-\beta}\frac{b}{a}>\chi_Z(0)$, which requires

$$
\begin{aligned}
\frac{\beta}{1-\beta}\frac{b}{a}&>\chi_Z(0)\\
\frac{\beta}{1-\beta}\frac{b}{a}&>\frac{\theta}{1-\theta}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL}{(1-\beta)aL}-\frac{1-\alpha}{1-\theta}\\
\psi_Z&<\frac{\left(\frac{\beta(1-\theta)}{\theta}\frac{b}{a}+\frac{(1-\alpha)(1-\beta)}{\theta}\right)aL+\beta aL}{[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L}\\
\psi_Z&<\frac{[\beta b+(1-\alpha)(1-\beta)a]\,L}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L}=\frac{\beta b+(1-\alpha)(1-\beta)a}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}}
\end{aligned}
$$

The requirement of $r^R_Z(g) > 0$ therefore restricts the admissible parameter range of ψ_Z to the set $\psi_Z \in \mathcal{K}_Z = \left(\frac{\delta[G+(1-\alpha)F]+\left[(1-\theta)\frac{G}{F}+(1-\alpha)+\frac{\beta}{1-\beta}\frac{b}{a}\theta\right](1-\beta)aL}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L}, \frac{\beta b+(1-\alpha)(1-\beta)a}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}} \right) \subset \mathbb{R}_{++}$. The final step is to prove that this set is non-empty, this is the case if

$$\begin{aligned}
\frac{\delta[G+(1-\alpha)F]+\left[(1-\theta)\frac{G}{F}+(1-\alpha)+\frac{\beta}{1-\beta}\frac{b}{a}\theta\right](1-\beta)aL}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L} &< \frac{\beta b+(1-\alpha)(1-\beta)a}{\theta[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}} \\
\delta(G+(1-\alpha)F)\frac{1}{L}+\left[(1-\theta)\frac{G}{F}+(1-\alpha)+\frac{\beta}{1-\beta}\frac{b}{a}\theta\right](1-\beta)a &< \beta b+(1-\alpha)(1-\beta)a \\
\delta[G+(1-\alpha)F]+(1-\beta)(1-\theta)\frac{G}{F}a &< \beta(1-\theta)bL \\
G+(1-\alpha)F &< (1-\beta)(1-\theta)\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right]\frac{a}{\delta}L \\
\frac{G}{F}+1-\alpha &< (1-\beta)(1-\theta)\underbrace{\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right]}_{>0}\frac{a}{\delta}\frac{L}{F}
\end{aligned}$$

This requirement is always satisfied if L is large enough. Formally, let this be the second parameter restriction I need to impose for existence.

Assumption 5 (Equilibrium condition II). *The interest rate equation $r^R_Z(g)$ is positive for all $g \in \mathcal{G} = [0,1]$ if and only if parameters be such that $\psi_Z \in \mathcal{K}_Z \subset \mathbb{R}_{++}$. The admissible set $\mathcal{K}_Z \neq \{\varnothing\}$ if and only if parameters are such that*

$$\frac{G}{F}+1-\alpha < (1-\beta)(1-\theta)\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right]\frac{a}{\delta}\frac{L}{F}$$

holds. This restriction is satisfied if L is large relative to F.

This completes (ii). It is easy to show that the conditions on ψ_Z imply that the ad-hoc constraint (A.20) always holds. Otherwise there would exist a $\tilde{g}$ for which $\chi_Z(\tilde{g}) < 0$ which can never be true. (A.20) is always fulfilled and therefore can be dropped.[33]

[33] Notice that equation (A.21) can be rewritten as

$$\begin{aligned}
&\theta\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L > \delta[G+(1-\alpha)F]+\left[(1-\theta)\frac{G}{F}+(1-\alpha)+\frac{\beta}{1-\beta}\frac{b}{a}\theta\right](1-\beta)aL \\
&\theta\left[\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)aL\right] > \delta\left[G+(1-\alpha)F\right]+(1-\alpha)(1-\beta)aL+\frac{G}{F}(1-\beta)aL \\
&\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)aL > \theta^{-1}\left[\delta\left[G+(1-\alpha)F\right]+(1-\alpha)(1-\beta)aL+\frac{G}{F}(1-\beta)aL\right]
\end{aligned}$$

of which the right-hand side is clearly positive, so

$$\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)aL > \theta^{-1}\left[\delta\left[G+(1-\alpha)F\right]+(1-\alpha)(1-\beta)aL+\frac{G}{F}(1-\beta)aL\right] > 0$$

(iii) Finally, I show there exists a unique intersection of r_Z^R and r^E. Define $\tilde{H}_Z : [0,1] \to \mathbb{R}$ with

$$\tilde{H}_Z(g) := r^E(g) - r_Z^R(g) = g + \rho - \frac{\chi_Z(g) - \frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F} - \chi_Z(g)} \frac{(1-\beta)aL}{\delta F}. \tag{A.22}$$

Notice that by the above derivation, $\tilde{H}_Z(g)$ is continuous on the compact Euclidean subset $\mathcal{G}$. Moreover, $r_Z^R(g)$ is continuous, increasing, and strictly convex on $\mathcal{G}$ while $r^E(g)$ is affine and increasing. Sufficient conditions for the existence of a *unique* fixed point are

$$\tilde{H}_Z(0) = \rho - r_Z^R(0) > 0 \qquad \Leftrightarrow \qquad \frac{(1-\beta)\chi_Z(0) - \beta\frac{b}{a}}{G - \chi_Z(0)F}\frac{aL}{\delta} < \rho$$
$$\tilde{H}_Z(1) = 1 + \rho - r_Z^R(1) < 0 \qquad \Leftrightarrow \qquad \frac{(1-\beta)\chi_Z(1) - \beta\frac{b}{a}}{G - \chi_Z(1)F}\frac{aL}{\delta} > 1 + \rho.$$

Figure 9 provides some intuition for uniqueness. Solving the first inequality for $\chi_Z(0)$ gives

$$\frac{(1-\beta)\chi_Z(0) - \beta\frac{b}{a}}{\underbrace{G - \chi_Z(0)F}_{<0 \text{ by } 4}}\frac{aL}{\delta} < \rho$$
$$(1-\beta)aL\chi_Z(0) - \beta bL > \delta\left(G - \chi_Z(0)F\right)\rho$$
$$\chi_Z(0) > \frac{\beta bL + \rho\delta G}{(1-\beta)aL + \rho\delta F}$$

Similarly, for the second condition one can get

$$\chi_Z(1) < \frac{\beta bL + (1+\rho)\delta G}{(1-\beta)aL + (1+\rho)\delta F}$$

From point (ii) of the derivation I know that $\frac{\partial \chi_Z(g)}{\partial g} < 0$, so clearly $\chi_Z(0) > \chi_Z(1)$. However, it is not known whether $\frac{\beta bL + \rho\delta G}{(1-\beta)aL + \rho\delta F} \lesseqgtr \frac{\beta bL + (1+\rho)\delta G}{(1-\beta)aL + (1+\rho)\delta F}$. Let me conjecture that

$$\frac{\beta bL + \rho\delta G}{(1-\beta)aL + \rho\delta F} > \frac{\beta bL + (1+\rho)\delta G}{(1-\beta)aL + (1+\rho)\delta F}$$
$$(\beta bL + \rho\delta G)[(1-\beta)aL + (1+\rho)\delta F] > [\beta bL + (1+\rho)\delta G][(1-\beta)aL + \rho\delta F]$$
$$\delta F\beta bL > \delta G(1-\beta)aL$$
$$\frac{\beta}{1-\beta}\frac{b}{a} > \frac{G}{F}$$

which implies that the ad-hoc constraint is met because it corresponds to the left-hand side of the above:

$$\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L - \left[\frac{\beta}{1-\beta}\frac{b}{a} - \frac{G}{F}\right](1-\beta)aL > 0.$$

Hence, if assumption 5 holds, that is, if $\psi_Z \in \mathcal{K}_Z$, then the ad-hoc constraint also holds. In other words, if an equilibrium exists $r_Z^R(g)$ has to be convex increasing. The ad-hoc is redundant.

which is just assumption 4. So it follows that $\frac{\beta bL+\rho\delta G}{(1-\beta)aL+\rho\delta F} > \frac{\beta bL+(1+\rho)\delta G}{(1-\beta)aL+(1+\rho)\delta F}$ under assumption 4. But then I know that

$$\chi_Z(0) > \frac{\beta bL+\rho\delta G}{(1-\beta)aL+\rho\delta F} > \frac{\beta bL+(1+\rho)\delta G}{(1-\beta)aL+(1+\rho)\delta F} > \chi_Z(1)$$
$$\frac{\beta}{1-\beta}\frac{b}{a} > \chi_Z(0) > \frac{\beta bL+\rho\delta G}{(1-\beta)aL+\rho\delta F} > \frac{\beta bL+(1+\rho)\delta G}{(1-\beta)aL+(1+\rho)\delta F} > \chi_Z(1) > \frac{G}{F}.$$

This is true by the fact that $\chi_Z(g)$ is decreasing in g. Therefore, by the above I have defined the set $\rho \in \mathcal{P} \equiv (\underline{\rho}, \bar{\rho})$ where the lower bound is $\underline{\rho} \equiv \inf\left\{\rho > 0 \middle| \chi_Z(1) < \frac{\beta bL+(1+\rho)\delta G}{(1-\beta)aL+(1+\rho)\delta F}\right\}$ and the upper bound is $\bar{\rho} \equiv \sup\left\{\rho > \underline{\rho} \middle| \chi_Z(0) > \frac{\beta bL+\rho\delta G}{(1-\beta)aL+\rho\delta F}\right\}$. And therefore, for any $\rho \in \mathcal{P} = \left(\frac{\chi_Z(0)-\frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F}-\chi_Z(0)}\frac{(1-\beta)aL}{\delta F}, \frac{\chi_Z(1)-\frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F}-\chi_Z(1)}\frac{(1-\beta)aL}{\delta F}\right)$ requirement (iii) holds.[34] Hence, given assumption 4 and 5, I have proved that $\tilde{H}_Z(0) > 0$ and $\tilde{H}_Z(1) < 0$, so via the (Bolzano) intermediate value theorem there exists an intersection on the compact set $\mathcal{G} = [0,1]$ and because $r^E(g)$ is increasing and affine, and $r_Z^R(g)$ is increasing and strictly convex, the solution is unique.

A.4.2 Proof of proposition 3

Take the derivative $\frac{\partial r_Z^R(g)}{\partial \phi} = \frac{\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}}{[G-\chi_Z(g)F]^2} \cdot \frac{(1-\beta)aLF}{\delta} \cdot \frac{\partial \chi_Z(g)}{\partial \phi}$. The first term is negative by assumption 4. The second term is positive and the third term is

$$\frac{\partial \chi_Z(g)}{\partial \phi} = \frac{\hat{\Psi}[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L}{\delta F g + (1-\beta)aL}\left(\frac{\zeta_Z}{\phi} - \frac{1-\zeta_Z}{1-\phi}\right) \lesseqgtr 0.$$

The first term is positive, the second is positive if $\zeta_Z > \phi$ and negative if $\zeta_Z < \phi$. The partial equilibrium effect is $\frac{\partial r_Z^R(g)}{\partial \phi} > 0$ if $\zeta_Z < \phi$ so the r_Z^R-curve shifts upwards (to the left). Conversely, the partial equilibrium effect is $\frac{\partial r_Z^R(g)}{\partial \phi} < 0$ if $\zeta_Z > \phi$ so the r_Z^R-curve shifts downwards (to the right). To prove the existence of a maximum, take $\frac{dg^*}{d\phi} = -\frac{\partial \chi_Z/\partial \phi}{\partial \chi_Z/\partial g^* - 1}$ which is equal to zero if $\zeta_Z = \phi$. Moreover, (4.17) is strictly convex on the positive orthant, so $\tilde{H}_Z(\cdot)$ is the sum of a convex and an affine function, hence convex - which is sufficient.

[34] $\mathcal{P} \neq \{\varnothing\}$ by $\chi_Z(0) > \chi_Z(1)$.

A.4.3 Proof of proposition 4

Take the derivative $\frac{\partial r_Z^R(g)}{\partial\theta} = \frac{\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}}{[G-\chi_Z(g)F]^2}\cdot\frac{(1-\beta)aLF}{\delta}\cdot\frac{\partial\chi_Z(g)}{\partial\theta}$. The first term is negative by assumption 4. The second term is positive and the third term is

$$\begin{aligned}
\frac{\partial\chi_Z(g)}{\partial\theta} &= \frac{1}{(1-\theta)^2}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg}{\delta Fg+(1-\beta)aL}-\frac{1-\alpha}{(1-\theta)^2}>0\\
\Leftrightarrow &\quad \frac{1}{(1-\theta)}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg}{\delta Fg+(1-\beta)aL}-\frac{1-\alpha}{1-\theta}>0\\
\Leftrightarrow &\quad \frac{1}{\theta}\tilde{\chi}_Z(g)-\frac{1-\alpha}{1-\theta}>0
\end{aligned}$$

where $\tilde{\chi}_Z(g) = \frac{\theta}{1-\theta}\frac{\psi_Z[(1-\phi)\delta]^{1-\zeta_Z}\phi^{\zeta_Z}L-\beta bL-\delta Gg}{\delta Fg+(1-\beta)aL}$. And because $\chi_Z(g)>0$ by proof A.4.1, this implies that $\tilde{\chi}_Z(g)>\frac{1-\alpha}{1-\theta}$. Moreover, $\theta\in(0,1)$, so $\frac{1}{\theta}\tilde{\chi}_Z(g)>\tilde{\chi}_Z(g)$ and therefore $\frac{1}{\theta}\tilde{\chi}_Z(g)>\frac{1-\alpha}{1-\theta}$. Hence, the partial equilibrium effect is $\frac{\partial r_Z^R(g)}{\partial\theta}<0$.

Now for the effect of a change in income concentration, take the derivative $\frac{\partial r_Z^R(g)}{\partial\beta} = -\frac{\frac{b}{a}+\chi_Z(g)}{G-\chi_Z(g)F}\frac{aL}{\delta} + \frac{\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}}{(G-\chi_Z(g)F)^2}\frac{(1-\beta)aLF}{\delta}\cdot\frac{\partial\chi_Z(g)}{\partial\beta}$. The first term can be positive or negative depending on $\frac{b}{a}\lesseqgtr\chi_Z(g)$. The second term is always negative by assumption 4. The third term, which is $\frac{\partial\chi_Z(g)}{\partial\beta}$, can also be positive or negative depending on $\frac{b}{a}\lesseqgtr\frac{\beta}{1-\beta}$. Therefore, the overall effect depends on the parameterization.

A.4.4 Proof of proposition 5

From (4.19) $\frac{A(t)}{N^m(t)}=\psi_{N^e}n$ and use (A.10) and (A.14) in the resource constraint

$$\begin{aligned}
\delta(1-\phi)\psi_{N^e}nL &= \delta Fg+(1-\beta)aL+[\delta(G-\alpha\phi F)g+(\beta b-\alpha\phi(1-\beta)a)L]z^m\\
\delta(1-\phi)\psi_{N^e}\left[\frac{\phi}{1-\phi}\frac{1}{\delta}(1-\alpha z^m)\right]L &= \delta Fg+(1-\beta)aL+[\delta(G-\alpha\phi F)g+(\beta b-\alpha\phi(1-\beta)a)L]z^m.
\end{aligned} \tag{A.23}$$

Solve for z^m and plug in (A.15) to get $\chi_{N^e}(g)=\frac{\theta}{1-\theta}\frac{\psi_{N^e}\phi(1-\alpha\phi)L+\delta Gg+\beta bL}{\psi^e\phi L-\delta Fg-(1-\beta)aL}-\frac{1-\alpha}{1-\theta}$. Notice

$$\begin{aligned}
\frac{\partial\chi_{N^e}(g)}{\partial g} &= \frac{\theta}{1-\theta}\frac{\psi_{N^e}\phi\left[\frac{G}{F}+1-\alpha\phi\right]L+\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)aL}{[\psi^e\phi L-\delta Fg-(1-\beta)aL]^2}\delta F>0.\\
\frac{\partial^2\chi_{N^e}(g)}{\partial g^2} &= \frac{2\delta F}{\psi_{N^e}\phi L-\delta Fg-(1-\beta)aL}\frac{\partial\chi_{N^e}(g)}{\partial g}>0
\end{aligned}$$

For existence check that $r_{N^e}^R(g)=\frac{\chi_N^e(g)-\frac{\beta}{1-\beta}\frac{b}{a}}{\frac{G}{F}-\chi_N^e(g)}\frac{(1-\beta)aL}{\delta F}>0\ \forall g\in\mathcal{G}$. So one needs to check $\frac{G}{F}<\chi_{N^e}<\frac{\beta}{1-\beta}\frac{b}{a}$. Get $\psi_{N^e}\in\mathcal{K}_{N^e}=\left(\frac{\left(\frac{\beta}{1-\beta}\frac{b}{a}\frac{1-\theta}{\theta}+\frac{1-\alpha}{\theta}\right)(\delta Fg+(1-\beta)aL)+\delta Gg+\beta bL}{\left(\frac{\beta}{1-\beta}\frac{b}{a}\frac{1-\theta}{\theta}+\frac{1-\alpha}{\theta}-1+\alpha\right)\phi L},\frac{\left(\frac{G}{F}\frac{1-\theta}{\theta}+\frac{1-\alpha}{\theta}\right)(\delta Fg+(1-\beta)aL)+\delta Gg+\beta bL}{\left(\frac{G}{F}\frac{1-\theta}{\theta}+\frac{1-\alpha}{\theta}-1+\alpha\right)\phi L}\right)\neq\{\emptyset\}$ by assumption 4. The final steps are to show convexity and to pick $0<\rho<r_{N^e}^R(0)$. Take

derivatives $\frac{\partial r^R_{N^e}(g)}{\partial g} = \frac{\left(\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}\right)}{\left[\frac{G}{F}-\chi^e_N(g)\right]^2}\frac{(1-\beta)aL}{\delta F}\frac{\partial \chi_{N^e}(g)}{\partial g} < 0$ which follows from $\frac{\partial \chi_{N^e}(g)}{\partial g} > 0$ and assumption 4. Then,

$$
\begin{aligned}
\frac{\partial^2 r^R_{N^e}(g)}{\partial g^2} &= \frac{(1-\beta)aL}{\delta F}\left(\frac{G}{F}-\frac{\beta}{1-\beta}\frac{b}{a}\right)\left[\frac{2\left(\frac{\partial \chi_{N^e}}{\partial g}\right)^2}{\left(\frac{G}{F}-\chi_{N^e}(g)\right)^3}+\frac{\frac{\partial^2 \chi_{N^e}(g)}{\partial g^2}}{\left(\frac{G}{F}-\chi_{N^e}(g)\right)^2}\right] > 0 \\
&\Leftrightarrow \frac{2\left(\frac{\partial \chi_{N^e}}{\partial g}\right)^2}{\left(\frac{G}{F}-\chi_{N^e}(g)\right)^3}+\frac{\frac{2\delta F}{\psi_{N^e}\phi L-\delta F g-(1-\beta)aL}\frac{\partial \chi_{N^e}(g)}{\partial g}}{\left(\frac{G}{F}-\chi_{N^e}(g)\right)^2} < 0 \\
&\Leftrightarrow \frac{\frac{\partial \chi_{N^e}}{\partial g}}{\frac{G}{F}-\chi_{N^e}(g)}+\frac{\delta F}{\psi_{N^e}\phi L-\delta F g-(1-\beta)aL} < 0 \\
&\Leftrightarrow \frac{\partial \chi_{N^e}}{\partial g}\left[\psi_{N^e}\phi L-\delta F g-(1-\beta)aL\right] > -\delta F\left[\frac{G}{F}-\chi_{N^e}(g)\right] \\
&\Leftrightarrow \frac{\theta}{1-\theta}\frac{\psi_{N^e}\phi\left[\frac{G}{F}+1-\alpha\phi\right]L+\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)aL}{\psi_{N^e}\phi L-\delta F g-(1-\beta)aL}\delta F > \delta F\left[\chi_{N^e}(g)-\frac{G}{F}\right] \\
&\Leftrightarrow \frac{\theta}{1-\theta}\frac{\psi_{N^e}\phi\left[\frac{G}{F}+1-\alpha\phi\right]L+\left[\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}\right](1-\beta)aL}{\psi_{N^e}\phi L-\delta F g-(1-\beta)aL} > \frac{\theta}{1-\theta}\frac{\psi_{N^e}\phi(1-\alpha\phi)L+\delta G g+\beta bL}{\psi_{N^e}\phi L-\delta F g-(1-\beta)aL}-\frac{1-\alpha}{1-\theta}-\frac{G}{F} \\
&\Leftrightarrow \frac{G}{F} > \alpha-1
\end{aligned}
$$

which is always true and hence $r^R_{N^e}(g)$ is convex. Finally pick $0<\rho<r^R_{N^e}(0)$.

A.4.5 Proof of proposition 6

Take the ratio of the no-arbitrage profit conditions (A.9) and (A.11) to get an expression for relative profits independent of r. That is, $\frac{\pi^m_2-\pi^m_1}{\pi^m_1} = \frac{\beta(p^m_l-a^m_l)+(1-\beta)\left(p^m_l+\frac{q^m_h-q^m_l}{q^m_h}p^m_1-a^m_h\right)-(1-\beta)(p^m_1-a^m_h)}{(1-\beta)(p^m_1-a^m_h)}$ and solve for p^m_l as

$$
p^m_l = (1-\beta)\left(\frac{G}{F}+\frac{q^m_l}{q^m_h}\right)p^m_1+\beta a^m_l-\frac{G}{F}(1-\beta)a^m_h \tag{A.24}
$$

Moreover, using (4.13) and plugging in for p^m_l and solving for p^m_1

$$
\begin{aligned}
\frac{1-\theta}{(1-\beta)\theta} &= \frac{\left(\frac{1}{z^m}-\alpha\phi-\frac{q^m_h-q^m_l}{q^m_h}\right)p^m_1}{(1-\beta)\left[\frac{G}{F}+\frac{q^m_l}{q^m_h}\right]p^m_1+\beta a^m_l-(1-\beta)\frac{G}{F}q^m_h} \\
p^m_1 &= \frac{\frac{\beta}{1-\beta}a^m_l-\frac{G}{F}a^m_h}{\frac{\theta}{1-\theta}\left(\frac{1}{z^m}-\alpha\phi-\frac{q^m_h-q^m_l}{q^m_h}\right)-\frac{G}{F}-\frac{q^m_l}{q^m_h}} = \frac{\frac{\beta}{1-\beta}\frac{b}{a}-\frac{G}{F}}{\frac{\theta}{1-\theta}\left[\frac{1}{z^m}-\alpha(1+\phi)\right]-\frac{G}{F}-(1-\alpha)}\frac{(1-\beta)a}{\delta}
\end{aligned}
$$

Taking the derivative reveals $\frac{\partial p_1^m}{\partial z^m} > 0$. Now use (A.24) to get the relative price

$$\frac{p_l^m}{p_1^m} = (1-\beta)\left(\frac{G}{F} + \frac{q_l^m}{q_h^m}\right) + \frac{\beta a_l^m - \frac{G}{F}(1-\beta)a_h^m}{p_1^m} \tag{A.25}$$

and taking the derivative of (A.25) with respect to z^m yields

$$\frac{\partial \chi(\cdot)}{\partial z^m} = \frac{\partial [p_l^m / p_1^m]}{\partial z^m} = -(1-\beta)\left[\frac{\beta}{1-\beta}\frac{b}{a} - \frac{G}{F}\right]\frac{a}{(p_l^m)^2}\frac{\partial p_1^m}{\partial z^m} < 0. \tag{A.26}$$

Thus relative prices need to fall if the number of stage-two products increases for the no-arbitrage condition to hold. Totally differentiating (i) equation (A.23) for the case where technical progress follows (4.19) and (ii) equation (A.16) for the case of (4.16) reveals that g increases in z^m for (ii) and decreases for (i). This completes the proof.

A.4.6 Proof of proposition 7

The argument is equivalent to A.4.3. Take the derivative $\frac{\partial r^R_{N^e}(g)}{\partial \theta} = \frac{\frac{G}{F} - \frac{\beta}{1-\beta}\frac{b}{a}}{[G - \chi_{N^e}(g)F]^2} \cdot \frac{(1-\beta)aLF}{\delta} \cdot \frac{\partial \chi_{N^e}(g)}{\partial \theta}$. The first term is negative by assumption 4. The second term is positive and the third term is $\frac{\partial \chi_{N^e}(g)}{\partial \theta} > 0$ by the same argument as in A.4.3. For a change in ϕ, assume $\alpha\phi < \frac{1}{2}$, and thus

$$\frac{\partial \chi_{N^e}(g)}{\partial \phi} = -\frac{\theta}{1-\theta}\frac{\alpha\phi^2\psi_{N^e}L + \left[(1-2\alpha\phi)\frac{G}{F} + 1\right]\delta F g + \left[\frac{\beta}{1-\beta}\frac{b}{a} + (1-\alpha\phi)\right](1-\beta)aL}{[\psi_{N^e}\phi L - \delta F g - (1-\beta)aL]^2}\psi_{N^e}L < 0.$$

B Transitional dynamics

In this section I briefly touch on the behavior of the economy off the steady state. I refrain from explicitly investigating the transitional dynamics because cost symmetry assumption in the main model implies that the dynamics will be equivalent to Foellmi et al. (2014). I start by showing that only on the BGP the co-existence of all three types of innovation is possible.

Proposition 8 (Dynamics)**.** *If the economy features all three types of innovation it is in the PSBGE.*

Proof. Suppose the economy is in an equilibrium but not in the steady state. That is, $\{n, z^e(t), z^m(t)\}$ are not constant but markets clear and the conditions for a separating equilibrium are still satisfied. Notice that cost symmetry implies that $p_1^e = \delta p_1^m$ and hence $k\mu_R^e(t) = \delta k\gamma\mu_R^m(t)$. Thus sector e moves along with m. The real interest rate is pinned down by the no-arbitrage conditions

$$r(t) = \left[q_l^m \mu_P^m(t) - (1-\beta) q_l^m \mu_R^m(t) - \frac{b}{\delta}\right] \frac{L}{G} = (1-\beta)\left[q_h^m \mu_R^m(t) - \frac{a}{\delta}\right] \frac{L}{F}$$

The Euler-Lagrange equations of rich and poor read

$$\frac{\dot{N}^m(t)}{N^m(t)} = (1-\beta)\left[q_h^m \mu_R^m(t) - \frac{a}{\delta}\right] \frac{L}{F} - \rho - \frac{\dot{\mu}_R^m(t)}{\mu_R^m(t)}$$
$$\frac{\dot{Z}^m(t)}{Z^m(t)} = (1-\beta)\left[q_h^m \mu_R^m(t) - \frac{a}{\delta}\right] \frac{L}{F} - \rho - \frac{\dot{\mu}_P^m(t)}{\mu_P^m(t)}$$

Having established that $\frac{\dot{\mu}_i^e(t)}{\mu_i^e(t)} = \frac{\dot{\mu}_i^m(t)}{\mu_i^m(t)}$ it follows that $\frac{\dot{X}_i^e(t)}{X_i^e(t)} = \frac{\dot{X}_i^m(t)}{X_i^m(t)}$. From $\frac{\dot{\mu}_i^e(t)}{\mu_i^e(t)} = \frac{\dot{\mu}_i^m(t)}{\mu_i^m(t)}$ and consumption baskets (4.8) it follows that $\frac{\dot{X}_R^e(t)}{X_R^e(t)} = \frac{\dot{X}_R^m(t)}{X_R^m(t)} = \frac{\dot{N}^m(t)}{N^m(t)}$ and $\frac{\dot{X}_P^e(t)}{X_P^e(t)} = \frac{\dot{X}_P^m}{X_P^m} = \frac{\dot{Z}^m(t)}{Z^m(t)} = \frac{\dot{Z}^e(t)}{Z^e(t)}$. Therefore, I can reduce this system of differential equations to get a single equation in $\mu_R^m(t)$ and $\frac{Z^m(t)}{N^m(t)}$. Use

$$\frac{\dot{N}^e(t)}{N^e(t)} = \frac{\dot{N}^m(t)}{N^m(t)} + (q_h^e - q_l^e)\frac{Z^e(t)}{N^e(t)}\left[\frac{\dot{N}^m(t)}{N^m(t)} - \frac{\dot{Z}^m(t)}{Z^m(t)}\right]$$

Re-arranging the Euler-Lagrange equation above gives $q_l^m \mu_P^m(t) = (1-\beta)\left(q_l^m + q_h^m \frac{G}{F}\right)\mu_R^m(t) + \beta a_l^m - (1-\beta) a_h^m \frac{G}{F}$. Differentiating and inserting into the Euler-Lagrange equation of the poor gives

$$\frac{\dot{\mu}_P^m(t)}{\mu_P^m(t)} = \frac{(1-\beta)\left(q_l^m + q_h^m \frac{G}{F}\right)}{(1-\beta)\left(q_l^m + q_h^m \frac{G}{F}\right)\mu_R^m(t) + \beta a_l^m - (1-\beta) a_h^m \frac{G}{F}} \dot{\mu}_R^m(t) = \frac{1}{\mu_R^m(t) + \frac{\beta b - (1-\beta) a \frac{G}{F}}{(1-\beta)\delta\left(q_l^m + q_h^m \frac{G}{F}\right)}} \dot{\mu}_R^m(t).$$

Consider the resource constraint (4.10). Assume all types of innovation cause growth,

$$D(z^m)L = \frac{\dot{N}^e(t)}{N^e(t)} n(t)\delta F + \frac{\dot{N}^m(t)}{N^m(t)} F + \frac{\dot{Z}^m(t)}{Z^m(t)} z^m(t) G + \frac{\dot{Z}^e(t)}{Z^e(t)} z^e(t) n(t)\delta G + \left[((1-\beta)\delta a + \beta\delta b z^e(t)) n(t) + (1-\beta) a + \beta b z^m(t)\right] \frac{L}{\delta}$$

where $D(z^m) = \frac{A(t)L}{N^m(t)} = \psi\phi^{\zeta_N e+\zeta_Z}[(1-\phi)\delta]^{1-\zeta_N e-\zeta_Z}\,(1-\alpha z^m)^{\zeta_N e}\,[z^m(t)]^{1-\zeta_N e-\zeta_N m}$ is the stock of technology. Recall the following relationships from appendix A.3 must hold at all time

$$n = \frac{\phi}{(1-\phi)\delta}(1-\alpha z^m) \quad \text{and} \quad z^e = \frac{1}{\alpha}\left[\frac{1}{1-\alpha z^m} - 1\right] \quad \text{and} \quad z^e n = \frac{\phi}{(1-\phi)\delta} z^m$$

I use the Euler equations and definitions of $\{n(t), z^m(t), z^e(t)\}$ to reduce this equation to

$$\begin{aligned} \frac{\dot{\mu}^m_R(t)}{\mu^m_R(t)} + \frac{\mathcal{Q}_2 z^m(t)}{\mu^m_R(t)+\mathcal{Q}_1}\dot{\mu}^m_R(t) &= \left[(1-\beta)\left[q^m_h\mu^m_R(t) - \frac{a}{\delta}\right]\frac{L}{F} - \rho\right][1+\mathcal{Q}_2 z^m(t)] \\ &+ (1-\beta)\frac{a}{\delta}[1+\mathcal{Q}_3 z^m(t)]\frac{L}{F} - \frac{D(z^m)}{\delta}\frac{L}{F}, \end{aligned} \tag{B.1}$$

where (B.1) is a Riccati equation in $\mu^m_R(t)$.[35] Notice that $\dot{\mu}^m_R(t)$ increases monotonically in $\mu^m_R(t)$. If $\mu^m_R(t)$ is below its steady-state value, it diverges to negative infinity without bound, and if it is above its steady-state value, it diverges to positive infinity. Hence, $\mu^m_R(t) = \mu^m_R$ for all t and since all willingnesses to pay are monotonically related the same holds for the set $\{\mu^m_P(t), \mu^e_R(t), \mu^m_P(t)\}$. □

The proposition implies that, when the economy has too few stage-two products $z^m(t)$ and $z^e(t)$ (that move *pari passu*), the transition process will be characterized by stage-two innovations only. Similarly, if there are too few stage-one products $N^m(t) - Z^m(t)$ and $N^e(t) - Z^e(t)$, the transition process will be characterized by product innovations only. Hence, all adjustments in the state variables $\{z^s(t)\}_{s\in S}$ occur by a *bang-bang* rule. This resembles models of directed technical change where off equilibrium only one type of innovation occurs (Acemoğlu & Zilibotti 2001). It can be shown that this bang-bang behavior implies that the transition from an old to a new steady state occurs in finite time. Moreover note that the cost symmetry assumption implies the perfect co-movement of marginal willingnesses to pay in both sectors and hence the co-movement of process and product innovation in both sectors. Note that in a situation off the equilibrium path, where either stage-one or stage-two innovation does not augment the stock of technology, the economy could be stuck in a poverty trap indefinitely. For further discussion, I refer to Foellmi et al. (2014).[36]

[35] D is as defined above. Constants are $\mathcal{Q}_1 = \frac{\beta b-(1-\beta)a\frac{G}{F}}{(1-\beta)\delta\left(q^m_l+q^m_h\frac{G}{F}\right)}$ and $\mathcal{Q}_2 = \frac{G}{F} - \alpha\phi$ and $\mathcal{Q}_3 = \frac{\beta}{1-\beta}\frac{b}{a} - \alpha\phi$.

[36] Note that to fully solve the model, one needs an equation in the remaining state variable $z^m(t)$ which can be obtained from the Euler-equations, as $\frac{\dot{z}^m(t)}{z^m(t)} = \frac{\dot{Z}^m(t)}{Z^m(t)} - \frac{\dot{N}^m(t)}{N^m(t)} = \frac{\dot{\mu}^m_R(t)}{\mu^m_R(t)+\mathcal{Q}_1} - \frac{\dot{\mu}^m_R(t)}{\mu^m_R(t)}$. The final part then would be to keep track of the changing wealth distribution (the changing ownership in firms).

C Quantitative exercise

In this section, I provide a quantitative example for the effect of a change in inequality through a change in income concentration captured by β. All parameter values can be found in the accompanying *Matlab* file.[37] Holding θ constant, a higher β implies that income becomes more concentrated for R-types and common measures of inequality such as the Gini coefficient indicate an increase in inequality. The numerical simulations show that for an economy with an initially low share of poor

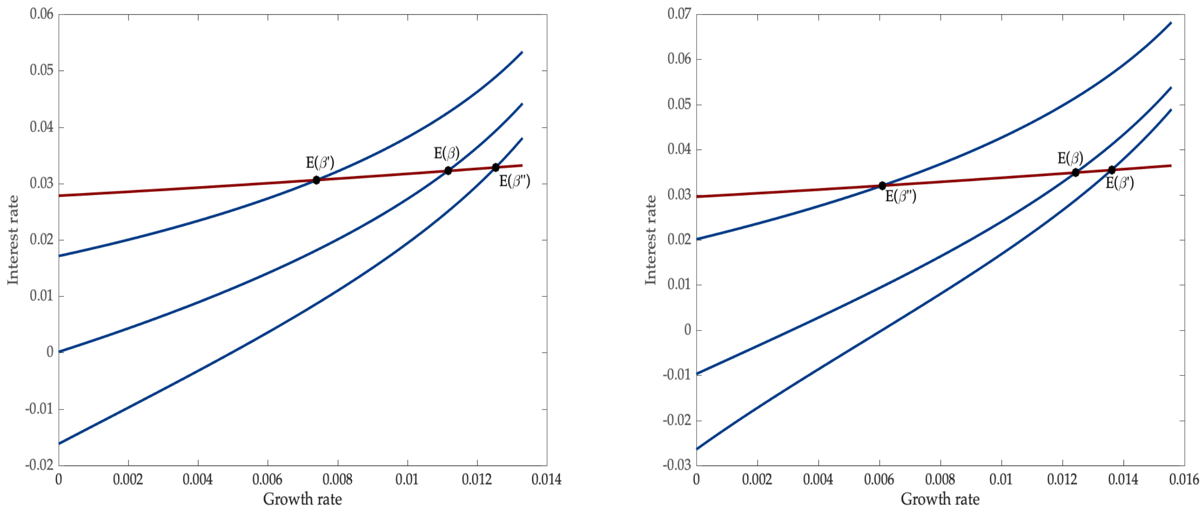

Figure 17: Change in income concentration with initial high β (left) and low β (right).

consumers, an increase might encourage growth. For an already large share of poor consumers, a further increase usually reduces growth but also might entail a break-down of equilibrium if pooling becomes too attractive. The effect of a change in the income concentration also depends on income gaps: If poor consumer's endowment is very low, an increase might discourage growth even if there are only few poor types initially. As an example, figure 17 shows comparative statics of a change in β for a low labor endowment of P-types ($\theta_\ell = 0.2$). The left panel assumes high initial income concentration (high share of poor consumers, $\beta = 0.7$). Then, a further increase ($\beta' = 0.9$) tends to reduce growth while a decrease in β raises growth rates ($\beta'' = 0.6$). In the right panel, I plot equilibrium with an initially lower income concentration ($\beta = 0.6$). An increase in β (to $\beta' = 0.7$) increases growth while a further decrease reduces growth (to $\beta'' = 0.5$). For the cases where labor endowment of P-types is higher, it is crucial to check that the conditions for a PSBGE are satisfied. For instance, if θ and initial β are high, typically the stage-one separation cannot be upheld (pooling becomes too attractive).

[37]The full set of codes is available upon request.

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